Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.3/97-1.2.1.3-d2

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**316**]. This is test number [97].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (316)	0.00 (0)
Mathematica	100.00 (316)	0.00 (0)
Maple	90.82 (287)	9.18 (29)
Reduce	86.71 (274)	13.29 (42)
Fricas	86.39 (273)	13.61 (43)
Giac	77.85 (246)	22.15 (70)
Mupad	41.14 (130)	58.86 (186)
Maxima	22.78 (72)	77.22 (244)
Sympy	6.96 (22)	93.04 (294)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

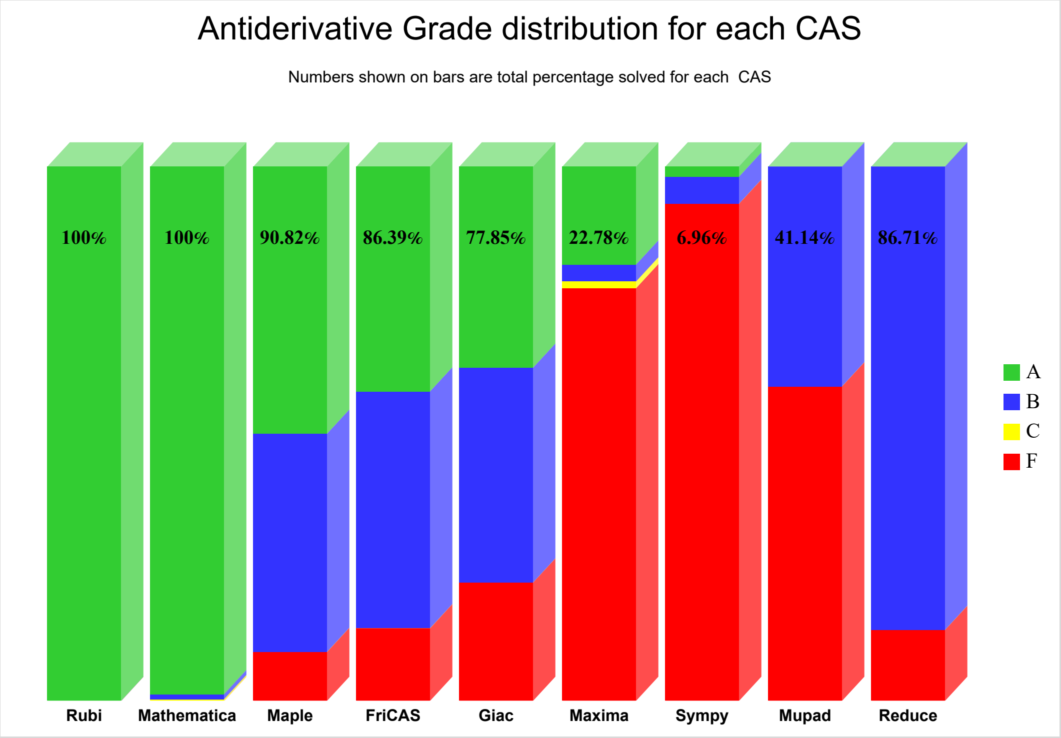
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

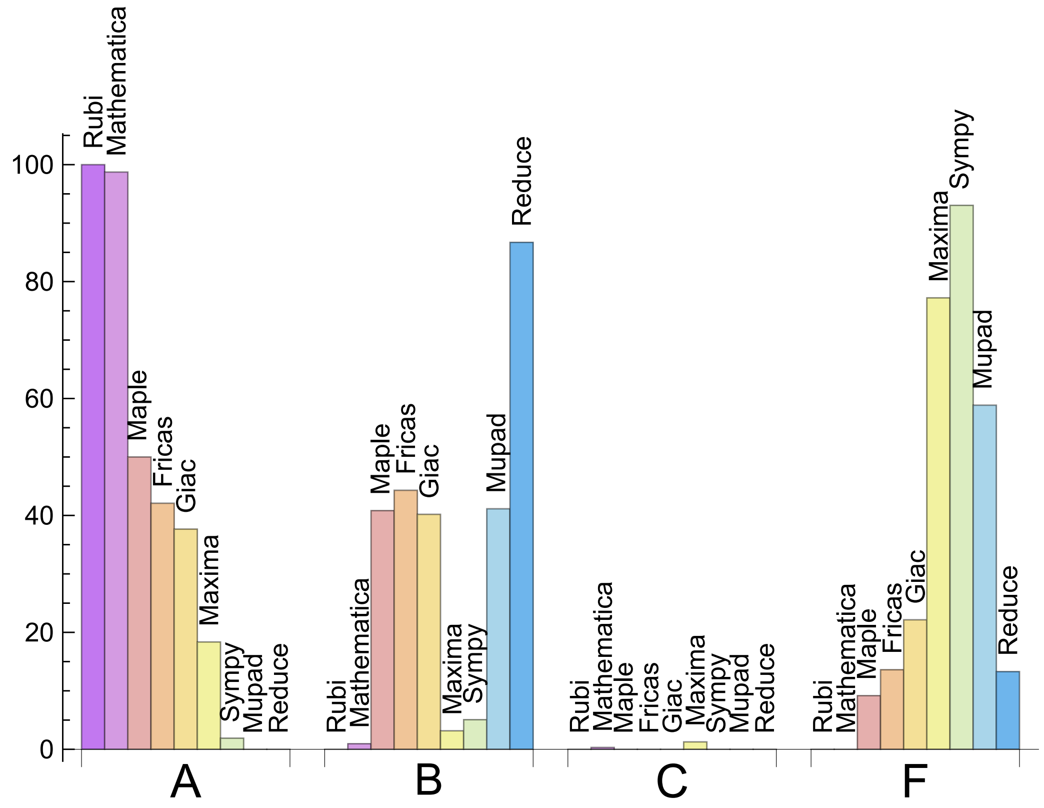
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	98.734	0.949	0.316	0.000
Maple	50.000	40.823	0.000	9.177
Fricas	42.089	44.304	0.000	13.608
Giac	37.658	40.190	0.000	22.152
Maxima	18.354	3.165	1.266	77.215
Sympy	1.899	5.063	0.000	93.038
Mupad	0.000	41.139	0.000	58.861
Reduce	0.000	86.709	0.000	13.291

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	29	100.00	0.00	0.00
Reduce	42	100.00	0.00	0.00
Fricas	43	65.12	34.88	0.00
Giac	70	48.57	24.29	27.14
Mupad	186	0.00	100.00	0.00
Maxima	244	63.93	0.00	36.07
Sympy	294	56.80	40.82	2.38

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.09
Giac	0.74
Rubi	0.76
Mathematica	1.28
Reduce	1.43
Maple	3.12
Fricas	4.52
Sympy	6.07
Mupad	11.62

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	194.82	0.87	165.00	0.82
Rubi	234.51	1.07	224.00	1.04
Maxima	286.43	1.26	199.00	1.07
Maple	701.78	2.61	361.00	1.57
Fricas	901.23	3.59	649.00	3.11
Giac	1187.98	4.16	407.50	1.84
Reduce	1379.80	4.95	521.00	2.45
Mupad	2462.73	9.15	273.00	1.52
Sympy	4900.55	13.44	1905.50	7.38

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

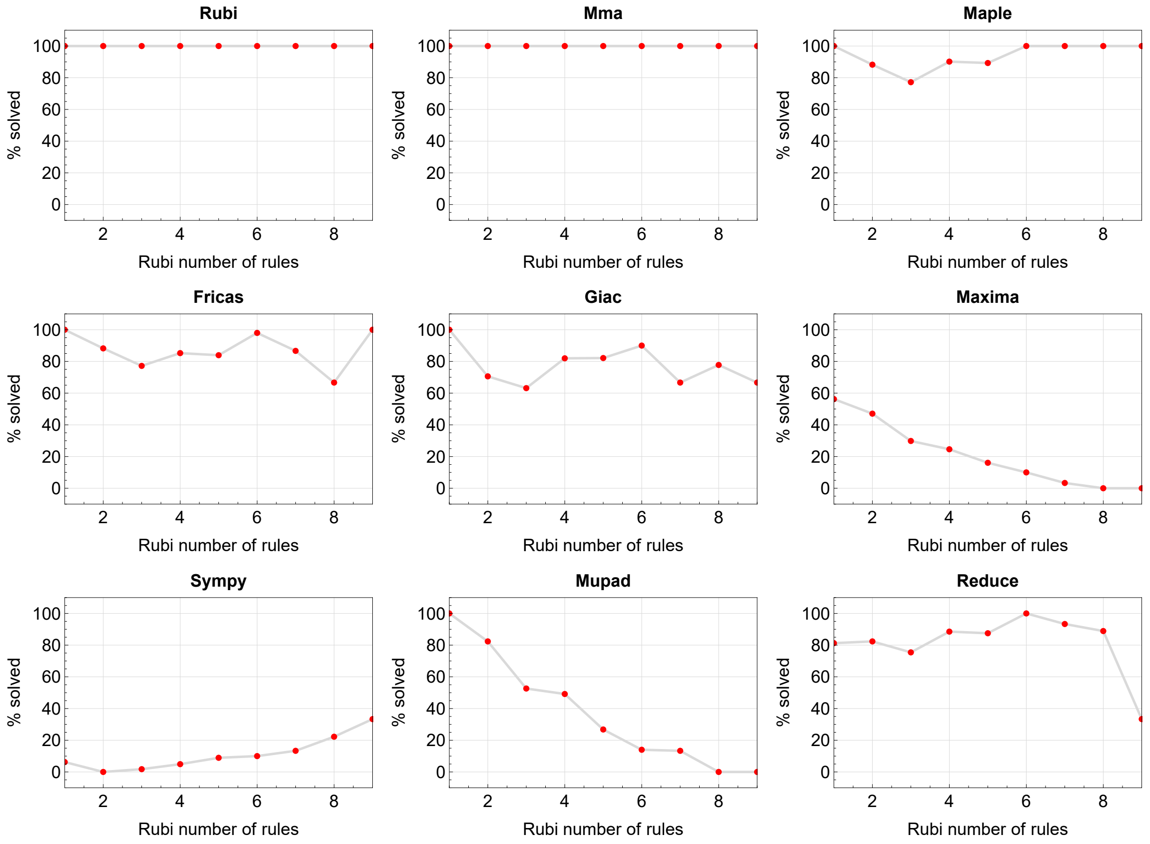


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

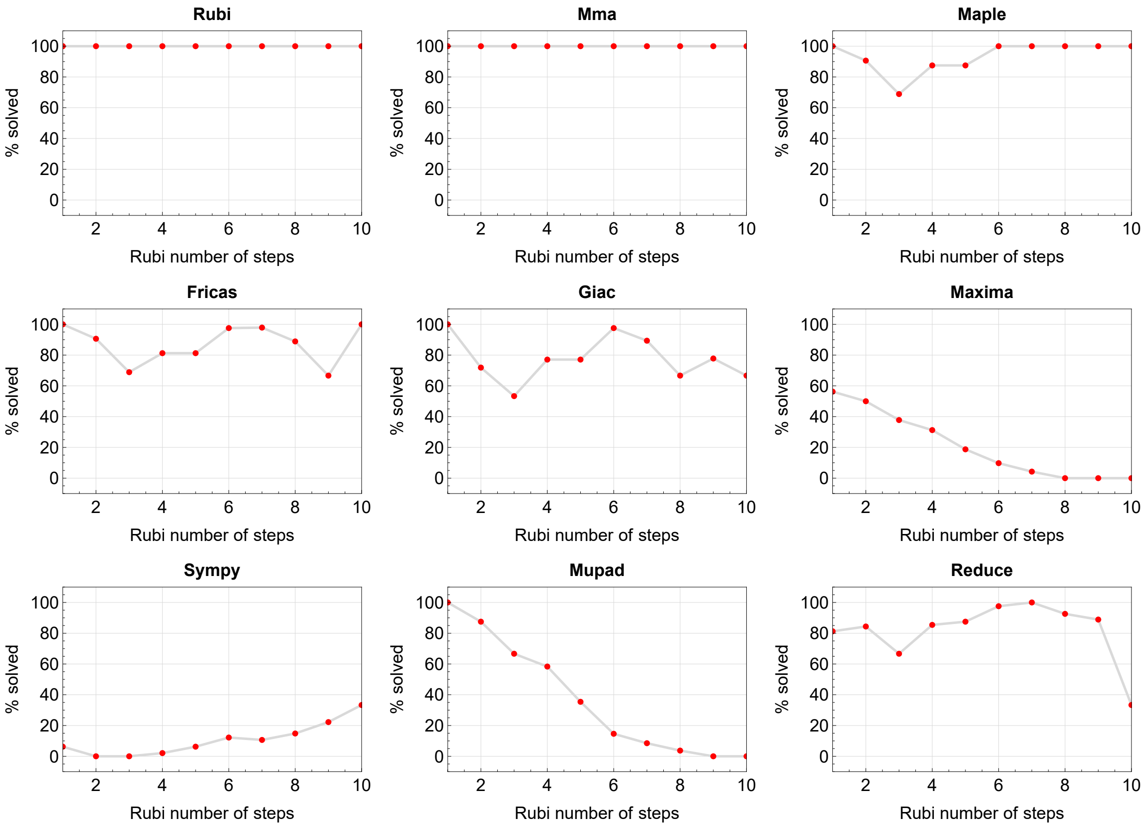


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

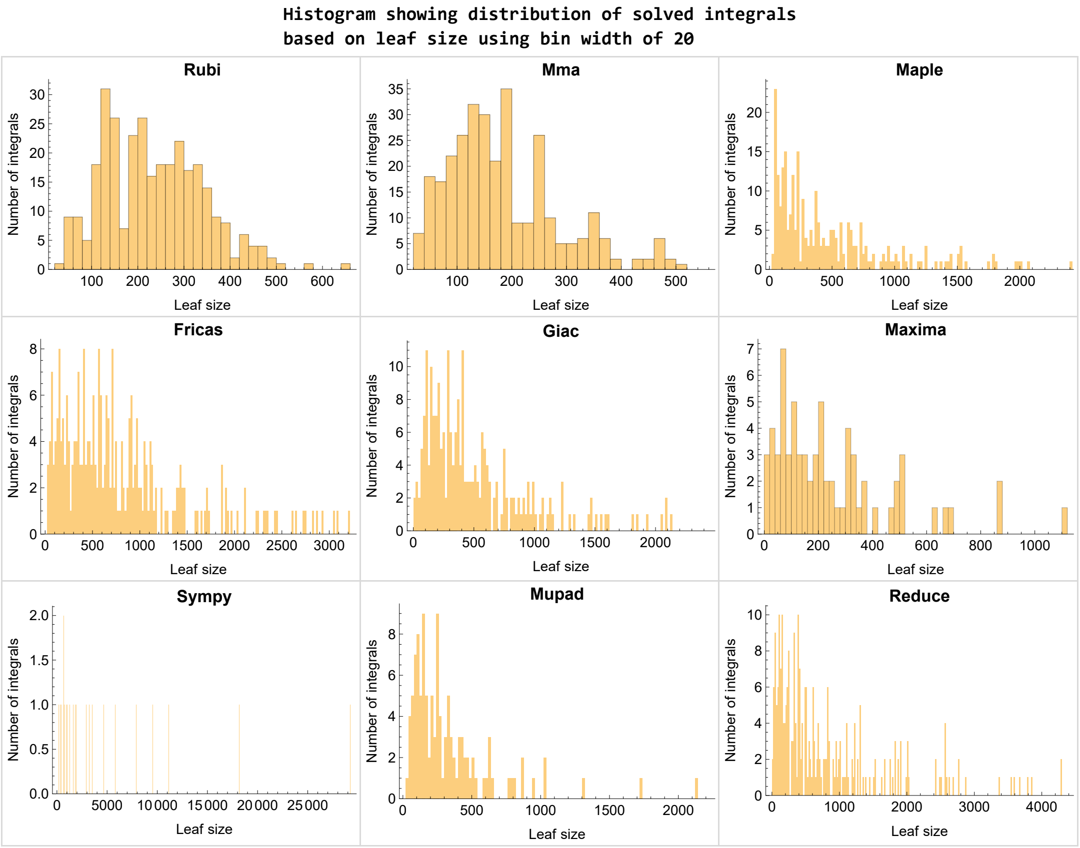
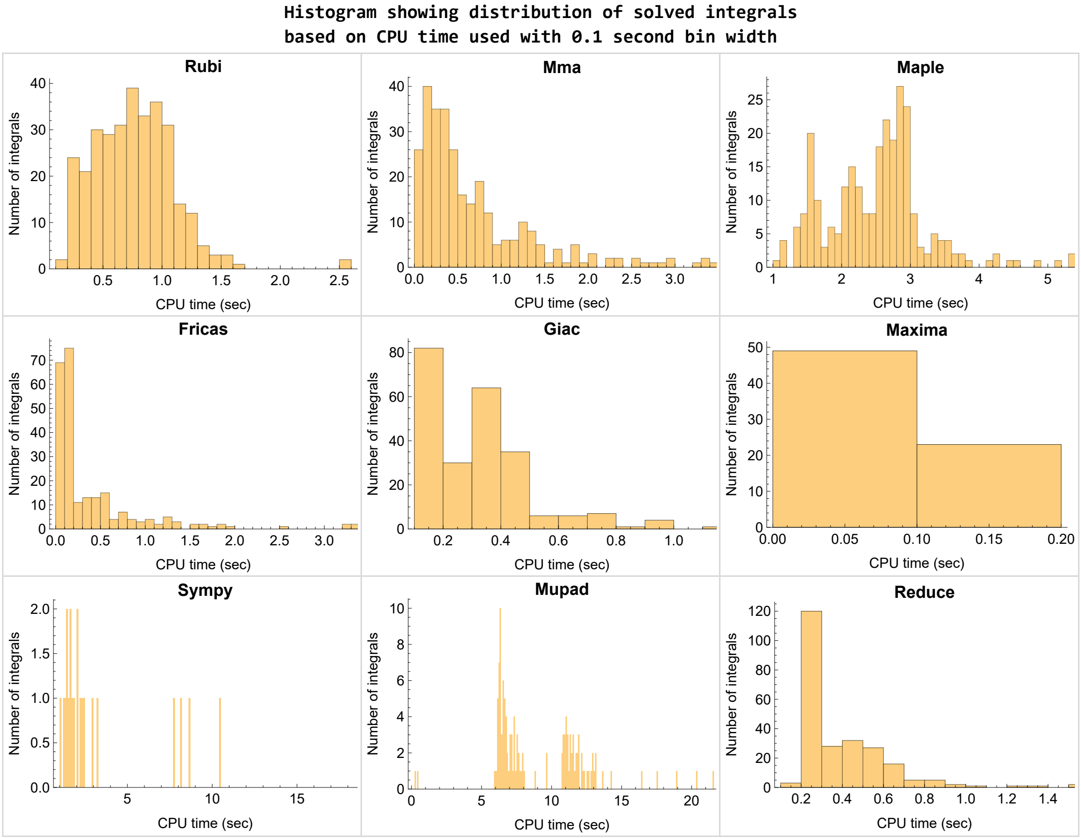


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.



Histogram showing distribution of solved integrals based on CPU time used with 0.1 second bin width

Rubi Mma Maple

Fricas Giac Maxima

Sympy Mupad Reduce

Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

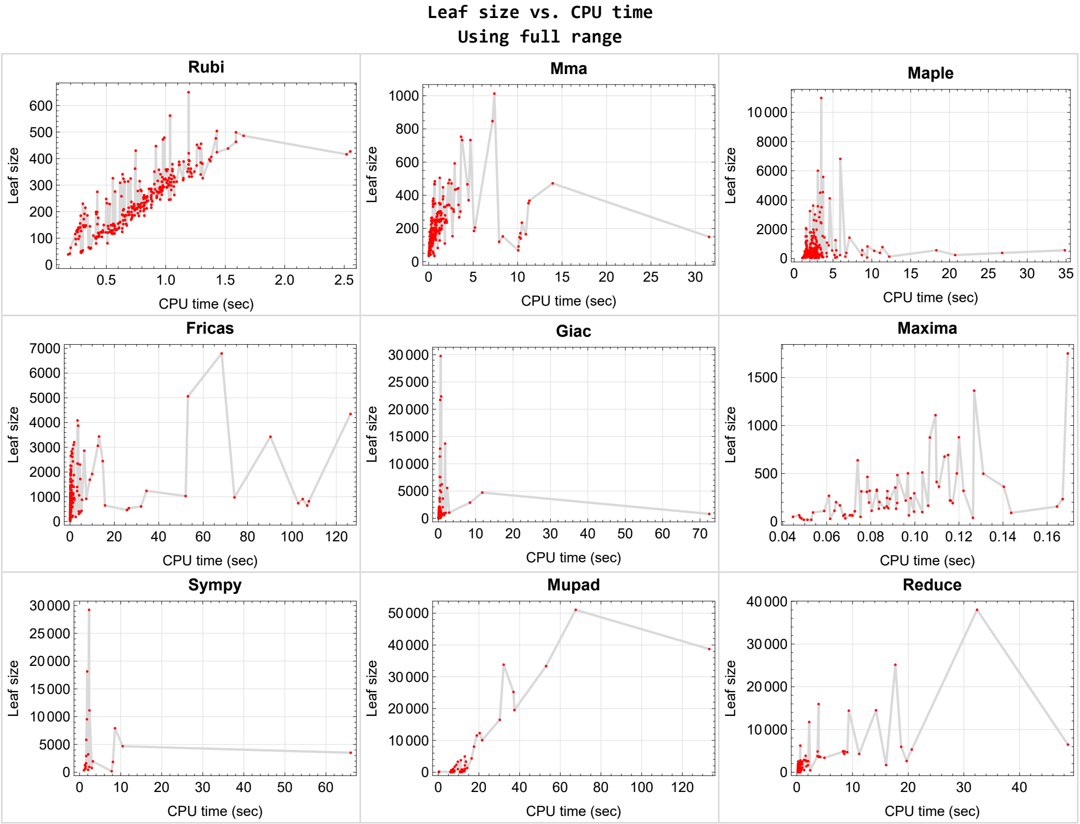


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{250, 251, 252, 253, 254, 255\}$

Mathematica $\{250, 251, 252, 253, 254, 255\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

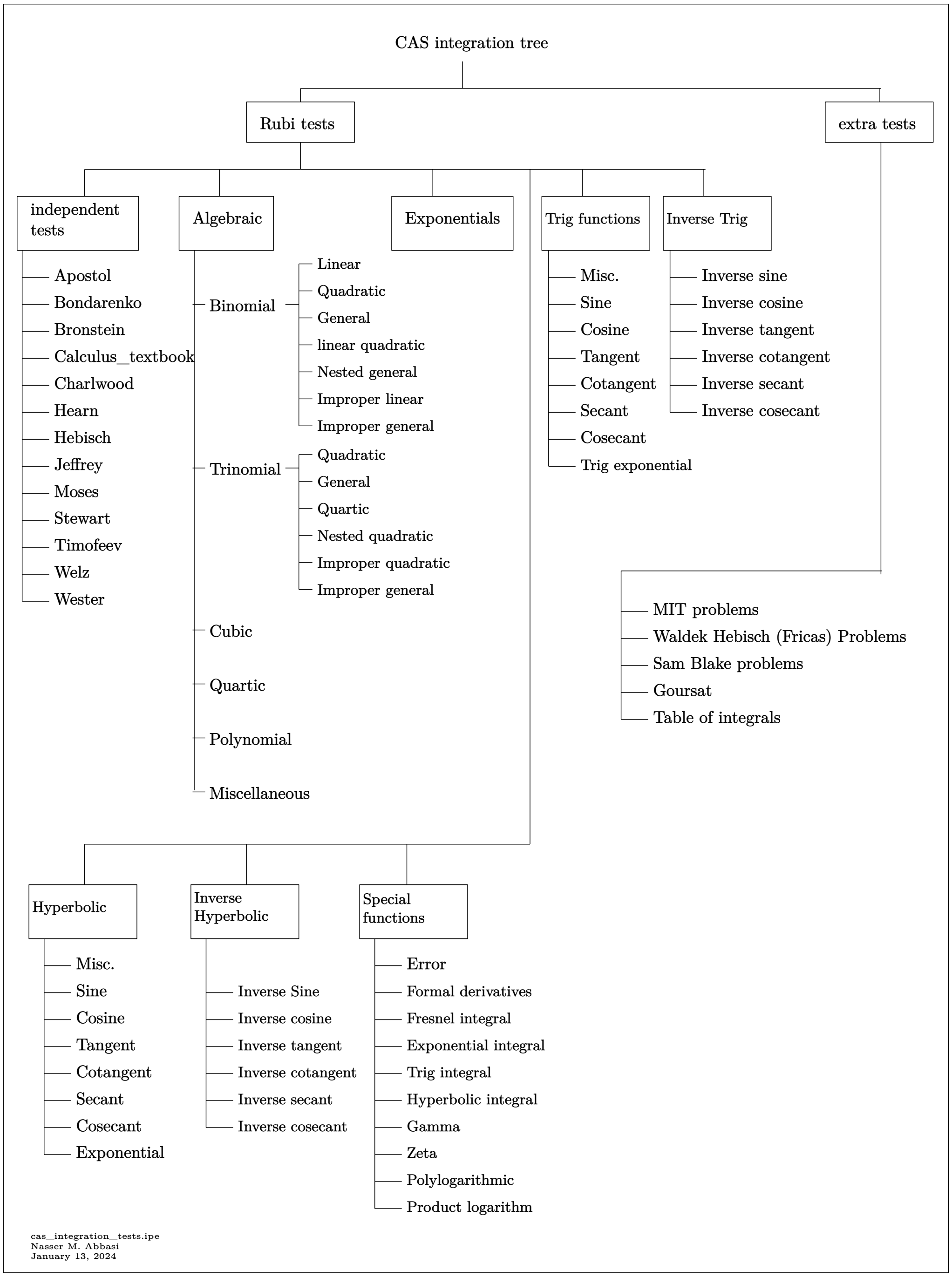
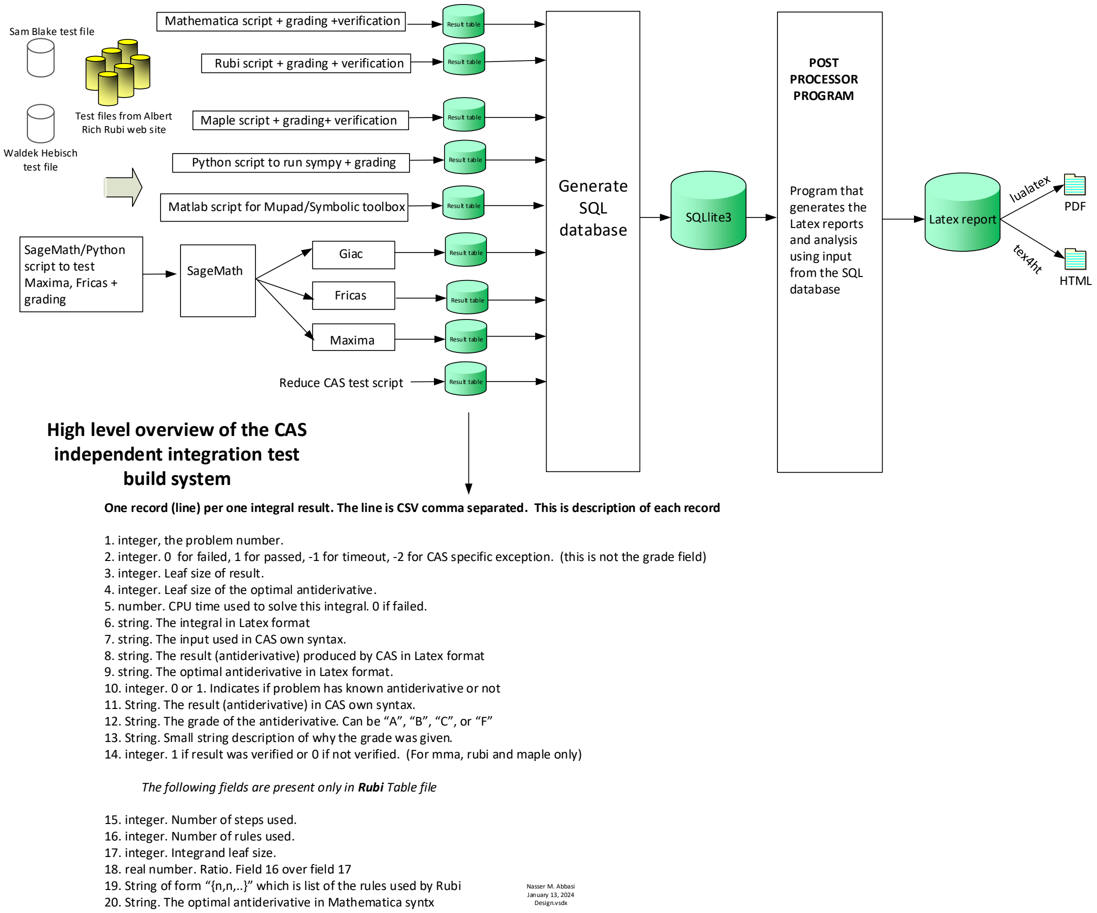


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316 }

B grade { 159, 161, 249 }

C grade { 187 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 58, 59, 60, 61, 62, 63, 64, 67, 70, 71, 72, 73, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 101, 102, 103, 104, 113, 114, 115, 124, 127, 128, 129, 130, 131, 132, 141, 142, 143, 144, 145, 146, 155, 156, 157, 158, 169, 170, 171, 172, 176, 177, 178, 179, 180, 184, 185, 190, 191, 195, 196, 197, 198, 199, 204, 205, 206, 207, 208, 214, 215, 216, 217, 218, 219, 226, 227, 228, 229, 232, 233, 234, 235, 236, 240, 241, 242, 243, 244, 249, 256, 257, 258, 261, 262, 270, 291, 292, 295, 296, 297, 298, 299, 307, 308, 309, 310 }

B grade { 10, 20, 21, 27, 28, 29, 31, 32, 33, 55, 56, 57, 65, 66, 68, 69, 74, 75, 76, 77, 78, 79, 91, 92, 98, 99, 100, 112, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 181, 182, 183, 186, 187, 188, 189, 192, 193, 194, 200, 201, 202, 203, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 230, 231, 237, 238, 239, 245, 246, 247, 248, 259, 260, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 300, 301, 302, 303, 304, 305, 306 }

C grade { }

F normal fail { 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 250, 251, 252, 253, 254, 255, 311, 312, 313, 314, 315, 316 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 11, 12, 13, 14, 15, 16, 17, 23, 24, 25, 27, 28, 29, 34, 35, 36, 37, 38, 42, 43, 44, 45, 49, 50, 51, 56, 57, 58, 60, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 84, 91, 92, 93, 100, 113, 114, 115, 124, 125, 127, 128, 129, 130, 131, 132, 139, 140, 141, 150, 151, 152, 153, 164, 165, 166, 167, 173, 174, 175, 176, 177, 178, 181, 182, 190, 195, 196, 197, 198, 199, 200, 209, 210, 220, 221, 222, 226, 227, 228, 229, 232, 233, 234, 235, 240, 241, 242, 243, 244, 249, 256, 259, 260, 261, 262, 267, 268, 269, 270, 275, 276, 277, 282, 283, 284, 285, 291, 292, 295, 296, 297, 298, 299, 302, 303, 304 }

B grade { 7, 8, 9, 10, 18, 19, 20, 21, 22, 26, 30, 31, 32, 33, 39, 40, 41, 46, 47, 48, 52, 53, 54, 55, 59, 61, 62, 63, 64, 70, 71, 72, 73, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 112, 133, 134, 135, 136, 137, 138, 142, 143, 144, 147, 148, 149, 154, 155, 156, 159, 160, 161, 162, 163, 168, 169, 179, 180, 183, 184, 185, 186, 187, 188, 189, 191, 192, 201, 202, 203, 204, 205, 206, 207, 208, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 230, 231, 236, 237, 238, 239, 245, 246, 247, 248, 257, 258, 263, 264, 265, 274, 280, 281, 286, 287, 288, 289, 290, 293, 294, 300, 301, 305, 306, 307, 308, 309, 310 }

C grade { }

F normal fail { 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 126, 250, 251, 252, 253, 254, 255, 311, 312, 313, 314, 315, 316 }

F(-1) timedout fail { 145, 146, 157, 158, 170, 171, 172, 193, 194, 266, 271, 272, 273, 278, 279 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 34, 35, 36, 37, 42, 43, 44, 45, 49, 50, 51, 52, 112, 113, 114, 115, 124, 125, 127, 128, 129, 130, 131, 195, 196, 197, 198, 207, 208, 226, 227, 228, 232, 233, 234, 235, 240, 241, 242, 243, 244, 256, 258 }

B grade { 163, 204, 205, 206, 214, 215, 216, 217, 218, 219 }

C grade { 295, 296, 297, 298 }

F normal fail { 6, 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 126, 132, 133, 134, 135, 199, 200, 201, 202, 203, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 229, 230, 231, 236, 237, 238, 239, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 264, 265, 266, 272, 273, 274, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316 }

F(-1) timedout fail { }

F(-2) exception fail { 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

Giac

A grade { 3, 4, 5, 6, 7, 8, 9, 16, 17, 18, 19, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 67, 68, 69, 76, 77, 78, 84, 85, 86, 88, 91, 92, 93, 94, 115, 124, 129, 130, 131, 132, 133, 139, 140, 152, 164, 165, 173, 174, 175, 199, 200, 201, 209, 210, 211, 220, 221, 222, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 241, 242, 243, 244, 245, 246, 247, 259, 260, 261, 262, 263, 267, 268, 269, 270, 275, 276, 277, 298, 299, 300, 301, 304, 305, 306, 307, 308 }

B grade { 1, 2, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 26, 32, 33, 56, 57, 61, 62, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 87, 89, 90, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 112, 113, 114, 127, 128, 134, 135, 136, 137, 138, 141, 143, 147, 148, 149, 150, 151, 153, 154, 159, 160, 161, 162, 163, 166, 167, 176, 177, 180, 181, 182, 183, 185, 187, 188, 189, 193, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 208, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 232, 239, 240, 248, 256, 257, 258, 264, 265, 266, 273, 274, 280, 287, 295, 296, 297, 309, 310 }

C grade { }

F normal fail { 105, 106, 107, 108, 109, 110, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 184, 186, 191, 192, 194, 249, 250, 251, 252, 253, 254, 255, 311, 312, 313, 314, 315, 316 }

F(-1) timedout fail { 111, 144, 145, 146, 156, 157, 158, 168, 169, 170, 171, 172, 272, 281, 294, 302, 303 }

F(-2) exception fail { 142, 155, 178, 179, 190, 271, 278, 279, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 34, 35, 36, 37, 42, 43, 44, 45, 49, 50, 51, 52, 61, 62, 63, 64, 70, 71, 72, 73, 80, 81, 82, 83, 87, 88, 89, 90, 94, 95, 96, 97, 101, 102, 103, 104, 112, 113, 114, 115, 124, 127, 128, 129, 130, 131, 136, 137, 138, 142, 143, 144, 145, 146, 155, 156, 157, 158, 169, 170, 171, 172, 177, 178, 179, 180, 183, 184, 185, 186, 190, 191, 192, 193, 194, 195, 196, 197, 198, 204, 205, 206, 207, 208, 214, 215, 216, 217, 218, 219, 226, 227, 228, 232, 233, 234, 235, 240, 241, 242, 243, 244, 256, 257, 258, 291, 292, 295, 296, 297, 298, 307, 308, 309, 310 }

C grade { }

F normal fail { }

F(-1) timedout fail { 6, 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 84, 85, 86, 91, 92, 93, 98, 99, 100, 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 132, 133, 134, 135, 139, 140, 141, 147, 148, 149, 150, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 181, 182, 187, 188, 189, 199, 200, 201, 202, 203, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 229, 230, 231, 236, 237, 238, 239, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 299, 300, 301, 302, 303, 304, 305, 306, 311, 312, 313, 314, 315, 316 }

F(-2) exception fail { }

Sympy

A grade { 150, 162, 163, 268, 269, 270 }

B grade { 136, 137, 138, 147, 148, 149, 159, 160, 161, 173, 174, 175, 256, 275, 276, 277 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 57, 58, 59, 60, 61, 66, 67, 68, 84, 85, 86, 87, 88, 93, 94, 95, 106, 107, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 151, 152, 153, 154, 155, 156, 157, 164, 165, 166, 167, 168, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 259, 260, 261, 262, 263, 264, 265, 271, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305 }

F(-1) timedout fail { 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 42, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 112, 113, 117, 119, 120, 121, 122, 123, 124, 125, 126, 158, 169, 171, 172, 193, 194, 213, 214, 215, 222, 223, 224, 225, 232, 233, 240, 241, 242, 243, 257, 258, 266, 267, 272, 273, 274, 287, 288, 293, 294, 302, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316 }

F(-2) exception fail { 111, 114, 115, 116, 118, 254, 255 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 256, 258, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310 }

C grade { }

F normal fail { 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 249, 250, 251, 252, 253, 254, 255, 257, 259, 267, 268, 274, 282, 294, 311, 312, 313, 314, 315, 316 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	351	195	273	320	375	0	668	349	347
N.S.	1	1.18	0.66	0.92	1.08	1.26	0.00	2.25	1.18	1.17
time (sec)	N/A	1.182	0.258	2.953	0.122	0.088	0.000	0.123	0.600	6.395

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	275	136	178	218	264	0	457	231	242
N.S.	1	1.18	0.58	0.76	0.93	1.13	0.00	1.95	0.99	1.03
time (sec)	N/A	0.962	0.182	2.914	0.116	0.082	0.000	0.123	0.426	6.248

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	199	90	106	133	173	0	284	136	157
N.S.	1	1.16	0.53	0.62	0.78	1.01	0.00	1.66	0.80	0.92
time (sec)	N/A	0.690	0.122	2.861	0.091	0.098	0.000	0.113	0.370	6.170

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	123	54	57	65	102	0	145	63	93
N.S.	1	1.14	0.50	0.53	0.60	0.94	0.00	1.34	0.58	0.86
time (sec)	N/A	0.472	0.074	2.630	0.071	0.085	0.000	0.108	0.562	6.036

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	40	18	57	0	18	25	49
N.S.	1	1.00	0.77	0.83	0.38	1.19	0.00	0.38	0.52	1.02
time (sec)	N/A	0.295	0.026	2.144	0.051	0.086	0.000	0.103	0.533	5.986

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	114	143	0	350	0	119	61	0
N.S.	1	1.00	0.92	1.15	0.00	2.82	0.00	0.96	0.49	0.00
time (sec)	N/A	0.634	0.176	2.833	0.000	0.095	0.000	0.111	0.318	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	151	0	563	0	150	159	0
N.S.	1	1.00	0.83	1.14	0.00	4.27	0.00	1.14	1.20	0.00
time (sec)	N/A	0.577	0.401	2.998	0.000	0.106	0.000	0.119	0.471	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	202	165	275	0	1057	0	243	383	0
N.S.	1	0.98	0.80	1.33	0.00	5.11	0.00	1.17	1.85	0.00
time (sec)	N/A	0.786	0.788	2.809	0.000	0.116	0.000	0.134	0.569	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	282	202	443	0	1733	0	420	691	0
N.S.	1	1.02	0.73	1.60	0.00	6.26	0.00	1.52	2.49	0.00
time (sec)	N/A	1.046	1.247	2.952	0.000	0.283	0.000	0.135	0.419	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	362	234	686	0	2611	0	628	1070	0
N.S.	1	1.04	0.67	1.98	0.00	7.52	0.00	1.81	3.08	0.00
time (sec)	N/A	1.211	1.822	2.821	0.000	1.007	0.000	0.149	0.292	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	351	195	275	413	472	0	1230	454	445
N.S.	1	1.18	0.66	0.93	1.39	1.59	0.00	4.14	1.53	1.50
time (sec)	N/A	1.241	0.328	2.963	0.110	0.101	0.000	0.172	0.566	6.566

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	275	137	180	294	340	0	852	313	310
N.S.	1	1.18	0.59	0.77	1.26	1.45	0.00	3.64	1.34	1.32
time (sec)	N/A	0.932	0.241	3.019	0.100	0.098	0.000	0.150	0.547	6.559

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	199	90	108	192	230	0	540	195	206
N.S.	1	1.16	0.53	0.63	1.12	1.35	0.00	3.16	1.14	1.20
time (sec)	N/A	0.683	0.159	2.903	0.117	0.099	0.000	0.138	0.363	6.386

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	123	54	59	107	137	0	290	99	109
N.S.	1	1.14	0.50	0.55	0.99	1.27	0.00	2.69	0.92	1.01
time (sec)	N/A	0.461	0.100	2.769	0.073	0.083	0.000	0.126	0.292	6.315

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	43	74	0	114	42	62
N.S.	1	1.00	0.77	0.88	0.90	1.54	0.00	2.38	0.88	1.29
time (sec)	N/A	0.289	0.034	2.087	0.048	0.080	0.000	0.122	0.550	6.122

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	185	132	253	0	440	0	217	147	0
N.S.	1	1.03	0.74	1.41	0.00	2.46	0.00	1.21	0.82	0.00
time (sec)	N/A	0.820	0.301	2.800	0.000	0.103	0.000	0.133	0.561	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	184	144	296	0	476	0	238	155	0
N.S.	1	1.03	0.81	1.66	0.00	2.67	0.00	1.34	0.87	0.00
time (sec)	N/A	0.775	0.529	2.745	0.000	0.122	0.000	0.135	0.339	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	194	135	266	0	841	0	236	324	0
N.S.	1	0.99	0.69	1.36	0.00	4.31	0.00	1.21	1.66	0.00
time (sec)	N/A	0.765	0.742	2.720	0.000	0.112	0.000	0.137	0.465	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	264	201	443	0	1435	0	392	601	0
N.S.	1	1.00	0.76	1.67	0.00	5.42	0.00	1.48	2.27	0.00
time (sec)	N/A	0.967	1.217	2.667	0.000	0.190	0.000	0.158	0.618	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	344	240	655	0	2239	0	604	962	0
N.S.	1	1.03	0.72	1.96	0.00	6.68	0.00	1.80	2.87	0.00
time (sec)	N/A	1.215	1.818	2.754	0.000	0.566	0.000	0.177	0.395	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	424	302	945	0	3205	0	866	1396	0
N.S.	1	1.05	0.75	2.33	0.00	7.91	0.00	2.14	3.45	0.00
time (sec)	N/A	1.430	3.570	2.816	0.000	1.879	0.000	0.204	0.304	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	351	205	275	498	567	0	2094	559	523
N.S.	1	1.18	0.69	0.93	1.68	1.91	0.00	7.05	1.88	1.76
time (sec)	N/A	1.205	0.327	2.975	0.131	0.093	0.000	0.213	0.563	6.728

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	275	147	180	362	416	0	1479	395	379
N.S.	1	1.18	0.63	0.77	1.55	1.78	0.00	6.32	1.69	1.62
time (sec)	N/A	0.946	0.250	2.971	0.140	0.096	0.000	0.183	0.615	6.671

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	199	100	108	243	284	0	958	254	259
N.S.	1	1.16	0.58	0.63	1.42	1.66	0.00	5.60	1.49	1.51
time (sec)	N/A	0.682	0.158	2.970	0.098	0.085	0.000	0.166	0.281	6.429

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	123	64	59	141	173	0	535	135	134
N.S.	1	1.14	0.59	0.55	1.31	1.60	0.00	4.95	1.25	1.24
time (sec)	N/A	0.454	0.109	2.829	0.086	0.091	0.000	0.147	0.407	6.359

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	60	91	0	218	59	79
N.S.	1	1.00	0.77	0.88	1.25	1.90	0.00	4.54	1.23	1.65
time (sec)	N/A	0.288	0.047	2.182	0.068	0.076	0.000	0.127	0.516	6.261

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	246	168	421	0	619	0	360	286	0
N.S.	1	1.04	0.71	1.78	0.00	2.62	0.00	1.53	1.21	0.00
time (sec)	N/A	1.040	0.356	2.826	0.000	0.107	0.000	0.137	0.440	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	245	183	513	0	704	0	392	345	0
N.S.	1	1.04	0.78	2.18	0.00	3.00	0.00	1.67	1.47	0.00
time (sec)	N/A	1.028	0.730	2.765	0.000	0.123	0.000	0.147	0.285	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	189	516	0	715	0	378	310	0
N.S.	1	1.00	0.77	2.10	0.00	2.91	0.00	1.54	1.26	0.00
time (sec)	N/A	0.953	0.830	2.873	0.000	0.203	0.000	0.160	0.456	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	256	171	431	0	1141	0	364	517	0
N.S.	1	1.01	0.68	1.70	0.00	4.51	0.00	1.44	2.04	0.00
time (sec)	N/A	0.926	1.193	2.891	0.000	0.130	0.000	0.181	0.539	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	326	244	655	0	1863	0	566	850	0
N.S.	1	1.01	0.76	2.03	0.00	5.77	0.00	1.75	2.63	0.00
time (sec)	N/A	1.149	2.020	2.910	0.000	0.325	0.000	0.184	0.469	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	406	301	914	0	2751	0	824	1265	0
N.S.	1	1.03	0.77	2.33	0.00	7.00	0.00	2.10	3.22	0.00
time (sec)	N/A	1.383	2.827	2.834	0.000	1.091	0.000	0.716	0.288	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	486	370	1251	0	3873	0	1132	1754	0
N.S.	1	1.05	0.80	2.70	0.00	8.37	0.00	2.44	3.79	0.00
time (sec)	N/A	1.655	4.493	2.868	0.000	3.669	0.000	0.240	0.636	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	275	136	170	218	193	0	311	155	218
N.S.	1	1.20	0.59	0.74	0.95	0.84	0.00	1.35	0.67	0.95
time (sec)	N/A	0.958	0.170	2.318	0.096	0.103	0.000	0.120	0.507	6.388

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	199	89	98	133	123	0	178	83	142
N.S.	1	1.18	0.53	0.58	0.79	0.73	0.00	1.05	0.49	0.84
time (sec)	N/A	0.711	0.111	2.344	0.084	0.083	0.000	0.116	0.315	6.378

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	123	53	49	65	71	0	88	33	88
N.S.	1	1.16	0.50	0.46	0.61	0.67	0.00	0.83	0.31	0.83
time (sec)	N/A	0.453	0.073	2.234	0.071	0.090	0.000	0.116	0.398	6.218

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	32	18	49	0	36	17	54
N.S.	1	1.00	0.76	0.70	0.39	1.07	0.00	0.78	0.37	1.17
time (sec)	N/A	0.287	0.015	2.267	0.050	0.118	0.000	0.127	0.541	6.117

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	77	0	253	0	67	59	0
N.S.	1	1.00	1.16	0.96	0.00	3.16	0.00	0.84	0.74	0.00
time (sec)	N/A	0.414	0.103	2.756	0.000	0.103	0.000	0.127	0.468	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	140	136	158	0	704	0	171	196	0
N.S.	1	0.99	0.96	1.12	0.00	4.99	0.00	1.21	1.39	0.00
time (sec)	N/A	0.608	0.384	2.960	0.000	0.105	0.000	0.129	0.278	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	220	163	275	0	1284	0	306	452	0
N.S.	1	1.03	0.77	1.29	0.00	6.03	0.00	1.44	2.12	0.00
time (sec)	N/A	0.806	0.482	2.904	0.000	0.117	0.000	0.143	0.409	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	300	191	440	0	2028	0	478	776	0
N.S.	1	1.07	0.68	1.57	0.00	7.24	0.00	1.71	2.77	0.00
time (sec)	N/A	1.028	0.750	2.839	0.000	0.487	0.000	0.151	0.534	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	262	134	179	165	216	0	216	156	252
N.S.	1	1.15	0.59	0.79	0.72	0.95	0.00	0.95	0.68	1.11
time (sec)	N/A	0.878	0.155	2.375	0.106	0.097	0.000	0.128	0.451	6.500

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	186	88	108	98	147	0	120	84	178
N.S.	1	1.11	0.53	0.65	0.59	0.88	0.00	0.72	0.50	1.07
time (sec)	N/A	0.641	0.108	2.232	0.103	0.085	0.000	0.132	0.309	6.323

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	150	51	58	48	96	0	48	35	118
N.S.	1	1.44	0.49	0.56	0.46	0.92	0.00	0.46	0.34	1.13
time (sec)	N/A	0.551	0.083	2.178	0.075	0.093	0.000	0.114	0.541	6.291

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	42	18	74	0	18	19	82
N.S.	1	1.00	0.76	0.91	0.39	1.61	0.00	0.39	0.41	1.78
time (sec)	N/A	0.289	0.013	2.171	0.053	0.077	0.000	0.111	0.628	6.238

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	109	118	0	553	0	83	107	0
N.S.	1	1.00	0.82	0.89	0.00	4.16	0.00	0.62	0.80	0.00
time (sec)	N/A	0.591	0.158	2.733	0.000	0.102	0.000	0.127	0.454	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	215	141	215	0	1067	0	187	288	0
N.S.	1	1.10	0.72	1.10	0.00	5.47	0.00	0.96	1.48	0.00
time (sec)	N/A	0.788	0.563	2.787	0.000	0.122	0.000	0.118	0.382	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	295	185	369	0	1863	0	316	571	0
N.S.	1	1.08	0.68	1.35	0.00	6.82	0.00	1.16	2.09	0.00
time (sec)	N/A	1.067	0.860	2.913	0.000	0.250	0.000	0.120	0.557	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	251	131	179	219	251	0	327	166	278
N.S.	1	1.09	0.57	0.78	0.95	1.09	0.00	1.42	0.72	1.21
time (sec)	N/A	0.833	0.184	2.259	0.116	0.229	0.000	0.142	0.493	6.513

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	217	87	108	138	180	0	172	95	206
N.S.	1	1.30	0.52	0.65	0.83	1.08	0.00	1.03	0.57	1.23
time (sec)	N/A	0.718	0.120	2.299	0.088	0.132	0.000	0.130	0.302	6.473

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	154	52	58	73	129	0	79	46	149
N.S.	1	1.45	0.49	0.55	0.69	1.22	0.00	0.75	0.43	1.41
time (sec)	N/A	0.559	0.086	2.120	0.068	0.118	0.000	0.126	0.418	6.357

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	28	107	0	39	29	110
N.S.	1	1.00	0.77	0.88	0.58	2.23	0.00	0.81	0.60	2.29
time (sec)	N/A	0.294	0.030	2.138	0.062	0.102	0.000	0.121	0.553	6.263

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	203	129	209	0	1015	0	220	294	0
N.S.	1	1.08	0.69	1.11	0.00	5.40	0.00	1.17	1.56	0.00
time (sec)	N/A	0.814	0.333	2.729	0.000	0.154	0.000	0.142	0.467	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	294	180	414	0	1907	0	417	650	0
N.S.	1	1.17	0.71	1.64	0.00	7.57	0.00	1.65	2.58	0.00
time (sec)	N/A	1.014	0.679	2.822	0.000	0.424	0.000	0.160	0.339	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	374	240	660	0	2935	0	577	1055	0
N.S.	1	1.12	0.72	1.98	0.00	8.79	0.00	1.73	3.16	0.00
time (sec)	N/A	1.260	1.234	2.904	0.000	1.194	0.000	0.203	0.626	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	397	235	732	0	1126	0	886	609	0
N.S.	1	1.10	0.65	2.03	0.00	3.12	0.00	2.45	1.69	0.00
time (sec)	N/A	1.367	0.828	2.645	0.000	1.399	0.000	0.256	0.569	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	319	188	504	0	908	0	518	412	0
N.S.	1	1.09	0.64	1.73	0.00	3.11	0.00	1.77	1.41	0.00
time (sec)	N/A	1.146	0.681	2.458	0.000	0.788	0.000	0.194	0.299	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	241	162	319	0	718	0	271	251	0
N.S.	1	1.10	0.74	1.45	0.00	3.26	0.00	1.23	1.14	0.00
time (sec)	N/A	0.851	0.600	2.538	0.000	0.585	0.000	0.153	0.521	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	167	144	188	0	577	0	208	129	0
N.S.	1	1.17	1.01	1.31	0.00	4.03	0.00	1.45	0.90	0.00
time (sec)	N/A	0.642	0.426	2.544	0.000	0.490	0.000	0.139	0.585	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	158	133	187	0	521	0	201	145	0
N.S.	1	1.18	0.99	1.40	0.00	3.89	0.00	1.50	1.08	0.00
time (sec)	N/A	0.612	0.306	2.582	0.000	0.468	0.000	0.208	0.407	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	53	0	169	0	115	148	136
N.S.	1	1.00	0.83	0.84	0.00	2.68	0.00	1.83	2.35	2.16
time (sec)	N/A	0.370	0.069	2.674	0.000	0.096	0.000	0.328	0.322	6.680

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	70	0	402	0	255	382	187
N.S.	1	1.00	0.53	0.54	0.00	3.12	0.00	1.98	2.96	1.45
time (sec)	N/A	0.536	0.151	2.582	0.000	0.143	0.000	0.616	0.618	6.765

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	211	105	119	0	748	0	459	698	289
N.S.	1	1.07	0.53	0.60	0.00	3.78	0.00	2.32	3.53	1.46
time (sec)	N/A	0.760	0.213	2.718	0.000	0.322	0.000	0.558	0.631	7.050

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	293	152	191	0	1179	0	726	1094	409
N.S.	1	1.10	0.57	0.72	0.00	4.42	0.00	2.72	4.10	1.53
time (sec)	N/A	0.984	0.341	2.632	0.000	0.882	0.000	0.966	0.723	7.185

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	391	244	732	0	1120	0	6262	609	0
N.S.	1	1.07	0.67	2.01	0.00	3.08	0.00	17.20	1.67	0.00
time (sec)	N/A	1.375	1.006	2.648	0.000	1.385	0.000	0.956	0.743	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	313	199	504	0	908	0	2098	412	0
N.S.	1	1.07	0.68	1.73	0.00	3.11	0.00	7.18	1.41	0.00
time (sec)	N/A	1.092	0.842	2.632	0.000	0.791	0.000	0.418	0.402	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	239	165	315	0	712	0	304	253	0
N.S.	1	1.12	0.77	1.47	0.00	3.33	0.00	1.42	1.18	0.00
time (sec)	N/A	0.887	0.653	2.639	0.000	0.564	0.000	0.183	0.498	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	227	158	373	0	663	0	165	339	0
N.S.	1	1.15	0.80	1.88	0.00	3.35	0.00	0.83	1.71	0.00
time (sec)	N/A	0.821	0.623	2.592	0.000	0.549	0.000	0.196	0.639	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	219	152	321	0	685	0	245	249	0
N.S.	1	1.15	0.80	1.69	0.00	3.61	0.00	1.29	1.31	0.00
time (sec)	N/A	0.770	0.506	2.605	0.000	0.558	0.000	0.232	0.482	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	55	0	232	0	111	244	232
N.S.	1	1.00	0.83	0.87	0.00	3.68	0.00	1.76	3.87	3.68
time (sec)	N/A	0.366	0.094	2.624	0.000	0.103	0.000	0.252	0.647	6.817

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	526	0	258	525	247
N.S.	1	1.00	0.53	0.77	0.00	4.08	0.00	2.00	4.07	1.91
time (sec)	N/A	0.543	0.197	2.638	0.000	0.165	0.000	0.341	0.816	6.940

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	211	105	169	0	918	0	466	902	377
N.S.	1	1.07	0.53	0.85	0.00	4.64	0.00	2.35	4.56	1.90
time (sec)	N/A	0.759	0.296	2.733	0.000	0.386	0.000	0.439	0.785	7.107

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	293	152	260	0	1420	0	743	1359	519
N.S.	1	1.10	0.57	0.97	0.00	5.32	0.00	2.78	5.09	1.94
time (sec)	N/A	1.013	0.398	2.611	0.000	1.224	0.000	0.867	1.711	7.033

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	463	303	1005	0	1392	0	13705	844	0
N.S.	1	1.06	0.69	2.31	0.00	3.19	0.00	31.43	1.94	0.00
time (sec)	N/A	1.590	1.349	2.597	0.000	3.223	0.000	1.836	0.838	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	385	244	732	0	1126	0	4837	609	0
N.S.	1	1.06	0.67	2.01	0.00	3.09	0.00	13.29	1.67	0.00
time (sec)	N/A	1.295	1.092	2.647	0.000	1.503	0.000	0.749	0.814	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	311	189	498	0	898	0	413	412	0
N.S.	1	1.11	0.68	1.78	0.00	3.21	0.00	1.48	1.47	0.00
time (sec)	N/A	1.035	0.915	2.682	0.000	0.869	0.000	0.236	0.495	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	299	199	625	0	915	0	253	617	0
N.S.	1	1.11	0.74	2.31	0.00	3.39	0.00	0.94	2.29	0.00
time (sec)	N/A	1.045	0.739	2.783	0.000	0.562	0.000	0.194	0.470	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	291	187	628	0	973	0	393	634	0
N.S.	1	1.12	0.72	2.42	0.00	3.74	0.00	1.51	2.44	0.00
time (sec)	N/A	1.006	0.723	2.708	0.000	0.538	0.000	0.236	0.316	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	280	188	501	0	933	0	489	505	0
N.S.	1	1.12	0.75	2.00	0.00	3.73	0.00	1.96	2.02	0.00
time (sec)	N/A	0.972	0.554	2.635	0.000	0.524	0.000	0.298	1.309	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	299	0	155	328	325
N.S.	1	1.00	0.83	1.00	0.00	4.75	0.00	2.46	5.21	5.16
time (sec)	N/A	0.364	0.118	2.714	0.000	0.144	0.000	0.372	0.654	6.691

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	639	0	344	668	315
N.S.	1	1.00	0.53	0.77	0.00	4.95	0.00	2.67	5.18	2.44
time (sec)	N/A	0.553	0.265	2.562	0.000	0.210	0.000	0.494	0.580	6.836

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	211	115	169	0	1101	0	600	1106	465
N.S.	1	1.07	0.58	0.85	0.00	5.56	0.00	3.03	5.59	2.35
time (sec)	N/A	0.749	0.293	2.583	0.000	0.470	0.000	0.657	1.005	7.048

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	293	162	260	0	1648	0	915	1624	627
N.S.	1	1.10	0.61	0.97	0.00	6.17	0.00	3.43	6.08	2.35
time (sec)	N/A	0.972	0.430	2.524	0.000	1.166	0.000	0.911	1.991	7.161

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	247	156	318	0	716	0	284	253	0
N.S.	1	1.12	0.71	1.45	0.00	3.25	0.00	1.29	1.15	0.00
time (sec)	N/A	0.878	0.522	3.066	0.000	0.555	0.000	0.165	0.235	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	169	138	191	0	582	0	190	129	0
N.S.	1	1.17	0.95	1.32	0.00	4.01	0.00	1.31	0.89	0.00
time (sec)	N/A	0.658	0.407	2.999	0.000	0.502	0.000	0.137	0.243	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	105	94	102	0	404	0	95	56	0
N.S.	1	1.30	1.16	1.26	0.00	4.99	0.00	1.17	0.69	0.00
time (sec)	N/A	0.463	0.230	3.050	0.000	0.482	0.000	0.123	0.255	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	45	0	114	0	112	71	100
N.S.	1	1.00	0.82	0.74	0.00	1.87	0.00	1.84	1.16	1.64
time (sec)	N/A	0.365	0.066	3.171	0.000	0.091	0.000	0.134	0.247	7.393

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	61	0	288	0	212	231	147
N.S.	1	1.00	0.53	0.47	0.00	2.23	0.00	1.64	1.79	1.14
time (sec)	N/A	0.554	0.144	2.975	0.000	0.111	0.000	0.164	0.260	7.490

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	211	105	111	0	572	0	407	498	242
N.S.	1	1.07	0.53	0.56	0.00	2.89	0.00	2.06	2.52	1.22
time (sec)	N/A	0.738	0.206	2.820	0.000	0.439	0.000	0.207	0.281	7.647

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	293	152	183	0	953	0	696	833	357
N.S.	1	1.10	0.57	0.69	0.00	3.57	0.00	2.61	3.12	1.34
time (sec)	N/A	0.953	0.288	3.059	0.000	1.675	0.000	0.269	0.341	7.923

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	312	183	638	0	971	0	406	398	0
N.S.	1	1.13	0.66	2.30	0.00	3.51	0.00	1.47	1.44	0.00
time (sec)	N/A	1.069	0.734	2.692	0.000	0.564	0.000	0.297	0.294	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	234	142	386	0	725	0	297	222	0
N.S.	1	1.15	0.70	1.90	0.00	3.57	0.00	1.46	1.09	0.00
time (sec)	N/A	0.818	0.484	2.664	0.000	0.515	0.000	0.217	0.289	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	161	117	200	0	569	0	110	102	0
N.S.	1	1.18	0.85	1.46	0.00	4.15	0.00	0.80	0.74	0.00
time (sec)	N/A	0.616	0.267	2.637	0.000	0.459	0.000	0.155	0.272	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	55	0	125	0	52	57	147
N.S.	1	1.00	0.82	0.90	0.00	2.05	0.00	0.85	0.93	2.41
time (sec)	N/A	0.366	0.062	2.842	0.000	0.093	0.000	0.123	0.250	7.348

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	64	70	0	325	0	285	153	151
N.S.	1	1.00	0.52	0.56	0.00	2.62	0.00	2.30	1.23	1.22
time (sec)	N/A	0.530	0.144	2.816	0.000	0.107	0.000	0.176	0.275	7.561

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	206	105	120	0	649	0	672	367	268
N.S.	1	1.07	0.55	0.62	0.00	3.38	0.00	3.50	1.91	1.40
time (sec)	N/A	0.741	0.212	2.852	0.000	0.200	0.000	0.328	0.270	7.881

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	288	150	192	0	1062	0	1476	664	414
N.S.	1	1.10	0.57	0.73	0.00	4.05	0.00	5.63	2.53	1.58
time (sec)	N/A	0.946	0.285	2.891	0.000	1.270	0.000	0.616	0.327	7.968

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	381	240	1010	0	1413	0	683	813	0
N.S.	1	1.12	0.70	2.96	0.00	4.14	0.00	2.00	2.38	0.00
time (sec)	N/A	1.308	0.934	2.658	0.000	0.969	0.000	0.774	0.363	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	303	188	642	0	1055	0	517	497	0
N.S.	1	1.14	0.71	2.42	0.00	3.98	0.00	1.95	1.88	0.00
time (sec)	N/A	1.022	0.627	2.755	0.000	0.690	0.000	0.490	0.284	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	227	134	333	0	755	0	368	193	0
N.S.	1	1.16	0.69	1.71	0.00	3.87	0.00	1.89	0.99	0.00
time (sec)	N/A	0.785	0.419	2.671	0.000	0.561	0.000	0.340	0.270	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	55	0	193	0	151	123	169
N.S.	1	1.00	0.83	0.87	0.00	3.06	0.00	2.40	1.95	2.68
time (sec)	N/A	0.371	0.071	2.644	0.000	0.109	0.000	0.299	0.241	7.307

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	67	72	0	318	0	268	179	246
N.S.	1	1.00	0.52	0.56	0.00	2.48	0.00	2.09	1.40	1.92
time (sec)	N/A	0.544	0.163	2.789	0.000	0.153	0.000	0.156	0.248	7.571

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	205	103	121	0	667	0	633	389	255
N.S.	1	1.06	0.53	0.63	0.00	3.46	0.00	3.28	2.02	1.32
time (sec)	N/A	0.738	0.216	3.188	0.000	0.303	0.000	0.267	0.264	7.564

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	285	152	191	0	1065	0	1119	760	416
N.S.	1	1.10	0.58	0.73	0.00	4.10	0.00	4.30	2.92	1.60
time (sec)	N/A	0.955	0.308	2.921	0.000	0.830	0.000	0.520	0.283	7.794

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	122	100	0	0	0	0	0	0	0
N.S.	1	1.09	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	0.340	0.000	0.000	0.000	0.000	0.000	0.454	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	120	100	0	0	0	0	0	0	0
N.S.	1	1.07	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.432	0.215	0.000	0.000	0.000	0.000	0.000	0.364	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	118	98	0	0	0	0	0	520	0
N.S.	1	1.07	0.89	0.00	0.00	0.00	0.00	0.00	4.73	0.00
time (sec)	N/A	0.440	0.212	0.000	0.000	0.000	0.000	0.000	0.308	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	110	98	0	0	0	0	0	0	0
N.S.	1	0.98	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.437	0.290	0.000	0.000	0.000	0.000	0.000	0.321	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	122	100	0	0	0	0	0	0	0
N.S.	1	1.27	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.495	0.413	0.000	0.000	0.000	0.000	0.000	0.400	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	122	110	0	0	0	0	0	0	0
N.S.	1	1.09	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	0.480	0.000	0.000	0.000	0.000	0.000	0.458	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	107	95	0	0	0	0	0	45	0
N.S.	1	1.08	0.96	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.441	0.116	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	324	134	527	331	705	0	1926	180	615
N.S.	1	1.25	0.52	2.03	1.27	2.71	0.00	7.41	0.69	2.37
time (sec)	N/A	0.997	0.437	10.310	0.083	0.106	0.000	0.211	0.268	6.360

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	237	131	235	193	350	0	929	131	327
N.S.	1	1.25	0.69	1.24	1.02	1.84	0.00	4.89	0.69	1.72
time (sec)	N/A	0.711	0.327	5.908	0.092	0.098	0.000	0.184	0.250	6.401

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	150	67	89	94	145	0	347	82	139
N.S.	1	1.25	0.56	0.74	0.78	1.21	0.00	2.89	0.68	1.16
time (sec)	N/A	0.479	0.181	3.747	0.054	0.111	0.000	0.222	0.267	6.107

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	57	33	57	0	81	38	57
N.S.	1	1.00	0.78	1.06	0.61	1.06	0.00	1.50	0.70	1.06
time (sec)	N/A	0.303	0.015	2.940	0.068	0.083	0.000	0.176	0.270	6.109

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	82	0	0	0	0	0	70	0
N.S.	1	1.04	0.86	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.415	0.111	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	101	84	0	0	0	0	0	107	0
N.S.	1	1.04	0.87	0.00	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.409	0.110	0.000	0.000	0.000	0.000	0.000	0.305	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	88	0	0	0	0	0	144	0
N.S.	1	1.04	0.87	0.00	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.421	0.122	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	94	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.433	0.292	0.000	0.000	0.000	0.000	0.000	0.364	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	44	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.416	0.197	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	76	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.416	0.221	0.000	0.000	0.000	0.000	0.000	0.296	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	113	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.410	0.290	0.000	0.000	0.000	0.000	0.000	0.290	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	150	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.425	0.362	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	52	64	49	66	0	106	48	63
N.S.	1	1.03	0.83	1.02	0.78	1.05	0.00	1.68	0.76	1.00
time (sec)	N/A	0.352	0.022	9.397	0.045	0.090	0.000	0.130	0.248	6.243

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	81	64	0	32	35	0	0	99	0
N.S.	1	1.04	0.82	0.00	0.41	0.45	0.00	0.00	1.27	0.00
time (sec)	N/A	0.510	0.042	0.000	0.049	0.083	0.000	0.000	0.238	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	222	145	0	0	0	0	0	0	0
N.S.	1	1.04	0.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.735	0.393	0.000	0.000	0.000	0.000	0.000	0.521	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	380	623	693	597	0	1159	608	653
N.S.	1	1.00	0.87	1.42	1.58	1.36	0.00	2.65	1.39	1.49
time (sec)	N/A	1.524	0.532	3.432	0.115	0.098	0.000	0.141	0.290	6.595

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	362	264	407	484	408	0	755	392	438
N.S.	1	1.03	0.75	1.16	1.38	1.16	0.00	2.14	1.11	1.24
time (sec)	N/A	1.190	0.338	3.562	0.092	0.093	0.000	0.138	0.231	6.786

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	286	169	237	309	256	0	439	222	279
N.S.	1	1.06	0.63	0.88	1.14	0.95	0.00	1.63	0.82	1.03
time (sec)	N/A	0.970	0.229	3.573	0.078	0.088	0.000	0.124	0.278	6.750

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	189	96	113	168	141	0	211	98	152
N.S.	1	1.02	0.52	0.61	0.90	0.76	0.00	1.13	0.53	0.82
time (sec)	N/A	0.634	0.131	3.561	0.066	0.084	0.000	0.116	0.264	6.653

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	110	54	51	65	73	0	89	35	85
N.S.	1	1.02	0.50	0.47	0.60	0.68	0.00	0.82	0.32	0.79
time (sec)	N/A	0.392	0.071	2.243	0.048	0.094	0.000	0.107	0.229	6.620

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	140	153	0	512	0	132	153	0
N.S.	1	1.00	1.01	1.10	0.00	3.68	0.00	0.95	1.10	0.00
time (sec)	N/A	0.607	0.247	2.983	0.000	0.121	0.000	0.120	0.240	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	170	154	337	0	897	0	254	444	0
N.S.	1	1.01	0.91	1.99	0.00	5.31	0.00	1.50	2.63	0.00
time (sec)	N/A	0.709	0.746	3.046	0.000	0.127	0.000	0.128	0.261	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	252	200	663	0	1705	0	533	927	0
N.S.	1	0.97	0.77	2.54	0.00	6.53	0.00	2.04	3.55	0.00
time (sec)	N/A	0.922	1.460	2.971	0.000	0.164	0.000	0.140	0.270	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	332	279	1132	0	2737	0	875	1527	0
N.S.	1	0.95	0.79	3.23	0.00	7.80	0.00	2.49	4.35	0.00
time (sec)	N/A	1.197	1.851	2.905	0.000	0.873	0.000	0.159	0.234	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	378	448	2477	0	1469	2919	755	2564	3311
N.S.	1	1.08	1.28	7.08	0.00	4.20	8.34	2.16	7.33	9.46
time (sec)	N/A	1.191	1.612	3.415	0.000	0.771	1.608	0.156	2.053	9.641

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	313	349	1375	0	1117	1608	568	1875	1732
N.S.	1	1.13	1.26	4.96	0.00	4.03	5.81	2.05	6.77	6.25
time (sec)	N/A	0.969	1.330	2.745	0.000	0.350	1.577	0.162	0.946	8.077

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	215	269	724	0	825	952	409	1300	801
N.S.	1	0.99	1.23	3.32	0.00	3.78	4.37	1.88	5.96	3.67
time (sec)	N/A	0.688	0.893	2.250	0.000	0.195	2.364	0.155	0.490	7.272

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	148	181	293	0	395	0	178	490	0
N.S.	1	1.03	1.27	2.05	0.00	2.76	0.00	1.24	3.43	0.00
time (sec)	N/A	0.673	0.895	2.447	0.000	0.111	0.000	0.142	0.265	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	176	129	372	0	399	0	294	692	0
N.S.	1	1.12	0.82	2.37	0.00	2.54	0.00	1.87	4.41	0.00
time (sec)	N/A	0.715	0.482	2.941	0.000	0.399	0.000	0.212	0.269	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	186	160	299	0	579	0	517	1004	0
N.S.	1	1.14	0.98	1.83	0.00	3.55	0.00	3.17	6.16	0.00
time (sec)	N/A	0.739	0.439	3.392	0.000	0.749	0.000	0.943	0.244	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	154	102	128	0	307	0	0	1197	1022
N.S.	1	1.12	0.74	0.93	0.00	2.24	0.00	0.00	8.74	7.46
time (sec)	N/A	0.667	0.250	4.206	0.000	3.979	0.000	0.000	0.345	7.304

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	236	166	236	0	540	0	1619	1918	2325
N.S.	1	1.12	0.79	1.12	0.00	2.57	0.00	7.71	9.13	11.07
time (sec)	N/A	0.836	0.296	5.193	0.000	26.503	0.000	0.167	0.663	8.894

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	317	244	382	0	817	0	0	2777	4962
N.S.	1	1.11	0.86	1.34	0.00	2.87	0.00	0.00	9.74	17.41
time (sec)	N/A	1.016	0.392	6.715	0.000	107.640	0.000	0.000	1.511	13.063

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	394	347	564	0	0	0	0	3784	10084
N.S.	1	1.09	0.96	1.57	0.00	0.00	0.00	0.00	10.51	28.01
time (sec)	N/A	1.194	0.516	8.764	0.000	0.000	0.000	0.000	3.704	21.584

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	476	466	782	0	0	0	0	4939	19572
N.S.	1	1.08	1.06	1.78	0.00	0.00	0.00	0.00	11.25	44.58
time (sec)	N/A	1.421	0.676	11.363	0.000	0.000	0.000	0.000	8.320	37.472

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	441	733	3509	0	2337	9527	1229	4284	0
N.S.	1	1.04	1.73	8.28	0.00	5.51	22.47	2.90	10.10	0.00
time (sec)	N/A	1.277	3.764	3.528	0.000	3.432	1.775	0.424	11.200	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	376	592	1977	0	1877	5834	983	3367	0
N.S.	1	1.07	1.69	5.63	0.00	5.35	16.62	2.80	9.59	0.00
time (sec)	N/A	1.112	2.922	2.883	0.000	1.738	1.622	0.380	4.957	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	278	473	1068	0	1473	3228	765	2564	0
N.S.	1	0.95	1.62	3.66	0.00	5.04	11.05	2.62	8.78	0.00
time (sec)	N/A	0.795	2.217	2.411	0.000	0.777	2.090	0.380	1.920	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	215	292	475	0	809	1958	408	1300	0
N.S.	1	0.99	1.34	2.18	0.00	3.71	8.98	1.87	5.96	0.00
time (sec)	N/A	0.788	1.177	2.549	0.000	0.190	3.226	0.359	0.462	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	262	220	564	0	567	0	1297	839	0
N.S.	1	1.24	1.04	2.67	0.00	2.69	0.00	6.15	3.98	0.00
time (sec)	N/A	0.818	0.830	2.864	0.000	0.216	0.000	0.663	0.332	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	258	217	732	0	635	0	405	1201	0
N.S.	1	1.17	0.99	3.33	0.00	2.89	0.00	1.84	5.46	0.00
time (sec)	N/A	0.843	1.479	3.388	0.000	0.596	0.000	0.395	0.317	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	260	187	900	0	593	0	582	1184	0
N.S.	1	1.18	0.85	4.09	0.00	2.70	0.00	2.65	5.38	0.00
time (sec)	N/A	0.925	0.620	4.451	0.000	1.093	0.000	1.605	0.355	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	221	237	563	0	879	0	1592	1667	0
N.S.	1	1.05	1.13	2.68	0.00	4.19	0.00	7.58	7.94	0.00
time (sec)	N/A	0.816	0.733	5.424	0.000	5.385	0.000	0.436	0.451	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	137	104	128	0	464	0	0	1837	3763
N.S.	1	0.99	0.75	0.93	0.00	3.36	0.00	0.00	13.31	27.27
time (sec)	N/A	0.603	0.304	6.592	0.000	25.682	0.000	0.000	0.747	11.117

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	213	168	236	0	739	0	0	2680	8039
N.S.	1	1.01	0.80	1.12	0.00	3.52	0.00	0.00	12.76	38.28
time (sec)	N/A	0.752	0.372	8.709	0.000	102.834	0.000	0.000	1.699	17.587

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	286	249	382	0	0	0	0	3670	16485
N.S.	1	1.00	0.87	1.34	0.00	0.00	0.00	0.00	12.88	57.84
time (sec)	N/A	0.868	0.503	11.036	0.000	0.000	0.000	0.000	4.036	30.258

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	359	349	564	0	0	0	0	4808	33375
N.S.	1	1.00	0.97	1.57	0.00	0.00	0.00	0.00	13.36	92.71
time (sec)	N/A	1.013	0.689	18.306	0.000	0.000	0.000	0.000	8.294	53.007

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	504	1013	4541	0	3437	29237	1818	6460	0
N.S.	1	1.01	2.03	9.12	0.00	6.90	58.71	3.65	12.97	0.00
time (sec)	N/A	1.430	7.388	3.687	0.000	13.078	2.298	0.486	48.694	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	439	847	2579	0	2861	18144	1512	5315	0
N.S.	1	1.03	1.99	6.07	0.00	6.73	42.69	3.56	12.51	0.00
time (sec)	N/A	1.285	7.179	2.856	0.000	6.430	1.837	0.409	20.642	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	341	733	1412	0	2345	11123	1234	4284	0
N.S.	1	0.93	2.00	3.86	0.00	6.41	30.39	3.37	11.70	0.00
time (sec)	N/A	0.956	4.701	2.405	0.000	3.320	2.402	0.410	8.499	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	280	471	655	0	1457	7917	758	2564	0
N.S.	1	0.96	1.61	2.24	0.00	4.99	27.11	2.60	8.78	0.00
time (sec)	N/A	0.957	2.571	2.501	0.000	0.725	8.626	0.398	1.734	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	330	376	752	1751	1097	4675	3630	1875	0
N.S.	1	1.16	1.32	2.64	6.14	3.85	16.40	12.74	6.58	0.00
time (sec)	N/A	0.971	1.600	2.808	0.169	0.373	10.487	1.220	0.868	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	327	292	920	0	813	0	410	1300	0
N.S.	1	1.12	1.00	3.15	0.00	2.78	0.00	1.40	4.45	0.00
time (sec)	N/A	1.029	1.457	3.257	0.000	0.377	0.000	0.400	0.442	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	309	258	1088	0	931	0	503	1826	0
N.S.	1	1.07	0.89	3.75	0.00	3.21	0.00	1.73	6.30	0.00
time (sec)	N/A	1.008	1.520	4.227	0.000	1.037	0.000	0.432	0.414	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	326	267	1256	0	951	0	1076	2027	0
N.S.	1	1.10	0.90	4.23	0.00	3.20	0.00	3.62	6.82	0.00
time (sec)	N/A	1.311	3.437	5.322	0.000	1.882	0.000	0.658	0.531	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	311	248	1424	0	917	0	818	2008	0
N.S.	1	1.11	0.88	5.07	0.00	3.26	0.00	2.91	7.15	0.00
time (sec)	N/A	1.046	1.024	7.117	0.000	7.313	0.000	72.477	0.717	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	270	298	825	0	1239	0	0	2439	0
N.S.	1	1.04	1.15	3.19	0.00	4.78	0.00	0.00	9.42	0.00
time (sec)	N/A	0.898	1.165	9.434	0.000	34.440	0.000	0.000	0.983	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	137	104	128	0	646	0	0	2583	12294
N.S.	1	0.99	0.75	0.93	0.00	4.68	0.00	0.00	18.72	89.09
time (sec)	N/A	0.635	0.354	12.240	0.000	106.886	0.000	0.000	1.884	20.371

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	213	169	236	0	0	0	0	3556	25236
N.S.	1	1.01	0.80	1.12	0.00	0.00	0.00	0.00	16.93	120.17
time (sec)	N/A	0.719	0.413	20.723	0.000	0.000	0.000	0.000	4.185	37.013

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	286	250	382	0	0	0	0	4677	51074
N.S.	1	1.00	0.88	1.34	0.00	0.00	0.00	0.00	16.41	179.21
time (sec)	N/A	0.864	0.589	26.774	0.000	0.000	0.000	0.000	9.057	67.586

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	359	351	564	0	0	0	0	5946	38717
N.S.	1	1.00	0.98	1.57	0.00	0.00	0.00	0.00	16.52	107.55
time (sec)	N/A	1.016	0.739	34.843	0.000	0.000	0.000	0.000	18.733	133.293

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	313	289	1445	0	825	1032	412	1300	0
N.S.	1	1.13	1.05	5.24	0.00	2.99	3.74	1.49	4.71	0.00
time (sec)	N/A	0.967	0.952	3.892	0.000	0.331	1.470	0.402	0.430	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	248	223	773	0	585	660	282	839	0
N.S.	1	1.22	1.10	3.81	0.00	2.88	3.25	1.39	4.13	0.00
time (sec)	N/A	0.814	0.723	3.136	0.000	0.174	1.437	0.400	0.314	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	149	181	380	0	403	416	190	492	0
N.S.	1	1.03	1.26	2.64	0.00	2.80	2.89	1.32	3.42	0.00
time (sec)	N/A	0.538	0.891	2.485	0.000	0.115	2.070	0.374	0.222	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	121	141	134	0	405	0	288	486	0
N.S.	1	1.03	1.21	1.15	0.00	3.46	0.00	2.46	4.15	0.00
time (sec)	N/A	0.513	0.323	2.599	0.000	0.344	0.000	0.394	0.248	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	89	115	0	182	0	302	627	101
N.S.	1	1.00	0.65	0.84	0.00	1.33	0.00	2.20	4.58	0.74
time (sec)	N/A	0.572	0.245	2.997	0.000	0.916	0.000	0.435	0.240	6.391

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	213	166	224	0	368	0	0	1262	471
N.S.	1	1.01	0.79	1.07	0.00	1.75	0.00	0.00	6.01	2.24
time (sec)	N/A	0.727	0.281	3.302	0.000	4.632	0.000	0.000	0.318	6.595

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	286	247	370	0	606	0	0	1998	624
N.S.	1	1.00	0.87	1.30	0.00	2.13	0.00	0.00	7.01	2.19
time (sec)	N/A	0.859	0.391	4.161	0.000	31.989	0.000	0.000	0.506	7.689

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	359	348	552	0	906	0	900	2874	949
N.S.	1	1.00	0.97	1.53	0.00	2.52	0.00	2.50	7.98	2.64
time (sec)	N/A	1.001	0.487	4.887	0.000	104.865	0.000	0.336	1.253	9.616

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	263	235	1535	0	745	0	495	918	0
N.S.	1	1.06	0.95	6.19	0.00	3.00	0.00	2.00	3.70	0.00
time (sec)	N/A	1.072	2.040	3.665	0.000	0.635	0.000	0.407	0.269	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	182	142	836	0	503	0	439	551	0
N.S.	1	1.08	0.85	4.98	0.00	2.99	0.00	2.61	3.28	0.00
time (sec)	N/A	0.655	0.391	3.007	0.000	0.410	0.000	0.358	0.238	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	129	139	421	0	483	0	374	424	344
N.S.	1	1.04	1.12	3.40	0.00	3.90	0.00	3.02	3.42	2.77
time (sec)	N/A	0.471	0.311	2.332	0.000	0.345	0.000	0.322	0.261	12.877

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	149	228	0	407	0	0	1113	872
N.S.	1	1.00	1.10	1.68	0.00	2.99	0.00	0.00	8.18	6.41
time (sec)	N/A	0.519	0.309	2.345	0.000	3.919	0.000	0.000	0.238	11.578

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	233	380	0	649	0	7557	1782	2126
N.S.	1	1.00	1.11	1.82	0.00	3.11	0.00	36.16	8.53	10.17
time (sec)	N/A	0.647	0.392	2.569	0.000	15.754	0.000	0.508	0.347	13.115

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	282	331	562	0	974	0	0	2565	4339
N.S.	1	0.99	1.17	1.98	0.00	3.43	0.00	0.00	9.03	15.28
time (sec)	N/A	0.790	0.497	2.881	0.000	74.106	0.000	0.000	0.677	16.416

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	340	139	6819	0	1233	0	1051	2003	0
N.S.	1	1.09	0.45	21.93	0.00	3.96	0.00	3.38	6.44	0.00
time (sec)	N/A	1.293	10.354	5.935	0.000	3.209	0.000	2.931	0.471	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	276	204	4116	0	881	0	997	1231	0
N.S.	1	1.20	0.89	17.90	0.00	3.83	0.00	4.33	5.35	0.00
time (sec)	N/A	0.878	0.780	4.579	0.000	1.946	0.000	2.019	0.296	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	186	187	2417	0	785	0	943	987	0
N.S.	1	1.08	1.08	13.97	0.00	4.54	0.00	5.45	5.71	0.00
time (sec)	N/A	0.658	0.584	3.492	0.000	1.216	0.000	1.517	0.261	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	146	100	123	0	227	0	0	628	107
N.S.	1	1.09	0.75	0.92	0.00	1.69	0.00	0.00	4.69	0.80
time (sec)	N/A	0.572	0.234	2.736	0.000	1.268	0.000	0.000	0.225	11.554

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	151	225	0	431	0	0	1117	795
N.S.	1	1.00	1.03	1.54	0.00	2.95	0.00	0.00	7.65	5.45
time (sec)	N/A	0.513	0.306	2.369	0.000	5.191	0.000	0.000	0.251	11.878

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	205	345	418	0	1028	0	0	2763	3326
N.S.	1	0.99	1.66	2.01	0.00	4.94	0.00	0.00	13.28	15.99
time (sec)	N/A	0.642	0.499	2.398	0.000	52.098	0.000	0.000	0.644	13.637

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	278	468	598	0	0	0	22347	3857	11539
N.S.	1	0.98	1.65	2.11	0.00	0.00	0.00	78.96	13.63	40.77
time (sec)	N/A	0.766	0.645	2.505	0.000	0.000	0.000	0.786	1.572	18.942

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	351	505	766	0	0	0	0	4847	33819
N.S.	1	0.98	1.41	2.14	0.00	0.00	0.00	0.00	13.54	94.47
time (sec)	N/A	0.904	1.245	2.856	0.000	0.000	0.000	0.000	3.722	32.173

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	322	262	359	501	499	0	3520	500	501
N.S.	1	0.94	0.76	1.05	1.46	1.45	0.00	10.26	1.46	1.46
time (sec)	N/A	0.995	0.304	2.892	0.119	0.091	0.000	0.367	0.251	11.570

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	254	179	227	354	352	0	2139	332	337
N.S.	1	0.95	0.67	0.85	1.33	1.32	0.00	8.01	1.24	1.26
time (sec)	N/A	0.822	0.221	2.911	0.091	0.093	0.000	0.304	0.225	11.205

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	187	119	131	236	233	0	804	200	219
N.S.	1	0.98	0.63	0.69	1.24	1.23	0.00	4.23	1.05	1.15
time (sec)	N/A	0.668	0.134	2.477	0.089	0.099	0.000	0.334	0.206	11.040

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	76	71	112	140	0	379	103	100
N.S.	1	1.02	0.66	0.61	0.97	1.21	0.00	3.27	0.89	0.86
time (sec)	N/A	0.546	0.095	2.802	0.064	0.083	0.000	0.327	0.232	10.953

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	183	152	322	0	421	0	157	166	0
N.S.	1	0.98	0.82	1.73	0.00	2.26	0.00	0.84	0.89	0.00
time (sec)	N/A	0.720	0.362	2.100	0.000	0.099	0.000	0.359	0.242	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	217	136	351	0	661	0	158	413	0
N.S.	1	1.16	0.73	1.88	0.00	3.53	0.00	0.84	2.21	0.00
time (sec)	N/A	0.783	0.402	2.134	0.000	0.100	0.000	0.291	0.215	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	219	181	622	0	1045	0	328	780	0
N.S.	1	1.00	0.82	2.83	0.00	4.75	0.00	1.49	3.55	0.00
time (sec)	N/A	0.775	0.787	2.207	0.000	0.112	0.000	0.361	0.216	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	284	246	1025	0	1606	0	587	1302	0
N.S.	1	0.96	0.83	3.46	0.00	5.43	0.00	1.98	4.40	0.00
time (sec)	N/A	0.911	1.384	2.202	0.000	0.155	0.000	0.379	0.269	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	360	330	1533	0	2258	0	1013	1916	0
N.S.	1	0.96	0.88	4.09	0.00	6.02	0.00	2.70	5.11	0.00
time (sec)	N/A	1.056	1.756	2.271	0.000	0.242	0.000	0.372	0.219	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	388	364	529	875	880	0	12772	944	863
N.S.	1	0.92	0.86	1.26	2.08	2.09	0.00	30.34	2.24	2.05
time (sec)	N/A	1.112	0.479	2.106	0.107	0.127	0.000	0.480	0.228	12.978

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	322	264	361	676	678	0	7604	704	637
N.S.	1	0.94	0.77	1.05	1.97	1.98	0.00	22.17	2.05	1.86
time (sec)	N/A	0.984	0.345	2.020	0.113	0.109	0.000	0.410	0.240	12.637

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	254	183	229	502	505	0	3758	500	441
N.S.	1	0.95	0.69	0.86	1.88	1.89	0.00	14.07	1.87	1.65
time (sec)	N/A	0.813	0.240	2.020	0.097	0.090	0.000	0.373	0.250	12.107

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	188	121	133	320	354	0	1123	332	239
N.S.	1	0.99	0.64	0.70	1.68	1.86	0.00	5.91	1.75	1.26
time (sec)	N/A	0.694	0.161	1.963	0.080	0.098	0.000	0.383	0.221	11.753

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	78	73	197	229	0	540	200	133
N.S.	1	1.02	0.67	0.63	1.70	1.97	0.00	4.66	1.72	1.15
time (sec)	N/A	0.558	0.119	2.122	0.079	0.088	0.000	0.354	0.233	11.450

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	241	194	593	0	658	0	316	392	0
N.S.	1	0.96	0.78	2.37	0.00	2.63	0.00	1.26	1.57	0.00
time (sec)	N/A	0.843	0.544	1.717	0.000	0.109	0.000	0.340	0.214	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	276	192	687	0	641	0	323	442	0
N.S.	1	1.07	0.74	2.66	0.00	2.48	0.00	1.25	1.71	0.00
time (sec)	N/A	0.924	0.707	1.625	0.000	0.115	0.000	0.372	0.252	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	270	192	657	0	1000	0	334	788	0
N.S.	1	1.05	0.75	2.56	0.00	3.89	0.00	1.30	3.07	0.00
time (sec)	N/A	0.939	0.785	1.538	0.000	0.119	0.000	0.388	0.213	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	272	255	991	0	1456	0	592	1239	0
N.S.	1	0.95	0.89	3.48	0.00	5.11	0.00	2.08	4.35	0.00
time (sec)	N/A	0.892	1.386	1.563	0.000	0.136	0.000	0.437	0.263	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	338	339	1509	0	2106	0	983	1836	0
N.S.	1	0.93	0.93	4.13	0.00	5.77	0.00	2.69	5.03	0.00
time (sec)	N/A	1.044	2.327	1.501	0.000	0.215	0.000	0.394	0.237	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	456	484	733	1364	1370	0	29765	1532	1307
N.S.	1	0.92	0.98	1.48	2.75	2.76	0.00	60.01	3.09	2.64
time (sec)	N/A	1.297	0.694	2.119	0.127	0.208	0.000	0.629	0.232	14.277

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	389	367	529	1108	1112	0	21712	1220	1023
N.S.	1	0.92	0.87	1.26	2.63	2.64	0.00	51.57	2.90	2.43
time (sec)	N/A	1.150	0.537	1.955	0.109	0.158	0.000	0.519	0.223	13.142

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	321	264	361	878	881	0	11336	944	769
N.S.	1	0.94	0.77	1.05	2.56	2.57	0.00	33.05	2.75	2.24
time (sec)	N/A	0.999	0.376	1.993	0.120	0.122	0.000	0.430	0.261	12.228

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	255	183	229	638	675	0	5116	704	491
N.S.	1	0.96	0.69	0.86	2.39	2.53	0.00	19.16	2.64	1.84
time (sec)	N/A	0.843	0.321	2.089	0.074	0.110	0.000	0.460	0.212	11.697

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	188	121	133	465	500	0	3083	500	320
N.S.	1	0.99	0.64	0.70	2.45	2.63	0.00	16.23	2.63	1.68
time (sec)	N/A	0.718	0.202	2.009	0.078	0.095	0.000	0.411	0.204	11.359

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	78	73	314	345	0	1540	331	170
N.S.	1	1.02	0.67	0.63	2.71	2.97	0.00	13.28	2.85	1.47
time (sec)	N/A	0.570	0.162	1.961	0.076	0.089	0.000	0.435	0.205	11.015

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	299	253	948	0	951	0	560	688	0
N.S.	1	0.95	0.80	3.00	0.00	3.01	0.00	1.77	2.18	0.00
time (sec)	N/A	0.975	0.677	1.578	0.000	0.108	0.000	0.405	0.252	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	334	254	1128	0	992	0	579	836	0
N.S.	1	1.01	0.77	3.41	0.00	3.00	0.00	1.75	2.53	0.00
time (sec)	N/A	1.067	0.964	1.497	0.000	0.111	0.000	0.426	0.225	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	329	254	1181	0	961	0	572	809	0
N.S.	1	0.96	0.74	3.46	0.00	2.82	0.00	1.68	2.37	0.00
time (sec)	N/A	1.074	1.260	1.542	0.000	0.127	0.000	0.393	0.213	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	323	257	1062	0	1390	0	596	1230	0
N.S.	1	0.99	0.79	3.27	0.00	4.28	0.00	1.83	3.78	0.00
time (sec)	N/A	1.051	1.415	1.500	0.000	0.167	0.000	0.409	0.211	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	325	343	1509	0	1912	0	956	1756	0
N.S.	1	0.92	0.97	4.27	0.00	5.42	0.00	2.71	4.97	0.00
time (sec)	N/A	1.030	2.244	1.491	0.000	0.183	0.000	0.424	0.219	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	391	442	2079	0	2652	0	1449	2434	0
N.S.	1	0.91	1.03	4.83	0.00	6.17	0.00	3.37	5.66	0.00
time (sec)	N/A	1.199	3.335	1.527	0.000	0.314	0.000	0.486	0.265	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	255	181	215	319	235	0	405	200	246
N.S.	1	0.96	0.68	0.81	1.20	0.89	0.00	1.53	0.75	0.93
time (sec)	N/A	0.839	0.196	1.863	0.083	0.080	0.000	0.330	0.216	11.422

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	118	119	201	143	0	188	104	149
N.S.	1	1.00	0.63	0.63	1.07	0.76	0.00	1.00	0.55	0.79
time (sec)	N/A	0.690	0.121	1.947	0.064	0.082	0.000	0.331	0.218	11.120

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	117	63	59	110	79	0	68	44	89
N.S.	1	1.03	0.55	0.52	0.96	0.69	0.00	0.60	0.39	0.78
time (sec)	N/A	0.557	0.088	1.473	0.059	0.077	0.000	0.340	0.222	10.911

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	137	153	0	444	0	81	138	0
N.S.	1	1.00	1.05	1.17	0.00	3.39	0.00	0.62	1.05	0.00
time (sec)	N/A	0.581	0.282	1.491	0.000	0.087	0.000	0.315	0.231	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	153	320	0	700	0	156	398	0
N.S.	1	1.00	1.00	2.09	0.00	4.58	0.00	1.02	2.60	0.00
time (sec)	N/A	0.644	0.579	1.678	0.000	0.096	0.000	0.295	0.216	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	232	196	622	0	1170	0	357	828	0
N.S.	1	1.00	0.84	2.67	0.00	5.02	0.00	1.53	3.55	0.00
time (sec)	N/A	0.797	0.771	1.555	0.000	0.132	0.000	0.323	0.249	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	353	245	361	317	374	0	692	334	398
N.S.	1	1.05	0.73	1.07	0.94	1.11	0.00	2.05	0.99	1.18
time (sec)	N/A	1.007	0.293	1.567	0.088	0.094	0.000	0.357	0.213	11.350

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	286	168	229	203	257	0	393	202	267
N.S.	1	1.09	0.64	0.87	0.77	0.98	0.00	1.49	0.77	1.02
time (sec)	N/A	0.879	0.197	1.515	0.084	0.088	0.000	0.358	0.244	11.045

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	219	105	133	112	165	0	196	106	167
N.S.	1	1.18	0.56	0.72	0.60	0.89	0.00	1.05	0.57	0.90
time (sec)	N/A	0.713	0.124	1.669	0.080	0.084	0.000	0.305	0.267	10.811

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	148	60	72	62	102	0	73	45	107
N.S.	1	1.32	0.54	0.64	0.55	0.91	0.00	0.65	0.40	0.96
time (sec)	N/A	0.555	0.078	1.621	0.072	0.084	0.000	0.294	0.216	10.865

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	138	199	0	778	0	121	198	0
N.S.	1	1.00	0.89	1.28	0.00	5.02	0.00	0.78	1.28	0.00
time (sec)	N/A	0.599	0.310	1.644	0.000	0.103	0.000	0.336	0.256	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	228	172	471	0	1353	0	288	550	0
N.S.	1	1.02	0.77	2.11	0.00	6.07	0.00	1.29	2.47	0.00
time (sec)	N/A	0.766	0.687	1.738	0.000	0.168	0.000	0.369	0.222	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	304	255	816	0	1978	0	467	992	0
N.S.	1	1.00	0.84	2.69	0.00	6.53	0.00	1.54	3.27	0.00
time (sec)	N/A	0.904	1.341	1.629	0.000	0.317	0.000	0.373	0.283	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	379	339	1216	0	2834	0	776	1500	0
N.S.	1	0.99	0.89	3.18	0.00	7.42	0.00	2.03	3.93	0.00
time (sec)	N/A	1.056	2.050	1.665	0.000	0.675	0.000	0.351	0.219	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	420	366	529	511	578	0	975	516	596
N.S.	1	1.01	0.88	1.27	1.23	1.39	0.00	2.34	1.24	1.43
time (sec)	N/A	1.193	0.486	1.581	0.103	0.258	0.000	0.387	0.224	12.565

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	353	263	361	363	428	0	616	348	435
N.S.	1	1.04	0.78	1.06	1.07	1.26	0.00	1.82	1.03	1.28
time (sec)	N/A	1.056	0.321	1.575	0.111	0.197	0.000	0.370	0.260	12.346

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	283	180	229	246	308	0	358	215	314
N.S.	1	1.08	0.68	0.87	0.94	1.17	0.00	1.36	0.82	1.19
time (sec)	N/A	0.875	0.224	1.678	0.087	0.137	0.000	0.356	0.230	12.167

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	218	117	132	157	216	0	178	119	214
N.S.	1	1.17	0.63	0.71	0.84	1.16	0.00	0.96	0.64	1.15
time (sec)	N/A	0.716	0.142	1.533	0.087	0.102	0.000	0.338	0.237	11.971

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	152	76	72	103	154	0	76	60	154
N.S.	1	1.33	0.67	0.63	0.90	1.35	0.00	0.67	0.53	1.35
time (sec)	N/A	0.564	0.098	1.563	0.100	0.091	0.000	0.362	0.257	11.920

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	229	185	477	0	1439	0	233	588	0
N.S.	1	1.04	0.84	2.16	0.00	6.51	0.00	1.05	2.66	0.00
time (sec)	N/A	0.734	0.695	1.538	0.000	0.211	0.000	0.377	0.239	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	316	248	896	0	2108	0	370	1124	0
N.S.	1	1.09	0.85	3.08	0.00	7.24	0.00	1.27	3.86	0.00
time (sec)	N/A	0.933	1.265	1.608	0.000	0.442	0.000	0.362	0.249	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	329	1520	0	3098	0	614	1902	0
N.S.	1	1.00	0.87	4.03	0.00	8.22	0.00	1.63	5.05	0.00
time (sec)	N/A	1.115	1.832	1.612	0.000	1.577	0.000	0.362	0.233	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	452	438	2003	0	4086	0	1067	2504	0
N.S.	1	0.98	0.95	4.36	0.00	8.90	0.00	2.32	5.46	0.00
time (sec)	N/A	1.262	3.219	1.576	0.000	3.368	0.000	0.391	0.222	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	149	42	0	25	0	0	32	0
N.S.	1	1.00	2.92	0.82	0.00	0.49	0.00	0.00	0.63	0.00
time (sec)	N/A	0.289	31.539	1.115	0.000	0.073	0.000	0.000	0.311	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	226	170	0	0	0	0	0	44	0
N.S.	1	1.30	0.98	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.879	1.193	0.000	0.000	0.000	0.000	0.000	200.097	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	215	149	0	0	0	0	0	58	0
N.S.	1	1.24	0.86	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.879	0.719	0.000	0.000	0.000	0.000	0.000	2.030	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	212	155	0	0	0	0	0	74	0
N.S.	1	1.24	0.91	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.840	0.750	0.000	0.000	0.000	0.000	0.000	0.472	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	210	176	0	0	0	0	0	242	0
N.S.	1	1.18	0.99	0.00	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.819	0.851	0.000	0.000	0.000	0.000	0.000	0.652	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	229	185	0	0	0	0	0	44	0
N.S.	1	1.27	1.03	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.528	1.110	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	232	193	0	0	0	0	0	44	0
N.S.	1	1.29	1.07	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.505	1.420	0.000	0.000	0.000	0.000	0.000	200.033	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	34	56	64	66	173	162	67	79
N.S.	1	1.00	0.81	1.33	1.52	1.57	4.12	3.86	1.60	1.88
time (sec)	N/A	0.194	0.653	5.302	0.097	0.088	7.797	0.372	0.206	10.725

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	91	0	132	0	412	254	138
N.S.	1	1.00	0.75	1.42	0.00	2.06	0.00	6.44	3.97	2.16
time (sec)	N/A	0.201	0.542	5.491	0.000	0.091	0.000	0.371	0.195	11.065

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	153	153	217	268	448	0	299	1017	224
N.S.	1	1.07	1.07	1.52	1.87	3.13	0.00	2.09	7.11	1.57
time (sec)	N/A	0.426	0.122	1.505	0.061	0.101	0.000	0.280	0.196	11.386

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	562	472	1323	0	1274	0	635	44	0
N.S.	1	1.07	0.90	2.51	0.00	2.42	0.00	1.20	0.08	0.00
time (sec)	N/A	1.036	13.948	2.259	0.000	0.637	0.000	0.344	200.043	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	341	367	691	0	812	0	366	846	0
N.S.	1	1.09	1.17	2.20	0.00	2.59	0.00	1.17	2.69	0.00
time (sec)	N/A	0.637	11.326	2.023	0.000	0.199	0.000	0.322	0.357	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	186	193	299	0	502	0	192	398	0
N.S.	1	1.09	1.13	1.75	0.00	2.94	0.00	1.12	2.33	0.00
time (sec)	N/A	0.342	1.307	1.812	0.000	0.100	0.000	0.321	0.256	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	131	112	131	0	337	0	114	135	0
N.S.	1	1.15	0.98	1.15	0.00	2.96	0.00	1.00	1.18	0.00
time (sec)	N/A	0.255	0.098	1.850	0.000	0.089	0.000	0.306	0.257	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	199	247	660	0	1689	0	216	410	0
N.S.	1	1.21	1.50	4.00	0.00	10.24	0.00	1.31	2.48	0.00
time (sec)	N/A	0.414	1.076	2.351	0.000	8.908	0.000	0.447	0.314	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	191	147	1748	0	1113	0	744	1453	0
N.S.	1	1.27	0.98	11.65	0.00	7.42	0.00	4.96	9.69	0.00
time (sec)	N/A	0.416	10.213	2.507	0.000	0.971	0.000	0.728	0.413	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	309	235	3574	0	2441	0	1848	11755	0
N.S.	1	1.20	0.91	13.85	0.00	9.46	0.00	7.16	45.56	0.00
time (sec)	N/A	0.616	10.480	3.046	0.000	14.732	0.000	0.387	2.216	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	479	352	5581	0	0	0	6024	25163	0
N.S.	1	1.20	0.88	13.95	0.00	0.00	0.00	15.06	62.91	0.00
time (sec)	N/A	0.988	11.223	3.740	0.000	0.000	0.000	0.553	17.695	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	716	650	753	1980	0	2430	0	1337	44	0
N.S.	1	0.91	1.05	2.77	0.00	3.39	0.00	1.87	0.06	0.00
time (sec)	N/A	1.192	3.647	2.475	0.000	1.280	0.000	0.417	200.035	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	430	493	1069	0	1592	3509	828	44	0
N.S.	1	0.96	1.10	2.38	0.00	3.55	7.82	1.84	0.10	0.00
time (sec)	N/A	0.747	2.341	2.197	0.000	0.583	66.022	0.350	200.036	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	275	299	491	0	960	1853	449	1003	0
N.S.	1	1.03	1.12	1.84	0.00	3.60	6.94	1.68	3.76	0.00
time (sec)	N/A	0.425	1.063	1.806	0.000	0.159	8.107	0.330	0.322	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	215	180	230	0	532	751	229	400	0
N.S.	1	1.16	0.97	1.24	0.00	2.86	4.04	1.23	2.15	0.00
time (sec)	N/A	0.324	0.106	1.859	0.000	0.096	2.958	0.315	0.232	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	358	302	1240	0	0	0	0	1106	0
N.S.	1	1.22	1.03	4.22	0.00	0.00	0.00	0.00	3.76	0.00
time (sec)	N/A	0.943	1.213	2.294	0.000	0.000	0.000	0.000	0.803	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	325	258	3015	0	0	0	0	2635	0
N.S.	1	1.25	0.99	11.55	0.00	0.00	0.00	0.00	10.10	0.00
time (sec)	N/A	0.704	1.671	2.671	0.000	0.000	0.000	0.000	19.715	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	447	322	6009	0	0	0	2052	14401	0
N.S.	1	1.23	0.88	16.51	0.00	0.00	0.00	5.64	39.56	0.00
time (sec)	N/A	0.917	2.572	3.046	0.000	0.000	0.000	1.177	9.330	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	326	314	10984	0	3425	0	5573	44	0
N.S.	1	1.20	1.15	40.38	0.00	12.59	0.00	20.49	0.16	0.00
time (sec)	N/A	0.556	2.787	3.487	0.000	90.330	0.000	2.403	200.075	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	362	353	1458	0	997	1282	527	1678	0
N.S.	1	1.04	1.01	4.19	0.00	2.86	3.68	1.51	4.82	0.00
time (sec)	N/A	0.742	1.193	2.761	0.000	0.414	1.288	0.406	16.025	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	248	223	773	0	585	660	282	839	0
N.S.	1	1.22	1.10	3.81	0.00	2.88	3.25	1.39	4.13	0.00
time (sec)	N/A	0.499	0.276	2.022	0.000	0.182	1.367	0.370	0.449	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	172	162	378	0	339	354	151	315	0
N.S.	1	1.41	1.33	3.10	0.00	2.78	2.90	1.24	2.58	0.00
time (sec)	N/A	0.311	0.416	1.743	0.000	0.100	1.061	0.369	0.340	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	258	276	482	0	0	0	0	2483	0
N.S.	1	1.22	1.31	2.28	0.00	0.00	0.00	0.00	11.77	0.00
time (sec)	N/A	0.628	0.988	2.141	0.000	0.000	0.000	0.000	0.471	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	314	279	746	0	0	0	0	6230	0
N.S.	1	1.30	1.16	3.10	0.00	0.00	0.00	0.00	25.85	0.00
time (sec)	N/A	0.684	1.375	2.168	0.000	0.000	0.000	0.000	0.620	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	313	254	1561	0	2302	0	4728	15967	0
N.S.	1	1.18	0.96	5.89	0.00	8.69	0.00	17.84	60.25	0.00
time (sec)	N/A	0.655	1.271	2.572	0.000	4.479	0.000	11.778	3.906	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	472	466	2826	0	5060	0	0	38018	0
N.S.	1	1.15	1.14	6.89	0.00	12.34	0.00	0.00	92.73	0.00
time (sec)	N/A	0.976	4.353	3.227	0.000	53.141	0.000	0.000	32.365	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	499	393	3171	0	1402	0	0	44	0
N.S.	1	1.10	0.87	7.00	0.00	3.09	0.00	0.00	0.10	0.00
time (sec)	N/A	1.590	1.637	3.309	0.000	1.648	0.000	0.000	200.050	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	315	244	1800	0	872	0	0	725	0
N.S.	1	1.09	0.84	6.21	0.00	3.01	0.00	0.00	2.50	0.00
time (sec)	N/A	0.781	0.856	2.518	0.000	0.753	0.000	0.000	0.662	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	199	151	970	0	512	0	0	324	0
N.S.	1	1.12	0.85	5.45	0.00	2.88	0.00	0.00	1.82	0.00
time (sec)	N/A	0.424	0.382	2.059	0.000	0.442	0.000	0.000	0.508	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	125	113	485	0	349	0	0	104	0
N.S.	1	1.13	1.02	4.37	0.00	3.14	0.00	0.00	0.94	0.00
time (sec)	N/A	0.253	0.062	1.833	0.000	0.124	0.000	0.000	0.455	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	177	139	953	0	758	0	0	438	0
N.S.	1	1.27	1.00	6.86	0.00	5.45	0.00	0.00	3.15	0.00
time (sec)	N/A	0.432	0.425	2.067	0.000	1.345	0.000	0.000	2.388	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	254	189	1795	0	1923	0	2888	4829	0
N.S.	1	1.21	0.90	8.55	0.00	9.16	0.00	13.75	23.00	0.00
time (sec)	N/A	0.877	1.248	2.168	0.000	9.975	0.000	8.485	8.601	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	427	342	3621	0	4347	0	0	14503	0
N.S.	1	1.25	1.00	10.59	0.00	12.71	0.00	0.00	42.41	0.00
time (sec)	N/A	2.552	3.360	2.447	0.000	126.402	0.000	0.000	14.209	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	332	345	4501	0	1718	0	0	2000	0
N.S.	1	1.12	1.16	15.15	0.00	5.78	0.00	0.00	6.73	0.00
time (sec)	N/A	1.218	1.301	3.339	0.000	4.632	0.000	0.000	0.617	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	276	203	2629	0	1058	0	0	943	0
N.S.	1	1.19	0.88	11.38	0.00	4.58	0.00	0.00	4.08	0.00
time (sec)	N/A	0.574	0.595	2.549	0.000	4.088	0.000	0.000	0.533	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	148	78	109	0	184	0	0	329	96
N.S.	1	1.06	0.56	0.78	0.00	1.31	0.00	0.00	2.35	0.69
time (sec)	N/A	0.330	0.190	2.098	0.000	2.538	0.000	0.000	0.552	10.932

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	59	85	0	156	0	0	168	72
N.S.	1	1.00	0.51	0.73	0.00	1.34	0.00	0.00	1.45	0.62
time (sec)	N/A	0.239	0.116	1.580	0.000	0.446	0.000	0.000	0.507	10.711

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	324	219	1783	0	3059	0	0	3593	0
N.S.	1	1.20	0.81	6.60	0.00	11.33	0.00	0.00	13.31	0.00
time (sec)	N/A	0.645	0.880	2.048	0.000	12.549	0.000	0.000	3.921	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	416	434	3247	0	6793	0	0	44	0
N.S.	1	0.98	1.02	7.66	0.00	16.02	0.00	0.00	0.10	0.00
time (sec)	N/A	2.520	2.982	2.033	0.000	68.333	0.000	0.000	200.058	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	109	127	233	146	0	585	144	185
N.S.	1	1.00	0.74	0.86	1.57	0.99	0.00	3.95	0.97	1.25
time (sec)	N/A	0.306	1.823	1.173	0.167	0.075	0.000	0.500	0.218	0.435

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	82	82	156	94	0	359	88	81
N.S.	1	1.00	0.77	0.77	1.46	0.88	0.00	3.36	0.82	0.76
time (sec)	N/A	0.275	1.016	1.187	0.165	0.074	0.000	0.272	0.225	0.278

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	77	59	48	90	55	0	184	43	47
N.S.	1	1.07	0.82	0.67	1.25	0.76	0.00	2.56	0.60	0.65
time (sec)	N/A	0.243	0.488	1.138	0.144	0.075	0.000	0.261	0.198	11.104

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	42	32	38	31	0	31	20	31
N.S.	1	1.00	1.08	0.82	0.97	0.79	0.00	0.79	0.51	0.79
time (sec)	N/A	0.180	0.229	1.043	0.126	0.068	0.000	0.232	0.247	10.817

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	97	130	166	0	309	0	99	112	0
N.S.	1	0.92	1.23	1.57	0.00	2.92	0.00	0.93	1.06	0.00
time (sec)	N/A	0.266	0.812	1.391	0.000	0.084	0.000	0.282	0.200	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	127	127	182	0	396	0	104	243	0
N.S.	1	1.15	1.15	1.65	0.00	3.60	0.00	0.95	2.21	0.00
time (sec)	N/A	0.289	1.235	1.330	0.000	0.082	0.000	0.295	0.208	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	140	153	370	0	684	0	153	410	0
N.S.	1	1.03	1.12	2.72	0.00	5.03	0.00	1.12	3.01	0.00
time (sec)	N/A	0.283	2.679	1.412	0.000	0.096	0.000	0.283	0.226	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	192	253	494	0	588	0	0	492	0
N.S.	1	0.87	1.15	2.25	0.00	2.67	0.00	0.00	2.24	0.00
time (sec)	N/A	0.316	1.949	1.355	0.000	0.117	0.000	0.000	0.233	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	156	205	350	0	488	0	0	347	0
N.S.	1	0.91	1.19	2.03	0.00	2.84	0.00	0.00	2.02	0.00
time (sec)	N/A	0.280	5.215	1.394	0.000	0.109	0.000	0.000	0.204	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	124	185	234	0	406	0	117	227	0
N.S.	1	0.97	1.45	1.83	0.00	3.17	0.00	0.91	1.77	0.00
time (sec)	N/A	0.260	5.095	1.310	0.000	0.109	0.000	0.415	0.234	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	141	171	272	0	454	0	117	117	0
N.S.	1	1.23	1.49	2.37	0.00	3.95	0.00	1.02	1.02	0.00
time (sec)	N/A	0.281	10.316	1.377	0.000	0.112	0.000	0.454	0.247	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	126	165	422	0	631	0	158	241	0
N.S.	1	1.06	1.39	3.55	0.00	5.30	0.00	1.33	2.03	0.00
time (sec)	N/A	0.250	10.929	1.497	0.000	0.111	0.000	0.442	0.207	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	68	58	0	200	0	122	118	172
N.S.	1	1.00	0.70	0.60	0.00	2.06	0.00	1.26	1.22	1.77
time (sec)	N/A	0.243	10.069	1.410	0.000	0.086	0.000	0.599	0.206	11.708

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	140	91	90	0	325	0	218	233	251
N.S.	1	0.97	0.63	0.62	0.00	2.24	0.00	1.50	1.61	1.73
time (sec)	N/A	0.265	10.101	1.522	0.000	0.094	0.000	0.728	0.238	11.875

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	185	120	135	0	476	0	341	383	336
N.S.	1	0.96	0.62	0.70	0.00	2.47	0.00	1.77	1.98	1.74
time (sec)	N/A	0.286	7.930	1.578	0.000	0.098	0.000	0.767	0.210	11.910

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	230	152	191	0	653	0	491	566	436
N.S.	1	0.95	0.63	0.79	0.00	2.71	0.00	2.04	2.35	1.81
time (sec)	N/A	0.303	8.336	1.451	0.000	0.112	0.000	0.415	0.220	12.910

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	135	0	0	0	0	0	38	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.363	0.793	0.000	0.000	0.000	0.000	0.000	200.038	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	124	0	0	0	0	0	36	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.311	0.498	0.000	0.000	0.000	0.000	0.000	200.033	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	123	0	0	0	0	0	31	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.265	0.317	0.000	0.000	0.000	0.000	0.000	200.033	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	132	0	0	0	0	0	38	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.319	0.433	0.000	0.000	0.000	0.000	0.000	200.058	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	133	0	0	0	0	0	38	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.322	0.418	0.000	0.000	0.000	0.000	0.000	200.045	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	148	138	0	0	0	0	0	38	0
N.S.	1	1.01	0.94	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.315	0.420	0.000	0.000	0.000	0.000	0.000	200.045	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [302] had the largest ratio of [.2285709999999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.18	46	0.109
2	A	4	4	1.18	46	0.087
3	A	3	3	1.16	46	0.065
4	A	2	2	1.14	44	0.045
5	A	1	1	1.00	39	0.026
6	A	4	3	1.00	46	0.065
7	A	4	3	1.00	46	0.065
8	A	5	4	0.98	46	0.087
9	A	6	5	1.02	46	0.109
10	A	7	6	1.04	46	0.130
11	A	5	5	1.18	46	0.109
12	A	4	4	1.18	46	0.087
13	A	3	3	1.16	46	0.065
14	A	2	2	1.14	44	0.045
15	A	1	1	1.00	39	0.026
16	A	5	4	1.03	46	0.087
17	A	5	4	1.03	46	0.087
18	A	5	4	0.99	46	0.087
19	A	6	5	1.00	46	0.109
20	A	7	6	1.03	46	0.130
21	A	8	7	1.05	46	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	5	1.18	46	0.109
23	A	4	4	1.18	46	0.087
24	A	3	3	1.16	46	0.065
25	A	2	2	1.14	44	0.045
26	A	1	1	1.00	39	0.026
27	A	6	5	1.04	46	0.109
28	A	6	5	1.04	46	0.109
29	A	6	5	1.00	46	0.109
30	A	6	5	1.01	46	0.109
31	A	7	6	1.01	46	0.130
32	A	8	7	1.03	46	0.152
33	A	9	8	1.05	46	0.174
34	A	4	4	1.20	46	0.087
35	A	3	3	1.18	46	0.065
36	A	2	2	1.16	44	0.045
37	A	1	1	1.00	39	0.026
38	A	3	2	1.00	46	0.043
39	A	4	3	0.99	46	0.065
40	A	5	4	1.03	46	0.087
41	A	6	5	1.07	46	0.109
42	A	4	4	1.15	46	0.087
43	A	3	3	1.11	46	0.065
44	A	2	2	1.44	44	0.045
45	A	1	1	1.00	39	0.026
46	A	4	3	1.00	46	0.065
47	A	5	4	1.10	46	0.087
48	A	6	5	1.08	46	0.109
49	A	4	4	1.09	46	0.087
50	A	3	3	1.30	46	0.065
51	A	2	2	1.45	44	0.045
52	A	1	1	1.00	39	0.026
53	A	5	4	1.08	46	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	6	5	1.17	46	0.109
55	A	7	6	1.12	46	0.130
56	A	8	7	1.10	48	0.146
57	A	7	6	1.09	48	0.125
58	A	6	5	1.10	48	0.104
59	A	5	4	1.17	48	0.083
60	A	5	4	1.18	48	0.083
61	A	1	1	1.00	48	0.021
62	A	2	2	1.00	48	0.042
63	A	3	3	1.07	48	0.062
64	A	4	4	1.10	48	0.083
65	A	8	7	1.07	48	0.146
66	A	7	6	1.07	48	0.125
67	A	6	5	1.12	48	0.104
68	A	6	5	1.15	48	0.104
69	A	6	5	1.15	48	0.104
70	A	1	1	1.00	48	0.021
71	A	2	2	1.00	48	0.042
72	A	3	3	1.07	48	0.062
73	A	4	4	1.10	48	0.083
74	A	9	8	1.06	48	0.167
75	A	8	7	1.06	48	0.146
76	A	7	6	1.11	48	0.125
77	A	7	6	1.11	48	0.125
78	A	7	6	1.12	48	0.125
79	A	7	6	1.12	48	0.125
80	A	1	1	1.00	48	0.021
81	A	2	2	1.00	48	0.042
82	A	3	3	1.07	48	0.062
83	A	4	4	1.10	48	0.083
84	A	6	5	1.12	48	0.104
85	A	5	4	1.17	48	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.30	48	0.062
87	A	1	1	1.00	48	0.021
88	A	2	2	1.00	48	0.042
89	A	3	3	1.07	48	0.062
90	A	4	4	1.10	48	0.083
91	A	7	6	1.13	48	0.125
92	A	6	5	1.15	48	0.104
93	A	5	4	1.18	48	0.083
94	A	1	1	1.00	48	0.021
95	A	2	2	1.00	48	0.042
96	A	3	3	1.07	48	0.062
97	A	4	4	1.10	48	0.083
98	A	8	7	1.12	48	0.146
99	A	7	6	1.14	48	0.125
100	A	6	5	1.16	48	0.104
101	A	1	1	1.00	48	0.021
102	A	2	2	1.00	48	0.042
103	A	3	3	1.06	48	0.062
104	A	4	4	1.10	48	0.083
105	A	3	3	1.09	46	0.065
106	A	3	3	1.07	46	0.065
107	A	3	3	1.07	46	0.065
108	A	3	3	0.98	46	0.065
109	A	3	3	1.27	46	0.065
110	A	3	3	1.09	46	0.065
111	A	3	3	1.08	44	0.068
112	A	4	4	1.25	44	0.091
113	A	3	3	1.25	44	0.068
114	A	2	2	1.25	42	0.048
115	A	1	1	1.00	37	0.027
116	A	2	2	1.04	44	0.045
117	A	2	2	1.04	44	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.04	44	0.045
119	A	3	3	1.00	46	0.065
120	A	3	3	1.00	46	0.065
121	A	3	3	1.00	46	0.065
122	A	3	3	1.00	46	0.065
123	A	3	3	1.00	46	0.065
124	A	1	1	1.03	47	0.021
125	A	3	3	1.04	73	0.041
126	A	4	4	1.04	46	0.087
127	A	6	6	1.00	46	0.130
128	A	5	5	1.03	46	0.109
129	A	4	4	1.06	46	0.087
130	A	3	3	1.02	44	0.068
131	A	2	2	1.02	39	0.051
132	A	4	3	1.00	46	0.065
133	A	4	3	1.01	46	0.065
134	A	5	4	0.97	46	0.087
135	A	6	5	0.95	46	0.109
136	A	8	7	1.08	44	0.159
137	A	7	6	1.13	44	0.136
138	A	5	4	0.99	42	0.095
139	A	5	4	1.03	44	0.091
140	A	7	6	1.12	44	0.136
141	A	6	5	1.14	44	0.114
142	A	3	3	1.12	44	0.068
143	A	4	4	1.12	44	0.091
144	A	5	5	1.11	44	0.114
145	A	6	6	1.09	44	0.136
146	A	7	7	1.08	44	0.159
147	A	9	8	1.04	44	0.182
148	A	8	7	1.07	44	0.159
149	A	6	5	0.95	42	0.119

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	6	5	0.99	44	0.114
151	A	6	5	1.24	44	0.114
152	A	6	5	1.17	44	0.114
153	A	7	6	1.18	44	0.136
154	A	7	6	1.05	44	0.136
155	A	2	2	0.99	44	0.045
156	A	3	3	1.01	44	0.068
157	A	4	4	1.00	44	0.091
158	A	5	5	1.00	44	0.114
159	A	10	9	1.01	44	0.205
160	A	9	8	1.03	44	0.182
161	A	7	6	0.93	42	0.143
162	A	7	6	0.96	44	0.136
163	A	7	6	1.16	44	0.136
164	A	8	7	1.12	44	0.159
165	A	7	6	1.07	44	0.136
166	A	8	7	1.10	44	0.159
167	A	8	7	1.11	44	0.159
168	A	8	7	1.04	44	0.159
169	A	2	2	0.99	44	0.045
170	A	3	3	1.01	44	0.068
171	A	4	4	1.00	44	0.091
172	A	5	5	1.00	44	0.114
173	A	7	6	1.13	44	0.136
174	A	6	5	1.22	44	0.114
175	A	4	3	1.03	42	0.071
176	A	4	3	1.03	44	0.068
177	A	2	2	1.00	44	0.045
178	A	3	3	1.01	44	0.068
179	A	4	4	1.00	44	0.091
180	A	5	5	1.00	44	0.114
181	A	7	6	1.06	44	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	7	6	1.08	44	0.136
183	A	5	4	1.04	42	0.095
184	A	2	2	1.00	44	0.045
185	A	3	3	1.00	44	0.068
186	A	4	4	0.99	44	0.091
187	A	8	7	1.09	44	0.159
188	A	8	7	1.20	44	0.159
189	A	5	4	1.08	44	0.091
190	A	3	3	1.09	44	0.068
191	A	2	2	1.00	42	0.048
192	A	3	3	0.99	44	0.068
193	A	4	4	0.98	44	0.091
194	A	5	5	0.98	44	0.114
195	A	5	5	0.94	46	0.109
196	A	4	4	0.95	46	0.087
197	A	3	3	0.98	46	0.065
198	A	2	2	1.02	46	0.043
199	A	5	4	0.98	46	0.087
200	A	5	4	1.16	46	0.087
201	A	5	4	1.00	46	0.087
202	A	6	5	0.96	46	0.109
203	A	7	6	0.96	46	0.130
204	A	6	6	0.92	46	0.130
205	A	5	5	0.94	46	0.109
206	A	4	4	0.95	46	0.087
207	A	3	3	0.99	46	0.065
208	A	2	2	1.02	46	0.043
209	A	6	5	0.96	46	0.109
210	A	6	5	1.07	46	0.109
211	A	6	5	1.05	46	0.109
212	A	6	5	0.95	46	0.109
213	A	7	6	0.93	46	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	7	7	0.92	46	0.152
215	A	6	6	0.92	46	0.130
216	A	5	5	0.94	46	0.109
217	A	4	4	0.96	46	0.087
218	A	3	3	0.99	46	0.065
219	A	2	2	1.02	46	0.043
220	A	7	6	0.95	46	0.130
221	A	7	6	1.01	46	0.130
222	A	7	6	0.96	46	0.130
223	A	7	6	0.99	46	0.130
224	A	7	6	0.92	46	0.130
225	A	8	7	0.91	46	0.152
226	A	4	4	0.96	46	0.087
227	A	3	3	1.00	46	0.065
228	A	2	2	1.03	46	0.043
229	A	4	3	1.00	46	0.065
230	A	4	3	1.00	46	0.065
231	A	5	4	1.00	46	0.087
232	A	5	5	1.05	46	0.109
233	A	4	4	1.09	46	0.087
234	A	3	3	1.18	46	0.065
235	A	2	2	1.32	46	0.043
236	A	4	3	1.00	46	0.065
237	A	5	4	1.02	46	0.087
238	A	6	5	1.00	46	0.109
239	A	7	6	0.99	46	0.130
240	A	6	6	1.01	46	0.130
241	A	5	5	1.04	46	0.109
242	A	4	4	1.08	46	0.087
243	A	3	3	1.17	46	0.065
244	A	2	2	1.33	46	0.043
245	A	5	4	1.04	46	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	6	5	1.09	46	0.109
247	A	7	6	1.00	46	0.130
248	A	8	7	0.98	46	0.152
249	A	2	2	1.00	25	0.080
250	A	5	5	1.30	44	0.114
251	A	5	5	1.24	44	0.114
252	A	5	5	1.24	44	0.114
253	A	5	5	1.18	44	0.114
254	A	5	5	1.27	44	0.114
255	A	5	5	1.29	44	0.114
256	A	1	1	1.00	61	0.016
257	A	1	1	1.00	60	0.017
258	A	2	2	1.07	42	0.048
259	A	9	8	1.07	44	0.182
260	A	7	6	1.09	44	0.136
261	A	5	4	1.09	42	0.095
262	A	4	3	1.15	37	0.081
263	A	8	7	1.21	44	0.159
264	A	5	4	1.27	44	0.091
265	A	7	6	1.20	44	0.136
266	A	9	8	1.20	44	0.182
267	A	10	9	0.91	44	0.205
268	A	8	7	0.96	44	0.159
269	A	6	5	1.03	42	0.119
270	A	5	4	1.16	37	0.108
271	A	9	8	1.22	44	0.182
272	A	8	7	1.25	44	0.159
273	A	9	8	1.23	44	0.182
274	A	6	5	1.20	44	0.114
275	A	8	7	1.04	46	0.152
276	A	6	5	1.22	44	0.114
277	A	5	4	1.41	39	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	8	7	1.22	46	0.152
279	A	8	7	1.30	46	0.152
280	A	6	5	1.18	46	0.109
281	A	8	7	1.15	46	0.152
282	A	10	9	1.10	44	0.205
283	A	7	6	1.09	44	0.136
284	A	7	6	1.12	42	0.143
285	A	4	3	1.13	37	0.081
286	A	5	4	1.27	44	0.091
287	A	6	5	1.21	44	0.114
288	A	8	7	1.25	44	0.159
289	A	8	7	1.12	44	0.159
290	A	7	6	1.19	44	0.136
291	A	3	3	1.06	42	0.071
292	A	2	2	1.00	37	0.054
293	A	7	6	1.20	44	0.136
294	A	8	7	0.98	44	0.159
295	A	3	3	1.00	33	0.091
296	A	3	3	1.00	33	0.091
297	A	4	4	1.07	31	0.129
298	A	2	2	1.00	26	0.077
299	A	7	6	0.92	33	0.182
300	A	7	6	1.15	33	0.182
301	A	7	6	1.03	33	0.182
302	A	9	8	0.87	35	0.229
303	A	8	7	0.91	35	0.200
304	A	7	6	0.97	35	0.171
305	A	7	6	1.23	35	0.171
306	A	7	6	1.06	35	0.171
307	A	4	4	1.00	35	0.114
308	A	5	5	0.97	35	0.143
309	A	6	6	0.96	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	7	7	0.95	35	0.200
311	A	4	4	1.00	36	0.111
312	A	4	4	1.00	34	0.118
313	A	3	2	1.00	29	0.069
314	A	4	4	1.00	36	0.111
315	A	4	4	1.00	36	0.111
316	A	4	4	1.01	36	0.111

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 297

$$
\begin{aligned}
\int \frac{(f+gx)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx
&= \frac{2(cdf-aeg)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^5d^5(d+ex)^{3/2}} \\
&\quad + \frac{8g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^5d^5(d+ex)^{5/2}} \\
&\quad + \frac{12g^2(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^5d^5(d+ex)^{7/2}} \\
&\quad + \frac{8g^3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^5d^5(d+ex)^{9/2}} \\
&\quad + \frac{2g^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^5d^5(d+ex)^{11/2}}
\end{aligned}
$$

output

```
2/3*(-a*e*g+c*d*f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^5/d^5/(e*x+
d)^(3/2)+8/5*g*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^
5/d^5/(e*x+d)^(5/2)+12/7*g^2*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(7/2)/c^5/d^5/(e*x+d)^(7/2)+8/9*g^3*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(9/2)/c^5/d^5/(e*x+d)^(9/2)+2/11*g^4*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(11/2)/c^5/d^5/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.66

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2((ae+cdx)(d+ex))^{3/2}\left(128a^4e^4g^4-64a^3cde^3g^3(11f+3gx)+48a^2c^2d^2e^2g^2(33f^2+22fgx+5g^2x^2)\right.}{3}$$

input
```
Integrate[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(
11*f + 3*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(33*f^2 + 22*f*g*x + 5*g^2*x^2) - 8
*a*c^3*d^3*e*g*(231*f^3 + 297*f^2*g*x + 165*f*g^2*x^2 + 35*g^3*x^3) + c^4*
d^4*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 1540*f*g^3*x^3 + 315*g^4
*x^4)))/(3465*c^5*d^5*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1253, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{8(cdf-aeg)\int \frac{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{11cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{8(cdf-aeg)\left(\frac{2(cdf-aeg)\int\frac{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{3cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}\right)}{11cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

$$\Big\downarrow 1253$$

$$\frac{8(cdf-aeg)\left(\frac{2(cdf-aeg)\left(\frac{4(cdf-aeg)\int\frac{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{7cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}\right)}{3cd}+\frac{2(f+gx)^3(x(ae^2}{9cd(}\right.}{11cd}$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

$$\Big\downarrow 1221$$

$$\frac{8(cdf-aeg)\left(\frac{2(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{5}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}\right)}{7cd}+\frac{2(f+gx)^2\left(x\right.}{}\right.}{3cd}\right.}{11cd}$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

$$\Big\downarrow 1122$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}+$$

$$\frac{8(cdf-aeg)\left(\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}+\frac{2(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)\right.}{}\right.}{}\right.}{3cd}\right.}{11cd}$$

input
```
Int[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x
],x]
```

output
```
(2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(11*c*d*(d +
 e*x)^(3/2)) + (8*(c*d*f - a*e*g)*((2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(3/2))/(9*c*d*(d + e*x)^(3/2)) + (2*(c*d*f - a*e*g)*((2*(f
 + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c*d*(d + e*x)^
(3/2)) + (4*(c*d*f - a*e*g)*((2*(5*f - (3*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*c*d*(d + e*x)^(3/2)) + (2*g*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])))/(7
*c*d)))/(3*c*d)))/(11*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.92

method	result
default	$2(cdx+ae)\left(315g^4x^4d^4c^4-280a\,c^3d^3e\,g^4x^3+1540c^4d^4f\,g^3x^3+240a^2c^2d^2e^2g^4x^2-1320a\,c^3d^3ef\,g^3x^2+2970c^4d^4f^2g^2x^2-192a^3cd\,e\right.$
gosper	$2(cdx+ae)\left(315g^4x^4d^4c^4-280a\,c^3d^3e\,g^4x^3+1540c^4d^4f\,g^3x^3+240a^2c^2d^2e^2g^4x^2-1320a\,c^3d^3ef\,g^3x^2+2970c^4d^4f^2g^2x^2-192a^3cd\,e\right.$
orering	$2\left(315g^4x^4d^4c^4-280a\,c^3d^3e\,g^4x^3+1540c^4d^4f\,g^3x^3+240a^2c^2d^2e^2g^4x^2-1320a\,c^3d^3ef\,g^3x^2+2970c^4d^4f^2g^2x^2-192a^3cd\,e^3g^4x+105\right.$

input
```
int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/3465*(c*d*x+a*e)*(315*c^4*d^4*g^4*x^4-280*a*c^3*d^3*e*g^4*x^3+1540*c^4*d
^4*f*g^3*x^3+240*a^2*c^2*d^2*e^2*g^4*x^2-1320*a*c^3*d^3*e*f*g^3*x^2+2970*c
^4*d^4*f^2*g^2*x^2-192*a^3*c*d*e^3*g^4*x+1056*a^2*c^2*d^2*e^2*f*g^3*x-2376
*a*c^3*d^3*e*f^2*g^2*x+2772*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-704*a^3*c*d*e^
3*f*g^3+1584*a^2*c^2*d^2*e^2*f^2*g^2-1848*a*c^3*d^3*e*f^3*g+1155*c^4*d^4*f
^4)*((e*x+d)*(c*d*x+a*e))^(1/2)/d^5/c^5/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.26

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\,(315\,c^5d^5g^4x^5+1155\,ac^4d^4ef^4-1848\,a^2c^3d^3e^2f^3g+1584\,a^3c^2d^2e^3f^2g^2-704\,a^4cde^4fg^3+128\,a^5e^5g^4}{}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
2/3465*(315*c^5*d^5*g^4*x^5 + 1155*a*c^4*d^4*e*f^4 - 1848*a^2*c^3*d^3*e^2*
f^3*g + 1584*a^3*c^2*d^2*e^3*f^2*g^2 - 704*a^4*c*d*e^4*f*g^3 + 128*a^5*e^5
*g^4 + 35*(44*c^5*d^5*f*g^3 + a*c^4*d^4*e*g^4)*x^4 + 10*(297*c^5*d^5*f^2*g
^2 + 22*a*c^4*d^4*e*f*g^3 - 4*a^2*c^3*d^3*e^2*g^4)*x^3 + 6*(462*c^5*d^5*f^
3*g + 99*a*c^4*d^4*e*f^2*g^2 - 44*a^2*c^3*d^3*e^2*f*g^3 + 8*a^3*c^2*d^2*e^
3*g^4)*x^2 + (1155*c^5*d^5*f^4 + 924*a*c^4*d^4*e*f^3*g - 792*a^2*c^3*d^3*e
^2*f^2*g^2 + 352*a^3*c^2*d^2*e^3*f*g^3 - 64*a^4*c*d*e^4*g^4)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F]

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^4}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**
(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**4/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.08

$$
\begin{aligned}
&\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx \\
&= \frac{2\,(cdx+ae)^{\frac{3}{2}}f^4}{3\,cd} + \frac{8\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}\,f^3g}{15\,c^2d^2} \\
&\quad + \frac{4\,(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3)\sqrt{cdx+ae}\,f^2g^2}{35\,c^3d^3} \\
&\quad + \frac{8\,(35\,c^4d^4x^4+5\,ac^3d^3ex^3-6\,a^2c^2d^2e^2x^2+8\,a^3cde^3x-16\,a^4e^4)\sqrt{cdx+ae}\,fg^3}{315\,c^4d^4} \\
&\quad + \frac{2\,(315\,c^5d^5x^5+35\,ac^4d^4ex^4-40\,a^2c^3d^3e^2x^3+48\,a^3c^2d^2e^3x^2-64\,a^4cde^4x+128\,a^5e^5)\sqrt{cdx+ae}\,g^4}{3465\,c^5d^5}
\end{aligned}
$$

input

```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output

```
2/3*(c*d*x + a*e)^(3/2)*f^4/(c*d) + 8/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^
2*e^2)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2) + 4/35*(15*c^3*d^3*x^3 + 3*a*c^2*
d^2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f^2*g^2/(c^3*d^
3) + 8/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8
*a^3*c*d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/3465*(3
15*c^5*d^5*x^5 + 35*a*c^4*d^4*e*x^4 - 40*a^2*c^3*d^3*e^2*x^3 + 48*a^3*c^2*
d^2*e^3*x^2 - 64*a^4*c*d*e^4*x + 128*a^5*e^5)*sqrt(c*d*x + a*e)*g^4/(c^5*d
^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(267) = 534.

Time = 0.12 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.25

$$
\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \text{Too large to display}
$$

input

```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="giac")
```

output

```
2/3465*(3465*sqrt(c*d*x + a*e)*a*e*f^4 - 1155*(3*sqrt(c*d*x + a*e)*a*e - (
c*d*x + a*e)^(3/2))*f^4 - 4620*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e)^(3
/2))*a*e*f^3*g/(c*d) + 924*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e
)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*f^3*g/(c*d) + 1386*(15*sqrt(c*d*x + a
*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*e*f^2*
g^2/(c^2*d^2) - 594*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)
*a^2*e^2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*f^2*g^2/(c^
2*d^2) - 396*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^
2 + 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a*e*f*g^3/(c^3*d^3
) + 44*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 +
378*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x
+ a*e)^(9/2))*f*g^3/(c^3*d^3) + 11*(315*sqrt(c*d*x + a*e)*a^4*e^4 - 420*(c
*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)^(5/2)*a^2*e^2 - 180*(c*d*x +
 a*e)^(7/2)*a*e + 35*(c*d*x + a*e)^(9/2))*a*e*g^4/(c^4*d^4) - 5*(693*sqrt(
c*d*x + a*e)*a^5*e^5 - 1155*(c*d*x + a*e)^(3/2)*a^4*e^4 + 1386*(c*d*x + a*
e)^(5/2)*a^3*e^3 - 990*(c*d*x + a*e)^(7/2)*a^2*e^2 + 385*(c*d*x + a*e)^(9/
2)*a*e - 63*(c*d*x + a*e)^(11/2))*g^4/(c^4*d^4))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.17

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$= \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{2\,g^4\,x^5}{11}+\frac{256\,a^5\,e^5\,g^4-1408\,a^4\,c\,d\,e^4\,f\,g^3+3168\,a^3\,c^2\,d^2\,e^3\,f^2\,g^2-3696\,a^2\,c^3\,d^3\,e^2\,f^3\,g+2310}{3465\,c^5\,d^5}\right.}{}$$

input

```
int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5)/11 + (256*a^5*
e^5*g^4 + 2310*a*c^4*d^4*e*f^4 - 3696*a^2*c^3*d^3*e^2*f^3*g - 1408*a^4*c*d
*e^4*f*g^3 + 3168*a^3*c^2*d^2*e^3*f^2*g^2)/(3465*c^5*d^5) + (x*(2310*c^5*d
^5*f^4 - 128*a^4*c*d*e^4*g^4 + 704*a^3*c^2*d^2*e^3*f*g^3 + 1848*a*c^4*d^4*
e*f^3*g - 1584*a^2*c^3*d^3*e^2*f^2*g^2))/(3465*c^5*d^5) + (4*g*x^2*(8*a^3*
e^3*g^3 + 462*c^3*d^3*f^3 + 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/
(1155*c^3*d^3) + (4*g^2*x^3*(297*c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*
f*g))/(693*c^2*d^2) + (2*g^3*x^4*(a*e*g + 44*c*d*f))/(99*c*d)))/(d + e*x)^
(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.18

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\sqrt{cdx+ae}\,(315c^5d^5g^4x^5+35a\,c^4d^4e\,g^4x^4+1540c^5d^5f\,g^3x^4-40a^2c^3d^3e^2g^4x^3+220a\,c^4d^4ef\,g^3x^3+29}{}$$

input
```
int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(128*a**5*e**5*g**4 - 704*a**4*c*d*e**4*f*g**3 - 64*a
**4*c*d*e**4*g**4*x + 1584*a**3*c**2*d**2*e**3*f**2*g**2 + 352*a**3*c**2*d
**2*e**3*f*g**3*x + 48*a**3*c**2*d**2*e**3*g**4*x**2 - 1848*a**2*c**3*d**3
*e**2*f**3*g - 792*a**2*c**3*d**3*e**2*f**2*g**2*x - 264*a**2*c**3*d**3*e*
*2*f*g**3*x**2 - 40*a**2*c**3*d**3*e**2*g**4*x**3 + 1155*a*c**4*d**4*e*f**
4 + 924*a*c**4*d**4*e*f**3*g*x + 594*a*c**4*d**4*e*f**2*g**2*x**2 + 220*a*
c**4*d**4*e*f*g**3*x**3 + 35*a*c**4*d**4*e*g**4*x**4 + 1155*c**5*d**5*f**4
*x + 2772*c**5*d**5*f**3*g*x**2 + 2970*c**5*d**5*f**2*g**2*x**3 + 1540*c**
5*d**5*f*g**3*x**4 + 315*c**5*d**5*g**4*x**5))/(3465*c**5*d**5)
```

3.2 $\int \frac{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 234

$$\begin{aligned}\int \frac{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx &= \frac{2(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^4d^4(d+ex)^{3/2}}\\ &\quad+\frac{6g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^4d^4(d+ex)^{5/2}}\\ &\quad+\frac{6g^2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^4d^4(d+ex)^{7/2}}\\ &\quad+\frac{2g^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^4d^4(d+ex)^{9/2}}\end{aligned}$$

output

```
2/3*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^4/(e*x+
d)^(3/2)+6/5*g*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^
4/d^4/(e*x+d)^(5/2)+6/7*g^2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(7/2)/c^4/d^4/(e*x+d)^(7/2)+2/9*g^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
9/2)/c^4/d^4/(e*x+d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.58

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2((ae+cdx)(d+ex))^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(3f+gx)-6ac^2d^2eg(21f^2+18fgx+5g^2x^2)+c^3d^3\right.}{315c^4d^4(d+ex)^{3/2}}$$

input
```
Integrate[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(
3*f + g*x) - 6*a*c^2*d^2*e*g*(21*f^2 + 18*f*g*x + 5*g^2*x^2) + c^3*d^3*(10
5*f^3 + 189*f^2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3)))/(315*c^4*d^4*(d + e*x)
^(3/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{2(cdf-aeg)\int \frac{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{3cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{2(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{7cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}\right)}{3cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{2(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{5}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}\right)}{7cd}+\frac{2(f+gx)^2\left(x\left(ae^2+c\right.\right.}{7cd(d-}\right.}{3cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}+\frac{2(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)}{15cd(d+ex)^{3/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+\right.}{5cde\sqrt{}}\right.}{7cd}\right.}{3cd}$$

input

```
Int[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x
],x]
```

output

```
(2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d*(d +
e*x)^(3/2)) + (2*(c*d*f - a*e*g)*((2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2))/(7*c*d*(d + e*x)^(3/2)) + (4*(c*d*f - a*e*g)*((2*(5*
f - (3*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/(15*c*d*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])))/(7*c*d)))/(3*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.76

method	result
default	$-\frac{2(cdx+ae)\left(-35x^3g^3d^3c^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+16a^3e^3g^3-72a^2cd\right.}{315d^4c^4\sqrt{ex+d}}$
gosper	$-\frac{2(cdx+ae)\left(-35x^3g^3d^3c^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+16a^3e^3g^3-72a^2cd\right.}{315d^4c^4\sqrt{ex+d}}$
orering	$-\frac{2\left(-35x^3g^3d^3c^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+16a^3e^3g^3-72a^2cd\,e^2f\,g^2+1\right.}{315d^4c^4\sqrt{ex+d}}$

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output

```
-2/315*(c*d*x+a*e)*(-35*c^3*d^3*g^3*x^3+30*a*c^2*d^2*e*g^3*x^2-135*c^3*d^3
*f*g^2*x^2-24*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-189*c^3*d^3*f^2*g*
x+16*a^3*e^3*g^3-72*a^2*c*d*e^2*f*g^2+126*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^
3)*((e*x+d)*(c*d*x+a*e))^(1/2)/d^4/c^4/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.13

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$= \frac{2\left(35\,c^4d^4g^3x^4+105\,ac^3d^3ef^3-126\,a^2c^2d^2e^2f^2g+72\,a^3cde^3fg^2-16\,a^4e^4g^3+5\left(27\,c^4d^4fg^2+ac^3d^3eg^3\right.\right.}{}$$

input

```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output

```
2/315*(35*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 126*a^2*c^2*d^2*e^2*f^2*
g + 72*a^3*c*d*e^3*f*g^2 - 16*a^4*e^4*g^3 + 5*(27*c^4*d^4*f*g^2 + a*c^3*d^
3*e*g^3)*x^3 + 3*(63*c^4*d^4*f^2*g + 9*a*c^3*d^3*e*f*g^2 - 2*a^2*c^2*d^2*e
^2*g^3)*x^2 + (105*c^4*d^4*f^3 + 63*a*c^3*d^3*e*f^2*g - 36*a^2*c^2*d^2*e^2
*f*g^2 + 8*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F]

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^3}{\sqrt{d+ex}}\,dx$$

input

```
integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**
(1/2),x)
```

output

```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\,(cdx+ae)^{\frac{3}{2}}f^3}{3\,cd} + \frac{2\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}f^2g}{5\,c^2d^2}$$
$$+ \frac{2\,(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3)\sqrt{cdx+ae}fg^2}{35\,c^3d^3}$$
$$+ \frac{2\,(35\,c^4d^4x^4+5\,ac^3d^3ex^3-6\,a^2c^2d^2e^2x^2+8\,a^3cde^3x-16\,a^4e^4)\sqrt{cdx+ae}g^3}{315\,c^4d^4}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)*f^3/(c*d) + 2/5*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2) + 2/35*(15*c^3*d^3*x^3 + 3*a*c^2*d
^2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f*g^2/(c^3*d^3)
+ 2/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*e*x^3 - 6*a^2*c^2*d^2*e^2*x^2 + 8*a^
3*c*d*e^3*x - 16*a^4*e^4)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(210) = 420.

Time = 0.12 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.95

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\Big(315\sqrt{cdx+ae}aef^3-105\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)f^3-\frac{315\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)aef^2g}{cd}+\frac{63\,(1}{}}{}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
2/315*(315*sqrt(c*d*x + a*e)*a*e*f^3 - 105*(3*sqrt(c*d*x + a*e)*a*e - (c*d
*x + a*e)^(3/2))*f^3 - 315*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e)^(3/2))
*a*e*f^2*g/(c*d) + 63*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/
2)*a*e + 3*(c*d*x + a*e)^(5/2))*f^2*g/(c*d) + 63*(15*sqrt(c*d*x + a*e)*a^2
*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*e*f*g^2/(c^2*
d^2) - 27*(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 +
 21*(c*d*x + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*f*g^2/(c^2*d^2) - 9*(
35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x
+ a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*a*e*g^3/(c^3*d^3) + (315*sqrt(c*
d*x + a*e)*a^4*e^4 - 420*(c*d*x + a*e)^(3/2)*a^3*e^3 + 378*(c*d*x + a*e)^(
5/2)*a^2*e^2 - 180*(c*d*x + a*e)^(7/2)*a*e + 35*(c*d*x + a*e)^(9/2))*g^3/(
c^3*d^3))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^3\,x^4}{9}-\frac{32\,a^4\,e^4\,g^3-144\,a^3\,c\,d\,e^3\,f\,g^2+252\,a^2\,c^2\,d^2\,e^2\,f^2\,g-210\,a\,c^3\,d^3\,e\,f^3}{315\,c^4\,d^4}+\frac{x\,(16\,a^3\,c\,d}{}\right.}{\sqrt{d+e}}$$

input
```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^3*x^4)/9 - (32*a^4*e^
4*g^3 - 210*a*c^3*d^3*e*f^3 + 252*a^2*c^2*d^2*e^2*f^2*g - 144*a^3*c*d*e^3*
f*g^2)/(315*c^4*d^4) + (x*(210*c^4*d^4*f^3 + 16*a^3*c*d*e^3*g^3 - 72*a^2*c
^2*d^2*e^2*f*g^2 + 126*a*c^3*d^3*e*f^2*g))/(315*c^4*d^4) + (2*g*x^2*(63*c^
2*d^2*f^2 - 2*a^2*e^2*g^2 + 9*a*c*d*e*f*g))/(105*c^2*d^2) + (2*g^2*x^3*(a*
e*g + 27*c*d*f))/(63*c*d)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.99

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\sqrt{cdx+ae}\,(35c^4d^4g^3x^4+5a\,c^3d^3e\,g^3x^3+135c^4d^4f\,g^2x^3-6a^2c^2d^2e^2g^3x^2+27a\,c^3d^3ef\,g^2x^2+189c^4d^4}{}$$

input

```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output

```
(2*sqrt(a*e + c*d*x)*( - 16*a**4*e**4*g**3 + 72*a**3*c*d*e**3*f*g**2 + 8*a
**3*c*d*e**3*g**3*x - 126*a**2*c**2*d**2*e**2*f**2*g - 36*a**2*c**2*d**2*e
**2*f*g**2*x - 6*a**2*c**2*d**2*e**2*g**3*x**2 + 105*a*c**3*d**3*e*f**3 +
63*a*c**3*d**3*e*f**2*g*x + 27*a*c**3*d**3*e*f*g**2*x**2 + 5*a*c**3*d**3*e
*g**3*x**3 + 105*c**4*d**4*f**3*x + 189*c**4*d**4*f**2*g*x**2 + 135*c**4*d
**4*f*g**2*x**3 + 35*c**4*d**4*g**3*x**4))/(315*c**4*d**4)
```

3.3 $\int \frac{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 171

$$\begin{aligned}\int \frac{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx &= \frac{2(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^3d^3(d+ex)^{3/2}} \\ &\quad+\frac{4g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^3d^3(d+ex)^{5/2}} \\ &\quad+\frac{2g^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^3d^3(d+ex)^{7/2}}\end{aligned}$$

output

```
2/3*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*x+
d)^(3/2)+4/5*g*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/
d^3/(e*x+d)^(5/2)+2/7*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/
(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.53

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2((ae+cdx)(d+ex))^{3/2}\left(8a^2e^2g^2-4acdeg(7f+3gx)+c^2d^2(35f^2+42fgx+15g^2x^2)\right)}{105c^3d^3(d+ex)^{3/2}}$$

input
```
Integrate[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(7*f + 3*g
*x) + c^2*d^2*(35*f^2 + 42*f*g*x + 15*g^2*x^2)))/(105*c^3*d^3*(d + e*x)^(3
/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \ 1253$$

$$\frac{4(cdf-aeg)\int \frac{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{7cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}$$

$$\Big\downarrow \ 1221$$

$$\frac{4(cdf-aeg)\left(\frac{1}{5}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}\right)}{7cd}+$$
$$\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}$$

$$\Big\downarrow \; 1122$$

$$\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}+$$
$$\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)}{15cd(d+ex)^{3/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}\right)}{7cd}$$

input

```
Int[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x
],x]
```

output

```
(2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c*d*(d +
e*x)^(3/2)) + (4*(c*d*f - a*e*g)*((2*(5*f - (3*d*g)/e - (2*a*e*g)/(c*d))*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*c*d*(d + e*x)^(3/2)) + (
2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
))/(7*c*d)
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2(cdx+ae)\left(15g^{2}x^{2}d^{2}c^{2}-12acde\,g^{2}x+42c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-28acdefg+35f^{2}c^{2}d^{2}\right)\sqrt{(ex+d)(cdx+ae)}}{105d^{3}c^{3}\sqrt{ex+d}}$	106
gosper	$\frac{2(cdx+ae)\left(15g^{2}x^{2}d^{2}c^{2}-12acde\,g^{2}x+42c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-28acdefg+35f^{2}c^{2}d^{2}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{105d^{3}c^{3}\sqrt{ex+d}}$	116
orering	$\frac{2\left(15g^{2}x^{2}d^{2}c^{2}-12acde\,g^{2}x+42c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-28acdefg+35f^{2}c^{2}d^{2}\right)(cdx+ae)\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{105d^{3}c^{3}\sqrt{ex+d}}$	117

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/105*(c*d*x+a*e)*(15*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+42*c^2*d^2*f*g*x+8*
a^2*e^2*g^2-28*a*c*d*e*f*g+35*c^2*d^2*f^2)*((e*x+d)*(c*d*x+a*e))^(1/2)/d^3
/c^3/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\left(15\,c^3d^3g^2x^3+35\,ac^2d^2ef^2-28\,a^2cde^2fg+8\,a^3e^3g^2+3\,(14\,c^3d^3fg+ac^2d^2eg^2)x^2+(35\,c^3d^3f^2+14\,a}{105\left(c^3d^3ex+c^3d^4\right)}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output
```
2/105*(15*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 28*a^2*c*d*e^2*f*g + 8*a^
3*e^3*g^2 + 3*(14*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 + 1
4*a*c^2*d^2*e*f*g - 4*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^2}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**
(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\begin{aligned}&\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx\\&=\frac{2\,(cdx+ae)^{\frac{3}{2}}f^2}{3\,cd}+\frac{4\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}fg}{15\,c^2d^2}\\&\quad+\frac{2\,(15\,c^3d^3x^3+3\,ac^2d^2ex^2-4\,a^2cde^2x+8\,a^3e^3)\sqrt{cdx+ae}g^2}{105\,c^3d^3}\end{aligned}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)*f^2/(c*d) + 4/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^
2*e^2)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) + 2/105*(15*c^3*d^3*x^3 + 3*a*c^2*d
^2*e*x^2 - 4*a^2*c*d*e^2*x + 8*a^3*e^3)*sqrt(c*d*x + a*e)*g^2/(c^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.66

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(105\sqrt{cdx+ae}aef^2-35\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)f^2-\frac{70\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)aefg}{cd}+\frac{14\left(15\sqrt{\vphantom{x}}\right.}{}\right.}{}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
2/105*(105*sqrt(c*d*x + a*e)*a*e*f^2 - 35*(3*sqrt(c*d*x + a*e)*a*e - (c*d*
x + a*e)^(3/2))*f^2 - 70*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e)^(3/2))*a
*e*f*g/(c*d) + 14*(15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a
*e + 3*(c*d*x + a*e)^(5/2))*f*g/(c*d) + 7*(15*sqrt(c*d*x + a*e)*a^2*e^2 -
10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x + a*e)^(5/2))*a*e*g^2/(c^2*d^2) - 3*
(35*sqrt(c*d*x + a*e)*a^3*e^3 - 35*(c*d*x + a*e)^(3/2)*a^2*e^2 + 21*(c*d*x
 + a*e)^(5/2)*a*e - 5*(c*d*x + a*e)^(7/2))*g^2/(c^2*d^2))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^3}{7}+\frac{16\,a^3\,e^3\,g^2-56\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,c^3\,d^3}+\frac{x\,\left(-8\,a^2\,c\,d\,e^2\,g^2+28\,a\,c^2\,d^2\,e\,f\,g+7\right.}{105\,c^3\,d^3}\right.}{\sqrt{d+e\,x}}$$

input
```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3)/7 + (16*a^3*e^
3*g^2 + 70*a*c^2*d^2*e*f^2 - 56*a^2*c*d*e^2*f*g)/(105*c^3*d^3) + (x*(70*c^
3*d^3*f^2 - 8*a^2*c*d*e^2*g^2 + 28*a*c^2*d^2*e*f*g))/(105*c^3*d^3) + (2*g*
x^2*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\sqrt{cdx+ae}\,(15c^3d^3g^2x^3+3a\,c^2d^2e\,g^2x^2+42c^3d^3f\,g\,x^2-4a^2cd\,e^2g^2x+14a\,c^2d^2efgx+35c^3d^3f^2x+8}{105c^3d^3}$$

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(8*a**3*e**3*g**2 - 28*a**2*c*d*e**2*f*g - 4*a**2*c*d
*e**2*g**2*x + 35*a*c**2*d**2*e*f**2 + 14*a*c**2*d**2*e*f*g*x + 3*a*c**2*d
**2*e*g**2*x**2 + 35*c**3*d**3*f**2*x + 42*c**3*d**3*f*g*x**2 + 15*c**3*d*
*3*g**2*x**3))/(105*c**3*d**3)
```

3.4 $\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 108

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(cdf-aeg)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3c^2d^2(d+ex)^{3/2}} + \frac{2g\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{5c^2d^2(d+ex)^{5/2}}$$

output
```
2/3*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)
^(3/2)+2/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}(-2aeg+cd(5f+3gx))}{15c^2d^2(d+ex)^{3/2}}$$

input
```
Integrate[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
 e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*a*e*g + c*d*(5*f + 3*g*x)))/(15*c^2
*d^2*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{5}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(-\frac{2aeg}{cd}-\frac{3dg}{e}+5f\right)}{15cd(d+ex)^{3/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}$$

input
```
Int[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x],
x]
```

output
```
(2*(5*f - (3*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(15*c*d*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{2(cdx+ae)(-3cdgx+2aeg-5dfc)\sqrt{(ex+d)(cdx+ae)}}{15c^{2}d^{2}\sqrt{ex+d}}$	57
gosper	$-\frac{2(cdx+ae)(-3cdgx+2aeg-5dfc)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{15c^{2}d^{2}\sqrt{ex+d}}$	67
orering	$-\frac{2(-3cdgx+2aeg-5dfc)(cdx+ae)\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{15c^{2}d^{2}\sqrt{ex+d}}$	68

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/15*(c*d*x+a*e)*(-3*c*d*g*x+2*a*e*g-5*c*d*f)*((e*x+d)*(c*d*x+a*e))^(1/2)
/c^2/d^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\,(3\,c^2d^2gx^2+5\,acdef-2\,a^2e^2g+(5\,c^2d^2f+acdeg)x)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{15\,(c^2d^2ex+c^2d^3)}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
 algorithm="fricas")
```

output
```
2/15*(3*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 2*a^2*e^2*g + (5*c^2*d^2*f + a*c*d*e
*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*
e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/
2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\,(cdx+ae)^{\frac{3}{2}}f}{3\,cd}+\frac{2\,(3\,c^2d^2x^2+acdex-2\,a^2e^2)\sqrt{cdx+ae}g}{15\,c^2d^2}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
 algorithm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)*f/(c*d) + 2/15*(3*c^2*d^2*x^2 + a*c*d*e*x - 2*a^2*
e^2)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.34

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\left(15\sqrt{cdx+ae}aef-5\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)f-\frac{5\left(3\sqrt{cdx+ae}ae-(cdx+ae)^{\frac{3}{2}}\right)aeg}{cd}+\frac{(15\sqrt{cdx+ae}a^2e}{}\right.}{15\,cd}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
 algorithm="giac")
```

output
```
2/15*(15*sqrt(c*d*x + a*e)*a*e*f - 5*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a
*e)^(3/2))*f - 5*(3*sqrt(c*d*x + a*e)*a*e - (c*d*x + a*e)^(3/2))*a*e*g/(c*
d) + (15*sqrt(c*d*x + a*e)*a^2*e^2 - 10*(c*d*x + a*e)^(3/2)*a*e + 3*(c*d*x
 + a*e)^(5/2))*g/(c*d))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\left(\frac{2\,g\,x^2}{5}-\frac{4\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,c^2\,d^2}+\frac{x\,(10\,f\,c^2\,d^2+2\,a\,e\,g\,c\,d)}{15\,c^2\,d^2}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

input
```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1
/2),x)
```

output
```
(((2*g*x^2)/5 - (4*a^2*e^2*g - 10*a*c*d*e*f)/(15*c^2*d^2) + (x*(10*c^2*d^2
*f + 2*a*c*d*e*g))/(15*c^2*d^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.58

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{cdx+ae}\,(3c^2d^2g\,x^2+acdegx+5c^2d^2fx-2a^2e^2g+5acdef)}{15c^2d^2}$$

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 2*a**2*e**2*g + 5*a*c*d*e*f + a*c*d*e*g*x + 5*c**
2*d**2*f*x + 3*c**2*d**2*g*x**2))/(15*c**2*d**2)
```

3.5 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

output
```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}}{3cd(d+ex)^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x (ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2(cdx+ae)\sqrt{(ex+d)(cdx+ae)}}{3cd\sqrt{ex+d}}$	40
gosper	$\frac{2(cdx+ae)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3cd\sqrt{ex+d}}$	50
orering	$\frac{2(cdx+ae)\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{3cd\sqrt{ex+d}}$	51

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/3*(c*d*x+a*e)*((e*x+d)*(c*d*x+a*e))^(1/2)/c/d/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdx+ae)\sqrt{ex+d}}{3\,(cdex+cd^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorit
hm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d
)/(c*d*e*x + c*d^2)
```

Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\,(cdx+ae)^{\frac{3}{2}}}{3\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorit
hm="maxima")
```

output
```
2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\,(cdx+ae)^{\frac{3}{2}}}{3\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorit
hm="giac")
```

output
```
2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{\left(\frac{2x}{3}+\frac{2\,a\,e}{3\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(1/2),x)
```

output

```
(((2*x)/3 + (2*a*e)/(3*c*d))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)
)/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\sqrt{cdx+ae}\,(cdx+ae)}{3cd}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output

```
(2*sqrt(a*e + c*d*x)*(a*e + c*d*x))/(3*c*d)
```

3.6 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 124

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}}$$

output

```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)-2*(-a*e*g+c*d*f)
^(1/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}-\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x] - Sqrt[c*d*f
 - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(g^(3/
2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = $\{1250, 1255, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)}dx$$

$$\Big\downarrow \color{blue}{1250}$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}$$

$$\Big\downarrow \color{blue}{1255}$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}$$

$$\Big\downarrow \color{blue}{218}$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)),
x]
```

output

```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqr
t[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1250

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)aeg-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-dfc)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g\sqrt{(aeg-dfc)g}}$	143

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f),x,method
=_RETURNVERBOSE)
```

output

```
-2*((e*x+d)*(c*d*x+a*e))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a*e*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*
f-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/
2)/g/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.82

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx$$
$$=\left[\frac{(ex+d)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}g\sqrt{-\frac{cdf-aeg}{g}}-(cdef-(cd^2+2\,ae^2)g)x}{egx^2+df+(ef+dg)x}\right)}{egx+dg}\right.$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f),x,
 algorithm="fricas")
```

output

```
[((e*x + d)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(
c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e
*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d
))/(e*g*x + d*g), 2*((e*x + d)*sqrt((c*d*f - a*e*g)/g)*arctan(-sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt((c*d*f - a*e*g)/g)/(
c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g*x + d*g)]
```

Sympy [F]

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\int \frac{\sqrt{\left(d+ex\right)\left(ae+cdx\right)}}{\sqrt{d+ex}\left(f+gx\right)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f),x,
 algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\frac{2\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}}{g}-\frac{(cde^2f-ae^3g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)}{\sqrt{cdfg-aeg^2}\,eg}\right)|e|}{e^2}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f),x,
 algorithm="giac")
```

output

```
2*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/g - (c*d*e^2*f - a*e^3*g)*arcta
n(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(
sqrt(c*d*f*g - a*e*g^2)*e*g))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(f+g\,x\right)\,\sqrt{d+e\,x}}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/
2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)}\,dx$$
$$=\frac{-2\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)+2\sqrt{cdx+ae}\,g}{g^2}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f),x)
```

output

```
(2*( - sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f))) + sqrt(a*e + c*d*x)*g))/g**2
```

3.7 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 132

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}(f+gx)} + \frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

output

```
-(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)+c*d*arcta
n(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*
x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}}{f+gx}+\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]/(f + g*x)) + (c*d*ArcTan[(Sqrt[g
]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e +
 c*d*x])))/(g^(3/2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1249, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^2}dx$$

$$\Big\downarrow \; 1249$$

$$\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

$$\Big\downarrow \; 1255$$

$$\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

$$\Big\downarrow \; 218$$

$$\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^2
),x]
```

output

```
-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)))
 + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt
[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1255

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)cdgx-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-dfc)g}\right)\sqrt{(ex+d)(cdx+ae)}}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g(gx+f)\sqrt{(aeg-dfc)g}}$	151

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^2,x,meth
od=_RETURNVERBOSE)
```

output
```
(-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g*x-arctanh(g*(
c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2))*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/
2)/g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(116) = 232$.

Time = 0.11 (sec) , antiderivative size = 563, normalized size of antiderivative = 4.27

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx$$

$$= \Bigg[-\frac{(cdegx^2 + cd^2f + (cdef + cd^2g)x)\sqrt{-cdfg + aeg^2}\log\Bigg(-\frac{cdegx^2 - cd^2f + 2\,adeg - (cdef - (cd^2 + 2\,ae^2)g)x - 2\,\sqrt{cdex^2}}{egx^2 + df + (ef + dg)}}{2\,(cd^2f^2g^2 - adefg^3 + (cdefg^3 - ae^2g^4)x^2 + (c}$$

$$-\frac{(cdegx^2 + cd^2f + (cdef + cd^2g)x)\sqrt{cdfg - aeg^2}\arctan\Bigg(-\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{cdfg - aeg^2}\sqrt{ex + d}}{cd^2f - adeg + (cdef - ae^2g)x}\Bigg) + \sqrt{}}{cd^2f^2g^2 - adefg^3 + (cdefg^3 - ae^2g^4)x^2 + (cdef^2g^2 - adeg^4 + (c}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^2,
x, algorithm="fricas")
```

output
```
[-1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e
*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^
2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*
e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c*d^2*f^2
*g^2 - a*d*e*f*g^3 + (c*d*e*f*g^3 - a*e^2*g^4)*x^2 + (c*d*e*f^2*g^2 - a*d*
e*g^4 + (c*d^2 - a*e^2)*f*g^3)*x), -((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c
*d^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d^2*f - a*d*e*g + (c
*d*e*f - a*e^2*g)*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f
*g - a*e*g^2)*sqrt(e*x + d))/(c*d^2*f^2*g^2 - a*d*e*f*g^3 + (c*d*e*f*g^3 -
 a*e^2*g^4)*x^2 + (c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - a*e^2)*f*g^3)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,(f + gx)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**2,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^2}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^2,
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx$$
$$= -\frac{\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,cde^2}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)g}-\frac{cde\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)}{\sqrt{cdfg-aeg^2}\,g}\right)|e|}{e^2}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^2,
x, algorithm="giac")
```

output

```
-(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*e^2/((c*d*e^2*f - a*e^3*g +
((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*g) - c*d*e*arctan(sqrt((e*x + d)*c*
d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*
g^2)*g))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{(f+g\,x)^2\,\sqrt{d+e\,x}}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(
1/2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^2}\,dx$$
$$= \frac{-\sqrt{g}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)cdf-\sqrt{g}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)cdgx-\sqrt{cdx+ae}\,a}{g^2\left(ae\,g^2x-cdf\,gx+aefg-cd\,f^2\right)}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^2,x)
```

output

```
( - sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqr
t( - a*e*g + c*d*f)))*c*d*f - sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*
e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*d*g*x - sqrt(a*e + c*d*x
)*a*e*g**2 + sqrt(a*e + c*d*x)*c*d*f*g)/(g**2*(a*e*f*g + a*e*g**2*x - c*d*
f**2 - c*d*f*g*x))
```

3.8 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 207

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx=-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}(f+gx)}+\frac{c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}$$

output

```
-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^2+1/4
*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(e*x+d)^(1/2
)/(g*x+f)+1/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\Big(\sqrt{g}\sqrt{cdf-aeg}\sqrt{ae+cdx}(2aeg+cd(-f+gx))+c^2d^2(f+gx)^2\arctan\Big(\frac{\sqrt{g}\sqrt{ae+cd}}{\sqrt{cdf-aeg}}}{4g^{3/2}(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^2}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^3),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x]*(2*a*e*g + c*d*(-f + g*x)) + c^2*d^2*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqr
t[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(4*g^(3/2)*(c*d*f - a*e*g)^(3/2)*Sq
rt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1249, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4g}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

$$\Big\downarrow \text{1254}$$

$$\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

$$\Big\downarrow \; 1255$$

$$\frac{cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^3
),x]
```

output

```
-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x
)^2) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*
Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f -
 a*e*g)^(3/2))))/(4*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.33

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\,\Big(\operatorname{arctanh}\Big(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\Big)c^{2}d^{2}g^{2}x^{2}+2\,\operatorname{arctanh}\Big(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\Big)c^{2}d^{2}fgx+\operatorname{arctanh}\Big(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\Big)c^{2}d^{2}f^{2}-cdg}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,(aeg-dfc)g(gx+f)^{2}\sqrt{(aeg-}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^3,x,meth
od=_RETURNVERBOSE)
```

output
```
1/4*((e*x+d)*(c*d*x+a*e))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^2*d^2*g^2*x^2+2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^2*d^2*f*g*x+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^2*d^2*f^2-c*d*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-2*((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e
)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+f)^2/(
(a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(181) = 362$.

Time = 0.12 (sec) , antiderivative size = 1057, normalized size of antiderivative = 5.11

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^3,
x, algorithm="fricas")
```

output
```
[1/8*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x
^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d
*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
 + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(c^2*d^2*f^2*g - 3*a*c*d*e*f*g
^2 + 2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g^2 - 2*a*c*d^2*e*f^3*
g^3 + a^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g^5 + a^2*e^3
*g^6)*x^3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 - 4*a*c*d*e^2)
*f^2*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c^2*d^2*e*f^4*g^2 + 2*a^2
*d*e^2*f*g^5 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^3 - (4*a*c*d^2*e - a^2*e^3)*f
^2*g^4)*x), -1/4*((c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^
2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*g^2
)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g
^2)*sqrt(e*x + d)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) + (c^2*d^2*
f^2*g - 3*a*c*d*e*f*g^2 + 2*a^2*e^2*g^3 - (c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^4*g
^2 - 2*a*c*d^2*e*f^3*g^3 + a^2*d*e^2*f^2*g^4 + (c^2*d^2*e*f^2*g^4 - 2*a*c*
d*e^2*f*g^5 + a^2*e^3*g^6)*x^3 + (2*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c
^2*d^3 - 4*a*c*d*e^2)*f^2*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f*g^5)*x^2 + (c...
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,(f + gx)^3}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**3,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^3,
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^3), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^3}\,dx$$

$$= \frac{\left(\frac{c^2d^2e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(cdfg-aeg^2)^{\frac{3}{2}}} - \frac{\sqrt{(ex+d)cde-cd^2e+ae^3}c^3d^3e^4f-\sqrt{(ex+d)cde-cd^2e+ae^3}ac^2d^2e^5g-((ex+d)cde-cd^2e+ae^3)}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)^2(cdfg-aeg^2)}\right.}{4\,e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^3,
x, algorithm="giac")
```

output
```
1/4*(c^2*d^2*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e))/(c*d*f*g - a*e*g^2)^(3/2) - (sqrt((e*x + d)*c*d*e - c*d
^2*e + a*e^3)*c^3*d^3*e^4*f - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^
2*d^2*e^5*g - ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^2*g)/((c
*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2*(c*d*f*g - a
*e*g^2)))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^3\,\sqrt{d+e\,x}}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(
1/2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^3}\,dx$$
$$=\frac{\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^2d^2f^2+2\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^2d^2fgx+\sqrt{g}\,\sqrt{-}}{4g^2\,(a^2e^2g^4x^2-2acdef\,g^3x^2+c^2d}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^3,x)
```

output

```
(sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt(
- a*e*g + c*d*f)))*c**2*d**2*f**2 + 2*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan(
(sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*f*g*x +
sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( -
 a*e*g + c*d*f)))*c**2*d**2*g**2*x**2 - 2*sqrt(a*e + c*d*x)*a**2*e**2*g**3
 + 3*sqrt(a*e + c*d*x)*a*c*d*e*f*g**2 - sqrt(a*e + c*d*x)*a*c*d*e*g**3*x -
 sqrt(a*e + c*d*x)*c**2*d**2*f**2*g + sqrt(a*e + c*d*x)*c**2*d**2*f*g**2*x
)/(4*g**2*(a**2*e**2*f**2*g**2 + 2*a**2*e**2*f*g**3*x + a**2*e**2*g**4*x**
2 - 2*a*c*d*e*f**3*g - 4*a*c*d*e*f**2*g**2*x - 2*a*c*d*e*f*g**3*x**2 + c**
2*d**2*f**4 + 2*c**2*d**2*f**3*g*x + c**2*d**2*f**2*g**2*x**2))
```

3.9 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 277

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx = &-\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3} \\ &+\frac{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} \\ &+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8g(cdf-aeg)^2\sqrt{d+ex}(f+gx)} \\ &+\frac{c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}\end{aligned}$$

output
```
-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^3+1/1
2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(e*x+d)^(1/
2)/(g*x+f)^2+1/8*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g
+c*d*f)^2/(e*x+d)^(1/2)/(g*x+f)+1/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e
*g+c*d*f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$$

$$= \frac{\sqrt{ae+cdx}\sqrt{d+ex}\bigl(\sqrt{g}\sqrt{cdf-aeg}\sqrt{ae+cdx}(-8a^2e^2g^2-2acdeg(-7f+gx)+c^2d^2(-3f^2+8fgx+3}{24g^{3/2}(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^3}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^4),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x]*(-8*a^2*e^2*g^2 - 2*a*c*d*e*g*(-7*f + g*x) + c^2*d^2*(-3*f^2 + 8*f*g
*x + 3*g^2*x^2)) + 3*c^3*d^3*(f + g*x)^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d*f - a*e*g]]))/(24*g^(3/2)*(c*d*f - a*e*g)^(5/2)*Sqrt[(a*e + c*d
*x)*(d + e*x)]*(f + g*x)^3)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1249, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6g}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

$$\Big\downarrow \text{1254}$$

$$\frac{cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

$$\Big\downarrow \; 1254$$

$$\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

$$\Big\downarrow \; 1255$$

$$\frac{cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+c}}{2\sqrt{d+ex}(f+gx)^2(cdf-}\right.}{6g}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^4
),x]
```

output

```
-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x
)^3) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g
)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
 e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(6*g)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\Big(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^3d^3g^3x^3+9\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^3d^3fg^2x^2+9\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^3d}{\quad}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^4,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/24*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f^2*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^3*d^3*f^3-3*c^2*d^2*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)
^(1/2)+2*a*c*d*e*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*c^2*d^2
*f*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+8*((a*e*g-c*d*f)*g)^(1/2)
*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2
)*a*c*d*e*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(e*
x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)^2/(c*d*x+a*e)
^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(245) = 490$.

Time = 0.28 (sec) , antiderivative size = 1733, normalized size of antiderivative = 6.26

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^4}\,dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^4,
x, algorithm="fricas")
```

output

```
[-1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^
3*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
(e*f + d*g)*x)) + 2*(3*c^3*d^3*f^3*g - 17*a*c^2*d^2*e*f^2*g^2 + 22*a^2*c*d
*e^2*f*g^3 - 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 - 2*(
4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^6*g^2 - 3*a*c^2*
d^3*e*f^5*g^3 + 3*a^2*c*d^2*e^2*f^4*g^4 - a^3*d*e^3*f^3*g^5 + (c^3*d^3*e*f
^3*g^5 - 3*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^4
+ (3*c^3*d^3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g
^5 - 3*(a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^2*g^6 + 3*(a^2*c*d^2*e^2 - a^3*e^4)
*f*g^7)*x^3 + 3*(c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 3*a*c^2*
d^2*e^2)*f^4*g^4 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e^
2 - a^3*e^4)*f^2*g^6)*x^2 + (c^3*d^3*e*f^6*g^2 - 3*a^3*d*e^3*f^2*g^6 + 3*(
c^3*d^4 - a*c^2*d^2*e^2)*f^5*g^3 - 3*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^4
 + (9*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^5)*x), -1/24*(3*(c^3*d^3*e*g^3*x^4 +
c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g +
 c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g...
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^4}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,(f + gx)^4}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**4,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^4}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^4,
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^4}\,dx$$

$$= \frac{\left(\frac{3\,c^3 d^3 e \arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)}{(c^2d^2f^2g-2\,acdefg^2+a^2e^2g^3)\sqrt{cdfg-aeg^2}} - \frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}\,c^5d^5e^6f^2-6\,\sqrt{(ex+d)cde-cd^2e+ae^3}\,ac^4d^4e^7fg+3\,\sqrt{(ex+d)cde-}}{(c^2d^2f^2g-2}\right.}{2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^4,
x, algorithm="giac")
```

output
```
1/24*(3*c^3*d^3*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c
*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g - 2*a*c*d*e*f*g^2 + a^2*e^2*g^3)*sqr
t(c*d*f*g - a*e*g^2)) - (3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^5*d^5
*e^6*f^2 - 6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^4*d^4*e^7*f*g + 3
*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^3*d^3*e^8*g^2 - 8*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^4*e^4*f*g + 8*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(3/2)*a*c^3*d^3*e^5*g^2 - 3*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(5/2)*c^3*d^3*e^2*g^2)/((c^2*d^2*f^2*g - 2*a*c*d*e*f*g^2 + a^2*e^2*g^
3)*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^3))*abs(e
)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^4\,\sqrt{d+e\,x}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(
1/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^4}\,dx$$

$$=\frac{-3\sqrt{g}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^3d^3f^3-9\sqrt{g}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^3d^3f^2gx-9}{24g^2\,(a^3}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^4,x)
```

output
```
( - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*s
qrt( - a*e*g + c*d*f)))*c**3*d**3*f**3 - 9*sqrt(g)*sqrt( - a*e*g + c*d*f)*
atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d**3*f**
2*g*x - 9*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(
g)*sqrt( - a*e*g + c*d*f)))*c**3*d**3*f*g**2*x**2 - 3*sqrt(g)*sqrt( - a*e*
g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*
*3*d**3*g**3*x**3 - 8*sqrt(a*e + c*d*x)*a**3*e**3*g**4 + 22*sqrt(a*e + c*d
*x)*a**2*c*d*e**2*f*g**3 - 2*sqrt(a*e + c*d*x)*a**2*c*d*e**2*g**4*x - 17*s
qrt(a*e + c*d*x)*a*c**2*d**2*e*f**2*g**2 + 10*sqrt(a*e + c*d*x)*a*c**2*d**
2*e*f*g**3*x + 3*sqrt(a*e + c*d*x)*a*c**2*d**2*e*g**4*x**2 + 3*sqrt(a*e +
c*d*x)*c**3*d**3*f**3*g - 8*sqrt(a*e + c*d*x)*c**3*d**3*f**2*g**2*x - 3*sq
rt(a*e + c*d*x)*c**3*d**3*f*g**3*x**2)/(24*g**2*(a**3*e**3*f**3*g**3 + 3*a
**3*e**3*f**2*g**4*x + 3*a**3*e**3*f*g**5*x**2 + a**3*e**3*g**6*x**3 - 3*a
**2*c*d*e**2*f**4*g**2 - 9*a**2*c*d*e**2*f**3*g**3*x - 9*a**2*c*d*e**2*f**
2*g**4*x**2 - 3*a**2*c*d*e**2*f*g**5*x**3 + 3*a*c**2*d**2*e*f**5*g + 9*a*c
**2*d**2*e*f**4*g**2*x + 9*a*c**2*d**2*e*f**3*g**3*x**2 + 3*a*c**2*d**2*e*
f**2*g**4*x**3 - c**3*d**3*f**6 - 3*c**3*d**3*f**5*g*x - 3*c**3*d**3*f**4*
g**2*x**2 - c**3*d**3*f**3*g**3*x**3))
```

3.10 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 347

$$\begin{aligned}\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx = &-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4} \\ &+\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} \\ &+\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{96g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} \\ &+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} \\ &+\frac{5c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}\end{aligned}$$

output

```
-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^4+1/2
4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(e*x+d)^(1/
2)/(g*x+f)^3+5/96*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*
g+c*d*f)^2/(e*x+d)^(1/2)/(g*x+f)^2+5/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)+5/64*c^4*d^4*arctan
(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x
+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(7/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx$$
$$=\frac{c^4d^4\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{g}\left(48a^3e^3g^3+8a^2cde^2g^2(-17f+gx)-2ac^2d^2eg\left(-59f^2+18fgx+5g^2x^2\right)+c^3d^3\left(-15f^3+73f^2gx+55fg^2\right.\right.}{c^4d^4(cdf-aeg)^3(f+gx)^4}\right.}{192g^{3/2}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^5),x]
```

output
```
(c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[g]*(48*a^3*e^3*g^3 + 8*a^2*c
*d*e^2*g^2*(-17*f + g*x) - 2*a*c^2*d^2*e*g*(-59*f^2 + 18*f*g*x + 5*g^2*x^2
) + c^3*d^3*(-15*f^3 + 73*f^2*g*x + 55*f*g^2*x^2 + 15*g^3*x^3)))/(c^4*d^4*
(c*d*f - a*e*g)^3*(f + g*x)^4) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sq
rt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x])))/(192*g^(3/
2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1249, 1254, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^5}dx$$
$$\Big\downarrow \text{1249}$$
$$\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$
$$\Big\downarrow \text{1254}$$

$$\frac{cd\left(\frac{5cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

$$\Big\downarrow \text{1254}$$

$$\frac{cd\left(\frac{5cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

$$\Big\downarrow \text{1254}$$

$$\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex}}{3\sqrt{d+ex}(f+gx)^3(cdf-ae}\right.}{8g}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

$$\Big\downarrow \text{1255}$$

$$cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{2\sqrt{d+ex}(f+gx)^2(cdf-}}{6(cdf-aeg)}\right.}{8g}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

$$\Big\downarrow \text{218}$$

$$\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cde}}{3\sqrt{d+ex}(f+gx)^3(cdf-a}}{8g}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^5
),x]
```

output
```
-1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x
)^4) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g
)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x
)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sq
rt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c
*d*f - a*e*g))))/(6*(c*d*f - a*e*g))))/(8*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(309) = 618$.

Time = 2.82 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.98

method	result
default	$\sqrt{(ex+d)(cdx+ae)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{4}d^{4}g^{4}x^{4}+60\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{4}d^{4}f\,g^{3}x^{3}+90\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{4}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^5,x,meth
od=_RETURNVERBOSE)
```

output
```
1/192*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^4*d^4*f^3*g*x-15*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*
e*g-c*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^4*d^4*f^4+10*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(
1/2)-55*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*a^2*
c*d*e^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*a*c^2*d^2*e*f*g
^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-73*c^3*d^3*f^2*g*x*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-48*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(
1/2)*a^3*e^3*g^3+136*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^2
*f*g^2-118*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+15*
((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2)/(g*x+f)^4/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^
2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1285 vs. $2(309) = 618$.

Time = 1.01 (sec) , antiderivative size = 2611, normalized size of antiderivative = 7.52

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^5,
x, algorithm="fricas")
```

output

```
[1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5
*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e
*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqr
t(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2
*(15*c^4*d^4*f^4*g - 133*a*c^3*d^3*e*f^3*g^2 + 254*a^2*c^2*d^2*e^2*f^2*g^3
 - 184*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*
e*g^5)*x^3 - 5*(11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*
e^2*g^5)*x^2 - (73*c^4*d^4*f^3*g^2 - 109*a*c^3*d^3*e*f^2*g^3 + 44*a^2*c^2*
d^2*e^2*f*g^4 - 8*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^8*g^2 - 4*a*c^3*d^4*e*f^7*g^3 + 6*a^2*c^
2*d^3*e^2*f^6*g^4 - 4*a^3*c*d^2*e^3*f^5*g^5 + a^4*d*e^4*f^4*g^6 + (c^4*d^4
*e*f^4*g^6 - 4*a*c^3*d^3*e^2*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c
*d*e^4*f*g^9 + a^4*e^5*g^10)*x^5 + (4*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 +
 (c^4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^6 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3
)*f^3*g^7 + 2*(3*a^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^8 - 4*(a^3*c*d^2*e
^3 - a^4*e^5)*f*g^9)*x^4 + 2*(3*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*
(c^4*d^5 - 6*a*c^3*d^3*e^2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3
)*f^4*g^6 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e...
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,(f + gx)^5}\,dx$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**5,x)
```

output

```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**5), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^5,
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. 2(309) = 618.

Time = 0.15 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^5}\,dx$$
$$= \frac{\left(\frac{15\,c^4d^4e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)}{(c^3d^3f^3g-3\,ac^2d^2ef^2g^2+3\,a^2cde^2fg^3-a^3e^3g^4)\sqrt{cdfg-aeg^2}} - \frac{15\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^7d^7e^8f^3-45\,\sqrt{(ex+d)cde-cd^2e+ae^3}ac^6d^6e}{}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^5,
x, algorithm="giac")
```

output

```
1/192*(15*c^4*d^4*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt
(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*f^3*g - 3*a*c^2*d^2*e*f^2*g^2 + 3*a^2*c*
d*e^2*f*g^3 - a^3*e^3*g^4)*sqrt(c*d*f*g - a*e*g^2)) - (15*sqrt((e*x + d)*c
*d*e - c*d^2*e + a*e^3)*c^7*d^7*e^8*f^3 - 45*sqrt((e*x + d)*c*d*e - c*d^2*
e + a*e^3)*a*c^6*d^6*e^9*f^2*g + 45*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*a^2*c^5*d^5*e^10*f*g^2 - 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*
c^4*d^4*e^11*g^3 - 73*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^6*d^6*e^
6*f^2*g + 146*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^5*d^5*e^7*f*g^
2 - 73*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^4*d^4*e^8*g^3 - 55*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^5*d^5*e^4*f*g^2 + 55*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^4*d^4*e^5*g^3 - 15*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(7/2)*c^4*d^4*e^2*g^3)/((c^3*d^3*f^3*g - 3*a*c^2*d^2*e*f
^2*g^2 + 3*a^2*c*d*e^2*f*g^3 - a^3*e^3*g^4)*(c*d*e^2*f - a*e^3*g + ((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)*g)^4))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^5\,\sqrt{d+e\,x}}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(
1/2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1070, normalized size of antiderivative = 3.08

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^5}\,dx=\text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^5,x)
```

output

```
(15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqr
t( - a*e*g + c*d*f)))*c**4*d**4*f**4 + 60*sqrt(g)*sqrt( - a*e*g + c*d*f)*a
tan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**4*d**4*f**3
*g*x + 90*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(
g)*sqrt( - a*e*g + c*d*f)))*c**4*d**4*f**2*g**2*x**2 + 60*sqrt(g)*sqrt( -
a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f))
)*c**4*d**4*f*g**3*x**3 + 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e
 + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**4*d**4*g**4*x**4 - 48*sq
rt(a*e + c*d*x)*a**4*e**4*g**5 + 184*sqrt(a*e + c*d*x)*a**3*c*d*e**3*f*g**
4 - 8*sqrt(a*e + c*d*x)*a**3*c*d*e**3*g**5*x - 254*sqrt(a*e + c*d*x)*a**2*
c**2*d**2*e**2*f**2*g**3 + 44*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*f*g**4
*x + 10*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*g**5*x**2 + 133*sqrt(a*e + c
*d*x)*a*c**3*d**3*e*f**3*g**2 - 109*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f**2*g
**3*x - 65*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f*g**4*x**2 - 15*sqrt(a*e + c*d
*x)*a*c**3*d**3*e*g**5*x**3 - 15*sqrt(a*e + c*d*x)*c**4*d**4*f**4*g + 73*s
qrt(a*e + c*d*x)*c**4*d**4*f**3*g**2*x + 55*sqrt(a*e + c*d*x)*c**4*d**4*f*
*2*g**3*x**2 + 15*sqrt(a*e + c*d*x)*c**4*d**4*f*g**4*x**3)/(192*g**2*(a**4
*e**4*f**4*g**4 + 4*a**4*e**4*f**3*g**5*x + 6*a**4*e**4*f**2*g**6*x**2 + 4
*a**4*e**4*f*g**7*x**3 + a**4*e**4*g**8*x**4 - 4*a**3*c*d*e**3*f**5*g**3 -
 16*a**3*c*d*e**3*f**4*g**4*x - 24*a**3*c*d*e**3*f**3*g**5*x**2 - 16*a*...
```

3.11 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 297

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2(cdf-aeg)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^5d^5(d+ex)^{5/2}}$$
$$+\frac{8g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^5d^5(d+ex)^{7/2}}$$
$$+\frac{4g^2(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{3c^5d^5(d+ex)^{9/2}}$$
$$+\frac{8g^3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^5d^5(d+ex)^{11/2}}$$
$$+\frac{2g^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{13/2}}{13c^5d^5(d+ex)^{13/2}}$$

output

```
2/5*(-a*e*g+c*d*f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^5/d^5/(e*x+
d)^(5/2)+8/7*g*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^
5/d^5/(e*x+d)^(7/2)+4/3*g^2*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(9/2)/c^5/d^5/(e*x+d)^(9/2)+8/11*g^3*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(11/2)/c^5/d^5/(e*x+d)^(11/2)+2/13*g^4*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(13/2)/c^5/d^5/(e*x+d)^(13/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.66

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(128a^4e^4g^4-64a^3cde^3g^3(13f\right.}{}$$

input
```
Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(
13*f + 5*g*x) + 16*a^2*c^2*d^2*e^2*g^2*(143*f^2 + 130*f*g*x + 35*g^2*x^2)
- 8*a*c^3*d^3*e*g*(429*f^3 + 715*f^2*g*x + 455*f*g^2*x^2 + 105*g^3*x^3) +
c^4*d^4*(3003*f^4 + 8580*f^3*g*x + 10010*f^2*g^2*x^2 + 5460*f*g^3*x^3 + 11
55*g^4*x^4)))/(15015*c^5*d^5*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1253, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^4 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{8(cdf-aeg)\int \frac{(f+gx)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{13cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\int \frac{(f+gx)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{11cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}\right)}{13cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1253$$

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{9cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^3(x(}{1}\right.}{13cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1221$$

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{7}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}\right)}{9cd}+\frac{2(f+gx)^2}{}\right.}{11cd}\right.}{13cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1122$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}+$$

$$\frac{8(cdf-aeg)\left(\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}+\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)\right.}{}\right.}{}\right.}{11cd}\right.}{13cd}$$

input
```
Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x]
```

output
```
(2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(13*c*d*(d +
 e*x)^(5/2)) + (8*(c*d*f - a*e*g)*((2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(11*c*d*(d + e*x)^(5/2)) + (6*(c*d*f - a*e*g)*((2*(
f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*c*d*(d + e*x)
^(5/2)) + (4*(c*d*f - a*e*g)*((2*(7*f - (5*d*g)/e - (2*a*e*g)/(c*d))*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*c*d*(d + e*x)^(5/2)) + (2*g*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))))
/(9*c*d)))/(11*c*d)))/(13*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.93

method	result
default	$\underline{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^2\left(1155g^4x^4d^4c^4-840a\,c^3d^3e\,g^4x^3+5460c^4d^4f\,g^3x^3+560a^2c^2d^2e^2g^4x^2-3640a\,c^3d^3ef\,g^3x^2+10010\right.}$
gosper	$\underline{2(cdx+ae)\left(1155g^4x^4d^4c^4-840a\,c^3d^3e\,g^4x^3+5460c^4d^4f\,g^3x^3+560a^2c^2d^2e^2g^4x^2-3640a\,c^3d^3ef\,g^3x^2+10010c^4d^4f^2g^2x^2-320a^3\right.}$
orering	$\underline{2\left(1155g^4x^4d^4c^4-840a\,c^3d^3e\,g^4x^3+5460c^4d^4f\,g^3x^3+560a^2c^2d^2e^2g^4x^2-3640a\,c^3d^3ef\,g^3x^2+10010c^4d^4f^2g^2x^2-320a^3cd\,e^3g^4x+\right.}$

input
```
int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/15015*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(1155*c^4*
d^4*g^4*x^4-840*a*c^3*d^3*e*g^4*x^3+5460*c^4*d^4*f*g^3*x^3+560*a^2*c^2*d^2
*e^2*g^4*x^2-3640*a*c^3*d^3*e*f*g^3*x^2+10010*c^4*d^4*f^2*g^2*x^2-320*a^3*
c*d*e^3*g^4*x+2080*a^2*c^2*d^2*e^2*f*g^3*x-5720*a*c^3*d^3*e*f^2*g^2*x+8580
*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-832*a^3*c*d*e^3*f*g^3+2288*a^2*c^2*d^2*e^
2*f^2*g^2-3432*a*c^3*d^3*e*f^3*g+3003*c^4*d^4*f^4)/d^5/c^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.59

$$\int \frac{(f+gx)^4 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(1155\,c^6d^6g^4x^6+3003\,a^2c^4d^4e^2f^4-3432\,a^3c^3d^3e^3f^3}{}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="fricas")
```

output
```
2/15015*(1155*c^6*d^6*g^4*x^6 + 3003*a^2*c^4*d^4*e^2*f^4 - 3432*a^3*c^3*d^
3*e^3*f^3*g + 2288*a^4*c^2*d^2*e^4*f^2*g^2 - 832*a^5*c*d*e^5*f*g^3 + 128*a
^6*e^6*g^4 + 210*(26*c^6*d^6*f*g^3 + 7*a*c^5*d^5*e*g^4)*x^5 + 35*(286*c^6*
d^6*f^2*g^2 + 208*a*c^5*d^5*e*f*g^3 + a^2*c^4*d^4*e^2*g^4)*x^4 + 20*(429*c
^6*d^6*f^3*g + 715*a*c^5*d^5*e*f^2*g^2 + 13*a^2*c^4*d^4*e^2*f*g^3 - 2*a^3*
c^3*d^3*e^3*g^4)*x^3 + 3*(1001*c^6*d^6*f^4 + 4576*a*c^5*d^5*e*f^3*g + 286*
a^2*c^4*d^4*e^2*f^2*g^2 - 104*a^3*c^3*d^3*e^3*f*g^3 + 16*a^4*c^2*d^2*e^4*g
^4)*x^2 + 2*(3003*a*c^5*d^5*e*f^4 + 858*a^2*c^4*d^4*e^2*f^3*g - 572*a^3*c^
3*d^3*e^3*f^2*g^2 + 208*a^4*c^2*d^2*e^4*f*g^3 - 32*a^5*c*d*e^5*g^4)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x + c^5*
d^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**
(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.39

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}f^4}{5\,cd}$$
$$+\frac{8\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}f^3g}{35\,c^2d^2}$$
$$+\frac{4\left(35\,c^4d^4x^4+50\,ac^3d^3ex^3+3\,a^2c^2d^2e^2x^2-4\,a^3cde^3x+8\,a^4e^4\right)\sqrt{cdx+ae}f^2g^2}{105\,c^3d^3}$$
$$+\frac{8\left(105\,c^5d^5x^5+140\,ac^4d^4ex^4+5\,a^2c^3d^3e^2x^3-6\,a^3c^2d^2e^3x^2+8\,a^4cde^4x-16\,a^5e^5\right)\sqrt{cdx+ae}fg^3}{1155\,c^4d^4}$$
$$+\frac{2\,(1155\,c^6d^6x^6+1470\,ac^5d^5ex^5+35\,a^2c^4d^4e^2x^4-40\,a^3c^3d^3e^3x^3+48\,a^4c^2d^2e^4x^2-64\,a^5cde^5x+128\,a^6e^6}{15015\,c^5d^5}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^4/(c*d) + 8/
35*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*
d*x + a*e)*f^3*g/(c^2*d^2) + 4/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 +
3*a^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f^2
*g^2/(c^3*d^3) + 8/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3
*d^3*e^2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(
c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/15015*(1155*c^6*d^6*x^6 + 1470*a*c^5*d^5*
e*x^5 + 35*a^2*c^4*d^4*e^2*x^4 - 40*a^3*c^3*d^3*e^3*x^3 + 48*a^4*c^2*d^2*e
^4*x^2 - 64*a^5*c*d*e^5*x + 128*a^6*e^6)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. 2(267) = 534.

Time = 0.17 (sec) , antiderivative size = 1230, normalized size of antiderivative = 4.14

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="giac")
```

output
```
2/45045*(15015*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*f^4*abs(e)/(c*d
*e^2) - 3003*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*f^4*abs(e)/(c*d*e^5) - 12012*(5*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)^(5/2))*a*f^3*g*abs(e)/(c^2*d^2*e^4) + 1716*(35*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*f^3*g*abs(e)/(c
^2*d^2*e^7) + 2574*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 -
 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(7/2))*a*f^2*g^2*abs(e)/(c^3*d^3*e^6) - 858*(105*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*
a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*f^2*g^2*abs(e)/(c^3*
d^3*e^9) - 572*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 18
9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(
9/2))*a*f*g^3*abs(e)/(c^4*d^4*e^8) + 52*(1155*((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^
3*e^9 + 2970*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((e*x + d)*c*d*e - c*...
```

Mupad [B] (verification not implemented)

Time = 6.57 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.50

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g^3\,x^5\,\left(7\,a\,e\,g+26\,c\,d\,f\right)}{143}\right.}{}$$

input
```
int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x)
```

output

```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^3*x^5*(7*a*e*g + 26*c
*d*f))/143 + (256*a^6*e^6*g^4 + 6006*a^2*c^4*d^4*e^2*f^4 - 6864*a^3*c^3*d^
3*e^3*f^3*g - 1664*a^5*c*d*e^5*f*g^3 + 4576*a^4*c^2*d^2*e^4*f^2*g^2)/(1501
5*c^5*d^5) + (x^2*(6006*c^6*d^6*f^4 + 96*a^4*c^2*d^2*e^4*g^4 - 624*a^3*c^3
*d^3*e^3*f*g^3 + 27456*a*c^5*d^5*e*f^3*g + 1716*a^2*c^4*d^4*e^2*f^2*g^2))/
(15015*c^5*d^5) + (x*(12012*a*c^5*d^5*e*f^4 - 128*a^5*c*d*e^5*g^4 + 3432*a
^2*c^4*d^4*e^2*f^3*g + 832*a^4*c^2*d^2*e^4*f*g^3 - 2288*a^3*c^3*d^3*e^3*f^
2*g^2))/(15015*c^5*d^5) + (2*c*d*g^4*x^6)/13 + (8*g*x^3*(429*c^3*d^3*f^3 -
 2*a^3*e^3*g^3 + 715*a*c^2*d^2*e*f^2*g + 13*a^2*c*d*e^2*f*g^2))/(3003*c^2*
d^2) + (2*g^2*x^4*(a^2*e^2*g^2 + 286*c^2*d^2*f^2 + 208*a*c*d*e*f*g))/(429*
c*d)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.53

$$\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{cdx+ae}\,(1155c^6d^6g^4x^6+1470a\,c^5d^5e\,g^4x^5+5460$$

input

```
int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output

```
(2*sqrt(a*e + c*d*x)*(128*a**6*e**6*g**4 - 832*a**5*c*d*e**5*f*g**3 - 64*a
**5*c*d*e**5*g**4*x + 2288*a**4*c**2*d**2*e**4*f**2*g**2 + 416*a**4*c**2*d
**2*e**4*f*g**3*x + 48*a**4*c**2*d**2*e**4*g**4*x**2 - 3432*a**3*c**3*d**3
*e**3*f**3*g - 1144*a**3*c**3*d**3*e**3*f**2*g**2*x - 312*a**3*c**3*d**3*e
**3*f*g**3*x**2 - 40*a**3*c**3*d**3*e**3*g**4*x**3 + 3003*a**2*c**4*d**4*e
**2*f**4 + 1716*a**2*c**4*d**4*e**2*f**3*g*x + 858*a**2*c**4*d**4*e**2*f**
2*g**2*x**2 + 260*a**2*c**4*d**4*e**2*f*g**3*x**3 + 35*a**2*c**4*d**4*e**2
*g**4*x**4 + 6006*a*c**5*d**5*e*f**4*x + 13728*a*c**5*d**5*e*f**3*g*x**2 +
 14300*a*c**5*d**5*e*f**2*g**2*x**3 + 7280*a*c**5*d**5*e*f*g**3*x**4 + 147
0*a*c**5*d**5*e*g**4*x**5 + 3003*c**6*d**6*f**4*x**2 + 8580*c**6*d**6*f**3
*g*x**3 + 10010*c**6*d**6*f**2*g**2*x**4 + 5460*c**6*d**6*f*g**3*x**5 + 11
55*c**6*d**6*g**4*x**6))/(15015*c**5*d**5)
```

3.12 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 234

$$\begin{aligned}\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx &= \frac{2(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^4d^4(d+ex)^{5/2}}\\ &+\frac{6g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^4d^4(d+ex)^{7/2}}\\ &+\frac{2g^2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{3c^4d^4(d+ex)^{9/2}}\\ &+\frac{2g^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^4d^4(d+ex)^{11/2}}\end{aligned}$$

output

```
2/5*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/(e*x+
d)^(5/2)+6/7*g*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^
4/d^4/(e*x+d)^(7/2)+2/3*g^2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(9/2)/c^4/d^4/(e*x+d)^(9/2)+2/11*g^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(11/2)/c^4/d^4/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.59

$$\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{5/2}\left(-16a^3e^3g^3+8a^2cde^2g^2(11f\right.}{}$$

input
```
Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(1
1*f + 5*g*x) - 2*a*c^2*d^2*e*g*(99*f^2 + 110*f*g*x + 35*g^2*x^2) + c^3*d^3
*(231*f^3 + 495*f^2*g*x + 385*f*g^2*x^2 + 105*g^3*x^3)))/(1155*c^4*d^4*(d
+ e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{6(cdf-aeg)\int \frac{(f+gx)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{11cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{9cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{7}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}\right)}{9cd}+\frac{2(f+gx)^2(x(ae}{9c}}{11cd}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}+$$

$$\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)}{35cd(d+ex)^{5/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+\right.}{7cde(d+}\right.}{9cd}\right.}{11cd}$$

input

```
Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x]
```

output

```
(2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d*(d +
 e*x)^(5/2)) + (6*(c*d*f - a*e*g)*((2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(9*c*d*(d + e*x)^(5/2)) + (4*(c*d*f - a*e*g)*((2*(7
*f - (5*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2))/(35*c*d*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))))/(9*c*d)))/(11*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.77

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^2\left(-105x^3g^3d^3c^3+70a\,c^2d^2e\,g^3x^2-385c^3d^3f\,g^2x^2-40a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-495c^3d^3f^2g\right.}{1155\sqrt{ex+d}\,d^4c^4}$
gosper	$-\frac{2(cdx+ae)\left(-105x^3g^3d^3c^3+70a\,c^2d^2e\,g^3x^2-385c^3d^3f\,g^2x^2-40a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-495c^3d^3f^2gx+16a^3e^3g^3-88a^2c\right.}{1155d^4c^4(ex+d)^{\frac{3}{2}}}$
orering	$-\frac{2\left(-105x^3g^3d^3c^3+70a\,c^2d^2e\,g^3x^2-385c^3d^3f\,g^2x^2-40a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-495c^3d^3f^2gx+16a^3e^3g^3-88a^2cd\,e^2f\,g^2+\right.}{1155d^4c^4(ex+d)^{\frac{3}{2}}}$

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output

```
-2/1155*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(-105*c^3*
d^3*g^3*x^3+70*a*c^2*d^2*e*g^3*x^2-385*c^3*d^3*f*g^2*x^2-40*a^2*c*d*e^2*g^
3*x+220*a*c^2*d^2*e*f*g^2*x-495*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-88*a^2*c*d*
e^2*f*g^2+198*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3)/d^4/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.45

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(105\,c^5d^5g^3x^5+231\,a^2c^3d^3e^2f^3-198\,a^3c^2d^2e^3f^2g+\right.}{}$$

input

```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="fricas")
```

output

```
2/1155*(105*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 198*a^3*c^2*d^2*e^
3*f^2*g + 88*a^4*c*d*e^4*f*g^2 - 16*a^5*e^5*g^3 + 35*(11*c^5*d^5*f*g^2 + 4
*a*c^4*d^4*e*g^3)*x^4 + 5*(99*c^5*d^5*f^2*g + 110*a*c^4*d^4*e*f*g^2 + a^2*
c^3*d^3*e^2*g^3)*x^3 + 3*(77*c^5*d^5*f^3 + 264*a*c^4*d^4*e*f^2*g + 11*a^2*
c^3*d^3*e^2*f*g^2 - 2*a^3*c^2*d^2*e^3*g^3)*x^2 + (462*a*c^4*d^4*e*f^3 + 99
*a^2*c^3*d^3*e^2*f^2*g - 44*a^3*c^2*d^2*e^3*f*g^2 + 8*a^4*c*d*e^4*g^3)*x)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c
^4*d^5)
```

Sympy [F]

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(f+gx\right)^3}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**
(3/2),x)
```

output

```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**3/(d + e*x)**(3/2), x
)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.26

$$\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\,\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}f^3}{5\,cd}$$
$$+\frac{6\,\left(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3\right)\sqrt{cdx+ae}f^2g}{35\,c^2d^2}$$
$$+\frac{2\,\left(35\,c^4d^4x^4+50\,ac^3d^3ex^3+3\,a^2c^2d^2e^2x^2-4\,a^3cde^3x+8\,a^4e^4\right)\sqrt{cdx+ae}fg^2}{105\,c^3d^3}$$
$$+\frac{2\,\left(105\,c^5d^5x^5+140\,ac^4d^4ex^4+5\,a^2c^3d^3e^2x^3-6\,a^3c^2d^2e^3x^2+8\,a^4cde^4x-16\,a^5e^5\right)\sqrt{cdx+ae}g^3}{1155\,c^4d^4}$$

input

```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="maxima")
```

output

```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^3/(c*d) + 6/
35*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*
d*x + a*e)*f^2*g/(c^2*d^2) + 2/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 +
3*a^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f*g
^2/(c^3*d^3) + 2/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*e*x^4 + 5*a^2*c^3*d
^3*e^2*x^3 - 6*a^3*c^2*d^2*e^3*x^2 + 8*a^4*c*d*e^4*x - 16*a^5*e^5)*sqrt(c*
d*x + a*e)*g^3/(c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(210) = 420.

Time = 0.15 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.64

$$\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="giac")
```

output
```
2/3465*(1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*f^3*abs(e)/(c*d*e
^2) - 231*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(5/2))*f^3*abs(e)/(c*d*e^5) - 693*(5*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(5/2))*a*f^2*g*abs(e)/(c^2*d^2*e^4) + 99*(35*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*
e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*f^2*g*abs(e)/(c^2*d^2*
e^7) + 99*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(7/2))*a*f*g^2*abs(e)/(c^3*d^3*e^6) - 33*(105*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*f*g^2*abs(e)/(c^3*d^3*e^9) - 11*
(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*
d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*a*g^3*abs
(e)/(c^4*d^4*e^8) + (1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^
12 - 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(9/2)*a*e^3 + 315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(11/2...
```

Mupad [B] (verification not implemented)

Time = 6.56 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^4\,(4\,a\,e\,g+11\,c\,d\,f)}{33}\right.}{}$$

input
```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^4*(4*a*e*g + 11*c
*d*f))/33 - (32*a^5*e^5*g^3 - 462*a^2*c^3*d^3*e^2*f^3 + 396*a^3*c^2*d^2*e^
3*f^2*g - 176*a^4*c*d*e^4*f*g^2)/(1155*c^4*d^4) + (x^2*(462*c^5*d^5*f^3 -
12*a^3*c^2*d^2*e^3*g^3 + 66*a^2*c^3*d^3*e^2*f*g^2 + 1584*a*c^4*d^4*e*f^2*g
))/(1155*c^4*d^4) + (2*c*d*g^3*x^5)/11 + (2*g*x^3*(a^2*e^2*g^2 + 99*c^2*d^
2*f^2 + 110*a*c*d*e*f*g))/(231*c*d) + (2*a*e*x*(8*a^3*e^3*g^3 + 462*c^3*d^
3*f^3 + 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*c^3*d^3)))/(d
+ e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.34

$$\int \frac{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{cdx+ae}\,(105c^5d^5g^3x^5+140a\,c^4d^4e\,g^3x^4+385c^5d^5}{}$$

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 16*a**5*e**5*g**3 + 88*a**4*c*d*e**4*f*g**2 + 8*a
**4*c*d*e**4*g**3*x - 198*a**3*c**2*d**2*e**3*f**2*g - 44*a**3*c**2*d**2*e
**3*f*g**2*x - 6*a**3*c**2*d**2*e**3*g**3*x**2 + 231*a**2*c**3*d**3*e**2*f
**3 + 99*a**2*c**3*d**3*e**2*f**2*g*x + 33*a**2*c**3*d**3*e**2*f*g**2*x**2
 + 5*a**2*c**3*d**3*e**2*g**3*x**3 + 462*a*c**4*d**4*e*f**3*x + 792*a*c**4
*d**4*e*f**2*g*x**2 + 550*a*c**4*d**4*e*f*g**2*x**3 + 140*a*c**4*d**4*e*g*
*3*x**4 + 231*c**5*d**5*f**3*x**2 + 495*c**5*d**5*f**2*g*x**3 + 385*c**5*d
**5*f*g**2*x**4 + 105*c**5*d**5*g**3*x**5))/(1155*c**4*d**4)
```

3.13 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 171

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^3d^3(d+ex)^{5/2}}+\frac{4g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^3d^3(d+ex)^{7/2}}+\frac{2g^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^3d^3(d+ex)^{9/2}}$$

output
```
2/5*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/(e*x+
d)^(5/2)+4/7*g*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/
d^3/(e*x+d)^(7/2)+2/9*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^3/d^3/
(e*x+d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.53

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(8a^2e^2g^2-4acdeg(9f+5gx)\right.}{315c^3d^3(d+ex)^{5/2}}$$

input
```
Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(9*f + 5*g
*x) + c^2*d^2*(63*f^2 + 90*f*g*x + 35*g^2*x^2)))/(315*c^3*d^3*(d + e*x)^(5
/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{4(cdf-aeg)\int \frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{9cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{4(cdf-aeg)\left(\frac{1}{7}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}\right)}{9cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)}{35cd(d+ex)^{5/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}\right)}{9cd}$$

input
```
Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x]
```

output
```
(2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*c*d*(d +
e*x)^(5/2)) + (4*(c*d*f - a*e*g)*((2*(7*f - (5*d*g)/e - (2*a*e*g)/(c*d))*(
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*c*d*(d + e*x)^(5/2)) + (
2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2
))))/(9*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^2\left(35g^2x^2d^2c^2-20acde\,g^2x+90c^2d^2fgx+8a^2e^2g^2-36acdefg+63f^2c^2d^2\right)}{315\sqrt{ex+d}\,d^3c^3}$	108
gosper	$\frac{2(cdx+ae)\left(35g^2x^2d^2c^2-20acde\,g^2x+90c^2d^2fgx+8a^2e^2g^2-36acdefg+63f^2c^2d^2\right)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{315d^3c^3(ex+d)^{\frac{3}{2}}}$	116
orering	$\frac{2\left(35g^2x^2d^2c^2-20acde\,g^2x+90c^2d^2fgx+8a^2e^2g^2-36acdefg+63f^2c^2d^2\right)(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{3}{2}}}{315d^3c^3(ex+d)^{\frac{3}{2}}}$	117

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/315*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(35*c^2*d^2*
g^2*x^2-20*a*c*d*e*g^2*x+90*c^2*d^2*f*g*x+8*a^2*e^2*g^2-36*a*c*d*e*f*g+63*
c^2*d^2*f^2)/d^3/c^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = 2\left(35\,c^4d^4g^2x^4+63\,a^2c^2d^2e^2f^2-36\,a^3cde^3fg+8\,a^4e^4\right.$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="fricas")
```

output
```
2/315*(35*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 36*a^3*c*d*e^3*f*g +
8*a^4*e^4*g^2 + 10*(9*c^4*d^4*f*g + 5*a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4
*f^2 + 48*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f
^2 + 9*a^2*c^2*d^2*e^2*f*g - 2*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,(f+gx)^2}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**
(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**2/(d + e*x)**(3/2), x
)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

$$\int \frac{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\,(c^2d^2x^2+2\,acdex+a^2e^2)\sqrt{cdx+ae}f^2}{5\,cd}$$
$$+\frac{4\,(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3)\sqrt{cdx+ae}fg}{35\,c^2d^2}$$
$$+\frac{2\,(35\,c^4d^4x^4+50\,ac^3d^3ex^3+3\,a^2c^2d^2e^2x^2-4\,a^3cde^3x+8\,a^4e^4)\sqrt{cdx+ae}g^2}{315\,c^3d^3}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/
35*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*
d*x + a*e)*f*g/(c^2*d^2) + 2/315*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*e*x^3 + 3*
a^2*c^2*d^2*e^2*x^2 - 4*a^3*c*d*e^3*x + 8*a^4*e^4)*sqrt(c*d*x + a*e)*g^2/(
c^3*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(153) = 306$.

Time = 0.14 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.16

$$\int \frac{(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{105\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}af^2|e|}{cde^2} - \frac{21\left(5\left((ex+d)cde-cd^2e+ae^3\right.\right.}{}\right.}{}$$

input

```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="giac")
```

output

```
2/315*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*f^2*abs(e)/(c*d*e^2
) - 21*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(5/2))*f^2*abs(e)/(c*d*e^5) - 42*(5*((e*x + d)*c*d
*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(5/2))*a*f*g*abs(e)/(c^2*d^2*e^4) + 6*(35*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 1
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*f*g*abs(e)/(c^2*d^2*e^7) + 3*
(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*
e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(7/2))*a*g^2*abs(e)/(c^3*d^3*e^6) - (105*((e*x + d)*c*d*e - c*d^2*e + a*e^
3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 +
 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(9/2))*g^2*abs(e)/(c^3*d^3*e^9))/e
```

Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.20

$$\int \frac{(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g\,x^3\,\left(5\,a\,e\,g+9\,c\,d\,f\right)}{63}+\right.}{}$$

input

```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g*x^3*(5*a*e*g + 9*c*d*
f))/63 + (16*a^4*e^4*g^2 + 126*a^2*c^2*d^2*e^2*f^2 - 72*a^3*c*d*e^3*f*g)/(
315*c^3*d^3) + (x^2*(126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 + 288*a*c^3*d
^3*e*f*g))/(315*c^3*d^3) + (2*c*d*g^2*x^4)/9 + (4*a*e*x*(63*c^2*d^2*f^2 -
2*a^2*e^2*g^2 + 9*a*c*d*e*f*g))/(315*c^2*d^2)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.14

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\sqrt{cdx+ae}\,(35c^4d^4g^2x^4+50a\,c^3d^3e\,g^2x^3+90c^4d^4fg}{}$$

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(8*a**4*e**4*g**2 - 36*a**3*c*d*e**3*f*g - 4*a**3*c*d
*e**3*g**2*x + 63*a**2*c**2*d**2*e**2*f**2 + 18*a**2*c**2*d**2*e**2*f*g*x
+ 3*a**2*c**2*d**2*e**2*g**2*x**2 + 126*a*c**3*d**3*e*f**2*x + 144*a*c**3*
d**3*e*f*g*x**2 + 50*a*c**3*d**3*e*g**2*x**3 + 63*c**4*d**4*f**2*x**2 + 90
*c**4*d**4*f*g*x**3 + 35*c**4*d**4*g**2*x**4))/(315*c**3*d**3)
```

3.14 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 108

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^2d^2(d+ex)^{5/2}}+\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^2d^2(d+ex)^{7/2}}$$

output

```
2/5*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)
^(5/2)+2/7*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}(-2aeg+cd(7f+5gx))}{35c^2d^2(d+ex)^{5/2}}$$

input

```
Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-2*a*e*g + c*d*(7*f + 5*g*x)))/(35*c^2
*d^2*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \textcolor{blue}{1221}$$

$$\frac{1}{7}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(-\frac{2aeg}{cd}-\frac{5dg}{e}+7f\right)}{35cd(d+ex)^{5/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}$$

input
```
Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3
/2),x]
```

output
```
(2*(7*f - (5*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(35*c*d*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{2\sqrt{\left(ex+d\right)\left(cdx+ae\right)}\,\left(cdx+ae\right)^{2}\left(-5cdgx+2aeg-7dfc\right)}{35\sqrt{ex+d}\,c^{2}d^{2}}$	59
gosper	$-\frac{2\left(cdx+ae\right)\left(-5cdgx+2aeg-7dfc\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{35c^{2}d^{2}\left(ex+d\right)^{\frac{3}{2}}}$	67
orering	$-\frac{2\left(-5cdgx+2aeg-7dfc\right)\left(cdx+ae\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{35c^{2}d^{2}\left(ex+d\right)^{\frac{3}{2}}}$	68

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/35*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(-5*c*d*g*x+
2*a*e*g-7*c*d*f)/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(5\,c^3d^3gx^3+7\,a^2cde^2f-2\,a^3e^3g+\left(7\,c^3d^3f+8\,ac^2d^2\right.\right.}{3}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
 algorithm="fricas")
```

output
```
2/35*(5*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 2*a^3*e^3*g + (7*c^3*d^3*f + 8*a
*c^2*d^2*e*g)*x^2 + (14*a*c^2*d^2*e*f + a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/
2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\,(c^2d^2x^2+2\,acdex+a^2e^2)\sqrt{cdx+ae}f}{5\,cd}+\frac{2\,(5\,c^3d^3x^3+8\,ac^2d^2ex^2+a^2cde^2x-2\,a^3e^3)\sqrt{cdx+ae}g}{35\,c^2d^2}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
 algorithm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f/(c*d) + 2/35
*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*g/(c^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(96) = 192$.

Time = 0.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.69

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{35\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}af|e|}{cde^2} - \frac{7\left(5\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}a\right.}{}\right.}{}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
 algorithm="giac")
```

output
```
2/105*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*f*abs(e)/(c*d*e^2) -
 7*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(5/2))*f*abs(e)/(c*d*e^5) - 7*(5*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*
a*g*abs(e)/(c^2*d^2*e^4) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(7/2))*g*abs(e)/(c^2*d^2*e^7))/e
```

Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(x^2\left(\frac{16\,a\,e\,g}{35}+\frac{2\,c\,d\,f}{5}\right)-\right.}{\sqrt{d+e}}$$

input
```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(3
/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x^2*((16*a*e*g)/35 + (2*c*
d*f)/5) - (4*a^3*e^3*g - 14*a^2*c*d*e^2*f)/(35*c^2*d^2) + (2*c*d*g*x^3)/7
+ (2*a*e*x*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{cdx+ae}\,\left(5c^3d^3g\,x^3+8a\,c^2d^2eg\,x^2+7c^3d^3f\,x^2+a^2\right.}{35c^2d^2}$$

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 2*a**3*e**3*g + 7*a**2*c*d*e**2*f + a**2*c*d*e**2
*g*x + 14*a*c**2*d**2*e*f*x + 8*a*c**2*d**2*e*g*x**2 + 7*c**3*d**3*f*x**2
+ 5*c**3*d**3*g*x**3))/(35*c**2*d**2)
```

3.15 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

output

```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}}{5cd(d+ex)^{5/2}}$$

input

```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output

```
(2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^{2}}{5\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{5cd(ex+d)^{\frac{3}{2}}}$	50
orering	$\frac{2(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{5cd(ex+d)^{\frac{3}{2}}}$	51

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2/5*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2/c/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+}}{5\left(cdex+cd^2\right)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorit
hm="fricas")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(c^2d^2x^2+2\,acdex+a^2e^2\right)\sqrt{cdx+ae}}{5\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorit
hm="maxima")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)/(c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(42) = 84.

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{5\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}a|e|}{cde^2}-\frac{\left(5\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}ae^3-3\left((ex+d)cde\right.\right.}{cde^5}\right.}{15\,e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorit
hm="giac")
```

output
```
2/15*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*abs(e)/(c*d*e^2) - (5*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d
^2*e + a*e^3)^(5/2))*abs(e)/(c*d*e^5))/e
```

Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{\left(\frac{4\,a\,e\,x}{5}+\frac{2\,c\,d\,x^2}{5}+\frac{2\,a^2\,e^2}{5\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x)
```

output

```
(((4*a*e*x)/5 + (2*c*d*x^2)/5 + (2*a^2*e^2)/(5*c*d))*(x*(a*e^2 + c*d^2) +
a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\sqrt{cdx+ae}\,\left(c^2d^2x^2+2acdex+a^2e^2\right)}{5cd}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output

```
(2*sqrt(a*e + c*d*x)*(a**2*e**2 + 2*a*c*d*e*x + c**2*d**2*x**2))/(5*c*d)
```

3.16 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 179

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = -\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}+\frac{2(cdf-aeg)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{5/2}}$$

output

```
-2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2
)+2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+2*(-a*e*g+c*
d*f)^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.74

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(4aeg+cd(-3f+gx))+3(\right.}{3g^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(4*a*e*g + c
*d*(-3*f + g*x)) + 3*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*
x])/Sqrt[c*d*f - a*e*g]]))/(3*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1250, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{g}$$

$$\Big\downarrow 1255$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}\right)}{g}$$

$$\Big\downarrow 218$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{g}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)),x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) -
((c*d*f - a*e*g)*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[
d + e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)))/g
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.41

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)a^{2}e^{2}g^{2}-6\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)acdefg+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{2}d^{2}f^{2}-\right.}{3\sqrt{ex+d}\,\sqrt{cdx+ae}\,g^{2}\sqrt{(aeg-dfc)g}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,method
=_RETURNVERBOSE)
```

output
```
-2/3*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*a^2*e^2*g^2-6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*a*c*d*e*f*g+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
*c^2*d^2*f^2-c*d*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-4*((a*e*g-c
*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*
e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/((a*e*g-c*d*f)*g)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.46

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=\left[-\frac{3\left(cd^2f-adeg+\left(cdef-ae^2g\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd}{}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
 algorithm="fricas")
```

output
```
[-1/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-(c*d*f - a*e*g)
/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c
*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 3*c*d*f + 4*a*e*g)*sqrt(e*x + d))/
(e*g^2*x + d*g^2), -2/3*(3*(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqr
t((c*d*f - a*e*g)/g)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(e*x + d)*g*sqrt((c*d*f - a*e*g)/g)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e
^2*g)*x)) - sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 3*c*d*f
 + 4*a*e*g)*sqrt(e*x + d))/(e*g^2*x + d*g^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=\int \frac{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
 algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.21

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx=\frac{2\left(c^2d^2f^2|e|-2\,acdefg|e|+a^2e^2g^2|e|\right)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae}}{\sqrt{cdfg-aeg^2}e}\right.}{\sqrt{cdfg-aeg^2}eg^2}-\frac{2\left(3\,\sqrt{(ex+d)cde-cd^2e+ae^3}cde^{10}fg|e|-3\,\sqrt{(ex+d)cde-cd^2e+ae^3}ae^{11}g^2|e|-((ex+d)cde-cd\right.}{3\,e^{12}g^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
 algorithm="giac")
```

output
```
2*(c^2*d^2*f^2*abs(e) - 2*a*c*d*e*f*g*abs(e) + a^2*e^2*g^2*abs(e))*arctan(
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sq
rt(c*d*f*g - a*e*g^2)*e*g^2) - 2/3*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e
^3)*c*d*e^10*f*g*abs(e) - 3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^11
*g^2*abs(e) - ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^8*g^2*abs(e))/(e
^12*g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{\left(f+g\,x\right)\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/
2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.82

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{-2\sqrt{g}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)aeg+2\sqrt{g}\,\sqrt{-aeg+cd}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x)
```

output
```
(2*( - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g
)*sqrt( - a*e*g + c*d*f)))*a*e*g + 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((
sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*d*f + 4*sqrt(a*e
+ c*d*x)*a*e*g**2 - 3*sqrt(a*e + c*d*x)*c*d*f*g + sqrt(a*e + c*d*x)*c*d*g*
*2*x))/(3*g**3)
```

3.17 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 178

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx &= \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} \\ &- \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)} \\ &- \frac{3cd\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{5/2}}\end{aligned}$$

output

```
3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)-(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)-3*c*d*(-a*e*g+c*d*f)
^(1/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-aeg+cd(3f+2gx))-3c\right.}{g^{5/2}\sqrt{(ae+cdx)(d+ex)}(f}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^2),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-(a*e*g) + c*
d*(3*f + 2*g*x)) - 3*c*d*Sqrt[c*d*f - a*e*g]*(f + g*x)*ArcTan[(Sqrt[g]*Sqr
t[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*
x)]*(f + g*x))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1249, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}dx$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{3cd\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

$$\Big\downarrow \textbf{1255}$$

$$\frac{3cd\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{3cd\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^2),x]
```

output
```
-((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f + g*
x))) + (3*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d +
 e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)))/(2*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.66

method	result
default	$\frac{\left(-3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)acde\,g^{2}x+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{2}d^{2}fgx-3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)acdefg+3\,\operatorname{arctanh}\left(\right.\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g^{2}(}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x,meth
od=_RETURNVERBOSE)
```

output

```
(-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*g^2*x+3*a
rctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g*x-3*arctan
h(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*f*g+3*arctanh(g*(c*
d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f^2+2*c*d*g*x*(c*d*x+a*e)^
(1/2)*((a*e*g-c*d*f)*g)^(1/2)-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*
e*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)*((e*x+d)*(c*d*x+a*e
))^(1/2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.67

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx=\left[\frac{3\left(cdegx^2+cd^2f+\left(cdef+cd^2g\right)x\right)\sqrt{-\frac{cdf-aeg}{g}}\log\left(-\frac{cdegx^2-cd}{}\right.}{}\right.$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,
x, algorithm="fricas")
```

output

```
[1/2*(3*(c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-(c*d*f - a*e
*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d
))/(e*g^3*x^2 + d*f*g^2 + (e*f*g^2 + d*g^3)*x), (3*(c*d*e*g*x^2 + c*d^2*f
+ (c*d*e*f + c*d^2*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(-sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt((c*d*f - a*e*g)/g)/(c*d^2*
f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e*g)*sqrt(e*x + d))/(e*g^3*x^2 + d*f*g^
2 + (e*f*g^2 + d*g^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx &= \frac{2\sqrt{(ex+d)cde-cd^2e+ae^3}cd|e|}{e^2g^2}\\ &\quad - \frac{3\,(c^2d^2f|e|-acdeg|e|)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}eg^2}\\ &\quad + \frac{\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2f|e|-\sqrt{(ex+d)cde-cd^2e+ae^3}acdeg|e|}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)g^2}\end{aligned}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,
x, algorithm="giac")
```

output
```
2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*abs(e)/(e^2*g^2) - 3*(c^2*d^
2*f*abs(e) - a*c*d*e*g*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e
^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^2) + (sqrt
((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*f*abs(e) - sqrt((e*x + d)*c*d*
e - c*d^2*e + a*e^3)*a*c*d*e*g*abs(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)*
c*d*e - c*d^2*e + a*e^3)*g)*g^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(
3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(
3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.87

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx=\frac{-3\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)cdf-3\sqrt{g}\,\sqrt{-aeg+cd}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x)
```

output

```
( - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*s
qrt( - a*e*g + c*d*f)))*c*d*f - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqr
t(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*d*g*x - sqrt(a*e + c
*d*x)*a*e*g**2 + 3*sqrt(a*e + c*d*x)*c*d*f*g + 2*sqrt(a*e + c*d*x)*c*d*g**
2*x)/(g**3*(f + g*x))
```

3.18 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 195

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx &= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)} \\ &\quad -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{3c^2d^2\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}\end{aligned}$$

output
```
-3/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)/(g*x+f)
-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^2+3/4
*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\Bigg(-\frac{\sqrt{g}(2aeg+cd(3f+5gx))}{(f+gx)^2}+\frac{3c^2d^2\arctan\Big(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}}{\sqrt{cdf-aeg}\sqrt{ae+cdx}}}{4g^{5/2}\sqrt{d+ex}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^3),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(2*a*e*g + c*d*(3*f + 5*g*x)))/
(f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f -
a*e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d + e*x
])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1249, 1249, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^2}dx}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

$$\Big\downarrow \text{1249}$$

$$\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

$$\Big\downarrow \text{1255}$$

$$\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^3),x]
```

output

```
-1/2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f +
 g*x)^2) + (3*c*d*(-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d
 + e*x]*(f + g*x))) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f -
a*e*g])))/(4*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{2}d^{2}g^{2}x^{2}+6\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{2}d^{2}fgx+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{2}d^{2}f\right.}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,g^{2}(gx+f)^{2}\sqrt{(aeg-}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/4*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^2*d^2*g^2*x^2+6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^2*d^2*f*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2))*c^2*d^2*f^2+5*c*d*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+2*(
(a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1/2)*(
c*d*x+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)^2/((a*
e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(169) = 338$.

Time $= 0.11$ (sec) , antiderivative size $= 841$, normalized size of antiderivative $= 4.31$

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{3/2}(f + gx)^3}\,dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,
x, algorithm="fricas")
```

output

```
[-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2
)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(
c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(
e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*f^2*g - a*c*d*e*
f*g^2 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^3*g^3 - a*d*e*f^2*g^4
 + (c*d*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 - a*d*e*g^6 + (c*d^2 -
 2*a*e^2)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5 + (2*c*d^2 - a*e^2)*
f^2*g^4)*x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g +
 c^2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(c*d*f*g - a*e*
g^2)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*
e*g^2)*sqrt(e*x + d)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) + (3*c^2
*d^2*f^2*g - a*c*d*e*f*g^2 - 2*a^2*e^2*g^3 + 5*(c^2*d^2*f*g^2 - a*c*d*e*g^
3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^
3*g^3 - a*d*e*f^2*g^4 + (c*d*e*f*g^5 - a*e^2*g^6)*x^3 + (2*c*d*e*f^2*g^4 -
 a*d*e*g^6 + (c*d^2 - 2*a*e^2)*f*g^5)*x^2 + (c*d*e*f^3*g^3 - 2*a*d*e*f*g^5
 + (2*c*d^2 - a*e^2)*f^2*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{3/2}(f + gx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.21

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx=\frac{3\,c^2d^2|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{4\sqrt{cdfg-aeg^2}eg^2}$$
$$-\frac{3\sqrt{(ex+d)cde-cd^2e+ae^3}c^3d^3e^2f|e|-3\sqrt{(ex+d)cde-cd^2e+ae^3}ac^2d^2e^3g|e|+5\,((ex+d)cde-c}{4\left(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g\right)^2g^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,
x, algorithm="giac")
```

output
```
3/4*c^2*d^2*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^2) - 1/4*(3*sqrt((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)*c^3*d^3*e^2*f*abs(e) - 3*sqrt((e*x + d)*c*d*
e - c*d^2*e + a*e^3)*a*c^2*d^2*e^3*g*abs(e) + 5*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*c^2*d^2*g*abs(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)*g)^2*g^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^3}\,dx = \int \frac{{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)}^{3/2}}{{\left(f + g\,x\right)}^3\,{\left(d + e\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(
3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(
3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.66

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^3}\,dx = \frac{-3\sqrt{g}\,\sqrt{-aeg + cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right) c^2 d^2 f^2 - 6\sqrt{g}\,\sqrt{-aeg +}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x)
```

output

```
( - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*s
qrt( - a*e*g + c*d*f)))*c**2*d**2*f**2 - 6*sqrt(g)*sqrt( - a*e*g + c*d*f)*
atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*f*g
*x - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*c**2*d**2*g**2*x**2 - 2*sqrt(a*e + c*d*x)*a**2*e*
*2*g**3 - sqrt(a*e + c*d*x)*a*c*d*e*f*g**2 - 5*sqrt(a*e + c*d*x)*a*c*d*e*g
**3*x + 3*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g + 5*sqrt(a*e + c*d*x)*c**2*d*
*2*f*g**2*x)/(4*g**3*(a*e*f**2*g + 2*a*e*f*g**2*x + a*e*g**3*x**2 - c*d*f*
*3 - 2*c*d*f**2*g*x - c*d*f*g**2*x**2))
```

3.19 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 265

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx = -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}$$

output

```
-1/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)/(g*x+f)
^2+1/8*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/
(e*x+d)^(1/2)/(g*x+f)-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d
)^(3/2)/(g*x+f)^3+1/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(3/2
)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.76

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}\sqrt{ae+cdx}(8a^2e^2g^2-2acde}{24g^{5/2}(cdf-aeg}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^4),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x]*(8*a^2*e^2*g^2 - 2*a*c*d*e*g*(f - 7*g*x) + c^2*d^2*(-3*f^2 - 8*f*g*x
 + 3*g^2*x^2)) + 3*c^3*d^3*(f + g*x)^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/
Sqrt[c*d*f - a*e*g]]))/(24*g^(5/2)*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x
)*(d + e*x)]*(f + g*x)^3)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1249, 1249, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^3}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$\Big\downarrow \text{1254}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$\Big\downarrow \text{1255}$$

$$\frac{cd\left(\frac{cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cd}}{2g\sqrt{d+ex}(f+gx)^2}\right.}{2g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^4),x]
```

output

```
-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f +
 g*x)^3) + (c*d*(-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[
d + e*x]*(f + g*x)^2) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/
((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/
(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*g)))/(2*g)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.67

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(3 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^3d^3g^3x^3+9 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^3d^3f\,g^2x^2+9 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^3d^3f\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x,meth
od=_RETURNVERBOSE)
```

output
```
1/24*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f*g^2*x^2+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^3*f^2*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^3*d^3*f^3-3*c^2*d^2*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)-14*a*c*d*e*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+8*c^2*d^2
*f*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*((a*e*g-c*d*f)*g)^(1/2)
*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)
*a*c*d*e*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(e*x
+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g^2/(g*x+f)^3/((a*e*g-c*d*f)*g)^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(233) = 466$.

Time = 0.19 (sec) , antiderivative size = 1435, normalized size of antiderivative = 5.42

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,
x, algorithm="fricas")
```

output
```
[1/48*(3*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g
^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3
*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*
d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (
e*f + d*g)*x)) - 2*(3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*e^2
*f*g^3 + 8*a^3*e^3*g^4 - 3*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*(4*c^
3*d^3*f^2*g^2 - 11*a*c^2*d^2*e*f*g^3 + 7*a^2*c*d*e^2*g^4)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^5*g^3 - 2*a*c*d^2
*e*f^4*g^4 + a^2*d*e^2*f^3*g^5 + (c^2*d^2*e*f^2*g^6 - 2*a*c*d*e^2*f*g^7 +
a^2*e^3*g^8)*x^4 + (3*c^2*d^2*e*f^3*g^5 + a^2*d*e^2*g^8 + (c^2*d^3 - 6*a*c
*d*e^2)*f^2*g^6 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g^7)*x^3 + 3*(c^2*d^2*e*f^4*
g^4 + a^2*d*e^2*f*g^7 + (c^2*d^3 - 2*a*c*d*e^2)*f^3*g^5 - (2*a*c*d^2*e - a
^2*e^3)*f^2*g^6)*x^2 + (c^2*d^2*e*f^5*g^3 + 3*a^2*d*e^2*f^2*g^6 + (3*c^2*d
^3 - 2*a*c*d*e^2)*f^4*g^4 - (6*a*c*d^2*e - a^2*e^3)*f^3*g^5)*x), -1/24*(3*
(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*g^3)*x^3 +
 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*
g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d^2*f - a*d*e*g + (c*d*e*
f - a*e^2*g)*x)) + (3*c^3*d^3*f^3*g - a*c^2*d^2*e*f^2*g^2 - 10*a^2*c*d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^4), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.48

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=\frac{c^3d^3|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{8\left(cdfg^2-aeg^3\right)\sqrt{cdfg-aeg^2}e}$$
$$-\frac{3\sqrt{(ex+d)cde-cd^2e+ae^3}c^5d^5e^4f^2|e|-6\sqrt{(ex+d)cde-cd^2e+ae^3}ac^4d^4e^5fg|e|+3\sqrt{(ex+d)cde}{24\,(cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,
x, algorithm="giac")
```

output
```
1/8*c^3*d^3*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/((c*d*f*g^2 - a*e*g^3)*sqrt(c*d*f*g - a*e*g^2)*e) -
 1/24*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^5*d^5*e^4*f^2*abs(e) -
6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^4*d^4*e^5*f*g*abs(e) + 3*sqr
t((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^3*d^3*e^6*g^2*abs(e) + 8*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^4*e^2*f*g*abs(e) - 8*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^3*d^3*e^3*g^2*abs(e) - 3*((e*x + d)*c*d
*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^3*g^2*abs(e))/((c*d*e^2*f - a*e^3*g + ((
e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^3*(c*d*f*g^2 - a*e*g^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(f+g\,x\right)}^4\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(
3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(
3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.27

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx = \frac{3\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^3d^3f^3+9\sqrt{g}\,\sqrt{-aeg+c}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x)
```

output

```
(3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt
( - a*e*g + c*d*f)))*c**3*d**3*f**3 + 9*sqrt(g)*sqrt( - a*e*g + c*d*f)*ata
n((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d**3*f**2*g
*x + 9*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*c**3*d**3*f*g**2*x**2 + 3*sqrt(g)*sqrt( - a*e*g +
 c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*
d**3*g**3*x**3 - 8*sqrt(a*e + c*d*x)*a**3*e**3*g**4 + 10*sqrt(a*e + c*d*x)
*a**2*c*d*e**2*f*g**3 - 14*sqrt(a*e + c*d*x)*a**2*c*d*e**2*g**4*x + sqrt(a
*e + c*d*x)*a*c**2*d**2*e*f**2*g**2 + 22*sqrt(a*e + c*d*x)*a*c**2*d**2*e*f
*g**3*x - 3*sqrt(a*e + c*d*x)*a*c**2*d**2*e*g**4*x**2 - 3*sqrt(a*e + c*d*x
)*c**3*d**3*f**3*g - 8*sqrt(a*e + c*d*x)*c**3*d**3*f**2*g**2*x + 3*sqrt(a*
e + c*d*x)*c**3*d**3*f*g**3*x**2)/(24*g**3*(a**2*e**2*f**3*g**2 + 3*a**2*e
**2*f**2*g**3*x + 3*a**2*e**2*f*g**4*x**2 + a**2*e**2*g**5*x**3 - 2*a*c*d*
e*f**4*g - 6*a*c*d*e*f**3*g**2*x - 6*a*c*d*e*f**2*g**3*x**2 - 2*a*c*d*e*f*
g**4*x**3 + c**2*d**2*f**5 + 3*c**2*d**2*f**4*g*x + 3*c**2*d**2*f**3*g**2*
x**2 + c**2*d**2*f**2*g**3*x**3))
```

3.20 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 335

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx &= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3} \\ &+ \frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^2} + \frac{3c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)} \\ &- \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4} + \frac{3c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}\end{aligned}$$

output
```
-1/8*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)/(g*x+f)
^3+1/32*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)
/(e*x+d)^(1/2)/(g*x+f)^2+3/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/g^2/(-a*e*g+c*d*f)^2/(e*x+d)^(1/2)/(g*x+f)-1/4*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^4+3/64*c^4*d^4*arctan(g^(1/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g
^(5/2)/(-a*e*g+c*d*f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.72

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\frac{c^4d^4((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{g}(-16a^3e^3g^3+24a^2cde^2g^2(f-gx)-2ac^2d^2eg}{c^4d^4(cdf-}\right.}{64g^{5/2}}$$

input

```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^5),x]
```

output

```
(c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[g]*(-16*a^3*e^3*g^3 + 24*a
^2*c*d*e^2*g^2*(f - g*x) - 2*a*c^2*d^2*e*g*(f^2 - 22*f*g*x + g^2*x^2) + c^
3*d^3*(-3*f^3 - 11*f^2*g*x + 11*f*g^2*x^2 + 3*g^3*x^3)))/(c^4*d^4*(c*d*f -
 a*e*g)^2*(a*e + c*d*x)*(f + g*x)^4) + (3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x
])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(5/2)*(a*e + c*d*x)^(3/2))))/(64
*g^(5/2)*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1249, 1249, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^4}dx}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow \text{1249}$$

$$\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow \text{1254}$$

$$\frac{3cd\left(\frac{cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow \text{1254}$$

$$\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex}}{3g\sqrt{d+ex}(f+gx)^3}\right.}{8g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow \text{1255}$$

$$\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{2\sqrt{d+ex}(f+gx)^2(cdf-}}{6g}\right.}{8g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$\Big\downarrow \text{218}$$

$$\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cde}}{3g\sqrt{d+ex}(f+gx)^3}\right.}{8g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^5),x]
```

output

```
-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f +
 g*x)^4) + (3*c*d*(-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqr
t[d + e*x]*(f + g*x)^3) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c
*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*
f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f -
a*e*g))))/(6*g)))/(8*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(297) = 594.

Time = 2.75 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.96

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{4}d^{4}g^{4}x^{4}+12\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{4}d^{4}f\,g^{3}x^{3}+18\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/64*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+12*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+12*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^4*d^4*f^3*g*x-3*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*
g-c*d*f)*g)^(1/2)+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c
^4*d^4*f^4+2*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)
-11*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+24*a^2*c*d
*e^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-44*a*c^2*d^2*e*f*g^2*
x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+11*c^3*d^3*f^2*g*x*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+16*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2
)*a^3*e^3*g^3-24*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^2*f*g
^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+3*((a*e*g
-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)/(g*x+f)^4/g^2/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(297) = 594$.

Time = 0.57 (sec) , antiderivative size = 2239, normalized size of antiderivative = 6.68

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,
x, algorithm="fricas")
```

output
```
[-1/128*(3*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5
*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e
*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqr
t(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2
*(3*c^4*d^4*f^4*g - a*c^3*d^3*e*f^3*g^2 - 26*a^2*c^2*d^2*e^2*f^2*g^3 + 40*
a^3*c*d*e^3*f*g^4 - 16*a^4*e^4*g^5 - 3*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5)*x
^3 - (11*c^4*d^4*f^2*g^3 - 13*a*c^3*d^3*e*f*g^4 + 2*a^2*c^2*d^2*e^2*g^5)*x
^2 + (11*c^4*d^4*f^3*g^2 - 55*a*c^3*d^3*e*f^2*g^3 + 68*a^2*c^2*d^2*e^2*f*g
^4 - 24*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d))/(c^3*d^4*f^7*g^3 - 3*a*c^2*d^3*e*f^6*g^4 + 3*a^2*c*d^2*e^2*f^
5*g^5 - a^3*d*e^3*f^4*g^6 + (c^3*d^3*e*f^3*g^7 - 3*a*c^2*d^2*e^2*f^2*g^8 +
 3*a^2*c*d*e^3*f*g^9 - a^3*e^4*g^10)*x^5 + (4*c^3*d^3*e*f^4*g^6 - a^3*d*e^
3*g^10 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^7 - 3*(a*c^2*d^3*e - 4*a^2*c*d
*e^3)*f^2*g^8 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^9)*x^4 + 2*(3*c^3*d^3*e*
f^5*g^5 - 2*a^3*d*e^3*f*g^9 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g^6 - 3*(2
*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^7 + 3*(2*a^2*c*d^2*e^2 - a^3*e^4)*f^2*
g^8)*x^3 + 2*(2*c^3*d^3*e*f^6*g^4 - 3*a^3*d*e^3*f^2*g^8 + 3*(c^3*d^4 - 2*a
*c^2*d^2*e^2)*f^5*g^5 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^6 + (9*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(297) = 594$.

Time = 0.18 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.80

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\frac{3\,c^4d^4|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2e}}\right)}{64\left(c^2d^2f^2g^2-2\,acdefg^3+a^2e^2g^4\right)\sqrt{cdfg-aeg^2e}}$$

$$-\,\frac{3\sqrt{(ex+d)cde-cd^2e+ae^3}c^7d^7e^6f^3|e|-9\sqrt{(ex+d)cde-cd^2e+ae^3}ac^6d^6e^7f^2g|e|+9\sqrt{(ex+d)cd}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,
x, algorithm="giac")
```

output

```
3/64*c^4*d^4*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt
(c*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g^2 - 2*a*c*d*e*f*g^3 + a^2*e^2*g^4)
*sqrt(c*d*f*g - a*e*g^2)*e) - 1/64*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e
^3)*c^7*d^7*e^6*f^3*abs(e) - 9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c
^6*d^6*e^7*f^2*g*abs(e) + 9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^
5*d^5*e^8*f*g^2*abs(e) - 3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^4
*d^4*e^9*g^3*abs(e) + 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^6*d^6
*e^4*f^2*g*abs(e) - 22*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^5*d^5
*e^5*f*g^2*abs(e) + 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^4*d
^4*e^6*g^3*abs(e) - 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^5*d^5*e
^2*f*g^2*abs(e) + 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^4*d^4*e
^3*g^3*abs(e) - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^4*d^4*g^3*ab
s(e))/((c^2*d^2*f^2*g^2 - 2*a*c*d*e*f*g^3 + a^2*e^2*g^4)*(c*d*e^2*f - a*e^
3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)^5\,\left(d+e\,x\right)^{3/2}}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(
3/2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(
3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 962, normalized size of antiderivative = 2.87

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx=\text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x)
```

output

```
( - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*s
qrt( - a*e*g + c*d*f)))*c**4*d**4*f**4 - 12*sqrt(g)*sqrt( - a*e*g + c*d*f)
*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**4*d**4*f*
*3*g*x - 18*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqr
t(g)*sqrt( - a*e*g + c*d*f)))*c**4*d**4*f**2*g**2*x**2 - 12*sqrt(g)*sqrt(
- a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f
)))*c**4*d**4*f*g**3*x**3 - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*
e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**4*d**4*g**4*x**4 - 16*s
qrt(a*e + c*d*x)*a**4*e**4*g**5 + 40*sqrt(a*e + c*d*x)*a**3*c*d*e**3*f*g**
4 - 24*sqrt(a*e + c*d*x)*a**3*c*d*e**3*g**5*x - 26*sqrt(a*e + c*d*x)*a**2*
c**2*d**2*e**2*f**2*g**3 + 68*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*f*g**4
*x - 2*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*g**5*x**2 - sqrt(a*e + c*d*x)
*a*c**3*d**3*e*f**3*g**2 - 55*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f**2*g**3*x
+ 13*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f*g**4*x**2 + 3*sqrt(a*e + c*d*x)*a*c
**3*d**3*e*g**5*x**3 + 3*sqrt(a*e + c*d*x)*c**4*d**4*f**4*g + 11*sqrt(a*e
+ c*d*x)*c**4*d**4*f**3*g**2*x - 11*sqrt(a*e + c*d*x)*c**4*d**4*f**2*g**3*
x**2 - 3*sqrt(a*e + c*d*x)*c**4*d**4*f*g**4*x**3)/(64*g**3*(a**3*e**3*f**4
*g**3 + 4*a**3*e**3*f**3*g**4*x + 6*a**3*e**3*f**2*g**5*x**2 + 4*a**3*e**3
*f*g**6*x**3 + a**3*e**3*g**7*x**4 - 3*a**2*c*d*e**2*f**5*g**2 - 12*a**2*c
*d*e**2*f**4*g**3*x - 18*a**2*c*d*e**2*f**3*g**4*x**2 - 12*a**2*c*d*e**...
```

3.21 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 405

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx &= -\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}(f+gx)^3} \\ &\quad+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^2(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2}+\frac{3c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128g^2(cdf-aeg)^3\sqrt{d+ex}(f+gx)} \\ &\quad-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{3c^5d^5\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}}\end{aligned}$$

output
```
-3/40*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)/(g*x+f
)^4+1/80*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f
)/(e*x+d)^(1/2)/(g*x+f)^3+1/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/g^2/(-a*e*g+c*d*f)^2/(e*x+d)^(1/2)/(g*x+f)^2+3/128*c^4*d^4*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)-1
/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^5+3/128
*c^5*d^5*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(7/2)
```

Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.75

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx=\frac{c^5d^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{g}(128a^4e^4g^4+16a^3cde^3g^3(-21f+11gx)+8a}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^6),x]
```

output
```
(c^5*d^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[g]*(128*a^4*e^4*g^4 + 16*a
^3*c*d*e^3*g^3*(-21*f + 11*g*x) + 8*a^2*c^2*d^2*e^2*g^2*(31*f^2 - 64*f*g*x
 + g^2*x^2) - 2*a*c^3*d^3*e*g*(5*f^3 - 233*f^2*g*x + 23*f*g^2*x^2 + 5*g^3*
x^3) + c^4*d^4*(-15*f^4 - 70*f^3*g*x + 128*f^2*g^2*x^2 + 70*f*g^3*x^3 + 15
*g^4*x^4)))/(c^5*d^5*(c*d*f - a*e*g)^3*(a*e + c*d*x)*(f + g*x)^5) + (15*Ar
cTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(7
/2)*(a*e + c*d*x)^(3/2))))/(640*g^(5/2)*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1249, 1249, 1254, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^5}dx}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$$\Big\downarrow \text{1249}$$

$$\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}\right)}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$$\Big\downarrow \; 1254$$

$$\frac{3cd\left(\frac{cd\left(\frac{5cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}\right)}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$$\Big\downarrow \; 1254$$

$$\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cd}}{4g\sqrt{d+ex}(f+gx)^4}\right.}{10g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$$\Big\downarrow \; 1254$$

$$3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{3\sqrt{d+ex}(f+gx)^3(cdf-ae}\right.}{8g}\right.}{10g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$$\Big\downarrow \text{1255}$$

$$3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+c}}{2\sqrt{d+ex}(f+gx)^2}\right.}{6(cdf-aeg)}\right.}{8g}\right.}{10g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

$$\Big\downarrow \; 218$$

$$\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}{3\sqrt{d+ex}(f+gx)^3(cdf-a}\right.}{8g}\right.}{10g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^6),x]
```

output
```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f +
 g*x)^5) + (3*c*d*(-1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqr
t[d + e*x]*(f + g*x)^4) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c*d*(Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2)
+ (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqr
t[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*
e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(6*(c*d*f - a*e*g))))/(8*g)))/(10*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(361) = 722$.

Time = 2.82 (sec) , antiderivative size = 945, normalized size of antiderivative = 2.33

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\ \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^5d^5g^5x^5+75\ \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^5d^5f\,g^4x^4+150\ \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x,meth
od=_RETURNVERBOSE)
```

output
```
1/640*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^5*d^5*g^5*x^5+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^5*d^5*f*g^4*x^4+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^5*d^5*f^2*g^3*x^3+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*
g-c*d*f)*g)^(1/2))*c^5*d^5*f^3*g^2*x^2-15*c^4*d^4*g^4*x^4*(c*d*x+a*e)^(1/2
)*((a*e*g-c*d*f)*g)^(1/2)+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^5*d^5*f^4*g*x+10*a*c^3*d^3*e*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)-70*c^4*d^4*f*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1
/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^5-8*
a^2*c^2*d^2*e^2*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+46*a*c^3
*d^3*e*f*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-128*c^4*d^4*f^2
*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-176*a^3*c*d*e^3*g^4*x*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+512*a^2*c^2*d^2*e^2*f*g^3*x*(c*d*
x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-466*a*c^3*d^3*e*f^2*g^2*x*(c*d*x+a*e)
^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+70*c^4*d^4*f^3*g*x*(c*d*x+a*e)^(1/2)*((a*e*
g-c*d*f)*g)^(1/2)-128*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^4*e^4*g^
4+336*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*c*d*e^3*f*g^3-248*(c*d
*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^3*d^3*e*f^3*g+15*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1582 vs. $2(361) = 722$.

Time = 1.88 (sec) , antiderivative size = 3205, normalized size of antiderivative = 7.91

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{3/2}(f + gx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,
x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{3/2}(f + gx)^6}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{(d + ex)^{3/2}(f + gx)^6}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^6}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,
x, algorithm="maxima")
```

output

```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(361) = 722$.

Time = 0.20 (sec) , antiderivative size = 866, normalized size of antiderivative = 2.14

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^6}\, dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,
x, algorithm="giac")
```

output

```
3/128*c^5*d^5*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*f^3*g^2 - 3*a*c^2*d^2*e*f^2*g^3 + 3*a^2
*c*d*e^2*f*g^4 - a^3*e^3*g^5)*sqrt(c*d*f*g - a*e*g^2)*e) - 1/640*(15*sqrt(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^9*d^9*e^8*f^4*abs(e) - 60*sqrt((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)*a*c^8*d^8*e^9*f^3*g*abs(e) + 90*sqrt((e*x + d
)*c*d*e - c*d^2*e + a*e^3)*a^2*c^7*d^7*e^10*f^2*g^2*abs(e) - 60*sqrt((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^6*d^6*e^11*f*g^3*abs(e) + 15*sqrt((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)*a^4*c^5*d^5*e^12*g^4*abs(e) + 70*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^8*d^8*e^6*f^3*g*abs(e) - 210*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^7*d^7*e^7*f^2*g^2*abs(e) + 210*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^6*d^6*e^8*f*g^3*abs(e) - 70*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*c^5*d^5*e^9*g^4*abs(e) - 128*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^7*d^7*e^4*f^2*g^2*abs(e) + 256*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^6*d^6*e^5*f*g^3*abs(e) - 128*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*c^5*d^5*e^6*g^4*abs(e) - 70*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^6*d^6*e^2*f*g^3*abs(e) + 70*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*c^5*d^5*e^3*g^4*abs(e) - 15*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*c^5*d^5*g^4*abs(e))/((c^3*d^3*f^3*g^2 -
 3*a*c^2*d^2*e*f^2*g^3 + 3*a^2*c*d*e^2*f*g^4 - a^3*e^3*g^5)*(c*d*e^2*f - a
*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^6\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(
3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(
3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1396, normalized size of antiderivative = 3.45

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x)
```

output

```
(15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqr
t( - a*e*g + c*d*f)))*c**5*d**5*f**5 + 75*sqrt(g)*sqrt( - a*e*g + c*d*f)*a
tan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**5*d**5*f**4
*g*x + 150*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt
(g)*sqrt( - a*e*g + c*d*f)))*c**5*d**5*f**3*g**2*x**2 + 150*sqrt(g)*sqrt(
- a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f
)))*c**5*d**5*f**2*g**3*x**3 + 75*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqr
t(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**5*d**5*f*g**4*x**4
+ 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sq
rt( - a*e*g + c*d*f)))*c**5*d**5*g**5*x**5 - 128*sqrt(a*e + c*d*x)*a**5*e*
*5*g**6 + 464*sqrt(a*e + c*d*x)*a**4*c*d*e**4*f*g**5 - 176*sqrt(a*e + c*d*
x)*a**4*c*d*e**4*g**6*x - 584*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**3*f**2*g
**4 + 688*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**3*f*g**5*x - 8*sqrt(a*e + c*
d*x)*a**3*c**2*d**2*e**3*g**6*x**2 + 258*sqrt(a*e + c*d*x)*a**2*c**3*d**3*
e**2*f**3*g**3 - 978*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*f**2*g**4*x + 5
4*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*f*g**5*x**2 + 10*sqrt(a*e + c*d*x)
*a**2*c**3*d**3*e**2*g**6*x**3 + 5*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f**4*g*
*2 + 536*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f**3*g**3*x - 174*sqrt(a*e + c*d*
x)*a*c**4*d**4*e*f**2*g**4*x**2 - 80*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f*g**
5*x**3 - 15*sqrt(a*e + c*d*x)*a*c**4*d**4*e*g**6*x**4 - 15*sqrt(a*e + c...
```

3.22 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 297

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(cdf-aeg)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^5d^5(d+ex)^{7/2}}$$
$$+\frac{8g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^5d^5(d+ex)^{9/2}}$$
$$+\frac{12g^2(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^5d^5(d+ex)^{11/2}}$$
$$+\frac{8g^3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{13/2}}{13c^5d^5(d+ex)^{13/2}}$$
$$+\frac{2g^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{15/2}}{15c^5d^5(d+ex)^{15/2}}$$

output

```
2/7*(-a*e*g+c*d*f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^5/d^5/(e*x+
d)^(7/2)+8/9*g*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^
5/d^5/(e*x+d)^(9/2)+12/11*g^2*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(11/2)/c^5/d^5/(e*x+d)^(11/2)+8/13*g^3*(-a*e*g+c*d*f)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(13/2)/c^5/d^5/(e*x+d)^(13/2)+2/15*g^4*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(15/2)/c^5/d^5/(e*x+d)^(15/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.69

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(128a^4e^4g^4-64a^3c$$

input
```
Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
 e*x)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^4*g^4 - 64*a^3
*c*d*e^3*g^3*(15*f + 7*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(65*f^2 + 70*f*g*x +
21*g^2*x^2) - 8*a*c^3*d^3*e*g*(715*f^3 + 1365*f^2*g*x + 945*f*g^2*x^2 + 23
1*g^3*x^3) + c^4*d^4*(6435*f^4 + 20020*f^3*g*x + 24570*f^2*g^2*x^2 + 13860
*f*g^3*x^3 + 3003*g^4*x^4)))/(45045*c^5*d^5*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1253, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1253}$$

$$\frac{8(cdf-aeg)\int \frac{(f+gx)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{15cd}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{ 1253}$$

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\int \frac{(f+gx)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{13cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}\right)}{15cd}+$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int \frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{11cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}\right)}{13cd}+\frac{2(f+gx)^3(x(}{1}\right.}{15cd}$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{9}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^2}{}\right.}{13cd}\right.}{15cd}$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}+$$

$$\frac{8(cdf-aeg)\left(\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}+\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)\right.}{}\right.}{}\right.}{13cd}\right.}{15cd}$$

input
```
Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x]
```

output
```
(2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(15*c*d*(d +
 e*x)^(7/2)) + (8*(c*d*f - a*e*g)*((2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(13*c*d*(d + e*x)^(7/2)) + (6*(c*d*f - a*e*g)*((2*(
f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*c*d*(d + e*x
)^(7/2)) + (4*(c*d*f - a*e*g)*((2*(9*f - (7*d*g)/e - (2*a*e*g)/(c*d))*(a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*c*d*(d + e*x)^(7/2)) + (2*g
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2)))
)/(11*c*d)))/(13*c*d)))/(15*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.93

method	result
default	$2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^3\left(3003g^4x^4d^4c^4-1848a\,c^3d^3e\,g^4x^3+13860c^4d^4f\,g^3x^3+1008a^2c^2d^2e^2g^4x^2-7560a\,c^3d^3ef\,g^3x^2+24\right.$
gosper	$2(cdx+ae)\left(3003g^4x^4d^4c^4-1848a\,c^3d^3e\,g^4x^3+13860c^4d^4f\,g^3x^3+1008a^2c^2d^2e^2g^4x^2-7560a\,c^3d^3ef\,g^3x^2+24570c^4d^4f^2g^2x^2-448\right.$
orering	$2\left(3003g^4x^4d^4c^4-1848a\,c^3d^3e\,g^4x^3+13860c^4d^4f\,g^3x^3+1008a^2c^2d^2e^2g^4x^2-7560a\,c^3d^3ef\,g^3x^2+24570c^4d^4f^2g^2x^2-448a^3cd\,e^3g^4\right.$

input
```
int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/45045*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(3003*c^4*
d^4*g^4*x^4-1848*a*c^3*d^3*e*g^4*x^3+13860*c^4*d^4*f*g^3*x^3+1008*a^2*c^2*
d^2*e^2*g^4*x^2-7560*a*c^3*d^3*e*f*g^3*x^2+24570*c^4*d^4*f^2*g^2*x^2-448*a
^3*c*d*e^3*g^4*x+3360*a^2*c^2*d^2*e^2*f*g^3*x-10920*a*c^3*d^3*e*f^2*g^2*x+
20020*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-960*a^3*c*d*e^3*f*g^3+3120*a^2*c^2*d
^2*e^2*f^2*g^2-5720*a*c^3*d^3*e*f^3*g+6435*c^4*d^4*f^4)/d^5/c^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(267) = 534$.

Time $= 0.09$ (sec) , antiderivative size $= 567$, normalized size of antiderivative $= 1.91$

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(3003\,c^7d^7g^4x^7+6435\,a^3c^4d^4e^3f^4-5720\,a^4c^3d^3e^4f^3\right.}{}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="fricas")
```

output
```
2/45045*(3003*c^7*d^7*g^4*x^7 + 6435*a^3*c^4*d^4*e^3*f^4 - 5720*a^4*c^3*d^
3*e^4*f^3*g + 3120*a^5*c^2*d^2*e^5*f^2*g^2 - 960*a^6*c*d*e^6*f*g^3 + 128*a
^7*e^7*g^4 + 231*(60*c^7*d^7*f*g^3 + 31*a*c^6*d^6*e*g^4)*x^6 + 63*(390*c^7
*d^7*f^2*g^2 + 540*a*c^6*d^6*e*f*g^3 + 71*a^2*c^5*d^5*e^2*g^4)*x^5 + 35*(5
72*c^7*d^7*f^3*g + 1794*a*c^6*d^6*e*f^2*g^2 + 636*a^2*c^5*d^5*e^2*f*g^3 +
a^3*c^4*d^4*e^3*g^4)*x^4 + 5*(1287*c^7*d^7*f^4 + 10868*a*c^6*d^6*e*f^3*g +
 8814*a^2*c^5*d^5*e^2*f^2*g^2 + 60*a^3*c^4*d^4*e^3*f*g^3 - 8*a^4*c^3*d^3*e
^4*g^4)*x^3 + 3*(6435*a*c^6*d^6*e*f^4 + 14300*a^2*c^5*d^5*e^2*f^3*g + 390*
a^3*c^4*d^4*e^3*f^2*g^2 - 120*a^4*c^3*d^3*e^4*f*g^3 + 16*a^5*c^2*d^2*e^5*g
^4)*x^2 + (19305*a^2*c^5*d^5*e^2*f^4 + 2860*a^3*c^4*d^4*e^3*f^3*g - 1560*a
^4*c^3*d^3*e^4*f^2*g^2 + 480*a^5*c^2*d^2*e^5*f*g^3 - 64*a^6*c*d*e^6*g^4)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x +
 c^5*d^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**
(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.68

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}f}{7\,cd}$$
$$+\frac{8\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)\sqrt{cdx+ae}f^3g}{63\,c^2d^2}$$
$$+\frac{4\left(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5\right)\sqrt{cdx+ae}f^2g^2}{231\,c^3d^3}$$
$$+\frac{8\left(231\,c^6d^6x^6+567\,ac^5d^5ex^5+371\,a^2c^4d^4e^2x^4+5\,a^3c^3d^3e^3x^3-6\,a^4c^2d^2e^4x^2+8\,a^5cde^5x-16\,a^6e^6\right)\sqrt{cd}}{3003\,c^4d^4}$$
$$+\frac{2\left(3003\,c^7d^7x^7+7161\,ac^6d^6ex^6+4473\,a^2c^5d^5e^2x^5+35\,a^3c^4d^4e^3x^4-40\,a^4c^3d^3e^4x^3+48\,a^5c^2d^2e^5x^2-64\right.}{45045\,c^5d^5}$$

input

```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="maxima")
```

output

```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)*f^4/(c*d) + 8/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^
2*d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^
2) + 4/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3
 + 3*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*
f^2*g^2/(c^3*d^3) + 8/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^
2*c^4*d^4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*
c*d*e^5*x - 16*a^6*e^6)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/45045*(3003*
c^7*d^7*x^7 + 7161*a*c^6*d^6*e*x^6 + 4473*a^2*c^5*d^5*e^2*x^5 + 35*a^3*c^4
*d^4*e^3*x^4 - 40*a^4*c^3*d^3*e^4*x^3 + 48*a^5*c^2*d^2*e^5*x^2 - 64*a^6*c*
d*e^6*x + 128*a^7*e^7)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(267) = 534$.

Time = 0.21 (sec) , antiderivative size = 2094, normalized size of antiderivative = 7.05

$$\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="giac")
```

output
```
2/45045*(15015*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*f^4*abs(e)/(c
*d*e) - 6006*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*a*f^4*abs(e)/(c*d*e^4) - 12012*(5*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*
e + a*e^3)^(5/2))*a^2*f^3*g*abs(e)/(c^2*d^2*e^3) + 429*(35*((e*x + d)*c*d*
e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*f^4*abs(e)/(
c*d*e^7) + 3432*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(7/2))*a*f^3*g*abs(e)/(c^2*d^2*e^6) + 2574*(35*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*a^2*f^2*
g^2*abs(e)/(c^3*d^3*e^5) - 572*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3
/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(9/2))*f^3*g*abs(e)/(c^2*d^2*e^9) - 1716*(105*((e*x + d)*c*
d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 -
 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*a*f^2*g^2*abs(e)/(c^3*d^3*e
^8) - 572*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*...
```

Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.76

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^5\,\left(71\,a^2\,e^2\,g^2+540\,a}{715}\right.}{}$$

input
```
int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^5*(71*a^2*e^2*g^2
 + 390*c^2*d^2*f^2 + 540*a*c*d*e*f*g))/715 + (256*a^7*e^7*g^4 + 12870*a^3*
c^4*d^4*e^3*f^4 - 11440*a^4*c^3*d^3*e^4*f^3*g - 1920*a^6*c*d*e^6*f*g^3 + 6
240*a^5*c^2*d^2*e^5*f^2*g^2)/(45045*c^5*d^5) + (x^3*(12870*c^7*d^7*f^4 - 8
0*a^4*c^3*d^3*e^4*g^4 + 600*a^3*c^4*d^4*e^3*f*g^3 + 108680*a*c^6*d^6*e*f^3
*g + 88140*a^2*c^5*d^5*e^2*f^2*g^2))/(45045*c^5*d^5) + (2*c^2*d^2*g^4*x^7)
/15 + (2*c*d*g^3*x^6*(31*a*e*g + 60*c*d*f))/195 + (2*g*x^4*(a^3*e^3*g^3 +
572*c^3*d^3*f^3 + 1794*a*c^2*d^2*e*f^2*g + 636*a^2*c*d*e^2*f*g^2))/(1287*c
*d) + (2*a^2*e^2*x*(19305*c^4*d^4*f^4 - 64*a^4*e^4*g^4 + 2860*a*c^3*d^3*e*
f^3*g + 480*a^3*c*d*e^3*f*g^3 - 1560*a^2*c^2*d^2*e^2*f^2*g^2))/(45045*c^4*
d^4) + (2*a*e*x^2*(16*a^4*e^4*g^4 + 6435*c^4*d^4*f^4 + 14300*a*c^3*d^3*e*f
^3*g - 120*a^3*c*d*e^3*f*g^3 + 390*a^2*c^2*d^2*e^2*f^2*g^2))/(15015*c^3*d^
3)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.88

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{cdx+ae}\,(3003c^7d^7g^4x^7+7161a\,c^6d^6e\,g^4x^6+1386$$

input
```
int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(128*a**7*e**7*g**4 - 960*a**6*c*d*e**6*f*g**3 - 64*a
**6*c*d*e**6*g**4*x + 3120*a**5*c**2*d**2*e**5*f**2*g**2 + 480*a**5*c**2*d
**2*e**5*f*g**3*x + 48*a**5*c**2*d**2*e**5*g**4*x**2 - 5720*a**4*c**3*d**3
*e**4*f**3*g - 1560*a**4*c**3*d**3*e**4*f**2*g**2*x - 360*a**4*c**3*d**3*e
**4*f*g**3*x**2 - 40*a**4*c**3*d**3*e**4*g**4*x**3 + 6435*a**3*c**4*d**4*e
**3*f**4 + 2860*a**3*c**4*d**4*e**3*f**3*g*x + 1170*a**3*c**4*d**4*e**3*f*
*2*g**2*x**2 + 300*a**3*c**4*d**4*e**3*f*g**3*x**3 + 35*a**3*c**4*d**4*e**
3*g**4*x**4 + 19305*a**2*c**5*d**5*e**2*f**4*x + 42900*a**2*c**5*d**5*e**2
*f**3*g*x**2 + 44070*a**2*c**5*d**5*e**2*f**2*g**2*x**3 + 22260*a**2*c**5*
d**5*e**2*f*g**3*x**4 + 4473*a**2*c**5*d**5*e**2*g**4*x**5 + 19305*a*c**6*
d**6*e*f**4*x**2 + 54340*a*c**6*d**6*e*f**3*g*x**3 + 62790*a*c**6*d**6*e*f
**2*g**2*x**4 + 34020*a*c**6*d**6*e*f*g**3*x**5 + 7161*a*c**6*d**6*e*g**4*
x**6 + 6435*c**7*d**7*f**4*x**3 + 20020*c**7*d**7*f**3*g*x**4 + 24570*c**7
*d**7*f**2*g**2*x**5 + 13860*c**7*d**7*f*g**3*x**6 + 3003*c**7*d**7*g**4*x
**7))/(45045*c**5*d**5)
```

3.23 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 234

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^4d^4(d+ex)^{7/2}}$$
$$+\frac{2g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{3c^4d^4(d+ex)^{9/2}}$$
$$+\frac{6g^2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^4d^4(d+ex)^{11/2}}$$
$$+\frac{2g^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{13/2}}{13c^4d^4(d+ex)^{13/2}}$$

output

```
2/7*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/(e*x+
d)^(7/2)+2/3*g*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^
4/d^4/(e*x+d)^(9/2)+6/11*g^2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(11/2)/c^4/d^4/(e*x+d)^(11/2)+2/13*g^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(13/2)/c^4/d^4/(e*x+d)^(13/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.63

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-16a^3e^3g^3+8a^2cd}{}$$

input
```
Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
 e*x)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*
c*d*e^2*g^2*(13*f + 7*g*x) - 2*a*c^2*d^2*e*g*(143*f^2 + 182*f*g*x + 63*g^2
*x^2) + c^3*d^3*(429*f^3 + 1001*f^2*g*x + 819*f*g^2*x^2 + 231*g^3*x^3)))/(
3003*c^4*d^4*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{6(cdf-aeg)\int \frac{(f+gx)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{13cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int\frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{11cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}\right)}{13cd}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{9}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^2(x(ae}{11c}}{13cd}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}+$$

$$\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)}{63cd(d+ex)^{7/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+}{9cde(d+}}{11cd}}{13cd}$$

input
```
Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x]
```

output
```
(2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*(d +
 e*x)^(7/2)) + (6*(c*d*f - a*e*g)*((2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(11*c*d*(d + e*x)^(7/2)) + (4*(c*d*f - a*e*g)*((2*(
9*f - (7*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(7/2))/(63*c*d*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))))/(11*c*d)))/(13*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.77

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^3(-231x^3g^3d^3c^3+126a\,c^2d^2e\,g^3x^2-819c^3d^3f\,g^2x^2-56a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-1001c^3d^3f}{3003\sqrt{ex+d}\,d^4c^4}$
gosper	$-\frac{2(cdx+ae)(-231x^3g^3d^3c^3+126a\,c^2d^2e\,g^3x^2-819c^3d^3f\,g^2x^2-56a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-1001c^3d^3f^2gx+16a^3e^3g^3-104}{3003d^4c^4(ex+d)^{\frac{5}{2}}}$
orering	$-\frac{2(-231x^3g^3d^3c^3+126a\,c^2d^2e\,g^3x^2-819c^3d^3f\,g^2x^2-56a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-1001c^3d^3f^2gx+16a^3e^3g^3-104a^2cd\,e^2f\,g}{3003d^4c^4(ex+d)^{\frac{5}{2}}}$

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/3003*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(-231*c^3*
d^3*g^3*x^3+126*a*c^2*d^2*e*g^3*x^2-819*c^3*d^3*f*g^2*x^2-56*a^2*c*d*e^2*g
^3*x+364*a*c^2*d^2*e*f*g^2*x-1001*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-104*a^2*c
*d*e^2*f*g^2+286*a*c^2*d^2*e*f^2*g-429*c^3*d^3*f^3)/d^4/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.78

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = 2\left(231\,c^6d^6g^3x^6+429\,a^3c^3d^3e^3f^3-286\,a^4c^2d^2e^4f^2g+\right.$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="fricas")
```

output
```
2/3003*(231*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 286*a^4*c^2*d^2*e^
4*f^2*g + 104*a^5*c*d*e^5*f*g^2 - 16*a^6*e^6*g^3 + 63*(13*c^6*d^6*f*g^2 +
9*a*c^5*d^5*e*g^3)*x^5 + 7*(143*c^6*d^6*f^2*g + 299*a*c^5*d^5*e*f*g^2 + 53
*a^2*c^4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 + 2717*a*c^5*d^5*e*f^2*g + 14
69*a^2*c^4*d^4*e^2*f*g^2 + 5*a^3*c^3*d^3*e^3*g^3)*x^3 + 3*(429*a*c^5*d^5*e
*f^3 + 715*a^2*c^4*d^4*e^2*f^2*g + 13*a^3*c^3*d^3*e^3*f*g^2 - 2*a^4*c^2*d^
2*e^4*g^3)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 + 143*a^3*c^3*d^3*e^3*f^2*g - 5
2*a^4*c^2*d^2*e^4*f*g^2 + 8*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**
(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.55

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3)\sqrt{cdx+ae}f}{7\,cd}$$
$$+\frac{2\,(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4)\sqrt{cdx+ae}f^2g}{21\,c^2d^2}$$
$$+\frac{2\,(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5)\sqrt{cdx+ae}fg^2}{231\,c^3d^3}$$
$$+\frac{2\,(231\,c^6d^6x^6+567\,ac^5d^5ex^5+371\,a^2c^4d^4e^2x^4+5\,a^3c^3d^3e^3x^3-6\,a^4c^2d^2e^4x^2+8\,a^5cde^5x-16\,a^6e^6)\sqrt{cd}}{3003\,c^4d^4}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="maxima")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)*f^3/(c*d) + 2/21*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^
2*d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^
2) + 2/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3
 + 3*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*
f*g^2/(c^3*d^3) + 2/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*e*x^5 + 371*a^2*
c^4*d^4*e^2*x^4 + 5*a^3*c^3*d^3*e^3*x^3 - 6*a^4*c^2*d^2*e^4*x^2 + 8*a^5*c*
d*e^5*x - 16*a^6*e^6)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. $2(210) = 420$.

Time = 0.18 (sec) , antiderivative size = 1479, normalized size of antiderivative = 6.32

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="giac")
```

output
```
2/45045*(15015*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*f^3*abs(e)/(c
*d*e) - 6006*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*a*f^3*abs(e)/(c*d*e^4) - 9009*(5*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(5/2))*a^2*f^2*g*abs(e)/(c^2*d^2*e^3) + 429*(35*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*f^3*abs(e)/(c
*d*e^7) + 2574*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(7/2))*a*f^2*g*abs(e)/(c^2*d^2*e^6) + 1287*(35*((e*x + d)*c
*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*a^2*f*g^2
*abs(e)/(c^3*d^3*e^5) - 429*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)
*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(9/2))*f^2*g*abs(e)/(c^2*d^2*e^9) - 858*(105*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*a*f*g^2*abs(e)/(c^3*d^3*e^8) -
143*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((e*x ...
```

Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.62

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g\,x^4\,\left(53\,a^2\,e^2\,g^2+299\,a}{429}\right.}{}$$

input
```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g*x^4*(53*a^2*e^2*g^2 +
 143*c^2*d^2*f^2 + 299*a*c*d*e*f*g))/429 - (32*a^6*e^6*g^3 - 858*a^3*c^3*d
^3*e^3*f^3 + 572*a^4*c^2*d^2*e^4*f^2*g - 208*a^5*c*d*e^5*f*g^2)/(3003*c^4*
d^4) + (x^3*(858*c^6*d^6*f^3 + 10*a^3*c^3*d^3*e^3*g^3 + 2938*a^2*c^4*d^4*e
^2*f*g^2 + 5434*a*c^5*d^5*e*f^2*g))/(3003*c^4*d^4) + (2*c^2*d^2*g^3*x^6)/1
3 + (6*c*d*g^2*x^5*(9*a*e*g + 13*c*d*f))/143 + (2*a^2*e^2*x*(8*a^3*e^3*g^3
 + 1287*c^3*d^3*f^3 + 143*a*c^2*d^2*e*f^2*g - 52*a^2*c*d*e^2*f*g^2))/(3003
*c^3*d^3) + (2*a*e*x^2*(429*c^3*d^3*f^3 - 2*a^3*e^3*g^3 + 715*a*c^2*d^2*e*
f^2*g + 13*a^2*c*d*e^2*f*g^2))/(1001*c^2*d^2)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.69

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\sqrt{cdx+ae}\left(231c^6d^6g^3x^6+567a\,c^5d^5e\,g^3x^5+819c^6d^6\right.}{}$$

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 16*a**6*e**6*g**3 + 104*a**5*c*d*e**5*f*g**2 + 8*
a**5*c*d*e**5*g**3*x - 286*a**4*c**2*d**2*e**4*f**2*g - 52*a**4*c**2*d**2*
e**4*f*g**2*x - 6*a**4*c**2*d**2*e**4*g**3*x**2 + 429*a**3*c**3*d**3*e**3*
f**3 + 143*a**3*c**3*d**3*e**3*f**2*g*x + 39*a**3*c**3*d**3*e**3*f*g**2*x*
*2 + 5*a**3*c**3*d**3*e**3*g**3*x**3 + 1287*a**2*c**4*d**4*e**2*f**3*x + 2
145*a**2*c**4*d**4*e**2*f**2*g*x**2 + 1469*a**2*c**4*d**4*e**2*f*g**2*x**3
 + 371*a**2*c**4*d**4*e**2*g**3*x**4 + 1287*a*c**5*d**5*e*f**3*x**2 + 2717
*a*c**5*d**5*e*f**2*g*x**3 + 2093*a*c**5*d**5*e*f*g**2*x**4 + 567*a*c**5*d
**5*e*g**3*x**5 + 429*c**6*d**6*f**3*x**3 + 1001*c**6*d**6*f**2*g*x**4 + 8
19*c**6*d**6*f*g**2*x**5 + 231*c**6*d**6*g**3*x**6))/(3003*c**4*d**4)
```

3.24 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 171

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^3d^3(d+ex)^{7/2}}+\frac{4g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^3d^3(d+ex)^{9/2}}+\frac{2g^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{11/2}}{11c^3d^3(d+ex)^{11/2}}$$

output
```
2/7*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/(e*x+
d)^(7/2)+4/9*g*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^3/
d^3/(e*x+d)^(9/2)+2/11*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(11/2)/c^3/d^
3/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.58

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\left(8a^2e^2g^2-4acdeg(1\right.}{693c^3d^3\sqrt{d+}}$$

input
```
Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
 e*x)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*
e*g*(11*f + 7*g*x) + c^2*d^2*(99*f^2 + 154*f*g*x + 63*g^2*x^2)))/(693*c^3*
d^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = $\{1253, 1221, 1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{4(cdf-aeg)\int \frac{(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx}{11cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{4(cdf-aeg)\left(\frac{1}{9}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}\right)}{11cd}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}+\frac{4(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)}{63cd(d+ex)^{7/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}\right)}{11cd}$$

input

```
Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x]
```

output

```
(2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*c*d*(d +
 e*x)^(7/2)) + (4*(c*d*f - a*e*g)*((2*(9*f - (7*d*g)/e - (2*a*e*g)/(c*d))*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*c*d*(d + e*x)^(7/2)) +
(2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/
2))))/(11*c*d)
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^3\left(63g^2x^2d^2c^2-28acde\,g^2x+154c^2d^2fgx+8a^2e^2g^2-44acdefg+99f^2c^2d^2\right)}{693\sqrt{ex+d}\,d^3c^3}$	108
gosper	$\frac{2(cdx+ae)\left(63g^2x^2d^2c^2-28acde\,g^2x+154c^2d^2fgx+8a^2e^2g^2-44acdefg+99f^2c^2d^2\right)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{693d^3c^3(ex+d)^{\frac{5}{2}}}$	116
orering	$\frac{2\left(63g^2x^2d^2c^2-28acde\,g^2x+154c^2d^2fgx+8a^2e^2g^2-44acdefg+99f^2c^2d^2\right)(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{5}{2}}}{693d^3c^3(ex+d)^{\frac{5}{2}}}$	117

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/693*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(63*c^2*d^2*
g^2*x^2-28*a*c*d*e*g^2*x+154*c^2*d^2*f*g*x+8*a^2*e^2*g^2-44*a*c*d*e*f*g+99
*c^2*d^2*f^2)/d^3/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.66

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(63\,c^5d^5g^2x^5+99\,a^3c^2d^2e^3f^2-44\,a^4cde^4fg+8\,a^5e^5\right.}{}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="fricas")
```

output
```
2/693*(63*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 44*a^4*c*d*e^4*f*g +
8*a^5*e^5*g^2 + 7*(22*c^5*d^5*f*g + 23*a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*
f^2 + 418*a*c^4*d^4*e*f*g + 113*a^2*c^3*d^3*e^2*g^2)*x^3 + 3*(99*a*c^4*d^4
*e*f^2 + 110*a^2*c^3*d^3*e^2*f*g + a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3
*d^3*e^2*f^2 + 22*a^3*c^2*d^2*e^3*f*g - 4*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Timed out}$$

input

```
integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**
(5/2),x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.42

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}f}{7\,cd}$$
$$+\frac{4\left(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4\right)\sqrt{cdx+ae}\,fg}{63\,c^2d^2}$$
$$+\frac{2\left(63\,c^5d^5x^5+161\,ac^4d^4ex^4+113\,a^2c^3d^3e^2x^3+3\,a^3c^2d^2e^3x^2-4\,a^4cde^4x+8\,a^5e^5\right)\sqrt{cdx+ae}\,g^2}{693\,c^3d^3}$$

input

```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="maxima")
```

output

```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)*f^2/(c*d) + 4/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^
2*d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2)
 + 2/693*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*e*x^4 + 113*a^2*c^3*d^3*e^2*x^3 +
 3*a^3*c^2*d^2*e^3*x^2 - 4*a^4*c*d*e^4*x + 8*a^5*e^5)*sqrt(c*d*x + a*e)*g^
2/(c^3*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(153) = 306$.

Time = 0.17 (sec) , antiderivative size = 958, normalized size of antiderivative = 5.60

$$\int \frac{(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),
x, algorithm="giac")
```

output

```
2/3465*(1155*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*f^2*abs(e)/(c*d
*e) - 462*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(5/2))*a*f^2*abs(e)/(c*d*e^4) - 462*(5*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(5/2))*a^2*f*g*abs(e)/(c^2*d^2*e^3) + 33*(35*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*
a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*f^2*abs(e)/(c*d*e^7)
 + 132*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(7/2))*a*f*g*abs(e)/(c^2*d^2*e^6) + 33*(35*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a
*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*a^2*g^2*abs(e)/(c^3*d
^3*e^5) - 22*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/
2))*f*g*abs(e)/(c^2*d^2*e^9) - 22*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(3/2)*a^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 1
35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e -
 c*d^2*e + a*e^3)^(9/2))*a*g^2*abs(e)/(c^3*d^3*e^8) + (1155*((e*x + d)*c*d
*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((e*x + d)*c*d*e - c*d^2*e ...
```

Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.51

$$\int \frac{(f+gx)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,a^5\,e^5\,g^2-88\,a^4\,c\,d\,e^4\,f}{693\,c^3}\right.}{}$$

input
```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^
(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*a^5*e^5*g^2 + 198*a^3*
c^2*d^2*e^3*f^2 - 88*a^4*c*d*e^4*f*g)/(693*c^3*d^3) + (x^3*(198*c^5*d^5*f^
2 + 226*a^2*c^3*d^3*e^2*g^2 + 836*a*c^4*d^4*e*f*g))/(693*c^3*d^3) + (2*c^2
*d^2*g^2*x^5)/11 + (2*c*d*g*x^4*(23*a*e*g + 22*c*d*f))/99 + (2*a^2*e^2*x*(
297*c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(693*c^2*d^2) + (2*a*e*
x^2*(a^2*e^2*g^2 + 99*c^2*d^2*f^2 + 110*a*c*d*e*f*g))/(231*c*d)))/(d + e*x
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.49

$$\int \frac{(f+gx)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{cdx+ae}\,(63c^5d^5g^2x^5+161a\,c^4d^4e\,g^2x^4+154c^5d^5}{}$$

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(8*a**5*e**5*g**2 - 44*a**4*c*d*e**4*f*g - 4*a**4*c*d
*e**4*g**2*x + 99*a**3*c**2*d**2*e**3*f**2 + 22*a**3*c**2*d**2*e**3*f*g*x
+ 3*a**3*c**2*d**2*e**3*g**2*x**2 + 297*a**2*c**3*d**3*e**2*f**2*x + 330*a
**2*c**3*d**3*e**2*f*g*x**2 + 113*a**2*c**3*d**3*e**2*g**2*x**3 + 297*a*c*
*4*d**4*e*f**2*x**2 + 418*a*c**4*d**4*e*f*g*x**3 + 161*a*c**4*d**4*e*g**2*
x**4 + 99*c**5*d**5*f**2*x**3 + 154*c**5*d**5*f*g*x**4 + 63*c**5*d**5*g**2
*x**5))/(693*c**3*d**3)
```

3.25 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 108

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^2d^2(d+ex)^{7/2}}+\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^2d^2(d+ex)^{9/2}}$$

output
```
2/7*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/(e*x+d)
^(7/2)+2/9*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^2/d^2/(e*x+d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-2aeg+cd(9f+7g}{63c^2d^2\sqrt{d+ex}}$$

input
```
Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(9*f + 7*
g*x)))/(63*c^2*d^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{9}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}}dx+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}\left(-\frac{2aeg}{cd}-\frac{7dg}{e}+9f\right)}{63cd(d+ex)^{7/2}}+\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}$$

input
```
Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5
/2),x]
```

output
```
(2*(9*f - (7*d*g)/e - (2*a*e*g)/(c*d))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(7/2))/(63*c*d*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{2\sqrt{\left(ex+d\right)\left(cdx+ae\right)}\, \left(cdx+ae\right)^{3} \left(-7cdgx+2aeg-9dfc\right)}{63\sqrt{ex+d}\, c^{2} d^{2}}$	59
gosper	$-\frac{2\left(cdx+ae\right)\left(-7cdgx+2aeg-9dfc\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{63c^{2}d^{2}\left(ex+d\right)^{\frac{5}{2}}}$	67
orering	$-\frac{2\left(-7cdgx+2aeg-9dfc\right)\left(cdx+ae\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{63c^{2}d^{2}\left(ex+d\right)^{\frac{5}{2}}}$	68

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/63*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(-7*c*d*g*x+
2*a*e*g-9*c*d*f)/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(7\,c^4d^4gx^4+9\,a^3cde^3f-2\,a^4e^4g+(9\,c^4d^4f+19\,ac^3d}{}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
 algorithm="fricas")
```

output
```
2/63*(7*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 2*a^4*e^4*g + (9*c^4*d^4*f + 19*
a*c^3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f + 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a
^2*c^2*d^2*e^2*f + a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/
2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.31

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3)\sqrt{cdx+ae}f}{7\,cd} + \frac{2\,(7\,c^4d^4x^4+19\,ac^3d^3ex^3+15\,a^2c^2d^2e^2x^2+a^3cde^3x-2\,a^4e^4)\sqrt{cdx+ae}g}{63\,c^2d^2}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
 algorithm="maxima")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)*f/(c*d) + 2/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*
d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(96) = 192$.

Time = 0.15 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.95

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{105\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}a^2f|e|}{cde}-\frac{42\left(5\left((ex+d)cde-cd^2e+ae^3\right)\right.}{}\right.}{}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
 algorithm="giac")
```

output
```
2/315*(105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*f*abs(e)/(c*d*e)
- 42*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d
*e - c*d^2*e + a*e^3)^(5/2))*a*f*abs(e)/(c*d*e^4) - 21*(5*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5
/2))*a^2*g*abs(e)/(c^2*d^2*e^3) + 3*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*f*abs(e)/(c*d*e^7) + 6*(35*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(5/2)*a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*a
*g*abs(e)/(c^2*d^2*e^6) - (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^3*e^9 - 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(9/2))*g*abs(e)/(c^2*d^2*e^9))/e
```

Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.24

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,c^2\,d^2\,g\,x^4}{9}+\frac{2\,a\,e\,x^2\,(5\,a}{2}\right.}{}$$

input
```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(5
/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*c^2*d^2*g*x^4)/9 + (2*a
*e*x^2*(5*a*e*g + 9*c*d*f))/21 + (2*c*d*x^3*(19*a*e*g + 9*c*d*f))/63 - (2*
a^3*e^3*(2*a*e*g - 9*c*d*f))/(63*c^2*d^2) + (2*a^2*e^2*x*(a*e*g + 27*c*d*f
))/(63*c*d)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.25

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{cdx+ae}\,(7c^4d^4g\,x^4+19a\,c^3d^3eg\,x^3+9c^4d^4f\,x^3+1}{}$$

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 2*a**4*e**4*g + 9*a**3*c*d*e**3*f + a**3*c*d*e**3
*g*x + 27*a**2*c**2*d**2*e**2*f*x + 15*a**2*c**2*d**2*e**2*g*x**2 + 27*a*c
**3*d**3*e*f*x**2 + 19*a*c**3*d**3*e*g*x**3 + 9*c**4*d**4*f*x**3 + 7*c**4*
d**4*g*x**4))/(63*c**2*d**2)
```

3.26 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{7/2}}{7cd(d+ex)^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \; 1122$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^{3}}{7\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{7cd(ex+d)^{\frac{5}{2}}}$	50
orering	$\frac{2(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{7cd(ex+d)^{\frac{5}{2}}}$	51

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
2/7*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3/c/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3)\sqrt{cdex^2+ade+(cd^2}}{7\,(cdex+cd^2)}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorit
hm="fricas")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(c^3d^3x^3+3\,ac^2d^2ex^2+3\,a^2cde^2x+a^3e^3\right)\sqrt{cdx+ae}}{7\,cd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorit
hm="maxima")
```

output
```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*x + a*e)/(c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(42) = 84.

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.54

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\left(\frac{35\left(\left(ex+d\right)cde-cd^2e+ae^3\right)^{\frac{3}{2}}a^2|e|}{cde}-\frac{14\left(5\left(\left(ex+d\right)cde-cd^2e+ae^3\right)^{\frac{3}{2}}ae^3-3\left(\left(ex+\right.\right.\right.}{cde^4}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorit
hm="giac")
```

output
```
2/105*(35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*abs(e)/(c*d*e) - 1
4*(5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(5/2))*a*abs(e)/(c*d*e^4) + (35*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*
a*e^3 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*abs(e)/(c*d*e^7))/e
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{6\,a^2\,e^2\,x}{7}+\frac{2\,c^2\,d^2\,x^3}{7}+\frac{2\,a^3\,e^3}{7\,c\,d}+6\right.}{\sqrt{d+e\,x}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((6*a^2*e^2*x)/7 + (2*c^2*d
^2*x^3)/7 + (2*a^3*e^3)/(7*c*d) + (6*a*c*d*e*x^2)/7))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2\sqrt{cdx+ae}\,\left(c^3d^3x^3+3a\,c^2d^2e\,x^2+3a^2cd\,e^2x+a^3e^3\right)}{7cd}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(a**3*e**3 + 3*a**2*c*d*e**2*x + 3*a*c**2*d**2*e*x**2
 + c**3*d**3*x**3))/(7*c*d)
```

3.27 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 236

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}$$
$$-\frac{2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}$$
$$-\frac{2(cdf-aeg)^{5/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{7/2}}$$

output
```
2*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/
2)-2/3*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^
(3/2)+2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)-2*(-a*e*
g+c*d*f)^(5/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*
e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\Big(\sqrt{g}\sqrt{ae+cdx}(23a^2e^2g^2+acdeg(-35f+1}{15g^{7/2}\sqrt{}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(23*a^2*e^2*
g^2 + a*c*d*e*g*(-35*f + 11*g*x) + c^2*d^2*(15*f^2 - 5*f*g*x + 3*g^2*x^2))
 - 15*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f
- a*e*g]]))/(15*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1250, 1250, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}dx$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}dx}{g}$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{g}\right)}{g}$$

$$\Big\downarrow \ 1250$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{g}\right.}{g}$$

$$\Big\downarrow \ 1255$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x\right.}{d+ex}}}{g}\right.}{g}\right.}{g}$$

$$\Big\downarrow \ 218$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{g}\right)}{g}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)) -
((c*d*f - a*e*g)*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(
d + e*x)^(3/2)) - ((c*d*f - a*e*g)*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x]
)])/g^(3/2)))/g))/g
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(208) = 416$.

Time = 2.83 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.78

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)a^{3}e^{3}g^{3}-45\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)a^{2}cd\,e^{2}f\,g^{2}+45\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)\right.}{}$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,method
=_RETURNVERBOSE)
```

output

```
-2/15*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*a^3*e^3*g^3-45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^3*f^3-3*c^2*d^2*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*
g)^(1/2)-11*a*c*d*e*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+5*c^2*
d^2*f*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-23*((a*e*g-c*d*f)*g)^(
1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+35*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^
(1/2)*a*c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2
)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.62

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx=\left[\frac{15\left(c^2d^3f^2-2\,acd^2efg+a^2de^2g^2+\left(c^2d^2ef^2-2\,acde^2fg+a^2\right.\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
 algorithm="fricas")
```

output
```
[1/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2
- 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g
*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g
)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*
f^2 - 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 11*a*c*d*e*g^2)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^3*x + d*
g^3), 2/15*(15*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e
*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(-s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt((c*d*f - a
*e*g)/g)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) + (3*c^2*d^2*g^2*x^2
 + 15*c^2*d^2*f^2 - 35*a*c*d*e*f*g + 23*a^2*e^2*g^2 - (5*c^2*d^2*f*g - 11*
a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))
/(e*g^3*x + d*g^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
 algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.53

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx=$$

$$-\frac{2\left(c^3d^3f^3|e|-3\,ac^2d^2ef^2g|e|+3\,a^2cde^2fg^2|e|-a^3e^3g^3|e|\right)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}eg^3}$$

$$+\frac{2\left(15\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2e^{28}f^2g^2|e|-30\,\sqrt{(ex+d)cde-cd^2e+ae^3}acde^{29}fg^3|e|+15\,\sqrt{(ex}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
 algorithm="giac")
```

output
```
-2*(c^3*d^3*f^3*abs(e) - 3*a*c^2*d^2*e*f^2*g*abs(e) + 3*a^2*c*d*e^2*f*g^2*
abs(e) - a^3*e^3*g^3*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) + 2/15*(1
5*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*e^28*f^2*g^2*abs(e) - 30
*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e^29*f*g^3*abs(e) + 15*sqrt
((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*e^30*g^4*abs(e) - 5*((e*x + d)*c*d
*e - c*d^2*e + a*e^3)^(3/2)*c*d*e^26*f*g^3*abs(e) + 5*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(3/2)*a*e^27*g^4*abs(e) + 3*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(5/2)*e^24*g^4*abs(e))/(e^30*g^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/
2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.21

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \frac{-2\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)a^2e^2g^2+4\sqrt{g}\,\sqrt{-aeg+}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x)
```

output

```
(2*( - 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(
g)*sqrt( - a*e*g + c*d*f)))*a**2*e**2*g**2 + 30*sqrt(g)*sqrt( - a*e*g + c*
d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*c*d*e*
f*g - 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g
)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*f**2 + 23*sqrt(a*e + c*d*x)*a**2*e**2
*g**3 - 35*sqrt(a*e + c*d*x)*a*c*d*e*f*g**2 + 11*sqrt(a*e + c*d*x)*a*c*d*e
*g**3*x + 15*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g - 5*sqrt(a*e + c*d*x)*c**2
*d**2*f*g**2*x + 3*sqrt(a*e + c*d*x)*c**2*d**2*g**3*x**2))/(15*g**4)
```

3.28 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 235

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx =\\ -\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}\\ +\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}\\ +\frac{5cd(cdf-aeg)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{7/2}}\end{aligned}$$

output

```
-5*c*d*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^
(1/2)+5/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)-(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)+5*c*d*(-a*e*
g+c*d*f)^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*
e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.78

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\Big(\sqrt{g}\sqrt{ae+cdx}(-3a^2e^2g^2+2acdeg(10f+7}{3g^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^2),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-3*a^2*e^2*g^
2 + 2*a*c*d*e*g*(10*f + 7*g*x) + c^2*d^2*(-15*f^2 - 10*f*g*x + 2*g^2*x^2))
 + 15*c*d*(c*d*f - a*e*g)^(3/2)*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x
])/Sqrt[c*d*f - a*e*g]]))/(3*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*
x))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1249, 1250, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$\Big\downarrow \text{1250}$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$\Big\downarrow \ 1250$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$\Big\downarrow \ 1255$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d}{g}\right.}{g}\right.}{2g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

$$\Big\downarrow \ 218$$

$$\frac{5cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^2),x]
```

output

```
-((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f + g*
x))) + (5*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d +
 e*x)^(3/2)) - ((c*d*f - a*e*g)*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])
/g^(3/2)))/g))/(2*g)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(209) = 418.

Time = 2.76 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.18

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)a^2cd\,e^2g^3x-30\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)a\,c^2d^2ef\,g^2x+15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/3*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a*c^2*d^2*e*f*g^2*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a^2*c*d*e^2*f*g^2-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*f^3-2*c^2*d^2*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*
d*f)*g)^(1/2)-14*a*c*d*e*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+1
0*c^2*d^2*f*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*((a*e*g-c*d*f)
*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2-20*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+
a*e)^(1/2)*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^
2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 704, normalized size of antiderivative = 3.00

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \left[-\frac{15\,(c^2d^3f^2-acd^2efg+(c^2d^2efg-acde^2g^2)x^2+(c^2d^2ef^2-}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,
x, algorithm="fricas")
```

output

```
[-1/6*(15*(c^2*d^3*f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x
^2 + (c^2*d^2*e*f^2 - a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt(-
(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g)
- (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*
(2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*e^2*g^2 - 2*(
5*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), -1/3*(15*(
c^2*d^3*f^2 - a*c*d^2*e*f*g + (c^2*d^2*e*f*g - a*c*d*e^2*g^2)*x^2 + (c^2*d
^2*e*f^2 - a*c*d^2*e*g^2 + (c^2*d^3 - a*c*d*e^2)*f*g)*x)*sqrt((c*d*f - a*e
*g)/g)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g
*sqrt((c*d*f - a*e*g)/g)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) - (2
*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 20*a*c*d*e*f*g - 3*a^2*e^2*g^2 - 2*(5*
c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(e*x + d))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \text{Timed out}$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**2,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \int \frac{\left(cdex^2 + ade + \left(cd^2 + ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^2}\,dx$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.67

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \frac{5\left(c^3d^3f^2|e|-2\,ac^2d^2efg|e|+a^2cde^2g^2|e|\right)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2}}{\sqrt{cdfg-aeg}}\right.}{\sqrt{cdfg-aeg^2}eg^3}$$

$$-\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}c^3d^3f^2|e|-2\sqrt{(ex+d)cde-cd^2e+ae^3}ac^2d^2efg|e|+\sqrt{(ex+d)cde-cd^2}}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)g^3}$$

$$-\frac{2\left(6\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2e^{10}fg^3|e|-6\sqrt{(ex+d)cde-cd^2e+ae^3}acde^{11}g^4|e|-((ex+d)cde\right.}{3\,e^{12}g^6}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,
x, algorithm="giac")
```

output
```
5*(c^3*d^3*f^2*abs(e) - 2*a*c^2*d^2*e*f*g*abs(e) + a^2*c*d*e^2*g^2*abs(e))
*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)
*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) - (sqrt((e*x + d)*c*d*e - c*d^2*e + a
*e^3)*c^3*d^3*f^2*abs(e) - 2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2
*d^2*e*f*g*abs(e) + sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c*d*e^2*g^
2*abs(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*g
^3) - 2/3*(6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*e^10*f*g^3*ab
s(e) - 6*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e^11*g^4*abs(e) - (
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*e^8*g^4*abs(e))/(e^12*g^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(
5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.47

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = \frac{-15\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)acdefg-15\sqrt{g}\,\sqrt{-ae}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x)
```

output
```
( - 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*a*c*d*e*f*g - 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*a
tan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*c*d*e*g**2*x
 + 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*s
qrt( - a*e*g + c*d*f)))*c**2*d**2*f**2 + 15*sqrt(g)*sqrt( - a*e*g + c*d*f)
*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*f*
g*x - 3*sqrt(a*e + c*d*x)*a**2*e**2*g**3 + 20*sqrt(a*e + c*d*x)*a*c*d*e*f*
g**2 + 14*sqrt(a*e + c*d*x)*a*c*d*e*g**3*x - 15*sqrt(a*e + c*d*x)*c**2*d**
2*f**2*g - 10*sqrt(a*e + c*d*x)*c**2*d**2*f*g**2*x + 2*sqrt(a*e + c*d*x)*c
**2*d**2*g**3*x**2)/(3*g**4*(f + g*x))
```

3.29 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 246

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx &= \frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}\\ &\quad-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}\\ &\quad-\frac{15c^2d^2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4g^{7/2}}\end{aligned}$$

output

```
15/4*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)-5/4
*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)-1/2
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^2-15/4*c^
2*d^2*(-a*e*g+c*d*f)^(1/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-2a^2e^2g^2-acdeg(5f+9gx)\right.}{4g^{7/2}\sqrt{}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^3),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-2*a^2*e^2*g^
2 - a*c*d*e*g*(5*f + 9*g*x) + c^2*d^2*(15*f^2 + 25*f*g*x + 8*g^2*x^2)) - 1
5*c^2*d^2*Sqrt[c*d*f - a*e*g]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x
])/Sqrt[c*d*f - a*e*g]]))/(4*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*
x)^2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1249, 1249, 1250, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}dx}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$$\Big\downarrow \text{1250}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$$\Big\downarrow \text{1255}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2e^2(cdf-aeg)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right.\right.}{g(d+e}\right.}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^3),x]
```

output

```
-1/2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^2) + (5*c*d*(-((a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d +
 e*x)^(3/2)*(f + g*x))) + (3*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/g^(3/2)))/(2*g)))/(4*g)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(214) = 428$.

Time = 2.87 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.10

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\,\left(15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)a\,c^{2}d^{2}e\,g^{3}x^{2}-15\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{3}d^{3}f\,g^{2}x^{2}+30\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/4*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f*g^2*x-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^3*d^3*f^3-8*c^2*d^2*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2)+9*a*c*d*e*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-
25*c^2*d^2*f*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+2*((a*e*g-c*d*f
)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+
a*e)^(1/2)*a*c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^
2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.91

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = \left[\frac{15\,(c^2d^2eg^2x^3+c^2d^3f^2+(2\,c^2d^2efg+c^2d^3g^2)x^2+(c^2d^2ef^2+}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,
x, algorithm="fricas")
```

output

```
[1/8*(15*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2
)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-(c*d*f - a*e*g)/g)*log(-(
c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a
*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(8*c^2*d^2*g^2*x^2 + 15*c
^2*d^2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d^2*f*g - 9*a*c*d*e*g
^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*g^5*x
^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 + 2*d*f*g^4)*x), 1/4
*(15*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^
2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt((c*d*f - a*e*g)/g)*arctan(-sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt((c*d*f - a*e
*g)/g)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) + (8*c^2*d^2*g^2*x^2 +
 15*c^2*d^2*f^2 - 5*a*c*d*e*f*g - 2*a^2*e^2*g^2 + (25*c^2*d^2*f*g - 9*a*c*
d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(e*
g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)*x^2 + (e*f^2*g^3 + 2*d*f*g^4)*x)
]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx=\text{Timed out}$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**3,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^3}\,dx$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^3), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.54

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx=\frac{2\sqrt{(ex+d)cde-cd^2e+ae^3}c^2d^2|e|}{e^2g^3}$$
$$-\frac{15\left(c^3d^3f|e|-ac^2d^2eg|e|\right)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{4\sqrt{cdfg-aeg^2}eg^3}$$
$$+\frac{7\sqrt{(ex+d)cde-cd^2e+ae^3}c^4d^4e^2f^2|e|-14\sqrt{(ex+d)cde-cd^2e+ae^3}ac^3d^3e^3fg|e|+7\sqrt{(ex+d)cde}}{4(cde^2f-ae^3g+((e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,
x, algorithm="giac")
```

output
```
2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*abs(e)/(e^2*g^3) - 15/4*
(c^3*d^3*f*abs(e) - a*c^2*d^2*e*g*abs(e))*arctan(sqrt((e*x + d)*c*d*e - c*
d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g
^3) + 1/4*(7*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*e^2*f^2*abs(e
) - 14*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^3*d^3*e^3*f*g*abs(e) +
7*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^2*d^2*e^4*g^2*abs(e) + 9*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*f*g*abs(e) - 9*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^2*d^2*e*g^2*abs(e))/((c*d*e^2*f - a*e^
3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2*g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^3\,\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(
5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.26

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx = \frac{-15\sqrt{g}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^2d^2f^2-30\sqrt{g}\,\sqrt{-aeg}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x)
```

output
```
( - 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*c**2*d**2*f**2 - 30*sqrt(g)*sqrt( - a*e*g + c*d*f
)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*f
*g*x - 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(
g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*g**2*x**2 - 2*sqrt(a*e + c*d*x)*a**2
*e**2*g**3 - 5*sqrt(a*e + c*d*x)*a*c*d*e*f*g**2 - 9*sqrt(a*e + c*d*x)*a*c*
d*e*g**3*x + 15*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g + 25*sqrt(a*e + c*d*x)*
c**2*d**2*f*g**2*x + 8*sqrt(a*e + c*d*x)*c**2*d**2*g**3*x**2)/(4*g**4*(f**
2 + 2*f*g*x + g**2*x**2))
```

3.30 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 253

$$\begin{aligned}
&\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = \\
&-\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2} \\
&-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{5c^3d^3\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}
\end{aligned}$$

output

```
-5/8*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)/(g*
x+f)-5/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g
*x+f)^2-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f
)^3+5/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a
*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(1/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.68

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}\left(8a^2e^2g^2+2acdeg(5f+13gx)+c^2d^2\left(15f^2+40fgx+\right.\right.}{(f+gx)^3}\right.}{24g^{7/2}\sqrt{d+ex}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^4),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(8*a^2*e^2*g^2 + 2*a*c*d*e*g*(5
*f + 13*g*x) + c^2*d^2*(15*f^2 + 40*f*g*x + 33*g^2*x^2)))/(f + g*x)^3) + (
15*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[
c*d*f - a*e*g]*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1249, 1249, 1249, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}dx}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^2}dx}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}\right)}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$$\Big\downarrow \text{ 1249}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}\right)}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$$\Big\downarrow \text{ 1255}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{(x\left(ae^2+cd^2\right)+ade+c}{2g(d+ex)^{3/2}(f+}\right.}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}\right)}{4g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}\right)}{6g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^4),x]
```

output
```
-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^3) + (5*c*d*(-1/2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(
d + e*x)^(3/2)*(f + g*x)^2) + (3*c*d*(-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x))) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^
(3/2)*Sqrt[c*d*f - a*e*g])))/(4*g)))/(6*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.70

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^3d^3g^3x^3+45\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^3d^3f\,g^2x^2+45\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/24*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^3*d^3*f^3+33*c^2*d^2*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*
f)*g)^(1/2)+26*a*c*d*e*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+40*
c^2*d^2*f*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+8*((a*e*g-c*d*f)*g
)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*
e)^(1/2)*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*
f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(221) = 442.

Time = 0.13 (sec) , antiderivative size = 1141, normalized size of antiderivative = 4.51

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,
x, algorithm="fricas")
```

output
```
[-1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4
*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c
^3*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*
a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
 (e*f + d*g)*x)) + 2*(15*c^3*d^3*f^3*g - 5*a*c^2*d^2*e*f^2*g^2 - 2*a^2*c*d
*e^2*f*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 2*
(20*c^3*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*d*e^2*g^4)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c*d^2*f^4*g^4 - a*d*e
*f^3*g^5 + (c*d*e*f*g^7 - a*e^2*g^8)*x^4 + (3*c*d*e*f^2*g^6 - a*d*e*g^8 +
(c*d^2 - 3*a*e^2)*f*g^7)*x^3 + 3*(c*d*e*f^3*g^5 - a*d*e*f*g^7 + (c*d^2 - a
*e^2)*f^2*g^6)*x^2 + (c*d*e*f^4*g^4 - 3*a*d*e*f^2*g^6 + (3*c*d^2 - a*e^2)*
f^3*g^5)*x), -1/24*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g
^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3
*e*f^3 + 3*c^3*d^4*f^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(-sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d^
2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) + (15*c^3*d^3*f^3*g - 5*a*c^2*d^2*
e*f^2*g^2 - 2*a^2*c*d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 33*(c^3*d^3*f*g^3 - a*c^
2*d^2*e*g^4)*x^2 + 2*(20*c^3*d^3*f^2*g^2 - 7*a*c^2*d^2*e*f*g^3 - 13*a^2*c*
d*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex+d\right)^{\frac{5}{2}}\left(gx+f\right)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^4), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.44

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx=\frac{5\,c^3d^3|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{8\sqrt{cdfg-aeg^2}eg^3}$$
$$-\frac{15\sqrt{(ex+d)cde-cd^2e+ae^3}c^5d^5e^4f^2|e|-30\sqrt{(ex+d)cde-cd^2e+ae^3}ac^4d^4e^5fg|e|+15\sqrt{(ex+d)}}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,
x, algorithm="giac")
```

output
```
5/8*c^3*d^3*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(
c*d*f*g - a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g^3) - 1/24*(15*sqrt((e*
x + d)*c*d*e - c*d^2*e + a*e^3)*c^5*d^5*e^4*f^2*abs(e) - 30*sqrt((e*x + d)
*c*d*e - c*d^2*e + a*e^3)*a*c^4*d^4*e^5*f*g*abs(e) + 15*sqrt((e*x + d)*c*d
*e - c*d^2*e + a*e^3)*a^2*c^3*d^3*e^6*g^2*abs(e) + 40*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(3/2)*c^4*d^4*e^2*f*g*abs(e) - 40*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(3/2)*a*c^3*d^3*e^3*g^2*abs(e) + 33*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)^(5/2)*c^3*d^3*g^2*abs(e))/((c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d
*e - c*d^2*e + a*e^3)*g)^3*g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx=\int \frac{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}{(f+g\,x)^4\,(d+e\,x)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(
5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.04

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx=\frac{-15\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^3d^3f^3-45\sqrt{g}\,\sqrt{-aeg}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x)
```

output
```
( - 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*c**3*d**3*f**3 - 45*sqrt(g)*sqrt( - a*e*g + c*d*f
)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d**3*f
**2*g*x - 45*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sq
rt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d**3*f*g**2*x**2 - 15*sqrt(g)*sqrt( -
a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f))
)*c**3*d**3*g**3*x**3 - 8*sqrt(a*e + c*d*x)*a**3*e**3*g**4 - 2*sqrt(a*e +
c*d*x)*a**2*c*d*e**2*f*g**3 - 26*sqrt(a*e + c*d*x)*a**2*c*d*e**2*g**4*x -
5*sqrt(a*e + c*d*x)*a*c**2*d**2*e*f**2*g**2 - 14*sqrt(a*e + c*d*x)*a*c**2*
d**2*e*f*g**3*x - 33*sqrt(a*e + c*d*x)*a*c**2*d**2*e*g**4*x**2 + 15*sqrt(a
*e + c*d*x)*c**3*d**3*f**3*g + 40*sqrt(a*e + c*d*x)*c**3*d**3*f**2*g**2*x
+ 33*sqrt(a*e + c*d*x)*c**3*d**3*f*g**3*x**2)/(24*g**4*(a*e*f**3*g + 3*a*e
*f**2*g**2*x + 3*a*e*f*g**3*x**2 + a*e*g**4*x**3 - c*d*f**4 - 3*c*d*f**3*g
*x - 3*c*d*f**2*g**2*x**2 - c*d*f*g**3*x**3))
```

3.31 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 323

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx &= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^2} \\ &+\frac{5c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3} \\ &-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}+\frac{5c^4d^4\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}\end{aligned}$$

output

```
-5/32*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)/(g
*x+f)^2+5/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c
*d*f)/(e*x+d)^(1/2)/(g*x+f)-5/24*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)/g^2/(e*x+d)^(3/2)/(g*x+f)^3-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
/g/(e*x+d)^(5/2)/(g*x+f)^4+5/64*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c
*d*f)^(3/2)
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx=\frac{c^4d^4((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(48a^3e^3g^3-8a^2cde^2g^2(f-17gx)+2ac^2d^2eg}{c^4d^4(c}\right.}{192g}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^5),x]
```

output
```
(c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(48*a^3*e^3*g^3 - 8*a^2
*c*d*e^2*g^2*(f - 17*g*x) + 2*a*c^2*d^2*e*g*(-5*f^2 - 18*f*g*x + 59*g^2*x^
2) - c^3*d^3*(15*f^3 + 55*f^2*g*x + 73*f*g^2*x^2 - 15*g^3*x^3)))/(c^4*d^4*
(c*d*f - a*e*g)*(a*e + c*d*x)^2*(f + g*x)^4) + (15*ArcTan[(Sqrt[g]*Sqrt[a*
e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(3/2)*(a*e + c*d*x)^(5/
2))))/(192*g^(7/2)*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1249, 1249, 1249, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}dx}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\left(\frac{cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^3}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$\Big\downarrow \; 1249$$

$$\frac{5cd\left(\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$\Big\downarrow \; 1254$$

$$\frac{5cd\left(\frac{cd\left(\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{3g(d+ex)^{3/2}(f+gx)^3}\right.}{8g}$$

$$-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$\Big\downarrow \; 1255$$

$$\frac{5cd\left(\frac{cd\left(\frac{cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}{2g\sqrt{d+ex}(f+gx)^2}}{2g}}{8g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

$$\Big\downarrow \ 218$$

$$\frac{5cd\left(\frac{cd\left(\frac{cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}\right)}{2g}-\frac{(x(ae^2+cd^2)+ade+cdex}{3g(d+ex)^{3/2}(f+gx)}}{8g}$$

$$\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^5),x]
```

output
```
-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^4) + (5*c*d*(-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(
d + e*x)^(3/2)*(f + g*x)^3) + (c*d*(-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)^2) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan
[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g
]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*g)))/(2*g)))/(8*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(285) = 570.

Time = 2.91 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.03

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{4}d^{4}g^{4}x^{4}+60\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{4}d^{4}f\,g^{3}x^{3}+90\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{4}}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x,meth
od=_RETURNVERBOSE)
```

output
```
1/192*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^4*d^4*f^3*g*x-15*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*
e*g-c*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^4*d^4*f^4-118*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)+73*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-136*a
^2*c*d*e^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*a*c^2*d^2*e*
f*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+55*c^3*d^3*f^2*g*x*(c*d*
x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-48*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e
)^(1/2)*a^3*e^3*g^3+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^
2*f*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+15*
((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/(c*d*
x+a*e)^(1/2)/(a*e*g-c*d*f)/g^3/(g*x+f)^4/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(285) = 570$.

Time = 0.32 (sec) , antiderivative size = 1863, normalized size of antiderivative = 5.77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,
x, algorithm="fricas")
```

output
```
[1/384*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 + (4*c^4*d^4*e*f*g^3 + c^4*d^5
*g^4)*x^4 + 2*(3*c^4*d^4*e*f^2*g^2 + 2*c^4*d^5*f*g^3)*x^3 + 2*(2*c^4*d^4*e
*f^3*g + 3*c^4*d^5*f^2*g^2)*x^2 + (c^4*d^4*e*f^4 + 4*c^4*d^5*f^3*g)*x)*sqr
t(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f -
 (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2
*(15*c^4*d^4*f^4*g - 5*a*c^3*d^3*e*f^3*g^2 - 2*a^2*c^2*d^2*e^2*f^2*g^3 - 5
6*a^3*c*d*e^3*f*g^4 + 48*a^4*e^4*g^5 - 15*(c^4*d^4*f*g^4 - a*c^3*d^3*e*g^5
)*x^3 + (73*c^4*d^4*f^2*g^3 - 191*a*c^3*d^3*e*f*g^4 + 118*a^2*c^2*d^2*e^2*
g^5)*x^2 + (55*c^4*d^4*f^3*g^2 - 19*a*c^3*d^3*e*f^2*g^3 - 172*a^2*c^2*d^2*
e^2*f*g^4 + 136*a^3*c*d*e^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*sqrt(e*x + d))/(c^2*d^3*f^6*g^4 - 2*a*c*d^2*e*f^5*g^5 + a^2*d*e^2*f^
4*g^6 + (c^2*d^2*e*f^2*g^8 - 2*a*c*d*e^2*f*g^9 + a^2*e^3*g^10)*x^5 + (4*c^
2*d^2*e*f^3*g^7 + a^2*d*e^2*g^10 + (c^2*d^3 - 8*a*c*d*e^2)*f^2*g^8 - 2*(a*
c*d^2*e - 2*a^2*e^3)*f*g^9)*x^4 + 2*(3*c^2*d^2*e*f^4*g^6 + 2*a^2*d*e^2*f*g
^9 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^7 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g^8
)*x^3 + 2*(2*c^2*d^2*e*f^5*g^5 + 3*a^2*d*e^2*f^2*g^8 + (3*c^2*d^3 - 4*a*c*
d*e^2)*f^4*g^6 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^7)*x^2 + (c^2*d^2*e*f^6*g
^4 + 4*a^2*d*e^2*f^3*g^7 + 2*(2*c^2*d^3 - a*c*d*e^2)*f^5*g^5 - (8*a*c*d^2*
e - a^2*e^3)*f^4*g^6)*x), -1/192*(15*(c^4*d^4*e*g^4*x^5 + c^4*d^5*f^4 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{5/2}}{(d + ex)^{5/2}(f + gx)^5}\, dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^5}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^5), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.75

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^5}\,dx = \frac{5\,c^4d^4|e|\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2e}}\right)}{64\,(cdfg^3 - aeg^4)\sqrt{cdfg - aeg^2e}}$$

$$- \frac{15\,\sqrt{(ex+d)cde - cd^2e + ae^3}c^7d^7e^6f^3|e| - 45\,\sqrt{(ex+d)cde - cd^2e + ae^3}ac^6d^6e^7f^2g|e| + 45\,\sqrt{(ex + d}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,
x, algorithm="giac")
```

output
```
5/64*c^4*d^4*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt
(c*d*f*g - a*e*g^2)*e))/((c*d*f*g^3 - a*e*g^4)*sqrt(c*d*f*g - a*e*g^2)*e)
- 1/192*(15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^7*d^7*e^6*f^3*abs(e)
 - 45*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^6*d^6*e^7*f^2*g*abs(e) +
 45*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^5*d^5*e^8*f*g^2*abs(e) -
 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^4*d^4*e^9*g^3*abs(e) + 5
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^6*d^6*e^4*f^2*g*abs(e) - 110
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^5*d^5*e^5*f*g^2*abs(e) + 55
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^4*d^4*e^6*g^3*abs(e) + 73
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^5*d^5*e^2*f*g^2*abs(e) - 73*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^4*d^4*e^3*g^3*abs(e) - 15*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^4*d^4*g^3*abs(e))/((c*d*f*g^3 - a
*e*g^4)*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(f+g\,x\right)}^5\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(
5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 850, normalized size of antiderivative = 2.63

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx=\frac{15\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^4d^4f^4+60\sqrt{g}\,\sqrt{-aeg+}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x)
```

output

```
(15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqr
t( - a*e*g + c*d*f)))*c**4*d**4*f**4 + 60*sqrt(g)*sqrt( - a*e*g + c*d*f)*a
tan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**4*d**4*f**3
*g*x + 90*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(
g)*sqrt( - a*e*g + c*d*f)))*c**4*d**4*f**2*g**2*x**2 + 60*sqrt(g)*sqrt( -
a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f))
)*c**4*d**4*f*g**3*x**3 + 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e
 + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**4*d**4*g**4*x**4 - 48*sq
rt(a*e + c*d*x)*a**4*e**4*g**5 + 56*sqrt(a*e + c*d*x)*a**3*c*d*e**3*f*g**4
 - 136*sqrt(a*e + c*d*x)*a**3*c*d*e**3*g**5*x + 2*sqrt(a*e + c*d*x)*a**2*c
**2*d**2*e**2*f**2*g**3 + 172*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*f*g**4
*x - 118*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*g**5*x**2 + 5*sqrt(a*e + c*
d*x)*a*c**3*d**3*e*f**3*g**2 + 19*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f**2*g**
3*x + 191*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f*g**4*x**2 - 15*sqrt(a*e + c*d*
x)*a*c**3*d**3*e*g**5*x**3 - 15*sqrt(a*e + c*d*x)*c**4*d**4*f**4*g - 55*sq
rt(a*e + c*d*x)*c**4*d**4*f**3*g**2*x - 73*sqrt(a*e + c*d*x)*c**4*d**4*f**
2*g**3*x**2 + 15*sqrt(a*e + c*d*x)*c**4*d**4*f*g**4*x**3)/(192*g**4*(a**2*
e**2*f**4*g**2 + 4*a**2*e**2*f**3*g**3*x + 6*a**2*e**2*f**2*g**4*x**2 + 4*
a**2*e**2*f*g**5*x**3 + a**2*e**2*g**6*x**4 - 2*a*c*d*e*f**5*g - 8*a*c*d*e
*f**4*g**2*x - 12*a*c*d*e*f**3*g**3*x**2 - 8*a*c*d*e*f**2*g**4*x**3 - 2...
```

3.32 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 393

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx &= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^2}\\ &\quad+\frac{3c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)}-\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}\\ &\quad-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}+\frac{3c^5d^5\arctan\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}}\end{aligned}$$

output

```
-1/16*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)/(g
*x+f)^3+1/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c
*d*f)/(e*x+d)^(1/2)/(g*x+f)^2+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/g^3/(-a*e*g+c*d*f)^2/(e*x+d)^(1/2)/(g*x+f)-1/8*c*d*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^4-1/5*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^5+3/128*c^5*d^5*arctan(g^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)
^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(5/2)
```

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.77

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \frac{c^5d^5((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(-128a^4e^4g^4+16a^3cde^3g^3(11f-21gx)-8a^2}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^6),x]
```

output
```
(c^5*d^5*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(-128*a^4*e^4*g^4 + 16*
a^3*c*d*e^3*g^3*(11*f - 21*g*x) - 8*a^2*c^2*d^2*e^2*g^2*(f^2 - 64*f*g*x +
31*g^2*x^2) - 2*a*c^3*d^3*e*g*(5*f^3 + 23*f^2*g*x - 233*f*g^2*x^2 + 5*g^3*
x^3) + c^4*d^4*(-15*f^4 - 70*f^3*g*x - 128*f^2*g^2*x^2 + 70*f*g^3*x^3 + 15
*g^4*x^4)))/(c^5*d^5*(c*d*f - a*e*g)^2*(a*e + c*d*x)^2*(f + g*x)^5) + (15*
ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^
(5/2)*(a*e + c*d*x)^(5/2))))/(640*g^(7/2)*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1249, 1249, 1249, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}dx}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^4}dx}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\Big\downarrow \ 1249$$

$$\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}\right)}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\Big\downarrow \ 1254$$

$$\frac{cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right)}{8g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex}{4g(d+ex)^{3/2}(f+gx)}\right.}{2g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\Big\downarrow \ 1254$$

$$cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}\right.}{8g}\right.}{2g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\Big\downarrow \; 1255$$

$$cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cde^2\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ad}}{2\sqrt{d+ex}(f+gx)^2(c}}{6g}}{8g}}{2g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

$$\Big\downarrow\ 218$$

$$cd\left(\frac{3cd\left(\frac{cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{3g\sqrt{d+ex}(f+gx)^3}}{8g}}{2g}\right.$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^6),x]
```

output

```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^5) + (c*d*(-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d
+ e*x)^(3/2)*(f + g*x)^4) + (3*c*d*(-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)^3) + (c*d*(Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*
(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2
))))/(4*(c*d*f - a*e*g))))/(6*g)))/(8*g)))/(2*g)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(349) = 698$.

Time = 2.83 (sec) , antiderivative size = 914, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{5}d^{5}g^{5}x^{5}+75\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{5}d^{5}f\,g^{4}x^{4}+150\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x,meth
od=_RETURNVERBOSE)
```

output
```
-1/640*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^5*d^5*g^5*x^5+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^5*d^5*f*g^4*x^4+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^5*d^5*f^2*g^3*x^3+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2))*c^5*d^5*f^3*g^2*x^2-15*c^4*d^4*g^4*x^4*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^5*d^5*f^4*g*x+10*a*c^3*d^3*e*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2)-70*c^4*d^4*f*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(
1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^5+2
48*a^2*c^2*d^2*e^2*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-466*a
*c^3*d^3*e*f*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+128*c^4*d^4
*f^2*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+336*a^3*c*d*e^3*g^4
*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-512*a^2*c^2*d^2*e^2*f*g^3*x*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+46*a*c^3*d^3*e*f^2*g^2*x*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+70*c^4*d^4*f^3*g*x*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)+128*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^4*e^4
*g^4-176*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*c*d*e^3*f*g^3+8*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^3*d^3*e*f^3*g+15*(c*d*x+a*e)^(1/2)*(
(a*e*g-c*d*f)*g)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1355 vs. 2(349) = 698.

Time = 1.09 (sec) , antiderivative size = 2751, normalized size of antiderivative = 7.00

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^6}\,dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,
x, algorithm="fricas")
```

output

```
[-1/1280*(15*(c^5*d^5*e*g^5*x^6 + c^5*d^6*f^5 + (5*c^5*d^5*e*f*g^4 + c^5*d
^6*g^5)*x^5 + 5*(2*c^5*d^5*e*f^2*g^3 + c^5*d^6*f*g^4)*x^4 + 10*(c^5*d^5*e*
f^3*g^2 + c^5*d^6*f^2*g^3)*x^3 + 5*(c^5*d^5*e*f^4*g + 2*c^5*d^6*f^3*g^2)*x
^2 + (c^5*d^5*e*f^5 + 5*c^5*d^6*f^4*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c
*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e
*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(15*c^5*d^5*f^5*g - 5*a*c^4*
d^4*e*f^4*g^2 - 2*a^2*c^3*d^3*e^2*f^3*g^3 - 184*a^3*c^2*d^2*e^3*f^2*g^4 +
304*a^4*c*d*e^4*f*g^5 - 128*a^5*e^5*g^6 - 15*(c^5*d^5*f*g^5 - a*c^4*d^4*e*
g^6)*x^4 - 10*(7*c^5*d^5*f^2*g^4 - 8*a*c^4*d^4*e*f*g^5 + a^2*c^3*d^3*e^2*g
^6)*x^3 + 2*(64*c^5*d^5*f^3*g^3 - 297*a*c^4*d^4*e*f^2*g^4 + 357*a^2*c^3*d^
3*e^2*f*g^5 - 124*a^3*c^2*d^2*e^3*g^6)*x^2 + 2*(35*c^5*d^5*f^4*g^2 - 12*a*
c^4*d^4*e*f^3*g^3 - 279*a^2*c^3*d^3*e^2*f^2*g^4 + 424*a^3*c^2*d^2*e^3*f*g^
5 - 168*a^4*c*d*e^4*g^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d))/(c^3*d^4*f^8*g^4 - 3*a*c^2*d^3*e*f^7*g^5 + 3*a^2*c*d^2*e^2*f^
6*g^6 - a^3*d*e^3*f^5*g^7 + (c^3*d^3*e*f^3*g^9 - 3*a*c^2*d^2*e^2*f^2*g^10
+ 3*a^2*c*d*e^3*f*g^11 - a^3*e^4*g^12)*x^6 + (5*c^3*d^3*e*f^4*g^8 - a^3*d*
e^3*g^12 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*f^3*g^9 - 3*(a*c^2*d^3*e - 5*a^2*c
*d*e^3)*f^2*g^10 + (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*g^11)*x^5 + 5*(2*c^3*d^
3*e*f^5*g^7 - a^3*d*e^3*f*g^11 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^4*g^8 - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^6}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(349) = 698.

Time = 0.72 (sec) , antiderivative size = 824, normalized size of antiderivative = 2.10

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,
x, algorithm="giac")
```

output

```
3/128*c^5*d^5*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2*g^3 - 2*a*c*d*e*f*g^4 + a^2*e^2*g^5
)*sqrt(c*d*f*g - a*e*g^2)*e) - 1/640*(15*sqrt((e*x + d)*c*d*e - c*d^2*e +
a*e^3)*c^9*d^9*e^8*f^4*abs(e) - 60*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)
*a*c^8*d^8*e^9*f^3*g*abs(e) + 90*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a
^2*c^7*d^7*e^10*f^2*g^2*abs(e) - 60*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*a^3*c^6*d^6*e^11*f*g^3*abs(e) + 15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^
3)*a^4*c^5*d^5*e^12*g^4*abs(e) + 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3
/2)*c^8*d^8*e^6*f^3*g*abs(e) - 210*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/
2)*a*c^7*d^7*e^7*f^2*g^2*abs(e) + 210*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^2*c^6*d^6*e^8*f*g^3*abs(e) - 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3
)^(3/2)*a^3*c^5*d^5*e^9*g^4*abs(e) + 128*((e*x + d)*c*d*e - c*d^2*e + a*e^
3)^(5/2)*c^7*d^7*e^4*f^2*g^2*abs(e) - 256*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(5/2)*a*c^6*d^6*e^5*f*g^3*abs(e) + 128*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(5/2)*a^2*c^5*d^5*e^6*g^4*abs(e) - 70*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(7/2)*c^6*d^6*e^2*f*g^3*abs(e) + 70*((e*x + d)*c*d*e - c*d^2*e + a*e^
3)^(7/2)*a*c^5*d^5*e^3*g^4*abs(e) - 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(9/2)*c^5*d^5*g^4*abs(e))/((c^2*d^2*f^2*g^3 - 2*a*c*d*e*f*g^4 + a^2*e^2*g
^5)*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^6\,\left(d+e\,x\right)^{5/2}}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(
5/2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(
5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1265, normalized size of antiderivative = 3.22

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = \text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x)
```

output

```
( - 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*c**5*d**5*f**5 - 75*sqrt(g)*sqrt( - a*e*g + c*d*f
)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**5*d**5*f
**4*g*x - 150*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(s
qrt(g)*sqrt( - a*e*g + c*d*f)))*c**5*d**5*f**3*g**2*x**2 - 150*sqrt(g)*sqr
t( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*
d*f)))*c**5*d**5*f**2*g**3*x**3 - 75*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((
sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**5*d**5*f*g**4*x*
*4 - 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)
*sqrt( - a*e*g + c*d*f)))*c**5*d**5*g**5*x**5 - 128*sqrt(a*e + c*d*x)*a**5
*e**5*g**6 + 304*sqrt(a*e + c*d*x)*a**4*c*d*e**4*f*g**5 - 336*sqrt(a*e + c
*d*x)*a**4*c*d*e**4*g**6*x - 184*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**3*f**
2*g**4 + 848*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**3*f*g**5*x - 248*sqrt(a*e
 + c*d*x)*a**3*c**2*d**2*e**3*g**6*x**2 - 2*sqrt(a*e + c*d*x)*a**2*c**3*d*
*3*e**2*f**3*g**3 - 558*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*f**2*g**4*x
+ 714*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*f*g**5*x**2 - 10*sqrt(a*e + c*
d*x)*a**2*c**3*d**3*e**2*g**6*x**3 - 5*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f**
4*g**2 - 24*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f**3*g**3*x - 594*sqrt(a*e + c
*d*x)*a*c**4*d**4*e*f**2*g**4*x**2 + 80*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f*
g**5*x**3 + 15*sqrt(a*e + c*d*x)*a*c**4*d**4*e*g**6*x**4 + 15*sqrt(a*e ...
```

3.33 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 463

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx &= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} \\ &+ \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{768g^3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} \\ &+ \frac{5c^5d^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512g^3(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} \\ &- \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} + \frac{5c^6d^6\arctan\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{512g^{7/2}(cdf-aeg)^{7/2}}\end{aligned}$$

output
```
-1/32*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)/(g
*x+f)^4+1/192*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+
c*d*f)/(e*x+d)^(1/2)/(g*x+f)^3+5/768*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/g^3/(-a*e*g+c*d*f)^2/(e*x+d)^(1/2)/(g*x+f)^2+5/512*c^5*d^5*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*
x+f)-1/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g
*x+f)^5-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f
)^6+5/512*c^6*d^6*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(
-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(7/2)
```

Mathematica [A] (verified)

Time = 4.49 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.80

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx=\frac{c^6d^6((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}(256a^5e^5g^5+640a^4cde^4g^4(-f+gx)+16a^3c^2}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^7),x]
```

output
```
(c^6*d^6*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(256*a^5*e^5*g^5 + 640*
a^4*c*d*e^4*g^4*(-f + g*x) + 16*a^3*c^2*d^2*e^3*g^3*(27*f^2 - 106*f*g*x +
27*g^2*x^2) + 8*a^2*c^3*d^3*e^2*g^2*(-f^3 + 159*f^2*g*x - 159*f*g^2*x^2 +
g^3*x^3) - 2*a*c^4*d^4*e*g*(5*f^4 + 28*f^3*g*x - 594*f^2*g^2*x^2 + 28*f*g^
3*x^3 + 5*g^4*x^4) + c^5*d^5*(-15*f^5 - 85*f^4*g*x - 198*f^3*g^2*x^2 + 198
*f^2*g^3*x^3 + 85*f*g^4*x^4 + 15*g^5*x^5)))/(c^6*d^6*(c*d*f - a*e*g)^3*(a*
e + c*d*x)^2*(f + g*x)^6) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*
d*f - a*e*g]])/((c*d*f - a*e*g)^(7/2)*(a*e + c*d*x)^(5/2))))/(1536*g^(7/2)
*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1249, 1249, 1249, 1254, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}dx}{12g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \; 1249$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^5}dx}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}\right)}{12g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \; 1249$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}\right)}{10g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}\right)}{12g}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \; 1254$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\left(\frac{5cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}\right)}{10g}-\frac{(x(ae^2+cd^2)+ade+cde}{5g(d+ex)^{3/2}(f+g}}{12g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \; 1254$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}\right)}{8g}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{4g\sqrt{d+ex}(f+gx)^4}\right.}{10g}\right.}{12g}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

$$\Big\downarrow \; 1254$$

$$5cd\Bigg(\frac{3cd\Bigg(\frac{cd\Bigg(\frac{5cd\left(\frac{3cd\left(\frac{cd\int\frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade}}{3\sqrt{d+ex}(f+gx)^3(cd}}{8g}}{10g}$$

$\downarrow$ 1255

$$5cd\left(\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int\frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+c\right.}}{2\sqrt{d+ex}(f}\right.}{6(cdf-aeg)}\right.}{8g}\right.}{10g}\right.$$

↓ 218

$$5cd\left(\frac{3cd\left(\frac{cd\left(\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ad}}{3\sqrt{d+ex}(f+gx)^3(c}}{8g}}{10g}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^7),x]
```

output

```
-1/6*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d + e*x)^(5/2)*(f +
 g*x)^6) + (5*c*d*(-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(
d + e*x)^(3/2)*(f + g*x)^5) + (3*c*d*(-1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)^4) + (c*d*(Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c*
d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(
(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(
Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(6*(c*d*f - a*e*g))
))/(8*g)))/(10*g)))/(12*g)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1249

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1254

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. 2(413) = 826.

Time = 2.87 (sec) , antiderivative size = 1251, normalized size of antiderivative = 2.70

method	result	size
default	Expression too large to display	1251

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x,meth
od=_RETURNVERBOSE)
```

output

```
1/1536*((e*x+d)*(c*d*x+a*e))^(1/2)*(-85*c^5*d^5*f*g^4*x^4*(c*d*x+a*e)^(1/2
)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^6*d^6*f^6+8*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c^3*d^
3*e^2*f^3*g^2-15*c^5*d^5*g^5*x^5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)
+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^6*d^6*f*g^5*x^5
+10*a*c^4*d^4*e*g^5*x^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*arcta
nh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^6*d^6*g^6*x^6+1272*a^2*c
^3*d^3*e^2*f*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1188*a*c^4*
d^4*e*f^2*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+1696*a^3*c^2*d
^2*e^3*f*g^4*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1272*a^2*c^3*d^3*
e^2*f^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+56*a*c^4*d^4*e*f^3
*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+56*a*c^4*d^4*e*f*g^4*x^3*
(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*a^2*c^3*d^3*e^2*g^5*x^3*(c*d*x
+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-432*a^3*c^2*d^2*e^3*g^5*x^2*(c*d*x+a*e
)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-640*a^4*c*d*e^4*g^5*x*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)-256*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^5*e^
5*g^5+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^5*d^5*f^5-198*c^5*d^5
*f^2*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+198*c^5*d^5*f^3*g^2
*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+85*c^5*d^5*f^4*g*x*(c*d*x+a
*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+640*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1916 vs. $2(413) = 826$.

Time = 3.67 (sec) , antiderivative size = 3873, normalized size of antiderivative = 8.37

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,
x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**7,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^7}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^7}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. 2(413) = 826.

Time = 0.24 (sec) , antiderivative size = 1132, normalized size of antiderivative = 2.44

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,
x, algorithm="giac")
```

output
```
5/512*c^6*d^6*abs(e)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqr
t(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*f^3*g^3 - 3*a*c^2*d^2*e*f^2*g^4 + 3*a^2
*c*d*e^2*f*g^5 - a^3*e^3*g^6)*sqrt(c*d*f*g - a*e*g^2)*e) - 1/1536*(15*sqrt
((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^11*d^11*e^10*f^5*abs(e) - 75*sqrt((e
*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^10*d^10*e^11*f^4*g*abs(e) + 150*sqrt(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^9*d^9*e^12*f^3*g^2*abs(e) - 150*s
qrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^8*d^8*e^13*f^2*g^3*abs(e) + 7
5*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^4*c^7*d^7*e^14*f*g^4*abs(e) -
15*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^5*c^6*d^6*e^15*g^5*abs(e) + 8
5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^10*d^10*e^8*f^4*g*abs(e) - 3
40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^9*d^9*e^9*f^3*g^2*abs(e)
+ 510*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c^8*d^8*e^10*f^2*g^3*a
bs(e) - 340*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*c^7*d^7*e^11*f*g
^4*abs(e) + 85*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*c^6*d^6*e^12*
g^5*abs(e) + 198*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^9*d^9*e^6*f^3
*g^2*abs(e) - 594*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^8*d^8*e^7*
f^2*g^3*abs(e) + 594*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*c^7*d^7
*e^8*f*g^4*abs(e) - 198*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*c^6*
d^6*e^9*g^5*abs(e) - 198*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^8*d^8
*e^4*f^2*g^3*abs(e) + 396*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(f+g\,x\right)}^7\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(
5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(
5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 1754, normalized size of antiderivative = 3.79

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x)
```

output

```
(15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqr
t( - a*e*g + c*d*f)))*c**6*d**6*f**6 + 90*sqrt(g)*sqrt( - a*e*g + c*d*f)*a
tan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**6*d**6*f**5
*g*x + 225*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt
(g)*sqrt( - a*e*g + c*d*f)))*c**6*d**6*f**4*g**2*x**2 + 300*sqrt(g)*sqrt(
- a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f
)))*c**6*d**6*f**3*g**3*x**3 + 225*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sq
rt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**6*d**6*f**2*g**4*x
**4 + 90*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g
)*sqrt( - a*e*g + c*d*f)))*c**6*d**6*f*g**5*x**5 + 15*sqrt(g)*sqrt( - a*e*
g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*
*6*d**6*g**6*x**6 - 256*sqrt(a*e + c*d*x)*a**6*e**6*g**7 + 896*sqrt(a*e +
c*d*x)*a**5*c*d*e**5*f*g**6 - 640*sqrt(a*e + c*d*x)*a**5*c*d*e**5*g**7*x -
 1072*sqrt(a*e + c*d*x)*a**4*c**2*d**2*e**4*f**2*g**5 + 2336*sqrt(a*e + c*
d*x)*a**4*c**2*d**2*e**4*f*g**6*x - 432*sqrt(a*e + c*d*x)*a**4*c**2*d**2*e
**4*g**7*x**2 + 440*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**3*f**3*g**4 - 2968
*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**3*f**2*g**5*x + 1704*sqrt(a*e + c*d*x
)*a**3*c**3*d**3*e**3*f*g**6*x**2 - 8*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**
3*g**7*x**3 + 2*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**2*f**4*g**3 + 1328*sqr
t(a*e + c*d*x)*a**2*c**4*d**4*e**2*f**3*g**4*x - 2460*sqrt(a*e + c*d*x)...
```

3.34 $\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 230

$$\begin{aligned}
&\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= \frac{2(cdf-aeg)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{c^4d^4\sqrt{d+ex}} \\
&\quad + \frac{2g(cdf-aeg)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{c^4d^4(d+ex)^{3/2}} \\
&\quad + \frac{6g^2(cdf-aeg)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{5c^4d^4(d+ex)^{5/2}} \\
&\quad + \frac{2g^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{7/2}}{7c^4d^4(d+ex)^{7/2}}
\end{aligned}$$

output

```
2*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/(e*x+d)
^(1/2)+2*g*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^
4/(e*x+d)^(3/2)+6/5*g^2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
5/2)/c^4/d^4/(e*x+d)^(5/2)+2/7*g^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)
/c^4/d^4/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{2\sqrt{(ae+cdx)(d+ex)}(-16a^3e^3g^3+8a^2cde^2g^2(7f+gx)-2ac^2d^2eg(35f^2+14fgx+3g^2x^2)+c^3d^3(35}{35c^4d^4\sqrt{d+ex}}$$

input

```
Integrate[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2],x]
```

output

```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(7*f
 + g*x) - 2*a*c^2*d^2*e*g*(35*f^2 + 14*f*g*x + 3*g^2*x^2) + c^3*d^3*(35*f^
3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3)))/(35*c^4*d^4*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$
$$\Big\downarrow \text{ 1253}$$
$$\frac{6(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{7cd}+\frac{2(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{7cd\sqrt{d+ex}}$$
$$\Big\downarrow \text{ 1253}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}\right)}{7cd}+\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2}}{5cd\sqrt{}}\right.}{7cd}\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}+$$
$$\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}+\frac{4(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+}}{3cde}\right.}{5cd}\right.}{7cd}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
],x]
```

output
```
(2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d
+ e*x]) + (6*(c*d*f - a*e*g)*((2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(5*c*d*Sqrt[d + e*x]) + (4*(c*d*f - a*e*g)*((2*(3*f - (d*g
)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d
*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(3*c*d*e)))/(5*c*d)))/(7*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.74

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-5x^3g^3d^3c^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e^3g^3-56a^2}{35\sqrt{ex+d}\,d^4c^4}$
gosper	$-\frac{2(cdx+ae)(-5x^3g^3d^3c^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e^3g^3-56a^2cd\,e^2f\,g^2-}{35d^4c^4\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}$
orering	$-\frac{2(-5x^3g^3d^3c^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e^3g^3-56a^2cd\,e^2f\,g^2+70a\,c^2d^2}{35d^4c^4\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}$

input
```
int((e*x+d)^(1/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output

```
-2/35/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-5*c^3*d^3*g^3*x^3+6*a*c^
2*d^2*e*g^3*x^2-21*c^3*d^3*f*g^2*x^2-8*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*
g^2*x-35*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-56*a^2*c*d*e^2*f*g^2+70*a*c^2*d^2*
e*f^2*g-35*c^3*d^3*f^3)/d^4/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\,(5\,c^3d^3g^3x^3+35\,c^3d^3f^3-70\,ac^2d^2ef^2g+56\,a^2cde^2fg^2-16\,a^3e^3g^3+3\,(7\,c^3d^3fg^2-2\,ac^2d^2eg^3)x^2+(}{35\,(c^4d^4ex+c^4d^5)}$$

input

```
integrate((e*x+d)^(1/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output

```
2/35*(5*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 70*a*c^2*d^2*e*f^2*g + 56*a^2*c
*d*e^2*f*g^2 - 16*a^3*e^3*g^3 + 3*(7*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^
2 + (35*c^3*d^3*f^2*g - 28*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^4*d^4*e*x + c^4*d^
5)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}\,(f+gx)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input

```
integrate((e*x+d)**(1/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output

```
Integral(sqrt(d + e*x)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= \frac{2\,\sqrt{cdx+ae}f^3}{cd} + \frac{2\,(c^2d^2x^2-acdex-2\,a^2e^2)f^2g}{\sqrt{cdx+ae}c^2d^2} \\
&\quad + \frac{2\,(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3)fg^2}{5\,\sqrt{cdx+ae}c^3d^3} \\
&\quad + \frac{2\,(5\,c^4d^4x^4-ac^3d^3ex^3+2\,a^2c^2d^2e^2x^2-8\,a^3cde^3x-16\,a^4e^4)g^3}{35\,\sqrt{cdx+ae}c^4d^4}
\end{aligned}
$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
2*sqrt(c*d*x + a*e)*f^3/(c*d) + 2*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f^
2*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4
*a^2*c*d*e^2*x + 8*a^3*e^3)*f*g^2/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(5*c^
4*d^4*x^4 - a*c^3*d^3*e*x^3 + 2*a^2*c^2*d^2*e^2*x^2 - 8*a^3*c*d*e^3*x - 16
*a^4*e^4)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.35

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= \frac{2\,e\left(\frac{35\,(c^3d^3f^3-3\,ac^2d^2ef^2g+3\,a^2cde^2fg^2-a^3e^3g^3)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^4d^4e} + \frac{35\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^2d^2e^4f^2g-70\,((ex+d)cde-}{}\right.}{}
\end{aligned}
$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output

```
2/35*e*(35*(c^3*d^3*f^3 - 3*a*c^2*d^2*e*f^2*g + 3*a^2*c*d*e^2*f*g^2 - a^3*
e^3*g^3)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^4*d^4*e) + (35*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^4*f^2*g - 70*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(3/2)*a*c*d*e^5*f*g^2 + 35*((e*x + d)*c*d*e - c*d^2*e
+ a*e^3)^(3/2)*a^2*e^6*g^3 + 21*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*
c*d*e^2*f*g^2 - 21*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3*g^3 + 5
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*g^3)/(c^4*d^4*e^7))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-112\,a^2\,c\,d\,e^2\,f\,g^2+140\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3\right)}{35\,c^4\,d^4\,e}-\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{7\,c\,d\,e}\right.}{x+\frac{d}{e}}$$

input

```
int(((f + g*x)^3*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output

```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*
e^3*g^3 - 70*c^3*d^3*f^3 + 140*a*c^2*d^2*e*f^2*g - 112*a^2*c*d*e^2*f*g^2))
/(35*c^4*d^4*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(7*c*d*e) + (6*g^2*x^2*(2*a*
e*g - 7*c*d*f)*(d + e*x)^(1/2))/(35*c^2*d^2*e) - (2*g*x*(d + e*x)^(1/2)*(8
*a^2*e^2*g^2 + 35*c^2*d^2*f^2 - 28*a*c*d*e*f*g))/(35*c^3*d^3*e)))/(x + d/e
)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}\,(5c^3d^3g^3x^3-6a\,c^2d^2e\,g^3x^2+21c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-28a\,c^2d^2ef\,g^2x+35c^3d^3f^2gx-}{35c^4d^4}$$

input

```
int((e*x+d)^(1/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(2*sqrt(a*e + c*d*x)*( - 16*a**3*e**3*g**3 + 56*a**2*c*d*e**2*f*g**2 + 8*a
**2*c*d*e**2*g**3*x - 70*a*c**2*d**2*e*f**2*g - 28*a*c**2*d**2*e*f*g**2*x
- 6*a*c**2*d**2*e*g**3*x**2 + 35*c**3*d**3*f**3 + 35*c**3*d**3*f**2*g*x +
21*c**3*d**3*f*g**2*x**2 + 5*c**3*d**3*g**3*x**3))/(35*c**4*d**4)
```

3.35 $\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 169

$$\begin{aligned}\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= \frac{2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3\sqrt{d+ex}}\\ &\quad+\frac{4g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^3d^3(d+ex)^{3/2}}\\ &\quad+\frac{2g^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^3d^3(d+ex)^{5/2}}\end{aligned}$$

output

```
2*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)
^(1/2)+4/3*g*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^
3/(e*x+d)^(3/2)+2/5*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/(e
*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\ = \frac{2\sqrt{(ae+cdx)(d+ex)}(8a^2e^2g^2-4acdeg(5f+gx)+c^2d^2(15f^2+10fgx+3g^2x^2))}{15c^3d^3\sqrt{d+ex}}$$

input

```
Integrate[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2],x]
```

output

```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(5*f + g*x)
+ c^2*d^2*(15*f^2 + 10*f*g*x + 3*g^2*x^2)))/(15*c^3*d^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{4(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}\right)}{5cd}+\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd\sqrt{d+ex}}+\frac{4(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}\right)}{5cd}$$

input

```
Int[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
],x]
```

output

```
(2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d
+ e*x]) + (4*(c*d*f - a*e*g)*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d
+ e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(5*c*d)
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(3g^2x^2d^2c^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15f^2c^2d^2)}{15\sqrt{ex+d}\,d^3c^3}$	98
gosper	$\frac{2(cdx+ae)(3g^2x^2d^2c^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15f^2c^2d^2)\sqrt{ex+d}}{15d^3c^3\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}$	116
orering	$\frac{2(3g^2x^2d^2c^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15f^2c^2d^2)(cdx+ae)\sqrt{ex+d}}{15d^3c^3\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}$	117

input
```
int((e*x+d)^(1/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/15/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*c^2*d^2*g^2*x^2-4*a*c*d*
e*g^2*x+10*c^2*d^2*f*g*x+8*a^2*e^2*g^2-20*a*c*d*e*f*g+15*c^2*d^2*f^2)/d^3/
c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2\,(3\,c^2d^2g^2x^2+15\,c^2d^2f^2-20\,acdefg+8\,a^2e^2g^2+2\,(5\,c^2d^2fg-2\,acdeg^2)x)\sqrt{cdex^2+ade+(cd^2+ae}}{15\,(c^3d^3ex+c^3d^4)}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
2/15*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g + 8*a^2*e^2*g^2
+ 2*(5*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}\,f^2}{cd} + \frac{4\,(c^2d^2x^2-acdex-2\,a^2e^2)fg}{3\sqrt{cdx+ae}\,c^2d^2} + \frac{2\,(3\,c^3d^3x^3-ac^2d^2ex^2+4\,a^2cde^2x+8\,a^3e^3)g^2}{15\sqrt{cdx+ae}\,c^3d^3}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
2*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*
f*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 +
4*a^2*c*d*e^2*x + 8*a^3*e^3)*g^2/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\,e\left(\frac{15\,(c^2d^2f^2-2\,acdefg+a^2e^2g^2)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^3d^3e}+\frac{10\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}\,cde^2fg-10\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}\,ae^3g^2+3}{c^3d^3e^5}\right.}{15\,|e|}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
2/15*e*(15*(c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e^2*g^2)*sqrt((e*x + d)*c*d*
e - c*d^2*e + a*e^3)/(c^3*d^3*e) + (10*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(3/2)*c*d*e^2*f*g - 10*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3*g^
2 + 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*g^2)/(c^3*d^3*e^5))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\left(\frac{\sqrt{d+e\,x}\,(16\,a^2\,e^2\,g^2-40\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2)}{15\,c^3\,d^3\,e}+\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{5\,c\,d\,e}-\frac{4\,g\,x\,(2\,a\,e\,g-5\,c\,d\,f)}{15\,c^2\,d^2\,e}\right.}{x+\frac{d}{e}}$$

input
```
int(((f + g*x)^2*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e
^2*g^2 + 30*c^2*d^2*f^2 - 40*a*c*d*e*f*g))/(15*c^3*d^3*e) + (2*g^2*x^2*(d
+ e*x)^(1/2))/(5*c*d*e) - (4*g*x*(2*a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*
c^2*d^2*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{2\sqrt{cdx+ae}\,(3c^2d^2g^2x^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15c^2d^2f^2)}{15c^3d^3}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(8*a**2*e**2*g**2 - 20*a*c*d*e*f*g - 4*a*c*d*e*g**2*x
 + 15*c**2*d**2*f**2 + 10*c**2*d**2*f*g*x + 3*c**2*d**2*g**2*x**2))/(15*c*
*3*d**3)
```

3.36 $\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 106

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2\sqrt{d+ex}}+\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^2d^2(d+ex)^{3/2}}$$

output
```
2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(
1/2)+2/3*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}(-2aeg+cd(3f+gx))}{3c^2d^2\sqrt{d+ex}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(3*f + g*x)))/(3*c^2*d^2*
Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \; 1221$$

$$\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}$$

$$\Big\downarrow \; 1122$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],
x]
```

output
```
(2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*c*d*e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

method	result	size
default	$-\frac{2 \sqrt{\left(e x +d \right) \left(c d x +a e \right)}\, \left(-c d g x +2 a e g -3 d f c \right)}{3 \sqrt{e x +d}\, c^{2} d^{2}}$	49
gosper	$-\frac{2 \left(c d x +a e \right) \left(-c d g x +2 a e g -3 d f c \right) \sqrt{e x +d}}{3 c^{2} d^{2} \sqrt{c d \,x^{2} e +a \,e^{2} x +c \,d^{2} x +a d e}}$	67
orering	$-\frac{2 \left(-c d g x +2 a e g -3 d f c \right) \left(c d x +a e \right) \sqrt{e x +d}}{3 c^{2} d^{2} \sqrt{a d e +\left(a \,e^{2}+c \,d^{2}\right) x +c d \,x^{2} e}}$	68

input
```
int((e*x+d)^(1/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-c*d*g*x+2*a*e*g-3*c*d*f)/
c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx+3\,cdf-2\,aeg)\sqrt{ex+d}}{3\left(c^2d^2ex+c^2d^3\right)}$$

input

```
integrate((e*x+d)^(1/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="fricas")
```

output

```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2*a*e
*g)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input

```
integrate((e*x+d)**(1/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/
2),x)
```

output

```
Integral(sqrt(d + e*x)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}f}{cd} + \frac{2\left(c^2d^2x^2-acdex-2\,a^2e^2\right)g}{3\sqrt{cdx+ae}c^2d^2}$$

input

```
integrate((e*x+d)^(1/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="maxima")
```

output
```
2*sqrt(c*d*x + a*e)*f/(c*d) + 2/3*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*g/
(sqrt(c*d*x + a*e)*c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}(cdf-aeg)}{c^2d^2e}+\frac{\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}g}{c^2d^2e^3}\right)}{3\,|e|}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
2/3*e*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(c*d*f - a*e*g)/(c^2*d^2*
e) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*g/(c^2*d^2*e^3))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{\left(\frac{(4\,a\,e\,g-6\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}-\frac{2\,g\,x\,\sqrt{d+e\,x}}{3\,c\,d\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

input
```
int(((f + g*x)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2),x)
```

output
```
-((((4*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*c^2*d^2*e) - (2*g*x*(d + e*x)^
(1/2))/(3*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}\,(cdgx-2aeg+3cdf)}{3c^2d^2}$$

input

```
int((e*x+d)^(1/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(2*sqrt(a*e + c*d*x)*( - 2*a*e*g + 3*c*d*f + c*d*g*x))/(3*c**2*d**2)
```

3.37 $\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}}{cd\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)])/(c*d*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

input
```
Int[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}}{\sqrt{ex+d}\,cd}$	32
gosper	$\frac{2(cdx+ae)\sqrt{ex+d}}{cd\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}$	50
orering	$\frac{2(cdx+ae)\sqrt{ex+d}}{cd\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}$	51

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/c/d
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{cdex+cd^2}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c*d*e*x + c*d
^2)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{cdx+ae}}{cd}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2*sqrt(c*d*x + a*e)/(c*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ex+d)cde-cd^2e+ae^3}}{cd|e|}$$

input
```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c*d*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c\,d\,e\,\left(x+\frac{d}{e}\right)}$$

input
```
int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c*d*e*(
x + d/e))
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{cdx+ae}}{cd}$$

input
```
int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x))/(c*d)
```

3.38 $\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 80

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

output
```
-2*arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*g+c*d*f)
^(1/2)*(e*x+d)^(1/2))/g^(1/2)/(-a*e*g+c*d*f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output

```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt
[c*d*f - a*e*g]])/(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x
)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1255$$

$$2e^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}$$

$$\Big\downarrow \ 218$$

$$\frac{2\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

input

```
Int[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),
x]
```

output

```
(2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*
f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*Sqrt[c*d*f - a*e*g])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{(aeg-dfc)g}}$	77

input
```
int((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2)*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.16

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\left[-\frac{\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-(cdef-(cd^2+2\,ae^2)g)x-2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{-cdfg+aeg^2}\sqrt{ex+d}}{egx^2+df+(ef+dg)x}\right)}{cdfg-aeg^2},\right.$$
$$\left.-\frac{2\,\arctan\left(-\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cd^2f-adeg+(cdef-ae^2g)x}\right)}{\sqrt{cdfg-aeg^2}}\right]$$

input

```
integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="fricas")
```

output

```
[-sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*
e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)
)/(c*d*f*g - a*e*g^2), -2*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d^2*f - a*d*e*g + (c*d*e*f -
a*e^2*g)*x))/sqrt(c*d*f*g - a*e*g^2)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)}\,dx$$

input

```
integrate((e*x+d)**(1/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/
2),x)
```

output

```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)}\,dx$$

input

```
integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="maxima")
```

output

```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{\sqrt{cdfg-aeg^2}|e|}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
2*e*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g
^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{(f+g\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)}{g\,(aeg-cdf)}$$

input

```
int((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 2*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*s
qrt( - a*e*g + c*d*f))))/(g*(a*e*g - c*d*f))
```

3.39 $\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 141

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}(f+gx)} - \frac{cd\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}$$

output

```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(g*x+
f)-c*d*arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*g+c*
d*f)^(1/2)*(e*x+d)^(1/2))/g^(1/2)/(-a*e*g+c*d*f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}(ae+cdx)+cd\sqrt{ae+cdx}(f+gx)\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{\sqrt{g}(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)}$$

input

```
Integrate[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]),x]
```

output

```
(Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*Sqrt[a*e +
 c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])
)/(Sqrt[g]*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]
*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2)
)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\,\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)cdgx+\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-dfc)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,(aeg-dfc)(gx+f)\sqrt{(aeg-dfc)g}}$	158

input

```
int((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output

```
((e*x+d)*(c*d*x+a*e))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c*d*g*x+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f
-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2
)/(a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(125) = 250$.

Time = 0.10 (sec) , antiderivative size = 704, normalized size of antiderivative = 4.99

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \left[\frac{(cdegx^2+cd^2f+(cdef+cd^2g)x)\sqrt{-cdfg+aeg^2}\log\left(-\frac{cdegx^2-cd^2f+2\,adeg-(cdef-(cd^2+2\,ae^2)g)x+2\,\sqrt{cdex^2+c}}{egx^2+df+(ef+dg)x}\right.}{2\,(c^2d^3f^3g-2\,acd^2ef^2g^2+a^2de^2fg^3+(c^2d^2ef^2g^2-2\,acde^2fg^3+a^2e^3g^4)x^2+(c^2}\right.$$

$$-\frac{(cdegx^2+cd^2f+(cdef+cd^2g)x)\sqrt{cdfg-aeg^2}\arctan\left(-\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdfg-aeg^2}\sqrt{ex+d}}{cd^2f-adeg+(cdef-ae^2g)x}\right)-\sqrt{}}{c^2d^3f^3g-2\,acd^2ef^2g^2+a^2de^2fg^3+(c^2d^2ef^2g^2-2\,acde^2fg^3+a^2e^3g^4)x^2+(c^2d^2ef^3g+a^2de^2g^4}$$

input

```
integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
[1/2*((c*d*e*g*x^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(-c*d*f*g + a*e*
g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2
)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e
*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^
3*g - 2*a*c*d^2*e*f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d
*e^2*f*g^3 + a^2*e^3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^
3 - 2*a*c*d*e^2)*f^2*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x), -((c*d*e*g*x
^2 + c*d^2*f + (c*d*e*f + c*d^2*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(-sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x +
 d)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) - sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d))/(c^2*d^3*f^3*g - 2
*a*c*d^2*e*f^2*g^2 + a^2*d*e^2*f*g^3 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*
g^3 + a^2*e^3*g^4)*x^2 + (c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 2*a
*c*d*e^2)*f^2*g^2 - (2*a*c*d^2*e - a^2*e^3)*f*g^3)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=e^2\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,cd}{(cde^2f-ae^3g+((ex+d)cde-cd^2e+ae^3)g)(cdf|e|-aeg|e|)}+\frac{cd\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}\,g}{\sqrt{cdfg-aeg^2}\,e}\right)}{\sqrt{cdfg-aeg^2}\,(cdf|e|-aeg|e|)e}\right)$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
e^2*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d/((c*d*e^2*f - a*e^3*g + (
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*(c*d*f*abs(e) - a*e*g*abs(e))) + c*d
*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)
*e))/(sqrt(c*d*f*g - a*e*g^2)*(c*d*f*abs(e) - a*e*g*abs(e))*e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output

```
int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{\sqrt{g}\,\sqrt{-aeg+cdf}\ atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)cdf+\sqrt{g}\,\sqrt{-aeg+cdf}\ atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)cdgx-\sqrt{cdx+ae}\,ae}{g\left(a^2e^2g^3x-2acdef\,g^2x+c^2d^2f^2gx+a^2e^2f\,g^2-2acde\,f^2g+c^2d^2f^3\right)}$$

input

```
int((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt(
- a*e*g + c*d*f)))*c*d*f + sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e +
 c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*d*g*x - sqrt(a*e + c*d*x)*a
*e*g**2 + sqrt(a*e + c*d*x)*c*d*f*g)/(g*(a**2*e**2*f*g**2 + a**2*e**2*g**3
*x - 2*a*c*d*e*f**2*g - 2*a*c*d*e*f*g**2*x + c**2*d**2*f**3 + c**2*d**2*f*
*2*g*x))
```

3.40 $\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 213

$$\begin{aligned}\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}(f+gx)^2}+\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}(f+gx)} \\ &\quad -\frac{3c^2d^2\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4\sqrt{g}(cdf-aeg)^{5/2}}\end{aligned}$$

output
```
1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(
g*x+f)^2+3/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/
(e*x+d)^(1/2)/(g*x+f)-3/4*c^2*d^2*arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)*(-a*e*g+c*d*f)^(1/2)*(e*x+d)^(1/2))/g^(1/2)/(-a*e*g+c*d*f
)^(5/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{d+ex}\left(\sqrt{g}\sqrt{cdf-aeg}(ae+cdx)(-2aeg+cd(5f+3gx))+3c^2d^2\sqrt{ae+cdx}(f+gx)^2\arctan\left(\frac{\sqrt{g}\sqrt{ae}}{\sqrt{cdf-}}\right.\right.}{4\sqrt{g}(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^2}$$

input

```
Integrate[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]),x]
```

output

```
(Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x)*(-2*a*e*g + c*d*
(5*f + 3*g*x)) + 3*c^2*d^2*Sqrt[a*e + c*d*x]*(f + g*x)^2*ArcTan[(Sqrt[g]*S
qrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2)*
Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$
$$\Big\downarrow \text{1254}$$
$$\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$
$$\Big\downarrow \text{1254}$$

$$\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

input

```
Int[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

output

```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*
x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*
d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqr
t[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^2d^2g^2x^2+6\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^2d^2fgx+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^2d^2f}{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,(aeg-dfc)^2(gx+f)^2\sqrt{(}}$

input
```
int((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output

```
-1/4/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*g^2*x^2+6*arctanh(g*(c*d*x+a*e)^(1/2)
/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2))*c^2*d^2*f^2-3*c*d*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f
)*g)^(1/2)+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g-5*((a*e*g-c*d
*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^2/(g
*x+f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(187) = 374$.

Time = 0.12 (sec) , antiderivative size = 1284, normalized size of antiderivative = 6.03

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output

```
[-1/8*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2
)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(
c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(
e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(5*c^2*d^2*f^2*g - 7*a*c*d*
e*f*g^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g - 3*a*c^2*d^3
*e*f^4*g^2 + 3*a^2*c*d^2*e^2*f^3*g^3 - a^3*d*e^3*f^2*g^4 + (c^3*d^3*e*f^3*
g^3 - 3*a*c^2*d^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^3 + (
2*c^3*d^3*e*f^4*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f^3*g^3
- 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^4 + (3*a^2*c*d^2*e^2 - 2*a^3*e^4)*
f*g^5)*x^2 + (c^3*d^3*e*f^5*g - 2*a^3*d*e^3*f*g^5 + (2*c^3*d^4 - 3*a*c^2*d
^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (6*a^2*c*d^2*e
^2 - a^3*e^4)*f^2*g^4)*x), -1/4*(3*(c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c
^2*d^2*e*f*g + c^2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*sqrt(
c*d*f*g - a*e*g^2)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g
)*x)) - (5*c^2*d^2*f^2*g - 7*a*c*d*e*f*g^2 + 2*a^2*e^2*g^3 + 3*(c^2*d^2*f*
g^2 - a*c*d*e*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x
 + d))/(c^3*d^4*f^5*g - 3*a*c^2*d^3*e*f^4*g^2 + 3*a^2*c*d^2*e^2*f^3*g^3...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^3}\,dx$$

input

```
integrate((e*x+d)**(1/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output

```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^3}\,dx$$

input

```
integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output

```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{1}{4}\left(\frac{3\,c^2d^2\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\sqrt{cdfg-aeg^2}e} + \frac{5\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^3d^3e^2f-5}{(c^2d^2ef^2|e|-2\,acde^2fg|e|}\right.$$

input

```
integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output

```
1/4*(3*c^2*d^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e))/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e
^3*g^2*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) + (5*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*c^3*d^3*e^2*f - 5*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c
^2*d^2*e^3*g + 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*g)/((c^
2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e^3*g^2*abs(e))*(c*d*e^2
*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2))*e^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{-3\sqrt{g}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^2d^2f^2-6\sqrt{g}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^2d^2fgx-3}{4g\left(a^3e^3g^5x^2-3a^2cd\,e^2f\,g^4x^2+3a\,c^2d^2e\,f^2g^3x^2-c^3d^3f^3g\right.}$$

input
```
int((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*s
qrt( - a*e*g + c*d*f)))*c**2*d**2*f**2 - 6*sqrt(g)*sqrt( - a*e*g + c*d*f)*
atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*f*g
*x - 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*c**2*d**2*g**2*x**2 - 2*sqrt(a*e + c*d*x)*a**2*e*
*2*g**3 + 7*sqrt(a*e + c*d*x)*a*c*d*e*f*g**2 + 3*sqrt(a*e + c*d*x)*a*c*d*e
*g**3*x - 5*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g - 3*sqrt(a*e + c*d*x)*c**2*
d**2*f*g**2*x)/(4*g*(a**3*e**3*f**2*g**3 + 2*a**3*e**3*f*g**4*x + a**3*e**
3*g**5*x**2 - 3*a**2*c*d*e**2*f**3*g**2 - 6*a**2*c*d*e**2*f**2*g**3*x - 3*
a**2*c*d*e**2*f*g**4*x**2 + 3*a*c**2*d**2*e*f**4*g + 6*a*c**2*d**2*e*f**3*
g**2*x + 3*a*c**2*d**2*e*f**2*g**3*x**2 - c**3*d**3*f**5 - 2*c**3*d**3*f**
4*g*x - c**3*d**3*f**3*g**2*x**2))
```

3.41 $\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 280

$$\begin{aligned}
&\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx \\
&= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^3} + \frac{5cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} \\
&\quad + \frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8(cdf-aeg)^3\sqrt{d+ex}(f+gx)} - \frac{5c^3d^3\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{g}(cdf-aeg)^{7/2}}
\end{aligned}$$

output
```
1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(
g*x+f)^3+5/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2
/(e*x+d)^(1/2)/(g*x+f)^2+5/8*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)-5/8*c^3*d^3*arctan(1/g^(1/2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*g+c*d*f)^(1/2)*(e*x+d)^(1/2))/g
^(1/2)/(-a*e*g+c*d*f)^(7/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{c^3d^3\sqrt{d+ex}\left(\frac{(ae+cdx)\left(8a^2e^2g^2-2acdeg(13f+5gx)+c^2d^2\left(33f^2+40fgx+15g^2x^2\right)\right)}{c^3d^3(cdf-aeg)^3(f+gx)^3}+\frac{15\sqrt{ae+cdx}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{7/2}}\right)}{24\sqrt{(ae+cdx)(d+ex)}}$$

input

```
Integrate[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]),x]
```

output

```
(c^3*d^3*Sqrt[d + e*x]*(((a*e + c*d*x)*(8*a^2*e^2*g^2 - 2*a*c*d*e*g*(13*f
+ 5*g*x) + c^2*d^2*(33*f^2 + 40*f*g*x + 15*g^2*x^2)))/(c^3*d^3*(c*d*f - a*
e*g)^3*(f + g*x)^3) + (15*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d
*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[g]*(c*d*f - a*e*g)^(7/2))))/(24*Sqrt[(a*e
 + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1254, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \ 1254$$

$$\frac{5cd\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6(cdf-aeg)}+\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \; 1254$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \; 1255$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+}}{2\sqrt{d+ex}(f+gx)^2(cdf}}{6(cdf-aeg)}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow \; 218$$

$$\frac{5cd\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

input

```
Int[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

output

```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*
x]*(f + g*x)^3) + (5*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(
c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*Arc
Tan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*
e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g)
)))/(6*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.57

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\,\bigg(15\,\mathrm{arctanh}\bigg(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\bigg)c^3d^3g^3x^3+45\,\mathrm{arctanh}\bigg(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\bigg)c^3d^3fg^2x^2+45\,\mathrm{arctanh}\bigg(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\bigg)c^3}{}$

input
```
int((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/24*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f^3-15*c^2*d^2*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f
)*g)^(1/2)+10*a*c*d*e*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-40*c
^2*d^2*f*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*((a*e*g-c*d*f)*g)
^(1/2)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+26*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e
)^(1/2)*a*c*d*e*f*g-33*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f
^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^3/(g*x+f)^3/((a*e*g-c*d*
f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(248) = 496$.

Time = 0.49 (sec) , antiderivative size = 2028, normalized size of antiderivative = 7.24

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
[1/48*(15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g^2 + c^3*d^4*
g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + (c^3*d^3*e*f^3 + 3*c^
3*d^4*f^2*g)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
(e*f + d*g)*x)) + 2*(33*c^3*d^3*f^3*g - 59*a*c^2*d^2*e*f^2*g^2 + 34*a^2*c*
d*e^2*f*g^3 - 8*a^3*e^3*g^4 + 15*(c^3*d^3*f*g^3 - a*c^2*d^2*e*g^4)*x^2 + 1
0*(4*c^3*d^3*f^2*g^2 - 5*a*c^2*d^2*e*f*g^3 + a^2*c*d*e^2*g^4)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g - 4*a*c^3
*d^4*e*f^6*g^2 + 6*a^2*c^2*d^3*e^2*f^5*g^3 - 4*a^3*c*d^2*e^3*f^4*g^4 + a^4
*d*e^4*f^3*g^5 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a^2*c^2*
d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^4 + (3*c^4*d^4*e*f^
5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^4 - 2*(2*a*c^3*
d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^5 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f
^2*g^6 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^7)*x^3 + 3*(c^4*d^4*e*f^6*g^2 +
 a^4*d*e^4*f*g^7 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^5*g^3 - 2*(2*a*c^3*d^4*e
- 3*a^2*c^2*d^2*e^3)*f^4*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g
^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^6)*x^2 + (c^4*d^4*e*f^7*g + 3*a^4*d
*e^4*f^2*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^2 - 6*(2*a*c^3*d^4*e -
a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^4}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^4}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^4), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{1}{24}\left(\frac{15\,c^3d^3\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2e}}\right)}{(c^3d^3e^2f^3|e|-3\,ac^2d^2e^3f^2g|e|+3\,a^2cde^4fg^2|e|-a^3e^5g^3|e|)\sqrt{cdfg-aeg^2e}}+\frac{33\,\sqrt{(ex+d)cde-}}{}\right.$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
1/24*(15*c^3*d^3*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*
d*f*g - a*e*g^2)*e))/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(
e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt(c*d*f*g - a*e*g
^2)*e) + (33*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^5*d^5*e^4*f^2 - 66*
sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^4*d^4*e^5*f*g + 33*sqrt((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^3*d^3*e^6*g^2 + 40*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(3/2)*c^4*d^4*e^2*f*g - 40*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*a*c^3*d^3*e^3*g^2 + 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2
)*c^3*d^3*g^2)/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3
*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*(c*d*e^2*f - a*e^3*g + ((e
*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^3))*e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.77

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{15\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^3d^3f^3+45\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c^3d^3f^2gx+45}{24g\left(a^4e^4g^7x^3-4a^3cd\,e^3f\,g^6x^3+6a^2c^2d^2e^2f\right.}$$

input

```
int((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(15*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqr
t( - a*e*g + c*d*f)))*c**3*d**3*f**3 + 45*sqrt(g)*sqrt( - a*e*g + c*d*f)*a
tan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d**3*f**2
*g*x + 45*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(
g)*sqrt( - a*e*g + c*d*f)))*c**3*d**3*f*g**2*x**2 + 15*sqrt(g)*sqrt( - a*e
*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c
**3*d**3*g**3*x**3 - 8*sqrt(a*e + c*d*x)*a**3*e**3*g**4 + 34*sqrt(a*e + c*
d*x)*a**2*c*d*e**2*f*g**3 + 10*sqrt(a*e + c*d*x)*a**2*c*d*e**2*g**4*x - 59
*sqrt(a*e + c*d*x)*a*c**2*d**2*e*f**2*g**2 - 50*sqrt(a*e + c*d*x)*a*c**2*d
**2*e*f*g**3*x - 15*sqrt(a*e + c*d*x)*a*c**2*d**2*e*g**4*x**2 + 33*sqrt(a*
e + c*d*x)*c**3*d**3*f**3*g + 40*sqrt(a*e + c*d*x)*c**3*d**3*f**2*g**2*x +
 15*sqrt(a*e + c*d*x)*c**3*d**3*f*g**3*x**2)/(24*g*(a**4*e**4*f**3*g**4 +
3*a**4*e**4*f**2*g**5*x + 3*a**4*e**4*f*g**6*x**2 + a**4*e**4*g**7*x**3 -
4*a**3*c*d*e**3*f**4*g**3 - 12*a**3*c*d*e**3*f**3*g**4*x - 12*a**3*c*d*e**
3*f**2*g**5*x**2 - 4*a**3*c*d*e**3*f*g**6*x**3 + 6*a**2*c**2*d**2*e**2*f**
5*g**2 + 18*a**2*c**2*d**2*e**2*f**4*g**3*x + 18*a**2*c**2*d**2*e**2*f**3*
g**4*x**2 + 6*a**2*c**2*d**2*e**2*f**2*g**5*x**3 - 4*a*c**3*d**3*e*f**6*g
- 12*a*c**3*d**3*e*f**5*g**2*x - 12*a*c**3*d**3*e*f**4*g**3*x**2 - 4*a*c**
3*d**3*e*f**3*g**4*x**3 + c**4*d**4*f**7 + 3*c**4*d**4*f**6*g*x + 3*c**4*d
**4*f**5*g**2*x**2 + c**4*d**4*f**4*g**3*x**3))
```

3.42 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 228

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(cdf-aeg)^3\sqrt{d+ex}}{c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{6g(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^4d^4\sqrt{d+ex}} + \frac{2g^2(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{c^4d^4(d+ex)^{3/2}} + \frac{2g^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^4d^4(d+ex)^{5/2}}$$

output

```
-2*(-a*e*g+c*d*f)^3*(e*x+d)^(1/2)/c^4/d^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+6*g*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d
^4/(e*x+d)^(1/2)+2*g^2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)/c^4/d^4/(e*x+d)^(3/2)+2/5*g^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/
c^4/d^4/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\sqrt{d+ex}(16a^3e^3g^3+8a^2cde^2g^2(-5f+gx)-2ac^2d^2eg(-15f^2}{5c^4d^4\sqrt{(ae+cd}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2),x]
```

output
```
(2*Sqrt[d + e*x]*(16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(-5*f + g*x) - 2*a*c^
2*d^2*e*g*(-15*f^2 + 10*f*g*x + g^2*x^2) + c^3*d^3*(-5*f^3 + 15*f^2*g*x +
5*f*g^2*x^2 + g^3*x^3)))/(5*c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1251, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1251}$$

$$\frac{6g\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1253}$$

$$\frac{6g\left(\frac{4(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{6g\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+}}{5cd\sqrt{d+ex}}\right.}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{6g\left(\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}+\frac{4(cdf-aeg)\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{5cd}\right)}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (6*g*((2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(5*c*d*Sqrt[d + e*x]) + (4*(c*d*f - a*e*g)*((2*(3*f - (d*g)/e - (2*a*e*g)
/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x])
 + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*
e)))/(5*c*d)))/(c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.79

method	result
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(x^3g^3d^3c^3-2a\,c^2d^2e\,g^3x^2+5c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-20a\,c^2d^2ef\,g^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2cd\,e^2f}{5\sqrt{ex+d}\,(cdx+ae)c^4d^4}$
gosper	$\frac{2(cdx+ae)\,(x^3g^3d^3c^3-2a\,c^2d^2e\,g^3x^2+5c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-20a\,c^2d^2ef\,g^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2cd\,e^2f\,g^2+30a\,c^2}{5d^4c^4(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$
orering	$\frac{2(x^3g^3d^3c^3-2a\,c^2d^2e\,g^3x^2+5c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-20a\,c^2d^2ef\,g^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2cd\,e^2f\,g^2+30a\,c^2d^2e\,f^2g-}{5d^4c^4(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/5/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(c^3*d^3*g^3*x^3-2*a*c^2*d^2
*e*g^3*x^2+5*c^3*d^3*f*g^2*x^2+8*a^2*c*d*e^2*g^3*x-20*a*c^2*d^2*e*f*g^2*x+
15*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-40*a^2*c*d*e^2*f*g^2+30*a*c^2*d^2*e*f^2*
g-5*c^3*d^3*f^3)/(c*d*x+a*e)/c^4/d^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(c^3d^3g^3x^3-5\,c^3d^3f^3+30\,ac^2d^2ef^2g-40\,a^2cde^2fg^2+16\,a^3e^3g\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output
```
2/5*(c^3*d^3*g^3*x^3 - 5*c^3*d^3*f^3 + 30*a*c^2*d^2*e*f^2*g - 40*a^2*c*d*e
^2*f*g^2 + 16*a^3*e^3*g^3 + (5*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 + (1
5*c^3*d^3*f^2*g - 20*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^5*d^5*e*x^2 + a*c^4*d^5*
e + (c^5*d^6 + a*c^4*d^4*e^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,f^3}{\sqrt{cdx+ae}cd} + \frac{6\,(cdx+2\,ae)f^2g}{\sqrt{cdx+ae}c^2d^2} + \frac{2\,(c^2d^2x^2-4\,acdex-8\,a^2e^2)fg^2}{\sqrt{cdx+ae}c^3d^3} + \frac{2\,(c^3d^3x^3-2\,ac^2d^2ex^2+8\,a^2cde^2x+16\,a^3e^3)g^3}{5\,\sqrt{cdx+ae}c^4d^4}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
-2*f^3/(sqrt(c*d*x + a*e)*c*d) + 6*(c*d*x + 2*a*e)*f^2*g/(sqrt(c*d*x + a*e
)*c^2*d^2) + 2*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*f*g^2/(sqrt(c*d*x +
 a*e)*c^3*d^3) + 2/5*(c^3*d^3*x^3 - 2*a*c^2*d^2*e*x^2 + 8*a^2*c*d*e^2*x +
16*a^3*e^3)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$
$$-\frac{2\,(c^3d^3f^3-3\,ac^2d^2ef^2g+3\,a^2cde^2fg^2-a^3e^3g^3)}{\sqrt{cdx+ae}c^4d^4}$$
$$+\frac{2\left(15\sqrt{cdx+ae}c^{18}d^{18}f^2g-30\sqrt{cdx+ae}ac^{17}d^{17}efg^2+15\sqrt{cdx+ae}a^2c^{16}d^{16}e^2g^3+5\,(cdx+ae)^{\frac{3}{2}}c^{17}d^{1}\right.}{5\,c^{20}d^{20}}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
-2*(c^3*d^3*f^3 - 3*a*c^2*d^2*e*f^2*g + 3*a^2*c*d*e^2*f*g^2 - a^3*e^3*g^3)
/(sqrt(c*d*x + a*e)*c^4*d^4) + 2/5*(15*sqrt(c*d*x + a*e)*c^18*d^18*f^2*g -
 30*sqrt(c*d*x + a*e)*a*c^17*d^17*e*f*g^2 + 15*sqrt(c*d*x + a*e)*a^2*c^16*
d^16*e^2*g^3 + 5*(c*d*x + a*e)^(3/2)*c^17*d^17*f*g^2 - 5*(c*d*x + a*e)^(3/
2)*a*c^16*d^16*e*g^3 + (c*d*x + a*e)^(5/2)*c^16*d^16*g^3)/(c^20*d^20)
```

Mupad [B] (verification not implemented)

Time = 6.50 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-80\,a^2\,c\,d\,e^2\,f\,g^2+6\right.}{5\,c^5\,d^5\,e}\right.}{}$$

input
```
int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(3/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e
^3*g^3 - 10*c^3*d^3*f^3 + 60*a*c^2*d^2*e*f^2*g - 80*a^2*c*d*e^2*f*g^2))/(5
*c^5*d^5*e) + (2*g^3*x^3*(d + e*x)^(1/2))/(5*c^2*d^2*e) - (2*g^2*x^2*(2*a*
e*g - 5*c*d*f)*(d + e*x)^(1/2))/(5*c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2)*(8*
a^2*e^2*g^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(5*c^4*d^4*e)))/(a/c + x^2
 + (x*(5*c^5*d^6 + 5*a*c^4*d^4*e^2))/(5*c^5*d^5*e))
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{\frac{2}{5}c^3d^3g^3x^3-\frac{4}{5}a\,c^2d^2e\,g^3x^2+2c^3d^3f\,g^2x^2+\frac{16}{5}a^2cd\,e^2g^3x-8a\,c^2}{\sqrt{}}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(16*a**3*e**3*g**3 - 40*a**2*c*d*e**2*f*g**2 + 8*a**2*c*d*e**2*g**3*x +
 30*a*c**2*d**2*e*f**2*g - 20*a*c**2*d**2*e*f*g**2*x - 2*a*c**2*d**2*e*g**
3*x**2 - 5*c**3*d**3*f**3 + 15*c**3*d**3*f**2*g*x + 5*c**3*d**3*f*g**2*x**
2 + c**3*d**3*g**3*x**3))/(5*sqrt(a*e + c*d*x)*c**4*d**4)
```

3.43 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 167

$$\begin{aligned}\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx &= -\frac{2(cdf-aeg)^2\sqrt{d+ex}}{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\ &+\frac{4g(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3\sqrt{d+ex}}\\ &+\frac{2g^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^3d^3(d+ex)^{3/2}}\end{aligned}$$

output
```
-2*(-a*e*g+c*d*f)^2*(e*x+d)^(1/2)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+4*g*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3
/(e*x+d)^(1/2)+2/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*
x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\sqrt{d+ex}\left(-8a^2e^2g^2-4acdeg(-3f+gx)+c^2d^2\left(-3f^2+6fgx\right.\right.}{3c^3d^3\sqrt{(ae+cdx)(d+ex)}}$$

input

```
Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2),x]
```

output

```
(2*Sqrt[d + e*x]*(-8*a^2*e^2*g^2 - 4*a*c*d*e*g*(-3*f + g*x) + c^2*d^2*(-3*
f^2 + 6*f*g*x + g^2*x^2)))/(3*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1251, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1251}$$

$$\frac{4g\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{4g\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{4g\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) + (4*g*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-g^2x^2d^2c^2+4acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-12acdefg+3f^2c^2d^2)}{3\sqrt{ex+d}\,(cdx+ae)c^3d^3}$	108
gosper	$-\frac{2(cdx+ae)(-g^2x^2d^2c^2+4acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-12acdefg+3f^2c^2d^2)(ex+d)^{\frac{3}{2}}}{3d^3c^3(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	116
orering	$-\frac{2(-g^2x^2d^2c^2+4acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-12acdefg+3f^2c^2d^2)(cdx+ae)(ex+d)^{\frac{3}{2}}}{3d^3c^3(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}$	117

input
```
int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-c^2*d^2*g^2*x^2+4*a*c*d*e
*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^2*g^2-12*a*c*d*e*f*g+3*c^2*d^2*f^2)/(c*d*x+
a*e)/c^3/d^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\,(c^2d^2g^2x^2-3\,c^2d^2f^2+12\,acdefg-8\,a^2e^2g^2+2\,(3\,c^2d^2fg-2}{3\,(c^4d^4ex^2+ac^3d^4e+(c^4d^5}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output
```
2/3*(c^2*d^2*g^2*x^2 - 3*c^2*d^2*f^2 + 12*a*c*d*e*f*g - 8*a^2*e^2*g^2 + 2*
(3*c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(e*x + d)/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^4*d^5 + a*c^3*d^3*e^2
)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^2}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x
)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2\,f^2}{\sqrt{cdx+ae}cd}+\frac{4\,(cdx+2\,ae)fg}{\sqrt{cdx+ae}c^2d^2}+\frac{2\,(c^2d^2x^2-4\,acdex-8\,a^2e^2)g^2}{3\,\sqrt{cdx+ae}c^3d^3}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
-2*f^2/(sqrt(c*d*x + a*e)*c*d) + 4*(c*d*x + 2*a*e)*f*g/(sqrt(c*d*x + a*e)*
c^2*d^2) + 2/3*(c^2*d^2*x^2 - 4*a*c*d*e*x - 8*a^2*e^2)*g^2/(sqrt(c*d*x + a
*e)*c^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\,(c^2d^2f^2-2\,acdefg+a^2e^2g^2)}{\sqrt{cdx+ae}c^3d^3} + \frac{2\left(6\,\sqrt{cdx+ae}c^7d^7fg-6\,\sqrt{cdx+ae}ac^6d^6eg^2+(cdx+ae)^{\frac{3}{2}}c^6d^6g^2\right)}{3\,c^9d^9}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
-2*(c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e^2*g^2)/(sqrt(c*d*x + a*e)*c^3*d^3)
 + 2/3*(6*sqrt(c*d*x + a*e)*c^7*d^7*f*g - 6*sqrt(c*d*x + a*e)*a*c^6*d^6*e*
g^2 + (c*d*x + a*e)^(3/2)*c^6*d^6*g^2)/(c^9*d^9)
```

Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,(16\,a^2\,e^2\,g^2-24\,a\,c\,d\,e\,f\,g+6\,c^2\,d^2\,f^2)}{3\,c^4\,d^4\,e}-\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}+\frac{4\,g\,x\,(2\,a\,e\,g-3\,c\,d\,f)\,\sqrt{d}}{3\,c^3\,d^3\,e}\right.}{\frac{a}{c}+x^2+\frac{x\,(3\,c^4\,d^5+3\,a\,c^3\,d^3\,e^2)}{3\,c^4\,d^4\,e}}$$

input
```
int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(3/2),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*
e^2*g^2 + 6*c^2*d^2*f^2 - 24*a*c*d*e*f*g))/(3*c^4*d^4*e) - (2*g^2*x^2*(d +
 e*x)^(1/2))/(3*c^2*d^2*e) + (4*g*x*(2*a*e*g - 3*c*d*f)*(d + e*x)^(1/2))/(
3*c^3*d^3*e)))/(a/c + x^2 + (x*(3*c^4*d^5 + 3*a*c^3*d^3*e^2))/(3*c^4*d^4*e
))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\frac{2}{3}c^2d^2g^2x^2-\frac{8}{3}acde\,g^2x+4c^2d^2fgx-\frac{16}{3}a^2e^2g^2+8acdefg-2c^2}{\sqrt{cdx+ae}\,c^3d^3}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 8*a**2*e**2*g**2 + 12*a*c*d*e*f*g - 4*a*c*d*e*g**2*x - 3*c**2*d**2*
f**2 + 6*c**2*d**2*f*g*x + c**2*d**2*g**2*x**2))/(3*sqrt(a*e + c*d*x)*c**3
*d**3)
```

3.44 $\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 104

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(cdf-aeg)\sqrt{d+ex}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2\sqrt{d+ex}}$$

output

```
-2*(-a*e*g+c*d*f)*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+2*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\sqrt{d+ex}(2aeg+cd(-f+gx))}{c^2d^2\sqrt{(ae+cdx)(d+ex)}}$$

input

```
Integrate[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2),x]
```

output

```
(2*Sqrt[d + e*x]*(2*a*e*g + c*d*(-f + g*x)))/(c^2*d^2*Sqrt[(a*e + c*d*x)*(
d + e*x)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1218, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1122}$$

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input

```
Int[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2),x]
```

output

```
(-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdgx+2aeg-dfc)}{\sqrt{ex+d}\,(cdx+ae)c^{2}d^{2}}$	58
gosper	$\frac{2(cdx+ae)(cdgx+2aeg-dfc)(ex+d)^{\frac{3}{2}}}{c^{2}d^{2}(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	66
orering	$\frac{2(cdgx+2aeg-dfc)(cdx+ae)(ex+d)^{\frac{3}{2}}}{c^{2}d^{2}(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}$	67

input
```
int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method
=_RETURNVERBOSE)
```

output
```
2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(c*d*g*x+2*a*e*g-c*d*f)/(c*d*x
+a*e)/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdgx-cdf+2\,aeg)\sqrt{ex+d}}{c^3d^3ex^2+ac^2d^3e+(c^3d^4+ac^2d^2e^2)x}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - c*d*f + 2*a*e*g)*
sqrt(e*x + d)/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/
2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2\,f}{\sqrt{cdx+ae}cd}+\frac{2\,(cdx+2\,ae)g}{\sqrt{cdx+ae}c^2d^2}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="maxima")
```

output
```
-2*f/(sqrt(c*d*x + a*e)*c*d) + 2*(c*d*x + 2*a*e)*g/(sqrt(c*d*x + a*e)*c^2*
d^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.46

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\sqrt{cdx+aeg}}{c^2d^2}-\frac{2\,(cdf-aeg)}{\sqrt{cdx+ae}c^2d^2}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="giac")
```

output
```
2*sqrt(c*d*x + a*e)*g/(c^2*d^2) - 2*(c*d*f - a*e*g)/(sqrt(c*d*x + a*e)*c^2
*d^2)
```

Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\left(\frac{(4\,a\,e\,g-2\,c\,d\,f)\,\sqrt{d+e\,x}}{c^3\,d^3\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a}}{\frac{a}{c}+x^2+\frac{x\,\left(c^3\,d^4+a\,c^2\,d^2\,e^2\right)}{c^3\,d^3\,e}}$$

input
```
int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2),x)
```

output
```
((((4*a*e*g - 2*c*d*f)*(d + e*x)^(1/2))/(c^3*d^3*e) + (2*g*x*(d + e*x)^(1/
2))/(c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(a/c + x^2
 + (x*(c^3*d^4 + a*c^2*d^2*e^2))/(c^3*d^3*e))
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.34

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2cdgx+4aeg-2cdf}{\sqrt{cdx+ae}\,c^2d^2}$$

input

```
int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(2*(2*a*e*g - c*d*f + c*d*g*x))/(sqrt(a*e + c*d*x)*c**2*d**2)
```

3.45 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$
$$\Big\downarrow \textcolor{blue}{1122}$$
$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}}{\sqrt{ex+d}\,(cdx+ae)cd}$	42
gosper	$-\frac{2(cdx+ae)(ex+d)^{\frac{3}{2}}}{cd(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	50
orering	$-\frac{2(cdx+ae)(ex+d)^{\frac{3}{2}}}{cd(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}$	51

input

```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output

```
-2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)/c/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x}$$

input

```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output

```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x^
2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2}{\sqrt{cdx+ae}cd}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
-2/(sqrt(c*d*x + a*e)*c*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2}{\sqrt{cdx+ae}cd}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
-2/(sqrt(c*d*x + a*e)*c*d)
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = -\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c^2\,d^2\,e\,\left(\frac{a}{c}+x^2+\frac{x\,\left(c^2\,d^3+a\,c\,d\,e^2\right)}{c^2\,d^2\,e}\right)}$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
-(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c^2*d^
2*e*(a/c + x^2 + (x*(c^2*d^3 + a*c*d*e^2))/(c^2*d^2*e)))
```

Reduce [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.41

$$\int \frac{(d+ex)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = -\frac{2}{\sqrt{cdx+ae}\,cd}$$

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 2)/(sqrt(a*e + c*d*x)*c*d)
```

3.46 $\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 133

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = & -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ & +\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{(cdf-aeg)^{3/2}}\end{aligned}$$

output

```
-2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*
g^(1/2)*arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*g+c
*d*f)^(1/2)*(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(\sqrt{cdf-aeg}+\sqrt{g}\sqrt{ae+cdx}\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input

```
Integrate[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)),x]
```

output

```
(-2*Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g] + Sqrt[g]*Sqrt[a*e + c*d*x]*ArcTan[
(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/((c*d*f - a*e*g)^(3/2)*
Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1252, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{g\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$-\frac{2e^2g\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow 218$$

$$-\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input

```
Int[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)),x]
```

output

```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(3/2)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1255

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(g \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)\sqrt{cdx+ae}-\sqrt{(aeg-dfc)g}\right)}{\sqrt{ex+d}\,(cdx+ae)(aeg-dfc)\sqrt{(aeg-dfc)g}}$	118

input
```
int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(g*arctanh(g*(c*d*x+a*e)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)-((a*e*g-c*d*f)*g)^(1/2))/(c*d
*x+a*e)/(a*e*g-c*d*f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(117) = 234.

Time = 0.10 (sec) , antiderivative size = 553, normalized size of antiderivative = 4.16

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[-\frac{(cdex^2+ade+(cd^2+ae^2)x)\sqrt{-\frac{g}{cdf-aeg}}\log\left(-\frac{cdeg}{}\right.}{acd^2e}\right.$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="fricas")
```

output

```
[-((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(
c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f
- (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c*d^2*e*f - a^2*d*e^2*g
 + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e
 + a^2*e^3)*g)*x), -2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d
*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f -
a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a*d*e*g + (c*d
^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d))/(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*
d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Timed out}$$

input

```
integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/
2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)}\,dx$$

input

```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\,g\arctan\left(\frac{\sqrt{cdx+aeg}}{\sqrt{cdfg-aeg^2}}\right)}{\sqrt{cdfg-aeg^2}(cdf-aeg)}-\frac{2}{(cdf-aeg)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
 algorithm="giac")
```

output
```
-2*g*arctan(sqrt(c*d*x + a*e)*g/sqrt(c*d*f*g - a*e*g^2))/(sqrt(c*d*f*g - a
*e*g^2)*(c*d*f - a*e*g)) - 2/((c*d*f - a*e*g)*sqrt(c*d*x + a*e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{{\left(d+e\,x\right)}^{3/2}}{\left(f+g\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

input
```
int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/
2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{-2\sqrt{g}\,\sqrt{cdx+ae}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)+2}{\sqrt{cdx+ae}\,\left(a^2e^2g^2-2acdefg+c^2d^2f^2\right)}$$

input

```
int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(2*( - sqrt(g)*sqrt(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c
*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f))) + a*e*g - c*d*f))/(sqrt(a*e + c
*d*x)*(a**2*e**2*g**2 - 2*a*c*d*e*f*g + c**2*d**2*f**2))
```

3.47 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 195

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{3cd\sqrt{d+ex}}{(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3cd\sqrt{g}\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{(cdf-aeg)^{5/2}}$$

output

```
-3*c*d*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)+(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)+3*c*d*g^(1/2)*arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)*(-a*e*g+c*d*f)^(1/2)*(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{\sqrt{d+ex}\left(\sqrt{cdf-aeg}(aeg+cd(2f+3gx))+3cd\sqrt{g}\sqrt{ae+cdx}(f+gx)\arctan\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)}$$

input

```
Integrate[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)),x]
```

output

```
-((Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g]*(a*e*g + c*d*(2*f + 3*g*x)) + 3*c*d*
Sqrt[g]*Sqrt[a*e + c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqr
t[c*d*f - a*e*g]]))/((c*d*f - a*e*g)^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(
f + g*x)))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1252, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{3g\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$-\frac{3g\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$-\frac{3g\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{3g\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2]) - (3*g*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d
*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt
[g]*(c*d*f - a*e*g)^(3/2))))/(c*d*f - a*e*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\Bigg(3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)cd\,g^{2}x+3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)cdfg-3\sqrt{(aeg-dfc)g}\,cdgx-}{\sqrt{ex+d}\,(cdx+ae)(aeg-dfc)^{2}(gx+f)\sqrt{(aeg-dfc)g}}$

input
```
int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*(c*d*x+a*e)^(1/2)*arctanh(g
*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g^2*x+3*(c*d*x+a*e)^(1/2)*
arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f*g-3*((a*e*g-c*d
*f)*g)^(1/2)*c*d*g*x-((a*e*g-c*d*f)*g)^(1/2)*a*e*g-2*((a*e*g-c*d*f)*g)^(1/
2)*c*d*f)/(c*d*x+a*e)/(a*e*g-c*d*f)^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(175) = 350$.

Time = 0.12 (sec) , antiderivative size = 1067, normalized size of antiderivative = 5.47

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output

```
[1/2*(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f + (c^2*d^3 + a*c*d*e
^2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x)*sqrt(-g/(c*d*f - a
*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)
) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) -
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + 2*c*d*f + a*e*g
)*sqrt(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^3*d*e^3*f*g^
2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3)*x^3 + (c^3
*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e + a^2*c*d*e^
3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3 + (c^3*d^4
+ a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (a^2*c*d^2*e^
2 - a^3*e^4)*f*g^2)*x), -(3*(c^2*d^2*e*g*x^3 + a*c*d^2*e*f + (c^2*d^2*e*f
+ (c^2*d^3 + a*c*d*e^2)*g)*x^2 + (a*c*d^2*e*g + (c^2*d^3 + a*c*d*e^2)*f)*x
)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*d*e*g*x^2 + a
*d*e*g + (c*d^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(3*c*d*g*x + 2*c*d*f + a*e*g)*sqrt(e*x + d))/(a*c^2*d^3*e*f^3 - 2*a^2*c
*d^2*e^2*f^2*g + a^3*d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^
2 + a^2*c*d*e^3*g^3)*x^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*
g - (2*a*c^2*d^3*e + a^2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Timed out}$$

input

```
integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=-\frac{3\,cdg\arctan\left(\frac{\sqrt{cdx+aeg}}{\sqrt{cdfg-aeg^2}}\right)}{(c^2d^2f^2-2\,acdefg+a^2e^2g^2)\sqrt{cdfg-aeg^2}}-\frac{2\,c^2d^2f-2\,acdeg+3\,(cdx+ae)cdg}{(c^2d^2f^2-2\,acdefg+a^2e^2g^2)\left(\sqrt{cdx+ae}cdf-\sqrt{cdx+ae}aeg+(cdx+ae)^{\frac{3}{2}}g\right)}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
-3*c*d*g*arctan(sqrt(c*d*x + a*e)*g/sqrt(c*d*f*g - a*e*g^2))/((c^2*d^2*f^2
 - 2*a*c*d*e*f*g + a^2*e^2*g^2)*sqrt(c*d*f*g - a*e*g^2)) - (2*c^2*d^2*f -
2*a*c*d*e*g + 3*(c*d*x + a*e)*c*d*g)/((c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e
^2*g^2)*(sqrt(c*d*x + a*e)*c*d*f - sqrt(c*d*x + a*e)*a*e*g + (c*d*x + a*e)
^(3/2)*g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{{\left(f+g\,x\right)}^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{3\sqrt{g}\,\sqrt{cdx+ae}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)cdf}{\sqrt{cdx+ae}\,\left(a^3e^3g^4x-3a^2cde\right.}$$

input
```
int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(3*sqrt(g)*sqrt(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x
)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*d*f + 3*sqrt(g)*sqrt(a*e + c*d*x)
*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g
+ c*d*f)))*c*d*g*x - a**2*e**2*g**2 - a*c*d*e*f*g - 3*a*c*d*e*g**2*x + 2*c
**2*d**2*f**2 + 3*c**2*d**2*f*g*x)/(sqrt(a*e + c*d*x)*(a**3*e**3*f*g**3 +
a**3*e**3*g**4*x - 3*a**2*c*d*e**2*f**2*g**2 - 3*a**2*c*d*e**2*f*g**3*x +
3*a*c**2*d**2*e*f**3*g + 3*a*c**2*d**2*e*f**2*g**2*x - c**3*d**3*f**4 - c*
*3*d**3*f**3*g*x))
```

3.48 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 273

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =& -\frac{15c^2d^2\sqrt{d+ex}}{4(cdf-aeg)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &+ \frac{\sqrt{d+ex}}{2(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &+ \frac{5cd\sqrt{d+ex}}{4(cdf-aeg)^2(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &+ \frac{15c^2d^2\sqrt{g}\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4(cdf-aeg)^{7/2}}\end{aligned}$$

output
```
-15/4*c^2*d^2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+1/2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)+5/4*c*d*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/4*c^2*d^2*g^(1/2)*arctan(1/g^(1/2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*g+c*d*f)^(1/2)*(e*x+d)^(1/2))/(
-a*e*g+c*d*f)^(7/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{d+ex}\Big(\sqrt{cdf-aeg}(-2a^2e^2g^2+acdeg(9f+5gx)+c^2d^2(8f^2+25fgx+15g^2x^2))+15c^2d^2\sqrt{g}\sqrt{ae+}{4(cdf-aeg)^{7/2}\sqrt{(ae+cdx)(d+ex)}(f+gx)^2}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)),x]
```

output
```
-1/4*(Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g]*(-2*a^2*e^2*g^2 + a*c*d*e*g*(9*f
+ 5*g*x) + c^2*d^2*(8*f^2 + 25*f*g*x + 15*g^2*x^2)) + 15*c^2*d^2*Sqrt[g]*S
qrt[a*e + c*d*x]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f
 - a*e*g]]))/((c*d*f - a*e*g)^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x
)^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1252, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 1252$$

$$-\frac{5g\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$-\frac{5g\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$-\frac{5g\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$-\frac{5g\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade}}{2\sqrt{d+ex}(f+gx)^2(cd}}{cdf-aeg}$$

$$\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{5g\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2]) - (5*g*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*
(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*Ar
cTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a
*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g
))))/(c*d*f - a*e*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\,\Big(15\,\operatorname{arctanh}\Big(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\Big)\sqrt{cdx+ae}\,c^{2}d^{2}g^{3}x^{2}+30\,\operatorname{arctanh}\Big(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\Big)\sqrt{cdx+ae}\,c^{2}d^{2}f\,g^{2}x-15\sqrt{(ae}}{}$

input
```
int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-1/4/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(
1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*g^3*x^2+30*arctanh
(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*f*
g^2*x-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2+15*arctanh(g*(c*d*x+a*e)^
(1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2*g-5*((a*e*g-c
*d*f)*g)^(1/2)*a*c*d*e*g^2*x-25*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+2*((
a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-9*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g-8*
((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3/(g*x+f)^2
/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(241) = 482$.

Time = 0.25 (sec) , antiderivative size = 1863, normalized size of antiderivative = 6.82

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output

```
[-1/8*(15*(c^3*d^3*e*g^2*x^4 + a*c^2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d
^4 + a*c^2*d^2*e^2)*g^2)*x^3 + (c^3*d^3*e*f^2 + a*c^2*d^3*e*g^2 + 2*(c^3*d
^4 + a*c^2*d^2*e^2)*f*g)*x^2 + (2*a*c^2*d^3*e*f*g + (c^3*d^4 + a*c^2*d^2*e
^2)*f^2)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e
*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*
x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*
x^2 + d*f + (e*f + d*g)*x)) + 2*(15*c^2*d^2*g^2*x^2 + 8*c^2*d^2*f^2 + 9*a*
c*d*e*f*g - 2*a^2*e^2*g^2 + 5*(5*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a*c^3*d^4*e*f^5 - 3*a^2*c
^2*d^3*e^2*f^4*g + 3*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*
e*f^3*g^2 - 3*a*c^3*d^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^
4*g^5)*x^4 + (2*c^4*d^4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*
(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4
)*f*g^4 - (a^3*c*d^2*e^3 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*
g^5 + (2*c^4*d^5 - a*c^3*d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e
^3)*f^3*g^2 + (3*a^2*c^2*d^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3
 - 2*a^4*e^5)*f*g^4)*x^2 - (2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*
f^5 + (a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c
*d*e^4)*f^3*g^2 - (5*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^3)*x), -1/4*(15*(c^3*d
^3*e*g^2*x^4 + a*c^2*d^3*e*f^2 + (2*c^3*d^3*e*f*g + (c^3*d^4 + a*c^2*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^3), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.16

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=&\\ -\frac{15\,c^2d^2g\arctan\left(\frac{\sqrt{cdx+ae}g}{\sqrt{cdfg-aeg^2}}\right)}{4\,(c^3d^3f^3-3\,ac^2d^2ef^2g+3\,a^2cde^2fg^2-a^3e^3g^3)\sqrt{cdfg-aeg^2}}&\\ -\frac{2\,c^2d^2}{(c^3d^3f^3-3\,ac^2d^2ef^2g+3\,a^2cde^2fg^2-a^3e^3g^3)\sqrt{cdx+ae}}&\\ -\frac{9\,\sqrt{cdx+ae}c^3d^3fg-9\,\sqrt{cdx+ae}ac^2d^2eg^2+7\,(cdx+ae)^{\frac{3}{2}}c^2d^2g^2}{4\,(c^3d^3f^3-3\,ac^2d^2ef^2g+3\,a^2cde^2fg^2-a^3e^3g^3)(cdf-aeg+(cdx+ae)g)^2}&\end{aligned}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
-15/4*c^2*d^2*g*arctan(sqrt(c*d*x + a*e)*g/sqrt(c*d*f*g - a*e*g^2))/((c^3*
d^3*f^3 - 3*a*c^2*d^2*e*f^2*g + 3*a^2*c*d*e^2*f*g^2 - a^3*e^3*g^3)*sqrt(c*
d*f*g - a*e*g^2)) - 2*c^2*d^2/((c^3*d^3*f^3 - 3*a*c^2*d^2*e*f^2*g + 3*a^2*
c*d*e^2*f*g^2 - a^3*e^3*g^3)*sqrt(c*d*x + a*e)) - 1/4*(9*sqrt(c*d*x + a*e)
*c^3*d^3*f*g - 9*sqrt(c*d*x + a*e)*a*c^2*d^2*e*g^2 + 7*(c*d*x + a*e)^(3/2)
*c^2*d^2*g^2)/((c^3*d^3*f^3 - 3*a*c^2*d^2*e*f^2*g + 3*a^2*c*d*e^2*f*g^2 -
a^3*e^3*g^3)*(c*d*f - a*e*g + (c*d*x + a*e)*g)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{{\left(f+g\,x\right)}^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{-15\sqrt{g}\,\sqrt{cdx+ae}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c}{4\sqrt{cdx+ae}\,\left(a^{4}e}$$

input
```
int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
( - 15*sqrt(g)*sqrt(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c
*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*f**2 - 30*sqrt(g)*sqr
t(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*c**2*d**2*f*g*x - 15*sqrt(g)*sqrt(a*e + c*d*x)*sq
rt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c
*d*f)))*c**2*d**2*g**2*x**2 - 2*a**3*e**3*g**3 + 11*a**2*c*d*e**2*f*g**2 +
 5*a**2*c*d*e**2*g**3*x - a*c**2*d**2*e*f**2*g + 20*a*c**2*d**2*e*f*g**2*x
 + 15*a*c**2*d**2*e*g**3*x**2 - 8*c**3*d**3*f**3 - 25*c**3*d**3*f**2*g*x -
 15*c**3*d**3*f*g**2*x**2)/(4*sqrt(a*e + c*d*x)*(a**4*e**4*f**2*g**4 + 2*a
**4*e**4*f*g**5*x + a**4*e**4*g**6*x**2 - 4*a**3*c*d*e**3*f**3*g**3 - 8*a*
*3*c*d*e**3*f**2*g**4*x - 4*a**3*c*d*e**3*f*g**5*x**2 + 6*a**2*c**2*d**2*e
**2*f**4*g**2 + 12*a**2*c**2*d**2*e**2*f**3*g**3*x + 6*a**2*c**2*d**2*e**2
*f**2*g**4*x**2 - 4*a*c**3*d**3*e*f**5*g - 8*a*c**3*d**3*e*f**4*g**2*x - 4
*a*c**3*d**3*e*f**3*g**3*x**2 + c**4*d**4*f**6 + 2*c**4*d**4*f**5*g*x + c*
*4*d**4*f**4*g**2*x**2))
```

3.49 $\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 230

$$\begin{aligned}
\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx &= \\
&-\frac{2(cdf-aeg)^3(d+ex)^{3/2}}{3c^4d^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{6g(cdf-aeg)^2\sqrt{d+ex}}{c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&+\frac{6g^2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^4d^4\sqrt{d+ex}} \\
&+\frac{2g^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^4d^4(d+ex)^{3/2}}
\end{aligned}$$

output

```
-2/3*(-a*e*g+c*d*f)^3*(e*x+d)^(3/2)/c^4/d^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)-6*g*(-a*e*g+c*d*f)^2*(e*x+d)^(1/2)/c^4/d^4/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)+6*g^2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/c^4/d^4/(e*x+d)^(1/2)+2/3*g^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/c^4/d^4/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2(d+ex)^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(f-gx)-6ac^2d^2eg(f^2-\right.}{3c^4d^4((ae+cdx)(d}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(f - g*x) - 6*a*c
^2*d^2*e*g*(f^2 - 6*f*g*x + g^2*x^2) + c^3*d^3*(-f^3 - 9*f^2*g*x + 9*f*g^2
*x^2 + g^3*x^3)))/(3*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1251, 1251, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{2g\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1251}$$

$$\frac{2g\left(\frac{4g\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{2g\left(\frac{4g\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2g\left(\frac{4g\left(\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)}{3cd\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cde}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)) + (2*g*((-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) + (4*g*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(c*d
)))/(c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.78

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-x^3g^3d^3c^3+6a\,c^2d^2e\,g^3x^2-9c^3d^3f\,g^2x^2+24a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x+9c^3d^3f^2gx+16a^3e^3g^3-24a^2cd}{3\sqrt{ex+d}\,(cdx+ae)^2c^4d^4}$
gosper	$-\frac{2(cdx+ae)(-x^3g^3d^3c^3+6a\,c^2d^2e\,g^3x^2-9c^3d^3f\,g^2x^2+24a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x+9c^3d^3f^2gx+16a^3e^3g^3-24a^2cd\,e^2f\,g^2+6}{3d^4c^4(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$
orering	$-\frac{2(-x^3g^3d^3c^3+6a\,c^2d^2e\,g^3x^2-9c^3d^3f\,g^2x^2+24a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x+9c^3d^3f^2gx+16a^3e^3g^3-24a^2cd\,e^2f\,g^2+6a\,c^2d^2e\,f^2}{3d^4c^4(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{5}{2}}}$

input
```
int((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output

```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-c^3*d^3*g^3*x^3+6*a*c^2*d
^2*e*g^3*x^2-9*c^3*d^3*f*g^2*x^2+24*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2
*x+9*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-24*a^2*c*d*e^2*f*g^2+6*a*c^2*d^2*e*f^2
*g+c^3*d^3*f^3)/(c*d*x+a*e)^2/c^4/d^4
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\,(c^3d^3g^3x^3 - c^3d^3f^3 - 6\,ac^2d^2ef^2g + 24\,a^2cde^2fg^2 - 16\,a^3e^3g^3 -}{3\,(c^6d^6ex^3}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output

```
2/3*(c^3*d^3*g^3*x^3 - c^3*d^3*f^3 - 6*a*c^2*d^2*e*f^2*g + 24*a^2*c*d*e^2*
f*g^2 - 16*a^3*e^3*g^3 + 3*(3*c^3*d^3*f*g^2 - 2*a*c^2*d^2*e*g^3)*x^2 - 3*(
3*c^3*d^3*f^2*g - 12*a*c^2*d^2*e*f*g^2 + 8*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^6*d^6*e*x^3 + a^2*c^4*d^
5*e^2 + (c^6*d^7 + 2*a*c^5*d^5*e^2)*x^2 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3
)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(5/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx =$$

$$-\frac{2\,(3\,cdx+2\,ae)f^2g}{(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}}+\frac{2\,(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2)fg^2}{(c^4d^4x+ac^3d^3e)\sqrt{cdx+ae}}$$
$$+\frac{2\,(c^3d^3x^3-6\,ac^2d^2ex^2-24\,a^2cde^2x-16\,a^3e^3)g^3}{3\,(c^5d^5x+ac^4d^4e)\sqrt{cdx+ae}}-\frac{2\,f^3}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
-2*(3*c*d*x + 2*a*e)*f^2*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
 2*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*f*g^2/((c^4*d^4*x + a*c^3*d^
3*e)*sqrt(c*d*x + a*e)) + 2/3*(c^3*d^3*x^3 - 6*a*c^2*d^2*e*x^2 - 24*a^2*c*
d*e^2*x - 16*a^3*e^3)*g^3/((c^5*d^5*x + a*c^4*d^4*e)*sqrt(c*d*x + a*e)) -
2/3*f^3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx =$$

$$-\frac{2\,(c^3d^3e^4f^3-3\,ac^2d^2e^5f^2g+3\,a^2cde^6fg^2-a^3e^7g^3+9\,((ex+d)cde-cd^2e+ae^3)c^2d^2e^2f^2g-18\,((ex+}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^4d^4|e}$$
$$+\frac{2\,\Big(9\,\sqrt{(ex+d)cde-cd^2e+ae^3}c^9d^9e^8fg^2-9\,\sqrt{(ex+d)cde-cd^2e+ae^3}ac^8d^8e^9g^3+((ex+d)cde-cd}{3\,c^{12}d^{12}e^8|e|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
-2/3*(c^3*d^3*e^4*f^3 - 3*a*c^2*d^2*e^5*f^2*g + 3*a^2*c*d*e^6*f*g^2 - a^3*
e^7*g^3 + 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*e^2*f^2*g - 18*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c*d*e^3*f*g^2 + 9*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)*a^2*e^4*g^3)/(((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4
*d^4*abs(e)) + 2/3*(9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^9*d^9*e^8*
f*g^2 - 9*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^8*d^8*e^9*g^3 + ((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^8*d^8*e^6*g^3)/(c^12*d^12*e^8*abs(
e))
```

Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(\frac{32\,a^3\,e^3\,g^3}{3}-16\,a^2\,c\,d\,e^2\,f\,g^2+4\,a\,c^2\,d^2\,e\,f^2\,g+\frac{2\,c^3\,d^3\,f^3}{3}\right)}{c^6\,d^6\,e}-\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{3\,c^3\,d^3\,e}+g^2\right.}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c^6\,d^7+2\,a\,c^5\,d^5\,e^2\right)}{c^6\,d^6\,e}}$$

input
```
int(((f + g*x)^3*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(5/2),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*((32*a^3
*e^3*g^3)/3 + (2*c^3*d^3*f^3)/3 + 4*a*c^2*d^2*e*f^2*g - 16*a^2*c*d*e^2*f*g
^2))/(c^6*d^6*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(3*c^3*d^3*e) + (g^2*x^2*(4
*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d + e*x)^(1/2)*(8
*a^2*e^2*g^2 + 3*c^2*d^2*f^2 - 12*a*c*d*e*f*g))/(c^5*d^5*e)))/(x^3 + (a^2*
e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^6*d^7 + 2*a*c^5*d
^5*e^2))/(c^6*d^6*e))
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\frac{2}{3}c^3d^3g^3x^3-4a\,c^2d^2e\,g^3x^2+6c^3d^3f\,g^2x^2-16a^2cd\,e^2g^3x+24a\,c}{\sqrt{cdx+}}$$

input
```
int((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*( - 16*a**3*e**3*g**3 + 24*a**2*c*d*e**2*f*g**2 - 24*a**2*c*d*e**2*g**3
*x - 6*a*c**2*d**2*e*f**2*g + 36*a*c**2*d**2*e*f*g**2*x - 6*a*c**2*d**2*e*
g**3*x**2 - c**3*d**3*f**3 - 9*c**3*d**3*f**2*g*x + 9*c**3*d**3*f*g**2*x**
2 + c**3*d**3*g**3*x**3))/(3*sqrt(a*e + c*d*x)*c**4*d**4*(a*e + c*d*x))
```

3.50 $\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 167

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(cdf-aeg)^2(d+ex)^{3/2}}{3c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{4g(cdf-aeg)\sqrt{d+ex}}{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3\sqrt{d+ex}}$$

output

```
-2/3*(-a*e*g+c*d*f)^2*(e*x+d)^(3/2)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)-4*g*(-a*e*g+c*d*f)*(e*x+d)^(1/2)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+2*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*
x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\left(8a^2e^2g^2-4acdeg(f-3gx)-c^2d^2(f^2+6fgx-3g\right.}{3c^3d^3((ae+cdx)(d+ex))^{3/2}}$$

input

```
Integrate[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2),x]
```

output

```
(2*(d + e*x)^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(f - 3*g*x) - c^2*d^2*(f^2
 + 6*f*g*x - 3*g^2*x^2)))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = $\{1251, 1218, 1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{4g\int \frac{(d+ex)^{3/2}(f+gx)}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1218}$$

$$\frac{4g\left(-\frac{\left(2ae^2g-cd(dg+ef)\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd(cd^2-ae^2)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{4g\left(-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{c^2d^2\sqrt{d+ex}(cd^2-ae^2)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2),x]
```

output

```
(-2*(d + e*x)^(3/2)*(f + g*x)^2)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)) + (4*g*((-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*
e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e
*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 -
a*e^2)*Sqrt[d + e*x])))/(3*c*d)
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 1251

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(3g^{2}x^{2}d^{2}c^{2}+12acde\,g^{2}x-6c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-4acdefg-f^{2}c^{2}d^{2})}{3\sqrt{ex+d}\,(cdx+ae)^{2}c^{3}d^{3}}$	108
gosper	$\frac{2(cdx+ae)(3g^{2}x^{2}d^{2}c^{2}+12acde\,g^{2}x-6c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-4acdefg-f^{2}c^{2}d^{2})(ex+d)^{\frac{5}{2}}}{3d^{3}c^{3}(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	116
orering	$\frac{2(3g^{2}x^{2}d^{2}c^{2}+12acde\,g^{2}x-6c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-4acdefg-f^{2}c^{2}d^{2})(cdx+ae)(ex+d)^{\frac{5}{2}}}{3d^{3}c^{3}(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{5}{2}}}$	117

input

```
int((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output

```
2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*c^2*d^2*g^2*x^2+12*a*c*d*
e*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^2*g^2-4*a*c*d*e*f*g-c^2*d^2*f^2)/(c*d*x+a*
e)^2/c^3/d^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2\left(3\,c^2d^2g^2x^2-c^2d^2f^2-4\,acdefg+8\,a^2e^2g^2-6\left(c^2d^2fg-2\,ac\right.\right.}{3\left(c^5d^5ex^3+a^2c^3d^4e^2+\left(c^5d^6+2\,ac^4d^4e^2\right)x^2\right.}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output

```
2/3*(3*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 - 4*a*c*d*e*f*g + 8*a^2*e^2*g^2 - 6*(
c^2*d^2*f*g - 2*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(e*x + d)/(c^5*d^5*e*x^3 + a^2*c^3*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*
e^2)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{4\,(3\,cdx+2\,ae)fg}{3\,\left(c^3d^3x+ac^2d^2e\right)\sqrt{cdx+ae}}+\frac{2\,\left(3\,c^2d^2x^2+12\,acdex+8\,a^2e^2\right)g^2}{3\,\left(c^4d^4x+ac^3d^3e\right)\sqrt{cdx+ae}}-\frac{2\,f^2}{3\,\left(c^2d^2x+acde\right)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
-4/3*(3*c*d*x + 2*a*e)*f*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
 2/3*(3*c^2*d^2*x^2 + 12*a*c*d*e*x + 8*a^2*e^2)*g^2/((c^4*d^4*x + a*c^3*d^
3*e)*sqrt(c*d*x + a*e)) - 2/3*f^2/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e)
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2\sqrt{(ex+d)cde-cd^2e+ae^3}g^2}{c^3d^3|e|} - \frac{2\left(c^2d^2e^4f^2-2\,acde^5fg+a^2e^6g^2+6\left((ex+d)cde-cd^2e+ae^3\right)cde^2fg-6\left((ex+d)cde-cd^2e+ae^3\right)\right.}{3\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}c^3d^3|e|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
2*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g^2/(c^3*d^3*abs(e)) - 2/3*(c^2*
d^2*e^4*f^2 - 2*a*c*d*e^5*f*g + a^2*e^6*g^2 + 6*((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)*c*d*e^2*f*g - 6*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*e^3*g^2)/(
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{c^3\,d^3\,e}-\frac{\sqrt{d+e\,x}\,\left(-16\,a^2\,e^2\right.}{}\right.}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(3\,c^5\right.}{}}$$

input
```
int(((f + g*x)^2*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(5/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^2*(d + e*x)^(1/2)
)/(c^3*d^3*e) - ((d + e*x)^(1/2)*(2*c^2*d^2*f^2 - 16*a^2*e^2*g^2 + 8*a*c*d
*e*f*g))/(3*c^5*d^5*e) + (4*g*x*(2*a*e*g - c*d*f)*(d + e*x)^(1/2))/(c^4*d^
4*e)))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(
3*c^5*d^6 + 6*a*c^4*d^4*e^2))/(3*c^5*d^5*e))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2c^2d^2g^2x^2+8acde\,g^2x-4c^2d^2fgx+\frac{16}{3}a^2e^2g^2-\frac{8}{3}acdefg-\frac{2}{3}c^2}{\sqrt{cdx+ae}\,c^3d^3\,(cdx+ae)}$$

input
```
int((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(8*a**2*e**2*g**2 - 4*a*c*d*e*f*g + 12*a*c*d*e*g**2*x - c**2*d**2*f**2
- 6*c**2*d**2*f*g*x + 3*c**2*d**2*g**2*x**2))/(3*sqrt(a*e + c*d*x)*c**3*d*
*3*(a*e + c*d*x))
```

3.51 $\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 106

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(cdf-aeg)(d+ex)^{3/2}}{3c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output

```
-2/3*(-a*e*g+c*d*f)*(e*x+d)^(3/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)-2*g*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(2aeg+cd(f+3gx))}{3c^2d^2((ae+cdx)(d+ex))^{3/2}}$$

input

```
Integrate[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2),x]
```

output

```
(-2*(d + e*x)^(3/2)*(2*a*e*g + c*d*(f + 3*g*x)))/(3*c^2*d^2*((a*e + c*d*x)
*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1218, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{\left(2ae^2g+cd(ef-3dg)\right)\int \frac{(d+ex)^{3/2}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3cd\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\sqrt{d+ex}\left(2ae^2g+cd(ef-3dg)\right)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2),x]
```

output

```
(-2*(c*d*f - a*e*g)*(d + e*x)^(5/2))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*Sqrt
[d + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(3cdgx+2aeg+dfc)}{3\sqrt{ex+d}\,(cdx+ae)^{2}c^{2}d^{2}}$	58
gosper	$-\frac{2(cdx+ae)(3cdgx+2aeg+dfc)(ex+d)^{\frac{5}{2}}}{3c^{2}d^{2}(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	66
orering	$-\frac{2(3cdgx+2aeg+dfc)(cdx+ae)(ex+d)^{\frac{5}{2}}}{3c^{2}d^{2}(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{5}{2}}}$	67

input
```
int((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*c*d*g*x+2*a*e*g+c*d*f)/(
c*d*x+a*e)^2/c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(3\,cdgx+cdf+2\,aeg)\sqrt{ex+d}}{3\left(c^4d^4ex^3+a^2c^2d^3e^2+(c^4d^5+2\,ac^3d^3e^2)x^2+(2\,ac^3d^4e+a^2c^2d^2e^3)x\right)}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="fricas")
```

output
```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + c*d*f + 2*a*
e*g)*sqrt(e*x + d)/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d
^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/
2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = -\frac{2\,(3\,cdx+2\,ae)g}{3\,(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}} - \frac{2\,f}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="maxima")
```

output
```
-2/3*(3*c*d*x + 2*a*e)*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) - 2
/3*f/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = -\frac{2\,(cde^4f-ae^5g+3\,((ex+d)cde-cd^2e+ae^3)e^2g)}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^2d^2|e|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="giac")
```

output
```
-2/3*(c*d*e^4*f - a*e^5*g + 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*e^2*g)/(
((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = -\frac{\left(\frac{\left(\frac{4\,a\,e\,g}{3}+\frac{2\,c\,d\,f}{3}\right)\sqrt{d+e\,x}}{c^4\,d^4\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^3\,d^3\,e}\right)\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,(2\,c\,d^2+a\,e^2)}{c^2\,d^2}+\frac{x^2\,(c^4\,d^5+2\,a\,c^3\,d^3\,e^2)}{c^4\,d^4\,e}}$$

input

```
int(((f + g*x)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2),x)
```

output

```
-(((((4*a*e*g)/3 + (2*c*d*f)/3)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d +
 e*x)^(1/2))/(c^3*d^3*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(
x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^4*d^5
+ 2*a*c^3*d^3*e^2))/(c^4*d^4*e))
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.43

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{-2cdgx-\frac{4}{3}aeg-\frac{2}{3}cdf}{\sqrt{cdx+ae}\,c^2d^2\,(cdx+ae)}$$

input

```
int((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(2*( - 2*a*e*g - c*d*f - 3*c*d*g*x))/(3*sqrt(a*e + c*d*x)*c**2*d**2*(a*e +
 c*d*x))
```

3.52 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

output
```
-2/3*(e*x+d)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}\,dx$$
$$\Big\downarrow \textcolor{blue}{1122}$$
$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}}{3\sqrt{ex+d}\,(cdx+ae)^2 cd}$	42
gosper	$-\frac{2(cdx+ae)(ex+d)^{\frac{5}{2}}}{3cd(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	50
orering	$-\frac{2(cdx+ae)(ex+d)^{\frac{5}{2}}}{3cd(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{5}{2}}}$	51

input

```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURN
VERBOSE)
```

output

```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)^2/c/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(42) = 84$.

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.23

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{3\,(c^3d^3ex^3+a^2cd^2e^2+(c^3d^4+2\,ac^2d^2e^2)x^2+(2\,ac^2d^3e+a^2cde^3)x)}$$

input

```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="fricas")
```

output

```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*
x^3 + a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^2 + (2*a*c^2*d^3*e + a
^2*c*d*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="maxima")
```

output
```
-2/3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\,e^4}{3\,((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}cd|e|}$$

input
```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorit
hm="giac")
```

output
```
-2/3*e^4/(((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.29

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^2\,c\,d^2\,e^2+a^2\,c\,d\,e^3\,x+2\,a\,c^2\,d^3\,e\,x+2\,a\,c^2\,d^2\,e^2\,x^2+c^3\,d^4\,x^2+c^3\,d^3\,e\,x^3\right)}$$

input

```
int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output

```
-(2*(d + e*x)^(1/2)*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(c^3
*d^4*x^2 + a^2*c*d^2*e^2 + c^3*d^3*e*x^3 + 2*a*c^2*d^3*e*x + a^2*c*d*e^3*x
 + 2*a*c^2*d^2*e^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2}{3\sqrt{cdx+ae}\,cd\,(cdx+ae)}$$

input

```
int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
( - 2)/(3*sqrt(a*e + c*d*x)*c*d*(a*e + c*d*x))
```

3.53 $\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 188

$$\begin{aligned}\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx &= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\ &\quad+\frac{2g\sqrt{d+ex}}{(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &\quad-\frac{2g^{3/2}\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{(cdf-aeg)^{5/2}}\end{aligned}$$

output
```
-2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+
2*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
-2*g^(3/2)*arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*
g+c*d*f)^(1/2)*(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}\left(\sqrt{cdf-aeg}(4aeg-cd(f-3gx))+3g^{3/2}}{3(cdf-aeg)^{5/2}((ae+cdx)(d}$$

input
```
Integrate[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2)),x]
```

output
```
(2*(d + e*x)^(3/2)*(Sqrt[c*d*f - a*e*g]*(4*a*e*g - c*d*(f - 3*g*x)) + 3*g^
(3/2)*(a*e + c*d*x)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f -
a*e*g]]))/(3*(c*d*f - a*e*g)^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1252, 1252, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{g\int \frac{(d+ex)^{3/2}}{(f+gx)(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1252}$$

$$-\frac{g\left(-\frac{g\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \; 1255$$

$$-\frac{g\left(-\frac{2e^2g\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \; 218$$

$$-\frac{g\left(-\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}-\frac{2\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2)),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)) - (g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a
*e*g)^(3/2)))/(c*d*f - a*e*g)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)cd\,g^2x+3\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)ae\,g^2\sqrt{cdx+ae}-3\sqrt{(aeg-dfc)g}\,cd\right.}{3\sqrt{ex+d}\,(cdx+ae)^2(aeg-dfc)^2\sqrt{(aeg-dfc)g}}$

input
```
int((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/3*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*(c*d*x+a*e)^(1/2)*arctanh(g*(c*d*x+a*e
)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g^2*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2))*a*e*g^2*(c*d*x+a*e)^(1/2)-3*((a*e*g-c*d*f)*g)^(1/2
)*c*d*g*x-4*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*c*d*f)/(
e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(166) = 332$.

Time = 0.15 (sec) , antiderivative size = 1015, normalized size of antiderivative = 5.40

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="fricas")
```

output

```
[1/3*(3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (
2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c
*d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f
- a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*
e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^
3*e^2*f^2 - 2*a^3*c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3
*d^3*e^2*f*g + a^2*c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2
 - 2*(a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*
e^4)*g^2)*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*
e^2 + a^3*c*d*e^4)*f*g + (2*a^3*c*d^2*e^3 + a^4*e^5)*g^2)*x), 2/3*(3*(c^2*
d^2*e*g*x^3 + a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e +
 a^2*e^3)*g*x)*sqrt(g/(c*d*f - a*e*g))*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(g/(c*d*f - a*e*g))/(c*
d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)) + sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(3*c*d*g*x - c*d*f + 4*a*e*g)*sqrt(e*x + d))/(a^2*c^2*d^3*
e^2*f^2 - 2*a^3*c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d
^3*e^2*f*g + a^2*c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 -
 2*(a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^
4)*g^2)*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\frac{2}{3}\,e^3\Bigg(\frac{3\,g^2\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2ef^2|e|-2\,acde^2fg|e|+a^2e^3g^2|e|)\sqrt{cdfg-a}}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
 algorithm="giac")
```

output

```
2/3*e^3*(3*g^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*
f*g - a*e*g^2)*e))/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g*abs(e) + a^2*e
^3*g^2*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - (c*d*e^2*f - a*e^3*g - 3*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)*g)/((c^2*d^2*e*f^2*abs(e) - 2*a*c*d*e^2*f*g
*abs(e) + a^2*e^3*g^2*abs(e))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(d+e\,x)^{5/2}}{(f+g\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

input

```
int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/
2)),x)
```

output

```
int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx=\frac{-2\sqrt{g}\,\sqrt{cdx+ae}\,\sqrt{-aeg+cdf}\;atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)aeg}{\sqrt{cdx+ae}\,\left(a^3cd\,e^3g^3x-3a^2c^2d\right.}$$

input

```
int((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(2*( - 3*sqrt(g)*sqrt(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e +
 c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*e*g - 3*sqrt(g)*sqrt(a*e +
c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( -
a*e*g + c*d*f)))*c*d*g*x + 4*a**2*e**2*g**2 - 5*a*c*d*e*f*g + 3*a*c*d*e*g*
*2*x + c**2*d**2*f**2 - 3*c**2*d**2*f*g*x))/(3*sqrt(a*e + c*d*x)*(a**4*e**
4*g**3 - 3*a**3*c*d*e**3*f*g**2 + a**3*c*d*e**3*g**3*x + 3*a**2*c**2*d**2*
e**2*f**2*g - 3*a**2*c**2*d**2*e**2*f*g**2*x - a*c**3*d**3*e*f**3 + 3*a*c*
*3*d**3*e*f**2*g*x - c**4*d**4*f**3*x))
```

3.54 $\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 252

$$\begin{aligned}
\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx &= \\
&-\frac{5cd(d+ex)^{3/2}}{3(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&+\frac{(d+ex)^{3/2}}{(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&+\frac{5cdg\sqrt{d+ex}}{(cdf-aeg)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&-\frac{5cdg^{3/2}\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{(cdf-aeg)^{7/2}}
\end{aligned}$$

output

```
-5/3*c*d*(e*x+d)^(3/2)/(-a*e*g+c*d*f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(3/2)+(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)+5*c*d*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)-5*c*d*g^(3/2)*arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)*(-a*e*g+c*d*f)^(1/2)*(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(7/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{(d+ex)^{3/2}\left(\sqrt{cdf-aeg}(3a^2e^2g^2+2acdeg(7f+10gx\right.}{3(cdf}$$

input
```
Integrate[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2)),x]
```

output
```
((d + e*x)^(3/2)*(Sqrt[c*d*f - a*e*g]*(3*a^2*e^2*g^2 + 2*a*c*d*e*g*(7*f +
10*g*x) + c^2*d^2*(-2*f^2 + 10*f*g*x + 15*g^2*x^2)) + 15*c*d*g^(3/2)*(a*e
+ c*d*x)^(3/2)*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a
*e*g]]))/(3*(c*d*f - a*e*g)^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x
))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1252, 1252, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{5g\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1252}$$

$$-\frac{5g\left(-\frac{3g\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$-\frac{5g\left(-\frac{3g\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$-\frac{5g\left(-\frac{3g\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2}{(f+gx)\sqrt{x(ae^2+c}}\right.}{3(cdf-aeg)}$$

$$\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{5g\left(-\frac{3g\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

input

```
Int[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)),x]
```

output

```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(3/2)) - (5*g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*
x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*g*(Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d
*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f
- a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))/(c*d*f - a*e*g
)))/(3*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.64

method	result
default	$\sqrt{(ex+d)(cdx+ae)}\,\Big(15\ \operatorname{arctanh}\Big(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\Big)\sqrt{cdx+ae}\,c^2d^2g^3x^2+15\ \operatorname{arctanh}\Big(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\Big)acde\,g^3x\sqrt{cdx+ae}+15\ \operatorname{arctanh}\Big($

input
```
int((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/3*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*g^3*x^2+15*arctanh(g*(c*d*x+a*e)^
(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*g^3*x*(c*d*x+a*e)^(1/2)+15*arctanh(
g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g
^2*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*f*g^2
*(c*d*x+a*e)^(1/2)-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2-20*((a*e*g-c
*d*f)*g)^(1/2)*a*c*d*e*g^2*x-10*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-3*((
a*e*g-c*d*f)*g)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g+2
*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c
*d*f)^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(226) = 452$.

Time = 0.42 (sec) , antiderivative size = 1907, normalized size of antiderivative = 7.57

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output
```
[-1/6*(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^3*e*f*g + (c^3*d
^4 + 2*a*c^2*d^2*e^2)*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f*g + (2*a*c
^2*d^3*e + a^2*c*d*e^3)*g^2)*x^2 + (a^2*c*d^2*e^2*g^2 + (2*a*c^2*d^3*e + a
^2*c*d*e^3)*f*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*d^2*f +
 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)
*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*
x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(15*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^
2 + 14*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 10*(c^2*d^2*f*g + 2*a*c*d*e*g^2)*x)*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(a^2*c^3*d^4*e^2
*f^4 - 3*a^3*c^2*d^3*e^3*f^3*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3
 + (c^5*d^5*e*f^3*g - 3*a*c^4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 -
a^3*c^2*d^2*e^4*g^4)*x^4 + (c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*
g - 3*(a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3
*c^2*d^2*e^4)*f*g^3 - (a^3*c^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d
^6 + 2*a*c^4*d^4*e^2)*f^4 - (a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a
^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e
^5)*f*g^3 - (2*a^4*c*d^2*e^4 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c
^4*d^5*e + a^2*c^3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*
f^3*g - 3*(a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e
^6)*f*g^3)*x), 1/3*(15*(c^3*d^3*e*g^2*x^4 + a^2*c*d^2*e^2*f*g + (c^3*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{1}{3}\left(\frac{3\sqrt{(ex+d)c}}{(c^3d^3e^2f^3|e|-3\,ac^2d^2e^3f^2g|e|+3\,a^2cde^4fg^2|e|-}\right.$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
1/3*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g^2/((c^3*d^3*e^2*f^3*a
bs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^
5*g^3*abs(e))*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g
)) + 15*c*d*g^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d
*f*g - a*e*g^2)*e))/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e
) + 3*a^2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*sqrt(c*d*f*g - a*e*g^
2)*e) - 2*(c^2*d^2*e^2*f - a*c*d*e^3*g - 6*((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*c*d*g)/((c^3*d^3*e^2*f^3*abs(e) - 3*a*c^2*d^2*e^3*f^2*g*abs(e) + 3*a^
2*c*d*e^4*f*g^2*abs(e) - a^3*e^5*g^3*abs(e))*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)))*e^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(d+e\,x)^{5/2}}{(f+g\,x)^2\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}\,dx$$

input
```
int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2)),x)
```

output
```
int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 650, normalized size of antiderivative = 2.58

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\frac{15\sqrt{g}\,\sqrt{cdx+ae}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)ac}{}$$

input
```
int((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(15*sqrt(g)*sqrt(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*
x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*c*d*e*f*g + 15*sqrt(g)*sqrt(a*e
+ c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt(
- a*e*g + c*d*f)))*a*c*d*e*g**2*x + 15*sqrt(g)*sqrt(a*e + c*d*x)*sqrt( - a
*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))
*c**2*d**2*f*g*x + 15*sqrt(g)*sqrt(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*ata
n((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*g**2*x
**2 - 3*a**3*e**3*g**3 - 11*a**2*c*d*e**2*f*g**2 - 20*a**2*c*d*e**2*g**3*x
 + 16*a*c**2*d**2*e*f**2*g + 10*a*c**2*d**2*e*f*g**2*x - 15*a*c**2*d**2*e*
g**3*x**2 - 2*c**3*d**3*f**3 + 10*c**3*d**3*f**2*g*x + 15*c**3*d**3*f*g**2
*x**2)/(3*sqrt(a*e + c*d*x)*(a**5*e**5*f*g**4 + a**5*e**5*g**5*x - 4*a**4*
c*d*e**4*f**2*g**3 - 3*a**4*c*d*e**4*f*g**4*x + a**4*c*d*e**4*g**5*x**2 +
6*a**3*c**2*d**2*e**3*f**3*g**2 + 2*a**3*c**2*d**2*e**3*f**2*g**3*x - 4*a*
*3*c**2*d**2*e**3*f*g**4*x**2 - 4*a**2*c**3*d**3*e**2*f**4*g + 2*a**2*c**3
*d**3*e**2*f**3*g**2*x + 6*a**2*c**3*d**3*e**2*f**2*g**3*x**2 + a*c**4*d**
4*e*f**5 - 3*a*c**4*d**4*e*f**4*g*x - 4*a*c**4*d**4*e*f**3*g**2*x**2 + c**
5*d**5*f**5*x + c**5*d**5*f**4*g*x**2))
```

3.55 $\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 334

$$\begin{aligned}\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx &= -\frac{35c^2d^2(d+ex)^{3/2}}{12(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\\ &\quad+\frac{(d+ex)^{3/2}}{2(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\\ &\quad+\frac{7cd(d+ex)^{3/2}}{4(cdf-aeg)^2(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\\ &\quad+\frac{35c^2d^2g\sqrt{d+ex}}{4(cdf-aeg)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\ &\quad-\frac{35c^2d^2g^{3/2}\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4(cdf-aeg)^{9/2}}\end{aligned}$$

output
```
-35/12*c^2*d^2*(e*x+d)^(3/2)/(-a*e*g+c*d*f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(3/2)+1/2*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)+7/4*c*d*(e*x+d)^(3/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+35/4*c^2*d^2*g*(e*x+d)^(1/2)/(-a*e*g+c*d
*f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-35/4*c^2*d^2*g^(3/2)*arctan(
1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*g+c*d*f)^(1/2)*(e*
x+d)^(1/2))/(-a*e*g+c*d*f)^(9/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{c^2d^2\sqrt{d+ex}\left(\frac{-6a^3e^3g^3+3a^2cde^2g^2(13f+7gx)+2ac^2d^2eg(40f^2+11}{c^2d^2(cdf-aeg}\right.}{}$$

input
```
Integrate[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2)),x]
```

output
```
(c^2*d^2*Sqrt[d + e*x]*((-6*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(13*f + 7*g*x)
 + 2*a*c^2*d^2*e*g*(40*f^2 + 119*f*g*x + 70*g^2*x^2) + c^3*d^3*(-8*f^3 + 5
6*f^2*g*x + 175*f*g^2*x^2 + 105*g^3*x^3))/(c^2*d^2*(c*d*f - a*e*g)^4*(a*e
+ c*d*x)*(f + g*x)^2) + (105*g^(3/2)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqr
t[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(9/2)))/(12*Sqrt[(a*
e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1252, 1252, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 1252$$

$$-\frac{7g\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1252$$

$$-\frac{7g\left(-\frac{5g\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$-\frac{7g\left(-\frac{5g\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$-\frac{7g\left(-\frac{5g\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{}}{(f+gx)^2\sqrt{x(ae^2+cd^2}}\right.}{3(cdf-aeg)}$$

$$\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \; 1255$$

$$-\frac{7g\left(-\frac{5g\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ad}}{2\sqrt{d+ex}(f+gx)^2(}\right.}{cdf-aeg}\right.}{3(cdf-aeg)}$$

$$\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \; 218$$

$$-\frac{7g\left(-\frac{5g\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{cdf-aeg}-\frac{2}{(f+gx)^2\sqrt{x(ae^2+cd}}\right.}{3(cdf-aeg)}$$

$$\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

input

```
Int[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)),x]
```

output

```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(3/2)) - (7*g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f +
g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (5*g*(Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2
) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*S
qrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f -
a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(c*d*f - a*e*g)))/(3*(c*d*f - a*e*g)
)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(296) = 592$.

Time = 2.90 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.98

method	result
default	$-\frac{\sqrt{(ex+d)(cdx+ae)}\,\Big(105\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{3}d^{3}g^{4}x^{3}\sqrt{cdx+ae}+105\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)a\,c^{2}d^{2}e\,g^{4}x^{2}\sqrt{cdx+ae}+210}{}$

input

```
int((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,meth
od=_RETURNVERBOSE)
```

output

```
-1/12*((e*x+d)*(c*d*x+a*e))^(1/2)*(105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^3*d^3*g^4*x^3*(c*d*x+a*e)^(1/2)+105*arctanh(g*(c*d*x+a
*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*g^4*x^2*(c*d*x+a*e)^(1/2)+2
10*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f*g^3*x^2*
(c*d*x+a*e)^(1/2)+210*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
*a*c^2*d^2*e*f*g^3*x*(c*d*x+a*e)^(1/2)+105*arctanh(g*(c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f^2*g^2*x*(c*d*x+a*e)^(1/2)-105*((a*e*g-c*d*
f)*g)^(1/2)*c^3*d^3*g^3*x^3+105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a*c^2*d^2*e*f^2*g^2*(c*d*x+a*e)^(1/2)-140*((a*e*g-c*d*f)*g)^(1/
2)*a*c^2*d^2*e*g^3*x^2-175*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f*g^2*x^2-21*((
a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*g^3*x-238*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*
d^2*e*f*g^2*x-56*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^2*g*x+6*((a*e*g-c*d*f)*
g)^(1/2)*a^3*e^3*g^3-39*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2-80*((a*e
*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+8*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3
)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4/(g*x+f)^2/((a*e*g-c*d*f)*g)^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1446 vs. $2(296) = 592$.

Time = 1.19 (sec) , antiderivative size = 2935, normalized size of antiderivative = 8.79

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output
```
[1/24*(105*(c^4*d^4*e*g^3*x^5 + a^2*c^2*d^3*e^2*f^2*g + (2*c^4*d^4*e*f*g^2
 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*g^3)*x^4 + (c^4*d^4*e*f^2*g + 2*(c^4*d^5 +
2*a*c^3*d^3*e^2)*f*g^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*g^3)*x^3 + (a^2
*c^2*d^3*e^2*g^3 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g + 2*(2*a*c^3*d^4*e +
a^2*c^2*d^2*e^3)*f*g^2)*x^2 + (2*a^2*c^2*d^3*e^2*f*g^2 + (2*a*c^3*d^4*e +
a^2*c^2*d^2*e^3)*f^2*g)*x)*sqrt(-g/(c*d*f - a*e*g))*log(-(c*d*e*g*x^2 - c*
d^2*f + 2*a*d*e*g + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f -
 a*e*g)*sqrt(e*x + d)*sqrt(-g/(c*d*f - a*e*g)) - (c*d*e*f - (c*d^2 + 2*a*e
^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(105*c^3*d^3*g^3*x^3 - 8*c^
3*d^3*f^3 + 80*a*c^2*d^2*e*f^2*g + 39*a^2*c*d*e^2*f*g^2 - 6*a^3*e^3*g^3 +
35*(5*c^3*d^3*f*g^2 + 4*a*c^2*d^2*e*g^3)*x^2 + 7*(8*c^3*d^3*f^2*g + 34*a*c
^2*d^2*e*f*g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(e*x + d))/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6
*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (
c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 -
4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g +
(c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^
3*g^3 + 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d
^4*e^3 + 3*a^4*c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)
*x^4 + (c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)^3), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\frac{1}{12}\left(\frac{105\,c^2d^2g^2\arctan\left(\sqrt{(}\right.}{(c^4d^4e^3f^4|e|-4\,ac^3d^3e^4f^3g|e|+6\,a^2c^2d^2e^5f^2g^2|e}\right.$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
1/12*(105*c^2*d^2*g^2*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sq
rt(c*d*f*g - a*e*g^2)*e))/((c^4*d^4*e^3*f^4*abs(e) - 4*a*c^3*d^3*e^4*f^3*g
*abs(e) + 6*a^2*c^2*d^2*e^5*f^2*g^2*abs(e) - 4*a^3*c*d*e^6*f*g^3*abs(e) +
a^4*e^7*g^4*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - 8*(c^3*d^3*e^2*f - a*c^2*
d^2*e^3*g - 9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2*g)/((c^4*d^4*e^3
*f^4*abs(e) - 4*a*c^3*d^3*e^4*f^3*g*abs(e) + 6*a^2*c^2*d^2*e^5*f^2*g^2*abs
(e) - 4*a^3*c*d*e^6*f*g^3*abs(e) + a^4*e^7*g^4*abs(e))*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)) + 3*(13*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^
3*d^3*e^2*f*g^2 - 13*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^2*d^2*e^3
*g^3 + 11*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*g^3)/((c^4*d^4
*e^3*f^4*abs(e) - 4*a*c^3*d^3*e^4*f^3*g*abs(e) + 6*a^2*c^2*d^2*e^5*f^2*g^2
*abs(e) - 4*a^3*c*d*e^6*f*g^3*abs(e) + a^4*e^7*g^4*abs(e))*(c*d*e^2*f - a*
e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2))*e^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{5/2}}{{\left(f+g\,x\right)}^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}} \,d x$$

input
```
int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2)),x)
```

output
```
int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 1055, normalized size of antiderivative = 3.16

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
( - 105*sqrt(g)*sqrt(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e +
c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*c**2*d**2*e*f**2*g - 210*sqr
t(g)*sqrt(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(
sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*c**2*d**2*e*f*g**2*x - 105*sqrt(g)*sqrt
(a*e + c*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*s
qrt( - a*e*g + c*d*f)))*a*c**2*d**2*e*g**3*x**2 - 105*sqrt(g)*sqrt(a*e + c
*d*x)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a
*e*g + c*d*f)))*c**3*d**3*f**2*g*x - 210*sqrt(g)*sqrt(a*e + c*d*x)*sqrt( -
 a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)
))*c**3*d**3*f*g**2*x**2 - 105*sqrt(g)*sqrt(a*e + c*d*x)*sqrt( - a*e*g + c
*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d*
*3*g**3*x**3 - 6*a**4*e**4*g**4 + 45*a**3*c*d*e**3*f*g**3 + 21*a**3*c*d*e*
*3*g**4*x + 41*a**2*c**2*d**2*e**2*f**2*g**2 + 217*a**2*c**2*d**2*e**2*f*g
**3*x + 140*a**2*c**2*d**2*e**2*g**4*x**2 - 88*a*c**3*d**3*e*f**3*g - 182*
a*c**3*d**3*e*f**2*g**2*x + 35*a*c**3*d**3*e*f*g**3*x**2 + 105*a*c**3*d**3
*e*g**4*x**3 + 8*c**4*d**4*f**4 - 56*c**4*d**4*f**3*g*x - 175*c**4*d**4*f*
*2*g**2*x**2 - 105*c**4*d**4*f*g**3*x**3)/(12*sqrt(a*e + c*d*x)*(a**6*e**6
*f**2*g**5 + 2*a**6*e**6*f*g**6*x + a**6*e**6*g**7*x**2 - 5*a**5*c*d*e**5*
f**3*g**4 - 9*a**5*c*d*e**5*f**2*g**5*x - 3*a**5*c*d*e**5*f*g**6*x**2 + a*
*5*c*d*e**5*g**7*x**3 + 10*a**4*c**2*d**2*e**4*f**4*g**3 + 15*a**4*c**2...
```

3.56 $\int \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 361

$$\begin{aligned}\int \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx &= \frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}}\\
&+\frac{5(cdf-aeg)^2\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{32c^3d^3(d+ex)^{3/2}}\\
&+\frac{5(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24c^2d^2(d+ex)^{3/2}}\\
&+\frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4cd(d+ex)^{3/2}}\\
&-\frac{5(cdf-aeg)^4\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}}\end{aligned}$$

output

```
5/64*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c^3/d^3/g/(e*x+d)^(1/2)+5/32*(-a*e*g+c*d*f)^2*(g*x+f)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*x+d)^(3/2)+5/24*(-a*e*g+c*d*f)*(g*x
+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)^(3/2)+1/
4*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)-
5/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/c^(7/2)/d^(7/2)/g^(3/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.65

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(15a^3e^3g^3-5\right.}{}$$

input
```
Integrate[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sq
rt[d + e*x],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(15*
a^3*e^3*g^3 - 5*a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g*(73*f^2 + 3
6*f*g*x + 8*g^2*x^2) + c^3*d^3*(15*f^3 + 118*f^2*g*x + 136*f*g^2*x^2 + 48*
g^3*x^3)) - (15*(c*d*f - a*e*g)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/
(Sqrt[g]*Sqrt[a*e + c*d*x])])/Sqrt[a*e + c*d*x]))/(192*c^(7/2)*d^(7/2)*g^(
3/2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1250, 1253, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{5/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \, 1250$$

$$\frac{(f+gx)^{7/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8g}$$

$$\Big\downarrow \, 1253$$

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cd}+\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cd\sqrt{d+ex}}\right)}{8g}$$

$$\Big\downarrow \; 1253$$

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{6cd}+\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+a}}{3cd\sqrt{d+ex}}\right.}{8g}$$

$$\Big\downarrow \; 1253$$

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae\right.}}{2cd}\right.}{6cd}\right.}{8g}$$

$$\Big\downarrow \; 1268$$

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2\right.}}{2cd\sqrt{}}\right.}{6cd}\right.}{8g}$$

$$\Big\downarrow 66$$

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}}{}\right.}{6cd}\right.}{8g}$$

$$\Big\downarrow 221$$

$$\frac{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{5(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{}}{}\right.}{6cd}\right.}{8g}$$

input

```
Int[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
 e*x],x]
```

output

```
((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*Sqrt[d
+ e*x]) - ((c*d*f - a*e*g)*(((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)*(((f + g*x)^(3/
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3
*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^
(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*c*
d)))/(6*c*d)))/(8*g)
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(309) = 618$.

Time = 2.64 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\,\left(-96c^3d^3g^3x^3\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}+15\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^4e^4g^4-6$

input
```
int((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output

```
-1/384*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-96*c^3*d^3*g^3*x^3*((c*
d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*(
(c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln
(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3+90*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x
+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2
-60*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*(
(c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-16*a*
c^2*d^2*e*g^3*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-272*c^3*d^3*f*
g^2*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+20*(c*d*g)^(1/2)*((c*d*x
+a*e)*(g*x+f))^(1/2)*a^2*c*d*e^2*g^3*x-72*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+
f))^(1/2)*a*c^2*d^2*e*f*g^2*x-236*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2
)*c^3*d^3*f^2*g*x-30*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a^3*e^3*g^3
+110*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a^2*c*d*e^2*f*g^2-146*(c*d*
g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a*c^2*d^2*e*f^2*g-30*(c*d*g)^(1/2)*((
c*d*x+a*e)*(g*x+f))^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/g/((c*d*x+a*e)*(g*x+f
))^(1/2)/c^3/d^3/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 1126, normalized size of antiderivative = 3.12

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output

```
[1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g + 73*a*c^3*d^3*e*f^2*g^2
- 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(17*c^4*d^4*f*g^3 + a*
c^3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*a*c^3*d^3*e*f*g^3 - 5*a^2*
c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*
e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a
*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4
*e^5*g^4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d
^2*e*f*g + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d
^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d
^4*e*g^2*x + c^4*d^5*g^2), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g
 + 73*a*c^3*d^3*e*f^2*g^2 - 55*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4
+ 8*(17*c^4*d^4*f*g^3 + a*c^3*d^3*e*g^4)*x^2 + 2*(59*c^4*d^4*f^2*g^2 + 18*
a*c^3*d^3*e*f*g^3 - 5*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^
4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*
g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 -
 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(1/2*sqrt(c*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx=\text{Timed out}$$

input

```
integrate((g*x+f)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+
d)**(1/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{5}{2}}}{\sqrt{ex+d}}\,dx$$

input

```
integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output

```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(5/2)/sqrt
(e*x + d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(309) = 618$.

Time $= 0.26$ (sec) , antiderivative size $= 886$, normalized size of antiderivative $= 2.45$

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
1/192*(192*((c*d*f*g - a*e*g^2)*log(abs(-sqrt(c*d*g)*sqrt(g*x + f) + sqrt(
(g*x + f)*c*d*g - c*d*f*g + a*e*g^2)))/sqrt(c*d*g) + sqrt((g*x + f)*c*d*g
- c*d*f*g + a*e*g^2)*sqrt(g*x + f))*f^3*abs(g)/g^2 + (sqrt((g*x + f)*c*d*g
 - c*d*f*g + a*e*g^2)*(2*(g*x + f)*(4*(g*x + f)*(6*(g*x + f)/g^3 - (25*c^6
*d^6*f*g^11 - a*c^5*d^5*e*g^12)/(c^6*d^6*g^14)) + (163*c^6*d^6*f^2*g^11 -
14*a*c^5*d^5*e*f*g^12 - 5*a^2*c^4*d^4*e^2*g^13)/(c^6*d^6*g^14)) - 3*(93*c^
6*d^6*f^3*g^11 - 15*a*c^5*d^5*e*f^2*g^12 - 9*a^2*c^4*d^4*e^2*f*g^13 - 5*a^
3*c^3*d^3*e^3*g^14)/(c^6*d^6*g^14))*sqrt(g*x + f) - 3*(35*c^4*d^4*f^4 - 20
*a*c^3*d^3*e*f^3*g - 6*a^2*c^2*d^2*e^2*f^2*g^2 - 4*a^3*c*d*e^3*f*g^3 - 5*a
^4*e^4*g^4)*log(abs(-sqrt(c*d*g)*sqrt(g*x + f) + sqrt((g*x + f)*c*d*g - c*
d*f*g + a*e*g^2)))/(sqrt(c*d*g)*c^3*d^3*g^2))*g*abs(g) + 144*(sqrt((g*x +
f)*c*d*g - c*d*f*g + a*e*g^2)*(2*g*x + 2*f - (5*c^2*d^2*f - a*c*d*e*g)/(c^
2*d^2))*sqrt(g*x + f) - (3*c^2*d^2*f^2*g - 2*a*c*d*e*f*g^2 - a^2*e^2*g^3)*
log(abs(-sqrt(c*d*g)*sqrt(g*x + f) + sqrt((g*x + f)*c*d*g - c*d*f*g + a*e*
g^2)))/(sqrt(c*d*g)*c*d))*f^2*abs(g)/g^2 + 24*(sqrt((g*x + f)*c*d*g - c*d*
f*g + a*e*g^2)*(2*(4*g*x + 4*f - (13*c^4*d^4*f - a*c^3*d^3*e*g)/(c^4*d^4))
*(g*x + f) + 3*(11*c^4*d^4*f^2 - 2*a*c^3*d^3*e*f*g - a^2*c^2*d^2*e^2*g^2)/
(c^4*d^4))*sqrt(g*x + f) + 3*(5*c^3*d^3*f^3*g - 3*a*c^2*d^2*e*f^2*g^2 - a^
2*c*d*e^2*f*g^3 - a^3*e^3*g^4)*log(abs(-sqrt(c*d*g)*sqrt(g*x + f) + sqrt((
g*x + f)*c*d*g - c*d*f*g + a*e*g^2)))/(sqrt(c*d*g)*c^2*d^2))*f*abs(g)/g...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{{\left(f+g\,x\right)}^{5/2}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}} \,d x$$

input
```
int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2),x)
```

output
```
int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.69

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{15\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^3cd\,e^3g^4-55\sqrt{gx+f}\,\sqrt{cdx+}}{}$$

input

```
int((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output

```
(15*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**3*g**4 - 55*sqrt(f + g*x)*
sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*f*g**3 - 10*sqrt(f + g*x)*sqrt(a*e +
 c*d*x)*a**2*c**2*d**2*e**2*g**4*x + 73*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*
c**3*d**3*e*f**2*g**2 + 36*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f
*g**3*x + 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*g**4*x**2 + 15*s
qrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*f**3*g + 118*sqrt(f + g*x)*sqrt(a
*e + c*d*x)*c**4*d**4*f**2*g**2*x + 136*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c*
*4*d**4*f*g**3*x**2 + 48*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*g**4*x*
*3 - 15*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*s
qrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**4*e**4*g**4 + 60*sqrt(g)*sqr
t(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x
))/sqrt(a*e*g - c*d*f))*a**3*c*d*e**3*f*g**3 - 90*sqrt(g)*sqrt(d)*sqrt(c)*
log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g
 - c*d*f))*a**2*c**2*d**2*e**2*f**2*g**2 + 60*sqrt(g)*sqrt(d)*sqrt(c)*log(
(sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c
*d*f))*a*c**3*d**3*e*f**3*g - 15*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**4*d
**4*f**4)/(192*c**4*d**4*g**2)
```

3.57 $\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 292

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}$$
$$+\frac{(cdf-aeg)\sqrt{f+gx}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{4c^2d^2(d+ex)^{3/2}}$$
$$+\frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$
$$-\frac{(cdf-aeg)^3\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}}$$

output
```
1/8*(-a*e*g+c*d*f)^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/c^2/d^2/g/(e*x+d)^(1/2)+1/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)^(3/2)+1/3*(g*x+f)^(3/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)-1/8*(-a*e*g+c*d*f)^3*arcta
nh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)
^(1/2)/(g*x+f)^(1/2))/c^(5/2)/d^(5/2)/g^(3/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.64

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(-3a^2e^2g^2+\right.}{}$$

input
```
Integrate[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sq
rt[d + e*x],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(-3*
a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + g*x) + c^2*d^2*(3*f^2 + 14*f*g*x + 8*g^2*
x^2)) - (3*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt
[g]*Sqrt[a*e + c*d*x])])/Sqrt[a*e + c*d*x]))/(24*c^(5/2)*d^(5/2)*g^(3/2)*S
qrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1250, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6g}$$

$$\Big\downarrow \text{1253}$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{6g}$$

$$\Big\downarrow \text{ 1253}$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1268}$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+}}{2cd\sqrt{d}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+c}}{2cd\sqrt{d+}}\right.}{6g}$$

input
```
Int[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
 e*x],x]
```

output
```
((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d
+ e*x]) - ((c*d*f - a*e*g)*(((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*
f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*c*d)))/(6*g)
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(248) = 496$.

Time = 2.46 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.73

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\left(3\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^{3}e^{3}g^{3}-9\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right.\right.}{}$

input
```
int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output

```
1/48*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+
d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*
g^3-9*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^
(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*
((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*
g-3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+16*c^2*d^2*g^2*x^2*((c*d*x+a*e)*(g*x+f))^(
1/2)*(c*d*g)^(1/2)+4*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*g^2
*x+28*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f*g*x-6*((c*d*x+a*
e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2+16*a*c*d*e*f*g*((c*d*x+a*e)*(g
*x+f))^(1/2)*(c*d*g)^(1/2)+6*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2
*d^2*f^2)/(e*x+d)^(1/2)/g/((c*d*x+a*e)*(g*x+f))^(1/2)/c^2/d^2/(c*d*g)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 908, normalized size of antiderivative = 3.11

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 - 3*a^
2*c*d*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 -
3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f
^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*
d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2
*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d
^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*
g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^2*x + c^3*d^4
*g^2), 1/48*(2*(8*c^3*d^3*g^3*x^2 + 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2
- 3*a^2*c*d*e^2*g^3 + 2*(7*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^3*d^4*
f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d
^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*s
qrt(-c*d*g)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
g*x + c*d*f + a*e*g)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^2*e*g
^2*x^3 + a*c*d^2*e*f*g + (c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
 (a*c*d^2*e*g^2 + (c^2*d^3 + a*c*d*e^2)*f*g)*x)))/(c^3*d^3*e*g^2*x + c^3*d
^4*g^2)]
```

Sympy [F]

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+
d)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt
(e*x + d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(248) = 496$.

Time = 0.19 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.77

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\frac{24\left(\frac{\left(cdfg-aeg^2\right)\log\left(\left|-\sqrt{cdg}\sqrt{gx+f}+\sqrt{(gx+f)cdg-cdfg+aeg^2}\right|\right)}{\sqrt{cdg}}+\sqrt{(gx+f)c}\right.}{g^2}}{}$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
1/24*(24*((c*d*f*g - a*e*g^2)*log(abs(-sqrt(c*d*g)*sqrt(g*x + f) + sqrt((g
*x + f)*c*d*g - c*d*f*g + a*e*g^2)))/sqrt(c*d*g) + sqrt((g*x + f)*c*d*g -
c*d*f*g + a*e*g^2)*sqrt(g*x + f))*f^2*abs(g)/g^2 + 12*(sqrt((g*x + f)*c*d*
g - c*d*f*g + a*e*g^2)*(2*g*x + 2*f - (5*c^2*d^2*f - a*c*d*e*g)/(c^2*d^2))
*sqrt(g*x + f) - (3*c^2*d^2*f^2*g - 2*a*c*d*e*f*g^2 - a^2*e^2*g^3)*log(abs
(-sqrt(c*d*g)*sqrt(g*x + f) + sqrt((g*x + f)*c*d*g - c*d*f*g + a*e*g^2)))/
(sqrt(c*d*g)*c*d))*f*abs(g)/g^2 + (sqrt((g*x + f)*c*d*g - c*d*f*g + a*e*g^
2)*(2*(4*g*x + 4*f - (13*c^4*d^4*f - a*c^3*d^3*e*g)/(c^4*d^4))*(g*x + f) +
 3*(11*c^4*d^4*f^2 - 2*a*c^3*d^3*e*f*g - a^2*c^2*d^2*e^2*g^2)/(c^4*d^4))*s
qrt(g*x + f) + 3*(5*c^3*d^3*f^3*g - 3*a*c^2*d^2*e*f^2*g^2 - a^2*c*d*e^2*f*
g^3 - a^3*e^3*g^4)*log(abs(-sqrt(c*d*g)*sqrt(g*x + f) + sqrt((g*x + f)*c*d
*g - c*d*f*g + a*e*g^2)))/(sqrt(c*d*g)*c^2*d^2))*abs(g)/g^2)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{(f+g\,x)^{3/2}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2),x)
```

output
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.41

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{-3\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2cd\,e^2g^3+8\sqrt{gx+f}\,\sqrt{cdx+}}{}$$

input
```
int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**2*g**3 + 8*sqrt(f + g*x)
*sqrt(a*e + c*d*x)*a*c**2*d**2*e*f*g**2 + 2*sqrt(f + g*x)*sqrt(a*e + c*d*x
)*a*c**2*d**2*e*g**3*x + 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*f**2*
g + 14*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*f*g**2*x + 8*sqrt(f + g*x
)*sqrt(a*e + c*d*x)*c**3*d**3*g**3*x**2 + 3*sqrt(g)*sqrt(d)*sqrt(c)*log((s
qrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d
*f))*a**3*e**3*g**3 - 9*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*c*d*e**2*f
*g**2 + 9*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)
*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*c**2*d**2*e*f**2*g - 3*sqrt
(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(
f + g*x))/sqrt(a*e*g - c*d*f))*c**3*d**3*f**3)/(24*c**3*d**3*g**2)
```

3.58 $\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 220

$$\begin{aligned}\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx &= -\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4\sqrt{d+ex}} \\ &\quad+\frac{\sqrt{f+gx}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{2cd(d+ex)^{3/2}} \\ &\quad-\frac{(cdf-aeg)^2\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}}\end{aligned}$$

output

```
-1/4*(a*e/c/d-f/g)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(
e*x+d)^(1/2)+1/2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d
/(e*x+d)^(3/2)-1/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/c^(3/2)/d^(
3/2)/g^(3/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{f+gx}(aeg+cd(f+2gx))-(cdf-aeg)^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+}}{\sqrt{g}\sqrt{ae+c}}\right.\right.}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt
[d + e*x],x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x
]*Sqrt[f + g*x]*(a*e*g + c*d*(f + 2*g*x)) - (c*d*f - a*e*g)^2*ArcTanh[(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*c^(3/2)*d^(3
/2)*g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {1250, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \; 1250$$
$$\frac{(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4g}$$
$$\Big\downarrow \; 1253$$

$$\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4g}$$

$$\Big\downarrow \ 1268$$

$$\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4g}$$

$$\Big\downarrow \ 66$$

$$\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4g}$$

$$\Big\downarrow \ 221$$

$$\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4g}$$

input

```
Int[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e
*x],x]
```

output

```
((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*g*Sqrt[d
+ e*x]) - ((c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqr
t[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g
*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])))/(4*g)
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.45

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\,\Bigg(\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^{2}e^{2}g^{2}-2\ln\Bigg(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}}{8}$

input
```
int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/8*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+d*
f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*e^2*g^
2-2*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*
e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2-4*((c*d*x+a*e)
*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g
)^(1/2)*a*e*g-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*f)/(e*x+d)^(
1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/c/d/g/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 718, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\left[\frac{4\,(2\,c^2d^2g^2x+c^2d^2fg+acdeg^2)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}+(c^2d^3f^2-2\,acd^2efg}{}\right.$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

output

```
[1/16*(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (c^2*d^3*f^2 - 2*a*
c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g
^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f
*g + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*
e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3
*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g
^2*x + c^2*d^3*g^2), 1/8*(2*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) +
(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*
e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(-c*d*g)*sqrt(e*x + d)
*sqrt(g*x + f)/(c^2*d^2*e*g^2*x^3 + a*c*d^2*e*f*g + (c^2*d^2*e*f*g + (c^2*
d^3 + a*c*d*e^2)*g^2)*x^2 + (a*c*d^2*e*g^2 + (c^2*d^3 + a*c*d*e^2)*f*g)*x)
))/(c^2*d^2*e*g^2*x + c^2*d^3*g^2)]
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\sqrt{f+gx}}{\sqrt{d+ex}}\,dx$$

input

```
integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+
d)**(1/2),x)
```

output

```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{gx+f}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e
*x + d), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$= \frac{\frac{4\left(\frac{\left(cdfg-aeg^2\right)\log\left(\left|-\sqrt{cdg}\sqrt{gx+f}+\sqrt{(gx+f)cdg-cdfg+aeg^2}\right|\right)}{\sqrt{cdg}}+\sqrt{(gx+f)cdg-cdfg+aeg^2}\sqrt{gx+f}\right)f|g|}{g^2}+\frac{\left(\sqrt{(gx+f)cdg-cdfg+aeg^2}\left(2\,gx+\right.\right.}{}}{4\,g}$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

output
```
1/4*(4*((c*d*f*g - a*e*g^2)*log(abs(-sqrt(c*d*g)*sqrt(g*x + f) + sqrt((g*x
 + f)*c*d*g - c*d*f*g + a*e*g^2)))/sqrt(c*d*g) + sqrt((g*x + f)*c*d*g - c*
d*f*g + a*e*g^2)*sqrt(g*x + f))*f*abs(g)/g^2 + (sqrt((g*x + f)*c*d*g - c*d
*f*g + a*e*g^2)*(2*g*x + 2*f - (5*c^2*d^2*f - a*c*d*e*g)/(c^2*d^2))*sqrt(g
*x + f) - (3*c^2*d^2*f^2*g - 2*a*c*d*e*f*g^2 - a^2*e^2*g^3)*log(abs(-sqrt(
c*d*g)*sqrt(g*x + f) + sqrt((g*x + f)*c*d*g - c*d*f*g + a*e*g^2)))/(sqrt(c
*d*g)*c*d))*abs(g)/g^2)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e
*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{gx+f}\,\sqrt{cdx+ae}\,acde\,g^2+\sqrt{gx+f}\,\sqrt{cdx+ae}\,c^2d^2fg+2\sqrt{gx+f}\,\sqrt{cdx+ae}\,c^2d^2g^2x-\sqrt{g}\,\sqrt{d}\,\sqrt{c}}{}$$

input
```
int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c*d*e*g**2 + sqrt(f + g*x)*sqrt(a*e + c
*d*x)*c**2*d**2*f*g + 2*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*g**2*x -
 sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*
sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*e**2*g**2 + 2*sqrt(g)*sqrt(d)*sqr
t(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(
a*e*g - c*d*f))*a*c*d*e*f*g - sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*
e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2*d**2
*f**2)/(4*c**2*d**2*g**2)
```

3.59 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 143

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}}$$

output

```
(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)-(-a*
e*g+c*d*f)*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)
/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/c^(1/2)/d^(1/2)/g^(3/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx &= \frac{\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}{g\sqrt{d+ex}} \\ &\quad + \frac{(-cdf+aeg)\sqrt{(ae+cdx)(d+ex)}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{ae+cdx}\sqrt{d+ex}}\end{aligned}$$

input

```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[
f + g*x]),x]
```

output

```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])/(g*Sqrt[d + e*x]) + ((-(c*d*
f) + a*e*g)*Sqrt[(a*e + c*d*x)*(d + e*x)]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*e +
c*d*x]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{\sqrt{f+gx}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2g}$$

$$\Big\downarrow \text{ 1268}$$

$$\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 66$$

$$\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 221$$

$$\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f + g*
x]),x]
```

output

```
(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*
x]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2
)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.31

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\,\sqrt{gx+f}\,\left(\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)aeg-\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdf+\right.}{2\sqrt{ex+d}\,\sqrt{(cdx+ae)(gx+f)}\,g\sqrt{cdg}}$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,
method=_RETURNVERBOSE)
```

output

```
1/2*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(g*x+f)^(1/2)*(ln(1/2*(2*c*d
*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2
))*a*e*g-ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))*c*d*f+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)
)/((c*d*x+a*e)*(g*x+f))^(1/2)/g/(c*d*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(119) = 238$.

Time = 0.49 (sec) , antiderivative size = 577, normalized size of antiderivative = 4.03

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}\sqrt{f + gx}}\,dx$$

$$= \left[\frac{4\,\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}\sqrt{gx + f}\,cdg - (cd^2f - adeg + (cdef - ae^2g)x)\sqrt{cdg}\,\log\left(-\frac{8}{}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1
/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
 + f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(
-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sq
rt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*
d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^2*x + c*d^2*g^2), 1/2*(2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c*d
*g + (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqr
t(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^2*e*g^2*x^3 + a*c*d^2*e*f*g +
 (c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (a*c*d^2*e*g^2 + (c^2*d
^3 + a*c*d*e^2)*f*g)*x)))/(c*d*e*g^2*x + c*d^2*g^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}\sqrt{f + gx}}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\sqrt{f + gx}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**(1/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*sqrt(
g*x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\left(\frac{(cdef - ae^2g)\log\left(\left|-\sqrt{(ex+d)cde - cd^2e + ae^3}\sqrt{cdg} + \sqrt{c^2d^2e^2f - acde^3g + ((ex+d)cde - cd^2e + ae^3)cdg}\right|\right)}{\sqrt{cdg}g} + \frac{\sqrt{c^2d^2e^2f - acde^3g + ((ex+d)cd}}{}\right.}{cde}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1
/2),x, algorithm="giac")
```

output
```
((c*d*e*f - a*e^2*g)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqrt
(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e +
a*e^3)*c*d*g)))/(sqrt(c*d*g)*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e
^3)/(c*d*e*g))*abs(c)*abs(d)/(c*d*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*
x)^(1/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*
x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$=\frac{\sqrt{gx+f}\,\sqrt{cdx+ae}\,cdg+\sqrt{g}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{gx+f}}{\sqrt{aeg-cdf}}\right)aeg-\sqrt{g}\,\sqrt{d}\,\sqrt{c}\log\bigg(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}}{\sqrt{aeg-cdf}}}{cd\,g^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

output
```
(sqrt(f + g*x)*sqrt(a*e + c*d*x)*c*d*g + sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt
(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f)
)*a*e*g - sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)
*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c*d*f)/(c*d*g**2)
```

3.60 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 134

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx=-\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}+\frac{2\sqrt{c}\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{g^{3/2}}$$

output
```
-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^(1/2)+2
*c^(1/2)*d^(1/2)*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c
^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/g^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\bigg(-\sqrt{g}\sqrt{ae+cdx}+\sqrt{c}\sqrt{d}\sqrt{f+gx}\,\text{arctanh}\bigg(}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(3/2)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]) + Sqrt[c]
*Sqrt[d]*Sqrt[f + g*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sq
rt[a*e + c*d*x])]))/(g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1249, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{g}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

$$\Big\downarrow \text{1268}$$

$$\frac{cd\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

$$\Big\downarrow \text{66}$$

$$\frac{2cd\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
3/2)),x]
```

output

```
(-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f +
 g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[
g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])]])/(g^(3/2)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1268

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.40

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\,\Bigg(\ln\Bigg(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\Bigg)cdgx+\ln\Bigg(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\Bigg)cdf-2\sqrt{(cd}}{\sqrt{cdg}\,\sqrt{(cdx+ae)(gx+f)}\,g\sqrt{gx+f}\,\sqrt{ex+d}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(c*d*x+a*e))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*
(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*g*x+ln(1/2*(2*c*d*g*x+a*e
*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*f
-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)/((c*d*x+a*e)*(
g*x+f))^(1/2)/g/(g*x+f)^(1/2)/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.89

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx=\Bigg[\frac{(egx^2+df+(ef+dg)x)\sqrt{\frac{cd}{g}}\log\Bigg(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2efg+a^2}{}}{}$$

$$-\frac{(egx^2+df+(ef+dg)x)\sqrt{-\frac{cd}{g}}\arctan\left(\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}\sqrt{-\frac{cd}{g}}g}{2\,cdegx^2+cd^2f+adeg+(cdef+(2\,cd^2+ae^2)g)x}\right)+2\,\sqrt{cdex^2+ade+(}}{eg^2x^2+dfg+(efg+dg^2)x}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(3
/2),x, algorithm="fricas")
```

output
```
[1/2*((e*g*x^2 + d*f + (e*f + d*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^
3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f
*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sq
rt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^
2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*
e^3)*g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(e*x + d)*sqrt(g*x + f))/(e*g^2*x^2 + d*f*g + (e*f*g + d*g^2)*x), -((e*g
*x^2 + d*f + (e*f + d*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*
x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(e*g^2*x^
2 + d*f*g + (e*f*g + d*g^2)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{3/2}}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,(f + gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**(3/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(3/2)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{3/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(3
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = -\frac{2\left(\frac{e^2|c||d|\log\left(\left|-\sqrt{(ex+d)cde-cd^2e+ae^3}\sqrt{cdg}+\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg}\right|\right)}{\sqrt{cdg}g|e|}+\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}e}{\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-c}}\right.}{e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(3
/2),x, algorithm="giac")
```

output
```
-2*(e^2*abs(c)*abs(d)*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqr
t(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)*c*d*g)))/(sqrt(c*d*g)*g*abs(e)) + sqrt((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)*e^2*abs(c)*abs(d)/(sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c
*d*e - c*d^2*e + a*e^3)*c*d*g)*g*abs(e)))*abs(e)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{d+e\,x}} \,d x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*
x)^(1/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*
x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx = \frac{-2\sqrt{gx+f}\,\sqrt{cdx+ae}\,g+2\sqrt{g}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{gx+f}}{\sqrt{aeg-cdf}}\right.}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(3/2),x)
```

output
```
(2*( - sqrt(f + g*x)*sqrt(a*e + c*d*x)*g + sqrt(g)*sqrt(d)*sqrt(c)*log((sq
rt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*
f))*f + sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*s
qrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*g*x - sqrt(g)*sqrt(d)*sqrt(c)*f
 - sqrt(g)*sqrt(d)*sqrt(c)*g*x))/(g**2*(f + g*x))
```

3.61 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

output

```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(
g*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

input

```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(5/2)),x]
```

output

```
(2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f
+ g*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = $\{1248\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{ 1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
5/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d*f - a*e*g)*(d +
e*x)^(3/2)*(f + g*x)^(3/2))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{2(cdx+ae)\sqrt{(ex+d)(cdx+ae)}}{3(gx+f)^{\frac{3}{2}}(aeg-dfc)\sqrt{ex+d}}$	53
gosper	$-\frac{2(cdx+ae)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3(gx+f)^{\frac{3}{2}}(aeg-dfc)\sqrt{ex+d}}$	63
orering	$-\frac{2(cdx+ae)\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{3(gx+f)^{\frac{3}{2}}(aeg-dfc)\sqrt{ex+d}}$	64

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/3*(c*d*x+a*e)/(g*x+f)^(3/2)/(a*e*g-c*d*f)*((e*x+d)*(c*d*x+a*e))^(1/2)/(
e*x+d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(55) = 110$.

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x(}}{3\,(cd^2f^3-adef^2g+(cdefg^2-ae^2g^3)x^3+(2\,cdef^2g-adeg^3+(c}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5
/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*sqrt(e*x + d
)*sqrt(g*x + f)/(c*d^2*f^3 - a*d*e*f^2*g + (c*d*e*f*g^2 - a*e^2*g^3)*x^3 +
 (2*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 - 2*a*e^2)*f*g^2)*x^2 + (c*d*e*f^3 -
2*a*d*e*f*g^2 + (2*c*d^2 - a*e^2)*f^2*g)*x)
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{5/2}}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{\sqrt{d + ex}\,(f + gx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**(5/2),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(5/2)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{5/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(55) = 110.

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{5/2}}\,dx = \frac{2\left(\left(ex + d\right)cde - cd^2e + ae^3\right)^{\frac{3}{2}}c^2d^2e^2g|c||d||e|}{3\left(c^2d^2e^2f - acde^3g + \left(\left(ex + d\right)cde - cd^2e + ae^3\right)cdg\right)^{\frac{3}{2}}\left(cde^2fg|e|\right.}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5
/2),x, algorithm="giac")
```

output
```
2/3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*e^2*g*abs(c)*abs(d)*
abs(e)/((c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)
*c*d*g)^(3/2)*(c*d*e^2*f*g*abs(e) - a*e^3*g^2*abs(e)))
```

Mupad [B] (verification not implemented)

Time = 6.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = -\frac{\left(\frac{2\,a\,e}{3\,a\,e\,g^2-3\,c\,d\,f\,g}+\frac{2\,c\,d\,x}{3\,a\,e\,g^2-3\,c\,d\,f\,g}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(3\,c\,d\,f^2-3\,a\,e\,f\,g\right)\,\sqrt{d+e\,x}}{3\,a\,e\,g^2-3\,c\,d\,f\,g}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(5/2)*(d + e*
x)^(1/2)),x)
```

output
```
-(((2*a*e)/(3*a*e*g^2 - 3*c*d*f*g) + (2*c*d*x)/(3*a*e*g^2 - 3*c*d*f*g))*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x*(f + g*x)^(1/2)*(d + e*x)^
(1/2) - ((f + g*x)^(1/2)*(3*c*d*f^2 - 3*a*e*f*g)*(d + e*x)^(1/2))/(3*a*e*g
^2 - 3*c*d*f*g))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,ae\,g^2}{3}-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,cd\,g^2x}{3}-\frac{2\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,cd\,f^2}{3}-4\sqrt{g}\,\sqrt{d}}{g^2\left(ae\,g^3x^2-cdf\,g^2x^2+2aef\,g^2x-2cd\,f^2gx+ae\,f\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5/2),x)
```

output
```
(2*( - sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*e*g**2 - sqrt(f + g*x)*sqrt(a*e +
 c*d*x)*c*d*g**2*x - sqrt(g)*sqrt(d)*sqrt(c)*c*d*f**2 - 2*sqrt(g)*sqrt(d)*
sqrt(c)*c*d*f*g*x - sqrt(g)*sqrt(d)*sqrt(c)*c*d*g**2*x**2))/(3*g**2*(a*e*f
**2*g + 2*a*e*f*g**2*x + a*e*g**3*x**2 - c*d*f**3 - 2*c*d*f**2*g*x - c*d*f
*g**2*x**2))
```

3.62 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

output

```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(
g*x+f)^(5/2)+4/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}(-3aeg+cd(5f+2gx))}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}}$$

input

```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(7/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-3*a*e*g + c*d*(5*f + 2*g*x)))/(15*(c*
d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}dx$$

$$\Big\downarrow \ 1254$$

$$\frac{2cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^{5/2}}dx}{5(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1248$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
7/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d +
e*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)(-2cdgx+3aeg-5dfc)}{15\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(aeg-dfc)^{2}}$	70
gosper	$-\frac{2(cdx+ae)(-2cdgx+3aeg-5dfc)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{15(gx+f)^{\frac{5}{2}}(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2})\sqrt{ex+d}}$	99
orering	$-\frac{2(-2cdgx+3aeg-5dfc)(cdx+ae)\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{15(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2})(gx+f)^{\frac{5}{2}}\sqrt{ex+d}}$	100

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(7/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/15*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5/2)*(c*d*x+a*e)*
(-2*c*d*g*x+3*a*e*g-5*c*d*f)/(a*e*g-c*d*f)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(113) = 226$.

Time = 0.14 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.12

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^{7/2}}\,dx = \frac{}{15\left(c^2d^3f^5 - 2\,acd^2ef^4g + a^2de^2f^3g^2 + \left(c^2d^2ef^2g^3 - 2\,acde^2fg^4 +\right.\right.}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(7
/2),x, algorithm="fricas")
```

output

```
2/15*(2*c^2*d^2*g*x^2 + 5*a*c*d*e*f - 3*a^2*e^2*g + (5*c^2*d^2*f - a*c*d*e
*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
+ f)/(c^2*d^3*f^5 - 2*a*c*d^2*e*f^4*g + a^2*d*e^2*f^3*g^2 + (c^2*d^2*e*f^2
*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^4 + (3*c^2*d^2*e*f^3*g^2 + a^2*d
*e^2*g^5 + (c^2*d^3 - 6*a*c*d*e^2)*f^2*g^3 - (2*a*c*d^2*e - 3*a^2*e^3)*f*g
^4)*x^3 + 3*(c^2*d^2*e*f^4*g + a^2*d*e^2*f*g^4 + (c^2*d^3 - 2*a*c*d*e^2)*f
^3*g^2 - (2*a*c*d^2*e - a^2*e^3)*f^2*g^3)*x^2 + (c^2*d^2*e*f^5 + 3*a^2*d*e
^2*f^2*g^3 + (3*c^2*d^3 - 2*a*c*d*e^2)*f^4*g - (6*a*c*d^2*e - a^2*e^3)*f^3
*g^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}(f + gx)^{7/2}}\,dx = \text{Timed out}$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**(7/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{7/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(7
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(113) = 226$.

Time = 0.62 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{7/2}}\,dx = \frac{2\left(\frac{2\,((ex+d)cde - cd^2e + ae^3)c^4d^4e^6g^3|c||d|}{c^2d^2e^4f^2g^2|e| - 2\,acde^5fg^3|e| + a^2e^6g^4|e|} + \frac{5\,(c^5d^5e^8fg^2|c||d| - ac^4d^4e^9g^3|c||d|)}{c^2d^2e^4f^2g^2|e| - 2\,acde^5fg^3|e| + a^2e^6g^4|e|}\right)}{15\,(c^2d^2e^2f - acde^3g + ((ex + d)cde - cd^2e +}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(7
/2),x, algorithm="giac")
```

output
```
2/15*(2*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*e^6*g^3*abs(c)*abs(d)/
(c^2*d^2*e^4*f^2*g^2*abs(e) - 2*a*c*d*e^5*f*g^3*abs(e) + a^2*e^6*g^4*abs(e
)) + 5*(c^5*d^5*e^8*f*g^2*abs(c)*abs(d) - a*c^4*d^4*e^9*g^3*abs(c)*abs(d))
/(c^2*d^2*e^4*f^2*g^2*abs(e) - 2*a*c*d*e^5*f*g^3*abs(e) + a^2*e^6*g^4*abs(
e)))*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*abs(e)/((c^2*d^2*e^2*f - a*
c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(5/2)*e^2)
```

Mupad [B] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx = \frac{\left(\frac{x\,(10\,c^2\,d^2\,f-2\,a\,c\,d\,e\,g)}{15\,g^2\,(a\,e\,g-c\,d\,f)^2}-\frac{6\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,g^2\,(a\,e\,g-c\,d\,f)^2}+\frac{4\,c^2\,d^2\,x^2}{15\,g\,(a\,e\,g-c\,d\,f)^2}\right)\sqrt{c\,d\,e\,x^2}}{x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{2\,f\,x\,\sqrt{f+g}}{g}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(7/2)*(d + e*
x)^(1/2)),x)
```

output
```
(((x*(10*c^2*d^2*f - 2*a*c*d*e*g))/(15*g^2*(a*e*g - c*d*f)^2) - (6*a^2*e^2
*g - 10*a*c*d*e*f)/(15*g^2*(a*e*g - c*d*f)^2) + (4*c^2*d^2*x^2)/(15*g*(a*e
*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g
*x)^(1/2)*(d + e*x)^(1/2) + (f^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (2
*f*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g)
```

Reduce [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2e^2g^3}{5}+\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,acdef\,g^2}{3}-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,acde\,g}{15}}{g^2\left(a^2e^2g^5x^3-2acdef\,g^4x^3+c^2d^2f^2g^3x^3+3a^2e^2f\,g\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(7/2),x)
```

output
```
(2*( - 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*e**2*g**3 + 5*sqrt(f + g*x)*
sqrt(a*e + c*d*x)*a*c*d*e*f*g**2 - sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c*d*e
*g**3*x + 5*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*f*g**2*x + 2*sqrt(f
+ g*x)*sqrt(a*e + c*d*x)*c**2*d**2*g**3*x**2 - 2*sqrt(g)*sqrt(d)*sqrt(c)*c
**2*d**2*f**3 - 6*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f**2*g*x - 6*sqrt(g)*s
qrt(d)*sqrt(c)*c**2*d**2*f*g**2*x**2 - 2*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2
*g**3*x**3))/(15*g**2*(a**2*e**2*f**3*g**2 + 3*a**2*e**2*f**2*g**3*x + 3*a
**2*e**2*f*g**4*x**2 + a**2*e**2*g**5*x**3 - 2*a*c*d*e*f**4*g - 6*a*c*d*e*
f**3*g**2*x - 6*a*c*d*e*f**2*g**3*x**2 - 2*a*c*d*e*f*g**4*x**3 + c**2*d**2
*f**5 + 3*c**2*d**2*f**4*g*x + 3*c**2*d**2*f**3*g**2*x**2 + c**2*d**2*f**2
*g**3*x**3))
```

3.63 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{16c^2d^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{3/2}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(
g*x+f)^(7/2)+8/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(3/2)/(g*x+f)^(5/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{2((ae+cdx)(d+ex))^{3/2}\left(15a^2e^2g^2-6acdeg(7f+2gx)+c^2d^2(35}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{7/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(9/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(15*a^2*e^2*g^2 - 6*a*c*d*e*g*(7*f + 2*
g*x) + c^2*d^2*(35*f^2 + 28*f*g*x + 8*g^2*x^2)))/(105*(c*d*f - a*e*g)^3*(d
 + e*x)^(3/2)*(f + g*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x \left(a e^2+c d^2\right)+a d e+c d e x^2}}{\sqrt{d+e x} (f+g x)^{9/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{4 c d \int \frac{\sqrt{c d e x^2+\left(c d^2+a e^2\right) x+a d e}}{\sqrt{d+e x} (f+g x)^{7/2}}dx}{7 (c d f-a e g)}+\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{7 (d+e x)^{3/2} (f+g x)^{7/2} (c d f-a e g)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{4 c d \left(\frac{2 c d \int \frac{\sqrt{c d e x^2+\left(c d^2+a e^2\right) x+a d e}}{\sqrt{d+e x} (f+g x)^{5/2}}dx}{5 (c d f-a e g)}+\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{5 (d+e x)^{3/2} (f+g x)^{5/2} (c d f-a e g)}\right)}{7 (c d f-a e g)}+\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{7 (d+e x)^{3/2} (f+g x)^{7/2} (c d f-a e g)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{7 (d+e x)^{3/2} (f+g x)^{7/2} (c d f-a e g)}+\frac{4 c d \left(\frac{4 c d \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{15 (d+e x)^{3/2} (f+g x)^{3/2} (c d f-a e g)^2}+\frac{2 \left(x \left(a e^2+c d^2\right)+a d e+c d e x^2\right)^{3/2}}{5 (d+e x)^{3/2} (f+g x)^{5/2} (c d f-a e g)}\right)}{7 (c d f-a e g)}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
9/2)),x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d +
e*x)^(3/2)*(f + g*x)^(7/2)) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(15*(c*d*f - a*e*g)^2*(d +
 e*x)^(3/2)*(f + g*x)^(3/2))))/(7*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 1248

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)\left(8g^{2}x^{2}d^{2}c^{2}-12acde\,g^{2}x+28c^{2}d^{2}fgx+15a^{2}e^{2}g^{2}-42acdefg+35f^{2}c^{2}d^{2}\right)}{105\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}\,(aeg-dfc)^{3}}$	119
gosper	$-\frac{2(cdx+ae)\left(8g^{2}x^{2}d^{2}c^{2}-12acde\,g^{2}x+28c^{2}d^{2}fgx+15a^{2}e^{2}g^{2}-42acdefg+35f^{2}c^{2}d^{2}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{105(gx+f)^{\frac{7}{2}}\,(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}d^{3}c^{3})\sqrt{ex+d}}$	169
orering	$-\frac{2\left(8g^{2}x^{2}d^{2}c^{2}-12acde\,g^{2}x+28c^{2}d^{2}fgx+15a^{2}e^{2}g^{2}-42acdefg+35f^{2}c^{2}d^{2}\right)(cdx+ae)\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{105(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}d^{3}c^{3})(gx+f)^{\frac{7}{2}}\sqrt{ex+d}}$	170

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(9/2),x,
method=_RETURNVERBOSE)
```

output

```
-2/105*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(7/2)*(c*d*x+a*e)
*(8*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+28*c^2*d^2*f*g*x+15*a^2*e^2*g^2-42*a*
c*d*e*f*g+35*c^2*d^2*f^2)/(a*e*g-c*d*f)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(174) = 348$.

Time = 0.32 (sec) , antiderivative size = 748, normalized size of antiderivative = 3.78

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{}{105\,(c^3d^4f^7-3\,ac^2d^3ef^6g+3\,a^2cd^2e^2f^5g^2-a^3de^3f^4g^3+(c^3d^3ef^3}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(9
/2),x, algorithm="fricas")
```

output

```
2/105*(8*c^3*d^3*g^2*x^3 + 35*a*c^2*d^2*e*f^2 - 42*a^2*c*d*e^2*f*g + 15*a^
3*e^3*g^2 + 4*(7*c^3*d^3*f*g - a*c^2*d^2*e*g^2)*x^2 + (35*c^3*d^3*f^2 - 14
*a*c^2*d^2*e*f*g + 3*a^2*c*d*e^2*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^7 - 3*a*c^2*d^3*e*f^6*g
+ 3*a^2*c*d^2*e^2*f^5*g^2 - a^3*d*e^3*f^4*g^3 + (c^3*d^3*e*f^3*g^4 - 3*a*c
^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^5 + (4*c^3*d^3*e
*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2
*d^3*e - 4*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^6)*x^4
 + 2*(3*c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 9*a*c^2*d^2*e
^2)*f^4*g^3 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^4 + 3*(2*a^2*c*d^2*e
^2 - a^3*e^4)*f^2*g^5)*x^3 + 2*(2*c^3*d^3*e*f^6*g - 3*a^3*d*e^3*f^2*g^5 +
3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g^2 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*
f^4*g^3 + (9*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^4)*x^2 + (c^3*d^3*e*f^7 - 4*
a^3*d*e^3*f^3*g^4 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g - 3*(4*a*c^2*d^3*e
 - a^2*c*d*e^3)*f^5*g^2 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}(gx+f)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(9
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(9/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(174) = 348.

Time = 0.56 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx=\frac{2\left((ex+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}\left(4\left(\frac{2\left((ex+d)cde-cd^2e+ae^3\right)c^6d^6e^8g}{c^3d^3e^6f^3g^3|e|-3\,ac^2d^2e^7f^2g^4|e|+3\,a^2cde^8f}\right.\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(9
/2),x, algorithm="giac")
```

output
```
2/105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*(4*(2*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)*c^6*d^6*e^8*g^5*abs(c)*abs(d)/(c^3*d^3*e^6*f^3*g^3*abs(e)
- 3*a*c^2*d^2*e^7*f^2*g^4*abs(e) + 3*a^2*c*d*e^8*f*g^5*abs(e) - a^3*e^9*g^
6*abs(e)) + 7*(c^7*d^7*e^10*f*g^4*abs(c)*abs(d) - a*c^6*d^6*e^11*g^5*abs(c
)*abs(d))/(c^3*d^3*e^6*f^3*g^3*abs(e) - 3*a*c^2*d^2*e^7*f^2*g^4*abs(e) + 3
*a^2*c*d*e^8*f*g^5*abs(e) - a^3*e^9*g^6*abs(e)))*((e*x + d)*c*d*e - c*d^2*
e + a*e^3) + 35*(c^8*d^8*e^12*f^2*g^3*abs(c)*abs(d) - 2*a*c^7*d^7*e^13*f*g
^4*abs(c)*abs(d) + a^2*c^6*d^6*e^14*g^5*abs(c)*abs(d))/(c^3*d^3*e^6*f^3*g^
3*abs(e) - 3*a*c^2*d^2*e^7*f^2*g^4*abs(e) + 3*a^2*c*d*e^8*f*g^5*abs(e) - a
^3*e^9*g^6*abs(e)))*abs(e)/((c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*
e - c*d^2*e + a*e^3)*c*d*g)^(7/2)*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{30\,a^3\,e^3\,g^2-84\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x\,\left(6\,a^2\,c\,d\,e^2\,g^2-28\,a\,c^2\,d^2\,e\,f\,g+70\,c^3\,d^3\,f^2\right)}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}}{g^2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(9/2)*(d + e*
x)^(1/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((30*a^3*e^3*g^2 + 70*a*c^
2*d^2*e*f^2 - 84*a^2*c*d*e^2*f*g)/(105*g^3*(a*e*g - c*d*f)^3) + (x*(70*c^3
*d^3*f^2 + 6*a^2*c*d*e^2*g^2 - 28*a*c^2*d^2*e*f*g))/(105*g^3*(a*e*g - c*d*
f)^3) + (16*c^3*d^3*x^3)/(105*g*(a*e*g - c*d*f)^3) - (8*c^2*d^2*x^2*(a*e*g
 - 7*c*d*f))/(105*g^2*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2)*(d + e*x)^
(1/2) + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f + g*x)^(1/
2)*(d + e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

Reduce [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^3e^3g^4}{7}+\frac{4\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2cd\,e^2f\,g^3}{5}-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2}{35}}{g^2\left(a^3e^3g^7x^4-3a^2cd\,e^2f\,g^6x^4+3a\,c^2d^2e\,f^2g^5x^4-c^3d^3f^3g^4\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(9/2),x)
```

output
```
(2*( - 15*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*e**3*g**4 + 42*sqrt(f + g*x
)*sqrt(a*e + c*d*x)*a**2*c*d*e**2*f*g**3 - 3*sqrt(f + g*x)*sqrt(a*e + c*d*
x)*a**2*c*d*e**2*g**4*x - 35*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e
*f**2*g**2 + 14*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e*f*g**3*x + 4
*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e*g**4*x**2 - 35*sqrt(f + g*x
)*sqrt(a*e + c*d*x)*c**3*d**3*f**2*g**2*x - 28*sqrt(f + g*x)*sqrt(a*e + c*
d*x)*c**3*d**3*f*g**3*x**2 - 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*g
**4*x**3 + 8*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f**4 + 32*sqrt(g)*sqrt(d)*s
qrt(c)*c**3*d**3*f**3*g*x + 48*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f**2*g**2
*x**2 + 32*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f*g**3*x**3 + 8*sqrt(g)*sqrt(
d)*sqrt(c)*c**3*d**3*g**4*x**4))/(105*g**2*(a**3*e**3*f**4*g**3 + 4*a**3*e
**3*f**3*g**4*x + 6*a**3*e**3*f**2*g**5*x**2 + 4*a**3*e**3*f*g**6*x**3 + a
**3*e**3*g**7*x**4 - 3*a**2*c*d*e**2*f**5*g**2 - 12*a**2*c*d*e**2*f**4*g**
3*x - 18*a**2*c*d*e**2*f**3*g**4*x**2 - 12*a**2*c*d*e**2*f**2*g**5*x**3 -
3*a**2*c*d*e**2*f*g**6*x**4 + 3*a*c**2*d**2*e*f**6*g + 12*a*c**2*d**2*e*f*
*5*g**2*x + 18*a*c**2*d**2*e*f**4*g**3*x**2 + 12*a*c**2*d**2*e*f**3*g**4*x
**3 + 3*a*c**2*d**2*e*f**2*g**5*x**4 - c**3*d**3*f**7 - 4*c**3*d**3*f**6*g
*x - 6*c**3*d**3*f**5*g**2*x**2 - 4*c**3*d**3*f**4*g**3*x**3 - c**3*d**3*f
**3*g**4*x**4))
```

3.64 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 267

$$\begin{aligned}\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} \\ &+ \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{5/2}} \\ &+ \frac{32c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{315(cdf-aeg)^4(d+ex)^{3/2}(f+gx)^{3/2}}\end{aligned}$$

output
```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(
g*x+f)^(9/2)+4/21*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(3/2)/(g*x+f)^(7/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(5/2)+32/315*c^3*d^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(3/2)/(g*
x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{3/2}\left(-35a^3e^3g^3+15a^2cde^2g^2(9f+2gx)-3\right.}{315(cdf-ae}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f +
g*x)^(11/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-35*a^3*e^3*g^3 + 15*a^2*c*d*e^2*g^2*(
9*f + 2*g*x) - 3*a*c^2*d^2*e*g*(63*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(
105*f^3 + 126*f^2*g*x + 72*f*g^2*x^2 + 16*g^3*x^3)))/(315*(c*d*f - a*e*g)^
4*(d + e*x)^(3/2)*(f + g*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1254, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^{9/2}}dx}{3(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\left(\frac{4cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^{7/2}}dx}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{3(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \color{blue}{1254}$$

$$\frac{2cd\left(\frac{4cd\left(\frac{2cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^{5/2}}dx}{5(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{3(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \color{blue}{1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}+\frac{2cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}\right)}{3(cdf-aeg)}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(
11/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d*f - a*e*g)*(d +
e*x)^(3/2)*(f + g*x)^(9/2)) + (2*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f + g*x)^(7/2)) + (4*c*d
*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d
+ e*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))))/(7*(c
*d*f - a*e*g))))/(3*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.72

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)\left(-16x^3g^3d^3c^3+24a\,c^2d^2e\,g^3x^2-72c^3d^3f\,g^2x^2-30a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-126c^3d^3f^2gx+\right.}{315\sqrt{ex+d}\,(gx+f)^{\frac{9}{2}}\,(aeg-dfc)^4}$
gosper	$-\frac{2(cdx+ae)\left(-16x^3g^3d^3c^3+24a\,c^2d^2e\,g^3x^2-72c^3d^3f\,g^2x^2-30a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-126c^3d^3f^2gx+35a^3e^3g^3-135a^2cd\right.}{315(gx+f)^{\frac{9}{2}}\left(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4d^4\right.}$
orering	$-\frac{2\left(-16x^3g^3d^3c^3+24a\,c^2d^2e\,g^3x^2-72c^3d^3f\,g^2x^2-30a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-126c^3d^3f^2gx+35a^3e^3g^3-135a^2cd\,e^2f\,g^2+\right.}{315(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4d^4c^4)(gx+}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(11/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/315*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(9/2)*(c*d*x+a*e)
*(-16*c^3*d^3*g^3*x^3+24*a*c^2*d^2*e*g^3*x^2-72*c^3*d^3*f*g^2*x^2-30*a^2*c
*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-126*c^3*d^3*f^2*g*x+35*a^3*e^3*g^3-13
5*a^2*c*d*e^2*f*g^2+189*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)/(a*e*g-c*d*f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(235) = 470$.

Time = 0.88 (sec) , antiderivative size = 1179, normalized size of antiderivative = 4.42

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1
1/2),x, algorithm="fricas")
```

output
```
2/315*(16*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 189*a^2*c^2*d^2*e^2*f^2*
g + 135*a^3*c*d*e^3*f*g^2 - 35*a^4*e^4*g^3 + 8*(9*c^4*d^4*f*g^2 - a*c^3*d^
3*e*g^3)*x^3 + 6*(21*c^4*d^4*f^2*g - 6*a*c^3*d^3*e*f*g^2 + a^2*c^2*d^2*e^2
*g^3)*x^2 + (105*c^4*d^4*f^3 - 63*a*c^3*d^3*e*f^2*g + 27*a^2*c^2*d^2*e^2*f
*g^2 - 5*a^3*c*d*e^3*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^9 - 4*a*c^3*d^4*e*f^8*g + 6*a^2*c^2*
d^3*e^2*f^7*g^2 - 4*a^3*c*d^2*e^3*f^6*g^3 + a^4*d*e^4*f^5*g^4 + (c^4*d^4*e
*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*a^3*c*d
*e^4*f*g^8 + a^4*e^5*g^9)*x^6 + (5*c^4*d^4*e*f^5*g^4 + a^4*d*e^4*g^9 + (c^
4*d^5 - 20*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)
*f^3*g^6 + 2*(3*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e
^3 - 5*a^4*e^5)*f*g^8)*x^5 + 5*(2*c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c
^4*d^5 - 8*a*c^3*d^3*e^2)*f^5*g^4 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^
4*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^3*g^6 - 2*(2*a^3*c*d^2*e^3
 - a^4*e^5)*f^2*g^7)*x^4 + 10*(c^4*d^4*e*f^7*g^2 + a^4*d*e^4*f^2*g^7 + (c^
4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f
^5*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*g^5 - (4*a^3*c*d^2*e^3
- a^4*e^5)*f^3*g^6)*x^3 + 5*(c^4*d^4*e*f^8*g + 2*a^4*d*e^4*f^3*g^6 + 2*(c^
4*d^5 - 2*a*c^3*d^3*e^2)*f^7*g^2 - 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f
^6*g^3 + 4*(3*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^4 - (8*a^3*c*d^2*e^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)/(g*x+
f)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{11/2}}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1
1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x
+ f)^(11/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(235) = 470$.

Time = 0.97 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{\sqrt{d + ex}(f + gx)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1
1/2),x, algorithm="giac")
```

output
```
2/315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*(2*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*(4*(2*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^8*d^8*e^10*g^7*ab
s(c)*abs(d)/(c^4*d^4*e^8*f^4*g^4*abs(e) - 4*a*c^3*d^3*e^9*f^3*g^5*abs(e) +
 6*a^2*c^2*d^2*e^10*f^2*g^6*abs(e) - 4*a^3*c*d*e^11*f*g^7*abs(e) + a^4*e^1
2*g^8*abs(e)) + 9*(c^9*d^9*e^12*f*g^6*abs(c)*abs(d) - a*c^8*d^8*e^13*g^7*a
bs(c)*abs(d))/(c^4*d^4*e^8*f^4*g^4*abs(e) - 4*a*c^3*d^3*e^9*f^3*g^5*abs(e)
 + 6*a^2*c^2*d^2*e^10*f^2*g^6*abs(e) - 4*a^3*c*d*e^11*f*g^7*abs(e) + a^4*e
^12*g^8*abs(e)))*((e*x + d)*c*d*e - c*d^2*e + a*e^3) + 63*(c^10*d^10*e^14*
f^2*g^5*abs(c)*abs(d) - 2*a*c^9*d^9*e^15*f*g^6*abs(c)*abs(d) + a^2*c^8*d^8
*e^16*g^7*abs(c)*abs(d))/(c^4*d^4*e^8*f^4*g^4*abs(e) - 4*a*c^3*d^3*e^9*f^3
*g^5*abs(e) + 6*a^2*c^2*d^2*e^10*f^2*g^6*abs(e) - 4*a^3*c*d*e^11*f*g^7*abs
(e) + a^4*e^12*g^8*abs(e))) + 105*(c^11*d^11*e^16*f^3*g^4*abs(c)*abs(d) -
3*a*c^10*d^10*e^17*f^2*g^5*abs(c)*abs(d) + 3*a^2*c^9*d^9*e^18*f*g^6*abs(c)
*abs(d) - a^3*c^8*d^8*e^19*g^7*abs(c)*abs(d))/(c^4*d^4*e^8*f^4*g^4*abs(e)
- 4*a*c^3*d^3*e^9*f^3*g^5*abs(e) + 6*a^2*c^2*d^2*e^10*f^2*g^6*abs(e) - 4*a
^3*c*d*e^11*f*g^7*abs(e) + a^4*e^12*g^8*abs(e)))*abs(e)/((c^2*d^2*e^2*f -
a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)^(9/2)*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.18 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}}\,dx=\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{x\,(-10\,a^3\,c\,d\,e^3\,g^3+54\,a^2\,c^2\,d^2\,e^2\,f\,g^2-12}{315\,g^4\,(a\,e\,g-c\,d}\right.}{x^4\,\sqrt{f+}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(11/2)*(d + e
*x)^(1/2)),x)
```

output

```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((x*(210*c^4*d^4*f^3 - 10*a
^3*c*d*e^3*g^3 + 54*a^2*c^2*d^2*e^2*f*g^2 - 126*a*c^3*d^3*e*f^2*g))/(315*g
^4*(a*e*g - c*d*f)^4) - (70*a^4*e^4*g^3 - 210*a*c^3*d^3*e*f^3 + 378*a^2*c^
2*d^2*e^2*f^2*g - 270*a^3*c*d*e^3*f*g^2)/(315*g^4*(a*e*g - c*d*f)^4) + (32
*c^4*d^4*x^4)/(315*g*(a*e*g - c*d*f)^4) + (4*c^2*d^2*x^2*(a^2*e^2*g^2 + 21
*c^2*d^2*f^2 - 6*a*c*d*e*f*g))/(105*g^3*(a*e*g - c*d*f)^4) - (16*c^3*d^3*x
^3*(a*e*g - 9*c*d*f))/(315*g^2*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2)*(
d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f +
 g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))
/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

Reduce [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 1094, normalized size of antiderivative = 4.10

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{11/2}}\,dx = \text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(11/2),x
)
```

output

```
(2*( - 35*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**4*e**4*g**5 + 135*sqrt(f + g*
x)*sqrt(a*e + c*d*x)*a**3*c*d*e**3*f*g**4 - 5*sqrt(f + g*x)*sqrt(a*e + c*d
*x)*a**3*c*d*e**3*g**5*x - 189*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**2*d
**2*e**2*f**2*g**3 + 27*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**
2*f*g**4*x + 6*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*g**5*x*
*2 + 105*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f**3*g**2 - 63*sqrt
(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f**2*g**3*x - 36*sqrt(f + g*x)*s
qrt(a*e + c*d*x)*a*c**3*d**3*e*f*g**4*x**2 - 8*sqrt(f + g*x)*sqrt(a*e + c*
d*x)*a*c**3*d**3*e*g**5*x**3 + 105*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d*
*4*f**3*g**2*x + 126*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*f**2*g**3*x
**2 + 72*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*f*g**4*x**3 + 16*sqrt(f
 + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*g**5*x**4 - 16*sqrt(g)*sqrt(d)*sqrt(c)
*c**4*d**4*f**5 - 80*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*f**4*g*x - 160*sqrt
(g)*sqrt(d)*sqrt(c)*c**4*d**4*f**3*g**2*x**2 - 160*sqrt(g)*sqrt(d)*sqrt(c)
*c**4*d**4*f**2*g**3*x**3 - 80*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*f*g**4*x*
*4 - 16*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*g**5*x**5))/(315*g**2*(a**4*e**4
*f**5*g**4 + 5*a**4*e**4*f**4*g**5*x + 10*a**4*e**4*f**3*g**6*x**2 + 10*a*
*4*e**4*f**2*g**7*x**3 + 5*a**4*e**4*f*g**8*x**4 + a**4*e**4*g**9*x**5 - 4
*a**3*c*d*e**3*f**6*g**3 - 20*a**3*c*d*e**3*f**5*g**4*x - 40*a**3*c*d*e**3
*f**4*g**5*x**2 - 40*a**3*c*d*e**3*f**3*g**6*x**3 - 20*a**3*c*d*e**3*f*...
```

3.65 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 364

$$\begin{aligned}
&\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \\
&-\frac{3(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}} \\
&+\frac{(cdf-aeg)^2\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{32c^2d^2g(d+ex)^{3/2}} \\
&+\frac{(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8c^2d^2(d+ex)^{5/2}} \\
&+\frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4cd(d+ex)^{5/2}} \\
&+\frac{3(cdf-aeg)^4\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}}
\end{aligned}$$

output
```
-3/64*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/c^2/d^2/g^2/(e*x+d)^(1/2)+1/32*(-a*e*g+c*d*f)^2*(g*x+f)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/g/(e*x+d)^(3/2)+1/8*(-a*e*g+c*d*f)*
(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)^(5/2
)+1/4*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5
/2)+3/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/c^(5/2)/d^(5/2)/g^(5/
2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.67

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(-3a^3e^3g^3+a^2cde^2}{}\right.}{}$$

input
```
Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/
(d + e*x)^(3/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(
-3*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g*(11*f^2 +
44*f*g*x + 24*g^2*x^2) + c^3*d^3*(-3*f^3 + 2*f^2*g*x + 24*f*g^2*x^2 + 16*g
^3*x^3)))/(a*e + c*d*x) + (3*(c*d*f - a*e*g)^4*ArcTanh[(Sqrt[c]*Sqrt[d]*Sq
rt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(3/2)))/(64*c^(5/
2)*d^(5/2)*g^(5/2)*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1250, 1250, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \ 1250$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\int \frac{(f+gx)^{3/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{8g}$$

$$\Big\downarrow \ 1250$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6g}\right)}{8g}$$

$$\Big\downarrow \ 1253$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ad}}{2cd\sqrt{d+ex}}\right.}{6g}\right.}{8g}$$

$$\Big\downarrow \ 1253$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-$$

$$\frac{3(cdf-aeg)\Bigg(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\Bigg(\frac{3(cdf-aeg)\Bigg(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{}}{}}{4cd}}{6g}}{8g}$$

$$\Big\downarrow \; 1268$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-$$

$$\frac{3(cdf-aeg)\Bigg(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\Bigg(\frac{3(cdf-aeg)\Bigg(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\Big(}}{}}{4cd}}{6g}}{8g}$$

$$\Big\downarrow \; 66$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+}}{\ }\right.}{4cd}\right.}{6g}\right.}{8g}$$

$$\Big\downarrow \text{221}$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+g}}{\ }\right.}{4cd}\right.}{6g}\right.}{8g}$$

input
```
Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2),x]
```

output

```
((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g*(d +
e*x)^(3/2)) - (3*(c*d*f - a*e*g)*(((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(((f + g*x)^(
3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) +
(3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*
x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(
c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*
c*d)))/(6*g)))/(8*g)
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(312) = 624$.

Time = 2.65 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.01

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\,\Big(32c^3d^3g^3x^3\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}+3\ln\Big(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\Big)a^4e^4g^4-12\ln\Big(}{}$

input
```
int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
1/128*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(32*c^3*d^3*g^3*x^3*((c*d*
x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*
d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-12*ln(1/
2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d
*g)^(1/2))*a^3*c*d*e^3*f*g^3+18*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*
e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-12
*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)
)/(c*d*g)^(1/2))*a*c^3*d^3*e*f^3*g+3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d
*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4+48*a*c^2*
d^2*e*g^3*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+48*c^3*d^3*f*g^2*x
^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+4*(c*d*g)^(1/2)*((c*d*x+a*e)*
(g*x+f))^(1/2)*a^2*c*d*e^2*g^3*x+88*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1
/2)*a*c^2*d^2*e*f*g^2*x+4*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*c^3*d^
3*f^2*g*x-6*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a^3*e^3*g^3+22*(c*d*
g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a^2*c*d*e^2*f*g^2+22*(c*d*g)^(1/2)*((
c*d*x+a*e)*(g*x+f))^(1/2)*a*c^2*d^2*e*f^2*g-6*(c*d*g)^(1/2)*((c*d*x+a*e)*(
g*x+f))^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/c^2/d^2/g^2/((c*d*x+a*e)*(g*x+f))
^(1/2)/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 1120, normalized size of antiderivative = 3.08

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="fricas")
```

output

```
[1/256*(4*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3*d^3*e*f^2*g^2 +
 11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^4*f*g^3 + a*c^3*
d^3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3 + a^2*c^2*d^2*e
^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f) + 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^
2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e
^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*
x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g +
 a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
+ c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*
g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c
*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^3*x
 + c^3*d^4*g^3), 1/128*(2*(16*c^4*d^4*g^4*x^3 - 3*c^4*d^4*f^3*g + 11*a*c^3
*d^3*e*f^2*g^2 + 11*a^2*c^2*d^2*e^2*f*g^3 - 3*a^3*c*d*e^3*g^4 + 24*(c^4*d^
4*f*g^3 + a*c^3*d^3*e*g^4)*x^2 + 2*(c^4*d^4*f^2*g^2 + 22*a*c^3*d^3*e*f*g^3
 + a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)*sqrt(g*x + f) - 3*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^
2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4
 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3
 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+
d)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(e
*x + d)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6262 vs. $2(312) = 624$.

Time = 0.96 (sec) , antiderivative size = 6262, normalized size of antiderivative = 17.20

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="giac")
```

output
```
1/192*(48*(4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*
g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d
)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2
*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e*
f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d
)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
 + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
(e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
)*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d
*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c
^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) -
 (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^
2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt
(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*
c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))*a*f*abs(e)^2/(e^3*g) - 8*(24*(
(c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqr
t(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)
*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)
*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g)/g^2
 - 24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\left(f+g\,x\right)^{3/2}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(d+e\,x\right)}^{3/2}}\,d$$

input
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2),x)
```

output
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.67

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{-3\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^3cd\,e^3g^4+11\sqrt{gx+f}\,\sqrt{cdx}}{}$$

input
```
int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
( - 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**3*g**4 + 11*sqrt(f + g*x
)*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*f*g**3 + 2*sqrt(f + g*x)*sqrt(a*e
+ c*d*x)*a**2*c**2*d**2*e**2*g**4*x + 11*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a
*c**3*d**3*e*f**2*g**2 + 44*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*
f*g**3*x + 24*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*g**4*x**2 - 3*
sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*f**3*g + 2*sqrt(f + g*x)*sqrt(a*
e + c*d*x)*c**4*d**4*f**2*g**2*x + 24*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4
*d**4*f*g**3*x**2 + 16*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*g**4*x**3
 + 3*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt
(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**4*e**4*g**4 - 12*sqrt(g)*sqrt(d
)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/
sqrt(a*e*g - c*d*f))*a**3*c*d*e**3*f*g**3 + 18*sqrt(g)*sqrt(d)*sqrt(c)*log
((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g -
c*d*f))*a**2*c**2*d**2*e**2*f**2*g**2 - 12*sqrt(g)*sqrt(d)*sqrt(c)*log((sq
rt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*
f))*a*c**3*d**3*e*f**3*g + 3*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e
 + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**4*d**4*
f**4)/(64*c**3*d**3*g**3)
```

3.66 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 292

$$\begin{aligned}\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx &= -\frac{(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}} \\ &\quad -\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12(d+ex)^{3/2}} \\ &\quad +\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3cd(d+ex)^{5/2}} \\ &\quad +\frac{(cdf-aeg)^3\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}}\end{aligned}$$

output

```
-1/8*(-a*e*g+c*d*f)^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c/d/g^2/(e*x+d)^(1/2)-1/12*(a*e/c/d-f/g)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)+1/3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)+1/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g
*x+f)^(1/2))/c^(3/2)/d^(3/2)/g^(5/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(3a^2e^2g^2+2acdeg(4f+}{ae+cd}\right.}{24c^{3/2}d^{3/2}g}$$

input

```
Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d
 + e*x)^(3/2),x]
```

output

```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(
3*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + 7*g*x) + c^2*d^2*(-3*f^2 + 2*f*g*x + 8*
g^2*x^2)))/(a*e + c*d*x) + (3*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*S
qrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(3/2)))/(24*c^(3
/2)*d^(3/2)*g^(5/2)*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1250, 1250, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int \frac{\sqrt{f+gx}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}dx}{2g}$$

$$\Big\downarrow \text{ 1250}$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4g}\right)}{2g}$$

$$\Big\downarrow 1253$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+a}}{cd\sqrt{d+ex}}\right.}{4g}\right.}{2g}$$

$$\Big\downarrow 1268$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade}}{cd\sqrt{d+ex}}\right.}{4g}\right.}{2g}$$

$$\Big\downarrow 66$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd\right.}}{cd\sqrt{d+}}\right.}{4g}\right.}{2g}$$

$$\Big\downarrow 221$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right.}}{cd\sqrt{d+e}}\right.}{4g}\right.}{2g}$$

input
```
Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x
)^(3/2),x]
```

output
```
((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d +
e*x)^(3/2)) - ((c*d*f - a*e*g)*(((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(2*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*((Sqrt[f + g*x]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*
f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*g)))/(2*g)
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(248) = 496$.

Time = 2.63 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\left(3\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^3e^3g^3-9\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{c}}{2\sqrt{cdg}}\right.\right.}{}$

input
```
int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x,
method=_RETURNVERBOSE)
```

output

```
-1/48*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g
+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3
*g^3-9*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+9*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2
*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2
*g-3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(
1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3-16*c^2*d^2*g^2*x^2*((c*d*x+a*e)*(g*x+f))^
(1/2)*(c*d*g)^(1/2)-28*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*g
^2*x-4*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f*g*x-6*((c*d*x+a
*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-16*a*c*d*e*f*g*((c*d*x+a*e)*(
g*x+f))^(1/2)*(c*d*g)^(1/2)+6*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^
2*d^2*f^2)/(e*x+d)^(1/2)/c/d/((c*d*x+a*e)*(g*x+f))^(1/2)/g^2/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 908, normalized size of antiderivative = 3.11

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2 + 3*a^
2*c*d*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*f^3 -
3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d^3*e*f
^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*sqrt(c*
d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2
*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f +
a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d
^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*
g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g^3*x + c^2*d^3
*g^3), 1/48*(2*(8*c^3*d^3*g^3*x^2 - 3*c^3*d^3*f^2*g + 8*a*c^2*d^2*e*f*g^2
+ 3*a^2*c*d*e^2*g^3 + 2*(c^3*d^3*f*g^2 + 7*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^3*d^4*
f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*d
^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*s
qrt(-c*d*g)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
g*x + c*d*f + a*e*g)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^2*e*g
^2*x^3 + a*c*d^2*e*f*g + (c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 +
 (a*c*d^2*e*g^2 + (c^2*d^3 + a*c*d*e^2)*f*g)*x)))/(c^2*d^2*e*g^3*x + c^2*d
^3*g^3)]
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\sqrt{f+gx}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+
d)**(3/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)/(d + e*x)**(3/2),
x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}\sqrt{gx+f}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(g*x + f)/(e*x
 + d)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2098 vs. $2(248) = 496$.

Time = 0.42 (sec) , antiderivative size = 2098, normalized size of antiderivative = 7.18

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3
/2),x, algorithm="giac")
```

output
```
1/24*(6*(4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g
- d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*
e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
 + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e*f*
abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*
e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
 d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*
d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e
*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c^2
*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) - (
3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^2*
e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c
*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*
d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))*a*abs(e)^2/(e^3*g) - (24*((c*d*e
^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*
g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g
)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g -
 d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g)/g^2 - 24*
((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{\sqrt{f+g\,x}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2),x)
```

output
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e
*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{3\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2cd\,e^2g^3+8\sqrt{gx+f}\,\sqrt{cdx+ae}}{}$$

input

```
int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output

```
(3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**2*g**3 + 8*sqrt(f + g*x)*sq
rt(a*e + c*d*x)*a*c**2*d**2*e*f*g**2 + 14*sqrt(f + g*x)*sqrt(a*e + c*d*x)*
a*c**2*d**2*e*g**3*x - 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*f**2*g
 + 2*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*f*g**2*x + 8*sqrt(f + g*x)*s
qrt(a*e + c*d*x)*c**3*d**3*g**3*x**2 - 3*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt
(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f)
)*a**3*e**3*g**3 + 9*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*c*d*e**2*f*g*
*2 - 9*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sq
rt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*c**2*d**2*e*f**2*g + 3*sqrt(g)
*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f +
 g*x))/sqrt(a*e*g - c*d*f))*c**3*d**3*f**3)/(24*c**2*d**2*g**3)
```

3.67 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 214

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = &-\frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}}\\ &+\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}\\ &+\frac{3(cdf-aeg)^2\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}g^{5/2}}\end{aligned}$$

output

```
-3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
g^2/(e*x+d)^(1/2)+1/2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/g/(e*x+d)^(3/2)+3/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/c^(1/2)/
d^(1/2)/g^(5/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.77

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\Big(\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{ae+cdx}\sqrt{f+gx}(5aeg+cd(-}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{(ae+cdx}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*S
qrt[f + g*x]),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x
]*Sqrt[f + g*x]*(5*a*e*g + c*d*(-3*f + 2*g*x)) + 3*(c*d*f - a*e*g)^2*ArcTa
nh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*Sqrt[
c]*Sqrt[d]*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {1250, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}\right)}{4g}$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{4g}$$

$$\Big\downarrow \text{66}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{4g}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{4g}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sqrt[f
+ g*x]),x]
```

output

```
(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2*g*(d + e*
x)^(3/2)) - (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*S
qrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f +
 g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])))/(4*g)
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\,\sqrt{gx+f}\,\left(3\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2e^2g^2-6\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right.\right.}{8}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/8*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)*(g*x+f)^(1/2)*(3*ln(1/2*(2*c
*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1
/2))*a^2*e^2*g^2-6*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(
1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+d
*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f
^2+4*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*g*x+10*((c*d*x+a*e)*(g*
x+f))^(1/2)*(c*d*g)^(1/2)*a*e*g-6*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2
)*c*d*f)/((c*d*x+a*e)*(g*x+f))^(1/2)/g^2/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 712, normalized size of antiderivative = 3.33

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \left[\frac{4\left(2\,c^2d^2g^2x-3\,c^2d^2fg+5\,acdeg^2\right)\sqrt{cdex^2+ade+\left(cd^2+ae^2\right.}}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g^2)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2
- 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2
*e^3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d
^2*e*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d
^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e
*g^3*x + c*d^2*g^3), 1/8*(2*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*e*g
^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f
) - 3*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*
a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(-c*d*g)*sqrt(e*
x + d)*sqrt(g*x + f)/(c^2*d^2*e*g^2*x^3 + a*c*d^2*e*f*g + (c^2*d^2*e*f*g +
 (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (a*c*d^2*e*g^2 + (c^2*d^3 + a*c*d*e^2)*f
*g)*x)))/(c*d*e*g^3*x + c*d^2*g^3)]
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\int \frac{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(1/2),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*sqrt(f + g*x))
, x)
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\int \frac{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*s
qrt(g*x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.42

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\frac{\Bigg(\sqrt{c^2d^2e^2f-acde^3g+((ex+d)cde-cd^2e+ae^3)cdg}\sqrt{(ex+}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
/2),x, algorithm="giac")
```

output

```
1/4*(sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*c*d*g)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*((e*x + d)*c*d*e - c*d
^2*e + a*e^3)*abs(e)/(c*d*e^2*g) - 3*(c*d*e^2*f*g*abs(e) - a*e^3*g^2*abs(e
))/(c*d*e^2*g^3)) - 3*(c^2*d^2*e^2*f^2*abs(e) - 2*a*c*d*e^3*f*g*abs(e) + a
^2*e^4*g^2*abs(e))*log(abs(-sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*sqrt(c
*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*
e^3)*c*d*g)))/(sqrt(c*d*g)*g^2))*abs(c)*abs(d)/(c*d*e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*
x)^(3/2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*
x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.18

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx=\frac{5\sqrt{gx+f}\,\sqrt{cdx+ae}\,acde\,g^2-3\sqrt{gx+f}\,\sqrt{cdx+ae}\,c^2d^2fg+2}{}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x)
```

output

```
(5*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c*d*e*g**2 - 3*sqrt(f + g*x)*sqrt(a*e
 + c*d*x)*c**2*d**2*f*g + 2*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*g**2
*x + 3*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sq
rt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*e**2*g**2 - 6*sqrt(g)*sqrt(
d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))
/sqrt(a*e*g - c*d*f))*a*c*d*e*f*g + 3*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)
*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c
**2*d**2*f**2)/(4*c*d*g**3)
```

3.68 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \frac{3cd\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}} - \frac{3\sqrt{c}\sqrt{d}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{g^{5/2}}$$

output

```
3*c*d*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1
/2)-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(1/2
)-3*c^(1/2)*d^(1/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/g^(5/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-2aeg+cd(3f+gx))-3\sqrt{}\right.}{g^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(3/2)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-2*a*e*g + c*
d*(3*f + g*x)) - 3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*Sqrt[f + g*x]*ArcTanh[(
Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(g^(5/2)*Sqr
t[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {1249, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}dx$$

$$\Big\downarrow \; 1249$$

$$\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$\Big\downarrow \; 1250$$

$$\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}\right)}{g}-$$
$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$\Big\downarrow \; 1268$$

$$\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{g}-$$
$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$\Big\downarrow \; 66$$

$$\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{g}-$$
$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

$$\Big\downarrow \; 221$$

$$\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{g}-$$
$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(3/2)),x]
```

output

```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(g*(d + e*x)^(3/2)*Sqrt
[f + g*x]) + (3*c*d*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x
]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(S
qrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/g
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(168) = 336$.

Time = 2.59 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.88

method	result
default	$\frac{\left(3 \ln \left(\frac{2 c d g x + a e g + d f c + 2 \sqrt{(c d x + a e)(g x + f)}\, \sqrt{c d g}}{2 \sqrt{c d g}}\right) a c d e\, g^{2} x - 3 \ln \left(\frac{2 c d g x + a e g + d f c + 2 \sqrt{(c d x + a e)(g x + f)}\, \sqrt{c d g}}{2 \sqrt{c d g}}\right) c^{2} d^{2} f g x + 3 \ln \left(\frac{2 c d g x + a e}{}\right.\right.}{}$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/2),x,
method=_RETURNVERBOSE)
```

output

```
1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)
^(1/2))/(c*d*g)^(1/2))*a*c*d*e*g^2*x-3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c
*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f*g*x+3*ln(
1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c
*d*g)^(1/2))*a*c*d*e*f*g-3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g
*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+2*((c*d*x+a*e)*(g*x
+f))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-4*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/
2)*a*e*g+6*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*f)*((e*x+d)*(c*d*
x+a*e))^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/g^2/(g*x+f)^(1/2)/
(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.35

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \left[\frac{4\,\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdgx + 3\,cdf - 2\,aeg)\sqrt{ex + d}}{}\right.$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3
/2),x, algorithm="fricas")
```

output

```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x + 3*c*d*f - 2
*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*
g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g)*x)*sqrt
(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*
e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g
 + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*
d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^3*x^2 + d*f*
g^2 + (e*f*g^2 + d*g^3)*x), 1/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(c*d*g*x + 3*c*d*f - 2*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c*d^2*
f^2 - a*d*e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*g^2 + (
c*d^2 - a*e^2)*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 +
 c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^3*x^2 + d*f
*g^2 + (e*f*g^2 + d*g^3)*x)]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \int \frac{((d+ex)(ae+cdx))^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{3}{2}}}\,dx$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(3/2),x)
```

output

```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*(f + g*x)**(3/
2)), x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3
/2),x, algorithm="maxima")
```

output

```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.83

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \frac{\sqrt{cdx+ae}\left(\frac{(cdx+ae)|c||d|}{g}+\frac{3\,(cdfg|c||d|-aeg^2|c||d|)}{g^3}\right)}{\sqrt{c^2d^2f-acdeg+(cdx+ae)cdg}} + \frac{3\,(cdf|c||d|-aeg|c||d|)\log\left(\left|-\sqrt{cdg}\sqrt{cdx+ae}+\sqrt{c^2d^2f-acdeg+(cdx+ae)cdg}\right|\right)}{\sqrt{cdg}g^2}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3
/2),x, algorithm="giac")
```

output

```
sqrt(c*d*x + a*e)*((c*d*x + a*e)*abs(c)*abs(d)/g + 3*(c*d*f*g*abs(c)*abs(d
) - a*e*g^2*abs(c)*abs(d))/g^3)/sqrt(c^2*d^2*f - a*c*d*e*g + (c*d*x + a*e)
*c*d*g) + 3*(c*d*f*abs(c)*abs(d) - a*e*g*abs(c)*abs(d))*log(abs(-sqrt(c*d*
g)*sqrt(c*d*x + a*e) + sqrt(c^2*d^2*f - a*c*d*e*g + (c*d*x + a*e)*c*d*g)))
/(sqrt(c*d*g)*g^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^{3/2}\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*
x)^(3/2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*
x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.71

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = \frac{-8\sqrt{gx+f}\,\sqrt{cdx+ae}\,ae\,g^2+12\sqrt{gx+f}\,\sqrt{cdx+ae}\,cdfg+4\sqrt{}}{}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/2),x)
```

output

```
( - 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*e*g**2 + 12*sqrt(f + g*x)*sqrt(a*e
 + c*d*x)*c*d*f*g + 4*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c*d*g**2*x + 12*sqrt
(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(
f + g*x))/sqrt(a*e*g - c*d*f))*a*e*f*g + 12*sqrt(g)*sqrt(d)*sqrt(c)*log((s
qrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d
*f))*a*e*g**2*x - 12*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c*d*f**2 - 12*sqrt
(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(
f + g*x))/sqrt(a*e*g - c*d*f))*c*d*f*g*x - 9*sqrt(g)*sqrt(d)*sqrt(c)*a*e*f
*g - 9*sqrt(g)*sqrt(d)*sqrt(c)*a*e*g**2*x + 9*sqrt(g)*sqrt(d)*sqrt(c)*c*d*
f**2 + 9*sqrt(g)*sqrt(d)*sqrt(c)*c*d*f*g*x)/(4*g**3*(f + g*x))
```

3.69 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 190

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx=-\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{2c^{3/2}d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{g^{5/2}}$$

output

```
-2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)/(g*x+f)^(
1/2)-2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(
3/2)+2*c^(3/2)*d^(3/2)*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/g^(5/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.80

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\Big(-\sqrt{g}\sqrt{ae+cdx}(aeg+cd(3f+4gx))+3c}{3g^{5/2}\sqrt{(ae+cdx)(d+ex)}(f+}$$

input

```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(5/2)),x]
```

output

```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]*(a*e*g + c
*d*(3*f + 4*g*x))) + 3*c^(3/2)*d^(3/2)*(f + g*x)^(3/2)*ArcTanh[(Sqrt[c]*Sq
rt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*g^(5/2)*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*(f + g*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {1249, 1249, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}dx$$

$$\Big\downarrow \textcolor{blue}{1249}$$

$$\frac{cd\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}(f+gx)^{3/2}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1249}$$

$$\frac{cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1268}$$

$$\frac{cd\left(\frac{cd\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \; 66$$

$$\frac{cd\left(\frac{2cd\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{cd\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(5/2)),x]
```

output

```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)*(f
 + g*x)^(3/2)) + (c*d*((-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g
*Sqrt[d + e*x]*Sqrt[f + g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[
d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])])/(g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/g
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(158) = 316$.

Time = 2.60 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(3\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2g^2x^2+6\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2}{3\sqrt{cdg}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
1/3*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x
+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*g^2*x^2+6*ln(1/
2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d
*g)^(1/2))*c^2*d^2*f*g*x+3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g
*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2-8*((c*d*x+a*e)*(g*x
+f))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/
2)*a*e*g-6*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*f)/(c*d*g)^(1/2)/
((c*d*x+a*e)*(g*x+f))^(1/2)/g^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 685, normalized size of antiderivative = 3.61

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = \left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}(4\,cdgx+3\,cdf+aeg)\sqrt{ex+}}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5
/2),x, algorithm="fricas")
```

output
```
[-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f
+ a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c
*d*e*f*g + c*d^2*g^2)*x^2 + (c*d*e*f^2 + 2*c*d^2*f*g)*x)*sqrt(c*d/g)*log(-
(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(
2*c*d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 +
 a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g +
(8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^4*x^3 + d*f^2*g^2 + (2*e*
f*g^3 + d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x), -1/3*(2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + 3*c*d*f + a*e*g)*sqrt(e*x + d)*sqr
t(g*x + f) + 3*(c*d*e*g^2*x^3 + c*d^2*f^2 + (2*c*d*e*f*g + c*d^2*g^2)*x^2
+ (c*d*e*f^2 + 2*c*d^2*f*g)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e
*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x
^3 + d*f^2*g^2 + (2*e*f*g^3 + d*g^4)*x^2 + (e*f^2*g^2 + 2*d*f*g^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.29

$$\begin{aligned}&\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx=\\&-\frac{2\,cd|c||d|\log\left(\left|-\sqrt{cdg}\sqrt{cdx+ae}+\sqrt{c^2d^2f-acdeg+(cdx+ae)cdg}\right|\right)}{\sqrt{cdg}g^2}\\&-\frac{2\sqrt{cdx+ae}\left(\frac{4\left(c^3d^3fg^2|c||d|-ac^2d^2eg^3|c||d|\right)(cdx+ae)}{cdfg^3-aeg^4}+\frac{3\left(c^4d^4f^2g|c||d|-2\,ac^3d^3efg^2|c||d|+a^2c^2d^2e^2g^3|c||d|\right)}{cdfg^3-aeg^4}\right)}{3\left(c^2d^2f-acdeg+(cdx+ae)cdg\right)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5
/2),x, algorithm="giac")
```

output
```
-2*c*d*abs(c)*abs(d)*log(abs(-sqrt(c*d*g)*sqrt(c*d*x + a*e) + sqrt(c^2*d^2
*f - a*c*d*e*g + (c*d*x + a*e)*c*d*g)))/(sqrt(c*d*g)*g^2) - 2/3*sqrt(c*d*x
 + a*e)*(4*(c^3*d^3*f*g^2*abs(c)*abs(d) - a*c^2*d^2*e*g^3*abs(c)*abs(d))*(
c*d*x + a*e)/(c*d*f*g^3 - a*e*g^4) + 3*(c^4*d^4*f^2*g*abs(c)*abs(d) - 2*a*
c^3*d^3*e*f*g^2*abs(c)*abs(d) + a^2*c^2*d^2*e^2*g^3*abs(c)*abs(d))/(c*d*f*
g^3 - a*e*g^4))/(c^2*d^2*f - a*c*d*e*g + (c*d*x + a*e)*c*d*g)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(f+g\,x\right)}^{5/2}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*
x)^(3/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*
x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.31

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx=\frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,ae\,g^2}{3}-2\sqrt{gx+f}\,\sqrt{cdx+ae}\,cdfg-\frac{8\sqrt{gx+f}\,\sqrt{cdx+}}{3}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/2),x)
```

output

```
(2*( - sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*e*g**2 - 3*sqrt(f + g*x)*sqrt(a*e
 + c*d*x)*c*d*f*g - 4*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c*d*g**2*x + 3*sqrt(
g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f
 + g*x))/sqrt(a*e*g - c*d*f))*c*d*f**2 + 6*sqrt(g)*sqrt(d)*sqrt(c)*log((sq
rt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*
f))*c*d*f*g*x + 3*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c*d*g**2*x**2))/(3*g*
*3*(f**2 + 2*f*g*x + g**2*x**2))
```

3.70 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

output

```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(
g*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

input

```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(7/2)),x]
```

output

```
(2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f
+ g*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}dx$$

$$\Big\downarrow \text{1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(7/2)),x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(5/2))
```

Defintions of rubi rules used

rule 1248

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(cdx+ae)^{2}}{5\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}\,(aeg-dfc)}$	55
gosper	$-\frac{2(cdx+ae)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{5(gx+f)^{\frac{5}{2}}\,(aeg-dfc)(ex+d)^{\frac{3}{2}}}$	63
orering	$-\frac{2(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{5(gx+f)^{\frac{5}{2}}\,(aeg-dfc)(ex+d)^{\frac{3}{2}}}$	64

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/5*((e*x+d)*(c*d*x+a*e))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5/2)*(c*d*x+a*e)^2
/(a*e*g-c*d*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(55) = 110$.

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = \frac{2\left(c^2d^2x^2+2\,a\right.}{5\left(cd^2f^4-adef^3g+(cdefg^3-ae^2g^4)x^4+(3\,cdef^2g^2-adeg^4+\right.}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7
/2),x, algorithm="fricas")
```

output
```
2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^4 - a*d*e*f^3*g + (c*d*e*
f*g^3 - a*e^2*g^4)*x^4 + (3*c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - 3*a*e^2)*
f*g^3)*x^3 + 3*(c*d*e*f^3*g - a*d*e*f*g^3 + (c*d^2 - a*e^2)*f^2*g^2)*x^2 +
 (c*d*e*f^4 - 3*a*d*e*f^2*g^2 + (3*c*d^2 - a*e^2)*f^3*g)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(55) = 110$.

Time $= 0.25$ (sec) , antiderivative size $= 111$, normalized size of antiderivative $= 1.76$

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2\left(c^5d^5fg^2|c||d|-ac^4d^4eg^3|c||d|\right)(cdx+ae)^{\frac{5}{2}}}{5\left(c^2d^2f^2g^2-2\,acdefg^3+a^2e^2g^4\right)(c^2d^2f-acdeg+(cdx+ae)c}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7
/2),x, algorithm="giac")
```

output
```
2/5*(c^5*d^5*f*g^2*abs(c)*abs(d) - a*c^4*d^4*e*g^3*abs(c)*abs(d))*(c*d*x +
 a*e)^(5/2)/((c^2*d^2*f^2*g^2 - 2*a*c*d*e*f*g^3 + a^2*e^2*g^4)*(c^2*d^2*f
- a*c*d*e*g + (c*d*x + a*e)*c*d*g)^(5/2))
```

Mupad [B] (verification not implemented)

Time = 6.82 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = -\frac{\left(\frac{2\,a^2\,e^2}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{2\,c^2\,d^2\,x^2}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{4\,a\,c\,d\,e\,x}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(5\,c\,d\,f^3-5\,a\,e\,f^2\,g\right)\,\sqrt{d+e\,x}}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{x\,\sqrt{f+g\,x}\,\left(10\,a\,e\,f\,g^2-10\,c\,d\,f^2\,g\right)\,\sqrt{d+e\,x}}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(7/2)*(d + e*
x)^(3/2)),x)
```

output
```
-(((2*a^2*e^2)/(5*a*e*g^3 - 5*c*d*f*g^2) + (2*c^2*d^2*x^2)/(5*a*e*g^3 - 5*
c*d*f*g^2) + (4*a*c*d*e*x)/(5*a*e*g^3 - 5*c*d*f*g^2))*(x*(a*e^2 + c*d^2) +
 a*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*
x)^(1/2)*(5*c*d*f^3 - 5*a*e*f^2*g)*(d + e*x)^(1/2))/(5*a*e*g^3 - 5*c*d*f*g
^2) + (x*(f + g*x)^(1/2)*(10*a*e*f*g^2 - 10*c*d*f^2*g)*(d + e*x)^(1/2))/(5
*a*e*g^3 - 5*c*d*f*g^2))
```

Reduce [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.87

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2e^2g^3}{5}-\frac{4\sqrt{gx+f}\,\sqrt{cdx+ae}\,acde\,g^3x}{5}-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,c^2}{5}}{g^3\left(ae\,g^4x^3-cdf\,g^3x^3+3aef\,g^3x^2-\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/2),x)
```

output

```
(2*( - sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*e**2*g**3 - 2*sqrt(f + g*x)*sq
rt(a*e + c*d*x)*a*c*d*e*g**3*x - sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2
*g**3*x**2 - sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f**3 - 3*sqrt(g)*sqrt(d)*sq
rt(c)*c**2*d**2*f**2*g*x - 3*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f*g**2*x**2
 - sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*g**3*x**3))/(5*g**3*(a*e*f**3*g + 3*a
*e*f**2*g**2*x + 3*a*e*f*g**3*x**2 + a*e*g**4*x**3 - c*d*f**4 - 3*c*d*f**3
*g*x - 3*c*d*f**2*g**2*x**2 - c*d*f*g**3*x**3))
```

3.71 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{5/2}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(
g*x+f)^(7/2)+4/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}(-5aeg+cd(7f+2gx))}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(9/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-5*a*e*g + c*d*(7*f + 2*g*x)))/(35*(c*
d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx$$

$$\Big\downarrow \; 1254$$

$$\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}dx}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

$$\Big\downarrow \; 1248$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(9/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(cdx+ae)(-2cdgx+5aeg-7dfc)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{35(gx+f)^{\frac{7}{2}}\left(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2}\right)(ex+d)^{\frac{3}{2}}}$	99
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,\left(-2g\,x^{2}d^{2}c^{2}+3acdegx-7c^{2}d^{2}fx+5a^{2}e^{2}g-7acdef\right)(cdx+ae)}{35\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}(aeg-dfc)^{2}}$	100
orering	$-\frac{2(-2cdgx+5aeg-7dfc)(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{35\left(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2}\right)(gx+f)^{\frac{7}{2}}(ex+d)^{\frac{3}{2}}}$	100

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/35*(c*d*x+a*e)*(-2*c*d*g*x+5*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(3/2)/(g*x+f)^(7/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)
^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(113) = 226.

Time = 0.16 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.08

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \frac{}{35\,(c^2d^3f^6-2\,acd^2ef^5g+a^2de^2f^4g^2+(c^2d^2ef^2g^4-2\,acde^2fg^5}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9
/2),x, algorithm="fricas")
```

output
```
2/35*(2*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 5*a^3*e^3*g + (7*c^3*d^3*f - a*c
^2*d^2*e*g)*x^2 + 2*(7*a*c^2*d^2*e*f - 4*a^2*c*d*e^2*g)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^6 - 2*
a*c*d^2*e*f^5*g + a^2*d*e^2*f^4*g^2 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d*e^2*f*g
^5 + a^2*e^3*g^6)*x^5 + (4*c^2*d^2*e*f^3*g^3 + a^2*d*e^2*g^6 + (c^2*d^3 -
8*a*c*d*e^2)*f^2*g^4 - 2*(a*c*d^2*e - 2*a^2*e^3)*f*g^5)*x^4 + 2*(3*c^2*d^2
*e*f^4*g^2 + 2*a^2*d*e^2*f*g^5 + 2*(c^2*d^3 - 3*a*c*d*e^2)*f^3*g^3 - (4*a*
c*d^2*e - 3*a^2*e^3)*f^2*g^4)*x^3 + 2*(2*c^2*d^2*e*f^5*g + 3*a^2*d*e^2*f^2
*g^4 + (3*c^2*d^3 - 4*a*c*d*e^2)*f^4*g^2 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g
^3)*x^2 + (c^2*d^2*e*f^6 + 4*a^2*d*e^2*f^3*g^3 + 2*(2*c^2*d^3 - a*c*d*e^2)
*f^5*g - (8*a*c*d^2*e - a^2*e^3)*f^4*g^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{9}{2}}}\,dx$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9
/2),x, algorithm="maxima")
```

output

```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(9/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(113) = 226$.

Time = 0.34 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.00

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx=\frac{2\,(cdx+ae)^{\frac{5}{2}}\left(\frac{2\,(c^7d^7fg^4|c||d|-ac^6d^6eg^5|c||d|)(cdx+ae)}{c^3d^3f^3g^3-3\,ac^2d^2ef^2g^4+3\,a^2cde^2fg^5-a^3e^3g^6}+\frac{7\,(c^8d^8f^2g^3|c||}{c^3d^3f^3g^3-}\right.}{35\,(c^2d^2f-acdeg+(cdx+ae)cdg)}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9
/2),x, algorithm="giac")
```

output

```
2/35*(c*d*x + a*e)^(5/2)*(2*(c^7*d^7*f*g^4*abs(c)*abs(d) - a*c^6*d^6*e*g^5
*abs(c)*abs(d))*(c*d*x + a*e)/(c^3*d^3*f^3*g^3 - 3*a*c^2*d^2*e*f^2*g^4 + 3
*a^2*c*d*e^2*f*g^5 - a^3*e^3*g^6) + 7*(c^8*d^8*f^2*g^3*abs(c)*abs(d) - 2*a
*c^7*d^7*e*f*g^4*abs(c)*abs(d) + a^2*c^6*d^6*e^2*g^5*abs(c)*abs(d))/(c^3*d
^3*f^3*g^3 - 3*a*c^2*d^2*e*f^2*g^4 + 3*a^2*c*d*e^2*f*g^5 - a^3*e^3*g^6))/(
c^2*d^2*f - a*c*d*e*g + (c*d*x + a*e)*c*d*g)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.91

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = -\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^2\,e^2\,\left(5\,a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{4\,c^3\,d^3\,x^3}{35\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,c^2\,d^2\,x^2\,\left(a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{4\,a\,c\,d\,e\,x\,(4\,a\,e}{35\,g^3\,(a\,e\,g-}\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(9/2)*(d + e*
x)^(3/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^2*e^2*(5*a*e*g - 7*c
*d*f))/(35*g^3*(a*e*g - c*d*f)^2) - (4*c^3*d^3*x^3)/(35*g^2*(a*e*g - c*d*f
)^2) + (2*c^2*d^2*x^2*(a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2) + (4*a
*c*d*e*x*(4*a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^
(1/2)*(d + e*x)^(1/2) + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x
^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)
^(1/2))/g^2)
```

Reduce [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 525, normalized size of antiderivative = 4.07

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^3e^3g^4}{7}+\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2cd\,e^2f\,g^3}{5}-\frac{16\sqrt{gx+f}\,\sqrt{cdx+a}}{35}}{g^3\left(a^2e^2g^6x^4-2acdef\,g^5x^4+c^2d\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/2),x)
```

output

```
(2*( - 5*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*e**3*g**4 + 7*sqrt(f + g*x)*
sqrt(a*e + c*d*x)*a**2*c*d*e**2*f*g**3 - 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)
*a**2*c*d*e**2*g**4*x + 14*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e*f
*g**3*x - sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e*g**4*x**2 + 7*sqrt
(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*f*g**3*x**2 + 2*sqrt(f + g*x)*sqrt(a
*e + c*d*x)*c**3*d**3*g**4*x**3 - 2*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f**4
 - 8*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f**3*g*x - 12*sqrt(g)*sqrt(d)*sqrt(
c)*c**3*d**3*f**2*g**2*x**2 - 8*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f*g**3*x
**3 - 2*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*g**4*x**4))/(35*g**3*(a**2*e**2*
f**4*g**2 + 4*a**2*e**2*f**3*g**3*x + 6*a**2*e**2*f**2*g**4*x**2 + 4*a**2*
e**2*f*g**5*x**3 + a**2*e**2*g**6*x**4 - 2*a*c*d*e*f**5*g - 8*a*c*d*e*f**4
*g**2*x - 12*a*c*d*e*f**3*g**3*x**2 - 8*a*c*d*e*f**2*g**4*x**3 - 2*a*c*d*e
*f*g**5*x**4 + c**2*d**2*f**6 + 4*c**2*d**2*f**5*g*x + 6*c**2*d**2*f**4*g*
*2*x**2 + 4*c**2*d**2*f**3*g**3*x**3 + c**2*d**2*f**2*g**4*x**4))
```

3.72 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{315(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{5/2}}$$

output

```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(
g*x+f)^(9/2)+8/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(5/2)/(g*x+f)^(7/2)+16/315*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\,(35a^2e^2g^2-10acdeg(9f+2gx)+c^2d^2}{315(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{9/2}}$$

input

```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(11/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(35*a^2*e^2*g^2 - 10*a*c*d*e*g*(9*f + 2
*g*x) + c^2*d^2*(63*f^2 + 36*f*g*x + 8*g^2*x^2)))/(315*(c*d*f - a*e*g)^3*(
d + e*x)^(5/2)*(f + g*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}dx$$

$$\Big\downarrow \ 1254$$

$$\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}dx}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1248$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{9(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(11/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(9/2)) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(35*(c*d*f - a*e*g)^2*(d +
 e*x)^(5/2)*(f + g*x)^(5/2))))/(9*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)\left(8g^2x^2d^2c^2-20acde\,g^2x+36c^2d^2fgx+35a^2e^2g^2-90acdefg+63f^2c^2d^2\right)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{315(gx+f)^{\frac{9}{2}}\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3d^3c^3\right)(ex+d)^{\frac{3}{2}}}$
orering	$-\frac{2\left(8g^2x^2d^2c^2-20acde\,g^2x+36c^2d^2fgx+35a^2e^2g^2-90acdefg+63f^2c^2d^2\right)(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{3}{2}}}{315\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3d^3c^3\right)(gx+f)^{\frac{9}{2}}(ex+d)^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,\left(8x^3g^2d^3c^3-12a\,c^2d^2e\,g^2x^2+36c^3d^3fg\,x^2+15a^2cd\,e^2g^2x-54a\,c^2d^2efgx+63c^3d^3f^2x+35a^3e^3g^2-90a^2cd}{315\sqrt{ex+d}\,(gx+f)^{\frac{9}{2}}(aeg-dfc)^3}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11/2),x
,method=_RETURNVERBOSE)
```

output

```
-2/315*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-20*a*c*d*e*g^2*x+36*c^2*d^2*f*g*x+35
*a^2*e^2*g^2-90*a*c*d*e*f*g+63*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^(3/2)/(g*x+f)^(9/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f^2
*g-c^3*d^3*f^3)/(e*x+d)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. 2(174) = 348.

Time = 0.39 (sec) , antiderivative size = 918, normalized size of antiderivative = 4.64

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
1/2),x, algorithm="fricas")
```

output

```
2/315*(8*c^4*d^4*g^2*x^4 + 63*a^2*c^2*d^2*e^2*f^2 - 90*a^3*c*d*e^3*f*g + 3
5*a^4*e^4*g^2 + 4*(9*c^4*d^4*f*g - a*c^3*d^3*e*g^2)*x^3 + 3*(21*c^4*d^4*f^
2 - 6*a*c^3*d^3*e*f*g + a^2*c^2*d^2*e^2*g^2)*x^2 + 2*(63*a*c^3*d^3*e*f^2 -
 72*a^2*c^2*d^2*e^2*f*g + 25*a^3*c*d*e^3*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^8 - 3*a*c^2*d^3*
e*f^7*g + 3*a^2*c*d^2*e^2*f^6*g^2 - a^3*d*e^3*f^5*g^3 + (c^3*d^3*e*f^3*g^5
 - 3*a*c^2*d^2*e^2*f^2*g^6 + 3*a^2*c*d*e^3*f*g^7 - a^3*e^4*g^8)*x^6 + (5*c
^3*d^3*e*f^4*g^4 - a^3*d*e^3*g^8 + (c^3*d^4 - 15*a*c^2*d^2*e^2)*f^3*g^5 -
3*(a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2*g^6 + (3*a^2*c*d^2*e^2 - 5*a^3*e^4)*f*
g^7)*x^5 + 5*(2*c^3*d^3*e*f^5*g^3 - a^3*d*e^3*f*g^7 + (c^3*d^4 - 6*a*c^2*d
^2*e^2)*f^4*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^3*g^5 + (3*a^2*c*d^2*e
^2 - 2*a^3*e^4)*f^2*g^6)*x^4 + 10*(c^3*d^3*e*f^6*g^2 - a^3*d*e^3*f^2*g^6 +
 (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^5*g^3 - 3*(a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g
^4 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^3*g^5)*x^3 + 5*(c^3*d^3*e*f^7*g - 2*a^3
*d*e^3*f^3*g^5 + (2*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g^2 - 3*(2*a*c^2*d^3*e
- a^2*c*d*e^3)*f^5*g^3 + (6*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^4)*x^2 + (c^3*d
^3*e*f^8 - 5*a^3*d*e^3*f^4*g^4 + (5*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^7*g - 3*(
5*a*c^2*d^3*e - a^2*c*d*e^3)*f^6*g^2 + (15*a^2*c*d^2*e^2 - a^3*e^4)*f^5*g^
3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \int \frac{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
1/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(11/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(174) = 348.

Time = 0.44 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.35

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \frac{2\,(cdx+ae)^{\frac{5}{2}}\left(4\,(cdx+ae)\left(\frac{2\,\left(c^9d^9fg^6|c||d|-ac^8d^8eg^7|c||d|\right)(cdx}{c^4d^4f^4g^4-4\,ac^3d^3ef^3g^5+6\,a^2c^2d^2e^2f^2g^6-4\,a^3c}\right.\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
1/2),x, algorithm="giac")
```

output
```
2/315*(c*d*x + a*e)^(5/2)*(4*(c*d*x + a*e)*(2*(c^9*d^9*f*g^6*abs(c)*abs(d)
 - a*c^8*d^8*e*g^7*abs(c)*abs(d))*(c*d*x + a*e)/(c^4*d^4*f^4*g^4 - 4*a*c^3
*d^3*e*f^3*g^5 + 6*a^2*c^2*d^2*e^2*f^2*g^6 - 4*a^3*c*d*e^3*f*g^7 + a^4*e^4
*g^8) + 9*(c^10*d^10*f^2*g^5*abs(c)*abs(d) - 2*a*c^9*d^9*e*f*g^6*abs(c)*ab
s(d) + a^2*c^8*d^8*e^2*g^7*abs(c)*abs(d))/(c^4*d^4*f^4*g^4 - 4*a*c^3*d^3*e
*f^3*g^5 + 6*a^2*c^2*d^2*e^2*f^2*g^6 - 4*a^3*c*d*e^3*f*g^7 + a^4*e^4*g^8))
 + 63*(c^11*d^11*f^3*g^4*abs(c)*abs(d) - 3*a*c^10*d^10*e*f^2*g^5*abs(c)*ab
s(d) + 3*a^2*c^9*d^9*e^2*f*g^6*abs(c)*abs(d) - a^3*c^8*d^8*e^3*g^7*abs(c)*
abs(d))/(c^4*d^4*f^4*g^4 - 4*a*c^3*d^3*e*f^3*g^5 + 6*a^2*c^2*d^2*e^2*f^2*g
^6 - 4*a^3*c*d*e^3*f*g^7 + a^4*e^4*g^8))/(c^2*d^2*f - a*c*d*e*g + (c*d*x +
 a*e)*c*d*g)^(9/2)
```

Mupad [B] (verification not implemented)

Time = 7.11 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.90

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\Big(\frac{70\,a^4\,e^4\,g^2-180\,a^3\,c\,d\,e^3\,f\,g+126\,a^2\,c^2\,d^2\,e^2\,f^2}{315\,g^4\,(a\,e\,g-c\,d\,f)^3}+\frac{x^2\,(6\,a^2\,c^2\,d^2\,e^2\,g^2-36\,a\,c^3\,d^3\,e\,f\,g+126\,c^4}{315\,g^4\,(a\,e\,g-c\,d\,f)^3}}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}-}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(11/2)*(d + e
*x)^(3/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((70*a^4*e^4*g^2 + 126*a^2
*c^2*d^2*e^2*f^2 - 180*a^3*c*d*e^3*f*g)/(315*g^4*(a*e*g - c*d*f)^3) + (x^2
*(126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 - 36*a*c^3*d^3*e*f*g))/(315*g^4*
(a*e*g - c*d*f)^3) + (16*c^4*d^4*x^4)/(315*g^2*(a*e*g - c*d*f)^3) - (8*c^3
*d^3*x^3*(a*e*g - 9*c*d*f))/(315*g^3*(a*e*g - c*d*f)^3) + (4*a*c*d*e*x*(25
*a^2*e^2*g^2 + 63*c^2*d^2*f^2 - 72*a*c*d*e*f*g))/(315*g^4*(a*e*g - c*d*f)^
3)))/(x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)
^(1/2))/g^4 + (4*f*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f +
g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/
2))/g^2)
```

Reduce [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 902, normalized size of antiderivative = 4.56

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx=\text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11/2),x
)
```

output

```
(2*( - 35*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**4*e**4*g**5 + 90*sqrt(f + g*x
)*sqrt(a*e + c*d*x)*a**3*c*d*e**3*f*g**4 - 50*sqrt(f + g*x)*sqrt(a*e + c*d
*x)*a**3*c*d*e**3*g**5*x - 63*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**2*d*
*2*e**2*f**2*g**3 + 144*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**
2*f*g**4*x - 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*g**5*x*
*2 - 126*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f**2*g**3*x + 18*sq
rt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f*g**4*x**2 + 4*sqrt(f + g*x)*
sqrt(a*e + c*d*x)*a*c**3*d**3*e*g**5*x**3 - 63*sqrt(f + g*x)*sqrt(a*e + c*
d*x)*c**4*d**4*f**2*g**3*x**2 - 36*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d*
*4*f*g**4*x**3 - 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*g**5*x**4 + 8
*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*f**5 + 40*sqrt(g)*sqrt(d)*sqrt(c)*c**4*
d**4*f**4*g*x + 80*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*f**3*g**2*x**2 + 80*s
qrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*f**2*g**3*x**3 + 40*sqrt(g)*sqrt(d)*sqrt(
c)*c**4*d**4*f*g**4*x**4 + 8*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*g**5*x**5))
/(315*g**3*(a**3*e**3*f**5*g**3 + 5*a**3*e**3*f**4*g**4*x + 10*a**3*e**3*f
**3*g**5*x**2 + 10*a**3*e**3*f**2*g**6*x**3 + 5*a**3*e**3*f*g**7*x**4 + a*
*3*e**3*g**8*x**5 - 3*a**2*c*d*e**2*f**6*g**2 - 15*a**2*c*d*e**2*f**5*g**3
*x - 30*a**2*c*d*e**2*f**4*g**4*x**2 - 30*a**2*c*d*e**2*f**3*g**5*x**3 - 1
5*a**2*c*d*e**2*f**2*g**6*x**4 - 3*a**2*c*d*e**2*f*g**7*x**5 + 3*a*c**2*d*
*2*e*f**7*g + 15*a*c**2*d**2*e*f**6*g**2*x + 30*a*c**2*d**2*e*f**5*g**3...
```

3.73 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 267

$$
\begin{aligned}
\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx &= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} \\
&+ \frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{7/2}} \\
&+ \frac{32c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155(cdf-aeg)^4(d+ex)^{5/2}(f+gx)^{5/2}}
\end{aligned}
$$

output

```
2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/
(g*x+f)^(11/2)+4/33*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*
d*f)^2/(e*x+d)^(5/2)/(g*x+f)^(9/2)+16/231*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(7/2)+32/1155*c^3*d
^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(5/2)/
(g*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}\left(-105a^3e^3g^3+35a^2cde^2g^2(11f+2gx)\right.}{1155(cdf\,-}$$

input

```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(
f + g*x)^(13/2)),x]
```

output

```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-105*a^3*e^3*g^3 + 35*a^2*c*d*e^2*g^2*
(11*f + 2*g*x) - 5*a*c^2*d^2*e*g*(99*f^2 + 44*f*g*x + 8*g^2*x^2) + c^3*d^3
*(231*f^3 + 198*f^2*g*x + 88*f*g^2*x^2 + 16*g^3*x^3)))/(1155*(c*d*f - a*e*
g)^4*(d + e*x)^(5/2)*(f + g*x)^(11/2))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1254, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}dx$$

$$\Big\downarrow \ 1254$$

$$\frac{6cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}dx}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$\frac{6cd\left(\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1254}$$

$$\frac{6cd\left(\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}dx}{7(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}+\frac{6cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}\right)}{9(cdf-aeg)}\right)}{11(cdf-aeg)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*
x)^(13/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d*f - a*e*g)*(d +
 e*x)^(5/2)*(f + g*x)^(11/2)) + (6*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f + g*x)^(9/2)) + (4*c
*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(
d + e*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))))/(9*
(c*d*f - a*e*g))))/(11*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97

method	result
gosper	$-\frac{2(cdx+ae)\left(-16x^{3}g^{3}d^{3}c^{3}+40a\,c^{2}d^{2}e\,g^{3}x^{2}-88c^{3}d^{3}f\,g^{2}x^{2}-70a^{2}cd\,e^{2}g^{3}x+220a\,c^{2}d^{2}ef\,g^{2}x-198c^{3}d^{3}f^{2}gx+105a^{3}e^{3}g^{3}-385a^{2}c\right.}{1155(gx+f)^{\frac{11}{2}}\left(a^{4}e^{4}g^{4}-4a^{3}cd\,e^{3}f\,g^{3}+6a^{2}c^{2}d^{2}e^{2}f^{2}g^{2}-4a\,c^{3}d^{3}e\,f^{3}g+f^{4}\right.}$
orering	$-\frac{2\left(-16x^{3}g^{3}d^{3}c^{3}+40a\,c^{2}d^{2}e\,g^{3}x^{2}-88c^{3}d^{3}f\,g^{2}x^{2}-70a^{2}cd\,e^{2}g^{3}x+220a\,c^{2}d^{2}ef\,g^{2}x-198c^{3}d^{3}f^{2}gx+105a^{3}e^{3}g^{3}-385a^{2}cd\,e^{2}f\,g^{2}+\right.}{1155\left(a^{4}e^{4}g^{4}-4a^{3}cd\,e^{3}f\,g^{3}+6a^{2}c^{2}d^{2}e^{2}f^{2}g^{2}-4a\,c^{3}d^{3}e\,f^{3}g+f^{4}d^{4}c^{4}\right)(gx+}$
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,\left(-16g^{3}x^{4}d^{4}c^{4}+24a\,c^{3}d^{3}e\,g^{3}x^{3}-88c^{4}d^{4}f\,g^{2}x^{3}-30a^{2}c^{2}d^{2}e^{2}g^{3}x^{2}+132a\,c^{3}d^{3}ef\,g^{2}x^{2}-198c^{4}d^{4}f^{2}g\,x^{2}+35a\right.}{1155\sqrt{ex+d}\,(}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/1155*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+40*a*c^2*d^2*e*g^3*x^2-88*c^3*d^3
*f*g^2*x^2-70*a^2*c*d*e^2*g^3*x+220*a*c^2*d^2*e*f*g^2*x-198*c^3*d^3*f^2*g*
x+105*a^3*e^3*g^3-385*a^2*c*d*e^2*f*g^2+495*a*c^2*d^2*e*f^2*g-231*c^3*d^3*
f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/(g*x+f)^(11/2)/(a^4*e^4*g^4-4
*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^4*f
^4)/(e*x+d)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. $2(235) = 470$.

Time = 1.22 (sec) , antiderivative size = 1420, normalized size of antiderivative = 5.32

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
3/2),x, algorithm="fricas")
```

output

```
2/1155*(16*c^5*d^5*g^3*x^5 + 231*a^2*c^3*d^3*e^2*f^3 - 495*a^3*c^2*d^2*e^3
*f^2*g + 385*a^4*c*d*e^4*f*g^2 - 105*a^5*e^5*g^3 + 8*(11*c^5*d^5*f*g^2 - a
*c^4*d^4*e*g^3)*x^4 + 2*(99*c^5*d^5*f^2*g - 22*a*c^4*d^4*e*f*g^2 + 3*a^2*c
^3*d^3*e^2*g^3)*x^3 + (231*c^5*d^5*f^3 - 99*a*c^4*d^4*e*f^2*g + 33*a^2*c^3
*d^3*e^2*f*g^2 - 5*a^3*c^2*d^2*e^3*g^3)*x^2 + 2*(231*a*c^4*d^4*e*f^3 - 396
*a^2*c^3*d^3*e^2*f^2*g + 275*a^3*c^2*d^2*e^3*f*g^2 - 70*a^4*c*d*e^4*g^3)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/
(c^4*d^5*f^10 - 4*a*c^3*d^4*e*f^9*g + 6*a^2*c^2*d^3*e^2*f^8*g^2 - 4*a^3*c*
d^2*e^3*f^7*g^3 + a^4*d*e^4*f^6*g^4 + (c^4*d^4*e*f^4*g^6 - 4*a*c^3*d^3*e^2
*f^3*g^7 + 6*a^2*c^2*d^2*e^3*f^2*g^8 - 4*a^3*c*d*e^4*f*g^9 + a^4*e^5*g^10)
*x^7 + (6*c^4*d^4*e*f^5*g^5 + a^4*d*e^4*g^10 + (c^4*d^5 - 24*a*c^3*d^3*e^2
)*f^4*g^6 - 4*(a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^3*g^7 + 6*(a^2*c^2*d^3*e
^2 - 4*a^3*c*d*e^4)*f^2*g^8 - 2*(2*a^3*c*d^2*e^3 - 3*a^4*e^5)*f*g^9)*x^6 +
 3*(5*c^4*d^4*e*f^6*g^4 + 2*a^4*d*e^4*f*g^9 + 2*(c^4*d^5 - 10*a*c^3*d^3*e^
2)*f^5*g^5 - 2*(4*a*c^3*d^4*e - 15*a^2*c^2*d^2*e^3)*f^4*g^6 + 4*(3*a^2*c^2
*d^3*e^2 - 5*a^3*c*d*e^4)*f^3*g^7 - (8*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^2*g^8)
*x^5 + 5*(4*c^4*d^4*e*f^7*g^3 + 3*a^4*d*e^4*f^2*g^8 + (3*c^4*d^5 - 16*a*c^
3*d^3*e^2)*f^6*g^4 - 12*(a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3)*f^5*g^5 + 2*(9*a
^2*c^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^4*g^6 - 4*(3*a^3*c*d^2*e^3 - a^4*e^5)*f^
3*g^7)*x^4 + 5*(3*c^4*d^4*e*f^8*g^2 + 4*a^4*d*e^4*f^3*g^7 + 4*(c^4*d^5 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{(d + ex)^{3/2}(f + gx)^{13/2}}\, dx = \text{Timed out}$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+
f)**(13/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
3/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(
g*x + f)^(13/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(235) = 470$.

Time = 0.87 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.78

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1
3/2),x, algorithm="giac")
```

output

```
2/1155*(c*d*x + a*e)^(5/2)*(2*(c*d*x + a*e)*(4*(c*d*x + a*e)*(2*(c^11*d^11
*f*g^8*abs(c)*abs(d) - a*c^10*d^10*e*g^9*abs(c)*abs(d))*(c*d*x + a*e)/(c^5
*d^5*f^5*g^5 - 5*a*c^4*d^4*e*f^4*g^6 + 10*a^2*c^3*d^3*e^2*f^3*g^7 - 10*a^3
*c^2*d^2*e^3*f^2*g^8 + 5*a^4*c*d*e^4*f*g^9 - a^5*e^5*g^10) + 11*(c^12*d^12
*f^2*g^7*abs(c)*abs(d) - 2*a*c^11*d^11*e*f*g^8*abs(c)*abs(d) + a^2*c^10*d^
10*e^2*g^9*abs(c)*abs(d))/(c^5*d^5*f^5*g^5 - 5*a*c^4*d^4*e*f^4*g^6 + 10*a^
2*c^3*d^3*e^2*f^3*g^7 - 10*a^3*c^2*d^2*e^3*f^2*g^8 + 5*a^4*c*d*e^4*f*g^9 -
 a^5*e^5*g^10)) + 99*(c^13*d^13*f^3*g^6*abs(c)*abs(d) - 3*a*c^12*d^12*e*f^
2*g^7*abs(c)*abs(d) + 3*a^2*c^11*d^11*e^2*f*g^8*abs(c)*abs(d) - a^3*c^10*d
^10*e^3*g^9*abs(c)*abs(d))/(c^5*d^5*f^5*g^5 - 5*a*c^4*d^4*e*f^4*g^6 + 10*a
^2*c^3*d^3*e^2*f^3*g^7 - 10*a^3*c^2*d^2*e^3*f^2*g^8 + 5*a^4*c*d*e^4*f*g^9
- a^5*e^5*g^10)) + 231*(c^14*d^14*f^4*g^5*abs(c)*abs(d) - 4*a*c^13*d^13*e*
f^3*g^6*abs(c)*abs(d) + 6*a^2*c^12*d^12*e^2*f^2*g^7*abs(c)*abs(d) - 4*a^3*
c^11*d^11*e^3*f*g^8*abs(c)*abs(d) + a^4*c^10*d^10*e^4*g^9*abs(c)*abs(d))/(
c^5*d^5*f^5*g^5 - 5*a*c^4*d^4*e*f^4*g^6 + 10*a^2*c^3*d^3*e^2*f^3*g^7 - 10*
a^3*c^2*d^2*e^3*f^2*g^8 + 5*a^4*c*d*e^4*f*g^9 - a^5*e^5*g^10))/(c^2*d^2*f
- a*c*d*e*g + (c*d*x + a*e)*c*d*g)^(11/2)
```

Mupad [B] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.94

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{210\,a^5\,e^5\,g^3-770\,a^4\,c\,d\,e^4\,f\,g^2+990\,a^3\,c^2\,d^2\,e^3\,f^2\,g-462\,a^2\,c^3\,d^3\,e^2\,f^3}{1155\,g^5\,\left(a\,e\,g-c\,d\,f\right)^4}-\frac{x^2\,\left(-10\,a^3\,c^2\,d^2\,e^3\right.}{}\right.}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+}$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(13/2)*(d + e
*x)^(3/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((210*a^5*e^5*g^3 - 462*a^
2*c^3*d^3*e^2*f^3 + 990*a^3*c^2*d^2*e^3*f^2*g - 770*a^4*c*d*e^4*f*g^2)/(11
55*g^5*(a*e*g - c*d*f)^4) - (x^2*(462*c^5*d^5*f^3 - 10*a^3*c^2*d^2*e^3*g^3
 + 66*a^2*c^3*d^3*e^2*f*g^2 - 198*a*c^4*d^4*e*f^2*g))/(1155*g^5*(a*e*g - c
*d*f)^4) - (32*c^5*d^5*x^5)/(1155*g^2*(a*e*g - c*d*f)^4) - (4*c^3*d^3*x^3*
(3*a^2*e^2*g^2 + 99*c^2*d^2*f^2 - 22*a*c*d*e*f*g))/(1155*g^4*(a*e*g - c*d*
f)^4) + (16*c^4*d^4*x^4*(a*e*g - 11*c*d*f))/(1155*g^3*(a*e*g - c*d*f)^4) +
 (4*a*c*d*e*x*(70*a^3*e^3*g^3 - 231*c^3*d^3*f^3 + 396*a*c^2*d^2*e*f^2*g -
275*a^2*c*d*e^2*f*g^2))/(1155*g^5*(a*e*g - c*d*f)^4)))/(x^5*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(
f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/
2))/g^4 + (10*f^2*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(
f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3)
```

Reduce [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 1359, normalized size of antiderivative = 5.09

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13/2),x
)
```

output

```
(2*( - 105*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**5*e**5*g**6 + 385*sqrt(f + g
*x)*sqrt(a*e + c*d*x)*a**4*c*d*e**4*f*g**5 - 140*sqrt(f + g*x)*sqrt(a*e +
c*d*x)*a**4*c*d*e**4*g**6*x - 495*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**
2*d**2*e**3*f**2*g**4 + 550*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**2
*e**3*f*g**5*x - 5*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**3*g**
6*x**2 + 231*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*f**3*g**3
 - 792*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*f**2*g**4*x + 3
3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*f*g**5*x**2 + 6*sqrt
(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*g**6*x**3 + 462*sqrt(f + g
*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f**3*g**3*x - 99*sqrt(f + g*x)*sqrt(a*
e + c*d*x)*a*c**4*d**4*e*f**2*g**4*x**2 - 44*sqrt(f + g*x)*sqrt(a*e + c*d*
x)*a*c**4*d**4*e*f*g**5*x**3 - 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**4*d*
*4*e*g**6*x**4 + 231*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**5*d**5*f**3*g**3*x
**2 + 198*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**5*d**5*f**2*g**4*x**3 + 88*sq
rt(f + g*x)*sqrt(a*e + c*d*x)*c**5*d**5*f*g**5*x**4 + 16*sqrt(f + g*x)*sqr
t(a*e + c*d*x)*c**5*d**5*g**6*x**5 - 16*sqrt(g)*sqrt(d)*sqrt(c)*c**5*d**5*
f**6 - 96*sqrt(g)*sqrt(d)*sqrt(c)*c**5*d**5*f**5*g*x - 240*sqrt(g)*sqrt(d)
*sqrt(c)*c**5*d**5*f**4*g**2*x**2 - 320*sqrt(g)*sqrt(d)*sqrt(c)*c**5*d**5*
f**3*g**3*x**3 - 240*sqrt(g)*sqrt(d)*sqrt(c)*c**5*d**5*f**2*g**4*x**4 - 96
*sqrt(g)*sqrt(d)*sqrt(c)*c**5*d**5*f*g**5*x**5 - 16*sqrt(g)*sqrt(d)*sqr...
```

3.74 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 436

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{3(cdf-aeg)^4\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}}$$

$$-\frac{(cdf-aeg)^3\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{64c^2d^2g^2(d+ex)^{3/2}}$$

$$+\frac{(cdf-aeg)^2\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{80c^2d^2g(d+ex)^{5/2}}$$

$$+\frac{3(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{40c^2d^2(d+ex)^{7/2}}$$

$$+\frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{5cd(d+ex)^{7/2}}$$

$$-\frac{3(cdf-aeg)^5\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}}$$

output

```
3/128*(-a*e*g+c*d*f)^4*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/c^2/d^2/g^3/(e*x+d)^(1/2)-1/64*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/g^2/(e*x+d)^(3/2)+1/80*(-a*e*g+c*d*
f)^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/g/(e*x+
d)^(5/2)+3/40*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(7/2)/c^2/d^2/(e*x+d)^(7/2)+1/5*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)-3/128*(-a*e*g+c*d*f)^5*arctanh(g^(1/2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f
)^(1/2))/c^(5/2)/d^(5/2)/g^(7/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.69

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(-15a^4e^4g^4+10a^3c}{}\right.}{}$$

input

```
Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/
(d + e*x)^(5/2),x]
```

output

```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(
-15*a^4*e^4*g^4 + 10*a^3*c*d*e^3*g^3*(7*f + g*x) + 2*a^2*c^2*d^2*e^2*g^2*(
64*f^2 + 233*f*g*x + 124*g^2*x^2) + 2*a*c^3*d^3*e*g*(-35*f^3 + 23*f^2*g*x
+ 256*f*g^2*x^2 + 168*g^3*x^3) + c^4*d^4*(15*f^4 - 10*f^3*g*x + 8*f^2*g^2*
x^2 + 176*f*g^3*x^3 + 128*g^4*x^4)))/(a*e + c*d*x)^2 - (15*(c*d*f - a*e*g)
^5*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(
a*e + c*d*x)^(5/2)))/(640*c^(5/2)*d^(5/2)*g^(7/2)*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1250, 1250, 1250, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\int \frac{(f+gx)^{3/2}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{2g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\int \frac{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{8g}\right)}{2g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+(cd^2+ae^2}}}{6g}\right.}{8g}\right.}{2g}$$

$$\Big\downarrow \text{1253}$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\int\frac{}{\sqrt{cde}}}{}\right.}{}\right.}{8g}\right.}{2g}$$

$$\Big\downarrow 1253$$

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{(cdf-}{}\right.}{}\right.}{}\right.}{}\right.}{2g}$$

↓ 1268

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+e}}{}\right.}{}\right.}{}\right.}{}\right.}{2g}$$

$\downarrow$ 66

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-$$

$$\frac{(cdf-aeg)\Bigg(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\Bigg(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\Bigg(\frac{3(cdf-aeg)\Bigg(\frac{\sqrt{d+e}}{}}{}}{}}{}}{2g}$$

$\downarrow$ 221

$$\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}-$$

$$\frac{(cdf-aeg)\left(\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+e}}{}\right.}{}\right.}{}\right.}{}\right.}{2g}$$

input
```
Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2),x]
```

output
```
((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d +
e*x)^(5/2)) - ((c*d*f - a*e*g)*(((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2))/(4*g*(d + e*x)^(3/2)) - (3*(c*d*f - a*e*g)*(((f + g*
x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x])
- ((c*d*f - a*e*g)*(((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*
g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sq
rt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])))/(4*c*d)))/(6*g)))/(8*g)))/(2*g)
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(376) = 752$.

Time = 2.60 (sec) , antiderivative size = 1005, normalized size of antiderivative = 2.31

method	result	size
default	Expression too large to display	1005

input

```
int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2),x,
method=_RETURNVERBOSE)
```

output

```
1/1280*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(256*c^4*d^4*g^4*x^4*((c*
d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+672*a*c^3*d^3*e*g^4*x^3*((c*d*x+a*e)
*(g*x+f))^(1/2)*(c*d*g)^(1/2)+352*c^4*d^4*f*g^3*x^3*((c*d*x+a*e)*(g*x+f))^
(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f
))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^5*e^5*g^5-75*ln(1/2*(2*c*d*g*x+a*
e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*
c*d*e^4*f*g^4+150*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1
/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*c^2*d^2*e^3*f^2*g^3-150*ln(1/2*(2*c*
d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))*a^2*c^3*d^3*e^2*f^3*g^2+75*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e
)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^4*d^4*e*f^4*g-15*ln(1/2
*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*
g)^(1/2))*c^5*d^5*f^5+496*a^2*c^2*d^2*e^2*g^4*x^2*((c*d*x+a*e)*(g*x+f))^(1
/2)*(c*d*g)^(1/2)+1024*a*c^3*d^3*e*f*g^3*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(
c*d*g)^(1/2)+16*c^4*d^4*f^2*g^2*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1
/2)+20*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^3*c*d*e^3*g^4*x+932*((c
*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*c^2*d^2*e^2*f*g^3*x+92*((c*d*x+
a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c^3*d^3*e*f^2*g^2*x-20*((c*d*x+a*e)*(g
*x+f))^(1/2)*(c*d*g)^(1/2)*c^4*d^4*f^3*g*x-30*((c*d*x+a*e)*(g*x+f))^(1/2)*
(c*d*g)^(1/2)*a^4*e^4*g^4+140*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 1392, normalized size of antiderivative = 3.19

$$\int \frac{(f+gx)^{3/2} \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="fricas")
```

output

```
[1/2560*(4*(128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3*g^
2 + 128*a^2*c^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d*e^
4*g^5 + 16*(11*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2*g^
3 + 64*a*c^4*d^4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f^3*
g^2 - 23*a*c^4*d^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2*d^2*e
^3*g^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(
g*x + f) - 15*(c^5*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*
g^2 - 10*a^3*c^2*d^3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 +
 (c^5*d^5*e*f^5 - 5*a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*
a^3*c^2*d^2*e^4*f^2*g^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*sqrt(c*d*g
)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^
2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e
*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3
+ a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g +
 (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d^3*e*g^4*x + c^3*d^4*g^
4), 1/1280*(2*(128*c^5*d^5*g^5*x^4 + 15*c^5*d^5*f^4*g - 70*a*c^4*d^4*e*f^3
*g^2 + 128*a^2*c^3*d^3*e^2*f^2*g^3 + 70*a^3*c^2*d^2*e^3*f*g^4 - 15*a^4*c*d
*e^4*g^5 + 16*(11*c^5*d^5*f*g^4 + 21*a*c^4*d^4*e*g^5)*x^3 + 8*(c^5*d^5*f^2
*g^3 + 64*a*c^4*d^4*e*f*g^4 + 31*a^2*c^3*d^3*e^2*g^5)*x^2 - 2*(5*c^5*d^5*f
^3*g^2 - 23*a*c^4*d^4*e*f^2*g^3 - 233*a^2*c^3*d^3*e^2*f*g^4 - 5*a^3*c^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+
d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^(3/2)/(e
*x + d)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13705 vs. 2(376) = 752.

Time = 1.84 (sec) , antiderivative size = 13705, normalized size of antiderivative = 31.43

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="giac")
```

output
```
1/1920*(480*(4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*
e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
 d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*
e*f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x +
 d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e
*x + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2
+ (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*
g))*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g -
 d*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4
*c^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
 - (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*
d^2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sq
rt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g
)*c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))*a^2*f*abs(e)^2/(e^2*g) - 80*
(24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g
)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d
*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
 + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g
)/g^2 - 24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{3/2}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^{5/2}}\,d$$

input
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2),x)
```

output
```
int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.94

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Too large to display}$$

input

```
int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

output

```
( - 15*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**4*c*d*e**4*g**5 + 70*sqrt(f + g*
x)*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**3*f*g**4 + 10*sqrt(f + g*x)*sqrt(a*
e + c*d*x)*a**3*c**2*d**2*e**3*g**5*x + 128*sqrt(f + g*x)*sqrt(a*e + c*d*x
)*a**2*c**3*d**3*e**2*f**2*g**3 + 466*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2
*c**3*d**3*e**2*f*g**4*x + 248*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**3*d
**3*e**2*g**5*x**2 - 70*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f**3
*g**2 + 46*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f**2*g**3*x + 512
*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f*g**4*x**2 + 336*sqrt(f +
g*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e*g**5*x**3 + 15*sqrt(f + g*x)*sqrt(a*e
 + c*d*x)*c**5*d**5*f**4*g - 10*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**5*d**5*
f**3*g**2*x + 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**5*d**5*f**2*g**3*x**2 +
 176*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**5*d**5*f*g**4*x**3 + 128*sqrt(f +
g*x)*sqrt(a*e + c*d*x)*c**5*d**5*g**5*x**4 + 15*sqrt(g)*sqrt(d)*sqrt(c)*lo
g((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g -
 c*d*f))*a**5*e**5*g**5 - 75*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e
 + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**4*c*d*e
**4*f*g**4 + 150*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) +
sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**3*c**2*d**2*e**3*f*
*2*g**3 - 150*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqr
t(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*c**3*d**3*e**2*f*...
```

3.75 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 364

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}}$$
$$-\frac{5(cdf-aeg)^2\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{96cdg^2(d+ex)^{3/2}}$$
$$-\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{24(d+ex)^{5/2}}$$
$$+\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{4cd(d+ex)^{7/2}}$$
$$-\frac{5(cdf-aeg)^4\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}}$$

output

```
5/64*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c/d/g^3/(e*x+d)^(1/2)-5/96*(-a*e*g+c*d*f)^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/g^2/(e*x+d)^(3/2)-1/24*(a*e/c/d-f/g)*(g*x+f)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)+1/4*(g*x+f)^(
1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)-5/64*(-a*e*
g+c*d*f)^4*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)
/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/c^(3/2)/d^(3/2)/g^(7/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\left(15a^3e^3g^3+a^2cde^2g^2(7\right.}{}\right.}{}$$

input
```
Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d
 + e*x)^(5/2),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[f + g*x]*(
15*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(73*f + 118*g*x) + a*c^2*d^2*e*g*(-55*f^2
 + 36*f*g*x + 136*g^2*x^2) + c^3*d^3*(15*f^3 - 10*f^2*g*x + 8*f*g^2*x^2 +
48*g^3*x^3)))/(a*e + c*d*x)^2 - (15*(c*d*f - a*e*g)^4*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(5/2)))/(1
92*c^(3/2)*d^(3/2)*g^(7/2)*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1250, 1250, 1250, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\int \frac{\sqrt{f+gx}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}}dx}{8g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\int\frac{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}dx}{2g}\right)}{8g}$$

$$\Big\downarrow \ 1250$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}\sqrt{f+}}{\sqrt{cdex^2+\left(cd^2+ae\right.}}}{4g}\right.}{2g}\right.}{8g}$$

$$\Big\downarrow \ 1253$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{(cdf-aeg)\int\frac{}{\sqrt{f+g}}}{}\right.}{}\right.}{2g}\right.}{8g}$$

$$\Big\downarrow \ 1268$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cd}{2cd\sqrt{x\left(ae}}\right.}{}\right.}{2g}\right.}{8g}$$

$$\Big\downarrow \ 66$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cd}{cd\sqrt{x}}\right.}{}\right.}{2g}\right.}{8g}$$

$$\Big\downarrow \ 221$$

$$\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}-\frac{(cdf-aeg)\left(\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cd}{c^{3/2}d^{3/2}\sqrt{}}\right.}{}\right.}{2g}\right.}{8g}$$

input

```
Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x
)^(5/2),x]
```

output

```
((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4*g*(d +
e*x)^(5/2)) - (5*(c*d*f - a*e*g)*(((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) - ((c*d*f - a*e*g)*(((f + g*
x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*g*Sqrt[d + e*x])
- ((c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*
x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(
c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*
g)))/(2*g)))/(8*g)
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(312) = 624$.

Time = 2.65 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.01

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\,\Bigl(-96c^3d^3g^3x^3\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}+15\ln\Bigl(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\Bigr)a^4e^4g^4-}{}$

input
```
int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2),x,
method=_RETURNVERBOSE)
```

output

```
-1/384*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-96*c^3*d^3*g^3*x^3*((c*
d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*(
(c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-60*ln
(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*a^3*c*d*e^3*f*g^3+90*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x
+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2
-60*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1
/2))/(c*d*g)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*(
(c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^4-272*a
*c^2*d^2*e*g^3*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-16*c^3*d^3*f*
g^2*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-236*(c*d*g)^(1/2)*((c*d*
x+a*e)*(g*x+f))^(1/2)*a^2*c*d*e^2*g^3*x-72*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x
+f))^(1/2)*a*c^2*d^2*e*f*g^2*x+20*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2
)*c^3*d^3*f^2*g*x-30*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a^3*e^3*g^3
-146*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a^2*c*d*e^2*f*g^2+110*(c*d*
g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a*c^2*d^2*e*f^2*g-30*(c*d*g)^(1/2)*((
c*d*x+a*e)*(g*x+f))^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/c/d/((c*d*x+a*e)*(g*x
+f))^(1/2)/g^3/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 1126, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="fricas")
```

output
```
[1/768*(4*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g - 55*a*c^3*d^3*e*f^2*g^2
+ 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4 + 8*(c^4*d^4*f*g^3 + 17*a*
c^3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a*c^3*d^3*e*f*g^3 - 59*a^2*
c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*
e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a
*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4
*e^5*g^4)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d
^2*e*f*g + a^2*d*e^2*g^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d
^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d
^2*e*g^4*x + c^2*d^3*g^4), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 15*c^4*d^4*f^3*g
 - 55*a*c^3*d^3*e*f^2*g^2 + 73*a^2*c^2*d^2*e^2*f*g^3 + 15*a^3*c*d*e^3*g^4
+ 8*(c^4*d^4*f*g^3 + 17*a*c^3*d^3*e*g^4)*x^2 - 2*(5*c^4*d^4*f^2*g^2 - 18*a
*c^3*d^3*e*f*g^3 - 59*a^2*c^2*d^2*e^2*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^
4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*
g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 -
 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*sqrt(-c*d*g)*arctan(1/2*sqrt(c*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+
d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}\sqrt{gx+f}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input

```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="maxima")
```

output

```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)/(e*x
 + d)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4837 vs. $2(312) = 624$.

Time = 0.75 (sec) , antiderivative size = 4837, normalized size of antiderivative = 13.29

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5
/2),x, algorithm="giac")
```

output
```
1/192*(48*(4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*
g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d
)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2
*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*e*
f*abs(g)/g^2 - 4*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d
)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
 + d)*e*g - d*e*g)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
(e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)
)*d*abs(g)/g + (sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d
*e*g)*c*d*g)*(2*e^2*f + 2*(e*x + d)*e*g - 2*d*e*g - (5*c^2*d^2*e^2*f - 4*c
^2*d^3*e*g - a*c*d*e^3*g)/(c^2*d^2))*sqrt(e^2*f + (e*x + d)*e*g - d*e*g) -
 (3*c^2*d^2*e^4*f^2*g - 4*c^2*d^3*e^3*f*g^2 - 2*a*c*d*e^5*f*g^2 + 4*a*c*d^
2*e^4*g^3 - a^2*e^6*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt
(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*
c*d*g)))/(sqrt(c*d*g)*c*d))*abs(g)/(e*g^2))*a^2*abs(e)^2/(e^2*g) - 16*(24*
((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sq
rt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g
)*c*d*g)))/sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d
)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g))*d*e*f*abs(g)/g^
2 - 24*((c*d*e^2*f*g - a*e^3*g^2)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx=\int \frac{\sqrt{f+g\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2),x)
```

output
```
int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e
*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{15\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^3cd\,e^3g^4+73\sqrt{gx+f}\,\sqrt{cdx+}}{}$$

input
```
int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
(15*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**3*g**4 + 73*sqrt(f + g*x)*
sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*f*g**3 + 118*sqrt(f + g*x)*sqrt(a*e
+ c*d*x)*a**2*c**2*d**2*e**2*g**4*x - 55*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a
*c**3*d**3*e*f**2*g**2 + 36*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*
f*g**3*x + 136*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*g**4*x**2 + 1
5*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*f**3*g - 10*sqrt(f + g*x)*sqrt
(a*e + c*d*x)*c**4*d**4*f**2*g**2*x + 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c*
*4*d**4*f*g**3*x**2 + 48*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**4*d**4*g**4*x*
*3 - 15*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*s
qrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**4*e**4*g**4 + 60*sqrt(g)*sqr
t(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x
))/sqrt(a*e*g - c*d*f))*a**3*c*d*e**3*f*g**3 - 90*sqrt(g)*sqrt(d)*sqrt(c)*
log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g
 - c*d*f))*a**2*c**2*d**2*e**2*f**2*g**2 + 60*sqrt(g)*sqrt(d)*sqrt(c)*log(
(sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c
*d*f))*a*c**3*d**3*e*f**3*g - 15*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**4*d
**4*f**4)/(192*c**2*d**2*g**4)
```

3.76 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 280

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx &= \frac{5(cdf-aeg)^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}}\\ &-\frac{5(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}}\\ &+\frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}\\ &-\frac{5(cdf-aeg)^3\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{8\sqrt{c}\sqrt{d}g^{7/2}}\end{aligned}$$

output

```
5/8*(-a*e*g+c*d*f)^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/g^3/(e*x+d)^(1/2)-5/12*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)+1/3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)-5/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x
+f)^(1/2))/c^(1/2)/d^(1/2)/g^(7/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}\sqrt{f+gx}(33a^2e^2g^2+2acdeg(-20f+13gx)+c^2d^2(1}{(ae+cdx)^2}\right.}{24g^{7/2}(d+ex)^5}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*S
qrt[f + g*x]),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*Sqrt[f + g*x]*(33*a^2*e^2*g^2 +
 2*a*c*d*e*g*(-20*f + 13*g*x) + c^2*d^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2)))/
(a*e + c*d*x)^2 - (15*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f +
g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)^(5/2)))
)/(24*g^(7/2)*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1250, 1250, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1250}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{6g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4g}\right)}{6g}$$

$$\Big\downarrow \text{1250}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+a\right.}}}{2g}\right.}{4g}\right.}{6g}$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{}{\sqrt{ae+cd}}}{2g\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}\right.}{4g}\right.}{6g}$$

$$\Big\downarrow \text{66}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(a}{f}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+c}}\right.}{4g}\right.}{6g}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-$$

$$\frac{5(cdf-aeg)\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+a}}\right.}{4g}\right.}{6g}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sqrt[f
+ g*x]),x]
```

output
```
(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*
x)^(5/2)) - (5*(c*d*f - a*e*g)*((Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(2*g*(d + e*x)^(3/2)) - (3*(c*d*f - a*e*g)*((Sqrt[f +
g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - ((c*
d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c
*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*g)))/(6*g)
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(236) = 472$.

Time = 2.68 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\,\sqrt{gx+f}\,\left(15\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^{3}e^{3}g^{3}-45\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{c}}{2\sqrt{cdg}}\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/48*((e*x+d)*(c*d*x+a*e))^(1/2)*(g*x+f)^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g
+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3
*g^3-45*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c
+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f
^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+16*c^2*d^2*g^2*x^2*((c*d*x+a*e)*(g*x+f
))^(1/2)*(c*d*g)^(1/2)+52*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*
e*g^2*x-20*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f*g*x+66*((c*
d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-80*a*c*d*e*f*g*((c*d*x+a
*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+30*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1
/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/g^3/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 898, normalized size of antiderivative = 3.21

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
/2),x, algorithm="fricas")
```

output

```
[1/96*(4*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g^2 + 33
*a^2*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^3*d^4
*f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3 + (c^3*
d^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g^3)*x)*
sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^
2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c
*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g +
 (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*
e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c*d*e*g^4*x + c*d^
2*g^4), 1/48*(2*(8*c^3*d^3*g^3*x^2 + 15*c^3*d^3*f^2*g - 40*a*c^2*d^2*e*f*g
^2 + 33*a^2*c*d*e^2*g^3 - 2*(5*c^3*d^3*f*g^2 - 13*a*c^2*d^2*e*g^3)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(
c^3*d^4*f^3 - 3*a*c^2*d^3*e*f^2*g + 3*a^2*c*d^2*e^2*f*g^2 - a^3*d*e^3*g^3
+ (c^3*d^3*e*f^3 - 3*a*c^2*d^2*e^2*f^2*g + 3*a^2*c*d*e^3*f*g^2 - a^3*e^4*g
^3)*x)*sqrt(-c*d*g)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2
*d^2*e*g^2*x^3 + a*c*d^2*e*f*g + (c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^
2)*x^2 + (a*c*d^2*e*g^2 + (c^2*d^3 + a*c*d*e^2)*f*g)*x)))/(c*d*e*g^4*x + c
*d^2*g^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*s
qrt(g*x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.48

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\frac{\left(\sqrt{c^2d^2e^2f-acde^3g+\left((ex+d)cde-cd^2e+ae^3\right)cdg}\sqrt{(ex+a}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
/2),x, algorithm="giac")
```

output
```
1/24*(sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^
3)*c*d*g)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(2*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)*(4*((e*x + d)*c*d*e - c*d^2*e + a*e^3)*abs(e)/(c*d*e^3*g) -
 5*(c*d*e^2*f*g^3*abs(e) - a*e^3*g^4*abs(e))/(c*d*e^3*g^5)) + 15*(c^2*d^2*
e^4*f^2*g^2*abs(e) - 2*a*c*d*e^5*f*g^3*abs(e) + a^2*e^6*g^4*abs(e))/(c*d*e
^3*g^5)) + 15*(c^3*d^3*e^3*f^3*abs(e) - 3*a*c^2*d^2*e^4*f^2*g*abs(e) + 3*a
^2*c*d*e^5*f*g^2*abs(e) - a^3*e^6*g^3*abs(e))*log(abs(-sqrt((e*x + d)*c*d*
e - c*d^2*e + a*e^3)*sqrt(c*d*g) + sqrt(c^2*d^2*e^2*f - a*c*d*e^3*g + ((e*
x + d)*c*d*e - c*d^2*e + a*e^3)*c*d*g)))/(sqrt(c*d*g)*g^3))*abs(c)*abs(d)/
(c*d*e^3*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*
x)^(5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*
x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.47

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\frac{33\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2cd\,e^2g^3-40\sqrt{gx+f}\,\sqrt{cdx+ae}\,a\,c^2d^2e}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x)
```

output
```
(33*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**2*g**3 - 40*sqrt(f + g*x)*
sqrt(a*e + c*d*x)*a*c**2*d**2*e*f*g**2 + 26*sqrt(f + g*x)*sqrt(a*e + c*d*x
)*a*c**2*d**2*e*g**3*x + 15*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*f**2
*g - 10*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*f*g**2*x + 8*sqrt(f + g*
x)*sqrt(a*e + c*d*x)*c**3*d**3*g**3*x**2 + 15*sqrt(g)*sqrt(d)*sqrt(c)*log(
(sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c
*d*f))*a**3*e**3*g**3 - 45*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e +
 c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*c*d*e**
2*f*g**2 + 45*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqr
t(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*c**2*d**2*e*f**2*g - 15
*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*
sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**3*d**3*f**3)/(24*c*d*g**4)
```

3.77 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 270

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}} + \frac{15\sqrt{c}\sqrt{d}(cdf-aeg)^2\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{4g^{7/2}}$$

output

```
-15/4*c*d*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/g^3/(e*x+d)^(1/2)+5/2*c*d*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/g^2/(e*x+d)^(3/2)-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(
e*x+d)^(5/2)/(g*x+f)^(1/2)+15/4*c^(1/2)*d^(1/2)*(-a*e*g+c*d*f)^2*arctanh(g
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/
2)/(g*x+f)^(1/2))/g^(7/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.74

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}(-8a^2e^2g^2+acdeg(25f+9g\right.}{4g^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(3/2)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-8*a^2*e^2*g^
2 + a*c*d*e*g*(25*f + 9*g*x) + c^2*d^2*(-15*f^2 - 5*f*g*x + 2*g^2*x^2)) +
15*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*ArcTanh[(Sqrt[c]*Sqrt[d
]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*g^(7/2)*Sqrt[(a*e + c*d
*x)*(d + e*x)]*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1249, 1250, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$$\Big\downarrow \text{1250}$$

$$\frac{5cd\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4g}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$$\Big\downarrow \ 1250$$

$$\frac{5cd\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}\right.}{4g}\right.}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$$\Big\downarrow \ 1268$$

$$\frac{5cd\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{4g}\right.}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$$\Big\downarrow \ 66$$

$$\frac{5cd\left(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae}}{\sqrt{f}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right.}{4g}\right.}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

$$\Big\downarrow \ 221$$

$$\frac{5cd\Bigg(\frac{\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}-\frac{3(cdf-aeg)\Bigg(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\big(\frac{\sqrt{g}\sqrt{ae+c}}{\sqrt{c}\sqrt{d}\sqrt{f+}}}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}}{4g}}{g}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(3/2)),x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(g*(d + e*x)^(5/2)*Sqrt
[f + g*x]) + (5*c*d*((Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2))/(2*g*(d + e*x)^(3/2)) - (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - ((c*d*f - a*e*g
)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])))/(4*g)))/g
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(228) = 456.

Time = 2.78 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.31

method	result
default	$\left(15\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^{2}cd\,e^{2}g^{3}x-30\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^{2}d^{2}ef\,g^{2}x+15\ln\right.$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/2),x,
method=_RETURNVERBOSE)
```

output

```
1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c
+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f
*g^2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c
+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f
*g^2-30*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c
+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3+4
*c^2*d^2*g^2*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+18*((c*d*x+a*e)
*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*g^2*x-10*((c*d*x+a*e)*(g*x+f))^(1/2)
*(c*d*g)^(1/2)*c^2*d^2*f*g*x-16*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*
a^2*e^2*g^2+50*a*c*d*e*f*g*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-30*((
c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((e*x+d)*(c*d*x+a*e))
^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(1/2)/(e*x+d)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 915, normalized size of antiderivative = 3.39

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx=\text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3
/2),x, algorithm="fricas")
```

output

```
[1/16*(4*(2*c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*
g^2 - (5*c^2*d^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2
*g + a^2*d*e^2*f*g^2 + (c^2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)
*x^2 + (c^2*d^2*e*f^3 + a^2*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2
*a*c*d^2*e - a^2*e^3)*f*g^2)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^
2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x + c*d*f*g + a
*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
 + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c
^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g
^2)*x)/(e*x + d)))/(e*g^4*x^2 + d*f*g^3 + (e*f*g^3 + d*g^4)*x), 1/8*(2*(2*
c^2*d^2*g^2*x^2 - 15*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 8*a^2*e^2*g^2 - (5*c^2
*d^2*f*g - 9*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^3*f^3 - 2*a*c*d^2*e*f^2*g + a^2*d*e
^2*f*g^2 + (c^2*d^2*e*f^2*g - 2*a*c*d*e^2*f*g^2 + a^2*e^3*g^3)*x^2 + (c^2*
d^2*e*f^3 + a^2*d*e^2*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f^2*g - (2*a*c*d^2*e -
 a^2*e^3)*f*g^2)*x)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*
d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(e*g^4*x^2 + d*f*g^
3 + (e*f*g^3 + d*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{3/2}}\,dx = \text{Timed out}$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(3/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.94

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = \frac{\sqrt{cdx+ae}\left((cdx+ae)\left(\frac{2\,(cdx+ae)|c||d|}{g}-\frac{5\,(cdfg^3|c||d|-aeg^4|c||d|)}{g^5}\right)-15\right.}{4\,\sqrt{c^2d^2f-acdeg+(cdx+ae)}} - \frac{15\,(c^2d^2f^2|c||d|-2\,acdefg|c||d|+a^2e^2g^2|c||d|)\log\left(\left|-\sqrt{cdg}\sqrt{cdx+ae}+\sqrt{c^2d^2f-acdeg+(cdx+ae)}\right.\right.}{4\,\sqrt{cdg}g^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3
/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*d*x + a*e)*((c*d*x + a*e)*(2*(c*d*x + a*e)*abs(c)*abs(d)/g - 5*
(c*d*f*g^3*abs(c)*abs(d) - a*e*g^4*abs(c)*abs(d))/g^5) - 15*(c^2*d^2*f^2*g
^2*abs(c)*abs(d) - 2*a*c*d*e*f*g^3*abs(c)*abs(d) + a^2*e^2*g^4*abs(c)*abs(
d))/g^5)/sqrt(c^2*d^2*f - a*c*d*e*g + (c*d*x + a*e)*c*d*g) - 15/4*(c^2*d^2
*f^2*abs(c)*abs(d) - 2*a*c*d*e*f*g*abs(c)*abs(d) + a^2*e^2*g^2*abs(c)*abs(
d))*log(abs(-sqrt(c*d*g)*sqrt(c*d*x + a*e) + sqrt(c^2*d^2*f - a*c*d*e*g +
(c*d*x + a*e)*c*d*g)))/(sqrt(c*d*g)*g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{3/2}}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(f+g\,x\right)}^{3/2}\,{\left(d+e\,x\right)}^{5/2}} \,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*
x)^(5/2)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*
x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.29

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{3/2}}\,dx = \frac{-8\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2e^2g^3 + 25\sqrt{gx+f}\,\sqrt{cdx+ae}\,acdef\,g^2}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/2),x)
```

output

```
( - 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*e**2*g**3 + 25*sqrt(f + g*x)*sq
rt(a*e + c*d*x)*a*c*d*e*f*g**2 + 9*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c*d*e
*g**3*x - 15*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g - 5*sqrt(f +
 g*x)*sqrt(a*e + c*d*x)*c**2*d**2*f*g**2*x + 2*sqrt(f + g*x)*sqrt(a*e + c*
d*x)*c**2*d**2*g**3*x**2 + 15*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*
e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*e**2
*f*g**2 + 15*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*e**2*g**3*x - 30*sqrt
(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(
f + g*x))/sqrt(a*e*g - c*d*f))*a*c*d*e*f**2*g - 30*sqrt(g)*sqrt(d)*sqrt(c)
*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*
g - c*d*f))*a*c*d*e*f*g**2*x + 15*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqr
t(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2*
d**2*f**3 + 15*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sq
rt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2*d**2*f**2*g*x - 10*
sqrt(g)*sqrt(d)*sqrt(c)*a**2*e**2*f*g**2 - 10*sqrt(g)*sqrt(d)*sqrt(c)*a**2
*e**2*g**3*x + 20*sqrt(g)*sqrt(d)*sqrt(c)*a*c*d*e*f**2*g + 20*sqrt(g)*sqrt
(d)*sqrt(c)*a*c*d*e*f*g**2*x - 10*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f**3 -
 10*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f**2*g*x)/(4*g**4*(f + g*x))
```

3.78 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 260

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} - \frac{5c^{3/2}d^{3/2}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{g^{7/2}}$$

output

```
5*c^2*d^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d
)^(1/2)-10/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)
/(g*x+f)^(1/2)-2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)
/(g*x+f)^(3/2)-5*c^(3/2)*d^(3/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))
/g^(7/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.72

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}\left(-2a^2e^2g^2-2acdeg(5f+7gx)+c^2d^2\left(15f^2+20fg\right.\right.}{(ae+cdx)^2(f+gx)^{3/2}}\right.}{3g^{7/2}(d+ex)^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(5/2)),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(-2*a^2*e^2*g^2 - 2*a*c*d*e*g*(
5*f + 7*g*x) + c^2*d^2*(15*f^2 + 20*f*g*x + 3*g^2*x^2)))/((a*e + c*d*x)^2*
(f + g*x)^(3/2)) - (15*c^(3/2)*d^(3/2)*(c*d*f - a*e*g)*ArcTanh[(Sqrt[c]*Sq
rt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(5/2)))/(
3*g^(7/2)*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1249, 1249, 1250, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}dx}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow \text{1249}$$

$$\frac{5cd\left(\frac{3cd\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}\sqrt{f+gx}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow 1250$$

$$\frac{5cd\left(\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2g}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow 1268$$

$$\frac{5cd\left(\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow 66$$

$$\frac{5cd\left(\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

$$\Big\downarrow 221$$

$$\frac{5cd\left(\frac{3cd\left(\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}\right)}{3g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(5/2)),x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*x)^(5/2)*(f
 + g*x)^(3/2)) + (5*c*d*((-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
)/(g*(d + e*x)^(3/2)*Sqrt[f + g*x]) + (3*c*d*((Sqrt[f + g*x]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*Sqrt[
a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sq
rt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])))/g))/(3*g)
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1250
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Simp[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1)))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(
p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*d^2 - b*d*e +
 a*e^2, 0] && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n,
0] &&  !(IntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(220) = 440.

Time = 2.71 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.42

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^2d^2e\,g^3x^2-15\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^3d^3f\,g^2x^2+30\ln\left(\right.\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/2),x,
method=_RETURNVERBOSE)
```

output

```
1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*ln(1/2*(2*c*d*g*x+a*e*g+d*f
*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f*g
^2*x^2+30*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f*g^2*x-30*ln(1/2*(2*c*d*g*x+a*e*g+d
*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f
^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+d*f
*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^3
+6*c^2*d^2*g^2*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-28*((c*d*x+a*
e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*g^2*x+40*((c*d*x+a*e)*(g*x+f))^(1/
2)*(c*d*g)^(1/2)*c^2*d^2*f*g*x-4*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)
*a^2*e^2*g^2-20*a*c*d*e*f*g*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+30*(
(c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((e*x+d)*(c*d*x+a*e)
)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(3/2)/(e*x+d
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 973, normalized size of antiderivative = 3.74

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5
/2),x, algorithm="fricas")
```

output
```
[1/12*(4*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g - 2*a^2*e^2*
g^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^3*f^3 - a*c*d^2*e*f^
2*g + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x^3 + (2*c^2*d^2*e*f^2*g - a*c*d^2
*e*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x^2 + (c^2*d^2*e*f^3 - 2*a*c*d^2*e
*f*g^2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c^2*d^2*e*g
^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*d*g^2*x +
c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^
2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e +
 a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^4 + d*g^5)
*x^2 + (e*f^2*g^3 + 2*d*f*g^4)*x), 1/6*(2*(3*c^2*d^2*g^2*x^2 + 15*c^2*d^2*
f^2 - 10*a*c*d*e*f*g - 2*a^2*e^2*g^2 + 2*(10*c^2*d^2*f*g - 7*a*c*d*e*g^2)*
x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)
 + 15*(c^2*d^3*f^3 - a*c*d^2*e*f^2*g + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x
^3 + (2*c^2*d^2*e*f^2*g - a*c*d^2*e*g^3 + (c^2*d^3 - 2*a*c*d*e^2)*f*g^2)*x
^2 + (c^2*d^2*e*f^3 - 2*a*c*d^2*e*f*g^2 + (2*c^2*d^3 - a*c*d*e^2)*f^2*g)*x
)*sqrt(-c*d/g)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e
*x + d)*sqrt(g*x + f)*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g +
(c*d*e*f + (2*c*d^2 + a*e^2)*g)*x))/(e*g^5*x^3 + d*f^2*g^3 + (2*e*f*g^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\int \frac{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5
/2),x, algorithm="maxima")
```

output

```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.51

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\frac{\sqrt{cdx+ae}\left((cdx+ae)\left(\frac{3\left(c^4d^4fg^4|c||d|-ac^3d^3eg^5|c||d|\right)(cdx+ae)}{c^2d^2fg^5-acdeg^6}+\frac{20\left(c^5d^5\right.}{}\right.\right.}{3}$$

$$+\frac{5\left(c^2d^2f|c||d|-acdeg|c||d|\right)\log\left(\left|-\sqrt{cdg}\sqrt{cdx+ae}+\sqrt{c^2d^2f-acdeg+(cdx+ae)cdg}\right|\right)}{\sqrt{cdg}g^3}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5
/2),x, algorithm="giac")
```

output

```
1/3*sqrt(c*d*x + a*e)*((c*d*x + a*e)*(3*(c^4*d^4*f*g^4*abs(c)*abs(d) - a*c
^3*d^3*e*g^5*abs(c)*abs(d))*(c*d*x + a*e)/(c^2*d^2*f*g^5 - a*c*d*e*g^6) +
20*(c^5*d^5*f^2*g^3*abs(c)*abs(d) - 2*a*c^4*d^4*e*f*g^4*abs(c)*abs(d) + a^
2*c^3*d^3*e^2*g^5*abs(c)*abs(d))/(c^2*d^2*f*g^5 - a*c*d*e*g^6)) + 15*(c^6*
d^6*f^3*g^2*abs(c)*abs(d) - 3*a*c^5*d^5*e*f^2*g^3*abs(c)*abs(d) + 3*a^2*c^
4*d^4*e^2*f*g^4*abs(c)*abs(d) - a^3*c^3*d^3*e^3*g^5*abs(c)*abs(d))/(c^2*d^
2*f*g^5 - a*c*d*e*g^6))/(c^2*d^2*f - a*c*d*e*g + (c*d*x + a*e)*c*d*g)^(3/2
) + 5*(c^2*d^2*f*abs(c)*abs(d) - a*c*d*e*g*abs(c)*abs(d))*log(abs(-sqrt(c*
d*g)*sqrt(c*d*x + a*e) + sqrt(c^2*d^2*f - a*c*d*e*g + (c*d*x + a*e)*c*d*g)
))/(sqrt(c*d*g)*g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(f+g\,x\right)}^{5/2}\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*
x)^(5/2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*
x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.44

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx=\frac{-4\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2e^2g^3-20\sqrt{gx+f}\,\sqrt{cdx+ae}\,acdef\,g^2}{}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/2),x)
```

output

```
( - 4*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*e**2*g**3 - 20*sqrt(f + g*x)*sq
rt(a*e + c*d*x)*a*c*d*e*f*g**2 - 28*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c*d*
e*g**3*x + 30*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g + 40*sqrt(f
 + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*f*g**2*x + 6*sqrt(f + g*x)*sqrt(a*e +
c*d*x)*c**2*d**2*g**3*x**2 + 30*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(
a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*c*d*e
*f**2*g + 60*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*c*d*e*f*g**2*x + 30*sqrt
(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(
f + g*x))/sqrt(a*e*g - c*d*f))*a*c*d*e*g**3*x**2 - 30*sqrt(g)*sqrt(d)*sqrt
(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a
*e*g - c*d*f))*c**2*d**2*f**3 - 60*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sq
rt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2
*d**2*f**2*g*x - 30*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2*d**2*f*g**2*x*
*2 + 5*sqrt(g)*sqrt(d)*sqrt(c)*a*c*d*e*f**2*g + 10*sqrt(g)*sqrt(d)*sqrt(c)
*a*c*d*e*f*g**2*x + 5*sqrt(g)*sqrt(d)*sqrt(c)*a*c*d*e*g**3*x**2 - 5*sqrt(g
)*sqrt(d)*sqrt(c)*c**2*d**2*f**3 - 10*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f*
*2*g*x - 5*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f*g**2*x**2)/(6*g**4*(f**2 +
2*f*g*x + g**2*x**2))
```

3.79 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 250

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx &= -\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} \\ &\quad -\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}} \\ &\quad +\frac{2c^{5/2}d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}\right)}{g^{7/2}}\end{aligned}$$

output
```
-2*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)/(g*x+
f)^(1/2)-2/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)
/(g*x+f)^(3/2)-2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)
/(g*x+f)^(5/2)+2*c^(5/2)*d^(5/2)*arctanh(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2))/g^(7/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.75

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(-\sqrt{g}\sqrt{ae+cdx}(3a^2e^2g^2+acdeg(5f+11}{15g^{7/2}\sqrt{(}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(7/2)),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]*(3*a^2*e^2
*g^2 + a*c*d*e*g*(5*f + 11*g*x) + c^2*d^2*(15*f^2 + 35*f*g*x + 23*g^2*x^2)
)) + 15*c^(5/2)*d^(5/2)*(f + g*x)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f +
g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(15*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e
*x)]*(f + g*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1249, 1249, 1249, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}dx$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \text{1249}$$

$$\frac{cd\left(\frac{cd\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}(f+gx)^{3/2}}dx}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \ 1249$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \ 1268$$

$$\frac{cd\left(\frac{cd\left(\frac{cd\sqrt{d+ex}\sqrt{ae+cdx}\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \ 66$$

$$\frac{cd\left(\frac{cd\left(\frac{2cd\sqrt{d+ex}\sqrt{ae+cdx}\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{cd\left(\frac{cd\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}\right)}{g}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(7/2)),x]
```

output
```
(-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)*(f
 + g*x)^(5/2)) + (c*d*((-2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/
(3*g*(d + e*x)^(3/2)*(f + g*x)^(3/2)) + (c*d*((-2*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f + g*x]) + (2*Sqrt[c]*Sqrt[d]*
Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[
c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])))/g))/g
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1249
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Simp[c*(m/(e*g*(n + 1)))   Int[(d + e*x)
^(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(210) = 420.

Time = 2.64 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.00

method	result
default	$\frac{\sqrt{(ex+d)(cdx+ae)}\left(15\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\sqrt{cdg}}{2\sqrt{cdg}}\right)c^3d^3g^3x^3+45\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\sqrt{cdg}}{2\sqrt{cdg}}\right)c^3\right.}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/2),x,
method=_RETURNVERBOSE)
```

output
```
1/15*((e*x+d)*(c*d*x+a*e))^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d
*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*g^3*x^3+45*ln
(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x
+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1
/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*
d*g)^(1/2))*c^3*d^3*f^3-46*c^2*d^2*g^2*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*
d*g)^(1/2)-22*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*g^2*x-70*(
(c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f*g*x-6*((c*d*x+a*e)*(g*x
+f))^(1/2)*(c*d*g)^(1/2)*a^2*e^2*g^2-10*a*c*d*e*f*g*((c*d*x+a*e)*(g*x+f))^
(1/2)*(c*d*g)^(1/2)-30*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f
^2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/g^3/(g*x+f)^(5/2)/(e*x+d)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 933, normalized size of antiderivative = 3.73

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx=\text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7
/2),x, algorithm="fricas")
```

output

```
[-1/30*(4*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e^2
*g^2 + (35*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(c^2*d^2*e*g^3*x^4 + c^2*d^
3*f^3 + (3*c^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2*d
^3*f*g^2)*x^2 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(c*d/g)*log(-(8*c
^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 4*(2*c*
d*g^2*x + c*d*f*g + a*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(e*x + d)*sqrt(g*x + f)*sqrt(c*d/g) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c
*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a
*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(e*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^
5 + d*g^6)*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x
), -1/15*(2*(23*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 5*a*c*d*e*f*g + 3*a^2*e
^2*g^2 + (35*c^2*d^2*f*g + 11*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^2*d^2*e*g^3*x^4 + c^2*
d^3*f^3 + (3*c^2*d^2*e*f*g^2 + c^2*d^3*g^3)*x^3 + 3*(c^2*d^2*e*f^2*g + c^2
*d^3*f*g^2)*x^2 + (c^2*d^2*e*f^3 + 3*c^2*d^3*f^2*g)*x)*sqrt(-c*d/g)*arctan
(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)
*sqrt(-c*d/g)*g/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 +
 a*e^2)*g)*x)))/(e*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)*x^3 + 3*(e*f^
2*g^4 + d*f*g^5)*x^2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\, dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\, dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{7}{2}}}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(210) = 420$.

Time = 0.30 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.96

$$\int \frac{\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{5/2}}{(d + ex)^{5/2}(f + gx)^{7/2}}\, dx =$$

$$-\frac{2\, c^2 d^2 |c||d| \log\left(\left|-\sqrt{cdg}\sqrt{cdx + ae} + \sqrt{c^2d^2f - acdeg + (cdx + ae)cdg}\right|\right)}{\sqrt{cdg}g^3}$$

$$-\frac{2\sqrt{cdx + ae}\left((cdx + ae)\left(\frac{23\,(c^6d^6f^2g^4|c||d| - 2\,ac^5d^5efg^5|c||d| + a^2c^4d^4e^2g^6|c||d|)\,(cdx + ae)}{c^2d^2f^2g^5 - 2\,acdefg^6 + a^2e^2g^7} + \frac{35\,(c^7d^7f^3g^3|c||d| - 3\,ac^6d^6ef^2g^4|c||}{c^2d^2f^2g^5 - 2}\right.\right.}{15\,(c^2d^2f - ac}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7
/2),x, algorithm="giac")
```

output

```
-2*c^2*d^2*abs(c)*abs(d)*log(abs(-sqrt(c*d*g)*sqrt(c*d*x + a*e) + sqrt(c^2
*d^2*f - a*c*d*e*g + (c*d*x + a*e)*c*d*g)))/(sqrt(c*d*g)*g^3) - 2/15*sqrt(
c*d*x + a*e)*((c*d*x + a*e)*(23*(c^6*d^6*f^2*g^4*abs(c)*abs(d) - 2*a*c^5*d
^5*e*f*g^5*abs(c)*abs(d) + a^2*c^4*d^4*e^2*g^6*abs(c)*abs(d))*(c*d*x + a*e
)/(c^2*d^2*f^2*g^5 - 2*a*c*d*e*f*g^6 + a^2*e^2*g^7) + 35*(c^7*d^7*f^3*g^3*
abs(c)*abs(d) - 3*a*c^6*d^6*e*f^2*g^4*abs(c)*abs(d) + 3*a^2*c^5*d^5*e^2*f*
g^5*abs(c)*abs(d) - a^3*c^4*d^4*e^3*g^6*abs(c)*abs(d))/(c^2*d^2*f^2*g^5 -
2*a*c*d*e*f*g^6 + a^2*e^2*g^7)) + 15*(c^8*d^8*f^4*g^2*abs(c)*abs(d) - 4*a*
c^7*d^7*e*f^3*g^3*abs(c)*abs(d) + 6*a^2*c^6*d^6*e^2*f^2*g^4*abs(c)*abs(d)
- 4*a^3*c^5*d^5*e^3*f*g^5*abs(c)*abs(d) + a^4*c^4*d^4*e^4*g^6*abs(c)*abs(d
))/(c^2*d^2*f^2*g^5 - 2*a*c*d*e*f*g^6 + a^2*e^2*g^7))/(c^2*d^2*f - a*c*d*e
*g + (c*d*x + a*e)*c*d*g)^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2+ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2+a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^{7/2}\,\left(d+e\,x\right)^{5/2}}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*
x)^(5/2)),x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*
x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.02

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2e^2g^3}{5}-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,acdef\,g^2}{3}-\frac{22\sqrt{gx+f}\,\sqrt{cdx+ae}\,a}{15}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/2),x)
```

output
```
(2*( - 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*e**2*g**3 - 5*sqrt(f + g*x)*
sqrt(a*e + c*d*x)*a*c*d*e*f*g**2 - 11*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c*
d*e*g**3*x - 15*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g - 35*sqrt
(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*f*g**2*x - 23*sqrt(f + g*x)*sqrt(a*e
 + c*d*x)*c**2*d**2*g**3*x**2 + 15*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sq
rt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2
*d**2*f**3 + 45*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + s
qrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2*d**2*f**2*g*x + 45
*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*
sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2*d**2*f*g**2*x**2 + 15*sqrt(g)*sqr
t(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x
))/sqrt(a*e*g - c*d*f))*c**2*d**2*g**3*x**3 + 5*sqrt(g)*sqrt(d)*sqrt(c)*c*
*2*d**2*f**3 + 15*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f**2*g*x + 15*sqrt(g)*
sqrt(d)*sqrt(c)*c**2*d**2*f*g**2*x**2 + 5*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**
2*g**3*x**3))/(15*g**4*(f**3 + 3*f**2*g*x + 3*f*g**2*x**2 + g**3*x**3))
```

3.80 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

output
```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(9/2)),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f
+ g*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = $\{1248\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx$$

$$\Big\downarrow \text{1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(9/2)),x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(7/2))
```

Defintions of rubi rules used

rule 1248

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2(cdx+ae)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{7(gx+f)^{\frac{7}{2}}(aeg-dfc)(ex+d)^{\frac{5}{2}}}$	63
orering	$-\frac{2(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{7(gx+f)^{\frac{7}{2}}(aeg-dfc)(ex+d)^{\frac{5}{2}}}$	64
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,\left(d^{2}c^{2}x^{2}+2acdex+e^{2}a^{2}\right)(cdx+ae)}{7\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}(aeg-dfc)}$	78

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/2),x,
method=_RETURNVERBOSE)
```

output

```
-2/7*(c*d*x+a*e)/(g*x+f)^(7/2)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*
d*e)^(5/2)/(e*x+d)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(55) = 110$.

Time = 0.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 4.75

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = \frac{}{7\,(cd^2f^5-adef^4g+(cdefg^4-ae^2g^5)x^5+(4\,cdef^2g^3-adeg^5+}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9
/2),x, algorithm="fricas")
```

output

```
2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c*d^2*f^5
 - a*d*e*f^4*g + (c*d*e*f*g^4 - a*e^2*g^5)*x^5 + (4*c*d*e*f^2*g^3 - a*d*e*
g^5 + (c*d^2 - 4*a*e^2)*f*g^4)*x^4 + 2*(3*c*d*e*f^3*g^2 - 2*a*d*e*f*g^4 +
(2*c*d^2 - 3*a*e^2)*f^2*g^3)*x^3 + 2*(2*c*d*e*f^4*g - 3*a*d*e*f^2*g^3 + (3
*c*d^2 - 2*a*e^2)*f^3*g^2)*x^2 + (c*d*e*f^5 - 4*a*d*e*f^3*g^2 + (4*c*d^2 -
 a*e^2)*f^4*g)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = \int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9
/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(9/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(55) = 110$.

Time = 0.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.46

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = \frac{2\,(c^8d^8f^2g^3|c||d| - 2\,ac^7d^7efg^4|c||d| + a^2c^6d^6e^2g^5|c||d|)}{7\,(c^3d^3f^3g^3 - 3\,ac^2d^2ef^2g^4 + 3\,a^2cde^2fg^5 - a^3e^3g^6)(c^2d^2f - acd}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9
/2),x, algorithm="giac")
```

output
```
2/7*(c^8*d^8*f^2*g^3*abs(c)*abs(d) - 2*a*c^7*d^7*e*f*g^4*abs(c)*abs(d) + a
^2*c^6*d^6*e^2*g^5*abs(c)*abs(d))*(c*d*x + a*e)^(7/2)/((c^3*d^3*f^3*g^3 -
3*a*c^2*d^2*e*f^2*g^4 + 3*a^2*c*d*e^2*f*g^5 - a^3*e^3*g^6)*(c^2*d^2*f - a*
c*d*e*g + (c*d*x + a*e)*c*d*g)^(7/2))
```

Mupad [B] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 325, normalized size of antiderivative = 5.16

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{2\,c^3\,d^3\,x^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a^2\,c\,d\,e^2\,x}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a\,c^2\,d^2\,e}{7\,a\,e\,g^4-7\,c}\right.}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,(7\,c\,d\,f^4-7\,a\,e\,f^3\,g)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{x^2\,\sqrt{f+g\,x}\,(21\,a\,e\,f\,g^3-21\,c\,d\,f^2\,g^2)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}-\frac{x\,\sqrt{f+g\,x}\,(21\,c\,d\,f^3}{7\,a\,e\,g^4}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(9/2)*(d + e*
x)^(5/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3)/(7*a*e*g^4 -
7*c*d*f*g^3) + (2*c^3*d^3*x^3)/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a^2*c*d*e^2*
x)/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a*c^2*d^2*e*x^2)/(7*a*e*g^4 - 7*c*d*f*g^
3)))/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^(1/2)*(7*c*d*f^4 -
7*a*e*f^3*g)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3) + (x^2*(f + g*x)^(
1/2)*(21*a*e*f*g^3 - 21*c*d*f^2*g^2)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f
*g^3) - (x*(f + g*x)^(1/2)*(21*c*d*f^3*g - 21*a*e*f^2*g^2)*(d + e*x)^(1/2)
)/(7*a*e*g^4 - 7*c*d*f*g^3))
```

Reduce [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 328, normalized size of antiderivative = 5.21

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^3e^3g^4}{7}-\frac{6\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2cd\,e^2g^4x}{7}-\frac{6\sqrt{gx+f}\,\sqrt{cdx+ae}}{7}}{g^4\,(ae\,g^5x^4-cdf\,g^4x^4+4a}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/2),x)
```

output

```
(2*( - sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*e**3*g**4 - 3*sqrt(f + g*x)*sq
rt(a*e + c*d*x)*a**2*c*d*e**2*g**4*x - 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a
*c**2*d**2*e*g**4*x**2 - sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*g**4*x*
*3 - sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f**4 - 4*sqrt(g)*sqrt(d)*sqrt(c)*c*
*3*d**3*f**3*g*x - 6*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f**2*g**2*x**2 - 4*
sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f*g**3*x**3 - sqrt(g)*sqrt(d)*sqrt(c)*c*
*3*d**3*g**4*x**4))/(7*g**4*(a*e*f**4*g + 4*a*e*f**3*g**2*x + 6*a*e*f**2*g
**3*x**2 + 4*a*e*f*g**4*x**3 + a*e*g**5*x**4 - c*d*f**5 - 4*c*d*f**4*g*x -
 6*c*d*f**3*g**2*x**2 - 4*c*d*f**2*g**3*x**3 - c*d*f*g**4*x**4))
```

3.81 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{7/2}}$$

output
```
2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(9/2)+4/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{7/2}(-7aeg+cd(9f+2gx))}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(11/2)),x]
```

output

```
(2*((a*e + c*d*x)*(d + e*x))^(7/2)*(-7*a*e*g + c*d*(9*f + 2*g*x)))/(63*(c*
d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1254, 1248\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(11/2)),x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(cdx+ae)(-2cdgx+7aeg-9dfc)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{63(gx+f)^{\frac{9}{2}}\left(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2}\right)(ex+d)^{\frac{5}{2}}}$	99
orering	$-\frac{2(-2cdgx+7aeg-9dfc)(cdx+ae)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{63\left(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2}\right)(gx+f)^{\frac{9}{2}}(ex+d)^{\frac{5}{2}}}$	100
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,\left(-2c^{3}d^{3}g\,x^{3}+3a\,c^{2}d^{2}eg\,x^{2}-9c^{3}d^{3}f\,x^{2}+12a^{2}cd\,e^{2}gx-18a\,c^{2}d^{2}efx+7a^{3}e^{3}g-9a^{2}cd\,e^{2}f\right)(cdx+ae)}{63\sqrt{ex+d}\,(gx+f)^{\frac{9}{2}}(aeg-dfc)^{2}}$	136

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/63*(c*d*x+a*e)*(-2*c*d*g*x+7*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+
a*d*e)^(5/2)/(g*x+f)^(9/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)
^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(113) = 226$.

Time = 0.21 (sec) , antiderivative size = 639, normalized size of antiderivative = 4.95

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \frac{}{63\,(c^2d^3f^7-2\,acd^2ef^6g+a^2de^2f^5g^2+(c^2d^2ef^2g^5-2\,acde^2fg^6}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
1/2),x, algorithm="fricas")
```

output
```
2/63*(2*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 7*a^4*e^4*g + (9*c^4*d^4*f - a*c
^3*d^3*e*g)*x^3 + 3*(9*a*c^3*d^3*e*f - 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2*
c^2*d^2*e^2*f - 19*a^3*c*d*e^3*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^7 - 2*a*c*d^2*e*f^6*g + a^2*
d*e^2*f^5*g^2 + (c^2*d^2*e*f^2*g^5 - 2*a*c*d*e^2*f*g^6 + a^2*e^3*g^7)*x^6
+ (5*c^2*d^2*e*f^3*g^4 + a^2*d*e^2*g^7 + (c^2*d^3 - 10*a*c*d*e^2)*f^2*g^5
- (2*a*c*d^2*e - 5*a^2*e^3)*f*g^6)*x^5 + 5*(2*c^2*d^2*e*f^4*g^3 + a^2*d*e^
2*f*g^6 + (c^2*d^3 - 4*a*c*d*e^2)*f^3*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f^2*g^
5)*x^4 + 10*(c^2*d^2*e*f^5*g^2 + a^2*d*e^2*f^2*g^5 + (c^2*d^3 - 2*a*c*d*e^
2)*f^4*g^3 - (2*a*c*d^2*e - a^2*e^3)*f^3*g^4)*x^3 + 5*(c^2*d^2*e*f^6*g + 2
*a^2*d*e^2*f^3*g^4 + 2*(c^2*d^3 - a*c*d*e^2)*f^5*g^2 - (4*a*c*d^2*e - a^2*
e^3)*f^4*g^3)*x^2 + (c^2*d^2*e*f^7 + 5*a^2*d*e^2*f^4*g^3 + (5*c^2*d^3 - 2*
a*c*d*e^2)*f^6*g - (10*a*c*d^2*e - a^2*e^3)*f^5*g^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
1/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(11/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(113) = 226.

Time = 0.49 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.67

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx=\frac{2\,(cdx+ae)^{\frac{7}{2}}\Bigg(\frac{2\,\left(c^{10}d^{10}f^2g^5|c||d|-2\,ac^9d^9efg^6|c||d|+a^2c^8d^8e^2g^7|c||d|\right)(cdx+ae)}{c^4d^4f^4g^4-4\,ac^3d^3ef^3g^5+6\,a^2c^2d^2e^2f^2g^6-4\,a^3cde^3fg^7+a^4e^4g^8}}{63\,\left(c^2d^2f-acdeg-\right.}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
1/2),x, algorithm="giac")
```

output
```
2/63*(c*d*x + a*e)^(7/2)*(2*(c^10*d^10*f^2*g^5*abs(c)*abs(d) - 2*a*c^9*d^9
*e*f*g^6*abs(c)*abs(d) + a^2*c^8*d^8*e^2*g^7*abs(c)*abs(d))*(c*d*x + a*e)/
(c^4*d^4*f^4*g^4 - 4*a*c^3*d^3*e*f^3*g^5 + 6*a^2*c^2*d^2*e^2*f^2*g^6 - 4*a
^3*c*d*e^3*f*g^7 + a^4*e^4*g^8) + 9*(c^11*d^11*f^3*g^4*abs(c)*abs(d) - 3*a
*c^10*d^10*e*f^2*g^5*abs(c)*abs(d) + 3*a^2*c^9*d^9*e^2*f*g^6*abs(c)*abs(d)
 - a^3*c^8*d^8*e^3*g^7*abs(c)*abs(d))/(c^4*d^4*f^4*g^4 - 4*a*c^3*d^3*e*f^3
*g^5 + 6*a^2*c^2*d^2*e^2*f^2*g^6 - 4*a^3*c*d*e^3*f*g^7 + a^4*e^4*g^8))/(c^
2*d^2*f - a*c*d*e*g + (c*d*x + a*e)*c*d*g)^(9/2)
```

Mupad [B] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.44

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d + ex)^{5/2}(f + gx)^{11/2}}\,dx =$$
$$-\frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}\,\left(\frac{2\,a^3\,e^3\,(7\,a\,e\,g - 9\,c\,d\,f)}{63\,g^4\,(a\,e\,g - c\,d\,f)^2} - \frac{4\,c^4\,d^4\,x^4}{63\,g^3\,(a\,e\,g - c\,d\,f)^2} + \frac{2\,c^3\,d^3\,x^3\,(a\,e\,g - 9\,c\,d\,f)}{63\,g^4\,(a\,e\,g - c\,d\,f)^2} + \frac{2\,a^2\,c\,d\,e^2\,x\,(19}{63\,g^4\,(a\,e}\right.}{x^4\,\sqrt{f + g\,x}\,\sqrt{d + e\,x} + \frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4} + \frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g} + \frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3} + 6\,f}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(11/2)*(d + e
*x)^(5/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3*(7*a*e*g - 9*c
*d*f))/(63*g^4*(a*e*g - c*d*f)^2) - (4*c^4*d^4*x^4)/(63*g^3*(a*e*g - c*d*f
)^2) + (2*c^3*d^3*x^3*(a*e*g - 9*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a
^2*c*d*e^2*x*(19*a*e*g - 27*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a*c^2*
d^2*e*x^2*(5*a*e*g - 9*c*d*f))/(21*g^4*(a*e*g - c*d*f)^2)))/(x^4*(f + g*x)
^(1/2)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*
x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x
)^(1/2))/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 668, normalized size of antiderivative = 5.18

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{(d + ex)^{5/2}(f + gx)^{11/2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^4e^4g^5}{9} + \frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^3cd\,e^3f\,g^4}{7} - \frac{38\sqrt{gx+f}\,\sqrt{cdx+a}}{63}}{g^4\left(a^2e^2g^7x^5 - 2acde\right.}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11/2),x
)
```

output

```
(2*( - 7*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**4*e**4*g**5 + 9*sqrt(f + g*x)*
sqrt(a*e + c*d*x)*a**3*c*d*e**3*f*g**4 - 19*sqrt(f + g*x)*sqrt(a*e + c*d*x
)*a**3*c*d*e**3*g**5*x + 27*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**2
*e**2*f*g**4*x - 15*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**2*e**2*g*
*5*x**2 + 27*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*f*g**4*x**2 - s
qrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e*g**5*x**3 + 9*sqrt(f + g*x)*s
qrt(a*e + c*d*x)*c**4*d**4*f*g**4*x**3 + 2*sqrt(f + g*x)*sqrt(a*e + c*d*x)
*c**4*d**4*g**5*x**4 - 2*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*f**5 - 10*sqrt(
g)*sqrt(d)*sqrt(c)*c**4*d**4*f**4*g*x - 20*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d*
*4*f**3*g**2*x**2 - 20*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*f**2*g**3*x**3 -
10*sqrt(g)*sqrt(d)*sqrt(c)*c**4*d**4*f*g**4*x**4 - 2*sqrt(g)*sqrt(d)*sqrt(
c)*c**4*d**4*g**5*x**5))/(63*g**4*(a**2*e**2*f**5*g**2 + 5*a**2*e**2*f**4*
g**3*x + 10*a**2*e**2*f**3*g**4*x**2 + 10*a**2*e**2*f**2*g**5*x**3 + 5*a**
2*e**2*f*g**6*x**4 + a**2*e**2*g**7*x**5 - 2*a*c*d*e*f**6*g - 10*a*c*d*e*f
**5*g**2*x - 20*a*c*d*e*f**4*g**3*x**2 - 20*a*c*d*e*f**3*g**4*x**3 - 10*a*
c*d*e*f**2*g**5*x**4 - 2*a*c*d*e*f*g**6*x**5 + c**2*d**2*f**7 + 5*c**2*d**
2*f**6*g*x + 10*c**2*d**2*f**5*g**2*x**2 + 10*c**2*d**2*f**4*g**3*x**3 + 5
*c**2*d**2*f**3*g**4*x**4 + c**2*d**2*f**2*g**5*x**5))
```

3.82 $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 198

$$\begin{aligned}\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx &= \frac{2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}\\ &\quad+\frac{8cd\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{99(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{16c^2d^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{693(cdf-aeg)^3(d+ex)^{7/2}(f+gx)^{7/2}}\end{aligned}$$

output
```
2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/
(g*x+f)^(11/2)+8/99*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*
d*f)^2/(e*x+d)^(7/2)/(g*x+f)^(9/2)+16/693*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

$$\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = \frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(63a^2e^2g^2-14acdeg(11f+2}{693(cdf-aeg)^3\sqrt{d+ex}(f+gx}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(13/2)),x]
```

output

```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(63*a^2*e^2*g^2 - 14*a*c*
d*e*g*(11*f + 2*g*x) + c^2*d^2*(99*f^2 + 44*f*g*x + 8*g^2*x^2)))/(693*(c*d
*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^(11/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}dx}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(13/2)),x]
```

output
```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d +
 e*x)^(7/2)*(f + g*x)^(11/2)) + (4*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (4*c
*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(63*(c*d*f - a*e*g)^2*(d
 + e*x)^(7/2)*(f + g*x)^(7/2))))/(11*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{2(cdx+ae)\left(8g^2x^2d^2c^2-28acde\,g^2x+44c^2d^2fgx+63a^2e^2g^2-154acdefg+99f^2c^2d^2\right)\left(cd\,x^2e+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{693(gx+f)^{\frac{11}{2}}\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3d^3c^3\right)(ex+d)^{\frac{5}{2}}}$
orering	$-\frac{2\left(8g^2x^2d^2c^2-28acde\,g^2x+44c^2d^2fgx+63a^2e^2g^2-154acdefg+99f^2c^2d^2\right)(cdx+ae)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{5}{2}}}{693\left(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3d^3c^3\right)(gx+f)^{\frac{11}{2}}(ex+d)^{\frac{5}{2}}}$
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,\left(8c^4d^4g^2x^4-12a\,c^3d^3e\,g^2x^3+44c^4d^4fg\,x^3+15a^2c^2d^2e^2g^2x^2-66a\,c^3d^3efg\,x^2+99c^4d^4f^2x^2+98a^3cd\,e^3g^2x\right.}{693\sqrt{ex+d}\,(gx+f)^{\frac{11}{2}}(aeg-dfc)^3}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/693*(c*d*x+a*e)*(8*c^2*d^2*g^2*x^2-28*a*c*d*e*g^2*x+44*c^2*d^2*f*g*x+63
*a^2*e^2*g^2-154*a*c*d*e*f*g+99*c^2*d^2*f^2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*
d*e)^(5/2)/(g*x+f)^(11/2)/(a^3*e^3*g^3-3*a^2*c*d*e^2*f*g^2+3*a*c^2*d^2*e*f
^2*g-c^3*d^3*f^3)/(e*x+d)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(174) = 348$.

Time = 0.47 (sec) , antiderivative size = 1101, normalized size of antiderivative = 5.56

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
3/2),x, algorithm="fricas")
```

output
```
2/693*(8*c^5*d^5*g^2*x^5 + 99*a^3*c^2*d^2*e^3*f^2 - 154*a^4*c*d*e^4*f*g +
63*a^5*e^5*g^2 + 4*(11*c^5*d^5*f*g - a*c^4*d^4*e*g^2)*x^4 + (99*c^5*d^5*f^
2 - 22*a*c^4*d^4*e*f*g + 3*a^2*c^3*d^3*e^2*g^2)*x^3 + (297*a*c^4*d^4*e*f^2
 - 330*a^2*c^3*d^3*e^2*f*g + 113*a^3*c^2*d^2*e^3*g^2)*x^2 + (297*a^2*c^3*d
^3*e^2*f^2 - 418*a^3*c^2*d^2*e^3*f*g + 161*a^4*c*d*e^4*g^2)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^9
- 3*a*c^2*d^3*e*f^8*g + 3*a^2*c*d^2*e^2*f^7*g^2 - a^3*d*e^3*f^6*g^3 + (c^3
*d^3*e*f^3*g^6 - 3*a*c^2*d^2*e^2*f^2*g^7 + 3*a^2*c*d*e^3*f*g^8 - a^3*e^4*g
^9)*x^7 + (6*c^3*d^3*e*f^4*g^5 - a^3*d*e^3*g^9 + (c^3*d^4 - 18*a*c^2*d^2*e
^2)*f^3*g^6 - 3*(a*c^2*d^3*e - 6*a^2*c*d*e^3)*f^2*g^7 + 3*(a^2*c*d^2*e^2 -
 2*a^3*e^4)*f*g^8)*x^6 + 3*(5*c^3*d^3*e*f^5*g^4 - 2*a^3*d*e^3*f*g^8 + (2*c
^3*d^4 - 15*a*c^2*d^2*e^2)*f^4*g^5 - 3*(2*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^3
*g^6 + (6*a^2*c*d^2*e^2 - 5*a^3*e^4)*f^2*g^7)*x^5 + 5*(4*c^3*d^3*e*f^6*g^3
 - 3*a^3*d*e^3*f^2*g^7 + 3*(c^3*d^4 - 4*a*c^2*d^2*e^2)*f^5*g^4 - 3*(3*a*c^
2*d^3*e - 4*a^2*c*d*e^3)*f^4*g^5 + (9*a^2*c*d^2*e^2 - 4*a^3*e^4)*f^3*g^6)*
x^4 + 5*(3*c^3*d^3*e*f^7*g^2 - 4*a^3*d*e^3*f^3*g^6 + (4*c^3*d^4 - 9*a*c^2*
d^2*e^2)*f^6*g^3 - 3*(4*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^5*g^4 + 3*(4*a^2*c*
d^2*e^2 - a^3*e^4)*f^4*g^5)*x^3 + 3*(2*c^3*d^3*e*f^8*g - 5*a^3*d*e^3*f^4*g
^5 + (5*c^3*d^4 - 6*a*c^2*d^2*e^2)*f^7*g^2 - 3*(5*a*c^2*d^3*e - 2*a^2*c*d*
e^3)*f^6*g^3 + (15*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^5*g^4)*x^2 + (c^3*d^3*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
3/2),x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(13/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(174) = 348.

Time = 0.66 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.03

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx=\frac{2\,(cdx+ae)^{\frac{7}{2}}\Bigg(4\,(cdx+ae)\Bigg(\frac{2\,\big(c^{12}d^{12}f^2g^7|c||d|-2\,ac^{11}d^{11}efg^8|c||d|+}{c^5d^5f^5g^5-5\,ac^4d^4ef^4g^6+10\,a^2c^3d^3e^2f^3g^7-10\,a}}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
3/2),x, algorithm="giac")
```

output
```
2/693*(c*d*x + a*e)^(7/2)*(4*(c*d*x + a*e)*(2*(c^12*d^12*f^2*g^7*abs(c)*ab
s(d) - 2*a*c^11*d^11*e*f*g^8*abs(c)*abs(d) + a^2*c^10*d^10*e^2*g^9*abs(c)*
abs(d))*(c*d*x + a*e)/(c^5*d^5*f^5*g^5 - 5*a*c^4*d^4*e*f^4*g^6 + 10*a^2*c^
3*d^3*e^2*f^3*g^7 - 10*a^3*c^2*d^2*e^3*f^2*g^8 + 5*a^4*c*d*e^4*f*g^9 - a^5
*e^5*g^10) + 11*(c^13*d^13*f^3*g^6*abs(c)*abs(d) - 3*a*c^12*d^12*e*f^2*g^7
*abs(c)*abs(d) + 3*a^2*c^11*d^11*e^2*f*g^8*abs(c)*abs(d) - a^3*c^10*d^10*e
^3*g^9*abs(c)*abs(d))/(c^5*d^5*f^5*g^5 - 5*a*c^4*d^4*e*f^4*g^6 + 10*a^2*c^
3*d^3*e^2*f^3*g^7 - 10*a^3*c^2*d^2*e^3*f^2*g^8 + 5*a^4*c*d*e^4*f*g^9 - a^5
*e^5*g^10)) + 99*(c^14*d^14*f^4*g^5*abs(c)*abs(d) - 4*a*c^13*d^13*e*f^3*g^
6*abs(c)*abs(d) + 6*a^2*c^12*d^12*e^2*f^2*g^7*abs(c)*abs(d) - 4*a^3*c^11*d
^11*e^3*f*g^8*abs(c)*abs(d) + a^4*c^10*d^10*e^4*g^9*abs(c)*abs(d))/(c^5*d^
5*f^5*g^5 - 5*a*c^4*d^4*e*f^4*g^6 + 10*a^2*c^3*d^3*e^2*f^3*g^7 - 10*a^3*c^
2*d^2*e^3*f^2*g^8 + 5*a^4*c*d*e^4*f*g^9 - a^5*e^5*g^10))/(c^2*d^2*f - a*c*
d*e*g + (c*d*x + a*e)*c*d*g)^(11/2)
```

Mupad [B] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.35

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{126\,a^5\,e^5\,g^2-308\,a^4\,c\,d\,e^4\,f\,g+198\,a^3\,c^2\,d^2\,e^3\,f^2}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x^3\,\left(6\,a^2\,c^3\,d^3\,e^2\,g^2-44\,a\,c^4\,d^4\,e\,f\,g+198\,c}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right.}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^5}+\frac{5\,f\,x^4}{}}$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(13/2)*(d + e
*x)^(5/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((126*a^5*e^5*g^2 + 198*a^
3*c^2*d^2*e^3*f^2 - 308*a^4*c*d*e^4*f*g)/(693*g^5*(a*e*g - c*d*f)^3) + (x^
3*(198*c^5*d^5*f^2 + 6*a^2*c^3*d^3*e^2*g^2 - 44*a*c^4*d^4*e*f*g))/(693*g^5
*(a*e*g - c*d*f)^3) + (16*c^5*d^5*x^5)/(693*g^3*(a*e*g - c*d*f)^3) - (8*c^
4*d^4*x^4*(a*e*g - 11*c*d*f))/(693*g^4*(a*e*g - c*d*f)^3) + (2*a^2*c*d*e^2
*x*(161*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 418*a*c*d*e*f*g))/(693*g^5*(a*e*g
- c*d*f)^3) + (2*a*c^2*d^2*e*x^2*(113*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 330*
a*c*d*e*f*g))/(693*g^5*(a*e*g - c*d*f)^3)))/(x^5*(f + g*x)^(1/2)*(d + e*x)
^(1/2) + (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (
10*f^2*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g^3)
```

Reduce [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 1106, normalized size of antiderivative = 5.59

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13/2),x
)
```

output

```
(2*( - 63*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**5*e**5*g**6 + 154*sqrt(f + g*
x)*sqrt(a*e + c*d*x)*a**4*c*d*e**4*f*g**5 - 161*sqrt(f + g*x)*sqrt(a*e + c
*d*x)*a**4*c*d*e**4*g**6*x - 99*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**2*
d**2*e**3*f**2*g**4 + 418*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e
**3*f*g**5*x - 113*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**2*e**3*g**
6*x**2 - 297*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*f**2*g**4
*x + 330*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*f*g**5*x**2 -
 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**2*g**6*x**3 - 297*sqr
t(f + g*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f**2*g**4*x**2 + 22*sqrt(f + g*
x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e*f*g**5*x**3 + 4*sqrt(f + g*x)*sqrt(a*e
+ c*d*x)*a*c**4*d**4*e*g**6*x**4 - 99*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**5
*d**5*f**2*g**4*x**3 - 44*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**5*d**5*f*g**5
*x**4 - 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**5*d**5*g**6*x**5 + 8*sqrt(g)*
sqrt(d)*sqrt(c)*c**5*d**5*f**6 + 48*sqrt(g)*sqrt(d)*sqrt(c)*c**5*d**5*f**5
*g*x + 120*sqrt(g)*sqrt(d)*sqrt(c)*c**5*d**5*f**4*g**2*x**2 + 160*sqrt(g)*
sqrt(d)*sqrt(c)*c**5*d**5*f**3*g**3*x**3 + 120*sqrt(g)*sqrt(d)*sqrt(c)*c**
5*d**5*f**2*g**4*x**4 + 48*sqrt(g)*sqrt(d)*sqrt(c)*c**5*d**5*f*g**5*x**5 +
 8*sqrt(g)*sqrt(d)*sqrt(c)*c**5*d**5*g**6*x**6))/(693*g**4*(a**3*e**3*f**6
*g**3 + 6*a**3*e**3*f**5*g**4*x + 15*a**3*e**3*f**4*g**5*x**2 + 20*a**3*e*
*3*f**3*g**6*x**3 + 15*a**3*e**3*f**2*g**7*x**4 + 6*a**3*e**3*f*g**8*x*...
```

3.83 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 267

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}}$$
$$+\frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429(cdf-aeg)^3(d+ex)^{7/2}(f+gx)^{9/2}}$$
$$+\frac{32c^3d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{3003(cdf-aeg)^4(d+ex)^{7/2}(f+gx)^{7/2}}$$

output

```
2/13*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/
(g*x+f)^(13/2)+12/143*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+
c*d*f)^2/(e*x+d)^(7/2)/(g*x+f)^(11/2)+16/429*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(9/2)+32/3003*c^
3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(7/
2)/(g*x+f)^(7/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.61

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx=\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}(-231a^3e^3g^3+63a^2cde^2g^2(13}{300}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(
f + g*x)^(15/2)),x]
```

output
```
(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-231*a^3*e^3*g^3 + 63*a^
2*c*d*e^2*g^2*(13*f + 2*g*x) - 7*a*c^2*d^2*e*g*(143*f^2 + 52*f*g*x + 8*g^2
*x^2) + c^3*d^3*(429*f^3 + 286*f^2*g*x + 104*f*g^2*x^2 + 16*g^3*x^3)))/(30
03*(c*d*f - a*e*g)^4*Sqrt[d + e*x]*(f + g*x)^(13/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1254, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}dx$$

$$\Big\downarrow \ 1254$$

$$\frac{6cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}dx}{13(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$\frac{6cd\left(\frac{4cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}dx}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}\right)}{13(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$\frac{6cd\left(\frac{4cd\left(\frac{2cd\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}dx}{9(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}\right)}{13(cdf-aeg)}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}$$

$$\Big\downarrow \ 1248$$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}+\frac{6cd\left(\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}+\frac{4cd\left(\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}\right)}{11(cdf-aeg)}\right)}{13(cdf-aeg)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*
x)^(15/2)),x]
```

output

```
(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d*f - a*e*g)*(d +
 e*x)^(7/2)*(f + g*x)^(13/2)) + (6*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f + g*x)^(11/2)) + (4
*c*d*((2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)
*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))))/(
11*(c*d*f - a*e*g))))/(13*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97

method	result
gosper	$-\frac{2(cdx+ae)(-16x^3g^3d^3c^3+56a\,c^2d^2e\,g^3x^2-104c^3d^3f\,g^2x^2-126a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-286c^3d^3f^2gx+231a^3e^3g^3-819a}{3003(gx+f)^{\frac{13}{2}}(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f}$
orering	$-\frac{2(-16x^3g^3d^3c^3+56a\,c^2d^2e\,g^3x^2-104c^3d^3f\,g^2x^2-126a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-286c^3d^3f^2gx+231a^3e^3g^3-819a^2cd\,e^2f\,g^2}{3003(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4d^4c^4)(gx}$
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-16c^5d^5g^3x^5+24a\,c^4d^4e\,g^3x^4-104c^5d^5f\,g^2x^4-30a^2c^3d^3e^2g^3x^3+156a\,c^4d^4ef\,g^2x^3-286c^5d^5f^2g\,x^3+35}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15/2),x
,method=_RETURNVERBOSE)
```

output
```
-2/3003*(c*d*x+a*e)*(-16*c^3*d^3*g^3*x^3+56*a*c^2*d^2*e*g^3*x^2-104*c^3*d^
3*f*g^2*x^2-126*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2*x-286*c^3*d^3*f^2*
g*x+231*a^3*e^3*g^3-819*a^2*c*d*e^2*f*g^2+1001*a*c^2*d^2*e*f^2*g-429*c^3*d
^3*f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/(g*x+f)^(13/2)/(a^4*e^4*g^
4-4*a^3*c*d*e^3*f*g^3+6*a^2*c^2*d^2*e^2*f^2*g^2-4*a*c^3*d^3*e*f^3*g+c^4*d^
4*f^4)/(e*x+d)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1648 vs. $2(235) = 470$.

Time = 1.17 (sec) , antiderivative size = 1648, normalized size of antiderivative = 6.17

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{15/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
5/2),x, algorithm="fricas")
```

output

```
2/3003*(16*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 1001*a^4*c^2*d^2*e^
4*f^2*g + 819*a^5*c*d*e^5*f*g^2 - 231*a^6*e^6*g^3 + 8*(13*c^6*d^6*f*g^2 -
a*c^5*d^5*e*g^3)*x^5 + 2*(143*c^6*d^6*f^2*g - 26*a*c^5*d^5*e*f*g^2 + 3*a^2
*c^4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 - 143*a*c^5*d^5*e*f^2*g + 39*a^2*
c^4*d^4*e^2*f*g^2 - 5*a^3*c^3*d^3*e^3*g^3)*x^3 + (1287*a*c^5*d^5*e*f^3 - 2
145*a^2*c^4*d^4*e^2*f^2*g + 1469*a^3*c^3*d^3*e^3*f*g^2 - 371*a^4*c^2*d^2*e
^4*g^3)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 - 2717*a^3*c^3*d^3*e^3*f^2*g + 209
3*a^4*c^2*d^2*e^4*f*g^2 - 567*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^11 - 4*a*c^3*d^
4*e*f^10*g + 6*a^2*c^2*d^3*e^2*f^9*g^2 - 4*a^3*c*d^2*e^3*f^8*g^3 + a^4*d*e
^4*f^7*g^4 + (c^4*d^4*e*f^4*g^7 - 4*a*c^3*d^3*e^2*f^3*g^8 + 6*a^2*c^2*d^2*
e^3*f^2*g^9 - 4*a^3*c*d*e^4*f*g^10 + a^4*e^5*g^11)*x^8 + (7*c^4*d^4*e*f^5*
g^6 + a^4*d*e^4*g^11 + (c^4*d^5 - 28*a*c^3*d^3*e^2)*f^4*g^7 - 2*(2*a*c^3*d
^4*e - 21*a^2*c^2*d^2*e^3)*f^3*g^8 + 2*(3*a^2*c^2*d^3*e^2 - 14*a^3*c*d*e^4
)*f^2*g^9 - (4*a^3*c*d^2*e^3 - 7*a^4*e^5)*f*g^10)*x^7 + 7*(3*c^4*d^4*e*f^6
*g^5 + a^4*d*e^4*f*g^10 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^5*g^6 - 2*(2*a*c^
3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^7 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)
*f^3*g^8 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^9)*x^6 + 7*(5*c^4*d^4*e*f^7
*g^4 + 3*a^4*d*e^4*f^2*g^9 + (3*c^4*d^5 - 20*a*c^3*d^3*e^2)*f^6*g^5 - 6*(2
*a*c^3*d^4*e - 5*a^2*c^2*d^2*e^3)*f^5*g^6 + 2*(9*a^2*c^2*d^3*e^2 - 10*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^{5/2}(f + gx)^{15/2}}\,dx = \text{Timed out}$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+
f)**(15/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{15}{2}}}\,dx$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
5/2),x, algorithm="maxima")
```

output

```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(
g*x + f)^(15/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(235) = 470$.

Time $= 0.91$ (sec) , antiderivative size $= 915$, normalized size of antiderivative $= 3.43$

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx=\text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1
5/2),x, algorithm="giac")
```

output

```
2/3003*(c*d*x + a*e)^(7/2)*(2*(c*d*x + a*e)*(4*(c*d*x + a*e)*(2*(c^14*d^14
*f^2*g^9*abs(c)*abs(d) - 2*a*c^13*d^13*e*f*g^10*abs(c)*abs(d) + a^2*c^12*d
^12*e^2*g^11*abs(c)*abs(d))*(c*d*x + a*e)/(c^6*d^6*f^6*g^6 - 6*a*c^5*d^5*e
*f^5*g^7 + 15*a^2*c^4*d^4*e^2*f^4*g^8 - 20*a^3*c^3*d^3*e^3*f^3*g^9 + 15*a^
4*c^2*d^2*e^4*f^2*g^10 - 6*a^5*c*d*e^5*f*g^11 + a^6*e^6*g^12) + 13*(c^15*d
^15*f^3*g^8*abs(c)*abs(d) - 3*a*c^14*d^14*e*f^2*g^9*abs(c)*abs(d) + 3*a^2*
c^13*d^13*e^2*f*g^10*abs(c)*abs(d) - a^3*c^12*d^12*e^3*g^11*abs(c)*abs(d))
/(c^6*d^6*f^6*g^6 - 6*a*c^5*d^5*e*f^5*g^7 + 15*a^2*c^4*d^4*e^2*f^4*g^8 - 2
0*a^3*c^3*d^3*e^3*f^3*g^9 + 15*a^4*c^2*d^2*e^4*f^2*g^10 - 6*a^5*c*d*e^5*f*
g^11 + a^6*e^6*g^12)) + 143*(c^16*d^16*f^4*g^7*abs(c)*abs(d) - 4*a*c^15*d^
15*e*f^3*g^8*abs(c)*abs(d) + 6*a^2*c^14*d^14*e^2*f^2*g^9*abs(c)*abs(d) - 4
*a^3*c^13*d^13*e^3*f*g^10*abs(c)*abs(d) + a^4*c^12*d^12*e^4*g^11*abs(c)*ab
s(d))/(c^6*d^6*f^6*g^6 - 6*a*c^5*d^5*e*f^5*g^7 + 15*a^2*c^4*d^4*e^2*f^4*g^
8 - 20*a^3*c^3*d^3*e^3*f^3*g^9 + 15*a^4*c^2*d^2*e^4*f^2*g^10 - 6*a^5*c*d*e
^5*f*g^11 + a^6*e^6*g^12)) + 429*(c^17*d^17*f^5*g^6*abs(c)*abs(d) - 5*a*c^
16*d^16*e*f^4*g^7*abs(c)*abs(d) + 10*a^2*c^15*d^15*e^2*f^3*g^8*abs(c)*abs(
d) - 10*a^3*c^14*d^14*e^3*f^2*g^9*abs(c)*abs(d) + 5*a^4*c^13*d^13*e^4*f*g^
10*abs(c)*abs(d) - a^5*c^12*d^12*e^5*g^11*abs(c)*abs(d))/(c^6*d^6*f^6*g^6
- 6*a*c^5*d^5*e*f^5*g^7 + 15*a^2*c^4*d^4*e^2*f^4*g^8 - 20*a^3*c^3*d^3*e^3*
f^3*g^9 + 15*a^4*c^2*d^2*e^4*f^2*g^10 - 6*a^5*c*d*e^5*f*g^11 + a^6*e^6*...
```

Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.35

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\Bigg(\frac{462\,a^6\,e^6\,g^3-1638\,a^5\,c\,d\,e^5\,f\,g^2+2002\,a^4\,c^2\,d^2\,e^4\,f^2\,g-858\,a^3\,c^3\,d^3\,e^3\,f^3}{3003\,g^6\,\left(a\,e\,g-c\,d\,f\right)^4}-\frac{x^3\,\left(-10\,a^3\,c^3\,d^3}{}}{}$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(15/2)*(d + e
*x)^(5/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((462*a^6*e^6*g^3 - 858*a^
3*c^3*d^3*e^3*f^3 + 2002*a^4*c^2*d^2*e^4*f^2*g - 1638*a^5*c*d*e^5*f*g^2)/(
3003*g^6*(a*e*g - c*d*f)^4) - (x^3*(858*c^6*d^6*f^3 - 10*a^3*c^3*d^3*e^3*g
^3 + 78*a^2*c^4*d^4*e^2*f*g^2 - 286*a*c^5*d^5*e*f^2*g))/(3003*g^6*(a*e*g -
 c*d*f)^4) - (32*c^6*d^6*x^6)/(3003*g^3*(a*e*g - c*d*f)^4) - (4*c^4*d^4*x^
4*(3*a^2*e^2*g^2 + 143*c^2*d^2*f^2 - 26*a*c*d*e*f*g))/(3003*g^5*(a*e*g - c
*d*f)^4) + (16*c^5*d^5*x^5*(a*e*g - 13*c*d*f))/(3003*g^4*(a*e*g - c*d*f)^4
) + (2*a^2*c*d*e^2*x*(567*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2717*a*c^2*d^2*
e*f^2*g - 2093*a^2*c*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4) + (2*a*c^2
*d^2*e*x^2*(371*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2145*a*c^2*d^2*e*f^2*g -
1469*a^2*c*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4)))/(x^6*(f + g*x)^(1/
2)*(d + e*x)^(1/2) + (f^6*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^6 + (6*f*x^5*
(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (6*f^5*x*(f + g*x)^(1/2)*(d + e*x)^(1
/2))/g^5 + (15*f^2*x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (20*f^3*x^3*
(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (15*f^4*x^2*(f + g*x)^(1/2)*(d + e*
x)^(1/2))/g^4)
```

Reduce [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 1624, normalized size of antiderivative = 6.08

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15/2),x
)
```

output

```
(2*( - 231*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**6*e**6*g**7 + 819*sqrt(f + g
*x)*sqrt(a*e + c*d*x)*a**5*c*d*e**5*f*g**6 - 567*sqrt(f + g*x)*sqrt(a*e +
c*d*x)*a**5*c*d*e**5*g**7*x - 1001*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**4*c*
*2*d**2*e**4*f**2*g**5 + 2093*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**4*c**2*d*
*2*e**4*f*g**6*x - 371*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**2*e**4
*g**7*x**2 + 429*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**3*f**3*
g**4 - 2717*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**3*f**2*g**5*
x + 1469*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**3*f*g**6*x**2 -
 5*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**3*g**7*x**3 + 1287*sq
rt(f + g*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**2*f**3*g**4*x - 2145*sqrt(
f + g*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**2*f**2*g**5*x**2 + 39*sqrt(f
+ g*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**2*f*g**6*x**3 + 6*sqrt(f + g*x)
*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**2*g**7*x**4 + 1287*sqrt(f + g*x)*sqrt
(a*e + c*d*x)*a*c**5*d**5*e*f**3*g**4*x**2 - 143*sqrt(f + g*x)*sqrt(a*e +
c*d*x)*a*c**5*d**5*e*f**2*g**5*x**3 - 52*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a
*c**5*d**5*e*f*g**6*x**4 - 8*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**5*d**5*e
*g**7*x**5 + 429*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**6*d**6*f**3*g**4*x**3
+ 286*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**6*d**6*f**2*g**5*x**4 + 104*sqrt(
f + g*x)*sqrt(a*e + c*d*x)*c**6*d**6*f*g**6*x**5 + 16*sqrt(f + g*x)*sqrt(a
*e + c*d*x)*c**6*d**6*g**7*x**6 - 16*sqrt(g)*sqrt(d)*sqrt(c)*c**6*d**6*...
```

3.84 $\int \frac{\sqrt{d+ex}\left(f+gx\right)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 220

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{3(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}} + \frac{3(cdf-aeg)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}}$$

output

```
3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c
^2/d^2/(e*x+d)^(1/2)+1/2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/c/d/(e*x+d)^(1/2)+3/4*(-a*e*g+c*d*f)^2*arctanh(c^(1/2)*d^(1/2)*(e*x+d
)^(1/2)*(g*x+f)^(1/2)/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(
5/2)/d^(5/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}(ae+cdx)\sqrt{f+gx}(-3aeg+cd(5f+2gx))+\frac{3(c}{}\right.}{4c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2],x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*Sqrt[f + g*x]*(-3*a*e*g + c*
d*(5*f + 2*g*x)) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]
*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/Sqrt[g]))/(4*c^(5/2)
*d^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.104, Rules used = {1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{3(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow 1268$$

$$\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow 66$$

$$\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

$$\Big\downarrow 221$$

$$\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

input

```
Int[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2],x]
```

output

```
((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d*Sqrt[
d + e*x]) + (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
 + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])))/(4*c*d)
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.45

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\left(3\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^{2}e^{2}g^{2}-6\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right.\right.}{8}$

input
```
int((e*x+d)^(1/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/8*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+d
*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*e^2*g
^2-6*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(
1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x
+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f^2+4*((c*d*x+a
*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-6*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*
d*g)^(1/2)*a*e*g+10*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*f)/(e*x+
d)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/c^2/d^2/(c*d*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.25

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \Bigg[\frac{4\left(2\,c^2d^2g^2x+5\,c^2d^2fg-3\,acdeg^2\right)\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e*g^2)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2
- 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2
*e^3*g^2)*x)*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d
^2*e*f*g + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*g*x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d
^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^3*d
^3*e*g*x + c^3*d^4*g), 1/8*(2*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*e
*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f) - 3*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 -
2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*sqrt(-c*d*g)*arctan(1/2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sqrt(-c*d*g)*sqrt(
e*x + d)*sqrt(g*x + f)/(c^2*d^2*e*g^2*x^3 + a*c*d^2*e*f*g + (c^2*d^2*e*f*g
 + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (a*c*d^2*e*g^2 + (c^2*d^3 + a*c*d*e^2)
*f*g)*x)))/(c^3*d^3*e*g*x + c^3*d^4*g)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}(f+gx)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{\Bigl(\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex}{}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output

```
1/4*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g
)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*a
bs(e)/(c*d*e^2*g) + 3*(c^2*d^2*e^2*f*abs(e) - a*c*d*e^3*g*abs(e))/(c^3*d^3
*e^2*g)) - 3*(c^2*d^2*e^2*f^2*abs(e) - 2*a*c*d*e^3*f*g*abs(e) + a^2*e^4*g^
2*abs(e))*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(
c*d*g)*c^2*d^2))*g/(e^2*abs(e)*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{3/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input

```
int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2),x)
```

output

```
int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{-3\sqrt{gx+f}\,\sqrt{cdx+ae}\,acde\,g^2+5\sqrt{gx+f}\,\sqrt{cdx+ae}\,c^2d^2fg+2}{}$$

input

```
int((e*x+d)^(1/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c*d*e*g**2 + 5*sqrt(f + g*x)*sqrt(
a*e + c*d*x)*c**2*d**2*f*g + 2*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*g
**2*x + 3*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)
*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*e**2*g**2 - 6*sqrt(g)*sq
rt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*
x))/sqrt(a*e*g - c*d*f))*a*c*d*e*f*g + 3*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt
(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f)
)*c**2*d**2*f**2)/(4*c**3*d**3*g)
```

3.85 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 145

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}}+\frac{(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}\sqrt{g}}$$

output

```
(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+(-
a*e*g+c*d*f)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/g^(1/2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}(ae+cdx)\sqrt{f+gx}+(cdf-aeg)\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{(ae+cdx)(d+ex)}}$$

input

```
Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output

```
(Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*(a*e + c*d*x)*Sqrt[f + g*x] + (c*d
*f - a*e*g)*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqr
t[g]*Sqrt[a*e + c*d*x])]))/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{66}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

input

```
Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output

```
(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d +
e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*
Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt
[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1253

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\,\Bigl(\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)aeg-\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cd}{2\sqrt{ex+d}\,\sqrt{(cdx+ae)(gx+f)}\,cd\sqrt{cdg}}$

input
```
int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/2/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(ln(1/2*(2*c*
d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))*a*e*g-ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*c*d*f-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2
))/((c*d*x+a*e)*(g*x+f))^(1/2)/c/d/(c*d*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(121) = 242.

Time = 0.50 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.01

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\Biggl[\frac{4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}cdg-(cd^2f-adeg+(cdef-ae^2g)x)\sqrt{cdg}\log\Bigl(-}{}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
 + f)*c*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(c*d*g)*log(
-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 - 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*sq
rt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*
d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*
c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^2*d^2*e*g*x + c^2*d^3*g), 1/2*(2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*c
*d*g - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*sqrt(-c*d*g)*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)*s
qrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^2*e*g^2*x^3 + a*c*d^2*e*f*g
 + (c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (a*c*d^2*e*g^2 + (c^2
*d^3 + a*c*d*e^2)*f*g)*x)))/(c^2*d^2*e*g*x + c^2*d^3*g)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*sqrt(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}\sqrt{gx+f}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=$$
$$-\frac{g\left(\frac{(cdef-ae^2g)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}cd}-\frac{\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex}{}\right.}{e|g|}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
-g*((c*d*e*f - a*e^2*g)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(
c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c
*d*g)))/(sqrt(c*d*g)*c*d) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)/(c*d*e*g))/(e
*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input

```
int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2),x)
```

output

```
int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{\sqrt{gx+f}\,\sqrt{cdx+ae}\,cdg-\sqrt{g}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{gx+f}}{\sqrt{aeg-cdf}}\right)aeg+\sqrt{g}\,\sqrt{d}\,\sqrt{c}\log\Bigg(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}}{\sqrt{aeg-cdf}}}{c^2d^2g}$$

input

```
int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(sqrt(f + g*x)*sqrt(a*e + c*d*x)*c*d*g - sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt
(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f)
)*a*e*g + sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)
*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c*d*f)/(c**2*d**2*g)
```

3.86 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 81

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}}$$

output
```
2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/g^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(1/2)/d^(1/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ae+cdx}\sqrt{d+ex}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])
/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[(a*e + c*d*x)
*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sq
rt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\,\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)}{\sqrt{ex+d}\,\sqrt{cdg}\,\sqrt{(cdx+ae)(gx+f)}}$	102

input
```
int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*ln(1/2*(2*c*d*g*
x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))/
(c*d*g)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(63) = 126$.

Time = 0.48 (sec) , antiderivative size = 404, normalized size of antiderivative = 4.99

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{\sqrt{cdg}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2efg+a^2de^2g^2+4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdgx+cdf+aeg)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+8\,(c^2d^2e}{ex+d}}{2\,cdg}\right.$$
$$\left. -\frac{\sqrt{-cdg}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdgx+cdf+aeg)\sqrt{-cdg}\sqrt{ex+d}\sqrt{gx+f}}{2\,(c^2d^2eg^2x^3+acd^2efg+(c^2d^2efg+(c^2d^3+acde^2)g^2)x^2+(acd^2eg^2+(c^2d^3+acde^2)fg)x)}\right)}{cdg} \right]$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(c*d*g)*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g
 + a^2*d*e^2*g^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*
x + c*d*f + a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(c^2*d^2*e*
f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a
*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d))/(c*d*g), -sqrt(
-c*d*g)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x
+ c*d*f + a*e*g)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^2*e*g^2*x
^3 + a*c*d^2*e*f*g + (c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (a*
c*d^2*e*g^2 + (c^2*d^3 + a*c*d*e^2)*f*g)*x))/(c*d*g)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
g*x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = -\frac{2\,e^2 g\log\left(\left|-\sqrt{e^2 f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2 fg+ae^3g^2+(e^2 f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}{\left|e\right|}^2\left|g\right|}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
-2*e^2*g*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-
c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c
*d*g)*abs(e)^2*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{g}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{gx+f}}{\sqrt{aeg-cdf}}\right)}{cdg}$$

input
```
int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(g)*sqrt(d)*sqrt(c)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c
)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f)))/(c*d*g)
```

3.87 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 61

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\sqrt{f+gx}}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(g*
x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}}{(cdf-aeg)\sqrt{d+ex}\sqrt{f+gx}}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)])/((c*d*f - a*e*g)*Sqrt[d + e*x]*Sqrt[f +
g*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \; 1248$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)}$$

input

```
Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output

```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)*Sqrt[d +
e*x]*Sqrt[f + g*x])
```

Defintions of rubi rules used

rule 1248

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}}{\sqrt{ex+d}\,\sqrt{gx+f}\,(aeg-dfc)}$	45
gosper	$-\frac{2(cdx+ae)\sqrt{ex+d}}{\sqrt{gx+f}\,(aeg-dfc)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}$	63
orering	$-\frac{2(cdx+ae)\sqrt{ex+d}}{\sqrt{gx+f}\,(aeg-dfc)\,\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}$	64

input
```
int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/(a*e*g-c*d*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(55) = 110$.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}}{cd^2f^2-adefg+(cdefg-ae^2g^2)x^2+(cdef^2-adeg^2}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/
(c*d^2*f^2 - a*d*e*f*g + (c*d*e*f*g - a*e^2*g^2)*x^2 + (c*d*e*f^2 - a*d*e*
g^2 + (c*d^2 - a*e^2)*f*g)*x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{4\sqrt{}}{\left(cde^2fg-ae^3g^2+\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}-\right.\right.}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
4*sqrt(c*d*g)*e^2*g/((c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*
g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d
)*e*g - d*e*g)*c*d*g))^2)*abs(g))
```

Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(x\,\sqrt{f+g\,x}-\frac{\sqrt{f+g\,x}\,\left(c\,d^2\,f-a\,d\,e\,g\right)}{a\,e^2\,g-c\,d\,e\,f}\right)\,\left(a\,e^2\,g-c\,d\,e\,f\right)}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
-(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((x*(f
+ g*x)^(1/2) - ((f + g*x)^(1/2)*(c*d^2*f - a*d*e*g))/(a*e^2*g - c*d*e*f))*
(a*e^2*g - c*d*e*f))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{-2\sqrt{gx+f}\,\sqrt{cdx+ae}\,g-2\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,f-2\sqrt{g}\,\sqrt{d}}{g\left(ae\,g^2x-cdf\,gx+aefg-cd\,f^2\right)}$$

input
```
int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 2*(sqrt(f + g*x)*sqrt(a*e + c*d*x)*g + sqrt(g)*sqrt(d)*sqrt(c)*f + sqr
t(g)*sqrt(d)*sqrt(c)*g*x))/(g*(a*e*f*g + a*e*g**2*x - c*d*f**2 - c*d*f*g*x
))
```

3.88 $\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 129

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}(f+gx)^{3/2}} + \frac{4cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}$$

output

```
2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(
g*x+f)^(3/2)+4/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^2/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-aeg+cd(3f+2gx))}{3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}}$$

input

```
Integrate[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a*e*g) + c*d*(3*f + 2*g*x)))/(3*(c*d*f
 - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d
+ e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-2cdgx+aeg-3dfc)}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}\,(aeg-dfc)^{2}}$	61
gosper	$-\frac{2(cdx+ae)(-2cdgx+aeg-3dfc)\sqrt{ex+d}}{3(gx+f)^{\frac{3}{2}}\,(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2})\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}$	98
orering	$-\frac{2(-2cdgx+aeg-3dfc)(cdx+ae)\sqrt{ex+d}}{3(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2})(gx+f)^{\frac{3}{2}}\,\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}$	99

input
```
int((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-2*c*d*g*x+a
*e*g-3*c*d*f)/(a*e*g-c*d*f)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(113) = 226$.

Time = 0.11 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{}{3\,(c^2d^3f^4-2\,acd^2ef^3g+a^2de^2f^2g^2+(c^2d^2ef^2g^2-2\,a}$$

input

```
integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output

```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 3*c*d*f - a*e
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(c^2*d^3*f^4 - 2*a*c*d^2*e*f^3*g + a^2*d*e
^2*f^2*g^2 + (c^2*d^2*e*f^2*g^2 - 2*a*c*d*e^2*f*g^3 + a^2*e^3*g^4)*x^3 + (
2*c^2*d^2*e*f^3*g + a^2*d*e^2*g^4 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^2 - 2*(a
*c*d^2*e - a^2*e^3)*f*g^3)*x^2 + (c^2*d^2*e*f^4 + 2*a^2*d*e^2*f*g^3 + 2*(c
^2*d^3 - a*c*d*e^2)*f^3*g - (4*a*c*d^2*e - a^2*e^3)*f^2*g^2)*x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^{\frac{5}{2}}}\,dx$$

input

```
integrate((e*x+d)**(1/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output

```
Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(5/2)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{8\Big(cde^2fg-ae^3g^2+3\Big(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cd}}{3\Big(cde^2fg-ae^3g^2+\Big(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{}}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
8/3*(c*d*e^2*f*g - a*e^3*g^2 + 3*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt
(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*
c*d*g))^2)*sqrt(c*d*g)*c*d*e^4*g^2/((c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f
 + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e
^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^3*abs(g))
```

Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{\left(\frac{(2\,a\,e\,g-6\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,e\,g\,{(a\,e\,g-c\,d\,f)}^2}-\frac{4\,c\,d\,x\,\sqrt{d+e\,x}}{3\,e\,{(a\,e\,g-c\,d\,f)}^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{d\,f\,\sqrt{f+g\,x}}{e\,g}+\frac{x\,\sqrt{f+g\,x}\,(d\,g+e\,f)}{e\,g}}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
-((((2*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*e*g*(a*e*g - c*d*f)^2) - (4*c*
d*x*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e +
 c*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (d*f*(f + g*x)^(1/2))/(e*g) + (x
*(f + g*x)^(1/2)*(d*g + e*f))/(e*g))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,aeg^2}{3}+2\sqrt{gx+f}\,\sqrt{cdx+ae}\,cdfg+}{g\,(a^2e^2g^4x^2-2acdefg^3x^2+c^2d^2f^2g^2x^2+2a^2e^2fg^3}$$

input
```
int((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*( - sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*e*g**2 + 3*sqrt(f + g*x)*sqrt(a*e
 + c*d*x)*c*d*f*g + 2*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c*d*g**2*x - 2*sqrt(
g)*sqrt(d)*sqrt(c)*c*d*f**2 - 4*sqrt(g)*sqrt(d)*sqrt(c)*c*d*f*g*x - 2*sqrt
(g)*sqrt(d)*sqrt(c)*c*d*g**2*x**2))/(3*g*(a**2*e**2*f**2*g**2 + 2*a**2*e**
2*f*g**3*x + a**2*e**2*g**4*x**2 - 2*a*c*d*e*f**3*g - 4*a*c*d*e*f**2*g**2*
x - 2*a*c*d*e*f*g**3*x**2 + c**2*d**2*f**4 + 2*c**2*d**2*f**3*g*x + c**2*d
**2*f**2*g**2*x**2))
```

3.89 $\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 198

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}} + \frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}$$

output
```
2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(
g*x+f)^(5/2)+8/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(1/2)/(g*x+f)^(3/2)+16/15*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(3a^2e^2g^2-2acdeg(5f+2gx)+}{15(cdf-aeg)^3\sqrt{d+ex}(f+g}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(5*f + 2*g*x
) + c^2*d^2*(15*f^2 + 20*f*g*x + 8*g^2*x^2)))/(15*(c*d*f - a*e*g)^3*Sqrt[d
 + e*x]*(f + g*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{4cd\left(\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{4cd\left(\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d
+ e*x]*(f + g*x)^(5/2)) + (4*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*S
qrt[f + g*x])))/(5*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(8g^2x^2d^2c^2-4acde\,g^2x+20c^2d^2fgx+3a^2e^2g^2-10acdefg+15f^2c^2d^2)}{15\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}\,(aeg-dfc)^3}$	111
gosper	$-\frac{2(cdx+ae)\left(8g^2x^2d^2c^2-4acde\,g^2x+20c^2d^2fgx+3a^2e^2g^2-10acdefg+15f^2c^2d^2\right)\sqrt{ex+d}}{15(gx+f)^{\frac{5}{2}}\,(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3d^3c^3)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}$	169
orering	$-\frac{2\left(8g^2x^2d^2c^2-4acde\,g^2x+20c^2d^2fgx+3a^2e^2g^2-10acdefg+15f^2c^2d^2\right)(cdx+ae)\sqrt{ex+d}}{15(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3d^3c^3)(gx+f)^{\frac{5}{2}}\,\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}$	170

input
```
int((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/15/(e*x+d)^(1/2)/(g*x+f)^(5/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(8*c^2*d^2*g
^2*x^2-4*a*c*d*e*g^2*x+20*c^2*d^2*f*g*x+3*a^2*e^2*g^2-10*a*c*d*e*f*g+15*c^
2*d^2*f^2)/(a*e*g-c*d*f)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(174) = 348.

Time = 0.44 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.89

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{}{15\,(c^3d^4f^6-3\,ac^2d^3ef^5g+3\,a^2cd^2e^2f^4g^2-a^3de^3f^3g^3}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output
```
2/15*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 3*a^2*e^2*g^2
+ 4*(5*c^2*d^2*f*g - a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^3*d^4*f^6 - 3*a*c^2*d^3*e*f^5*g + 3*
a^2*c*d^2*e^2*f^4*g^2 - a^3*d*e^3*f^3*g^3 + (c^3*d^3*e*f^3*g^3 - 3*a*c^2*d
^2*e^2*f^2*g^4 + 3*a^2*c*d*e^3*f*g^5 - a^3*e^4*g^6)*x^4 + (3*c^3*d^3*e*f^4
*g^2 - a^3*d*e^3*g^6 + (c^3*d^4 - 9*a*c^2*d^2*e^2)*f^3*g^3 - 3*(a*c^2*d^3*
e - 3*a^2*c*d*e^3)*f^2*g^4 + 3*(a^2*c*d^2*e^2 - a^3*e^4)*f*g^5)*x^3 + 3*(c
^3*d^3*e*f^5*g - a^3*d*e^3*f*g^5 + (c^3*d^4 - 3*a*c^2*d^2*e^2)*f^4*g^2 - 3
*(a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^3 + (3*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^4
)*x^2 + (c^3*d^3*e*f^6 - 3*a^3*d*e^3*f^2*g^4 + 3*(c^3*d^4 - a*c^2*d^2*e^2)
*f^5*g - 3*(3*a*c^2*d^3*e - a^2*c*d*e^3)*f^4*g^2 + (9*a^2*c*d^2*e^2 - a^3*
e^4)*f^3*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(174) = 348$.

Time = 0.21 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{32\left(c^2d^2e^4f^2g^2-2\,acde^5fg^3+a^2e^6g^4+5\left(\sqrt{e^2f+(e\right.\right.}{}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
32/15*(c^2*d^2*e^4*f^2*g^2 - 2*a*c*d*e^5*f*g^3 + a^2*e^6*g^4 + 5*(sqrt(e^2
*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
(e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c*d*e^2*f*g - 5*(sqrt(e^2*f + (e
*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f
+ (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*e^3*g^2 + 10*(sqrt(e^2*f + (e*x + d)*
e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
 d)*e*g - d*e*g)*c*d*g))^4)*sqrt(c*d*g)*c^2*d^2*e^6*g^3/((c*d*e^2*f*g - a*
e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2
*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^5*abs(g))
```

Mupad [B] (verification not implemented)

Time = 7.65 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx =$$

$$-\frac{\left(\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2-20\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2\right)}{15\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^2\,d^2\,x^2\,\sqrt{d+e\,x}}{15\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c\,d\,x\,\left(a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{15\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)}}{x^3\,\sqrt{f+g\,x}+\frac{d\,f^2\,\sqrt{f+g\,x}}{e\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(d\,g+2\,e\,f\right)}{e\,g}+\frac{f\,x\,\sqrt{f+g\,x}\,\left(2\,d\,g+e\,f\right)}{e\,g^2}}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
-((((d + e*x)^(1/2)*(6*a^2*e^2*g^2 + 30*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(15
*e*g^2*(a*e*g - c*d*f)^3) + (16*c^2*d^2*x^2*(d + e*x)^(1/2))/(15*e*(a*e*g
- c*d*f)^3) - (8*c*d*x*(a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*e*g*(a*e*g -
 c*d*f)^3))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^
(1/2) + (d*f^2*(f + g*x)^(1/2))/(e*g^2) + (x^2*(f + g*x)^(1/2)*(d*g + 2*e*
f))/(e*g) + (f*x*(f + g*x)^(1/2)*(2*d*g + e*f))/(e*g^2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.52

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2e^2g^3}{5}+\frac{4\sqrt{gx+f}\,\sqrt{cdx+ae}\,acdef\,g^2}{3}+8}{g\,(a^3e^3g^6x^3-3a^2cd\,e^2f\,g^5x^3+3a\,c^2d^2e\,f^2g^4x^3-c^3d^3f}$$

input

```
int((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(2*( - 3*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**2*e**2*g**3 + 10*sqrt(f + g*x)
*sqrt(a*e + c*d*x)*a*c*d*e*f*g**2 + 4*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c*
d*e*g**3*x - 15*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g - 20*sqrt
(f + g*x)*sqrt(a*e + c*d*x)*c**2*d**2*f*g**2*x - 8*sqrt(f + g*x)*sqrt(a*e
+ c*d*x)*c**2*d**2*g**3*x**2 + 8*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f**3 +
24*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*f**2*g*x + 24*sqrt(g)*sqrt(d)*sqrt(c)
*c**2*d**2*f*g**2*x**2 + 8*sqrt(g)*sqrt(d)*sqrt(c)*c**2*d**2*g**3*x**3))/(
15*g*(a**3*e**3*f**3*g**3 + 3*a**3*e**3*f**2*g**4*x + 3*a**3*e**3*f*g**5*x
**2 + a**3*e**3*g**6*x**3 - 3*a**2*c*d*e**2*f**4*g**2 - 9*a**2*c*d*e**2*f*
*3*g**3*x - 9*a**2*c*d*e**2*f**2*g**4*x**2 - 3*a**2*c*d*e**2*f*g**5*x**3 +
 3*a*c**2*d**2*e*f**5*g + 9*a*c**2*d**2*e*f**4*g**2*x + 9*a*c**2*d**2*e*f*
*3*g**3*x**2 + 3*a*c**2*d**2*e*f**2*g**4*x**3 - c**3*d**3*f**6 - 3*c**3*d*
*3*f**5*g*x - 3*c**3*d**3*f**4*g**2*x**2 - c**3*d**3*f**3*g**3*x**3))
```

3.90 $\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 267

$$\begin{aligned}\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= \frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}(f+gx)^{7/2}}\\ &+\frac{12cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}}+\frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}}\\ &+\frac{32c^3d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}\end{aligned}$$

output

```
2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(
g*x+f)^(7/2)+12/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d
*f)^2/(e*x+d)^(1/2)/(g*x+f)^(5/2)+16/35*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(3/2)+32/35*c^3*d^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x
+f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-5a^3e^3g^3+3a^2cde^2g^2(7f+2g}{35(}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-5*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(7*f
+ 2*g*x) - a*c^2*d^2*e*g*(35*f^2 + 28*f*g*x + 8*g^2*x^2) + c^3*d^3*(35*f^3
 + 70*f^2*g*x + 56*f*g^2*x^2 + 16*g^3*x^3)))/(35*(c*d*f - a*e*g)^4*Sqrt[d
+ e*x]*(f + g*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1254, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{7(cdf-aeg)}+\frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\left(\frac{4cd\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$\frac{6cd\left(\frac{4cd\left(\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{6cd\left(\frac{4cd\left(\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{7(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

input

```
Int[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]),x]
```

output

```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d*f - a*e*g)*Sqrt[d
+ e*x]*(f + g*x)^(7/2)) + (6*c*d*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (4*c*d*((2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3
*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])))/(5*(c*d*f - a*e*g))))/(7
*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-16x^3g^3d^3c^3+8a\,c^2d^2e\,g^3x^2-56c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-70c^3d^3f^2gx+5a^3e^3g^3-21a^2}{35\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}\,(aeg-dfc)^4}$
gosper	$-\frac{2(cdx+ae)(-16x^3g^3d^3c^3+8a\,c^2d^2e\,g^3x^2-56c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-70c^3d^3f^2gx+5a^3e^3g^3-21a^2cd\,e^2f\,g^2-}{35(gx+f)^{\frac{7}{2}}\,(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4d^4c^4)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x}}$
orering	$-\frac{2(-16x^3g^3d^3c^3+8a\,c^2d^2e\,g^3x^2-56c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-70c^3d^3f^2gx+5a^3e^3g^3-21a^2cd\,e^2f\,g^2+35a\,c^2d^2}{35(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4d^4c^4)(gx+f)^{\frac{7}{2}}\,\sqrt{ade+(a\,e^2+c\,d^2)x+c}}$

input
```
int((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/35/(e*x+d)^(1/2)/(g*x+f)^(7/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-16*c^3*d^3
*g^3*x^3+8*a*c^2*d^2*e*g^3*x^2-56*c^3*d^3*f*g^2*x^2-6*a^2*c*d*e^2*g^3*x+28
*a*c^2*d^2*e*f*g^2*x-70*c^3*d^3*f^2*g*x+5*a^3*e^3*g^3-21*a^2*c*d*e^2*f*g^2
+35*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)/(a*e*g-c*d*f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(235) = 470$.

Time = 1.67 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.57

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="fricas")
```

output

```
2/35*(16*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 35*a*c^2*d^2*e*f^2*g + 21*a^2*
c*d*e^2*f*g^2 - 5*a^3*e^3*g^3 + 8*(7*c^3*d^3*f*g^2 - a*c^2*d^2*e*g^3)*x^2
+ 2*(35*c^3*d^3*f^2*g - 14*a*c^2*d^2*e*f*g^2 + 3*a^2*c*d*e^2*g^3)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^
5*f^8 - 4*a*c^3*d^4*e*f^7*g + 6*a^2*c^2*d^3*e^2*f^6*g^2 - 4*a^3*c*d^2*e^3*
f^5*g^3 + a^4*d*e^4*f^4*g^4 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5
 + 6*a^2*c^2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^5 + (4
*c^4*d^4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 16*a*c^3*d^3*e^2)*f^4*g^4
- 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 8*a
^3*c*d*e^4)*f^2*g^6 - 4*(a^3*c*d^2*e^3 - a^4*e^5)*f*g^7)*x^4 + 2*(3*c^4*d^
4*e*f^6*g^2 + 2*a^4*d*e^4*f*g^7 + 2*(c^4*d^5 - 6*a*c^3*d^3*e^2)*f^5*g^3 -
2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^4 + 12*(a^2*c^2*d^3*e^2 - a^3*
c*d*e^4)*f^3*g^5 - (8*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^6)*x^3 + 2*(2*c^4*d
^4*e*f^7*g + 3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^6*g^2 -
 12*(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 4*a^3
*c*d*e^4)*f^4*g^4 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x^2 + (c^4*d^4*
e*f^8 + 4*a^4*d*e^4*f^3*g^5 + 4*(c^4*d^5 - a*c^3*d^3*e^2)*f^7*g - 2*(8*a*c
^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^2 + 4*(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4
)*f^5*g^3 - (16*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{9}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
+ f)^(9/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(235) = 470$.

Time = 0.27 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{64\left(c^3d^3e^6f^3g^3-3\,ac^2d^2e^7f^2g^4+3\,a^2cde^8fg^5-a^3e^9g\right.}{}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2),x, algorithm="giac")
```

output
```
64/35*(c^3*d^3*e^6*f^3*g^3 - 3*a*c^2*d^2*e^7*f^2*g^4 + 3*a^2*c*d*e^8*f*g^5
 - a^3*e^9*g^6 + 7*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt
(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c^2*
d^2*e^4*f^2*g^2 - 14*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sq
rt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*
c*d*e^5*f*g^3 + 7*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a^2*e
^6*g^4 + 21*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e
^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*c*d*e^2*f*g
 - 21*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g
 + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*a*e^3*g^2 + 35*(s
qrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3
*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^6)*sqrt(c*d*g)*c^3*d^3*e^8*
g^4/((c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(
c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c
*d*g))^2)^7*abs(g))
```

Mupad [B] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(10\,a^3\,e^3\,g^3-42\,a^2\,c\,d\,e^2\,f\,g^2+70\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3\right)}{35\,e\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{32\,c^3\,d^3\,x^3\,\sqrt{d+e\,x}}{35\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\right.}{x^4\,\sqrt{f+g\,x}+\frac{d\,f^3\,\sqrt{f+g\,x}}{e\,g^3}+\frac{x^3\,\sqrt{f+g\,x}\,\left(d\,g+3\,e\,f\right)}{e\,g}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,(d\,g}{e\,g^2}}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(9/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(10*a^3*
e^3*g^3 - 70*c^3*d^3*f^3 + 70*a*c^2*d^2*e*f^2*g - 42*a^2*c*d*e^2*f*g^2))/(
35*e*g^3*(a*e*g - c*d*f)^4) - (32*c^3*d^3*x^3*(d + e*x)^(1/2))/(35*e*(a*e*
g - c*d*f)^4) - (4*c*d*x*(d + e*x)^(1/2)*(3*a^2*e^2*g^2 + 35*c^2*d^2*f^2 -
 14*a*c*d*e*f*g))/(35*e*g^2*(a*e*g - c*d*f)^4) + (16*c^2*d^2*x^2*(a*e*g -
7*c*d*f)*(d + e*x)^(1/2))/(35*e*g*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2
) + (d*f^3*(f + g*x)^(1/2))/(e*g^3) + (x^3*(f + g*x)^(1/2)*(d*g + 3*e*f))/
(e*g) + (3*f*x^2*(f + g*x)^(1/2)*(d*g + e*f))/(e*g^2) + (f^2*x*(f + g*x)^(
1/2)*(3*d*g + e*f))/(e*g^3))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 833, normalized size of antiderivative = 3.12

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{-\frac{2\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^3e^3g^4}{7}+\frac{6\sqrt{gx+f}\,\sqrt{cdx+ae}\,a^2c}{5}}{g\left(a^4e^4g^8x^4-4a^3cd\,e^3f\,g^7x^4+6a^2c^2d^2e^2f^2g^6x^4-4a\,c\right.}$$

input
```
int((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(2*( - 5*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a**3*e**3*g**4 + 21*sqrt(f + g*x)
*sqrt(a*e + c*d*x)*a**2*c*d*e**2*f*g**3 + 6*sqrt(f + g*x)*sqrt(a*e + c*d*x
)*a**2*c*d*e**2*g**4*x - 35*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e*
f**2*g**2 - 28*sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e*f*g**3*x - 8*
sqrt(f + g*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e*g**4*x**2 + 35*sqrt(f + g*x)
*sqrt(a*e + c*d*x)*c**3*d**3*f**3*g + 70*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c
**3*d**3*f**2*g**2*x + 56*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*f*g**3
*x**2 + 16*sqrt(f + g*x)*sqrt(a*e + c*d*x)*c**3*d**3*g**4*x**3 - 16*sqrt(g
)*sqrt(d)*sqrt(c)*c**3*d**3*f**4 - 64*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f*
*3*g*x - 96*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d**3*f**2*g**2*x**2 - 64*sqrt(g)*
sqrt(d)*sqrt(c)*c**3*d**3*f*g**3*x**3 - 16*sqrt(g)*sqrt(d)*sqrt(c)*c**3*d*
*3*g**4*x**4))/(35*g*(a**4*e**4*f**4*g**4 + 4*a**4*e**4*f**3*g**5*x + 6*a*
*4*e**4*f**2*g**6*x**2 + 4*a**4*e**4*f*g**7*x**3 + a**4*e**4*g**8*x**4 - 4
*a**3*c*d*e**3*f**5*g**3 - 16*a**3*c*d*e**3*f**4*g**4*x - 24*a**3*c*d*e**3
*f**3*g**5*x**2 - 16*a**3*c*d*e**3*f**2*g**6*x**3 - 4*a**3*c*d*e**3*f*g**7
*x**4 + 6*a**2*c**2*d**2*e**2*f**6*g**2 + 24*a**2*c**2*d**2*e**2*f**5*g**3
*x + 36*a**2*c**2*d**2*e**2*f**4*g**4*x**2 + 24*a**2*c**2*d**2*e**2*f**3*g
**5*x**3 + 6*a**2*c**2*d**2*e**2*f**2*g**6*x**4 - 4*a*c**3*d**3*e*f**7*g -
 16*a*c**3*d**3*e*f**6*g**2*x - 24*a*c**3*d**3*e*f**5*g**3*x**2 - 16*a*c**
3*d**3*e*f**4*g**4*x**3 - 4*a*c**3*d**3*e*f**3*g**5*x**4 + c**4*d**4*f*...
```

3.91 $\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 277

$$
\begin{aligned}
\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = &-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&+\frac{15g(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^3d^3\sqrt{d+ex}} \\
&+\frac{5g(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2c^2d^2\sqrt{d+ex}} \\
&+\frac{15\sqrt{g}(cdf-aeg)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{4c^{7/2}d^{7/2}}
\end{aligned}
$$

output

```
-2*(e*x+d)^(1/2)*(g*x+f)^(5/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+15/4*g*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/c^3/d^3/(e*x+d)^(1/2)+5/2*g*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+15/4*g^(1/2)*(-a*e*g+c*d*f)^2*arctanh(c^(
1/2)*d^(1/2)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}(-15a^2e^2g^2-5acdeg(-5f+gx)+c^2d\right.}{4c^{7/2}d^7}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2),x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(-15*a^2*e^2*g^2 - 5*a*c*d*e
*g*(-5*f + g*x) + c^2*d^2*(-8*f^2 + 9*f*g*x + 2*g^2*x^2)) + 15*Sqrt[g]*(c*
d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(
Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*c^(7/2)*d^(7/2)*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1251, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{5g\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{5g\left(\frac{3(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1253$$

$$\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex}}{2cd\sqrt{d+ex}}}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1268$$

$$\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 66$$

$$\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+}}{2cd\sqrt{d+ex}}}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 221$$

$$\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+c}}{2cd\sqrt{d+ex}}}{cd}$$

$$\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^(5/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (5*g*(((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)*((Sqrt[f + g*x]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g
)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])))/(4*c*d)))/(c*d)
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(235) = 470$.

Time = 2.69 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.30

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a^2cd\,e^2g^3x-30\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^2d^2ef\,g^2x+15\ln\left(\right.\right.}{}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,
method=_RETURNVERBOSE)
```

output

```
1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c
+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f
*g^2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c
+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^3-3
0*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2
))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*g^2+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c
*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*f^2*g+4
*c^2*d^2*g^2*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-10*((c*d*x+a*e)
*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*g^2*x+18*((c*d*x+a*e)*(g*x+f))^(1/2)
*(c*d*g)^(1/2)*c^2*d^2*f*g*x-30*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*
a^2*e^2*g^2+50*a*c*d*e*f*g*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-16*((
c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((e*x+d)*(c*d*x+a*e))
^(1/2)*(g*x+f)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/(c*d*x+a*e)
/c^3/d^3/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 971, normalized size of antiderivative = 3.51

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25*a*c*d*e*f*g - 15*a^2*e^2*
g^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(a*c^2*d^3*e*f^2 - 2*a^2*c*d^2
*e^2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*c^2*d^2*e^2*f*g + a^2*c*d*
e^3*g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 2*(a*c^2*d^3*e + a^2*c*d*e
^3)*f*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*
e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f
*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d
*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
 f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*
a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c
^4*d^5 + a*c^3*d^3*e^2)*x), 1/8*(2*(2*c^2*d^2*g^2*x^2 - 8*c^2*d^2*f^2 + 25
*a*c*d*e*f*g - 15*a^2*e^2*g^2 + (9*c^2*d^2*f*g - 5*a*c*d*e*g^2)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a*c
^2*d^3*e*f^2 - 2*a^2*c*d^2*e^2*f*g + a^3*d*e^3*g^2 + (c^3*d^3*e*f^2 - 2*a*
c^2*d^2*e^2*f*g + a^2*c*d*e^3*g^2)*x^2 + ((c^3*d^4 + a*c^2*d^2*e^2)*f^2 -
2*(a*c^2*d^3*e + a^2*c*d*e^3)*f*g + (a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt
(-g/(c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
 d)*sqrt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g +
(c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*x^2 + a*c^3*d^4*e + (c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output

```
integrate((e*x + d)^(3/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=$$

$$-\frac{\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex+d)eg-deg}\Big((e^2f+(ex+d)eg-deg)}{4\,(cde^2fg-ae^3g^2-(e^2f+(ex+d}$$

$$-\frac{15\,(c^2d^2f^2g^2-2\,acdefg^3+a^2e^2g^4)\log\Big(\Big|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2}{4\,\sqrt{cdg}c^3d^3|g|}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output

```
-1/4*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g
)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*(2*
(e^2*f + (e*x + d)*e*g - d*e*g)*g^2/(c*d*e^4*abs(g)) + 5*(c^4*d^4*e^2*f*g^
2 - a*c^3*d^3*e^3*g^3)/(c^5*d^5*e^4*abs(g))) - 15*(c^4*d^4*e^4*f^2*g^2 - 2
*a*c^3*d^3*e^5*f*g^3 + a^2*c^2*d^2*e^6*g^4)/(c^5*d^5*e^4*abs(g)))/(c*d*e^2
*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g) - 15/4*(c^2*d^2*
f^2*g^2 - 2*a*c*d*e*f*g^3 + a^2*e^2*g^4)*log(abs(-sqrt(e^2*f + (e*x + d)*e
*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(f+g\,x)^{5/2}\,(d+e\,x)^{3/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input

```
int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2),x)
```

output

```
int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{15\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{gx+f}}{\sqrt{aeg-cdf}}\right)a^2e^2g^2-30}{}$$

input

```
int((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(15*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*e**2*g**2 -
30*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*c*d*e*f*g + 15*s
qrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*d*x) +
sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2*d**2*f**2 - 10*sq
rt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**2*g**2 + 20*sqrt(g)*sqrt(d
)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d*e*f*g - 10*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*c**2*d**2*f**2 - 15*sqrt(f + g*x)*a**2*c*d*e**2*g**2 + 25*sqr
t(f + g*x)*a*c**2*d**2*e*f*g - 5*sqrt(f + g*x)*a*c**2*d**2*e*g**2*x - 8*sq
rt(f + g*x)*c**3*d**3*f**2 + 9*sqrt(f + g*x)*c**3*d**3*f*g*x + 2*sqrt(f +
g*x)*c**3*d**3*g**2*x**2)/(4*sqrt(a*e + c*d*x)*c**4*d**4)
```

3.92 $\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 203

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{3g\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2\sqrt{d+ex}} + \frac{3\sqrt{g}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{c^{5/2}d^{5/2}}$$

output

```
-2*(e*x+d)^(1/2)*(g*x+f)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+3*g*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)
^(1/2)+3*g^(1/2)*(-a*e*g+c*d*f)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)*(g*x
+f)^(1/2)/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{d+ex}\Big(\sqrt{c}\sqrt{d}\sqrt{f+gx}(-2cdf+3aeg+cdgx)+3\sqrt{g}(cdf-a}{c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+ex}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2),x]
```

output
```
(Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(-2*c*d*f + 3*a*e*g + c*d*g*
x) + 3*Sqrt[g]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*
Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(c^(5/2)*d^(5/2)*Sqrt[(a*e +
 c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {1251, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 1251$$

$$\frac{3g\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1253$$

$$\frac{3g\left(\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1268$$

$$\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 66$$

$$\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input

```
Int[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2),x]
```

output

```
(-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (3*g*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x
]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c
^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(c*d
)
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(173) = 346$.

Time = 2.66 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\left(3\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)acde\,g^2x-3\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^2d^2fgx+3\ln\left(\frac{2cdgx+}{}\right.\right.}{}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g
)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*g^2*x-3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((
c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*f*g*x+3*ln
(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(
c*d*g)^(1/2))*a^2*e^2*g^2-3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(
g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*f*g-2*((c*d*x+a*e)*(g*
x+f))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-6*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1
/2)*a*e*g+4*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*f)*((e*x+d)*(c*d
*x+a*e))^(1/2)*(g*x+f)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*x+a*e)/(c*d*
g)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 725, normalized size of antiderivative = 3.57

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\,(cdgx-2\,cdf+3\,aeg)\sqrt{ex+d}}{}\right.$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output

```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3
*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*
d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e
^3)*g)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^
2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^
2 - 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2
 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x
+ d)))/(c^3*d^3*e*x^2 + a*c^2*d^3*e + (c^3*d^4 + a*c^2*d^2*e^2)*x), 1/2*(2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*g*x - 2*c*d*f + 3*a*e*g)
*sqrt(e*x + d)*sqrt(g*x + f) - 3*(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f
 - a*c*d*e^2*g)*x^2 + ((c^2*d^3 + a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*
x)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)*sqrt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d
*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x))/(c^3*d^3*e*x^2 + a*c^2*d^3*e +
 (c^3*d^4 + a*c^2*d^2*e^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Timed out}$$

input

```
integrate((e*x+d)**(3/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex+d)eg-deg}\,(\frac{(e^2f+(ex+d)eg-deg)g^2}{cde^2|g|} - \frac{3\,(c^2}{}}{cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg}$$

$$-\frac{3\,(cdfg^2-aeg^3)\log\,(\,|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-de}}{\sqrt{cdg}c^2d^2|g|}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
-sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*sq
rt(e^2*f + (e*x + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*g^2/(c*
d*e^2*abs(g)) - 3*(c^2*d^2*e^2*f*g^2 - a*c*d*e^3*g^3)/(c^3*d^3*e^2*abs(g))
)/(c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g) - 3*(c
*d*f*g^2 - a*e*g^3)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*
g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g
)))/(sqrt(c*d*g)*c^2*d^2*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{{\left(f+g\,x\right)}^{3/2}\,{\left(d+e\,x\right)}^{3/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

input
```
int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2),x)
```

output
```
int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{-12\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,\log\left(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{gx+f}}{\sqrt{aeg-cdf}}\right)aeg+12}{}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 12*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*e*g + 12*sqr
t(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*d*x) + sq
rt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c*d*f + 9*sqrt(g)*sqrt(d
)*sqrt(c)*sqrt(a*e + c*d*x)*a*e*g - 9*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c
*d*x)*c*d*f + 12*sqrt(f + g*x)*a*c*d*e*g - 8*sqrt(f + g*x)*c**2*d**2*f + 4
*sqrt(f + g*x)*c**2*d**2*g*x)/(4*sqrt(a*e + c*d*x)*c**3*d**3)
```

3.93 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 137

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{g}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{c^{3/2}d^{3/2}}$$

output
```
-2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+2*g^(1/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/g^(1/2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}-\sqrt{g}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input

```
Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2),x]
```

output

```
(-2*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x] - Sqrt[g]*Sqrt[a*e + c*d*
x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/
(c^(3/2)*d^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1251, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{g\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1268}$$

$$\frac{g\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{2g\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqr
t[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.46

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\left(\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)cdgx+\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)aeg\right.}{\sqrt{cdg}\,(cdx+ae)\sqrt{(cdx+ae)(gx+f)}\,dc\sqrt{ex+d}}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2
*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c*d*g*x+ln(1/2*
(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g
)^(1/2))*a*e*g-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2)/
(c*d*x+a*e)/((c*d*x+a*e)*(g*x+f))^(1/2)/d/c/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 569, normalized size of antiderivative = 4.15

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \left[\frac{(cdex^2+ade+(cd^2+ae^2)x)\sqrt{\frac{g}{cd}}\log\left(-\frac{8\,c^2d^2eg^2x^3+c^2d^3f^2+6\,acd^2}{}\right.}{}\right.$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output

```
[1/2*((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^
2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e
*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c
*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x
 + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (
8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e +
(c^2*d^3 + a*c*d*e^2)*x), -((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-
g/(c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d
)*sqrt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c
*d*e*f + (2*c*d^2 + a*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f))/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3
 + a*c*d*e^2)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output

```
Integral((d + e*x)**(3/2)*sqrt(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2),
x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,g^2\log\left(\left|-\sqrt{cdg}\sqrt{gx+f}+\sqrt{(gx+f)cdg-cdfg+aeg^2}\right|\right)}{\sqrt{cdg}cd|g|} - \frac{2\sqrt{gx+f}g^2}{\sqrt{(gx+f)cdg-cdfg+aeg^2}cd|g|}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
-2*g^2*log(abs(-sqrt(c*d*g)*sqrt(g*x + f) + sqrt((g*x + f)*c*d*g - c*d*f*g
 + a*e*g^2)))/(sqrt(c*d*g)*c*d*abs(g)) - 2*sqrt(g*x + f)*g^2/(sqrt((g*x +
f)*c*d*g - c*d*f*g + a*e*g^2)*c*d*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{\sqrt{f+g\,x}\,{(d+e\,x)}^{3/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{gx+f}}{\sqrt{aeg-cdf}}\right)-2\sqrt{g}\,\sqrt{d}\,\sqrt{}}{\sqrt{cdx+ae}\,c^2d^2}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f)) - sqrt(g)*sqrt(d)
*sqrt(c)*sqrt(a*e + c*d*x) - sqrt(f + g*x)*c*d))/(sqrt(a*e + c*d*x)*c**2*d
**2)
```

3.94 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 61

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

output

```
-2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(cdf-aeg)\sqrt{(ae+cdx)(d+ex)}}$$

input

```
Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)),x]
```

output

```
(-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d +
e*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = $\{1248\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \; 1248$$

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

input

```
Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2)),x]
```

output

```
(-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1248

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}}{\sqrt{ex+d}\,(cdx+ae)(aeg-dfc)}$	55
gosper	$\frac{2(cdx+ae)\sqrt{gx+f}\,(ex+d)^{\frac{3}{2}}}{(aeg-dfc)(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	63
orering	$\frac{2(cdx+ae)\sqrt{gx+f}\,(ex+d)^{\frac{3}{2}}}{(aeg-dfc)(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}$	64

input

```
int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,
method=_RETURNVERBOSE)
```

output

```
2/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)/(a*e
*g-c*d*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(55) = 110$.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}}{acd^2ef-a^2de^2g+(c^2d^2ef-acde^2g)x^2+((c^2d^3+acde^2)f-(acd^2e+a^2e^3)g)x}$$

input

```
integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output

```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)
/(a*c*d^2*e*f - a^2*d*e^2*g + (c^2*d^2*e*f - a*c*d*e^2*g)*x^2 + ((c^2*d^3
+ a*c*d*e^2)*f - (a*c*d^2*e + a^2*e^3)*g)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x))
, x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}\,\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*s
qrt(g*x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{gx+f}g^2}{\sqrt{(gx+f)cdg-cdfg+aeg^2}(cdf|g|-aeg|g|)}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
-2*sqrt(g*x + f)*g^2/(sqrt((g*x + f)*c*d*g - c*d*f*g + a*e*g^2)*(c*d*f*abs
(g) - a*e*g*abs(g)))
```

Mupad [B] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.41

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\frac{\left(\frac{2\,f\,\sqrt{d+e\,x}}{c\,d\,e\,(a\,e\,g-c\,d\,f)}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c\,d\,e\,(a\,e\,g-c\,d\,f)}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2}}{x^2\,\sqrt{f+g\,x}+\frac{a\,\sqrt{f+g\,x}}{c}+\frac{x\,\sqrt{f+g\,x}\,(c\,d^2+a\,e^2)}{c\,d\,e}}$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2)),x)
```

output
```
(((2*f*(d + e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)) + (2*g*x*(d + e*x)^(1/2))/
(c*d*e*(a*e*g - c*d*f)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x
^2*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1/2)*(a*e^2 + c
*d^2))/(c*d*e))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\frac{2\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}+2\sqrt{gx+f}\,cd}{\sqrt{cdx+ae}\,cd\,(aeg-cdf)}$$

input
```
int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x) + sqrt(f + g*x)*c*d))/(sqrt(
a*e + c*d*x)*c*d*(a*e*g - c*d*f))
```

3.95 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 124

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{4g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}$$

output

```
-2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)-4*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(
e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(aeg+cd(f+2gx))}{(cdf-aeg)^2\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*(a*e*g + c*d*(f + 2*g*x)))/((c*d*f - a*e*g)^2*Sqrt[(a*e
+ c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1252, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{2g\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$-\frac{4g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

input

```
Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)),x]
```

output

```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Defintions of rubi rules used

rule 1248

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2\sqrt{\left(e x +d \right) \left(c d x +a e \right)}\, \left(2 c d g x +a e g +d f c \right)}{\sqrt{e x +d}\, \sqrt{g x +f}\, \left(c d x +a e \right) \left(a e g -d f c \right)^{2}}$	70
gosper	$-\frac{2 \left(c d x +a e \right) \left(2 c d g x +a e g +d f c \right) \left(e x +d \right)^{\frac{3}{2}}}{\sqrt{g x +f}\, \left(a^{2} e^{2} g^{2}-2 a c d e f g +f^{2} c^{2} d^{2}\right) \left(c d \,x^{2} e +a \,e^{2} x +c \,d^{2} x +a d e \right)^{\frac{3}{2}}}$	97
orering	$-\frac{2 \left(2 c d g x +a e g +d f c \right) \left(c d x +a e \right) \left(e x +d \right)^{\frac{3}{2}}}{\left(a^{2} e^{2} g^{2}-2 a c d e f g +f^{2} c^{2} d^{2}\right) \sqrt{g x +f}\, \left(a d e +\left(a \,e^{2}+c \,d^{2}\right) x +c d \,x^{2} e \right)^{\frac{3}{2}}}$	98

input
```
int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(2*c*d*g*x+a*e*
g+c*d*f)/(c*d*x+a*e)/(a*e*g-c*d*f)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(112) = 224$.

Time = 0.11 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.62

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{cdex^2+a}}{ac^2d^3ef^3-2\,a^2cd^2e^2f^2g+a^3de^3fg^2+(c^3d^3ef^2g-2\,ac^2d^2e^2fg^2+a^2cde^3g^3)x^3+(c^3d^3ef^3+(c^3d^4-ac}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + c*d*f + a*e*g)
*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^2*d^3*e*f^3 - 2*a^2*c*d^2*e^2*f^2*g + a^
3*d*e^3*f*g^2 + (c^3*d^3*e*f^2*g - 2*a*c^2*d^2*e^2*f*g^2 + a^2*c*d*e^3*g^3
)*x^3 + (c^3*d^3*e*f^3 + (c^3*d^4 - a*c^2*d^2*e^2)*f^2*g - (2*a*c^2*d^3*e
+ a^2*c*d*e^3)*f*g^2 + (a^2*c*d^2*e^2 + a^3*e^4)*g^3)*x^2 + (a^3*d*e^3*g^3
 + (c^3*d^4 + a*c^2*d^2*e^2)*f^3 - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*f^2*g - (
a^2*c*d^2*e^2 - a^3*e^4)*f*g^2)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**(3/
2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(112) = 224$.

Time = 0.18 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.30

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=2\Bigg(\frac{\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-de}}{(c^2d^2e^2f^2|g|-2\,acde^3fg|g|+a^2e^4g^2|g|)(cde^2fg}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
2*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)*
sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g^2/((c^2*d^2*e^2*f^2*abs(g) - 2*a
*c*d*e^3*f*g*abs(g) + a^2*e^4*g^2*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 - (e^2*
f + (e*x + d)*e*g - d*e*g)*c*d*g)) - 2*sqrt(c*d*g)*g^2/((c*d*e^2*f*g - a*e
^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*
f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)*(c*d*f*abs(g)
 - a*e*g*abs(g))))*e^2
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{\left(\frac{4\,g\,x\,\sqrt{d+e\,x}}{e\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{\left(2\,a\,e\,g+2\,c\,d\,f\right)\,\sqrt{d+e\,x}}{c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{a\,\sqrt{f+g\,x}}{c}+\frac{x\,\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)}{c\,d\,e}}$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2)),x)
```

output
```
-(((4*g*x*(d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^2) + ((2*a*e*g + 2*c*d*f)*(d
 + e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)
^(1/2)*(a*e^2 + c*d^2))/(c*d*e))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{-4\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,f-4\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}}{\sqrt{cdx+ae}\,(a^2e^2g^3x-2acdef\,g^2x+c^2}$$

input
```
int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 2*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*f - 2*sqrt(g)*sqrt(d)*s
qrt(c)*sqrt(a*e + c*d*x)*g*x - sqrt(f + g*x)*a*e*g - sqrt(f + g*x)*c*d*f -
 2*sqrt(f + g*x)*c*d*g*x))/(sqrt(a*e + c*d*x)*(a**2*e**2*f*g**2 + a**2*e**
2*g**3*x - 2*a*c*d*e*f**2*g - 2*a*c*d*e*f*g**2*x + c**2*d**2*f**3 + c**2*d
**2*f**2*g*x))
```

3.96 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 192

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =\\ -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\ -\frac{8g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{3/2}}-\frac{16cdg\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}\sqrt{f+gx}}\end{aligned}$$

output

```
-2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)-8/3*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2
/(e*x+d)^(1/2)/(g*x+f)^(3/2)-16/3*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(-a^2e^2g^2+2acdeg(3f+2gx)+c^2d^2\left(3f^2+12fgx+8g^2x^2\right)\right)}{3(cdf-aeg)^3\sqrt{(ae+cdx)(d+ex)}(f+gx)^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*(-(a^2*e^2*g^2) + 2*a*c*d*e*g*(3*f + 2*g*x) + c^2*d^2*(3
*f^2 + 12*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d
+ e*x)]*(f + g*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1252, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 1252$$

$$-\frac{4g\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \ 1254$$

$$-\frac{4g\left(\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$-\frac{4g\left(\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) - (4*g*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt
[f + g*x])))/(c*d*f - a*e*g)
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,\left(-8g^{2}x^{2}d^{2}c^{2}-4acde\,g^{2}x-12c^{2}d^{2}fgx+a^{2}e^{2}g^{2}-6acdefg-3f^{2}c^{2}d^{2}\right)}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}\,(cdx+ae)(aeg-dfc)^{3}}$	120
gosper	$-\frac{2(cdx+ae)\left(-8g^{2}x^{2}d^{2}c^{2}-4acde\,g^{2}x-12c^{2}d^{2}fgx+a^{2}e^{2}g^{2}-6acdefg-3f^{2}c^{2}d^{2}\right)(ex+d)^{\frac{3}{2}}}{3(gx+f)^{\frac{3}{2}}\,(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}d^{3}c^{3})(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	168
orering	$-\frac{2\left(-8g^{2}x^{2}d^{2}c^{2}-4acde\,g^{2}x-12c^{2}d^{2}fgx+a^{2}e^{2}g^{2}-6acdefg-3f^{2}c^{2}d^{2}\right)(cdx+ae)(ex+d)^{\frac{3}{2}}}{3(a^{3}e^{3}g^{3}-3a^{2}cd\,e^{2}f\,g^{2}+3a\,c^{2}d^{2}e\,f^{2}g-f^{3}d^{3}c^{3})(gx+f)^{\frac{3}{2}}\,(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}$	169

input
```
int((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-8*c^2*d^2*g
^2*x^2-4*a*c*d*e*g^2*x-12*c^2*d^2*f*g*x+a^2*e^2*g^2-6*a*c*d*e*f*g-3*c^2*d^
2*f^2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(170) = 340$.

Time = 0.20 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.38

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{}{3\left(ac^3d^4ef^5-3\,a^2c^2d^3e^2f^4g+3\,a^3cd^2e^3f^3g^2-a^4de^4f^2g^3+\left(c^4d^4ef^3g^2-3\,ac^3d^3e^2f^2g^3+3\,a^2c^2d^2e^3fg^4\right.\right.}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output
```
-2/3*(8*c^2*d^2*g^2*x^2 + 3*c^2*d^2*f^2 + 6*a*c*d*e*f*g - a^2*e^2*g^2 + 4*
(3*c^2*d^2*f*g + a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^3*d^4*e*f^5 - 3*a^2*c^2*d^3*e^2*f^4*g
+ 3*a^3*c*d^2*e^3*f^3*g^2 - a^4*d*e^4*f^2*g^3 + (c^4*d^4*e*f^3*g^2 - 3*a*c
^3*d^3*e^2*f^2*g^3 + 3*a^2*c^2*d^2*e^3*f*g^4 - a^3*c*d*e^4*g^5)*x^4 + (2*c
^4*d^4*e*f^4*g + (c^4*d^5 - 5*a*c^3*d^3*e^2)*f^3*g^2 - 3*(a*c^3*d^4*e - a^
2*c^2*d^2*e^3)*f^2*g^3 + (3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^4 - (a^3*c*
d^2*e^3 + a^4*e^5)*g^5)*x^3 + (c^4*d^4*e*f^5 - a^4*d*e^4*g^5 + (2*c^4*d^5
- a*c^3*d^3*e^2)*f^4*g - (5*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*f^3*g^2 + (3*
a^2*c^2*d^3*e^2 + 5*a^3*c*d*e^4)*f^2*g^3 + (a^3*c*d^2*e^3 - 2*a^4*e^5)*f*g
^4)*x^2 - (2*a^4*d*e^4*f*g^4 - (c^4*d^5 + a*c^3*d^3*e^2)*f^5 + (a*c^3*d^4*
e + 3*a^2*c^2*d^2*e^3)*f^4*g + 3*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^2 -
 (5*a^3*c*d^2*e^3 - a^4*e^5)*f^2*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(170) = 340.

Time = 0.33 (sec) , antiderivative size = 672, normalized size of antiderivative = 3.50

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2}{3}\left(\frac{3\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+}{(c^3d^3e^3f^3|g|-3\,ac^2d^2e^4f^2g|g|+3\,a^2cde^5fg^2|g|}\right.$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d
*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*c^2*d^2*g^2/((c^3*d^3*e^3*f^3*abs(
g) - 3*a*c^2*d^2*e^4*f^2*g*abs(g) + 3*a^2*c*d*e^5*f*g^2*abs(g) - a^3*e^6*g
^3*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*
g)) - 2*(5*sqrt(c*d*g)*c^3*d^3*e^4*f^2*g^4 - 10*sqrt(c*d*g)*a*c^2*d^2*e^5*
f*g^5 + 5*sqrt(c*d*g)*a^2*c*d*e^6*g^6 + 12*sqrt(c*d*g)*(sqrt(e^2*f + (e*x
+ d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (
e*x + d)*e*g - d*e*g)*c*d*g))^2*c^2*d^2*e^2*f*g^3 - 12*sqrt(c*d*g)*(sqrt(e
^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2
+ (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a*c*d*e^3*g^4 + 3*sqrt(c*d*g)*
(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e
^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*c*d*g^2)/((c^2*d^2*e*f^
2*abs(g) - 2*a*c*d*e^2*f*g*abs(g) + a^2*e^3*g^2*abs(g))*(c*d*e^2*f*g - a*e
^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*
f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2)^3))*e^3
```

Mupad [B] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\left(\frac{8\,x\,(a\,e\,g+3\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,e\,(a\,e\,g-c\,d\,f)^3}+\frac{\sqrt{d+e\,x}\,\left(-2\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g+6\,c^2\right.}{3\,c\,d\,e\,g\,(a\,e\,g-c\,d\,f)^3}\right.}{x^3\,\sqrt{f+g\,x}+\frac{a\,f\,\sqrt{f+g\,x}}{c\,g}+\frac{x\,\sqrt{f+g\,x}\,(c\,f}{c}}$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2)),x)
```

output
```
(((8*x*(a*e*g + 3*c*d*f)*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3) + ((d +
e*x)^(1/2)*(6*c^2*d^2*f^2 - 2*a^2*e^2*g^2 + 12*a*c*d*e*f*g))/(3*c*d*e*g*(a
*e*g - c*d*f)^3) + (16*c*d*g*x^2*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3))
*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2) + (a*
f*(f + g*x)^(1/2))/(c*g) + (x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + a*d*e*g
))/(c*d*e*g) + (x^2*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + c*d*e*f))/(c*d*e*
g))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{-\frac{16\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,cd\,f^2}{3}-\frac{32\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,cdf\,gx}{3}-\frac{16}{}}{\sqrt{cdx+ae}\,\left(a^3e^3g^5x^2-3a^2cd\,e^2f\,g^4x^2+3a\,c^2d^2e\,f^2\right.}$$

input
```
int((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 8*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*f**2 - 16*sqrt(g)*s
qrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*f*g*x - 8*sqrt(g)*sqrt(d)*sqrt(c)*sqr
t(a*e + c*d*x)*c*d*g**2*x**2 - sqrt(f + g*x)*a**2*e**2*g**2 + 6*sqrt(f + g
*x)*a*c*d*e*f*g + 4*sqrt(f + g*x)*a*c*d*e*g**2*x + 3*sqrt(f + g*x)*c**2*d*
*2*f**2 + 12*sqrt(f + g*x)*c**2*d**2*f*g*x + 8*sqrt(f + g*x)*c**2*d**2*g**
2*x**2))/(3*sqrt(a*e + c*d*x)*(a**3*e**3*f**2*g**3 + 2*a**3*e**3*f*g**4*x
+ a**3*e**3*g**5*x**2 - 3*a**2*c*d*e**2*f**3*g**2 - 6*a**2*c*d*e**2*f**2*g
**3*x - 3*a**2*c*d*e**2*f*g**4*x**2 + 3*a*c**2*d**2*e*f**4*g + 6*a*c**2*d*
*2*e*f**3*g**2*x + 3*a*c**2*d**2*e*f**2*g**3*x**2 - c**3*d**3*f**5 - 2*c**
3*d**3*f**4*g*x - c**3*d**3*f**3*g**2*x**2))
```

3.97 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 262

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{12g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^2\sqrt{d+ex}(f+gx)^{5/2}} - \frac{16cdg\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}} - \frac{32c^2d^2g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}$$

output

```
-2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)-12/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^
2/(e*x+d)^(1/2)/(g*x+f)^(5/2)-16/5*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(3/2)-32/5*c^2*d^2*g*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(a^3e^3g^3-a^2cde^2g^2(5f+2gx)+ac^2d^2eg\left(15f^2+20fgx+8g^2x^2\right)+c^3d^3\left(5f^3+30f^2gx+40fg\right.\right.}{5(cdf-aeg)^4\sqrt{(ae+cdx)(d+ex)}(f+gx)^{5/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*(a^3*e^3*g^3 - a^2*c*d*e^2*g^2*(5*f + 2*g*x) + a*c^2*d^2
*e*g*(15*f^2 + 20*f*g*x + 8*g^2*x^2) + c^3*d^3*(5*f^3 + 30*f^2*g*x + 40*f*
g^2*x^2 + 16*g^3*x^3)))/(5*(c*d*f - a*e*g)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]
*(f + g*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1252, 1254, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{6g\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1254}$$

$$-\frac{6g\left(\frac{4cd\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1254}$$

$$-\frac{6g\left(\frac{4cd\left(\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{ 1248}$$

$$-\frac{6g\left(\frac{4cd\left(\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{5(cdf-aeg)}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input

```
Int[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2)),x]
```

output

```
(-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) - (6*g*((2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(5*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (4*c*d*((2*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f
 + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])))/(5*(c*d*f - a*e*g))))/(c*d*
f - a*e*g)
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(16x^{3}g^{3}d^{3}c^{3}+8a\,c^{2}d^{2}e\,g^{3}x^{2}+40c^{3}d^{3}f\,g^{2}x^{2}-2a^{2}cd\,e^{2}g^{3}x+20a\,c^{2}d^{2}ef\,g^{2}x+30c^{3}d^{3}f^{2}gx+a^{3}e^{3}g^{3}-5a^{2}cd\,e}{5\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(cdx+ae)(aeg-dfc)^{4}}$
gosper	$-\frac{2(cdx+ae)(16x^{3}g^{3}d^{3}c^{3}+8a\,c^{2}d^{2}e\,g^{3}x^{2}+40c^{3}d^{3}f\,g^{2}x^{2}-2a^{2}cd\,e^{2}g^{3}x+20a\,c^{2}d^{2}ef\,g^{2}x+30c^{3}d^{3}f^{2}gx+a^{3}e^{3}g^{3}-5a^{2}cd\,e^{2}f\,g^{2}+15a}{5(gx+f)^{\frac{5}{2}}(a^{4}e^{4}g^{4}-4a^{3}cd\,e^{3}f\,g^{3}+6a^{2}c^{2}d^{2}e^{2}f^{2}g^{2}-4a\,c^{3}d^{3}e\,f^{3}g+f^{4}d^{4}c^{4})(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ad}$
orering	$-\frac{2(16x^{3}g^{3}d^{3}c^{3}+8a\,c^{2}d^{2}e\,g^{3}x^{2}+40c^{3}d^{3}f\,g^{2}x^{2}-2a^{2}cd\,e^{2}g^{3}x+20a\,c^{2}d^{2}ef\,g^{2}x+30c^{3}d^{3}f^{2}gx+a^{3}e^{3}g^{3}-5a^{2}cd\,e^{2}f\,g^{2}+15a\,c^{2}d^{2}e\,f^{2}}{5(a^{4}e^{4}g^{4}-4a^{3}cd\,e^{3}f\,g^{3}+6a^{2}c^{2}d^{2}e^{2}f^{2}g^{2}-4a\,c^{3}d^{3}e\,f^{3}g+f^{4}d^{4}c^{4})(gx+f)^{\frac{5}{2}}(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}}$

input

```
int((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,
method=_RETURNVERBOSE)
```

output

```
-2/5/(e*x+d)^(1/2)/(g*x+f)^(5/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(16*c^3*d^3*g
^3*x^3+8*a*c^2*d^2*e*g^3*x^2+40*c^3*d^3*f*g^2*x^2-2*a^2*c*d*e^2*g^3*x+20*a
*c^2*d^2*e*f*g^2*x+30*c^3*d^3*f^2*g*x+a^3*e^3*g^3-5*a^2*c*d*e^2*f*g^2+15*a
*c^2*d^2*e*f^2*g+5*c^3*d^3*f^3)/(c*d*x+a*e)/(a*e*g-c*d*f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1062 vs. $2(232) = 464$.

Time = 1.27 (sec) , antiderivative size = 1062, normalized size of antiderivative = 4.05

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="fricas")
```

output

```
-2/5*(16*c^3*d^3*g^3*x^3 + 5*c^3*d^3*f^3 + 15*a*c^2*d^2*e*f^2*g - 5*a^2*c*
d*e^2*f*g^2 + a^3*e^3*g^3 + 8*(5*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 2*
(15*c^3*d^3*f^2*g + 10*a*c^2*d^2*e*f*g^2 - a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a*c^4*d^5*e*
f^7 - 4*a^2*c^3*d^4*e^2*f^6*g + 6*a^3*c^2*d^3*e^3*f^5*g^2 - 4*a^4*c*d^2*e^
4*f^4*g^3 + a^5*d*e^5*f^3*g^4 + (c^5*d^5*e*f^4*g^3 - 4*a*c^4*d^4*e^2*f^3*g
^4 + 6*a^2*c^3*d^3*e^3*f^2*g^5 - 4*a^3*c^2*d^2*e^4*f*g^6 + a^4*c*d*e^5*g^7
)*x^5 + (3*c^5*d^5*e*f^5*g^2 + (c^5*d^6 - 11*a*c^4*d^4*e^2)*f^4*g^3 - 2*(2
*a*c^4*d^5*e - 7*a^2*c^3*d^3*e^3)*f^3*g^4 + 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d
^2*e^4)*f^2*g^5 - (4*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^6 + (a^4*c*d^2*e^4
 + a^5*e^6)*g^7)*x^4 + (3*c^5*d^5*e*f^6*g + a^5*d*e^5*g^7 + 3*(c^5*d^6 - 3
*a*c^4*d^4*e^2)*f^5*g^2 - (11*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^4*g^3 + 2
*(7*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g^4 - 3*(2*a^3*c^2*d^3*e^3 +
3*a^4*c*d*e^5)*f^2*g^5 - (a^4*c*d^2*e^4 - 3*a^5*e^6)*f*g^6)*x^3 + (c^5*d^5
*e*f^7 + 3*a^5*d*e^5*f*g^6 + (3*c^5*d^6 - a*c^4*d^4*e^2)*f^6*g - 3*(3*a*c^
4*d^5*e + 2*a^2*c^3*d^3*e^3)*f^5*g^2 + 2*(3*a^2*c^3*d^4*e^2 + 7*a^3*c^2*d^
2*e^4)*f^4*g^3 + (6*a^3*c^2*d^3*e^3 - 11*a^4*c*d*e^5)*f^3*g^4 - 3*(3*a^4*c
*d^2*e^4 - a^5*e^6)*f^2*g^5)*x^2 + (3*a^5*d*e^5*f^2*g^5 + (c^5*d^6 + a*c^4
*d^4*e^2)*f^7 - (a*c^4*d^5*e + 4*a^2*c^3*d^3*e^3)*f^6*g - 6*(a^2*c^3*d^4*e
^2 - a^3*c^2*d^2*e^4)*f^5*g^2 + 2*(7*a^3*c^2*d^3*e^3 - 2*a^4*c*d*e^5)*f...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Timed out}$$

input

```
integrate((e*x+d)**(3/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(3/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(
g*x + f)^(7/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1476 vs. $2(232) = 464$.

Time = 0.62 (sec) , antiderivative size = 1476, normalized size of antiderivative = 5.63

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2),x, algorithm="giac")
```

output
```
2/5*(5*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d
*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*c^3*d^3*g^2/((c^4*d^4*e^4*f^4*abs(
g) - 4*a*c^3*d^3*e^5*f^3*g*abs(g) + 6*a^2*c^2*d^2*e^6*f^2*g^2*abs(g) - 4*a
^3*c*d*e^7*f*g^3*abs(g) + a^4*e^8*g^4*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 - (
e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)) - 2*(11*sqrt(c*d*g)*c^6*d^6*e^8*f^4
*g^6 - 44*sqrt(c*d*g)*a*c^5*d^5*e^9*f^3*g^7 + 66*sqrt(c*d*g)*a^2*c^4*d^4*e
^10*f^2*g^8 - 44*sqrt(c*d*g)*a^3*c^3*d^3*e^11*f*g^9 + 11*sqrt(c*d*g)*a^4*c
^2*d^2*e^12*g^10 + 50*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqr
t(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)
*c*d*g))^2*c^5*d^5*e^6*f^3*g^5 - 150*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e
*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
d)*e*g - d*e*g)*c*d*g))^2*a*c^4*d^4*e^7*f^2*g^6 + 150*sqrt(c*d*g)*(sqrt(e^
2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
 (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a^2*c^3*d^3*e^8*f*g^7 - 50*sqrt
(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f
*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*a^3*c^2*d^2*e^9
*g^8 + 80*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - s
qrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^4*c
^4*d^4*e^4*f^2*g^4 - 160*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*
sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - ...
```

Mupad [B] (verification not implemented)

Time = 7.97 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,x\,\sqrt{d+e\,x}\,\left(-a^2\,e^2\,g^2+10\,a\,c\,d\,e\,f\,g+15\,c^2\,d^2\,f^2\right)}{5\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{\sqrt{d+e\,x}\,(\frac{2\,a^3\,e^3\,g^3}{5}-2\,a^2\,c\,d\,e^2\,f\,g^2+6\,c}{c\,d\,e\,g^2\,(a\,e\,g-c\,d}\right.}{x^4\,\sqrt{f+g\,x}+\frac{a\,f^2\,\sqrt{f+g\,x}}{c\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(2\,c\,d^2\,f\,g+c\,d\,e\,f^2+a\,d\,e\,g^2+2\,a\,e^2\,f\,g\right)}{c\,d\,e\,g^2}+\frac{x^3\,\sqrt{f+g\,x}\,(c\,g\,d^2+}{c\,d\,e}}$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2)),x)
```

output
```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*x*(d + e*x)^(1/2)*(15*
c^2*d^2*f^2 - a^2*e^2*g^2 + 10*a*c*d*e*f*g))/(5*e*g*(a*e*g - c*d*f)^4) + (
(d + e*x)^(1/2)*((2*a^3*e^3*g^3)/5 + 2*c^3*d^3*f^3 + 6*a*c^2*d^2*e*f^2*g -
 2*a^2*c*d*e^2*f*g^2))/(c*d*e*g^2*(a*e*g - c*d*f)^4) + (32*c^2*d^2*g*x^3*(
d + e*x)^(1/2))/(5*e*(a*e*g - c*d*f)^4) + (16*c*d*x^2*(a*e*g + 5*c*d*f)*(d
 + e*x)^(1/2))/(5*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (a*f^2*(f
+ g*x)^(1/2))/(c*g^2) + (x^2*(f + g*x)^(1/2)*(a*d*e*g^2 + c*d*e*f^2 + 2*a*
e^2*f*g + 2*c*d^2*f*g))/(c*d*e*g^2) + (x^3*(f + g*x)^(1/2)*(a*e^2*g + c*d^
2*g + 2*c*d*e*f))/(c*d*e*g) + (f*x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + 2*
a*d*e*g))/(c*d*e*g^2))
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 664, normalized size of antiderivative = 2.53

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{\frac{32\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c^2d^2f^3}{5}+\frac{96\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c^2d^2}{5}}{\sqrt{cdx+ae}\,(a^4e^4g^7x^3-4a^3cd\,e^3f\,g^6x^3+6a^2c^2d^2e^2f}$$

input
```
int((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(2*(16*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**2*f**3 + 48*sqrt(
g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g*x + 48*sqrt(g)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**2*f*g**2*x**2 + 16*sqrt(g)*sqrt(d)*sq
rt(c)*sqrt(a*e + c*d*x)*c**2*d**2*g**3*x**3 - sqrt(f + g*x)*a**3*e**3*g**3
 + 5*sqrt(f + g*x)*a**2*c*d*e**2*f*g**2 + 2*sqrt(f + g*x)*a**2*c*d*e**2*g*
*3*x - 15*sqrt(f + g*x)*a*c**2*d**2*e*f**2*g - 20*sqrt(f + g*x)*a*c**2*d**
2*e*f*g**2*x - 8*sqrt(f + g*x)*a*c**2*d**2*e*g**3*x**2 - 5*sqrt(f + g*x)*c
**3*d**3*f**3 - 30*sqrt(f + g*x)*c**3*d**3*f**2*g*x - 40*sqrt(f + g*x)*c**
3*d**3*f*g**2*x**2 - 16*sqrt(f + g*x)*c**3*d**3*g**3*x**3))/(5*sqrt(a*e +
c*d*x)*(a**4*e**4*f**3*g**4 + 3*a**4*e**4*f**2*g**5*x + 3*a**4*e**4*f*g**6
*x**2 + a**4*e**4*g**7*x**3 - 4*a**3*c*d*e**3*f**4*g**3 - 12*a**3*c*d*e**3
*f**3*g**4*x - 12*a**3*c*d*e**3*f**2*g**5*x**2 - 4*a**3*c*d*e**3*f*g**6*x*
*3 + 6*a**2*c**2*d**2*e**2*f**5*g**2 + 18*a**2*c**2*d**2*e**2*f**4*g**3*x
+ 18*a**2*c**2*d**2*e**2*f**3*g**4*x**2 + 6*a**2*c**2*d**2*e**2*f**2*g**5*
x**3 - 4*a*c**3*d**3*e*f**6*g - 12*a*c**3*d**3*e*f**5*g**2*x - 12*a*c**3*d
**3*e*f**4*g**3*x**2 - 4*a*c**3*d**3*e*f**3*g**4*x**3 + c**4*d**4*f**7 + 3
*c**4*d**4*f**6*g*x + 3*c**4*d**4*f**5*g**2*x**2 + c**4*d**4*f**4*g**3*x**
3))
```

3.98 $\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 341

$$\begin{aligned}\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = &-\frac{2(d+ex)^{3/2}(f+gx)^{7/2}}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}-\frac{14g\sqrt{d+ex}(f+gx)^{5/2}}{3c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\\&+\frac{35g^2(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^4d^4\sqrt{d+ex}}\\&+\frac{35g^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{6c^3d^3\sqrt{d+ex}}\\&+\frac{35g^{3/2}(cdf-aeg)^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{9/2}d^{9/2}}\end{aligned}$$

output

```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(7/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)-14/3*g*(e*x+d)^(1/2)*(g*x+f)^(5/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)+35/4*g^2*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)/c^4/d^4/(e*x+d)^(1/2)+35/6*g^2*(g*x+f)^(3/2)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)+35/4*g^(3/2)*(-a*e*g+c*d*
f)^2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/g^(1/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{(d+ex)^{5/2}\left(-\sqrt{c}\sqrt{d}(ae+cdx)\sqrt{f+gx}(105a^3e^3g^3+35a^2cde^2g}{}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^(7/2))/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(5/2),x]
```

output
```
((d + e*x)^(5/2)*(-(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*Sqrt[f + g*x]*(105*a^3*e
^3*g^3 + 35*a^2*c*d*e^2*g^2*(-5*f + 4*g*x) + 7*a*c^2*d^2*e*g*(8*f^2 - 34*f
*g*x + 3*g^2*x^2) + c^3*d^3*(8*f^3 + 80*f^2*g*x - 39*f*g^2*x^2 - 6*g^3*x^3
))) + 105*g^(3/2)*(c*d*f - a*e*g)^2*(a*e + c*d*x)^(5/2)*ArcTanh[(Sqrt[c]*S
qrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(12*c^(9/2)*d^(9/2)*(
(a*e + c*d*x)*(d + e*x))^(5/2))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1251, 1251, 1253, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1251}$$

$$\frac{7g\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{7/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1251}$$

$$\frac{7g\left(\frac{5g\int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{7/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1253$$

$$\frac{7g\left(\frac{5g\left(\frac{3(cdf-aeg)\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{7/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1253$$

$$\frac{7g\left(\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}\right.}{3cd}$$

$$\frac{2(d+ex)^{3/2}(f+gx)^{7/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1268$$

$$\frac{7g\left(\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2cd\sqrt{d+ex}}\right)}{cd}-\right.}{3cd}$$

$$\frac{2(d+ex)^{3/2}(f+gx)^{7/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow 66$$

$$\frac{7g\left(\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{2cd\sqrt{d+ex}}\right.}{cd}\right.}{3cd}$$

$$\frac{2(d+ex)^{3/2}(f+gx)^{7/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow 221$$

$$\frac{7g\left(\frac{5g\left(\frac{3(cdf-aeg)\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{4cd}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{2cd\sqrt{d+ex}}\right.}{cd}\right.}{3cd}$$

$$\frac{2(d+ex)^{3/2}(f+gx)^{7/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x)^(7/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^(7/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2)) + (7*g*((-2*Sqrt[d + e*x]*(f + g*x)^(5/2))/(c*d*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g*(((f + g*x)^(3/2)*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)
*((Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d
+ e*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g
]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sq
rt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(4*c*d)))/(c*d)))/(3*
c*d)
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1253

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. 2(289) = 578.

Time = 2.66 (sec) , antiderivative size = 1010, normalized size of antiderivative = 2.96

method	result	size
default	Expression too large to display	1010

input

```
int((e*x+d)^(5/2)*(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,
method=_RETURNVERBOSE)
```

output

```
1/24*(105*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d
*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*g^4*x^2-210*ln(1/2*(2*c*d*g*x+a*
e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^
3*d^3*e*f*g^3*x^2+105*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f)
)^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+210*ln(1/2*(2*c*
d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/
2))*a^3*c*d*e^3*g^4*x-420*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*
x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f*g^3*x+210*ln(1
/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*
d*g)^(1/2))*a*c^3*d^3*e*f^2*g^2*x+12*c^3*d^3*g^3*x^3*((c*d*x+a*e)*(g*x+f))
^(1/2)*(c*d*g)^(1/2)+105*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x
+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^4*e^4*g^4-210*ln(1/2*(2*c*d*g*x
+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a
^3*c*d*e^3*f*g^3+105*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))
^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-42*a*c^2*d^2*
e*g^3*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+78*c^3*d^3*f*g^2*x^2*(
(c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-280*(c*d*g)^(1/2)*((c*d*x+a*e)*(g
*x+f))^(1/2)*a^2*c*d*e^2*g^3*x+476*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/
2)*a*c^2*d^2*e*f*g^2*x-160*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*c^3*d
^3*f^2*g*x-210*(c*d*g)^(1/2)*((c*d*x+a*e)*(g*x+f))^(1/2)*a^3*e^3*g^3+35...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(289) = 578.

Time = 0.97 (sec) , antiderivative size = 1413, normalized size of antiderivative = 4.14

$$\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
[1/48*(4*(6*c^3*d^3*g^3*x^3 - 8*c^3*d^3*f^3 - 56*a*c^2*d^2*e*f^2*g + 175*a
^2*c*d*e^2*f*g^2 - 105*a^3*e^3*g^3 + 3*(13*c^3*d^3*f*g^2 - 7*a*c^2*d^2*e*g
^3)*x^2 - 2*(40*c^3*d^3*f^2*g - 119*a*c^2*d^2*e*f*g^2 + 70*a^2*c*d*e^2*g^3
)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x +
f) + 105*(a^2*c^2*d^3*e^2*f^2*g - 2*a^3*c*d^2*e^3*f*g^2 + a^4*d*e^4*g^3 +
(c^4*d^4*e*f^2*g - 2*a*c^3*d^3*e^2*f*g^2 + a^2*c^2*d^2*e^3*g^3)*x^3 + ((c^
4*d^5 + 2*a*c^3*d^3*e^2)*f^2*g - 2*(a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3)*f*g^2
 + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^3)*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*
d^2*e^3)*f^2*g - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g^2 + (2*a^3*c*d^2*
e^3 + a^4*e^5)*g^3)*x)*sqrt(g/(c*d))*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f
^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d
*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (
c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*
g^2)*x)/(e*x + d)))/(c^6*d^6*e*x^3 + a^2*c^4*d^5*e^2 + (c^6*d^7 + 2*a*c^5*
d^5*e^2)*x^2 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*x), 1/24*(2*(6*c^3*d^3*g^
3*x^3 - 8*c^3*d^3*f^3 - 56*a*c^2*d^2*e*f^2*g + 175*a^2*c*d*e^2*f*g^2 - 105
*a^3*e^3*g^3 + 3*(13*c^3*d^3*f*g^2 - 7*a*c^2*d^2*e*g^3)*x^2 - 2*(40*c^3*d^
3*f^2*g - 119*a*c^2*d^2*e*f*g^2 + 70*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 105*(a^2*c^2*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{7}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)*(g*x + f)^(7/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(289) = 578.

Time = 0.77 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.00

$$\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\frac{\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex}}{}$$

$$-\frac{35\,(c^2d^2f^2g^3-2\,acdefg^4+a^2e^2g^5)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2}\right.\right.}{4\sqrt{cdg}c^4d^4|g|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output

```
1/12*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g
)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*(3*
(e^2*f + (e*x + d)*e*g - d*e*g)*(2*(c^7*d^7*e^2*f*g^5 - a*c^6*d^6*e^3*g^6)
*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^8*d^8*e^6*f*g*abs(g) - a*c^7*d^7*e^7*g
^2*abs(g)) + 7*(c^7*d^7*e^4*f^2*g^5 - 2*a*c^6*d^6*e^5*f*g^6 + a^2*c^5*d^5*
e^6*g^7)/(c^8*d^8*e^6*f*g*abs(g) - a*c^7*d^7*e^7*g^2*abs(g))) - 140*(c^7*d
^7*e^6*f^3*g^5 - 3*a*c^6*d^6*e^7*f^2*g^6 + 3*a^2*c^5*d^5*e^8*f*g^7 - a^3*c
^4*d^4*e^9*g^8)/(c^8*d^8*e^6*f*g*abs(g) - a*c^7*d^7*e^7*g^2*abs(g))) + 105
*(c^7*d^7*e^8*f^4*g^5 - 4*a*c^6*d^6*e^9*f^3*g^6 + 6*a^2*c^5*d^5*e^10*f^2*g
^7 - 4*a^3*c^4*d^4*e^11*f*g^8 + a^4*c^3*d^3*e^12*g^9)/(c^8*d^8*e^6*f*g*abs
(g) - a*c^7*d^7*e^7*g^2*abs(g)))/(c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)^2 - 35/4*(c^2*d^2*f^2*g^3 - 2*a*c*d*e*f*g^4 + a^2
*e^2*g^5)*log(abs(-sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(
c*d*g)*c^4*d^4*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(f+g\,x)^{7/2}\,(d+e\,x)^{5/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e)^{5/2}}\,dx$$

input

```
int(((f + g*x)^(7/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2),x)
```

output

```
int(((f + g*x)^(7/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.38

$$\int \frac{(d+ex)^{5/2}(f+gx)^{7/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^(5/2)*(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(840*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**3*e**3*g**3 -
 1680*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*c*d*e**2*f
*g**2 + 840*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*
e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*c*d*
e**2*g**3*x + 840*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*s
qrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*c
**2*d**2*e*f**2*g - 1680*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sq
rt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*
f))*a*c**2*d**2*e*f*g**2*x + 840*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)
*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*
g - c*d*f))*c**3*d**3*f**2*g*x + 175*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*
d*x)*a**3*e**3*g**3 - 350*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c
*d*e**2*f*g**2 + 175*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d*e*
*2*g**3*x + 175*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**2*e*f*
*2*g - 350*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**2*e*f*g**2*
x + 175*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**3*f**2*g*x - 840
*sqrt(f + g*x)*a**3*c*d*e**3*g**3 + 1400*sqrt(f + g*x)*a**2*c**2*d**2*e**2
*f*g**2 - 1120*sqrt(f + g*x)*a**2*c**2*d**2*e**2*g**3*x - 448*sqrt(f + ...
```

3.99 $\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 265

$$\begin{aligned}
&\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \\
&-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&+\frac{5g^2\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3\sqrt{d+ex}} \\
&+\frac{5g^{3/2}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{c^{7/2}d^{7/2}}
\end{aligned}$$

output

```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(5/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)-10/3*g*(e*x+d)^(1/2)*(g*x+f)^(3/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)+5*g^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^
3/d^3/(e*x+d)^(1/2)+5*g^(3/2)*(-a*e*g+c*d*f)*arctanh(c^(1/2)*d^(1/2)*(e*x+
d)^(1/2)*(g*x+f)^(1/2)/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^
(7/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{(d+ex)^{5/2}\left(\sqrt{c}\sqrt{d}(ae+cdx)\sqrt{f+gx}(15a^2e^2g^2-10acdeg(f-\right.}{3}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(5/2),x]
```

output
```
((d + e*x)^(5/2)*(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)*Sqrt[f + g*x]*(15*a^2*e^2*
g^2 - 10*a*c*d*e*g*(f - 2*g*x) + c^2*d^2*(-2*f^2 - 14*f*g*x + 3*g^2*x^2))
+ 15*g^(3/2)*(c*d*f - a*e*g)*(a*e + c*d*x)^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*
Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*c^(7/2)*d^(7/2)*((a*e + c
*d*x)*(d + e*x))^(5/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1251, 1251, 1253, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1251}$$

$$\frac{5g\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1251}$$

$$\frac{5g\left(\frac{3g\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{5g\left(\frac{3g\left(\frac{(cdf-aeg)\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1268}$$

$$\frac{5g\left(\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{5g\left(\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\int \frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{5g\left(\frac{3g\left(\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}\right)}{cd}-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2),x]
```

output

```
(-2*(d + e*x)^(3/2)*(f + g*x)^(5/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2)) + (5*g*((-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*g*((Sqrt[f + g*x]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x]) + ((c*d*f - a*e*g)*Sqrt[
a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sq
rt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])))/(c*d)))/(3*c*d)
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(225) = 450$.

Time = 2.76 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.42

method	result
default	$-\frac{\left(15\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)a\,c^{2}d^{2}e\,g^{3}x^{2}-15\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^{3}d^{3}f\,g^{2}x^{2}+30\right.}{}$

input
```
int((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,
method=_RETURNVERBOSE)
```

output

```
-1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*ln(1/2*(2*c*d*g*x+a*e*g+d*
f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^3*d^3*f*
g^2*x^2+30*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*
d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+d*
f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a*c^2*d^2*
e*f*g^2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(
c*d*g)^(1/2))/(c*d*g)^(1/2))*a^3*e^3*g^3-15*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+
2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*c*d*e^2*f*
g^2-6*c^2*d^2*g^2*x^2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)-40*((c*d*x
+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*a*c*d*e*g^2*x+28*((c*d*x+a*e)*(g*x+f))^
(1/2)*(c*d*g)^(1/2)*c^2*d^2*f*g*x-30*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(
1/2)*a^2*e^2*g^2+20*a*c*d*e*f*g*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)+
4*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c^2*d^2*f^2)*((e*x+d)*(c*d*x+a
*e))^(1/2)*(g*x+f)^(1/2)/((c*d*x+a*e)*(g*x+f))^(1/2)/(c*d*g)^(1/2)/(c*d*x+
a*e)^2/c^3/d^3/(e*x+d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(225) = 450$.

Time = 0.69 (sec) , antiderivative size = 1055, normalized size of antiderivative = 3.98

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output

```
[1/12*(4*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a^2*e^2*
g^2 - 2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g - a^3*d*
e^3*g^2 + (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a*c^2*d^
2*e^2)*f*g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3*e + a^
2*c*d*e^3)*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(g/(c*d))*log(-(8
*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^
2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 - 4*(2*c^2*d^2*g*x + c^2*d^2*
f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*s
qrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*
f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^5*d^5*e*x^3 + a^2*c^3
*d^4*e^2 + (c^5*d^6 + 2*a*c^4*d^4*e^2)*x^2 + (2*a*c^4*d^5*e + a^2*c^3*d^3*
e^3)*x), 1/6*(2*(3*c^2*d^2*g^2*x^2 - 2*c^2*d^2*f^2 - 10*a*c*d*e*f*g + 15*a
^2*e^2*g^2 - 2*(7*c^2*d^2*f*g - 10*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) - 15*(a^2*c*d^2*e^2*f*g -
 a^3*d*e^3*g^2 + (c^3*d^3*e*f*g - a*c^2*d^2*e^2*g^2)*x^3 + ((c^3*d^4 + 2*a
*c^2*d^2*e^2)*f*g - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g^2)*x^2 + ((2*a*c^2*d^3
*e + a^2*c*d*e^3)*f*g - (2*a^2*c*d^2*e^2 + a^3*e^4)*g^2)*x)*sqrt(-g/(c*d))
*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g
*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate((e*x+d)**(5/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(225) = 450$.

Time $= 0.49$ (sec) , antiderivative size $= 517$, normalized size of antiderivative $= 1.95$

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex}{}$$

$$-\frac{5\left(cdfg^3-aeg^4\right)\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-de}\right.\right.}{\sqrt{cdg}c^3d^3|g|}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output
```
1/3*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)
*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*((e^2*f + (e*x + d)*e*g - d*e*g)*(3*(
c^5*d^5*e^2*f*g^5 - a*c^4*d^4*e^3*g^6)*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^
6*d^6*e^4*f*g*abs(g) - a*c^5*d^5*e^5*g^2*abs(g)) - 20*(c^5*d^5*e^4*f^2*g^5
 - 2*a*c^4*d^4*e^5*f*g^6 + a^2*c^3*d^3*e^6*g^7)/(c^6*d^6*e^4*f*g*abs(g) -
a*c^5*d^5*e^5*g^2*abs(g))) + 15*(c^5*d^5*e^6*f^3*g^5 - 3*a*c^4*d^4*e^7*f^2
*g^6 + 3*a^2*c^3*d^3*e^8*f*g^7 - a^3*c^2*d^2*e^9*g^8)/(c^6*d^6*e^4*f*g*abs
(g) - a*c^5*d^5*e^5*g^2*abs(g)))/(c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)^2 - 5*(c*d*f*g^3 - a*e*g^4)*log(abs(-sqrt(e^2*f +
 (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2
*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^3*d^3*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(f+g\,x)^{5/2}\,(d+e\,x)^{5/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e)^{5/2}}\,dx$$

input
```
int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2),x)
```

output
```
int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\frac{-30\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,\log\left(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{gx+f}}{\sqrt{aeg-cdf}}\right)a^2e^2g^2+}{}$$

input
```
int((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
( - 30*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a**2*e**2*g**2
 + 30*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*c*d*e*f*g - 3
0*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*c*d*e*g**2*x + 30
*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*d*x)
+ sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c**2*d**2*f*g*x - 5*
sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**2*g**2 + 5*sqrt(g)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d*e*f*g - 5*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*a*c*d*e*g**2*x + 5*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*
c**2*d**2*f*g*x + 30*sqrt(f + g*x)*a**2*c*d*e**2*g**2 - 20*sqrt(f + g*x)*a
*c**2*d**2*e*f*g + 40*sqrt(f + g*x)*a*c**2*d**2*e*g**2*x - 4*sqrt(f + g*x)
*c**3*d**3*f**2 - 28*sqrt(f + g*x)*c**3*d**3*f*g*x + 6*sqrt(f + g*x)*c**3*
d**3*g**2*x**2)/(6*sqrt(a*e + c*d*x)*c**4*d**4*(a*e + c*d*x))
```

3.100 $\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 195

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2g^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{5/2}d^{5/2}}$$

output

```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)-2*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)+2*g^(3/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/g^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=$$

$$-\frac{2(d+ex)^{3/2}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}(3aeg+cd(f+4gx))-3g^{3/2}(ae+cdx)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{3c^{5/2}d^{5/2}((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(3*a*e*g + c*d*(f + 4*g
*x)) - 3*g^(3/2)*(a*e + c*d*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*c^(5/2)*d^(5/2)*((a*e + c*d*x)*(d + e
*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {1251, 1251, 1268, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \textcolor{blue}{1251}$$

$$\frac{g\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1251}$$

$$\frac{g\left(\frac{g\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1268}$$

$$\frac{g\left(\frac{g\sqrt{d+ex}\sqrt{ae+cdx}\int\frac{1}{\sqrt{ae+cdx}\sqrt{f+gx}}dx}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 66$$

$$\frac{g\left(\frac{2g\sqrt{d+ex}\sqrt{ae+cdx}\int\frac{1}{cd-\frac{g(ae+cdx)}{f+gx}}d\frac{\sqrt{ae+cdx}}{\sqrt{f+gx}}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{g\left(\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{cd}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2),x]
```

output

```
(-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2)) + (g*((-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*
x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(
c^(3/2)*d^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(c*d)
```

Defintions of rubi rules used

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1251
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[e*g*(n/(c*(p + 1)))   Int[(
d + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p,
 0] && LtQ[p, -1] && GtQ[n, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(163) = 326$.

Time = 2.67 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.71

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\,\left(3\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{cdg}}{2\sqrt{cdg}}\right)c^{2}d^{2}g^{2}x^{2}+6\ln\left(\frac{2cdgx+aeg+dfc+2\sqrt{(cdx+ae)(gx+f)}\,\sqrt{c}}{2\sqrt{cdg}}\right.\right.}{3\sqrt{cdg}}$

input
```
int((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,
method=_RETURNVERBOSE)
```

output

```
1/3*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+d
*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*c^2*d^2*g
^2*x^2+6*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*
g)^(1/2))/(c*d*g)^(1/2))*a*c*d*e*g^2*x+3*ln(1/2*(2*c*d*g*x+a*e*g+d*f*c+2*(
(c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2))/(c*d*g)^(1/2))*a^2*e^2*g^2-8*((c
*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*g*x-6*((c*d*x+a*e)*(g*x+f))^(1/
2)*(c*d*g)^(1/2)*a*e*g-2*((c*d*x+a*e)*(g*x+f))^(1/2)*(c*d*g)^(1/2)*c*d*f)/
(c*d*g)^(1/2)/(c*d*x+a*e)^2/((c*d*x+a*e)*(g*x+f))^(1/2)/d^2/c^2/(e*x+d)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.87

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(4\,cdgx+cdf+3\,aeg)\sqrt{ex+}}{}\right.$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
[-1/6*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*d*g*x + c*d*f +
3*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) - 3*(c^2*d^2*e*g*x^3 + a^2*d*e^2*g +
(c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*x)*sqrt(g/(c*d))
*log(-(8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2
 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + 4*(2*c^2*d^2*g*x +
c^2*d^2*f + a*c*d*e*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*
x + d)*sqrt(g*x + f)*sqrt(g/(c*d)) + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c
*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)/(e*x + d)))/(c^4*d^4*e*x^3 +
 a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*
c^2*d^2*e^3)*x), -1/3*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(4*c*
d*g*x + c*d*f + 3*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^2*e*g*x^3
+ a^2*d*e^2*g + (c^2*d^3 + 2*a*c*d*e^2)*g*x^2 + (2*a*c*d^2*e + a^2*e^3)*g*
x)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(e*x + d)*sqrt(g*x + f)*c*d*sqrt(-g/(c*d))/(2*c*d*e*g*x^2 + c*d^2*f + a*d
*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e
^2 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output

```
integrate((e*x + d)^(5/2)*(g*x + f)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(163) = 326$.

Time = 0.34 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=$$

$$-\frac{2\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\sqrt{e^2f+(ex+d)eg-deg}\Big(\frac{4\left(c^3d^3e^2fg^4-ac^2d^2e^3g^5\right)(e^2f+}{c^4d^4e^2f|g|-ac^3d^3e}}{3\left(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg\right)^2}$$

$$-\frac{2\,g^3\log\left(\left|-\sqrt{e^2f+(ex+d)eg-deg}\sqrt{cdg}+\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}\right|\right)}{\sqrt{cdg}c^2d^2|g|}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output

```
-2/3*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g
)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(4*(c^3*d^3*e^2*f*g^4 - a*c^2*d^2*e^
3*g^5)*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^4*d^4*e^2*f*abs(g) - a*c^3*d^3*e
^3*g*abs(g)) - 3*(c^3*d^3*e^4*f^2*g^4 - 2*a*c^2*d^2*e^5*f*g^5 + a^2*c*d*e^
6*g^6)/(c^4*d^4*e^2*f*abs(g) - a*c^3*d^3*e^3*g*abs(g)))/(c*d*e^2*f*g - a*e
^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)^2 - 2*g^3*log(abs(-sqrt(e^
2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 +
 (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)))/(sqrt(c*d*g)*c^2*d^2*abs(g))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\int \frac{{\left(f+g\,x\right)}^{3/2}\,{\left(d+e\,x\right)}^{5/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

input
```
int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2),x)
```

output
```
int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\frac{2\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{g}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{gx+f}}{\sqrt{aeg-cdf}}\right)aeg+2\sqrt{g}\,\sqrt{}}{}$$

input
```
int((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(3*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*a*e*g + 3*sqrt(
g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(g)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(f + g*x))/sqrt(a*e*g - c*d*f))*c*d*g*x - 3*sqrt(f + g*x)*
a*c*d*e*g - sqrt(f + g*x)*c**2*d**2*f - 4*sqrt(f + g*x)*c**2*d**2*g*x))/(3
*sqrt(a*e + c*d*x)*c**3*d**3*(a*e + c*d*x))
```

3.101 $\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 63

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

output
```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*((a*e + c*d*x)*(d
+ e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = $\{1248\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1248$$

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

input

```
Int[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2),x]
```

output

```
(-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2))
```

Defintions of rubi rules used

rule 1248

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2(gx+f)^{\frac{3}{2}}\sqrt{(ex+d)(cdx+ae)}}{3\sqrt{ex+d}\,(cdx+ae)^{2}(aeg-dfc)}$	55
gosper	$\frac{2(cdx+ae)(gx+f)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}{3(aeg-dfc)(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	63
orering	$\frac{2(cdx+ae)(gx+f)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}{3(aeg-dfc)(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{5}{2}}}$	64

input
```
int((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
2/3/(e*x+d)^(1/2)*(g*x+f)^(3/2)*((e*x+d)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)^2/
(a*e*g-c*d*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(55) = 110.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}{3\left(a^2cd^2e^2f-a^3de^3g+(c^3d^3ef-ac^2d^2e^2g)x^3+((c^3d^4+2\,ac^2d^2e^2)f-(ac^2d^3e+2\,a^2cde^3)g)x^2+((2\,a\right.}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x + f)^(
3/2)/(a^2*c*d^2*e^2*f - a^3*d*e^3*g + (c^3*d^3*e*f - a*c^2*d^2*e^2*g)*x^3
+ ((c^3*d^4 + 2*a*c^2*d^2*e^2)*f - (a*c^2*d^3*e + 2*a^2*c*d*e^3)*g)*x^2 +
((2*a*c^2*d^3*e + a^2*c*d*e^3)*f - (2*a^2*c*d^2*e^2 + a^3*e^4)*g)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}\sqrt{gx+f}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)*sqrt(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(55) = 110$.

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.40

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=$$
$$-\frac{2\sqrt{-cde^2fg+ae^3g^2+(e^2f+(ex+d)eg-deg)cdg}(e^2f+(ex+d)eg-deg)^{\frac{3}{2}}cde^2g^4|e|}{3\,(c^2d^2e^2f|e||g|-acde^3g|e||g|)(cde^2fg-ae^3g^2-(e^2f+(ex+d)eg-deg)cdg)^2}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output

```
-2/3*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g
)*(e^2*f + (e*x + d)*e*g - d*e*g)^(3/2)*c*d*e^2*g^4*abs(e)/((c^2*d^2*e^2*f
*abs(e)*abs(g) - a*c*d*e^3*g*abs(e)*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 - (e^
2*f + (e*x + d)*e*g - d*e*g)*c*d*g)^2)
```

Mupad [B] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.68

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{\left(\frac{2\,f\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,(a\,e\,g-c\,d\,f)}+\frac{2\,g\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,(a\,e\,g-c\,d\,f)}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}}$$

input

```
int(((f + g*x)^(1/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^(5/2),x)
```

output

```
(((2*f*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)) + (2
*g*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)))*(x*(a
*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a
*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(2*a*e^2 + c*d^2))/(c*d*e))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{\frac{2\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,aeg}{3}+\frac{2\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,cdgx}{3}+\frac{2\sqrt{gx+f}\,c^2d^2f}{3}+2\sqrt{gx+}}{\sqrt{cdx+ae}\,c^2d^2\left(acdegx-c^2d^2fx+a^2e^2g-acdef\right)}$$

input

```
int((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(2*(sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e*g + sqrt(g)*sqrt(d)*sqrt
(c)*sqrt(a*e + c*d*x)*c*d*g*x + sqrt(f + g*x)*c**2*d**2*f + sqrt(f + g*x)*
c**2*d**2*g*x))/(3*sqrt(a*e + c*d*x)*c**2*d**2*(a**2*e**2*g - a*c*d*e*f +
a*c*d*e*g*x - c**2*d**2*f*x))
```

3.102 $\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 128

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output

```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)+4/3*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)^2/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\sqrt{f+gx}(-3aeg+cd(f-2gx))}{3(cdf-aeg)^2((ae+cdx)(d+ex))^{3/2}}$$

input

```
Integrate[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2)),x]
```

output

```
(-2*(d + e*x)^(3/2)*Sqrt[f + g*x]*(-3*a*e*g + c*d*(f - 2*g*x)))/(3*(c*d*f
- a*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1252, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(x\,(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{2g\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}\,(cdf-aeg)}$$

$$\Big\downarrow \text{1248}$$

$$\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}\,(cdf-aeg)}$$

input

```
Int[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2)),x]
```

output

```
(-2*(d + e*x)^(3/2)*Sqrt[f + g*x])/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(3*(c*d*f -
 a*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1248

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{2\sqrt{gx+f}\,\sqrt{(ex+d)(cdx+ae)}\,(2cdgx+3aeg-dfc)}{3\sqrt{ex+d}\,(cdx+ae)^{2}(aeg-dfc)^{2}}$	72
gosper	$\frac{2(cdx+ae)\sqrt{gx+f}\,(2cdgx+3aeg-dfc)(ex+d)^{\frac{5}{2}}}{3(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2})(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	99
orering	$\frac{2(2cdgx+3aeg-dfc)(cdx+ae)\sqrt{gx+f}\,(ex+d)^{\frac{5}{2}}}{3(a^{2}e^{2}g^{2}-2acdefg+f^{2}c^{2}d^{2})(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{5}{2}}}$	100

input
```
int((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
2/3/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(2*c*d*g*x+3*a
*e*g-c*d*f)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(112) = 224$.

Time = 0.15 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.48

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{}{3\,(a^2c^2d^3e^2f^2-2\,a^3cd^2e^3fg+a^4de^4g^2+(c^4d^4ef^2-2\,c}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x - c*d*f + 3*a*e
*g)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*c*d^2*e^3*f*g
 + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*c^2*d^2*e^3*
g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e + 2*a^2*c^2*d
^2*e^3)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*g^2)*x^2 + ((2*a*c^3*d^4*e
 + a^2*c^2*d^2*e^3)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f*g + (2*a^3
*c*d^2*e^3 + a^4*e^5)*g^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*s
qrt(g*x + f)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(112) = 224.

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\frac{2\left(\frac{2\,(e^2f+(ex+d)eg-deg)c^2d^2g^4}{c^3d^3e^2f^2|g|-2\,ac^2d^2e^3fg|g|+a^2cde^4g^2|g|}-\frac{3\,(c^2d^2e^2fg^4-a}{c^3d^3e^2f^2|g|-2\,ac^2d^2e^3f}\right.}{3\,(cde^2}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output
```
2/3*(2*(e^2*f + (e*x + d)*e*g - d*e*g)*c^2*d^2*g^4/(c^3*d^3*e^2*f^2*abs(g)
 - 2*a*c^2*d^2*e^3*f*g*abs(g) + a^2*c*d*e^4*g^2*abs(g)) - 3*(c^2*d^2*e^2*f
*g^4 - a*c*d*e^3*g^5)/(c^3*d^3*e^2*f^2*abs(g) - 2*a*c^2*d^2*e^3*f*g*abs(g)
 + a^2*c*d*e^4*g^2*abs(g)))*sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x
+ d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*e^2/(c*d*e^2*
f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)^2
```

Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\Bigg(\frac{4\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c\,d\,e\,(a\,e\,g-c\,d\,f)^2}-}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,(c\,d^2}{c\,d\,e}}$$

input
```
int((d + e*x)^(5/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2)),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^2*x^2*(d + e*x)^(1/2)
)/(3*c*d*e*(a*e*g - c*d*f)^2) - ((2*c*d*f^2 - 6*a*e*f*g)*(d + e*x)^(1/2))/
(3*c^2*d^2*e*(a*e*g - c*d*f)^2) + (x*(6*a*e*g^2 + 2*c*d*f*g)*(d + e*x)^(1/
2))/(3*c^2*d^2*e*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2) + (a^2*e*(f + g
*x)^(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(c*d*e) + (a*
x*(f + g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{-\frac{4\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,aeg}{3}-\frac{4\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,cdgx}{3}+2\sqrt{gx+}}{\sqrt{cdx+ae}\,cd\,(a^2cd\,e^2g^2x-2a\,c^2d^2efgx+c^3d^3f^2x+}$$

input
```
int((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*( - 2*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e*g - 2*sqrt(g)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*g*x + 3*sqrt(f + g*x)*a*c*d*e*g - sqrt(f
+ g*x)*c**2*d**2*f + 2*sqrt(f + g*x)*c**2*d**2*g*x))/(3*sqrt(a*e + c*d*x)*
c*d*(a**3*e**3*g**2 - 2*a**2*c*d*e**2*f*g + a**2*c*d*e**2*g**2*x + a*c**2*
d**2*e*f**2 - 2*a*c**2*d**2*e*f*g*x + c**3*d**3*f**2*x))
```

3.103 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 193

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}}{(cdf-aeg)\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}$$
$$-\frac{8cd(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$
$$+\frac{16cdg\sqrt{d+ex}\sqrt{f+gx}}{3(cdf-aeg)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)-8/3*c*d*(e*x+d)^(3/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)^2/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+16/3*c*d*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e
*g+c*d*f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2(d+ex)^{3/2}\left(3a^2e^2g^2+6acdeg(f+2gx)+c^2d^2(-f\right.}{3(cdf-aeg)^3((ae+cdx)(d+ex))^{3/2}\sqrt{\,}}$$

input
```
Integrate[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2)),x]
```

output
```
(2*(d + e*x)^(3/2)*(3*a^2*e^2*g^2 + 6*a*c*d*e*g*(f + 2*g*x) + c^2*d^2*(-f^
2 + 4*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*((a*e + c*d*x)*(d + e*x))^
(3/2)*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1252, 1252, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{4g\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1252}$$

$$-\frac{4g\left(-\frac{2g\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}-\frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \textcolor{blue}{1248}$$

$$-\frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}-\frac{4g\left(-\frac{4g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{3(cdf-aeg)}$$

input
```
Int[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2)),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(3/2)) - (4*g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqr
t[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f
+ g*x])))/(3*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(8g^2x^2d^2c^2+12acde\,g^2x+4c^2d^2fgx+3a^2e^2g^2+6acdefg-f^2c^2d^2)}{3\sqrt{ex+d}\,\sqrt{gx+f}\,(cdx+ae)^2(aeg-dfc)^3}$	121
gosper	$-\frac{2(cdx+ae)(8g^2x^2d^2c^2+12acde\,g^2x+4c^2d^2fgx+3a^2e^2g^2+6acdefg-f^2c^2d^2)(ex+d)^{\frac{5}{2}}}{3\sqrt{gx+f}\,(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3d^3c^3)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	169
orering	$-\frac{2(8g^2x^2d^2c^2+12acde\,g^2x+4c^2d^2fgx+3a^2e^2g^2+6acdefg-f^2c^2d^2)(cdx+ae)(ex+d)^{\frac{5}{2}}}{3(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3d^3c^3)\sqrt{gx+f}\,(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{5}{2}}}$	170

input

```
int((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,
method=_RETURNVERBOSE)
```

output

```
-2/3/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(8*c^2*d^2*g^
2*x^2+12*a*c*d*e*g^2*x+4*c^2*d^2*f*g*x+3*a^2*e^2*g^2+6*a*c*d*e*f*g-c^2*d^2
*f^2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(171) = 342$.

Time = 0.30 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.46

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \frac{}{3\,(a^2c^3d^4e^2f^4-3\,a^3c^2d^3e^3f^3g+3\,a^4cd^2e^4f^2g^2-a^5d}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output
```
2/3*(8*c^2*d^2*g^2*x^2 - c^2*d^2*f^2 + 6*a*c*d*e*f*g + 3*a^2*e^2*g^2 + 4*(
c^2*d^2*f*g + 3*a*c*d*e*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^3*d^4*e^2*f^4 - 3*a^3*c^2*d^3*e^3*f^3
*g + 3*a^4*c*d^2*e^4*f^2*g^2 - a^5*d*e^5*f*g^3 + (c^5*d^5*e*f^3*g - 3*a*c^
4*d^4*e^2*f^2*g^2 + 3*a^2*c^3*d^3*e^3*f*g^3 - a^3*c^2*d^2*e^4*g^4)*x^4 + (
c^5*d^5*e*f^4 + (c^5*d^6 - a*c^4*d^4*e^2)*f^3*g - 3*(a*c^4*d^5*e + a^2*c^3
*d^3*e^3)*f^2*g^2 + (3*a^2*c^3*d^4*e^2 + 5*a^3*c^2*d^2*e^4)*f*g^3 - (a^3*c
^2*d^3*e^3 + 2*a^4*c*d*e^5)*g^4)*x^3 + ((c^5*d^6 + 2*a*c^4*d^4*e^2)*f^4 -
(a*c^4*d^5*e + 5*a^2*c^3*d^3*e^3)*f^3*g - 3*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2
*e^4)*f^2*g^2 + (5*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f*g^3 - (2*a^4*c*d^2*e^4
 + a^5*e^6)*g^4)*x^2 - (a^5*d*e^5*g^4 - (2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*
f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2*d^3*e^3 +
 a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(171) = 342$.

Time = 0.27 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.28

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2}{3}\left(\frac{}{(c^2d^2ef^2|g|-2\,acde^2fg|g|+a^2e^3g^2|g|)\left(cde^2fg\right.}\right.$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output

```
2/3*(6*sqrt(c*d*g)*g^3/((c^2*d^2*e*f^2*abs(g) - 2*a*c*d*e^2*f*g*abs(g) + a
^2*e^3*g^2*abs(g))*(c*d*e^2*f*g - a*e^3*g^2 + (sqrt(e^2*f + (e*x + d)*e*g
- d*e*g)*sqrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*
e*g - d*e*g)*c*d*g))^2)) + sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x +
 d)*e*g - d*e*g)*c*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(5*(c^5*d^5*e*
f^2*g^4*abs(g) - 2*a*c^4*d^4*e^2*f*g^5*abs(g) + a^2*c^3*d^3*e^3*g^6*abs(g)
)*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^6*d^6*e^4*f^5*g^2 - 5*a*c^5*d^5*e^5*f
^4*g^3 + 10*a^2*c^4*d^4*e^6*f^3*g^4 - 10*a^3*c^3*d^3*e^7*f^2*g^5 + 5*a^4*c
^2*d^2*e^8*f*g^6 - a^5*c*d*e^9*g^7) - 6*(c^5*d^5*e^3*f^3*g^4*abs(g) - 3*a*
c^4*d^4*e^4*f^2*g^5*abs(g) + 3*a^2*c^3*d^3*e^5*f*g^6*abs(g) - a^3*c^2*d^2*
e^6*g^7*abs(g))/(c^6*d^6*e^4*f^5*g^2 - 5*a*c^5*d^5*e^5*f^4*g^3 + 10*a^2*c^
4*d^4*e^6*f^3*g^4 - 10*a^3*c^3*d^3*e^7*f^2*g^5 + 5*a^4*c^2*d^2*e^8*f*g^6 -
 a^5*c*d*e^9*g^7))/(c*d*e^2*f*g - a*e^3*g^2 - (e^2*f + (e*x + d)*e*g - d*e
*g)*c*d*g)^2)*e^3
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx =$$

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g-2\,c^2\,d^2\,f^2\right)}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{8\,g\,x\,\left(3\,a\,e\,g+c\,d\,f\right)\,\sqrt{d+}}{3\,c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right.}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}}$$

input

```
int((d + e*x)^(5/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2)),x)
```

output

```
-((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g^2*x^2*(d + e*x)^(1/
2))/(3*e*(a*e*g - c*d*f)^3) + ((d + e*x)^(1/2)*(6*a^2*e^2*g^2 - 2*c^2*d^2*
f^2 + 12*a*c*d*e*f*g))/(3*c^2*d^2*e*(a*e*g - c*d*f)^3) + (8*g*x*(3*a*e*g +
 c*d*f)*(d + e*x)^(1/2))/(3*c*d*e*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2
) + (a^2*e*(f + g*x)^(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^
2))/(c*d*e) + (a*x*(f + g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.02

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{\frac{16\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,aefg}{3}+\frac{16\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,ae\,g^2x}{3}+\frac{16\sqrt{g}}{}}{\sqrt{cdx+ae}\,\left(a^3cd\,e^3g^4x^2-3a^2c^2d^2e^2f\,g^3x\right.}$$

input
```
int((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(8*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e*f*g + 8*sqrt(g)*sqrt(d
)*sqrt(c)*sqrt(a*e + c*d*x)*a*e*g**2*x + 8*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*
e + c*d*x)*c*d*f*g*x + 8*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*g**
2*x**2 - 3*sqrt(f + g*x)*a**2*e**2*g**2 - 6*sqrt(f + g*x)*a*c*d*e*f*g - 12
*sqrt(f + g*x)*a*c*d*e*g**2*x + sqrt(f + g*x)*c**2*d**2*f**2 - 4*sqrt(f +
g*x)*c**2*d**2*f*g*x - 8*sqrt(f + g*x)*c**2*d**2*g**2*x**2))/(3*sqrt(a*e +
 c*d*x)*(a**4*e**4*f*g**3 + a**4*e**4*g**4*x - 3*a**3*c*d*e**3*f**2*g**2 -
 2*a**3*c*d*e**3*f*g**3*x + a**3*c*d*e**3*g**4*x**2 + 3*a**2*c**2*d**2*e**
2*f**3*g - 3*a**2*c**2*d**2*e**2*f*g**3*x**2 - a*c**3*d**3*e*f**4 + 2*a*c*
*3*d**3*e*f**3*g*x + 3*a*c**3*d**3*e*f**2*g**2*x**2 - c**4*d**4*f**4*x - c
**4*d**4*f**3*g*x**2))
```

3.104 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 260

$$\begin{aligned}
&\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \\
&-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&+\frac{4g\sqrt{d+ex}}{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&+\frac{16g^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^3\sqrt{d+ex}(f+gx)^{3/2}}+\frac{32cdg^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3(cdf-aeg)^4\sqrt{d+ex}\sqrt{f+gx}}
\end{aligned}$$

output

```
-2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)+4*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^(3/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+16/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(3/2)+32/3*c*d*g^2*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2(d+ex)^{3/2}\left(-a^3e^3g^3+3a^2cde^2g^2(3f+2gx)+3ac^2\right.}{3(cdf-aeg)^4(}$$

input

```
Integrate[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2)),x]
```

output

```
(2*(d + e*x)^(3/2)*(-(a^3*e^3*g^3) + 3*a^2*c*d*e^2*g^2*(3*f + 2*g*x) + 3*a
*c^2*d^2*e*g*(3*f^2 + 12*f*g*x + 8*g^2*x^2) + c^3*d^3*(-f^3 + 6*f^2*g*x +
24*f*g^2*x^2 + 16*g^3*x^3)))/(3*(c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x)
)^(3/2)*(f + g*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1252, 1252, 1254, 1248}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1252}$$

$$-\frac{2g\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1252}$$

$$-\frac{2g\left(-\frac{4g\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \textbf{1254}$$

$$-\frac{2g\left(-\frac{4g\left(\frac{2cd\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(cdf-aeg)}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \textbf{1248}$$

$$-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}-\frac{2g\left(-\frac{4g\left(\frac{4cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}\right)}{cdf-aeg}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}\right)}{cdf-aeg}$$

input
```
Int[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2)),x]
```

output
```
(-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*g*((-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(
f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (4*g*((2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3
*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])))/(c*d*f - a*e*g)))/(c*d*f
 - a*e*g)
```

Defintions of rubi rules used

rule 1248
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && EqQ[m + p, 0] && EqQ[m - n - 2, 0]
```

rule 1252
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Si
mp[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m
 - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, f, g, n}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[p,
-1] && RationalQ[n]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-16x^3g^3d^3c^3-24a\,c^2d^2e\,g^3x^2-24c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x-6c^3d^3f^2gx+a^3e^3g^3-9a^2cd}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}(cdx+ae)^2(aeg-dfc)^4}$
gosper	$-\frac{2(cdx+ae)(-16x^3g^3d^3c^3-24a\,c^2d^2e\,g^3x^2-24c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x-6c^3d^3f^2gx+a^3e^3g^3-9a^2cd\,e^2f\,g^2-9}{3(gx+f)^{\frac{3}{2}}(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4d^4c^4)(cd\,x^2e+a\,e^2x+c\,d^2x+ade}$
orering	$-\frac{2(-16x^3g^3d^3c^3-24a\,c^2d^2e\,g^3x^2-24c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x-6c^3d^3f^2gx+a^3e^3g^3-9a^2cd\,e^2f\,g^2-9a\,c^2d^2e\,f^2}{3(a^4e^4g^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4d^4c^4)(gx+f)^{\frac{3}{2}}(ade+(a\,e^2+c\,d^2)x+cd\,x^2}$

input
```
int((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,
method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-16*c^3*d^3*
g^3*x^3-24*a*c^2*d^2*e*g^3*x^2-24*c^3*d^3*f*g^2*x^2-6*a^2*c*d*e^2*g^3*x-36
*a*c^2*d^2*e*f*g^2*x-6*c^3*d^3*f^2*g*x+a^3*e^3*g^3-9*a^2*c*d*e^2*f*g^2-9*a
*c^2*d^2*e*f^2*g+c^3*d^3*f^3)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(230) = 460$.

Time = 0.83 (sec) , antiderivative size = 1065, normalized size of antiderivative = 4.10

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="fricas")
```

output

```
2/3*(16*c^3*d^3*g^3*x^3 - c^3*d^3*f^3 + 9*a*c^2*d^2*e*f^2*g + 9*a^2*c*d*e^
2*f*g^2 - a^3*e^3*g^3 + 24*(c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 6*(c^3*
d^3*f^2*g + 6*a*c^2*d^2*e*f*g^2 + a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^4*d^5*e^2*f^6 -
 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3
*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 +
6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)
*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5
*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e
^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d
^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^
4*c^2*d^3*e^4*f*g^5 - 3*(2*a*c^5*d^6*e + 3*a^2*c^4*d^4*e^3)*f^4*g^2 + 4*(a
^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3*d^4*e^3 - 9*a^4*c
^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^3 - (6*a^2*c^4*d^
4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 + 2*a*c^5*d^5*e^2
)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4*(4*a^3*c^3*d^4
*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*f^
2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*f^6 -
2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3*d^4*e^3 + 3
*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f^3*g^3 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input

```
integrate((e*x+d)**(5/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**
2)**(5/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input

```
integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="maxima")
```

output

```
integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(
g*x + f)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1119 vs. $2(230) = 460$.

Time = 0.52 (sec) , antiderivative size = 1119, normalized size of antiderivative = 4.30

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2),x, algorithm="giac")
```

output
```
2/3*e^4*(sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c
*d*g)*sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*(8*(c^7*d^7*e^2*f^3*g^4*abs(g) -
 3*a*c^6*d^6*e^3*f^2*g^5*abs(g) + 3*a^2*c^5*d^5*e^4*f*g^6*abs(g) - a^3*c^4
*d^4*e^5*g^7*abs(g))*(e^2*f + (e*x + d)*e*g - d*e*g)/(c^8*d^8*e^6*f^7*g^2
- 7*a*c^7*d^7*e^7*f^6*g^3 + 21*a^2*c^6*d^6*e^8*f^5*g^4 - 35*a^3*c^5*d^5*e^
9*f^4*g^5 + 35*a^4*c^4*d^4*e^10*f^3*g^6 - 21*a^5*c^3*d^3*e^11*f^2*g^7 + 7*
a^6*c^2*d^2*e^12*f*g^8 - a^7*c*d*e^13*g^9) - 9*(c^7*d^7*e^4*f^4*g^4*abs(g)
 - 4*a*c^6*d^6*e^5*f^3*g^5*abs(g) + 6*a^2*c^5*d^5*e^6*f^2*g^6*abs(g) - 4*a
^3*c^4*d^4*e^7*f*g^7*abs(g) + a^4*c^3*d^3*e^8*g^8*abs(g))/(c^8*d^8*e^6*f^7
*g^2 - 7*a*c^7*d^7*e^7*f^6*g^3 + 21*a^2*c^6*d^6*e^8*f^5*g^4 - 35*a^3*c^5*d
^5*e^9*f^4*g^5 + 35*a^4*c^4*d^4*e^10*f^3*g^6 - 21*a^5*c^3*d^3*e^11*f^2*g^7
 + 7*a^6*c^2*d^2*e^12*f*g^8 - a^7*c*d*e^13*g^9))/(c*d*e^2*f*g - a*e^3*g^2
- (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g)^2 + 4*(4*sqrt(c*d*g)*c^3*d^3*e^4*
f^2*g^5 - 8*sqrt(c*d*g)*a*c^2*d^2*e^5*f*g^6 + 4*sqrt(c*d*g)*a^2*c*d*e^6*g^
7 + 9*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d*g) - sqrt(
-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*g))^2*c^2*d
^2*e^2*f*g^4 - 9*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(c*d
*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d*e*g)*c*d*
g))^2*a*c*d*e^3*g^5 + 3*sqrt(c*d*g)*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*s
qrt(c*d*g) - sqrt(-c*d*e^2*f*g + a*e^3*g^2 + (e^2*f + (e*x + d)*e*g - d...
```

Mupad [B] (verification not implemented)

Time = 7.79 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\Bigg(\frac{16\,g\,x^2\,(a\,e\,g+c\,d\,f)\,\sqrt{d}}{e\,(a\,e\,g-c\,d\,f)^4}}{x^4\,\sqrt{f+g\,x}+\frac{x^2\,\sqrt{f+g\,x}\,(g\,a^2\,e^3+2\,g\,a\,c}{c^2\,d^2}}$$

input
```
int((d + e*x)^(5/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2)),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g*x^2*(a*e*g + c*d*f)*
(d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^4) - ((d + e*x)^(1/2)*(2*a^3*e^3*g^3 +
 2*c^3*d^3*f^3 - 18*a*c^2*d^2*e*f^2*g - 18*a^2*c*d*e^2*f*g^2))/(3*c^2*d^2*
e*g*(a*e*g - c*d*f)^4) + (32*c*d*g^2*x^3*(d + e*x)^(1/2))/(3*e*(a*e*g - c*
d*f)^4) + (4*x*(d + e*x)^(1/2)*(a^2*e^2*g^2 + c^2*d^2*f^2 + 6*a*c*d*e*f*g)
)/(c*d*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (x^2*(f + g*x)^(1/2)*
(a^2*e^3*g + c^2*d^3*f + 2*a*c*d*e^2*f + 2*a*c*d^2*e*g))/(c^2*d^2*e*g) + (
a*x*(f + g*x)^(1/2)*(a*e^2*f + 2*c*d^2*f + a*d*e*g))/(c^2*d^2*g) + (a^2*e*
f*(f + g*x)^(1/2))/(c^2*d*g) + (x^3*(f + g*x)^(1/2)*(2*a*e^2*g + c*d^2*g +
 c*d*e*f))/(c*d*e*g))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.92

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{-\frac{32\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,acde\,f^2g}{3}-\frac{64\sqrt{g}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,acdef\,g^2x}{3}}{\sqrt{cdx+ae}\,\left(a^4cd\,e^4g^6x^3-4a^3c^2d^2e^3f\,g^5x^3\right.}$$

input
```
int((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(2*( - 16*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d*e*f**2*g - 32*sq
rt(g)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d*e*f*g**2*x - 16*sqrt(g)*sqrt
(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d*e*g**3*x**2 - 16*sqrt(g)*sqrt(d)*sqrt(
c)*sqrt(a*e + c*d*x)*c**2*d**2*f**2*g*x - 32*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*c**2*d**2*f*g**2*x**2 - 16*sqrt(g)*sqrt(d)*sqrt(c)*sqrt(a*e +
 c*d*x)*c**2*d**2*g**3*x**3 - sqrt(f + g*x)*a**3*e**3*g**3 + 9*sqrt(f + g*
x)*a**2*c*d*e**2*f*g**2 + 6*sqrt(f + g*x)*a**2*c*d*e**2*g**3*x + 9*sqrt(f
+ g*x)*a*c**2*d**2*e*f**2*g + 36*sqrt(f + g*x)*a*c**2*d**2*e*f*g**2*x + 24
*sqrt(f + g*x)*a*c**2*d**2*e*g**3*x**2 - sqrt(f + g*x)*c**3*d**3*f**3 + 6*
sqrt(f + g*x)*c**3*d**3*f**2*g*x + 24*sqrt(f + g*x)*c**3*d**3*f*g**2*x**2
+ 16*sqrt(f + g*x)*c**3*d**3*g**3*x**3))/(3*sqrt(a*e + c*d*x)*(a**5*e**5*f
**2*g**4 + 2*a**5*e**5*f*g**5*x + a**5*e**5*g**6*x**2 - 4*a**4*c*d*e**4*f*
*3*g**3 - 7*a**4*c*d*e**4*f**2*g**4*x - 2*a**4*c*d*e**4*f*g**5*x**2 + a**4
*c*d*e**4*g**6*x**3 + 6*a**3*c**2*d**2*e**3*f**4*g**2 + 8*a**3*c**2*d**2*e
**3*f**3*g**3*x - 2*a**3*c**2*d**2*e**3*f**2*g**4*x**2 - 4*a**3*c**2*d**2*
e**3*f*g**5*x**3 - 4*a**2*c**3*d**3*e**2*f**5*g - 2*a**2*c**3*d**3*e**2*f*
*4*g**2*x + 8*a**2*c**3*d**3*e**2*f**3*g**3*x**2 + 6*a**2*c**3*d**3*e**2*f
**2*g**4*x**3 + a*c**4*d**4*e*f**6 - 2*a*c**4*d**4*e*f**5*g*x - 7*a*c**4*d
**4*e*f**4*g**2*x**2 - 4*a*c**4*d**4*e*f**3*g**3*x**3 + c**5*d**5*f**6*x +
 2*c**5*d**5*f**5*g*x**2 + c**5*d**5*f**4*g**2*x**3))
```

3.105 $\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 112

$$\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{5cd(d+ex}$$

output
```
2/5*(g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*hypergeom([5/2, -n],
[7/2],-g*(c*d*x+a*e)/(-a*e*g+c*d*f))/c/d/(e*x+d)^(5/2)/((c*d*(g*x+f)/(-a*e
*g+c*d*f))^n)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2((ae+cdx)(d+ex))^{5/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hyper}}{5cd(d+ex)^{5/2}}$$

input
```
Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x)^(3/2),x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(f + g*x)^n*Hypergeometric2F1[5/2, -n,
7/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*c*d*(d + e*x)^(5/2)*((c*d*(
f + g*x))/(c*d*f - a*e*g))^n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 1268}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\int (ae+cdx)^{3/2}(f+gx)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{(f+gx)^n\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int (ae+cdx)^{3/2}\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{2(f+gx)^n(ae+cdx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{5}{2},-n,\frac{7}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{5cd\sqrt{d+ex}}$$

input
```
Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x]
```

output
```
(2*(a*e + c*d*x)^2*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
*Hypergeometric2F1[5/2, -n, 7/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(5
*c*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(gx+f)^n \left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}dx$$

input
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

input

```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="fricas")
```

output

```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*(g*x +
f)^n/sqrt(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{Timed out}$$

input

```
integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**
(3/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

input

```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
 d)^(3/2), x)
```

Giac [F]

$$\int \frac{(f+gx)^n\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(gx+f)^n}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),
x, algorithm="giac")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(e*x +
 d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\int \frac{(f+g\,x)^n\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(d+e\,x\right)^{3/2}}\,dx$$

input
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2),x)
```

output
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^
(3/2), x)
```

Reduce [F]

$$\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\text{too large to display}$$

input

```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

output

```
(2*(4*(f + g*x)**n*sqrt(a*e + c*d*x)*a**2*e**2*f*g*n**2 + 12*(f + g*x)**n*
sqrt(a*e + c*d*x)*a**2*e**2*f*g*n + 3*(f + g*x)**n*sqrt(a*e + c*d*x)*a**2*
e**2*f*g + 4*(f + g*x)**n*sqrt(a*e + c*d*x)*a**2*e**2*g**2*n**2*x + 12*(f
+ g*x)**n*sqrt(a*e + c*d*x)*a**2*e**2*g**2*n*x - 4*(f + g*x)**n*sqrt(a*e +
 c*d*x)*a*c*d*e*f**2*n + 4*(f + g*x)**n*sqrt(a*e + c*d*x)*a*c*d*e*f*g*n**2
*x + 2*(f + g*x)**n*sqrt(a*e + c*d*x)*a*c*d*e*f*g*n*x + 6*(f + g*x)**n*sqr
t(a*e + c*d*x)*a*c*d*e*f*g*x + 4*(f + g*x)**n*sqrt(a*e + c*d*x)*a*c*d*e*g*
*2*n**2*x**2 + 6*(f + g*x)**n*sqrt(a*e + c*d*x)*a*c*d*e*g**2*n*x**2 + 2*(f
 + g*x)**n*sqrt(a*e + c*d*x)*c**2*d**2*f**2*n*x + 2*(f + g*x)**n*sqrt(a*e
+ c*d*x)*c**2*d**2*f*g*n*x**2 + 3*(f + g*x)**n*sqrt(a*e + c*d*x)*c**2*d**2
*f*g*x**2 + 24*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(8*a**2*e**2*f*g*n**
3 + 32*a**2*e**2*f*g*n**2 + 30*a**2*e**2*f*g*n + 8*a**2*e**2*g**2*n**3*x +
 32*a**2*e**2*g**2*n**2*x + 30*a**2*e**2*g**2*n*x + 4*a*c*d*e*f**2*n**2 +
16*a*c*d*e*f**2*n + 15*a*c*d*e*f**2 + 8*a*c*d*e*f*g*n**3*x + 36*a*c*d*e*f*
g*n**2*x + 46*a*c*d*e*f*g*n*x + 15*a*c*d*e*f*g*x + 8*a*c*d*e*g**2*n**3*x**
2 + 32*a*c*d*e*g**2*n**2*x**2 + 30*a*c*d*e*g**2*n*x**2 + 4*c**2*d**2*f**2*
n**2*x + 16*c**2*d**2*f**2*n*x + 15*c**2*d**2*f**2*x + 4*c**2*d**2*f*g*n**
2*x**2 + 16*c**2*d**2*f*g*n*x**2 + 15*c**2*d**2*f*g*x**2),x)*a**4*e**4*g**
4*n**4 + 96*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(8*a**2*e**2*f*g*n**3 +
 32*a**2*e**2*f*g*n**2 + 30*a**2*e**2*f*g*n + 8*a**2*e**2*g**2*n**3*x +...
```

3.106 $\int \frac{(f+gx)^n\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 112

$$\int \frac{(f+gx)^n\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(d+ex)^{3/2}}$$

output
```
2/3*(g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*hypergeom([3/2, -n],
[5/2],-g*(c*d*x+a*e)/(-a*e*g+c*d*f))/c/d/(e*x+d)^(3/2)/((c*d*(g*x+f)/(-a*e
*g+c*d*f))^n)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)^n\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2((ae+cdx)(d+ex))^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{3cd(d+ex)^{3/2}}$$

input
```
Integrate[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
 + e*x],x]
```

output
```
(2*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x)^n*Hypergeometric2F1[3/2, -n,
5/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*(d + e*x)^(3/2)*((c*d*(
f + g*x))/(c*d*f - a*e*g))^n)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \textcolor{blue}{1268}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\int \sqrt{ae+cdx}(f+gx)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \textcolor{blue}{80}$$

$$\frac{(f+gx)^n\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \sqrt{ae+cdx}\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^ndx}{\sqrt{d+ex}\sqrt{ae+cdx}}$$

$$\Big\downarrow \textcolor{blue}{79}$$

$$\frac{2(f+gx)^n(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd\sqrt{d+ex}}$$

input
```
Int[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x
],x]
```

output
```
(2*(a*e + c*d*x)*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*H
ypergeometric2F1[3/2, -n, 5/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(3*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(gx+f)^n \sqrt{ade+(a\,e^2+c\,d^2)\,x+cd\,x^2e}}{\sqrt{ex+d}}dx$$

input
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(f+gx)^n\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^n}{\sqrt{ex+d}}\,dx$$

input

```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="fricas")
```

output

```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
+ d), x)
```

Sympy [F]

$$\int \frac{(f+gx)^n\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^n}{\sqrt{d+ex}}\,dx$$

input

```
integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**
(1/2),x)
```

output

```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**n/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{(f+gx)^n\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^n}{\sqrt{ex+d}}\,dx$$

input

```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
 + d), x)
```

Giac [F]

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^n}{\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),
x, algorithm="giac")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(e*x
 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \int \frac{(f+g\,x)^n\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2),x)
```

output
```
int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^
(1/2), x)
```

Reduce [F]

$$\int \frac{(f+gx)^n \sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \text{too large to display}$$

input

```
int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

output

```
(2*(2*(f + g*x)**n*sqrt(a*e + c*d*x)*a*e*f*n + (f + g*x)**n*sqrt(a*e + c*d
*x)*a*e*f + 2*(f + g*x)**n*sqrt(a*e + c*d*x)*a*e*g*n*x + (f + g*x)**n*sqrt
(a*e + c*d*x)*c*d*f*x + 4*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(4*a**2*e
**2*f*g*n**2 + 6*a**2*e**2*f*g*n + 4*a**2*e**2*g**2*n**2*x + 6*a**2*e**2*g
**2*n*x + 2*a*c*d*e*f**2*n + 3*a*c*d*e*f**2 + 4*a*c*d*e*f*g*n**2*x + 8*a*c
*d*e*f*g*n*x + 3*a*c*d*e*f*g*x + 4*a*c*d*e*g**2*n**2*x**2 + 6*a*c*d*e*g**2
*n*x**2 + 2*c**2*d**2*f**2*n*x + 3*c**2*d**2*f**2*x + 2*c**2*d**2*f*g*n*x*
*2 + 3*c**2*d**2*f*g*x**2),x)*a**3*e**3*g**3*n**3 + 6*int(((f + g*x)**n*sq
rt(a*e + c*d*x)*x)/(4*a**2*e**2*f*g*n**2 + 6*a**2*e**2*f*g*n + 4*a**2*e**2
*g**2*n**2*x + 6*a**2*e**2*g**2*n*x + 2*a*c*d*e*f**2*n + 3*a*c*d*e*f**2 +
4*a*c*d*e*f*g*n**2*x + 8*a*c*d*e*f*g*n*x + 3*a*c*d*e*f*g*x + 4*a*c*d*e*g**
2*n**2*x**2 + 6*a*c*d*e*g**2*n*x**2 + 2*c**2*d**2*f**2*n*x + 3*c**2*d**2*f
**2*x + 2*c**2*d**2*f*g*n*x**2 + 3*c**2*d**2*f*g*x**2),x)*a**3*e**3*g**3*n
**2 - 8*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(4*a**2*e**2*f*g*n**2 + 6*a
**2*e**2*f*g*n + 4*a**2*e**2*g**2*n**2*x + 6*a**2*e**2*g**2*n*x + 2*a*c*d*
e*f**2*n + 3*a*c*d*e*f**2 + 4*a*c*d*e*f*g*n**2*x + 8*a*c*d*e*f*g*n*x + 3*a
*c*d*e*f*g*x + 4*a*c*d*e*g**2*n**2*x**2 + 6*a*c*d*e*g**2*n*x**2 + 2*c**2*d
**2*f**2*n*x + 3*c**2*d**2*f**2*x + 2*c**2*d**2*f*g*n*x**2 + 3*c**2*d**2*f
*g*x**2),x)*a**2*c*d*e**2*f*g**2*n**3 - 10*int(((f + g*x)**n*sqrt(a*e + c*
d*x)*x)/(4*a**2*e**2*f*g*n**2 + 6*a**2*e**2*f*g*n + 4*a**2*e**2*g**2*n*...
```

3.107 $\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 110

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{d+ex}}$$

output
```
2*(g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*hypergeom([1/2, -n],[3
/2],-g*(c*d*x+a*e)/(-a*e*g+c*d*f))/c/d/(e*x+d)^(1/2)/((c*d*(g*x+f)/(-a*e*g
+c*d*f))^n)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{cd\sqrt{d+ex}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/
2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[d + e*x]*((c*d*(f + g*
x))/(c*d*f - a*e*g))^n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = $\{1268, 80, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{(f+gx)^n}{\sqrt{ae+cdx}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt{d+ex}(f+gx)^n\sqrt{ae+cdx}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{\sqrt{ae+cdx}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
],x]
```

output

```
(2*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
 -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e
*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{\sqrt{ex+d}\,(gx+f)^{n}}{\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}dx$$

input

```
int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x)
```

output

```
int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{d+ex}\,(f+gx)^n}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)**n/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(f+g\,x)^n\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{2(gx+f)^n\sqrt{cdx+ae}\,f+4\left(\int \frac{(gx+f)^n\sqrt{cdx+ae}\,x}{2acde\,g^2n\,x^2+2a^2e^2g^2nx+2acdefgnx+c^2d^2fg\,x^2+2a^2e^2fgn+acdefgx+c^2d^2f^2x+acde\,f^2}dx\right)}{}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(2*((f + g*x)**n*sqrt(a*e + c*d*x)*f + 2*int(((f + g*x)**n*sqrt(a*e + c*d*
x)*x)/(2*a**2*e**2*f*g*n + 2*a**2*e**2*g**2*n*x + a*c*d*e*f**2 + 2*a*c*d*e
*f*g*n*x + a*c*d*e*f*g*x + 2*a*c*d*e*g**2*n*x**2 + c**2*d**2*f**2*x + c**2
*d**2*f*g*x**2),x)*a**2*e**2*g**3*n**2 - 2*int(((f + g*x)**n*sqrt(a*e + c*
d*x)*x)/(2*a**2*e**2*f*g*n + 2*a**2*e**2*g**2*n*x + a*c*d*e*f**2 + 2*a*c*d
*e*f*g*n*x + a*c*d*e*f*g*x + 2*a*c*d*e*g**2*n*x**2 + c**2*d**2*f**2*x + c*
*2*d**2*f*g*x**2),x)*a*c*d*e*f*g**2*n**2 + int(((f + g*x)**n*sqrt(a*e + c*
d*x)*x)/(2*a**2*e**2*f*g*n + 2*a**2*e**2*g**2*n*x + a*c*d*e*f**2 + 2*a*c*d
*e*f*g*n*x + a*c*d*e*f*g*x + 2*a*c*d*e*g**2*n*x**2 + c**2*d**2*f**2*x + c*
*2*d**2*f*g*x**2),x)*a*c*d*e*f*g**2*n - int(((f + g*x)**n*sqrt(a*e + c*d*x
)*x)/(2*a**2*e**2*f*g*n + 2*a**2*e**2*g**2*n*x + a*c*d*e*f**2 + 2*a*c*d*e*
f*g*n*x + a*c*d*e*f*g*x + 2*a*c*d*e*g**2*n*x**2 + c**2*d**2*f**2*x + c**2*
d**2*f*g*x**2),x)*c**2*d**2*f**2*g*n))/(2*a*e*g*n + c*d*f)
```

3.108 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 112

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-n,-\frac{1}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

output
```
-2/3*(e*x+d)^(3/2)*(g*x+f)^n*hypergeom([-3/2, -n],[-1/2],-g*(c*d*x+a*e)/(-
a*e*g+c*d*f))/c/d/((c*d*(g*x+f)/(-a*e*g+c*d*f))^n)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-n,\frac{1}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{cd\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, (g*(a*e + c
*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f +
g*x))/(c*d*f - a*e*g))^n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{(f+gx)^n}{(ae+cdx)^{3/2}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt{d+ex}(f+gx)^n\sqrt{ae+cdx}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{3/2}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-n,\frac{1}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((g*(a*e +
 c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g))^n*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{3}{2}}\,(gx+f)^{n}}{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x)
```

output
```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x +
f)^n/(c^2*d^2*e*x^3 + a^2*d*e^2 + (c^2*d^3 + 2*a*c*d*e^2)*x^2 + (2*a*c*d^2
*e + a^2*e^3)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),
x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(f+g\,x)^n\,(d+e\,x)^{3/2}}{\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(3/2),x)
```

output
```
int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{too large to display}$$

input

```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(2*((f + g*x)**n*sqrt(a*e + c*d*x)*f + 2*int(((f + g*x)**n*sqrt(a*e + c*d*
x)*x)/(2*a**3*e**3*f*g*n + 2*a**3*e**3*g**2*n*x - a**2*c*d*e**2*f**2 + 4*a
**2*c*d*e**2*f*g*n*x - a**2*c*d*e**2*f*g*x + 4*a**2*c*d*e**2*g**2*n*x**2 -
 2*a*c**2*d**2*e*f**2*x + 2*a*c**2*d**2*e*f*g*n*x**2 - 2*a*c**2*d**2*e*f*g
*x**2 + 2*a*c**2*d**2*e*g**2*n*x**3 - c**3*d**3*f**2*x**2 - c**3*d**3*f*g*
x**3),x)*a**3*e**3*g**3*n**2 - 2*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(2
*a**3*e**3*f*g*n + 2*a**3*e**3*g**2*n*x - a**2*c*d*e**2*f**2 + 4*a**2*c*d*
e**2*f*g*n*x - a**2*c*d*e**2*f*g*x + 4*a**2*c*d*e**2*g**2*n*x**2 - 2*a*c**
2*d**2*e*f**2*x + 2*a*c**2*d**2*e*f*g*n*x**2 - 2*a*c**2*d**2*e*f*g*x**2 +
2*a*c**2*d**2*e*g**2*n*x**3 - c**3*d**3*f**2*x**2 - c**3*d**3*f*g*x**3),x)
*a**2*c*d*e**2*f*g**2*n**2 - int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(2*a**
3*e**3*f*g*n + 2*a**3*e**3*g**2*n*x - a**2*c*d*e**2*f**2 + 4*a**2*c*d*e**2
*f*g*n*x - a**2*c*d*e**2*f*g*x + 4*a**2*c*d*e**2*g**2*n*x**2 - 2*a*c**2*d*
*2*e*f**2*x + 2*a*c**2*d**2*e*f*g*n*x**2 - 2*a*c**2*d**2*e*f*g*x**2 + 2*a*
c**2*d**2*e*g**2*n*x**3 - c**3*d**3*f**2*x**2 - c**3*d**3*f*g*x**3),x)*a**
2*c*d*e**2*f*g**2*n + 2*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(2*a**3*e**
3*f*g*n + 2*a**3*e**3*g**2*n*x - a**2*c*d*e**2*f**2 + 4*a**2*c*d*e**2*f*g*
n*x - a**2*c*d*e**2*f*g*x + 4*a**2*c*d*e**2*g**2*n*x**2 - 2*a*c**2*d**2*e*
f**2*x + 2*a*c**2*d**2*e*f*g*n*x**2 - 2*a*c**2*d**2*e*f*g*x**2 + 2*a*c**2*
d**2*e*g**2*n*x**3 - c**3*d**3*f**2*x**2 - c**3*d**3*f*g*x**3),x)*a**2*...
```

3.109 $\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 96

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{1+n}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2}+n,-\frac{1}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

output

```
-2/3*(e*x+d)^(3/2)*(g*x+f)^(1+n)*hypergeom([1, -1/2+n],[-1/2],-g*(c*d*x+a*
e)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-n,-\frac{1}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{3cd((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, (g*(a*e
+ c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*((c*
d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{(f+gx)^n}{(ae+cdx)^{5/2}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt{d+ex}(f+gx)^n\sqrt{ae+cdx}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{5/2}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-n,-\frac{1}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2),x]
```

output

```
(-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((g*(a*e
+ c*d*x))/(c*d*f - a*e*g))])/(3*c*d*(a*e + c*d*x)*((c*d*(f + g*x))/(c*d*f
- a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{5}{2}} (gx+f)^{n}}{(ade+(a\,e^{2}+c\,d^{2})\,x+cd\,x^{2}e)^{\frac{5}{2}}}dx$$

input

```
int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x)
```

output

```
int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x +
f)^n/(c^3*d^3*e*x^4 + a^3*d*e^3 + (c^3*d^4 + 3*a*c^2*d^2*e^2)*x^3 + 3*(a*c
^2*d^3*e + a^2*c*d*e^3)*x^2 + (3*a^2*c*d^2*e^2 + a^3*e^4)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{5}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),
x, algorithm="giac")
```

output
```
integrate((e*x + d)^(5/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(f+g\,x)^n\,(d+e\,x)^{5/2}}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(5/2),x)
```

output
```
int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{too large to display}$$

input

```
int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(2*((f + g*x)**n*sqrt(a*e + c*d*x)*f + 2*int(((f + g*x)**n*sqrt(a*e + c*d*
x)*x)/(2*a**4*e**4*f*g*n + 2*a**4*e**4*g**2*n*x - 3*a**3*c*d*e**3*f**2 + 6
*a**3*c*d*e**3*f*g*n*x - 3*a**3*c*d*e**3*f*g*x + 6*a**3*c*d*e**3*g**2*n*x*
*2 - 9*a**2*c**2*d**2*e**2*f**2*x + 6*a**2*c**2*d**2*e**2*f*g*n*x**2 - 9*a
**2*c**2*d**2*e**2*f*g*x**2 + 6*a**2*c**2*d**2*e**2*g**2*n*x**3 - 9*a*c**3
*d**3*e*f**2*x**2 + 2*a*c**3*d**3*e*f*g*n*x**3 - 9*a*c**3*d**3*e*f*g*x**3
+ 2*a*c**3*d**3*e*g**2*n*x**4 - 3*c**4*d**4*f**2*x**3 - 3*c**4*d**4*f*g*x*
*4),x)*a**4*e**4*g**3*n**2 - 2*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(2*a
**4*e**4*f*g*n + 2*a**4*e**4*g**2*n*x - 3*a**3*c*d*e**3*f**2 + 6*a**3*c*d*
e**3*f*g*n*x - 3*a**3*c*d*e**3*f*g*x + 6*a**3*c*d*e**3*g**2*n*x**2 - 9*a**
2*c**2*d**2*e**2*f**2*x + 6*a**2*c**2*d**2*e**2*f*g*n*x**2 - 9*a**2*c**2*d
**2*e**2*f*g*x**2 + 6*a**2*c**2*d**2*e**2*g**2*n*x**3 - 9*a*c**3*d**3*e*f*
*2*x**2 + 2*a*c**3*d**3*e*f*g*n*x**3 - 9*a*c**3*d**3*e*f*g*x**3 + 2*a*c**3
*d**3*e*g**2*n*x**4 - 3*c**4*d**4*f**2*x**3 - 3*c**4*d**4*f*g*x**4),x)*a**
3*c*d*e**3*f*g**2*n**2 - 3*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(2*a**4*
e**4*f*g*n + 2*a**4*e**4*g**2*n*x - 3*a**3*c*d*e**3*f**2 + 6*a**3*c*d*e**3
*f*g*n*x - 3*a**3*c*d*e**3*f*g*x + 6*a**3*c*d*e**3*g**2*n*x**2 - 9*a**2*c*
*2*d**2*e**2*f**2*x + 6*a**2*c**2*d**2*e**2*f*g*n*x**2 - 9*a**2*c**2*d**2*
e**2*f*g*x**2 + 6*a**2*c**2*d**2*e**2*g**2*n*x**3 - 9*a*c**3*d**3*e*f**2*x
**2 + 2*a*c**3*d**3*e*f*g*n*x**3 - 9*a*c**3*d**3*e*f*g*x**3 + 2*a*c**3*...
```

3.110 $\int \frac{(d+ex)^{7/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 112

$$\int \frac{(d+ex)^{7/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx = -\frac{2(d+ex)^{5/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{5}{2},-n,-\frac{3}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}$$

output
```
-2/5*(e*x+d)^(5/2)*(g*x+f)^n*hypergeom([-5/2, -n],[-3/2],-g*(c*d*x+a*e)/(-
a*e*g+c*d*f))/c/d/((c*d*(g*x+f)/(-a*e*g+c*d*f))^n)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{7/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx = -\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{5}{2},-n,-\frac{3}{2},\frac{g(ae+cdx)}{-cdf+aeg}\right)}{5cd(ae+cdx)^2\sqrt{(ae+cdx)(d+ex)}}$$

input

```
Integrate[((d + e*x)^(7/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(7/2),x]
```

output

```
(-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-5/2, -n, -3/2, (g*(a*e +
c*d*x))/(-(c*d*f) + a*e*g)])/(5*c*d*(a*e + c*d*x)^2*Sqrt[(a*e + c*d*x)*(d
+ e*x)]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}(f+gx)^n}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{(f+gx)^n}{(ae+cdx)^{7/2}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt{d+ex}(f+gx)^n\sqrt{ae+cdx}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{7/2}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2\sqrt{d+ex}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{5}{2},-n,-\frac{3}{2},-\frac{g(ae+cdx)}{cdf-aeg}\right)}{5cd(ae+cdx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input

```
Int[((d + e*x)^(7/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(7/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-5/2, -n, -3/2, -((g*(a*e
+ c*d*x))/(c*d*f - a*e*g))])/(5*c*d*(a*e + c*d*x)^2*((c*d*(f + g*x))/(c*d*
f - a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{7}{2}} (gx+f)^{n}}{(ade+(a\,e^{2}+c\,d^{2})\,x+cd\,x^{2}e)^{\frac{7}{2}}}dx$$

input
```
int((e*x+d)^(7/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2),x)
```

output
```
int((e*x+d)^(7/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^{7/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^(7/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x +
f)^n/(c^4*d^4*e*x^5 + a^4*d*e^4 + (c^4*d^5 + 4*a*c^3*d^3*e^2)*x^4 + 2*(2*a
*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*x^3 + 2*(3*a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4
)*x^2 + (4*a^3*c*d^2*e^3 + a^4*e^5)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{7/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^(7/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(7/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(7/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{7/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^(7/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),
x, algorithm="giac")
```

output
```
integrate((e*x + d)^(7/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}(f+gx)^n}{(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}\,dx = \int \frac{(f+g\,x)^n\,(d+e\,x)^{7/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{7/2}}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^(7/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(7/2),x)
```

output
```
int(((f + g*x)^n*(d + e*x)^(7/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(7/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{7/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx=\text{too large to display}$$

input

```
int((e*x+d)^(7/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x)
```

output

```
(2*((f + g*x)**n*sqrt(a*e + c*d*x)*f + 2*int(((f + g*x)**n*sqrt(a*e + c*d*
x)*x)/(2*a**5*e**5*f*g*n + 2*a**5*e**5*g**2*n*x - 5*a**4*c*d*e**4*f**2 + 8
*a**4*c*d*e**4*f*g*n*x - 5*a**4*c*d*e**4*f*g*x + 8*a**4*c*d*e**4*g**2*n*x*
*2 - 20*a**3*c**2*d**2*e**3*f**2*x + 12*a**3*c**2*d**2*e**3*f*g*n*x**2 - 2
0*a**3*c**2*d**2*e**3*f*g*x**2 + 12*a**3*c**2*d**2*e**3*g**2*n*x**3 - 30*a
**2*c**3*d**3*e**2*f**2*x**2 + 8*a**2*c**3*d**3*e**2*f*g*n*x**3 - 30*a**2*
c**3*d**3*e**2*f*g*x**3 + 8*a**2*c**3*d**3*e**2*g**2*n*x**4 - 20*a*c**4*d*
*4*e*f**2*x**3 + 2*a*c**4*d**4*e*f*g*n*x**4 - 20*a*c**4*d**4*e*f*g*x**4 +
2*a*c**4*d**4*e*g**2*n*x**5 - 5*c**5*d**5*f**2*x**4 - 5*c**5*d**5*f*g*x**5
),x)*a**5*e**5*g**3*n**2 - 2*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(2*a**
5*e**5*f*g*n + 2*a**5*e**5*g**2*n*x - 5*a**4*c*d*e**4*f**2 + 8*a**4*c*d*e*
*4*f*g*n*x - 5*a**4*c*d*e**4*f*g*x + 8*a**4*c*d*e**4*g**2*n*x**2 - 20*a**3
*c**2*d**2*e**3*f**2*x + 12*a**3*c**2*d**2*e**3*f*g*n*x**2 - 20*a**3*c**2*
d**2*e**3*f*g*x**2 + 12*a**3*c**2*d**2*e**3*g**2*n*x**3 - 30*a**2*c**3*d**
3*e**2*f**2*x**2 + 8*a**2*c**3*d**3*e**2*f*g*n*x**3 - 30*a**2*c**3*d**3*e*
*2*f*g*x**3 + 8*a**2*c**3*d**3*e**2*g**2*n*x**4 - 20*a*c**4*d**4*e*f**2*x*
*3 + 2*a*c**4*d**4*e*f*g*n*x**4 - 20*a*c**4*d**4*e*f*g*x**4 + 2*a*c**4*d**
4*e*g**2*n*x**5 - 5*c**5*d**5*f**2*x**4 - 5*c**5*d**5*f*g*x**5),x)*a**4*c*
d*e**4*f*g**2*n**2 - 5*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(2*a**5*e**5
*f*g*n + 2*a**5*e**5*g**2*n*x - 5*a**4*c*d*e**4*f**2 + 8*a**4*c*d*e**4*...
```

3.111 $\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal result

Integrand size = 44, antiderivative size = 99

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$
$$-\frac{(d+ex)^{-1+m}(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}\text{Hypergeometric2F1}\left(1,2-m+n,2+n\right.}{(cdf-aeg)(1+n)}$$

output
```
-(e*x+d)^(-1+m)*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)*hype
rgeom([1, 2-m+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\frac{\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{1+n}\text{Hypergeometric2F1}\left(m,1+n,2+n,\frac{cd(f+g}{cdf-ae}\right.}{g(1+n)}$$

input
```
Integrate[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m,x]
```

output
```
(((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hy
pergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 +
 n)*((a*e + c*d*x)*(d + e*x))^m)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m(f+gx)^n\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int (ae+cdx)^{-m}(f+gx)^ndx$$

$$\Big\downarrow \text{80}$$

$$(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int (f+gx)^n\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{(d+ex)^m(f+gx)^{n+1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(m,n+1,n+2,\frac{cd}{c}\right.}{g(n+1)}$$

input
```
Int[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hy
pergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 +
 n)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int (ex+d)^m (gx+f)^n \left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{-m}dx$$

input
```
int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{(ex+d)^m(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
integral((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(g*x+f)**n/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (d+ex)^m(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{(ex+d)^m(gx+f)^n}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output
```
integrate((e*x + d)^m*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^
m, x)
```

Giac [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int (d+ex)^m(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx\\&=\int \frac{(f+g\,x)^n\,(d+e\,x)^m}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx\end{aligned}$$

input
```
int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x
)
```

Reduce [F]

$$\int (d+ex)^m (f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$$
$$=\int \frac{(gx+f)^n (ex+d)^m}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx$$

input
```
int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int(((f + g*x)**n*(d + e*x)**m)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)
**m,x)
```

3.112 $\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-}$

Optimal result

Integrand size = 44, antiderivative size = 260

$$\begin{aligned}
&\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx \\
&= \frac{(cdf-aeg)^3 (d+ex)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^4 d^4 (1-m)} \\
&\quad + \frac{3g(cdf-aeg)^2 (d+ex)^{-2+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{2-m}}{c^4 d^4 (2-m)} \\
&\quad + \frac{3g^2(cdf-aeg) (d+ex)^{-3+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3-m}}{c^4 d^4 (3-m)} \\
&\quad + \frac{g^3 (d+ex)^{-4+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{4-m}}{c^4 d^4 (4-m)}
\end{aligned}$$

output
```
(-a*e*g+c*d*f)^3*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^
4/d^4/(1-m)+3*g*(-a*e*g+c*d*f)^2*(e*x+d)^(-2+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(2-m)/c^4/d^4/(2-m)+3*g^2*(-a*e*g+c*d*f)*(e*x+d)^(-3+m)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3-m)/c^4/d^4/(3-m)+g^3*(e*x+d)^(-4+m)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(4-m)/c^4/d^4/(4-m)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.52

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\left(-\frac{(cdf-aeg)^3}{-1+m}-\frac{3g(cdf-aeg)^2(ae+cdx)}{-2+m}+\frac{3g^2(-cdf+aeg)(ae+cdx)^2}{-3+m}-\frac{g^3(ae+c}{-4+}\right.}{c^4d^4}$$

input
```
Integrate[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m,x]
```

output
```
((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(-((c*d*f - a*e*g)^3
/(-1 + m)) - (3*g*(c*d*f - a*e*g)^2*(a*e + c*d*x))/(-2 + m) + (3*g^2*(-(c*
d*f) + a*e*g)*(a*e + c*d*x)^2)/(-3 + m) - (g^3*(a*e + c*d*x)^3)/(-4 + m)))
/(c^4*d^4)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,dx$$

$$\Big\downarrow \text{1253}$$

$$\frac{3(cdf-aeg)\int (d+ex)^m(f+gx)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cd(4-m)}+\frac{(f+gx)^3(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

$$\Big\downarrow \text{1253}$$

$$\frac{3(cdf-aeg)\left(\frac{2(cdf-aeg)\int (d+ex)^m(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cd(3-m)}+\frac{(f+gx)^2(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(3-m)}\right)}{cd(4-m)}-$$

$$\frac{(f+gx)^3(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{3(cdf-aeg)\Bigg(\frac{2(cdf-aeg)\Bigg(\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{\big(ae^2g+cd(dg(1-m)-ef(2-m))\big)\int (d+ex)^m\big(cdex^2+\big(cd^2+ae^2\big)x+ade\big)^{-m}}{cde(2-m)}}{cd(3-m)}}{cd(4-m)}$$

$$\frac{(f+gx)^3(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{3(cdf-aeg)\Bigg(\frac{2(cdf-aeg)\Bigg(\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\big(ae^2g+cd(dg(1-m)-ef(2-m)}{c^2d^2e(1-m)(2-m)}}{cd(3-m)}}{cd(4-m)}$$

$$\frac{(f+gx)^3(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

input
```
Int[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((d + e*x)^(-1 + m)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1
 - m))/(c*d*(4 - m)) + (3*(c*d*f - a*e*g)*(((d + e*x)^(-1 + m)*(f + g*x)^2
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(3 - m)) + (2*(c*d*
f - a*e*g)*(-(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 +
 m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2
 - m))) + (g*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/
(c*d*e*(2 - m))))/(c*d*(3 - m))))/(c*d*(4 - m))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(260) = 520$.

Time = 10.31 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.03

method	result
gosper	$-\frac{(ex+d)^m (c^3d^3g^3m^3x^3+3c^3d^3f\,g^2m^3x^2-6c^3d^3g^3m^2x^3+3a\,c^2d^2e\,g^3m^2x^2+3c^3d^3f^2g\,m^3x-21c^3d^3f\,g^2m^2x^2+11c^3d^3g^3}{}$
orering	$-\frac{(c^3d^3g^3m^3x^3+3c^3d^3f\,g^2m^3x^2-6c^3d^3g^3m^2x^3+3a\,c^2d^2e\,g^3m^2x^2+3c^3d^3f^2g\,m^3x-21c^3d^3f\,g^2m^2x^2+11c^3d^3g^3m\,x^3+6a}{}$
risch	$-\frac{(c^4d^4g^3m^3x^4+a\,c^3d^3e\,g^3m^3x^3+3c^4d^4f\,g^2m^3x^3-6c^4d^4g^3m^2x^4+3a\,c^3d^3ef\,g^2m^3x^2-3a\,c^3d^3e\,g^3m^2x^3+3c^4d^4f^2g\,m^3x^2}{}$
parallelrisch	Expression too large to display

input
```
int((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x,method=_RE
TURNVERBOSE)
```

output
```
-(e*x+d)^m*(c^3*d^3*g^3*m^3*x^3+3*c^3*d^3*f*g^2*m^3*x^2-6*c^3*d^3*g^3*m^2*
x^3+3*a*c^2*d^2*e*g^3*m^2*x^2+3*c^3*d^3*f^2*g*m^3*x-21*c^3*d^3*f*g^2*m^2*x
^2+11*c^3*d^3*g^3*m*x^3+6*a*c^2*d^2*e*f*g^2*m^2*x-9*a*c^2*d^2*e*g^3*m*x^2+
c^3*d^3*f^3*m^3-24*c^3*d^3*f^2*g*m^2*x+42*c^3*d^3*f*g^2*m*x^2-6*c^3*d^3*g^
3*x^3+6*a^2*c*d*e^2*g^3*m*x+3*a*c^2*d^2*e*f^2*g*m^2-30*a*c^2*d^2*e*f*g^2*m
*x+6*a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f^3*m^2+57*c^3*d^3*f^2*g*m*x-24*c^3*d^3
*f*g^2*x^2+6*a^2*c*d*e^2*f*g^2*m-6*a^2*c*d*e^2*g^3*x-21*a*c^2*d^2*e*f^2*g*
m+24*a*c^2*d^2*e*f*g^2*x+26*c^3*d^3*f^3*m-36*c^3*d^3*f^2*g*x+6*a^3*e^3*g^3
-24*a^2*c*d*e^2*f*g^2+36*a*c^2*d^2*e*f^2*g-24*c^3*d^3*f^3)*(c*d*x+a*e)/((c
*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)/d^4/c^4/(m^4-10*m^3+35*m^2-50*m+24)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(254) = 508$.

Time = 0.11 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.71

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$

$$-\frac{(ac^3d^3ef^3m^3 - 24\,ac^3d^3ef^3 + 36\,a^2c^2d^2e^2f^2g - 24\,a^3cde^3fg^2 + 6\,a^4e^4g^3 + (c^4d^4g^3m^3 - 6\,c^4d^4g^3m^2 +}{}$$

input
```
integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
-(a*c^3*d^3*e*f^3*m^3 - 24*a*c^3*d^3*e*f^3 + 36*a^2*c^2*d^2*e^2*f^2*g - 24
*a^3*c*d*e^3*f*g^2 + 6*a^4*e^4*g^3 + (c^4*d^4*g^3*m^3 - 6*c^4*d^4*g^3*m^2
+ 11*c^4*d^4*g^3*m - 6*c^4*d^4*g^3)*x^4 - (24*c^4*d^4*f*g^2 - (3*c^4*d^4*f
*g^2 + a*c^3*d^3*e*g^3)*m^3 + 3*(7*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m^2 -
2*(21*c^4*d^4*f*g^2 + a*c^3*d^3*e*g^3)*m)*x^3 - 3*(3*a*c^3*d^3*e*f^3 - a^2
*c^2*d^2*e^2*f^2*g)*m^2 - 3*(12*c^4*d^4*f^2*g - (c^4*d^4*f^2*g + a*c^3*d^3
*e*f*g^2)*m^3 + (8*c^4*d^4*f^2*g + 5*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g
^3)*m^2 - (19*c^4*d^4*f^2*g + 4*a*c^3*d^3*e*f*g^2 - a^2*c^2*d^2*e^2*g^3)*m
)*x^2 + (26*a*c^3*d^3*e*f^3 - 21*a^2*c^2*d^2*e^2*f^2*g + 6*a^3*c*d*e^3*f*g
^2)*m - (24*c^4*d^4*f^3 - (c^4*d^4*f^3 + 3*a*c^3*d^3*e*f^2*g)*m^3 + 3*(3*c
^4*d^4*f^3 + 7*a*c^3*d^3*e*f^2*g - 2*a^2*c^2*d^2*e^2*f*g^2)*m^2 - 2*(13*c^
4*d^4*f^3 + 18*a*c^3*d^3*e*f^2*g - 12*a^2*c^2*d^2*e^2*f*g^2 + 3*a^3*c*d*e^
3*g^3)*m)*x)*(e*x + d)^m/((c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 -
 50*c^4*d^4*m + 24*c^4*d^4)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.27

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$= -\frac{(cdx+ae)f^3}{(cdx+ae)^m cd(m-1)} - \frac{3\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)f^2g}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2}$$
$$- \frac{3\left((m^2-3\,m+2)c^3d^3x^3+(m^2-m)ac^2d^2ex^2+2\,a^2cde^2mx+2\,a^3e^3\right)fg^2}{(m^3-6\,m^2+11\,m-6)(cdx+ae)^m c^3d^3}$$
$$- \frac{((m^3-6\,m^2+11\,m-6)c^4d^4x^4+(m^3-3\,m^2+2\,m)ac^3d^3ex^3+3\,(m^2-m)a^2c^2d^2e^2x^2+6\,a^3cde^3mx}{(m^4-10\,m^3+35\,m^2-50\,m+24)(cdx+ae)^m c^4d^4}$$

input

```
integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output

```
-(c*d*x + a*e)*f^3/((c*d*x + a*e)^m*c*d*(m - 1)) - 3*(c^2*d^2*(m - 1)*x^2
+ a*c*d*e*m*x + a^2*e^2)*f^2*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) -
 3*((m^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^
2*m*x + 2*a^3*e^3)*f*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3
) - ((m^3 - 6*m^2 + 11*m - 6)*c^4*d^4*x^4 + (m^3 - 3*m^2 + 2*m)*a*c^3*d^3*
e*x^3 + 3*(m^2 - m)*a^2*c^2*d^2*e^2*x^2 + 6*a^3*c*d*e^3*m*x + 6*a^4*e^4)*g
^3/((m^4 - 10*m^3 + 35*m^2 - 50*m + 24)*(c*d*x + a*e)^m*c^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. 2(254) = 508.

Time = 0.21 (sec) , antiderivative size = 1926, normalized size of antiderivative = 7.41

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
-((e*x + d)^m*c^4*d^4*g^3*m^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(e*x + d))
 + 3*(e*x + d)^m*c^4*d^4*f*g^2*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x
+ d)) + (e*x + d)^m*a*c^3*d^3*e*g^3*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log
(e*x + d)) - 6*(e*x + d)^m*c^4*d^4*g^3*m^2*x^4*e^(-m*log(c*d*x + a*e) - m*
log(e*x + d)) + 3*(e*x + d)^m*c^4*d^4*f^2*g*m^3*x^2*e^(-m*log(c*d*x + a*e)
 - m*log(e*x + d)) + 3*(e*x + d)^m*a*c^3*d^3*e*f*g^2*m^3*x^2*e^(-m*log(c*d
*x + a*e) - m*log(e*x + d)) - 21*(e*x + d)^m*c^4*d^4*f*g^2*m^2*x^3*e^(-m*l
og(c*d*x + a*e) - m*log(e*x + d)) - 3*(e*x + d)^m*a*c^3*d^3*e*g^3*m^2*x^3*
e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 11*(e*x + d)^m*c^4*d^4*g^3*m*x^
4*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*c^4*d^4*f^3*m^3*x
*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 3*(e*x + d)^m*a*c^3*d^3*e*f^2*
g*m^3*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 24*(e*x + d)^m*c^4*d^4*
f^2*g*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 15*(e*x + d)^m*a*
c^3*d^3*e*f*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + 3*(e*x
+ d)^m*a^2*c^2*d^2*e^2*g^3*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)
) + 42*(e*x + d)^m*c^4*d^4*f*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x
+ d)) + 2*(e*x + d)^m*a*c^3*d^3*e*g^3*m*x^3*e^(-m*log(c*d*x + a*e) - m*log
(e*x + d)) - 6*(e*x + d)^m*c^4*d^4*g^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(
e*x + d)) + (e*x + d)^m*a*c^3*d^3*e*f^3*m^3*e^(-m*log(c*d*x + a*e) - m*log
(e*x + d)) - 9*(e*x + d)^m*c^4*d^4*f^3*m^2*x*e^(-m*log(c*d*x + a*e) - m...
```

Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 615, normalized size of antiderivative = 2.37

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$

$$-\frac{\frac{g^3\,x^4\,{\left(d+e\,x\right)}^m\,\left(m^3-6\,m^2+11\,m-6\right)}{m^4-10\,m^3+35\,m^2-50\,m+24}+\frac{x\,{\left(d+e\,x\right)}^m\,\left(6\,a^3\,c\,d\,e^3\,g^3\,m+6\,a^2\,c^2\,d^2\,e^2\,f\,g^2\,m^2-24\,a^2\,c^2\,d^2\,e^2\,f\,g^2\,m+3\,a\,c^3\,d^3\,e\,f^2\,g\,m^3-21\,a}{c^4\,d^4\,\left(m^4-10\,m^3+35\right.}}{}$$

input
```
int(((f + g*x)^3*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
-((g^3*x^4*(d + e*x)^m*(11*m - 6*m^2 + m^3 - 6))/(35*m^2 - 50*m - 10*m^3 +
 m^4 + 24) + (x*(d + e*x)^m*(26*c^4*d^4*f^3*m - 24*c^4*d^4*f^3 - 9*c^4*d^4
*f^3*m^2 + c^4*d^4*f^3*m^3 + 6*a^3*c*d*e^3*g^3*m - 24*a^2*c^2*d^2*e^2*f*g^
2*m + 36*a*c^3*d^3*e*f^2*g*m + 6*a^2*c^2*d^2*e^2*f*g^2*m^2 - 21*a*c^3*d^3*
e*f^2*g*m^2 + 3*a*c^3*d^3*e*f^2*g*m^3))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 +
 m^4 + 24)) + (a*e*(d + e*x)^m*(6*a^3*e^3*g^3 - 24*c^3*d^3*f^3 + 26*c^3*d^
3*f^3*m - 9*c^3*d^3*f^3*m^2 + c^3*d^3*f^3*m^3 + 36*a*c^2*d^2*e*f^2*g - 24*
a^2*c*d*e^2*f*g^2 - 21*a*c^2*d^2*e*f^2*g*m + 6*a^2*c*d*e^2*f*g^2*m + 3*a*c
^2*d^2*e*f^2*g*m^2))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)) + (3*g*
x^2*(m - 1)*(d + e*x)^m*(12*c^2*d^2*f^2 + a^2*e^2*g^2*m - 7*c^2*d^2*f^2*m
+ c^2*d^2*f^2*m^2 - 4*a*c*d*e*f*g*m + a*c*d*e*f*g*m^2))/(c^2*d^2*(35*m^2 -
 50*m - 10*m^3 + m^4 + 24)) + (g^2*x^3*(d + e*x)^m*(a*e*g*m - 12*c*d*f + 3
*c*d*f*m)*(m^2 - 3*m + 2))/(c*d*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)))/(x*(
a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
```

Reduce [F]

$$
\begin{aligned}
&\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx \\
&= \left(\int \frac{(ex+d)^m}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right) f^3 \\
&\quad + \left(\int \frac{(ex+d)^m\,x^3}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right) g^3 \\
&\quad + 3\left(\int \frac{(ex+d)^m\,x^2}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right) f\,g^2 \\
&\quad + 3\left(\int \frac{(ex+d)^m\,x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right) f^2g
\end{aligned}
$$

input
```
int((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)**m/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m,x)*f**3 + i
nt(((d + e*x)**m*x**3)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m,x)*g*
*3 + 3*int(((d + e*x)**m*x**2)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)*
*m,x)*f*g**2 + 3*int(((d + e*x)**m*x)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e
*x**2)**m,x)*f**2*g
```

3.113 $\int (d+ex)^m (f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-}$

Optimal result

Integrand size = 44, antiderivative size = 190

$$\begin{aligned}
&\int (d+ex)^m (f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx \\
&= \frac{(cdf-aeg)^2 (d+ex)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^3d^3(1-m)} \\
&\quad + \frac{2g(cdf-aeg)(d+ex)^{-2+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{2-m}}{c^3d^3(2-m)} \\
&\quad + \frac{g^2(d+ex)^{-3+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3-m}}{c^3d^3(3-m)}
\end{aligned}$$

output

```
(-a*e*g+c*d*f)^2*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^
3/d^3/(1-m)+2*g*(-a*e*g+c*d*f)*(e*x+d)^(-2+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(2-m)/c^3/d^3/(2-m)+g^2*(e*x+d)^(-3+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3-m)/c^3/d^3/(3-m)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.69

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$
$$-\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\left(2a^2e^2g^2+2acdeg(f(-3+m)+g(-1+m)x)+c^2d^2(f^2(6-\right.}{c^3d^3(-3+m)(-2+m)(-1+m)}$$

input

```
Integrate[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m,x]
```

output

```
-(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(2*a^2*e^2*g^2 + 2
*a*c*d*e*g*(f*(-3 + m) + g*(-1 + m)*x) + c^2*d^2*(f^2*(6 - 5*m + m^2) + 2*
f*g*(3 - 4*m + m^2)*x + g^2*(2 - 3*m + m^2)*x^2)))/(c^3*d^3*(-3 + m)*(-2 +
 m)*(-1 + m)))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}dx$$

$$\Big\downarrow \, 1253$$

$$\frac{2(cdf-aeg)\int (d+ex)^m(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cd(3-m)}+\frac{(f+gx)^2(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(3-m)}$$

$$\Big\downarrow \, 1221$$

$$\frac{2(cdf-aeg)\left(\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)\int(d+ex)^m\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cde(2-m)}\right.}{cd(3-m)}$$

$$\frac{(f+gx)^2(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(3-m)}$$

$$\Big\downarrow 1122$$

$$\frac{2(cdf-aeg)\left(\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^2d^2e(1-m)(2-m)}\right.}{cd(3-m)}$$

$$\frac{(f+gx)^2(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(3-m)}$$

input

```
Int[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output

```
((d + e*x)^(-1 + m)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1
 - m))/(c*d*(3 - m)) + (2*(c*d*f - a*e*g)*(-(((a*e^2*g + c*d*(d*g*(1 - m)
- e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g*(d + e*x)^m*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 - m))))/(c*d*(3 - m))
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

Maple [A] (verified)

Time = 5.91 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.24

method	result
gosper	$-\frac{(cdx+ae)\left(c^2d^2g^2m^2x^2+2c^2d^2fg\,m^2x-3c^2d^2g^2m\,x^2+2acde\,g^2mx+c^2d^2f^2m^2-8c^2d^2fgmx+2g^2x^2d^2c^2+2acdefgm-2a\right.}{d^3c^3(m^3-6m^2+11m}$
orering	$-\frac{\left(c^2d^2g^2m^2x^2+2c^2d^2fg\,m^2x-3c^2d^2g^2m\,x^2+2acde\,g^2mx+c^2d^2f^2m^2-8c^2d^2fgmx+2g^2x^2d^2c^2+2acdefgm-2acde\,g^2x-\right.}{d^3c^3(m^3-6m^2+11m}$
risch	$-\frac{\left(c^3d^3g^2m^2x^3+a\,c^2d^2e\,g^2m^2x^2+2c^3d^3fg\,m^2x^2-3c^3d^3g^2m\,x^3+2a\,c^2d^2efg\,m^2x-a\,c^2d^2e\,g^2m\,x^2+c^3d^3f^2m^2x-8c^3d^3fg\right.}{}$
parallelrisch	$\frac{\left(-x^2(ex+d)^m a\,c^2d^2e^2g^2m^2-2x^2(ex+d)^m c^3d^3efg\,m^2+x^2(ex+d)^m a\,c^2d^2e^2g^2m-2x(ex+d)^m a\,c^2d^2e^2fg\,m^2+6x(ex+d)^m\right.}{}$

input

```
int((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x,method=_RE
TURNVERBOSE)
```

output

```
-(c*d*x+a*e)*(c^2*d^2*g^2*m^2*x^2+2*c^2*d^2*f*g*m^2*x-3*c^2*d^2*g^2*m*x^2+
2*a*c*d*e*g^2*m*x+c^2*d^2*f^2*m^2-8*c^2*d^2*f*g*m*x+2*c^2*d^2*g^2*x^2+2*a*
c*d*e*f*g*m-2*a*c*d*e*g^2*x-5*c^2*d^2*f^2*m+6*c^2*d^2*f*g*x+2*a^2*e^2*g^2-
6*a*c*d*e*f*g+6*c^2*d^2*f^2)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^
m)/d^3/c^3/(m^3-6*m^2+11*m-6)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.84

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(ac^2d^2ef^2m^2+6\,ac^2d^2ef^2-6\,a^2cde^2fg+2\,a^3e^3g^2+(c^3d^3g^2m^2-3\,c^3d^3g^2m+2\,c^3d^3g^2)x^3+(6\,c^3d^3}{}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
-(a*c^2*d^2*e*f^2*m^2 + 6*a*c^2*d^2*e*f^2 - 6*a^2*c*d*e^2*f*g + 2*a^3*e^3*
g^2 + (c^3*d^3*g^2*m^2 - 3*c^3*d^3*g^2*m + 2*c^3*d^3*g^2)*x^3 + (6*c^3*d^3
*f*g + (2*c^3*d^3*f*g + a*c^2*d^2*e*g^2)*m^2 - (8*c^3*d^3*f*g + a*c^2*d^2*
e*g^2)*m)*x^2 - (5*a*c^2*d^2*e*f^2 - 2*a^2*c*d*e^2*f*g)*m + (6*c^3*d^3*f^2
 + (c^3*d^3*f^2 + 2*a*c^2*d^2*e*f*g)*m^2 - (5*c^3*d^3*f^2 + 6*a*c^2*d^2*e*
f*g - 2*a^2*c*d*e^2*g^2)*m)*x)*(e*x + d)^m/((c^3*d^3*m^3 - 6*c^3*d^3*m^2 +
 11*c^3*d^3*m - 6*c^3*d^3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.02

$$\begin{aligned}\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx\\ =-\frac{(cdx+ae)f^2}{(cdx+ae)^m cd(m-1)}-\frac{2\left(c^2d^2(m-1)x^2+acdemx+a^2e^2\right)fg}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2}\\ -\frac{((m^2-3\,m+2)c^3d^3x^3+(m^2-m)ac^2d^2ex^2+2\,a^2cde^2mx+2\,a^3e^3)g^2}{(m^3-6\,m^2+11\,m-6)(cdx+ae)^m c^3d^3}\end{aligned}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output
```
-(c*d*x + a*e)*f^2/((c*d*x + a*e)^m*c*d*(m - 1)) - 2*(c^2*d^2*(m - 1)*x^2
+ a*c*d*e*m*x + a^2*e^2)*f*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - (
(m^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*e*x^2 + 2*a^2*c*d*e^2*m*
x + 2*a^3*e^3)*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. 2(186) = 372.

Time = 0.18 (sec) , antiderivative size = 929, normalized size of antiderivative = 4.89

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
-((e*x + d)^m*c^3*d^3*g^2*m^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(e*x + d))
 + 2*(e*x + d)^m*c^3*d^3*f*g*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x +
d)) + (e*x + d)^m*a*c^2*d^2*e*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(e
*x + d)) - 3*(e*x + d)^m*c^3*d^3*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(
e*x + d)) + (e*x + d)^m*c^3*d^3*f^2*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(e
*x + d)) + 2*(e*x + d)^m*a*c^2*d^2*e*f*g*m^2*x*e^(-m*log(c*d*x + a*e) - m*
log(e*x + d)) - 8*(e*x + d)^m*c^3*d^3*f*g*m*x^2*e^(-m*log(c*d*x + a*e) - m
*log(e*x + d)) - (e*x + d)^m*a*c^2*d^2*e*g^2*m*x^2*e^(-m*log(c*d*x + a*e)
- m*log(e*x + d)) + 2*(e*x + d)^m*c^3*d^3*g^2*x^3*e^(-m*log(c*d*x + a*e) -
 m*log(e*x + d)) + (e*x + d)^m*a*c^2*d^2*e*f^2*m^2*e^(-m*log(c*d*x + a*e)
- m*log(e*x + d)) - 5*(e*x + d)^m*c^3*d^3*f^2*m*x*e^(-m*log(c*d*x + a*e) -
 m*log(e*x + d)) - 6*(e*x + d)^m*a*c^2*d^2*e*f*g*m*x*e^(-m*log(c*d*x + a*e
) - m*log(e*x + d)) + 2*(e*x + d)^m*a^2*c*d*e^2*g^2*m*x*e^(-m*log(c*d*x +
a*e) - m*log(e*x + d)) + 6*(e*x + d)^m*c^3*d^3*f*g*x^2*e^(-m*log(c*d*x + a
*e) - m*log(e*x + d)) - 5*(e*x + d)^m*a*c^2*d^2*e*f^2*m*e^(-m*log(c*d*x +
a*e) - m*log(e*x + d)) + 2*(e*x + d)^m*a^2*c*d*e^2*f*g*m*e^(-m*log(c*d*x +
 a*e) - m*log(e*x + d)) + 6*(e*x + d)^m*c^3*d^3*f^2*x*e^(-m*log(c*d*x + a*
e) - m*log(e*x + d)) + 6*(e*x + d)^m*a*c^2*d^2*e*f^2*e^(-m*log(c*d*x + a*e
) - m*log(e*x + d)) - 6*(e*x + d)^m*a^2*c*d*e^2*f*g*e^(-m*log(c*d*x + a*e)
 - m*log(e*x + d)) + 2*(e*x + d)^m*a^3*e^3*g^2*e^(-m*log(c*d*x + a*e) -...
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.72

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=$$

$$-\frac{\frac{g^2\,x^3\,{\left(d+e\,x\right)}^m\,\left(m^2-3\,m+2\right)}{m^3-6\,m^2+11\,m-6}+\frac{x\,{\left(d+e\,x\right)}^m\,\left(2\,a^2\,c\,d\,e^2\,g^2\,m+2\,a\,c^2\,d^2\,e\,f\,g\,m^2-6\,a\,c^2\,d^2\,e\,f\,g\,m+c^3\,d^3\,f^2\,m^2-5\,c^3\,d^3\,f^2\,m+6\,c^3\,d^3\,f^2\right)}{c^3\,d^3\,\left(m^3-6\,m^2+11\,m-6\right)}-}{{\left(c\,d\,e\,x^2+(c\,d}}$$

input
```
int(((f + g*x)^2*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
-((g^2*x^3*(d + e*x)^m*(m^2 - 3*m + 2))/(11*m - 6*m^2 + m^3 - 6) + (x*(d +
 e*x)^m*(6*c^3*d^3*f^2 - 5*c^3*d^3*f^2*m + c^3*d^3*f^2*m^2 + 2*a^2*c*d*e^2
*g^2*m + 2*a*c^2*d^2*e*f*g*m^2 - 6*a*c^2*d^2*e*f*g*m))/(c^3*d^3*(11*m - 6*
m^2 + m^3 - 6)) + (a*e*(d + e*x)^m*(2*a^2*e^2*g^2 + 6*c^2*d^2*f^2 - 5*c^2*
d^2*f^2*m + c^2*d^2*f^2*m^2 - 6*a*c*d*e*f*g + 2*a*c*d*e*f*g*m))/(c^3*d^3*(
11*m - 6*m^2 + m^3 - 6)) + (g*x^2*(m - 1)*(d + e*x)^m*(a*e*g*m - 6*c*d*f +
 2*c*d*f*m))/(c*d*(11*m - 6*m^2 + m^3 - 6)))/(x*(a*e^2 + c*d^2) + a*d*e +
c*d*e*x^2)^m
```

Reduce [F]

$$\begin{aligned}\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx\\ =\left(\int \frac{(ex+d)^m}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right)f^2\\ +\left(\int \frac{(ex+d)^m\,x^2}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right)g^2\\ +2\left(\int \frac{(ex+d)^m\,x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right)fg\end{aligned}$$

input
```
int((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)**m/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m,x)*f**2 + i
nt(((d + e*x)**m*x**2)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m,x)*g*
*2 + 2*int(((d + e*x)**m*x)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m,
x)*f*g
```

3.114 $\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal result

Integrand size = 42, antiderivative size = 120

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(cdf-aeg)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2(1-m)} + \frac{g(d+ex)^{-2+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{2-m}}{c^2d^2(2-m)}$$

output
```
(-a*e*g+c*d*f)*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^2/
d^2/(1-m)+g*(e*x+d)^(-2+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(2-m)/c^2/d^2
/(2-m)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}(aeg+cd(f(-2+m)+g(-1+m)x))}{c^2d^2(-2+m)(-1+m)}$$

input
```
Integrate[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m,x]
```

output
```
-(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(a*e*g + c*d*(f*(-
2 + m) + g*(-1 + m)*x)))/(c^2*d^2*(-2 + m)*(-1 + m)))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{g(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)\int (d+ex)^m \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{-m}dx}{cde(2-m)}$$

$$\Big\downarrow \text{1122}$$

$$\frac{g(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^2d^2e(1-m)(2-m)}$$

input
```
Int[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
-(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g
*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 -
m))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

method	result
gosper	$-\frac{(ex+d)^{m} (cdgmx+cfmd-cdgx+aeg-2dfc) (cdx+ae) \left(cd\,x^{2} e+a\,e^{2} x+c\,d^{2} x+ade\right)^{-m}}{c^{2} d^{2} \left(m^{2}-3m+2\right)}$
orering	$-\frac{(cdgmx+cfmd-cdgx+aeg-2dfc) (cdx+ae) (ex+d)^{m} \left(ade+\left(a\,e^{2}+c\,d^{2}\right) x+cd\,x^{2} e\right)^{-m}}{c^{2} d^{2} \left(m^{2}-3m+2\right)}$
risch	$-\frac{\left(g\,x^{2} c^{2} d^{2} m+acdegmx+c^{2} d^{2} fmx-g\,x^{2} c^{2} d^{2}+acdefm-2c^{2} d^{2} fx+a^{2} e^{2} g-2acdef\right) (cdx+ae)^{-m} \mathrm{e}^{i\pi\ \operatorname{csgn}(i(ex+d)(cdx+ae))}}{c^{2} d^{2} (-2+m) (m-1)}$
parallelrisch	$-\frac{(x^{2} (ex+d)^{m} c^{2} d^{2} eg\,m^{2}-x^{2} (ex+d)^{m} c^{2} d^{2} egm+x (ex+d)^{m} acd\,e^{2} g\,m^{2}+x (ex+d)^{m} c^{2} d^{2} ef\,m^{2}-2x (ex+d)^{m} c^{2} d^{2} efm+(ex+}{me\,c^{2} d^{2} (-2+m) (m-1)}$

input
```
int((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x,method=_RETU
RNVERBOSE)
```

output
```
-(e*x+d)^m*(c*d*g*m*x+c*d*f*m-c*d*g*x+a*e*g-2*c*d*f)*(c*d*x+a*e)/((c*d*e*x
^2+a*e^2*x+c*d^2*x+a*d*e)^m)/c^2/d^2/(m^2-3*m+2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(acdefm-2\,acdef+a^2e^2g+(c^2d^2gm-c^2d^2g)x^2-(2\,c^2d^2f-(c^2d^2f+acdeg)m)x)(ex+d)^m}{(c^2d^2m^2-3\,c^2d^2m+2\,c^2d^2)(cdex^2+ade+(cd^2+ae^2)x)^m}$$

input
```
integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="fricas")
```

output
```
-(a*c*d*e*f*m - 2*a*c*d*e*f + a^2*e^2*g + (c^2*d^2*g*m - c^2*d^2*g)*x^2 -
(2*c^2*d^2*f - (c^2*d^2*f + a*c*d*e*g)*m)*x)*(e*x + d)^m/((c^2*d^2*m^2 - 3
*c^2*d^2*m + 2*c^2*d^2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)**m*(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(cdx+ae)f}{(cdx+ae)^m cd(m-1)} - \frac{(c^2d^2(m-1)x^2+acdemx+a^2e^2)g}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2}$$

input
```
integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="maxima")
```

output
```
-(c*d*x + a*e)*f/((c*d*x + a*e)^m*c*d*(m - 1)) - (c^2*d^2*(m - 1)*x^2 + a*
c*d*e*m*x + a^2*e^2)*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(118) = 236$.

Time = 0.22 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.89

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx =$$

$$-\frac{(ex+d)^m c^2d^2gmx^2e^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+d)^m c^2d^2fmxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+}{}$$

input
```
integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="giac")
```

output
```
-((e*x + d)^m*c^2*d^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (
e*x + d)^m*c^2*d^2*f*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x +
 d)^m*a*c*d*e*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - (e*x + d)^m
*c^2*d^2*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*a*c*
d*e*f*m*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 2*(e*x + d)^m*c^2*d^2*f
*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) - 2*(e*x + d)^m*a*c*d*e*f*e^(-
m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m*a^2*e^2*g*e^(-m*log(c*d
*x + a*e) - m*log(e*x + d)))/(c^2*d^2*m^2 - 3*c^2*d^2*m + 2*c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.16

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{\frac{g\,x^2\,(m-1)\,(d+e\,x)^m}{m^2-3\,m+2}+\frac{x\,(d+e\,x)^m\,(a\,e\,g\,m-2\,c\,d\,f+c\,d\,f\,m)}{c\,d\,(m^2-3\,m+2)}+\frac{a\,e\,(d+e\,x)^m\,(a\,e\,g-2\,c\,d\,f+c\,d\,f\,m)}{c^2\,d^2\,(m^2-3\,m+2)}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

input
```
int(((f + g*x)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
-((g*x^2*(m - 1)*(d + e*x)^m)/(m^2 - 3*m + 2) + (x*(d + e*x)^m*(a*e*g*m -
2*c*d*f + c*d*f*m))/(c*d*(m^2 - 3*m + 2)) + (a*e*(d + e*x)^m*(a*e*g - 2*c*
d*f + c*d*f*m))/(c^2*d^2*(m^2 - 3*m + 2)))/(x*(a*e^2 + c*d^2) + a*d*e + c*
d*e*x^2)^m
```

Reduce [F]

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \left(\int \frac{(ex+d)^m}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right)f + \left(\int \frac{(ex+d)^m\,x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right)g$$

input
```
int((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)**m/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m,x)*f + int(
((d + e*x)**m*x)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m,x)*g
```

3.115 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx$

Optimal result

Integrand size = 37, antiderivative size = 54

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

output
```
(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(1-m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = -\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}}{cd(-1+m)}$$

input
```
Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
-(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m))/(c*d*(-1 + m)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = $\{1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}dx$$

$$\Big\downarrow \; 1122$$

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

input
```
Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*
(1 - m))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result
gosper	$-\frac{(cdx+ae)(ex+d)^{m}\left(c d \,x^{2} e+a \,e^{2} x+c \,d^{2} x+a d e\right)^{-m}}{c d(m-1)}$
orering	$-\frac{(cdx+ae)(ex+d)^{m}\left(a d e+\left(a \,e^{2}+c \,d^{2}\right) x+c d \,x^{2} e\right)^{-m}}{c d(m-1)}$
parallelrisch	$\frac{\left(-x(ex+d)^{m} c d e-(ex+d)^{m} a \,e^{2}\right)\left(c d \,x^{2} e+a \,e^{2} x+c \,d^{2} x+a d e\right)^{-m}}{d e c(m-1)}$
norman	$\left(-\frac{x \,{\mathrm e}^{m \ln(ex+d)}}{m-1}-\frac{a e \,{\mathrm e}^{m \ln(ex+d)}}{c d(m-1)}\right) {\mathrm e}^{-m \ln\left(a d e+\left(a \,e^{2}+c \,d^{2}\right) x+c d \,x^{2} e\right)}$
risch	$-\frac{(cdx+ae)(cdx+ae)^{-m} {\mathrm e}^{\frac{i \pi \,\mathrm{csgn}(i(ex+d)(cdx+ae)) m(-\mathrm{csgn}(i(ex+d)(cdx+ae))+\mathrm{csgn}(i(cdx+ae)))(-\mathrm{csgn}(i(ex+d)(cdx+ae))+\mathrm{csgn}(i}{2}}}{c d(m-1)}$

input
```
int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x,method=_RETURNVERBOS
E)
```

output
```
-(c*d*x+a*e)/c/d/(m-1)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(cdx+ae)(ex+d)^m}{(cdm-cd)(cdex^2+ade+(cd^2+ae^2)x)^m}$$

input
```
integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fr
icas")
```

output
```
-(c*d*x + a*e)*(e*x + d)^m/((c*d*m - c*d)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)^m)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=-\frac{cdx+ae}{(cdx+ae)^m cd(m-1)}$$

input
```
integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="ma
xima")
```

output
```
-(c*d*x + a*e)/((c*d*x + a*e)^m*c*d*(m - 1))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(ex+d)^m cdxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(ex+d)^m aee^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cd}$$

input
```
integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="gi
ac")
```

output
```
-((e*x + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)) + (e*x + d)^m
*a*e*e^(-m*log(c*d*x + a*e) - m*log(e*x + d)))/(c*d*m - c*d)
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(a\,e+c\,d\,x)\,\left(d+e\,x\right)^m}{c\,d\,\left(m-1\right)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

input
```
int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
-((a*e + c*d*x)*(d + e*x)^m)/(c*d*(m - 1)*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^m)
```

Reduce [F]

$$\int (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(ex+d)^m}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m}dx$$

input
```
int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)**m/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m,x)
```

3.116 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 95

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}\text{Hypergeometric2F1}\left(1,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)(1-m)}$$

output
```
(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)*hypergeom([1, 1-m],
[2-m],-g*(c*d*x+a*e)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1-m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = -\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\text{Hypergeometric2F1}\left(1,1-m,2-m,\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)(-1+m)}$$

input
```
Integrate[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
),x]
```

output
```
-(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2F1[
1, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)*(
-1 + m)))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1268, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{f+gx}dx$$

$$\Big\downarrow \text{ 1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{f+gx}dx$$

$$\Big\downarrow \text{ 78}$$

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(1,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

input
```
Int[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

output
```
((a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[1, 1 - m, 2 - m, -((g*(a*e +
c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)*(1 - m)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{m} \left(ade+\left(a\,e^{2}+c\,d^{2}\right) x+cd\,x^{2}e\right)^{-m}}{gx+f}dx$$

input
```
int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(ex+d)^{m}}{(gx+f)(cdex^{2}+ade+(cd^{2}+ae^{2})x)^{m}}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="fricas")
```

output
```
integral((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m/(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(ex+d)^m}{(gx+f)(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="maxima")
```

output
```
integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
), x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(ex+d)^m}{(gx+f)(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input

```
integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algor
ithm="giac")
```

output

```
integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(d+e\,x)^m}{(f+g\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input

```
int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

output

```
int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \int \frac{(ex+d)^m}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m f+\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m gx}dx$$

input

```
int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output

```
int((d + e*x)**m/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*f + (a*d*e
 + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*g*x),x)
```

3.117 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 97

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \frac{cd(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}\text{Hypergeometric2F1}\left(2,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)^2(1-m)}$$

output

```
c*d*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)*hypergeom([2, 1
-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)^2/(1-m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = -\frac{cd(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\text{Hypergeometric2F1}\left(2,1-m,2-m,\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)^2(-1+m)}$$

input
```
Integrate[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m),x]
```

output
```
-((c*d*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric
2F1[2, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*
g)^2*(-1 + m)))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1268, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{(f+gx)^2}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{(f+gx)^2}dx$$

$$\Big\downarrow \text{78}$$

$$\frac{cd(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(2,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

input
```
Int[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

output
```
(c*d*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[2, 1 - m, 2 - m, -((g*(a*
e + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^2*(1 - m)*(a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{m} \left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{-m}}{\left(gx+f\right)^{2}}dx$$

input
```
int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \int \frac{(ex+d)^{m}}{(gx+f)^{2}(cdex^2+ade+(cd^2+ae^2)x)^{m}}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
integral((e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)^m), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m/(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx$$
$$=\int \frac{(ex+d)^m}{(gx+f)^2(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output
```
integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \int \frac{(ex+d)^m}{(gx+f)^2(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx = \int \frac{(d+e\,x)^m}{(f+g\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x
)
```

Reduce [F]

$$\int \frac{(d+ex)^m \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}}{(f+gx)^2}\,dx$$

$$= \int \frac{(ex+d)^m}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m f^2+2\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m fgx+\left(cde\,x^2+a\,e^2x+c\,d^2x\right.}$$

input

```
int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output

```
int((d + e*x)**m/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*f**2 + 2*(
a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*f*g*x + (a*d*e + a*e**2*x + c
*d**2*x + c*d*e*x**2)**m*g**2*x**2),x)
```

3.118 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 101

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \frac{c^2d^2(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}\text{Hypergeometric2F1}\left(3,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)^3(1-m)}$$

output

```
c^2*d^2*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)*hypergeom([
3, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)^3/(1-m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = -\frac{c^2d^2(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\text{Hypergeometric2F1}\left(3,1-m,2-m,\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)^3(-1+m)}$$

input

```
Integrate[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m),x]
```

output

```
-((c^2*d^2*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeome
tric2F1[3, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f -
a*e*g)^3*(-1 + m)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{1268, 78\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{(f+gx)^3}dx$$

$$\Big\downarrow \text{ 1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{(f+gx)^3}dx$$

$$\Big\downarrow \text{ 78}$$

$$\frac{c^2d^2(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(3,1-m,2-m,-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^3}$$

input

```
Int[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

output

```
(c^2*d^2*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[3, 1 - m, 2 - m, -((g
*(a*e + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^3*(1 - m)*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{m} \left(ade+\left(a\,e^{2}+c\,d^{2}\right) x+cd\,x^{2} e\right)^{-m}}{\left(gx+f\right)^{3}}dx$$

input
```
int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \int \frac{(ex+d)^{m}}{(gx+f)^{3}(cdex^{2}+ade+(cd^{2}+ae^{2})x)^{m}}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="fricas")
```

output
```
integral((e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m/(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),
x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$$
$$= \int \frac{(ex+d)^m}{(gx+f)^3(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="maxima")
```

output
```
integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \int \frac{(ex+d)^m}{(gx+f)^3(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, alg
orithm="giac")
```

output
```
integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx = \int \frac{(d+e\,x)^m}{(f+g\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x
)
```

Reduce [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$$

$$= \int \frac{(ex+d)^m}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m f^3+3\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m f^2gx+3\left(cde\,x^2+a\,e^2x+c\right.}$$

input
```
int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((d + e*x)**m/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*f**3 + 3*(
a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*f**2*g*x + 3*(a*d*e + a*e**2*
x + c*d**2*x + c*d*e*x**2)**m*f*g**2*x**2 + (a*d*e + a*e**2*x + c*d**2*x +
 c*d*e*x**2)**m*g**3*x**3),x)
```

3.119 $\int (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)$

Optimal result

Integrand size = 46, antiderivative size = 105

$$\int (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \, dx = \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m (f+gx)^{5/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \text{Hypergeometric2}}{5g}$$

output
```
2/5*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(5/2)*hypergeom([5
/2, m],[7/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^m)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} \, dx = \frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m ((ae+cdx)(d+ex))^{-m} (f+gx)^{5/2} \text{Hypergeometric2F1}\left(\frac{5}{2}, m, \frac{7}{2}\right.}{5g}$$

input
```
Integrate[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^m,x]
```

output
```
(2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hy
pergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*((a*e
+ c*d*x)*(d + e*x))^m)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^{3/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int (ae+cdx)^{-m}(f+gx)^{3/2}dx$$

$$\Big\downarrow \text{80}$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int (f+gx)^{3/2}\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2(f+gx)^{5/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(\frac{5}{2},m,\frac{7}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

input
```
Int[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m,x]
```

output
```
(2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hy
pergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \left(ex+d\right)^{m} \left(gx+f\right)^{\frac{3}{2}} \left(ade+\left(a\,e^{2}+c\,d^{2}\right) x+cd\,x^{2}e\right)^{-m}dx$$

input
```
int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
integral((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate((g*x + f)^(3/2)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(f+g\,x)^{3/2}\,(d+e\,x)^m}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m,x)
```

output
```
int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m, x)
```

Reduce [F]

$$\int (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \left(\int \frac{\sqrt{gx+f}\,(ex+d)^m\,x}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right)g + \left(\int \frac{\sqrt{gx+f}\,(ex+d)^m}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx\right)f$$

input

```
int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output

```
int((sqrt(f + g*x)*(d + e*x)**m*x)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x*
*2)**m,x)*g + int((sqrt(f + g*x)*(d + e*x)**m)/(a*d*e + a*e**2*x + c*d**2*
x + c*d*e*x**2)**m,x)*f
```

3.120 $\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal result

Integrand size = 46, antiderivative size = 105

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(\frac{3}{2},m,\frac{5}{2},\frac{cd(}{cd}\right.}{3g}$$

output
```
2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(3/2)*hypergeom([3
/2, m],[5/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^m)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},m,\frac{5}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

input
```
Integrate[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m,x]
```

output
```
(2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hy
pergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e
+ c*d*x)*(d + e*x))^m)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{f+gx}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \int (ae+cdx)^{-m}\sqrt{f+gx}dx$$

$$\Big\downarrow \text{80}$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \int \sqrt{f+gx}\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2(f+gx)^{3/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(\frac{3}{2},m,\frac{5}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

input
```
Int[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,
x]
```

output
```
(2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hy
pergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int (ex+d)^m \sqrt{gx+f}\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{-m}dx$$

input
```
int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \int \frac{\sqrt{gx+f}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \int \frac{\sqrt{gx+f}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m, x)
```

Giac [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int\frac{\sqrt{gx+f}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int\frac{\sqrt{f+g\,x}\,(d+e\,x)^m}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m,x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m, x)
```

Reduce [F]

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=\int \frac{\sqrt{gx+f}\,(ex+d)^m}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m}dx$$

input
```
int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((sqrt(f + g*x)*(d + e*x)**m)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2
)**m,x)
```

3.121 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 103

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},m,\frac{3}{2},\frac{cd(f+g}{cdf-a}\right.}{g}$$

output
```
2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(1/2)*hypergeom([1/2
, m],[3/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}\sqrt{f+gx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},m,\frac{3}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

input
```
Integrate[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^m),x]
```

output
```
(2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Sqrt[f + g*x]*Hype
rgeometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*
d*x)*(d + e*x))^m)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{80}$$

$$(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int \frac{\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2\sqrt{f+gx}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2},m,\frac{3}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

input
```
Int[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),
x]
```

output
```
(2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Sqrt[f + g*x]*Hype
rgeometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{m} \left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{-m}}{\sqrt{gx+f}}dx$$

input
```
int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(ex+d)^m}{\sqrt{gx+f}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
integral((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m/(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(ex+d)^m}{\sqrt{gx+f}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m(ade+(cd^2+ae^2)\,x+cdex^2)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(ex+d)^m}{\sqrt{gx+f}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(ade+(cd^2+ae^2)\,x+cdex^2)^{-m}}{\sqrt{f+gx}}\,dx = \int \frac{(d+e\,x)^m}{\sqrt{f+g\,x}\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
),x)
```

output
```
int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$$
$$=\int \frac{\sqrt{gx+f}\,(ex+d)^m}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m f+\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m gx}dx$$

input

```
int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output

```
int((sqrt(f + g*x)*(d + e*x)**m)/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**
2)**m*f + (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*g*x),x)
```

3.122 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 103

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(-\frac{1}{2},m,\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

output
```
-2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-1/2, m],[1/2],c
*d*(g*x+f)/(-a*e*g+c*d*f))/g/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^m)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = -\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}\text{Hypergeometric2F1}\left(-\frac{1}{2},m,\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

input
```
Integrate[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m),x]
```

output
```
(-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1
[-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*x)*(d + e*
x))^m*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{80}$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int \frac{\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{2(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(-\frac{1}{2},m,\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

input
```
Int[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
),x]
```

output
```
(-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1
[-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*Sqrt[f + g*x]*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])    Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])    Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(a e^2+c d^2\right) x+cd x^2 e\right)^{-m}}{(gx+f)^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input

```
integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output

```
integral(sqrt(g*x + f)*(e*x + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\text{Timed out}$$

input

```
integrate((e*x+d)**m/(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input

```
integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output

```
integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{3}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate((e*x + d)^m/((g*x + f)^(3/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\int \frac{(d+e\,x)^m}{(f+g\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
),x)
```

output
```
int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx=\int \frac{\sqrt{g}}{(cde\,x^2+a\,e^2x+c\,d^2x+ade)^m\,f^2+2\,(cde\,x^2+a}$$

input
```
int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int((sqrt(f + g*x)*(d + e*x)**m)/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**
2)**m*f**2 + 2*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*f*g*x + (a*d*
e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*g**2*x**2),x)
```

3.123 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 105

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx = -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\text{Hypergeometric2F1}\left(-\frac{3}{2},m,-\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

output
```
-2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-3/2, m],[-1/2
],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^m)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx = -\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m((ae+cdx)(d+ex))^{-m}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},m,-\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

input
```
Integrate[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m),x]
```

output
```
(-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1
[-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e + c*d*x)*(d +
 e*x))^m*(f + g*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int \frac{(ae+cdx)^{-m}}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{80}$$

$$(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\int \frac{\left(-\frac{cdxg}{cdf-aeg}-\frac{aeg}{cdf-aeg}\right)^{-m}}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{2(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m \text{Hypergeometric2F1}\left(-\frac{3}{2},m,-\frac{1}{2},\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

input
```
Int[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
),x]
```

output
```
(-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1
[-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(f + g*x)^(3/2)*(a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^m \left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{-m}}{(gx+f)^{\frac{5}{2}}}dx$$

input
```
int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
integral(sqrt(g*x + f)*(e*x + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f
^3)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m/(g*x+f)**(5/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^m), x)
```

Giac [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(ex+d)^m}{(gx+f)^{\frac{5}{2}}(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
integrate((e*x + d)^m/((g*x + f)^(5/2)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{(d+e\,x)^m}{(f+g\,x)^{5/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
),x)
```

output
```
int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx=\int \frac{}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m f^3+3\left(cde\,x^2+a\right.}$$

input
```
int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output

```
int((sqrt(f + g*x)*(d + e*x)**m)/((a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**
2)**m*f**3 + 3*(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*f**2*g*x + 3*
(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**2)**m*f*g**2*x**2 + (a*d*e + a*e**
2*x + c*d**2*x + c*d*e*x**2)**m*g**3*x**3),x)
```

3.124 $\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)$

Optimal result

Integrand size = 47, antiderivative size = 63

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(ae+cdx)^{1+n}(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{cd(1-m+n)}$$

output

```
(c*d*x+a*e)^(1+n)*(e*x+d)^m/c/d/(1-m+n)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(ae+cdx)^{1+n}(d+ex)^m((ae+cdx)(d+ex))^{-m}}{cd(1-m+n)}$$

input

```
Integrate[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^m,x]
```

output
```
((a*e + c*d*x)^(1 + n)*(d + e*x)^m)/(c*d*(1 - m + n)*((a*e + c*d*x)*(d + e
*x))^m)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = $\{1247\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} (ae+cdx)^n \, dx$$

$$\Big\downarrow \; 1247$$

$$\frac{(d+ex)^{m-1} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m} (ae+cdx)^n}{cd(-m+n+1)}$$

input
```
Int[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m,x]
```

output
```
((a*e + c*d*x)^n*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(1 - m))/(c*d*(1 - m + n))
```

Defintions of rubi rules used

rule 1247
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && EqQ[c
*e*f + c*d*g - b*e*g, 0] && NeQ[m - n - 1, 0]
```

Maple [A] (verified)

Time = 9.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result
gosper	$-\frac{(ex+d)^{m} (cdx+ae)^{1+n} \left(cd\,x^{2} e+a\,e^{2} x+c\,d^{2} x+ade\right)^{-m}}{cd(m-n-1)}$
orering	$-\frac{(cdx+ae) (cdx+ae)^{n} (ex+d)^{m} \left(ade+\left(a\,e^{2}+c\,d^{2}\right) x+cd\,x^{2} e\right)^{-m}}{cd(m-n-1)}$
parallelrisch	$-\frac{\left(x (ex+d)^{m} (cdx+ae)^{n} cdem+(ex+d)^{m} (cdx+ae)^{n} a\,e^{2} m\right) \left(cd\,x^{2} e+a\,e^{2} x+c\,d^{2} x+ade\right)^{-m}}{mdec(m-n-1)}$
risch	$-\frac{(cdx+ae)^{n} (cdx+ae) (cdx+ae)^{-m} \mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(i(ex+d)(cdx+ae)) m(-\mathrm{csgn}(i(ex+d)(cdx+ae))+\mathrm{csgn}(i(cdx+ae)))(-\mathrm{csgn}(i(ex+d)(cdx+}{2}}}{cd(m-n-1)}$

input
```
int((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^m),x,method
=_RETURNVERBOSE)
```

output
```
-1/c/d/(m-n-1)*(e*x+d)^m*(c*d*x+a*e)^(1+n)/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d
*e)^m)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int (ae+cdx)^n (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(cdx+ae)(cdx+ae)^n (ex+d)^m e^{(-m\log(cdx+ae)-m\log(ex+d))}}{cdm-cdn-cd}$$

input
```
integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="fricas")
```

output
```
-(c*d*x + a*e)*(c*d*x + a*e)^n*(e*x + d)^m*e^(-m*log(c*d*x + a*e) - m*log(
e*x + d))/(c*d*m - c*d*n - c*d)
```

Sympy [F(-1)]

Timed out.

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

input
```
integrate((c*d*x+a*e)**n*(e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*
*m),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$$
$$=-\frac{(cdx+ae)e^{(-m\log(cdx+ae)+n\log(cdx+ae))}}{cd(m-n-1)}$$

input
```
integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="maxima")
```

output
```
-(c*d*x + a*e)*e^(-m*log(c*d*x + a*e) + n*log(c*d*x + a*e))/(c*d*(m - n -
1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=$$
$$-\frac{(cdx+ae)^n(ex+d)^m cdxe^{(-m\log(cdx+ae)-m\log(ex+d))}+(cdx+ae)^n(ex+d)^m aee^{(-m\log(cdx+ae)-m\log(ex+}}{cdm-cdn-cd}$$

input
```
integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
 algorithm="giac")
```

output
```
-((c*d*x + a*e)^n*(e*x + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(e*x + d
)) + (c*d*x + a*e)^n*(e*x + d)^m*a*e*e^(-m*log(c*d*x + a*e) - m*log(e*x +
d)))/(c*d*m - c*d*n - c*d)
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{{\left(a\,e+c\,d\,x\right)}^{n+1}\,{\left(d+e\,x\right)}^m}{c\,d\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m\,\left(n-m+1\right)}$$

input
```
int(((a*e + c*d*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
m,x)
```

output
```
((a*e + c*d*x)^(n + 1)*(d + e*x)^m)/(c*d*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^m*(n - m + 1))
```

Reduce [F]

$$\int (ae+cdx)^n(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \int \frac{\left(ex+d\right)^{m}\left(cdx+ae\right)^{n}}{\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{m}}dx$$

input
```
int((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output
```
int(((d + e*x)**m*(a*e + c*d*x)**n)/(a*d*e + a*e**2*x + c*d**2*x + c*d*e*x
**2)**m,x)
```

3.125 $\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m} ($

Optimal result

Integrand size = 73, antiderivative size = 78

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{(d+ex)^m\left(-ae^3g-cde^2gx\right)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\log(ae+cdx)}{cde^2g}$$

output
```
-(e*x+d)^m*(-c*d*e^2*g*x-a*e^3*g)^m*ln(c*d*x+a*e)/c/d/e^2/g/((a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = -\frac{\left(-e^2g(ae+cdx)\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}\log(ae+cdx)}{cde^2g}$$

input
```
Integrate[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1
 + m))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
-(((-(e^2*g*(a*e + c*d*x)))^m*(d + e*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*((a
*e + c*d*x)*(d + e*x))^m))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.041$, Rules used = $\{1268, 37, 16\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-eg\left(ae^2+cd^2\right)+cd^2eg-cde^2gx\right)^{m-1}dx$$

$$\Big\downarrow \text{1268}$$

$$(d+ex)^m(ae+cdx)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\int (ae+cdx)^{-m}\left(-age^3-cdgxe^2\right)^{m-1}dx$$

$$\Big\downarrow \text{37}$$

$$(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-ae^3g-cde^2gx\right)^{m-1}\int \frac{1}{ae+cdx}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\log(ae+cdx)\left(-ae^3g-cde^2gx\right)^{m-1}}{cd}$$

input
```
Int[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1 + m))
/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

output
```
((a*e + c*d*x)*(d + e*x)^m*(-(a*e^3*g) - c*d*e^2*g*x)^(-1 + m)*Log[a*e + c
*d*x])/(c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int (ex+d)^m \left(c\,d^2eg-e\left(a\,e^2+c\,d^2\right)g-cd\,e^2gx\right)^{m-1}\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{-m}dx$$

input
```
int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(m-1)/((a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

output
```
int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(m-1)/((a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^m),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

$$\begin{aligned}\int (d+ex)^m\left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}&\left(ade\right.\\ \left.+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx&=-\frac{\log\left(cdx+ae\right)}{cde^2g\left(-\frac{1}{e^2g}\right)^m}\end{aligned}$$

input
```
integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fricas")
```

output
```
-log(c*d*x + a*e)/(c*d*e^2*g*(-1/(e^2*g))^m)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m*(c*d**2*e*g-e*(a*e**2+c*d**2)*g-c*d*e**2*g*x)**(-1+m)
/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.41

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=-\frac{e^{2m-2}(-g)^m\log\left(cdx+ae\right)}{cdg}$$

input
```
integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="maxima")
```

output
```
-e^(2*m - 2)*(-g)^m*log(c*d*x + a*e)/(c*d*g)
```

Giac [F]

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(-cde^2gx+cd^2eg-(cd^2+ae^2)eg)^{m-1}(ex+d)^m}{(cdex^2+ade+(cd^2+ae^2)x)^m}\,dx$$

input
```
integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="giac")
```

output
```
integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m - 1)*(e*x +
d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\int \frac{(d+e\,x)^m\,\left(c\,d^2\,e\,g-e\,g\,\left(c\,d^2+a\,e^2\right)-c\,d\,e^2\,g\,x\right)^{m-1}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

input
```
int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

output
```
int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=$$
$$-\frac{\int \frac{(ex+d)^m\left(-cd\,e^2gx-a\,e^3g\right)^m}{\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m ae+\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^m cdx}dx}{e^2g}$$

input

```
int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

output

```
( - int(((d + e*x)**m*( - a*e**3*g - c*d*e**2*g*x)**m)/((a*d*e + a*e**2*x
+ c*d**2*x + c*d*e*x**2)**m*a*e + (a*d*e + a*e**2*x + c*d**2*x + c*d*e*x**
2)**m*c*d*x),x))/(e**2*g)
```

3.126 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 214

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2e(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{2(2ae^2g(1+n)+cd(ef-dg(3+2n)))(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\ \text{Hypergeo}}{c^2d^2g(3+2n)\sqrt{d+ex}}$$

output
```
2*e*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(3+2*n)/(e
*x+d)^(1/2)-2*(2*a*e^2*g*(1+n)+c*d*(e*f-d*g*(3+2*n)))*(g*x+f)^n*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*hypergeom([1/2, -n],[3/2],-g*(c*d*x+a*e)/(-a
*e*g+c*d*f))/c^2/d^2/g/(3+2*n)/(e*x+d)^(1/2)/((c*d*(g*x+f)/(-a*e*g+c*d*f))
^n)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}(f+gx)^n\left(cde(f+gx)+(-2ae^2g(1+n)+\right.}{c^2d^2}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*(c*d*e*(f + g*x) + ((-2*a*e^2*g
*(1 + n) + c*d*(-(e*f) + d*g*(3 + 2*n)))*Hypergeometric2F1[1/2, -n, 3/2, (
g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
)/(c^2*d^2*g*(3/2 + n)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1258, 1268, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1258}$$

$$\frac{2e(f+gx)^{n+1}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{(2ae^2g(n+1)+cd(ef-dg(2n+3)))\int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cdg(2n+3)}$$

$$\Big\downarrow \text{1268}$$

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\int\frac{(f+gx)^n}{\sqrt{ae+cdx}}dx}{cdg(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 80$$

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{\sqrt{d+ex}(f+gx)^n\sqrt{ae+cdx}\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\int\frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{\sqrt{ae+cdx}}dx}{cdg(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 79$$

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\text{Hypergeometric2F1}\Big(\frac{1}{2},-n,}{c^2d^2g(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input

```
Int[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output

```
(2*e*(f + g*x)^(1 + n)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g
*(3 + 2*n)*Sqrt[d + e*x]) - (2*(2*a*e^2*g*(1 + n) + c*d*(e*f - d*g*(3 + 2*
n)))*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/
2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c^2*d^2*g*(3 + 2*n)*((c*d*(f +
g*x))/(c*d*f - a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1258
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{\frac{3}{2}} (gx+f)^{n}}{\sqrt{ade+(a\,e^{2}+c\,d^{2})\,x+cd\,x^{2}e}}dx$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x)
```

output
```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*(g*x +
f)^n/(c*d*x + a*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(g*x + f)^n/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(f+g\,x)^n\,(d+e\,x)^{3/2}}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{too large to display}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(2*( - 2*(f + g*x)**n*sqrt(a*e + c*d*x)*a*e**2*f + 2*(f + g*x)**n*sqrt(a*e
 + c*d*x)*a*e**2*g*n*x + 2*(f + g*x)**n*sqrt(a*e + c*d*x)*c*d**2*f*n + 3*(
f + g*x)**n*sqrt(a*e + c*d*x)*c*d**2*f + (f + g*x)**n*sqrt(a*e + c*d*x)*c*
d*e*f*x - 8*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(4*a**2*e**2*f*g*n**2 +
 6*a**2*e**2*f*g*n + 4*a**2*e**2*g**2*n**2*x + 6*a**2*e**2*g**2*n*x + 2*a*
c*d*e*f**2*n + 3*a*c*d*e*f**2 + 4*a*c*d*e*f*g*n**2*x + 8*a*c*d*e*f*g*n*x +
 3*a*c*d*e*f*g*x + 4*a*c*d*e*g**2*n**2*x**2 + 6*a*c*d*e*g**2*n*x**2 + 2*c*
*2*d**2*f**2*n*x + 3*c**2*d**2*f**2*x + 2*c**2*d**2*f*g*n*x**2 + 3*c**2*d*
*2*f*g*x**2),x)*a**3*e**4*g**3*n**4 - 20*int(((f + g*x)**n*sqrt(a*e + c*d*
x)*x)/(4*a**2*e**2*f*g*n**2 + 6*a**2*e**2*f*g*n + 4*a**2*e**2*g**2*n**2*x
+ 6*a**2*e**2*g**2*n*x + 2*a*c*d*e*f**2*n + 3*a*c*d*e*f**2 + 4*a*c*d*e*f*g
*n**2*x + 8*a*c*d*e*f*g*n*x + 3*a*c*d*e*f*g*x + 4*a*c*d*e*g**2*n**2*x**2 +
 6*a*c*d*e*g**2*n*x**2 + 2*c**2*d**2*f**2*n*x + 3*c**2*d**2*f**2*x + 2*c**
2*d**2*f*g*n*x**2 + 3*c**2*d**2*f*g*x**2),x)*a**3*e**4*g**3*n**3 - 12*int(
((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(4*a**2*e**2*f*g*n**2 + 6*a**2*e**2*f*g
*n + 4*a**2*e**2*g**2*n**2*x + 6*a**2*e**2*g**2*n*x + 2*a*c*d*e*f**2*n + 3
*a*c*d*e*f**2 + 4*a*c*d*e*f*g*n**2*x + 8*a*c*d*e*f*g*n*x + 3*a*c*d*e*f*g*x
 + 4*a*c*d*e*g**2*n**2*x**2 + 6*a*c*d*e*g**2*n*x**2 + 2*c**2*d**2*f**2*n*x
 + 3*c**2*d**2*f**2*x + 2*c**2*d**2*f*g*n*x**2 + 3*c**2*d**2*f*g*x**2),x)*
a**3*e**4*g**3*n**2 + 8*int(((f + g*x)**n*sqrt(a*e + c*d*x)*x)/(4*a**2*...
```

3.127 $\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 438

$$\begin{aligned}
\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx &= \frac{2(cd^2-ae^2)(cdf-aeg)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^6d^6\sqrt{d+ex}} \\
&- \frac{2(cdf-aeg)^3(5ae^2g-cd(ef+4dg))(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3c^6d^6(d+ex)^{3/2}} \\
&- \frac{4g(cdf-aeg)^2(5ae^2g-cd(2ef+3dg))(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5c^6d^6(d+ex)^{5/2}} \\
&- \frac{4g^2(cdf-aeg)(5ae^2g-cd(3ef+2dg))(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{7c^6d^6(d+ex)^{7/2}} \\
&- \frac{2g^3(5ae^2g-cd(4ef+dg))(ade+(cd^2+ae^2)x+cdex^2)^{9/2}}{9c^6d^6(d+ex)^{9/2}} \\
&+ \frac{2eg^4(ade+(cd^2+ae^2)x+cdex^2)^{11/2}}{11c^6d^6(d+ex)^{11/2}}
\end{aligned}$$

output

```
2*(-a*e^2+c*d^2)*(-a*e*g+c*d*f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
c^6/d^6/(e*x+d)^(1/2)-2/3*(-a*e*g+c*d*f)^3*(5*a*e^2*g-c*d*(4*d*g+e*f))*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^6/d^6/(e*x+d)^(3/2)-4/5*g*(-a*e*g+c
*d*f)^2*(5*a*e^2*g-c*d*(3*d*g+2*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2)/c^6/d^6/(e*x+d)^(5/2)-4/7*g^2*(-a*e*g+c*d*f)*(5*a*e^2*g-c*d*(2*d*g+3*e
*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^6/d^6/(e*x+d)^(7/2)-2/9*g^3
*(5*a*e^2*g-c*d*(d*g+4*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(9/2)/c^6/d
^6/(e*x+d)^(9/2)+2/11*e*g^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(11/2)/c^6/d
^6/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-1280a^5e^6g^4+128a^4cde^4g^3(44ef+11dg}{}$$

input

```
Integrate[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output

```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-1280*a^5*e^6*g^4 + 128*a^4*c*d*e^4*g^3*
(44*e*f + 11*d*g + 5*e*g*x) - 32*a^3*c^2*d^2*e^3*g^2*(22*d*g*(9*f + g*x) +
 e*(297*f^2 + 88*f*g*x + 15*g^2*x^2)) + 16*a^2*c^3*d^3*e^2*g*(33*d*g*(21*f
^2 + 6*f*g*x + g^2*x^2) + e*(462*f^3 + 297*f^2*g*x + 132*f*g^2*x^2 + 25*g^
3*x^3)) - 2*a*c^4*d^4*e*(44*d*g*(105*f^3 + 63*f^2*g*x + 27*f*g^2*x^2 + 5*g
^3*x^3) + e*(1155*f^4 + 1848*f^3*g*x + 1782*f^2*g^2*x^2 + 880*f*g^3*x^3 +
175*g^4*x^4)) + c^5*d^5*(11*d*(315*f^4 + 420*f^3*g*x + 378*f^2*g^2*x^2 + 1
80*f*g^3*x^3 + 35*g^4*x^4) + e*x*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x
^2 + 1540*f*g^3*x^3 + 315*g^4*x^4))))/(3465*c^6*d^6*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1258, 1253, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \; 1258$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\int \frac{\sqrt{d+ex}(f+gx)^4}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2e(f+gx)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$$\Big\downarrow \; 1253$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\Bigg(\frac{8(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{9cd}+\frac{2(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{9cd\sqrt{d+ex}}$$
$$\frac{2e(f+gx)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$$\Big\downarrow \; 1253$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\left(\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{7cd}+\frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cd\sqrt{d+ex}}\right)}{9cd}\right.$$
$$\frac{2e(f+gx)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$$\Big\downarrow \; 1253$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\left(\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cd}}{5cd\sqrt{d+ex}}\right.}{7cd}\right.}{9cd}\right.$$

$$\frac{2e(f+gx)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\left(\frac{8(cdf-aeg)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{}}{}\right.}{5cd}\right.}{7cd}\right.}{9}\right.$$

$$\frac{2e(f+gx)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2e(f+gx)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{11cdg\sqrt{d+ex}}+$$

$$\frac{1}{11}\left(-\frac{10ae^2}{cd}+11d-\frac{ef}{g}\right)\left(\frac{2(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cd\sqrt{d+ex}}+\frac{8(cdf-aeg)\left(\frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade}}{7cd\sqrt{d+ex}}\right.}{}\right.$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output
```
(2*e*(f + g*x)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(11*c*d*g*Sq
rt[d + e*x]) + ((11*d - (10*a*e^2)/(c*d) - (e*f)/g)*((2*(f + g*x)^4*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(9*c*d*Sqrt[d + e*x]) + (8*(c*d*f -
 a*e*g)*((2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*
d*Sqrt[d + e*x]) + (6*(c*d*f - a*e*g)*((2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d + e*x]) + (4*(c*d*f - a*e*g)*((2*(3
*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(3*c*d*e)))/(5*c*d)))/(7*c*d)))/(9*c*d)))/11
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1258
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.42

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-315e\,g^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4d^5e\,g^4x^3}{}$
gosper	$-\frac{2(cdx+ae)(-315e\,g^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4d^5e\,g^4x^3+1760a\,c^4}{}$
orering	$-\frac{2(-315e\,g^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4d^5e\,g^4x^3+1760a\,c^4d^4e^2f\,g^3}{}$

input
```
int((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/3465/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-315*c^5*d^5*e*g^4*x^5+
350*a*c^4*d^4*e^2*g^4*x^4-385*c^5*d^6*g^4*x^4-1540*c^5*d^5*e*f*g^3*x^4-400
*a^2*c^3*d^3*e^3*g^4*x^3+440*a*c^4*d^5*e*g^4*x^3+1760*a*c^4*d^4*e^2*f*g^3*
x^3-1980*c^5*d^6*f*g^3*x^3-2970*c^5*d^5*e*f^2*g^2*x^3+480*a^3*c^2*d^2*e^4*
g^4*x^2-528*a^2*c^3*d^4*e^2*g^4*x^2-2112*a^2*c^3*d^3*e^3*f*g^3*x^2+2376*a*
c^4*d^5*e*f*g^3*x^2+3564*a*c^4*d^4*e^2*f^2*g^2*x^2-4158*c^5*d^6*f^2*g^2*x^
2-2772*c^5*d^5*e*f^3*g*x^2-640*a^4*c*d*e^5*g^4*x+704*a^3*c^2*d^3*e^3*g^4*x
+2816*a^3*c^2*d^2*e^4*f*g^3*x-3168*a^2*c^3*d^4*e^2*f*g^3*x-4752*a^2*c^3*d^
3*e^3*f^2*g^2*x+5544*a*c^4*d^5*e*f^2*g^2*x+3696*a*c^4*d^4*e^2*f^3*g*x-4620
*c^5*d^6*f^3*g*x-1155*c^5*d^5*e*f^4*x+1280*a^5*e^6*g^4-1408*a^4*c*d^2*e^4*
g^4-5632*a^4*c*d*e^5*f*g^3+6336*a^3*c^2*d^3*e^3*f*g^3+9504*a^3*c^2*d^2*e^4
*f^2*g^2-11088*a^2*c^3*d^4*e^2*f^2*g^2-7392*a^2*c^3*d^3*e^3*f^3*g+9240*a*c
^4*d^5*e*f^3*g+2310*a*c^4*d^4*e^2*f^4-3465*c^5*d^6*f^4)/c^6/d^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,(315\,c^5d^5eg^4x^5+1155\,(3\,c^5d^6-2\,ac^4d^4e^2)\,f^4-1848\,(5\,ac^4d^5e-}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
2/3465*(315*c^5*d^5*e*g^4*x^5 + 1155*(3*c^5*d^6 - 2*a*c^4*d^4*e^2)*f^4 - 1
848*(5*a*c^4*d^5*e - 4*a^2*c^3*d^3*e^3)*f^3*g + 1584*(7*a^2*c^3*d^4*e^2 -
6*a^3*c^2*d^2*e^4)*f^2*g^2 - 704*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*f*g^3
 + 128*(11*a^4*c*d^2*e^4 - 10*a^5*e^6)*g^4 + 35*(44*c^5*d^5*e*f*g^3 + (11*
c^5*d^6 - 10*a*c^4*d^4*e^2)*g^4)*x^4 + 10*(297*c^5*d^5*e*f^2*g^2 + 22*(9*c
^5*d^6 - 8*a*c^4*d^4*e^2)*f*g^3 - 4*(11*a*c^4*d^5*e - 10*a^2*c^3*d^3*e^3)*
g^4)*x^3 + 6*(462*c^5*d^5*e*f^3*g + 99*(7*c^5*d^6 - 6*a*c^4*d^4*e^2)*f^2*g
^2 - 44*(9*a*c^4*d^5*e - 8*a^2*c^3*d^3*e^3)*f*g^3 + 8*(11*a^2*c^3*d^4*e^2
- 10*a^3*c^2*d^2*e^4)*g^4)*x^2 + (1155*c^5*d^5*e*f^4 + 924*(5*c^5*d^6 - 4*
a*c^4*d^4*e^2)*f^3*g - 792*(7*a*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^2*g^2 + 3
52*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*f*g^3 - 64*(11*a^3*c^2*d^3*e^3
- 10*a^4*c*d*e^5)*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e*x + d)/(c^6*d^6*e*x + c^6*d^7)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^4}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)**4/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f^4}{3\,\sqrt{cdx+ae}c^2d^2}$$
$$+\frac{8\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)f^3g}{15\,\sqrt{cdx+ae}c^3d^3}$$
$$+\frac{4\,(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2}{35\,\sqrt{cdx+ae}c^4d^4}$$
$$+\frac{8\,(35\,c^5d^5ex^5-144\,a^4cd^2e^4+128\,a^5e^6+5\,(9\,c^5d^6-ac^4d^4e^2)x^4-(9\,ac^4d^5e-8\,a^2c^3d^3e^3)x^3+2\,(9\,a^2c^3d^4}{315\,\sqrt{cdx+ae}c^5d^5}$$
$$+\frac{2\,(315\,c^6d^6ex^6+1408\,a^5cd^2e^5-1280\,a^6e^7+35\,(11\,c^6d^7-ac^5d^5e^2)x^5-5\,(11\,ac^5d^6e-10\,a^2c^4d^4e^3)x^4+}{3465\,\sqrt{cdx}}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*
f^4/(sqrt(c*d*x + a*e)*c^2*d^2) + 8/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2
 + 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*
d*e^3)*x)*f^3*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 4/35*(15*c^4*d^4*e*x^4 + 56*
a^3*c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*
d^4*e - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*
f^2*g^2/(sqrt(c*d*x + a*e)*c^4*d^4) + 8/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*
d^2*e^4 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e
 - 8*a^2*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2
- 8*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*f*g^3/(sqrt(c*d*x + a*e)*c^5*d^
5) + 2/3465*(315*c^6*d^6*e*x^6 + 1408*a^5*c*d^2*e^5 - 1280*a^6*e^7 + 35*(1
1*c^6*d^7 - a*c^5*d^5*e^2)*x^5 - 5*(11*a*c^5*d^6*e - 10*a^2*c^4*d^4*e^3)*x
^4 + 8*(11*a^2*c^4*d^5*e^2 - 10*a^3*c^3*d^3*e^4)*x^3 - 16*(11*a^3*c^3*d^4*
e^3 - 10*a^4*c^2*d^2*e^5)*x^2 + 64*(11*a^4*c^2*d^3*e^4 - 10*a^5*c*d*e^6)*x
)*g^4/(sqrt(c*d*x + a*e)*c^6*d^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(404) = 808$.

Time $= 0.14$ (sec) , antiderivative size $= 1159$, normalized size of antiderivative $= 2.65$

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output

```
2/3465*e*(3465*(c^5*d^6*f^4 - a*c^4*d^4*e^2*f^4 - 4*a*c^4*d^5*e*f^3*g + 4*
a^2*c^3*d^3*e^3*f^3*g + 6*a^2*c^3*d^4*e^2*f^2*g^2 - 6*a^3*c^2*d^2*e^4*f^2*
g^2 - 4*a^3*c^2*d^3*e^3*f*g^3 + 4*a^4*c*d*e^5*f*g^3 + a^4*c*d^2*e^4*g^4 -
a^5*e^6*g^4)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^6*d^6*e) + (1155*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^4*e^8*f^4 + 4620*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^5*e^7*f^3*g - 9240*((e*x + d)*c*d*e
- c*d^2*e + a*e^3)^(3/2)*a*c^3*d^3*e^9*f^3*g - 13860*((e*x + d)*c*d*e - c*
d^2*e + a*e^3)^(3/2)*a*c^3*d^4*e^8*f^2*g^2 + 20790*((e*x + d)*c*d*e - c*d^
2*e + a*e^3)^(3/2)*a^2*c^2*d^2*e^10*f^2*g^2 + 13860*((e*x + d)*c*d*e - c*d
^2*e + a*e^3)^(3/2)*a^2*c^2*d^3*e^9*f*g^3 - 18480*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(3/2)*a^3*c*d*e^11*f*g^3 - 4620*((e*x + d)*c*d*e - c*d^2*e + a
*e^3)^(3/2)*a^3*c*d^2*e^10*g^4 + 5775*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^4*e^12*g^4 + 2772*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^
3*e^6*f^3*g + 4158*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^4*e^5*f
^2*g^2 - 12474*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^2*d^2*e^7*f^2
*g^2 - 8316*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^2*d^3*e^6*f*g^3
+ 16632*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*c*d*e^8*f*g^3 + 4158
*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*c*d^2*e^7*g^4 - 6930*((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9*g^4 + 2970*((e*x + d)*c*d*e -
c*d^2*e + a*e^3)^(7/2)*c^2*d^2*e^4*f^2*g^2 + 1980*((e*x + d)*c*d*e - c*...
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^4\,x^5\,\sqrt{d+e\,x}}{11\,c\,d}-\frac{\sqrt{d+e\,x}\,\left(2560\,a^5\,e^6\,g^4\right.}{}\right.}{}$$

input
```
int(((f + g*x)^4*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5*(d + e*x)^(1/2)
)/(11*c*d) - ((d + e*x)^(1/2)*(2560*a^5*e^6*g^4 - 6930*c^5*d^6*f^4 + 4620*
a*c^4*d^4*e^2*f^4 - 2816*a^4*c*d^2*e^4*g^4 - 14784*a^2*c^3*d^3*e^3*f^3*g +
 12672*a^3*c^2*d^3*e^3*f*g^3 + 18480*a*c^4*d^5*e*f^3*g - 11264*a^4*c*d*e^5
*f*g^3 - 22176*a^2*c^3*d^4*e^2*f^2*g^2 + 19008*a^3*c^2*d^2*e^4*f^2*g^2))/(
3465*c^6*d^6*e) + (x*(d + e*x)^(1/2)*(2310*c^5*d^5*e*f^4 + 9240*c^5*d^6*f^
3*g - 1408*a^3*c^2*d^3*e^3*g^4 + 1280*a^4*c*d*e^5*g^4 - 7392*a*c^4*d^4*e^2
*f^3*g - 11088*a*c^4*d^5*e*f^2*g^2 + 6336*a^2*c^3*d^4*e^2*f*g^3 - 5632*a^3
*c^2*d^2*e^4*f*g^3 + 9504*a^2*c^3*d^3*e^3*f^2*g^2))/(3465*c^6*d^6*e) + (x^
2*(d + e*x)^(1/2)*(8316*c^5*d^6*f^2*g^2 + 1056*a^2*c^3*d^4*e^2*g^4 - 960*a
^3*c^2*d^2*e^4*g^4 + 5544*c^5*d^5*e*f^3*g - 7128*a*c^4*d^4*e^2*f^2*g^2 + 4
224*a^2*c^3*d^3*e^3*f*g^3 - 4752*a*c^4*d^5*e*f*g^3))/(3465*c^6*d^6*e) + (4
*g^2*x^3*(d + e*x)^(1/2)*(40*a^2*e^3*g^2 + 297*c^2*d^2*e*f^2 + 198*c^2*d^3
*f*g - 44*a*c*d^2*e*g^2 - 176*a*c*d*e^2*f*g))/(693*c^3*d^3*e) + (2*g^3*x^4
*(d + e*x)^(1/2)*(11*c*d^2*g - 10*a*e^2*g + 44*c*d*e*f))/(99*c^2*d^2*e)))/
(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}\,(315c^5d^5e\,g^4x^5-350a\,c^4d^4e^2g^4x^4+385c^5d^6g^4x^4+15}{}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(2*sqrt(a*e + c*d*x)*( - 1280*a**5*e**6*g**4 + 1408*a**4*c*d**2*e**4*g**4
+ 5632*a**4*c*d*e**5*f*g**3 + 640*a**4*c*d*e**5*g**4*x - 6336*a**3*c**2*d*
*3*e**3*f*g**3 - 704*a**3*c**2*d**3*e**3*g**4*x - 9504*a**3*c**2*d**2*e**4
*f**2*g**2 - 2816*a**3*c**2*d**2*e**4*f*g**3*x - 480*a**3*c**2*d**2*e**4*g
**4*x**2 + 11088*a**2*c**3*d**4*e**2*f**2*g**2 + 3168*a**2*c**3*d**4*e**2*
f*g**3*x + 528*a**2*c**3*d**4*e**2*g**4*x**2 + 7392*a**2*c**3*d**3*e**3*f*
*3*g + 4752*a**2*c**3*d**3*e**3*f**2*g**2*x + 2112*a**2*c**3*d**3*e**3*f*g
**3*x**2 + 400*a**2*c**3*d**3*e**3*g**4*x**3 - 9240*a*c**4*d**5*e*f**3*g -
 5544*a*c**4*d**5*e*f**2*g**2*x - 2376*a*c**4*d**5*e*f*g**3*x**2 - 440*a*c
**4*d**5*e*g**4*x**3 - 2310*a*c**4*d**4*e**2*f**4 - 3696*a*c**4*d**4*e**2*
f**3*g*x - 3564*a*c**4*d**4*e**2*f**2*g**2*x**2 - 1760*a*c**4*d**4*e**2*f*
g**3*x**3 - 350*a*c**4*d**4*e**2*g**4*x**4 + 3465*c**5*d**6*f**4 + 4620*c*
*5*d**6*f**3*g*x + 4158*c**5*d**6*f**2*g**2*x**2 + 1980*c**5*d**6*f*g**3*x
**3 + 385*c**5*d**6*g**4*x**4 + 1155*c**5*d**5*e*f**4*x + 2772*c**5*d**5*e
*f**3*g*x**2 + 2970*c**5*d**5*e*f**2*g**2*x**3 + 1540*c**5*d**5*e*f*g**3*x
**4 + 315*c**5*d**5*e*g**4*x**5))/(3465*c**6*d**6)
```

3.128 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 352

$$\begin{aligned}
\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx &= \frac{2\left(cd^2-ae^2\right)(cdf-aeg)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^5d^5\sqrt{d+ex}} \\
&- \frac{2(cdf-aeg)^2\left(4ae^2g-cd(ef+3dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^5d^5(d+ex)^{3/2}} \\
&- \frac{6g(cdf-aeg)\left(2ae^2g-cd(ef+dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^5d^5(d+ex)^{5/2}} \\
&- \frac{2g^2\left(4ae^2g-cd(3ef+dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^5d^5(d+ex)^{7/2}} \\
&+ \frac{2eg^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{9/2}}{9c^5d^5(d+ex)^{9/2}}
\end{aligned}$$

output
```
2*(-a*e^2+c*d^2)*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
c^5/d^5/(e*x+d)^(1/2)-2/3*(-a*e*g+c*d*f)^2*(4*a*e^2*g-c*d*(3*d*g+e*f))*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^5/d^5/(e*x+d)^(3/2)-6/5*g*(-a*e*g+c
*d*f)*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^
5/d^5/(e*x+d)^(5/2)-2/7*g^2*(4*a*e^2*g-c*d*(d*g+3*e*f))*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(7/2)/c^5/d^5/(e*x+d)^(7/2)+2/9*e*g^3*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(9/2)/c^5/d^5/(e*x+d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(128a^4e^5g^3-16a^3cde^3g^2(27ef+9dg+4eg$$

input

```
Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output

```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^5*g^3 - 16*a^3*c*d*e^3*g^2*(27
*e*f + 9*d*g + 4*e*g*x) + 24*a^2*c^2*d^2*e^2*g*(3*d*g*(7*f + g*x) + e*(21*
f^2 + 9*f*g*x + 2*g^2*x^2)) - 2*a*c^3*d^3*e*(9*d*g*(35*f^2 + 14*f*g*x + 3*
g^2*x^2) + e*(105*f^3 + 126*f^2*g*x + 81*f*g^2*x^2 + 20*g^3*x^3)) + c^4*d^
4*(9*d*(35*f^3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3) + e*x*(105*f^3 + 1
89*f^2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3))))/(315*c^5*d^5*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1258, 1253, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1258}$$

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}dx+\frac{2e(f+gx)^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\left(\frac{6(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{7cd}+\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}\right)$$

$$\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

$$\Big\downarrow 1253$$

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\int\frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}\right)}{7cd}+\right.$$

$$\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

$$\Big\downarrow 1221$$

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\left(\frac{6(cdf-aeg)\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+a}}{3cde}\right.}{5cd}\right.}{7cd}\right.$$

$$\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

$$\Big\downarrow 1122$$

$$\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}+$$

$$\frac{1}{9}\left(-\frac{8ae^2}{cd}+9d-\frac{ef}{g}\right)\left(\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cd\sqrt{d+ex}}+\frac{6(cdf-aeg)\left(\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cd}}{5cd\sqrt{d+ex}}\right.}{}\right.$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output
```
(2*e*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(9*c*d*g*Sqr
t[d + e*x]) + ((9*d - (8*a*e^2)/(c*d) - (e*f)/g)*((2*(f + g*x)^3*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d + e*x]) + (6*(c*d*f - a*
e*g)*((2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*S
qrt[d + e*x]) + (4*(c*d*f - a*e*g)*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(5*c
*d)))/(7*c*d)))/9
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1258
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.16

method	result
default	$2\sqrt{(ex+d)(cdx+ae)}\,(35e\,g^3x^4d^4c^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x^2-162a\,c^3d^3$
gosper	$2(cdx+ae)(35e\,g^3x^4d^4c^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x^2-162a\,c^3d^3e^2f\,g^2x^2+$
orering	$2(35e\,g^3x^4d^4c^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x^2-162a\,c^3d^3e^2f\,g^2x^2+189c^4d^5$

input
```
int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/315/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(35*c^4*d^4*e*g^3*x^4-40*a
*c^3*d^3*e^2*g^3*x^3+45*c^4*d^5*g^3*x^3+135*c^4*d^4*e*f*g^2*x^3+48*a^2*c^2
*d^2*e^3*g^3*x^2-54*a*c^3*d^4*e*g^3*x^2-162*a*c^3*d^3*e^2*f*g^2*x^2+189*c^
4*d^5*f*g^2*x^2+189*c^4*d^4*e*f^2*g*x^2-64*a^3*c*d*e^4*g^3*x+72*a^2*c^2*d^
3*e^2*g^3*x+216*a^2*c^2*d^2*e^3*f*g^2*x-252*a*c^3*d^4*e*f*g^2*x-252*a*c^3*
d^3*e^2*f^2*g*x+315*c^4*d^5*f^2*g*x+105*c^4*d^4*e*f^3*x+128*a^4*e^5*g^3-14
4*a^3*c*d^2*e^3*g^3-432*a^3*c*d*e^4*f*g^2+504*a^2*c^2*d^3*e^2*f*g^2+504*a^
2*c^2*d^2*e^3*f^2*g-630*a*c^3*d^4*e*f^2*g-210*a*c^3*d^3*e^2*f^3+315*c^4*d^
5*f^3)/d^5/c^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(35\,c^4d^4eg^3x^4+105\,(3\,c^4d^5-2\,ac^3d^3e^2)f^3-126\,(5\,ac^3d^4e-4\,a^2}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
2/315*(35*c^4*d^4*e*g^3*x^4 + 105*(3*c^4*d^5 - 2*a*c^3*d^3*e^2)*f^3 - 126*
(5*a*c^3*d^4*e - 4*a^2*c^2*d^2*e^3)*f^2*g + 72*(7*a^2*c^2*d^3*e^2 - 6*a^3*
c*d*e^4)*f*g^2 - 16*(9*a^3*c*d^2*e^3 - 8*a^4*e^5)*g^3 + 5*(27*c^4*d^4*e*f*
g^2 + (9*c^4*d^5 - 8*a*c^3*d^3*e^2)*g^3)*x^3 + 3*(63*c^4*d^4*e*f^2*g + 9*(
7*c^4*d^5 - 6*a*c^3*d^3*e^2)*f*g^2 - 2*(9*a*c^3*d^4*e - 8*a^2*c^2*d^2*e^3)
*g^3)*x^2 + (105*c^4*d^4*e*f^3 + 63*(5*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^2*g -
36*(7*a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f*g^2 + 8*(9*a^2*c^2*d^3*e^2 - 8*a^
3*c*d*e^4)*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x +
d)/(c^5*d^5*e*x + c^5*d^6)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f^3}{3\,\sqrt{cdx+ae}c^2d^2}$$
$$+\frac{2\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)f^2g}{5\,\sqrt{cdx+ae}c^3d^3}$$
$$+\frac{2\,(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2}{35\,\sqrt{cdx+ae}c^4d^4}$$
$$+\frac{2\,(35\,c^5d^5ex^5-144\,a^4cd^2e^4+128\,a^5e^6+5\,(9\,c^5d^6-ac^4d^4e^2)x^4-(9\,ac^4d^5e-8\,a^2c^3d^3e^3)x^3+2\,(9\,a^2c^3d^4}{315\,\sqrt{cdx+ae}c^5d^5}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output

```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*
f^3/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2
+ 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d
*e^3)*x)*f^2*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(15*c^4*d^4*e*x^4 + 56*a
^3*c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d
^4*e - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f
*g^2/(sqrt(c*d*x + a*e)*c^4*d^4) + 2/315*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2
*e^4 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e -
8*a^2*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8
*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*g^3/(sqrt(c*d*x + a*e)*c^5*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(324) = 648.

Time = 0.14 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.14

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
2/315*e*(315*(c^4*d^5*f^3 - a*c^3*d^3*e^2*f^3 - 3*a*c^3*d^4*e*f^2*g + 3*a^
2*c^2*d^2*e^3*f^2*g + 3*a^2*c^2*d^3*e^2*f*g^2 - 3*a^3*c*d*e^4*f*g^2 - a^3*
c*d^2*e^3*g^3 + a^4*e^5*g^3)*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)/(c^5*
d^5*e) + (105*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*e^6*f^3 +
315*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^4*e^5*f^2*g - 630*((e*
x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^2*d^2*e^7*f^2*g - 630*((e*x + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^2*d^3*e^6*f*g^2 + 945*((e*x + d)*c*d*e
 - c*d^2*e + a*e^3)^(3/2)*a^2*c*d*e^8*f*g^2 + 315*((e*x + d)*c*d*e - c*d^2
*e + a*e^3)^(3/2)*a^2*c*d^2*e^7*g^3 - 420*((e*x + d)*c*d*e - c*d^2*e + a*e
^3)^(3/2)*a^3*e^9*g^3 + 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^2*
d^2*e^4*f^2*g + 189*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^2*d^3*e^3*
f*g^2 - 567*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c*d*e^5*f*g^2 - 18
9*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c*d^2*e^4*g^3 + 378*((e*x +
d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6*g^3 + 135*((e*x + d)*c*d*e - c*d
^2*e + a*e^3)^(7/2)*c*d*e^2*f*g^2 + 45*((e*x + d)*c*d*e - c*d^2*e + a*e^3)
^(7/2)*c*d^2*e*g^3 - 180*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3*g
^3 + 35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*g^3)/(c^5*d^5*e^8))/abs(
e)
```

Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(256\,a^4\,e^5\,g^3-288\,a^3\,c\,d^2\,e^3\,g^3-8\right.}{}\right.}{}$$

input
```
int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(256*a^4*
e^5*g^3 + 630*c^4*d^5*f^3 - 420*a*c^3*d^3*e^2*f^3 - 288*a^3*c*d^2*e^3*g^3
+ 1008*a^2*c^2*d^2*e^3*f^2*g + 1008*a^2*c^2*d^3*e^2*f*g^2 - 1260*a*c^3*d^4
*e*f^2*g - 864*a^3*c*d*e^4*f*g^2))/(315*c^5*d^5*e) + (2*g^3*x^4*(d + e*x)^
(1/2))/(9*c*d) + (x*(d + e*x)^(1/2)*(210*c^4*d^4*e*f^3 + 630*c^4*d^5*f^2*g
 + 144*a^2*c^2*d^3*e^2*g^3 - 128*a^3*c*d*e^4*g^3 - 504*a*c^3*d^3*e^2*f^2*g
 + 432*a^2*c^2*d^2*e^3*f*g^2 - 504*a*c^3*d^4*e*f*g^2))/(315*c^5*d^5*e) + (
2*g*x^2*(d + e*x)^(1/2)*(16*a^2*e^3*g^2 + 63*c^2*d^2*e*f^2 + 63*c^2*d^3*f*
g - 18*a*c*d^2*e*g^2 - 54*a*c*d*e^2*f*g))/(105*c^3*d^3*e) + (2*g^2*x^3*(d
+ e*x)^(1/2)*(9*c*d^2*g - 8*a*e^2*g + 27*c*d*e*f))/(63*c^2*d^2*e)))/(x + d
/e)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = 2\sqrt{cdx+ae}\,(35c^4d^4e\,g^3x^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(128*a**4*e**5*g**3 - 144*a**3*c*d**2*e**3*g**3 - 432
*a**3*c*d*e**4*f*g**2 - 64*a**3*c*d*e**4*g**3*x + 504*a**2*c**2*d**3*e**2*
f*g**2 + 72*a**2*c**2*d**3*e**2*g**3*x + 504*a**2*c**2*d**2*e**3*f**2*g +
216*a**2*c**2*d**2*e**3*f*g**2*x + 48*a**2*c**2*d**2*e**3*g**3*x**2 - 630*
a*c**3*d**4*e*f**2*g - 252*a*c**3*d**4*e*f*g**2*x - 54*a*c**3*d**4*e*g**3*
x**2 - 210*a*c**3*d**3*e**2*f**3 - 252*a*c**3*d**3*e**2*f**2*g*x - 162*a*c
**3*d**3*e**2*f*g**2*x**2 - 40*a*c**3*d**3*e**2*g**3*x**3 + 315*c**4*d**5*
f**3 + 315*c**4*d**5*f**2*g*x + 189*c**4*d**5*f*g**2*x**2 + 45*c**4*d**5*g
**3*x**3 + 105*c**4*d**4*e*f**3*x + 189*c**4*d**4*e*f**2*g*x**2 + 135*c**4
*d**4*e*f*g**2*x**3 + 35*c**4*d**4*e*g**3*x**4))/(315*c**5*d**5)
```

3.129 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 270

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\left(cd^2-ae^2\right)(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^4d^4\sqrt{d+ex}} - \frac{2(cdf-aeg)\left(3ae^2g-cd(ef+2dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3c^4d^4(d+ex)^{3/2}} - \frac{2g\left(3ae^2g-cd(2ef+dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5c^4d^4(d+ex)^{5/2}} + \frac{2eg^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7c^4d^4(d+ex)^{7/2}}$$

output

```
2*(-a*e^2+c*d^2)*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
c^4/d^4/(e*x+d)^(1/2)-2/3*(-a*e*g+c*d*f)*(3*a*e^2*g-c*d*(2*d*g+e*f))*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^4/(e*x+d)^(3/2)-2/5*g*(3*a*e^2*g-
c*d*(d*g+2*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/(e*x+d)^(
5/2)+2/7*e*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/(e*x+d)^(7/
2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}(-48a^3e^4g^2+8a^2cde^2g(14ef+7dg+3egx)}{}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*a^3*e^4*g^2 + 8*a^2*c*d*e^2*g*(14*e*
f + 7*d*g + 3*e*g*x) - 2*a*c^2*d^2*e*(14*d*g*(5*f + g*x) + e*(35*f^2 + 28*
f*g*x + 9*g^2*x^2)) + c^3*d^3*(7*d*(15*f^2 + 10*f*g*x + 3*g^2*x^2) + e*x*(
35*f^2 + 42*f*g*x + 15*g^2*x^2))))/(105*c^4*d^4*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1258, 1253, 1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1258}$$

$$\frac{1}{7}\left(-\frac{6ae^2}{cd}+7d-\frac{ef}{g}\right)\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}dx+\frac{2e(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{7cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1253}$$

$$\frac{1}{7}\left(-\frac{6ae^2}{cd}+7d-\frac{ef}{g}\right)\left(\frac{4(cdf-aeg)\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{5cd}+\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd\sqrt{d+ex}}\right)$$

$$\frac{2e(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{7}\left(-\frac{6ae^2}{cd}+7d-\frac{ef}{g}\right)\left(\frac{4(cdf-aeg)\left(\frac{1}{3}\left(-\frac{2aeg}{cd}-\frac{dg}{e}+3f\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+a}}{3cde}\right.}{5cd}\right.$$

$$\frac{2e(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cdg\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2e(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{7cdg\sqrt{d+ex}}+$$

$$\frac{1}{7}\left(-\frac{6ae^2}{cd}+7d-\frac{ef}{g}\right)\left(\frac{2(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cd\sqrt{d+ex}}+\frac{4(cdf-aeg)\left(\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-\frac{2a}{c}\right.}{3cd\sqrt{d+ex}}\right.}{5}\right.$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2],x]
```

output
```
(2*e*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*g*Sqr
t[d + e*x]) + ((7*d - (6*a*e^2)/(c*d) - (e*f)/g)*((2*(f + g*x)^2*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d + e*x]) + (4*(c*d*f - a*
e*g)*((2*(3*f - (d*g)/e - (2*a*e*g)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)))/(5*c*d)))/7
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1253
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Simp[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1)))   Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (Intege
rQ[2*p] || IntegerQ[n])
```

rule 1258
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.88

method	result
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-15e\,g^2x^3d^3c^3+18a\,c^2d^2e^2g^2x^2-21c^3d^4g^2x^2-42c^3d^3efg\,x^2-24a^2cd\,e^3g^2x+28a\,c^2d^3e\,g^2x+56a\,c^2d^2e^2f}{105\sqrt{ex+d}\,d^4}$
gosper	$-\frac{2(cdx+ae)(-15e\,g^2x^3d^3c^3+18a\,c^2d^2e^2g^2x^2-21c^3d^4g^2x^2-42c^3d^3efg\,x^2-24a^2cd\,e^3g^2x+28a\,c^2d^3e\,g^2x+56a\,c^2d^2e^2fgx-70c^3}{105d^4c^4\sqrt{cd\,x^2e+a\,e^2x+}}$
orering	$-\frac{2(-15e\,g^2x^3d^3c^3+18a\,c^2d^2e^2g^2x^2-21c^3d^4g^2x^2-42c^3d^3efg\,x^2-24a^2cd\,e^3g^2x+28a\,c^2d^3e\,g^2x+56a\,c^2d^2e^2fgx-70c^3d^4fgx-35}{105d^4c^4\sqrt{ade+(a\,e^2+c\,d^2}}$

input

```
int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output

```
-2/105/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-15*c^3*d^3*e*g^2*x^3+18
*a*c^2*d^2*e^2*g^2*x^2-21*c^3*d^4*g^2*x^2-42*c^3*d^3*e*f*g*x^2-24*a^2*c*d*
e^3*g^2*x+28*a*c^2*d^3*e*g^2*x+56*a*c^2*d^2*e^2*f*g*x-70*c^3*d^4*f*g*x-35*
c^3*d^3*e*f^2*x+48*a^3*e^4*g^2-56*a^2*c*d^2*e^2*g^2-112*a^2*c*d*e^3*f*g+14
0*a*c^2*d^3*e*f*g+70*a*c^2*d^2*e^2*f^2-105*c^3*d^4*f^2)/d^4/c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,(15\,c^3d^3eg^2x^3+35\,(3\,c^3d^4-2\,ac^2d^2e^2)\,f^2-28\,(5\,ac^2d^3e-4\,a^2cd}{}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output

```
2/105*(15*c^3*d^3*e*g^2*x^3 + 35*(3*c^3*d^4 - 2*a*c^2*d^2*e^2)*f^2 - 28*(5
*a*c^2*d^3*e - 4*a^2*c*d*e^3)*f*g + 8*(7*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^2 +
3*(14*c^3*d^3*e*f*g + (7*c^3*d^4 - 6*a*c^2*d^2*e^2)*g^2)*x^2 + (35*c^3*d^3
*e*f^2 + 14*(5*c^3*d^4 - 4*a*c^2*d^2*e^2)*f*g - 4*(7*a*c^2*d^3*e - 6*a^2*c
*d*e^3)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/
(c^4*d^4*e*x + c^4*d^5)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^2}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input

```
integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output

```
Integral((d + e*x)**(3/2)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f^2}{3\,\sqrt{cdx+ae}c^2d^2}$$
$$+\frac{4\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)fg}{15\,\sqrt{cdx+ae}c^3d^3}$$
$$+\frac{2\,(15\,c^4d^4ex^4+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2}{105\,\sqrt{cdx+ae}c^4d^4}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output

```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*
f^2/(sqrt(c*d*x + a*e)*c^2*d^2) + 4/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2
 + 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*
d*e^3)*x)*f*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/105*(15*c^4*d^4*e*x^4 + 56*a
^3*c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d
^4*e - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*g
^2/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{105\,(c^3d^4f^2-ac^2d^2e^2f^2-2\,ac^2d^3efg+2\,a^2cde^3fg+a^2cd^2e^2g^2-a^3e^4g^2)\,\sqrt{(ex+d)cde}}{c^4d^4e}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
2/105*e*(105*(c^3*d^4*f^2 - a*c^2*d^2*e^2*f^2 - 2*a*c^2*d^3*e*f*g + 2*a^2*
c*d*e^3*f*g + a^2*c*d^2*e^2*g^2 - a^3*e^4*g^2)*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)/(c^4*d^4*e) + (35*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c
^2*d^2*e^4*f^2 + 70*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^3*e^3*
f*g - 140*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c*d*e^5*f*g - 70*((e
*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c*d^2*e^4*g^2 + 105*((e*x + d)*c*
d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6*g^2 + 42*((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)^(5/2)*c*d*e^2*f*g + 21*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c
*d^2*e*g^2 - 63*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3*g^2 + 15*(
(e*x + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*g^2)/(c^4*d^4*e^6))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\left(\frac{2\,g^2\,x^3\,\sqrt{d+e\,x}}{7\,c\,d}-\frac{\sqrt{d+e\,x}\,(96\,a^3\,e^4\,g^2-1}{}\right.}{}$$

input
```
int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^
(1/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3*(d + e*x)^(1/2)
)/(7*c*d) - ((d + e*x)^(1/2)*(96*a^3*e^4*g^2 - 210*c^3*d^4*f^2 + 140*a*c^2
*d^2*e^2*f^2 - 112*a^2*c*d^2*e^2*g^2 + 280*a*c^2*d^3*e*f*g - 224*a^2*c*d*e
^3*f*g))/(105*c^4*d^4*e) + (x*(d + e*x)^(1/2)*(70*c^3*d^3*e*f^2 + 140*c^3*
d^4*f*g - 56*a*c^2*d^3*e*g^2 + 48*a^2*c*d*e^3*g^2 - 112*a*c^2*d^2*e^2*f*g)
)/(105*c^4*d^4*e) + (2*g*x^2*(d + e*x)^(1/2)*(7*c*d^2*g - 6*a*e^2*g + 14*c
*d*e*f))/(35*c^2*d^2*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}\,(15c^3d^3e\,g^2x^3-18a\,c^2d^2e^2g^2x^2+21c^3d^4g^2x^2+42c^3d^3}{}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 48*a**3*e**4*g**2 + 56*a**2*c*d**2*e**2*g**2 + 11
2*a**2*c*d*e**3*f*g + 24*a**2*c*d*e**3*g**2*x - 140*a*c**2*d**3*e*f*g - 28
*a*c**2*d**3*e*g**2*x - 70*a*c**2*d**2*e**2*f**2 - 56*a*c**2*d**2*e**2*f*g
*x - 18*a*c**2*d**2*e**2*g**2*x**2 + 105*c**3*d**4*f**2 + 70*c**3*d**4*f*g
*x + 21*c**3*d**4*g**2*x**2 + 35*c**3*d**3*e*f**2*x + 42*c**3*d**3*e*f*g*x
**2 + 15*c**3*d**3*e*g**2*x**3))/(105*c**4*d**4)
```

3.130 $\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 186

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2(cd^2-ae^2)\,(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{c^3d^3\sqrt{d+ex}}-\frac{2(2ae^2g-cd(ef+dg))\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3c^3d^3(d+ex)^{3/2}}+\frac{2eg(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{5c^3d^3(d+ex)^{5/2}}$$

output
```
2*(-a*e^2+c*d^2)*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^
3/d^3/(e*x+d)^(1/2)-2/3*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)/c^3/d^3/(e*x+d)^(3/2)+2/5*e*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(5/2)/c^3/d^3/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}\left(8a^2e^3g-2acde(5ef+5dg+2egx)+c^2d^2(\right.}{15c^3d^3\sqrt{d+ex}}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^3*g - 2*a*c*d*e*(5*e*f + 5*d*g +
 2*e*g*x) + c^2*d^2*(5*d*(3*f + g*x) + e*x*(5*f + 3*g*x))))/(15*c^3*d^3*Sq
rt[d + e*x])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{1}{5}\left(-\frac{4aeg}{cd}-\frac{dg}{e}+5f\right)\int \frac{(d+ex)^{3/2}}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2g(d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cde}$$

$$\Big\downarrow \text{1128}$$

$$\frac{1}{5}\left(-\frac{4aeg}{cd}-\frac{dg}{e}+5f\right)\left(\frac{2\left(d^2-\frac{ae^2}{c}\right)\int\frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3d}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}\right)+$$
$$\frac{2g(d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cde}$$

$$\Big\downarrow \text{1122}$$

$$\frac{1}{5}\left(\frac{4\left(d^2-\frac{ae^2}{c}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}\right)\left(-\frac{4aeg}{cd}-\frac{dg}{e}+5\right.$$
$$\frac{2g(d+ex)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5cde}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
],x]
```

output
```
(2*g*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*e
) + ((5*f - (d*g)/e - (4*a*e*g)/(c*d))*((4*(d^2 - (a*e^2)/c)*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d)))/5
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{2\sqrt{(ex+d)(cdx+ae)}\,(3eg\,x^{2}d^{2}c^{2}-4acd\,e^{2}gx+5c^{2}d^{3}gx+5c^{2}d^{2}efx+8a^{2}e^{3}g-10ac\,d^{2}eg-10acd\,e^{2}f+15d^{3}f\,c^{2})}{15\sqrt{ex+d}\,d^{3}c^{3}}$	113
gosper	$\frac{2(cdx+ae)(3eg\,x^{2}d^{2}c^{2}-4acd\,e^{2}gx+5c^{2}d^{3}gx+5c^{2}d^{2}efx+8a^{2}e^{3}g-10ac\,d^{2}eg-10acd\,e^{2}f+15d^{3}f\,c^{2})\sqrt{ex+d}}{15d^{3}c^{3}\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}$	131
orering	$\frac{2(3eg\,x^{2}d^{2}c^{2}-4acd\,e^{2}gx+5c^{2}d^{3}gx+5c^{2}d^{2}efx+8a^{2}e^{3}g-10ac\,d^{2}eg-10acd\,e^{2}f+15d^{3}f\,c^{2})(cdx+ae)\sqrt{ex+d}}{15d^{3}c^{3}\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}$	132

input
```
int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
2/15/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(3*c^2*d^2*e*g*x^2-4*a*c*d*
e^2*g*x+5*c^2*d^3*g*x+5*c^2*d^2*e*f*x+8*a^2*e^3*g-10*a*c*d^2*e*g-10*a*c*d*
e^2*f+15*c^2*d^3*f)/d^3/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,(3\,c^2d^2egx^2+5\,(3\,c^2d^3-2\,acde^2)f-2\,(5\,acd^2e-4\,a^2e^3)g+(5\,c}{15\,(c^3d^3}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="fricas")
```

output

```
2/15*(3*c^2*d^2*e*g*x^2 + 5*(3*c^2*d^3 - 2*a*c*d*e^2)*f - 2*(5*a*c*d^2*e -
 4*a^2*e^3)*g + (5*c^2*d^2*e*f + (5*c^2*d^3 - 4*a*c*d*e^2)*g)*x)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^3*d^3*e*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input

```
integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/
2),x)
```

output

```
Integral((d + e*x)**(3/2)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f}{3\,\sqrt{cdx+ae}c^2d^2}+\frac{2\,(3\,c^3d^3ex^3-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)g}{15\,\sqrt{cdx+ae}c^3d^3}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="maxima")
```

output

```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*
f/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*
e^3)*x)*g/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,e\left(\frac{15\,(c^2d^3f-acde^2f-acd^2eg+a^2e^3g)\sqrt{(ex+d)cde-cd^2e+ae^3}}{c^3d^3e}+\frac{5\,((ex+d)cde-cd^2e}{}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
2/15*e*(15*(c^2*d^3*f - a*c*d*e^2*f - a*c*d^2*e*g + a^2*e^3*g)*sqrt((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)/(c^3*d^3*e) + (5*((e*x + d)*c*d*e - c*d^2*e +
 a*e^3)^(3/2)*c*d*e^2*f + 5*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d^
2*e*g - 10*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3*g + 3*((e*x + d
)*c*d*e - c*d^2*e + a*e^3)^(5/2)*g)/(c^3*d^3*e^4))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,g\,a^2\,e^3-20\,g\,a\,c\,d^2\,e-20\,f\,a\,c\,d\right.}{15\,c^3\,d^3\,e}\right.}{x+\frac{d}{e}}$$

input
```
int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2),x)
```

output
```
((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e
^3*g + 30*c^2*d^3*f - 20*a*c*d*e^2*f - 20*a*c*d^2*e*g))/(15*c^3*d^3*e) + (
2*g*x^2*(d + e*x)^(1/2))/(5*c*d) + (2*x*(d + e*x)^(1/2)*(5*c*d^2*g - 4*a*e
^2*g + 5*c*d*e*f))/(15*c^2*d^2*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{cdx+ae}\,(3c^2d^2eg\,x^2-4acd\,e^2gx+5c^2d^3gx+5c^2d^2efx+8a^2e}{15c^3d^3}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*(8*a**2*e**3*g - 10*a*c*d**2*e*g - 10*a*c*d*e**2*f -
4*a*c*d*e**2*g*x + 15*c**2*d**3*f + 5*c**2*d**3*g*x + 5*c**2*d**2*e*f*x +
3*c**2*d**2*e*g*x**2))/(15*c**3*d**3)
```

3.131 $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 108

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{c^2d^2\sqrt{d+ex}}+\frac{2e(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3c^2d^2(d+ex)^{3/2}}$$

output
```
2*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(
1/2)+2/3*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{(ae+cdx)(d+ex)}(-2ae^2+cd(3d+ex))}{3c^2d^2\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*
Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$
$$\Big\downarrow \; 1128$$
$$\frac{2\left(d^2-\frac{ae^2}{c}\right)\int \frac{\sqrt{d+ex}}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3d}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$
$$\Big\downarrow \; 1122$$
$$\frac{4\left(d^2-\frac{ae^2}{c}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}$$

input
```
Int[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(4*(d^2 - (a*e^2)/c)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d^2
*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c*d)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\,(-cdxe+2a\,e^{2}-3c\,d^{2})}{3\sqrt{ex+d}\,c^{2}d^{2}}$	51
gosper	$-\frac{2(cdx+ae)(-cdxe+2a\,e^{2}-3c\,d^{2})\sqrt{ex+d}}{3c^{2}d^{2}\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}$	69
orering	$-\frac{2(-cdxe+2a\,e^{2}-3c\,d^{2})(cdx+ae)\sqrt{ex+d}}{3c^{2}d^{2}\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}$	70

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))^(1/2)*(-c*d*e*x+2*a*e^2-3*c*d^2)/
c^2/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdex+3\,cd^2-2\,ae^2)\sqrt{ex+d}}{3\,(c^2d^2ex+c^2d^3)}$$

input

```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output

```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e
^2)*sqrt(e*x + d)/(c^2*d^2*e*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input

```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output

```
Integral((d + e*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,(c^2d^2ex^2+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)}{3\,\sqrt{cdx+ae}c^2d^2}$$

input

```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
2/3*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)/
(sqrt(c*d*x + a*e)*c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\,e\left(\frac{3\,\sqrt{(ex+d)cde-cd^2e+ae^3}\,(cd^2-ae^2)}{c^2d^2e}+\frac{((ex+d)cde-cd^2e+ae^3)^{\frac{3}{2}}}{c^2d^2e^2}\right)}{3\,|e|}$$

input
```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
2/3*e*(3*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*(c*d^2 - a*e^2)/(c^2*d^2*
e) + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)/(c^2*d^2*e^2))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{\left(\frac{2\,x\,\sqrt{d+e\,x}}{3\,c\,d}-\frac{\left(4\,a\,e^2-6\,c\,d^2\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

input
```
int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(((2*x*(d + e*x)^(1/2))/(3*c*d) - ((4*a*e^2 - 6*c*d^2)*(d + e*x)^(1/2))/(3
*c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.32

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{cdx+ae}\,(cdex-2a\,e^2+3c\,d^2)}{3c^2d^2}$$

input
```
int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(a*e + c*d*x)*( - 2*a*e**2 + 3*c*d**2 + c*d*e*x))/(3*c**2*d**2)
```

3.132 $\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 139

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2e\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg\sqrt{d+ex}} + \frac{2(ef-dg)\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

output
```
2*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^(1/2)+2*(-d*g+e*
f)*arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*g+c*d*f)
^(1/2)*(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{d+ex}\left(e\sqrt{g}\sqrt{cdf-aeg}(ae+cdx)+cd(-ef+dg)\sqrt{a}\right.}{cdg^{3/2}\sqrt{cdf-aeg}\sqrt{(ae+cdx)(d}}$$

input

```
Integrate[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]),x]
```

output

```
(2*Sqrt[d + e*x]*(e*Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*(-(e*f
) + d*g)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f -
 a*e*g]]))/(c*d*g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1258, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1258}$$

$$\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{(ef-dg)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}$$

$$\Big\downarrow \text{1255}$$

$$\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2e^2(ef-dg)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{g}$$

$$\Big\downarrow \text{218}$$

$$\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

input

```
Int[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
),x]
```

output

```
(2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*Sqrt[d + e*x]) -
(2*(e*f - d*g)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1255

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1258

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] &&
!LtQ[n, -1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{2\sqrt{(ex+d)(cdx+ae)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c\,d^{2}g-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)cdef-e\sqrt{cdx+ae}\,\sqrt{(aeg-dfc)g}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,dcg\sqrt{(aeg-dfc)g}}$	153

input
```
int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-2*((e*x+d)*(c*d*x+a*e))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c*d^2*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*
d*e*f-e*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*
e)^(1/2)/d/c/g/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.68

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \left[\frac{(cd^2ef-cd^3g+(cde^2f-cd^2eg)x)\sqrt{-cdfg+aeg^2}\log\Big(}{}\right.$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="fricas")
```

output
```
[((c*d^2*e*f - c*d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(-c*d*f*g + a*e*g^
2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*
g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g
^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(c*d*e*f*g - a*e^2
*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*
f*g^2 - a*c*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x), 2*((c*d^2*e*
f - c*d^3*g + (c*d*e^2*f - c*d^2*e*g)*x)*sqrt(c*d*f*g - a*e*g^2)*arctan(-s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*
x + d)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)) + (c*d*e*f*g - a*e^2*g
^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f*
g^2 - a*c*d^2*e*g^3 + (c^2*d^2*e*f*g^2 - a*c*d*e^2*g^3)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/
2),x)
```

output
```
Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*
x + f)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}e}{g|e|}-\frac{(cde^3f-cd^2e^2g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+}}{\sqrt{cdfg-aeg^2e}}\right.}{\sqrt{cdfg-aeg^2}eg|e|}\right.}{cd}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
 algorithm="giac")
```

output
```
2*(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*e/(g*abs(e)) - (c*d*e^3*f - c*d
^2*e^2*g)*arctan(sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g -
 a*e*g^2)*e))/(sqrt(c*d*f*g - a*e*g^2)*e*g*abs(e)))/(c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{-2\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)c\,d^2g+2\sqrt{g}\,\sqrt{-a}}{}$$

input
```
int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*( - sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*c*d**2*g + sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((s
qrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*d*e*f + sqrt(a*e +
 c*d*x)*a*e**2*g**2 - sqrt(a*e + c*d*x)*c*d*e*f*g))/(c*d*g**2*(a*e*g - c*d
*f))
```

3.133 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 169

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}(f+gx)} + \frac{(2ae^2g-cd(ef+dg))\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}$$

output

```
-(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(e*x+
d)^(1/2)/(g*x+f)+(2*a*e^2*g-c*d*(d*g+e*f))*arctan(1/g^(1/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*g+c*d*f)^(1/2)*(e*x+d)^(1/2))/g^(3/2)/(-a*
e*g+c*d*f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{\sqrt{d+ex}\left(-\frac{\sqrt{g}(-ef+dg)(ae+cdx)}{(-cdf+aeg)(f+gx)}+\frac{(-2ae^2g+cd(ef+dg))\sqrt{ae+cdx}}{(cdf-aeg)^{3/2}}\right.}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]),x]
```

output
```
(Sqrt[d + e*x]*(-((Sqrt[g]*(-(e*f) + d*g)*(a*e + c*d*x))/((-(c*d*f) + a*e*
g)*(f + g*x))) + ((-2*a*e^2*g + c*d*(e*f + d*g))*Sqrt[a*e + c*d*x]*ArcTan[
(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(3/2)))/
(g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1257, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1257}$$

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

$$\Big\downarrow \text{1255}$$

$$-\frac{e^2\left(2ae^2g-cd(dg+ef)\right)\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\sqrt{d+ex}}}{g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

$$\Big\downarrow \ 218$$

$$-\frac{\left(2ae^2g-cd(dg+ef)\right)\arctan\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)(cdf-aeg)}$$

input

```
Int[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

output

```
-(((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f - a*
e*g)*Sqrt[d + e*x]*(f + g*x))) - ((2*a*e^2*g - c*d*(e*f + d*g))*ArcTan[(Sq
rt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sq
rt[d + e*x])])/(g^(3/2)*(c*d*f - a*e*g)^(3/2))
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1255

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1257

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Simp[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m - 1)*(f + g*x)^(n +
1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] && LtQ[n, -1] && Integer
Q[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(153) = 306$.

Time = 3.05 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.99

method	result
default	$\frac{\left(-2 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right) a\, e^{2} g^{2} x+\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right) c\, d^{2} g^{2} x+\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right) cdefgx-2 \operatorname{arctanh}\bigg(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right.}{\sqrt{ex+d}\,\sqrt{cdx+}}$

input

```
int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output

```
(-2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*e^2*g^2*x+arcta
nh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d^2*g^2*x+arctanh(g*(c*d
*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*e*f*g*x-2*arctanh(g*(c*d*x+a*e)
^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*e^2*f*g+arctanh(g*(c*d*x+a*e)^(1/2)/((a*
e*g-c*d*f)*g)^(1/2))*c*d^2*f*g+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c*d*e*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*d*g+((a*e*g-
c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*e*f)/(e*x+d)^(1/2)*((e*x+d)*(c*d*x+a*e))
^(1/2)/(c*d*x+a*e)^(1/2)/g/(a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(153) = 306$.

Time = 0.13 (sec) , antiderivative size = 897, normalized size of antiderivative = 5.31

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output

```
[-1/2*((c*d^2*e*f^2 + (c*d^3 - 2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e -
2*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*
d*e^2)*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*
d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (
e*f + d*g)*x)) + 2*(c*d*e*f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2
*a*c*d^2*e*f^2*g^3 + a^2*d*e^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*
g^4 + a^2*e^3*g^5)*x^2 + (c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2
*a*c*d*e^2)*f^2*g^3 - (2*a*c*d^2*e - a^2*e^3)*f*g^4)*x), -((c*d^2*e*f^2 +
(c*d^3 - 2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2*a*e^3)*g^2)*x^2 + (c
*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*e^2)*g^2)*x)*sqrt(c*
d*f*g - a*e*g^2)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*
x)) + (c*d*e*f^2*g + a*d*e*g^3 - (c*d^2 + a*e^2)*f*g^2)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^2*d^3*f^3*g^2 - 2*a*c*d^2*e*f^
2*g^3 + a^2*d*e^2*f*g^4 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3
*g^5)*x^2 + (c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 2*a*c*d*e^2)*f
^2*g^3 - (2*a*c*d^2*e - a^2*e^3)*f*g^4)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*
x + f)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{e\left(\frac{(c^2d^2e^2f+c^2d^3eg-2\,acde^3g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(cdfg|e|-aeg^2|e|)\sqrt{cdfg-aeg^2}e}-\right.}{cd}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output
```
e*((c^2*d^2*e^2*f + c^2*d^3*e*g - 2*a*c*d*e^3*g)*arctan(sqrt((e*x + d)*c*d
*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c*d*f*g*abs(e) - a*
e*g^2*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - (sqrt((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)*c^2*d^2*e^2*f - sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^3*
e*g)/((c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)*(c*d*f
*g*abs(e) - a*e*g^2*abs(e))))/(c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^2\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.63

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{-2\sqrt{g}\,\sqrt{-aeg+cdf}\,atan\left(\frac{\sqrt{cdx+ae}\,g}{\sqrt{g}\,\sqrt{-aeg+cdf}}\right)a\,e^2fg-2\sqrt{g}\,\sqrt{}}{}$$

input
```
int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 2*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*s
qrt( - a*e*g + c*d*f)))*a*e**2*f*g - 2*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan
((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*e**2*g**2*x + s
qrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( -
a*e*g + c*d*f)))*c*d**2*f*g + sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*
e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*d**2*g**2*x + sqrt(g)*sq
rt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c
*d*f)))*c*d*e*f**2 + sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x
)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*d*e*f*g*x - sqrt(a*e + c*d*x)*a*d
*e*g**3 + sqrt(a*e + c*d*x)*a*e**2*f*g**2 + sqrt(a*e + c*d*x)*c*d**2*f*g**
2 - sqrt(a*e + c*d*x)*c*d*e*f**2*g)/(g**2*(a**2*e**2*f*g**2 + a**2*e**2*g*
*3*x - 2*a*c*d*e*f**2*g - 2*a*c*d*e*f*g**2*x + c**2*d**2*f**3 + c**2*d**2*
f**2*g*x))
```

3.134 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 261

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= \\ &-\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}(f+gx)^2} \\ &-\frac{(4ae^2g-cd(ef+3dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4g(cdf-aeg)^2\sqrt{d+ex}(f+gx)} \\ &+\frac{cd(4ae^2g-cd(ef+3dg))\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}\end{aligned}$$

output

```
-1/2*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(
e*x+d)^(1/2)/(g*x+f)^2-1/4*(4*a*e^2*g-c*d*(3*d*g+e*f))*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(e*x+d)^(1/2)/(g*x+f)+1/4*c*d*(4*a
*e^2*g-c*d*(3*d*g+e*f))*arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)*(-a*e*g+c*d*f)^(1/2)*(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{cd\sqrt{d+ex}\left(\frac{\sqrt{g}(ae+cdx)(-2aeg(dg+e(f+2gx))+cd(ef(-f+gx)+dg(5f}{cd(cdf-aeg)^2(f+gx)^2}\right.}{4g^{3/2}\sqrt{(ae+cd}}$$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]),x]
```

output
```
(c*d*Sqrt[d + e*x]*((Sqrt[g]*(a*e + c*d*x)*(-2*a*e*g*(d*g + e*(f + 2*g*x))
 + c*d*(e*f*(-f + g*x) + d*g*(5*f + 3*g*x))))/(c*d*(c*d*f - a*e*g)^2*(f +
g*x)^2) + ((-4*a*e^2*g + c*d*(e*f + 3*d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt
[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(5/2)))/(4*g^
(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1257, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \, 1257$$

$$-\frac{\left(4ae^2g-cd(3dg+ef)\right)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \, 1254$$

$$-\frac{\left(4ae^2g-cd(3dg+ef)\right)\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \; 1255$$

$$-\frac{\left(4ae^2g-cd(3dg+ef)\right)\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right.}{4g(cdf-aeg)}$$

$$\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

$$\Big\downarrow \; 218$$

$$-\frac{\left(4ae^2g-cd(3dg+ef)\right)\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2(cdf-aeg)}$$

input

```
Int[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

output

```
-1/2*((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f -
 a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) - ((4*a*e^2*g - c*d*(e*f + 3*d*g))*(Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*(f
 + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))))
/(4*g*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1257
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Simp[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m - 1)*(f + g*x)^(n +
1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] && LtQ[n, -1] && Integer
Q[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(235) = 470$.

Time = 2.97 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.54

method	result
default	$\sqrt{(ex+d)(cdx+ae)}\left(4 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)acd e^{2} g^{3} x^{2}-3 \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{2} d^{3} g^{3} x^{2}-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-dfc)g}}\right)c^{2} d^{2} e f g\right.$

input
```
int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/4*((e*x+d)*(c*d*x+a*e))^(1/2)*(4*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*a*c*d*e^2*g^3*x^2-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^2*d^3*g^3*x^2-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^2*d^2*e*f*g^2*x^2+8*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*a*c*d*e^2*f*g^2*x-6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^2*d^3*f*g^2*x-2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*c^2*d^2*e*f^2*g*x+4*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*a*c*d*e^2*f^2*g-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
*c^2*d^3*f^2*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^
2*e*f^3-4*a*e^2*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*c*d^2*g^
2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+c*d*e*f*g*x*(c*d*x+a*e)^(1/2
)*((a*e*g-c*d*f)*g)^(1/2)-2*a*d*e*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)-2*a*e^2*f*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+5*c*d^2*f*g*(c
*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-c*d*e*f^2*(c*d*x+a*e)^(1/2)*((a*e*
g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^2/g/(a*e*
g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(235) = 470$.

Time = 0.16 (sec) , antiderivative size = 1705, normalized size of antiderivative = 6.53

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output
```
[1/8*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*
g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2
*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*g^3)*x^2 + (c^2*
d^2*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*d^4 - 4*a*c*d^2
*e^2)*f*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a
*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x + 2*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f +
(e*f + d*g)*x)) - 2*(c^2*d^2*e*f^3*g - 2*a^2*d*e^2*g^4 - (5*c^2*d^3 - a*c*
d*e^2)*f^2*g^2 + (7*a*c*d^2*e - 2*a^2*e^3)*f*g^3 - (c^2*d^2*e*f^2*g^2 + (3
*c^2*d^3 - 5*a*c*d*e^2)*f*g^3 - (3*a*c*d^2*e - 4*a^2*e^3)*g^4)*x)*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^3*d^4*f^5*g^2 - 3*a*
c^2*d^3*e*f^4*g^3 + 3*a^2*c*d^2*e^2*f^3*g^4 - a^3*d*e^3*f^2*g^5 + (c^3*d^3
*e*f^3*g^4 - 3*a*c^2*d^2*e^2*f^2*g^5 + 3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*
x^3 + (2*c^3*d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 6*a*c^2*d^2*e^2)*f
^3*g^4 - 3*(a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 2*a^
3*e^4)*f*g^6)*x^2 + (c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 -
3*a*c^2*d^2*e^2)*f^4*g^3 - 3*(2*a*c^2*d^3*e - a^2*c*d*e^3)*f^3*g^4 + (6*a^
2*c*d^2*e^2 - a^3*e^4)*f^2*g^5)*x), -1/4*((c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*
a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)
*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^...
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^3}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*
x + f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(235) = 470$.

Time = 0.14 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{e^2\left(\frac{(c^3d^3ef+3\,c^3d^4g-4\,ac^2d^2e^2g)\arctan\left(\frac{\sqrt{(ex+d)cde-cd^2e+ae^3}g}{\sqrt{cdfg-aeg^2}e}\right)}{(c^2d^2f^2g|e|-2\,acdefg^2|e|+a^2e^2g^3|e|)\sqrt{cdfg-aeg^2}e}-\sqrt{}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output

```
1/4*e^2*((c^3*d^3*e*f + 3*c^3*d^4*g - 4*a*c^2*d^2*e^2*g)*arctan(sqrt((e*x
+ d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c^2*d^2*f^2
*g*abs(e) - 2*a*c*d*e*f*g^2*abs(e) + a^2*e^2*g^3*abs(e))*sqrt(c*d*f*g - a*
e*g^2)*e) - (sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^4*e^3*f^2 - 5*s
qrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*c^4*d^5*e^2*f*g + 3*sqrt((e*x + d)*
c*d*e - c*d^2*e + a*e^3)*a*c^3*d^3*e^4*f*g + 5*sqrt((e*x + d)*c*d*e - c*d^
2*e + a*e^3)*a*c^3*d^4*e^3*g^2 - 4*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)
*a^2*c^2*d^2*e^5*g^2 - ((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*e
*f*g - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^4*g^2 + 4*((e*x +
 d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^2*d^2*e^2*g^2)/((c^2*d^2*f^2*g*abs(
e) - 2*a*c*d*e*f*g^2*abs(e) + a^2*e^2*g^3*abs(e))*(c*d*e^2*f - a*e^3*g + (
(e*x + d)*c*d*e - c*d^2*e + a*e^3)*g)^2))/(c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{{\left(f+g\,x\right)}^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}} \,d x$$

input

```
int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output

```
int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 927, normalized size of antiderivative = 3.55

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Too large to display}$$

input

```
int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
(4*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt
( - a*e*g + c*d*f)))*a*c*d*e**2*f**2*g + 8*sqrt(g)*sqrt( - a*e*g + c*d*f)*
atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*c*d*e**2*f*
g**2*x + 4*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt
(g)*sqrt( - a*e*g + c*d*f)))*a*c*d*e**2*g**3*x**2 - 3*sqrt(g)*sqrt( - a*e*
g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c*
*2*d**3*f**2*g - 6*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*
g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**3*f*g**2*x - 3*sqrt(g)*sqrt(
- a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f
)))*c**2*d**3*g**3*x**2 - sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e +
c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*e*f**3 - 2*sqrt(g)*s
qrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g +
c*d*f)))*c**2*d**2*e*f**2*g*x - sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(
a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**2*d**2*e*f*g**2*x**2
- 2*sqrt(a*e + c*d*x)*a**2*d*e**2*g**4 - 2*sqrt(a*e + c*d*x)*a**2*e**3*f*g
**3 - 4*sqrt(a*e + c*d*x)*a**2*e**3*g**4*x + 7*sqrt(a*e + c*d*x)*a*c*d**2*
e*f*g**3 + 3*sqrt(a*e + c*d*x)*a*c*d**2*e*g**4*x + sqrt(a*e + c*d*x)*a*c*d
*e**2*f**2*g**2 + 5*sqrt(a*e + c*d*x)*a*c*d*e**2*f*g**3*x - 5*sqrt(a*e + c
*d*x)*c**2*d**3*f**2*g**2 - 3*sqrt(a*e + c*d*x)*c**2*d**3*f*g**3*x + sqrt(
a*e + c*d*x)*c**2*d**2*e*f**3*g - sqrt(a*e + c*d*x)*c**2*d**2*e*f**2*g*...
```

3.135 $\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 351

$$\begin{aligned}
\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \\
& -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}(f+gx)^3} \\
& -\frac{(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)^2\sqrt{d+ex}(f+gx)^2} \\
& -\frac{cd(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g(cdf-aeg)^3\sqrt{d+ex}(f+gx)} \\
& +\frac{c^2d^2(6ae^2g-cd(ef+5dg))\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}
\end{aligned}$$

output

```
-1/3*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(
e*x+d)^(1/2)/(g*x+f)^3-1/12*(6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(e*x+d)^(1/2)/(g*x+f)^2-1/8*c*d*(
6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e
*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)+1/8*c^2*d^2*(6*a*e^2*g-c*d*(5*d*g+e*f))*
arctan(1/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*(-a*e*g+c*d*f)^(1
/2)*(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(7/2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{c^2d^2\sqrt{d+ex}\left(\frac{\sqrt{g}(ae+cdx)(4a^2e^2g^2(2dg+e(f+3gx))-2acdeg(dg(13f+}{}\right.}{}$$

input

```
Integrate[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]),x]
```

output

```
(c^2*d^2*Sqrt[d + e*x]*((Sqrt[g]*(a*e + c*d*x)*(4*a^2*e^2*g^2*(2*d*g + e*(
f + 3*g*x)) - 2*a*c*d*e*g*(d*g*(13*f + 5*g*x) + e*(8*f^2 + 25*f*g*x + 9*g^
2*x^2)) + c^2*d^2*(e*f*(-3*f^2 + 8*f*g*x + 3*g^2*x^2) + d*g*(33*f^2 + 40*f
*g*x + 15*g^2*x^2))))/(c^2*d^2*(c*d*f - a*e*g)^3*(f + g*x)^3) + (3*(-6*a*e
^2*g + c*d*(e*f + 5*d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d
*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(7/2)))/(24*g^(3/2)*Sqrt[(a*e +
 c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1257, 1254, 1254, 1255, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \; 1257$$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow 1254$$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\left(\frac{3cd\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g(cdf-aeg)}-\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow 1254$$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\left(\frac{3cd\left(\frac{cd\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(cdf-aeg)}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right)}{6g(cdf-aeg)}$$

$$\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow 1255$$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\left(\frac{3cd\left(\frac{cde^2\int \frac{1}{(cdf-aeg)e^2+\frac{g\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)e^2}{d+ex}}d\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{\sqrt{d+ex}}}{cdf-aeg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right.}{4(cdf-aeg)}\right.}{6g(cdf-aeg)}$$

$$\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

$$\Big\downarrow 218$$

$$-\frac{\left(6ae^2g-cd(5dg+ef)\right)\left(\frac{3cd\left(\frac{cd\arctan\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)}\right)}{4(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)}\right.}{6g(cdf-aeg)}$$

$$\frac{(ef-dg)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]),x]
```

output
```
-1/3*((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f -
 a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) - ((6*a*e^2*g - c*d*(e*f + 5*d*g))*(Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x]*
(f + g*x)^2) + (3*c*d*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f
 - a*e*g)*Sqrt[d + e*x]*(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g
]*(c*d*f - a*e*g)^(3/2))))/(4*(c*d*f - a*e*g))))/(6*g*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1254
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Simp[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^m
*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p, 0] && LtQ[n, -1
] && IntegerQ[2*p]
```

rule 1255
```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2   Subst[Int[1/(c*(e*f + d*g) - b*e
*g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 1257
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Simp[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g)))   Int[(d + e*x)^(m - 1)*(f + g*x)^(n +
1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + p - 1, 0] && LtQ[n, -1] && Integer
Q[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. $2(319) = 638$.

Time = 2.90 (sec) , antiderivative size = 1132, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	1132

input
```
int((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-1/24*((e*x+d)*(c*d*x+a*e))^(1/2)*(54*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*a*c^2*d^2*e^2*f*g^3*x^2+8*c^2*d^2*e*f^2*g*x*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)-26*a*c*d^2*e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2)-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3
*d^3*e*f*g^3*x^3-9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^
3*d^3*e*f^2*g^2*x^2-18*a*c*d*e^2*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*
g)^(1/2)+3*c^2*d^2*e*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1
0*a*c*d^2*e*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-15*arctanh(g*(
c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^4*g^4*x^3-15*arctanh(g*(c*
d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^4*f^3*g-3*arctanh(g*(c*d*x+a
*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*e*f^4-50*a*c*d*e^2*f*g^2*x*(c*d
*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e
*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e^2*f^3*g+40*c^2*d^3*f*g^2*x*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)-9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^3*d^3*e*f^3*g*x+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*a*c^2*d^2*e^2*g^4*x^3+54*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*a*c^2*d^2*e^2*f^2*g^2*x-16*a*c*d*e^2*f^2*g*(c*d*x+a*e)^(1/2)*(
(a*e*g-c*d*f)*g)^(1/2)+12*a^2*e^3*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g
)^(1/2)+8*a^2*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+4*a^2*e^
3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+33*c^2*d^3*f^2*g*(c*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. 2(319) = 638.

Time = 0.87 (sec) , antiderivative size = 2737, normalized size of antiderivative = 7.80

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="fricas")
```

output

```
[-1/48*(3*(c^3*d^4*e*f^4 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^3*g + (c^3*d^3*
e^2*f*g^3 + (5*c^3*d^4*e - 6*a*c^2*d^2*e^3)*g^4)*x^4 + (3*c^3*d^3*e^2*f^2*
g^2 + 2*(8*c^3*d^4*e - 9*a*c^2*d^2*e^3)*f*g^3 + (5*c^3*d^5 - 6*a*c^2*d^3*e
^2)*g^4)*x^3 + 3*(c^3*d^3*e^2*f^3*g + 6*(c^3*d^4*e - a*c^2*d^2*e^3)*f^2*g^
2 + (5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f*g^3)*x^2 + (c^3*d^3*e^2*f^4 + 2*(4*c^3
*d^4*e - 3*a*c^2*d^2*e^3)*f^3*g + 3*(5*c^3*d^5 - 6*a*c^2*d^3*e^2)*f^2*g^2)
*x)*sqrt(-c*d*f*g + a*e*g^2)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*
d*e*f - (c*d^2 + 2*a*e^2)*g)*x - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x + d))/(e*g*x^2 + d*f + (e*f + d*g)*
x)) + 2*(3*c^3*d^3*e*f^4*g + 8*a^3*d*e^3*g^5 - (33*c^3*d^4 - 13*a*c^2*d^2*
e^2)*f^3*g^2 + (59*a*c^2*d^3*e - 20*a^2*c*d*e^3)*f^2*g^3 - 2*(17*a^2*c*d^2
*e^2 - 2*a^3*e^4)*f*g^4 - 3*(c^3*d^3*e*f^2*g^3 + (5*c^3*d^4 - 7*a*c^2*d^2*
e^2)*f*g^4 - (5*a*c^2*d^3*e - 6*a^2*c*d*e^3)*g^5)*x^2 - 2*(4*c^3*d^3*e*f^3
*g^2 + (20*c^3*d^4 - 29*a*c^2*d^2*e^2)*f^2*g^3 - (25*a*c^2*d^3*e - 31*a^2*
c*d*e^3)*f*g^4 + (5*a^2*c*d^2*e^2 - 6*a^3*e^4)*g^5)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d))/(c^4*d^5*f^7*g^2 - 4*a*c^3*d^4*e*f
^6*g^3 + 6*a^2*c^2*d^3*e^2*f^5*g^4 - 4*a^3*c*d^2*e^3*f^4*g^5 + a^4*d*e^4*f
^3*g^6 + (c^4*d^4*e*f^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*
f^2*g^7 - 4*a^3*c*d*e^4*f*g^8 + a^4*e^5*g^9)*x^4 + (3*c^4*d^4*e*f^5*g^4 +
a^4*d*e^4*g^9 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4*e...
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^4}\,dx$$

input

```
integrate((e*x+d)**(3/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
(1/2),x)
```

output

```
Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^4}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*
x + f)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(319) = 638$.

Time $= 0.16$ (sec) , antiderivative size $= 875$, normalized size of antiderivative $= 2.49$

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),
x, algorithm="giac")
```

output

```
1/24*e^3*(3*(c^4*d^4*e*f + 5*c^4*d^5*g - 6*a*c^3*d^3*e^2*g)*arctan(sqrt((e
*x + d)*c*d*e - c*d^2*e + a*e^3)*g/(sqrt(c*d*f*g - a*e*g^2)*e))/((c^3*d^3*
e*f^3*g*abs(e) - 3*a*c^2*d^2*e^2*f^2*g^2*abs(e) + 3*a^2*c*d*e^3*f*g^3*abs(
e) - a^3*e^4*g^4*abs(e))*sqrt(c*d*f*g - a*e*g^2)*e) - (3*sqrt((e*x + d)*c*
d*e - c*d^2*e + a*e^3)*c^6*d^6*e^5*f^3 - 33*sqrt((e*x + d)*c*d*e - c*d^2*e
 + a*e^3)*c^6*d^7*e^4*f^2*g + 24*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a
*c^5*d^5*e^6*f^2*g + 66*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a*c^5*d^6*
e^5*f*g^2 - 57*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^4*d^4*e^7*f*g
^2 - 33*sqrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^4*d^5*e^6*g^3 + 30*s
qrt((e*x + d)*c*d*e - c*d^2*e + a*e^3)*a^3*c^3*d^3*e^8*g^3 - 8*((e*x + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*c^5*d^5*e^3*f^2*g - 40*((e*x + d)*c*d*e - c
*d^2*e + a*e^3)^(3/2)*c^5*d^6*e^2*f*g^2 + 56*((e*x + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)*a*c^4*d^4*e^4*f*g^2 + 40*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a*c^4*d^5*e^3*g^3 - 48*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2
*c^3*d^3*e^5*g^3 - 3*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^4*d^4*e*f
*g^2 - 15*((e*x + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^4*d^5*g^3 + 18*((e*x
 + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^3*d^3*e^2*g^3)/((c^3*d^3*e*f^3*g*
abs(e) - 3*a*c^2*d^2*e^2*f^2*g^2*abs(e) + 3*a^2*c*d*e^3*f*g^3*abs(e) - a^3
*e^4*g^4*abs(e))*(c*d*e^2*f - a*e^3*g + ((e*x + d)*c*d*e - c*d^2*e + a*e^3
)*g)^3))/(c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input

```
int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)),x)
```

output

```
int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1527, normalized size of antiderivative = 4.35

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 18*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*
sqrt( - a*e*g + c*d*f)))*a*c**2*d**2*e**2*f**3*g - 54*sqrt(g)*sqrt( - a*e*
g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*
c**2*d**2*e**2*f**2*g**2*x - 54*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(
a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*a*c**2*d**2*e**2*f*g**3*
x**2 - 18*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(
g)*sqrt( - a*e*g + c*d*f)))*a*c**2*d**2*e**2*g**4*x**3 + 15*sqrt(g)*sqrt(
- a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f
)))*c**3*d**4*f**3*g + 45*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e +
c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d**4*f**2*g**2*x + 45*sqr
t(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*
e*g + c*d*f)))*c**3*d**4*f*g**3*x**2 + 15*sqrt(g)*sqrt( - a*e*g + c*d*f)*a
tan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d**4*g**4
*x**3 + 3*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(
g)*sqrt( - a*e*g + c*d*f)))*c**3*d**3*e*f**4 + 9*sqrt(g)*sqrt( - a*e*g + c
*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d*
*3*e*f**3*g*x + 9*sqrt(g)*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g
)/(sqrt(g)*sqrt( - a*e*g + c*d*f)))*c**3*d**3*e*f**2*g**2*x**2 + 3*sqrt(g)
*sqrt( - a*e*g + c*d*f)*atan((sqrt(a*e + c*d*x)*g)/(sqrt(g)*sqrt( - a*e*g
+ c*d*f)))*c**3*d**3*e*f*g**3*x**3 - 8*sqrt(a*e + c*d*x)*a**3*d*e**3*g*...
```

3.136 $\int (d+ex)^3(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 350

$$
\begin{aligned}
&\int (d+ex)^3(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx \\
&= \frac{7(2cd-be)^3(4cef+2cdg-3beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{512c^5e} \\
&\quad - \frac{(4cef+2cdg-3beg)(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{20c^2e^2} \\
&\quad - \frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{6ce^2} \\
&\quad - \frac{7(2cd-be)(4cef+2cdg-3beg)(16cd-5be+6cex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{960c^4e^2} \\
&\quad + \frac{7(2cd-be)^5(4cef+2cdg-3beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{512c^{11/2}e^2}
\end{aligned}
$$

output

```
7/512*(-b*e+2*c*d)^3*(-3*b*e*g+2*c*d*g+4*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(1/2)/c^5/e-1/20*(-3*b*e*g+2*c*d*g+4*c*e*f)*(e*x+d)^2*(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^2/e^2-1/6*g*(e*x+d)^3*(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(3/2)/c/e^2-7/960*(-b*e+2*c*d)*(-3*b*e*g+2*c*d*g+4*c*e*
f)*(6*c*e*x-5*b*e+16*c*d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^4/e^2+7
/512*(-b*e+2*c*d)^5*(-3*b*e*g+2*c*d*g+4*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(
-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(11/2)/e^2
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.28

$$\int (d+ex)^3(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$= \frac{(2cd-be)^5\sqrt{(d+ex)(-be+c(d-ex))}\left(\frac{\sqrt{c}(315b^5e^5g-210b^4ce^4(2ef+14dg+egx)+56b^3c^2e^3(193d^2g+e^2x(5f+3gx)+de(65f}{}\right.}{}$$

input
```
Integrate[(d + e*x)^3*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],
x]
```

output
```
((2*c*d - b*e)^5*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*((Sqrt[c]*(315*b^5
*e^5*g - 210*b^4*c*e^4*(2*e*f + 14*d*g + e*g*x) + 56*b^3*c^2*e^3*(193*d^2*
g + e^2*x*(5*f + 3*g*x) + d*e*(65*f + 31*g*x)) - 32*c^5*(176*d^5*g - 36*d*
e^4*x^3*(5*f + 4*g*x) - 8*e^5*x^4*(6*f + 5*g*x) - 2*d^3*e^2*x*(15*f + 16*g
*x) - 2*d^2*e^3*x^2*(112*f + 85*g*x) + d^4*e*(272*f + 105*g*x)) + 16*b*c^4
*e*(1047*d^4*g + 4*e^4*x^3*(3*f + 2*g*x) + 4*d*e^3*x^2*(23*f + 14*g*x) + 2
*d^2*e^2*x*(179*f + 95*g*x) + 2*d^3*e*(559*f + 227*g*x)) - 16*b^2*c^3*e^2*
(1213*d^3*g + 19*d*e^2*x*(7*f + 4*g*x) + e^3*x^2*(14*f + 9*g*x) + d^2*e*(7
49*f + 335*g*x))))/(2*c*d - b*e)^5 - (105*(4*c*e*f + 2*c*d*g - 3*b*e*g)*Ar
cTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/(Sqrt[d + e*x]*Sqrt
[-(b*e) + c*(d - e*x)])))/(7680*c^(11/2)*e^2)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1221, 1134, 1134, 1160, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}\,dx$$
$$\Big\downarrow \; 1221$$

$$\frac{(-3beg+2cdg+4cef)\int (d+ex)^3\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{4ce}-\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{6ce^2}$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{(-3beg+2cdg+4cef)\left(\frac{7(2cd-be)\int (d+ex)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{10c}-\frac{(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce}\right)}{4ce}-\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{6ce^2}$$

$$\Big\downarrow \text{ 1134}$$

$$(-3beg+2cdg+4cef)\left(\frac{7(2cd-be)\left(\frac{5(2cd-be)\int (d+ex)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{8c}-\frac{(d+ex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{4ce}\right)}{10c}-\frac{(d+ex)^2(d(c$$

$$4ce$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{6ce^2}$$

$$\Big\downarrow \text{ 1160}$$

$$(-3beg+2cdg+4cef)\left(\frac{7(2cd-be)\left(\frac{5(2cd-be)\left(\frac{(2cd-be)\int\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{2c}-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3ce}\right)}{8c}-\frac{(d+ex)\left(d(cd-be)-\right.}{4c}\right.}{10c}$$

$$4ce$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{6ce^2}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(-3beg+2cdg+4cef)\left(\frac{7(2cd-be)\left(\frac{5(2cd-be)\left(\frac{(2cd-be)\left(\frac{(2cd-be)^2\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{2c}-\frac{\left(c\right.}{}}{8c}\right.}{10c}\right.}{4ce}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{6ce^2}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{(-3beg+2cdg+4cef)\Bigg(\frac{7(2cd-be)\Bigg(\frac{5(2cd-be)\Bigg(\frac{(2cd-be)\Bigg(\frac{(2cd-be)^2\int\frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}+}{2c}}{8c}}{10c}}{4c}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{6ce^2}$$

$$\Big\downarrow\ 217$$

$$\frac{\left(\frac{7(2cd-be)\left(\frac{5(2cd-be)\left(\frac{(2cd-be)\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{2c}-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/}}{3ce}}{8c}}{10c}}{4ce}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{6ce^2}$$

input

```
Int[(d + e*x)^3*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output

```
-1/6*(g*(d + e*x)^3*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(c*e^2) +
 ((4*c*e*f + 2*c*d*g - 3*b*e*g)*(-1/5*((d + e*x)^2*(d*(c*d - b*e) - b*e^2*
x - c*e^2*x^2)^(3/2))/(c*e) + (7*(2*c*d - b*e)*(-1/4*((d + e*x)*(d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(c*e) + (5*(2*c*d - b*e)*(-1/3*(d*(c*d
- b*e) - b*e^2*x - c*e^2*x^2)^(3/2)/(c*e) + ((2*c*d - b*e)*(((b + 2*c*x)*S
qrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b*e)^2*ArcTan[
(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(8
*c^(3/2)*e)))/(2*c)))/(8*c)))/(10*c)))/(4*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2476 vs. $2(326) = 652$.

Time = 3.42 (sec) , antiderivative size = 2477, normalized size of antiderivative = 7.08

method	result	size
default	Expression too large to display	2477

input
```
int((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
d^3*f*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2
)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)
^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+e^2*(3*d*g+e*f
)*(-1/5*x^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-7/10*b/c*(-1/4*x*
(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-5/8*b/c*(-1/3*(-c*e^2*x^2-b*e
^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-1/2*b/c*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e
^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/
e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d
*e+c*d^2)^(1/2))))+1/4*(-b*d*e+c*d^2)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2
*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e
^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2
*x-b*d*e+c*d^2)^(1/2))))+2/5*(-b*d*e+c*d^2)/c/e^2*(-1/3*(-c*e^2*x^2-b*e^2*
x-b*d*e+c*d^2)^(3/2)/c/e^2-1/2*b/c*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*
x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2
/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+
c*d^2)^(1/2)))))+3*d*e*(d*g+e*f)*(-1/4*x*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^
(3/2)/c/e^2-5/8*b/c*(-1/3*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-1/2
*b/c*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)
-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^
(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))+1/4*(-b*d*e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(326) = 652$.

Time = 0.77 (sec) , antiderivative size = 1469, normalized size of antiderivative = 4.20

$$\int (d+ex)^3(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output
```
[-1/30720*(105*(4*(32*c^6*d^5*e - 80*b*c^5*d^4*e^2 + 80*b^2*c^4*d^3*e^3 -
40*b^3*c^3*d^2*e^4 + 10*b^4*c^2*d*e^5 - b^5*c*e^6)*f + (64*c^6*d^6 - 256*b
*c^5*d^5*e + 400*b^2*c^4*d^4*e^2 - 320*b^3*c^3*d^3*e^3 + 140*b^4*c^2*d^2*e
^4 - 32*b^5*c*d*e^5 + 3*b^6*e^6)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2
*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2
 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*(1280*c^6*e^5*g*x^5 + 128*(12*c^6*
e^5*f + (36*c^6*d*e^4 + b*c^5*e^5)*g)*x^4 + 16*(12*(30*c^6*d*e^4 + b*c^5*e
^5)*f + (340*c^6*d^2*e^3 + 56*b*c^5*d*e^4 - 9*b^2*c^4*e^5)*g)*x^3 + 8*(4*(
224*c^6*d^2*e^3 + 46*b*c^5*d*e^4 - 7*b^2*c^4*e^5)*f + (128*c^6*d^3*e^2 + 3
80*b*c^5*d^2*e^3 - 152*b^2*c^4*d*e^4 + 21*b^3*c^3*e^5)*g)*x^2 - 4*(2176*c^
6*d^4*e - 4472*b*c^5*d^3*e^2 + 2996*b^2*c^4*d^2*e^3 - 910*b^3*c^3*d*e^4 +
105*b^4*c^2*e^5)*f - (5632*c^6*d^5 - 16752*b*c^5*d^4*e + 19408*b^2*c^4*d^3
*e^2 - 10808*b^3*c^3*d^2*e^3 + 2940*b^4*c^2*d*e^4 - 315*b^5*c*e^5)*g + 2*(
4*(120*c^6*d^3*e^2 + 716*b*c^5*d^2*e^3 - 266*b^2*c^4*d*e^4 + 35*b^3*c^3*e^
5)*f - (1680*c^6*d^4*e - 3632*b*c^5*d^3*e^2 + 2680*b^2*c^4*d^2*e^3 - 868*b
^3*c^3*d*e^4 + 105*b^4*c^2*e^5)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 -
b*d*e))/(c^6*e^2), -1/15360*(105*(4*(32*c^6*d^5*e - 80*b*c^5*d^4*e^2 + 80*
b^2*c^4*d^3*e^3 - 40*b^3*c^3*d^2*e^4 + 10*b^4*c^2*d*e^5 - b^5*c*e^6)*f + (
64*c^6*d^6 - 256*b*c^5*d^5*e + 400*b^2*c^4*d^4*e^2 - 320*b^3*c^3*d^3*e^3 +
 140*b^4*c^2*d^2*e^4 - 32*b^5*c*d*e^5 + 3*b^6*e^6)*g)*sqrt(c)*arctan(1/...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2919 vs. $2(343) = 686$.

Time = 1.61 (sec) , antiderivative size = 2919, normalized size of antiderivative = 8.34

$$\int (d+ex)^3(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**3*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output

```
Piecewise((sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(e**3*g*x**5/6 -
 x**4*(-b*e**5*g/12 - 3*c*d*e**4*g - c*e**5*f)/(5*c*e**2) - x**3*(-4*b*d*e
**4*g - b*e**5*f - 9*b*(-b*e**5*g/12 - 3*c*d*e**4*g - c*e**5*f)/(10*c) - 2
*c*d**2*e**3*g - 3*c*d*e**4*f - e**3*g*(-5*b*d*e + 5*c*d**2)/6)/(4*c*e**2)
 - x**2*(-6*b*d**2*e**3*g - 4*b*d*e**4*f - 7*b*(-4*b*d*e**4*g - b*e**5*f -
 9*b*(-b*e**5*g/12 - 3*c*d*e**4*g - c*e**5*f)/(10*c) - 2*c*d**2*e**3*g - 3
*c*d*e**4*f - e**3*g*(-5*b*d*e + 5*c*d**2)/6)/(8*c) + 2*c*d**3*e**2*g - 2*
c*d**2*e**3*f + (-4*b*d*e + 4*c*d**2)*(-b*e**5*g/12 - 3*c*d*e**4*g - c*e**
5*f)/(5*c*e**2))/(3*c*e**2) - x*(-4*b*d**3*e**2*g - 6*b*d**2*e**3*f - 5*b*
(-6*b*d**2*e**3*g - 4*b*d*e**4*f - 7*b*(-4*b*d*e**4*g - b*e**5*f - 9*b*(-b
*e**5*g/12 - 3*c*d*e**4*g - c*e**5*f)/(10*c) - 2*c*d**2*e**3*g - 3*c*d*e**
4*f - e**3*g*(-5*b*d*e + 5*c*d**2)/6)/(8*c) + 2*c*d**3*e**2*g - 2*c*d**2*e
**3*f + (-4*b*d*e + 4*c*d**2)*(-b*e**5*g/12 - 3*c*d*e**4*g - c*e**5*f)/(5*
c*e**2))/(6*c) + 3*c*d**4*e*g + 2*c*d**3*e**2*f + (-3*b*d*e + 3*c*d**2)*(-
4*b*d*e**4*g - b*e**5*f - 9*b*(-b*e**5*g/12 - 3*c*d*e**4*g - c*e**5*f)/(10
*c) - 2*c*d**2*e**3*g - 3*c*d*e**4*f - e**3*g*(-5*b*d*e + 5*c*d**2)/6)/(4*
c*e**2))/(2*c*e**2) - (-b*d**4*e*g - 4*b*d**3*e**2*f - 3*b*(-4*b*d**3*e**2
*g - 6*b*d**2*e**3*f - 5*b*(-6*b*d**2*e**3*g - 4*b*d*e**4*f - 7*b*(-4*b*d*
e**4*g - b*e**5*f - 9*b*(-b*e**5*g/12 - 3*c*d*e**4*g - c*e**5*f)/(10*c) -
2*c*d**2*e**3*g - 3*c*d*e**4*f - e**3*g*(-5*b*d*e + 5*c*d**2)/6)/(8*c) ...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^3(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(326) = 652$.

Time = 0.16 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.16

$$\int (d+ex)^3(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$= \frac{1}{7680}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\Bigg(2\Bigg(4\Bigg(2\Bigg(8\Bigg(10\,e^3gx+\frac{12\,c^5e^{11}f+36\,c^5de^{10}g+bc^4e^{11}g}{c^5e^8}\Bigg)x+\frac{360\,c}{}$$
$$-\,7\,(128\,c^6d^5ef-320\,bc^5d^4e^2f+320\,b^2c^4d^3e^3f-160\,b^3c^3d^2e^4f+40\,b^4c^2de^5f-4\,b^5ce^6f+64\,c^6d^6g-$$

input
```
integrate((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
1/7680*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(2*(8*(10*e^3*g*x
+ (12*c^5*e^11*f + 36*c^5*d*e^10*g + b*c^4*e^11*g)/(c^5*e^8))*x + (360*c^5
*d*e^10*f + 12*b*c^4*e^11*f + 340*c^5*d^2*e^9*g + 56*b*c^4*d*e^10*g - 9*b^
2*c^3*e^11*g)/(c^5*e^8))*x + (896*c^5*d^2*e^9*f + 184*b*c^4*d*e^10*f - 28*
b^2*c^3*e^11*f + 128*c^5*d^3*e^8*g + 380*b*c^4*d^2*e^9*g - 152*b^2*c^3*d*e
^10*g + 21*b^3*c^2*e^11*g)/(c^5*e^8))*x + (480*c^5*d^3*e^8*f + 2864*b*c^4*
d^2*e^9*f - 1064*b^2*c^3*d*e^10*f + 140*b^3*c^2*e^11*f - 1680*c^5*d^4*e^7*
g + 3632*b*c^4*d^3*e^8*g - 2680*b^2*c^3*d^2*e^9*g + 868*b^3*c^2*d*e^10*g -
 105*b^4*c*e^11*g)/(c^5*e^8))*x - (8704*c^5*d^4*e^7*f - 17888*b*c^4*d^3*e^
8*f + 11984*b^2*c^3*d^2*e^9*f - 3640*b^3*c^2*d*e^10*f + 420*b^4*c*e^11*f +
 5632*c^5*d^5*e^6*g - 16752*b*c^4*d^4*e^7*g + 19408*b^2*c^3*d^3*e^8*g - 10
808*b^3*c^2*d^2*e^9*g + 2940*b^4*c*d*e^10*g - 315*b^5*e^11*g)/(c^5*e^8)) -
 7/1024*(128*c^6*d^5*e*f - 320*b*c^5*d^4*e^2*f + 320*b^2*c^4*d^3*e^3*f - 1
60*b^3*c^3*d^2*e^4*f + 40*b^4*c^2*d*e^5*f - 4*b^5*c*e^6*f + 64*c^6*d^6*g -
 256*b*c^5*d^5*e*g + 400*b^2*c^4*d^4*e^2*g - 320*b^3*c^3*d^3*e^3*g + 140*b
^4*c^2*d^2*e^4*g - 32*b^5*c*d*e^5*g + 3*b^6*e^6*g)*log(abs(-b*e^2 + 2*(sqr
t(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e))
)/(sqrt(-c)*c^5*e*abs(e))
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 3311, normalized size of antiderivative = 9.46

$$\int (d+ex)^3(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx=\text{Too large to display}$$

input
```
int((f + g*x)*(d + e*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output

```
d^3*f*(x/2 + b/(4*c))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) + (3*d*f
*(c*d^2 - b*d*e)*((x/2 + b/(4*c))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1
/2) - (log(b*e^2 - 2*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2)
 + 2*c*e^2*x)*((b^2*e^4)/4 + c*e^2*(c*d^2 - b*d*e)))/(2*(-c*e^2)^(3/2))))/
(4*c) - (7*b*e^3*f*((5*b*((log(b*e^2 - 2*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e -
 c*d + c*e*x))^(1/2) + 2*c*e^2*x)*(b^3*e^6 + 4*b*c*e^4*(c*d^2 - b*d*e)))/(
16*(-c*e^2)^(5/2)) - ((8*c*e^2*(c*e^2*x^2 - c*d^2 + b*d*e) - 3*b^2*e^4 + 2
*b*c*e^4*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(24*c^2*e^4)))/(8
*c) + ((c*d^2 - b*d*e)*((x/2 + b/(4*c))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2
*x)^(1/2) - (log(b*e^2 - 2*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))
^(1/2) + 2*c*e^2*x)*((b^2*e^4)/4 + c*e^2*(c*d^2 - b*d*e)))/(2*(-c*e^2)^(3/
2))))/(4*c*e^2) - (x*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(4*c*e^2
)))/(10*c) + (3*b*e^3*g*((7*b*((5*b*((log(b*e^2 - 2*(-c*e^2)^(1/2)*(-(d +
e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e^2*x)*(b^3*e^6 + 4*b*c*e^4*(c*d^2 -
 b*d*e)))/(16*(-c*e^2)^(5/2)) - ((8*c*e^2*(c*e^2*x^2 - c*d^2 + b*d*e) - 3*
b^2*e^4 + 2*b*c*e^4*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(24*c^
2*e^4)))/(8*c) + ((c*d^2 - b*d*e)*((x/2 + b/(4*c))*(c*d^2 - c*e^2*x^2 - b*
d*e - b*e^2*x)^(1/2) - (log(b*e^2 - 2*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*
d + c*e*x))^(1/2) + 2*c*e^2*x)*((b^2*e^4)/4 + c*e^2*(c*d^2 - b*d*e)))/(2*(
-c*e^2)^(3/2))))/(4*c*e^2) - (x*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(...
```

Reduce [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 2564, normalized size of antiderivative = 7.33

$$\int (d+ex)^3(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*(315*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*b**7*e**7*g - 3990*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b
*e + 2*c*d))*b**6*c*d*e**6*g - 420*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*b**6*c*e**7*f + 21420*sqrt(c)*asinh((sqrt( - b
*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**2*d**2*e**5*g + 5040*sq
rt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**2
*d*e**6*f - 63000*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
 + 2*c*d))*b**4*c**3*d**3*e**4*g - 25200*sqrt(c)*asinh((sqrt( - b*e + c*d
- c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**3*d**2*e**5*f + 109200*sqrt(c)*a
sinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**4*d**4*e
**3*g + 67200*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2
*c*d))*b**3*c**4*d**3*e**4*f - 110880*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**5*d**5*e**2*g - 100800*sqrt(c)*asin
h((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**5*d**4*e**3
*f + 60480*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*
d))*b*c**6*d**6*e*g + 80640*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/s
qrt( - b*e + 2*c*d))*b*c**6*d**5*e**2*f - 13440*sqrt(c)*asinh((sqrt( - b*e
 + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**7*d**7*g - 26880*sqrt(c)*asinh
((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**7*d**6*e*f + 315*
sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d ...
```

3.137 $\int (d+ex)^2(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 277

$$\begin{aligned}\int (d+ex)^2(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx &= \frac{(2cd-be)^2(10cef+4cdg-7beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{128c^4e}\\ &\quad-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2}\\ &\quad-\frac{(10cef+4cdg-7beg)(16cd-5be+6cex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{240c^3e^2}\\ &\quad+\frac{(2cd-be)^4(10cef+4cdg-7beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{128c^{9/2}e^2}\end{aligned}$$

output

```
1/128*(-b*e+2*c*d)^2*(-7*b*e*g+4*c*d*g+10*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b
*e^2*x-c*e^2*x^2)^(1/2)/c^4/e-1/5*g*(e*x+d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*
x^2)^(3/2)/c/e^2-1/240*(-7*b*e*g+4*c*d*g+10*c*e*f)*(6*c*e*x-5*b*e+16*c*d)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^3/e^2+1/128*(-b*e+2*c*d)^4*(-7*b*
e*g+4*c*d*g+10*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(1/2))/c^(9/2)/e^2
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.26

$$\int (d+ex)^2(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$=\frac{(-2cd+be)^4\sqrt{(d+ex)(-be+c(d-ex))}\Bigg(\frac{\sqrt{c}(-105b^4e^4g+10b^3ce^3(15ef+76dg+7egx)-16c^4(56d^4g-20de^3x^2(4f+3gx)+10}{}}{}$$

input
```
Integrate[(d + e*x)^2*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],
x]
```

output
```
((-2*c*d + b*e)^4*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*((Sqrt[c]*(-105*b
^4*e^4*g + 10*b^3*c*e^3*(15*e*f + 76*d*g + 7*e*g*x) - 16*c^4*(56*d^4*g - 2
0*d*e^3*x^2*(4*f + 3*g*x) + 10*d^3*e*(8*f + 3*g*x) - 6*e^4*x^3*(5*f + 4*g*
x) - d^2*e^2*x*(45*f + 32*g*x)) - 4*b^2*c^2*e^2*(499*d^2*g + e^2*x*(25*f +
 14*g*x) + 2*d*e*(125*f + 54*g*x)) + 8*b*c^3*e*(274*d^3*g + 2*e^3*x^2*(5*f
 + 3*g*x) + 2*d*e^2*x*(35*f + 18*g*x) + d^2*e*(285*f + 109*g*x))))/(-2*c*d
 + b*e)^4 - (15*(10*c*e*f + 4*c*d*g - 7*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e
*x]/(Sqrt[c]*Sqrt[d + e*x])])/(Sqrt[d + e*x]*Sqrt[-(b*e) + c*(d - e*x)])))
/(1920*c^(9/2)*e^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1221, 1134, 1160, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}\,dx$$

$$\Big\downarrow \; 1221$$

$$\frac{(-7beg+4cdg+10cef)\int (d+ex)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{10ce}-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2}$$

$$\Big\downarrow \ 1134$$

$$\frac{(-7beg+4cdg+10cef)\left(\frac{5(2cd-be)\int (d+ex)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{8c}-\frac{(d+ex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{4ce}\right)}{10ce}-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2}$$

$$\Big\downarrow \ 1160$$

$$\frac{(-7beg+4cdg+10cef)\left(\frac{5(2cd-be)\left(\frac{(2cd-be)\int\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{2c}-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3ce}\right)}{8c}-\frac{(d+ex)\bigl(d(cd-be)-be^2x}{4ce}\right.}{10ce}-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2}$$

$$\Big\downarrow \ 1087$$

$$\frac{(-7beg+4cdg+10cef)\left(\frac{5(2cd-be)\left(\frac{(2cd-be)\left(\frac{(2cd-be)^2\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{2c}-\bigl(d(cd-be)-\right.}{8c}\right.}{10ce}-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2}$$

$$\Big\downarrow \ 1092$$

$$\frac{(-7beg+4cdg+10cef)\left(\frac{5(2cd-be)\left(\frac{(2cd-be)\left(\frac{(2cd-be)^2\int\frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}+\frac{(b+2cx)\sqrt{d(}}{}\right.}{2c}\right.}{8c}\right.}{10ce}$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2}$$

$$\Big\downarrow 217$$

$$\frac{\left(\frac{5(2cd-be)\left(\frac{(2cd-be)\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{2c}-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3ce}\right)}{8c}-\frac{(d}{}\right.}{10ce}$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2}$$

input

```
Int[(d + e*x)^2*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
-1/5*(g*(d + e*x)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(c*e^2) +
 ((10*c*e*f + 4*c*d*g - 7*b*e*g)*(-1/4*((d + e*x)*(d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2)^(3/2))/(c*e) + (5*(2*c*d - b*e)*(-1/3*(d*(c*d - b*e) - b*e^2
*x - c*e^2*x^2)^(3/2)/(c*e) + ((2*c*d - b*e)*(((b + 2*c*x)*Sqrt[d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b*e)^2*ArcTan[(e*(b + 2*c*x)
)/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(8*c^(3/2)*e)))/
(2*c)))/(8*c)))/(10*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(257) = 514$.

Time = 2.74 (sec) , antiderivative size = 1375, normalized size of antiderivative = 4.96

method	result	size
default	Expression too large to display	1375

input
```
int((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
d^2*f*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2
)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)
^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+e*(2*d*g+e*f)*
(-1/4*x*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-5/8*b/c*(-1/3*(-c*e^2
*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-1/2*b/c*(-1/4*(-2*c*e^2*x-b*e^2)/c/e
^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2
*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e
^2*x-b*d*e+c*d^2)^(1/2))))+1/4*(-b*d*e+c*d^2)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^
2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^
2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x
^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))+d*(d*g+2*e*f)*(-1/3*(-c*e^2*x^2-b*e^2*x-b
*d*e+c*d^2)^(3/2)/c/e^2-1/2*b/c*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2
-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c
*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d
^2)^(1/2))))+e^2*g*(-1/5*x^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-
7/10*b/c*(-1/4*x*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-5/8*b/c*(-1/
3*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-1/2*b/c*(-1/4*(-2*c*e^2*x-b
*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c
*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^
2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))+1/4*(-b*d*e+c*d^2)/c/e^2*(-1/4*(-2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(257) = 514.

Time = 0.35 (sec) , antiderivative size = 1117, normalized size of antiderivative = 4.03

$$\int (d+ex)^2(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output

```
[1/7680*(15*(10*(16*c^5*d^4*e - 32*b*c^4*d^3*e^2 + 24*b^2*c^3*d^2*e^3 - 8*
b^3*c^2*d*e^4 + b^4*c*e^5)*f + (64*c^5*d^5 - 240*b*c^4*d^4*e + 320*b^2*c^3
*d^3*e^2 - 200*b^3*c^2*d^2*e^3 + 60*b^4*c*d*e^4 - 7*b^5*e^5)*g)*sqrt(-c)*l
og(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 4*sqrt(
-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) + 4*(384*c
^5*e^4*g*x^4 + 48*(10*c^5*e^4*f + (20*c^5*d*e^3 + b*c^4*e^4)*g)*x^3 + 8*(1
0*(16*c^5*d*e^3 + b*c^4*e^4)*f + (64*c^5*d^2*e^2 + 36*b*c^4*d*e^3 - 7*b^2*
c^3*e^4)*g)*x^2 - 10*(128*c^5*d^3*e - 228*b*c^4*d^2*e^2 + 100*b^2*c^3*d*e^
3 - 15*b^3*c^2*e^4)*f - (896*c^5*d^4 - 2192*b*c^4*d^3*e + 1996*b^2*c^3*d^2
*e^2 - 760*b^3*c^2*d*e^3 + 105*b^4*c*e^4)*g + 2*(10*(36*c^5*d^2*e^2 + 28*b
*c^4*d*e^3 - 5*b^2*c^3*e^4)*f - (240*c^5*d^3*e - 436*b*c^4*d^2*e^2 + 216*b
^2*c^3*d*e^3 - 35*b^3*c^2*e^4)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b
*d*e))/(c^5*e^2), -1/3840*(15*(10*(16*c^5*d^4*e - 32*b*c^4*d^3*e^2 + 24*b^
2*c^3*d^2*e^3 - 8*b^3*c^2*d*e^4 + b^4*c*e^5)*f + (64*c^5*d^5 - 240*b*c^4*d
^4*e + 320*b^2*c^3*d^3*e^2 - 200*b^3*c^2*d^2*e^3 + 60*b^4*c*d*e^4 - 7*b^5*
e^5)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c
*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*(38
4*c^5*e^4*g*x^4 + 48*(10*c^5*e^4*f + (20*c^5*d*e^3 + b*c^4*e^4)*g)*x^3 + 8
*(10*(16*c^5*d*e^3 + b*c^4*e^4)*f + (64*c^5*d^2*e^2 + 36*b*c^4*d*e^3 - 7*b
^2*c^3*e^4)*g)*x^2 - 10*(128*c^5*d^3*e - 228*b*c^4*d^2*e^2 + 100*b^2*c^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(265) = 530$.

Time = 1.58 (sec) , antiderivative size = 1608, normalized size of antiderivative = 5.81

$$\int (d+ex)^2(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**2*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output

```
Piecewise((sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(e**2*g*x**4/5 -
 x**3*(-b*e**4*g/10 - 2*c*d*e**3*g - c*e**4*f)/(4*c*e**2) - x**2*(-3*b*d*e
**3*g - b*e**4*f - 7*b*(-b*e**4*g/10 - 2*c*d*e**3*g - c*e**4*f)/(8*c) - 2*
c*d*e**3*f - e**2*g*(-4*b*d*e + 4*c*d**2)/5)/(3*c*e**2) - x*(-3*b*d**2*e**
2*g - 3*b*d*e**3*f - 5*b*(-3*b*d*e**3*g - b*e**4*f - 7*b*(-b*e**4*g/10 - 2
*c*d*e**3*g - c*e**4*f)/(8*c) - 2*c*d*e**3*f - e**2*g*(-4*b*d*e + 4*c*d**2
)/5)/(6*c) + 2*c*d**3*e*g + (-3*b*d*e + 3*c*d**2)*(-b*e**4*g/10 - 2*c*d*e*
*3*g - c*e**4*f)/(4*c*e**2))/(2*c*e**2) - (-b*d**3*e*g - 3*b*d**2*e**2*f -
 3*b*(-3*b*d**2*e**2*g - 3*b*d*e**3*f - 5*b*(-3*b*d*e**3*g - b*e**4*f - 7*
b*(-b*e**4*g/10 - 2*c*d*e**3*g - c*e**4*f)/(8*c) - 2*c*d*e**3*f - e**2*g*(
-4*b*d*e + 4*c*d**2)/5)/(6*c) + 2*c*d**3*e*g + (-3*b*d*e + 3*c*d**2)*(-b*e
**4*g/10 - 2*c*d*e**3*g - c*e**4*f)/(4*c*e**2))/(4*c) + c*d**4*g + 2*c*d**
3*e*f + (-2*b*d*e + 2*c*d**2)*(-3*b*d*e**3*g - b*e**4*f - 7*b*(-b*e**4*g/1
0 - 2*c*d*e**3*g - c*e**4*f)/(8*c) - 2*c*d*e**3*f - e**2*g*(-4*b*d*e + 4*c
*d**2)/5)/(3*c*e**2))/(c*e**2)) + (-b*d**3*e*f - b*(-b*d**3*e*g - 3*b*d**2
*e**2*f - 3*b*(-3*b*d**2*e**2*g - 3*b*d*e**3*f - 5*b*(-3*b*d*e**3*g - b*e*
*4*f - 7*b*(-b*e**4*g/10 - 2*c*d*e**3*g - c*e**4*f)/(8*c) - 2*c*d*e**3*f -
 e**2*g*(-4*b*d*e + 4*c*d**2)/5)/(6*c) + 2*c*d**3*e*g + (-3*b*d*e + 3*c*d*
*2)*(-b*e**4*g/10 - 2*c*d*e**3*g - c*e**4*f)/(4*c*e**2))/(4*c) + c*d**4*g
+ 2*c*d**3*e*f + (-2*b*d*e + 2*c*d**2)*(-3*b*d*e**3*g - b*e**4*f - 7*b*...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^2(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(257) = 514$.

Time = 0.16 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.05

$$\int (d+ex)^2(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$= \frac{1}{1920}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\Bigg(2\Bigg(4\Bigg(6\Bigg(8e^2gx+\frac{10c^4e^8f+20c^4de^7g+bc^3e^8g}{c^4e^6}\Bigg)x+\frac{160c^4de^7f+}{}$$
$$-\frac{(160c^5d^4ef-320bc^4d^3e^2f+240b^2c^3d^2e^3f-80b^3c^2de^4f+10b^4ce^5f+64c^5d^5g-240bc^4d^4eg+320}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
1/1920*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(6*(8*e^2*g*x + (1
0*c^4*e^8*f + 20*c^4*d*e^7*g + b*c^3*e^8*g)/(c^4*e^6))*x + (160*c^4*d*e^7*
f + 10*b*c^3*e^8*f + 64*c^4*d^2*e^6*g + 36*b*c^3*d*e^7*g - 7*b^2*c^2*e^8*g
)/(c^4*e^6))*x + (360*c^4*d^2*e^6*f + 280*b*c^3*d*e^7*f - 50*b^2*c^2*e^8*f
 - 240*c^4*d^3*e^5*g + 436*b*c^3*d^2*e^6*g - 216*b^2*c^2*d*e^7*g + 35*b^3*
c*e^8*g)/(c^4*e^6))*x - (1280*c^4*d^3*e^5*f - 2280*b*c^3*d^2*e^6*f + 1000*
b^2*c^2*d*e^7*f - 150*b^3*c*e^8*f + 896*c^4*d^4*e^4*g - 2192*b*c^3*d^3*e^5
*g + 1996*b^2*c^2*d^2*e^6*g - 760*b^3*c*d*e^7*g + 105*b^4*e^8*g)/(c^4*e^6)
) - 1/256*(160*c^5*d^4*e*f - 320*b*c^4*d^3*e^2*f + 240*b^2*c^3*d^2*e^3*f -
 80*b^3*c^2*d*e^4*f + 10*b^4*c*e^5*f + 64*c^5*d^5*g - 240*b*c^4*d^4*e*g +
320*b^2*c^3*d^3*e^2*g - 200*b^3*c^2*d^2*e^3*g + 60*b^4*c*d*e^4*g - 7*b^5*e
^5*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d
^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqrt(-c)*c^4*e*abs(e))
```

Mupad [B] (verification not implemented)

Time = 8.08 (sec) , antiderivative size = 1732, normalized size of antiderivative = 6.25

$$\int (d+ex)^2(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input
```
int((f + g*x)*(d + e*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output

```
d^2*f*(x/2 + b/(4*c))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) - (f*x*(
c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(4*c) - (g*x^2*(c*d^2 - c*e^2*
x^2 - b*d*e - b*e^2*x)^(3/2))/(5*c) + (f*(c*d^2 - b*d*e)*((x/2 + b/(4*c))*
(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) - (log(b*e^2 - 2*(-c*e^2)^(1/2
)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e^2*x)*((b^2*e^4)/4 + c*e^2
*(c*d^2 - b*d*e)))/(2*(-c*e^2)^(3/2))))/(4*c) - (g*(2*c*d^2 - 2*b*d*e)*((l
og(b*e^2 - 2*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e
^2*x)*(b^3*e^6 + 4*b*c*e^4*(c*d^2 - b*d*e)))/(16*(-c*e^2)^(5/2)) - ((8*c*e
^2*(c*e^2*x^2 - c*d^2 + b*d*e) - 3*b^2*e^4 + 2*b*c*e^4*x)*(c*d^2 - c*e^2*x
^2 - b*d*e - b*e^2*x)^(1/2))/(24*c^2*e^4)))/(5*c) - (7*b*e^2*g*((5*b*((log
(b*e^2 - 2*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e^2
*x)*(b^3*e^6 + 4*b*c*e^4*(c*d^2 - b*d*e)))/(16*(-c*e^2)^(5/2)) - ((8*c*e^2
*(c*e^2*x^2 - c*d^2 + b*d*e) - 3*b^2*e^4 + 2*b*c*e^4*x)*(c*d^2 - c*e^2*x^2
 - b*d*e - b*e^2*x)^(1/2))/(24*c^2*e^4)))/(8*c) + ((c*d^2 - b*d*e)*((x/2 +
 b/(4*c))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) - (log(b*e^2 - 2*(-c
*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e^2*x)*((b^2*e^4)
/4 + c*e^2*(c*d^2 - b*d*e)))/(2*(-c*e^2)^(3/2))))/(4*c*e^2) - (x*(c*d^2 -
c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(4*c*e^2)))/(10*c) - (d^2*f*log(b*e^2
- 2*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e^2*x)*((b
^2*e^4)/4 + c*e^2*(c*d^2 - b*d*e)))/(2*(-c*e^2)^(3/2)) + (5*b*e^2*f*((l...
```

Reduce [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 1875, normalized size of antiderivative = 6.77

$$\int (d+ex)^2(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*( - 105*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*
d))*b**6*e**6*g + 1110*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
- b*e + 2*c*d))*b**5*c*d*e**5*g + 150*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c*e**6*f - 4800*sqrt(c)*asinh((sqrt( -
 b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**2*d**2*e**4*g - 1500*
sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c*
*2*d*e**5*f + 10800*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b
*e + 2*c*d))*b**3*c**3*d**3*e**3*g + 6000*sqrt(c)*asinh((sqrt( - b*e + c*d
 - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**3*d**2*e**4*f - 13200*sqrt(c)*a
sinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**4*d**4*e
**2*g - 12000*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2
*c*d))*b**2*c**4*d**3*e**3*f + 8160*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e
*x)*i)/sqrt( - b*e + 2*c*d))*b*c**5*d**5*e*g + 12000*sqrt(c)*asinh((sqrt(
- b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**5*d**4*e**2*f - 1920*sq
rt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**6*d**6
*g - 4800*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*c**6*d**5*e*f - 105*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d
)*sqrt( - b*e + c*d - c*e*x)*b**4*c*e**4*g + 760*sqrt(d + e*x)*sqrt(b*e -
2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d*e**3*g
+ 150*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e ...
```

3.138 $\int (d+ex)(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 218

$$\begin{aligned}\int (d+ex)(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx &= \frac{(2cd-be)(8cef+2cdg-5beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{64c^3e} \\ &\quad + \frac{(5beg-8c(ef+dg)-6cegx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{24c^2e^2} \\ &\quad + \frac{(2cd-be)^3(8cef+2cdg-5beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{64c^{7/2}e^2}\end{aligned}$$

output

```
1/64*(-b*e+2*c*d)*(-5*b*e*g+2*c*d*g+8*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2
*x-c*e^2*x^2)^(1/2)/c^3/e+1/24*(5*b*e*g-8*c*(d*g+e*f)-6*c*e*g*x)*(d*(-b*e+
c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^2/e^2+1/64*(-b*e+2*c*d)^3*(-5*b*e*g+2*c*d*
g+8*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/
c^(7/2)/e^2
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.23

$$\int (d+ex)(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$=\frac{(2cd-be)^3\sqrt{(d+ex)(-be+c(d-ex))}\Bigg(\frac{\sqrt{c}(15b^3e^3g-2b^2ce^2(12ef+38dg+5egx)-8c^3(8d^3g-4de^2x(3f+2gx)-2e^3x^2(4f+3g}{(2cd-be)^3}}{192c^{7/2}e^2}$$

input

```
Integrate[(d + e*x)*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output

```
((2*c*d - b*e)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*((Sqrt[c]*(15*b^3*
e^3*g - 2*b^2*c*e^2*(12*e*f + 38*d*g + 5*e*g*x) - 8*c^3*(8*d^3*g - 4*d*e^2
*x*(3*f + 2*g*x) - 2*e^3*x^2*(4*f + 3*g*x) + d^2*e*(8*f + 3*g*x)) + 4*b*c^
2*e*(29*d^2*g + 2*e^2*x*(2*f + g*x) + 2*d*e*(14*f + 5*g*x))))/(2*c*d - b*e
)^3 - (3*(8*c*e*f + 2*c*d*g - 5*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqr
t[c]*Sqrt[d + e*x])])/(Sqrt[d + e*x]*Sqrt[-(b*e) + c*(d - e*x)])))/(192*c^
(7/2)*e^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1225, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}\,dx$$
$$\Big\downarrow \; 1225$$
$$\frac{(2cd-be)(-5beg+2cdg+8cef)\int \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16c^2e}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}(5beg-8c(dg+ef)-6cegx)}{24c^2e^2}$$
$$\Big\downarrow \; 1087$$

$$\frac{(2cd-be)(-5beg+2cdg+8cef)\left(\frac{(2cd-be)^2\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c^2e}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}(5beg-8c(dg+ef)-6cegx)}{24c^2e^2}$$

$$\Big\downarrow \; 1092$$

$$\frac{(2cd-be)(-5beg+2cdg+8cef)\left(\frac{(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}+\frac{(b+2cx)\sqrt{d(cd-b}}{4c}\right.}{16c^2e}$$
$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}(5beg-8c(dg+ef)-6cegx)}{24c^2e^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(2cd-be)\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)(-5beg+2cdg+8cef)}{16c^2e}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}(5beg-8c(dg+ef)-6cegx)}{24c^2e^2}$$

input
```
Int[(d + e*x)*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
((5*b*e*g - 8*c*(e*f + d*g) - 6*c*e*g*x)*(d*(c*d - b*e) - b*e^2*x - c*e^2*
x^2)^(3/2))/(24*c^2*e^2) + ((2*c*d - b*e)*(8*c*e*f + 2*c*d*g - 5*b*e*g)*((
(b + 2*c*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b
*e)^2*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e
^2*x^2])])/(8*c^(3/2)*e)))/(16*c^2*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(202) = 404$.

Time = 2.25 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.32

method	result
default	$df\left(-\frac{\left(-2c\,e^{2}x-b\,e^{2}\right)\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}{4c\,e^{2}}-\frac{\left(-4c\,e^{2}\left(-bde+c\,d^{2}\right)-b^{2}e^{4}\right)\arctan\left(\frac{\sqrt{c\,e^{2}}\,\left(x+\frac{b}{2c}\right)}{\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}\right)}{8c\,e^{2}\sqrt{c\,e^{2}}}\right)+($

input
```
int((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
d*f*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-
1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(
1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+(d*g+e*f)*(-1/3*
(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^2-1/2*b/c*(-1/4*(-2*c*e^2*x-b*e
^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d
^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*
x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))+e*g*(-1/4*x*(-c*e^2*x^2-b*e^2*x-b*d*e+c*
d^2)^(3/2)/c/e^2-5/8*b/c*(-1/3*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/c/e^
2-1/2*b/c*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^
(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*
e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))+1/4*(-b*d
*e+c*d^2)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c
*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arct
an((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(202) = 404$.

Time = 0.19 (sec) , antiderivative size = 825, normalized size of antiderivative = 3.78

$$\int (d+ex)(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algori
thm="fricas")
```

output

```
[-1/768*(3*(8*(8*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^
4)*f + (16*c^4*d^4 - 64*b*c^3*d^3*e + 72*b^2*c^2*d^2*e^2 - 32*b^3*c*d*e^3
+ 5*b^4*e^4)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b
*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x +
 b*e)*sqrt(-c)) - 4*(48*c^4*e^3*g*x^3 + 8*(8*c^4*e^3*f + (8*c^4*d*e^2 + b*
c^3*e^3)*g)*x^2 - 8*(8*c^4*d^2*e - 14*b*c^3*d*e^2 + 3*b^2*c^2*e^3)*f - (64
*c^4*d^3 - 116*b*c^3*d^2*e + 76*b^2*c^2*d*e^2 - 15*b^3*c*e^3)*g + 2*(8*(6*
c^4*d*e^2 + b*c^3*e^3)*f - (12*c^4*d^2*e - 20*b*c^3*d*e^2 + 5*b^2*c^2*e^3)
*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^4*e^2), -1/384*(3*(8
*(8*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^4)*f + (16*c^
4*d^4 - 64*b*c^3*d^3*e + 72*b^2*c^2*d^2*e^2 - 32*b^3*c*d*e^3 + 5*b^4*e^4)*
g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x
+ b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*(48*c^4*
e^3*g*x^3 + 8*(8*c^4*e^3*f + (8*c^4*d*e^2 + b*c^3*e^3)*g)*x^2 - 8*(8*c^4*d
^2*e - 14*b*c^3*d*e^2 + 3*b^2*c^2*e^3)*f - (64*c^4*d^3 - 116*b*c^3*d^2*e +
 76*b^2*c^2*d*e^2 - 15*b^3*c*e^3)*g + 2*(8*(6*c^4*d*e^2 + b*c^3*e^3)*f - (
12*c^4*d^2*e - 20*b*c^3*d*e^2 + 5*b^2*c^2*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e
^2*x + c*d^2 - b*d*e))/(c^4*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(209) = 418$.

Time = 2.36 (sec) , antiderivative size = 952, normalized size of antiderivative = 4.37

$$\int (d+ex)(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x)
```

output
```
Piecewise((sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(e*g*x**3/4 - x*
*2*(-b*e**3*g/8 - c*d*e**2*g - c*e**3*f)/(3*c*e**2) - x*(-2*b*d*e**2*g - b
*e**3*f - 5*b*(-b*e**3*g/8 - c*d*e**2*g - c*e**3*f)/(6*c) + c*d**2*e*g - c
*d*e**2*f - e*g*(-3*b*d*e + 3*c*d**2)/4)/(2*c*e**2) - (-b*d**2*e*g - 2*b*d
*e**2*f - 3*b*(-2*b*d*e**2*g - b*e**3*f - 5*b*(-b*e**3*g/8 - c*d*e**2*g -
c*e**3*f)/(6*c) + c*d**2*e*g - c*d*e**2*f - e*g*(-3*b*d*e + 3*c*d**2)/4)/(
4*c) + c*d**3*g + c*d**2*e*f + (-2*b*d*e + 2*c*d**2)*(-b*e**3*g/8 - c*d*e*
*2*g - c*e**3*f)/(3*c*e**2))/(c*e**2)) + (-b*d**2*e*f - b*(-b*d**2*e*g - 2
*b*d*e**2*f - 3*b*(-2*b*d*e**2*g - b*e**3*f - 5*b*(-b*e**3*g/8 - c*d*e**2*
g - c*e**3*f)/(6*c) + c*d**2*e*g - c*d*e**2*f - e*g*(-3*b*d*e + 3*c*d**2)/
4)/(4*c) + c*d**3*g + c*d**2*e*f + (-2*b*d*e + 2*c*d**2)*(-b*e**3*g/8 - c*
d*e**2*g - c*e**3*f)/(3*c*e**2))/(2*c) + c*d**3*f + (-b*d*e + c*d**2)*(-2*
b*d*e**2*g - b*e**3*f - 5*b*(-b*e**3*g/8 - c*d*e**2*g - c*e**3*f)/(6*c) +
c*d**2*e*g - c*d*e**2*f - e*g*(-3*b*d*e + 3*c*d**2)/4)/(2*c*e**2))*Piecewi
se((log(-b*e**2 - 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*
d**2 - c*e**2*x**2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0
)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True))
, Ne(c*e**2, 0)), (-2*(g*(-b*d*e - b*e**2*x + c*d**2)**(7/2)/(7*b**2*e**3)
 + (-b*d*e - b*e**2*x + c*d**2)**(5/2)*(b*d*e*g - b*e**2*f - 2*c*d**2*g)/(
5*b**2*e**3) + (-b*d*e - b*e**2*x + c*d**2)**(3/2)*(-b*c*d**3*e*g + b*c...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(202) = 404$.

Time = 0.16 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.88

$$\int (d+ex)(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$

$$= \frac{1}{192}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\Bigg(2\bigg(4\left(6\,egx+\frac{8\,c^3e^5f+8\,c^3de^4g+bc^2e^5g}{c^3e^4}\right)x+\frac{48\,c^3de^4f+8\,bc^2e^5f}{}$$

$$-\frac{(64\,c^4d^3ef-96\,bc^3d^2e^2f+48\,b^2c^2de^3f-8\,b^3ce^4f+16\,c^4d^4g-64\,bc^3d^3eg+72\,b^2c^2d^2e^2g-32\,b^3cde^3}{128\,\sqrt{-c}c^3e|e|}$$

input

```
integrate((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algori
thm="giac")
```

output

```
1/192*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(6*e*g*x + (8*c^3*e
^5*f + 8*c^3*d*e^4*g + b*c^2*e^5*g)/(c^3*e^4))*x + (48*c^3*d*e^4*f + 8*b*c
^2*e^5*f - 12*c^3*d^2*e^3*g + 20*b*c^2*d*e^4*g - 5*b^2*c*e^5*g)/(c^3*e^4))
*x - (64*c^3*d^2*e^3*f - 112*b*c^2*d*e^4*f + 24*b^2*c*e^5*f + 64*c^3*d^3*e
^2*g - 116*b*c^2*d^2*e^3*g + 76*b^2*c*d*e^4*g - 15*b^3*e^5*g)/(c^3*e^4)) -
 1/128*(64*c^4*d^3*e*f - 96*b*c^3*d^2*e^2*f + 48*b^2*c^2*d*e^3*f - 8*b^3*c
*e^4*f + 16*c^4*d^4*g - 64*b*c^3*d^3*e*g + 72*b^2*c^2*d^2*e^2*g - 32*b^3*c
*d*e^3*g + 5*b^4*e^4*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x
^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqrt(-c)*c^3*e*abs(e))
```

Mupad [B] (verification not implemented)

Time = 7.27 (sec) , antiderivative size = 801, normalized size of antiderivative = 3.67

$$\int (d+ex)(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$

$$= d\,f\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}$$

$$+\frac{5\,b\,e\,g\,\left(\frac{\ln\left(b\,e^2-2\,\sqrt{-c\,e^2}\,\sqrt{-\left(d+e\,x\right)\,\left(b\,e-c\,d+c\,e\,x\right)}+2\,c\,e^2\,x\right)\,\left(b^3\,e^6+4\,b\,c\,e^4\,\left(c\,d^2-b\,d\,e\right)\right)}{16\,{\left(-c\,e^2\right)}^{5/2}}-\frac{\left(8\,c\,e^2\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^2\right)-3\,b^2\,e^4+2}{24\,c^2\,e}\right.}{8\,c}$$

$$-\frac{d\,g\,\ln\left(b\,e^2-2\,\sqrt{-c\,e^2}\,\sqrt{-\left(d+e\,x\right)\,\left(b\,e-c\,d+c\,e\,x\right)}+2\,c\,e^2\,x\right)\,\left(b^3\,e^6+4\,b\,c\,e^4\,\left(c\,d^2-b\,d\,e\right)\right)}{16\,{\left(-c\,e^2\right)}^{5/2}}$$

$$-\frac{e\,f\,\ln\left(b\,e^2-2\,\sqrt{-c\,e^2}\,\sqrt{-\left(d+e\,x\right)\,\left(b\,e-c\,d+c\,e\,x\right)}+2\,c\,e^2\,x\right)\,\left(b^3\,e^6+4\,b\,c\,e^4\,\left(c\,d^2-b\,d\,e\right)\right)}{16\,{\left(-c\,e^2\right)}^{5/2}}$$

$$+\frac{f\,\left(8\,c\,e^2\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^2\right)-3\,b^2\,e^4+2\,b\,c\,e^4\,x\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{24\,c^2\,e^3}$$

$$+\frac{g\,\left(c\,d^2-b\,d\,e\right)\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}-\frac{\ln\left(b\,e^2-2\,\sqrt{-c\,e^2}\,\sqrt{-\left(d+e\,x\right)\,\left(b\,e-c\,d+c\,e\,x\right)}+2\,c\,e^2\,x\right)\,\left(\right.}{2\,{\left(-c\,e^2\right)}^{3/2}}\right.}{4\,c\,e}$$

$$-\frac{d\,f\,\ln\left(b\,e^2-2\,\sqrt{-c\,e^2}\,\sqrt{-\left(d+e\,x\right)\,\left(b\,e-c\,d+c\,e\,x\right)}+2\,c\,e^2\,x\right)\,\left(\frac{b^2\,e^4}{4}+c\,e^2\,\left(c\,d^2-b\,d\,e\right)\right)}{2\,{\left(-c\,e^2\right)}^{3/2}}$$

$$-\frac{g\,x\,{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)}^{3/2}}{4\,c\,e}$$

$$+\frac{d\,g\,\left(8\,c\,e^2\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^2\right)-3\,b^2\,e^4+2\,b\,c\,e^4\,x\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{24\,c^2\,e^4}$$

input
```
int((f + g*x)*(d + e*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output

```
d*f*(x/2 + b/(4*c))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) + (5*b*e*g
*((log(b*e^2 - 2*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2
*c*e^2*x)*(b^3*e^6 + 4*b*c*e^4*(c*d^2 - b*d*e)))/(16*(-c*e^2)^(5/2)) - ((8
*c*e^2*(c*e^2*x^2 - c*d^2 + b*d*e) - 3*b^2*e^4 + 2*b*c*e^4*x)*(c*d^2 - c*e
^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(24*c^2*e^4)))/(8*c) - (d*g*log(b*e^2 - 2
*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e^2*x)*(b^3*e
^6 + 4*b*c*e^4*(c*d^2 - b*d*e)))/(16*(-c*e^2)^(5/2)) - (e*f*log(b*e^2 - 2*
(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e^2*x)*(b^3*e^
6 + 4*b*c*e^4*(c*d^2 - b*d*e)))/(16*(-c*e^2)^(5/2)) + (f*(8*c*e^2*(c*e^2*x
^2 - c*d^2 + b*d*e) - 3*b^2*e^4 + 2*b*c*e^4*x)*(c*d^2 - c*e^2*x^2 - b*d*e
- b*e^2*x)^(1/2))/(24*c^2*e^3) + (g*(c*d^2 - b*d*e)*((x/2 + b/(4*c))*(c*d^
2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) - (log(b*e^2 - 2*(-c*e^2)^(1/2)*(-(
d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e^2*x)*((b^2*e^4)/4 + c*e^2*(c*d
^2 - b*d*e)))/(2*(-c*e^2)^(3/2))))/(4*c*e) - (d*f*log(b*e^2 - 2*(-c*e^2)^(
1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))^(1/2) + 2*c*e^2*x)*((b^2*e^4)/4 + c*
e^2*(c*d^2 - b*d*e)))/(2*(-c*e^2)^(3/2)) - (g*x*(c*d^2 - c*e^2*x^2 - b*d*e
 - b*e^2*x)^(3/2))/(4*c*e) + (d*g*(8*c*e^2*(c*e^2*x^2 - c*d^2 + b*d*e) - 3
*b^2*e^4 + 2*b*c*e^4*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(24*c
^2*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 1300, normalized size of antiderivative = 5.96

$$\int (d+ex)(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*(15*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b**5*e**5*g - 126*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
 + 2*c*d))*b**4*c*d*e**4*g - 24*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b**4*c*e**5*f + 408*sqrt(c)*asinh((sqrt( - b*e +
c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**2*d**2*e**3*g + 192*sqrt(c)*
asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**2*d*e**
4*f - 624*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*b**2*c**3*d**3*e**2*g - 576*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i
)/sqrt( - b*e + 2*c*d))*b**2*c**3*d**2*e**3*f + 432*sqrt(c)*asinh((sqrt( -
 b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*d**4*e*g + 768*sqrt(c)
*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*d**3*e*
*2*f - 96*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*c**5*d**5*g - 384*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
b*e + 2*c*d))*c**5*d**4*e*f + 15*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b
*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**3*g - 76*sqrt(d + e*x)*sq
rt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*
d*e**2*g - 24*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( -
 b*e + c*d - c*e*x)*b**2*c**2*e**3*f - 10*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*
sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**3*g*x + 116*s
qrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d -...
```

3.139 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{d+ex}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 143

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{d+ex}\,dx = \frac{(4cef-4cdg+beg+2cegx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^2} + \frac{(2cd-be)(4cef-2cdg-beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4c^{3/2}e^2}$$

output
```
1/4*(2*c*e*g*x+b*e*g-4*c*d*g+4*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/
2)/c/e^2+1/4*(-b*e+2*c*d)*(-b*e*g-2*c*d*g+4*c*e*f)*arctan(c^(1/2)*(e*x+d)/
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(3/2)/e^2
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{d+ex}\,dx = \frac{\sqrt{d+ex}\sqrt{cd-be-cex}\Big(\sqrt{d+ex}\sqrt{cd-be-cex}(beg+2c(2ef-2dg+egx))+\sqrt{-\frac{1}{c}}(2cd-be)(-4ce}{4ce^2\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x),
x]
```

output
```
(Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*x]*(Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e
*x]*(b*e*g + 2*c*(2*e*f - 2*d*g + e*g*x)) + Sqrt[-c^(-1)]*(2*c*d - b*e)*(-
4*c*e*f + 2*c*d*g + b*e*g)*Log[Sqrt[d + e*x] + (-c^(-1))^(3/2)*c*Sqrt[c*d
- b*e - c*e*x]]))/(4*c*e^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1215, 1225, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{d+ex}dx$$

$$\Big\downarrow \textcolor{blue}{1215}$$

$$\int \frac{(f+gx)\left(\frac{cd^2-bde}{d}-cex\right)}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \textcolor{blue}{1225}$$

$$\frac{(2cd-be)(-beg-2cdg+4cef)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8ce}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(beg-4cdg+4cef+2cegx)}{4ce^2}$$

$$\Big\downarrow \textcolor{blue}{1092}$$

$$\frac{(2cd-be)(-beg-2cdg+4cef)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4ce}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(beg-4cdg+4cef+2cegx)}{4ce^2}$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-beg-2cdg+4cef)}{8c^{3/2}e^2}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(beg-4cdg+4cef+2cegx)}{4ce^2}$$

input

```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x),x]
```

output

```
((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*
e^2*x^2])/(4*c*e^2) + ((2*c*d - b*e)*(4*c*e*f - 2*c*d*g - b*e*g)*ArcTan[(e
*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(8*c
^(3/2)*e^2)
```

Defintions of rubi rules used

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215

```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(131) = 262$.

Time = 2.45 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.05

method	result
default	$\frac{g\left(-\frac{\left(-2c\,e^{2}x-b\,e^{2}\right)\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}{4c\,e^{2}}-\frac{\left(-4c\,e^{2}\left(-bde+c\,d^{2}\right)-b^{2}e^{4}\right)\arctan\left(\frac{\sqrt{c\,e^{2}}\,\left(x+\frac{b}{2c}\right)}{\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}\right)}{8c\,e^{2}\sqrt{c\,e^{2}}}\right)}{e}-\frac{(dg-ef)\left(\sqrt{-}\right.}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d),x,method=_RETUR
NVERBOSE)
```

output
```
g/e*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-
1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(
1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))-(d*g-e*f)/e^2*((
-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/2*(-b*e^2+2*c*d*e)/(c*e
^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*
(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.76

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{d+ex}\,dx$$

$$= \left[-\frac{(4\,(2\,c^2de-bce^2)f-(4\,c^2d^2-b^2e^2)g)\sqrt{-c}\log\left(8\,c^2e^2x^2+8\,bce^2x-4\,c^2d^2+4\,bcde+b^2e^2-4\,\sqrt{-c}\right.}{}\right.$$

$$-\frac{(4\,(2\,c^2de-bce^2)f-(4\,c^2d^2-b^2e^2)g)\sqrt{c}\arctan\left(\frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,cex+be)\sqrt{c}}{2\,(c^2e^2x^2+bce^2x-c^2d^2+bcde)}\right)-2\,(2\,c^2egx+4\,c^2e}{8\,c^2e^2}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d),x, algori
thm="fricas")
```

output

```
[-1/16*((4*(2*c^2*d*e - b*c*e^2)*f - (4*c^2*d^2 - b^2*e^2)*g)*sqrt(-c)*log
(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqrt(-c
*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*(2*c^2*e
*g*x + 4*c^2*e*f - (4*c^2*d - b*c*e)*g)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2
- b*d*e))/(c^2*e^2), -1/8*((4*(2*c^2*d*e - b*c*e^2)*f - (4*c^2*d^2 - b^2*e
^2)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*
e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*(2*c
^2*e*g*x + 4*c^2*e*f - (4*c^2*d - b*c*e)*g)*sqrt(-c*e^2*x^2 - b*e^2*x + c*
d^2 - b*d*e))/(c^2*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{d+ex}\,dx = \int \frac{\sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\left(f+gx\right)}{d+ex}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d),x)
```

output

```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d),x, algori
thm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$
\begin{aligned}
&\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{d+ex}\,dx \\
&= \frac{1}{4}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left(\frac{2\,gx}{e}+\frac{4\,ce^2f-4\,cdeg+be^2g}{ce^3}\right) \\
&\quad - \frac{(8\,c^2def-4\,bce^2f-4\,c^2d^2g+b^2e^2g)\log\big(\big|-be^2+2\,\big(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-be^2x+cd^2-bde}\big)\sqrt{-c}|e}{8\,\sqrt{-c}ce|e|}
\end{aligned}
$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d),x, algori
thm="giac")
```

output
```
1/4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*g*x/e + (4*c*e^2*f - 4*c
*d*e*g + b*e^2*g)/(c*e^3)) - 1/8*(8*c^2*d*e*f - 4*b*c*e^2*f - 4*c^2*d^2*g
+ b^2*e^2*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*
x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqrt(-c)*c*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$
\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{d+ex}\,dx = \int \frac{(f+g\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{d+e\,x}\,dx
$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.43

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{d+ex}\,dx$$
$$= \frac{i\left(\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^3e^3g-2\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2cd\,e^2g-4\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2c\,e^3}{}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d),x)
```

output

```
(i*(sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**
3*e**3*g - 2*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*
c*d))*b**2*c*d*e**2*g - 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqr
t( - b*e + 2*c*d))*b**2*c*e**3*f - 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*
e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e*g + 16*sqrt(c)*asinh((sqrt( -
b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**2*f + 8*sqrt(c)*as
inh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**3*g - 16*
sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d*
*2*e*f + sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
+ c*d - c*e*x)*b*c*e*g - 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2
*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d*g + 4*sqrt(d + e*x)*sqrt(b*e - 2*c
*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*e*f + 2*sqrt(d +
e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**
2*e*g*x))/(4*c**2*e**2*(b*e - 2*c*d))
```

3.140 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 157

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^2}\,dx = \frac{g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)} + \frac{(2cef-4cdg+beg)\arctan\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{c}(d+ex)}\right)}{\sqrt{c}e^2}$$

output

```
g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2-2*(-d*g+e*f)*(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)+(b*e*g-4*c*d*g+2*c*e*f)*arctan(1/c^(1/2
)/(e*x+d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(1/2)/e^2
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^2}\,dx$$
$$=\frac{\sqrt{(d+ex)(-be+c(d-ex))}\left(\frac{-2ef+3dg+egx}{d+ex}+\frac{(2cef-4cdg+beg)\arctan\left(\frac{\sqrt{cd-be-cex}}{\sqrt{c}\sqrt{d+ex}}\right)}{\sqrt{c}\sqrt{d+ex}\sqrt{-be+c(d-ex)}}\right)}{e^2}$$

input

```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
2,x]
```

output

```
(Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*((-2*e*f + 3*d*g + e*g*x)/(d + e*x
) + ((2*c*e*f - 4*c*d*g + b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*S
qrt[d + e*x])])/(Sqrt[c]*Sqrt[d + e*x]*Sqrt[-(b*e) + c*(d - e*x)])))/e^2
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1216, 1211, 27, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^2}\,dx$$
$$\Big\downarrow \text{ 1216}$$
$$\int \frac{(f+gx)\left(\frac{cd^2-bde}{d}-cex\right)^2}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 1211}$$
$$-\frac{\int \frac{ce^2(cef-2cdg+beg+cegx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{ce^3}-\frac{2(ef-dg)(-be+cd-cex)}{e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow 27$$

$$-\frac{\int \frac{cef-2cdg+beg+cegx}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{e}-\frac{2(ef-dg)(-be+cd-cex)}{e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow 1160$$

$$-\frac{\frac{1}{2}(beg-4cdg+2cef)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}}{e}-\frac{2(ef-dg)(-be+cd-cex)}{e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow 1092$$

$$-\frac{(beg-4cdg+2cef)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)-\frac{g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}}{e}-\frac{2(ef-dg)(-be+cd-cex)}{e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow 217$$

$$-\frac{\frac{\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(beg-4cdg+2cef)}{2\sqrt{c}e}-\frac{g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}}{e}-\frac{2(ef-dg)(-be+cd-cex)}{e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input

```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^2,x]
```

output

```
(-2*(e*f - d*g)*(c*d - b*e - c*e*x))/(e^2*Sqrt[d*(c*d - b*e) - b*e^2*x - c
*e^2*x^2]) - (-((g*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/e) + ((2*c*e
*f - 4*c*d*g + b*e*g)*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2])])/(2*Sqrt[c]*e))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 1216
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
1/2))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(147) = 294.

Time = 2.94 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.37

method	result
default	$\frac{g\left(\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+(-b\,e^{2}+2dec)\left(x+\frac{d}{e}\right)}+\frac{\left(-b\,e^{2}+2dec\right)\arctan\left(\frac{\sqrt{c\,e^{2}}\left(x+\frac{d}{e}-\frac{-b\,e^{2}+2dec}{2c\,e^{2}}\right)}{\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)}}\right)}{2\sqrt{c\,e^{2}}}\right)}{e^{2}}-\frac{(dg-ef)\left(-\frac{2\left(-c\,e$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^2,x,method=_RET
URNVERBOSE)
```

output
```
g/e^2*((-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/2*(-b*e^2+2*c*d
*e)/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/
(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)))-(d*g-e*f)/e^3*(-2/(-b*
e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)-2
*c*e^2/(-b*e^2+2*c*d*e)*((-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)
+1/2*(-b*e^2+2*c*d*e)/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^
2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.54

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^2}\,dx$$

$$=\left[-\frac{(2\,cdef-(4\,cd^2-bde)g+(2\,ce^2f-(4\,cde-be^2)g)x)\sqrt{-c}\log\left(8\,c^2e^2x^2+8\,bce^2x-4\,c^2d^2+4\,bcde\right.}{4}\right.$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^2,x, algo
rithm="fricas")
```

output

```
[-1/4*((2*c*d*e*f - (4*c*d^2 - b*d*e)*g + (2*c*e^2*f - (4*c*d*e - b*e^2)*g
)*x)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^
2*e^2 + 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(
-c)) - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(c*e*g*x - 2*c*e*f + 3
*c*d*g))/(c*e^3*x + c*d*e^2), 1/2*((2*c*d*e*f - (4*c*d^2 - b*d*e)*g + (2*c
*e^2*f - (4*c*d*e - b*e^2)*g)*x)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^
2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^
2*d^2 + b*c*d*e)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(c*e*g*x
- 2*c*e*f + 3*c*d*g))/(c*e^3*x + c*d*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^2}\,dx=\int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{(d+ex)^2}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**2,x
)
```

output

```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^2}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^2,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.87

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^2}\,dx$$

$$= \left(\frac{\left(2\,cef\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)-4\,cdg\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)+beg\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)\right)\arctan\left(\frac{\sqrt{-c+\frac{2\,cd}{ex+d}-\frac{be}{ex+d}}}{\sqrt{c}}\right)}{\sqrt{c}e^3}-\right.$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^2,x, algo
rithm="giac")
```

output
```
((2*c*e*f*sgn(1/(e*x + d))*sgn(e) - 4*c*d*g*sgn(1/(e*x + d))*sgn(e) + b*e*
g*sgn(1/(e*x + d))*sgn(e))*arctan(sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d
))/sqrt(c))/(sqrt(c)*e^3) - 2*(sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*
e*f*sgn(1/(e*x + d))*sgn(e) - sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d
*g*sgn(1/(e*x + d))*sgn(e))/e^3 + (2*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*
x + d))*d*g*sgn(1/(e*x + d))*sgn(e) - b*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e
*x + d))*e*g*sgn(1/(e*x + d))*sgn(e))/((2*c*d/(e*x + d) - b*e/(e*x + d))*e
^3))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^2}\,dx = \int \frac{(f+g\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{{(d+e\,x)}^2}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2,x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2, x
)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.41

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^2}\,dx$$
$$= \frac{i\left(-4\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2d\,e^2g-4\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2e^3gx+24\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)bc\right.}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^2,x)
```

output
```
(i*( - 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*b**2*d*e**2*g - 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b
*e + 2*c*d))*b**2*e**3*g*x + 24*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b*c*d**2*e*g - 8*sqrt(c)*asinh((sqrt( - b*e + c*d
 - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*d*e**2*f + 24*sqrt(c)*asinh((sqrt(
- b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*d*e**2*g*x - 8*sqrt(c)*a
sinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*e**3*f*x - 3
2*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*
d**3*g + 16*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c
*d))*c**2*d**2*e*f - 32*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
 - b*e + 2*c*d))*c**2*d**2*e*g*x + 16*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*d*e**2*f*x + 12*sqrt(d + e*x)*sqrt(b*e
 - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c*d*g - 8*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
c*e*f + 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
 + c*d - c*e*x)*c*e*g*x + sqrt(c)*b**2*d*e**2*g + sqrt(c)*b**2*e**3*g*x -
12*sqrt(c)*b*c*d**2*e*g + 8*sqrt(c)*b*c*d*e**2*f - 12*sqrt(c)*b*c*d*e**2*g
*x + 8*sqrt(c)*b*c*e**3*f*x + 20*sqrt(c)*c**2*d**3*g - 16*sqrt(c)*c**2*d**
2*e*f + 20*sqrt(c)*c**2*d**2*e*g*x - 16*sqrt(c)*c**2*d*e**2*f*x))/(4*c*e**
2*(b*d*e + b*e**2*x - 2*c*d**2 - 2*c*d*e*x))
```

3.141 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 163

$$\begin{aligned}\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3}\,dx &= -\frac{2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(2cd-be)(d+ex)^3}\\ &\quad -\frac{2\sqrt{c}g\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e^2}\end{aligned}$$

output

```
-2*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)-2/3*(-d*g+e*f)*(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)/(e*x+d)^3-2*c^(1/2)*g
*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3}\,dx = \frac{2\sqrt{(d+ex)(-be+c(d-ex))}\left(\frac{be(2dg+e(f+3gx))-c(5d^2g-e^2fx+de(f+7gx))}{(2cd-be)(d+ex)^2}+\frac{3\sqrt{c}g\arctan\left(\frac{\sqrt{cd-be-cex}}{\sqrt{c}\sqrt{d+ex}}\right)}{\sqrt{d+ex}\sqrt{-be+c(d-ex)}}\right)}{3e^2}$$

input

```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
3,x]
```

output

```
(2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*((b*e*(2*d*g + e*(f + 3*g*x)) -
c*(5*d^2*g - e^2*f*x + d*e*(f + 7*g*x)))/((2*c*d - b*e)*(d + e*x)^2) + (3*
Sqrt[c]*g*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/(Sqrt[d
 + e*x]*Sqrt[-(b*e) + c*(d - e*x)])))/(3*e^2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1216, 1218, 1124, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1216}$$

$$\int \frac{(f+gx)\left(\frac{cd^2-bde}{d}-cex\right)^3}{(-bde-be^2x+cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{g\int \frac{(cd-be-cex)^2}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{e}-\frac{2(ef-dg)(-be+cd-cex)^3}{3e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 1124$$

$$\frac{g\left(-c\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{2(-be+cd-cex)}{e\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e}-\frac{2(ef-dg)(-be+cd-cex)^3}{3e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 1092$$

$$\frac{g\left(-2c\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)-\frac{2(-be+cd-cex)}{e\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e}-\frac{2(ef-dg)(-be+cd-cex)^3}{3e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 217$$

$$\frac{g\left(-\frac{\sqrt{c}\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e}-\frac{2(-be+cd-cex)}{e\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e}-\frac{2(ef-dg)(-be+cd-cex)^3}{3e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input

```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^3,x]
```

output

```
(-2*(e*f - d*g)*(c*d - b*e - c*e*x)^3)/(3*e^2*(2*c*d - b*e)*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(3/2)) + (g*((-2*(c*d - b*e - c*e*x))/(e*Sqrt[d*(c
*d - b*e) - b*e^2*x - c*e^2*x^2]) - (Sqrt[c]*ArcTan[(e*(b + 2*c*x))/(2*Sqr
t[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]])/e))/e
```

Defintions of rubi rules used

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1216

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
1/2))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IntegerQ[n]
```

rule 1218

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.83

method	result
default	$\frac{g\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{2}}-\frac{2c\,e^{2}\left(\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)}+\frac{\left(-b\,e^{2}+2dec\right)\arctan\left(\frac{\sqrt{c\,e^{2}}\left(x+\frac{d}{e}-\right.}{\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(\right.}}\right.}{2\sqrt{c\,e^{2}}}\right.}{-b\,e^{2}+2dec}\right.}{e^{3}}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^3,x,method=_RET
URNVERBOSE)
```

output
```
g/e^3*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x
+d/e))^(3/2)-2*c*e^2/(-b*e^2+2*c*d*e)*((-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*
(x+d/e))^(1/2)+1/2*(-b*e^2+2*c*d*e)/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+
d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e)
)^(1/2))))+2/3*(d*g-e*f)/e^4/(-b*e^2+2*c*d*e)/(x+d/e)^3*(-c*e^2*(x+d/e)^2+
(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.55

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3}\,dx$$
$$=\left[\frac{3\left((2\,cde^2-be^3)gx^2+2\,(2\,cd^2e-bde^2)gx+(2\,cd^3-bd^2e)g\right)\sqrt{-c}\log\left(8\,c^2e^2x^2+8\,bce^2x-4\,c^2d^2+4\,b\right.}{6}\right.$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^3,x, algo
rithm="fricas")
```

output
```
[1/6*(3*((2*c*d*e^2 - b*e^3)*g*x^2 + 2*(2*c*d^2*e - b*d*e^2)*g*x + (2*c*d^
3 - b*d^2*e)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b
*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x +
 b*e)*sqrt(-c)) - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((c*d*e - b
*e^2)*f + (5*c*d^2 - 2*b*d*e)*g - (c*e^2*f - (7*c*d*e - 3*b*e^2)*g)*x))/(2
*c*d^3*e^2 - b*d^2*e^3 + (2*c*d*e^4 - b*e^5)*x^2 + 2*(2*c*d^2*e^3 - b*d*e^
4)*x), 1/3*(3*((2*c*d*e^2 - b*e^3)*g*x^2 + 2*(2*c*d^2*e - b*d*e^2)*g*x + (
2*c*d^3 - b*d^2*e)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2
 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c
*d*e)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((c*d*e - b*e^2)*f +
 (5*c*d^2 - 2*b*d*e)*g - (c*e^2*f - (7*c*d*e - 3*b*e^2)*g)*x))/(2*c*d^3*e^
2 - b*d^2*e^3 + (2*c*d*e^4 - b*e^5)*x^2 + 2*(2*c*d^2*e^3 - b*d*e^4)*x)]
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{(d+ex)^3}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**3,x
)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^3,x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(151) = 302$.

Time = 0.94 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.17

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3}\,dx$$

$$= \frac{cg\log\left(\left|-bc^2d^4e^2+2\left(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-be^2x+cd^2-bde}\right)\sqrt{-c}c^2d^4|e|+4\left(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-\right.\right.\right.}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^3,x, algo
rithm="giac")
```

output

```
1/5*c*g*log(abs(-b*c^2*d^4*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e
^2*x + c*d^2 - b*d*e))*sqrt(-c)*c^2*d^4*abs(e) + 4*(sqrt(-c*e^2)*x - sqrt(
-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b*sqrt(-c)*c*d^3*e*abs(e) + 8*(sqrt
(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*c^2*d^3*e + 6*(
sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*b*c*d^2*e^2
 - 12*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*sqrt
(-c)*c*d^2*abs(e) - 4*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2
- b*d*e))^3*b*sqrt(-c)*d*e*abs(e) - 8*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 -
b*e^2*x + c*d^2 - b*d*e))^4*c*d*e - (sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*
e^2*x + c*d^2 - b*d*e))^4*b*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*
e^2*x + c*d^2 - b*d*e))^5*sqrt(-c)*abs(e)))/(sqrt(-c)*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3}\,dx = \int \frac{(f+g\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{{(d+e\,x)}^3}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^3,x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^3, x
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1004, normalized size of antiderivative = 6.16

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^3,x)
```

output

```
(2*i*( - 3*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*
d))*b**2*d**2*e**2*g - 6*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt
( - b*e + 2*c*d))*b**2*d*e**3*g*x - 3*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*e**4*g*x**2 + 12*sqrt(c)*asinh((sqrt(
- b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*d**3*e*g + 24*sqrt(c)*as
inh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*d**2*e**2*g*x
 + 12*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b
*c*d*e**3*g*x**2 - 12*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
 b*e + 2*c*d))*c**2*d**4*g - 24*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*c**2*d**3*e*g*x - 12*sqrt(c)*asinh((sqrt( - b*e +
 c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*d**2*e**2*g*x**2 - 2*sqrt(d +
e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*d
*e*g - sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b*e**2*f - 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*
c*d)*sqrt( - b*e + c*d - c*e*x)*b*e**2*g*x + 5*sqrt(d + e*x)*sqrt(b*e - 2*
c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c*d**2*g + sqrt(d + e
*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c*d*
e*f + 7*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*c*d*e*g*x - sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*
c*d)*sqrt( - b*e + c*d - c*e*x)*c*e**2*f*x - sqrt(c)*b**2*d**2*e**2*g -...
```

3.142 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 137

$$
\begin{aligned}
\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^4}\,dx
&= -\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5e^2(2cd-be)(d+ex)^4} \\
&\quad -\frac{2(2cef+8cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{15e^2(2cd-be)^2(d+ex)^3}
\end{aligned}
$$

output

```
-2/5*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^4-2/15*(-5*b*e*g+8*c*d*g+2*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
3/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$
\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^4}\,dx
$$

$$
= \frac{2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}(-be(3ef+2dg+5egx)+2c(d^2g+e^2fx+4de(f+gx}{15e^2(-2cd+be)^2(d+ex)^3}
$$

input
```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
4,x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-(b*e*(3
*e*f + 2*d*g + 5*e*g*x)) + 2*c*(d^2*g + e^2*f*x + 4*d*e*(f + g*x))))/(15*e
^2*(-2*c*d + b*e)^2*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1216, 1218, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{1216}$$

$$\int \frac{(f+gx)\left(\frac{cd^2-bde}{d}-cex\right)^4}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{(5beg-2c(4dg+ef))\int \frac{(cd-be-cex)^3}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{5e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^4}{5e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{1123}$$

$$\frac{2(-be+cd-cex)^3(5beg-2c(4dg+ef))}{15e^2(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{2(ef-dg)(-be+cd-cex)^4}{5e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}$$

input
```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^4,x]
```

output

```
(-2*(e*f - d*g)*(c*d - b*e - c*e*x)^4)/(5*e^2*(2*c*d - b*e)*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(5/2)) + (2*(5*b*e*g - 2*c*(e*f + 4*d*g))*(c*d - b
*e - c*e*x)^3)/(15*e^2*(2*c*d - b*e)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^
2)^(3/2))
```

Defintions of rubi rules used

rule 1123

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1216

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
1/2))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IntegerQ[n]
```

rule 1218

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

method	result
gosper	$-\frac{2(cex+be-cd)(5be^2gx-8cdegx-2ce^2fx+2bdeg+3be^2f-2cd^2g-8cdef)\sqrt{-x^2ce^2-xbe^2-bde+cd^2}}{15(ex+d)^3e^2(b^2e^2-4bcde+4c^2d^2)}$
orering	$-\frac{2(cex+be-cd)(5be^2gx-8cdegx-2ce^2fx+2bdeg+3be^2f-2cd^2g-8cdef)\sqrt{-x^2ce^2-xbe^2-bde+cd^2}}{15(ex+d)^3e^2(b^2e^2-4bcde+4c^2d^2)}$
trager	$-\frac{2(5bce^3gx^2-8c^2de^2gx^2-2fc^2e^3x^2+5b^2e^3gx-11bcde^2gx+bce^3fx+6c^2d^2egx-6c^2de^2fx+2b^2de^2g+3b^2e^3f-4bcd^2eg-11bc}{15(b^2e^2-4bcde+4c^2d^2)e^2(ex+d)^3}$
default	$-\frac{2g\left(-ce^2\left(x+\frac{d}{e}\right)^2+\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^4(-be^2+2dec)\left(x+\frac{d}{e}\right)^3}-\frac{(dg-ef)(-\frac{2\left(-ce^2\left(x+\frac{d}{e}\right)^2+\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)^4}-\frac{4ce^2(-ce^2\left(x+\frac{d}{e}\right)^2+(-}{15(-be^2+2de})}{e^5}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^4,x,method=_RET
URNVERBOSE)
```

output
```
-2/15*(c*e*x+b*e-c*d)*(5*b*e^2*g*x-8*c*d*e*g*x-2*c*e^2*f*x+2*b*d*e*g+3*b*e
^2*f-2*c*d^2*g-8*c*d*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^3
/e^2/(b^2*e^2-4*b*c*d*e+4*c^2*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(129) = 258$.

Time = 3.98 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.24

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^4}\,dx$$
$$=\frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}((2c^2e^3f+(8c^2de^2-5bce^3)g)x^2-(8c^2d^2e-11bcde^2+3b^2e^3)f-2(c^2}{15(4c^2d^5e^2-4bcd^4e^3+b^2d^3e^4+(4c^2d^2e^5-4bcde^6+b^2e^7)x^3+3(4c^2d^3e^4-}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^4,x, algo
rithm="fricas")
```

output

```
2/15*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((2*c^2*e^3*f + (8*c^2*d*e
^2 - 5*b*c*e^3)*g)*x^2 - (8*c^2*d^2*e - 11*b*c*d*e^2 + 3*b^2*e^3)*f - 2*(c
^2*d^3 - 2*b*c*d^2*e + b^2*d*e^2)*g + ((6*c^2*d*e^2 - b*c*e^3)*f - (6*c^2*
d^2*e - 11*b*c*d*e^2 + 5*b^2*e^3)*g)*x)/(4*c^2*d^5*e^2 - 4*b*c*d^4*e^3 + b
^2*d^3*e^4 + (4*c^2*d^2*e^5 - 4*b*c*d*e^6 + b^2*e^7)*x^3 + 3*(4*c^2*d^3*e^
4 - 4*b*c*d^2*e^5 + b^2*d*e^6)*x^2 + 3*(4*c^2*d^4*e^3 - 4*b*c*d^3*e^4 + b^
2*d^2*e^5)*x)
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{(d+ex)^4}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**4,x
)
```

output

```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^4,x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^4,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,3,0,0]%%%},[6,1]%%%}+%%%{%%{[%%%{-6,[0,2,1,0]%%%}
,0]:[1,0,
```

Mupad [B] (verification not implemented)

Time = 7.30 (sec) , antiderivative size = 1022, normalized size of antiderivative = 7.46

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^4,x)
```

output

```
(((48*c^3*d^2*g - 16*c^3*d*e*f + 12*b*c^2*e^2*f + 20*b^2*c*e^2*g - 64*b*c^
2*d*e*g)/(15*e^2*(b*e - 2*c*d)^3) - (d*((4*c^2*(7*b*e*g - 12*c*d*g + 2*c*e
*f))/(15*e*(b*e - 2*c*d)^3) - (8*c^3*d*g)/(15*e*(b*e - 2*c*d)^3)))/e)*(c*d
^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((2*f*(b*e - c*d))/(
5*b*e^2 - 10*c*d*e) - (d*((2*b*e*g - 2*c*d*g + 2*c*e*f)/(5*b*e^2 - 10*c*d*
e) - (2*c*d*g)/(5*b*e^2 - 10*c*d*e)))/e)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^
2*x)^(1/2))/(d + e*x)^3 - (((d*((4*c^2*e*f - 8*c^2*d*g + 6*b*c*e*g)/(5*(3*
b*e^2 - 6*c*d*e)*(b*e - 2*c*d)) - (4*c^2*d*g)/(5*(3*b*e^2 - 6*c*d*e)*(b*e
- 2*c*d))))/e - (2*b*(b*e*g - 2*c*d*g + c*e*f))/(5*(3*b*e^2 - 6*c*d*e)*(b*
e - 2*c*d)))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2 + ((
(d*((8*c^2*(6*b*e*g - 11*c*d*g + c*e*f))/(15*e*(b*e - 2*c*d)^3) - (8*c^3*d
*g)/(15*e*(b*e - 2*c*d)^3)))/e - (8*c*(b*e - c*d)*(5*b*e*g - 10*c*d*g + c*
e*f))/(15*e^2*(b*e - 2*c*d)^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2
))/(d + e*x) + (((d*((4*c*(4*b*e*g - 7*c*d*g + c*e*f))/(5*(3*b*e^2 - 6*c*d
*e)*(b*e - 2*c*d)) - (4*c^2*d*g)/(5*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d))))/e
 - (4*(b*e - c*d)*(3*b*e*g - 6*c*d*g + c*e*f))/(5*e*(3*b*e^2 - 6*c*d*e)*(b
*e - 2*c*d)))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2 + (
((d*((8*c^3*e*f - 24*c^3*d*g + 16*b*c^2*e*g)/(15*e*(b*e - 2*c*d)^3) - (8*c
^3*d*g)/(15*e*(b*e - 2*c*d)^3)))/e - (2*b*c*(3*b*e*g - 6*c*d*g + 2*c*e*f))
/(15*e*(b*e - 2*c*d)^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(...
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1197, normalized size of antiderivative = 8.74

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^4,x)
```

output

```
(2*i*( - 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*b**2*d*e**2*g - 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
 b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*f - 5*sqrt(d + e*x)*sqr
t(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*g
*x + 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b*c*d**2*e*g + 11*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e
 + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*f + 11*sqrt(d + e*x)*sqrt(
b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*g*
x - sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
 - c*e*x)*b*c*e**3*f*x - 5*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2
*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*e**3*g*x**2 - 2*sqrt(d + e*x)*sqrt(b*
e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d**3*g - 8
*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d -
c*e*x)*c**2*d**2*e*f - 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c
*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*e*g*x + 6*sqrt(d + e*x)*sqrt(b*e
- 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d*e**2*f*x +
 8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
- c*e*x)*c**2*d*e**2*g*x**2 + 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*
e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*e**3*f*x**2 + sqrt(c)*b**2*c*d*
*3*e**2*g + 3*sqrt(c)*b**2*c*d**2*e**3*g*x + 3*sqrt(c)*b**2*c*d*e**4*g*...
```

3.143 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 210

$$
\begin{aligned}
&\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^5}\,dx \\
&= -\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{7e^2(2cd-be)(d+ex)^5} \\
&\quad -\frac{2(4cef+10cdg-7beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{35e^2(2cd-be)^2(d+ex)^4} \\
&\quad -\frac{4c(4cef+10cdg-7beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{105e^2(2cd-be)^3(d+ex)^3}
\end{aligned}
$$

output

```
-2/7*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^5-2/35*(-7*b*e*g+10*c*d*g+4*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(3/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^4-4/105*c*(-7*b*e*g+10*c*d*g+4*c*e*f)*(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.79

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^5}\,dx =$$

$$-\frac{2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}(3b^2e^2(5ef+2dg+7egx)+4c^2(5d^3g+2e^3fx^2+5de}{105e^2(-2cd+be)^3(d-}$$

input
```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
5,x]
```

output
```
(-2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(3*b^2*e
^2*(5*e*f + 2*d*g + 7*e*g*x) + 4*c^2*(5*d^3*g + 2*e^3*f*x^2 + 5*d*e^2*x*(2
*f + g*x) + d^2*e*(23*f + 25*g*x)) - 2*b*c*e*(13*d^2*g + e^2*x*(6*f + 7*g*
x) + d*e*(36*f + 50*g*x))))/(105*e^2*(-2*c*d + b*e)^3*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1216, 1218, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^5}dx$$

$$\Big\downarrow \text{ 1216}$$

$$\int \frac{(f+gx)\left(\frac{cd^2-bde}{d}-cex\right)^5}{(-bde-be^2x+cd^2-ce^2x^2)^{9/2}}dx$$

$$\Big\downarrow \text{ 1218}$$

$$\frac{(-7beg+10cdg+4cef)\int \frac{(cd-be-cex)^4}{(-cx^2e^2-bxe^2+d(cd-be))^{7/2}}dx}{7e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^5}{7e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}$$

$$\Big\downarrow 1129$$

$$\frac{(-7beg+10cdg+4cef)\left(-\frac{2\int \frac{(cd-be-cex)^5}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{7/2}}dx}{3(2cd-be)}-\frac{2(-be+cd-cex)^4}{3e(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{5/2}}\right)}{7e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^5}{7e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}$$

$$\Big\downarrow 1123$$

$$\frac{\left(\frac{4(-be+cd-cex)^5}{15e(2cd-be)^2(d(cd-be)-be^2x-ce^2x^2)^{5/2}}-\frac{2(-be+cd-cex)^4}{3e(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{5/2}}\right)(-7beg+10cdg+4cef)}{7e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^5}{7e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}$$

input

```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^5,x]
```

output

```
(-2*(e*f - d*g)*(c*d - b*e - c*e*x)^5)/(7*e^2*(2*c*d - b*e)*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(7/2)) + ((4*c*e*f + 10*c*d*g - 7*b*e*g)*((-2*(c*d
 - b*e - c*e*x)^4)/(3*e*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2
)^(5/2)) + (4*(c*d - b*e - c*e*x)^5)/(15*e*(2*c*d - b*e)^2*(d*(c*d - b*e)
- b*e^2*x - c*e^2*x^2)^(5/2))))/(7*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1216
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
1/2))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IntegerQ[n]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 5.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.12

method	result
gosper	$-\frac{2(cex+be-cd)(-14bce^3gx^2+20c^2de^2gx^2+8fc^2e^3x^2+21b^2e^3gx-100bcde^2gx-12bce^3fx+100c^2d^2egx+40c^2de^2fx+6b^2de^2g}{105(ex+d)^4(b^3e^3-6de^2b^2c+12d^2ebc^2-8d^3c^3)}$
orering	$-\frac{2(cex+be-cd)(-14bce^3gx^2+20c^2de^2gx^2+8fc^2e^3x^2+21b^2e^3gx-100bcde^2gx-12bce^3fx+100c^2d^2egx+40c^2de^2fx+6b^2de^2g}{105(ex+d)^4(b^3e^3-6de^2b^2c+12d^2ebc^2-8d^3c^3)}$
trager	$-\frac{2(-14bc^2e^4gx^3+20c^3de^3gx^3+8c^3e^4fx^3+7b^2ce^4gx^2-66bc^2de^3gx^2-4bc^2e^4fx^2+80c^3d^2e^2gx^2+32c^3de^3fx^2+21b^3e^4gx-1}{}$
default	$\frac{g\left(-\frac{2\left(-ce^2\left(x+\frac{d}{e}\right)^2+\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)^4}-\frac{4ce^2\left(-ce^2\left(x+\frac{d}{e}\right)^2+\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15\left(-be^2+2dec\right)^2\left(x+\frac{d}{e}\right)^3}\right)}{e^5}-\frac{(dg-ef)(-\frac{2(-ce^2(x+\frac{d}{e})^2+(-}{7(-be^2+2d}}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^5,x,method=_RET
URNVERBOSE)
```

output
```
-2/105*(c*e*x+b*e-c*d)*(-14*b*c*e^3*g*x^2+20*c^2*d*e^2*g*x^2+8*c^2*e^3*f*x
^2+21*b^2*e^3*g*x-100*b*c*d*e^2*g*x-12*b*c*e^3*f*x+100*c^2*d^2*e*g*x+40*c^
2*d*e^2*f*x+6*b^2*d*e^2*g+15*b^2*e^3*f-26*b*c*d^2*e*g-72*b*c*d*e^2*f+20*c^
2*d^3*g+92*c^2*d^2*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^4/(
b^3*e^3-6*b^2*c*d*e^2+12*b*c^2*d^2*e-8*c^3*d^3)/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(198) = 396.

Time = 26.50 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.57

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^5}\,dx$$
$$= \frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,(4\,c^3e^4f+(10\,c^3de^3-7\,bc^2e^4)g)x^3+(4\,(8\,c^3de^3-bc^2e^4)f+(80\,c^3d^2e}{105\,(8\,c^3d^7e^2-12\,bc^2d^6e^3+6\,b^2cd^5e^4-b^3d^4e^5+(8\,c^3d^3e^6-12}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^5,x, algo
rithm="fricas")
```

output
```
2/105*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*c^3*e^4*f + (10*c^3
*d*e^3 - 7*b*c^2*e^4)*g)*x^3 + (4*(8*c^3*d*e^3 - b*c^2*e^4)*f + (80*c^3*d^
2*e^2 - 66*b*c^2*d*e^3 + 7*b^2*c*e^4)*g)*x^2 - (92*c^3*d^3*e - 164*b*c^2*d
^2*e^2 + 87*b^2*c*d*e^3 - 15*b^3*e^4)*f - 2*(10*c^3*d^4 - 23*b*c^2*d^3*e +
 16*b^2*c*d^2*e^2 - 3*b^3*d*e^3)*g + ((52*c^3*d^2*e^2 - 20*b*c^2*d*e^3 + 3
*b^2*c*e^4)*f - (80*c^3*d^3*e - 174*b*c^2*d^2*e^2 + 115*b^2*c*d*e^3 - 21*b
^3*e^4)*g)*x)/(8*c^3*d^7*e^2 - 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^
4*e^5 + (8*c^3*d^3*e^6 - 12*b*c^2*d^2*e^7 + 6*b^2*c*d*e^8 - b^3*e^9)*x^4 +
 4*(8*c^3*d^4*e^5 - 12*b*c^2*d^3*e^6 + 6*b^2*c*d^2*e^7 - b^3*d*e^8)*x^3 +
6*(8*c^3*d^5*e^4 - 12*b*c^2*d^4*e^5 + 6*b^2*c*d^3*e^6 - b^3*d^2*e^7)*x^2 +
 4*(8*c^3*d^6*e^3 - 12*b*c^2*d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x)
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^5}\,dx=\int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{(d+ex)^5}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**5,x
)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^5}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^5,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1619 vs. 2(198) = 396.

Time = 0.17 (sec) , antiderivative size = 1619, normalized size of antiderivative = 7.71

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^5}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^5,x, algo
rithm="giac")
```

output

```
-2/105*(2*(4*sqrt(-c)*c^3*e*f + 10*sqrt(-c)*c^3*d*g - 7*b*sqrt(-c)*c^2*e*g
)*sgn(1/(e*x + d))*sgn(e)/(8*c^3*d^3*e^3 - 12*b*c^2*d^2*e^4 + 6*b^2*c*d*e^
5 - b^3*e^6) - (6*(5*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^3*sqrt(-c + 2*c
*d/(e*x + d) - b*e/(e*x + d)) - 21*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^2
*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 35*c^3*sqrt(-c + 2*c*d/(e*
x + d) - b*e/(e*x + d)) - 35*c^2*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3
/2))*c*d*e^3*f*sgn(1/(e*x + d))*sgn(e)/(8*c^3*d^3*e^3 - 12*b*c^2*d^2*e^4 +
 6*b^2*c*d*e^5 - b^3*e^6) - 3*(5*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^3*s
qrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 21*(c - 2*c*d/(e*x + d) + b*e/
(e*x + d))^2*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 35*c^3*sqrt(-c
 + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 35*c^2*(-c + 2*c*d/(e*x + d) - b*e/(
e*x + d))^(3/2))*b*e^4*f*sgn(1/(e*x + d))*sgn(e)/(8*c^3*d^3*e^3 - 12*b*c^2
*d^2*e^4 + 6*b^2*c*d*e^5 - b^3*e^6) - 6*(5*(c - 2*c*d/(e*x + d) + b*e/(e*x
 + d))^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 21*(c - 2*c*d/(e*x +
 d) + b*e/(e*x + d))^2*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 35*c
^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 35*c^2*(-c + 2*c*d/(e*x +
d) - b*e/(e*x + d))^(3/2))*c*d^2*e^2*g*sgn(1/(e*x + d))*sgn(e)/(8*c^3*d^3*
e^3 - 12*b*c^2*d^2*e^4 + 6*b^2*c*d*e^5 - b^3*e^6) + 3*(5*(c - 2*c*d/(e*x +
 d) + b*e/(e*x + d))^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 21*(c
- 2*c*d/(e*x + d) + b*e/(e*x + d))^2*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/...
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 2325, normalized size of antiderivative = 11.07

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^5,x)
```

output

```
(((d*((16*c^4*e*f - 144*c^4*d*g + 80*b*c^3*e*g)/(105*e*(b*e - 2*c*d)^4) -
(16*c^4*d*g)/(105*e*(b*e - 2*c*d)^4)))/e - (4*b*c^2*(9*b*e*g - 18*c*d*g +
2*c*e*f))/(105*e*(b*e - 2*c*d)^4))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(
1/2))/(d + e*x) - (((d*((16*c^4*e*f - 64*c^4*d*g + 40*b*c^3*e*g)/(105*e*(b
*e - 2*c*d)^4) - (16*c^4*d*g)/(105*e*(b*e - 2*c*d)^4)))/e - (8*b*c^2*(2*b*
e*g - 4*c*d*g + c*e*f))/(105*e*(b*e - 2*c*d)^4))*(c*d^2 - c*e^2*x^2 - b*d*
e - b*e^2*x)^(1/2))/(d + e*x) + (((d*((16*c^4*e*f - 176*c^4*d*g + 96*b*c^3
*e*g)/(105*e*(b*e - 2*c*d)^4) - (16*c^4*d*g)/(105*e*(b*e - 2*c*d)^4)))/e -
 (4*b*c^2*(11*b*e*g - 22*c*d*g + 2*c*e*f))/(105*e*(b*e - 2*c*d)^4))*(c*d^2
 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((16*c^4*e*f - 256
*c^4*d*g + 136*b*c^3*e*g)/(105*e*(b*e - 2*c*d)^4) - (16*c^4*d*g)/(105*e*(b
*e - 2*c*d)^4)))/e - (8*b*c^2*(8*b*e*g - 16*c*d*g + c*e*f))/(105*e*(b*e -
2*c*d)^4))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*(
(4*c^2*e*f - 8*c^2*d*g + 6*b*c*e*g)/(7*(5*b*e^2 - 10*c*d*e)*(b*e - 2*c*d))
 - (4*c^2*d*g)/(7*(5*b*e^2 - 10*c*d*e)*(b*e - 2*c*d))))/e - (2*b*(b*e*g -
2*c*d*g + c*e*f))/(7*(5*b*e^2 - 10*c*d*e)*(b*e - 2*c*d)))*(c*d^2 - c*e^2*x
^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^3 - (((2*f*(b*e - c*d))/(7*b*e^2 -
14*c*d*e) - (d*((2*b*e*g - 2*c*d*g + 2*c*e*f)/(7*b*e^2 - 14*c*d*e) - (2*c*
d*g)/(7*b*e^2 - 14*c*d*e)))/e)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)
)/(d + e*x)^4 - (((d*((4*c^2*(9*b*e*g - 16*c*d*g + 2*c*e*f))/(35*(3*b*e...
```

Reduce [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 1918, normalized size of antiderivative = 9.13

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^5,x)
```

output

```
(2*i*( - 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*b**3*d*e**3*g - 15*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
- b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*f - 21*sqrt(d + e*x)*s
qrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*e**4
*g*x + 32*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
 + c*d - c*e*x)*b**2*c*d**2*e**2*g + 87*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**3*f + 115*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
b**2*c*d*e**3*g*x - 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)
*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**4*f*x - 7*sqrt(d + e*x)*sqrt(b*e - 2
*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**4*g*x**2 -
 46*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
 - c*e*x)*b*c**2*d**3*e*g - 164*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*
e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**2*e**2*f - 174*sqrt(d + e*
x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**
2*d**2*e**2*g*x + 20*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*
sqrt( - b*e + c*d - c*e*x)*b*c**2*d*e**3*f*x + 66*sqrt(d + e*x)*sqrt(b*e -
 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d*e**3*g*x*
*2 + 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b*c**2*e**4*f*x**2 + 14*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq...
```

3.144 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 285

$$\begin{aligned}
&\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^6}\,dx \\
&= -\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{9e^2(2cd-be)(d+ex)^6} \\
&\quad -\frac{2(2cef+4cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{21e^2(2cd-be)^2(d+ex)^5} \\
&\quad -\frac{8c(2cef+4cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{105e^2(2cd-be)^3(d+ex)^4} \\
&\quad -\frac{16c^2(2cef+4cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{315e^2(2cd-be)^4(d+ex)^3}
\end{aligned}$$

output

```
-2/9*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^6-2/21*(-3*b*e*g+4*c*d*g+2*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
3/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^5-8/105*c*(-3*b*e*g+4*c*d*g+2*c*e*f)*(d*(-
b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^4-16/315*c^2*
(-3*b*e*g+4*c*d*g+2*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*
e+2*c*d)^4/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^6}\,dx$$
$$= \frac{2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}(-5b^3e^3(7ef+2dg+9egx)+6b^2ce^2(11d^2g+e^2x(5f+}{}$$

input

```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
6,x]
```

output

```
(2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-5*b^3*e
^3*(7*e*f + 2*d*g + 9*e*g*x) + 6*b^2*c*e^2*(11*d^2*g + e^2*x*(5*f + 6*g*x)
 + d*e*(40*f + 52*g*x)) - 12*b*c^2*e*(12*d^3*g + 2*e^3*x^2*(f + g*x) + 2*d
*e^2*x*(7*f + 8*g*x) + d^2*e*(47*f + 61*g*x)) + 8*c^3*(11*d^4*g + 2*e^4*f*
x^3 + 4*d*e^3*x^2*(3*f + g*x) + 3*d^2*e^2*x*(11*f + 8*g*x) + d^3*e*(58*f +
 66*g*x))))/(315*e^2*(-2*c*d + b*e)^4*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1216, 1218, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^6}dx$$

$$\Big\downarrow \text{1216}$$

$$\int \frac{(f+gx)\left(\frac{cd^2-bde}{d}-cex\right)^6}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{11/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{(3beg-2c(2dg+ef))\int \frac{(cd-be-cex)^5}{(-cx^2e^2-bxe^2+d(cd-be))^{9/2}}dx}{3e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^6}{9e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(3beg-2c(2dg+ef))\left(-\frac{4\int \frac{(cd-be-cex)^6}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{9/2}}dx}{3(2cd-be)}-\frac{2(-be+cd-cex)^5}{3e(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{7/2}}\right)}{3e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^6}{9e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(3beg-2c(2dg+ef))\left(-\frac{4\left(-\frac{2\int \frac{(cd-be-cex)^7}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{9/2}}dx}{5(2cd-be)}-\frac{2(-be+cd-cex)^6}{5e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}\right)}{3(2cd-be)}-\frac{2(-be+cd-}{3e(2cd-be)(d(cd-be)-}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)(-be+cd-cex)^6}{9e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{2(ef-dg)(-be+cd-cex)^6}{9e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}-$$

$$\frac{\left(-\frac{2(-be+cd-cex)^5}{3e(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{7/2}}-\frac{4\left(\frac{4(-be+cd-cex)^7}{35e(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}-\frac{2(-be+cd-cex)^6}{5e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}\right)}{3(2cd-be)}\right)(3beg}{3e(2cd-be)}$$

input
```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^6,x]
```

output

```
(-2*(e*f - d*g)*(c*d - b*e - c*e*x)^6)/(9*e^2*(2*c*d - b*e)*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(9/2)) - ((3*b*e*g - 2*c*(e*f + 2*d*g))*((-2*(c*d
- b*e - c*e*x)^5)/(3*e*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)
^(7/2)) - (4*((-2*(c*d - b*e - c*e*x)^6)/(5*e*(2*c*d - b*e)*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(7/2)) + (4*(c*d - b*e - c*e*x)^7)/(35*e*(2*c*d -
b*e)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))))/(3*(2*c*d - b*e))))/
(3*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1216

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
1/2))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IntegerQ[n]
```

rule 1218

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 6.72 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.34

method	result
gosper	$-\frac{2(cex+be-cd)\left(24b\,c^{2}e^{4}g\,x^{3}-32c^{3}d\,e^{3}g\,x^{3}-16c^{3}e^{4}f\,x^{3}-36b^{2}c\,e^{4}g\,x^{2}+192b\,c^{2}d\,e^{3}g\,x^{2}+24b\,c^{2}e^{4}f\,x^{2}-192c^{3}d^{2}e^{2}g\,x^{2}-96c^{3}d\,e^{3}\right.}{}$
orering	$-\frac{2(cex+be-cd)\left(24b\,c^{2}e^{4}g\,x^{3}-32c^{3}d\,e^{3}g\,x^{3}-16c^{3}e^{4}f\,x^{3}-36b^{2}c\,e^{4}g\,x^{2}+192b\,c^{2}d\,e^{3}g\,x^{2}+24b\,c^{2}e^{4}f\,x^{2}-192c^{3}d^{2}e^{2}g\,x^{2}-96c^{3}d\,e^{3}\right.}{}$
default	$\frac{g\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{7\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{5}}+\frac{4c\,e^{2}\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{4}}-\frac{4c\,e^{2}\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right.\right.}{15\left(-b\,e^{2}+2dec\right)^{2}\left(x+\frac{d}{e}\right.}\right.}{7\left(-b\,e^{2}+2dec\right)}\right.}{e^{6}}$
trager	$-\frac{2\left(24b\,c^{3}e^{5}g\,x^{4}-32c^{4}d\,e^{4}g\,x^{4}-16c^{4}e^{5}f\,x^{4}-12b^{2}c^{2}e^{5}g\,x^{3}+136b\,c^{3}d\,e^{4}g\,x^{3}+8b\,c^{3}e^{5}f\,x^{3}-160c^{4}d^{2}e^{3}g\,x^{3}-80c^{4}d\,e^{4}f\,x^{3}+9b^{3}c\,e^{5}g\right.}{}$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^6,x,method=_RET
URNVERBOSE)
```

output

```
-2/315*(c*e*x+b*e-c*d)*(24*b*c^2*e^4*g*x^3-32*c^3*d*e^3*g*x^3-16*c^3*e^4*f
*x^3-36*b^2*c*e^4*g*x^2+192*b*c^2*d*e^3*g*x^2+24*b*c^2*e^4*f*x^2-192*c^3*d
^2*e^2*g*x^2-96*c^3*d*e^3*f*x^2+45*b^3*e^4*g*x-312*b^2*c*d*e^3*g*x-30*b^2*
c*e^4*f*x+732*b*c^2*d^2*e^2*g*x+168*b*c^2*d*e^3*f*x-528*c^3*d^3*e*g*x-264*
c^3*d^2*e^2*f*x+10*b^3*d*e^3*g+35*b^3*e^4*f-66*b^2*c*d^2*e^2*g-240*b^2*c*d
*e^3*f+144*b*c^2*d^3*e*g+564*b*c^2*d^2*e^2*f-88*c^3*d^4*g-464*c^3*d^3*e*f)
*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^5/e^2/(b^4*e^4-8*b^3*c*d*e
^3+24*b^2*c^2*d^2*e^2-32*b*c^3*d^3*e+16*c^4*d^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(269) = 538$.

Time = 107.64 (sec) , antiderivative size = 817, normalized size of antiderivative = 2.87

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^6,x, algo
rithm="fricas")
```

output

```
2/315*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(8*(2*c^4*e^5*f + (4*c^4*
d*e^4 - 3*b*c^3*e^5)*g)*x^4 + 4*(2*(10*c^4*d*e^4 - b*c^3*e^5)*f + (40*c^4*
d^2*e^3 - 34*b*c^3*d*e^4 + 3*b^2*c^2*e^5)*g)*x^3 + 3*(2*(28*c^4*d^2*e^3 -
8*b*c^3*d*e^4 + b^2*c^2*e^5)*f + (112*c^4*d^3*e^2 - 116*b*c^3*d^2*e^3 + 28
*b^2*c^2*d*e^4 - 3*b^3*c*e^5)*g)*x^2 - (464*c^4*d^4*e - 1028*b*c^3*d^3*e^2
 + 804*b^2*c^2*d^2*e^3 - 275*b^3*c*d*e^4 + 35*b^4*e^5)*f - 2*(44*c^4*d^5 -
 116*b*c^3*d^4*e + 105*b^2*c^2*d^3*e^2 - 38*b^3*c*d^2*e^3 + 5*b^4*d*e^4)*g
 + ((200*c^4*d^3*e^2 - 132*b*c^3*d^2*e^3 + 42*b^2*c^2*d*e^4 - 5*b^3*c*e^5)
*f - (440*c^4*d^4*e - 1116*b*c^3*d^3*e^2 + 978*b^2*c^2*d^2*e^3 - 347*b^3*c
*d*e^4 + 45*b^4*e^5)*g)*x)/(16*c^4*d^9*e^2 - 32*b*c^3*d^8*e^3 + 24*b^2*c^2
*d^7*e^4 - 8*b^3*c*d^6*e^5 + b^4*d^5*e^6 + (16*c^4*d^4*e^7 - 32*b*c^3*d^3*
e^8 + 24*b^2*c^2*d^2*e^9 - 8*b^3*c*d*e^10 + b^4*e^11)*x^5 + 5*(16*c^4*d^5*
e^6 - 32*b*c^3*d^4*e^7 + 24*b^2*c^2*d^3*e^8 - 8*b^3*c*d^2*e^9 + b^4*d*e^10
)*x^4 + 10*(16*c^4*d^6*e^5 - 32*b*c^3*d^5*e^6 + 24*b^2*c^2*d^4*e^7 - 8*b^3
*c*d^3*e^8 + b^4*d^2*e^9)*x^3 + 10*(16*c^4*d^7*e^4 - 32*b*c^3*d^6*e^5 + 24
*b^2*c^2*d^5*e^6 - 8*b^3*c*d^4*e^7 + b^4*d^3*e^8)*x^2 + 5*(16*c^4*d^8*e^3
- 32*b*c^3*d^7*e^4 + 24*b^2*c^2*d^6*e^5 - 8*b^3*c*d^5*e^6 + b^4*d^4*e^7)*x
)
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^6}\,dx = \int \frac{\sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\left(f+gx\right)}{\left(d+ex\right)^6}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**6,x
)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^6,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^6}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^6,x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 4962, normalized size of antiderivative = 17.41

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^6,x)
```

output

```
(((d*((32*c^5*e*f - 160*c^5*d*g + 96*b*c^4*e*g)/(945*e*(b*e - 2*c*d)^5) -
(32*c^5*d*g)/(945*e*(b*e - 2*c*d)^5)))/e - (8*b*c^3*(5*b*e*g - 10*c*d*g +
2*c*e*f))/(945*e*(b*e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(
1/2))/(d + e*x) - (((d*((32*c^5*e*f - 320*c^5*d*g + 176*b*c^4*e*g)/(945*e*
(b*e - 2*c*d)^5) - (32*c^5*d*g)/(945*e*(b*e - 2*c*d)^5)))/e - (16*b*c^3*(5
*b*e*g - 10*c*d*g + c*e*f))/(945*e*(b*e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2 -
b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((32*c^5*e*f - 384*c^5*d*g + 208*
b*c^4*e*g)/(945*e*(b*e - 2*c*d)^5) - (32*c^5*d*g)/(945*e*(b*e - 2*c*d)^5))
)/e - (16*b*c^3*(6*b*e*g - 12*c*d*g + c*e*f))/(945*e*(b*e - 2*c*d)^5))*(c*
d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((32*c^5*e*f -
448*c^5*d*g + 240*b*c^4*e*g)/(945*e*(b*e - 2*c*d)^5) - (32*c^5*d*g)/(945*e
*(b*e - 2*c*d)^5)))/e - (16*b*c^3*(7*b*e*g - 14*c*d*g + c*e*f))/(945*e*(b*
e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + ((
(d*((32*c^5*e*f - 544*c^5*d*g + 288*b*c^4*e*g)/(945*e*(b*e - 2*c*d)^5) - (
32*c^5*d*g)/(945*e*(b*e - 2*c*d)^5)))/e - (8*b*c^3*(17*b*e*g - 34*c*d*g +
2*c*e*f))/(945*e*(b*e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(
1/2))/(d + e*x) + (((d*((32*c^5*e*f - 608*c^5*d*g + 320*b*c^4*e*g)/(945*e*
(b*e - 2*c*d)^5) - (32*c^5*d*g)/(945*e*(b*e - 2*c*d)^5)))/e - (8*b*c^3*(19
*b*e*g - 38*c*d*g + 2*c*e*f))/(945*e*(b*e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2
- b*d*e - b*e^2*x)^(1/2))/(d + e*x) + (((d*((32*c^5*e*f - 672*c^5*d*g +...
```

Reduce [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 2777, normalized size of antiderivative = 9.74

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^6,x)
```

output

```
(2*i*( - 10*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b
*e + c*d - c*e*x)*b**4*d*e**4*g - 35*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
 - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**5*f - 45*sqrt(d + e*x)*
sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**
5*g*x + 76*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*b**3*c*d**2*e**3*g + 275*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*
sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*d*e**4*f + 347*sqrt
(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x
)*b**3*c*d*e**4*g*x - 5*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*
d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**5*f*x - 9*sqrt(d + e*x)*sqrt(b*e -
 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**5*g*x**2
 - 210*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b**2*c**2*d**3*e**2*g - 804*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*s
qrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**2*e**3*f - 978
*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d -
c*e*x)*b**2*c**2*d**2*e**3*g*x + 42*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
- b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d*e**4*f*x + 84*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
b**2*c**2*d*e**4*g*x**2 + 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**5*f*x**2 + 12*sqrt(d + e...
```

3.145 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 360

$$
\begin{aligned}
&\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^7}\,dx\\
&=-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{11e^2(2cd-be)(d+ex)^7}\\
&\quad-\frac{2(8cef+14cdg-11beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{99e^2(2cd-be)^2(d+ex)^6}\\
&\quad-\frac{4c(8cef+14cdg-11beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{231e^2(2cd-be)^3(d+ex)^5}\\
&\quad-\frac{16c^2(8cef+14cdg-11beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{1155e^2(2cd-be)^4(d+ex)^4}\\
&\quad-\frac{32c^3(8cef+14cdg-11beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3465e^2(2cd-be)^5(d+ex)^3}
\end{aligned}
$$

output

```
-2/11*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)/(
e*x+d)^7-2/99*(-11*b*e*g+14*c*d*g+8*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(3/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^6-4/231*c*(-11*b*e*g+14*c*d*g+8*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^5-16/115
5*c^2*(-11*b*e*g+14*c*d*g+8*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/
e^2/(-b*e+2*c*d)^4/(e*x+d)^4-32/3465*c^3*(-11*b*e*g+14*c*d*g+8*c*e*f)*(d*(
-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)^5/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^7}\,dx = $$

$$-\frac{2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}(35b^4e^4(9ef+2dg+11egx)-10b^3ce^3(61d^2g+e^2x(2}{}$$

input

```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
7,x]
```

output

```
(-2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(35*b^4*
e^4*(9*e*f + 2*d*g + 11*e*g*x) - 10*b^3*c*e^3*(61*d^2*g + e^2*x*(28*f + 33
*g*x) + d*e*(280*f + 346*g*x)) + 16*c^4*(91*d^5*g + 8*e^5*f*x^4 + 14*d*e^4
*x^3*(4*f + g*x) + 7*d^3*e^2*x*(52*f + 45*g*x) + 2*d^2*e^3*x^2*(90*f + 49*
g*x) + d^4*e*(547*f + 637*g*x)) + 12*b^2*c^2*e^2*(167*d^3*g + 2*e^3*x^2*(1
0*f + 11*g*x) + d*e^2*x*(180*f + 211*g*x) + d^2*e*(790*f + 986*g*x)) - 8*b
*c^3*e*(365*d^4*g + 2*e^4*x^3*(12*f + 11*g*x) + 4*d*e^3*x^2*(48*f + 49*g*x
) + 3*d^2*e^2*x*(244*f + 277*g*x) + 2*d^3*e*(912*f + 1141*g*x))))/(3465*e^
2*(-2*c*d + b*e)^5*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1216, 1218, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^7}dx$$

$$\Big\downarrow \text{1216}$$

$$\int \frac{(f+gx)\left(\frac{cd^2-bde}{d}-cex\right)^7}{(-bde-be^2x+cd^2-ce^2x^2)^{13/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(-11beg+14cdg+8cef)\int \frac{(cd-be-cex)^6}{(-cx^2e^2-bxe^2+d(cd-be))^{11/2}}dx}{11e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^7}{11e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-11beg+14cdg+8cef)\left(-\frac{2\int \frac{(cd-be-cex)^7}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{11/2}}dx}{2cd-be}-\frac{2(-be+cd-cex)^6}{3e(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{9/2}}\right)}{11e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^7}{11e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-11beg+14cdg+8cef)\left(-\frac{2\left(-\frac{4\int \frac{(cd-be-cex)^8}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{11/2}}dx}{5(2cd-be)}-\frac{2(-be+cd-cex)^7}{5e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}\right)}{2cd-be}-\frac{2(-be+}{3e(2cd-be)(d(cd-b}\right.}{11e(2cd-be)}$$

$$\frac{2(ef-dg)(-be+cd-cex)^7}{11e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}$$

$$\Big\downarrow 1129$$

$$\frac{(-11beg+14cdg+8cef)\left(-\frac{2\left(-\frac{4\left(-\frac{2\int \frac{(cd-be-cex)^9}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{11/2}}dx}{7(2cd-be)}-\frac{2(-be+cd-cex)^8}{7e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}\right)}{5(2cd-be)}-\frac{2(-be+}{5e(2cd-be)\left(d(cd-\right.}\right.}{2cd-be}\right.}{11e(2cd-be)}$$

$$\frac{2(ef-dg)(-be+cd-cex)^7}{11e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}$$

$$\Big\downarrow 1123$$

$$\frac{\left(-\frac{2(-be+cd-cex)^6}{3e(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{9/2}}-\frac{2\left(-\frac{2(-be+cd-cex)^7}{5e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}-\frac{4\left(\frac{4(-be+cd-cex)^9}{63e(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}-\frac{}{7e(2cd-}\right.}{5(2cd-be)}\right.}{2cd-be}\right.}{11e(2cd-be)}$$

$$\frac{2(ef-dg)(-be+cd-cex)^7}{11e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}$$

input
```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^7,x]
```

output
```
(-2*(e*f - d*g)*(c*d - b*e - c*e*x)^7)/(11*e^2*(2*c*d - b*e)*(d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2)^(11/2)) + ((8*c*e*f + 14*c*d*g - 11*b*e*g)*((-2*(
c*d - b*e - c*e*x)^6)/(3*e*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*
x^2)^(9/2)) - (2*((-2*(c*d - b*e - c*e*x)^7)/(5*e*(2*c*d - b*e)*(d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2)^(9/2)) - (4*((-2*(c*d - b*e - c*e*x)^8)/(7*e*(
2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(9/2)) + (4*(c*d - b*e
- c*e*x)^9)/(63*e*(2*c*d - b*e)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(9
/2))))/(5*(2*c*d - b*e))))/(2*c*d - b*e)))/(11*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1216
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
1/2))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IntegerQ[n]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 8.76 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.57

method	result
gosper	$-\frac{2(cex+be-cd)\left(-176bc^3e^5gx^4+224c^4de^4gx^4+128c^4e^5fx^4+264b^2c^2e^5gx^3-1568bc^3de^4gx^3-192bc^3e^5fx^3+1568c^4d^2e^3gx^3\right.}{}$
orering	$-\frac{2(cex+be-cd)\left(-176bc^3e^5gx^4+224c^4de^4gx^4+128c^4e^5fx^4+264b^2c^2e^5gx^3-1568bc^3de^4gx^3-192bc^3e^5fx^3+1568c^4d^2e^3gx^3\right.}{}$
default	$\frac{g\left(-\frac{2\left(-ce^2\left(x+\frac{d}{e}\right)^2+\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{9\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)^6}+\frac{2ce^2\left(-\frac{2\left(-ce^2\left(x+\frac{d}{e}\right)^2+\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{7\left(-be^2+2dec\right)\left(x+\frac{d}{e}\right)^5}+\frac{4ce^2\left(-\frac{2\left(-ce^2\left(x+\frac{d}{e}\right)^2+\left(-be^2+\right.\right.}{5\left(-be^2+2dec\right)\left(\right.}\right.}{}\right.}{3\left(-be^2+2dec\right)}\right.}{e^7}$
trager	$-\frac{2\left(-176bc^4e^6gx^5+224c^5de^5gx^5+128c^5e^6fx^5+88b^2c^3e^6gx^4-1168bc^4de^5gx^4-64bc^4e^6fx^4+1344c^5d^2e^4gx^4+768c^5de^5fx^4\right.}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^7,x,method=_RET
URNVERBOSE)
```

output
```
-2/3465*(c*e*x+b*e-c*d)*(-176*b*c^3*e^5*g*x^4+224*c^4*d*e^4*g*x^4+128*c^4*
e^5*f*x^4+264*b^2*c^2*e^5*g*x^3-1568*b*c^3*d*e^4*g*x^3-192*b*c^3*e^5*f*x^3
+1568*c^4*d^2*e^3*g*x^3+896*c^4*d*e^4*f*x^3-330*b^3*c*e^5*g*x^2+2532*b^2*c
^2*d*e^4*g*x^2+240*b^2*c^2*e^5*f*x^2-6648*b*c^3*d^2*e^3*g*x^2-1536*b*c^3*d
*e^4*f*x^2+5040*c^4*d^3*e^2*g*x^2+2880*c^4*d^2*e^3*f*x^2+385*b^4*e^5*g*x-3
460*b^3*c*d*e^4*g*x-280*b^3*c*e^5*f*x+11832*b^2*c^2*d^2*e^3*g*x+2160*b^2*c
^2*d*e^4*f*x-18256*b*c^3*d^3*e^2*g*x-5856*b*c^3*d^2*e^3*f*x+10192*c^4*d^4*
e*g*x+5824*c^4*d^3*e^2*f*x+70*b^4*d*e^4*g+315*b^4*e^5*f-610*b^3*c*d^2*e^3*
g-2800*b^3*c*d*e^4*f+2004*b^2*c^2*d^3*e^2*g+9480*b^2*c^2*d^2*e^3*f-2920*b*
c^3*d^4*e*g-14592*b*c^3*d^3*e^2*f+1456*c^4*d^5*g+8752*c^4*d^4*e*f)*(-c*e^2
*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^6/e^2/(b^5*e^5-10*b^4*c*d*e^4+40*b
^3*c^2*d^2*e^3-80*b^2*c^3*d^3*e^2+80*b*c^4*d^4*e-32*c^5*d^5)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^7}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^7,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^7}\,dx=\int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{(d+ex)^7}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**7,x
)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^7,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^7}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^7,x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 21.58 (sec) , antiderivative size = 10084, normalized size of antiderivative = 28.01

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^7}\,dx = \text{Too large to display}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^7,x)
```

output
```
(((d*((8*c^2*(12*b*e*g - 23*c*d*g + c*e*f))/(99*(7*b*e^2 - 14*c*d*e)*(b*e
- 2*c*d)^2) - (8*c^3*d*g)/(99*(7*b*e^2 - 14*c*d*e)*(b*e - 2*c*d)^2)))/e -
(8*c*(b*e - c*d)*(11*b*e*g - 22*c*d*g + c*e*f))/(99*e*(7*b*e^2 - 14*c*d*e)
*(b*e - 2*c*d)^2))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^
4 - (((d*((64*c^6*e*f - 1152*c^6*d*g + 608*b*c^5*e*g)/(10395*e*(b*e - 2*c*
d)^6) - (64*c^6*d*g)/(10395*e*(b*e - 2*c*d)^6)))/e - (32*b*c^4*(9*b*e*g -
18*c*d*g + c*e*f))/(10395*e*(b*e - 2*c*d)^6))*(c*d^2 - c*e^2*x^2 - b*d*e -
 b*e^2*x)^(1/2))/(d + e*x) - (((d*((64*c^6*e*f - 1280*c^6*d*g + 672*b*c^5*
e*g)/(10395*e*(b*e - 2*c*d)^6) - (64*c^6*d*g)/(10395*e*(b*e - 2*c*d)^6)))/
e - (32*b*c^4*(10*b*e*g - 20*c*d*g + c*e*f))/(10395*e*(b*e - 2*c*d)^6))*(c
*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (2*((d*((64*c^6*e*f
 - 1408*c^6*d*g + 736*b*c^5*e*g)/(10395*e*(b*e - 2*c*d)^6) - (64*c^6*d*g)/
(10395*e*(b*e - 2*c*d)^6)))/e - (32*b*c^4*(11*b*e*g - 22*c*d*g + c*e*f))/(
10395*e*(b*e - 2*c*d)^6))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d
+ e*x) - (((d*((64*c^6*e*f - 1536*c^6*d*g + 800*b*c^5*e*g)/(10395*e*(b*e -
 2*c*d)^6) - (64*c^6*d*g)/(10395*e*(b*e - 2*c*d)^6)))/e - (32*b*c^4*(12*b*
e*g - 24*c*d*g + c*e*f))/(10395*e*(b*e - 2*c*d)^6))*(c*d^2 - c*e^2*x^2 - b
*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((64*c^6*e*f - 1664*c^6*d*g + 864*
b*c^5*e*g)/(10395*e*(b*e - 2*c*d)^6) - (64*c^6*d*g)/(10395*e*(b*e - 2*c*d)
^6)))/e - (32*b*c^4*(13*b*e*g - 26*c*d*g + c*e*f))/(10395*e*(b*e - 2*c*...
```

Reduce [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 3784, normalized size of antiderivative = 10.51

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^7}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^7,x)
```

output

```
(2*i*( - 70*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b
*e + c*d - c*e*x)*b**5*d*e**5*g - 315*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt
( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*e**6*f - 385*sqrt(d + e*x
)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*e
**6*g*x + 680*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( -
 b*e + c*d - c*e*x)*b**4*c*d**2*e**4*g + 3115*sqrt(d + e*x)*sqrt(b*e - 2*c
*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*d*e**5*f + 3775
*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d -
c*e*x)*b**4*c*d*e**5*g*x - 35*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e
+ 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*e**6*f*x - 55*sqrt(d + e*x)*sqr
t(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*e**6
*g*x**2 - 2614*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt(
- b*e + c*d - c*e*x)*b**3*c**2*d**3*e**3*g - 12280*sqrt(d + e*x)*sqrt(b*e
- 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d**2*e*
*4*f - 14682*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( -
b*e + c*d - c*e*x)*b**3*c**2*d**2*e**4*g*x + 360*sqrt(d + e*x)*sqrt(b*e -
2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d*e**5*f*
x + 598*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*b**3*c**2*d*e**5*g*x**2 + 40*sqrt(d + e*x)*sqrt(b*e - 2*c*d)
*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*e**6*f*x**2 ...
```

3.146 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^8}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 439

$$\begin{aligned}
&\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^8}\,dx \\
&= -\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{13e^2(2cd-be)(d+ex)^8} \\
&\quad -\frac{2(10cef+16cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{143e^2(2cd-be)^2(d+ex)^7} \\
&\quad +\frac{16c(13beg-2c(5ef+8dg))\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{1287e^2(2cd-be)^3(d+ex)^6} \\
&\quad -\frac{32c^2(10cef+16cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3003e^2(2cd-be)^4(d+ex)^5} \\
&\quad +\frac{128c^3(13beg-2c(5ef+8dg))\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{15015e^2(2cd-be)^5(d+ex)^4} \\
&\quad -\frac{256c^4(10cef+16cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{45045e^2(2cd-be)^6(d+ex)^3}
\end{aligned}$$

output
```
-2/13*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)/(
e*x+d)^8-2/143*(-13*b*e*g+16*c*d*g+10*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(3/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^7+16/1287*c*(13*b*e*g-2*c*(8*d*g+5*e*
f))*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^6-32
/3003*c^2*(-13*b*e*g+16*c*d*g+10*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
3/2)/e^2/(-b*e+2*c*d)^4/(e*x+d)^5+128/15015*c^3*(13*b*e*g-2*c*(8*d*g+5*e*f
))*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(-b*e+2*c*d)^5/(e*x+d)^4-256
/45045*c^4*(-13*b*e*g+16*c*d*g+10*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(3/2)/e^2/(-b*e+2*c*d)^6/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^8}\,dx$$
$$= \frac{2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}(-315b^5e^5(11ef+2dg+13egx)-40b^3c^2e^3(736d^3g+2}{}$$

input
```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
8,x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-315*b^5
*e^5*(11*e*f + 2*d*g + 13*e*g*x) - 40*b^3*c^2*e^3*(736*d^3*g + 2*e^3*x^2*(
35*f + 39*g*x) + 85*d^2*e*(49*f + 59*g*x) + 2*d*e^2*x*(385*f + 446*g*x)) +
 70*b^4*c*e^4*(97*d^2*g + e^2*x*(45*f + 52*g*x) + d*e*(540*f + 644*g*x)) +
 32*c^5*(911*d^6*g + 40*e^6*f*x^5 + 64*d*e^5*x^4*(5*f + g*x) + 32*d^3*e^3*
x^2*(85*f + 59*g*x) + 4*d^2*e^4*x^3*(295*f + 128*g*x) + 8*d^5*e*(775*f + 9
11*g*x) + d^4*e^2*x*(4555*f + 4352*g*x)) + 48*b^2*c^3*e^2*(1337*d^4*g + 2*
e^4*x^3*(25*f + 26*g*x) + 4*d*e^3*x^2*(125*f + 137*g*x) + d^2*e^2*x*(2425*
f + 2802*g*x) + d^3*e*(7750*f + 9418*g*x)) - 16*b*c^4*e*(4378*d^5*g + 8*e^
5*x^4*(15*f + 13*g*x) + 44*d^2*e^3*x^2*(105*f + 109*g*x) + 8*d*e^4*x^3*(13
5*f + 128*g*x) + 4*d^3*e^2*x*(3195*f + 3616*g*x) + d^4*e*(26445*f + 32291*
g*x))))/(45045*e^2*(-2*c*d + b*e)^6*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1216, 1218, 1129, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^8}dx$$

$$\Big\downarrow \text{1216}$$

$$\int \frac{(f+gx)\left(\frac{cd^2-bde}{d}-cex\right)^8}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{15/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(-13beg+16cdg+10cef)\int \frac{(cd-be-cex)^7}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{13/2}}dx}{13e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^8}{13e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-13beg+16cdg+10cef)\left(-\frac{8\int \frac{(cd-be-cex)^8}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{13/2}}dx}{3(2cd-be)}-\frac{2(-be+cd-cex)^7}{3e(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{11/2}}\right)}{13e(2cd-be)}-\frac{2(ef-dg)(-be+cd-cex)^8}{13e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-13beg+16cdg+10cef)\left(-\frac{8\left(-\frac{6\int\frac{(cd-be-cex)^9}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{13/2}}dx}{5(2cd-be)}-\frac{2(-be+cd-cex)^8}{5e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}\right)}{3(2cd-be)}-\frac{2(-b}{3e(2cd-be)(d(cd}\right.}{13e(2cd-be)}$$

$$\frac{2(ef-dg)(-be+cd-cex)^8}{13e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-13beg+16cdg+10cef)\left(-\frac{8\left(-\frac{6\left(-\frac{4\int\frac{(cd-be-cex)^{10}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{13/2}}dx}{7(2cd-be)}-\frac{2(-be+cd-cex)^9}{7e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}\right)}{5(2cd-be)}-\frac{2(-}{5e(2cd-be)\left(d(cc\right.}\right.}{3(2cd-be)}\right.}{13e(2cd-be)}$$

$$\frac{2(ef-dg)(-be+cd-cex)^8}{13e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-13beg+16cdg+10cef)\left(-\frac{8\left(-\frac{6\left(-\frac{4\left(-\frac{2\int \frac{(cd-be-cex)^{11}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{13/2}}dx}{9(2cd-be)}-\frac{2(-be+cd-cex)^{10}}{9e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}\right)}{7(2cd-be)}-\frac{}{7e(2cd-be)}\right.}{5(2cd-be)}\right.}{3(2cd-be)}\right.}{13e(2cd-be)}$$

$$\frac{2(ef-dg)(-be+cd-cex)^8}{13e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}$$

$$\Big\downarrow \ 1123$$

$$\frac{\left(-\frac{2(-be+cd-cex)^7}{3e(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{11/2}}-\frac{8\left(-\frac{2(-be+cd-cex)^8}{5e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}-\frac{6\left(-\frac{2(-be+cd-cex)^9}{7e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}-\frac{4\left(-\right.}{9}\right.}{}\right.}{3(2cd-be}\right.}{13e(2cd-be)}$$

$$\frac{2(ef-dg)(-be+cd-cex)^8}{13e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}$$

input
```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^8,x]
```

output
```
(-2*(e*f - d*g)*(c*d - b*e - c*e*x)^8)/(13*e^2*(2*c*d - b*e)*(d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2)^(13/2)) + ((10*c*e*f + 16*c*d*g - 13*b*e*g)*((-2*
(c*d - b*e - c*e*x)^7)/(3*e*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2
*x^2)^(11/2)) - (8*((-2*(c*d - b*e - c*e*x)^8)/(5*e*(2*c*d - b*e)*(d*(c*d
- b*e) - b*e^2*x - c*e^2*x^2)^(11/2)) - (6*((-2*(c*d - b*e - c*e*x)^9)/(7*
e*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(11/2)) - (4*((-2*(c
*d - b*e - c*e*x)^10)/(9*e*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*
x^2)^(11/2)) + (4*(c*d - b*e - c*e*x)^11)/(99*e*(2*c*d - b*e)^2*(d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2)^(11/2))))/(7*(2*c*d - b*e))))/(5*(2*c*d - b*e)
)))/(3*(2*c*d - b*e))))/(13*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1216
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_.)*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
1/2))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IntegerQ[n]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 11.36 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.78

method	result
gosper	$-\frac{2(cex+be-cd)\left(1664b\,c^4e^6g\,x^5-2048c^5d\,e^5g\,x^5-1280c^5e^6f\,x^5-2496b^2c^3e^6g\,x^4+16384b\,c^4d\,e^5g\,x^4+1920b\,c^4e^6f\,x^4-16384c^5d^2\right.}{}$
orering	$-\frac{2(cex+be-cd)\left(1664b\,c^4e^6g\,x^5-2048c^5d\,e^5g\,x^5-1280c^5e^6f\,x^5-2496b^2c^3e^6g\,x^4+16384b\,c^4d\,e^5g\,x^4+1920b\,c^4e^6f\,x^4-16384c^5d^2\right.}{}$
default	$\frac{g\left(-\frac{2\left(-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{11\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)^7}+\frac{8c\,e^2\left(-\frac{2\left(-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{9\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)^6}+\frac{2c\,e^2\left(-\frac{2\left(-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+\right.\right.}{7\left(-b\,e^2+2dec\right)\left(\right.}\right.}{}\right.}{}\right.}{e^8}$
trager	Expression too large to display

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^8,x,method=_RET
URNVERBOSE)
```

output
```
-2/45045*(c*e*x+b*e-c*d)*(1664*b*c^4*e^6*g*x^5-2048*c^5*d*e^5*g*x^5-1280*c
^5*e^6*f*x^5-2496*b^2*c^3*e^6*g*x^4+16384*b*c^4*d*e^5*g*x^4+1920*b*c^4*e^6
*f*x^4-16384*c^5*d^2*e^4*g*x^4-10240*c^5*d*e^5*f*x^4+3120*b^3*c^2*e^6*g*x^
3-26304*b^2*c^3*d*e^5*g*x^3-2400*b^2*c^3*e^6*f*x^3+76736*b*c^4*d^2*e^4*g*x
^3+17280*b*c^4*d*e^5*f*x^3-60416*c^5*d^3*e^3*g*x^3-37760*c^5*d^2*e^4*f*x^3
-3640*b^4*c*e^6*g*x^2+35680*b^3*c^2*d*e^5*g*x^2+2800*b^3*c^2*e^6*f*x^2-134
496*b^2*c^3*d^2*e^4*g*x^2-24000*b^2*c^3*d*e^5*f*x^2+231424*b*c^4*d^3*e^3*g
*x^2+73920*b*c^4*d^2*e^4*f*x^2-139264*c^5*d^4*e^2*g*x^2-87040*c^5*d^3*e^3*
f*x^2+4095*b^5*e^6*g*x-45080*b^4*c*d*e^5*g*x-3150*b^4*c*e^6*f*x+200600*b^3
*c^2*d^2*e^4*g*x+30800*b^3*c^2*d*e^5*f*x-452064*b^2*c^3*d^3*e^3*g*x-116400
*b^2*c^3*d^2*e^4*f*x+516656*b*c^4*d^4*e^2*g*x+204480*b*c^4*d^3*e^3*f*x-233
216*c^5*d^5*e*g*x-145760*c^5*d^4*e^2*f*x+630*b^5*d*e^5*g+3465*b^5*e^6*f-67
90*b^4*c*d^2*e^4*g-37800*b^4*c*d*e^5*f+29440*b^3*c^2*d^3*e^3*g+166600*b^3*
c^2*d^2*e^4*f-64176*b^2*c^3*d^4*e^2*g-372000*b^2*c^3*d^3*e^3*f+70048*b*c^4
*d^5*e*g+423120*b*c^4*d^4*e^2*f-29152*c^5*d^6*g-198400*c^5*d^5*e*f)*(-c*e^
2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^7/e^2/(b^6*e^6-12*b^5*c*d*e^5+60*
b^4*c^2*d^2*e^4-160*b^3*c^3*d^3*e^3+240*b^2*c^4*d^4*e^2-192*b*c^5*d^5*e+64
*c^6*d^6)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^8}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^8,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^8}\,dx=\int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{(d+ex)^8}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**8,x
)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**8, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^8}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^8,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^8}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^8,x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 37.47 (sec) , antiderivative size = 19572, normalized size of antiderivative = 44.58

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^8}\,dx = \text{Too large to display}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^8,x)
```

output
```
(((d*((8*c^2*(14*b*e*g - 27*c*d*g + c*e*f))/(143*(9*b*e^2 - 18*c*d*e)*(b*e
 - 2*c*d)^2) - (8*c^3*d*g)/(143*(9*b*e^2 - 18*c*d*e)*(b*e - 2*c*d)^2)))/e
- (8*c*(b*e - c*d)*(13*b*e*g - 26*c*d*g + c*e*f))/(143*e*(9*b*e^2 - 18*c*d
*e)*(b*e - 2*c*d)^2))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*
x)^5 - (((2*f*(b*e - c*d))/(13*b*e^2 - 26*c*d*e) - (d*((2*b*e*g - 2*c*d*g
+ 2*c*e*f)/(13*b*e^2 - 26*c*d*e) - (2*c*d*g)/(13*b*e^2 - 26*c*d*e)))/e)*(c
*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^7 - (((d*((4*c^2*(15*
b*e*g - 28*c*d*g + 2*c*e*f))/(143*(9*b*e^2 - 18*c*d*e)*(b*e - 2*c*d)^2) -
(8*c^3*d*g)/(143*(9*b*e^2 - 18*c*d*e)*(b*e - 2*c*d)^2)))/e - (144*c^3*d^2*
g - 48*c^3*d*e*f + 28*b*c^2*e^2*f + 52*b^2*c*e^2*g - 176*b*c^2*d*e*g)/(143
*e*(9*b*e^2 - 18*c*d*e)*(b*e - 2*c*d)^2))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e
^2*x)^(1/2))/(d + e*x)^5 - (((d*((4*c^2*e*f - 8*c^2*d*g + 6*b*c*e*g)/(13*(
11*b*e^2 - 22*c*d*e)*(b*e - 2*c*d)) - (4*c^2*d*g)/(13*(11*b*e^2 - 22*c*d*e
)*(b*e - 2*c*d))))/e - (2*b*(b*e*g - 2*c*d*g + c*e*f))/(13*(11*b*e^2 - 22*
c*d*e)*(b*e - 2*c*d)))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e
*x)^6 - (((d*((16*c^4*e*f - 64*c^4*d*g + 40*b*c^3*e*g)/(1287*(7*b*e^2 - 14
*c*d*e)*(b*e - 2*c*d)^3) - (16*c^4*d*g)/(1287*(7*b*e^2 - 14*c*d*e)*(b*e -
2*c*d)^3)))/e - (8*b*c^2*(2*b*e*g - 4*c*d*g + c*e*f))/(1287*(7*b*e^2 - 14*
c*d*e)*(b*e - 2*c*d)^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d +
 e*x)^4 + (((d*((16*c^4*e*f - 240*c^4*d*g + 128*b*c^3*e*g)/(1287*(7*b*e...
```

Reduce [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 4939, normalized size of antiderivative = 11.25

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^8}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^8,x)
```

output
```
(2*i*( - 630*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( -
b*e + c*d - c*e*x)*b**6*d*e**6*g - 3465*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**6*e**7*f - 4095*sqrt(d +
e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**
6*e**7*g*x + 7420*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqr
t( - b*e + c*d - c*e*x)*b**5*c*d**2*e**5*g + 41265*sqrt(d + e*x)*sqrt(b*e
- 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*c*d*e**6*f +
 48545*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b**5*c*d*e**6*g*x - 315*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
 - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*c*e**7*f*x - 455*sqrt(d +
e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**
5*c*e**7*g*x**2 - 36230*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*
d)*sqrt( - b*e + c*d - c*e*x)*b**4*c**2*d**3*e**4*g - 204400*sqrt(d + e*x)
*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*
*2*d**2*e**5*f - 238890*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*
d)*sqrt( - b*e + c*d - c*e*x)*b**4*c**2*d**2*e**5*g*x + 3850*sqrt(d + e*x)
*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*
*2*d*e**6*f*x + 5760*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*
sqrt( - b*e + c*d - c*e*x)*b**4*c**2*d*e**6*g*x**2 + 350*sqrt(d + e*x)*sqr
t(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c**...
```

3.147 $\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 424

$$
\begin{aligned}
&\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x\right.\\
&\left.-ce^2x^2\right)^{3/2}\,dx=\frac{9(2cd-be)^5(16cef+6cdg-11beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{16384c^6e}\\
&+\frac{3(2cd-be)^3(16cef+6cdg-11beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2048c^5e}\\
&-\frac{(16cef+6cdg-11beg)(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{112c^2e^2}\\
&-\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{8ce^2}\\
&-\frac{3(2cd-be)(16cef+6cdg-11beg)(24cd-7be+10cex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{4480c^4e^2}\\
&+\frac{9(2cd-be)^7(16cef+6cdg-11beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{16384c^{13/2}e^2}
\end{aligned}
$$

output
```
9/16384*(-b*e+2*c*d)^5*(-11*b*e*g+6*c*d*g+16*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d
)-b*e^2*x-c*e^2*x^2)^(1/2)/c^6/e+3/2048*(-b*e+2*c*d)^3*(-11*b*e*g+6*c*d*g+
16*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^5/e-1/112*(-1
1*b*e*g+6*c*d*g+16*c*e*f)*(e*x+d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)
/c^2/e^2-1/8*g*(e*x+d)^3*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c/e^2-3/44
80*(-b*e+2*c*d)*(-11*b*e*g+6*c*d*g+16*c*e*f)*(10*c*e*x-7*b*e+24*c*d)*(d*(-
b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^4/e^2+9/16384*(-b*e+2*c*d)^7*(-11*b*e*
g+6*c*d*g+16*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(1/2))/c^(13/2)/e^2
```

Mathematica [A] (verified)

Time = 3.76 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.73

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \frac{(2cd-be)^7((d+ex)(-be+c(d-ex)))^{3/2}\left(-\frac{\sqrt{c}(-3465b^7e^7g+210b^6ce^6(24ef+218dg+11egx)-84b^5c^2}{}\right.}{}$$

input
```
Integrate[(d + e*x)^3*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2
),x]
```

output

```
((2*c*d - b*e)^7*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*(-((Sqrt[c]*(-34
65*b^7*e^7*g + 210*b^6*c*e^6*(24*e*f + 218*d*g + 11*e*g*x) - 84*b^5*c^2*e^
5*(3057*d^2*g + 2*e^2*x*(20*f + 11*g*x) + d*e*(760*f + 334*g*x)) + 128*c^7
*(1664*d^7*g + 320*d*e^6*x^5*(7*f + 6*g*x) + 80*e^7*x^6*(8*f + 7*g*x) - 16
*d^3*e^4*x^3*(175*f + 136*g*x) + 8*d^2*e^5*x^4*(208*f + 175*g*x) - 8*d^5*e
^2*x*(245*f + 176*g*x) + d^6*e*(2944*f + 945*g*x) - 2*d^4*e^3*x^2*(2624*f
+ 1925*g*x)) + 24*b^4*c^3*e^4*(32924*d^3*g + 2*e^3*x^2*(56*f + 33*g*x) + 8
*d*e^2*x*(203*f + 107*g*x) + 3*d^2*e*(4704*f + 1963*g*x)) + 64*b*c^6*e*(-1
3647*d^6*g + 80*e^6*x^5*(20*f + 17*g*x) + 6*d^4*e^2*x*(-116*f + 123*g*x) +
 48*d*e^5*x^4*(164*f + 135*g*x) + 8*d^3*e^3*x^2*(1574*f + 1187*g*x) + 8*d^
2*e^4*x^3*(1882*f + 1483*g*x) - 2*d^5*e*(9812*f + 3263*g*x)) - 16*b^3*c^4*
e^3*(89587*d^4*g + 8*e^4*x^3*(18*f + 11*g*x) + 8*d*e^3*x^2*(222*f + 125*g*
x) + 12*d^2*e^2*x*(960*f + 479*g*x) + 4*d^3*e*(15072*f + 5887*g*x)) + 32*b
^2*c^5*e^2*(47490*d^5*g + 8*e^5*x^4*(8*f + 5*g*x) + 16*d*e^4*x^3*(43*f + 2
5*g*x) + 12*d^2*e^3*x^2*(308*f + 163*g*x) + 8*d^3*e^2*x*(1748*f + 809*g*x)
 + d^4*e*(48712*f + 17401*g*x))))/((2*c*d - b*e)^7*(d + e*x)*(-(b*e) + c*(
d - e*x)))) - (315*(16*c*e*f + 6*c*d*g - 11*b*e*g)*ArcTan[Sqrt[c*d - b*e -
 c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/((d + e*x)^(3/2)*(-(b*e) + c*(d - e*x))^
(3/2))))/(573440*c^(13/2)*e^2)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1221, 1134, 1134, 1160, 1087, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-11beg+6cdg+16cef)\int (d+ex)^3\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{16ce}-\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{8ce^2}$$

$$\Big\downarrow \text{1134}$$

$$\frac{(-11beg+6cdg+16cef)\left(\frac{9(2cd-be)\int (d+ex)^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{14c}-\frac{(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce}\right)}{16ce}-\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{8ce^2}$$

$$\Big\downarrow \text{1134}$$

$$\frac{(-11beg+6cdg+16cef)\left(\frac{9(2cd-be)\left(\frac{7(2cd-be)\int (d+ex)\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{12c}-\frac{(d+ex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{6ce}\right)}{14c}-\frac{(d+e}{}\right.}{16ce}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{8ce^2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(-11beg+6cdg+16cef)\left(\frac{9(2cd-be)\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\int \left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{2c}-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5ce}\right)}{12c}-\frac{(d+ex)\left(d(c\right.}{}\right.}{14c}\right.}{16ce}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{8ce^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(-11beg+6cdg+16cef)\left(\frac{9(2cd-be)\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\left(\frac{3(2cd-be)^2\int\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/}}{8c}\right.}{2c}\right.}{12c}\right.}{14c}\right.}{16ce}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{8ce^2}$$

$$\Big\downarrow \color{blue}{1087}$$

$$\frac{(-11beg+6cdg+16cef)\left(\frac{9(2cd-be)\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be}}{4c}\right.}{16c}\right.}{2c}\right.}{12c}\right.}{}\right.}{}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{8ce^2}$$

$$\Big\downarrow \text{1092}$$

$$(-11beg+6cdg+16cef)\left(\frac{9(2cd-be)\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int\frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2c}{\sqrt{-cx^2e^2-bxe^2}}\right.}{4c}\right.}{16c}\right.}{2}\right.}{}\right.}{}\right.$$

↓ 217

$$\left(\frac{9(2cd-be)\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\frac{(b+2cx)\left(d(cd-\right.}{}\right.}{2c}\right.}{12c}\right.}{14c}\right.$$

input
```
Int[(d + e*x)^3*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),x]
```

output
```
-1/8*(g*(d + e*x)^3*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(c*e^2) +
 ((16*c*e*f + 6*c*d*g - 11*b*e*g)*(-1/7*((d + e*x)^2*(d*(c*d - b*e) - b*e^
2*x - c*e^2*x^2)^(5/2))/(c*e) + (9*(2*c*d - b*e)*(-1/6*((d + e*x)*(d*(c*d
- b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(c*e) + (7*(2*c*d - b*e)*(-1/5*(d*(c*
d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2)/(c*e) + ((2*c*d - b*e)*(((b + 2*c*x)
*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(8*c) + (3*(2*c*d - b*e)^2*(
((b + 2*c*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d -
b*e)^2*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*
e^2*x^2])])/(8*c^(3/2)*e)))/(16*c)))/(2*c)))/(12*c)))/(14*c)))/(16*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3508 vs. $2(396) = 792$.

Time = 3.53 (sec) , antiderivative size = 3509, normalized size of antiderivative = 8.28

method	result	size
default	Expression too large to display	3509

input
```
int((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=_RET
URNVERBOSE)
```

output

```
d^3*f*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2
)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/
e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^
2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*
e^2*x-b*d*e+c*d^2)^(1/2))))+e^2*(3*d*g+e*f)*(-1/7*x^2*(-c*e^2*x^2-b*e^2*x-
b*d*e+c*d^2)^(5/2)/c/e^2-9/14*b/c*(-1/6*x*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)
^(5/2)/c/e^2-7/12*b/c*(-1/5*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/c/e^2-1
/2*b/c*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/
2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c
/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b
^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b
*e^2*x-b*d*e+c*d^2)^(1/2))))))+1/6*(-b*d*e+c*d^2)/c/e^2*(-1/8*(-2*c*e^2*x-b
*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+
c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b
*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2
)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))
)))+2/7*(-b*d*e+c*d^2)/c/e^2*(-1/5*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/
c/e^2-1/2*b/c*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d
^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b
*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(396) = 792$.

Time = 3.43 (sec) , antiderivative size = 2337, normalized size of antiderivative = 5.51

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="fricas")
```

output

```
[-1/2293760*(315*(16*(128*c^8*d^7*e - 448*b*c^7*d^6*e^2 + 672*b^2*c^6*d^5*
e^3 - 560*b^3*c^5*d^4*e^4 + 280*b^4*c^4*d^3*e^5 - 84*b^5*c^3*d^2*e^6 + 14*
b^6*c^2*d*e^7 - b^7*c*e^8)*f + (768*c^8*d^8 - 4096*b*c^7*d^7*e + 8960*b^2*
c^6*d^6*e^2 - 10752*b^3*c^5*d^5*e^3 + 7840*b^4*c^4*d^4*e^4 - 3584*b^5*c^3*
d^3*e^5 + 1008*b^6*c^2*d^2*e^6 - 160*b^7*c*d*e^7 + 11*b^8*e^8)*g)*sqrt(-c)
*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqr
t(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) + 4*(716
80*c^8*e^7*g*x^7 + 5120*(16*c^8*e^7*f + (48*c^8*d*e^6 + 17*b*c^7*e^7)*g)*x
^6 + 1280*(16*(14*c^8*d*e^6 + 5*b*c^7*e^7)*f + (140*c^8*d^2*e^5 + 324*b*c^
7*d*e^6 + b^2*c^6*e^7)*g)*x^5 + 128*(16*(104*c^8*d^2*e^5 + 246*b*c^7*d*e^6
 + b^2*c^6*e^7)*f - (2176*c^8*d^3*e^4 - 5932*b*c^7*d^2*e^5 - 100*b^2*c^6*d
*e^6 + 11*b^3*c^5*e^7)*g)*x^4 - 16*(16*(1400*c^8*d^3*e^4 - 3764*b*c^7*d^2*
e^5 - 86*b^2*c^6*d*e^6 + 9*b^3*c^5*e^7)*f + (30800*c^8*d^4*e^3 - 37984*b*c
^7*d^3*e^4 - 3912*b^2*c^6*d^2*e^5 + 1000*b^3*c^5*d*e^6 - 99*b^4*c^4*e^7)*g
)*x^3 - 8*(16*(5248*c^8*d^4*e^3 - 6296*b*c^7*d^3*e^4 - 924*b^2*c^6*d^2*e^5
 + 222*b^3*c^5*d*e^6 - 21*b^4*c^4*e^7)*f + (22528*c^8*d^5*e^2 - 5904*b*c^7
*d^4*e^3 - 25888*b^2*c^6*d^3*e^4 + 11496*b^3*c^5*d^2*e^5 - 2568*b^4*c^4*d*
e^6 + 231*b^5*c^3*e^7)*g)*x^2 + 16*(23552*c^8*d^6*e - 78496*b*c^7*d^5*e^2
+ 97424*b^2*c^6*d^4*e^3 - 60288*b^3*c^5*d^3*e^4 + 21168*b^4*c^4*d^2*e^5 -
3990*b^5*c^3*d*e^6 + 315*b^6*c^2*e^7)*f + (212992*c^8*d^7 - 873408*b*c^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9527 vs. $2(418) = 836$.

Time = 1.77 (sec) , antiderivative size = 9527, normalized size of antiderivative = 22.47

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**3*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x
)
```

output

```
Piecewise((sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(-c*e**5*g*x**7/
8 - x**6*(17*b*c*e**7*g/16 + 3*c**2*d*e**6*g + c**2*e**7*f)/(7*c*e**2) - x
**5*(b**2*e**7*g + 8*b*c*d*e**6*g + 2*b*c*e**7*f - 13*b*(17*b*c*e**7*g/16
+ 3*c**2*d*e**6*g + c**2*e**7*f)/(14*c) + c**2*d**2*e**5*g + 3*c**2*d*e**6
*f + c*e**5*g*(-7*b*d*e + 7*c*d**2)/8)/(6*c*e**2) - x**4*(5*b**2*d*e**6*g
+ b**2*e**7*f + 10*b*c*d**2*e**5*g + 8*b*c*d*e**6*f - 11*b*(b**2*e**7*g +
8*b*c*d*e**6*g + 2*b*c*e**7*f - 13*b*(17*b*c*e**7*g/16 + 3*c**2*d*e**6*g +
 c**2*e**7*f)/(14*c) + c**2*d**2*e**5*g + 3*c**2*d*e**6*f + c*e**5*g*(-7*b
*d*e + 7*c*d**2)/8)/(12*c) - 5*c**2*d**3*e**4*g + c**2*d**2*e**5*f + (-6*b
*d*e + 6*c*d**2)*(17*b*c*e**7*g/16 + 3*c**2*d*e**6*g + c**2*e**7*f)/(7*c*e
**2))/(5*c*e**2) - x**3*(10*b**2*d**2*e**5*g + 5*b**2*d*e**6*f + 10*b*c*d*
*2*e**5*f - 9*b*(5*b**2*d*e**6*g + b**2*e**7*f + 10*b*c*d**2*e**5*g + 8*b*
c*d*e**6*f - 11*b*(b**2*e**7*g + 8*b*c*d*e**6*g + 2*b*c*e**7*f - 13*b*(17*
b*c*e**7*g/16 + 3*c**2*d*e**6*g + c**2*e**7*f)/(14*c) + c**2*d**2*e**5*g +
 3*c**2*d*e**6*f + c*e**5*g*(-7*b*d*e + 7*c*d**2)/8)/(12*c) - 5*c**2*d**3*
e**4*g + c**2*d**2*e**5*f + (-6*b*d*e + 6*c*d**2)*(17*b*c*e**7*g/16 + 3*c*
*2*d*e**6*g + c**2*e**7*f)/(7*c*e**2))/(10*c) - 5*c**2*d**4*e**3*g - 5*c**
2*d**3*e**4*f + (-5*b*d*e + 5*c*d**2)*(b**2*e**7*g + 8*b*c*d*e**6*g + 2*b*
c*e**7*f - 13*b*(17*b*c*e**7*g/16 + 3*c**2*d*e**6*g + c**2*e**7*f)/(14*c)
+ c**2*d**2*e**5*g + 3*c**2*d*e**6*f + c*e**5*g*(-7*b*d*e + 7*c*d**2)/8...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(396) = 792$.

Time = 0.42 (sec) , antiderivative size = 1229, normalized size of antiderivative = 2.90

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="giac")
```

output

```
-1/573440*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(2*(8*(10*(4*(1
4*c*e^5*g*x + (16*c^8*e^17*f + 48*c^8*d*e^16*g + 17*b*c^7*e^17*g)/(c^7*e^1
2))*x + (224*c^8*d*e^16*f + 80*b*c^7*e^17*f + 140*c^8*d^2*e^15*g + 324*b*c
^7*d*e^16*g + b^2*c^6*e^17*g)/(c^7*e^12))*x + (1664*c^8*d^2*e^15*f + 3936*
b*c^7*d*e^16*f + 16*b^2*c^6*e^17*f - 2176*c^8*d^3*e^14*g + 5932*b*c^7*d^2*
e^15*g + 100*b^2*c^6*d*e^16*g - 11*b^3*c^5*e^17*g)/(c^7*e^12))*x - (22400*
c^8*d^3*e^14*f - 60224*b*c^7*d^2*e^15*f - 1376*b^2*c^6*d*e^16*f + 144*b^3*
c^5*e^17*f + 30800*c^8*d^4*e^13*g - 37984*b*c^7*d^3*e^14*g - 3912*b^2*c^6*
d^2*e^15*g + 1000*b^3*c^5*d*e^16*g - 99*b^4*c^4*e^17*g)/(c^7*e^12))*x - (8
3968*c^8*d^4*e^13*f - 100736*b*c^7*d^3*e^14*f - 14784*b^2*c^6*d^2*e^15*f +
 3552*b^3*c^5*d*e^16*f - 336*b^4*c^4*e^17*f + 22528*c^8*d^5*e^12*g - 5904*
b*c^7*d^4*e^13*g - 25888*b^2*c^6*d^3*e^14*g + 11496*b^3*c^5*d^2*e^15*g - 2
568*b^4*c^4*d*e^16*g + 231*b^5*c^3*e^17*g)/(c^7*e^12))*x - (125440*c^8*d^5
*e^12*f + 22272*b*c^7*d^4*e^13*f - 223744*b^2*c^6*d^3*e^14*f + 92160*b^3*c
^5*d^2*e^15*f - 19488*b^4*c^4*d*e^16*f + 1680*b^5*c^3*e^17*f - 60480*c^8*d
^6*e^11*g + 208832*b*c^7*d^5*e^12*g - 278416*b^2*c^6*d^4*e^13*g + 188384*b
^3*c^5*d^3*e^14*g - 70668*b^4*c^4*d^2*e^15*g + 14028*b^5*c^3*d*e^16*g - 11
55*b^6*c^2*e^17*g)/(c^7*e^12))*x + (376832*c^8*d^6*e^11*f - 1255936*b*c^7*
d^5*e^12*f + 1558784*b^2*c^6*d^4*e^13*f - 964608*b^3*c^5*d^3*e^14*f + 3386
88*b^4*c^4*d^2*e^15*f - 63840*b^5*c^3*d*e^16*f + 5040*b^6*c^2*e^17*f + ...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\int (f+g\,x)\,\left(d+e\,x\right)^3\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}\,dx$$

input
```
int((f + g*x)*(d + e*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),x)
```

output
```
int((f + g*x)*(d + e*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 4284, normalized size of antiderivative = 10.10

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(i*(3465*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*b**9*e**9*g - 57330*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
 b*e + 2*c*d))*b**8*c*d*e**8*g - 5040*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*b**8*c*e**9*f + 418320*sqrt(c)*asinh((sqrt(
 - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**7*c**2*d**2*e**7*g + 806
40*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**7
*c**2*d*e**8*f - 1764000*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt
( - b*e + 2*c*d))*b**6*c**3*d**3*e**6*g - 564480*sqrt(c)*asinh((sqrt( - b*
e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**6*c**3*d**2*e**7*f + 4727520*
sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c*
*4*d**4*e**5*g + 2257920*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt
( - b*e + 2*c*d))*b**5*c**4*d**3*e**6*f - 8326080*sqrt(c)*asinh((sqrt( - b
*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**5*d**5*e**4*g - 5644800
*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c
**5*d**4*e**5*f + 9596160*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqr
t( - b*e + 2*c*d))*b**3*c**6*d**6*e**3*g + 9031680*sqrt(c)*asinh((sqrt( -
b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**6*d**5*e**4*f - 693504
0*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*
c**7*d**7*e**2*g - 9031680*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sq
rt( - b*e + 2*c*d))*b**2*c**7*d**6*e**3*f + 2822400*sqrt(c)*asinh((sqrt...
```

3.148 $\int (d+ex)^2(f+gx) \left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 44, antiderivative size = 351

$$
\begin{aligned}
&\int (d+ex)^2(f+gx) \left(cd^2-bde-be^2x\right.\\
&\left.-ce^2x^2\right)^{3/2} \, dx = \frac{(2cd-be)^4(14cef+4cdg-9beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{1024c^5e}\\
&+\frac{(2cd-be)^2(14cef+4cdg-9beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{384c^4e}\\
&-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2}\\
&-\frac{(14cef+4cdg-9beg)(24cd-7be+10cex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{840c^3e^2}\\
&+\frac{(2cd-be)^6(14cef+4cdg-9beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{1024c^{11/2}e^2}
\end{aligned}
$$

output
```
1/1024*(-b*e+2*c*d)^4*(-9*b*e*g+4*c*d*g+14*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(1/2)/c^5/e+1/384*(-b*e+2*c*d)^2*(-9*b*e*g+4*c*d*g+14*c
*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^4/e-1/7*g*(e*x+d)
^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c/e^2-1/840*(-9*b*e*g+4*c*d*g+14
*c*e*f)*(10*c*e*x-7*b*e+24*c*d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^3
/e^2+1/1024*(-b*e+2*c*d)^6*(-9*b*e*g+4*c*d*g+14*c*e*f)*arctan(c^(1/2)*(e*x
+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(11/2)/e^2
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.69

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\frac{(-2cd+be)^6((d+ex)(-be+c(d-ex)))^{3/2}\Bigg(-\frac{\sqrt{c}(945b^6e^6g-210b^5ce^5(7ef+50dg+3egx)+64c^6(432d^6}{}}{}$$

input

```
Integrate[(d + e*x)^2*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2
),x]
```

output

```
((-2*c*d + b*e)^6*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*(-((Sqrt[c]*(94
5*b^6*e^6*g - 210*b^5*c*e^5*(7*e*f + 50*d*g + 3*e*g*x) + 64*c^6*(432*d^6*g
 + 112*d*e^5*x^4*(6*f + 5*g*x) + 42*d^5*e*(16*f + 5*g*x) + 40*e^6*x^5*(7*f
 + 6*g*x) - 2*d^2*e^4*x^3*(35*f + 24*g*x) - 28*d^3*e^3*x^2*(48*f + 35*g*x)
 - 3*d^4*e^2*x*(315*f + 208*g*x)) + 28*b^4*c^2*e^4*(1708*d^2*g + e^2*x*(35
*f + 18*g*x) + d*e*(560*f + 226*g*x)) + 48*b^2*c^4*e^2*(3037*d^4*g + 2*e^4
*x^3*(7*f + 4*g*x) + 4*d*e^3*x^2*(35*f + 18*g*x) + 14*d^2*e^2*x*(52*f + 23
*g*x) + 4*d^3*e*(763*f + 255*g*x)) - 16*b^3*c^3*e^3*(7090*d^3*g + e^3*x^2*
(49*f + 27*g*x) + 4*d*e^2*x*(147*f + 71*g*x) + 2*d^2*e*(2107*f + 786*g*x))
 + 32*b*c^5*e*(-3054*d^5*g - 123*d^4*e*(35*f + 11*g*x) + 12*d^3*e^2*x*(91*
f + 75*g*x) + 8*e^5*x^4*(91*f + 75*g*x) + 4*d*e^4*x^3*(707*f + 556*g*x) +
2*d^2*e^3*x^2*(1911*f + 1409*g*x))))/((-2*c*d + b*e)^6*(d + e*x)*(-(b*e) +
 c*(d - e*x)))) - (105*(14*c*e*f + 4*c*d*g - 9*b*e*g)*ArcTan[Sqrt[c*d - b*
e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/((d + e*x)^(3/2)*(-(b*e) + c*(d - e*x
))^(3/2))))/(107520*c^(11/2)*e^2)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1221, 1134, 1160, 1087, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-9beg+4cdg+14cef)\int (d+ex)^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{14ce}-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2}$$

$$\Big\downarrow \text{1134}$$

$$\frac{(-9beg+4cdg+14cef)\left(\frac{7(2cd-be)\int (d+ex)\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{12c}-\frac{(d+ex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{6ce}\right)}{14ce}-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(-9beg+4cdg+14cef)\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\int\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{2c}-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5ce}\right)}{12c}-\frac{(d+ex)(d(cd-be)-b}{6ce}\right.}{14ce}-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(-9beg+4cdg+14cef)\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\left(\frac{3(2cd-be)^2\int\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{8c}\right)}{2c}-\frac{(d(cd-}{}\right.}{12c}\right.}{14ce}-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(-9beg+4cdg+14cef)\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+}{2c}}{12c}}{14ce}$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2}$$

$$\Big\downarrow \; 1092$$

$$\frac{(-9beg+4cdg+14cef)\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}\right.}{16c}\right.}{2c}\right.}{12c}\right.}{1}$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\left(\frac{7(2cd-be)\left(\frac{(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-\right.}{8c}\right.}{2c}\right.}{12c}\right.}{14ce}$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2}$$

input
```
Int[(d + e*x)^2*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),x]
```

output
```
-1/7*(g*(d + e*x)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(c*e^2) +
 ((14*c*e*f + 4*c*d*g - 9*b*e*g)*(-1/6*((d + e*x)*(d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2)^(5/2))/(c*e) + (7*(2*c*d - b*e)*(-1/5*(d*(c*d - b*e) - b*e^2
*x - c*e^2*x^2)^(5/2)/(c*e) + ((2*c*d - b*e)*(((b + 2*c*x)*(d*(c*d - b*e)
- b*e^2*x - c*e^2*x^2)^(3/2))/(8*c) + (3*(2*c*d - b*e)^2*(((b + 2*c*x)*Sqr
t[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b*e)^2*ArcTan[(e
*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(8*c
^(3/2)*e)))/(16*c)))/(2*c)))/(12*c)))/(14*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. 2(327) = 654.

Time = 2.88 (sec) , antiderivative size = 1977, normalized size of antiderivative = 5.63

method	result	size
default	Expression too large to display	1977

input
```
int((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
d^2*f*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2
)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/
e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^
2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*
e^2*x-b*d*e+c*d^2)^(1/2))))+e*(2*d*g+e*f)*(-1/6*x*(-c*e^2*x^2-b*e^2*x-b*d*
e+c*d^2)^(5/2)/c/e^2-7/12*b/c*(-1/5*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)
/c/e^2-1/2*b/c*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*
d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-
b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+
c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e
^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))))+1/6*(-b*d*e+c*d^2)/c/e^2*(-1/8*(-2*c
*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*
(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b
*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e
^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2
)^(1/2)))))+d*(d*g+2*e*f)*(-1/5*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/c/e
^2-1/2*b/c*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)
^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^
2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^
2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(327) = 654$.

Time = 1.74 (sec) , antiderivative size = 1877, normalized size of antiderivative = 5.35

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="fricas")
```

output

```
[1/430080*(105*(14*(64*c^7*d^6*e - 192*b*c^6*d^5*e^2 + 240*b^2*c^5*d^4*e^3
 - 160*b^3*c^4*d^3*e^4 + 60*b^4*c^3*d^2*e^5 - 12*b^5*c^2*d*e^6 + b^6*c*e^7
)*f + (256*c^7*d^7 - 1344*b*c^6*d^6*e + 2688*b^2*c^5*d^5*e^2 - 2800*b^3*c^
4*d^4*e^3 + 1680*b^4*c^3*d^3*e^4 - 588*b^5*c^2*d^2*e^5 + 112*b^6*c*d*e^6 -
 9*b^7*e^7)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*
c*d*e + b^2*e^2 + 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x +
b*e)*sqrt(-c)) - 4*(15360*c^7*e^6*g*x^6 + 1280*(14*c^7*e^6*f + (28*c^7*d*e
^5 + 15*b*c^6*e^6)*g)*x^5 + 128*(14*(24*c^7*d*e^5 + 13*b*c^6*e^6)*f - (24*
c^7*d^2*e^4 - 556*b*c^6*d*e^5 - 3*b^2*c^5*e^6)*g)*x^4 - 16*(14*(20*c^7*d^2
*e^4 - 404*b*c^6*d*e^5 - 3*b^2*c^5*e^6)*f + (3920*c^7*d^3*e^3 - 5636*b*c^6
*d^2*e^4 - 216*b^2*c^5*d*e^5 + 27*b^3*c^4*e^6)*g)*x^3 - 8*(14*(768*c^7*d^3
*e^3 - 1092*b*c^6*d^2*e^4 - 60*b^2*c^5*d*e^5 + 7*b^3*c^4*e^6)*f + (4992*c^
7*d^4*e^2 - 3600*b*c^6*d^3*e^3 - 1932*b^2*c^5*d^2*e^4 + 568*b^3*c^4*d*e^5
- 63*b^4*c^3*e^6)*g)*x^2 + 14*(3072*c^7*d^5*e - 9840*b*c^6*d^4*e^2 + 10464
*b^2*c^5*d^3*e^3 - 4816*b^3*c^4*d^2*e^4 + 1120*b^4*c^3*d*e^5 - 105*b^5*c^2
*e^6)*f + (27648*c^7*d^6 - 97728*b*c^6*d^5*e + 145776*b^2*c^5*d^4*e^2 - 11
3440*b^3*c^4*d^3*e^3 + 47824*b^4*c^3*d^2*e^4 - 10500*b^5*c^2*d*e^5 + 945*b
^6*c*e^6)*g - 2*(14*(2160*c^7*d^4*e^2 - 1248*b*c^6*d^3*e^3 - 1248*b^2*c^5*
d^2*e^4 + 336*b^3*c^4*d*e^5 - 35*b^4*c^3*e^6)*f - (6720*c^7*d^5*e - 21648*
b*c^6*d^4*e^2 + 24480*b^2*c^5*d^3*e^3 - 12576*b^3*c^4*d^2*e^4 + 3164*b^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5834 vs. $2(337) = 674$.

Time = 1.62 (sec) , antiderivative size = 5834, normalized size of antiderivative = 16.62

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**2*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x
)
```

output

```
Piecewise((sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(-c*e**4*g*x**6/
7 - x**5*(15*b*c*e**6*g/14 + 2*c**2*d*e**5*g + c**2*e**6*f)/(6*c*e**2) - x
**4*(b**2*e**6*g + 6*b*c*d*e**5*g + 2*b*c*e**6*f - 11*b*(15*b*c*e**6*g/14
+ 2*c**2*d*e**5*g + c**2*e**6*f)/(12*c) - c**2*d**2*e**4*g + 2*c**2*d*e**5
*f + c*e**4*g*(-6*b*d*e + 6*c*d**2)/7)/(5*c*e**2) - x**3*(4*b**2*d*e**5*g
+ b**2*e**6*f + 4*b*c*d**2*e**4*g + 6*b*c*d*e**5*f - 9*b*(b**2*e**6*g + 6*
b*c*d*e**5*g + 2*b*c*e**6*f - 11*b*(15*b*c*e**6*g/14 + 2*c**2*d*e**5*g + c
**2*e**6*f)/(12*c) - c**2*d**2*e**4*g + 2*c**2*d*e**5*f + c*e**4*g*(-6*b*d
*e + 6*c*d**2)/7)/(10*c) - 4*c**2*d**3*e**3*g - c**2*d**2*e**4*f + (-5*b*d
*e + 5*c*d**2)*(15*b*c*e**6*g/14 + 2*c**2*d*e**5*g + c**2*e**6*f)/(6*c*e**
2))/(4*c*e**2) - x**2*(6*b**2*d**2*e**4*g + 4*b**2*d*e**5*f - 4*b*c*d**3*e
**3*g + 4*b*c*d**2*e**4*f - 7*b*(4*b**2*d*e**5*g + b**2*e**6*f + 4*b*c*d**
2*e**4*g + 6*b*c*d*e**5*f - 9*b*(b**2*e**6*g + 6*b*c*d*e**5*g + 2*b*c*e**6
*f - 11*b*(15*b*c*e**6*g/14 + 2*c**2*d*e**5*g + c**2*e**6*f)/(12*c) - c**2
*d**2*e**4*g + 2*c**2*d*e**5*f + c*e**4*g*(-6*b*d*e + 6*c*d**2)/7)/(10*c)
- 4*c**2*d**3*e**3*g - c**2*d**2*e**4*f + (-5*b*d*e + 5*c*d**2)*(15*b*c*e*
*6*g/14 + 2*c**2*d*e**5*g + c**2*e**6*f)/(6*c*e**2))/(8*c) - c**2*d**4*e**
2*g - 4*c**2*d**3*e**3*f + (-4*b*d*e + 4*c*d**2)*(b**2*e**6*g + 6*b*c*d*e*
*5*g + 2*b*c*e**6*f - 11*b*(15*b*c*e**6*g/14 + 2*c**2*d*e**5*g + c**2*e**6
*f)/(12*c) - c**2*d**2*e**4*g + 2*c**2*d*e**5*f + c*e**4*g*(-6*b*d*e + ...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(327) = 654$.

Time = 0.38 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.80

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="giac")
```

output

```
-1/107520*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(2*(8*(10*(12*c
*e^4*g*x + (14*c^7*e^14*f + 28*c^7*d*e^13*g + 15*b*c^6*e^14*g)/(c^6*e^10))
*x + (336*c^7*d*e^13*f + 182*b*c^6*e^14*f - 24*c^7*d^2*e^12*g + 556*b*c^6*
d*e^13*g + 3*b^2*c^5*e^14*g)/(c^6*e^10))*x - (280*c^7*d^2*e^12*f - 5656*b*
c^6*d*e^13*f - 42*b^2*c^5*e^14*f + 3920*c^7*d^3*e^11*g - 5636*b*c^6*d^2*e^
12*g - 216*b^2*c^5*d*e^13*g + 27*b^3*c^4*e^14*g)/(c^6*e^10))*x - (10752*c^
7*d^3*e^11*f - 15288*b*c^6*d^2*e^12*f - 840*b^2*c^5*d*e^13*f + 98*b^3*c^4*
e^14*f + 4992*c^7*d^4*e^10*g - 3600*b*c^6*d^3*e^11*g - 1932*b^2*c^5*d^2*e^
12*g + 568*b^3*c^4*d*e^13*g - 63*b^4*c^3*e^14*g)/(c^6*e^10))*x - (30240*c^
7*d^4*e^10*f - 17472*b*c^6*d^3*e^11*f - 17472*b^2*c^5*d^2*e^12*f + 4704*b^
3*c^4*d*e^13*f - 490*b^4*c^3*e^14*f - 6720*c^7*d^5*e^9*g + 21648*b*c^6*d^4
*e^10*g - 24480*b^2*c^5*d^3*e^11*g + 12576*b^3*c^4*d^2*e^12*g - 3164*b^4*c
^3*d*e^13*g + 315*b^5*c^2*e^14*g)/(c^6*e^10))*x + (43008*c^7*d^5*e^9*f - 1
37760*b*c^6*d^4*e^10*f + 146496*b^2*c^5*d^3*e^11*f - 67424*b^3*c^4*d^2*e^1
2*f + 15680*b^4*c^3*d*e^13*f - 1470*b^5*c^2*e^14*f + 27648*c^7*d^6*e^8*g -
 97728*b*c^6*d^5*e^9*g + 145776*b^2*c^5*d^4*e^10*g - 113440*b^3*c^4*d^3*e^
11*g + 47824*b^4*c^3*d^2*e^12*g - 10500*b^5*c^2*d*e^13*g + 945*b^6*c*e^14*
g)/(c^6*e^10)) - 1/2048*(896*c^7*d^6*e*f - 2688*b*c^6*d^5*e^2*f + 3360*b^2
*c^5*d^4*e^3*f - 2240*b^3*c^4*d^3*e^4*f + 840*b^4*c^3*d^2*e^5*f - 168*b^5*
c^2*d*e^6*f + 14*b^6*c*e^7*f + 256*c^7*d^7*g - 1344*b*c^6*d^6*e*g + 268...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\int (f+g\,x)\,\left(d+e\,x\right)^2\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}\,dx$$

input

```
int((f + g*x)*(d + e*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),x)
```

output

```
int((f + g*x)*(d + e*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 4.96 (sec) , antiderivative size = 3367, normalized size of antiderivative = 9.59

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input

```
int((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(i*( - 945*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*
d))*b**8*e**8*g + 13650*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
 - b*e + 2*c*d))*b**7*c*d*e**7*g + 1470*sqrt(c)*asinh((sqrt( - b*e + c*d -
 c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**7*c*e**8*f - 85260*sqrt(c)*asinh((sqrt
( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**6*c**2*d**2*e**6*g - 20
580*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**
6*c**2*d*e**7*f + 299880*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt
( - b*e + 2*c*d))*b**5*c**3*d**3*e**5*g + 123480*sqrt(c)*asinh((sqrt( - b*
e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**3*d**2*e**6*f - 646800*s
qrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**
4*d**4*e**4*g - 411600*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
- b*e + 2*c*d))*b**4*c**4*d**3*e**5*f + 870240*sqrt(c)*asinh((sqrt( - b*e
+ c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**5*d**5*e**3*g + 823200*sqr
t(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**5*
d**4*e**4*f - 705600*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
b*e + 2*c*d))*b**2*c**6*d**6*e**2*g - 987840*sqrt(c)*asinh((sqrt( - b*e +
c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**6*d**5*e**3*f + 309120*sqrt(
c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**7*d**7*
e*g + 658560*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*
c*d))*b*c**7*d**6*e**2*f - 53760*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e...
```

3.149 $\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 292

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \frac{(2cd-be)^3(12cef+2cdg-7beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{512c^4e} + \frac{(2cd-be)(12cef+2cdg-7beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{192c^3e} + \frac{(7beg-12c(ef+dg)-10cegx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{60c^2e^2} + \frac{(2cd-be)^5(12cef+2cdg-7beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{512c^{9/2}e^2}$$

output

```
1/512*(-b*e+2*c*d)^3*(-7*b*e*g+2*c*d*g+12*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b
*e^2*x-c*e^2*x^2)^(1/2)/c^4/e+1/192*(-b*e+2*c*d)*(-7*b*e*g+2*c*d*g+12*c*e*
f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^3/e+1/60*(7*b*e*g-12
*c*(d*g+e*f)-10*c*e*g*x)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^2/e^2+1/
512*(-b*e+2*c*d)^5*(-7*b*e*g+2*c*d*g+12*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(
-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(9/2)/e^2
```

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.62

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\frac{(2cd-be)^5((d+ex)(-be+c(d-ex)))^{3/2}\left(-\frac{\sqrt{c}(-105b^5e^5g+10b^4ce^4(18ef+94dg+7egx)-8b^3c^2e^3(40}{}\right.}{}$$

input
```
Integrate[(d + e*x)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),
x]
```

output
```
((2*c*d - b*e)^5*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*(-((Sqrt[c]*(-10
5*b^5*e^5*g + 10*b^4*c*e^4*(18*e*f + 94*d*g + 7*e*g*x) - 8*b^3*c^2*e^3*(40
7*d^2*g + e^2*x*(15*f + 7*g*x) + 3*d*e*(65*f + 23*g*x)) + 48*b^2*c^3*e^2*(
111*d^3*g + e^3*x^2*(2*f + g*x) + d*e^2*x*(19*f + 8*g*x) + d^2*e*(107*f +
33*g*x)) + 32*c^5*(48*d^5*g + 12*d*e^4*x^3*(5*f + 4*g*x) + 8*e^5*x^4*(6*f
+ 5*g*x) + 3*d^4*e*(16*f + 5*g*x) - 6*d^3*e^2*x*(25*f + 16*g*x) - 2*d^2*e^
3*x^2*(48*f + 35*g*x)) + 16*b*c^4*e*(-273*d^4*g - 6*d^3*e*(57*f + 17*g*x)
+ 4*e^4*x^3*(33*f + 26*g*x) + 6*d^2*e^2*x*(43*f + 29*g*x) + 4*d*e^3*x^2*(9
3*f + 68*g*x))))/((2*c*d - b*e)^5*(d + e*x)*(-(b*e) + c*(d - e*x)))) + (15
*(7*b*e*g - 2*c*(6*e*f + d*g))*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqr
t[d + e*x])])/((d + e*x)^(3/2)*(-(b*e) + c*(d - e*x))^(3/2))))/(7680*c^(9/
2)*e^2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {1225, 1087, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{(2cd-be)(-7beg+2cdg+12cef)\int \left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{24c^2e}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(7beg-12c(dg+ef)-10cegx)}{60c^2e^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2cd-be)(-7beg+2cdg+12cef)\left(\frac{3(2cd-be)^2\int \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{8c}\right)}{24c^2e}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(7beg-12c(dg+ef)-10cegx)}{60c^2e^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2cd-be)(-7beg+2cdg+12cef)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+(b+2\right.}{24c^2e}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(7beg-12c(dg+ef)-10cegx)}{60c^2e^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(2cd-be)(-7beg+2cdg+12cef)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}+(b+2cx\right.}{16c}\right.}{24c^2e}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(7beg-12c(dg+ef)-10cegx)}{60c^2e^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\frac{(b+2cx)(d(cd-be)-be^2x-ce}{8c}\right.}{24c^2e}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(7beg-12c(dg+ef)-10cegx)}{60c^2e^2}$$

input
```
Int[(d + e*x)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),x]
```

output
```
((7*b*e*g - 12*c*(e*f + d*g) - 10*c*e*g*x)*(d*(c*d - b*e) - b*e^2*x - c*e^
2*x^2)^(5/2))/(60*c^2*e^2) + ((2*c*d - b*e)*(12*c*e*f + 2*c*d*g - 7*b*e*g)
*(((b + 2*c*x)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(8*c) + (3*(2*
c*d - b*e)^2*(((b + 2*c*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c
) + ((2*c*d - b*e)^2*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e)
- b*e^2*x - c*e^2*x^2])])/(8*c^(3/2)*e)))/(16*c)))/(24*c^2*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(272) = 544$.

Time = 2.41 (sec) , antiderivative size = 1068, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1068

input
```
int((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=_RETUR
NVERBOSE)
```

output

```
d*f*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-
3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^
2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*
e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^
2*x-b*d*e+c*d^2)^(1/2))))+(d*g+e*f)*(-1/5*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)
^(5/2)/c/e^2-1/2*b/c*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*
d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*
e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-
b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)
/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))))+e*g*(-1/6*x*(-c*e^2*x^2-b*e^2*
x-b*d*e+c*d^2)^(5/2)/c/e^2-7/12*b/c*(-1/5*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)
^(5/2)/c/e^2-1/2*b/c*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*
d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*
e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-
b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)
/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))))+1/6*(-b*d*e+c*d^2)/c/e^2*(-1/8
*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*
c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2
*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^
2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(272) = 544$.

Time = 0.78 (sec) , antiderivative size = 1473, normalized size of antiderivative = 5.04

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algori
thm="fricas")
```

output
```
[-1/30720*(15*(12*(32*c^6*d^5*e - 80*b*c^5*d^4*e^2 + 80*b^2*c^4*d^3*e^3 -
40*b^3*c^3*d^2*e^4 + 10*b^4*c^2*d*e^5 - b^5*c*e^6)*f + (64*c^6*d^6 - 384*b
*c^5*d^5*e + 720*b^2*c^4*d^4*e^2 - 640*b^3*c^3*d^3*e^3 + 300*b^4*c^2*d^2*e
^4 - 72*b^5*c*d*e^5 + 7*b^6*e^6)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2
*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2
 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) + 4*(1280*c^6*e^5*g*x^5 + 128*(12*c^6*
e^5*f + (12*c^6*d*e^4 + 13*b*c^5*e^5)*g)*x^4 + 16*(12*(10*c^6*d*e^4 + 11*b
*c^5*e^5)*f - (140*c^6*d^2*e^3 - 272*b*c^5*d*e^4 - 3*b^2*c^4*e^5)*g)*x^3 -
 8*(12*(32*c^6*d^2*e^3 - 62*b*c^5*d*e^4 - b^2*c^4*e^5)*f + (384*c^6*d^3*e^
2 - 348*b*c^5*d^2*e^3 - 48*b^2*c^4*d*e^4 + 7*b^3*c^3*e^5)*g)*x^2 + 12*(128
*c^6*d^4*e - 456*b*c^5*d^3*e^2 + 428*b^2*c^4*d^2*e^3 - 130*b^3*c^3*d*e^4 +
 15*b^4*c^2*e^5)*f + (1536*c^6*d^5 - 4368*b*c^5*d^4*e + 5328*b^2*c^4*d^3*e
^2 - 3256*b^3*c^3*d^2*e^3 + 940*b^4*c^2*d*e^4 - 105*b^5*c*e^5)*g - 2*(12*(
200*c^6*d^3*e^2 - 172*b*c^5*d^2*e^3 - 38*b^2*c^4*d*e^4 + 5*b^3*c^3*e^5)*f
- (240*c^6*d^4*e - 816*b*c^5*d^3*e^2 + 792*b^2*c^4*d^2*e^3 - 276*b^3*c^3*d
*e^4 + 35*b^4*c^2*e^5)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(
c^5*e^2), -1/15360*(15*(12*(32*c^6*d^5*e - 80*b*c^5*d^4*e^2 + 80*b^2*c^4*d
^3*e^3 - 40*b^3*c^3*d^2*e^4 + 10*b^4*c^2*d*e^5 - b^5*c*e^6)*f + (64*c^6*d^
6 - 384*b*c^5*d^5*e + 720*b^2*c^4*d^4*e^2 - 640*b^3*c^3*d^3*e^3 + 300*b^4*
c^2*d^2*e^4 - 72*b^5*c*d*e^5 + 7*b^6*e^6)*g)*sqrt(c)*arctan(1/2*sqrt(-c...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3228 vs. $2(280) = 560$.

Time = 2.09 (sec) , antiderivative size = 3228, normalized size of antiderivative = 11.05

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x)
```

output

```
Piecewise((sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(-c*e**3*g*x**5/
6 - x**4*(13*b*c*e**5*g/12 + c**2*d*e**4*g + c**2*e**5*f)/(5*c*e**2) - x**
3*(b**2*e**5*g + 4*b*c*d*e**4*g + 2*b*c*e**5*f - 9*b*(13*b*c*e**5*g/12 + c
**2*d*e**4*g + c**2*e**5*f)/(10*c) - 2*c**2*d**2*e**3*g + c**2*d*e**4*f +
c*e**3*g*(-5*b*d*e + 5*c*d**2)/6)/(4*c*e**2) - x**2*(3*b**2*d*e**4*g + b**
2*e**5*f + 4*b*c*d*e**4*f - 7*b*(b**2*e**5*g + 4*b*c*d*e**4*g + 2*b*c*e**5
*f - 9*b*(13*b*c*e**5*g/12 + c**2*d*e**4*g + c**2*e**5*f)/(10*c) - 2*c**2*
d**2*e**3*g + c**2*d*e**4*f + c*e**3*g*(-5*b*d*e + 5*c*d**2)/6)/(8*c) - 2*
c**2*d**3*e**2*g - 2*c**2*d**2*e**3*f + (-4*b*d*e + 4*c*d**2)*(13*b*c*e**5
*g/12 + c**2*d*e**4*g + c**2*e**5*f)/(5*c*e**2))/(3*c*e**2) - x*(3*b**2*d*
*2*e**3*g + 3*b**2*d*e**4*f - 4*b*c*d**3*e**2*g - 5*b*(3*b**2*d*e**4*g + b
**2*e**5*f + 4*b*c*d*e**4*f - 7*b*(b**2*e**5*g + 4*b*c*d*e**4*g + 2*b*c*e*
*5*f - 9*b*(13*b*c*e**5*g/12 + c**2*d*e**4*g + c**2*e**5*f)/(10*c) - 2*c**
2*d**2*e**3*g + c**2*d*e**4*f + c*e**3*g*(-5*b*d*e + 5*c*d**2)/6)/(8*c) -
2*c**2*d**3*e**2*g - 2*c**2*d**2*e**3*f + (-4*b*d*e + 4*c*d**2)*(13*b*c*e*
*5*g/12 + c**2*d*e**4*g + c**2*e**5*f)/(5*c*e**2))/(6*c) + c**2*d**4*e*g -
 2*c**2*d**3*e**2*f + (-3*b*d*e + 3*c*d**2)*(b**2*e**5*g + 4*b*c*d*e**4*g
+ 2*b*c*e**5*f - 9*b*(13*b*c*e**5*g/12 + c**2*d*e**4*g + c**2*e**5*f)/(10*
c) - 2*c**2*d**2*e**3*g + c**2*d*e**4*f + c*e**3*g*(-5*b*d*e + 5*c*d**2)/6
)/(4*c*e**2))/(2*c*e**2) - (b**2*d**3*e**2*g + 3*b**2*d**2*e**3*f - 2*b...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algori
thm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(272) = 544$.

Time = 0.38 (sec) , antiderivative size = 765, normalized size of antiderivative = 2.62

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{1}{7680}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left(2\left(4\left(2\left(8\left(10\,ce^3gx+\frac{12\,c^6e^{11}f+12\,c^6de^{10}g+13\,bc^5e^{11}g}{c^5e^8}\right)x+\right.\right.\right.\right.\right.$$

$$-\,(384\,c^6d^5ef-960\,bc^5d^4e^2f+960\,b^2c^4d^3e^3f-480\,b^3c^3d^2e^4f+120\,b^4c^2de^5f-12\,b^5ce^6f+64\,c^6d^6g-$$

input

```
integrate((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algori
thm="giac")
```

output

```
-1/7680*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(2*(8*(10*c*e^3*g
*x + (12*c^6*e^11*f + 12*c^6*d*e^10*g + 13*b*c^5*e^11*g)/(c^5*e^8))*x + (1
20*c^6*d*e^10*f + 132*b*c^5*e^11*f - 140*c^6*d^2*e^9*g + 272*b*c^5*d*e^10*
g + 3*b^2*c^4*e^11*g)/(c^5*e^8))*x - (384*c^6*d^2*e^9*f - 744*b*c^5*d*e^10
*f - 12*b^2*c^4*e^11*f + 384*c^6*d^3*e^8*g - 348*b*c^5*d^2*e^9*g - 48*b^2*
c^4*d*e^10*g + 7*b^3*c^3*e^11*g)/(c^5*e^8))*x - (2400*c^6*d^3*e^8*f - 2064
*b*c^5*d^2*e^9*f - 456*b^2*c^4*d*e^10*f + 60*b^3*c^3*e^11*f - 240*c^6*d^4*
e^7*g + 816*b*c^5*d^3*e^8*g - 792*b^2*c^4*d^2*e^9*g + 276*b^3*c^3*d*e^10*g
 - 35*b^4*c^2*e^11*g)/(c^5*e^8))*x + (1536*c^6*d^4*e^7*f - 5472*b*c^5*d^3*
e^8*f + 5136*b^2*c^4*d^2*e^9*f - 1560*b^3*c^3*d*e^10*f + 180*b^4*c^2*e^11*
f + 1536*c^6*d^5*e^6*g - 4368*b*c^5*d^4*e^7*g + 5328*b^2*c^4*d^3*e^8*g - 3
256*b^3*c^3*d^2*e^9*g + 940*b^4*c^2*d*e^10*g - 105*b^5*c*e^11*g)/(c^5*e^8)
) - 1/1024*(384*c^6*d^5*e*f - 960*b*c^5*d^4*e^2*f + 960*b^2*c^4*d^3*e^3*f
- 480*b^3*c^3*d^2*e^4*f + 120*b^4*c^2*d*e^5*f - 12*b^5*c*e^6*f + 64*c^6*d^
6*g - 384*b*c^5*d^5*e*g + 720*b^2*c^4*d^4*e^2*g - 640*b^3*c^3*d^3*e^3*g +
300*b^4*c^2*d^2*e^4*g - 72*b^5*c*d*e^5*g + 7*b^6*e^6*g)*log(abs(-b*e^2 + 2
*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*ab
s(e)))/(sqrt(-c)*c^4*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\int (f+g\,x)\,(d+e\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}\,dx$$

input
```
int((f + g*x)*(d + e*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),x)
```

output
```
int((f + g*x)*(d + e*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 2564, normalized size of antiderivative = 8.78

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(i*(105*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*b**7*e**7*g - 1290*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b
*e + 2*c*d))*b**6*c*d*e**6*g - 180*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*b**6*c*e**7*f + 6660*sqrt(c)*asinh((sqrt( - b*
e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**2*d**2*e**5*g + 2160*sqr
t(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**2*
d*e**6*f - 18600*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
+ 2*c*d))*b**4*c**3*d**3*e**4*g - 10800*sqrt(c)*asinh((sqrt( - b*e + c*d -
 c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**3*d**2*e**5*f + 30000*sqrt(c)*asi
nh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**4*d**4*e**
3*g + 28800*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c
*d))*b**3*c**4*d**3*e**4*f - 27360*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**5*d**5*e**2*g - 43200*sqrt(c)*asinh((s
qrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**5*d**4*e**3*f +
 12480*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b*c**6*d**6*e*g + 34560*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
 - b*e + 2*c*d))*b*c**6*d**5*e**2*f - 1920*sqrt(c)*asinh((sqrt( - b*e + c*
d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**7*d**7*g - 11520*sqrt(c)*asinh((sqr
t( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**7*d**6*e*f + 105*sqrt(
d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e...
```

3.150 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 218

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{d+ex}\,dx = \frac{(2cd-be)(8cef-2cdg-3beg)(b+2cx)\sqrt{d(cd-be)-b}}{64c^2e}$$

$$+\frac{(8cef-8cdg+3beg+6cegx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{24ce^2}$$

$$+\frac{(2cd-be)^3(8cef-2cdg-3beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{64c^{5/2}e^2}$$

output
```
1/64*(-b*e+2*c*d)*(-3*b*e*g-2*c*d*g+8*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2
*x-c*e^2*x^2)^(1/2)/c^2/e+1/24*(6*c*e*g*x+3*b*e*g-8*c*d*g+8*c*e*f)*(d*(-b*
e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c/e^2+1/64*(-b*e+2*c*d)^3*(-3*b*e*g-2*c*d*
g+8*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/
c^(5/2)/e^2
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.34

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{d+ex}\,dx=\frac{(2cd-be)^3((d+ex)(-be+c(d-ex)))^{3/2}\left(-\frac{\sqrt{c}(-9b^3e^3g+6}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
),x]
```

output
```
((2*c*d - b*e)^3*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*(-((Sqrt[c]*(-9*
b^3*e^3*g + 6*b^2*c*e^2*(4*e*f + 6*d*g + e*g*x) + 8*c^3*(8*d^3*g - 4*d*e^2
*x*(3*f + 2*g*x) + 2*e^3*x^2*(4*f + 3*g*x) - d^2*e*(8*f + 3*g*x)) + 4*b*c^
2*e*(-19*d^2*g + 2*d*e*(2*f + g*x) + 2*e^2*x*(14*f + 9*g*x))))/((2*c*d - b
*e)^3*(d + e*x)*(-(b*e) + c*(d - e*x)))) - (3*(8*c*e*f - 2*c*d*g - 3*b*e*g
)*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/((d + e*x)^(3/2
)*(-(b*e) + c*(d - e*x))^(3/2))))/(192*c^(5/2)*e^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1215, 1225, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{1215}$$

$$\int (f+gx)\left(\frac{cd^2-bde}{d}-cex\right)\sqrt{-bde-be^2x+cd^2-ce^2x^2}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{(2cd-be)(-3beg-2cdg+8cef)\int \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16ce}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}$$

$$\Big\downarrow 1087$$

$$\frac{(2cd-be)(-3beg-2cdg+8cef)\left(\frac{(2cd-be)^2\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16ce}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}$$

$$\Big\downarrow 1092$$

$$\frac{(2cd-be)(-3beg-2cdg+8cef)\left(\frac{(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}+\frac{(b+2cx)\sqrt{d(cd-b}}{4c}\right.}{16ce}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}$$

$$\Big\downarrow 217$$

$$\frac{(2cd-be)\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)(-3beg-2cdg+8cef)}{16ce}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x),x]
```

output

```
((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(d*(c*d - b*e) - b*e^2*x - c*e^
2*x^2)^(3/2))/(24*c*e^2) + ((2*c*d - b*e)*(8*c*e*f - 2*c*d*g - 3*b*e*g)*((
(b + 2*c*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b
*e)^2*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e
^2*x^2])])/(8*c^(3/2)*e)))/(16*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(202) = 404$.

Time $= 2.55$ (sec) , antiderivative size $= 475$, normalized size of antiderivative $= 2.18$

method	result
default	$\frac{g\left(-\frac{\left(-2c\,e^{2}x-b\,e^{2}\right)\left(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}\right)^{\frac{3}{2}}}{8c\,e^{2}}-\frac{3\left(-4c\,e^{2}\left(-bde+c\,d^{2}\right)-b^{2}e^{4}\right)\left(-\frac{\left(-2c\,e^{2}x-b\,e^{2}\right)\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}{4c\,e^{2}}-\frac{(-4c\,e^{2}(}{}\right.}{16c\,e^{2}}\right.}{e}$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d),x,method=_RETUR
NVERBOSE)
```

output

```
g/e*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-
3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^
2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*
e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^
2*x-b*d*e+c*d^2)^(1/2))))-(d*g-e*f)/e^2*(1/3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c
*d*e)*(x+d/e))^(3/2)+1/2*(-b*e^2+2*c*d*e)*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*
d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2
+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*
c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 809, normalized size of antiderivative = 3.71

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{d+ex}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d),x, algori
thm="fricas")
```

output
```
[-1/768*(3*(8*(8*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^
4)*f - (16*c^4*d^4 - 24*b^2*c^2*d^2*e^2 + 16*b^3*c*d*e^3 - 3*b^4*e^4)*g)*s
qrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2
- 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) +
 4*(48*c^4*e^3*g*x^3 + 8*(8*c^4*e^3*f - (8*c^4*d*e^2 - 9*b*c^3*e^3)*g)*x^2
 - 8*(8*c^4*d^2*e - 2*b*c^3*d*e^2 - 3*b^2*c^2*e^3)*f + (64*c^4*d^3 - 76*b*
c^3*d^2*e + 36*b^2*c^2*d*e^2 - 9*b^3*c*e^3)*g - 2*(8*(6*c^4*d*e^2 - 7*b*c^
3*e^3)*f + (12*c^4*d^2*e - 4*b*c^3*d*e^2 - 3*b^2*c^2*e^3)*g)*x)*sqrt(-c*e^
2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^3*e^2), -1/384*(3*(8*(8*c^4*d^3*e - 1
2*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^4)*f - (16*c^4*d^4 - 24*b^2*c^
2*d^2*e^2 + 16*b^3*c*d*e^3 - 3*b^4*e^4)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*
x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*
e^2*x - c^2*d^2 + b*c*d*e)) + 2*(48*c^4*e^3*g*x^3 + 8*(8*c^4*e^3*f - (8*c^
4*d*e^2 - 9*b*c^3*e^3)*g)*x^2 - 8*(8*c^4*d^2*e - 2*b*c^3*d*e^2 - 3*b^2*c^2
*e^3)*f + (64*c^4*d^3 - 76*b*c^3*d^2*e + 36*b^2*c^2*d*e^2 - 9*b^3*c*e^3)*g
 - 2*(8*(6*c^4*d*e^2 - 7*b*c^3*e^3)*f + (12*c^4*d^2*e - 4*b*c^3*d*e^2 - 3*
b^2*c^2*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^3*e^2)]
```

Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 1958, normalized size of antiderivative = 8.98

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d),x)
```

output
```
-b*e*f*Piecewise(((b/(4*c) + x/2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2
*x**2) + (b**2*e**2/(8*c) - b*d*e/2 + c*d**2/2)*Piecewise((log(-b*e**2 - 2
*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)
)/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*c) + x)*
log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)), Ne(c*e**2, 0)), (
-2*(-b*d*e - b*e**2*x + c*d**2)**(3/2)/(3*b*e**2), Ne(b*e**2, 0)), (x*sqrt
(-b*d*e + c*d**2), True)) - b*e*g*Piecewise(((-b*(-b*d*e + c*d**2)/(12*c)
- b*(b**2*e**2/(8*c) - b*d*e/3 + c*d**2/3)/(2*c))*Piecewise((log(-b*e**2 -
 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**
2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*c) + x
)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)) + (b*x/(12*c) +
x**2/3 - (b**2*e**2/(8*c) - b*d*e/3 + c*d**2/3)/(c*e**2))*sqrt(-b*d*e - b*
e**2*x + c*d**2 - c*e**2*x**2), Ne(c*e**2, 0)), (2*((b*d*e - c*d**2)*(-b*d
*e - b*e**2*x + c*d**2)**(3/2)/3 + (-b*d*e - b*e**2*x + c*d**2)**(5/2)/5)/
(b**2*e**4), Ne(b*e**2, 0)), (x**2*sqrt(-b*d*e + c*d**2)/2, True)) + c*d*f
*Piecewise(((b/(4*c) + x/2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)
 + (b**2*e**2/(8*c) - b*d*e/2 + c*d**2/2)*Piecewise((log(-b*e**2 - 2*c*e**
2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2))/sqrt
(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*c) + x)*log(b/
(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)), Ne(c*e**2, 0)), (-2*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(202) = 404$.

Time = 0.36 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.87

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{d+ex}\,dx =$$
$$-\frac{1}{192}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left(2\left(4\left(6\,cegx+\frac{8\,c^4e^5f-8\,c^4de^4g+9\,bc^3e^5g}{c^3e^4}\right)x-\frac{48\,c^4de^4f-56\,bc\right.\right.\right.$$
$$-\frac{\left(64\,c^4d^3ef-96\,bc^3d^2e^2f+48\,b^2c^2de^3f-8\,b^3ce^4f-16\,c^4d^4g+24\,b^2c^2d^2e^2g-16\,b^3cde^3g+3\,b^4e^4g\right)\log}{128\,\sqrt{-c}c^2e|e|}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d),x, algori
thm="giac")
```

output

```
-1/192*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(6*c*e*g*x + (8*c^
4*e^5*f - 8*c^4*d*e^4*g + 9*b*c^3*e^5*g)/(c^3*e^4))*x - (48*c^4*d*e^4*f -
56*b*c^3*e^5*f + 12*c^4*d^2*e^3*g - 4*b*c^3*d*e^4*g - 3*b^2*c^2*e^5*g)/(c^
3*e^4))*x - (64*c^4*d^2*e^3*f - 16*b*c^3*d*e^4*f - 24*b^2*c^2*e^5*f - 64*c
^4*d^3*e^2*g + 76*b*c^3*d^2*e^3*g - 36*b^2*c^2*d*e^4*g + 9*b^3*c*e^5*g)/(c
^3*e^4)) - 1/128*(64*c^4*d^3*e*f - 96*b*c^3*d^2*e^2*f + 48*b^2*c^2*d*e^3*f
 - 8*b^3*c*e^4*f - 16*c^4*d^4*g + 24*b^2*c^2*d^2*e^2*g - 16*b^3*c*d*e^3*g
+ 3*b^4*e^4*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^
2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqrt(-c)*c^2*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{d+ex}\,dx = \int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{d+e\,x}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x),x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 1300, normalized size of antiderivative = 5.96

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d),x)
```

output
```
(i*(9*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b
**5*e**5*g - 66*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
 2*c*d))*b**4*c*d*e**4*g - 24*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)
/sqrt( - b*e + 2*c*d))*b**4*c*e**5*f + 168*sqrt(c)*asinh((sqrt( - b*e + c*
d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**2*d**2*e**3*g + 192*sqrt(c)*as
inh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**2*d*e**4*
f - 144*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*b**2*c**3*d**3*e**2*g - 576*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/
sqrt( - b*e + 2*c*d))*b**2*c**3*d**2*e**3*f - 48*sqrt(c)*asinh((sqrt( - b*
e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*d**4*e*g + 768*sqrt(c)*as
inh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*d**3*e**2*
f + 96*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
c**5*d**5*g - 384*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
 + 2*c*d))*c**5*d**4*e*f + 9*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**3*g - 36*sqrt(d + e*x)*sqrt(b
*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d*e*
*2*g - 24*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
 + c*d - c*e*x)*b**2*c**2*e**3*f - 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
 - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**3*g*x + 76*sqrt(d
+ e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x...
```

3.151 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 211

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^2}\,dx = \frac{(6cef-4cdg-beg)(8cd-5be-2cex)\sqrt{d(cd-be)-be^2}}{24ce^2}$$

$$-\frac{g(cd-be-cex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$+\frac{(2cd-be)^2(6cef-4cdg-beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e^2}$$

output

```
1/24*(-b*e*g-4*c*d*g+6*c*e*f)*(-2*c*e*x-5*b*e+8*c*d)*(d*(-b*e+c*d)-b*e^2*x
-c*e^2*x^2)^(1/2)/c/e^2-1/3*g*(-c*e*x-b*e+c*d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e
^2*x^2)^(1/2)/c/e^2+1/8*(-b*e+2*c*d)^2*(-b*e*g-4*c*d*g+6*c*e*f)*arctan(c^(
1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(3/2)/e^2
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.04

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^2}\,dx=\frac{\sqrt{d+ex}\sqrt{cd-be-cex}\Big(-\sqrt{c}\sqrt{d+ex}\sqrt{cd-be-cex}(3b}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^2,x]
```

output
```
(Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*x]*(-(Sqrt[c]*Sqrt[d + e*x]*Sqrt[c*d -
 b*e - c*e*x]*(3*b^2*e^2*g + 2*b*c*e*(15*e*f - 14*d*g + 7*e*g*x) + 4*c^2*(
10*d^2*g - 6*d*e*(2*f + g*x) + e^2*x*(3*f + 2*g*x)))) + 3*(-2*c*d + b*e)^2
*(-6*c*e*f + 4*c*d*g + b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt
[d + e*x])]))/(24*c^(3/2)*e^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1220, 1131, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-beg-4cdg+6cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{d+ex}dx}{e(2cd-be)}+\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(-beg-4cdg+6cef)\left(\frac{1}{2}(2cd-be)\int \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e}\right)}{e(2cd-be)}+$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow 1087$$

$$\frac{(-beg-4cdg+6cef)\left(\frac{1}{2}(2cd-be)\left(\frac{(2cd-be)^2\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)+\frac{(d(cd-be}{}\right.}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow 1092$$

$$\frac{(-beg-4cdg+6cef)\left(\frac{1}{2}(2cd-be)\left(\frac{(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}+(b+2cx)\sqrt{d(c}\right.\right.}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow 217$$

$$\frac{\left(\frac{1}{2}(2cd-be)\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e}\right)}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^2(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^2,x]
```

output

```
(2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(e^2*(2*c*d -
b*e)*(d + e*x)^2) + ((6*c*e*f - 4*c*d*g - b*e*g)*((d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2)^(3/2)/(3*e) + ((2*c*d - b*e)*(((b + 2*c*x)*Sqrt[d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b*e)^2*ArcTan[(e*(b + 2*c*x))/
(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(8*c^(3/2)*e)))/2)
)/(e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(195) = 390.

Time = 2.86 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.67

method	result
default	$\frac{g\Bigg(\frac{\left(-c e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(-b e^{2}+2dec\right)\Bigg(-\frac{\left(-2c e^{2}\left(x+\frac{d}{e}\right)-b e^{2}+2dec\right)\sqrt{-c e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b e^{2}+2dec\right)\left(x+\frac{d}{e}\right)}}{4c e^{2}}+\frac{\big(-b e}{}}{2}}{e^{2}}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^2,x,method=_RET
URNVERBOSE)
```

output

```
g/e^2*(1/3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+1/2*(-b*e^2+2
*c*d*e)*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b
*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*ar
ctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-
b*e^2+2*c*d*e)*(x+d/e))^(1/2))))-(d*g-e*f)/e^3*(2/(-b*e^2+2*c*d*e)/(x+d/e)
^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+6*c*e^2/(-b*e^2+2*c*d
*e)*(1/3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+1/2*(-b*e^2+2*c
*d*e)*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e
^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arct
an((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*
e^2+2*c*d*e)*(x+d/e))^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.69

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^2}\,dx = \Bigg[\frac{3\,(6\,(4\,c^3d^2e-4\,bc^2de^2+b^2ce^3)f-(16\,c^3d^3-12\,bc^2d^2e}{}$$

$$-\frac{3\,(6\,(4\,c^3d^2e-4\,bc^2de^2+b^2ce^3)f-(16\,c^3d^3-12\,bc^2d^2e+b^3e^3)g)\sqrt{c}\arctan\Bigg(\frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,cex+be}{2\,(c^2e^2x^2+bce^2x-c^2d^2+bcde)}}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^2,x, algo
rithm="fricas")
```

output

```
[1/96*(3*(6*(4*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*f - (16*c^3*d^3 - 12
*b*c^2*d^2*e + b^3*e^3)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^
2*d^2 + 4*b*c*d*e + b^2*e^2 + 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)
*(2*c*e*x + b*e)*sqrt(-c)) - 4*(8*c^3*e^2*g*x^2 - 6*(8*c^3*d*e - 5*b*c^2*e
^2)*f + (40*c^3*d^2 - 28*b*c^2*d*e + 3*b^2*c*e^2)*g + 2*(6*c^3*e^2*f - (12
*c^3*d*e - 7*b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/
(c^2*e^2), -1/48*(3*(6*(4*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*f - (16*c
^3*d^3 - 12*b*c^2*d^2*e + b^3*e^3)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 -
 b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x
 - c^2*d^2 + b*c*d*e)) + 2*(8*c^3*e^2*g*x^2 - 6*(8*c^3*d*e - 5*b*c^2*e^2)*
f + (40*c^3*d^2 - 28*b*c^2*d*e + 3*b^2*c*e^2)*g + 2*(6*c^3*e^2*f - (12*c^3
*d*e - 7*b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^2
*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^2}\,dx=\int \frac{(-(d+ex)\,(be-cd+cex))^{\frac{3}{2}}\,(f+gx)}{(d+ex)^2}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**2,x
)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**2, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^2}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^2,x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1297 vs. $2(194) = 388$.

Time = 0.66 (sec) , antiderivative size = 1297, normalized size of antiderivative = 6.15

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^2,x, algo
rithm="giac")
```

output

```
-1/24*(3*(24*c^3*d^2*e*f*sgn(1/(e*x + d))*sgn(e) - 24*b*c^2*d*e^2*f*sgn(1/
(e*x + d))*sgn(e) + 6*b^2*c*e^3*f*sgn(1/(e*x + d))*sgn(e) - 16*c^3*d^3*g*s
gn(1/(e*x + d))*sgn(e) + 12*b*c^2*d^2*e*g*sgn(1/(e*x + d))*sgn(e) - b^3*e^
3*g*sgn(1/(e*x + d))*sgn(e))*arctan(sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x +
 d))/sqrt(c))/(c^(3/2)*e^3) - (120*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^2
*c^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^2*e*f*sgn(1/(e*x + d))*s
gn(e) + 72*c^5*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^2*e*f*sgn(1/(e
*x + d))*sgn(e) + 192*c^4*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*d^2
*e*f*sgn(1/(e*x + d))*sgn(e) - 120*b*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))
^2*c^2*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d*e^2*f*sgn(1/(e*x + d))
*sgn(e) - 72*b*c^4*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d*e^2*f*sgn(
1/(e*x + d))*sgn(e) - 192*b*c^3*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/
2)*d*e^2*f*sgn(1/(e*x + d))*sgn(e) + 30*b^2*(c - 2*c*d/(e*x + d) + b*e/(e*
x + d))^2*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*e^3*f*sgn(1/(e*x +
d))*sgn(e) + 18*b^2*c^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*e^3*f*s
gn(1/(e*x + d))*sgn(e) + 48*b^2*c^2*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))
^(3/2)*e^3*f*sgn(1/(e*x + d))*sgn(e) - 144*(c - 2*c*d/(e*x + d) + b*e/(e*x
 + d))^2*c^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^3*g*sgn(1/(e*x +
 d))*sgn(e) - 48*c^5*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^3*g*sgn(
1/(e*x + d))*sgn(e) - 128*c^4*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^2}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{{\left(d+e\,x\right)}^2}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^2,x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^2, x
)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.98

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^2}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^2,x)
```

output

```
(i*( - 3*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*b**4*e**4*g + 6*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
 + 2*c*d))*b**3*c*d*e**3*g + 18*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b**3*c*e**4*f + 36*sqrt(c)*asinh((sqrt( - b*e + c
*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d**2*e**2*g - 108*sqrt(c)*a
sinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d*e**3
*f - 120*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*b*c**3*d**3*e*g + 216*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
 - b*e + 2*c*d))*b*c**3*d**2*e**2*f + 96*sqrt(c)*asinh((sqrt( - b*e + c*d
- c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**4*g - 144*sqrt(c)*asinh((sqrt( -
 b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**3*e*f - 3*sqrt(d + e*
x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*
c*e**2*g + 28*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( -
 b*e + c*d - c*e*x)*b*c**2*d*e*g - 30*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt
( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*e**2*f - 14*sqrt(d + e*
x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**
2*e**2*g*x - 40*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt(
 - b*e + c*d - c*e*x)*c**3*d**2*g + 48*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqr
t( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**3*d*e*f + 24*sqrt(d + e*x)
*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**3...
```

3.152 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 220

$$
\begin{aligned}
&\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^3}\,dx = \\
&-\frac{(4cef-6cdg+beg)(8cd-5be-2cex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4e^2(2cd-be)} \\
&-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(2cd-be)(d+ex)^3} \\
&-\frac{3(2cd-be)(4cef-6cdg+beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4\sqrt{c}e^2}
\end{aligned}
$$

output
```
-1/4*(b*e*g-6*c*d*g+4*c*e*f)*(-2*c*e*x-5*b*e+8*c*d)*(d*(-b*e+c*d)-b*e^2*x-
c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)-2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2
*x^2)^(5/2)/e^2/(-b*e+2*c*d)/(e*x+d)^3-3/4*(-b*e+2*c*d)*(b*e*g-6*c*d*g+4*c
*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(1/
2)/e^2
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.99

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^3}\,dx=\frac{((d+ex)(-be+c(d-ex)))^{3/2}\left(\frac{\sqrt{cd-be-cex}(be(8ef-13dg-5egx}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^3,x]
```

output
```
(((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*((Sqrt[c*d - b*e - c*e*x]*(b*e*(
8*e*f - 13*d*g - 5*e*g*x) + 2*c*(14*d^2*g + 5*d*e*(-2*f + g*x) - e^2*x*(2*
f + g*x))))/(d + e*x)^2 - (6*(2*c*d - b*e)*(-4*c*e*f + 6*c*d*g - b*e*g)*Ar
cTan[(Sqrt[c]*Sqrt[d + e*x])/(Sqrt[2*c*d - b*e] - Sqrt[c*d - b*e - c*e*x])
])/(Sqrt[c]*(d + e*x)^(3/2))))/(4*e^2*(-(b*e) + c*(d - e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1220, 1131, 1131, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(beg-6cdg+4cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^2}dx}{e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{1131}$$

$$-\frac{(beg-6cdg+4cef)\left(\frac{3}{4}(2cd-be)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{d+ex}dx+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e(d+ex)}\right)}{e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{1131}$$

$$-\frac{(beg-6cdg+4cef)\left(\frac{3}{4}(2cd-be)\left(\frac{1}{2}(2cd-be)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}\right)+\frac{(d(cd-be)}{2}\right.}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{(beg-6cdg+4cef)\left(\frac{3}{4}(2cd-be)\left((2cd-be)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)+\sqrt{d(cd}\right.\right.}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\left(\frac{3}{4}(2cd-be)\left(\frac{(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2\sqrt{c}e}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}\right)+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e(d+ex)}\right)(beg}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^3(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^3,x]
```

output

```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(e^2*(2*c*d -
 b*e)*(d + e*x)^3) - ((4*c*e*f - 6*c*d*g + b*e*g)*((d*(c*d - b*e) - b*e^2*
x - c*e^2*x^2)^(3/2)/(2*e*(d + e*x)) + (3*(2*c*d - b*e)*(Sqrt[d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2]/e + ((2*c*d - b*e)*ArcTan[(e*(b + 2*c*x))/(2*Sqrt
[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(2*Sqrt[c]*e)))/4))/(e*(2
*c*d - b*e))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. 2(206) = 412.

Time = 3.39 (sec) , antiderivative size = 732, normalized size of antiderivative = 3.33

method	result
default	$\frac{g\left(\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{2}}+\frac{6c\,e^{2}\left(\frac{\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(-b\,e^{2}+2dec\right)\left(-\frac{\left(-2c\,e^{2}\left(x+\frac{d}{e}\right)-b\,e^{2}+\right.}{}\right.}{}\right.}{}\right.}{e^{3}}$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^3,x,method=_RET
URNVERBOSE)
```

output

```
g/e^3*(2/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+
d/e))^(5/2)+6*c*e^2/(-b*e^2+2*c*d*e)*(1/3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*
e)*(x+d/e))^(3/2)+1/2*(-b*e^2+2*c*d*e)*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e
*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*
c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d
*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2))))-(d*g-e*f)
/e^4*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+
d/e))^(5/2)-4*c*e^2/(-b*e^2+2*c*d*e)*(2/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2
*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+6*c*e^2/(-b*e^2+2*c*d*e)*(1/3*(
-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+1/2*(-b*e^2+2*c*d*e)*(-1/
4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e
)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)
^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*
e)*(x+d/e))^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.89

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^3}\,dx = \Bigg[\frac{3\left(4\left(2\,c^2d^2e-bcde^2\right)f-\left(12\,c^2d^3-8\,bcd^2e+b^2de^2\right)g+\right.}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^3,x, algo
rithm="fricas")
```

output
```
[1/16*(3*(4*(2*c^2*d^2*e - b*c*d*e^2)*f - (12*c^2*d^3 - 8*b*c*d^2*e + b^2*
d*e^2)*g + (4*(2*c^2*d*e^2 - b*c*e^3)*f - (12*c^2*d^2*e - 8*b*c*d*e^2 + b^
2*e^3)*g)*x)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*
d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*
e)*sqrt(-c)) - 4*(2*c^2*e^2*g*x^2 + 4*(5*c^2*d*e - 2*b*c*e^2)*f - (28*c^2*
d^2 - 13*b*c*d*e)*g + (4*c^2*e^2*f - 5*(2*c^2*d*e - b*c*e^2)*g)*x)*sqrt(-c
*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c*e^3*x + c*d*e^2), 1/8*(3*(4*(2*c^2
*d^2*e - b*c*d*e^2)*f - (12*c^2*d^3 - 8*b*c*d^2*e + b^2*d*e^2)*g + (4*(2*c
^2*d*e^2 - b*c*e^3)*f - (12*c^2*d^2*e - 8*b*c*d*e^2 + b^2*e^3)*g)*x)*sqrt(
c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*s
qrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*(2*c^2*e^2*g*x^2
 + 4*(5*c^2*d*e - 2*b*c*e^2)*f - (28*c^2*d^2 - 13*b*c*d*e)*g + (4*c^2*e^2*
f - 5*(2*c^2*d*e - b*c*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*
e))/(c*e^3*x + c*d*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^3}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}(f+gx)}{\left(d+ex\right)^3}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**3,x
)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**3, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^3,x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.84

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^3}\,dx =$$

$$-\frac{1}{4}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left(\frac{2\,cgx}{e}+\frac{4\,c^2e^3f-12\,c^2de^2g+5\,bce^3g}{ce^4}\right)$$

$$+\frac{(8\,c^2def-4\,bce^2f-12\,c^2d^2g+8\,bcdeg-b^2e^2g)\log\Big(\Big|bcd^2e^2-2\,(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-be^2x+cd^2-b}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^3,x, algo
rithm="giac")
```

output

```
-1/4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*g*x/e + (4*c^2*e^3*f
- 12*c^2*d*e^2*g + 5*b*c*e^3*g)/(c*e^4)) + 1/8*(8*c^2*d*e*f - 4*b*c*e^2*f
- 12*c^2*d^2*g + 8*b*c*d*e*g - b^2*e^2*g)*log(abs(b*c*d^2*e^2 - 2*(sqrt(-c
*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*c*d^2*abs(e
) - 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b*sqrt
(-c)*d*e*abs(e) - 4*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 -
b*d*e))^2*c*d*e - (sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*
d*e))^2*b*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*
d*e))^3*sqrt(-c)*abs(e)))/(sqrt(-c)*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^3}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^3,x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^3, x
)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1201, normalized size of antiderivative = 5.46

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^3}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^3,x)
```

output

```
(i*(3*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b
**3*d*e**3*g + 3*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
+ 2*c*d))*b**3*e**4*g*x - 30*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/
sqrt( - b*e + 2*c*d))*b**2*c*d**2*e**2*g + 12*sqrt(c)*asinh((sqrt( - b*e +
 c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d*e**3*f - 30*sqrt(c)*asinh(
(sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d*e**3*g*x + 1
2*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*
c*e**4*f*x + 84*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
 2*c*d))*b*c**2*d**3*e*g - 48*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)
/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e**2*f + 84*sqrt(c)*asinh((sqrt( - b*e
+ c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e**2*g*x - 48*sqrt(c)*
asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**3*f
*x - 72*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*c**3*d**4*g + 48*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
 + 2*c*d))*c**3*d**3*e*f - 72*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)
/sqrt( - b*e + 2*c*d))*c**3*d**3*e*g*x + 48*sqrt(c)*asinh((sqrt( - b*e + c
*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**2*e**2*f*x - 13*sqrt(d + e*x)
*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*d*e
*g + 8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b*c*e**2*f - 5*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e...
```

3.153 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 220

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^4}\,dx &= -\frac{cg\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2}\\ &+\frac{2(cef-3cdg+beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)}\\ &-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^3}\\ &-\frac{\sqrt{c}(2cef-8cdg+3beg)\arctan\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{c}(d+ex)}\right)}{e^2}\end{aligned}$$

output

```
-c*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2+2*(b*e*g-3*c*d*g+c*e*f)*(d
*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)-2/3*(-d*g+e*f)*(d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^3-c^(1/2)*(3*b*e*g-8*c*d*g+2*c*e*
f)*arctan(1/c^(1/2)/(e*x+d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^4}\,dx = \frac{((d+ex)(-be+c(d-ex)))^{3/2}\left(\frac{c(-19d^2g+de(4f-26gx)+e^2x(8f}{(d+ex)^3(-be+}\right.}{3}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^4,x]
```

output
```
(((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*((c*(-19*d^2*g + d*e*(4*f - 26*g
*x) + e^2*x*(8*f - 3*g*x)) + 2*b*e*(2*d*g + e*(f + 3*g*x)))/((d + e*x)^3*(
-(b*e) + c*(d - e*x))) - (3*Sqrt[c]*(2*c*e*f - 8*c*d*g + 3*b*e*g)*ArcTan[S
qrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/((d + e*x)^(3/2)*(-(b*e)
+ c*(d - e*x))^(3/2))))/(3*e^2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1220, 1125, 27, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(3beg-8cdg+2cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^3}dx}{3e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow \text{1125}$$

$$-\frac{(3beg-8cdg+2cef)\left(-\frac{\int \frac{ce^4(3cd-2be-cex)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{e^4}-\frac{2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)}{3e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{(3beg-8cdg+2cef)\left(-c\int \frac{3cd-2be-cex}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)}{3e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow \text{ 1160}$$

$$-\frac{(3beg-8cdg+2cef)\left(-c\left(\frac{3}{2}(2cd-be)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}\right)-\frac{2(2cd-be)\sqrt{d(cd-be}}{e(d+ex}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow \text{ 1092}$$

$$-\frac{(3beg-8cdg+2cef)\left(-c\left(3(2cd-be)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)+\frac{\sqrt{d(cd-be)-be^2}}{e}\right.\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{\left(-c\left(\frac{3(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2\sqrt{c}e}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}\right)-\frac{2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)(3beg}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^4(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^4,x]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(3*e^2*(2*c*d
 - b*e)*(d + e*x)^4) - ((2*c*e*f - 8*c*d*g + 3*b*e*g)*((-2*(2*c*d - b*e)*S
qrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e*(d + e*x)) - c*(Sqrt[d*(c*d -
 b*e) - b*e^2*x - c*e^2*x^2]/e + (3*(2*c*d - b*e)*ArcTan[(e*(b + 2*c*x))/(
2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(2*Sqrt[c]*e))))/(3
*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(206) = 412$.

Time = 4.45 (sec) , antiderivative size = 900, normalized size of antiderivative = 4.09

method	result
	$\left(4c\,e^{2}\left(\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{2}}+\frac{6c\,e^{2}\left(\frac{\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right.\right.}{3}\right.}{}\right.\right.$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^4,x,method=_RET
URNVERBOSE)
```

output
```
g/e^4*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x
+d/e))^(5/2)-4*c*e^2/(-b*e^2+2*c*d*e)*(2/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^
2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+6*c*e^2/(-b*e^2+2*c*d*e)*(1/3*
(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+1/2*(-b*e^2+2*c*d*e)*(-1
/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*
e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2
)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d
*e)*(x+d/e))^(1/2))))))-(d*g-e*f)/e^5*(-2/3/(-b*e^2+2*c*d*e)/(x+d/e)^4*(-c
*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)-2/3*c*e^2/(-b*e^2+2*c*d*e)*
(-2/(-b*e^2+2*c*d*e)/(x+d/e)^3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))
^(5/2)-4*c*e^2/(-b*e^2+2*c*d*e)*(2/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d
/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+6*c*e^2/(-b*e^2+2*c*d*e)*(1/3*(-c*e^
2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+1/2*(-b*e^2+2*c*d*e)*(-1/4*(-2
*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+
d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2
)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x
+d/e))^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.70

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^4}\,dx = \Bigg[\frac{3\,(2\,cd^2ef+(2\,ce^3f-(8\,cde^2-3\,be^3)g)x^2-(8\,cd^3-3\,b}{}$$

$$-\frac{3\,(2\,cd^2ef+(2\,ce^3f-(8\,cde^2-3\,be^3)g)x^2-(8\,cd^3-3\,bd^2e)g+2\,(2\,cde^2f-(8\,cd^2e-3\,bde^2)g)x)\sqrt{c}\,\mathrm{ar}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^4,x, algo
rithm="fricas")
```

output

```
[1/12*(3*(2*c*d^2*e*f + (2*c*e^3*f - (8*c*d*e^2 - 3*b*e^3)*g)*x^2 - (8*c*d
^3 - 3*b*d^2*e)*g + 2*(2*c*d*e^2*f - (8*c*d^2*e - 3*b*d*e^2)*g)*x)*sqrt(-c
)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 4*sq
rt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*(3*
c*e^2*g*x^2 - 2*(2*c*d*e + b*e^2)*f + (19*c*d^2 - 4*b*d*e)*g - 2*(4*c*e^2*
f - (13*c*d*e - 3*b*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))
/(e^4*x^2 + 2*d*e^3*x + d^2*e^2), -1/6*(3*(2*c*d^2*e*f + (2*c*e^3*f - (8*c
*d*e^2 - 3*b*e^3)*g)*x^2 - (8*c*d^3 - 3*b*d^2*e)*g + 2*(2*c*d*e^2*f - (8*c
*d^2*e - 3*b*d*e^2)*g)*x)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c
*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 +
 b*c*d*e)) + 2*(3*c*e^2*g*x^2 - 2*(2*c*d*e + b*e^2)*f + (19*c*d^2 - 4*b*d*
e)*g - 2*(4*c*e^2*f - (13*c*d*e - 3*b*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x
 + c*d^2 - b*d*e))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}{(d+ex)^4}\,dx=\int \frac{\left(-(d+ex)\,(be-cd+cex)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^4}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**4,x
)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**4, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}{(d+ex)^4}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^4,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(206) = 412$.

Time = 1.60 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.65

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^4}\,dx = -\frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}cg}{e^2}$$
$$+\frac{\left(2\sqrt{-c}cef-8\sqrt{-c}cdg+3b\sqrt{-c}eg\right)\log\left(\left|b\sqrt{-c}c^2d^4e^2+2\left(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-be^2x+cd^2-bde}\right)c\right.\right.}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^4,x, algo
rithm="giac")
```

output
```
-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*c*g/e^2 + 1/10*(2*sqrt(-c)*c*e
*f - 8*sqrt(-c)*c*d*g + 3*b*sqrt(-c)*e*g)*log(abs(b*sqrt(-c)*c^2*d^4*e^2 +
 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*c^3*d^4*a
bs(e) + 4*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b*
c^2*d^3*e*abs(e) - 8*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 -
 b*d*e))^2*sqrt(-c)*c^2*d^3*e - 6*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^
2*x + c*d^2 - b*d*e))^2*b*sqrt(-c)*c*d^2*e^2 - 12*(sqrt(-c*e^2)*x - sqrt(-
c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*c^2*d^2*abs(e) - 4*(sqrt(-c*e^2)*x
 - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*b*c*d*e*abs(e) + 8*(sqrt(
-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^4*sqrt(-c)*c*d*e +
 (sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^4*b*sqrt(-c
)*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^5*
c*abs(e)))/(e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^4}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{(d+e\,x)^4}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^4,x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^4, x
)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1184, normalized size of antiderivative = 5.38

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^4}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^4,x)
```

output

```
(i*(18*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b**2*d**2*e**2*g + 36*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
 b*e + 2*c*d))*b**2*d*e**3*g*x + 18*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e
*x)*i)/sqrt( - b*e + 2*c*d))*b**2*e**4*g*x**2 - 84*sqrt(c)*asinh((sqrt( -
b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*d**3*e*g + 12*sqrt(c)*asin
h((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*d**2*e**2*f - 1
68*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*
d**2*e**2*g*x + 24*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*
e + 2*c*d))*b*c*d*e**3*f*x - 84*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b*c*d*e**3*g*x**2 + 12*sqrt(c)*asinh((sqrt( - b*e
 + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*e**4*f*x**2 + 96*sqrt(c)*asin
h((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*d**4*g - 24*sq
rt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*d**3
*e*f + 192*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*
d))*c**2*d**3*e*g*x - 48*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt
( - b*e + 2*c*d))*c**2*d**2*e**2*f*x + 96*sqrt(c)*asinh((sqrt( - b*e + c*d
 - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*d**2*e**2*g*x**2 - 24*sqrt(c)*asin
h((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*d*e**3*f*x**2
+ 8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
 - c*e*x)*b*d*e*g + 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c...
```

3.154 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 210

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx = {} & \frac{2cg\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)} \\ & -\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^3}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(2cd-be)(d+ex)^5} \\ & +\frac{2c^{3/2}g\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e^2}\end{aligned}$$

output

```
2*c*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)-2/3*g*(d*(-b*e+c*
d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^3-2/5*(-d*g+e*f)*(d*(-b*e+c*d)-b*e
^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)/(e*x+d)^5+2*c^(3/2)*g*arctan(c^(1/2
)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.13

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx = \frac{2((d+ex)(-be+c(d-ex)))^{3/2}\Bigg(\frac{-b^2e^2(3ef+2dg+5egx)-2bce(3}{}}{}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^5,x]
```

output

```
(2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*((-(b^2*e^2*(3*e*f + 2*d*g + 5
*e*g*x)) - 2*b*c*e*(3*d^2*g + d*e*(-3*f + 7*g*x) + e^2*x*(3*f + 10*g*x)) +
 c^2*(23*d^3*g - 3*e^3*f*x^2 - 3*d^2*e*(f - 18*g*x) + d*e^2*x*(6*f + 43*g*
x)))/((2*c*d - b*e)*(d + e*x)^4*(-(b*e) + c*(d - e*x))) - (15*c^(3/2)*g*Ar
cTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/((d + e*x)^(3/2)*(-
(b*e) + c*(d - e*x))^(3/2))))/(15*e^2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1220, 1130, 1125, 27, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^5}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{g\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^4}dx}{e}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \text{1130}$$

$$\frac{g\left(-c\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^2}dx-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \; 1125$$

$$\frac{g\left(-c\left(-\frac{\int \frac{ce^2}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{e^2}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \; 27$$

$$\frac{g\left(-c\left(-c\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \; 1092$$

$$\frac{g\left(-c\left(-2c\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\big(d(cd-be)-be^2x-c}{3e(d+ex)^3}}{e}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \; 217$$

$$\frac{g\left(-c\left(-\frac{\sqrt{c}\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^3}\right)}{e}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(d+ex)^5(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^5,x]
```

output

```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(5*e^2*(2*c*d
 - b*e)*(d + e*x)^5) + (g*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)
)/(3*e*(d + e*x)^3) - c*((-2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e
*(d + e*x)) - (Sqrt[c]*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2])])/e)))/e
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(194) = 388$.

Time $= 5.42$ (sec) , antiderivative size $= 563$, normalized size of antiderivative $= 2.68$

method	result
	$4c\,e^{2}\left(\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2\right.\right.}{\left(-b\,e^{2}+2dec\right)\left(x+\right.}\right.$ $2c\,e^{2}\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{3}}-\right.$ $g\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{3\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{4}}-\right.$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^5,x,method=_RET
URNVERBOSE)
```

output

```
g/e^5*(-2/3/(-b*e^2+2*c*d*e)/(x+d/e)^4*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*
(x+d/e))^(5/2)-2/3*c*e^2/(-b*e^2+2*c*d*e)*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^3*(
-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)-4*c*e^2/(-b*e^2+2*c*d*e)*
(2/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^
(5/2)+6*c*e^2/(-b*e^2+2*c*d*e)*(1/3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+
d/e))^(3/2)+1/2*(-b*e^2+2*c*d*e)*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/
e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)
^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/
e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2))))))+2/5*(d*g-e*f)
/e^6/(-b*e^2+2*c*d*e)/(x+d/e)^5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e)
)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(194) = 388$.

Time = 5.39 (sec) , antiderivative size = 879, normalized size of antiderivative = 4.19

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^5,x, algo
rithm="fricas")
```

output

```
[1/30*(15*((2*c^2*d*e^3 - b*c*e^4)*g*x^3 + 3*(2*c^2*d^2*e^2 - b*c*d*e^3)*g
*x^2 + 3*(2*c^2*d^3*e - b*c*d^2*e^2)*g*x + (2*c^2*d^4 - b*c*d^3*e)*g)*sqrt
(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 4
*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*
sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((3*c^2*e^3*f - (43*c^2*d*e^2 -
 20*b*c*e^3)*g)*x^2 + 3*(c^2*d^2*e - 2*b*c*d*e^2 + b^2*e^3)*f - (23*c^2*d^
3 - 6*b*c*d^2*e - 2*b^2*d*e^2)*g - (6*(c^2*d*e^2 - b*c*e^3)*f + (54*c^2*d^
2*e - 14*b*c*d*e^2 - 5*b^2*e^3)*g)*x))/(2*c*d^4*e^2 - b*d^3*e^3 + (2*c*d*e
^5 - b*e^6)*x^3 + 3*(2*c*d^2*e^4 - b*d*e^5)*x^2 + 3*(2*c*d^3*e^3 - b*d^2*e
^4)*x), -1/15*(15*((2*c^2*d*e^3 - b*c*e^4)*g*x^3 + 3*(2*c^2*d^2*e^2 - b*c*
d*e^3)*g*x^2 + 3*(2*c^2*d^3*e - b*c*d^2*e^2)*g*x + (2*c^2*d^4 - b*c*d^3*e)
*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x
 + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) + 2*sqrt(-c
*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((3*c^2*e^3*f - (43*c^2*d*e^2 - 20*b*c
*e^3)*g)*x^2 + 3*(c^2*d^2*e - 2*b*c*d*e^2 + b^2*e^3)*f - (23*c^2*d^3 - 6*b
*c*d^2*e - 2*b^2*d*e^2)*g - (6*(c^2*d*e^2 - b*c*e^3)*f + (54*c^2*d^2*e - 1
4*b*c*d*e^2 - 5*b^2*e^3)*g)*x))/(2*c*d^4*e^2 - b*d^3*e^3 + (2*c*d*e^5 - b*
e^6)*x^3 + 3*(2*c*d^2*e^4 - b*d*e^5)*x^2 + 3*(2*c*d^3*e^3 - b*d^2*e^4)*x)]
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx = \int \frac{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}\left(f+gx\right)}{\left(d+ex\right)^5}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**5,x
)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**5, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^5,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1592 vs. $2(194) = 388$.

Time = 0.44 (sec) , antiderivative size = 1592, normalized size of antiderivative = 7.58

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^5,x, algo
rithm="giac")
```

output

```
-2/15*(15*c^(3/2)*g*arctan(sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))/sqrt
(c))*sgn(1/(e*x + d))*sgn(e)/e^3 - (30*c^(5/2)*d*g*arctan(sqrt(-c)/sqrt(c)
) - 15*b*c^(3/2)*e*g*arctan(sqrt(-c)/sqrt(c)) + 3*sqrt(-c)*c^2*e*f - 43*sq
rt(-c)*c^2*d*g + 20*b*sqrt(-c)*c*e*g)*sgn(1/(e*x + d))*sgn(e)/(2*c*d*e^3 -
 b*e^4) + (48*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^2*c^4*sqrt(-c + 2*c*d/
(e*x + d) - b*e/(e*x + d))*d^4*e^13*f*sgn(1/(e*x + d))*sgn(e) - 96*b*(c -
2*c*d/(e*x + d) + b*e/(e*x + d))^2*c^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*
x + d))*d^3*e^14*f*sgn(1/(e*x + d))*sgn(e) + 72*b^2*(c - 2*c*d/(e*x + d) +
 b*e/(e*x + d))^2*c^2*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^2*e^15*
f*sgn(1/(e*x + d))*sgn(e) - 24*b^3*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^2
*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d*e^16*f*sgn(1/(e*x + d))*sg
n(e) + 3*b^4*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^2*sqrt(-c + 2*c*d/(e*x
+ d) - b*e/(e*x + d))*e^17*f*sgn(1/(e*x + d))*sgn(e) - 48*(c - 2*c*d/(e*x
+ d) + b*e/(e*x + d))^2*c^4*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^5
*e^12*g*sgn(1/(e*x + d))*sgn(e) - 480*c^6*sqrt(-c + 2*c*d/(e*x + d) - b*e/
(e*x + d))*d^5*e^12*g*sgn(1/(e*x + d))*sgn(e) + 160*c^5*(-c + 2*c*d/(e*x +
 d) - b*e/(e*x + d))^(3/2)*d^5*e^12*g*sgn(1/(e*x + d))*sgn(e) + 96*b*(c -
2*c*d/(e*x + d) + b*e/(e*x + d))^2*c^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*
x + d))*d^4*e^13*g*sgn(1/(e*x + d))*sgn(e) + 1200*b*c^5*sqrt(-c + 2*c*d/(e
*x + d) - b*e/(e*x + d))*d^4*e^13*g*sgn(1/(e*x + d))*sgn(e) - 400*b*c^4...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{{\left(d+e\,x\right)}^5}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^5,x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^5, x
)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 1667, normalized size of antiderivative = 7.94

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^5,x)
```

output

```
(2*i*(15*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*b**2*c*d**3*e**2*g + 45*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqr
t( - b*e + 2*c*d))*b**2*c*d**2*e**3*g*x + 45*sqrt(c)*asinh((sqrt( - b*e +
c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d*e**4*g*x**2 + 15*sqrt(c)*as
inh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*e**5*g*x**
3 - 60*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b*c**2*d**4*e*g - 180*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
 b*e + 2*c*d))*b*c**2*d**3*e**2*g*x - 180*sqrt(c)*asinh((sqrt( - b*e + c*d
 - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e**3*g*x**2 - 60*sqrt(c)*as
inh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**4*g*x
**3 + 60*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*c**3*d**5*g + 180*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b
*e + 2*c*d))*c**3*d**4*e*g*x + 180*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**3*e**2*g*x**2 + 60*sqrt(c)*asinh((sqrt
( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**2*e**3*g*x**3 + 2*
sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c
*e*x)*b**2*d*e**2*g + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*
d)*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*f + 5*sqrt(d + e*x)*sqrt(b*e - 2*c
*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*g*x + 6*sqrt
(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*...
```

3.155 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 138

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^6}\,dx = -\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7e^2(2cd-be)(d+ex)^6} + \frac{2(7beg-2c(ef+6dg))\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{35e^2(2cd-be)^2(d+ex)^5}$$

output

```
-2/7*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^6+2/35*(7*b*e*g-2*c*(6*d*g+e*f))*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5
/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^6}\,dx =$$
$$-\frac{2(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}(-be(5ef+2dg+7egx)+2c(d^2g+e^2fx+6de(f+gx}{35e^2(-2cd+be)^2(d+ex)^4}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^6,x]
```

output
```
(-2*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-(b*e
*(5*e*f + 2*d*g + 7*e*g*x)) + 2*c*(d^2*g + e^2*f*x + 6*d*e*(f + g*x))))/(3
5*e^2*(-2*c*d + b*e)^2*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^6}dx$$
$$\Big\downarrow \color{blue}{1220}$$
$$\frac{(-7beg+12cdg+2cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^5}dx}{7e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7e^2(d+ex)^6(2cd-be)}$$
$$\Big\downarrow \color{blue}{1123}$$

$$-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(-7beg+12cdg+2cef)}{35e^2(d+ex)^5(2cd-be)^2}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7e^2(d+ex)^6(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^6,x]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(7*e^2*(2*c*d
 - b*e)*(d + e*x)^6) - (2*(2*c*e*f + 12*c*d*g - 7*b*e*g)*(d*(c*d - b*e) -
b*e^2*x - c*e^2*x^2)^(5/2))/(35*e^2*(2*c*d - b*e)^2*(d + e*x)^5)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

method	result
gosper	$-\frac{2(cex+be-cd)\left(7b\,e^{2}gx-12cdegx-2c\,e^{2}fx+2bdeg+5b\,e^{2}f-2c\,d^{2}g-12cdef\right)\left(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}\right)^{\frac{3}{2}}}{35(ex+d)^{5}e^{2}\left(b^{2}e^{2}-4bcde+4c^{2}d^{2}\right)}$
orering	$-\frac{2(cex+be-cd)\left(7b\,e^{2}gx-12cdegx-2c\,e^{2}fx+2bdeg+5b\,e^{2}f-2c\,d^{2}g-12cdef\right)\left(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}\right)^{\frac{3}{2}}}{35(ex+d)^{5}e^{2}\left(b^{2}e^{2}-4bcde+4c^{2}d^{2}\right)}$
default	$-\frac{2g\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5e^{6}\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{5}}-\frac{(dg-ef)\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{7\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{6}}-\frac{4c\,e^{2}\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+(-}{35\left(-b\,e^{2}+2de\right.}\right.}{e^{7}}$
trager	$\frac{2\left(7b\,c^{2}e^{4}g\,x^{3}-12c^{3}d\,e^{3}g\,x^{3}-2c^{3}e^{4}f\,x^{3}+14b^{2}c\,e^{4}g\,x^{2}-36b\,c^{2}d\,e^{3}g\,x^{2}+b\,c^{2}e^{4}f\,x^{2}+22c^{3}d^{2}e^{2}g\,x^{2}-8c^{3}d\,e^{3}f\,x^{2}+7b^{3}e^{4}gx-22b^{2}cd\,e^{3}\right.}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^6,x,method=_RET
URNVERBOSE)
```

output
```
-2/35*(c*e*x+b*e-c*d)*(7*b*e^2*g*x-12*c*d*e*g*x-2*c*e^2*f*x+2*b*d*e*g+5*b*
e^2*f-2*c*d^2*g-12*c*d*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)
^5/e^2/(b^2*e^2-4*b*c*d*e+4*c^2*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(130) = 260$.

Time = 25.68 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.36

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^6}\,dx =$$
$$-\frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left(\left(2\,c^3e^4f+\left(12\,c^3de^3-7\,bc^2e^4\right)g\right)x^3+\left(\left(8\,c^3de^3-bc^2e^4\right)f-2\left(11\,c^3d^2e^2-\right.\right.\right.}{35\left(4\,c^2d^6e^2-4\,bcd^5e^3+b^2d^4e^4+\left(\right.\right.}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^6,x, algo
rithm="fricas")
```

output

```
-2/35*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((2*c^3*e^4*f + (12*c^3*d
*e^3 - 7*b*c^2*e^4)*g)*x^3 + ((8*c^3*d*e^3 - b*c^2*e^4)*f - 2*(11*c^3*d^2*
e^2 - 18*b*c^2*d*e^3 + 7*b^2*c*e^4)*g)*x^2 + (12*c^3*d^3*e - 29*b*c^2*d^2*
e^2 + 22*b^2*c*d*e^3 - 5*b^3*e^4)*f + 2*(c^3*d^4 - 3*b*c^2*d^3*e + 3*b^2*c
*d^2*e^2 - b^3*d*e^3)*g - (2*(11*c^3*d^2*e^2 - 15*b*c^2*d*e^3 + 4*b^2*c*e^
4)*f - (8*c^3*d^3*e - 23*b*c^2*d^2*e^2 + 22*b^2*c*d*e^3 - 7*b^3*e^4)*g)*x)
/(4*c^2*d^6*e^2 - 4*b*c*d^5*e^3 + b^2*d^4*e^4 + (4*c^2*d^2*e^6 - 4*b*c*d*e
^7 + b^2*e^8)*x^4 + 4*(4*c^2*d^3*e^5 - 4*b*c*d^2*e^6 + b^2*d*e^7)*x^3 + 6*
(4*c^2*d^4*e^4 - 4*b*c*d^3*e^5 + b^2*d^2*e^6)*x^2 + 4*(4*c^2*d^5*e^3 - 4*b
*c*d^4*e^4 + b^2*d^3*e^5)*x)
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^6}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^6}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**6,x
)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**6, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^6}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^6,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^6,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,4,0,0]%%%},[8,1]%%%}+%%%{%%{[%%%{-8,[0,3,1,0]%%%}
,0]:[1,0,
```

Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 3763, normalized size of antiderivative = 27.27

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^6,x)
```

output
```
(((d*((d*((16*c^4*(6*b*e*g - 10*c*d*g + c*e*f))/(105*(b*e - 2*c*d)^4) - (1
6*c^5*d*g)/(105*(b*e - 2*c*d)^4)))/e - (608*c^5*d^2*g + 196*b^2*c^3*e^2*g
- 160*c^5*d*e*f + 96*b*c^4*e^2*f - 688*b*c^4*d*e*g)/(105*e*(b*e - 2*c*d)^4
)))/e + (4*b*c^2*(19*b^2*e^2*g + 76*c^2*d^2*g + 11*b*c*e^2*f - 20*c^2*d*e*
f - 76*b*c*d*e*g))/(105*e*(b*e - 2*c*d)^4))*(c*d^2 - c*e^2*x^2 - b*d*e - b
*e^2*x)^(1/2))/(d + e*x) - (((d*((d*((8*c^4*(7*b*e*g - 10*c*d*g + 2*c*e*f)
)/(105*(b*e - 2*c*d)^4) - (16*c^5*d*g)/(105*(b*e - 2*c*d)^4)))/e - (208*c^
5*d^2*g + 76*b^2*c^3*e^2*g - 80*c^5*d*e*f + 56*b*c^4*e^2*f - 248*b*c^4*d*e
*g)/(105*e*(b*e - 2*c*d)^4)))/e + (2*b*c^2*(13*b^2*e^2*g + 52*c^2*d^2*g +
12*b*c*e^2*f - 20*c^2*d*e*f - 52*b*c*d*e*g))/(105*e*(b*e - 2*c*d)^4))*(c*d
^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + (((d*((d*((16*c^4*(7*
b*e*g - 12*c*d*g + c*e*f))/(105*(b*e - 2*c*d)^4) - (16*c^5*d*g)/(105*(b*e
- 2*c*d)^4)))/e - (768*c^5*d^2*g + 244*b^2*c^3*e^2*g - 192*c^5*d*e*f + 112
*b*c^4*e^2*f - 864*b*c^4*d*e*g)/(105*e*(b*e - 2*c*d)^4)))/e + (4*b*c^2*(24
*b^2*e^2*g + 96*c^2*d^2*g + 13*b*c*e^2*f - 24*c^2*d*e*f - 96*b*c*d*e*g))/(
105*e*(b*e - 2*c*d)^4))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d +
e*x) - (((d*((d*((8*c^4*(19*b*e*g - 34*c*d*g + 2*c*e*f))/(105*(b*e - 2*c*d
)^4) - (16*c^5*d*g)/(105*(b*e - 2*c*d)^4)))/e - (1728*c^5*d^2*g + 504*b^2*
c^3*e^2*g - 272*c^5*d*e*f + 152*b*c^4*e^2*f - 1864*b*c^4*d*e*g)/(105*e*(b*
e - 2*c*d)^4)))/e + (8*b*c^2*(27*b^2*e^2*g + 108*c^2*d^2*g + 9*b*c*e^2*...
```

Reduce [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 1837, normalized size of antiderivative = 13.31

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^6}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^6,x)
```

output

```
(2*i*(2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*b**3*d*e**3*g + 5*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*
e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*f + 7*sqrt(d + e*x)*sqrt(b
*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*g*x
- 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
 - c*e*x)*b**2*c*d**2*e**2*g - 22*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**3*f - 22*sqrt(d + e*x)
*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*
d*e**3*g*x + 8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt(
- b*e + c*d - c*e*x)*b**2*c*e**4*f*x + 14*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*
sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**4*g*x**2 + 6*sqr
t(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*
x)*b*c**2*d**3*e*g + 29*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*
d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**2*e**2*f + 23*sqrt(d + e*x)*sqrt(b
*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**2*e*
*2*g*x - 30*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b
*e + c*d - c*e*x)*b*c**2*d*e**3*f*x - 36*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*s
qrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d*e**3*g*x**2 + sqrt
(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x
)*b*c**2*e**4*f*x**2 + 7*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + ...
```

3.156 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 210

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx = &-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e^2(2cd-be)(d+ex)^7}\\ &-\frac{2(4cef+14cdg-9beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{63e^2(2cd-be)^2(d+ex)^6}\\ &-\frac{4c(4cef+14cdg-9beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{315e^2(2cd-be)^3(d+ex)^5}\end{aligned}$$

output
```
-2/9*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^7-2/63*(-9*b*e*g+14*c*d*g+4*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(5/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^6-4/315*c*(-9*b*e*g+14*c*d*g+4*c*e*f)*(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx = \frac{2(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}(5b^2e^2(7e$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^7,x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(5*b^2*
e^2*(7*e*f + 2*d*g + 9*e*g*x) + 4*c^2*(7*d^3*g + 2*e^3*f*x^2 + 7*d*e^2*x*(
2*f + g*x) + d^2*e*(47*f + 49*g*x)) - 2*b*c*e*(19*d^2*g + e^2*x*(10*f + 9*
g*x) + d*e*(80*f + 98*g*x))))/(315*e^2*(-2*c*d + b*e)^3*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^7}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-9beg+14cdg+4cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^6}dx}{9e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e^2(d+ex)^7(2cd-be)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-9beg+14cdg+4cef)\left(\frac{2c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^5}dx}{7(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7e(d+ex)^6(2cd-be)}\right)}{9e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e^2(d+ex)^7(2cd-be)}$$

$$\Big\downarrow 1123$$

$$\frac{\left(-\frac{4c\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{35e(d+ex)^5(2cd-be)^2}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7e(d+ex)^6(2cd-be)}\right)(-9beg+14cdg+4cef)}{9e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e^2(d+ex)^7(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^7,x]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(9*e^2*(2*c*d
 - b*e)*(d + e*x)^7) + ((4*c*e*f + 14*c*d*g - 9*b*e*g)*((-2*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(5/2))/(7*e*(2*c*d - b*e)*(d + e*x)^6) - (4*c*(d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(35*e*(2*c*d - b*e)^2*(d + e*x)^5
)))/(9*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 8.71 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.12

method	result
gosper	$-\frac{2(cex+be-cd)(-18bc e^3 g x^2+28c^2 d e^2 g x^2+8f c^2 e^3 x^2+45b^2 e^3 gx-196bcd e^2 gx-20bc e^3 fx+196c^2 d^2 egx+56c^2 d e^2 fx+10b^2 d e^2}{315(ex+d)^6(b^3 e^3-6d e^2 b^2 c+12d^2 eb c^2-8d^3}$
orering	$-\frac{2(cex+be-cd)(-18bc e^3 g x^2+28c^2 d e^2 g x^2+8f c^2 e^3 x^2+45b^2 e^3 gx-196bcd e^2 gx-20bc e^3 fx+196c^2 d^2 egx+56c^2 d e^2 fx+10b^2 d e^2}{315(ex+d)^6(b^3 e^3-6d e^2 b^2 c+12d^2 eb c^2-8d^3}$
default	$\frac{g\left(-\frac{2\left(-c e^2\left(x+\frac{d}{e}\right)^2+\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{7\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)^6}-\frac{4c e^2\left(-c e^2\left(x+\frac{d}{e}\right)^2+\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{35\left(-b e^2+2dec\right)^2\left(x+\frac{d}{e}\right)^5}\right)}{e^7}-\frac{(dg-ef)\left(-\frac{2\left(-c e^2\left(x+\frac{d}{e}\right)^2+\left(-\right.\right.}{9\left(-b e^2+2d\right.}\right.}{}$
trager	$\frac{2(-18b c^3 e^5 g x^4+28c^4 d e^4 g x^4+8c^4 e^5 f x^4+9b^2 c^2 e^5 g x^3-104b c^3 d e^4 g x^3-4b c^3 e^5 f x^3+140c^4 d^2 e^3 g x^3+40c^4 d e^4 f x^3+72b^3 c e^5 g x}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^7,x,method=_RET
URNVERBOSE)
```

output
```
-2/315*(c*e*x+b*e-c*d)*(-18*b*c*e^3*g*x^2+28*c^2*d*e^2*g*x^2+8*c^2*e^3*f*x
^2+45*b^2*e^3*g*x-196*b*c*d*e^2*g*x-20*b*c*e^3*f*x+196*c^2*d^2*e*g*x+56*c^
2*d*e^2*f*x+10*b^2*d*e^2*g+35*b^2*e^3*f-38*b*c*d^2*e*g-160*b*c*d*e^2*f+28*
c^2*d^3*g+188*c^2*d^2*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^
6/(b^3*e^3-6*b^2*c*d*e^2+12*b*c^2*d^2*e-8*c^3*d^3)/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(198) = 396$.

Time = 102.83 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.52

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx =$$

$$-\frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,(4\,c^4e^5f+(14\,c^4de^4-9\,bc^3e^5)g)x^4+(4\,(10\,c^4de^4-bc^3e^5)f+(140\,c^4d^2}{315\,(8\,c^3}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^7,x, algo
rithm="fricas")
```

output
```
-2/315*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*c^4*e^5*f + (14*c^
4*d*e^4 - 9*b*c^3*e^5)*g)*x^4 + (4*(10*c^4*d*e^4 - b*c^3*e^5)*f + (140*c^4
*d^2*e^3 - 104*b*c^3*d*e^4 + 9*b^2*c^2*e^5)*g)*x^3 + 3*((28*c^4*d^2*e^3 -
8*b*c^3*d*e^4 + b^2*c^2*e^5)*f - 8*(14*c^4*d^3*e^2 - 28*b*c^3*d^2*e^3 + 17
*b^2*c^2*d*e^4 - 3*b^3*c*e^5)*g)*x^2 + (188*c^4*d^4*e - 536*b*c^3*d^3*e^2
+ 543*b^2*c^2*d^2*e^3 - 230*b^3*c*d*e^4 + 35*b^4*e^5)*f + 2*(14*c^4*d^5 -
47*b*c^3*d^4*e + 57*b^2*c^2*d^3*e^2 - 29*b^3*c*d^2*e^3 + 5*b^4*d*e^4)*g -
(2*(160*c^4*d^3*e^2 - 282*b*c^3*d^2*e^3 + 147*b^2*c^2*d*e^4 - 25*b^3*c*e^5
)*f - (140*c^4*d^4*e - 456*b*c^3*d^3*e^2 + 537*b^2*c^2*d^2*e^3 - 266*b^3*c
*d*e^4 + 45*b^4*e^5)*g)*x)/(8*c^3*d^8*e^2 - 12*b*c^2*d^7*e^3 + 6*b^2*c*d^6
*e^4 - b^3*d^5*e^5 + (8*c^3*d^3*e^7 - 12*b*c^2*d^2*e^8 + 6*b^2*c*d*e^9 - b
^3*e^10)*x^5 + 5*(8*c^3*d^4*e^6 - 12*b*c^2*d^3*e^7 + 6*b^2*c*d^2*e^8 - b^3
*d*e^9)*x^4 + 10*(8*c^3*d^5*e^5 - 12*b*c^2*d^4*e^6 + 6*b^2*c*d^3*e^7 - b^3
*d^2*e^8)*x^3 + 10*(8*c^3*d^6*e^4 - 12*b*c^2*d^5*e^5 + 6*b^2*c*d^4*e^6 - b
^3*d^3*e^7)*x^2 + 5*(8*c^3*d^7*e^3 - 12*b*c^2*d^6*e^4 + 6*b^2*c*d^5*e^5 -
b^3*d^4*e^6)*x)
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}(f+gx)}{\left(d+ex\right)^7}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**7,x
)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**7, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^7,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^7,x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 17.59 (sec) , antiderivative size = 8039, normalized size of antiderivative = 38.28

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx=\text{Too large to display}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^7,x)
```

output
```
(((d*((d*((32*c^5*(4*b*e*g - 6*c*d*g + c*e*f))/(945*(b*e - 2*c*d)^5) - (32
*c^6*d*g)/(945*(b*e - 2*c*d)^5)))/e - (608*c^6*d^2*g + 208*b^2*c^4*e^2*g -
 192*c^6*d*e*f + 128*b*c^5*e^2*f - 704*b*c^5*d*e*g)/(945*e*(b*e - 2*c*d)^5
)))/e + (4*b*c^3*(19*b^2*e^2*g + 76*c^2*d^2*g + 14*b*c*e^2*f - 24*c^2*d*e*
f - 76*b*c*d*e*g))/(945*e*(b*e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2 - b*d*e - b
*e^2*x)^(1/2))/(d + e*x) - (((d*((d*((16*c^5*(13*b*e*g - 22*c*d*g + 2*c*e*
f))/(945*(b*e - 2*c*d)^5) - (32*c^6*d*g)/(945*(b*e - 2*c*d)^5)))/e - (1568
*c^6*d^2*g + 488*b^2*c^4*e^2*g - 352*c^6*d*e*f + 208*b*c^5*e^2*f - 1744*b*
c^5*d*e*g)/(945*e*(b*e - 2*c*d)^5)))/e + (4*b*c^3*(49*b^2*e^2*g + 196*c^2*
d^2*g + 24*b*c*e^2*f - 44*c^2*d*e*f - 196*b*c*d*e*g))/(945*e*(b*e - 2*c*d)
^5))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((d*((1
6*c^5*(15*b*e*g - 26*c*d*g + 2*c*e*f))/(945*(b*e - 2*c*d)^5) - (32*c^6*d*g
)/(945*(b*e - 2*c*d)^5)))/e - (1952*c^6*d^2*g + 600*b^2*c^4*e^2*g - 416*c^
6*d*e*f + 240*b*c^5*e^2*f - 2160*b*c^5*d*e*g)/(945*e*(b*e - 2*c*d)^5)))/e
+ (4*b*c^3*(61*b^2*e^2*g + 244*c^2*d^2*g + 28*b*c*e^2*f - 52*c^2*d*e*f - 2
44*b*c*d*e*g))/(945*e*(b*e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2
*x)^(1/2))/(d + e*x) - (((d*((d*((16*c^5*(17*b*e*g - 30*c*d*g + 2*c*e*f))/
(945*(b*e - 2*c*d)^5) - (32*c^6*d*g)/(945*(b*e - 2*c*d)^5)))/e - (2336*c^6
*d^2*g + 712*b^2*c^4*e^2*g - 480*c^6*d*e*f + 272*b*c^5*e^2*f - 2576*b*c^5*
d*e*g)/(945*e*(b*e - 2*c*d)^5)))/e + (4*b*c^3*(73*b^2*e^2*g + 292*c^2*d...
```

Reduce [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 2680, normalized size of antiderivative = 12.76

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^7}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^7,x)
```

output

```
(2*i*(10*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
+ c*d - c*e*x)*b**4*d*e**4*g + 35*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**5*f + 45*sqrt(d + e*x)*sqr
t(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**5*g
*x - 58*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*b**3*c*d**2*e**3*g - 230*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqr
t( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*d*e**4*f - 266*sqrt(d
+ e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b
**3*c*d*e**4*g*x + 50*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)
*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**5*f*x + 72*sqrt(d + e*x)*sqrt(b*e -
2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**5*g*x**2
+ 114*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c
*d - c*e*x)*b**2*c**2*d**3*e**2*g + 543*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**2*e**3*f + 537*
sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c
*e*x)*b**2*c**2*d**2*e**3*g*x - 294*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
- b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d*e**4*f*x - 408*sqrt(
d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)
*b**2*c**2*d*e**4*g*x**2 + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**5*f*x**2 + 9*sqrt(d + e...
```

3.157 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^8}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 285

$$\begin{aligned}
&\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^8}\,dx = \\
&-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11e^2(2cd-be)(d+ex)^8} \\
&-\frac{2(6cef+16cdg-11beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{99e^2(2cd-be)^2(d+ex)^7} \\
&-\frac{8c(6cef+16cdg-11beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{693e^2(2cd-be)^3(d+ex)^6} \\
&-\frac{16c^2(6cef+16cdg-11beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3465e^2(2cd-be)^4(d+ex)^5}
\end{aligned}$$

output

```
-2/11*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)/(
e*x+d)^8-2/99*(-11*b*e*g+16*c*d*g+6*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(5/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^7-8/693*c*(-11*b*e*g+16*c*d*g+6*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^6-16/346
5*c^2*(-11*b*e*g+16*c*d*g+6*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/
e^2/(-b*e+2*c*d)^4/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.87

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^8}\,dx =$$
$$-\frac{2(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}(-35b^3e^3(9ef+2dg+11egx)+8c^3(61d^4g+6e^4fx^3+}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^8,x]
```

output
```
(-2*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-35*b
^3*e^3*(9*e*f + 2*d*g + 11*e*g*x) + 8*c^3*(61*d^4*g + 6*e^4*f*x^3 + 16*d*e
^3*x^2*(3*f + g*x) + 8*d^3*e*(57*f + 61*g*x) + d^2*e^2*x*(183*f + 128*g*x)
) + 10*b^2*c*e^2*(43*d^2*g + e^2*x*(21*f + 22*g*x) + d*e*(210*f + 254*g*x)
) - 4*b*c^2*e*(212*d^3*g + 2*e^3*x^2*(15*f + 11*g*x) + 2*d*e^2*x*(135*f +
128*g*x) + d^2*e*(1185*f + 1391*g*x))))/(3465*e^2*(-2*c*d + b*e)^4*(d + e*
x)^6)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^8}dx$$
$$\Big\downarrow \ 1220$$
$$\frac{(-11beg+16cdg+6cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^7}dx}{11e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11e^2(d+ex)^8(2cd-be)}$$

$$\Big\downarrow \color{blue}{1129}$$

$$\frac{(-11beg+16cdg+6cef)\left(\frac{4c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^6}dx}{9(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e(d+ex)^7(2cd-be)}\right)}{11e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11e^2(d+ex)^8(2cd-be)}$$

$$\Big\downarrow \color{blue}{1129}$$

$$\frac{(-11beg+16cdg+6cef)\left(\frac{4c\left(\frac{2c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^5}dx}{7(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7e(d+ex)^6(2cd-be)}\right)}{9(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e(d+ex)^7(2cd-be)}\right)}{11e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11e^2(d+ex)^8(2cd-be)}$$

$$\Big\downarrow \color{blue}{1123}$$

$$\frac{\left(\frac{4c\left(-\frac{4c\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{35e(d+ex)^5(2cd-be)^2}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7e(d+ex)^6(2cd-be)}\right)}{9(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e(d+ex)^7(2cd-be)}\right)(-11beg+16cdg+6cef)}{11e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11e^2(d+ex)^8(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^8,x]
```

output

```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(11*e^2*(2*c*
d - b*e)*(d + e*x)^8) + ((6*c*e*f + 16*c*d*g - 11*b*e*g)*((-2*(d*(c*d - b*
e) - b*e^2*x - c*e^2*x^2)^(5/2))/(9*e*(2*c*d - b*e)*(d + e*x)^7) + (4*c*((
-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(7*e*(2*c*d - b*e)*(d + e*
x)^6) - (4*c*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(35*e*(2*c*d - b
*e)^2*(d + e*x)^5)))/(9*(2*c*d - b*e))))/(11*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 11.04 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.34

method	result
gosper	$-\frac{2(cex+be-cd)\left(88b\,c^{2}e^{4}g\,x^{3}-128c^{3}d\,e^{3}g\,x^{3}-48c^{3}e^{4}f\,x^{3}-220b^{2}c\,e^{4}g\,x^{2}+1024b\,c^{2}d\,e^{3}g\,x^{2}+120b\,c^{2}e^{4}f\,x^{2}-1024c^{3}d^{2}e^{2}g\,x^{2}-384}{}$
orering	$-\frac{2(cex+be-cd)\left(88b\,c^{2}e^{4}g\,x^{3}-128c^{3}d\,e^{3}g\,x^{3}-48c^{3}e^{4}f\,x^{3}-220b^{2}c\,e^{4}g\,x^{2}+1024b\,c^{2}d\,e^{3}g\,x^{2}+120b\,c^{2}e^{4}f\,x^{2}-1024c^{3}d^{2}e^{2}g\,x^{2}-384}{}$
default	$\frac{g\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{9\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{7}}+\frac{4c\,e^{2}\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{7\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{6}}-\frac{4c\,e^{2}\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right.}{35\left(-b\,e^{2}+2dec\right)^{2}\left(x+\frac{d}{e}\right)}\right.}{9\left(-b\,e^{2}+2dec\right)}\right.}{e^{8}}$
trager	$\frac{2\left(88b\,c^{4}e^{6}g\,x^{5}-128c^{5}d\,e^{5}g\,x^{5}-48c^{5}e^{6}f\,x^{5}-44b^{2}c^{3}e^{6}g\,x^{4}+592b\,c^{4}d\,e^{5}g\,x^{4}+24b\,c^{4}e^{6}f\,x^{4}-768c^{5}d^{2}e^{4}g\,x^{4}-288c^{5}d\,e^{5}f\,x^{4}+33b^{3}c^{2}\right.}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^8,x,method=_RET
URNVERBOSE)
```

output
```
-2/3465*(c*e*x+b*e-c*d)*(88*b*c^2*e^4*g*x^3-128*c^3*d*e^3*g*x^3-48*c^3*e^4
*f*x^3-220*b^2*c*e^4*g*x^2+1024*b*c^2*d*e^3*g*x^2+120*b*c^2*e^4*f*x^2-1024
*c^3*d^2*e^2*g*x^2-384*c^3*d*e^3*f*x^2+385*b^3*e^4*g*x-2540*b^2*c*d*e^3*g*
x-210*b^2*c*e^4*f*x+5564*b*c^2*d^2*e^2*g*x+1080*b*c^2*d*e^3*f*x-3904*c^3*d
^3*e*g*x-1464*c^3*d^2*e^2*f*x+70*b^3*d*e^3*g+315*b^3*e^4*f-430*b^2*c*d^2*e
^2*g-2100*b^2*c*d*e^3*f+848*b*c^2*d^3*e*g+4740*b*c^2*d^2*e^2*f-488*c^3*d^4
*g-3648*c^3*d^3*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^7/e^2/
(b^4*e^4-8*b^3*c*d*e^3+24*b^2*c^2*d^2*e^2-32*b*c^3*d^3*e+16*c^4*d^4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^8}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^8,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^8}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^8}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**8,x
)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**8, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^8}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^8,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^8}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^8,x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 30.26 (sec) , antiderivative size = 16485, normalized size of antiderivative = 57.84

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^8}\,dx = \text{Too large to display}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^8,x)
```

output

```
(((d*((d*((64*c^6*(7*b*e*g - 12*c*d*g + c*e*f))/(10395*(b*e - 2*c*d)^6) -
(64*c^7*d*g)/(10395*(b*e - 2*c*d)^6)))/e - (3904*c^7*d^2*g + 1184*b^2*c^5*
e^2*g - 768*c^7*d*e*f + 448*b*c^6*e^2*f - 4288*b*c^6*d*e*g)/(10395*e*(b*e
- 2*c*d)^6)))/e + (8*b*c^4*(61*b^2*e^2*g + 244*c^2*d^2*g + 26*b*c*e^2*f -
48*c^2*d*e*f - 244*b*c*d*e*g))/(10395*e*(b*e - 2*c*d)^6))*(c*d^2 - c*e^2*x
^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((d*((32*c^6*(9*b*e*g - 14*c
*d*g + 2*c*e*f))/(10395*(b*e - 2*c*d)^6) - (64*c^7*d*g)/(10395*(b*e - 2*c*
d)^6)))/e - (1664*c^7*d^2*g + 544*b^2*c^5*e^2*g - 448*c^7*d*e*f + 288*b*c^
6*e^2*f - 1888*b*c^6*d*e*g)/(10395*e*(b*e - 2*c*d)^6)))/e + (16*b*c^4*(13*
b^2*e^2*g + 52*c^2*d^2*g + 8*b*c*e^2*f - 14*c^2*d*e*f - 52*b*c*d*e*g))/(10
395*e*(b*e - 2*c*d)^6))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d +
e*x) + (((d*((d*((64*c^6*(8*b*e*g - 14*c*d*g + c*e*f))/(10395*(b*e - 2*c*d
)^6) - (64*c^7*d*g)/(10395*(b*e - 2*c*d)^6)))/e - (4800*c^7*d^2*g + 1440*b
^2*c^5*e^2*g - 896*c^7*d*e*f + 512*b*c^6*e^2*f - 5248*b*c^6*d*e*g)/(10395*
e*(b*e - 2*c*d)^6)))/e + (8*b*c^4*(75*b^2*e^2*g + 300*c^2*d^2*g + 30*b*c*e
^2*f - 56*c^2*d*e*f - 300*b*c*d*e*g))/(10395*e*(b*e - 2*c*d)^6))*(c*d^2 -
c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + (((d*((d*((64*c^6*(9*b*e*g
 - 16*c*d*g + c*e*f))/(10395*(b*e - 2*c*d)^6) - (64*c^7*d*g)/(10395*(b*e -
 2*c*d)^6)))/e - (5696*c^7*d^2*g + 1696*b^2*c^5*e^2*g - 1024*c^7*d*e*f + 5
76*b*c^6*e^2*f - 6208*b*c^6*d*e*g)/(10395*e*(b*e - 2*c*d)^6)))/e + (8*b...
```

Reduce [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 3670, normalized size of antiderivative = 12.88

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^8}\,dx=\text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^8,x)
```

output
```
(2*i*(70*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
+ c*d - c*e*x)*b**5*d*e**5*g + 315*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
 b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*e**6*f + 385*sqrt(d + e*x)*s
qrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*e**6
*g*x - 570*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*b**4*c*d**2*e**4*g - 2730*sqrt(d + e*x)*sqrt(b*e - 2*c*d)
*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*d*e**5*f - 3170*sq
rt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e
*x)*b**4*c*d*e**5*g*x + 420*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*e**6*f*x + 550*sqrt(d + e*x)*sqrt
(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*e**6*
g*x**2 + 1778*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( -
 b*e + c*d - c*e*x)*b**3*c**2*d**3*e**3*g + 9255*sqrt(d + e*x)*sqrt(b*e -
2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d**2*e**4
*f + 10029*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*b**3*c**2*d**2*e**4*g*x - 3330*sqrt(d + e*x)*sqrt(b*e - 2
*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d*e**5*f*x
 - 4316*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*b**3*c**2*d*e**5*g*x**2 + 15*sqrt(d + e*x)*sqrt(b*e - 2*c*d)
*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*e**6*f*x**2 ...
```

3.158 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 360

$$
\begin{aligned}
&\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx = \\
&-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13e^2(2cd-be)(d+ex)^9} \\
&-\frac{2(8cef+18cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{143e^2(2cd-be)^2(d+ex)^8} \\
&-\frac{4c(8cef+18cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{429e^2(2cd-be)^3(d+ex)^7} \\
&-\frac{16c^2(8cef+18cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3003e^2(2cd-be)^4(d+ex)^6} \\
&-\frac{32c^3(8cef+18cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{15015e^2(2cd-be)^5(d+ex)^5}
\end{aligned}
$$

output
```
-2/13*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)/(
e*x+d)^9-2/143*(-13*b*e*g+18*c*d*g+8*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^
2)^(5/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^8-4/429*c*(-13*b*e*g+18*c*d*g+8*c*e*f)
*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^7-16/30
03*c^2*(-13*b*e*g+18*c*d*g+8*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)
/e^2/(-b*e+2*c*d)^4/(e*x+d)^6-32/15015*c^3*(-13*b*e*g+18*c*d*g+8*c*e*f)*(d
*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)^5/(e*x+d)^5
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx = \frac{2(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}(105b^4e^4(}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^9,x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(105*b^
4*e^4*(11*e*f + 2*d*g + 13*e*g*x) - 70*b^3*c*e^3*(25*d^2*g + e^2*x*(12*f +
 13*g*x) + 2*d*e*(72*f + 85*g*x)) + 20*b^2*c^2*e^2*(271*d^3*g + 2*e^3*x^2*
(14*f + 13*g*x) + d*e^2*x*(308*f + 323*g*x) + 2*d^2*e*(833*f + 977*g*x)) +
 16*c^4*(213*d^5*g + 8*e^5*f*x^4 + 18*d*e^4*x^3*(4*f + g*x) + 2*d^2*e^3*x^
2*(154*f + 81*g*x) + 3*d^3*e^2*x*(284*f + 231*g*x) + d^4*e*(1763*f + 1917*
g*x)) - 8*b*c^3*e*(911*d^4*g + 2*e^4*x^3*(20*f + 13*g*x) + 4*d*e^3*x^2*(10
0*f + 81*g*x) + d^2*e^2*x*(1940*f + 1901*g*x) + d^3*e*(6200*f + 7134*g*x))
))/(15015*e^2*(-2*c*d + b*e)^5*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx$$

$$\Big\downarrow \textcolor{blue}{1220}$$

$$\frac{(-13beg+18cdg+8cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^8}dx}{13e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13e^2(d+ex)^9(2cd-be)}$$

$$\Big\downarrow \textcolor{blue}{1129}$$

$$\frac{(-13beg+18cdg+8cef)\left(\frac{6c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^7}dx}{11(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11e(d+ex)^8(2cd-be)}\right)}{13e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13e^2(d+ex)^9(2cd-be)}$$

$$\Big\downarrow \textcolor{blue}{1129}$$

$$\frac{(-13beg+18cdg+8cef)\left(\frac{6c\left(\frac{4c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^6}dx}{9(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e(d+ex)^7(2cd-be)}\right)}{11(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11e(d+ex)^8(2cd-be)}\right)}{13e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13e^2(d+ex)^9(2cd-be)}$$

$$\Big\downarrow \textcolor{blue}{1129}$$

$$\frac{(-13beg+18cdg+8cef)\left(\frac{6c\left(\frac{4c\left(\frac{2c\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^5}dx}{7(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7e(d+ex)^6(2cd-be)}\right)}{9(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e(d+ex)^7(2cd-be)}\right)}{11(2cd-be)}\right.}{13e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13e^2(d+ex)^9(2cd-be)}$$

$$\Big\downarrow \text{1123}$$

$$\frac{\left(\frac{6c\left(\frac{4c\left(-\frac{4c\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{35e(d+ex)^5(2cd-be)^2}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7e(d+ex)^6(2cd-be)}\right)}{9(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9e(d+ex)^7(2cd-be)}\right)}{11(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11e(d+ex)^8(2cd-be)}\right)(}{13e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13e^2(d+ex)^9(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^9,x]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(13*e^2*(2*c*
d - b*e)*(d + e*x)^9) + ((8*c*e*f + 18*c*d*g - 13*b*e*g)*((-2*(d*(c*d - b*
e) - b*e^2*x - c*e^2*x^2)^(5/2))/(11*e*(2*c*d - b*e)*(d + e*x)^8) + (6*c*(
(-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(9*e*(2*c*d - b*e)*(d + e
*x)^7) + (4*c*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(7*e*(2*c*
d - b*e)*(d + e*x)^6) - (4*c*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/
(35*e*(2*c*d - b*e)^2*(d + e*x)^5)))/(9*(2*c*d - b*e))))/(11*(2*c*d - b*e)
)))/(13*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 18.31 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.57

method	result
gosper	$-\frac{2(cex+be-cd)(-208b c^3 e^5 g x^4+288c^4 d e^4 g x^4+128c^4 e^5 f x^4+520b^2 c^2 e^5 g x^3-2592b c^3 d e^4 g x^3-320b c^3 e^5 f x^3+2592c^4 d^2 e^3 g x^3}{}$
orering	$-\frac{2(cex+be-cd)(-208b c^3 e^5 g x^4+288c^4 d e^4 g x^4+128c^4 e^5 f x^4+520b^2 c^2 e^5 g x^3-2592b c^3 d e^4 g x^3-320b c^3 e^5 f x^3+2592c^4 d^2 e^3 g x^3}{}$
default	$\frac{g\left(-\frac{2\left(-c e^2\left(x+\frac{d}{e}\right)^2+\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{11\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)^8}+\frac{6c e^2\left(-\frac{2\left(-c e^2\left(x+\frac{d}{e}\right)^2+\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{9\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)^7}+\frac{4c e^2\left(-\frac{2\left(-c e^2\left(x+\frac{d}{e}\right)^2+\left(-b e^2+\right.\right.}{7\left(-b e^2+2dec\right)\left(\right.}\right.}{}\right.}{11\left(-b e^2+2dec\right)}\right.}{e^9}$
trager	$\frac{2(-208b c^5 e^7 g x^6+288c^6 d e^6 g x^6+128c^6 e^7 f x^6+104b^2 c^4 e^7 g x^5-1600b c^5 d e^6 g x^5-64b c^5 e^7 f x^5+2016c^6 d^2 e^5 g x^5+896c^6 d e^6 f x^5-}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^9,x,method=_RET
URNVERBOSE)
```

output
```
-2/15015*(c*e*x+b*e-c*d)*(-208*b*c^3*e^5*g*x^4+288*c^4*d*e^4*g*x^4+128*c^4
*e^5*f*x^4+520*b^2*c^2*e^5*g*x^3-2592*b*c^3*d*e^4*g*x^3-320*b*c^3*e^5*f*x^
3+2592*c^4*d^2*e^3*g*x^3+1152*c^4*d*e^4*f*x^3-910*b^3*c*e^5*g*x^2+6460*b^2
*c^2*d*e^4*g*x^2+560*b^2*c^2*e^5*f*x^2-15208*b*c^3*d^2*e^3*g*x^2-3200*b*c^
3*d*e^4*f*x^2+11088*c^4*d^3*e^2*g*x^2+4928*c^4*d^2*e^3*f*x^2+1365*b^4*e^5*
g*x-11900*b^3*c*d*e^4*g*x-840*b^3*c*e^5*f*x+39080*b^2*c^2*d^2*e^3*g*x+6160
*b^2*c^2*d*e^4*f*x-57072*b*c^3*d^3*e^2*g*x-15520*b*c^3*d^2*e^3*f*x+30672*c
^4*d^4*e*g*x+13632*c^4*d^3*e^2*f*x+210*b^4*d*e^4*g+1155*b^4*e^5*f-1750*b^3
*c*d^2*e^3*g-10080*b^3*c*d*e^4*f+5420*b^2*c^2*d^3*e^2*g+33320*b^2*c^2*d^2*
e^3*f-7288*b*c^3*d^4*e*g-49600*b*c^3*d^3*e^2*f+3408*c^4*d^5*g+28208*c^4*d^
4*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^8/e^2/(b^5*e^5-10*b^
4*c*d*e^4+40*b^3*c^2*d^2*e^3-80*b^2*c^3*d^3*e^2+80*b*c^4*d^4*e-32*c^5*d^5)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^9,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**9,x
)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^9,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^9,x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 53.01 (sec) , antiderivative size = 33375, normalized size of antiderivative = 92.71

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx = \text{Too large to display}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^9,x)
```

output
```
(((d*((d*((128*c^7*(5*b*e*g - 8*c*d*g + c*e*f))/(135135*(b*e - 2*c*d)^7) -
 (128*c^8*d*g)/(135135*(b*e - 2*c*d)^7)))/e - (4352*c^8*d^2*g + 1376*b^2*c
^6*e^2*g - 1024*c^8*d*e*f + 640*b*c^7*e^2*f - 4864*b*c^7*d*e*g)/(135135*e*
(b*e - 2*c*d)^7)))/e + (32*b*c^5*(17*b^2*e^2*g + 68*c^2*d^2*g + 9*b*c*e^2*
f - 16*c^2*d*e*f - 68*b*c*d*e*g))/(135135*e*(b*e - 2*c*d)^7))*(c*d^2 - c*e
^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((d*((64*c^7*(15*b*e*g -
 26*c*d*g + 2*c*e*f))/(135135*(b*e - 2*c*d)^7) - (128*c^8*d*g)/(135135*(b*
e - 2*c*d)^7)))/e - (9472*c^8*d^2*g + 2816*b^2*c^6*e^2*g - 1664*c^8*d*e*f
+ 960*b*c^7*e^2*f - 10304*b*c^7*d*e*g)/(135135*e*(b*e - 2*c*d)^7)))/e + (3
2*b*c^5*(37*b^2*e^2*g + 148*c^2*d^2*g + 14*b*c*e^2*f - 26*c^2*d*e*f - 148*
b*c*d*e*g))/(135135*e*(b*e - 2*c*d)^7))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2
*x)^(1/2))/(d + e*x) - (((d*((d*((64*c^7*(17*b*e*g - 30*c*d*g + 2*c*e*f))/
(135135*(b*e - 2*c*d)^7) - (128*c^8*d*g)/(135135*(b*e - 2*c*d)^7)))/e - (1
1520*c^8*d^2*g + 3392*b^2*c^6*e^2*g - 1920*c^8*d*e*f + 1088*b*c^7*e^2*f -
12480*b*c^7*d*e*g)/(135135*e*(b*e - 2*c*d)^7)))/e + (32*b*c^5*(45*b^2*e^2*
g + 180*c^2*d^2*g + 16*b*c*e^2*f - 30*c^2*d*e*f - 180*b*c*d*e*g))/(135135*
e*(b*e - 2*c*d)^7))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)
 - (((d*((d*((64*c^7*(19*b*e*g - 34*c*d*g + 2*c*e*f))/(135135*(b*e - 2*c*d
)^7) - (128*c^8*d*g)/(135135*(b*e - 2*c*d)^7)))/e - (13568*c^8*d^2*g + 396
8*b^2*c^6*e^2*g - 2176*c^8*d*e*f + 1216*b*c^7*e^2*f - 14656*b*c^7*d*e*g...
```

Reduce [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 4808, normalized size of antiderivative = 13.36

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^9}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^9,x)
```

output

```
(2*i*(210*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
 + c*d - c*e*x)*b**6*d*e**6*g + 1155*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
 - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**6*e**7*f + 1365*sqrt(d + e*x
)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**6*e
**7*g*x - 2170*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt(
- b*e + c*d - c*e*x)*b**5*c*d**2*e**5*g - 12390*sqrt(d + e*x)*sqrt(b*e - 2
*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*c*d*e**6*f - 14
210*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
 - c*e*x)*b**5*c*d*e**6*g*x + 1470*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
 b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*c*e**7*f*x + 1820*sqrt(d + e
*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5
*c*e**7*g*x**2 + 9130*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)
*sqrt( - b*e + c*d - c*e*x)*b**4*c**2*d**3*e**4*g + 54635*sqrt(d + e*x)*sq
rt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c**2*
d**2*e**5*f + 60325*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*s
qrt( - b*e + c*d - c*e*x)*b**4*c**2*d**2*e**5*g*x - 14630*sqrt(d + e*x)*sq
rt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c**2*
d*e**6*f*x - 18040*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*b**4*c**2*d*e**6*g*x**2 + 35*sqrt(d + e*x)*sqrt(b
*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c**2*e...
```

3.159 $\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 498

$$
\begin{aligned}
&\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x\right.\\
&\left.-ce^2x^2\right)^{5/2}\,dx=\frac{11(2cd-be)^7(20cef+6cdg-13beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{131072c^7e}\\
&+\frac{11(2cd-be)^5(20cef+6cdg-13beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{49152c^6e}\\
&+\frac{11(2cd-be)^3(20cef+6cdg-13beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{15360c^5e}\\
&-\frac{(20cef+6cdg-13beg)(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{180c^2e^2}\\
&-\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{10ce^2}\\
&-\frac{11(2cd-be)(20cef+6cdg-13beg)(32cd-9be+14cex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{40320c^4e^2}\\
&+\frac{11(2cd-be)^9(20cef+6cdg-13beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{131072c^{15/2}e^2}
\end{aligned}
$$

output
```
11/131072*(-b*e+2*c*d)^7*(-13*b*e*g+6*c*d*g+20*c*e*f)*(2*c*x+b)*(d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^7/e+11/49152*(-b*e+2*c*d)^5*(-13*b*e*g+6*c*
d*g+20*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^6/e+11/15
360*(-b*e+2*c*d)^3*(-13*b*e*g+6*c*d*g+20*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(5/2)/c^5/e-1/180*(-13*b*e*g+6*c*d*g+20*c*e*f)*(e*x+d)^2*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^2/e^2-1/10*g*(e*x+d)^3*(d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c/e^2-11/40320*(-b*e+2*c*d)*(-13*b*e*g+6*c*d*
g+20*c*e*f)*(14*c*e*x-9*b*e+32*c*d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)
/c^4/e^2+11/131072*(-b*e+2*c*d)^9*(-13*b*e*g+6*c*d*g+20*c*e*f)*arctan(c^(1
/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(15/2)/e^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1013 vs. $2(498) = 996$.

Time = 7.39 (sec) , antiderivative size = 1013, normalized size of antiderivative = 2.03

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \frac{(2cd-be)^9((d+ex)(-be+c(d-ex)))^{5/2}\left(69300c^{10}ef+20790c^{10}dg-45045bc^9eg+\right.}{}$$

$$-\frac{11(2cd-be)^9(20cef+6cdg-13beg)((d+ex)(-be+c(d-ex)))^{5/2}\arctan\left(\frac{\sqrt{cd-be-cex}}{\sqrt{c}\sqrt{d+ex}}\right)}{131072c^{15/2}e^2(d+ex)^{5/2}(cd-be-cex)^{5/2}}$$

input
```
Integrate[(d + e*x)^3*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2
),x]
```

output

```
((2*c*d - b*e)^9*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*(69300*c^10*e*f
+ 20790*c^10*d*g - 45045*b*c^9*e*g + (669900*c^9*e*f*(c*d - b*e - c*e*x))/
(d + e*x) + (200970*c^9*d*g*(c*d - b*e - c*e*x))/(d + e*x) - (435435*b*c^8
*e*g*(c*d - b*e - c*e*x))/(d + e*x) + (2901360*c^8*e*f*(c*d - b*e - c*e*x)
^2)/(d + e*x)^2 + (870408*c^8*d*g*(c*d - b*e - c*e*x)^2)/(d + e*x)^2 - (18
85884*b*c^7*e*g*(c*d - b*e - c*e*x)^2)/(d + e*x)^2 - (4391280*c^7*e*f*(c*d
 - b*e - c*e*x)^3)/(d + e*x)^3 - (9574920*c^7*d*g*(c*d - b*e - c*e*x)^3)/(
d + e*x)^3 + (6983100*b*c^6*e*g*(c*d - b*e - c*e*x)^3)/(d + e*x)^3 - (1390
1000*c^6*e*f*(c*d - b*e - c*e*x)^4)/(d + e*x)^4 - (4170300*c^6*d*g*(c*d -
b*e - c*e*x)^4)/(d + e*x)^4 + (9035650*b*c^5*e*g*(c*d - b*e - c*e*x)^4)/(d
 + e*x)^4 - (12313400*c^5*e*f*(c*d - b*e - c*e*x)^5)/(d + e*x)^5 - (369402
0*c^5*d*g*(c*d - b*e - c*e*x)^5)/(d + e*x)^5 + (8003710*b*c^4*e*g*(c*d - b
*e - c*e*x)^5)/(d + e*x)^5 - (7405200*c^4*e*f*(c*d - b*e - c*e*x)^6)/(d +
e*x)^6 - (2221560*c^4*d*g*(c*d - b*e - c*e*x)^6)/(d + e*x)^6 + (4813380*b*
c^3*e*g*(c*d - b*e - c*e*x)^6)/(d + e*x)^6 - (2901360*c^3*e*f*(c*d - b*e -
 c*e*x)^7)/(d + e*x)^7 - (870408*c^3*d*g*(c*d - b*e - c*e*x)^7)/(d + e*x)^
7 + (1885884*b*c^2*e*g*(c*d - b*e - c*e*x)^7)/(d + e*x)^7 - (669900*c^2*e*
f*(c*d - b*e - c*e*x)^8)/(d + e*x)^8 - (200970*c^2*d*g*(c*d - b*e - c*e*x)
^8)/(d + e*x)^8 + (435435*b*c*e*g*(c*d - b*e - c*e*x)^8)/(d + e*x)^8 - (69
300*c*e*f*(c*d - b*e - c*e*x)^9)/(d + e*x)^9 - (20790*c*d*g*(c*d - b*e ...
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {1221, 1134, 1134, 1160, 1087, 1087, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-13beg+6cdg+20cef)\int (d+ex)^3\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{20ce}-\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{10ce^2}$$

$$\Big\downarrow \text{1134}$$

$$\frac{(-13beg+6cdg+20cef)\left(\frac{11(2cd-be)\int (d+ex)^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{18c}-\frac{(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce}\right)}{20ce}-$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{10ce^2}$$

$$\Big\downarrow \text{1134}$$

$$\frac{(-13beg+6cdg+20cef)\Bigg(\frac{11(2cd-be)\left(\frac{9(2cd-be)\int (d+ex)\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{16c}-\frac{(d+ex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{8ce}\right)}{18c}-(d+}{20ce}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{10ce^2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(-13beg+6cdg+20cef)\Bigg(\frac{11(2cd-be)\Bigg(\frac{9(2cd-be)\left(\frac{(2cd-be)\int \left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{2c}-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7ce}\right)}{16c}-(d+ex)\Big(d(}{18c}}{20ce}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{10ce^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(-13beg+6cdg+20cef)\left(\frac{11(2cd-be)\left(\frac{9(2cd-be)\left(\frac{(2cd-be)\left(\frac{5(2cd-be)^2\int\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{24c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x\right.}{12c}\right.}{2c}\right.}{16c}\right.}{18c}\right.}{20ce}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{10ce^2}$$

$$\Big\downarrow \, 1087$$

$$\frac{(-13beg+6cdg+20cef)\left(\frac{11(2cd-be)\left(\frac{9(2cd-be)\left(\frac{(2cd-be)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\int\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16c}+\frac{(b+2cx)\left(d(cd-be)-b\right.}{8c}\right.}{24c}\right.}{2c}\right.}{16c}\right.}{}\right.}{}$$

$$\frac{g(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{10ce^2}$$

$$\Big\downarrow \text{1087}$$

$$(-13beg+6cdg+20cef)\left(\frac{11(2cd-be)\left(\frac{9(2cd-be)\left(\frac{(2cd-be)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)}{}\right.}{16c}\right.}{24c}\right.}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 1092

$$11(2cd-be)\left(9(2cd-be)\left((2cd-be)\left(5(2cd-be)^2\left(3(2cd-be)^2\left(\frac{(2cd-be)^2\int\frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}\,d\left(-\right.}{4c}\right.\right.\right.\right.\right.\right.$$

$$16$$

↓ 217

$$\left(11(2cd-be)\left(\frac{9(2cd-be)\left(\frac{(2cd-be)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\right.}{24c}\right.}{2c}\right.}{16c}\right.\right.$$

input
```
Int[(d + e*x)^3*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output
```
-1/10*(g*(d + e*x)^3*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(c*e^2)
+ ((20*c*e*f + 6*c*d*g - 13*b*e*g)*(-1/9*((d + e*x)^2*(d*(c*d - b*e) - b*e
^2*x - c*e^2*x^2)^(7/2))/(c*e) + (11*(2*c*d - b*e)*(-1/8*((d + e*x)*(d*(c*
d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(c*e) + (9*(2*c*d - b*e)*(-1/7*(d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2)/(c*e) + ((2*c*d - b*e)*(((b + 2*c*
x)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(12*c) + (5*(2*c*d - b*e)^
2*(((b + 2*c*x)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(8*c) + (3*(2
*c*d - b*e)^2*(((b + 2*c*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*
c) + ((2*c*d - b*e)^2*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2])])/(8*c^(3/2)*e)))/(16*c)))/(24*c)))/(2*c)))/(16*c
)))/(18*c)))/(20*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4540 vs. $2(466) = 932$.

Time = 3.69 (sec) , antiderivative size = 4541, normalized size of antiderivative = 9.12

method	result	size
default	Expression too large to display	4541

input
```
int((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
d^3*f*(-1/12*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/
2)-5/24*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/8*(-2*c*e^2*x-b*e^2)/c
/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-
b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*
d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arcta
n((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))))+e^2
*(3*d*g+e*f)*(-1/9*x^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2)/c/e^2-11/18*
b/c*(-1/8*x*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2)/c/e^2-9/16*b/c*(-1/7*(-
c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2)/c/e^2-1/2*b/c*(-1/12*(-2*c*e^2*x-b*e^
2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)-5/24*(-4*c*e^2*(-b*d*e+c*d
^2)-b^2*e^4)/c/e^2*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*
e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^
2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*
d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(
-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))))+1/8*(-b*d*e+c*d^2)/c/e^2*(-1/12
*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)-5/24*(-4*
c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2
*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e
^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1
/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1712 vs. $2(466) = 932$.

Time = 13.08 (sec) , antiderivative size = 3437, normalized size of antiderivative = 6.90

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="fricas")
```

output

```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29237 vs. 2(490) = 980.

Time = 2.30 (sec) , antiderivative size = 29237, normalized size of antiderivative = 58.71

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)**3*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x
)
```

output
```
Piecewise((sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(c**2*e**7*g*x**
9/10 - x**8*(-41*b*c**2*e**9*g/20 - 3*c**3*d*e**8*g - c**3*e**9*f)/(9*c*e*
*2) - x**7*(-3*b**2*c*e**9*g - 12*b*c**2*d*e**8*g - 3*b*c**2*e**9*f - 17*b
*(-41*b*c**2*e**9*g/20 - 3*c**3*d*e**8*g - c**3*e**9*f)/(18*c) - 3*c**3*d*
e**8*f - c**2*e**7*g*(-9*b*d*e + 9*c*d**2)/10)/(8*c*e**2) - x**6*(-b**3*e*
*9*g - 15*b**2*c*d*e**8*g - 3*b**2*c*e**9*f - 12*b*c**2*d**2*e**7*g - 12*b
*c**2*d*e**8*f - 15*b*(-3*b**2*c*e**9*g - 12*b*c**2*d*e**8*g - 3*b*c**2*e*
*9*f - 17*b*(-41*b*c**2*e**9*g/20 - 3*c**3*d*e**8*g - c**3*e**9*f)/(18*c)
- 3*c**3*d*e**8*f - c**2*e**7*g*(-9*b*d*e + 9*c*d**2)/10)/(16*c) + 8*c**3*
d**3*e**6*g + (-8*b*d*e + 8*c*d**2)*(-41*b*c**2*e**9*g/20 - 3*c**3*d*e**8*
g - c**3*e**9*f)/(9*c*e**2))/(7*c*e**2) - x**5*(-6*b**3*d*e**8*g - b**3*e*
*9*f - 27*b**2*c*d**2*e**7*g - 15*b**2*c*d*e**8*f + 12*b*c**2*d**3*e**6*g
- 12*b*c**2*d**2*e**7*f - 13*b*(-b**3*e**9*g - 15*b**2*c*d*e**8*g - 3*b**2
*c*e**9*f - 12*b*c**2*d**2*e**7*g - 12*b*c**2*d*e**8*f - 15*b*(-3*b**2*c*e
**9*g - 12*b*c**2*d*e**8*g - 3*b*c**2*e**9*f - 17*b*(-41*b*c**2*e**9*g/20
- 3*c**3*d*e**8*g - c**3*e**9*f)/(18*c) - 3*c**3*d*e**8*f - c**2*e**7*g*(-
9*b*d*e + 9*c*d**2)/10)/(16*c) + 8*c**3*d**3*e**6*g + (-8*b*d*e + 8*c*d**2
)*(-41*b*c**2*e**9*g/20 - 3*c**3*d*e**8*g - c**3*e**9*f)/(9*c*e**2))/(14*c
) + 6*c**3*d**4*e**5*g + 8*c**3*d**3*e**6*f + (-7*b*d*e + 7*c*d**2)*(-3*b*
*2*c*e**9*g - 12*b*c**2*d*e**8*g - 3*b*c**2*e**9*f - 17*b*(-41*b*c**2*e...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1818 vs. $2(466) = 932$.

Time $= 0.49$ (sec) , antiderivative size $= 1818$, normalized size of antiderivative $= 3.65$

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="giac")
```

output

```
1/41287680*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(2*(8*(2*(4*(1
4*(16*(18*c^2*e^7*g*x + (20*c^11*e^23*f + 60*c^11*d*e^22*g + 41*b*c^10*e^2
3*g)/(c^9*e^16))*x + (1080*c^11*d*e^22*f + 740*b*c^10*e^23*f + 324*c^11*d^
2*e^21*g + 2976*b*c^10*d*e^22*g + 383*b^2*c^9*e^23*g)/(c^9*e^16))*x + (512
0*c^11*d^2*e^21*f + 47800*b*c^10*d*e^22*f + 6180*b^2*c^9*e^23*f - 30720*c^
11*d^3*e^20*g + 59396*b*c^10*d^2*e^21*g + 31264*b^2*c^9*d*e^22*g + 15*b^3*
c^8*e^23*g)/(c^9*e^16))*x - (144480*c^11*d^3*e^20*f - 278160*b*c^10*d^2*e^
21*f - 147720*b^2*c^9*d*e^22*f - 100*b^3*c^8*e^23*f + 140112*c^11*d^4*e^19
*g + 16176*b*c^10*d^3*e^20*g - 298968*b^2*c^9*d^2*e^21*g - 780*b^3*c^8*d*e
^22*g + 65*b^4*c^7*e^23*g)/(c^9*e^16))*x - (337920*c^11*d^4*e^19*f + 46560
*b*c^10*d^3*e^20*f - 730320*b^2*c^9*d^2*e^21*f - 2760*b^3*c^8*d*e^22*f + 2
20*b^4*c^7*e^23*f - 92160*c^11*d^5*e^18*g + 762000*b*c^10*d^4*e^19*g - 733
200*b^2*c^9*d^3*e^20*g - 9960*b^3*c^8*d^2*e^21*g + 1860*b^4*c^7*d*e^22*g -
 143*b^5*c^6*e^23*g)/(c^9*e^16))*x + (981120*c^11*d^5*e^18*f - 7859520*b*c
^10*d^4*e^19*f + 7487040*b^2*c^9*d^3*e^20*f + 151520*b^3*c^8*d^2*e^21*f -
26840*b^4*c^7*d*e^22*f + 1980*b^5*c^6*e^23*f + 2358720*c^11*d^6*e^17*g - 6
092160*b*c^10*d^5*e^18*g + 3484080*b^2*c^9*d^4*e^19*g + 339840*b^3*c^8*d^3
*e^20*g - 106540*b^4*c^7*d^2*e^21*g + 18040*b^5*c^6*d*e^22*g - 1287*b^6*c^
5*e^23*g)/(c^9*e^16))*x + (6553600*c^11*d^6*e^17*f - 16717440*b*c^10*d^5*e
^18*f + 9107520*b^2*c^9*d^4*e^19*f + 1415360*b^3*c^8*d^3*e^20*f - 41712...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\int \left(f+g\,x\right)\,{\left(d+e\,x\right)}^3\,{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)}^{5/2}\,dx$$

input

```
int((f + g*x)*(d + e*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),x)
```

output

```
int((f + g*x)*(d + e*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 48.69 (sec) , antiderivative size = 6460, normalized size of antiderivative = 12.97

$$\int (d+ex)^3(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^3*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output
```
(i*(45045*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*b**11*e**11*g - 921690*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqr
t( - b*e + 2*c*d))*b**10*c*d*e**10*g - 69300*sqrt(c)*asinh((sqrt( - b*e +
c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**10*c*e**11*f + 8523900*sqrt(c)*as
inh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**9*c**2*d**2*e*
*9*g + 1386000*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
2*c*d))*b**9*c**2*d*e**10*f - 46985400*sqrt(c)*asinh((sqrt( - b*e + c*d -
c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**8*c**3*d**3*e**8*g - 12474000*sqrt(c)*a
sinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**8*c**3*d**2*e
**9*f + 171309600*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
 + 2*c*d))*b**7*c**4*d**4*e**7*g + 66528000*sqrt(c)*asinh((sqrt( - b*e + c
*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**7*c**4*d**3*e**8*f - 433097280*sqr
t(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**6*c**5*
d**5*e**6*g - 232848000*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
 - b*e + 2*c*d))*b**6*c**5*d**4*e**7*f + 773055360*sqrt(c)*asinh((sqrt( -
b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**6*d**6*e**5*g + 558835
200*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**
5*c**6*d**5*e**6*f - 971308800*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i
)/sqrt( - b*e + 2*c*d))*b**4*c**7*d**7*e**4*g - 931392000*sqrt(c)*asinh((s
qrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**7*d**6*e**5*...
```

3.160 $\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 425

$$\begin{aligned}
&\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x\right.\\
&\left.-ce^2x^2\right)^{5/2}\,dx=\frac{5(2cd-be)^6(18cef+4cdg-11beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{32768c^6e}\\
&+\frac{5(2cd-be)^4(18cef+4cdg-11beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{12288c^5e}\\
&+\frac{(2cd-be)^2(18cef+4cdg-11beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{768c^4e}\\
&-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce^2}\\
&-\frac{(18cef+4cdg-11beg)(32cd-9be+14cex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{2016c^3e^2}\\
&+\frac{5(2cd-be)^8(18cef+4cdg-11beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{32768c^{13/2}e^2}
\end{aligned}$$

output
```
5/32768*(-b*e+2*c*d)^6*(-11*b*e*g+4*c*d*g+18*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d
)-b*e^2*x-c*e^2*x^2)^(1/2)/c^6/e+5/12288*(-b*e+2*c*d)^4*(-11*b*e*g+4*c*d*g
+18*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^5/e+1/768*(-
b*e+2*c*d)^2*(-11*b*e*g+4*c*d*g+18*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2*x-
c*e^2*x^2)^(5/2)/c^4/e-1/9*g*(e*x+d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7
/2)/c/e^2-1/2016*(-11*b*e*g+4*c*d*g+18*c*e*f)*(14*c*e*x-9*b*e+32*c*d)*(d*(
-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^3/e^2+5/32768*(-b*e+2*c*d)^8*(-11*b*e
*g+4*c*d*g+18*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^
2)^(1/2))/c^(13/2)/e^2
```

Mathematica [A] (verified)

Time = 7.18 (sec) , antiderivative size = 847, normalized size of antiderivative = 1.99

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\frac{(-2cd+be)^8((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{\sqrt{c}(5670c^9ef(d+ex)^8+1260c^9dg(d+ex)^8-3465bc^8eg(d+e}{}\right.}{}$$

input
```
Integrate[(d + e*x)^2*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2
),x]
```

output
```
((-2*c*d + b*e)^8*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((Sqrt[c]*(5670
*c^9*e*f*(d + e*x)^8 + 1260*c^9*d*g*(d + e*x)^8 - 3465*b*c^8*e*g*(d + e*x)
^8 - 49140*c^8*e*f*(d + e*x)^7*(-(c*d) + b*e + c*e*x) + 30030*b*c^7*e*g*(d
 + e*x)^7*(-(c*d) + b*e + c*e*x) + 188244*c^7*e*f*(d + e*x)^6*(-(c*d) + b*
e + c*e*x)^2 + 41832*c^7*d*g*(d + e*x)^6*(-(c*d) + b*e + c*e*x)^2 - 115038
*b*c^6*e*g*(d + e*x)^6*(-(c*d) + b*e + c*e*x)^2 + 172188*c^6*e*f*(d + e*x)
^5*(-(c*d) + b*e + c*e*x)^3 - 334602*b*c^5*e*g*(d + e*x)^5*(-(c*d) + b*e +
 c*e*x)^3 - 589824*c^5*e*f*(d + e*x)^4*(-(c*d) + b*e + c*e*x)^4 - 131072*c
^5*d*g*(d + e*x)^4*(-(c*d) + b*e + c*e*x)^4 + 360448*b*c^4*e*g*(d + e*x)^4
*(-(c*d) + b*e + c*e*x)^4 + 417636*c^4*e*f*(d + e*x)^3*(-(c*d) + b*e + c*e
*x)^5 - 255222*b*c^3*e*g*(d + e*x)^3*(-(c*d) + b*e + c*e*x)^5 - 188244*c^3
*e*f*(d + e*x)^2*(-(c*d) + b*e + c*e*x)^6 - 41832*c^3*d*g*(d + e*x)^2*(-(c
*d) + b*e + c*e*x)^6 + 115038*b*c^2*e*g*(d + e*x)^2*(-(c*d) + b*e + c*e*x)
^6 + 49140*c^2*e*f*(d + e*x)*(-(c*d) + b*e + c*e*x)^7 - 30030*b*c*e*g*(d +
 e*x)*(-(c*d) + b*e + c*e*x)^7 - 5670*c*e*f*(-(c*d) + b*e + c*e*x)^8 - 126
0*c*d*g*(-(c*d) + b*e + c*e*x)^8 + 3465*b*e*g*(-(c*d) + b*e + c*e*x)^8 + 1
0920*c^8*d*g*(d + e*x)^7*(-(b*e) + c*(d - e*x)) - 497016*c^6*d*g*(d + e*x)
^5*(-(b*e) + c*(d - e*x))^3 - 92808*c^4*d*g*(d + e*x)^3*(-(b*e) + c*(d - e
*x))^5 - 10920*c^2*d*g*(d + e*x)*(-(b*e) + c*(d - e*x))^7))/((2*c*d - b*e)
^9*(d + e*x)^2*(-(c*d) + b*e + c*e*x)^2) - (315*(18*c*e*f + 4*c*d*g - 1...
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1221, 1134, 1160, 1087, 1087, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}dx$$

$$\Big\downarrow \ 1221$$

$$\frac{(-11beg+4cdg+18cef)\int (d+ex)^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{18ce}-\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce^2}$$

$$\Big\downarrow \ 1134$$

$$\frac{(-11beg+4cdg+18cef)\left(\frac{9(2cd-be)\int (d+ex)\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{16c}-\frac{(d+ex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{8ce}\right)}{18ce}-$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce^2}$$

$$\Big\downarrow 1160$$

$$\frac{(-11beg+4cdg+18cef)\Bigg(\frac{9(2cd-be)\left(\frac{(2cd-be)\int\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{2c}-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7ce}\right)}{16c}-\frac{(d+ex)\big(d(cd-be)-}{8ce}}{18ce}$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce^2}$$

$$\Big\downarrow 1087$$

$$\frac{(-11beg+4cdg+18cef)\Bigg(\frac{9(2cd-be)\Bigg(\frac{(2cd-be)\left(\frac{5(2cd-be)^2\int\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{24c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{12c}\right)}{2c}-\frac{\big(d}{}}{16c}}{18ce}$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce^2}$$

$$\Big\downarrow 1087$$

$$\frac{(-11beg+4cdg+18cef)\left(\frac{9(2cd-be)\left(\frac{(2cd-be)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\int\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^3}{8c}\right.}{24c}\right.}{2c}\right.}{16c}\right.}{18ce}$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce^2}$$

$$\Big\downarrow 1087$$

$$\frac{(-11beg+4cdg+18cef)\left(\frac{9(2cd-be)\left(\frac{(2cd-be)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-b}}{4c}\right.}{16c}\right.}{24c}\right.}{2c}\right.}{}\right.}{}$$

$$\frac{g(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce^2}$$

$$\Big\downarrow \text{1092}$$

$$(-11beg+4cdg+18cef)\left(\frac{9(2cd-be)\left(\frac{(2cd-be)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int\frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2}{\sqrt{-cx^2e^2-bxe}}\right.}{4c}\right.}{16c}\right.}{2}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{217}$$

$$\left(\frac{9(2cd-be)\left(\frac{(2cd-be)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\frac{(b+2cx)\left(d(cd\right.}{}\right.}{24c}\right.}{2c}\right.}{16c}\right.$$

input
```
Int[(d + e*x)^2*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output
```
-1/9*(g*(d + e*x)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(c*e^2) +
 ((18*c*e*f + 4*c*d*g - 11*b*e*g)*(-1/8*((d + e*x)*(d*(c*d - b*e) - b*e^2*
x - c*e^2*x^2)^(7/2))/(c*e) + (9*(2*c*d - b*e)*(-1/7*(d*(c*d - b*e) - b*e^
2*x - c*e^2*x^2)^(7/2)/(c*e) + ((2*c*d - b*e)*(((b + 2*c*x)*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(5/2))/(12*c) + (5*(2*c*d - b*e)^2*(((b + 2*c*x)*(
d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(8*c) + (3*(2*c*d - b*e)^2*(((
b + 2*c*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b*
e)^2*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^
2*x^2])])/(8*c^(3/2)*e)))/(16*c)))/(24*c)))/(2*c)))/(16*c)))/(18*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2578 vs. $2(397) = 794$.

Time $= 2.86$ (sec) , antiderivative size $= 2579$, normalized size of antiderivative $= 6.07$

method	result	size
default	Expression too large to display	2579

input
```
int((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
d^2*f*(-1/12*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/
2)-5/24*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/8*(-2*c*e^2*x-b*e^2)/c
/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-
b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*
d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arcta
n((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))))+e*(
2*d*g+e*f)*(-1/8*x*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2)/c/e^2-9/16*b/c*(
-1/7*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2)/c/e^2-1/2*b/c*(-1/12*(-2*c*e^2
*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)-5/24*(-4*c*e^2*(-b*
d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2
*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(
-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e
^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2
*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))))+1/8*(-b*d*e+c*d^2)/c/e^2
*(-1/12*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)-5/
24*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*
(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e
^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^
(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*
e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))))+d*(d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. 2(397) = 794.

Time = 6.43 (sec) , antiderivative size = 2861, normalized size of antiderivative = 6.73

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="fricas")
```

output
```
[1/8257536*(315*(18*(256*c^9*d^8*e - 1024*b*c^8*d^7*e^2 + 1792*b^2*c^7*d^6
*e^3 - 1792*b^3*c^6*d^5*e^4 + 1120*b^4*c^5*d^4*e^5 - 448*b^5*c^4*d^3*e^6 +
 112*b^6*c^3*d^2*e^7 - 16*b^7*c^2*d*e^8 + b^8*c*e^9)*f + (1024*c^9*d^9 - 6
912*b*c^8*d^8*e + 18432*b^2*c^7*d^7*e^2 - 26880*b^3*c^6*d^6*e^3 + 24192*b^
4*c^5*d^5*e^4 - 14112*b^5*c^4*d^4*e^5 + 5376*b^6*c^3*d^3*e^6 - 1296*b^7*c^
2*d^2*e^7 + 180*b^8*c*d*e^8 - 11*b^9*e^9)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 +
8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 4*sqrt(-c*e^2*x^2 - b*e^2*
x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) + 4*(229376*c^9*e^8*g*x^8 + 1
4336*(18*c^9*e^8*f + (36*c^9*d*e^7 + 37*b*c^8*e^8)*g)*x^7 + 1024*(18*(32*c
^9*d*e^7 + 33*b*c^8*e^8)*f - (320*c^9*d^2*e^6 - 1796*b*c^8*d*e^7 - 309*b^2
*c^7*e^8)*g)*x^6 - 256*(54*(28*c^9*d^2*e^6 - 156*b*c^8*d*e^7 - 27*b^2*c^7*
e^8)*f + (5712*c^9*d^3*e^5 - 5484*b*c^8*d^2*e^6 - 5928*b^2*c^7*d*e^7 - 5*b
^3*c^6*e^8)*g)*x^5 - 128*(18*(768*c^9*d^3*e^5 - 732*b*c^8*d^2*e^6 - 804*b^
2*c^7*d*e^7 - b^3*c^6*e^8)*f + (3072*c^9*d^4*e^4 + 15504*b*c^8*d^3*e^5 - 2
2044*b^2*c^7*d^2*e^6 - 120*b^3*c^6*d*e^7 + 11*b^4*c^5*e^8)*g)*x^4 - 16*(18
*(1680*c^9*d^4*e^4 + 8928*b*c^8*d^3*e^5 - 12552*b^2*c^7*d^2*e^6 - 104*b^3*
c^6*d*e^7 + 9*b^4*c^5*e^8)*f - (79296*c^9*d^5*e^3 - 232272*b*c^8*d^4*e^4 +
 148416*b^2*c^7*d^3*e^5 + 5704*b^3*c^6*d^2*e^6 - 1180*b^4*c^5*d*e^7 + 99*b
^5*c^4*e^8)*g)*x^3 + 8*(54*(4096*c^9*d^5*e^3 - 11920*b*c^8*d^4*e^4 + 7456*
b^2*c^7*d^3*e^5 + 456*b^3*c^6*d^2*e^6 - 88*b^4*c^5*d*e^7 + 7*b^5*c^4*e^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18144 vs. $2(413) = 826$.

Time = 1.84 (sec) , antiderivative size = 18144, normalized size of antiderivative = 42.69

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)**2*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x
)
```

output

```
Piecewise((sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(c**2*e**6*g*x**
8/9 - x**7*(-37*b*c**2*e**8*g/18 - 2*c**3*d*e**7*g - c**3*e**8*f)/(8*c*e**
2) - x**6*(-3*b**2*c*e**8*g - 9*b*c**2*d*e**7*g - 3*b*c**2*e**8*f - 15*b*(
-37*b*c**2*e**8*g/18 - 2*c**3*d*e**7*g - c**3*e**8*f)/(16*c) + 2*c**3*d**2
*e**6*g - 2*c**3*d*e**7*f - c**2*e**6*g*(-8*b*d*e + 8*c*d**2)/9)/(7*c*e**2
) - x**5*(-b**3*e**8*g - 12*b**2*c*d*e**7*g - 3*b**2*c*e**8*f - 3*b*c**2*d
**2*e**6*g - 9*b*c**2*d*e**7*f - 13*b*(-3*b**2*c*e**8*g - 9*b*c**2*d*e**7*
g - 3*b*c**2*e**8*f - 15*b*(-37*b*c**2*e**8*g/18 - 2*c**3*d*e**7*g - c**3*
e**8*f)/(16*c) + 2*c**3*d**2*e**6*g - 2*c**3*d*e**7*f - c**2*e**6*g*(-8*b*
d*e + 8*c*d**2)/9)/(14*c) + 6*c**3*d**3*e**5*g + 2*c**3*d**2*e**6*f + (-7*
b*d*e + 7*c*d**2)*(-37*b*c**2*e**8*g/18 - 2*c**3*d*e**7*g - c**3*e**8*f)/(
8*c*e**2))/(6*c*e**2) - x**4*(-5*b**3*d*e**7*g - b**3*e**8*f - 15*b**2*c*d
**2*e**6*g - 12*b**2*c*d*e**7*f + 15*b*c**2*d**3*e**5*g - 3*b*c**2*d**2*e*
*6*f - 11*b*(-b**3*e**8*g - 12*b**2*c*d*e**7*g - 3*b**2*c*e**8*f - 3*b*c**
2*d**2*e**6*g - 9*b*c**2*d*e**7*f - 13*b*(-3*b**2*c*e**8*g - 9*b*c**2*d*e*
*7*g - 3*b*c**2*e**8*f - 15*b*(-37*b*c**2*e**8*g/18 - 2*c**3*d*e**7*g - c*
*3*e**8*f)/(16*c) + 2*c**3*d**2*e**6*g - 2*c**3*d*e**7*f - c**2*e**6*g*(-8
*b*d*e + 8*c*d**2)/9)/(14*c) + 6*c**3*d**3*e**5*g + 2*c**3*d**2*e**6*f + (
-7*b*d*e + 7*c*d**2)*(-37*b*c**2*e**8*g/18 - 2*c**3*d*e**7*g - c**3*e**8*f
)/(8*c*e**2))/(12*c) + 6*c**3*d**3*e**5*f + (-6*b*d*e + 6*c*d**2)*(-3*b...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1512 vs. $2(397) = 794$.

Time = 0.41 (sec) , antiderivative size = 1512, normalized size of antiderivative = 3.56

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="giac")
```

output

```
1/2064384*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(2*(8*(2*(4*(14
*(16*c^2*e^6*g*x + (18*c^10*e^20*f + 36*c^10*d*e^19*g + 37*b*c^9*e^20*g)/(
c^8*e^14))*x + (576*c^10*d*e^19*f + 594*b*c^9*e^20*f - 320*c^10*d^2*e^18*g
 + 1796*b*c^9*d*e^19*g + 309*b^2*c^8*e^20*g)/(c^8*e^14))*x - (1512*c^10*d^
2*e^18*f - 8424*b*c^9*d*e^19*f - 1458*b^2*c^8*e^20*f + 5712*c^10*d^3*e^17*
g - 5484*b*c^9*d^2*e^18*g - 5928*b^2*c^8*d*e^19*g - 5*b^3*c^7*e^20*g)/(c^8
*e^14))*x - (13824*c^10*d^3*e^17*f - 13176*b*c^9*d^2*e^18*f - 14472*b^2*c^
8*d*e^19*f - 18*b^3*c^7*e^20*f + 3072*c^10*d^4*e^16*g + 15504*b*c^9*d^3*e^
17*g - 22044*b^2*c^8*d^2*e^18*g - 120*b^3*c^7*d*e^19*g + 11*b^4*c^6*e^20*g
)/(c^8*e^14))*x - (30240*c^10*d^4*e^16*f + 160704*b*c^9*d^3*e^17*f - 22593
6*b^2*c^8*d^2*e^18*f - 1872*b^3*c^7*d*e^19*f + 162*b^4*c^6*e^20*f - 79296*
c^10*d^5*e^15*g + 232272*b*c^9*d^4*e^16*g - 148416*b^2*c^8*d^3*e^17*g - 57
04*b^3*c^7*d^2*e^18*g + 1180*b^4*c^6*d*e^19*g - 99*b^5*c^5*e^20*g)/(c^8*e^
14))*x + (221184*c^10*d^5*e^15*f - 643680*b*c^9*d^4*e^16*f + 402624*b^2*c^
8*d^3*e^17*f + 24624*b^3*c^7*d^2*e^18*f - 4752*b^4*c^6*d*e^19*f + 378*b^5*
c^5*e^20*f + 106496*c^10*d^6*e^14*g - 192192*b*c^9*d^5*e^15*g + 52752*b^2*
c^8*d^4*e^16*g + 46144*b^3*c^7*d^3*e^17*g - 16104*b^4*c^6*d^2*e^18*g + 298
8*b^5*c^5*d*e^19*g - 231*b^6*c^4*e^20*g)/(c^8*e^14))*x + (669312*c^10*d^6*
e^14*f - 1123200*b*c^9*d^5*e^15*f + 116640*b^2*c^8*d^4*e^16*f + 459072*b^3
*c^7*d^3*e^17*f - 147528*b^4*c^6*d^2*e^18*f + 25704*b^5*c^5*d*e^19*f - ...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\int (f+g\,x)\,\left(d+e\,x\right)^2\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}\,dx$$

input
```
int((f + g*x)*(d + e*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),x)
```

output
```
int((f + g*x)*(d + e*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 5315, normalized size of antiderivative = 12.51

$$\int (d+ex)^2(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^2*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(i*( - 3465*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c
*d))*b**10*e**10*g + 63630*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sq
rt( - b*e + 2*c*d))*b**9*c*d*e**9*g + 5670*sqrt(c)*asinh((sqrt( - b*e + c*
d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**9*c*e**10*f - 521640*sqrt(c)*asinh(
(sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**8*c**2*d**2*e**8*g
 - 102060*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*b**8*c**2*d*e**9*f + 2509920*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b**7*c**3*d**3*e**7*g + 816480*sqrt(c)*asinh((sqr
t( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**7*c**3*d**2*e**8*f - 7
832160*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b**6*c**4*d**4*e**6*g - 3810240*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b**6*c**4*d**3*e**7*f + 16511040*sqrt(c)*asinh((s
qrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**5*d**5*e**5*g +
 11430720*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*b**5*c**5*d**4*e**6*f - 23708160*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e
*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**6*d**6*e**4*g - 22861440*sqrt(c)*asin
h((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**6*d**5*e**5
*f + 22740480*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2
*c*d))*b**3*c**7*d**7*e**3*g + 30481920*sqrt(c)*asinh((sqrt( - b*e + c*d -
 c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**7*d**6*e**4*f - 13789440*sqrt(...
```

3.161 $\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 366

$$
\begin{aligned}
\int (d+ex)(f+gx)&\left(cd^2-bde-be^2x\right.\\
\left.-ce^2x^2\right)^{5/2}\,dx&=\frac{5(2cd-be)^5(16cef+2cdg-9beg)(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{16384c^5e}\\
&+\frac{5(2cd-be)^3(16cef+2cdg-9beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{6144c^4e}\\
&+\frac{(2cd-be)(16cef+2cdg-9beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{384c^3e}\\
&+\frac{(9beg-16c(ef+dg)-14cegx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{112c^2e^2}\\
&+\frac{5(2cd-be)^7(16cef+2cdg-9beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{16384c^{11/2}e^2}
\end{aligned}
$$

output

```
5/16384*(-b*e+2*c*d)^5*(-9*b*e*g+2*c*d*g+16*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)
-b*e^2*x-c*e^2*x^2)^(1/2)/c^5/e+5/6144*(-b*e+2*c*d)^3*(-9*b*e*g+2*c*d*g+16
*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^4/e+1/384*(-b*e
+2*c*d)*(-9*b*e*g+2*c*d*g+16*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*
x^2)^(5/2)/c^3/e+1/112*(9*b*e*g-16*c*(d*g+e*f)-14*c*e*g*x)*(d*(-b*e+c*d)-b
*e^2*x-c*e^2*x^2)^(7/2)/c^2/e^2+5/16384*(-b*e+2*c*d)^7*(-9*b*e*g+2*c*d*g+1
6*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^
(11/2)/e^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 733 vs. $2(366) = 732$.

Time = 4.70 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.00

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\frac{(2cd-be)^7((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{\sqrt{c}(945b^7e^7g-210b^6ce^6(8ef+58dg+3egx)+28b^5c^2e^5(2363d}{}\right.}{}$$

input
```
Integrate[(d + e*x)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),
x]
```

output
```
((2*c*d - b*e)^7*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((Sqrt[c]*(945*b
^7*e^7*g - 210*b^6*c*e^6*(8*e*f + 58*d*g + 3*e*g*x) + 28*b^5*c^2*e^5*(2363
*d^2*g + 38*d*e*(20*f + 7*g*x) + 2*e^2*x*(20*f + 9*g*x)) - 128*c^7*(384*d^
7*g - 64*d*e^6*x^5*(7*f + 6*g*x) - 48*e^7*x^6*(8*f + 7*g*x) + 3*d^6*e*(128
*f + 35*g*x) - 24*d^5*e^2*x*(77*f + 48*g*x) + 16*d^3*e^4*x^3*(91*f + 72*g*
x) + 8*d^2*e^5*x^4*(144*f + 119*g*x) - 2*d^4*e^3*x^2*(576*f + 413*g*x)) +
64*b*c^6*e*(2967*d^6*g - 8*d^2*e^4*x^3*(30*f + 19*g*x) + 16*e^6*x^5*(116*f
 + 99*g*x) + 6*d^5*e*(692*f + 181*g*x) + 16*d*e^5*x^4*(284*f + 235*g*x) -
24*d^3*e^3*x^2*(374*f + 269*g*x) - 6*d^4*e^2*x*(1156*f + 739*g*x)) + 16*b^
3*c^4*e^3*(20779*d^4*g + 24*e^4*x^3*(2*f + g*x) + 8*d*e^3*x^2*(74*f + 33*g
*x) + 20*d^2*e^2*x*(192*f + 73*g*x) + 4*d^3*e*(5024*f + 1431*g*x)) + 32*b^
2*c^5*e^2*(-10434*d^5*g + 1224*d^3*e^2*x*(4*f + 3*g*x) + 8*e^5*x^4*(296*f
+ 243*g*x) + 16*d*e^4*x^3*(583*f + 455*g*x) - 3*d^4*e*(4616*f + 1227*g*x)
+ 4*d^2*e^3*x^2*(3276*f + 2375*g*x)) - 8*b^4*c^3*e^4*(24372*d^3*g + 2*e^3*
x^2*(56*f + 27*g*x) + 8*d*e^2*x*(203*f + 85*g*x) + d^2*e*(14112*f + 4523*g
*x))))/((2*c*d - b*e)^7*(d + e*x)^2*(-(c*d) + b*e + c*e*x)^2) + (105*(9*b*
e*g - 2*c*(8*e*f + d*g))*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d +
e*x])])/((d + e*x)^(5/2)*(-(b*e) + c*(d - e*x))^(5/2))))/(344064*c^(11/2)*
e^2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1225, 1087, 1087, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}dx$$

$$\Big\downarrow \ 1225$$

$$\frac{(2cd-be)(-9beg+2cdg+16cef)\int \left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{32c^2e}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}(9beg-16c(dg+ef)-14cegx)}{112c^2e^2}$$

$$\Big\downarrow \ 1087$$

$$\frac{(2cd-be)(-9beg+2cdg+16cef)\left(\frac{5(2cd-be)^2\int \left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{24c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{12c}\right)}{32c^2e}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}(9beg-16c(dg+ef)-14cegx)}{112c^2e^2}$$

$$\Big\downarrow \ 1087$$

$$\frac{(2cd-be)(-9beg+2cdg+16cef)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\int \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{8c}\right)}{24c}+\right.}{32c^2e}+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}(9beg-16c(dg+ef)-14cegx)}{112c^2e^2}$$

$$\Big\downarrow \ 1087$$

$$\frac{(2cd-be)(-9beg+2cdg+16cef)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right.}{16c}\right.}{24c}\right.}{32c^2e}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}(9beg-16c(dg+ef)-14cegx)}{112c^2e^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(2cd-be)(-9beg+2cdg+16cef)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int\frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right.}{4c}\right.}{16c}\right.}{24c}\right.}{32c^2e}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}(9beg-16c(dg+ef)-14cegx)}{112c^2e^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(2cd-be)\left(\frac{5(2cd-be)^2\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\frac{(b+2cx)\left(d(cd-be)-b\right.}{8c}\right.}{24c}\right.}{32c^2e}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}(9beg-16c(dg+ef)-14cegx)}{112c^2e^2}$$

input
```
Int[(d + e*x)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output
```
((9*b*e*g - 16*c*(e*f + d*g) - 14*c*e*g*x)*(d*(c*d - b*e) - b*e^2*x - c*e^
2*x^2)^(7/2))/(112*c^2*e^2) + ((2*c*d - b*e)*(16*c*e*f + 2*c*d*g - 9*b*e*g
)*(((b + 2*c*x)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(12*c) + (5*(
2*c*d - b*e)^2*(((b + 2*c*x)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/
(8*c) + (3*(2*c*d - b*e)^2*(((b + 2*c*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*
e^2*x^2])/(4*c) + ((2*c*d - b*e)^2*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[
d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(8*c^(3/2)*e)))/(16*c)))/(24*c)))/
(32*c^2*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. $2(342) = 684$.

Time = 2.40 (sec) , antiderivative size = 1412, normalized size of antiderivative = 3.86

method	result	size
default	Expression too large to display	1412

input
```
int((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RETUR
NVERBOSE)
```

output

```
d*f*(-1/12*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)
-5/24*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/8*(-2*c*e^2*x-b*e^2)/c/e
^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^
2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^
2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan(
(c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))))+(d*g+
e*f)*(-1/7*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2)/c/e^2-1/2*b/c*(-1/12*(-2
*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)-5/24*(-4*c*e^
2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2
-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(
-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(
-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*
(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))))+e*g*(-1/8*x*(-c*e^
2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2)/c/e^2-9/16*b/c*(-1/7*(-c*e^2*x^2-b*e^2*x-
b*d*e+c*d^2)^(7/2)/c/e^2-1/2*b/c*(-1/12*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x
^2-b*e^2*x-b*d*e+c*d^2)^(5/2)-5/24*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2
*(-1/8*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/1
6*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(
-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4
)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. $2(342) = 684$.

Time = 3.32 (sec) , antiderivative size = 2345, normalized size of antiderivative = 6.41

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algori
thm="fricas")
```

output

```
[-1/1376256*(105*(16*(128*c^8*d^7*e - 448*b*c^7*d^6*e^2 + 672*b^2*c^6*d^5*
e^3 - 560*b^3*c^5*d^4*e^4 + 280*b^4*c^4*d^3*e^5 - 84*b^5*c^3*d^2*e^6 + 14*
b^6*c^2*d*e^7 - b^7*c*e^8)*f + (256*c^8*d^8 - 2048*b*c^7*d^7*e + 5376*b^2*
c^6*d^6*e^2 - 7168*b^3*c^5*d^5*e^3 + 5600*b^4*c^4*d^4*e^4 - 2688*b^5*c^3*d
^3*e^5 + 784*b^6*c^2*d^2*e^6 - 128*b^7*c*d*e^7 + 9*b^8*e^8)*g)*sqrt(-c)*lo
g(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqrt(-
c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*(43008*
c^8*e^7*g*x^7 + 3072*(16*c^8*e^7*f + (16*c^8*d*e^6 + 33*b*c^7*e^7)*g)*x^6
+ 256*(16*(14*c^8*d*e^6 + 29*b*c^7*e^7)*f - (476*c^8*d^2*e^5 - 940*b*c^7*d
*e^6 - 243*b^2*c^6*e^7)*g)*x^5 - 128*(16*(72*c^8*d^2*e^5 - 142*b*c^7*d*e^6
 - 37*b^2*c^6*e^7)*f + (1152*c^8*d^3*e^4 + 76*b*c^7*d^2*e^5 - 1820*b^2*c^6
*d*e^6 - 3*b^3*c^5*e^7)*g)*x^4 - 16*(16*(728*c^8*d^3*e^4 + 60*b*c^7*d^2*e^
5 - 1166*b^2*c^6*d*e^6 - 3*b^3*c^5*e^7)*f - (6608*c^8*d^4*e^3 - 25824*b*c^
7*d^3*e^4 + 19000*b^2*c^6*d^2*e^5 + 264*b^3*c^5*d*e^6 - 27*b^4*c^4*e^7)*g)
*x^3 + 8*(16*(1152*c^8*d^4*e^3 - 4488*b*c^7*d^3*e^4 + 3276*b^2*c^6*d^2*e^5
 + 74*b^3*c^5*d*e^6 - 7*b^4*c^4*e^7)*f + (18432*c^8*d^5*e^2 - 35472*b*c^7*
d^4*e^3 + 14688*b^2*c^6*d^3*e^4 + 2920*b^3*c^5*d^2*e^5 - 680*b^4*c^4*d*e^6
 + 63*b^5*c^3*e^7)*g)*x^2 - 16*(3072*c^8*d^6*e - 16608*b*c^7*d^5*e^2 + 276
96*b^2*c^6*d^4*e^3 - 20096*b^3*c^5*d^3*e^4 + 7056*b^4*c^4*d^2*e^5 - 1330*b
^5*c^3*d*e^6 + 105*b^6*c^2*e^7)*f - (49152*c^8*d^7 - 189888*b*c^7*d^6*e...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11123 vs. $2(357) = 714$.

Time = 2.40 (sec) , antiderivative size = 11123, normalized size of antiderivative = 30.39

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x)
```

output

```
Piecewise((sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(c**2*e**5*g*x**
7/8 - x**6*(-33*b*c**2*e**7*g/16 - c**3*d*e**6*g - c**3*e**7*f)/(7*c*e**2)
 - x**5*(-3*b**2*c*e**7*g - 6*b*c**2*d*e**6*g - 3*b*c**2*e**7*f - 13*b*(-3
3*b*c**2*e**7*g/16 - c**3*d*e**6*g - c**3*e**7*f)/(14*c) + 3*c**3*d**2*e**
5*g - c**3*d*e**6*f - c**2*e**5*g*(-7*b*d*e + 7*c*d**2)/8)/(6*c*e**2) - x*
*4*(-b**3*e**7*g - 9*b**2*c*d*e**6*g - 3*b**2*c*e**7*f + 3*b*c**2*d**2*e**
5*g - 6*b*c**2*d*e**6*f - 11*b*(-3*b**2*c*e**7*g - 6*b*c**2*d*e**6*g - 3*b
*c**2*e**7*f - 13*b*(-33*b*c**2*e**7*g/16 - c**3*d*e**6*g - c**3*e**7*f)/(
14*c) + 3*c**3*d**2*e**5*g - c**3*d*e**6*f - c**2*e**5*g*(-7*b*d*e + 7*c*d
**2)/8)/(12*c) + 3*c**3*d**3*e**4*g + 3*c**3*d**2*e**5*f + (-6*b*d*e + 6*c
*d**2)*(-33*b*c**2*e**7*g/16 - c**3*d*e**6*g - c**3*e**7*f)/(7*c*e**2))/(5
*c*e**2) - x**3*(-4*b**3*d*e**6*g - b**3*e**7*f - 6*b**2*c*d**2*e**5*g - 9
*b**2*c*d*e**6*f + 12*b*c**2*d**3*e**4*g + 3*b*c**2*d**2*e**5*f - 9*b*(-b*
*3*e**7*g - 9*b**2*c*d*e**6*g - 3*b**2*c*e**7*f + 3*b*c**2*d**2*e**5*g - 6
*b*c**2*d*e**6*f - 11*b*(-3*b**2*c*e**7*g - 6*b*c**2*d*e**6*g - 3*b*c**2*e
**7*f - 13*b*(-33*b*c**2*e**7*g/16 - c**3*d*e**6*g - c**3*e**7*f)/(14*c) +
 3*c**3*d**2*e**5*g - c**3*d*e**6*f - c**2*e**5*g*(-7*b*d*e + 7*c*d**2)/8)
/(12*c) + 3*c**3*d**3*e**4*g + 3*c**3*d**2*e**5*f + (-6*b*d*e + 6*c*d**2)*
(-33*b*c**2*e**7*g/16 - c**3*d*e**6*g - c**3*e**7*f)/(7*c*e**2))/(10*c) -
3*c**3*d**4*e**3*g + 3*c**3*d**3*e**4*f + (-5*b*d*e + 5*c*d**2)*(-3*b**...
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algori
thm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1234 vs. 2(342) = 684.

Time = 0.41 (sec) , antiderivative size = 1234, normalized size of antiderivative = 3.37

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algori
thm="giac")
```

output

```
1/344064*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(2*(8*(2*(12*(14
*c^2*e^5*g*x + (16*c^9*e^17*f + 16*c^9*d*e^16*g + 33*b*c^8*e^17*g)/(c^7*e^
12))*x + (224*c^9*d*e^16*f + 464*b*c^8*e^17*f - 476*c^9*d^2*e^15*g + 940*b
*c^8*d*e^16*g + 243*b^2*c^7*e^17*g)/(c^7*e^12))*x - (1152*c^9*d^2*e^15*f -
 2272*b*c^8*d*e^16*f - 592*b^2*c^7*e^17*f + 1152*c^9*d^3*e^14*g + 76*b*c^8
*d^2*e^15*g - 1820*b^2*c^7*d*e^16*g - 3*b^3*c^6*e^17*g)/(c^7*e^12))*x - (1
1648*c^9*d^3*e^14*f + 960*b*c^8*d^2*e^15*f - 18656*b^2*c^7*d*e^16*f - 48*b
^3*c^6*e^17*f - 6608*c^9*d^4*e^13*g + 25824*b*c^8*d^3*e^14*g - 19000*b^2*c
^7*d^2*e^15*g - 264*b^3*c^6*d*e^16*g + 27*b^4*c^5*e^17*g)/(c^7*e^12))*x +
(18432*c^9*d^4*e^13*f - 71808*b*c^8*d^3*e^14*f + 52416*b^2*c^7*d^2*e^15*f
+ 1184*b^3*c^6*d*e^16*f - 112*b^4*c^5*e^17*f + 18432*c^9*d^5*e^12*g - 3547
2*b*c^8*d^4*e^13*g + 14688*b^2*c^7*d^3*e^14*g + 2920*b^3*c^6*d^2*e^15*g -
680*b^4*c^5*d*e^16*g + 63*b^5*c^4*e^17*g)/(c^7*e^12))*x + (118272*c^9*d^5*
e^12*f - 221952*b*c^8*d^4*e^13*f + 78336*b^2*c^7*d^3*e^14*f + 30720*b^3*c^
6*d^2*e^15*f - 6496*b^4*c^5*d*e^16*f + 560*b^5*c^4*e^17*f - 6720*c^9*d^6*e
^11*g + 34752*b*c^8*d^5*e^12*g - 58896*b^2*c^7*d^4*e^13*g + 45792*b^3*c^6*
d^3*e^14*g - 18092*b^4*c^5*d^2*e^15*g + 3724*b^5*c^4*d*e^16*g - 315*b^6*c^
3*e^17*g)/(c^7*e^12))*x - (49152*c^9*d^6*e^11*f - 265728*b*c^8*d^5*e^12*f
+ 443136*b^2*c^7*d^4*e^13*f - 321536*b^3*c^6*d^3*e^14*f + 112896*b^4*c^5*d
^2*e^15*f - 21280*b^5*c^4*d*e^16*f + 1680*b^6*c^3*e^17*f + 49152*c^9*d^...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\int (f+g\,x)\,(d+e\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}\,dx$$

input
```
int((f + g*x)*(d + e*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),x)
```

output
```
int((f + g*x)*(d + e*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 4284, normalized size of antiderivative = 11.70

$$\int (d+ex)(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(i*(945*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*b**9*e**9*g - 15330*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
b*e + 2*c*d))*b**8*c*d*e**8*g - 1680*sqrt(c)*asinh((sqrt( - b*e + c*d - c*
e*x)*i)/sqrt( - b*e + 2*c*d))*b**8*c*e**9*f + 109200*sqrt(c)*asinh((sqrt(
- b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**7*c**2*d**2*e**7*g + 2688
0*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**7*
c**2*d*e**8*f - 446880*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
- b*e + 2*c*d))*b**6*c**3*d**3*e**6*g - 188160*sqrt(c)*asinh((sqrt( - b*e
+ c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**6*c**3*d**2*e**7*f + 1152480*sq
rt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**4
*d**4*e**5*g + 752640*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
 b*e + 2*c*d))*b**5*c**4*d**3*e**6*f - 1928640*sqrt(c)*asinh((sqrt( - b*e
+ c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**5*d**5*e**4*g - 1881600*sq
rt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**5
*d**4*e**5*f + 2069760*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
- b*e + 2*c*d))*b**3*c**6*d**6*e**3*g + 3010560*sqrt(c)*asinh((sqrt( - b*e
 + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**6*d**5*e**4*f - 1344000*s
qrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**
7*d**7*e**2*g - 3010560*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
 - b*e + 2*c*d))*b**2*c**7*d**6*e**3*f + 456960*sqrt(c)*asinh((sqrt( - ...
```

3.162 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 292

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{d+ex}\,dx=\frac{(2cd-be)^3(12cef-2cdg-5beg)(b+2cx)\sqrt{d(cd-be)-}}{512c^3e}$$

$$+\frac{(2cd-be)(12cef-2cdg-5beg)(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{192c^2e}$$

$$+\frac{(12cef-12cdg+5beg+10cegx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{60ce^2}$$

$$+\frac{(2cd-be)^5(12cef-2cdg-5beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{512c^{7/2}e^2}$$

output

```
1/512*(-b*e+2*c*d)^3*(-5*b*e*g-2*c*d*g+12*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b
*e^2*x-c*e^2*x^2)^(1/2)/c^3/e+1/192*(-b*e+2*c*d)*(-5*b*e*g-2*c*d*g+12*c*e*
f)*(2*c*x+b)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^2/e+1/60*(10*c*e*g*x
+5*b*e*g-12*c*d*g+12*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c/e^2+1
/512*(-b*e+2*c*d)^5*(-5*b*e*g-2*c*d*g+12*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(7/2)/e^2
```

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.61

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{(2cd-be)^5((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{\sqrt{c}(75b^5e^5g-10b^4}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
),x]
```

output
```
((2*c*d - b*e)^5*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((Sqrt[c]*(75*b^
5*e^5*g - 10*b^4*c*e^4*(18*e*f + 62*d*g + 5*e*g*x) + 40*b^3*c^2*e^3*(47*d^
2*g + e^2*x*(3*f + g*x) + d*e*(39*f + 9*g*x)) - 32*c^5*(48*d^5*g + 12*d*e^
4*x^3*(5*f + 4*g*x) - 8*e^5*x^4*(6*f + 5*g*x) - 3*d^4*e*(16*f + 5*g*x) - 6
*d^3*e^2*x*(25*f + 16*g*x) + 2*d^2*e^3*x^2*(48*f + 35*g*x)) + 16*b*c^4*e*(
207*d^4*g + 4*d*e^3*x^2*(3*f + 2*g*x) - 6*d^3*e*(7*f + 3*g*x) + 4*e^4*x^3*
(63*f + 50*g*x) - 6*d^2*e^2*x*(107*f + 67*g*x)) - 48*b^2*c^3*e^2*(67*d^3*g
 + d^2*e*(43*f + 9*g*x) - e^3*x^2*(62*f + 45*g*x) - d*e^2*x*(109*f + 68*g*
x))))/((2*c*d - b*e)^5*(d + e*x)^2*(-(c*d) + b*e + c*e*x)^2) - (15*(12*c*e
*f - 2*c*d*g - 5*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e
*x])])/((d + e*x)^(5/2)*(-(b*e) + c*(d - e*x))^(5/2))))/(7680*c^(7/2)*e^2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1215, 1225, 1087, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{d+ex}dx$$

$$\Big\downarrow \; 1215$$

$$\int (f+gx)\left(\frac{cd^2-bde}{d}-cex\right)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(5beg-12cdg+12cef+10cegx)}{60ce^2}-\frac{(2cd-be)(5beg-2c(6ef-dg))\int \left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{24ce}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(5beg-12cdg+12cef+10cegx)}{60ce^2}-\frac{(2cd-be)(5beg-2c(6ef-dg))\left(\frac{3(2cd-be)^2\int \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{8c}\right)}{24ce}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(5beg-12cdg+12cef+10cegx)}{60ce^2}-\frac{(2cd-be)(5beg-2c(6ef-dg))\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\frac{(b+2cx}{}\right.}{24ce}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(5beg-12cdg+12cef+10cegx)}{60ce^2}-\frac{(2cd-be)(5beg-2c(6ef-dg))\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}+\frac{(b+2cx)\sqrt{}}{}\right.}{16c}\right.}{24ce}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(5beg-12cdg+12cef+10cegx)}{60ce^2}-$$

$$\frac{(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce\right.}{8c}\right.}{24ce}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x),x]
```

output
```
((12*c*e*f - 12*c*d*g + 5*b*e*g + 10*c*e*g*x)*(d*(c*d - b*e) - b*e^2*x - c
*e^2*x^2)^(5/2))/(60*c*e^2) - ((2*c*d - b*e)*(5*b*e*g - 2*c*(6*e*f - d*g))
*(((b + 2*c*x)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(8*c) + (3*(2*
c*d - b*e)^2*(((b + 2*c*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c
) + ((2*c*d - b*e)^2*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e)
- b*e^2*x - c*e^2*x^2])])/(8*c^(3/2)*e)))/(16*c)))/(24*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(272) = 544$.

Time = 2.50 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.24

method	result
default	$g\left(-\frac{\left(-2c\,e^2x-b\,e^2\right)\left(-x^2c\,e^2-xb\,e^2-bde+c\,d^2\right)^{\frac{5}{2}}}{12c\,e^2}-\frac{5\left(-4c\,e^2\left(-bde+c\,d^2\right)-b^2e^4\right)\left(-\frac{\left(-2c\,e^2x-b\,e^2\right)\left(-x^2c\,e^2-xb\,e^2-bde+c\,d^2\right)^{\frac{3}{2}}}{8c\,e^2}-\frac{3\left(-4c\right.}{}\right.}{}\right.$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d),x,method=_RETUR
NVERBOSE)
```

output
```
g/e*(-1/12*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)
-5/24*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2*(-1/8*(-2*c*e^2*x-b*e^2)/c/e
^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-3/16*(-4*c*e^2*(-b*d*e+c*d^2)-b^
2*e^4)/c/e^2*(-1/4*(-2*c*e^2*x-b*e^2)/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^
2)^(1/2)-1/8*(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/c/e^2/(c*e^2)^(1/2)*arctan(
(c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))))-(d*g-
e*f)/e^2*(1/5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+1/2*(-b*e^
2+2*c*d*e)*(-1/8*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+
(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+3/16*(-b*e^2+2*c*d*e)^2/c/e^2*(-1/4*(-2*c*
e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e
))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(
x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/
e))^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(272) = 544$.

Time = 0.72 (sec) , antiderivative size = 1457, normalized size of antiderivative = 4.99

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d),x, algori
thm="fricas")
```

output
```
[-1/30720*(15*(12*(32*c^6*d^5*e - 80*b*c^5*d^4*e^2 + 80*b^2*c^4*d^3*e^3 -
40*b^3*c^3*d^2*e^4 + 10*b^4*c^2*d*e^5 - b^5*c*e^6)*f - (64*c^6*d^6 - 240*b
^2*c^4*d^4*e^2 + 320*b^3*c^3*d^3*e^3 - 180*b^4*c^2*d^2*e^4 + 48*b^5*c*d*e^
5 - 5*b^6*e^6)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4
*b*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x
 + b*e)*sqrt(-c)) - 4*(1280*c^6*e^5*g*x^5 + 128*(12*c^6*e^5*f - (12*c^6*d*
e^4 - 25*b*c^5*e^5)*g)*x^4 - 16*(12*(10*c^6*d*e^4 - 21*b*c^5*e^5)*f + (140
*c^6*d^2*e^3 - 8*b*c^5*d*e^4 - 135*b^2*c^4*e^5)*g)*x^3 - 8*(12*(32*c^6*d^2
*e^3 - 2*b*c^5*d*e^4 - 31*b^2*c^4*e^5)*f - (384*c^6*d^3*e^2 - 804*b*c^5*d^
2*e^3 + 408*b^2*c^4*d*e^4 + 5*b^3*c^3*e^5)*g)*x^2 + 12*(128*c^6*d^4*e - 56
*b*c^5*d^3*e^2 - 172*b^2*c^4*d^2*e^3 + 130*b^3*c^3*d*e^4 - 15*b^4*c^2*e^5)
*f - (1536*c^6*d^5 - 3312*b*c^5*d^4*e + 3216*b^2*c^4*d^3*e^2 - 1880*b^3*c^
3*d^2*e^3 + 620*b^4*c^2*d*e^4 - 75*b^5*c*e^5)*g + 2*(12*(200*c^6*d^3*e^2 -
 428*b*c^5*d^2*e^3 + 218*b^2*c^4*d*e^4 + 5*b^3*c^3*e^5)*f + (240*c^6*d^4*e
 - 144*b*c^5*d^3*e^2 - 216*b^2*c^4*d^2*e^3 + 180*b^3*c^3*d*e^4 - 25*b^4*c^
2*e^5)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^4*e^2), -1/153
60*(15*(12*(32*c^6*d^5*e - 80*b*c^5*d^4*e^2 + 80*b^2*c^4*d^3*e^3 - 40*b^3*
c^3*d^2*e^4 + 10*b^4*c^2*d*e^5 - b^5*c*e^6)*f - (64*c^6*d^6 - 240*b^2*c^4*
d^4*e^2 + 320*b^3*c^3*d^3*e^3 - 180*b^4*c^2*d^2*e^4 + 48*b^5*c*d*e^5 - 5*b
^6*e^6)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e...
```

Sympy [A] (verification not implemented)

Time = 8.63 (sec) , antiderivative size = 7917, normalized size of antiderivative = 27.11

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d),x)
```

output

```
b**2*d*e**2*f*Piecewise(((b/(4*c) + x/2)*sqrt(-b*d*e - b*e**2*x + c*d**2 -
 c*e**2*x**2) + (b**2*e**2/(8*c) - b*d*e/2 + c*d**2/2)*Piecewise((log(-b*e
**2 - 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**
2*x**2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*c
) + x)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)), Ne(c*e**2,
 0)), (-2*(-b*d*e - b*e**2*x + c*d**2)**(3/2)/(3*b*e**2), Ne(b*e**2, 0)),
(x*sqrt(-b*d*e + c*d**2), True)) + b**2*d*e**2*g*Piecewise((((-b*(-b*d*e +
c*d**2)/(12*c) - b*(b**2*e**2/(8*c) - b*d*e/3 + c*d**2/3)/(2*c))*Piecewise
((log(-b*e**2 - 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d*
*2 - c*e**2*x**2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0))
, ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)) +
 (b*x/(12*c) + x**2/3 - (b**2*e**2/(8*c) - b*d*e/3 + c*d**2/3)/(c*e**2))*s
qrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2), Ne(c*e**2, 0)), (2*((b*d*e
- c*d**2)*(-b*d*e - b*e**2*x + c*d**2)**(3/2)/3 + (-b*d*e - b*e**2*x + c*d
**2)**(5/2)/5)/(b**2*e**4), Ne(b*e**2, 0)), (x**2*sqrt(-b*d*e + c*d**2)/2,
 True)) + b**2*e**3*f*Piecewise((((-b*(-b*d*e + c*d**2)/(12*c) - b*(b**2*e*
*2/(8*c) - b*d*e/3 + c*d**2/3)/(2*c))*Piecewise((log(-b*e**2 - 2*c*e**2*x
+ 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2))/sqrt(-c*
e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*c) + x)*log(b/(2*c
) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)) + (b*x/(12*c) + x**2/3 - ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d),x, algori
thm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(272) = 544$.

Time = 0.40 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.60

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{1}{7680}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left(2\left(4\left(2\left(8\left(10\,c^2e^3g\right.\right.\right.\right.\right.$$

$$-\frac{(384\,c^6d^5ef-960\,bc^5d^4e^2f+960\,b^2c^4d^3e^3f-480\,b^3c^3d^2e^4f+120\,b^4c^2de^5f-12\,b^5ce^6f-64\,c^6d^6g+24}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d),x, algori
thm="giac")
```

output

```
1/7680*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(2*(8*(10*c^2*e^3*
g*x + (12*c^7*e^11*f - 12*c^7*d*e^10*g + 25*b*c^6*e^11*g)/(c^5*e^8))*x - (
120*c^7*d*e^10*f - 252*b*c^6*e^11*f + 140*c^7*d^2*e^9*g - 8*b*c^6*d*e^10*g
 - 135*b^2*c^5*e^11*g)/(c^5*e^8))*x - (384*c^7*d^2*e^9*f - 24*b*c^6*d*e^10
*f - 372*b^2*c^5*e^11*f - 384*c^7*d^3*e^8*g + 804*b*c^6*d^2*e^9*g - 408*b^
2*c^5*d*e^10*g - 5*b^3*c^4*e^11*g)/(c^5*e^8))*x + (2400*c^7*d^3*e^8*f - 51
36*b*c^6*d^2*e^9*f + 2616*b^2*c^5*d*e^10*f + 60*b^3*c^4*e^11*f + 240*c^7*d
^4*e^7*g - 144*b*c^6*d^3*e^8*g - 216*b^2*c^5*d^2*e^9*g + 180*b^3*c^4*d*e^1
0*g - 25*b^4*c^3*e^11*g)/(c^5*e^8))*x + (1536*c^7*d^4*e^7*f - 672*b*c^6*d^
3*e^8*f - 2064*b^2*c^5*d^2*e^9*f + 1560*b^3*c^4*d*e^10*f - 180*b^4*c^3*e^1
1*f - 1536*c^7*d^5*e^6*g + 3312*b*c^6*d^4*e^7*g - 3216*b^2*c^5*d^3*e^8*g +
 1880*b^3*c^4*d^2*e^9*g - 620*b^4*c^3*d*e^10*g + 75*b^5*c^2*e^11*g)/(c^5*e
^8)) - 1/1024*(384*c^6*d^5*e*f - 960*b*c^5*d^4*e^2*f + 960*b^2*c^4*d^3*e^3
*f - 480*b^3*c^3*d^2*e^4*f + 120*b^4*c^2*d*e^5*f - 12*b^5*c*e^6*f - 64*c^6
*d^6*g + 240*b^2*c^4*d^4*e^2*g - 320*b^3*c^3*d^3*e^3*g + 180*b^4*c^2*d^2*e
^4*g - 48*b^5*c*d*e^5*g + 5*b^6*e^6*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e^2)*x
- sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqrt(-c)*
c^3*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{d+ex}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 2564, normalized size of antiderivative = 8.78

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d),x)
```

output

```
(i*(75*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b**7*e**7*g - 870*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
 + 2*c*d))*b**6*c*d*e**6*g - 180*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)
*i)/sqrt( - b*e + 2*c*d))*b**6*c*e**7*f + 4140*sqrt(c)*asinh((sqrt( - b*e
+ c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**2*d**2*e**5*g + 2160*sqrt(
c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c**2*d*
e**6*f - 10200*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
2*c*d))*b**4*c**3*d**3*e**4*g - 10800*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**3*d**2*e**5*f + 13200*sqrt(c)*asinh
((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**4*d**4*e**3*
g + 28800*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*b**3*c**4*d**3*e**4*f - 7200*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b**2*c**5*d**5*e**2*g - 43200*sqrt(c)*asinh((sqrt
( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**5*d**4*e**3*f - 96
0*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**
6*d**6*e*g + 34560*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*
e + 2*c*d))*b*c**6*d**5*e**2*f + 1920*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*c**7*d**7*g - 11520*sqrt(c)*asinh((sqrt( -
b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**7*d**6*e*f + 75*sqrt(d + e*
x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*...
```

3.163 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2} dx$

Optimal result

Integrand size = 44, antiderivative size = 285

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \frac{(2cd-be)^2(10cef-4cdg-3beg)(b+2cx)\sqrt{d(cd-be)-}}{128c^2e}$$

$$+\frac{(10cef-4cdg-3beg)(16cd-11be-6cex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{240ce^2}$$

$$-\frac{g(cd-be-cex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2}$$

$$+\frac{(2cd-be)^4(10cef-4cdg-3beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{128c^{5/2}e^2}$$

output

```
1/128*(-b*e+2*c*d)^2*(-3*b*e*g-4*c*d*g+10*c*e*f)*(2*c*x+b)*(d*(-b*e+c*d)-b
*e^2*x-c*e^2*x^2)^(1/2)/c^2/e+1/240*(-3*b*e*g-4*c*d*g+10*c*e*f)*(-6*c*e*x-
11*b*e+16*c*d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c/e^2-1/5*g*(-c*e*x-
b*e+c*d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c/e^2+1/128*(-b*e+2*c*d)
^4*(-3*b*e*g-4*c*d*g+10*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*
x-c*e^2*x^2)^(1/2))/c^(5/2)/e^2
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.32

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \frac{(-2cd+be)^4(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex}}{}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^2,x]
```

output

```
((-2*c*d + b*e)^4*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d -
 e*x))]*((Sqrt[c]*(-45*b^4*e^4*g + 30*b^3*c*e^3*(5*e*f + 8*d*g + e*g*x) -
16*c^4*(56*d^4*g + 20*d*e^3*x^2*(4*f + 3*g*x) - 10*d^3*e*(8*f + 3*g*x) - 6
*e^4*x^3*(5*f + 4*g*x) - d^2*e^2*x*(45*f + 32*g*x)) + 8*b*c^3*e*(174*d^3*g
 + 2*e^3*x^2*(85*f + 63*g*x) - d^2*e*(195*f + 71*g*x) - 2*d*e^2*x*(125*f +
 82*g*x)) + 4*b^2*c^2*e^2*(-199*d^2*g + d*e*(70*f + 32*g*x) + e^2*x*(295*f
 + 186*g*x))))/((-2*c*d + b*e)^4*(-(c*d) + b*e + c*e*x)^2) - (15*(10*c*e*f
 - 4*c*d*g - 3*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x
])])/(Sqrt[d + e*x]*(-(b*e) + c*(d - e*x))^(5/2))))/(1920*c^(5/2)*e^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1220, 1131, 1087, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(-3beg-4cdg+10cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{d+ex}dx}{3e(2cd-be)}+\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(-3beg-4cdg+10cef)\left(\frac{1}{2}(2cd-be)\int \left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e}\right)}{3e(2cd-be)}+\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(-3beg-4cdg+10cef)\left(\frac{1}{2}(2cd-be)\left(\frac{3(2cd-be)^2\int \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{8c}\right)+\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(-3beg-4cdg+10cef)\left(\frac{1}{2}(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\right.\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{(-3beg-4cdg+10cef)\left(\frac{1}{2}(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}+\right.}{16c}\right.\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow 217$$

$$\frac{\left(\frac{1}{2}(2cd-be)\left(\frac{3(2cd-be)^2\left(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\right)}{16c}+\frac{(b+2cx)\left(d(cd-be)-be^2\right.}{8c}\right.\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^2(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

output
```
(2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(3*e^2*(2*c*d
 - b*e)*(d + e*x)^2) + ((10*c*e*f - 4*c*d*g - 3*b*e*g)*((d*(c*d - b*e) - b*
e^2*x - c*e^2*x^2)^(5/2)/(5*e) + ((2*c*d - b*e)*(((b + 2*c*x)*(d*(c*d - b*
e) - b*e^2*x - c*e^2*x^2)^(3/2))/(8*c) + (3*(2*c*d - b*e)^2*(((b + 2*c*x)*
Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b*e)^2*ArcTan
[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(
8*c^(3/2)*e)))/(16*c)))/2))/(3*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(265) = 530$.

Time = 2.81 (sec) , antiderivative size = 752, normalized size of antiderivative = 2.64

method	result
default	$g\Bigg(\frac{(-c\,e^{2}(x+\frac{d}{e})^{2}+(-b\,e^{2}+2dec)(x+\frac{d}{e}))^{\frac{5}{2}}}{5}+\frac{(-b\,e^{2}+2dec)\Bigg(-\frac{(-2c\,e^{2}(x+\frac{d}{e})-b\,e^{2}+2dec)(-c\,e^{2}(x+\frac{d}{e})^{2}+(-b\,e^{2}+2dec)(x+\frac{d}{e}))^{\frac{3}{2}}}{8c\,e^{2}}+\frac{3(}{}}{}$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^2,x,method=_RET
URNVERBOSE)
```

output

```
g/e^2*(1/5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+1/2*(-b*e^2+2
*c*d*e)*(-1/8*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b
*e^2+2*c*d*e)*(x+d/e))^(3/2)+3/16*(-b*e^2+2*c*d*e)^2/c/e^2*(-1/4*(-2*c*e^2
*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^
(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d
/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))
^(1/2)))))-(d*g-e*f)/e^3*(2/3/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2
+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)+10/3*c*e^2/(-b*e^2+2*c*d*e)*(1/5*(-c*e^2*
(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+1/2*(-b*e^2+2*c*d*e)*(-1/8*(-2*c
*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/
e))^(3/2)+3/16*(-b*e^2+2*c*d*e)^2/c/e^2*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*
e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2
*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*
d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2))))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(264) = 528.

Time = 0.37 (sec) , antiderivative size = 1097, normalized size of antiderivative = 3.85

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^2,x, algo
rithm="fricas")
```

output

```
[1/7680*(15*(10*(16*c^5*d^4*e - 32*b*c^4*d^3*e^2 + 24*b^2*c^3*d^2*e^3 - 8*
b^3*c^2*d*e^4 + b^4*c*e^5)*f - (64*c^5*d^5 - 80*b*c^4*d^4*e + 40*b^3*c^2*d
^2*e^3 - 20*b^4*c*d*e^4 + 3*b^5*e^5)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c
*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 4*sqrt(-c*e^2*x^2 - b*e^2*x + c
*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) + 4*(384*c^5*e^4*g*x^4 + 48*(10*c^
5*e^4*f - (20*c^5*d*e^3 - 21*b*c^4*e^4)*g)*x^3 - 8*(10*(16*c^5*d*e^3 - 17*
b*c^4*e^4)*f - (64*c^5*d^2*e^2 - 164*b*c^4*d*e^3 + 93*b^2*c^3*e^4)*g)*x^2
+ 10*(128*c^5*d^3*e - 156*b*c^4*d^2*e^2 + 28*b^2*c^3*d*e^3 + 15*b^3*c^2*e^
4)*f - (896*c^5*d^4 - 1392*b*c^4*d^3*e + 796*b^2*c^3*d^2*e^2 - 240*b^3*c^2
*d*e^3 + 45*b^4*c*e^4)*g + 2*(10*(36*c^5*d^2*e^2 - 100*b*c^4*d*e^3 + 59*b^
2*c^3*e^4)*f + (240*c^5*d^3*e - 284*b*c^4*d^2*e^2 + 64*b^2*c^3*d*e^3 + 15*
b^3*c^2*e^4)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^3*e^2),
-1/3840*(15*(10*(16*c^5*d^4*e - 32*b*c^4*d^3*e^2 + 24*b^2*c^3*d^2*e^3 - 8*
b^3*c^2*d*e^4 + b^4*c*e^5)*f - (64*c^5*d^5 - 80*b*c^4*d^4*e + 40*b^3*c^2*d
^2*e^3 - 20*b^4*c*d*e^4 + 3*b^5*e^5)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2
 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2
*x - c^2*d^2 + b*c*d*e)) - 2*(384*c^5*e^4*g*x^4 + 48*(10*c^5*e^4*f - (20*c
^5*d*e^3 - 21*b*c^4*e^4)*g)*x^3 - 8*(10*(16*c^5*d*e^3 - 17*b*c^4*e^4)*f -
(64*c^5*d^2*e^2 - 164*b*c^4*d*e^3 + 93*b^2*c^3*e^4)*g)*x^2 + 10*(128*c^5*d
^3*e - 156*b*c^4*d^2*e^2 + 28*b^2*c^3*d*e^3 + 15*b^3*c^2*e^4)*f - (896*...
```

Sympy [A] (verification not implemented)

Time = 10.49 (sec) , antiderivative size = 4675, normalized size of antiderivative = 16.40

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**2,x
)
```

output
```
b**2*e**2*f*Piecewise(((b/(4*c) + x/2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c
*e**2*x**2) + (b**2*e**2/(8*c) - b*d*e/2 + c*d**2/2)*Piecewise((log(-b*e**
2 - 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*
x**2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*c)
+ x)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)), Ne(c*e**2, 0
)), (-2*(-b*d*e - b*e**2*x + c*d**2)**(3/2)/(3*b*e**2), Ne(b*e**2, 0)), (x
*sqrt(-b*d*e + c*d**2), True)) + b**2*e**2*g*Piecewise(((-b*(-b*d*e + c*d*
*2)/(12*c) - b*(b**2*e**2/(8*c) - b*d*e/3 + c*d**2/3)/(2*c))*Piecewise((lo
g(-b*e**2 - 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 -
 c*e**2*x**2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((
b/(2*c) + x)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)) + (b*
x/(12*c) + x**2/3 - (b**2*e**2/(8*c) - b*d*e/3 + c*d**2/3)/(c*e**2))*sqrt(
-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2), Ne(c*e**2, 0)), (2*((b*d*e - c*
d**2)*(-b*d*e - b*e**2*x + c*d**2)**(3/2)/3 + (-b*d*e - b*e**2*x + c*d**2)
**(5/2)/5)/(b**2*e**4), Ne(b*e**2, 0)), (x**2*sqrt(-b*d*e + c*d**2)/2, Tru
e)) - 2*b*c*d*e*f*Piecewise(((b/(4*c) + x/2)*sqrt(-b*d*e - b*e**2*x + c*d*
*2 - c*e**2*x**2) + (b**2*e**2/(8*c) - b*d*e/2 + c*d**2/2)*Piecewise((log(
-b*e**2 - 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c
*e**2*x**2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/
(2*c) + x)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)), Ne(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(264) = 528$.

Time = 0.17 (sec) , antiderivative size = 1751, normalized size of antiderivative = 6.14

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^2,x, algo
rithm="maxima")
```

output

```
5/4*b*c^3*d^3*f*arcsin(2*c*e*x/(2*c*d - b*e) + 4*c*d/(2*c*d - b*e) - b*e/(
2*c*d - b*e))/(-c)^(3/2) - 5/8*c^4*d^4*f*arcsin(2*c*e*x/(2*c*d - b*e) + 4*
c*d/(2*c*d - b*e) - b*e/(2*c*d - b*e))/((-c)^(3/2)*e) - 15/16*b^2*c^2*d^2*
e*f*arcsin(2*c*e*x/(2*c*d - b*e) + 4*c*d/(2*c*d - b*e) - b*e/(2*c*d - b*e)
)/(-c)^(3/2) + 5/16*b^3*c*d*e^2*f*arcsin(2*c*e*x/(2*c*d - b*e) + 4*c*d/(2*
c*d - b*e) - b*e/(2*c*d - b*e))/(-c)^(3/2) - 5/128*b^4*e^3*f*arcsin(2*c*e*
x/(2*c*d - b*e) + 4*c*d/(2*c*d - b*e) - b*e/(2*c*d - b*e))/(-c)^(3/2) + 1/
4*c^4*d^5*g*arcsin(2*c*e*x/(2*c*d - b*e) + 4*c*d/(2*c*d - b*e) - b*e/(2*c*
d - b*e))/((-c)^(3/2)*e^2) - 5/16*b*c^3*d^4*g*arcsin(2*c*e*x/(2*c*d - b*e)
 + 4*c*d/(2*c*d - b*e) - b*e/(2*c*d - b*e))/((-c)^(3/2)*e) + 5/32*b^3*c*d^
2*e*g*arcsin(2*c*e*x/(2*c*d - b*e) + 4*c*d/(2*c*d - b*e) - b*e/(2*c*d - b*
e))/(-c)^(3/2) - 5/64*b^4*d*e^2*g*arcsin(2*c*e*x/(2*c*d - b*e) + 4*c*d/(2*
c*d - b*e) - b*e/(2*c*d - b*e))/(-c)^(3/2) + 3/256*b^5*e^3*g*arcsin(2*c*e*
x/(2*c*d - b*e) + 4*c*d/(2*c*d - b*e) - b*e/(2*c*d - b*e))/((-c)^(3/2)*c)
+ 5/8*sqrt(c*e^2*x^2 + 4*c*d*e*x - b*e^2*x + 3*c*d^2 - b*d*e)*c^2*d^2*f*x
- 5/8*sqrt(c*e^2*x^2 + 4*c*d*e*x - b*e^2*x + 3*c*d^2 - b*d*e)*b*c*d*e*f*x
+ 5/32*sqrt(c*e^2*x^2 + 4*c*d*e*x - b*e^2*x + 3*c*d^2 - b*d*e)*b^2*e^2*f*x
 + 1/16*sqrt(c*e^2*x^2 + 4*c*d*e*x - b*e^2*x + 3*c*d^2 - b*d*e)*b*c*d^2*g*
x - 1/4*sqrt(c*e^2*x^2 + 4*c*d*e*x - b*e^2*x + 3*c*d^2 - b*d*e)*c^2*d^3*g*
x/e + 1/8*sqrt(c*e^2*x^2 + 4*c*d*e*x - b*e^2*x + 3*c*d^2 - b*d*e)*b^2*d...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3630 vs. 2(264) = 528.

Time = 1.22 (sec) , antiderivative size = 3630, normalized size of antiderivative = 12.74

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^2,x, algo
rithm="giac")
```

output

```
-1/1920*(15*(160*c^5*d^4*e*f*sgn(1/(e*x + d))*sgn(e) - 320*b*c^4*d^3*e^2*f
*sgn(1/(e*x + d))*sgn(e) + 240*b^2*c^3*d^2*e^3*f*sgn(1/(e*x + d))*sgn(e) -
 80*b^3*c^2*d*e^4*f*sgn(1/(e*x + d))*sgn(e) + 10*b^4*c*e^5*f*sgn(1/(e*x +
d))*sgn(e) - 64*c^5*d^5*g*sgn(1/(e*x + d))*sgn(e) + 80*b*c^4*d^4*e*g*sgn(1
/(e*x + d))*sgn(e) - 40*b^3*c^2*d^2*e^3*g*sgn(1/(e*x + d))*sgn(e) + 20*b^4
*c*d*e^4*g*sgn(1/(e*x + d))*sgn(e) - 3*b^5*e^5*g*sgn(1/(e*x + d))*sgn(e))*
arctan(sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))/sqrt(c))/(c^(5/2)*e^3) +
 (2400*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^4*c^5*sqrt(-c + 2*c*d/(e*x +
d) - b*e/(e*x + d))*d^4*e*f*sgn(1/(e*x + d))*sgn(e) + 9280*(c - 2*c*d/(e*x
 + d) + b*e/(e*x + d))^3*c^6*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^
4*e*f*sgn(1/(e*x + d))*sgn(e) - 20480*(c - 2*c*d/(e*x + d) + b*e/(e*x + d)
)^2*c^7*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^4*e*f*sgn(1/(e*x + d)
)*sgn(e) - 2400*c^9*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^4*e*f*sgn
(1/(e*x + d))*sgn(e) - 11200*c^8*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3
/2)*d^4*e*f*sgn(1/(e*x + d))*sgn(e) - 4800*b*(c - 2*c*d/(e*x + d) + b*e/(e
*x + d))^4*c^4*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^3*e^2*f*sgn(1/
(e*x + d))*sgn(e) - 18560*b*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^3*c^5*sq
rt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^3*e^2*f*sgn(1/(e*x + d))*sgn(e)
 + 40960*b*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^2*c^6*sqrt(-c + 2*c*d/(e*
x + d) - b*e/(e*x + d))*d^3*e^2*f*sgn(1/(e*x + d))*sgn(e) + 4800*b*c^8*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{{\left(d+e\,x\right)}^2}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^2,x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^2, x
)
```

Reduce [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 1875, normalized size of antiderivative = 6.58

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^2}\,dx=\text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^2,x)
```

output

```
(i*( - 45*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*b**6*e**6*g + 390*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
b*e + 2*c*d))*b**5*c*d*e**5*g + 150*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e
*x)*i)/sqrt( - b*e + 2*c*d))*b**5*c*e**6*f - 1200*sqrt(c)*asinh((sqrt( - b
*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**2*d**2*e**4*g - 1500*sq
rt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c**2
*d*e**5*f + 1200*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
+ 2*c*d))*b**3*c**3*d**3*e**3*g + 6000*sqrt(c)*asinh((sqrt( - b*e + c*d -
c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**3*d**2*e**4*f + 1200*sqrt(c)*asinh
((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**4*d**4*e**2*
g - 12000*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*b**2*c**4*d**3*e**3*f - 3360*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b*c**5*d**5*e*g + 12000*sqrt(c)*asinh((sqrt( - b*
e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**5*d**4*e**2*f + 1920*sqrt(c
)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**6*d**6*g -
 4800*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c
**6*d**5*e*f - 45*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqr
t( - b*e + c*d - c*e*x)*b**4*c*e**4*g + 240*sqrt(d + e*x)*sqrt(b*e - 2*c*d
)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d*e**3*g + 150
*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d...
```

3.164 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 292

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^3}\,dx = \frac{5(2cd-be)(8cef-6cdg-beg)(8cd-5be-2cex)\sqrt{d(cd}{192ce^2}$$

$$+\frac{(8cef-6cdg-beg)(cd-be-cex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{24ce^2}$$

$$-\frac{g(cd-be-cex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^2}$$

$$+\frac{5(2cd-be)^3(8cef-6cdg-beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{64c^{3/2}e^2}$$

output
```
5/192*(-b*e+2*c*d)*(-b*e*g-6*c*d*g+8*c*e*f)*(-2*c*e*x-5*b*e+8*c*d)*(d*(-b*
e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c/e^2+1/24*(-b*e*g-6*c*d*g+8*c*e*f)*(-c*e*
x-b*e+c*d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c/e^2-1/4*g*(-c*e*x-b*
e+c*d)^3*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c/e^2+5/64*(-b*e+2*c*d)^3*
(-b*e*g-6*c*d*g+8*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^
2*x^2)^(1/2))/c^(3/2)/e^2
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^3}\,dx=\frac{(2cd-be)^3((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{\sqrt{c}(15b^3e^3g+2b^2c}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^3,x]
```

output
```
((2*c*d - b*e)^3*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((Sqrt[c]*(15*b^
3*e^3*g + 2*b^2*c*e^2*(132*e*f - 118*d*g + 59*e*g*x) - 8*c^3*(72*d^3*g + 1
2*d*e^2*x*(3*f + 2*g*x) - 2*e^3*x^2*(4*f + 3*g*x) - d^2*e*(88*f + 45*g*x))
 + 4*b*c^2*e*(173*d^2*g + 2*e^2*x*(26*f + 17*g*x) - 2*d*e*(106*f + 51*g*x)
)))/((2*c*d - b*e)^3*(d + e*x)^2*(-(c*d) + b*e + c*e*x)^2) - (15*(8*c*e*f
- 6*c*d*g - b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])]
)/((d + e*x)^(5/2)*(-(b*e) + c*(d - e*x))^(5/2))))/(192*c^(3/2)*e^2)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1220, 1127, 1134, 1160, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-beg-6cdg+8cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^2}dx}{e(2cd-be)}+\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{1127}$$

$$\frac{(-beg-6cdg+8cef)\int (cd-be-cex)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{e(2cd-be)}+\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{1134}$$

$$\frac{(-beg-6cdg+8cef)\left(\frac{5}{8}(2cd-be)\int (cd-be-cex)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx+\frac{(-be+cd-cex)(d(cd-be)-be}{4e}}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(-beg-6cdg+8cef)\left(\frac{5}{8}(2cd-be)\left(\frac{1}{2}(2cd-be)\int \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e}\right)\right.}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(-beg-6cdg+8cef)\left(\frac{5}{8}(2cd-be)\left(\frac{1}{2}(2cd-be)\left(\frac{(2cd-be)^2\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x}}{4c}\right.\right.\right.}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(-beg-6cdg+8cef)\left(\frac{5}{8}(2cd-be)\left(\frac{1}{2}(2cd-be)\left(\frac{(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}\right.\right.\right.}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\Bigg(\frac{5}{8}(2cd-be)\Bigg(\frac{1}{2}(2cd-be)\Bigg(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\Bigg)+\frac{(d(cd-be)-}{}}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^3(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

output
```
(2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(e^2*(2*c*d -
b*e)*(d + e*x)^3) + ((8*c*e*f - 6*c*d*g - b*e*g)*(((c*d - b*e - c*e*x)*(d*
(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(4*e) + (5*(2*c*d - b*e)*((d*(c*
d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)/(3*e) + ((2*c*d - b*e)*(((b + 2*c*x)
*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b*e)^2*ArcTa
n[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/
(8*c^(3/2)*e)))/2))/8))/(e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1127
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Int[(a + b*x + c*x^2)^(m + p)/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m] && RationalQ
[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(272) = 544$.

Time = 3.26 (sec) , antiderivative size = 920, normalized size of antiderivative = 3.15

method	result
	$\Bigg(10c\,e^2 \Bigg(\frac{\left(-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5} + \frac{\left(-b\,e^2+2dec\right) \Bigg(-\frac{\left(-2c\,e^2\left(x+\frac{d}{e}\right)-b\,e^2\right.}{}}{}$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^3,x,method=_RET
URNVERBOSE)
```

output

```
g/e^3*(2/3/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(
x+d/e))^(7/2)+10/3*c*e^2/(-b*e^2+2*c*d*e)*(1/5*(-c*e^2*(x+d/e)^2+(-b*e^2+2
*c*d*e)*(x+d/e))^(5/2)+1/2*(-b*e^2+2*c*d*e)*(-1/8*(-2*c*e^2*(x+d/e)-b*e^2+
2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+3/16*(-b*
e^2+2*c*d*e)^2/c/e^2*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*
(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*
e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2
*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2))))))-(d*g-e*f)/e^4*(2/(-b*e^2+2
*c*d*e)/(x+d/e)^3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)+8*c*e^
2/(-b*e^2+2*c*d*e)*(2/3/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e
^2+2*c*d*e)*(x+d/e))^(7/2)+10/3*c*e^2/(-b*e^2+2*c*d*e)*(1/5*(-c*e^2*(x+d/e
)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+1/2*(-b*e^2+2*c*d*e)*(-1/8*(-2*c*e^2*(
x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3
/2)+3/16*(-b*e^2+2*c*d*e)^2/c/e^2*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c
/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e
)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c
/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.78

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^3}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^3,x, algo
rithm="fricas")
```

output

```
[-1/768*(15*(8*(8*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e
^4)*f - (48*c^4*d^4 - 64*b*c^3*d^3*e + 24*b^2*c^2*d^2*e^2 - b^4*e^4)*g)*sq
rt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 -
 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) -
4*(48*c^4*e^3*g*x^3 + 8*(8*c^4*e^3*f - (24*c^4*d*e^2 - 17*b*c^3*e^3)*g)*x^
2 + 8*(88*c^4*d^2*e - 106*b*c^3*d*e^2 + 33*b^2*c^2*e^3)*f - (576*c^4*d^3 -
 692*b*c^3*d^2*e + 236*b^2*c^2*d*e^2 - 15*b^3*c*e^3)*g - 2*(8*(18*c^4*d*e^
2 - 13*b*c^3*e^3)*f - (180*c^4*d^2*e - 204*b*c^3*d*e^2 + 59*b^2*c^2*e^3)*g
)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^2*e^2), -1/384*(15*(8*
(8*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e^4)*f - (48*c^4
*d^4 - 64*b*c^3*d^3*e + 24*b^2*c^2*d^2*e^2 - b^4*e^4)*g)*sqrt(c)*arctan(1/
2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*
e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*(48*c^4*e^3*g*x^3 + 8*(8*c^4
*e^3*f - (24*c^4*d*e^2 - 17*b*c^3*e^3)*g)*x^2 + 8*(88*c^4*d^2*e - 106*b*c^
3*d*e^2 + 33*b^2*c^2*e^3)*f - (576*c^4*d^3 - 692*b*c^3*d^2*e + 236*b^2*c^2
*d*e^2 - 15*b^3*c*e^3)*g - 2*(8*(18*c^4*d*e^2 - 13*b*c^3*e^3)*f - (180*c^4
*d^2*e - 204*b*c^3*d*e^2 + 59*b^2*c^2*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x
 + c*d^2 - b*d*e))/(c^2*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^3}\,dx = \int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{5}{2}}(f+gx)}{(d+ex)^3}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**3,x
)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**3, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^3,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.40

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^3}\,dx = \frac{1}{192}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\Bigg(2\Bigg(4\Bigg(6\,c^2egx+\frac{8\,c^5e^5}{}$$

$$-\frac{5\left(64\,c^4d^3ef-96\,bc^3d^2e^2f+48\,b^2c^2de^3f-8\,b^3ce^4f-48\,c^4d^4g+64\,bc^3d^3eg-24\,b^2c^2d^2e^2g+b^4e^4g\right)\log}{128\,\sqrt{-c}ce|e|}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^3,x, algo
rithm="giac")
```

output
```
1/192*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(6*c^2*e*g*x + (8*c
^5*e^5*f - 24*c^5*d*e^4*g + 17*b*c^4*e^5*g)/(c^3*e^4))*x - (144*c^5*d*e^4*
f - 104*b*c^4*e^5*f - 180*c^5*d^2*e^3*g + 204*b*c^4*d*e^4*g - 59*b^2*c^3*e
^5*g)/(c^3*e^4))*x + (704*c^5*d^2*e^3*f - 848*b*c^4*d*e^4*f + 264*b^2*c^3*
e^5*f - 576*c^5*d^3*e^2*g + 692*b*c^4*d^2*e^3*g - 236*b^2*c^3*d*e^4*g + 15
*b^3*c^2*e^5*g)/(c^3*e^4)) - 5/128*(64*c^4*d^3*e*f - 96*b*c^3*d^2*e^2*f +
48*b^2*c^2*d*e^3*f - 8*b^3*c*e^4*f - 48*c^4*d^4*g + 64*b*c^3*d^3*e*g - 24*
b^2*c^2*d^2*e^2*g + b^4*e^4*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-
c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqrt(-c)*c*e*abs(
e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^3}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^3,x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^3, x
)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 1300, normalized size of antiderivative = 4.45

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^3}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^3,x)
```

output

```
(i*(15*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b**5*e**5*g - 30*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e
+ 2*c*d))*b**4*c*d*e**4*g - 120*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b**4*c*e**5*f - 360*sqrt(c)*asinh((sqrt( - b*e +
c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**2*d**2*e**3*g + 960*sqrt(c)*
asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**2*d*e**
4*f + 1680*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*
d))*b**2*c**3*d**3*e**2*g - 2880*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)
*i)/sqrt( - b*e + 2*c*d))*b**2*c**3*d**2*e**3*f - 2640*sqrt(c)*asinh((sqrt
( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*d**4*e*g + 3840*sqr
t(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*d**
3*e**2*f + 1440*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
 2*c*d))*c**5*d**5*g - 1920*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/s
qrt( - b*e + 2*c*d))*c**5*d**4*e*f + 15*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**3*g - 236*sqrt(d +
 e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*
*2*c**2*d*e**2*g + 264*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d
)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**3*f + 118*sqrt(d + e*x)*sqrt(b*e
 - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**3*g
*x + 692*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b...
```

3.165 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 290

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = &-\frac{5(6cef-8cdg+beg)(8cd-5be-2cex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{24e^2}\\&-\frac{(6cef-8cdg+beg)(cd-be-cex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e^2(2cd-be)}\\&-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(2cd-be)(d+ex)^4}\\&-\frac{5(2cd-be)^2(6cef-8cdg+beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8\sqrt{c}e^2}\end{aligned}$$

output
```
-5/24*(b*e*g-8*c*d*g+6*c*e*f)*(-2*c*e*x-5*b*e+8*c*d)*(d*(-b*e+c*d)-b*e^2*x
-c*e^2*x^2)^(1/2)/e^2-1/3*(b*e*g-8*c*d*g+6*c*e*f)*(-c*e*x-b*e+c*d)^2*(d*(-
b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)-2*(-d*g+e*f)*(d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)/(e*x+d)^4-5/8*(-b*e+2*c*d)^2
*(b*e*g-8*c*d*g+6*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^
2*x^2)^(1/2))/c^(1/2)/e^2
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \frac{((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{3b^2e^2(-16ef+27dg+11egx)+2bc}{}\right.}{}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^4,x]
```

output

```
(((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((3*b^2*e^2*(-16*e*f + 27*d*g +
11*e*g*x) + 2*b*c*e*(-176*d^2*g + d*e*(123*f - 67*g*x) + e^2*x*(27*f + 13*
g*x)) + 4*c^2*(94*d^3*g + e^3*x^2*(3*f + 2*g*x) - d*e^2*x*(21*f + 10*g*x)
+ d^2*e*(-72*f + 34*g*x)))/((d + e*x)^3*(-(c*d) + b*e + c*e*x)^2) + (15*(-
2*c*d + b*e)^2*(6*c*e*f - 8*c*d*g + b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/
(Sqrt[c]*Sqrt[d + e*x])])/(Sqrt[c]*(d + e*x)^(5/2)*(-(b*e) + c*(d - e*x))^
(5/2))))/(24*e^2)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1220, 1130, 1131, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(beg-8cdg+6cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^3}dx}{e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow 1130$$

$$-\frac{(beg-8cdg+6cef)\left(5c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{d+ex}dx+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e(d+ex)^2}\right)}{e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow 1131$$

$$-\frac{(beg-8cdg+6cef)\Bigg(5c\bigg(\frac{1}{2}(2cd-be)\int \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx+\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e}\bigg)+\frac{2(d(cd-}{}}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow 1087$$

$$-\frac{(beg-8cdg+6cef)\Bigg(5c\Bigg(\frac{1}{2}(2cd-be)\Bigg(\frac{(2cd-be)^2\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\Bigg)+\frac{(d(c}{}}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow 1092$$

$$-\frac{(beg-8cdg+6cef)\Bigg(5c\Bigg(\frac{1}{2}(2cd-be)\Bigg(\frac{(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c}+\frac{(b+2cx)}{}}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow 217$$

$$-\frac{\Bigg(5c\Bigg(\frac{1}{2}(2cd-be)\Bigg(\frac{(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{3/2}e}+\frac{(b+2cx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c}\Bigg)+\frac{(d(cd-be)-be^2x-ce^2}{3e}}{e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^4(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

output

```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(e^2*(2*c*d -
 b*e)*(d + e*x)^4) - ((6*c*e*f - 8*c*d*g + b*e*g)*((2*(d*(c*d - b*e) - b*e
^2*x - c*e^2*x^2)^(5/2))/(e*(d + e*x)^2) + 5*c*((d*(c*d - b*e) - b*e^2*x -
 c*e^2*x^2)^(3/2)/(3*e) + ((2*c*d - b*e)*(((b + 2*c*x)*Sqrt[d*(c*d - b*e)
- b*e^2*x - c*e^2*x^2])/(4*c) + ((2*c*d - b*e)^2*ArcTan[(e*(b + 2*c*x))/(2
*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(8*c^(3/2)*e)))/2)))
/(e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1130

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(272) = 544$.

Time = 4.23 (sec) , antiderivative size = 1088, normalized size of antiderivative = 3.75

method	result	size
default	Expression too large to display	1088

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^4,x,method=_RET
URNVERBOSE)
```

output
```
g/e^4*(2/(-b*e^2+2*c*d*e)/(x+d/e)^3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+
d/e))^(7/2)+8*c*e^2/(-b*e^2+2*c*d*e)*(2/3/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e
^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)+10/3*c*e^2/(-b*e^2+2*c*d*e)*(
1/5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+1/2*(-b*e^2+2*c*d*e)
*(-1/8*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*
c*d*e)*(x+d/e))^(3/2)+3/16*(-b*e^2+2*c*d*e)^2/c/e^2*(-1/4*(-2*c*e^2*(x+d/e
)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1
/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*
(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2))
)))))-(d*g-e*f)/e^5*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^4*(-c*e^2*(x+d/e)^2+(-b*e
^2+2*c*d*e)*(x+d/e))^(7/2)-6*c*e^2/(-b*e^2+2*c*d*e)*(2/(-b*e^2+2*c*d*e)/(x
+d/e)^3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)+8*c*e^2/(-b*e^2+
2*c*d*e)*(2/3/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e
)*(x+d/e))^(7/2)+10/3*c*e^2/(-b*e^2+2*c*d*e)*(1/5*(-c*e^2*(x+d/e)^2+(-b*e^
2+2*c*d*e)*(x+d/e))^(5/2)+1/2*(-b*e^2+2*c*d*e)*(-1/8*(-2*c*e^2*(x+d/e)-b*e
^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+3/16*(
-b*e^2+2*c*d*e)^2/c/e^2*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e
^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/
(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*
e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 931, normalized size of antiderivative = 3.21

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^4}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^4,x, algo
rithm="fricas")
```

output

```
[-1/96*(15*(6*(4*c^3*d^3*e - 4*b*c^2*d^2*e^2 + b^2*c*d*e^3)*f - (32*c^3*d^
4 - 36*b*c^2*d^3*e + 12*b^2*c*d^2*e^2 - b^3*d*e^3)*g + (6*(4*c^3*d^2*e^2 -
 4*b*c^2*d*e^3 + b^2*c*e^4)*f - (32*c^3*d^3*e - 36*b*c^2*d^2*e^2 + 12*b^2*
c*d*e^3 - b^3*e^4)*g)*x)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*
d^2 + 4*b*c*d*e + b^2*e^2 + 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(
2*c*e*x + b*e)*sqrt(-c)) - 4*(8*c^3*e^3*g*x^3 + 2*(6*c^3*e^3*f - (20*c^3*d
*e^2 - 13*b*c^2*e^3)*g)*x^2 - 6*(48*c^3*d^2*e - 41*b*c^2*d*e^2 + 8*b^2*c*e
^3)*f + (376*c^3*d^3 - 352*b*c^2*d^2*e + 81*b^2*c*d*e^2)*g - (6*(14*c^3*d*
e^2 - 9*b*c^2*e^3)*f - (136*c^3*d^2*e - 134*b*c^2*d*e^2 + 33*b^2*c*e^3)*g)
*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c*e^3*x + c*d*e^2), 1/48*
(15*(6*(4*c^3*d^3*e - 4*b*c^2*d^2*e^2 + b^2*c*d*e^3)*f - (32*c^3*d^4 - 36*
b*c^2*d^3*e + 12*b^2*c*d^2*e^2 - b^3*d*e^3)*g + (6*(4*c^3*d^2*e^2 - 4*b*c^
2*d*e^3 + b^2*c*e^4)*f - (32*c^3*d^3*e - 36*b*c^2*d^2*e^2 + 12*b^2*c*d*e^3
 - b^3*e^4)*g)*x)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b
*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e
)) + 2*(8*c^3*e^3*g*x^3 + 2*(6*c^3*e^3*f - (20*c^3*d*e^2 - 13*b*c^2*e^3)*g
)*x^2 - 6*(48*c^3*d^2*e - 41*b*c^2*d*e^2 + 8*b^2*c*e^3)*f + (376*c^3*d^3 -
 352*b*c^2*d^2*e + 81*b^2*c*d*e^2)*g - (6*(14*c^3*d*e^2 - 9*b*c^2*e^3)*f -
 (136*c^3*d^2*e - 134*b*c^2*d*e^2 + 33*b^2*c*e^3)*g)*x)*sqrt(-c*e^2*x^2 -
b*e^2*x + c*d^2 - b*d*e))/(c*e^3*x + c*d*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^4}\,dx=\int \frac{\left(-(d+ex)\,(be-cd+cex)\right)^{\frac{5}{2}}(f+gx)}{(d+ex)^4}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**4,x
)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**4, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^4,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.73

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^4}\,dx = \frac{1}{24}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\,\Bigg(2\,\Bigg(4\,c^2gx+\frac{6\,c^4e^4f-2}{}$$

$$+\frac{5\,(24\,c^3d^2ef-24\,bc^2de^2f+6\,b^2ce^3f-32\,c^3d^3g+36\,bc^2d^2eg-12\,b^2cde^2g+b^3e^3g)\log\Big(\Big|bcd^2e^2-2\,(\sqrt{-}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^4,x, algo
rithm="giac")
```

output
```
1/24*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*c^2*g*x + (6*c^4*e^4
*f - 24*c^4*d*e^3*g + 13*b*c^3*e^4*g)/(c^2*e^4))*x - (96*c^4*d*e^3*f - 54*
b*c^3*e^4*f - 184*c^4*d^2*e^2*g + 160*b*c^3*d*e^3*g - 33*b^2*c^2*e^4*g)/(c
^2*e^4)) + 5/48*(24*c^3*d^2*e*f - 24*b*c^2*d*e^2*f + 6*b^2*c*e^3*f - 32*c^
3*d^3*g + 36*b*c^2*d^2*e*g - 12*b^2*c*d*e^2*g + b^3*e^3*g)*log(abs(b*c*d^2
*e^2 - 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqr
t(-c)*c*d^2*abs(e) - 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2
 - b*d*e))*b*sqrt(-c)*d*e*abs(e) - 4*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b
*e^2*x + c*d^2 - b*d*e))^2*c*d*e - (sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e
^2*x + c*d^2 - b*d*e))^2*b*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e
^2*x + c*d^2 - b*d*e))^3*sqrt(-c)*abs(e)))/(sqrt(-c)*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^4}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{{(d+e\,x)}^4}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^4,x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^4, x
)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1826, normalized size of antiderivative = 6.30

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^4}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^4,x)
```

output

```
(i*( - 120*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*
d))*b**4*d*e**4*g - 120*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
 - b*e + 2*c*d))*b**4*e**5*g*x + 1680*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c*d**2*e**3*g - 720*sqrt(c)*asinh((sqr
t( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c*d*e**4*f + 1680*sq
rt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c*d*
e**4*g*x - 720*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
2*c*d))*b**3*c*e**5*f*x - 7200*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i
)/sqrt( - b*e + 2*c*d))*b**2*c**2*d**3*e**2*g + 4320*sqrt(c)*asinh((sqrt(
- b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d**2*e**3*f - 7200
*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c
**2*d**2*e**3*g*x + 4320*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt
( - b*e + 2*c*d))*b**2*c**2*d*e**4*f*x + 12480*sqrt(c)*asinh((sqrt( - b*e
+ c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**3*d**4*e*g - 8640*sqrt(c)*asi
nh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**3*d**3*e**2*f
 + 12480*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*b*c**3*d**3*e**2*g*x - 8640*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)
/sqrt( - b*e + 2*c*d))*b*c**3*d**2*e**3*f*x - 7680*sqrt(c)*asinh((sqrt( -
b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**5*g + 5760*sqrt(c)*asi
nh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**4*e*f -...
```

3.166 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 297

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx = \frac{5c(4cef-10cdg+3beg)(8cd-5be-2cex)\sqrt{d(cd-be)-}}{12e^2(2cd-be)}$$
$$+\frac{2(4cef-10cdg+3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(2cd-be)(d+ex)^3}$$
$$-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(2cd-be)(d+ex)^5}$$
$$+\frac{5\sqrt{c}(2cd-be)(4cef-10cdg+3beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4e^2}$$

output

```
5/12*c*(3*b*e*g-10*c*d*g+4*c*e*f)*(-2*c*e*x-5*b*e+8*c*d)*(d*(-b*e+c*d)-b*e
^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)+2/3*(3*b*e*g-10*c*d*g+4*c*e*f)*(d*(
-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(-b*e+2*c*d)/(e*x+d)^3-2/3*(-d*g+e*
f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)/(e*x+d)^5+5/4*c
^(1/2)*(-b*e+2*c*d)*(3*b*e*g-10*c*d*g+4*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(
-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 3.44 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx = \frac{((d+ex)(-be+c(d-ex)))^{5/2}\big(-\sqrt{cd-be-cex}(bce(-1}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^5,x]
```

output
```
(((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*(-(Sqrt[c*d - b*e - c*e*x]*(b*c*
e*(-147*d^2*g + d*e*(24*f - 206*g*x) + e^2*x*(56*f - 27*g*x)) + 8*b^2*e^2*
(2*d*g + e*(f + 3*g*x)) + 2*c^2*(118*d^3*g - 23*d^2*e*(2*f - 7*g*x) - 3*e^
3*x^2*(2*f + g*x) + 4*d*e^2*x*(-17*f + 6*g*x)))) + 15*Sqrt[-c]*(2*c*d - b*
e)*(4*c*e*f - 10*c*d*g + 3*b*e*g)*(d + e*x)^(3/2)*Log[-(Sqrt[-c]*Sqrt[d +
e*x]) + Sqrt[c*d - b*e - c*e*x]]))/(12*e^2*(d + e*x)^4*(-(b*e) + c*(d - e*
x))^(5/2))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1220, 1125, 2192, 27, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^5}dx$$

$$\Big\downarrow \text{ 1220}$$

$$-\frac{(3beg-10cdg+4cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^4}dx}{3e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \text{ 1125}$$

$$-\frac{(3beg-10cdg+4cef)\left(-\frac{\int \frac{c^3x^2e^8-c^2(4cd-3be)xe^7+c\left(7c^2d^2-9bced+3b^2e^2\right)e^6}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{e^6}-\frac{2(2cd-be)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)}{3e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \; 2192$$

$$-\frac{(3beg-10cdg+4cef)\left(-\frac{-\frac{\int -\frac{c^2e^8(2(5cd-3be)(3cd-2be)-ce(16cd-9be)x)}{2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2ce^2}-\frac{1}{2}c^2e^6x\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^6}-\frac{2(2cd-be)^2\sqrt{d(cd-}}{e(d+}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \; 27$$

$$-\frac{(3beg-10cdg+4cef)\left(-\frac{\frac{1}{4}ce^6\int \frac{2(5cd-3be)(3cd-2be)-ce(16cd-9be)x}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{1}{2}c^2e^6x\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^6}-\frac{2(2cd-be)^2\sqrt{d(cd-be)}}{e(d+ex)}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \; 1160$$

$$-\frac{(3beg-10cdg+4cef)\left(-\frac{\frac{1}{4}ce^6\left(\frac{15}{2}(2cd-be)^2\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{(16cd-9be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}\right)-\frac{1}{2}c^2e^6x\sqrt{d(cd-be}}{e^6}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \; 1092$$

$$-\frac{(3beg-10cdg+4cef)\left(-\frac{\frac{1}{4}ce^6\left(15(2cd-be)^2\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)+\frac{(16cd-9be)\sqrt{d(cd-be}}{e}\right.}{e^6}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow 217$$

$$-\frac{\left(-\frac{\frac{1}{4}ce^6\left(\frac{15(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2\sqrt{c}e}+\frac{(16cd-9be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}\right)-\frac{1}{2}c^2e^6x\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^6}-\frac{2(2c}{}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^5(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^5,x]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(3*e^2*(2*c*d
 - b*e)*(d + e*x)^5) - ((4*c*e*f - 10*c*d*g + 3*b*e*g)*((-2*(2*c*d - b*e)^
2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e*(d + e*x)) - (-1/2*(c^2*e^
6*x*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) + (c*e^6*(((16*c*d - 9*b*e)
*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/e + (15*(2*c*d - b*e)^2*ArcTan
[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(
2*Sqrt[c]*e)))/4)/e^6))/(3*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. $2(277) = 554$.

Time = 5.32 (sec) , antiderivative size = 1256, normalized size of antiderivative = 4.23

method	result	size
default	Expression too large to display	1256

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^5,x,method=_RET
URNVERBOSE)
```

output

```
g/e^5*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^4*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x
+d/e))^(7/2)-6*c*e^2/(-b*e^2+2*c*d*e)*(2/(-b*e^2+2*c*d*e)/(x+d/e)^3*(-c*e^
2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)+8*c*e^2/(-b*e^2+2*c*d*e)*(2/3/
(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/
2)+10/3*c*e^2/(-b*e^2+2*c*d*e)*(1/5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+
d/e))^(5/2)+1/2*(-b*e^2+2*c*d*e)*(-1/8*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/
e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+3/16*(-b*e^2+2*c*d*e
)^2/c/e^2*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(
-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*
arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+
(-b*e^2+2*c*d*e)*(x+d/e))^(1/2))))))))-(d*g-e*f)/e^6*(-2/3/(-b*e^2+2*c*d*e
)/(x+d/e)^5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)-4/3*c*e^2/(-
b*e^2+2*c*d*e)*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^4*(-c*e^2*(x+d/e)^2+(-b*e^2+2*
c*d*e)*(x+d/e))^(7/2)-6*c*e^2/(-b*e^2+2*c*d*e)*(2/(-b*e^2+2*c*d*e)/(x+d/e)
^3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)+8*c*e^2/(-b*e^2+2*c*d
*e)*(2/3/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+
d/e))^(7/2)+10/3*c*e^2/(-b*e^2+2*c*d*e)*(1/5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c
*d*e)*(x+d/e))^(5/2)+1/2*(-b*e^2+2*c*d*e)*(-1/8*(-2*c*e^2*(x+d/e)-b*e^2+2*
d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+3/16*(-b*e^
2+2*c*d*e)^2/c/e^2*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2...
```

Fricas [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 951, normalized size of antiderivative = 3.20

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^5,x, algo
rithm="fricas")
```

output

```
[-1/48*(15*((4*(2*c^2*d*e^3 - b*c*e^4)*f - (20*c^2*d^2*e^2 - 16*b*c*d*e^3
+ 3*b^2*e^4)*g)*x^2 + 4*(2*c^2*d^3*e - b*c*d^2*e^2)*f - (20*c^2*d^4 - 16*b
*c*d^3*e + 3*b^2*d^2*e^2)*g + 2*(4*(2*c^2*d^2*e^2 - b*c*d*e^3)*f - (20*c^2
*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*g)*x)*sqrt(-c)*log(8*c^2*e^2*x^2 +
8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*
x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*(6*c^2*e^3*g*x^3 + 3*(4*c
^2*e^3*f - (16*c^2*d*e^2 - 9*b*c*e^3)*g)*x^2 + 4*(23*c^2*d^2*e - 6*b*c*d*e
^2 - 2*b^2*e^3)*f - (236*c^2*d^3 - 147*b*c*d^2*e + 16*b^2*d*e^2)*g + 2*(4*
(17*c^2*d*e^2 - 7*b*c*e^3)*f - (161*c^2*d^2*e - 103*b*c*d*e^2 + 12*b^2*e^3
)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(e^4*x^2 + 2*d*e^3*x +
 d^2*e^2), -1/24*(15*((4*(2*c^2*d*e^3 - b*c*e^4)*f - (20*c^2*d^2*e^2 - 16*
b*c*d*e^3 + 3*b^2*e^4)*g)*x^2 + 4*(2*c^2*d^3*e - b*c*d^2*e^2)*f - (20*c^2*
d^4 - 16*b*c*d^3*e + 3*b^2*d^2*e^2)*g + 2*(4*(2*c^2*d^2*e^2 - b*c*d*e^3)*f
 - (20*c^2*d^3*e - 16*b*c*d^2*e^2 + 3*b^2*d*e^3)*g)*x)*sqrt(c)*arctan(1/2*
sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^
2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*(6*c^2*e^3*g*x^3 + 3*(4*c^2*e^
3*f - (16*c^2*d*e^2 - 9*b*c*e^3)*g)*x^2 + 4*(23*c^2*d^2*e - 6*b*c*d*e^2 -
2*b^2*e^3)*f - (236*c^2*d^3 - 147*b*c*d^2*e + 16*b^2*d*e^2)*g + 2*(4*(17*c
^2*d*e^2 - 7*b*c*e^3)*f - (161*c^2*d^2*e - 103*b*c*d*e^2 + 12*b^2*e^3)*g)*
x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(e^4*x^2 + 2*d*e^3*x + d...
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{5}{2}}(f+gx)}{(d+ex)^5}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**5,x
)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**5, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^5,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(277) = 554$.

Time = 0.66 (sec) , antiderivative size = 1076, normalized size of antiderivative = 3.62

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^5,x, algo
rithm="giac")
```

output

```
-1/12*(15*(8*c^3*d*e*f*sgn(1/(e*x + d))*sgn(e) - 4*b*c^2*e^2*f*sgn(1/(e*x
+ d))*sgn(e) - 20*c^3*d^2*g*sgn(1/(e*x + d))*sgn(e) + 16*b*c^2*d*e*g*sgn(1
/(e*x + d))*sgn(e) - 3*b^2*c*e^2*g*sgn(1/(e*x + d))*sgn(e))*arctan(sqrt(-c
 + 2*c*d/(e*x + d) - b*e/(e*x + d))/sqrt(c))/(sqrt(c)*e^3) - 3*(8*c^4*sqrt
(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d*e*f*sgn(1/(e*x + d))*sgn(e) + 8*c
^3*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*d*e*f*sgn(1/(e*x + d))*sgn
(e) - 4*b*c^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*e^2*f*sgn(1/(e*x
+ d))*sgn(e) - 4*b*c^2*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*e^2*f*
sgn(1/(e*x + d))*sgn(e) - 36*c^4*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)
)*d^2*g*sgn(1/(e*x + d))*sgn(e) - 44*c^3*(-c + 2*c*d/(e*x + d) - b*e/(e*x
+ d))^(3/2)*d^2*g*sgn(1/(e*x + d))*sgn(e) + 32*b*c^3*sqrt(-c + 2*c*d/(e*x
+ d) - b*e/(e*x + d))*d*e*g*sgn(1/(e*x + d))*sgn(e) + 40*b*c^2*(-c + 2*c*d
/(e*x + d) - b*e/(e*x + d))^(3/2)*d*e*g*sgn(1/(e*x + d))*sgn(e) - 7*b^2*c^
2*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*e^2*g*sgn(1/(e*x + d))*sgn(e)
 - 9*b^2*c*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*e^2*g*sgn(1/(e*x +
 d))*sgn(e))/((2*c*d/(e*x + d) - b*e/(e*x + d))^2*e^3) - 8*(12*c^2*sqrt(-c
 + 2*c*d/(e*x + d) - b*e/(e*x + d))*d*e^7*f*sgn(1/(e*x + d))*sgn(e) - 2*c*
(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*d*e^7*f*sgn(1/(e*x + d))*sgn(
e) - 6*b*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*e^8*f*sgn(1/(e*x + d
))*sgn(e) + b*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*e^8*f*sgn(1/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{{(d+e\,x)}^5}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^5,x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^5, x
)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 2027, normalized size of antiderivative = 6.82

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^5,x)
```

output

```
(i*( - 360*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*
d))*b**3*d**2*e**3*g - 720*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sq
rt( - b*e + 2*c*d))*b**3*d*e**4*g*x - 360*sqrt(c)*asinh((sqrt( - b*e + c*d
 - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*e**5*g*x**2 + 2640*sqrt(c)*asinh((
sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d**3*e**2*g - 4
80*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2
*c*d**2*e**3*f + 5280*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
 b*e + 2*c*d))*b**2*c*d**2*e**3*g*x - 960*sqrt(c)*asinh((sqrt( - b*e + c*d
 - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d*e**4*f*x + 2640*sqrt(c)*asinh(
(sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d*e**4*g*x**2
- 480*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b
**2*c*e**5*f*x**2 - 6240*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt
( - b*e + 2*c*d))*b*c**2*d**4*e*g + 1920*sqrt(c)*asinh((sqrt( - b*e + c*d
- c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**3*e**2*f - 12480*sqrt(c)*asinh
((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**3*e**2*g*x
 + 3840*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*b*c**2*d**2*e**3*f*x - 6240*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/
sqrt( - b*e + 2*c*d))*b*c**2*d**2*e**3*g*x**2 + 1920*sqrt(c)*asinh((sqrt(
- b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**4*f*x**2 + 4800*
sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3...
```

3.167 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 281

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx &= \frac{c^2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2} \\ &\quad - \frac{2c(cef-5cdg+2beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)} \\ &\quad + \frac{2(cef-3cdg+beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^3} \\ &\quad - \frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(d+ex)^5} \\ &\quad + \frac{c^{3/2}(2cef-12cdg+5beg)\arctan\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{c}(d+ex)}\right)}{e^2}\end{aligned}$$

output
```
c^2*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2-2*c*(2*b*e*g-5*c*d*g+c*e*
f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)+2/3*(b*e*g-3*c*d*g+c
*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^3-2/5*(-d*g+e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(e*x+d)^5+c^(3/2)*(5*b*e*g-12*c
*d*g+2*c*e*f)*arctan(1/c^(1/2)/(e*x+d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1
/2))/e^2
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx = \frac{((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{-2b^2e^2(3ef+2dg+5egx)-2bce(16}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^6,x]
```

output
```
(((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((-2*b^2*e^2*(3*e*f + 2*d*g + 5*
e*g*x) - 2*b*c*e*(16*d^2*g - d*e*(f - 39*g*x) + e^2*x*(11*f + 35*g*x)) + c
^2*(141*d^3*g + e^3*x^2*(-46*f + 15*g*x) + 3*d*e^2*x*(-16*f + 77*g*x) + d^
2*e*(-26*f + 333*g*x)))/((d + e*x)^5*(-(c*d) + b*e + c*e*x)^2) + (15*c^(3/
2)*(5*b*e*g + 2*c*(e*f - 6*d*g))*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*S
qrt[d + e*x])])/((d + e*x)^(5/2)*(-(b*e) + c*(d - e*x))^(5/2))))/(15*e^2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1220, 1130, 1125, 27, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^6}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(5beg-12cdg+2cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^5}dx}{5e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^6(2cd-be)}$$

$$\Big\downarrow \text{1130}$$

$$-\frac{(5beg-12cdg+2cef)\left(-\frac{5}{3}c\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^3}dx-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e(d+ex)^4}\right)}{5e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^6(2cd-be)}$$

$$\Big\downarrow \text{ 1125}$$

$$-\frac{(5beg-12cdg+2cef)\left(-\frac{5}{3}c\left(-\frac{\int\frac{ce^4(3cd-2be-cex)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{e^4}-\frac{2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\left(d(cd-be)-be^2x-ce\right.}{3e(d+ex)^4}\right.}{5e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^6(2cd-be)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{(5beg-12cdg+2cef)\left(-\frac{5}{3}c\left(-c\int\frac{3cd-2be-cex}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\left(d(cd-be)-be^2x\right.}{3e(d+ex}\right.}{5e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^6(2cd-be)}$$

$$\Big\downarrow \text{ 1160}$$

$$-\frac{(5beg-12cdg+2cef)\left(-\frac{5}{3}c\left(-c\left(\frac{3}{2}(2cd-be)\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}\right)-\frac{2(2cd-be)\sqrt{}}{}\right.\right.}{5e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^6(2cd-be)}$$

$$\Big\downarrow \text{ 1092}$$

$$-\frac{(5beg-12cdg+2cef)\left(-\frac{5}{3}c\left(-c\left(3(2cd-be)\int\frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)+\frac{\sqrt{d(cd}}{}\right.\right.\right.}{5e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^6(2cd-be)}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{\left(-\frac{5}{3}c\left(-c\left(\frac{3(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2\sqrt{c}e}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}\right)-\frac{2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)\right.}{5e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^6(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^6,x]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(5*e^2*(2*c*d
 - b*e)*(d + e*x)^6) - ((2*c*e*f - 12*c*d*g + 5*b*e*g)*((-2*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(5/2))/(3*e*(d + e*x)^4) - (5*c*((-2*(2*c*d - b*e)
*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e*(d + e*x)) - c*(Sqrt[d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2]/e + (3*(2*c*d - b*e)*ArcTan[(e*(b + 2*c*x))
/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(2*Sqrt[c]*e))))/
3))/(5*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1423 vs. $2(263) = 526$.

Time = 7.12 (sec) , antiderivative size = 1424, normalized size of antiderivative = 5.07

method	result	size
default	Expression too large to display	1424

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^6,x,method=_RET
URNVERBOSE)
```

output
```
g/e^6*(-2/3/(-b*e^2+2*c*d*e)/(x+d/e)^5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*
(x+d/e))^(7/2)-4/3*c*e^2/(-b*e^2+2*c*d*e)*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^4*(
-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)-6*c*e^2/(-b*e^2+2*c*d*e)*
(2/(-b*e^2+2*c*d*e)/(x+d/e)^3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^
(7/2)+8*c*e^2/(-b*e^2+2*c*d*e)*(2/3/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+
d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)+10/3*c*e^2/(-b*e^2+2*c*d*e)*(1/5*(-
c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(5/2)+1/2*(-b*e^2+2*c*d*e)*(-1/8
*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)
*(x+d/e))^(3/2)+3/16*(-b*e^2+2*c*d*e)^2/c/e^2*(-1/4*(-2*c*e^2*(x+d/e)-b*e^
2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b
*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^
2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)))))))
)-(d*g-e*f)/e^7*(-2/5/(-b*e^2+2*c*d*e)/(x+d/e)^6*(-c*e^2*(x+d/e)^2+(-b*e^2
+2*c*d*e)*(x+d/e))^(7/2)-2/5*c*e^2/(-b*e^2+2*c*d*e)*(-2/3/(-b*e^2+2*c*d*e)
/(x+d/e)^5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)-4/3*c*e^2/(-b
*e^2+2*c*d*e)*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^4*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c
*d*e)*(x+d/e))^(7/2)-6*c*e^2/(-b*e^2+2*c*d*e)*(2/(-b*e^2+2*c*d*e)/(x+d/e)^
3*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)+8*c*e^2/(-b*e^2+2*c*d*
e)*(2/3/(-b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d
/e))^(7/2)+10/3*c*e^2/(-b*e^2+2*c*d*e)*(1/5*(-c*e^2*(x+d/e)^2+(-b*e^2+2...
```

Fricas [A] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.26

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^6,x, algo
rithm="fricas")
```

output

```
[1/60*(15*(2*c^2*d^3*e*f + (2*c^2*e^4*f - (12*c^2*d*e^3 - 5*b*c*e^4)*g)*x^
3 + 3*(2*c^2*d*e^3*f - (12*c^2*d^2*e^2 - 5*b*c*d*e^3)*g)*x^2 - (12*c^2*d^4
 - 5*b*c*d^3*e)*g + 3*(2*c^2*d^2*e^2*f - (12*c^2*d^3*e - 5*b*c*d^2*e^2)*g)
*x)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2
*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-
c)) + 4*(15*c^2*e^3*g*x^3 - (46*c^2*e^3*f - 7*(33*c^2*d*e^2 - 10*b*c*e^3)*
g)*x^2 - 2*(13*c^2*d^2*e - b*c*d*e^2 + 3*b^2*e^3)*f + (141*c^2*d^3 - 32*b*
c*d^2*e - 4*b^2*d*e^2)*g - (2*(24*c^2*d*e^2 + 11*b*c*e^3)*f - (333*c^2*d^2
*e - 78*b*c*d*e^2 - 10*b^2*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 -
b*d*e))/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2), 1/30*(15*(2*c^2*d
^3*e*f + (2*c^2*e^4*f - (12*c^2*d*e^3 - 5*b*c*e^4)*g)*x^3 + 3*(2*c^2*d*e^3
*f - (12*c^2*d^2*e^2 - 5*b*c*d*e^3)*g)*x^2 - (12*c^2*d^4 - 5*b*c*d^3*e)*g
+ 3*(2*c^2*d^2*e^2*f - (12*c^2*d^3*e - 5*b*c*d^2*e^2)*g)*x)*sqrt(c)*arctan
(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c
^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) + 2*(15*c^2*e^3*g*x^3 - (46*c
^2*e^3*f - 7*(33*c^2*d*e^2 - 10*b*c*e^3)*g)*x^2 - 2*(13*c^2*d^2*e - b*c*d*
e^2 + 3*b^2*e^3)*f + (141*c^2*d^3 - 32*b*c*d^2*e - 4*b^2*d*e^2)*g - (2*(24
*c^2*d*e^2 + 11*b*c*e^3)*f - (333*c^2*d^2*e - 78*b*c*d*e^2 - 10*b^2*e^3)*g
)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(e^5*x^3 + 3*d*e^4*x^2 +
3*d^2*e^3*x + d^3*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{5}{2}}(f+gx)}{(d+ex)^6}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**6,x
)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**6, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^6,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(263) = 526.

Time = 72.48 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.91

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx = \frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}\,c^2g}{e^2} - \frac{\big(2\sqrt{-c}c^2ef-12\sqrt{-c}c^2dg+5b\sqrt{-c}ceg\big)\log\big(\big|-b\sqrt{-c}c^3d^6e^2-2\,\big(\sqrt{-c}e^2x-\sqrt{-ce^2x^2-be^2x+cd^2-}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^6,x, algo
rithm="giac")
```

output

```
sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*c^2*g/e^2 - 1/14*(2*sqrt(-c)*c^
2*e*f - 12*sqrt(-c)*c^2*d*g + 5*b*sqrt(-c)*c*e*g)*log(abs(-b*sqrt(-c)*c^3*
d^6*e^2 - 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*
c^4*d^6*abs(e) - 6*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b
*d*e))*b*c^3*d^5*e*abs(e) + 12*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x
 + c*d^2 - b*d*e))^2*sqrt(-c)*c^3*d^5*e + 15*(sqrt(-c*e^2)*x - sqrt(-c*e^2
*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*b*sqrt(-c)*c^2*d^4*e^2 + 30*(sqrt(-c*e^
2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*c^3*d^4*abs(e) + 20*(
sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*b*c^2*d^3*e
*abs(e) - 40*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))
^4*sqrt(-c)*c^2*d^3*e - 15*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c
*d^2 - b*d*e))^4*b*sqrt(-c)*c*d^2*e^2 - 30*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x
^2 - b*e^2*x + c*d^2 - b*d*e))^5*c^2*d^2*abs(e) - 6*(sqrt(-c*e^2)*x - sqrt
(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^5*b*c*d*e*abs(e) + 12*(sqrt(-c*e^2
)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^6*sqrt(-c)*c*d*e + (sqrt
(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^6*b*sqrt(-c)*e^2
+ 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^7*c*abs(
e)))/(e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{{(d+e\,x)}^6}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^6,x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^6, x
)
```

Reduce [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 2008, normalized size of antiderivative = 7.15

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^6,x)
```

output
```
(i*( - 300*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*
d))*b**2*c*d**3*e**2*g - 900*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/
sqrt( - b*e + 2*c*d))*b**2*c*d**2*e**3*g*x - 900*sqrt(c)*asinh((sqrt( - b*
e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d*e**4*g*x**2 - 300*sqrt(
c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*e**5*
g*x**3 + 1320*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2
*c*d))*b*c**2*d**4*e*g - 120*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/
sqrt( - b*e + 2*c*d))*b*c**2*d**3*e**2*f + 3960*sqrt(c)*asinh((sqrt( - b*e
 + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**3*e**2*g*x - 360*sqrt(c
)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e
**3*f*x + 3960*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
2*c*d))*b*c**2*d**2*e**3*g*x**2 - 360*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**4*f*x**2 + 1320*sqrt(c)*asinh((
sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**4*g*x**3 -
 120*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*
c**2*e**5*f*x**3 - 1440*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt(
 - b*e + 2*c*d))*c**3*d**5*g + 240*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**4*e*f - 4320*sqrt(c)*asinh((sqrt( - b*
e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**4*e*g*x + 720*sqrt(c)*as
inh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**3*e**2...
```

3.168 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 259

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx = -\frac{2c^2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)}+\frac{2cg(d(cd-be)-be^2x-ce^2x^2)^{3/2}}{3e^2(d+ex)^3} - \frac{2g(d(cd-be)-be^2x-ce^2x^2)^{5/2}}{5e^2(d+ex)^5}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7e^2(2cd-be)(d+ex)^7} - \frac{2c^{5/2}g\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e^2}$$

output

```
-2*c^2*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)+2/3*c*g*(d*(-b
*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^3-2/5*g*(d*(-b*e+c*d)-b*e^2*x
-c*e^2*x^2)^(5/2)/e^2/(e*x+d)^5-2/7*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2
*x^2)^(7/2)/e^2/(-b*e+2*c*d)/(e*x+d)^7-2*c^(5/2)*g*arctan(c^(1/2)*(e*x+d)/
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx=\frac{2((d+ex)(-be+c(d-ex)))^{5/2}\left(-\frac{-210c^3dg(d+ex)^3+105bc^2eg(}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^7,x]
```

output
```
(2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*(-((-210*c^3*d*g*(d + e*x)^3 +
 105*b*c^2*e*g*(d + e*x)^3 + 35*b*c*e*g*(d + e*x)^2*(-(c*d) + b*e + c*e*x)
 - 42*c*d*g*(d + e*x)*(-(c*d) + b*e + c*e*x)^2 + 21*b*e*g*(d + e*x)*(-(c*d
) + b*e + c*e*x)^2 + 15*e*f*(-(c*d) + b*e + c*e*x)^3 + 70*c^2*d*g*(d + e*x
)^2*(-(b*e) + c*(d - e*x)) + 15*d*g*(-(b*e) + c*(d - e*x))^3)/((-2*c*d + b
*e)*(d + e*x)^6*(-(c*d) + b*e + c*e*x)^2)) + (105*c^(5/2)*g*ArcTan[Sqrt[c*
d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/((d + e*x)^(5/2)*(-(b*e) + c*(d
 - e*x))^(5/2))))/(105*e^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {1220, 1130, 1130, 1125, 27, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^7}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{g\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^6}dx}{e}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7e^2(d+ex)^7(2cd-be)}$$

$$\Big\downarrow \text{1130}$$

$$\frac{g\left(-c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^4}dx-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e(d+ex)^5}\right)}{e}-$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7e^2(d+ex)^7(2cd-be)}$$

$$\Big\downarrow 1130$$

$$\frac{g\left(-c\left(-c\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^2}dx-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^3}\right)-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e(d+ex)^5}\right)}{e}-$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7e^2(d+ex)^7(2cd-be)}$$

$$\Big\downarrow 1125$$

$$\frac{g\left(-c\left(-c\left(-\frac{\int \frac{ce^2}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{e^2}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^3}\right)-\frac{2\left(d(cd-be)-be^2x-c\right.}{5e(d+ex)^5}\right.}{e}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7e^2(d+ex)^7(2cd-be)}$$

$$\Big\downarrow 27$$

$$\frac{g\left(-c\left(-c\left(-c\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^3}\right)-\frac{2\left(d(cd-be)-be^2x\right.}{5e(d+ex}\right.}{e}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7e^2(d+ex)^7(2cd-be)}$$

$$\Big\downarrow 1092$$

$$\frac{g\left(-c\left(-c\left(-2c\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\left(d(cd-be)-be\right.}{3e(d-}\right.\right.}{e}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7e^2(d+ex)^7(2cd-be)}$$

$$\Big\downarrow 217$$

$$\frac{g\left(-c\left(-c\left(-\frac{\sqrt{c}\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)}\right)-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^3}\right)-\frac{2\big(d(cd-b}{}\right.}{e}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7e^2(d+ex)^7(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^7,x]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(7*e^2*(2*c*d
 - b*e)*(d + e*x)^7) + (g*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2)
)/(5*e*(d + e*x)^5) - c*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/
(3*e*(d + e*x)^3) - c*((-2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e*(
d + e*x)) - (Sqrt[c]*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e)
- b*e^2*x - c*e^2*x^2])])/e))))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(239) = 478.

Time = 9.43 (sec) , antiderivative size = 825, normalized size of antiderivative = 3.19

method	result
	$\left(\quad \left(\quad \left(\right.\right.\right.$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^7,x,method=_RET
URNVERBOSE)
```

output
```
g/e^7*(-2/5/(-b*e^2+2*c*d*e)/(x+d/e)^6*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*
(x+d/e))^(7/2)-2/5*c*e^2/(-b*e^2+2*c*d*e)*(-2/3/(-b*e^2+2*c*d*e)/(x+d/e)^5
*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)-4/3*c*e^2/(-b*e^2+2*c*d
*e)*(-2/(-b*e^2+2*c*d*e)/(x+d/e)^4*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d
/e))^(7/2)-6*c*e^2/(-b*e^2+2*c*d*e)*(2/(-b*e^2+2*c*d*e)/(x+d/e)^3*(-c*e^2*
(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)+8*c*e^2/(-b*e^2+2*c*d*e)*(2/3/(-
b*e^2+2*c*d*e)/(x+d/e)^2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)
+10/3*c*e^2/(-b*e^2+2*c*d*e)*(1/5*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/
e))^(5/2)+1/2*(-b*e^2+2*c*d*e)*(-1/8*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^
2*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+3/16*(-b*e^2+2*c*d*e)^
2/c/e^2*(-1/4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/c/e^2*(-c*e^2*(x+d/e)^2+(-b
*e^2+2*c*d*e)*(x+d/e))^(1/2)+1/8*(-b*e^2+2*c*d*e)^2/c/e^2/(c*e^2)^(1/2)*ar
ctan((c*e^2)^(1/2)*(x+d/e-1/2*(-b*e^2+2*c*d*e)/c/e^2)/(-c*e^2*(x+d/e)^2+(-
b*e^2+2*c*d*e)*(x+d/e))^(1/2)))))))))+2/7*(d*g-e*f)/e^8/(-b*e^2+2*c*d*e)/
(x+d/e)^7*(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(239) = 478$.

Time = 34.44 (sec) , antiderivative size = 1239, normalized size of antiderivative = 4.78

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^7,x, algo
rithm="fricas")
```

output

```
[1/210*(105*((2*c^3*d*e^4 - b*c^2*e^5)*g*x^4 + 4*(2*c^3*d^2*e^3 - b*c^2*d*
e^4)*g*x^3 + 6*(2*c^3*d^3*e^2 - b*c^2*d^2*e^3)*g*x^2 + 4*(2*c^3*d^4*e - b*
c^2*d^3*e^2)*g*x + (2*c^3*d^5 - b*c^2*d^4*e)*g)*sqrt(-c)*log(8*c^2*e^2*x^2
 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e
^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) + 4*sqrt(-c*e^2*x^2 - b*e^
2*x + c*d^2 - b*d*e)*((15*c^3*e^4*f - (337*c^3*d*e^3 - 161*b*c^2*e^4)*g)*x
^3 - (45*(c^3*d*e^3 - b*c^2*e^4)*f + (613*c^3*d^2*e^2 - 130*b*c^2*d*e^3 -
77*b^2*c*e^4)*g)*x^2 - 15*(c^3*d^3*e - 3*b*c^2*d^2*e^2 + 3*b^2*c*d*e^3 - b
^3*e^4)*f - (167*c^3*d^4 - 60*b*c^2*d^3*e + 4*b^2*c*d^2*e^2 - 6*b^3*d*e^3)
*g + (45*(c^3*d^2*e^2 - 2*b*c^2*d*e^3 + b^2*c*e^4)*f - (563*c^3*d^3*e - 20
9*b*c^2*d^2*e^2 + 17*b^2*c*d*e^3 - 21*b^3*e^4)*g)*x))/(2*c*d^5*e^2 - b*d^4
*e^3 + (2*c*d*e^6 - b*e^7)*x^4 + 4*(2*c*d^2*e^5 - b*d*e^6)*x^3 + 6*(2*c*d^
3*e^4 - b*d^2*e^5)*x^2 + 4*(2*c*d^4*e^3 - b*d^3*e^4)*x), 1/105*(105*((2*c^
3*d*e^4 - b*c^2*e^5)*g*x^4 + 4*(2*c^3*d^2*e^3 - b*c^2*d*e^4)*g*x^3 + 6*(2*
c^3*d^3*e^2 - b*c^2*d^2*e^3)*g*x^2 + 4*(2*c^3*d^4*e - b*c^2*d^3*e^2)*g*x +
 (2*c^3*d^5 - b*c^2*d^4*e)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x
 + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d
^2 + b*c*d*e)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((15*c^3*e^4
*f - (337*c^3*d*e^3 - 161*b*c^2*e^4)*g)*x^3 - (45*(c^3*d*e^3 - b*c^2*e^4)*
f + (613*c^3*d^2*e^2 - 130*b*c^2*d*e^3 - 77*b^2*c*e^4)*g)*x^2 - 15*(c^3...
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{5}{2}}(f+gx)}{\left(d+ex\right)^7}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**7,x
)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**7, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^7,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^7,x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx=\int \frac{(f+g\,x)\,{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)}^{5/2}}{{(d+e\,x)}^7}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^7,x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^7, x
)
```

Reduce [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 2439, normalized size of antiderivative = 9.42

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^7}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^7,x)
```

output
```
(2*i*( - 105*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*
c*d))*b**2*c**2*d**4*e**2*g - 420*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x
)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d**3*e**3*g*x - 630*sqrt(c)*asinh((sq
rt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d**2*e**4*g*x*
*2 - 420*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*b**2*c**2*d*e**5*g*x**3 - 105*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*b**2*c**2*e**6*g*x**4 + 420*sqrt(c)*asinh((sqrt(
- b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**3*d**5*e*g + 1680*sqrt(
c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**3*d**4*
e**2*g*x + 2520*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
 2*c*d))*b*c**3*d**3*e**3*g*x**2 + 1680*sqrt(c)*asinh((sqrt( - b*e + c*d -
 c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**3*d**2*e**4*g*x**3 + 420*sqrt(c)*asi
nh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**3*d*e**5*g*x*
*4 - 420*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*c**4*d**6*g - 1680*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
b*e + 2*c*d))*c**4*d**5*e*g*x - 2520*sqrt(c)*asinh((sqrt( - b*e + c*d - c*
e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**4*e**2*g*x**2 - 1680*sqrt(c)*asinh((
sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**3*e**3*g*x**3
- 420*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c
**4*d**2*e**4*g*x**4 - 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + ...
```

3.169 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^8} dx$

Optimal result

Integrand size = 44, antiderivative size = 138

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^8} dx = -\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9e^2(2cd-be)(d+ex)^8} + \frac{2(9beg-2c(ef+8dg))\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{63e^2(2cd-be)^2(d+ex)^7}$$

output
```
-2/9*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^8+2/63*(9*b*e*g-2*c*(8*d*g+e*f))*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7
/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^7
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^8}\,dx = \frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(-be(7ef}{63e^2(-2cd+be)^2(}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^8,x]
```

output

```
(2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-(b*e*
(7*e*f + 2*d*g + 9*e*g*x)) + 2*c*(d^2*g + e^2*f*x + 8*d*e*(f + g*x))))/(63
*e^2*(-2*c*d + b*e)^2*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^8}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(-9beg+16cdg+2cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^7}dx}{9e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9e^2(d+ex)^8(2cd-be)}$$

$$\Big\downarrow \text{ 1123}$$

$$-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}(-9beg+16cdg+2cef)}{63e^2(d+ex)^7(2cd-be)^2}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9e^2(d+ex)^8(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^8,x]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(9*e^2*(2*c*d
 - b*e)*(d + e*x)^8) - (2*(2*c*e*f + 16*c*d*g - 9*b*e*g)*(d*(c*d - b*e) -
b*e^2*x - c*e^2*x^2)^(7/2))/(63*e^2*(2*c*d - b*e)^2*(d + e*x)^7)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 12.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

method	result
gosper	$-\frac{2(cex+be-cd)\left(9b\,e^{2}gx-16cdegx-2c\,e^{2}fx+2bdeg+7b\,e^{2}f-2c\,d^{2}g-16cdef\right)\left(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}\right)^{\frac{5}{2}}}{63(ex+d)^{7}e^{2}\left(b^{2}e^{2}-4bcde+4c^{2}d^{2}\right)}$
orering	$-\frac{2(cex+be-cd)\left(9b\,e^{2}gx-16cdegx-2c\,e^{2}fx+2bdeg+7b\,e^{2}f-2c\,d^{2}g-16cdef\right)\left(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}\right)^{\frac{5}{2}}}{63(ex+d)^{7}e^{2}\left(b^{2}e^{2}-4bcde+4c^{2}d^{2}\right)}$
default	$-\frac{2g\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{7e^{8}\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{7}}-\frac{(dg-ef)\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{9\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{8}}-\frac{4c\,e^{2}\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-\right.\right.}{63\left(-b\,e^{2}+2de\right.}\right.}{e^{9}}$
trager	$-\frac{2\left(9b\,c^{3}e^{5}g\,x^{4}-16c^{4}d\,e^{4}g\,x^{4}-2c^{4}e^{5}f\,x^{4}+27b^{2}c^{2}e^{5}g\,x^{3}-73b\,c^{3}d\,e^{4}g\,x^{3}+b\,c^{3}e^{5}f\,x^{3}+46c^{4}d^{2}e^{3}g\,x^{3}-10c^{4}d\,e^{4}f\,x^{3}+27b^{3}c\,e^{5}g\,x^{2}-\right.}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^8,x,method=_RET
URNVERBOSE)
```

output
```
-2/63*(c*e*x+b*e-c*d)*(9*b*e^2*g*x-16*c*d*e*g*x-2*c*e^2*f*x+2*b*d*e*g+7*b*
e^2*f-2*c*d^2*g-16*c*d*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)
^7/e^2/(b^2*e^2-4*b*c*d*e+4*c^2*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(130) = 260$.

Time = 106.89 (sec) , antiderivative size = 646, normalized size of antiderivative = 4.68

$$\int \frac{(f+gx)\left(cd^{2}-bde-be^{2}x-ce^{2}x^{2}\right)^{5/2}}{(d+ex)^{8}}\,dx=\frac{2\sqrt{-ce^{2}x^{2}-be^{2}x+cd^{2}-bde}\left(\left(2\,c^{4}e^{5}f+\left(16\,c^{4}de^{4}-9\,bc^{3}\right.\right.\right.}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^8,x, algo
rithm="fricas")
```

output
```
2/63*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((2*c^4*e^5*f + (16*c^4*d*
e^4 - 9*b*c^3*e^5)*g)*x^4 + ((10*c^4*d*e^4 - b*c^3*e^5)*f - (46*c^4*d^2*e^
3 - 73*b*c^3*d*e^4 + 27*b^2*c^2*e^5)*g)*x^3 - 3*((14*c^4*d^2*e^3 - 19*b*c^
3*d*e^4 + 5*b^2*c^2*e^5)*f - (14*c^4*d^3*e^2 - 37*b*c^3*d^2*e^3 + 32*b^2*c
^2*d*e^4 - 9*b^3*c*e^5)*g)*x^2 - (16*c^4*d^4*e - 55*b*c^3*d^3*e^2 + 69*b^2
*c^2*d^2*e^3 - 37*b^3*c*d*e^4 + 7*b^4*e^5)*f - 2*(c^4*d^5 - 4*b*c^3*d^4*e
+ 6*b^2*c^2*d^3*e^2 - 4*b^3*c*d^2*e^3 + b^4*d*e^4)*g + ((46*c^4*d^3*e^2 -
111*b*c^3*d^2*e^3 + 84*b^2*c^2*d*e^4 - 19*b^3*c*e^5)*f - (10*c^4*d^4*e - 3
9*b*c^3*d^3*e^2 + 57*b^2*c^2*d^2*e^3 - 37*b^3*c*d*e^4 + 9*b^4*e^5)*g)*x)/(
4*c^2*d^7*e^2 - 4*b*c*d^6*e^3 + b^2*d^5*e^4 + (4*c^2*d^2*e^7 - 4*b*c*d*e^8
 + b^2*e^9)*x^5 + 5*(4*c^2*d^3*e^6 - 4*b*c*d^2*e^7 + b^2*d*e^8)*x^4 + 10*(
4*c^2*d^4*e^5 - 4*b*c*d^3*e^6 + b^2*d^2*e^7)*x^3 + 10*(4*c^2*d^5*e^4 - 4*b
*c*d^4*e^5 + b^2*d^3*e^6)*x^2 + 5*(4*c^2*d^6*e^3 - 4*b*c*d^5*e^4 + b^2*d^4
*e^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^8}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**8,x
)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^8}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^8,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^8}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^8,x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.37 (sec) , antiderivative size = 12294, normalized size of antiderivative = 89.09

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^8}\,dx=\text{Too large to display}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^8,x)
```

output

```
(((d*((336*b^2*c^5*e^3*f - 2016*c^7*d^3*g + 384*b^3*c^4*e^3*g + 800*c^7*d^
2*e*f - 1024*b*c^6*d*e^2*f + 3424*b*c^6*d^2*e*g - 1968*b^2*c^5*d*e^2*g)/(9
45*e*(b*e - 2*c*d)^5) - (d*((16*c^5*(21*b^2*e^2*g + 50*c^2*d^2*g + 10*b*c*
e^2*f - 14*c^2*d*e*f - 64*b*c*d*e*g))/(945*(b*e - 2*c*d)^5) - (d*((32*c^6*
e*(5*b*e*g - 7*c*d*g + c*e*f))/(945*(b*e - 2*c*d)^5) - (32*c^7*d*e*g)/(945
*(b*e - 2*c*d)^5)))/e))/e))/e - (132*b^3*c^4*e^3*f + 126*b^4*c^3*e^3*g - 1
008*b*c^6*d^3*g + 400*b*c^6*d^2*e*f - 456*b^2*c^5*d*e^2*f + 1512*b^2*c^5*d
^2*e*g - 756*b^3*c^4*d*e^2*g)/(945*e*(b*e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2
- b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((696*b^2*c^5*e^3*f - 6016*c^7*
d^3*g + 1044*b^3*c^4*e^3*g + 1920*c^7*d^2*e*f - 2304*b*c^6*d*e^2*f + 9984*
b*c^6*d^2*e*g - 5568*b^2*c^5*d*e^2*g)/(945*e*(b*e - 2*c*d)^5) - (d*((8*c^5
*(29*b^2*e^2*g + 80*c^2*d^2*g + 10*b*c*e^2*f - 16*c^2*d*e*f - 96*b*c*d*e*g
))/(315*(b*e - 2*c*d)^5) - (d*((16*c^6*e*(15*b*e*g - 24*c*d*g + 2*c*e*f))/
(945*(b*e - 2*c*d)^5) - (32*c^7*d*e*g)/(945*(b*e - 2*c*d)^5)))/e))/e))/e -
 (292*b^3*c^4*e^3*f + 376*b^4*c^3*e^3*g - 3008*b*c^6*d^3*g + 960*b*c^6*d^2
*e*f - 1056*b^2*c^5*d*e^2*f + 4512*b^2*c^5*d^2*e*g - 2256*b^3*c^4*d*e^2*g)
/(945*e*(b*e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d
+ e*x) - (((d*((840*b^2*c^5*e^3*f - 7616*c^7*d^3*g + 1308*b^3*c^4*e^3*g +
2368*c^7*d^2*e*f - 2816*b*c^6*d*e^2*f + 12608*b*c^6*d^2*e*g - 7008*b^2*c^5
*d*e^2*g)/(945*e*(b*e - 2*c*d)^5) - (d*((8*c^5*(105*b^2*e^2*g + 296*c^2...
```

Reduce [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 2583, normalized size of antiderivative = 18.72

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^8}\,dx=\text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^8,x)
```

output

```
(2*i*( - 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*b**4*d*e**4*g - 7*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
 b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**5*f - 9*sqrt(d + e*x)*sqr
t(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**5*g
*x + 8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b**3*c*d**2*e**3*g + 37*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
 - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*d*e**4*f + 37*sqrt(d + e
*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3
*c*d*e**4*g*x - 19*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*b**3*c*e**5*f*x - 27*sqrt(d + e*x)*sqrt(b*e - 2*c
*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**5*g*x**2 - 1
2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d -
 c*e*x)*b**2*c**2*d**3*e**2*g - 69*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
 b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**2*e**3*f - 57*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
b**2*c**2*d**2*e**3*g*x + 84*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d*e**4*f*x + 96*sqrt(d + e*x)
*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*
*2*d*e**4*g*x**2 - 15*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)
*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**5*f*x**2 - 27*sqrt(d + e*x)*sq...
```

3.170 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 210

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx = &-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e^2(2cd-be)(d+ex)^9}\\ &-\frac{2(4cef+18cdg-11beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{99e^2(2cd-be)^2(d+ex)^8}\\ &-\frac{4c(4cef+18cdg-11beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{693e^2(2cd-be)^3(d+ex)^7}\end{aligned}$$

output

```
-2/11*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)/(
e*x+d)^9-2/99*(-11*b*e*g+18*c*d*g+4*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(7/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^8-4/693*c*(-11*b*e*g+18*c*d*g+4*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^7
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.80

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx =$$
$$-\frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(7b^2e^2(9ef+2dg+11egx)-2bce(25d^2g+e^2x(14f+1}{693e^2(-2cd+be)^3(d-}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^9,x]
```

output
```
(-2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(7*b^2
*e^2*(9*e*f + 2*d*g + 11*e*g*x) - 2*b*c*e*(25*d^2*g + e^2*x*(14*f + 11*g*x
) + 2*d*e*(70*f + 81*g*x)) + 4*c^2*(9*d^3*g + 2*e^3*f*x^2 + 9*d*e^2*x*(2*f
 + g*x) + d^2*e*(79*f + 81*g*x))))/(693*e^2*(-2*c*d + b*e)^3*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-11beg+18cdg+4cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^8}dx}{11e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e^2(d+ex)^9(2cd-be)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-11beg+18cdg+4cef)\left(\frac{2c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^7}dx}{9(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9e(d+ex)^8(2cd-be)}\right)}{11e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e^2(d+ex)^9(2cd-be)}$$

$$\Big\downarrow 1123$$

$$\frac{\left(-\frac{4c\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{63e(d+ex)^7(2cd-be)^2}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9e(d+ex)^8(2cd-be)}\right)(-11beg+18cdg+4cef)}{11e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e^2(d+ex)^9(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^9,x]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(11*e^2*(2*c*
d - b*e)*(d + e*x)^9) + ((4*c*e*f + 18*c*d*g - 11*b*e*g)*((-2*(d*(c*d - b*
e) - b*e^2*x - c*e^2*x^2)^(7/2))/(9*e*(2*c*d - b*e)*(d + e*x)^8) - (4*c*(d
*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(63*e*(2*c*d - b*e)^2*(d + e*x)
^7)))/(11*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 20.72 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.12

method	result
gosper	$-\frac{2(cex+be-cd)\left(-22bc\,e^{3}g\,x^{2}+36c^{2}d\,e^{2}g\,x^{2}+8f\,c^{2}e^{3}x^{2}+77b^{2}e^{3}gx-324bcd\,e^{2}gx-28bc\,e^{3}fx+324c^{2}d^{2}egx+72c^{2}d\,e^{2}fx+14b^{2}d\,e^{2}}{693(ex+d)^{8}\left(b^{3}e^{3}-6d\,e^{2}b^{2}c+12d^{2}eb\,c^{2}-8d^{3}}$
orering	$-\frac{2(cex+be-cd)\left(-22bc\,e^{3}g\,x^{2}+36c^{2}d\,e^{2}g\,x^{2}+8f\,c^{2}e^{3}x^{2}+77b^{2}e^{3}gx-324bcd\,e^{2}gx-28bc\,e^{3}fx+324c^{2}d^{2}egx+72c^{2}d\,e^{2}fx+14b^{2}d\,e^{2}}{693(ex+d)^{8}\left(b^{3}e^{3}-6d\,e^{2}b^{2}c+12d^{2}eb\,c^{2}-8d^{3}}$
default	$\frac{g\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{9\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{8}}-\frac{4c\,e^{2}\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{63\left(-b\,e^{2}+2dec\right)^{2}\left(x+\frac{d}{e}\right)^{7}}\right)}{e^{9}}-\frac{(dg-ef)\left(-\frac{2\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-\right.\right.}{11\left(-b\,e^{2}+2\right.}\right.}{}$
trager	$-\frac{2\left(-22b\,c^{4}e^{6}g\,x^{5}+36c^{5}d\,e^{5}g\,x^{5}+8c^{5}e^{6}f\,x^{5}+11b^{2}c^{3}e^{6}g\,x^{4}-150b\,c^{4}d\,e^{5}g\,x^{4}-4b\,c^{4}e^{6}f\,x^{4}+216c^{5}d^{2}e^{4}g\,x^{4}+48c^{5}d\,e^{5}f\,x^{4}+165b^{3}c^{2}}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^9,x,method=_RET
URNVERBOSE)
```

output
```
-2/693*(c*e*x+b*e-c*d)*(-22*b*c*e^3*g*x^2+36*c^2*d*e^2*g*x^2+8*c^2*e^3*f*x
^2+77*b^2*e^3*g*x-324*b*c*d*e^2*g*x-28*b*c*e^3*f*x+324*c^2*d^2*e*g*x+72*c^
2*d*e^2*f*x+14*b^2*d*e^2*g+63*b^2*e^3*f-50*b*c*d^2*e*g-280*b*c*d*e^2*f+36*
c^2*d^3*g+316*c^2*d^2*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^
8/(b^3*e^3-6*b^2*c*d*e^2+12*b*c^2*d^2*e-8*c^3*d^3)/e^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^9,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx=\int \frac{(-(d+ex)\,(be-cd+cex))^{\frac{5}{2}}\,(f+gx)}{(d+ex)^9}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**9,x
)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**9, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^9,x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx=\text{Timed out}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^9,x, algo
rithm="giac")
```

output

```
Timed out
```

Mupad [B] (verification not implemented)

Time = 37.01 (sec) , antiderivative size = 25236, normalized size of antiderivative = 120.17

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx=\text{Too large to display}$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^9,x)
```

output

```
(((d*((1632*b^2*c^6*e^3*f - 16704*c^8*d^3*g + 2784*b^3*c^5*e^3*g + 4672*c^
8*d^2*e*f - 5504*b*c^7*d*e^2*f + 27392*b*c^7*d^2*e*g - 15072*b^2*c^6*d*e^2
*g)/(10395*e*(b*e - 2*c*d)^6) - (d*((32*c^6*(51*b^2*e^2*g + 146*c^2*d^2*g
+ 16*b*c*e^2*f - 26*c^2*d*e*f - 172*b*c*d*e*g))/(10395*(b*e - 2*c*d)^6) -
(d*((64*c^7*e*(8*b*e*g - 13*c*d*g + c*e*f))/(10395*(b*e - 2*c*d)^6) - (64*
c^8*d*e*g)/(10395*(b*e - 2*c*d)^6)))/e))/e))/e - (696*b^3*c^5*e^3*f + 1044
*b^4*c^4*e^3*g - 8352*b*c^7*d^3*g + 2336*b*c^7*d^2*e*f - 2544*b^2*c^6*d*e^
2*f + 12528*b^2*c^6*d^2*e*g - 6264*b^3*c^5*d*e^2*g)/(10395*e*(b*e - 2*c*d)
^6))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + (((d*((1952*
b^2*c^6*e^3*f - 20928*c^8*d^3*g + 3456*b^3*c^5*e^3*g + 5696*c^8*d^2*e*f -
6656*b*c^7*d*e^2*f + 34240*b*c^7*d^2*e*g - 18784*b^2*c^6*d*e^2*g)/(10395*e
*(b*e - 2*c*d)^6) - (d*((32*c^6*(61*b^2*e^2*g + 178*c^2*d^2*g + 18*b*c*e^2
*f - 30*c^2*d*e*f - 208*b*c*d*e*g))/(10395*(b*e - 2*c*d)^6) - (d*((64*c^7*
e*(9*b*e*g - 15*c*d*g + c*e*f))/(10395*(b*e - 2*c*d)^6) - (64*c^8*d*e*g)/(
10395*(b*e - 2*c*d)^6)))/e))/e))/e - (840*b^3*c^5*e^3*f + 1308*b^4*c^4*e^3
*g - 10464*b*c^7*d^3*g + 2848*b*c^7*d^2*e*f - 3088*b^2*c^6*d*e^2*f + 15696
*b^2*c^6*d^2*e*g - 7848*b^3*c^5*d*e^2*g)/(10395*e*(b*e - 2*c*d)^6))*(c*d^2
 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + (((d*((2272*b^2*c^6*e^3
*f - 25152*c^8*d^3*g + 4128*b^3*c^5*e^3*g + 6720*c^8*d^2*e*f - 7808*b*c^7*
d*e^2*f + 41088*b*c^7*d^2*e*g - 22496*b^2*c^6*d*e^2*g)/(10395*e*(b*e - ...
```

Reduce [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 3556, normalized size of antiderivative = 16.93

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^9}\,dx=\text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^9,x)
```

output

```
(2*i*( - 14*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b
*e + c*d - c*e*x)*b**5*d*e**5*g - 63*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
 - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*e**6*f - 77*sqrt(d + e*x)*
sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*e**
6*g*x + 92*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*b**4*c*d**2*e**4*g + 469*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*
sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*d*e**5*f + 513*sqrt
(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x
)*b**4*c*d*e**5*g*x - 161*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*
c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*e**6*f*x - 209*sqrt(d + e*x)*sqrt(b
*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c*e**6*g*
x**2 - 228*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*b**3*c**2*d**3*e**3*g - 1345*sqrt(d + e*x)*sqrt(b*e - 2*c
*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d**2*e**4*f
- 1293*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b**3*c**2*d**2*e**4*g*x + 1062*sqrt(d + e*x)*sqrt(b*e - 2*c*d
)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d*e**5*f*x + 1
290*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
 - c*e*x)*b**3*c**2*d*e**5*g*x**2 - 113*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*e**6*f*x**2 - 1...
```

3.171 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 285

$$\begin{aligned}
&\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx = \\
&-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13e^2(2cd-be)(d+ex)^{10}} \\
&-\frac{2(6cef+20cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{143e^2(2cd-be)^2(d+ex)^9} \\
&-\frac{8c(6cef+20cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{1287e^2(2cd-be)^3(d+ex)^8} \\
&-\frac{16c^2(6cef+20cdg-13beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9009e^2(2cd-be)^4(d+ex)^7}
\end{aligned}$$

output

```
-2/13*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)/(
e*x+d)^10-2/143*(-13*b*e*g+20*c*d*g+6*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(7/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^9-8/1287*c*(-13*b*e*g+20*c*d*g+6*c*e*
f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^8-16/
9009*c^2*(-13*b*e*g+20*c*d*g+6*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/
2)/e^2/(-b*e+2*c*d)^4/(e*x+d)^7
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx = \frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(-63b^3e^3(}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^10,x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-63*b^
3*e^3*(11*e*f + 2*d*g + 13*e*g*x) + 14*b^2*c*e^2*(53*d^2*g + e^2*x*(27*f +
 26*g*x) + 4*d*e*(81*f + 94*g*x)) + 8*c^3*(97*d^4*g + 6*e^4*f*x^3 + 20*d*e
^3*x^2*(3*f + g*x) + 10*d^3*e*(93*f + 97*g*x) + d^2*e^2*x*(291*f + 200*g*x
)) - 4*b*c^2*e*(348*d^3*g + 2*e^3*x^2*(21*f + 13*g*x) + 2*d*e^2*x*(231*f +
 200*g*x) + d^2*e*(2499*f + 2801*g*x))))/(9009*e^2*(-2*c*d + b*e)^4*(d + e
*x)^7)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-13beg+20cdg+6cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^9}dx}{13e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13e^2(d+ex)^{10}(2cd-be)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-13beg+20cdg+6cef)\left(\frac{4c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^8}dx}{11(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e(d+ex)^9(2cd-be)}\right)}{13e(2cd-be)}-$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13e^2(d+ex)^{10}(2cd-be)}$$

$$\Big\downarrow \ 1129$$

$$\frac{(-13beg+20cdg+6cef)\left(\frac{4c\left(\frac{2c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^7}dx}{9(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9e(d+ex)^8(2cd-be)}\right)}{11(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e(d+ex)^9(2cd-be)}\right)}{13e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13e^2(d+ex)^{10}(2cd-be)}$$

$$\Big\downarrow \ 1123$$

$$\frac{\left(\frac{4c\left(-\frac{4c\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{63e(d+ex)^7(2cd-be)^2}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9e(d+ex)^8(2cd-be)}\right)}{11(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e(d+ex)^9(2cd-be)}\right)(-13beg+20cdg+6cef)}{13e(2cd-be)}-$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13e^2(d+ex)^{10}(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^10,x
]
```

output

```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(13*e^2*(2*c*
d - b*e)*(d + e*x)^10) + ((6*c*e*f + 20*c*d*g - 13*b*e*g)*((-2*(d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(11*e*(2*c*d - b*e)*(d + e*x)^9) + (4*c*
((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(9*e*(2*c*d - b*e)*(d +
e*x)^8) - (4*c*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(63*e*(2*c*d -
 b*e)^2*(d + e*x)^7)))/(11*(2*c*d - b*e))))/(13*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 26.77 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.34

method	result
gosper	$-\frac{2(cex+be-cd)\left(104b\,c^2e^4g\,x^3-160c^3d\,e^3g\,x^3-48c^3e^4f\,x^3-364b^2c\,e^4g\,x^2+1600b\,c^2d\,e^3g\,x^2+168b\,c^2e^4f\,x^2-1600c^3d^2e^2g\,x^2-48\right.}{}$
orering	$-\frac{2(cex+be-cd)\left(104b\,c^2e^4g\,x^3-160c^3d\,e^3g\,x^3-48c^3e^4f\,x^3-364b^2c\,e^4g\,x^2+1600b\,c^2d\,e^3g\,x^2+168b\,c^2e^4f\,x^2-1600c^3d^2e^2g\,x^2-48\right.}{}$
default	$\frac{g\left(-\frac{2\left(-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{11\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)^9}+\frac{4c\,e^2\left(-\frac{2\left(-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{9\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)^8}-\frac{4c\,e^2\left(-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right.\right.}{63\left(-b\,e^2+2dec\right)^2\left(x+\frac{d}{e}\right.}\right.}{11\left(-b\,e^2+2dec\right)}\right.}{e^{10}}$
trager	$-\frac{2\left(104b\,c^5e^7g\,x^6-160c^6d\,e^6g\,x^6-48c^6e^7f\,x^6-52b^2c^4e^7g\,x^5+808b\,c^5d\,e^6g\,x^5+24b\,c^5e^7f\,x^5-1120c^6d^2e^5g\,x^5-336c^6d\,e^6f\,x^5+39\right.}{}$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^10,x,method=_RE
TURNVERBOSE)
```

output

```
-2/9009*(c*e*x+b*e-c*d)*(104*b*c^2*e^4*g*x^3-160*c^3*d*e^3*g*x^3-48*c^3*e^
4*f*x^3-364*b^2*c*e^4*g*x^2+1600*b*c^2*d*e^3*g*x^2+168*b*c^2*e^4*f*x^2-160
0*c^3*d^2*e^2*g*x^2-480*c^3*d*e^3*f*x^2+819*b^3*e^4*g*x-5264*b^2*c*d*e^3*g
*x-378*b^2*c*e^4*f*x+11204*b*c^2*d^2*e^2*g*x+1848*b*c^2*d*e^3*f*x-7760*c^3
*d^3*e*g*x-2328*c^3*d^2*e^2*f*x+126*b^3*d*e^3*g+693*b^3*e^4*f-742*b^2*c*d^
2*e^2*g-4536*b^2*c*d*e^3*f+1392*b*c^2*d^3*e*g+9996*b*c^2*d^2*e^2*f-776*c^3
*d^4*g-7440*c^3*d^3*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^9/
e^2/(b^4*e^4-8*b^3*c*d*e^3+24*b^2*c^2*d^2*e^2-32*b*c^3*d^3*e+16*c^4*d^4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^10,x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**10,
x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^10,x, alg
orithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^10,x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 67.59 (sec) , antiderivative size = 51074, normalized size of antiderivative = 179.21

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx=\text{Too large to display}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^10,x
)
```

output
```
(((d*((2016*b^2*c^7*e^3*f - 17664*c^9*d^3*g + 3040*b^3*c^6*e^3*g + 5376*c^
9*d^2*e*f - 6528*b*c^8*d*e^2*f + 29184*b*c^8*d^2*e*g - 16224*b^2*c^7*d*e^2
*g)/(135135*e*(b*e - 2*c*d)^7) - (d*((32*c^7*(21*b^2*e^2*g + 56*c^2*d^2*g
+ 8*b*c*e^2*f - 12*c^2*d*e*f - 68*b*c*d*e*g))/(45045*(b*e - 2*c*d)^7) - (d
*((128*c^8*e*(6*b*e*g - 9*c*d*g + c*e*f))/(135135*(b*e - 2*c*d)^7) - (128*
c^9*d*e*g)/(135135*(b*e - 2*c*d)^7)))/e))/e))/e - (16*b*c^5*(69*b^3*e^3*g
- 552*c^3*d^3*g + 52*b^2*c*e^3*f + 168*c^3*d^2*e*f - 186*b*c^2*d*e^2*f + 8
28*b*c^2*d^2*e*g - 414*b^2*c*d*e^2*g))/(135135*e*(b*e - 2*c*d)^7))*(c*d^2
- c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((d*((d*((4*c^4*e^2
*(19*b*e*g - 32*c*d*g + 2*c*e*f))/(143*(9*b*e^2 - 18*c*d*e)*(b*e - 2*c*d)^
2) - (8*c^5*d*e^2*g)/(143*(9*b*e^2 - 18*c*d*e)*(b*e - 2*c*d)^2)))/e - (4*c
^3*e*(51*b^2*e^2*g + 134*c^2*d^2*g + 19*b*c*e^2*f - 32*c^2*d*e*f - 166*b*c
*d*e*g))/(143*(9*b*e^2 - 18*c*d*e)*(b*e - 2*c*d)^2)))/e + (204*b^2*c^3*e^4
*f + 236*b^3*c^2*e^4*g + 536*c^5*d^2*e^2*f - 1216*c^5*d^3*e*g - 664*b*c^4*
d*e^3*f + 2092*b*c^4*d^2*e^2*g - 1212*b^2*c^3*d*e^3*g)/(143*e*(9*b*e^2 - 1
8*c*d*e)*(b*e - 2*c*d)^2)))/e - (960*c^5*d^4*g + 156*b^3*c^2*e^4*f + 100*b
^4*c*e^4*g - 576*c^5*d^3*e*f - 2240*b*c^4*d^3*e*g + 1132*b*c^4*d^2*e^2*f -
 732*b^2*c^3*d*e^3*f - 720*b^3*c^2*d*e^3*g + 1920*b^2*c^3*d^2*e^2*g)/(143*
e*(9*b*e^2 - 18*c*d*e)*(b*e - 2*c*d)^2))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^
2*x)^(1/2))/(d + e*x)^5 - (((d*((3776*b^2*c^7*e^3*f - 44544*c^9*d^3*g +...
```

Reduce [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 4677, normalized size of antiderivative = 16.41

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{10}}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^10,x)
```

output

```
(2*i*( - 126*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( -
b*e + c*d - c*e*x)*b**6*d*e**6*g - 693*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqr
t( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**6*e**7*f - 819*sqrt(d + e*
x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**6*
e**7*g*x + 1120*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt(
 - b*e + c*d - c*e*x)*b**5*c*d**2*e**5*g + 6615*sqrt(d + e*x)*sqrt(b*e - 2
*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*c*d*e**6*f + 73
43*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
- c*e*x)*b**5*c*d*e**6*g*x - 1701*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*c*e**7*f*x - 2093*sqrt(d + e*
x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*
c*e**7*g*x**2 - 3996*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*
sqrt( - b*e + c*d - c*e*x)*b**4*c**2*d**3*e**4*g - 25683*sqrt(d + e*x)*sqr
t(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c**2*d
**2*e**5*f - 26471*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*b**4*c**2*d**2*e**5*g*x + 14784*sqrt(d + e*x)*sqr
t(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c**2*d
*e**6*f*x + 17636*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqr
t( - b*e + c*d - c*e*x)*b**4*c**2*d*e**6*g*x**2 - 1113*sqrt(d + e*x)*sqrt(
b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*c**2*...
```

3.172 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 360

$$\begin{aligned}
&\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx = \\
&-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15e^2(2cd-be)(d+ex)^{11}} \\
&-\frac{2(8cef+22cdg-15beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{195e^2(2cd-be)^2(d+ex)^{10}} \\
&-\frac{4c(8cef+22cdg-15beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{715e^2(2cd-be)^3(d+ex)^9} \\
&-\frac{16c^2(8cef+22cdg-15beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{6435e^2(2cd-be)^4(d+ex)^8} \\
&-\frac{32c^3(8cef+22cdg-15beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{45045e^2(2cd-be)^5(d+ex)^7}
\end{aligned}$$

output

```
-2/15*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)/(
e*x+d)^11-2/195*(-15*b*e*g+22*c*d*g+8*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(7/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^10-4/715*c*(-15*b*e*g+22*c*d*g+8*c*e*
f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^9-16/
6435*c^2*(-15*b*e*g+22*c*d*g+8*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/
2)/e^2/(-b*e+2*c*d)^4/(e*x+d)^8-32/45045*c^3*(-15*b*e*g+22*c*d*g+8*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/e^2/(-b*e+2*c*d)^5/(e*x+d)^7
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.98

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx =$$

$$-\frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(231b^4e^4(13ef+2dg+15egx)+84b^2c^2e^2(133d^3g+2e^3}{}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^11,x]
```

output

```
(-2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(231*b
^4*e^4*(13*e*f + 2*d*g + 15*e*g*x) + 84*b^2*c^2*e^2*(133*d^3*g + 2*e^3*x^2
*(6*f + 5*g*x) + 3*d*e^2*x*(52*f + 51*g*x) + 6*d^2*e*(167*f + 189*g*x)) -
42*b^3*c*e^3*(89*d^2*g + e^2*x*(44*f + 45*g*x) + d*e*(616*f + 706*g*x)) +
16*c^4*(407*d^5*g + 8*e^5*f*x^4 + 22*d*e^4*x^3*(4*f + g*x) + 11*d^3*e^2*x*
(148*f + 117*g*x) + 2*d^2*e^3*x^2*(234*f + 121*g*x) + d^4*e*(4243*f + 4477
*g*x)) - 8*b*c^3*e*(1801*d^4*g + 2*e^4*x^3*(28*f + 15*g*x) + 4*d*e^3*x^2*(
168*f + 121*g*x) + 3*d^2*e^2*x*(1316*f + 1201*g*x) + 2*d^3*e*(7672*f + 848
1*g*x))))/(45045*e^2*(-2*c*d + b*e)^5*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(-15beg+22cdg+8cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{10}}dx}{15e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15e^2(d+ex)^{11}(2cd-be)}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(-15beg+22cdg+8cef)\left(\frac{6c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^9}dx}{13(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13e(d+ex)^{10}(2cd-be)}\right)}{15e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15e^2(d+ex)^{11}(2cd-be)}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(-15beg+22cdg+8cef)\left(\frac{6c\left(\frac{4c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^8}dx}{11(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e(d+ex)^9(2cd-be)}\right)}{13(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13e(d+ex)^{10}(2cd-be)}\right)}{15e(2cd-be)}-\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15e^2(d+ex)^{11}(2cd-be)}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(-15beg+22cdg+8cef)\left(\frac{6c\left(\frac{4c\left(\frac{2c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^7}dx}{9(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9e(d+ex)^8(2cd-be)}\right)}{11(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e(d+ex)^9(2cd-be)}\right)}{13(2cd-be)}\right.}{15e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15e^2(d+ex)^{11}(2cd-be)}$$

$$\Big\downarrow \; 1123$$

$$\frac{\left(\frac{6c\left(\frac{4c\left(-\frac{4c\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{63e(d+ex)^7(2cd-be)^2}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9e(d+ex)^8(2cd-be)}\right)}{11(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11e(d+ex)^9(2cd-be)}\right)}{13(2cd-be)}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13e(d+ex)^{10}(2cd-be)}\right)(}{15e(2cd-be)}$$

$$\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15e^2(d+ex)^{11}(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^11,x
]
```

output
```
(-2*(e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(15*e^2*(2*c*
d - b*e)*(d + e*x)^11) + ((8*c*e*f + 22*c*d*g - 15*b*e*g)*((-2*(d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(13*e*(2*c*d - b*e)*(d + e*x)^10) + (6*c
*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(11*e*(2*c*d - b*e)*(d
+ e*x)^9) + (4*c*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(9*e*(2
*c*d - b*e)*(d + e*x)^8) - (4*c*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2
))/(63*e*(2*c*d - b*e)^2*(d + e*x)^7)))/(11*(2*c*d - b*e))))/(13*(2*c*d -
b*e))))/(15*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 34.84 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.57

method	result
gosper	$-\frac{2(cex+be-cd)(-240b c^3 e^5 g x^4+352c^4 d e^4 g x^4+128c^4 e^5 f x^4+840b^2 c^2 e^5 g x^3-3872b c^3 d e^4 g x^3-448b c^3 e^5 f x^3+3872c^4 d^2 e^3 g x^3}{}$
orering	$-\frac{2(cex+be-cd)(-240b c^3 e^5 g x^4+352c^4 d e^4 g x^4+128c^4 e^5 f x^4+840b^2 c^2 e^5 g x^3-3872b c^3 d e^4 g x^3-448b c^3 e^5 f x^3+3872c^4 d^2 e^3 g x^3}{}$
default	$\frac{g\left(-\frac{2\left(-c e^2 \left(x+\frac{d}{e}\right)^2+\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{13\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)^{10}}+\frac{6c e^2\left(-\frac{2\left(-c e^2 \left(x+\frac{d}{e}\right)^2+\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{11\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)^{9}}+\frac{4c e^2\left(-\frac{2\left(-c e^2 \left(x+\frac{d}{e}\right)^2+\left(-b e^2+\right.\right.}{9\left(-b e^2+2dec\right)\left(\right.}\right.}{}\right.}{13\left(-b e^2+2dec\right)}\right.}{e^{11}}$
trager	Expression too large to display

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^11,x,method=_RE
TURNVERBOSE)
```

output

```
-2/45045*(c*e*x+b*e-c*d)*(-240*b*c^3*e^5*g*x^4+352*c^4*d*e^4*g*x^4+128*c^4
*e^5*f*x^4+840*b^2*c^2*e^5*g*x^3-3872*b*c^3*d*e^4*g*x^3-448*b*c^3*e^5*f*x^
3+3872*c^4*d^2*e^3*g*x^3+1408*c^4*d*e^4*f*x^3-1890*b^3*c*e^5*g*x^2+12852*b
^2*c^2*d*e^4*g*x^2+1008*b^2*c^2*e^5*f*x^2-28824*b*c^3*d^2*e^3*g*x^2-5376*b
*c^3*d*e^4*f*x^2+20592*c^4*d^3*e^2*g*x^2+7488*c^4*d^2*e^3*f*x^2+3465*b^4*e
^5*g*x-29652*b^3*c*d*e^4*g*x-1848*b^3*c*e^5*f*x+95256*b^2*c^2*d^2*e^3*g*x+
13104*b^2*c^2*d*e^4*f*x-135696*b*c^3*d^3*e^2*g*x-31584*b*c^3*d^2*e^3*f*x+7
1632*c^4*d^4*e*g*x+26048*c^4*d^3*e^2*f*x+462*b^4*d*e^4*g+3003*b^4*e^5*f-37
38*b^3*c*d^2*e^3*g-25872*b^3*c*d*e^4*f+11172*b^2*c^2*d^3*e^2*g+84168*b^2*c
^2*d^2*e^3*f-14408*b*c^3*d^4*e*g-122752*b*c^3*d^3*e^2*f+6512*c^4*d^5*g+678
88*c^4*d^4*e*f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^10/e^2/(b^5
*e^5-10*b^4*c*d*e^4+40*b^3*c^2*d^2*e^3-80*b^2*c^3*d^3*e^2+80*b*c^4*d^4*e-3
2*c^5*d^5)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx=\text{Timed out}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^11,x, alg
orithm="fricas")
```

output

```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx=\text{Timed out}$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**11,
x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^11,x, alg
orithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^11,x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 133.29 (sec) , antiderivative size = 38717, normalized size of antiderivative = 107.55

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx = \text{Too large to display}$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^11,x
)
```

output

```
(16*c^3*d^3*f*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(15*b*d^8*e^2 +
 15*b*e^10*x^8 - 30*c*d^9*e + 420*b*d^6*e^4*x^2 + 840*b*d^5*e^5*x^3 + 1050
*b*d^4*e^6*x^4 + 840*b*d^3*e^7*x^5 + 420*b*d^2*e^8*x^6 - 840*c*d^7*e^3*x^2
 - 1680*c*d^6*e^4*x^3 - 2100*c*d^5*e^5*x^4 - 1680*c*d^4*e^6*x^5 - 840*c*d^
3*e^7*x^6 - 240*c*d^2*e^8*x^7 + 120*b*d^7*e^3*x + 120*b*d*e^9*x^7 - 240*c*
d^8*e^2*x - 30*c*d*e^9*x^8) - (976*c^4*d^4*g*(c*d^2 - c*e^2*x^2 - b*d*e -
b*e^2*x)^(1/2))/(15*(13*b^2*d^7*e^4 + 52*c^2*d^9*e^2 + 13*b^2*e^11*x^7 + 9
1*b^2*d^6*e^5*x + 91*b^2*d*e^10*x^6 + 364*c^2*d^8*e^3*x + 273*b^2*d^5*e^6*
x^2 + 455*b^2*d^4*e^7*x^3 + 455*b^2*d^3*e^8*x^4 + 273*b^2*d^2*e^9*x^5 + 10
92*c^2*d^7*e^4*x^2 + 1820*c^2*d^6*e^5*x^3 + 1820*c^2*d^5*e^6*x^4 + 1092*c^
2*d^4*e^7*x^5 + 364*c^2*d^3*e^8*x^6 + 52*c^2*d^2*e^9*x^7 - 52*b*c*d^8*e^3
- 364*b*c*d^7*e^4*x - 52*b*c*d*e^10*x^7 - 1092*b*c*d^6*e^5*x^2 - 1820*b*c*
d^5*e^6*x^3 - 1820*b*c*d^4*e^7*x^4 - 1092*b*c*d^3*e^8*x^5 - 364*b*c*d^2*e^
9*x^6)) - (2*b^3*e^3*f*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(15*b*
d^8*e^2 + 15*b*e^10*x^8 - 30*c*d^9*e + 420*b*d^6*e^4*x^2 + 840*b*d^5*e^5*x
^3 + 1050*b*d^4*e^6*x^4 + 840*b*d^3*e^7*x^5 + 420*b*d^2*e^8*x^6 - 840*c*d^
7*e^3*x^2 - 1680*c*d^6*e^4*x^3 - 2100*c*d^5*e^5*x^4 - 1680*c*d^4*e^6*x^5 -
 840*c*d^3*e^7*x^6 - 240*c*d^2*e^8*x^7 + 120*b*d^7*e^3*x + 120*b*d*e^9*x^7
 - 240*c*d^8*e^2*x - 30*c*d*e^9*x^8) - (32*b^4*e^4*g*(c*d^2 - c*e^2*x^2 -
b*d*e - b*e^2*x)^(1/2))/(15*(13*b^2*d^7*e^4 + 52*c^2*d^9*e^2 + 13*b^2*e...
```

Reduce [B] (verification not implemented)

Time = 18.73 (sec) , antiderivative size = 5946, normalized size of antiderivative = 16.52

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^11,x)
```

output

```
(2*i*( - 462*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( -
b*e + c*d - c*e*x)*b**7*d*e**7*g - 3003*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**7*e**8*f - 3465*sqrt(d +
e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**
7*e**8*g*x + 5124*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqr
t( - b*e + c*d - c*e*x)*b**6*c*d**2*e**6*g + 34881*sqrt(d + e*x)*sqrt(b*e
- 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**6*c*d*e**7*f +
 38661*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b**6*c*d*e**7*g*x - 7161*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt
( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**6*c*e**8*f*x - 8505*sqrt(d
+ e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b
**6*c*e**8*g*x**2 - 23772*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*
c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*c**2*d**3*e**5*g - 170793*sqrt(d + e*
x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*
c**2*d**2*e**6*f - 180621*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*
c*d)*sqrt( - b*e + c*d - c*e*x)*b**5*c**2*d**2*e**6*g*x + 76986*sqrt(d + e
*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**5
*c**2*d*e**7*f*x + 89838*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c
*d)*sqrt( - b*e + c*d - c*e*x)*b**5*c**2*d*e**7*g*x**2 - 4473*sqrt(d + e*x
)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**...
```

3.173 $\int \frac{(d+ex)^3(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 276

$$
\begin{aligned}
&\int \frac{(d+ex)^3(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx \\
&= -\frac{(8cef+6cdg-7beg)(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{24c^2e^2} \\
&\quad -\frac{g(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^2} \\
&\quad -\frac{5(2cd-be)(8cef+6cdg-7beg)(8cd-3be+2cex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{192c^4e^2} \\
&\quad +\frac{5(2cd-be)^3(8cef+6cdg-7beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{64c^{9/2}e^2}
\end{aligned}
$$

output
```
-1/24*(-7*b*e*g+6*c*d*g+8*c*e*f)*(e*x+d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(1/2)/c^2/e^2-1/4*g*(e*x+d)^3*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c/e
^2-5/192*(-b*e+2*c*d)*(-7*b*e*g+6*c*d*g+8*c*e*f)*(2*c*e*x-3*b*e+8*c*d)*(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^4/e^2+5/64*(-b*e+2*c*d)^3*(-7*b*e*g+
6*c*d*g+8*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
1/2))/c^(9/2)/e^2
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^3(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{(2cd-be)^3\left(-\frac{\sqrt{c}(d+ex)(-be+c(d-ex))\left(-105b^3e^3g+10b^2ce^2(12ef+58dg+7egx)-4bc^2e\left(259d^2g+2e^2x(10f+7gx)+2de(70f+39gx)\right)+\right.}{(2cd-be)^3}\right.}{192c^{9/2}e^2}$$

input
```
Integrate[((d + e*x)^3*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
],x]
```

output
```
((2*c*d - b*e)^3*(-((Sqrt[c]*(d + e*x)*(-(b*e) + c*(d - e*x))*(-105*b^3*e^
3*g + 10*b^2*c*e^2*(12*e*f + 58*d*g + 7*e*g*x) - 4*b*c^2*e*(259*d^2*g + 2*
e^2*x*(10*f + 7*g*x) + 2*d*e*(70*f + 39*g*x)) + 8*c^3*(72*d^3*g + 12*d*e^2
*x*(3*f + 2*g*x) + 2*e^3*x^2*(4*f + 3*g*x) + d^2*e*(88*f + 45*g*x))))/(2*c
*d - b*e)^3) - 15*(8*c*e*f + 6*c*d*g - 7*b*e*g)*Sqrt[d + e*x]*Sqrt[c*d - b
*e - c*e*x]*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])]))/(192
*c^(9/2)*e^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1221, 1134, 1134, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-7beg+6cdg+8cef)\int \frac{(d+ex)^3}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8ce}-\frac{g(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^2}$$

$$\Big\downarrow \text{1134}$$

$$\frac{(-7beg+6cdg+8cef)\left(\frac{5(2cd-be)\int\frac{(d+ex)^2}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{6c}-\frac{(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)}{8ce}-\frac{g(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^2}$$

$$\Big\downarrow \ 1134$$

$$\frac{(-7beg+6cdg+8cef)\left(\frac{5(2cd-be)\left(\frac{3(2cd-be)\int\frac{d+ex}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}\right)}{6c}-\frac{(d+ex)^2\sqrt{d(cd-be)-}}{3ce}\right.}{8ce}$$
$$\frac{g(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^2}$$

$$\Big\downarrow \ 1160$$

$$\frac{(-7beg+6cdg+8cef)\left(\frac{5(2cd-be)\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2c}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be}}{2ce}\right.}{6c}\right.}{8ce}$$
$$\frac{g(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^2}$$

$$\Big\downarrow \ 1092$$

$$\frac{(-7beg+6cdg+8cef)\left(\frac{5(2cd-be)\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{c}-\frac{\sqrt{d(cd-be)-b}}{ce}\right.}{4c}\right.}{6c}\right.}{8ce}$$

$$\frac{g(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^2}$$

$$\Big\downarrow 217$$

$$\frac{\left(\frac{5(2cd-be)\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2c^{3/2}e}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}\right)}{6c}-\frac{(d+ex)}{}\right.}{8ce}$$

$$\frac{g(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^2}$$

input
```
Int[((d + e*x)^3*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
-1/4*(g*(d + e*x)^3*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(c*e^2) + (
(8*c*e*f + 6*c*d*g - 7*b*e*g)*(-1/3*((d + e*x)^2*Sqrt[d*(c*d - b*e) - b*e^
2*x - c*e^2*x^2])/(c*e) + (5*(2*c*d - b*e)*(-1/2*((d + e*x)*Sqrt[d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2])/(c*e) + (3*(2*c*d - b*e)*(-(Sqrt[d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2]/(c*e)) + ((2*c*d - b*e)*ArcTan[(e*(b + 2*c*x))/(2
*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(2*c^(3/2)*e)))/(4*c
)))/(6*c)))/(8*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1444 vs. $2(256) = 512$.

Time = 3.89 (sec) , antiderivative size = 1445, normalized size of antiderivative = 5.24

method	result	size
default	Expression too large to display	1445

input
```
int((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
d^3*f/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b
*d*e+c*d^2)^(1/2))+e^2*(3*d*g+e*f)*(-1/3*x^2/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d
*e+c*d^2)^(1/2)-5/6*b/c*(-1/2*x/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/
2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2
)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^
(1/2)))+1/2*(-b*d*e+c*d^2)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2
*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+2/3*(-b*d*e+c*d^2)/c/e^2*(-
1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arcta
n((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))+3*d*
e*(d*g+e*f)*(-1/2*x/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-3/4*b/c*(
-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arct
an((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+1/2*
(-b*d*e+c*d^2)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^
2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+d^2*(d*g+3*e*f)*(-1/c/e^2*(-c*e^2*x^2-b
*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/
2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+g*e^3*(-1/4*x^3/c/e^2*(-c*
e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-7/8*b/c*(-1/3*x^2/c/e^2*(-c*e^2*x^2-b*e
^2*x-b*d*e+c*d^2)^(1/2)-5/6*b/c*(-1/2*x/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*
d^2)^(1/2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/
c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*...
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.99

$$\int \frac{(d+ex)^3(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output

```
[-1/768*(15*(8*(8*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^3*c*e
^4)*f + (48*c^4*d^4 - 128*b*c^3*d^3*e + 120*b^2*c^2*d^2*e^2 - 48*b^3*c*d*e
^3 + 7*b^4*e^4)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 +
4*b*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*
x + b*e)*sqrt(-c)) + 4*(48*c^4*e^3*g*x^3 + 8*(8*c^4*e^3*f + (24*c^4*d*e^2
- 7*b*c^3*e^3)*g)*x^2 + 8*(88*c^4*d^2*e - 70*b*c^3*d*e^2 + 15*b^2*c^2*e^3)
*f + (576*c^4*d^3 - 1036*b*c^3*d^2*e + 580*b^2*c^2*d*e^2 - 105*b^3*c*e^3)*
g + 2*(8*(18*c^4*d*e^2 - 5*b*c^3*e^3)*f + (180*c^4*d^2*e - 156*b*c^3*d*e^2
 + 35*b^2*c^2*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^5*
e^2), -1/384*(15*(8*(8*c^4*d^3*e - 12*b*c^3*d^2*e^2 + 6*b^2*c^2*d*e^3 - b^
3*c*e^4)*f + (48*c^4*d^4 - 128*b*c^3*d^3*e + 120*b^2*c^2*d^2*e^2 - 48*b^3*
c*d*e^3 + 7*b^4*e^4)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d
^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b
*c*d*e)) + 2*(48*c^4*e^3*g*x^3 + 8*(8*c^4*e^3*f + (24*c^4*d*e^2 - 7*b*c^3*
e^3)*g)*x^2 + 8*(88*c^4*d^2*e - 70*b*c^3*d*e^2 + 15*b^2*c^2*e^3)*f + (576*
c^4*d^3 - 1036*b*c^3*d^2*e + 580*b^2*c^2*d*e^2 - 105*b^3*c*e^3)*g + 2*(8*(
18*c^4*d*e^2 - 5*b*c^3*e^3)*f + (180*c^4*d^2*e - 156*b*c^3*d*e^2 + 35*b^2*
c^2*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^5*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1032 vs. $2(270) = 540$.

Time = 1.47 (sec) , antiderivative size = 1032, normalized size of antiderivative = 3.74

$$\int \frac{(d+ex)^3(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**3*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output
```
Piecewise(((-b*(-3*b*(-5*b*(-7*b*e**3*g/(8*c) + 3*d*e**2*g + e**3*f)/(6*c)
 + 3*d**2*e*g + 3*d*e**2*f + e*g*(-3*b*d*e + 3*c*d**2)/(4*c))/(4*c) + d**3
*g + 3*d**2*e*f + (-2*b*d*e + 2*c*d**2)*(-7*b*e**3*g/(8*c) + 3*d*e**2*g +
e**3*f)/(3*c*e**2))/(2*c) + d**3*f + (-b*d*e + c*d**2)*(-5*b*(-7*b*e**3*g/
(8*c) + 3*d*e**2*g + e**3*f)/(6*c) + 3*d**2*e*g + 3*d*e**2*f + e*g*(-3*b*d
*e + 3*c*d**2)/(4*c))/(2*c*e**2))*Piecewise((log(-b*e**2 - 2*c*e**2*x + 2*
sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2))/sqrt(-c*e**2
), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*c) + x)*log(b/(2*c) +
x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)) + sqrt(-b*d*e - b*e**2*x + c*d**
2 - c*e**2*x**2)*(-e*g*x**3/(4*c) - x**2*(-7*b*e**3*g/(8*c) + 3*d*e**2*g +
 e**3*f)/(3*c*e**2) - x*(-5*b*(-7*b*e**3*g/(8*c) + 3*d*e**2*g + e**3*f)/(6
*c) + 3*d**2*e*g + 3*d*e**2*f + e*g*(-3*b*d*e + 3*c*d**2)/(4*c))/(2*c*e**2
) - (-3*b*(-5*b*(-7*b*e**3*g/(8*c) + 3*d*e**2*g + e**3*f)/(6*c) + 3*d**2*e
*g + 3*d*e**2*f + e*g*(-3*b*d*e + 3*c*d**2)/(4*c))/(4*c) + d**3*g + 3*d**2
*e*f + (-2*b*d*e + 2*c*d**2)*(-7*b*e**3*g/(8*c) + 3*d*e**2*g + e**3*f)/(3*
c*e**2))/(c*e**2)), Ne(c*e**2, 0)), (-2*(g*(-b*d*e - b*e**2*x + c*d**2)**(
9/2)/(9*b**4*e**5) + (-b*d*e - b*e**2*x + c*d**2)**(7/2)*(b*d*e*g - b*e**2
*f - 4*c*d**2*g)/(7*b**4*e**5) + (-b*d*e - b*e**2*x + c*d**2)**(5/2)*(-3*b
*c*d**3*e*g + 3*b*c*d**2*e**2*f + 6*c**2*d**4*g)/(5*b**4*e**5) + (-b*d*e -
 b*e**2*x + c*d**2)**(3/2)*(3*b*c**2*d**5*e*g - 3*b*c**2*d**4*e**2*f - ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^3(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$

$$-\frac{1}{192}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\bigg(2\bigg(4\bigg(\frac{6\,egx}{c}+\frac{8\,c^3e^5f+24\,c^3de^4g-7\,bc^2e^5g}{c^4e^4}\bigg)x+\frac{144\,c^3de^4f-4}{}$$

$$-\frac{5\,(64\,c^4d^3ef-96\,bc^3d^2e^2f+48\,b^2c^2de^3f-8\,b^3ce^4f+48\,c^4d^4g-128\,bc^3d^3eg+120\,b^2c^2d^2e^2g-48\,b^3}{128\,\sqrt{-c}c^4e|e|}$$

input

```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output

```
-1/192*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(6*e*g*x/c + (8*c^
3*e^5*f + 24*c^3*d*e^4*g - 7*b*c^2*e^5*g)/(c^4*e^4))*x + (144*c^3*d*e^4*f
- 40*b*c^2*e^5*f + 180*c^3*d^2*e^3*g - 156*b*c^2*d*e^4*g + 35*b^2*c*e^5*g)
/(c^4*e^4))*x + (704*c^3*d^2*e^3*f - 560*b*c^2*d*e^4*f + 120*b^2*c*e^5*f +
 576*c^3*d^3*e^2*g - 1036*b*c^2*d^2*e^3*g + 580*b^2*c*d*e^4*g - 105*b^3*e^
5*g)/(c^4*e^4)) - 5/128*(64*c^4*d^3*e*f - 96*b*c^3*d^2*e^2*f + 48*b^2*c^2*
d*e^3*f - 8*b^3*c*e^4*f + 48*c^4*d^4*g - 128*b*c^3*d^3*e*g + 120*b^2*c^2*d
^2*e^2*g - 48*b^3*c*d*e^3*g + 7*b^4*e^4*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e^2
)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqrt(
-c)*c^4*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{(f+g\,x)\,{(d+e\,x)}^3}{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input

```
int(((f + g*x)*(d + e*x)^3)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output

```
int(((f + g*x)*(d + e*x)^3)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 1300, normalized size of antiderivative = 4.71

$$\int \frac{(d+ex)^3(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
(i*(105*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*b**5*e**5*g - 930*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*
e + 2*c*d))*b**4*c*d*e**4*g - 120*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x
)*i)/sqrt( - b*e + 2*c*d))*b**4*c*e**5*f + 3240*sqrt(c)*asinh((sqrt( - b*e
 + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**2*d**2*e**3*g + 960*sqrt(
c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**2*d*
e**4*f - 5520*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2
*c*d))*b**2*c**3*d**3*e**2*g - 2880*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e
*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**3*d**2*e**3*f + 4560*sqrt(c)*asinh((s
qrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*d**4*e*g + 3840*
sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*
d**3*e**2*f - 1440*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*
e + 2*c*d))*c**5*d**5*g - 1920*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i
)/sqrt( - b*e + 2*c*d))*c**5*d**4*e*f + 105*sqrt(d + e*x)*sqrt(b*e - 2*c*d
)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**3*g - 580*sqrt
(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x
)*b**2*c**2*d*e**2*g - 120*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2
*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**3*f - 70*sqrt(d + e*x)*sqrt(
b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**
3*g*x + 1036*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt(...
```

3.174 $\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 203

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx &= -\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}\\ &\quad-\frac{(6cef+4cdg-5beg)(8cd-3be+2cex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{24c^3e^2}\\ &\quad+\frac{(2cd-be)^2(6cef+4cdg-5beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{7/2}e^2}\end{aligned}$$

output

```
-1/3*g*(e*x+d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c/e^2-1/24*(-5*b*e
*g+4*c*d*g+6*c*e*f)*(2*c*e*x-3*b*e+8*c*d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)
^(1/2)/c^3/e^2+1/8*(-b*e+2*c*d)^2*(-5*b*e*g+4*c*d*g+6*c*e*f)*arctan(c^(1/2
)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(7/2)/e^2
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\frac{(-2cd+be)^2\Bigl(-\frac{\sqrt{c}(d+ex)(-be+c(d-ex))\left(15b^2e^2g-2bce(9ef+26dg+5egx)+4c^2\left(10d^2g+6de(2f+gx)+e^2x(3f+2gx)\right)\right)}{(-2cd+be)^2}-3(6cef-}{24c^{7/2}e^2\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[((d + e*x)^2*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
],x]
```

output
```
((-2*c*d + b*e)^2*(-((Sqrt[c]*(d + e*x)*(-(b*e) + c*(d - e*x))*(15*b^2*e^2
*g - 2*b*c*e*(9*e*f + 26*d*g + 5*e*g*x) + 4*c^2*(10*d^2*g + 6*d*e*(2*f + g
*x) + e^2*x*(3*f + 2*g*x))))/(-2*c*d + b*e)^2) - 3*(6*c*e*f + 4*c*d*g - 5*
b*e*g)*Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*x]*ArcTan[Sqrt[c*d - b*e - c*e*x
]/(Sqrt[c]*Sqrt[d + e*x])]))/(24*c^(7/2)*e^2*Sqrt[(d + e*x)*(-(b*e) + c*(d
 - e*x))])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1221, 1134, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{(-5beg+4cdg+6cef)\int \frac{(d+ex)^2}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{6ce}-\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$\Big\downarrow \text{ 1134}$$

$$\frac{(-5beg+4cdg+6cef)\left(\frac{3(2cd-be)\int \frac{d+ex}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}\right)}{6ce}-\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$\Big\downarrow 1160$$

$$\frac{(-5beg+4cdg+6cef)\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2c}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce}}{2ce}\right.}{6ce}-\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$\Big\downarrow 1092$$

$$\frac{(-5beg+4cdg+6cef)\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{c}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right.}{4c}\right.}{6ce}-\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$\Big\downarrow 217$$

$$\frac{\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2c^{3/2}e}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}\right)(-5beg+4c}{6ce}-\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

input
```
Int[((d + e*x)^2*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
-1/3*(g*(d + e*x)^2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(c*e^2) + (
(6*c*e*f + 4*c*d*g - 5*b*e*g)*(-1/2*((d + e*x)*Sqrt[d*(c*d - b*e) - b*e^2*
x - c*e^2*x^2])/(c*e) + (3*(2*c*d - b*e)*(-(Sqrt[d*(c*d - b*e) - b*e^2*x -
 c*e^2*x^2]/(c*e)) + ((2*c*d - b*e)*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt
[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(2*c^(3/2)*e)))/(4*c)))/(6*c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(187) = 374$.

Time = 3.14 (sec) , antiderivative size = 773, normalized size of antiderivative = 3.81

method	result
default	$\frac{d^2 f \arctan\left(\frac{\sqrt{c\,e^2}\,\left(x+\frac{b}{2c}\right)}{\sqrt{-x^2 c\,e^2-x b\,e^2-b d e+c\,d^2}}\right)}{\sqrt{c\,e^2}}+e(2dg+ef)\left(-\frac{x\sqrt{-x^2 c\,e^2-x b\,e^2-b d e+c\,d^2}}{2c\,e^2}-\frac{3b\left(-\frac{\sqrt{-x^2 c\,e^2-x b\,e^2-b d e+c}}{c\,e^2}\right.}{}\right.$

input
```
int((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
d^2*f/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b
*d*e+c*d^2)^(1/2))+e*(2*d*g+e*f)*(-1/2*x/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c
*d^2)^(1/2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b
/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*
e+c*d^2)^(1/2)))+1/2*(-b*d*e+c*d^2)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/
2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+d*(d*g+2*e*f)*(-1/
c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arctan(
(c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+e^2*g*(
-1/3*x^2/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-5/6*b/c*(-1/2*x/c/e^
2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e
^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*
b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+1/2*(-b*d*e+c*d^2)/c/e^2/(c*
e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^
2)^(1/2)))+2/3*(-b*d*e+c*d^2)/c/e^2*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*
d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*
x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.88

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \Bigg[\frac{3\,(6\,(4\,c^3d^2e-4\,bc^2de^2+b^2ce^3)f+(16\,c^3d^3-36\,bc^2d^2e+24\,b^2cde^2-5\,b^3e^3)g)\sqrt{-c}\log\left(8\,c^2e^2x^2+8\,b\right.}{}$$

$$-\frac{3\,(6\,(4\,c^3d^2e-4\,bc^2de^2+b^2ce^3)f+(16\,c^3d^3-36\,bc^2d^2e+24\,b^2cde^2-5\,b^3e^3)g)\sqrt{c}\arctan\left(\frac{\sqrt{-ce^2x^2-be}}{2\,(c^2e^2x}\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output

```
[1/96*(3*(6*(4*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*f + (16*c^3*d^3 - 36
*b*c^2*d^2*e + 24*b^2*c*d*e^2 - 5*b^3*e^3)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 +
 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 4*sqrt(-c*e^2*x^2 - b*e^2
*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*(8*c^3*e^2*g*x^2 + 6*(8*
c^3*d*e - 3*b*c^2*e^2)*f + (40*c^3*d^2 - 52*b*c^2*d*e + 15*b^2*c*e^2)*g +
2*(6*c^3*e^2*f + (12*c^3*d*e - 5*b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*
x + c*d^2 - b*d*e))/(c^4*e^2), -1/48*(3*(6*(4*c^3*d^2*e - 4*b*c^2*d*e^2 +
b^2*c*e^3)*f + (16*c^3*d^3 - 36*b*c^2*d^2*e + 24*b^2*c*d*e^2 - 5*b^3*e^3)*
g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x
+ b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) + 2*(8*c^3*e
^2*g*x^2 + 6*(8*c^3*d*e - 3*b*c^2*e^2)*f + (40*c^3*d^2 - 52*b*c^2*d*e + 15
*b^2*c*e^2)*g + 2*(6*c^3*e^2*f + (12*c^3*d*e - 5*b*c^2*e^2)*g)*x)*sqrt(-c*
e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^4*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(194) = 388$.

Time = 1.44 (sec) , antiderivative size = 660, normalized size of antiderivative = 3.25

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \begin{cases} \left(-\frac{b\left(-\frac{3b\left(-\frac{5be^2g}{6c}+2deg+e^2f\right)}{4c}+d^2g+2def+\frac{g\left(-2bde+2cd^2\right)}{3c}\right)}{2c}+d^2f+\frac{\left(-bde+cd^2\right)\left(-\frac{5be^2g}{6c}+2deg+e^2f\right)}{2ce^2}\right)\left(\begin{cases}\frac{\log\left(-be^2-2ce\right.}{} \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}\right.}{\sqrt{-ce^2\left(\frac{b}{2c}+\right.}}\end{cases}\right. \\ -\frac{2\left(-\frac{g\left(-bde-be^2x+cd^2\right)^{\frac{7}{2}}}{7b^3e^4}-\frac{\left(-bde-be^2x+cd^2\right)^{\frac{5}{2}}\left(bdeg-be^2f-3cd^2g\right)}{5b^3e^4}-\frac{\left(-bde-be^2x+cd^2\right)^{\frac{3}{2}}\left(-2bcd^3eg+2bcd^2e^2f+3c^2d^4g\right)}{3b^3e^4}-\frac{\sqrt{-bde-be^2x+cd^2}\left(\right.}{}\right.}{be^2} \\ \frac{d^2fx+\frac{e^2gx^4}{4}+\frac{x^3\cdot\left(2deg+e^2f\right)}{3}+\frac{x^2\left(d^2g+2def\right)}{2}}{\sqrt{-bde+cd^2}} \end{cases}$$

input

```
integrate((e*x+d)**2*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output

```
Piecewise(((-b*(-3*b*(-5*b*e**2*g/(6*c) + 2*d*e*g + e**2*f)/(4*c) + d**2*g
 + 2*d*e*f + g*(-2*b*d*e + 2*c*d**2)/(3*c))/(2*c) + d**2*f + (-b*d*e + c*d
**2)*(-5*b*e**2*g/(6*c) + 2*d*e*g + e**2*f)/(2*c*e**2))*Piecewise((log(-b*
e**2 - 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e*
*2*x**2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*
c) + x)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)) + (-g*x**2
/(3*c) - x*(-5*b*e**2*g/(6*c) + 2*d*e*g + e**2*f)/(2*c*e**2) - (-3*b*(-5*b
*e**2*g/(6*c) + 2*d*e*g + e**2*f)/(4*c) + d**2*g + 2*d*e*f + g*(-2*b*d*e +
 2*c*d**2)/(3*c))/(c*e**2))*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)
, Ne(c*e**2, 0)), (-2*(-g*(-b*d*e - b*e**2*x + c*d**2)**(7/2)/(7*b**3*e**4
) - (-b*d*e - b*e**2*x + c*d**2)**(5/2)*(b*d*e*g - b*e**2*f - 3*c*d**2*g)/
(5*b**3*e**4) - (-b*d*e - b*e**2*x + c*d**2)**(3/2)*(-2*b*c*d**3*e*g + 2*b
*c*d**2*e**2*f + 3*c**2*d**4*g)/(3*b**3*e**4) - sqrt(-b*d*e - b*e**2*x + c
*d**2)*(b*c**2*d**5*e*g - b*c**2*d**4*e**2*f - c**3*d**6*g)/(b**3*e**4))/(
b*e**2), Ne(b*e**2, 0)), ((d**2*f*x + e**2*g*x**4/4 + x**3*(2*d*e*g + e**2
*f)/3 + x**2*(d**2*g + 2*d*e*f)/2)/sqrt(-b*d*e + c*d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$

$$-\frac{1}{24}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\Bigg(2\left(\frac{4\,gx}{c}+\frac{6\,c^2e^3f+12\,c^2de^2g-5\,bce^3g}{c^3e^3}\right)x+\frac{48\,c^2de^2f-18\,bce^3f}{}$$

$$-\frac{(24\,c^3d^2ef-24\,bc^2de^2f+6\,b^2ce^3f+16\,c^3d^3g-36\,bc^2d^2eg+24\,b^2cde^2g-5\,b^3e^3g)\log\left(\left|-be^2+2\,(\sqrt{}\right.\right.}{16\sqrt{-c}c^3e|e|}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
-1/24*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*g*x/c + (6*c^2*e^3*
f + 12*c^2*d*e^2*g - 5*b*c*e^3*g)/(c^3*e^3))*x + (48*c^2*d*e^2*f - 18*b*c*
e^3*f + 40*c^2*d^2*e*g - 52*b*c*d*e^2*g + 15*b^2*e^3*g)/(c^3*e^3)) - 1/16*
(24*c^3*d^2*e*f - 24*b*c^2*d*e^2*f + 6*b^2*c*e^3*f + 16*c^3*d^3*g - 36*b*c
^2*d^2*e*g + 24*b^2*c*d*e^2*g - 5*b^3*e^3*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e
^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqr
t(-c)*c^3*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{(f+g\,x)\,{(d+e\,x)}^2}{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^2)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^2)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 839, normalized size of antiderivative = 4.13

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*( - 15*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*b**4*e**4*g + 102*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
b*e + 2*c*d))*b**3*c*d*e**3*g + 18*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*b**3*c*e**4*f - 252*sqrt(c)*asinh((sqrt( - b*e
 + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d**2*e**2*g - 108*sqrt(
c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d*
e**3*f + 264*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*
c*d))*b*c**3*d**3*e*g + 216*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/s
qrt( - b*e + 2*c*d))*b*c**3*d**2*e**2*f - 96*sqrt(c)*asinh((sqrt( - b*e +
c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**4*g - 144*sqrt(c)*asinh((sqr
t( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**3*e*f - 15*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
b**2*c*e**2*g + 52*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*b*c**2*d*e*g + 18*sqrt(d + e*x)*sqrt(b*e - 2*c*d)
*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*e**2*f + 10*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
b*c**2*e**2*g*x - 40*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*
sqrt( - b*e + c*d - c*e*x)*c**3*d**2*g - 48*sqrt(d + e*x)*sqrt(b*e - 2*c*d
)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**3*d*e*f - 24*sqrt(d +
 e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)...
```

3.175 $\int \frac{(d+ex)(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 144

$$\int \frac{(d+ex)(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{(3beg-4c(ef+dg)-2cegx)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c^2e^2} + \frac{(2cd-be)(4cef+2cdg-3beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4c^{5/2}e^2}$$

output

```
1/4*(3*b*e*g-4*c*(d*g+e*f)-2*c*e*g*x)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/
2)/c^2/e^2+1/4*(-b*e+2*c*d)*(-3*b*e*g+2*c*d*g+4*c*e*f)*arctan(c^(1/2)*(e*x
+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(5/2)/e^2
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{\sqrt{d+ex}\Bigg(-\frac{\sqrt{d+ex}(-be+c(d-ex))(-3beg+2c(2ef+2dg+egx))}{c^2} + \left(-\frac{1}{c}\right)^{7/2} c(2cd-be)(4cef+2cdg-3beg)\sqrt{cd-be}}{4e^2\sqrt{(d+ex)(-be+c(d-ex))}}$$

input

```
Integrate[((d + e*x)*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],
x]
```

output

```
(Sqrt[d + e*x]*(-((Sqrt[d + e*x]*(-(b*e) + c*(d - e*x))*(-3*b*e*g + 2*c*(2
*e*f + 2*d*g + e*g*x)))/c^2) + (-c^(-1))^(7/2)*c*(2*c*d - b*e)*(4*c*e*f +
2*c*d*g - 3*b*e*g)*Sqrt[c*d - b*e - c*e*x]*Log[Sqrt[d + e*x] + (-c^(-1))^(
3/2)*c*Sqrt[c*d - b*e - c*e*x]]))/(4*e^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e
*x))])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1225, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(f+gx)}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \; 1225$$

$$\frac{(2cd-be)(-3beg+2cdg+4cef)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8c^2e}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(3beg-4c(dg+ef)-2cegx)}{4c^2e^2}$$

$$\Big\downarrow \; 1092$$

$$\frac{(2cd-be)(-3beg+2cdg+4cef)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{4c^2e}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(3beg-4c(dg+ef)-2cegx)}{4c^2e^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-3beg+2cdg+4cef)}{8c^{5/2}e^2}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(3beg-4c(dg+ef)-2cegx)}{4c^2e^2}$$

input
```
Int[((d + e*x)*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
((3*b*e*g - 4*c*(e*f + d*g) - 2*c*e*g*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*
e^2*x^2])/(4*c^2*e^2) + ((2*c*d - b*e)*(4*c*e*f + 2*c*d*g - 3*b*e*g)*ArcTa
n[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/
(8*c^(5/2)*e^2)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(132) = 264$.

Time = 2.48 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.64

method	result
default	$\frac{df \arctan\left(\frac{\sqrt{c\,e^2}\left(x+\frac{b}{2c}\right)}{\sqrt{-x^2 c\,e^2-xb\,e^2-bde+c\,d^2}}\right)}{\sqrt{c\,e^2}}+(dg+ef)\left(-\frac{\sqrt{-x^2 c\,e^2-xb\,e^2-bde+c\,d^2}}{c\,e^2}-\frac{b\arctan\left(\frac{\sqrt{c\,e^2}\left(x+\frac{b}{2c}\right)}{\sqrt{-x^2 c\,e^2-xb\,e^2-bde+c\,d^2}}\right.}{2c\sqrt{c\,e^2}}\right.$

input
```
int((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
d*f/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d
*e+c*d^2)^(1/2))+(d*g+e*f)*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2
)-1/2*b/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2
*x-b*d*e+c*d^2)^(1/2)))+e*g*(-1/2*x/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)
^(1/2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c
*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d
^2)^(1/2)))+1/2*(-b*d*e+c*d^2)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x
+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.80

$$\int \frac{(d+ex)(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$= \left[-\frac{(4\,(2\,c^2de-bce^2)f+(4\,c^2d^2-8\,bcde+3\,b^2e^2)g)\sqrt{-c}\log\left(8\,c^2e^2x^2+8\,bce^2x-4\,c^2d^2+4\,bcde+b^2e\right.}{}\right.$$
$$-\frac{(4\,(2\,c^2de-bce^2)f+(4\,c^2d^2-8\,bcde+3\,b^2e^2)g)\sqrt{c}\arctan\left(\frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,cex+be)\sqrt{c}}{2\,(c^2e^2x^2+bce^2x-c^2d^2+bcde)}\right)+2\,(2\,c^2}{8\,c^3e^2}$$

input
```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algori
thm="fricas")
```

output
```
[-1/16*((4*(2*c^2*d*e - b*c*e^2)*f + (4*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*g
)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e
^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)
) + 4*(2*c^2*e*g*x + 4*c^2*e*f + (4*c^2*d - 3*b*c*e)*g)*sqrt(-c*e^2*x^2 -
b*e^2*x + c*d^2 - b*d*e))/(c^3*e^2), -1/8*((4*(2*c^2*d*e - b*c*e^2)*f + (4
*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 -
b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x
- c^2*d^2 + b*c*d*e)) + 2*(2*c^2*e*g*x + 4*c^2*e*f + (4*c^2*d - 3*b*c*e)*g
)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^3*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(136) = 272$.

Time = 2.07 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.89

$$\int \frac{(d+ex)(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \begin{cases} \left(-\frac{gx}{2ce}-\frac{-\frac{3beg}{4c}+dg+ef}{ce^2}\right)\sqrt{-bde-be^2x+cd^2-ce^2x^2}+\left(-\frac{b\left(-\frac{3beg}{4c}+dg+ef\right)}{2c}+df+\frac{g(-bde+cd^2)}{2ce}\right)\left(\begin{cases}\frac{\log\left(-\right.}{} \\ \frac{\left(\frac{b}{2c}+\right.}{\sqrt{-}}\end{cases}\right. \\ -\frac{2\left(\frac{g\left(-bde-be^2x+cd^2\right)^{\frac{5}{2}}}{5b^2e^3}+\frac{\left(-bde-be^2x+cd^2\right)^{\frac{3}{2}}\left(bdeg-be^2f-2cd^2g\right)}{3b^2e^3}+\frac{\sqrt{-bde-be^2x+cd^2}\left(-bcd^3eg+bcd^2e^2f+c^2d^4g\right)}{b^2e^3}\right)}{be^2} \\ \frac{dfx+\frac{egx^3}{3}+\frac{x^2(dg+ef)}{2}}{\sqrt{-bde+cd^2}} \end{cases}$$

input
```
integrate((e*x+d)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x)
```

output

```
Piecewise(((-g*x/(2*c*e) - (-3*b*e*g/(4*c) + d*g + e*f)/(c*e**2))*sqrt(-b*
d*e - b*e**2*x + c*d**2 - c*e**2*x**2) + (-b*(-3*b*e*g/(4*c) + d*g + e*f)/
(2*c) + d*f + g*(-b*d*e + c*d**2)/(2*c*e))*Piecewise((log(-b*e**2 - 2*c*e*
*2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2))/sqr
t(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*c) + x)*log(b
/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)), Ne(c*e**2, 0)), (-2*(g
*(-b*d*e - b*e**2*x + c*d**2)**(5/2)/(5*b**2*e**3) + (-b*d*e - b*e**2*x +
c*d**2)**(3/2)*(b*d*e*g - b*e**2*f - 2*c*d**2*g)/(3*b**2*e**3) + sqrt(-b*d
*e - b*e**2*x + c*d**2)*(-b*c*d**3*e*g + b*c*d**2*e**2*f + c**2*d**4*g)/(b
**2*e**3))/(b*e**2), Ne(b*e**2, 0)), ((d*f*x + e*g*x**3/3 + x**2*(d*g + e*
f)/2)/sqrt(-b*d*e + c*d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algori
thm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.32

$$\begin{aligned}
&\int \frac{(d+ex)(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx \\
&= -\frac{1}{4}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left(\frac{2\,gx}{ce}+\frac{4\,ce^2f+4\,cdeg-3\,be^2g}{c^2e^3}\right) \\
&\quad -\frac{(8\,c^2def-4\,bce^2f+4\,c^2d^2g-8\,bcdeg+3\,b^2e^2g)\log\left(\left|-be^2+2\,(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-be^2x+cd^2-}\right.\right.}{8\,\sqrt{-c}c^2e|e|}
\end{aligned}$$

input
```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algori
thm="giac")
```

output
```
-1/4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*g*x/(c*e) + (4*c*e^2*f
+ 4*c*d*e*g - 3*b*e^2*g)/(c^2*e^3)) - 1/8*(8*c^2*d*e*f - 4*b*c*e^2*f + 4*c
^2*d^2*g - 8*b*c*d*e*g + 3*b^2*e^2*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e^2)*x -
 sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqrt(-c)*c
^2*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{(f+g\,x)\,(d+e\,x)}{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input
```
int(((f + g*x)*(d + e*x))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output
```
int(((f + g*x)*(d + e*x))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.42

$$\int \frac{(d+ex)(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{i\left(3\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^3e^3g-14\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2cd\,e^2g-4\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2c\right.}{}$$

input
```
int((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*(3*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b
**3*e**3*g - 14*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
 2*c*d))*b**2*c*d*e**2*g - 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/
sqrt( - b*e + 2*c*d))*b**2*c*e**3*f + 20*sqrt(c)*asinh((sqrt( - b*e + c*d
- c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e*g + 16*sqrt(c)*asinh((sqrt
( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**2*f - 8*sqrt(c
)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**3*g -
 16*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**
3*d**2*e*f + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt(
- b*e + c*d - c*e*x)*b*c*e*g - 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b
*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d*g - 4*sqrt(d + e*x)*sqrt(b*e
 - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*e*f - 2*sqr
t(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*
x)*c**2*e*g*x))/(4*c**3*e**2*(b*e - 2*c*d))
```

3.176 $\int \frac{f+gx}{(d+ex)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 117

$$\int \frac{f+gx}{(d+ex)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(2cd-be)(d+ex)}+\frac{2g\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{\sqrt{c}e^2}$$

output
```
-2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e*x
+d)+2*g*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(
1/2)/e^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

$$\int \frac{f+gx}{(d+ex)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\frac{-2\sqrt{c}(ef-dg)(-cd+be+cex)+2(2cd-be)g\sqrt{d+ex}\sqrt{cd-be-cex}\arctan\left(\frac{\sqrt{cd-be-cex}}{\sqrt{c}\sqrt{d+ex}}\right)}{\sqrt{c}e^2(-2cd+be)\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[(f + g*x)/((d + e*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),
x]
```

output
```
(-2*Sqrt[c]*(e*f - d*g)*(-(c*d) + b*e + c*e*x) + 2*(2*c*d - b*e)*g*Sqrt[d
+ e*x]*Sqrt[c*d - b*e - c*e*x]*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqr
t[d + e*x])])/(Sqrt[c]*e^2*(-2*c*d + b*e)*Sqrt[(d + e*x)*(-(b*e) + c*(d -
e*x))])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1220, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \textcolor{blue}{1220}$$

$$\frac{g\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{e}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)(2cd-be)}$$

$$\Big\downarrow \textcolor{blue}{1092}$$

$$\frac{2g\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{e}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)(2cd-be)}$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{g\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{\sqrt{c}e^2}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)(2cd-be)}$$

input
```
Int[(f + g*x)/((d + e*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output
```
(-2*(e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e^2*(2*c*d - b
*e)*(d + e*x)) + (g*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) -
 b*e^2*x - c*e^2*x^2])])/(Sqrt[c]*e^2)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{g \arctan\left(\frac{\sqrt{c\,e^{2}}\,\left(x+\frac{b}{2c}\right)}{\sqrt{-x^{2} c\,e^{2}-x b\,e^{2}-b d e+c\,d^{2}}}\right)}{e\sqrt{c\,e^{2}}}+\frac{2(d g-e f)\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2 d e c\right)\left(x+\frac{d}{e}\right)}}{e^{2}\left(-b\,e^{2}+2 d e c\right)\left(x+\frac{d}{e}\right)}$	134

input
```
int((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
g/e/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d
*e+c*d^2)^(1/2))+2*(d*g-e*f)/e^2/(-b*e^2+2*c*d*e)/(x+d/e)*(-c*e^2*(x+d/e)^
2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.46

$$\int \frac{f+gx}{(d+ex)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \Bigg[-\frac{((2\,cde-be^2)gx+(2\,cd^2-bde)g)\sqrt{-c}\log\big(8\,c^2e^2x^2+8\,bce^2x-4\,c^2d^2+4\,bcde+b^2e^2-4\,\sqrt{-ce^2x^2}}{2\,(2\,c^2d^2e^2-bcde^3+(2\,c^2de}$$

$$-\frac{((2\,cde-be^2)gx+(2\,cd^2-bde)g)\sqrt{c}\arctan\left(\frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,cex+be)\sqrt{c}}{2\,(c^2e^2x^2+bce^2x-c^2d^2+bcde)}\right)+2\,\sqrt{-ce^2x^2-be^2x+}}{2\,c^2d^2e^2-bcde^3+(2\,c^2de^3-bce^4)x}$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algori
thm="fricas")
```

output
```
[-1/2*(((2*c*d*e - b*e^2)*g*x + (2*c*d^2 - b*d*e)*g)*sqrt(-c)*log(8*c^2*e^
2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2
- b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) + 4*sqrt(-c*e^2*x^2 -
 b*e^2*x + c*d^2 - b*d*e)*(c*e*f - c*d*g))/(2*c^2*d^2*e^2 - b*c*d*e^3 + (2
*c^2*d*e^3 - b*c*e^4)*x), -(((2*c*d*e - b*e^2)*g*x + (2*c*d^2 - b*d*e)*g)*
sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b
*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) + 2*sqrt(-c*e^2
*x^2 - b*e^2*x + c*d^2 - b*d*e)*(c*e*f - c*d*g))/(2*c^2*d^2*e^2 - b*c*d*e^
3 + (2*c^2*d*e^3 - b*c*e^4)*x)]
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{f+gx}{\sqrt{-(d+ex)(be-cd+cex)}(d+ex)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x)
```

output
```
Integral((f + g*x)/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(109) = 218$.

Time = 0.39 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.46

$$\int \frac{f+gx}{(d+ex)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$

$$-\frac{g\log\left(\left|bcd^2e^2-2\left(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-be^2x+cd^2-bde}\right)\sqrt{-c}cd^2|e|-2\left(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-b\right.\right.\right.}{}$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algori
thm="giac")
```

output
```
-1/3*g*log(abs(b*c*d^2*e^2 - 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x
 + c*d^2 - b*d*e))*sqrt(-c)*c*d^2*abs(e) - 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2
*x^2 - b*e^2*x + c*d^2 - b*d*e))*b*sqrt(-c)*d*e*abs(e) - 4*(sqrt(-c*e^2)*x
 - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*c*d*e - (sqrt(-c*e^2)*x -
 sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*b*e^2 + 2*(sqrt(-c*e^2)*x -
 sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*sqrt(-c)*abs(e)))/(sqrt(-c)
*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{f+g\,x}{(d+e\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input
```
int((f + g*x)/((d + e*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),x)
```

output
```
int((f + g*x)/((d + e*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.15

$$\int \frac{f+gx}{(d+ex)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2i\left(\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2d\,e^2g+\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2e^3gx-4\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)bc\,d^2e}{}$$

input
```
int((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(2*i*(sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b
**2*d*e**2*g + sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
2*c*d))*b**2*e**3*g*x - 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqr
t( - b*e + 2*c*d))*b*c*d**2*e*g - 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e
*x)*i)/sqrt( - b*e + 2*c*d))*b*c*d*e**2*g*x + 4*sqrt(c)*asinh((sqrt( - b*e
 + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*d**3*g + 4*sqrt(c)*asinh((sq
rt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*d**2*e*g*x - sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
c*d*g + sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*c*e*f + sqrt(c)*b*c*d**2*e*g - sqrt(c)*b*c*d*e**2*f + sqrt(c
)*b*c*d*e**2*g*x - sqrt(c)*b*c*e**3*f*x - 2*sqrt(c)*c**2*d**3*g + 2*sqrt(c
)*c**2*d**2*e*f - 2*sqrt(c)*c**2*d**2*e*g*x + 2*sqrt(c)*c**2*d*e**2*f*x))/
(c*e**2*(b**2*d*e**2 + b**2*e**3*x - 4*b*c*d**2*e - 4*b*c*d*e**2*x + 4*c**
2*d**3 + 4*c**2*d**2*e*x))
```

3.177 $\int \frac{f+gx}{(d+ex)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 137

$$\begin{aligned}\int \frac{f+gx}{(d+ex)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx &= -\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e^2(2cd-be)(d+ex)^2}\\ &\quad -\frac{2(2cef+4cdg-3beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e^2(2cd-be)^2(d+ex)}\end{aligned}$$

output

```
-2/3*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^2-2/3*(-3*b*e*g+4*c*d*g+2*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1
/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int \frac{f+gx}{(d+ex)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{2\sqrt{(d+ex)(-be+c(d-ex))}(2c(d^2g+e^2fx+2de(f+gx))-be(2dg+e(f+3gx)))}{3e^2(-2cd+be)^2(d+ex)^2}$$

input

```
Integrate[(f + g*x)/((d + e*x)^2*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]
),x]
```

output

```
(-2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(2*c*(d^2*g + e^2*f*x + 2*d*e*(
f + g*x)) - b*e*(2*d*g + e*(f + 3*g*x))))/(3*e^2*(-2*c*d + b*e)^2*(d + e*x
)^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^2\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \color{blue}{1220}$$

$$\frac{(-3beg+4cdg+2cef)\int \frac{1}{(d+ex)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e^2(d+ex)^2(2cd-be)}$$

$$\Big\downarrow \color{blue}{1123}$$

$$-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e^2(d+ex)^2(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}(-3beg+4cdg+2cef)}{3e^2(d+ex)(2cd-be)^2}$$

input

```
Int[(f + g*x)/((d + e*x)^2*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output

```
(-2*(e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*e^2*(2*c*d -
 b*e)*(d + e*x)^2) - (2*(2*c*e*f + 4*c*d*g - 3*b*e*g)*Sqrt[d*(c*d - b*e) -
 b*e^2*x - c*e^2*x^2])/(3*e^2*(2*c*d - b*e)^2*(d + e*x))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.84

method	result
trager	$\frac{2\left(3b\,e^{2}gx-4cdegx-2c\,e^{2}fx+2bdeg+b\,e^{2}f-2c\,d^{2}g-4cdef\right)\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}{3\left(b^{2}e^{2}-4bcde+4c^{2}d^{2}\right)\left(ex+d\right)^{2}e^{2}}$
gosper	$-\frac{2\left(cex+be-cd\right)\left(3b\,e^{2}gx-4cdegx-2c\,e^{2}fx+2bdeg+b\,e^{2}f-2c\,d^{2}g-4cdef\right)}{3\left(ex+d\right)e^{2}\left(b^{2}e^{2}-4bcde+4c^{2}d^{2}\right)\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}$
orering	$-\frac{2\left(cex+be-cd\right)\left(3b\,e^{2}gx-4cdegx-2c\,e^{2}fx+2bdeg+b\,e^{2}f-2c\,d^{2}g-4cdef\right)}{3\left(ex+d\right)e^{2}\left(b^{2}e^{2}-4bcde+4c^{2}d^{2}\right)\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}$
default	$-\frac{2g\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)}}{e^{2}\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)}-\frac{\left(dg-ef\right)\left(-\frac{2\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)}}{3\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{2}}-\frac{4c\,e^{2}\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+}}{3\left(-b\,e^{2}+2dec\right)^{2}\left(x+}\right.}{e^{3}}$

input
```
int((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*(3*b*e^2*g*x-4*c*d*e*g*x-2*c*e^2*f*x+2*b*d*e*g+b*e^2*f-2*c*d^2*g-4*c*d
*e*f)/(b^2*e^2-4*b*c*d*e+4*c^2*d^2)/(e*x+d)^2/e^2*(-c*e^2*x^2-b*e^2*x-b*d*
e+c*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33

$$\int \frac{f+gx}{(d+ex)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left((4\,cde-be^2)f+2\,(cd^2-bde)g+(2\,ce^2f+(4\,cde-3\,be^2)g)x\right)}{3\,(4\,c^2d^4e^2-4\,bcd^3e^3+b^2d^2e^4+(4\,c^2d^2e^4-4\,bcde^5+b^2e^6)x^2+2\,(4\,c^2d^3e^3-4\,bcd^2e^4+b^2de^5)x)}$$

input
```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output
```
-2/3*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((4*c*d*e - b*e^2)*f + 2*(
c*d^2 - b*d*e)*g + (2*c*e^2*f + (4*c*d*e - 3*b*e^2)*g)*x)/(4*c^2*d^4*e^2 -
 4*b*c*d^3*e^3 + b^2*d^2*e^4 + (4*c^2*d^2*e^4 - 4*b*c*d*e^5 + b^2*e^6)*x^2
 + 2*(4*c^2*d^3*e^3 - 4*b*c*d^2*e^4 + b^2*d*e^5)*x)
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{f+gx}{\sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\left(d+ex\right)^2}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**2/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output
```
Integral((f + g*x)/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(129) = 258.

Time = 0.43 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.20

$$\int \frac{f+gx}{(d+ex)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$= \frac{2\left(\frac{(2\sqrt{-c}cef+4\sqrt{-c}cdg-3b\sqrt{-c}eg)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{4c^2d^2e-4bcde^2+b^2e^3} - \frac{\frac{\left(3c\sqrt{-c+\frac{2cd}{ex+d}-\frac{be}{ex+d}}+\left(-c+\frac{2cd}{ex+d}-\frac{be}{ex+d}\right)^{\frac{3}{2}}\right)e^2f}{2cde-be^2} - \frac{\left(3c\sqrt{-c+\frac{2cd}{ex+d}-\frac{be}{ex+d}}+(-c\right.}{2cde-be}}{2cde\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)-be^2\operatorname{sgn}\left(\frac{1}{ex+d}\right)}\right.}{3|e|}$$

input
```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
2/3*((2*sqrt(-c)*c*e*f + 4*sqrt(-c)*c*d*g - 3*b*sqrt(-c)*e*g)*sgn(1/(e*x +
 d))*sgn(e)/(4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3) - ((3*c*sqrt(-c + 2*c*d/
(e*x + d) - b*e/(e*x + d)) + (-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2))
*e^2*f/(2*c*d*e - b*e^2) - (3*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))
 + (-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2))*d*e*g/(2*c*d*e - b*e^2) +
 3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*g)/(2*c*d*e*sgn(1/(e*x + d))
*sgn(e) - b*e^2*sgn(1/(e*x + d))*sgn(e)))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

$$\int \frac{f+gx}{(d+ex)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{2\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,(2\,c\,d^2\,g-b\,e^2\,f-3\,b\,e^2\,g\,x+2\,c\,e^2\,f\,x-2\,b\,d\,e\,g+4\,c\,d\,e\,f+4\,c\,d\,e\,g}{3\,e^2\,{(b\,e-2\,c\,d)}^2\,{(d+e\,x)}^2}$$

input
```
int((f + g*x)/((d + e*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),x)
```

output
```
-(2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*(2*c*d^2*g - b*e^2*f - 3*b
*e^2*g*x + 2*c*e^2*f*x - 2*b*d*e*g + 4*c*d*e*f + 4*c*d*e*g*x))/(3*e^2*(b*e
 - 2*c*d)^2*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 627, normalized size of antiderivative = 4.58

$$\int \frac{f+gx}{(d+ex)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2i(2\sqrt{ex+d}\,\sqrt{be-2cd}\,\sqrt{-be+2cd}\,\sqrt{-cex-be+cd}\,bdeg+\sqrt{ex+d}\,\sqrt{be-2cd}\,\sqrt{-be+2cd}\,\sqrt{-cex-}{}$$

input
```
int((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(2*i*(2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*b*d*e*g + sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*
d)*sqrt( - b*e + c*d - c*e*x)*b*e**2*f + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)
*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*e**2*g*x - 2*sqrt(d + e
*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c*d*
*2*g - 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
+ c*d - c*e*x)*c*d*e*f - 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2
*c*d)*sqrt( - b*e + c*d - c*e*x)*c*d*e*g*x - 2*sqrt(d + e*x)*sqrt(b*e - 2*
c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c*e**2*f*x + sqrt(c)*
b**2*d**2*e**2*g + 2*sqrt(c)*b**2*d*e**3*g*x + sqrt(c)*b**2*e**4*g*x**2 -
2*sqrt(c)*b*c*d**3*e*g - 2*sqrt(c)*b*c*d**2*e**2*f - 4*sqrt(c)*b*c*d**2*e*
*2*g*x - 4*sqrt(c)*b*c*d*e**3*f*x - 2*sqrt(c)*b*c*d*e**3*g*x**2 - 2*sqrt(c
)*b*c*e**4*f*x**2 + 4*sqrt(c)*c**2*d**3*e*f + 8*sqrt(c)*c**2*d**2*e**2*f*x
 + 4*sqrt(c)*c**2*d*e**3*f*x**2))/(3*e**2*(b**3*d**2*e**3 + 2*b**3*d*e**4*
x + b**3*e**5*x**2 - 6*b**2*c*d**3*e**2 - 12*b**2*c*d**2*e**3*x - 6*b**2*c
*d*e**4*x**2 + 12*b*c**2*d**4*e + 24*b*c**2*d**3*e**2*x + 12*b*c**2*d**2*e
**3*x**2 - 8*c**3*d**5 - 16*c**3*d**4*e*x - 8*c**3*d**3*e**2*x**2))
```

3.178 $\int \frac{f+gx}{(d+ex)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 210

$$\begin{aligned}
&\int \frac{f+gx}{(d+ex)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx \\
&= -\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e^2(2cd-be)(d+ex)^3} \\
&\quad -\frac{2(4cef+6cdg-5beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{15e^2(2cd-be)^2(d+ex)^2} \\
&\quad -\frac{4c(4cef+6cdg-5beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{15e^2(2cd-be)^3(d+ex)}
\end{aligned}$$

output

```
-2/5*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^3-2/15*(-5*b*e*g+6*c*d*g+4*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
1/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^2-4/15*c*(-5*b*e*g+6*c*d*g+4*c*e*f)*(d*(-b
*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.79

$$\int \frac{f+gx}{(d+ex)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{2(-cd+be+cex)\left(b^2e^2(3ef+2dg+5egx)-2bce(7d^2g+e^2x(2f+5gx)+2de(4f+9gx))+4c^2(3}{15e^2(-2cd+be)^3(d+ex)^2\sqrt{(d+ex)(-be+c(d-e}}$$

input

```
Integrate[(f + g*x)/((d + e*x)^3*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]
),x]
```

output

```
(-2*(-(c*d) + b*e + c*e*x)*(b^2*e^2*(3*e*f + 2*d*g + 5*e*g*x) - 2*b*c*e*(7
*d^2*g + e^2*x*(2*f + 5*g*x) + 2*d*e*(4*f + 9*g*x)) + 4*c^2*(3*d^3*g + 2*e
^3*f*x^2 + 3*d*e^2*x*(2*f + g*x) + d^2*e*(7*f + 9*g*x))))/(15*e^2*(-2*c*d
+ b*e)^3*(d + e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^3\sqrt{-bde-be^2x+cd^2-ce^2x^2}}\,dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(-5beg+6cdg+4cef)\int \frac{1}{(d+ex)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{5e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(-5beg+6cdg+4cef)\left(\frac{2c\int \frac{1}{(d+ex)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)^2(2cd-be)}\right)}{5e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e^2(d+ex)^3(2cd-be)}$$

$$\Big\downarrow \textcolor{blue}{1123}$$

$$\frac{\left(-\frac{4c\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)(2cd-be)^2}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)^2(2cd-be)}\right)(-5beg+6cdg+4cef)}{5e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e^2(d+ex)^3(2cd-be)}$$

input
```
Int[(f + g*x)/((d + e*x)^3*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output
```
(-2*(e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(5*e^2*(2*c*d -
 b*e)*(d + e*x)^3) + ((4*c*e*f + 6*c*d*g - 5*b*e*g)*((-2*Sqrt[d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2])/(3*e*(2*c*d - b*e)*(d + e*x)^2) - (4*c*Sqrt[d*(c
*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*e*(2*c*d - b*e)^2*(d + e*x))))/(5*e*(
2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.07

method	result
trager	$\frac{2\left(-10bc\,e^3 g\,x^2+12c^2 d\,e^2 g\,x^2+8f\,c^2 e^3 x^2+5b^2 e^3 gx-36bcd\,e^2 gx-4bc\,e^3 fx+36c^2 d^2 egx+24c^2 d\,e^2 fx+2b^2 d\,e^2 g+3b^2 e^3 f-14bc\,d^2 eg\right.}{15(b^3 e^3-6d\,e^2 b^2 c+12d^2 eb\,c^2-8d^3 c^3)e^2 (ex+d)^3}$
gosper	$-\frac{2(cex+be-cd)\left(-10bc\,e^3 g\,x^2+12c^2 d\,e^2 g\,x^2+8f\,c^2 e^3 x^2+5b^2 e^3 gx-36bcd\,e^2 gx-4bc\,e^3 fx+36c^2 d^2 egx+24c^2 d\,e^2 fx+2b^2 d\,e^2 g+3b^2\right.}{15(ex+d)^2 (b^3 e^3-6d\,e^2 b^2 c+12d^2 eb\,c^2-8d^3 c^3)e^2 \sqrt{-x^2 c\,e^2-xb\,e^2-bde+c\,d^2}}$
orering	$-\frac{2(cex+be-cd)\left(-10bc\,e^3 g\,x^2+12c^2 d\,e^2 g\,x^2+8f\,c^2 e^3 x^2+5b^2 e^3 gx-36bcd\,e^2 gx-4bc\,e^3 fx+36c^2 d^2 egx+24c^2 d\,e^2 fx+2b^2 d\,e^2 g+3b^2\right.}{15(ex+d)^2 (b^3 e^3-6d\,e^2 b^2 c+12d^2 eb\,c^2-8d^3 c^3)e^2 \sqrt{-x^2 c\,e^2-xb\,e^2-bde+c\,d^2}}$
default	$\frac{g\left(-\frac{2\sqrt{-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)}}{3\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)^2}-\frac{4c\,e^2\sqrt{-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)}}{3\left(-b\,e^2+2dec\right)^2\left(x+\frac{d}{e}\right)}\right)}{e^3}-\frac{(dg-ef)\left(-\frac{2\sqrt{-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+\right.}}{5\left(-b\,e^2+2dec\right)\left(x+\right.}\right.}{}$

input
```
int((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/15*(-10*b*c*e^3*g*x^2+12*c^2*d*e^2*g*x^2+8*c^2*e^3*f*x^2+5*b^2*e^3*g*x-3
6*b*c*d*e^2*g*x-4*b*c*e^3*f*x+36*c^2*d^2*e*g*x+24*c^2*d*e^2*f*x+2*b^2*d*e^
2*g+3*b^2*e^3*f-14*b*c*d^2*e*g-16*b*c*d*e^2*f+12*c^2*d^3*g+28*c^2*d^2*e*f)
/(b^3*e^3-6*b^2*c*d*e^2+12*b*c^2*d^2*e-8*c^3*d^3)/e^2/(e*x+d)^3*(-c*e^2*x^
2-b*e^2*x-b*d*e+c*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.75

$$\int \frac{f+gx}{(d+ex)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$
$$-\frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,(4\,c^2e^3f+(6\,c^2de^2-5\,bce^3)g)x^2+(28\,c^2d^2e-16\,bcde^2+3\,b^2e^3)f+}{15\,(8\,c^3d^6e^2-12\,bc^2d^5e^3+6\,b^2cd^4e^4-b^3d^3e^5+(8\,c^3d^3e^5-12\,bc^2d^2e^6+6\,b^2cde^7-b^3e^8)x^3+3\,(8\,c^3d}$$

input
```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output
```
-2/15*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*c^2*e^3*f + (6*c^2*
d*e^2 - 5*b*c*e^3)*g)*x^2 + (28*c^2*d^2*e - 16*b*c*d*e^2 + 3*b^2*e^3)*f +
2*(6*c^2*d^3 - 7*b*c*d^2*e + b^2*d*e^2)*g + (4*(6*c^2*d*e^2 - b*c*e^3)*f +
 (36*c^2*d^2*e - 36*b*c*d*e^2 + 5*b^2*e^3)*g)*x)/(8*c^3*d^6*e^2 - 12*b*c^2
*d^5*e^3 + 6*b^2*c*d^4*e^4 - b^3*d^3*e^5 + (8*c^3*d^3*e^5 - 12*b*c^2*d^2*e
^6 + 6*b^2*c*d*e^7 - b^3*e^8)*x^3 + 3*(8*c^3*d^4*e^4 - 12*b*c^2*d^3*e^5 +
6*b^2*c*d^2*e^6 - b^3*d*e^7)*x^2 + 3*(8*c^3*d^5*e^3 - 12*b*c^2*d^4*e^4 + 6
*b^2*c*d^3*e^5 - b^3*d^2*e^6)*x)
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\int \frac{f+gx}{\sqrt{-(d+ex)\,(be-cd+cex)}\,(d+ex)^3}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**3/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output
```
Integral((f + g*x)/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[6,3,13,0]%%%}+%%%{-12,[5,4,12,1]%%%}+%%%{60,[4,5,11
,2]%%%}+%
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.24

$$\begin{aligned}\int \frac{f+gx}{(d+ex)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx &= \frac{\left(\frac{8\,c^2\,d\,g+16\,c^2\,e\,f-8\,b\,c\,e\,g}{15\,e^2\,(b\,e-2\,c\,d)^3}-\frac{8\,c^2\,d\,g}{15\,e^2\,(b\,e-2\,c\,d)^3}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{d+e\,x}\\ &\quad-\frac{\left(\frac{2\,b\,g}{5\,(3\,b\,e^2-6\,c\,d\,e)\,(b\,e-2\,c\,d)}-\frac{4\,c\,d\,g}{5\,e\,(3\,b\,e^2-6\,c\,d\,e)\,(b\,e-2\,c\,d)}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{(d+e\,x)^2}\\ &\quad-\frac{\left(\frac{12\,c\,d\,g-12\,b\,e\,g+8\,c\,e\,f}{5\,e\,(3\,b\,e^2-6\,c\,d\,e)\,(b\,e-2\,c\,d)}+\frac{4\,c\,d\,g}{5\,e\,(3\,b\,e^2-6\,c\,d\,e)\,(b\,e-2\,c\,d)}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{(d+e\,x)^2}\\ &\quad+\frac{\left(\frac{2\,f}{5\,b\,e^2-10\,c\,d\,e}-\frac{2\,d\,g}{e\,(5\,b\,e^2-10\,c\,d\,e)}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{(d+e\,x)^3}\\ &\quad-\frac{\left(\frac{4\,c\,g\,(3\,b\,e-4\,c\,d)}{15\,e^2\,(b\,e-2\,c\,d)^3}-\frac{8\,c^2\,d\,g}{15\,e^2\,(b\,e-2\,c\,d)^3}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{d+e\,x}\end{aligned}$$

input
```
int((f + g*x)/((d + e*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),x)
```

output
```
(((8*c^2*d*g + 16*c^2*e*f - 8*b*c*e*g)/(15*e^2*(b*e - 2*c*d)^3) - (8*c^2*d
*g)/(15*e^2*(b*e - 2*c*d)^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))
/(d + e*x) - (((2*b*g)/(5*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)) - (4*c*d*g)/(
5*e*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2
*x)^(1/2))/(d + e*x)^2 - (((12*c*d*g - 12*b*e*g + 8*c*e*f)/(5*e*(3*b*e^2 -
 6*c*d*e)*(b*e - 2*c*d)) + (4*c*d*g)/(5*e*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d
)))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2 + (((2*f)/(5*
b*e^2 - 10*c*d*e) - (2*d*g)/(e*(5*b*e^2 - 10*c*d*e)))*(c*d^2 - c*e^2*x^2 -
 b*d*e - b*e^2*x)^(1/2))/(d + e*x)^3 - (((4*c*g*(3*b*e - 4*c*d))/(15*e^2*(
b*e - 2*c*d)^3) - (8*c^2*d*g)/(15*e^2*(b*e - 2*c*d)^3))*(c*d^2 - c*e^2*x^2
 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1262, normalized size of antiderivative = 6.01

$$\int \frac{f+gx}{(d+ex)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
(2*i*(2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*b**2*d*e**2*g + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*
e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*f + 5*sqrt(d + e*x)*sqrt(b
*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*g*x
- 14*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*
d - c*e*x)*b*c*d**2*e*g - 16*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*f - 36*sqrt(d + e*x)*sqrt(b*
e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*g*x
- 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
 - c*e*x)*b*c*e**3*f*x - 10*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*e**3*g*x**2 + 12*sqrt(d + e*x)*sqrt(
b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d**3*g +
 28*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
 - c*e*x)*c**2*d**2*e*f + 36*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*e*g*x + 24*sqrt(d + e*x)*sqrt
(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d*e**2*
f*x + 12*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
+ c*d - c*e*x)*c**2*d*e**2*g*x**2 + 8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt
( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*e**3*f*x**2 - 10*sqrt(c)*
b**2*c*d**3*e**2*g - 30*sqrt(c)*b**2*c*d**2*e**3*g*x - 30*sqrt(c)*b**2*...
```

3.179 $\int \frac{f+gx}{(d+ex)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 285

$$\begin{aligned}
&\int \frac{f+gx}{(d+ex)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx \\
&= -\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7e^2(2cd-be)(d+ex)^4} \\
&\quad -\frac{2(6cef+8cdg-7beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{35e^2(2cd-be)^2(d+ex)^3} \\
&\quad -\frac{8c(6cef+8cdg-7beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{105e^2(2cd-be)^3(d+ex)^2} \\
&\quad -\frac{16c^2(6cef+8cdg-7beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{105e^2(2cd-be)^4(d+ex)}
\end{aligned}$$

output

```
-2/7*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^4-2/35*(-7*b*e*g+8*c*d*g+6*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
1/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^3-8/105*c*(-7*b*e*g+8*c*d*g+6*c*e*f)*(d*(-
b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^2-16/105*c^2*
(-7*b*e*g+8*c*d*g+6*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*
e+2*c*d)^4/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.87

$$\int \frac{f+gx}{(d+ex)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$= \frac{2(-cd+be+cex)\left(-3b^3e^3(5ef+2dg+7egx)+8c^3(13d^4g+6e^4fx^3+8de^3x^2(3f+gx)+4d^3e(9f+1\right.}{}$$

input
```
Integrate[(f + g*x)/((d + e*x)^4*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]
),x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)*(-3*b^3*e^3*(5*e*f + 2*d*g + 7*e*g*x) + 8*c^3*(1
3*d^4*g + 6*e^4*f*x^3 + 8*d*e^3*x^2*(3*f + g*x) + 4*d^3*e*(9*f + 13*g*x) +
 d^2*e^2*x*(39*f + 32*g*x)) + 2*b^2*c*e^2*(23*d^2*g + e^2*x*(9*f + 14*g*x)
 + d*e*(54*f + 82*g*x)) - 4*b*c^2*e*(36*d^3*g + 2*e^3*x^2*(3*f + 7*g*x) +
2*d*e^2*x*(15*f + 32*g*x) + d^2*e*(69*f + 131*g*x))))/(105*e^2*(-2*c*d + b
*e)^4*(d + e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^4\sqrt{-bde-be^2x+cd^2-ce^2x^2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-7beg+8cdg+6cef)\int \frac{1}{(d+ex)^3\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{7e(2cd-be)} - \frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-7beg+8cdg+6cef)\left(\frac{4c\int \frac{1}{(d+ex)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{5(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e(d+ex)^3(2cd-be)}\right)}{7e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow \; 1129$$

$$\frac{(-7beg+8cdg+6cef)\left(\frac{4c\left(\frac{2c\int \frac{1}{(d+ex)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)^2(2cd-be)}\right)}{5(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e(d+ex)^3(2cd-be)}\right)}{7e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7e^2(d+ex)^4(2cd-be)}$$

$$\Big\downarrow \; 1123$$

$$\frac{\left(\frac{4c\left(-\frac{4c\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)(2cd-be)^2}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)^2(2cd-be)}\right)}{5(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e(d+ex)^3(2cd-be)}\right)(-7beg+8cdg+6cef)}{7e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7e^2(d+ex)^4(2cd-be)}$$

input

```
Int[(f + g*x)/((d + e*x)^4*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output

```
(-2*(e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(7*e^2*(2*c*d -
 b*e)*(d + e*x)^4) + ((6*c*e*f + 8*c*d*g - 7*b*e*g)*((-2*Sqrt[d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2])/(5*e*(2*c*d - b*e)*(d + e*x)^3) + (4*c*((-2*Sqrt
[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*e*(2*c*d - b*e)*(d + e*x)^2) - (
4*c*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*e*(2*c*d - b*e)^2*(d + e
*x))))/(5*(2*c*d - b*e))))/(7*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.30

method	result
trager	$\frac{2(56bc^2e^4gx^3-64c^3de^3gx^3-48c^3e^4fx^3-28b^2ce^4gx^2+256bc^2de^3gx^2+24bc^2e^4fx^2-256c^3d^2e^2gx^2-192c^3de^3fx^2+21b^3e^4gx}{}$
gosper	$-\frac{2(cex+be-cd)(56bc^2e^4gx^3-64c^3de^3gx^3-48c^3e^4fx^3-28b^2ce^4gx^2+256bc^2de^3gx^2+24bc^2e^4fx^2-256c^3d^2e^2gx^2-192c^3de^3}{105(ex}$
orering	$-\frac{2(cex+be-cd)(56bc^2e^4gx^3-64c^3de^3gx^3-48c^3e^4fx^3-28b^2ce^4gx^2+256bc^2de^3gx^2+24bc^2e^4fx^2-256c^3d^2e^2gx^2-192c^3de^3}{105(ex}$
default	$\frac{g(-\frac{2\sqrt{-ce^2(x+\frac{d}{e})^2+(-be^2+2dec)(x+\frac{d}{e})}}{5(-be^2+2dec)(x+\frac{d}{e})^3}+\frac{4ce^2(-\frac{2\sqrt{-ce^2(x+\frac{d}{e})^2+(-be^2+2dec)(x+\frac{d}{e})}}{3(-be^2+2dec)(x+\frac{d}{e})^2}-\frac{4ce^2\sqrt{-ce^2(x+\frac{d}{e})^2+(-be^2+2dec)(x+}}{3(-be^2+2dec)^2(x+\frac{d}{e})}}{5(-be^2+2dec)}}{e^4}$

input
```
int((g*x+f)/(e*x+d)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/105*(56*b*c^2*e^4*g*x^3-64*c^3*d*e^3*g*x^3-48*c^3*e^4*f*x^3-28*b^2*c*e^4
*g*x^2+256*b*c^2*d*e^3*g*x^2+24*b*c^2*e^4*f*x^2-256*c^3*d^2*e^2*g*x^2-192*
c^3*d*e^3*f*x^2+21*b^3*e^4*g*x-164*b^2*c*d*e^3*g*x-18*b^2*c*e^4*f*x+524*b*
c^2*d^2*e^2*g*x+120*b*c^2*d*e^3*f*x-416*c^3*d^3*e*g*x-312*c^3*d^2*e^2*f*x+
6*b^3*d*e^3*g+15*b^3*e^4*f-46*b^2*c*d^2*e^2*g-108*b^2*c*d*e^3*f+144*b*c^2*
d^3*e*g+276*b*c^2*d^2*e^2*f-104*c^3*d^4*g-288*c^3*d^3*e*f)/(b^4*e^4-8*b^3*
c*d*e^3+24*b^2*c^2*d^2*e^2-32*b*c^3*d^3*e+16*c^4*d^4)/e^2/(e*x+d)^4*(-c*e^
2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(269) = 538$.

Time = 31.99 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.13

$$\int \frac{f+gx}{(d+ex)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$

$$-\frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(8(6c^3e^4f+(8c^3de^3-7bc^2e^4)g)x^3+4(6(8c^3de^3-bc^2e^4)f+(64c^3d}{105(16c^4d^8e^2-32bc^3d^7e^3+24b^2c^2d^6e^4-8b^3cd^5e^5+b^4d^4e^6+(16c^4d^4e^6-32bc^3d^3e^7+24b^2c^2d}$$

input
```
integrate((g*x+f)/(e*x+d)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output
```
-2/105*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(8*(6*c^3*e^4*f + (8*c^3
*d*e^3 - 7*b*c^2*e^4)*g)*x^3 + 4*(6*(8*c^3*d*e^3 - b*c^2*e^4)*f + (64*c^3*
d^2*e^2 - 64*b*c^2*d*e^3 + 7*b^2*c*e^4)*g)*x^2 + 3*(96*c^3*d^3*e - 92*b*c^
2*d^2*e^2 + 36*b^2*c*d*e^3 - 5*b^3*e^4)*f + 2*(52*c^3*d^4 - 72*b*c^2*d^3*e
 + 23*b^2*c*d^2*e^2 - 3*b^3*d*e^3)*g + (6*(52*c^3*d^2*e^2 - 20*b*c^2*d*e^3
 + 3*b^2*c*e^4)*f + (416*c^3*d^3*e - 524*b*c^2*d^2*e^2 + 164*b^2*c*d*e^3 -
 21*b^3*e^4)*g)*x)/(16*c^4*d^8*e^2 - 32*b*c^3*d^7*e^3 + 24*b^2*c^2*d^6*e^4
 - 8*b^3*c*d^5*e^5 + b^4*d^4*e^6 + (16*c^4*d^4*e^6 - 32*b*c^3*d^3*e^7 + 24
*b^2*c^2*d^2*e^8 - 8*b^3*c*d*e^9 + b^4*e^10)*x^4 + 4*(16*c^4*d^5*e^5 - 32*
b*c^3*d^4*e^6 + 24*b^2*c^2*d^3*e^7 - 8*b^3*c*d^2*e^8 + b^4*d*e^9)*x^3 + 6*
(16*c^4*d^6*e^4 - 32*b*c^3*d^5*e^5 + 24*b^2*c^2*d^4*e^6 - 8*b^3*c*d^3*e^7
+ b^4*d^2*e^8)*x^2 + 4*(16*c^4*d^7*e^3 - 32*b*c^3*d^6*e^4 + 24*b^2*c^2*d^5
*e^5 - 8*b^3*c*d^4*e^6 + b^4*d^3*e^7)*x)
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{f+gx}{\sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\left(d+ex\right)^4}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**4/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output
```
Integral((f + g*x)/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(e*x+d)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[8,4,16,0]%%%}+%%%{-16,[7,5,15,1]%%%}+%%%{112,[6,6,1
4,2]%%%}+
```

Mupad [B] (verification not implemented)

Time = 7.69 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.19

$$\int \frac{f+gx}{(d+ex)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\frac{\left(\frac{40\,c^2\,d\,g+48\,c^2\,e\,f-40\,b\,c\,e\,g}{35\,e\,(3\,b\,e^2-6\,c\,d\,e)\,(b\,e-2\,c\,d)^2}-\frac{8\,c^2\,d\,g}{35\,e\,(3\,b\,e^2-6\,c\,d\,e)\,(b\,e-2\,c\,d)^2}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{(d+e\,x)^2}$$
$$-\frac{\left(\frac{8\,c\,g\,(2\,b\,e-3\,c\,d)}{35\,e\,(3\,b\,e^2-6\,c\,d\,e)\,(b\,e-2\,c\,d)^2}-\frac{8\,c^2\,d\,g}{35\,e\,(3\,b\,e^2-6\,c\,d\,e)\,(b\,e-2\,c\,d)^2}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{(d+e\,x)^2}$$
$$-\frac{\left(\frac{2\,b\,g}{7\,(5\,b\,e^2-10\,c\,d\,e)\,(b\,e-2\,c\,d)}-\frac{4\,c\,d\,g}{7\,e\,(5\,b\,e^2-10\,c\,d\,e)\,(b\,e-2\,c\,d)}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{(d+e\,x)^3}$$
$$-\frac{\left(\frac{16\,c\,d\,g-16\,b\,e\,g+12\,c\,e\,f}{7\,e\,(5\,b\,e^2-10\,c\,d\,e)\,(b\,e-2\,c\,d)}+\frac{4\,c\,d\,g}{7\,e\,(5\,b\,e^2-10\,c\,d\,e)\,(b\,e-2\,c\,d)}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{(d+e\,x)^3}$$
$$+\frac{\left(\frac{2\,f}{7\,b\,e^2-14\,c\,d\,e}-\frac{2\,d\,g}{e\,(7\,b\,e^2-14\,c\,d\,e)}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{(d+e\,x)^4}$$
$$-\frac{\left(\frac{112\,c^3\,d\,g+96\,c^3\,e\,f-112\,b\,c^2\,e\,g}{105\,e^2\,(b\,e-2\,c\,d)^4}+\frac{16\,c^3\,d\,g}{105\,e^2\,(b\,e-2\,c\,d)^4}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{d+e\,x}$$

input
```
int((f + g*x)/((d + e*x)^4*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),x)
```

output
```
(((40*c^2*d*g + 48*c^2*e*f - 40*b*c*e*g)/(35*e*(3*b*e^2 - 6*c*d*e)*(b*e -
2*c*d)^2) - (8*c^2*d*g)/(35*e*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)^2))*(c*d^2
 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2 - (((8*c*g*(2*b*e - 3*c
*d))/(35*e*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)^2) - (8*c^2*d*g)/(35*e*(3*b*e
^2 - 6*c*d*e)*(b*e - 2*c*d)^2))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2
))/(d + e*x)^2 - (((2*b*g)/(7*(5*b*e^2 - 10*c*d*e)*(b*e - 2*c*d)) - (4*c*d
*g)/(7*e*(5*b*e^2 - 10*c*d*e)*(b*e - 2*c*d)))*(c*d^2 - c*e^2*x^2 - b*d*e -
 b*e^2*x)^(1/2))/(d + e*x)^3 - (((16*c*d*g - 16*b*e*g + 12*c*e*f)/(7*e*(5*
b*e^2 - 10*c*d*e)*(b*e - 2*c*d)) + (4*c*d*g)/(7*e*(5*b*e^2 - 10*c*d*e)*(b*
e - 2*c*d)))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^3 + ((
(2*f)/(7*b*e^2 - 14*c*d*e) - (2*d*g)/(e*(7*b*e^2 - 14*c*d*e)))*(c*d^2 - c*
e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^4 - (((112*c^3*d*g + 96*c^3*e*
f - 112*b*c^2*e*g)/(105*e^2*(b*e - 2*c*d)^4) + (16*c^3*d*g)/(105*e^2*(b*e
- 2*c*d)^4))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 1998, normalized size of antiderivative = 7.01

$$\int \frac{f+gx}{(d+ex)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)/(e*x+d)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(2*i*(6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*b**3*d*e**3*g + 15*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b
*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*f + 21*sqrt(d + e*x)*sqrt
(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*g*
x - 46*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b**2*c*d**2*e**2*g - 108*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt
( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**3*f - 164*sqrt(d +
 e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*
*2*c*d*e**3*g*x - 18*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*
sqrt( - b*e + c*d - c*e*x)*b**2*c*e**4*f*x - 28*sqrt(d + e*x)*sqrt(b*e - 2
*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**4*g*x**2 +
 144*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*
d - c*e*x)*b*c**2*d**3*e*g + 276*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b
*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**2*e**2*f + 524*sqrt(d + e
*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*
*2*d**2*e**2*g*x + 120*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d
)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d*e**3*f*x + 256*sqrt(d + e*x)*sqrt(b*
e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d*e**3*g
*x**2 + 24*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*b*c**2*e**4*f*x**2 + 56*sqrt(d + e*x)*sqrt(b*e - 2*c*d...
```

3.180 $\int \frac{f+gx}{(d+ex)^5\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 360

$$\begin{aligned}\int \frac{f+gx}{(d+ex)^5\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx &= -\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{9e^2(2cd-be)(d+ex)^5}\\ &\quad-\frac{2(8cef+10cdg-9beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{63e^2(2cd-be)^2(d+ex)^4}\\ &\quad-\frac{4c(8cef+10cdg-9beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{105e^2(2cd-be)^3(d+ex)^3}\\ &\quad-\frac{16c^2(8cef+10cdg-9beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{315e^2(2cd-be)^4(d+ex)^2}\\ &\quad-\frac{32c^3(8cef+10cdg-9beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{315e^2(2cd-be)^5(d+ex)}\end{aligned}$$

output

```
-2/9*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^5-2/63*(-9*b*e*g+10*c*d*g+8*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(1/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^4-4/105*c*(-9*b*e*g+10*c*d*g+8*c*e*f)*(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^3-16/315*c^
2*(-9*b*e*g+10*c*d*g+8*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(
-b*e+2*c*d)^4/(e*x+d)^2-32/315*c^3*(-9*b*e*g+10*c*d*g+8*c*e*f)*(d*(-b*e+c*
d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^5/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.97

$$\int \frac{f+gx}{(d+ex)^5\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2(cd-be-cex)\left(315c^4ef+315c^4dg-315bc^3eg+\frac{420c^3ef(cd-be-cex)}{d+ex}+\frac{210c^3dg(cd-be-cex)}{d+ex}-\frac{315bc^2eg(cd-be-}{d+ex}\right.}{315e^2(-}$$

input
```
Integrate[(f + g*x)/((d + e*x)^5*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]
),x]
```

output
```
(2*(c*d - b*e - c*e*x)*(315*c^4*e*f + 315*c^4*d*g - 315*b*c^3*e*g + (420*c
^3*e*f*(c*d - b*e - c*e*x))/(d + e*x) + (210*c^3*d*g*(c*d - b*e - c*e*x))/
(d + e*x) - (315*b*c^2*e*g*(c*d - b*e - c*e*x))/(d + e*x) + (378*c^2*e*f*(
c*d - b*e - c*e*x)^2)/(d + e*x)^2 - (189*b*c*e*g*(c*d - b*e - c*e*x)^2)/(d
 + e*x)^2 + (180*c*e*f*(c*d - b*e - c*e*x)^3)/(d + e*x)^3 - (90*c*d*g*(c*d
 - b*e - c*e*x)^3)/(d + e*x)^3 - (45*b*e*g*(c*d - b*e - c*e*x)^3)/(d + e*x
)^3 + (35*e*f*(c*d - b*e - c*e*x)^4)/(d + e*x)^4 - (35*d*g*(c*d - b*e - c*
e*x)^4)/(d + e*x)^4))/(315*e^2*(-2*c*d + b*e)^5*Sqrt[(d + e*x)*(-(b*e) + c
*(d - e*x))])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^5\sqrt{-bde-be^2x+cd^2-ce^2x^2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-9beg+10cdg+8cef)\int \frac{1}{(d+ex)^4\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{9e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{9e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(-9beg+10cdg+8cef)\left(\frac{6c\int \frac{1}{(d+ex)^3\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{7(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7e(d+ex)^4(2cd-be)}\right)}{9e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{9e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(-9beg+10cdg+8cef)\left(\frac{6c\left(\frac{4c\int \frac{1}{(d+ex)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{5(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e(d+ex)^3(2cd-be)}\right)}{7(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7e(d+ex)^4(2cd-be)}\right)}{9e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{9e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(-9beg+10cdg+8cef)\left(\frac{6c\left(\frac{4c\left(\frac{2c\int \frac{1}{(d+ex)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)^2(2cd-be)}\right)}{5(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e(d+ex)^3(2cd-be)}\right)}{7(2cd-be)}-2\right.}{9e(2cd-be)}-\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{9e^2(d+ex)^5(2cd-be)}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{\left(\frac{6c\left(\frac{4c\left(-\frac{4c\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)(2cd-be)^2}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)^2(2cd-be)}\right)}{5(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5e(d+ex)^3(2cd-be)}\right)}{7(2cd-be)}-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7e(d+ex)^4(2cd-be)}\right)(-9beg+1}{9e(2cd-be)}$$

$$\frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{9e^2(d+ex)^5(2cd-be)}$$

input

```
Int[(f + g*x)/((d + e*x)^5*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output

```
(-2*(e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(9*e^2*(2*c*d -
 b*e)*(d + e*x)^5) + ((8*c*e*f + 10*c*d*g - 9*b*e*g)*((-2*Sqrt[d*(c*d - b*
e) - b*e^2*x - c*e^2*x^2])/(7*e*(2*c*d - b*e)*(d + e*x)^4) + (6*c*((-2*Sqr
t[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(5*e*(2*c*d - b*e)*(d + e*x)^3) +
(4*c*((-2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*e*(2*c*d - b*e)*(d
 + e*x)^2) - (4*c*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*e*(2*c*d -
 b*e)^2*(d + e*x))))/(5*(2*c*d - b*e))))/(7*(2*c*d - b*e))))/(9*e*(2*c*d -
 b*e))
```

Defintions of rubi rules used

rule 1123

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.53

method	result
trager	$\frac{2(-144b c^3 e^5 g x^4+160c^4 d e^4 g x^4+128c^4 e^5 f x^4+72b^2 c^2 e^5 g x^3-800b c^3 d e^4 g x^3-64b c^3 e^5 f x^3+800c^4 d^2 e^3 g x^3+640c^4 d e^4 f x^3-54b}{}$
gosper	$-\frac{2(cex+be-cd)(-144b c^3 e^5 g x^4+160c^4 d e^4 g x^4+128c^4 e^5 f x^4+72b^2 c^2 e^5 g x^3-800b c^3 d e^4 g x^3-64b c^3 e^5 f x^3+800c^4 d^2 e^3 g x^3+640}{}$
orering	$-\frac{2(cex+be-cd)(-144b c^3 e^5 g x^4+160c^4 d e^4 g x^4+128c^4 e^5 f x^4+72b^2 c^2 e^5 g x^3-800b c^3 d e^4 g x^3-64b c^3 e^5 f x^3+800c^4 d^2 e^3 g x^3+640}{}$
default	$\frac{g\left(-\frac{2\sqrt{-c e^2\left(x+\frac{d}{e}\right)^2+\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)}}{7\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)^4}+\frac{6c e^2\left(-\frac{2\sqrt{-c e^2\left(x+\frac{d}{e}\right)^2+\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)}}{5\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)^3}+\frac{4c e^2\left(-\frac{2\sqrt{-c e^2\left(x+\frac{d}{e}\right)^2+\left(-b e^2+2dec\right)}}{3\left(-b e^2+2dec\right)\left(x+\frac{d}{e}\right)^2}\right.}{}\right.}{7\left(-b e^2+2dec\right)}\right.}{e^5}$

input
```
int((g*x+f)/(e*x+d)^5/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
2/315*(-144*b*c^3*e^5*g*x^4+160*c^4*d*e^4*g*x^4+128*c^4*e^5*f*x^4+72*b^2*c
^2*e^5*g*x^3-800*b*c^3*d*e^4*g*x^3-64*b*c^3*e^5*f*x^3+800*c^4*d^2*e^3*g*x^
3+640*c^4*d*e^4*f*x^3-54*b^3*c*e^5*g*x^2+492*b^2*c^2*d*e^4*g*x^2+48*b^2*c^
2*e^5*f*x^2-1992*b*c^3*d^2*e^3*g*x^2-384*b*c^3*d*e^4*f*x^2+1680*c^4*d^3*e^
2*g*x^2+1344*c^4*d^2*e^3*f*x^2+45*b^4*e^5*g*x-428*b^3*c*d*e^4*g*x-40*b^3*c
*e^5*f*x+1608*b^2*c^2*d^2*e^3*g*x+336*b^2*c^2*d*e^4*f*x-3120*b*c^3*d^3*e^2
*g*x-1056*b*c^3*d^2*e^3*f*x+2000*c^4*d^4*e*g*x+1600*c^4*d^3*e^2*f*x+10*b^4
*d*e^4*g+35*b^4*e^5*f-94*b^3*c*d^2*e^3*g-320*b^3*c*d*e^4*f+348*b^2*c^2*d^3
*e^2*g+1128*b^2*c^2*d^2*e^3*f-664*b*c^3*d^4*e*g-1856*b*c^3*d^3*e^2*f+400*c
^4*d^5*g+1328*c^4*d^4*e*f)/(b^5*e^5-10*b^4*c*d*e^4+40*b^3*c^2*d^2*e^3-80*b
^2*c^3*d^3*e^2+80*b*c^4*d^4*e-32*c^5*d^5)/e^2/(e*x+d)^5*(-c*e^2*x^2-b*e^2*
x-b*d*e+c*d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. 2(340) = 680.

Time = 104.87 (sec) , antiderivative size = 906, normalized size of antiderivative = 2.52

$$\int \frac{f+gx}{(d+ex)^5\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)/(e*x+d)^5/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output

```
-2/315*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(16*(8*c^4*e^5*f + (10*c
^4*d*e^4 - 9*b*c^3*e^5)*g)*x^4 + 8*(8*(10*c^4*d*e^4 - b*c^3*e^5)*f + (100*
c^4*d^2*e^3 - 100*b*c^3*d*e^4 + 9*b^2*c^2*e^5)*g)*x^3 + 6*(8*(28*c^4*d^2*e
^3 - 8*b*c^3*d*e^4 + b^2*c^2*e^5)*f + (280*c^4*d^3*e^2 - 332*b*c^3*d^2*e^3
 + 82*b^2*c^2*d*e^4 - 9*b^3*c*e^5)*g)*x^2 + (1328*c^4*d^4*e - 1856*b*c^3*d
^3*e^2 + 1128*b^2*c^2*d^2*e^3 - 320*b^3*c*d*e^4 + 35*b^4*e^5)*f + 2*(200*c
^4*d^5 - 332*b*c^3*d^4*e + 174*b^2*c^2*d^3*e^2 - 47*b^3*c*d^2*e^3 + 5*b^4*
d*e^4)*g + (8*(200*c^4*d^3*e^2 - 132*b*c^3*d^2*e^3 + 42*b^2*c^2*d*e^4 - 5*
b^3*c*e^5)*f + (2000*c^4*d^4*e - 3120*b*c^3*d^3*e^2 + 1608*b^2*c^2*d^2*e^3
 - 428*b^3*c*d*e^4 + 45*b^4*e^5)*g)*x)/(32*c^5*d^10*e^2 - 80*b*c^4*d^9*e^3
 + 80*b^2*c^3*d^8*e^4 - 40*b^3*c^2*d^7*e^5 + 10*b^4*c*d^6*e^6 - b^5*d^5*e^
7 + (32*c^5*d^5*e^7 - 80*b*c^4*d^4*e^8 + 80*b^2*c^3*d^3*e^9 - 40*b^3*c^2*d
^2*e^10 + 10*b^4*c*d*e^11 - b^5*e^12)*x^5 + 5*(32*c^5*d^6*e^6 - 80*b*c^4*d
^5*e^7 + 80*b^2*c^3*d^4*e^8 - 40*b^3*c^2*d^3*e^9 + 10*b^4*c*d^2*e^10 - b^5
*d*e^11)*x^4 + 10*(32*c^5*d^7*e^5 - 80*b*c^4*d^6*e^6 + 80*b^2*c^3*d^5*e^7
- 40*b^3*c^2*d^4*e^8 + 10*b^4*c*d^3*e^9 - b^5*d^2*e^10)*x^3 + 10*(32*c^5*d
^8*e^4 - 80*b*c^4*d^7*e^5 + 80*b^2*c^3*d^6*e^6 - 40*b^3*c^2*d^5*e^7 + 10*b
^4*c*d^4*e^8 - b^5*d^3*e^9)*x^2 + 5*(32*c^5*d^9*e^3 - 80*b*c^4*d^8*e^4 + 8
0*b^2*c^3*d^7*e^5 - 40*b^3*c^2*d^6*e^6 + 10*b^4*c*d^5*e^7 - b^5*d^4*e^8)*x
)
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^5\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\int \frac{f+gx}{\sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\left(d+ex\right)^5}\,dx$$

input

```
integrate((g*x+f)/(e*x+d)**5/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output

```
Integral((f + g*x)/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)**5), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^5\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)/(e*x+d)^5/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(340) = 680$.

Time = 0.34 (sec) , antiderivative size = 900, normalized size of antiderivative = 2.50

$$\int \frac{f+gx}{(d+ex)^5\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)/(e*x+d)^5/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
2/315*(16*(8*sqrt(-c)*c^4*e*f + 10*sqrt(-c)*c^4*d*g - 9*b*sqrt(-c)*c^3*e*g
)*sgn(1/(e*x + d))*sgn(e)/(32*c^5*d^5*e - 80*b*c^4*d^4*e^2 + 80*b^2*c^3*d^
3*e^3 - 40*b^3*c^2*d^2*e^4 + 10*b^4*c*d*e^5 - b^5*e^6) + (9*(5*(c - 2*c*d/
(e*x + d) + b*e/(e*x + d))^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) -
21*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^2*c*sqrt(-c + 2*c*d/(e*x + d) - b
*e/(e*x + d)) - 35*c^3*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 35*c^2
*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2))*e^2*g/(8*c^3*d^3*e^3 - 12*b
*c^2*d^2*e^4 + 6*b^2*c*d*e^5 - b^3*e^6) - (35*(c - 2*c*d/(e*x + d) + b*e/(
e*x + d))^4*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) - 180*(c - 2*c*d/(e
*x + d) + b*e/(e*x + d))^3*c*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) +
378*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^2*c^2*sqrt(-c + 2*c*d/(e*x + d)
- b*e/(e*x + d)) + 315*c^4*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) + 42
0*c^3*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2))*e^4*f/(2*c*d*e - b*e^2
)^4 + (35*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^4*sqrt(-c + 2*c*d/(e*x + d
) - b*e/(e*x + d)) - 180*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^3*c*sqrt(-c
 + 2*c*d/(e*x + d) - b*e/(e*x + d)) + 378*(c - 2*c*d/(e*x + d) + b*e/(e*x
+ d))^2*c^2*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d)) + 315*c^4*sqrt(-c +
 2*c*d/(e*x + d) - b*e/(e*x + d)) + 420*c^3*(-c + 2*c*d/(e*x + d) - b*e/(e
*x + d))^(3/2))*d*e^3*g/(2*c*d*e - b*e^2)^4)/(2*c*d*sgn(1/(e*x + d))*sgn(e
) - b*e*sgn(1/(e*x + d))*sgn(e)))/abs(e)
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 949, normalized size of antiderivative = 2.64

$$\int \frac{f+gx}{(d+ex)^5\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\text{Too large to display}$$

input
```
int((f + g*x)/((d + e*x)^5*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),x)
```

output

```
(((88*c^2*d*g + 96*c^2*e*f - 88*b*c*e*g)/(63*e*(5*b*e^2 - 10*c*d*e)*(b*e -
 2*c*d)^2) - (8*c^2*d*g)/(63*e*(5*b*e^2 - 10*c*d*e)*(b*e - 2*c*d)^2))*(c*d
^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^3 - (((32*c^3*g*(4*b*e
- 7*c*d))/(945*e^2*(b*e - 2*c*d)^5) - (32*c^4*d*g)/(945*e^2*(b*e - 2*c*d)^
5))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((4*c*g*(5*b
*e - 8*c*d))/(63*e*(5*b*e^2 - 10*c*d*e)*(b*e - 2*c*d)^2) - (8*c^2*d*g)/(63
*e*(5*b*e^2 - 10*c*d*e)*(b*e - 2*c*d)^2))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e
^2*x)^(1/2))/(d + e*x)^3 - (((400*c^3*d*g + 384*c^3*e*f - 400*b*c^2*e*g)/(
315*e*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)^3) + (16*c^3*d*g)/(315*e*(3*b*e^2
- 6*c*d*e)*(b*e - 2*c*d)^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/
(d + e*x)^2 - (((2*b*g)/(9*(7*b*e^2 - 14*c*d*e)*(b*e - 2*c*d)) - (4*c*d*g)
/(9*e*(7*b*e^2 - 14*c*d*e)*(b*e - 2*c*d)))*(c*d^2 - c*e^2*x^2 - b*d*e - b*
e^2*x)^(1/2))/(d + e*x)^4 - (((20*c*d*g - 20*b*e*g + 16*c*e*f)/(9*e*(7*b*e
^2 - 14*c*d*e)*(b*e - 2*c*d)) + (4*c*d*g)/(9*e*(7*b*e^2 - 14*c*d*e)*(b*e -
 2*c*d)))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^4 + (((2*
f)/(9*b*e^2 - 18*c*d*e) - (2*d*g)/(e*(9*b*e^2 - 18*c*d*e)))*(c*d^2 - c*e^2
*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^5 + (((16*c^3*d*g)/(315*e*(3*b*e^
2 - 6*c*d*e)*(b*e - 2*c*d)^3) + (16*c^2*g*(2*b*e - 5*c*d))/(315*e*(3*b*e^2
 - 6*c*d*e)*(b*e - 2*c*d)^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))
/(d + e*x)^2 + (((736*c^4*d*g + 768*c^4*e*f - 736*b*c^3*e*g)/(945*e^2*(...
```

Reduce [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 2874, normalized size of antiderivative = 7.98

$$\int \frac{f+gx}{(d+ex)^5\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)/(e*x+d)^5/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(2*i*(10*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e
+ c*d - c*e*x)*b**4*d*e**4*g + 35*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**5*f + 45*sqrt(d + e*x)*sqr
t(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**5*g
*x - 94*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*b**3*c*d**2*e**3*g - 320*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqr
t( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*d*e**4*f - 428*sqrt(d
+ e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b
**3*c*d*e**4*g*x - 40*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)
*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**5*f*x - 54*sqrt(d + e*x)*sqrt(b*e -
2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*e**5*g*x**2
+ 348*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c
*d - c*e*x)*b**2*c**2*d**3*e**2*g + 1128*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*s
qrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**2*e**3*f + 160
8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d -
 c*e*x)*b**2*c**2*d**2*e**3*g*x + 336*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt
( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d*e**4*f*x + 492*sqr
t(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*
x)*b**2*c**2*d*e**4*g*x**2 + 48*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*
e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*e**5*f*x**2 + 72*sqrt(d...
```

3.181 $\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 248

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx &= \frac{2(2cd-be)(cef+cdg-beg)(d+ex)}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} \\ &+ \frac{(4cef+12cdg-7beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c^3e^2} \\ &+ \frac{gx\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2c^2e} \\ &- \frac{3(2cd-be)(4cef+6cdg-5beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4c^{7/2}e^2}\end{aligned}$$

output

```
2*(-b*e+2*c*d)*(-b*e*g+c*d*g+c*e*f)*(e*x+d)/c^3/e^2/(d*(-b*e+c*d)-b*e^2*x-
c*e^2*x^2)^(1/2)+1/4*(-7*b*e*g+12*c*d*g+4*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e
^2*x^2)^(1/2)/c^3/e^2+1/2*g*x*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^2/e
-3/4*(-b*e+2*c*d)*(-5*b*e*g+6*c*d*g+4*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b
*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(7/2)/e^2
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^3(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{(-be+c(d-ex))^{3/2}\Bigg(\frac{\sqrt{c}(d+ex)^2\left(15b^2e^2g+bce(-12ef-43dg+5egx)+2c^2\left(14d^2g+\right.\right.}{\sqrt{-be+c(d-ex)}}}{4c^{7/}}$$

input

```
Integrate[((d + e*x)^3*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3
/2),x]
```

output

```
((-(b*e) + c*(d - e*x))^(3/2)*((Sqrt[c]*(d + e*x)^2*(15*b^2*e^2*g + b*c*e*
(-12*e*f - 43*d*g + 5*e*g*x) + 2*c^2*(14*d^2*g + 5*d*e*(2*f - g*x) - e^2*x
*(2*f + g*x))))/Sqrt[-(b*e) + c*(d - e*x)] + 6*(2*c*d - b*e)*(4*c*e*f + 6*
c*d*g - 5*b*e*g)*(d + e*x)^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(Sqrt[2*c*
d - b*e] - Sqrt[c*d - b*e - c*e*x])]))/(4*c^(7/2)*e^2*((d + e*x)*(-(b*e) +
 c*(d - e*x)))^(3/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1211, 2192, 27, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{2(d+ex)(2cd-be)(-beg+cdg+cef)}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\int \frac{c^2gx^2e^6+c(cef+3cdg-beg)xe^5+\left(d(3ef+4dg)c^2-be(ef+4dg)c+b^2e^2g\right)e^4}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{c^3e^5}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2(d+ex)(2cd-be)(-beg+cdg+cef)}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{-\frac{\int -\frac{ce^6(2(3cd-be)(2cef+3cdg-2beg)-ce(7beg-4c(ef+3dg))x)}{2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2ce^2}-\frac{1}{2}ce^4gx\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^3e^5}$$

$$\Big\downarrow \; 27$$

$$\frac{2(d+ex)(2cd-be)(-beg+cdg+cef)}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{1}{4}e^4\int \frac{2(3cd-be)(2cef+3cdg-2beg)-ce(7beg-4c(ef+3dg))x}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{1}{2}ce^4gx\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^3e^5}$$

$$\Big\downarrow \; 1160$$

$$\frac{2(d+ex)(2cd-be)(-beg+cdg+cef)}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{1}{4}e^4\left(\frac{3}{2}(2cd-be)(-5beg+6cdg+4cef)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(-7beg+12cdg+4cef)}{e}\right)-}{c^3e^5}$$

$$\Big\downarrow \; 1092$$

$$\frac{2(d+ex)(2cd-be)(-beg+cdg+cef)}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{1}{4}e^4\left(3(2cd-be)(-5beg+6cdg+4cef)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)-\frac{\sqrt{d(cd-be)-be^2x}}{}\right.}{c^3e^5}$$

$$\Big\downarrow \; 217$$

$$\frac{2(d+ex)(2cd-be)(-beg+cdg+cef)}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{1}{4}e^4\left(\frac{3(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-5beg+6cdg+4cef)}{2\sqrt{c}e}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(-7beg+12cdg+4cef)}{e}\right)-\frac{1}{2}ce^4gx}{c^3e^5}$$

input
```
Int[((d + e*x)^3*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),x]
```

output
```
(2*(2*c*d - b*e)*(c*e*f + c*d*g - b*e*g)*(d + e*x))/(c^3*e^2*Sqrt[d*(c*d -
 b*e) - b*e^2*x - c*e^2*x^2]) - (-1/2*(c*e^4*g*x*Sqrt[d*(c*d - b*e) - b*e^
2*x - c*e^2*x^2]) + (e^4*(-(((4*c*e*f + 12*c*d*g - 7*b*e*g)*Sqrt[d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2])/e) + (3*(2*c*d - b*e)*(4*c*e*f + 6*c*d*g - 5*
b*e*g)*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*
e^2*x^2])])/(2*Sqrt[c]*e)))/4)/(c^3*e^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. $2(230) = 460$.

Time = 3.66 (sec) , antiderivative size = 1535, normalized size of antiderivative = 6.19

method	result	size
default	Expression too large to display	1535

input
```
int((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2*d^3*f*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b
*e^2*x-b*d*e+c*d^2)^(1/2)+e^2*(3*d*g+e*f)*(-x^2/c/e^2/(-c*e^2*x^2-b*e^2*x-
b*d*e+c*d^2)^(1/2)-3/2*b/c*(x/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)
-1/2*b/c*(1/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-b/c*(-2*c*e^2*x-b
*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(
1/2))-1/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b
*e^2*x-b*d*e+c*d^2)^(1/2)))+2*(-b*d*e+c*d^2)/c/e^2*(1/c/e^2/(-c*e^2*x^2-b*
e^2*x-b*d*e+c*d^2)^(1/2)-b/c*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b
^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+3*d*e*(d*g+e*f)*(x/c/e^2/
(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c*(1/c/e^2/(-c*e^2*x^2-b*e^2*
x-b*d*e+c*d^2)^(1/2)-b/c*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e
^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))-1/c/e^2/(c*e^2)^(1/2)*arctan((
c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+d^2*(d*g
+3*e*f)*(1/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-b/c*(-2*c*e^2*x-b*
e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1
/2))+g*e^3*(-1/2*x^3/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-5/4*b/c*
(-x^2/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-3/2*b/c*(x/c/e^2/(-c*e^
2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c*(1/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*
e+c*d^2)^(1/2)-b/c*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-
c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))-1/c/e^2/(c*e^2)^(1/2)*arctan((c*e...
```

Fricas [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 745, normalized size of antiderivative = 3.00

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \left[\frac{3\left(4\left(2\,c^3d^2e-3\,bc^2de^2+b^2ce^3\right)f+\left(12\,c^3d^3-28\,bc^2d^2e+21\,b^2cd\right.\right.}{}\right.$$

$$-\frac{3\left(4\left(2\,c^3d^2e-3\,bc^2de^2+b^2ce^3\right)f+\left(12\,c^3d^3-28\,bc^2d^2e+21\,b^2cde^2-5\,b^3e^3\right)g-\left(4\left(2\,c^3de^2-bc^2e^3\right)f+\right.\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="fricas")
```

output

```
[1/16*(3*(4*(2*c^3*d^2*e - 3*b*c^2*d*e^2 + b^2*c*e^3)*f + (12*c^3*d^3 - 28
*b*c^2*d^2*e + 21*b^2*c*d*e^2 - 5*b^3*e^3)*g - (4*(2*c^3*d*e^2 - b*c^2*e^3
)*f + (12*c^3*d^2*e - 16*b*c^2*d*e^2 + 5*b^2*c*e^3)*g)*x)*sqrt(-c)*log(8*c
^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 4*sqrt(-c*e^2
*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) + 4*(2*c^3*e^2*g
*x^2 - 4*(5*c^3*d*e - 3*b*c^2*e^2)*f - (28*c^3*d^2 - 43*b*c^2*d*e + 15*b^2
*c*e^2)*g + (4*c^3*e^2*f + 5*(2*c^3*d*e - b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2
 - b*e^2*x + c*d^2 - b*d*e))/(c^5*e^3*x - c^5*d*e^2 + b*c^4*e^3), -1/8*(3*
(4*(2*c^3*d^2*e - 3*b*c^2*d*e^2 + b^2*c*e^3)*f + (12*c^3*d^3 - 28*b*c^2*d^
2*e + 21*b^2*c*d*e^2 - 5*b^3*e^3)*g - (4*(2*c^3*d*e^2 - b*c^2*e^3)*f + (12
*c^3*d^2*e - 16*b*c^2*d*e^2 + 5*b^2*c*e^3)*g)*x)*sqrt(c)*arctan(1/2*sqrt(-
c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2
+ b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*(2*c^3*e^2*g*x^2 - 4*(5*c^3*d*e - 3*
b*c^2*e^2)*f - (28*c^3*d^2 - 43*b*c^2*d*e + 15*b^2*c*e^2)*g + (4*c^3*e^2*f
 + 5*(2*c^3*d*e - b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d
*e))/(c^5*e^3*x - c^5*d*e^2 + b*c^4*e^3)]
```

Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{(d+ex)^3\,(f+gx)}{(-\,(d+ex)\,(be-cd+cex))^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)**3*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x
)
```

output

```
Integral((d + e*x)**3*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(3/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(230) = 460$.

Time = 0.41 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.00

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{1}{4}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left(\frac{2\,gx}{c^2e}+\frac{4\,c^6e^3f+12\,c^6de^2g-7\,bc^5}{c^8e^4}\right.$$
$$+\frac{(8\,c^2def-4\,bce^2f+12\,c^2d^2g-16\,bcdeg+5\,b^2e^2g)\log\left(\left|bc^2d^2e^2-2\,b^2cde^3+b^3e^4-2\,(\sqrt{-ce^2}x-\sqrt{-c}\right.\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="giac")
```

output
```
1/4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*g*x/(c^2*e) + (4*c^6*e^3
*f + 12*c^6*d*e^2*g - 7*b*c^5*e^3*g)/(c^8*e^4)) + 1/8*(8*c^2*d*e*f - 4*b*c
*e^2*f + 12*c^2*d^2*g - 16*b*c*d*e*g + 5*b^2*e^2*g)*log(abs(b*c^2*d^2*e^2
- 2*b^2*c*d*e^3 + b^3*e^4 - 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x
+ c*d^2 - b*d*e))*sqrt(-c)*c^2*d^2*abs(e) + 6*(sqrt(-c*e^2)*x - sqrt(-c*e^
2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b*sqrt(-c)*c*d*e*abs(e) - 4*(sqrt(-c*e^2
)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b^2*sqrt(-c)*e^2*abs(e)
+ 4*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*c^2*d*
e - 5*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*b*c*
e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*sq
rt(-c)*c*abs(e)))/(sqrt(-c)*c^3*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{(f+g\,x)\,(d+e\,x)^3}{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^3)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^3)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 918, normalized size of antiderivative = 3.70

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(i*( - 15*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*
e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*e**3*g + 78*sqrt(c)*sqrt( - b*e + c*d -
 c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*
d*e**2*g + 12*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d
- c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*e**3*f - 132*sqrt(c)*sqrt( - b*e
+ c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b*c**2*d**2*e*g - 48*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e
 + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**2*f + 72*sqrt(c)*sqrt
( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
2*c*d))*c**3*d**3*g + 48*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( -
 b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**2*e*f + 10*sqrt(c)*sq
rt( - b*e + c*d - c*e*x)*b**3*e**3*g - 51*sqrt(c)*sqrt( - b*e + c*d - c*e*
x)*b**2*c*d*e**2*g - 9*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**3*f +
84*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**2*e*g + 36*sqrt(c)*sqrt( -
 b*e + c*d - c*e*x)*b*c**2*d*e**2*f - 44*sqrt(c)*sqrt( - b*e + c*d - c*e*x
)*c**3*d**3*g - 36*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d**2*e*f + 15*s
qrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b**2*c*e**2*g - 43*sqr
t(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b*c**2*d*e*g - 12*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b*c**2*e**2*f + 5*sqrt(d +
e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b*c**2*e**2*g*x + 28*sqrt(d...
```

3.182 $\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 168

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = -\frac{2(beg-c(ef+dg))(d+ex)}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}+\frac{g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^2e^2} - \frac{(2cef+4cdg-3beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{c^{5/2}e^2}$$

output

```
-2*(b*e*g-c*(d*g+e*f))*(e*x+d)/c^2/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1
/2)+g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^2/e^2-(-3*b*e*g+4*c*d*g+2*c
*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(5/
2)/e^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^2(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{\sqrt{c}(d+ex)(-3beg+c(2ef+3dg-egx))+(2cef+4cdg-3beg)}{c^{5/2}e^2\sqrt{(d+ex)(-be+c(d-}}$$

input

```
Integrate[((d + e*x)^2*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3
/2),x]
```

output

```
(Sqrt[c]*(d + e*x)*(-3*b*e*g + c*(2*e*f + 3*d*g - e*g*x)) + (2*c*e*f + 4*c
*d*g - 3*b*e*g)*Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*x]*ArcTan[Sqrt[c*d - b*
e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/(c^(5/2)*e^2*Sqrt[(d + e*x)*(-(b*e) +
 c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1211, 25, 27, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1211}$$

$$\frac{\int -\frac{e^2(cef+2cdg-beg+cegx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{c^2e^3}-\frac{2(d+ex)(beg-c(dg+ef))}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{e^2(cef+2cdg-beg+cegx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{c^2e^3}-\frac{2(d+ex)(beg-c(dg+ef))}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{cef+2cdg-beg+cegx}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{c^2e}-\frac{2(d+ex)(beg-c(dg+ef))}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \ 1160$$

$$-\frac{\frac{1}{2}(-3beg+4cdg+2cef)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}}{c^2e}-\frac{2(d+ex)(beg-c(dg+ef))}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \ 1092$$

$$-\frac{(-3beg+4cdg+2cef)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)-\frac{g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}}{c^2e}-\frac{2(d+ex)(beg-c(dg+ef))}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \ 217$$

$$-\frac{\frac{\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-3beg+4cdg+2cef)}{2\sqrt{c}e}-\frac{g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}}{c^2e}-\frac{2(d+ex)(beg-c(dg+ef))}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input

```
Int[((d + e*x)^2*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),x]
```

output

```
(-2*(b*e*g - c*(e*f + d*g))*(d + e*x))/(c^2*e^2*Sqrt[d*(c*d - b*e) - b*e^2
*x - c*e^2*x^2]) - (-((g*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/e) + (
(2*c*e*f + 4*c*d*g - 3*b*e*g)*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*
d - b*e) - b*e^2*x - c*e^2*x^2])])/(2*Sqrt[c]*e))/(c^2*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(158) = 316$.

Time = 3.01 (sec) , antiderivative size = 836, normalized size of antiderivative = 4.98

method	result
default	$\frac{2d^{2}f\left(-2c\,e^{2}x-b\,e^{2}\right)}{\left(-4c\,e^{2}\left(-bde+c\,d^{2}\right)-b^{2}e^{4}\right)\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}+e(2dg+ef)\Bigg(\frac{x}{c\,e^{2}\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}-$

input
```
int((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2*d^2*f*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b
*e^2*x-b*d*e+c*d^2)^(1/2)+e*(2*d*g+e*f)*(x/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e
+c*d^2)^(1/2)-1/2*b/c*(1/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-b/c*
(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b
*d*e+c*d^2)^(1/2))-1/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/
(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+d*(d*g+2*e*f)*(1/c/e^2/(-c*e^2*x^
2-b*e^2*x-b*d*e+c*d^2)^(1/2)-b/c*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^
2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))+e^2*g*(-x^2/c/e^2/(-c*
e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-3/2*b/c*(x/c/e^2/(-c*e^2*x^2-b*e^2*x-b*
d*e+c*d^2)^(1/2)-1/2*b/c*(1/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-b
/c*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*
x-b*d*e+c*d^2)^(1/2))-1/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/
c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+2*(-b*d*e+c*d^2)/c/e^2*(1/c/e^
2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-b/c*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*
(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.99

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \left[-\frac{(2\,(c^2de-bce^2)f+(4\,c^2d^2-7\,bcde+3\,b^2e^2)g-(2\,c^2e^2f+(4\,c}{}\right.$$

$$-\frac{(2\,(c^2de-bce^2)f+(4\,c^2d^2-7\,bcde+3\,b^2e^2)g-(2\,c^2e^2f+(4\,c^2de-3\,bce^2)g)x)\sqrt{c}\arctan\left(\frac{\sqrt{-ce^2x^2-be^2x}}{2\,(c^2e^2x^2+}\right.}{2\,(c^4e^3x-c^4de^2+bc}$$

input

```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="fricas")
```

output

```
[-1/4*((2*(c^2*d*e - b*c*e^2)*f + (4*c^2*d^2 - 7*b*c*d*e + 3*b^2*e^2)*g -
(2*c^2*e^2*f + (4*c^2*d*e - 3*b*c*e^2)*g)*x)*sqrt(-c)*log(8*c^2*e^2*x^2 +
8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*
x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*(c^2*e*g*x - 2*c^2*e*f -
3*(c^2*d - b*c*e)*g)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^4*e^3*
x - c^4*d*e^2 + b*c^3*e^3), -1/2*((2*(c^2*d*e - b*c*e^2)*f + (4*c^2*d^2 -
7*b*c*d*e + 3*b^2*e^2)*g - (2*c^2*e^2*f + (4*c^2*d*e - 3*b*c*e^2)*g)*x)*sq
rt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e
)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*(c^2*e*g*x -
2*c^2*e*f - 3*(c^2*d - b*c*e)*g)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e
))/(c^4*e^3*x - c^4*d*e^2 + b*c^3*e^3)]
```

Sympy [F]

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \int \frac{(d+ex)^2\,(f+gx)}{(-\,(d+ex)\,(be-cd+cex))^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)**2*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x
)
```

output

```
Integral((d + e*x)**2*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(3/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(158) = 316$.

Time = 0.36 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.61

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{\sqrt{-ce^2x^2-be^2x+cd^2-bdeg}}{c^2e^2}$$
$$+\frac{(2\,cef+4\,cdg-3\,beg)\log\Big(\Big|-b\sqrt{-c}c^2d^2e^2+2\,b^2\sqrt{-c}cde^3-b^3\sqrt{-c}e^4-2\,\big(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-be^2x}}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="giac")
```

output
```
sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*g/(c^2*e^2) + 1/6*(2*c*e*f + 4*
c*d*g - 3*b*e*g)*log(abs(-b*sqrt(-c)*c^2*d^2*e^2 + 2*b^2*sqrt(-c)*c*d*e^3
- b^3*sqrt(-c)*e^4 - 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2
 - b*d*e))*c^3*d^2*abs(e) + 6*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x
+ c*d^2 - b*d*e))*b*c^2*d*e*abs(e) - 4*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 -
 b*e^2*x + c*d^2 - b*d*e))*b^2*c*e^2*abs(e) - 4*(sqrt(-c*e^2)*x - sqrt(-c*
e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*sqrt(-c)*c^2*d*e + 5*(sqrt(-c*e^2)*x
 - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*b*sqrt(-c)*c*e^2 + 2*(sqr
t(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*c^2*abs(e)))/(
sqrt(-c)*c^2*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\int \frac{(f+g\,x)\,\left(d+e\,x\right)^2}{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^2)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^2)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.28

$$\int \frac{(d+ex)^2(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{i\left(12\sqrt{c}\,\sqrt{-cex-be+cd}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2e^2g-40\sqrt{c}\,\sqrt{-ce}\right.}{}$$

input
```
int((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(i*(12*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x
)*i)/sqrt( - b*e + 2*c*d))*b**2*e**2*g - 40*sqrt(c)*sqrt( - b*e + c*d - c*
e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*d*e*g
- 8*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i
)/sqrt( - b*e + 2*c*d))*b*c*e**2*f + 32*sqrt(c)*sqrt( - b*e + c*d - c*e*x)
*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**2*d**2*g +
16*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)
/sqrt( - b*e + 2*c*d))*c**2*d*e*f - 9*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b
**2*e**2*g + 28*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c*d*e*g + 8*sqrt(c)*s
qrt( - b*e + c*d - c*e*x)*b*c*e**2*f - 20*sqrt(c)*sqrt( - b*e + c*d - c*e*
x)*c**2*d**2*g - 16*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**2*d*e*f - 12*sqr
t(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b*c*e*g + 12*sqrt(d + e*
x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*c**2*d*g + 8*sqrt(d + e*x)*sqrt(
b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*c**2*e*f - 4*sqrt(d + e*x)*sqrt(b*e - 2*
c*d)*sqrt( - b*e + 2*c*d)*c**2*e*g*x))/(4*sqrt( - b*e + c*d - c*e*x)*c**3*
e**2*(b*e - 2*c*d))
```

3.183 $\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 124

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{2(cef+cdg-beg)(d+ex)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}} - \frac{2g\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{c^{3/2}e^2}$$

output
```
2*(-b*e*g+c*d*g+c*e*f)*(e*x+d)/c/e^2/(-b*e+2*c*d)/(d*(-b*e+c*d)-b*e^2*x-c*
e^2*x^2)^(1/2)-2*g*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)
^(1/2))/c^(3/2)/e^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = -\frac{2\left(\sqrt{c}(cef+cdg-beg)(d+ex)+(2cd-be)g\sqrt{d+ex}\sqrt{cd-be-cex}\arctan\left(\frac{\sqrt{cd-be-cex}}{\sqrt{c}\sqrt{d+ex}}\right)\right)}{c^{3/2}e^2(-2cd+be)\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[((d + e*x)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2
),x]
```

output
```
(-2*(Sqrt[c]*(c*e*f + c*d*g - b*e*g)*(d + e*x) + (2*c*d - b*e)*g*Sqrt[d +
e*x]*Sqrt[c*d - b*e - c*e*x]*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[
d + e*x])]))/(c^(3/2)*e^2*(-2*c*d + b*e)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e
*x))])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1211, 27, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \textcolor{blue}{1211}$$

$$\frac{2(d+ex)(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\int \frac{g}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{ce}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{2(d+ex)(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{g\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{ce}$$

$$\Big\downarrow \textcolor{blue}{1092}$$

$$\frac{2(d+ex)(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2g\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{ce}$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{2(d+ex)(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{g\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{c^{3/2}e^2}$$

input
```
Int[((d + e*x)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x))/(c*e^2*(2*c*d - b*e)*Sqrt[d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2]) - (g*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*
(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(c^(3/2)*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(116) = 232$.

Time = 2.33 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.40

method	result
default	$\frac{2df(-2c\,e^2x-b\,e^2)}{(-4c\,e^2(-bde+c\,d^2)-b^2e^4)\sqrt{-x^2c\,e^2-xb\,e^2-bde+c\,d^2}}+(dg+ef)\,(\frac{1}{c\,e^2\sqrt{-x^2c\,e^2-xb\,e^2-bde+c\,d^2}}-\frac{}{c(-4c\,e^2(-bde+c\,d}$

input
```
int((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*d*f*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e
^2*x-b*d*e+c*d^2)^(1/2)+(d*g+e*f)*(1/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2
)^(1/2)-b/c*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x
^2-b*e^2*x-b*d*e+c*d^2)^(1/2))+e*g*(x/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^
2)^(1/2)-1/2*b/c*(1/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-b/c*(-2*c
*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+
c*d^2)^(1/2))-1/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e
^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(116) = 232$.

Time = 0.35 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.90

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \Bigg[\frac{((2\,c^2de-bce^2)gx-(2\,c^2d^2-3\,bcde+b^2e^2)g)\sqrt{-c}\log\left(8\,c^2e^2x^2\right.}{}$$

$$-\frac{((2\,c^2de-bce^2)gx-(2\,c^2d^2-3\,bcde+b^2e^2)g)\sqrt{c}\arctan\left(\frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,cex+be)\sqrt{c}}{2\,(c^2e^2x^2+bce^2x-c^2d^2+bcde)}\right)-2\sqrt{-ce^2x^2}}{2\,c^4d^2e^2-3\,bc^3de^3+b^2c^2e^4-(2\,c^4de^3-bc^3e^4)x}$$

input
```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algori
thm="fricas")
```

output

```
[1/2*(((2*c^2*d*e - b*c*e^2)*g*x - (2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*g)*sq
rt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 +
 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) +
4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(c^2*e*f + (c^2*d - b*c*e)*g)
)/(2*c^4*d^2*e^2 - 3*b*c^3*d*e^3 + b^2*c^2*e^4 - (2*c^4*d*e^3 - b*c^3*e^4)
*x), -(((2*c^2*d*e - b*c*e^2)*g*x - (2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*g)*s
qrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*
e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*sqrt(-c*e^2*
x^2 - b*e^2*x + c*d^2 - b*d*e)*(c^2*e*f + (c^2*d - b*c*e)*g))/(2*c^4*d^2*e
^2 - 3*b*c^3*d*e^3 + b^2*c^2*e^4 - (2*c^4*d*e^3 - b*c^3*e^4)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{(d+ex)\,(f+gx)}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x)
```

output

```
Integral((d + e*x)*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algori
thm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(116) = 232$.

Time = 0.32 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.02

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{g\log\left(\left|bc^2d^2e^2-2\,b^2cde^3+b^3e^4-2\left(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-be^2x+}\right.\right.\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algori
thm="giac")
```

output
```
1/3*g*log(abs(b*c^2*d^2*e^2 - 2*b^2*c*d*e^3 + b^3*e^4 - 2*(sqrt(-c*e^2)*x
- sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*c^2*d^2*abs(e) + 6*
(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b*sqrt(-c)*c
*d*e*abs(e) - 4*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*
e))*b^2*sqrt(-c)*e^2*abs(e) + 4*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*
x + c*d^2 - b*d*e))^2*c^2*d*e - 5*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^
2*x + c*d^2 - b*d*e))^2*b*c*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*
e^2*x + c*d^2 - b*d*e))^3*sqrt(-c)*c*abs(e)))/(sqrt(-c)*c*e*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.88 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.77

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{4\,c\,d^3\,g+2\,b\,d\,e^2\,f-4\,b\,d^2\,e\,g-2\,b\,d\,e^2\,g\,x+4\,c\,d\,e^2\,f\,x}{\left(b^2\,e^4+4\,c\,e^2\,\left(c\,d^2-b\,d\,e\right)\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}$$
$$+\frac{e\,g\,\ln\left(b\,e^2-2\,\sqrt{-c\,e^2}\,\sqrt{-\left(d+e\,x\right)\,\left(b\,e-c\,d+c\,e\,x\right)}+2\,c\,e^2\,x\right)}{{\left(-c\,e^2\right)}^{3/2}}$$
$$-\frac{e\,f\,\left(-4\,c\,d^2+4\,b\,d\,e+2\,b\,x\,e^2\right)}{\left(b^2\,e^4+4\,c\,e^2\,\left(c\,d^2-b\,d\,e\right)\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}$$
$$+\frac{g\,\left(x\,\left(\frac{b^2\,e^4}{2}+c\,e^2\,\left(c\,d^2-b\,d\,e\right)\right)-\frac{b\,e^2\,\left(c\,d^2-b\,d\,e\right)}{2}\right)}{c\,e\,\left(\frac{b^2\,e^4}{4}+c\,e^2\,\left(c\,d^2-b\,d\,e\right)\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}$$

input
```
int(((f + g*x)*(d + e*x))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),x)
```

output
```
(4*c*d^3*g + 2*b*d*e^2*f - 4*b*d^2*e*g - 2*b*d*e^2*g*x + 4*c*d*e^2*f*x)/((
b^2*e^4 + 4*c*e^2*(c*d^2 - b*d*e))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(
1/2)) + (e*g*log(b*e^2 - 2*(-c*e^2)^(1/2)*(-(d + e*x)*(b*e - c*d + c*e*x))
^(1/2) + 2*c*e^2*x))/(-c*e^2)^(3/2) - (e*f*(4*b*d*e - 4*c*d^2 + 2*b*e^2*x)
)/((b^2*e^4 + 4*c*e^2*(c*d^2 - b*d*e))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*
x)^(1/2)) + (g*(x*((b^2*e^4)/2 + c*e^2*(c*d^2 - b*d*e)) - (b*e^2*(c*d^2 -
b*d*e))/2))/(c*e*((b^2*e^4)/4 + c*e^2*(c*d^2 - b*d*e))*(c*d^2 - c*e^2*x^2
- b*d*e - b*e^2*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.42

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{2i\left(-\sqrt{c}\sqrt{-cex-be+cd}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2e^2g+4\sqrt{c}\sqrt{-ce}$$

input
```
int((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
(2*i*( - sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e
*x)*i)/sqrt( - b*e + 2*c*d))*b**2*e**2*g + 4*sqrt(c)*sqrt( - b*e + c*d - c
*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*d*e*g
 - 4*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*
i)/sqrt( - b*e + 2*c*d))*c**2*d**2*g + sqrt(c)*sqrt( - b*e + c*d - c*e*x)*
b**2*e**2*g - 3*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c*d*e*g - sqrt(c)*sqr
t( - b*e + c*d - c*e*x)*b*c*e**2*f + 2*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*
c**2*d**2*g + 2*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**2*d*e*f + sqrt(d + e
*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b*c*e*g - sqrt(d + e*x)*sqrt(b*
e - 2*c*d)*sqrt( - b*e + 2*c*d)*c**2*d*g - sqrt(d + e*x)*sqrt(b*e - 2*c*d)
*sqrt( - b*e + 2*c*d)*c**2*e*f))/(sqrt( - b*e + c*d - c*e*x)*c**2*e**2*(b*
*2*e**2 - 4*b*c*d*e + 4*c**2*d**2))
```

3.184 $\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 136

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{2(4cef+2cdg-3beg)(b+2cx)}{3e(2cd-be)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2(ef-dg)}{3e^2(2cd-be)(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

output

```
2/3*(-3*b*e*g+2*c*d*g+4*c*e*f)*(2*c*x+b)/e/(-b*e+2*c*d)^3/(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(1/2)-2/3*(-d*g+e*f)/e^2/(-b*e+2*c*d)/(e*x+d)/(d*(-b*e+c*
d)-b*e^2*x-c*e^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{-8c^2(d^3g+2e^3fx^2+d^2e(-f+gx)+de^2x(2f+gx))+4}{3e^2(-2cd+be)^3(d}$$

input

```
Integrate[(f + g*x)/((d + e*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2)
),x]
```

output
```
(-8*c^2*(d^3*g + 2*e^3*f*x^2 + d^2*e*(-f + g*x) + d*e^2*x*(2*f + g*x)) + 4
*b*c*e*(d^2*g + d*e*(-4*f + 2*g*x) + e^2*x*(-2*f + 3*g*x)) + 2*b^2*e^2*(2*
d*g + e*(f + 3*g*x)))/(3*e^2*(-2*c*d + b*e)^3*(d + e*x)*Sqrt[(d + e*x)*(-(
b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1220, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-3beg+2cdg+4cef)\int \frac{1}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3e(2cd-be)}-\frac{2(ef-dg)}{3e^2(d+ex)(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(b+2cx)(-3beg+2cdg+4cef)}{3e(2cd-be)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2(ef-dg)}{3e^2(d+ex)(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input
```
Int[(f + g*x)/((d + e*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2)),x]
```

output
```
(2*(4*c*e*f + 2*c*d*g - 3*b*e*g)*(b + 2*c*x))/(3*e*(2*c*d - b*e)^3*Sqrt[d*
(c*d - b*e) - b*e^2*x - c*e^2*x^2]) - (2*(e*f - d*g))/(3*e^2*(2*c*d - b*e)
*(d + e*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.68

method	result
gosper	$-\frac{2(cex+be-cd)\left(6bc\,e^{3}g\,x^{2}-4c^{2}d\,e^{2}g\,x^{2}-8f\,c^{2}e^{3}x^{2}+3b^{2}e^{3}gx+4bcd\,e^{2}gx-4bc\,e^{3}fx-4c^{2}d^{2}egx-8c^{2}d\,e^{2}fx+2b^{2}d\,e^{2}g+b^{2}e^{3}f+2bc}{3(b^{3}e^{3}-6d\,e^{2}b^{2}c+12d^{2}eb\,c^{2}-8d^{3}c^{3})e^{2}(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2})^{\frac{3}{2}}}$
orering	$-\frac{2(cex+be-cd)\left(6bc\,e^{3}g\,x^{2}-4c^{2}d\,e^{2}g\,x^{2}-8f\,c^{2}e^{3}x^{2}+3b^{2}e^{3}gx+4bcd\,e^{2}gx-4bc\,e^{3}fx-4c^{2}d^{2}egx-8c^{2}d\,e^{2}fx+2b^{2}d\,e^{2}g+b^{2}e^{3}f+2bc}{3(b^{3}e^{3}-6d\,e^{2}b^{2}c+12d^{2}eb\,c^{2}-8d^{3}c^{3})e^{2}(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2})^{\frac{3}{2}}}$
trager	$-\frac{2\left(6bc\,e^{3}g\,x^{2}-4c^{2}d\,e^{2}g\,x^{2}-8f\,c^{2}e^{3}x^{2}+3b^{2}e^{3}gx+4bcd\,e^{2}gx-4bc\,e^{3}fx-4c^{2}d^{2}egx-8c^{2}d\,e^{2}fx+2b^{2}d\,e^{2}g+b^{2}e^{3}f+2bc\,d^{2}eg-8bcd\,e}{3e^{2}(b^{2}e^{2}-4bcde+4c^{2}d^{2})(ex+d)^{2}(be-2cd)(cex+be-cd)}$
default	$\frac{2g\left(-2c\,e^{2}x-b\,e^{2}\right)}{e(-4c\,e^{2}(-bde+c\,d^{2})-b^{2}e^{4})\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}-\frac{(dg-ef)\left(-\frac{2}{3\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)}}\right.}{e^{2}}$

input
```
int((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
-2/3*(c*e*x+b*e-c*d)*(6*b*c*e^3*g*x^2-4*c^2*d*e^2*g*x^2-8*c^2*e^3*f*x^2+3*
b^2*e^3*g*x+4*b*c*d*e^2*g*x-4*b*c*e^3*f*x-4*c^2*d^2*e*g*x-8*c^2*d*e^2*f*x+
2*b^2*d*e^2*g+b^2*e^3*f+2*b*c*d^2*e*g-8*b*c*d*e^2*f-4*c^2*d^3*g+4*c^2*d^2*
e*f)/(b^3*e^3-6*b^2*c*d*e^2+12*b*c^2*d^2*e-8*c^3*d^3)/e^2/(-c*e^2*x^2-b*e^
2*x-b*d*e+c*d^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(128) = 256$.

Time = 3.92 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.99

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2(4c}{3(8c^4d^6e^2-20bc^3d^5e^3+18b^2c^2d^4e^4-7b^3cd^3e^5+b^4d^2e^6}$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algori
thm="fricas")
```

output
```
2/3*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*c^2*e^3*f + (2*c^2*d*
e^2 - 3*b*c*e^3)*g)*x^2 - (4*c^2*d^2*e - 8*b*c*d*e^2 + b^2*e^3)*f + 2*(2*c
^2*d^3 - b*c*d^2*e - b^2*d*e^2)*g + (4*(2*c^2*d*e^2 + b*c*e^3)*f + (4*c^2*
d^2*e - 4*b*c*d*e^2 - 3*b^2*e^3)*g)*x)/(8*c^4*d^6*e^2 - 20*b*c^3*d^5*e^3 +
 18*b^2*c^2*d^4*e^4 - 7*b^3*c*d^3*e^5 + b^4*d^2*e^6 - (8*c^4*d^3*e^5 - 12*
b*c^3*d^2*e^6 + 6*b^2*c^2*d*e^7 - b^3*c*e^8)*x^3 - (8*c^4*d^4*e^4 - 4*b*c^
3*d^3*e^5 - 6*b^2*c^2*d^2*e^6 + 5*b^3*c*d*e^7 - b^4*e^8)*x^2 + (8*c^4*d^5*
e^3 - 28*b*c^3*d^4*e^4 + 30*b^2*c^2*d^3*e^5 - 13*b^3*c*d^2*e^6 + 2*b^4*d*e
^7)*x)
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \int \frac{f+gx}{\left(-(d+ex)(be-cd+cex)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x)
```

output
```
Integral((f + g*x)/((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\int \frac{gx+f}{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algori
thm="giac")
```

output
```
integrate((g*x + f)/((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(e*x + d
)), x)
```

Mupad [B] (verification not implemented)

Time = 11.58 (sec) , antiderivative size = 872, normalized size of antiderivative = 6.41

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((f + g*x)/((d + e*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2)),x)
```

output

```
(((2*b*g)/(3*e*(b*e - 2*c*d)^3) - (4*c*d*g)/(3*e^2*(b*e - 2*c*d)^3))*(c*d^
2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + (((2*d*g)/(3*b^2*e^4 +
 12*c^2*d^2*e^2 - 12*b*c*d*e^3) - (2*e*f)/(3*b^2*e^4 + 12*c^2*d^2*e^2 - 12
*b*c*d*e^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2 + ((
x*(((e*(b*e - c*d) + c*d*e)*((4*c^3*e*(3*b*g - 2*c*f))/(3*(b*e - 2*c*d)^2*
(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e)) + (4*b*c^3*e*g)/(3*(b*e - 2*c*d)^2*
(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e)) - (8*c^3*g*(e*(b*e - c*d) + c*d*e))
/(3*e*(b*e - 2*c*d)^2*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e))))/(c*e^2) - (
2*c^2*(8*b^2*e^2*g + 8*c^2*d^2*g - 10*b*c*e^2*f + 16*c^2*d*e*f - 16*b*c*d*
e*g))/(3*e*(b*e - 2*c*d)^2*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e)) - (2*b*c
^2*e*(3*b*g - 2*c*f))/(3*(b*e - 2*c*d)^2*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*
d*e)) + (8*c^3*d*g*(b*e - c*d))/(3*e*(b*e - 2*c*d)^2*(4*c^3*d^2 + b^2*c*e^
2 - 4*b*c^2*d*e))) + (d*(b*e - c*d)*((4*c^3*e*(3*b*g - 2*c*f))/(3*(b*e - 2
*c*d)^2*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e)) + (4*b*c^3*e*g)/(3*(b*e - 2
*c*d)^2*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e)) - (8*c^3*g*(e*(b*e - c*d) +
 c*d*e))/(3*e*(b*e - 2*c*d)^2*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e))))/(c*
e^2) - (b*c*(8*b^2*e^2*g + 8*c^2*d^2*g - 10*b*c*e^2*f + 16*c^2*d*e*f - 16*
b*c*d*e*g))/(3*e*(b*e - 2*c*d)^2*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e))*(
c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/((d + e*x)*(b*e - c*d + c*e*x)
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1113, normalized size of antiderivative = 8.18

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(2*i*( - 6*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*d**2*e**2*g - 12*sqrt(c
)*sqrt( - b*e + c*d - c*e*x)*b**2*d*e**3*g*x - 6*sqrt(c)*sqrt( - b*e + c*d
 - c*e*x)*b**2*e**4*g*x**2 + 16*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c*d**
3*e*g + 8*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c*d**2*e**2*f + 32*sqrt(c)*
sqrt( - b*e + c*d - c*e*x)*b*c*d**2*e**2*g*x + 16*sqrt(c)*sqrt( - b*e + c*
d - c*e*x)*b*c*d*e**3*f*x + 16*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c*d*e*
*3*g*x**2 + 8*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c*e**4*f*x**2 - 8*sqrt(
c)*sqrt( - b*e + c*d - c*e*x)*c**2*d**4*g - 16*sqrt(c)*sqrt( - b*e + c*d -
 c*e*x)*c**2*d**3*e*f - 16*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**2*d**3*e*
g*x - 32*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*e**2*f*x - 8*sqrt(c)
*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*e**2*g*x**2 - 16*sqrt(c)*sqrt( - b*e
 + c*d - c*e*x)*c**2*d*e**3*f*x**2 + 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqr
t( - b*e + 2*c*d)*b**2*d*e**2*g + sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( -
b*e + 2*c*d)*b**2*e**3*f + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
 2*c*d)*b**2*e**3*g*x + 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*
c*d)*b*c*d**2*e*g - 8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)
*b*c*d*e**2*f + 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b*c
*d*e**2*g*x - 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b*c*e
**3*f*x + 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b*c*e**3*
g*x**2 - 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*c**2*d*...
```

3.185 $\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 209

$$\begin{aligned}\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx &= \frac{8c(6cef+4cdg-5beg)(b+2cx)}{15e(2cd-be)^4\sqrt{d(cd-be)-be^2x-ce^2x^2}}\\ &- \frac{2(ef-dg)}{5e^2(2cd-be)(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}\\ &- \frac{2(6cef+4cdg-5beg)}{15e^2(2cd-be)^2(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\end{aligned}$$

output

```
8/15*c*(-5*b*e*g+4*c*d*g+6*c*e*f)*(2*c*x+b)/e/(-b*e+2*c*d)^4/(d*(-b*e+c*d)
-b*e^2*x-c*e^2*x^2)^(1/2)-2/5*(-d*g+e*f)/e^2/(-b*e+2*c*d)/(e*x+d)^2/(d*(-b
*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-2/15*(-5*b*e*g+4*c*d*g+6*c*e*f)/e^2/(-b*e
+2*c*d)^2/(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

$$\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{2(b^3e^3(3ef+2dg+5egx)+4bc^2e(4d^3g+2de^2x(9f-8g$$

input
```
Integrate[(f + g*x)/((d + e*x)^2*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/
2)),x]
```

output
```
(2*(b^3*e^3*(3*e*f + 2*d*g + 5*e*g*x) + 4*b*c^2*e*(4*d^3*g + 2*d*e^2*x*(9*
f - 8*g*x) + 2*e^3*x^2*(3*f - 5*g*x) + 7*d^2*e*(3*f + g*x)) + 8*c^3*(d^4*g
 + 6*e^4*f*x^3 + 4*d*e^3*x^2*(3*f + g*x) + d^3*e*(-6*f + 2*g*x) + d^2*e^2*
x*(3*f + 8*g*x)) - 2*b^2*c*e^2*(13*d^2*g + 4*d*e*(3*f + 8*g*x) + e^2*x*(3*
f + 10*g*x))))/(15*e^2*(-2*c*d + b*e)^4*(d + e*x)^2*Sqrt[(d + e*x)*(-(b*e)
 + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068, Rules used = {1220, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^2\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-5beg+4cdg+6cef)\int \frac{1}{(d+ex)\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{5e(2cd-be)}-\frac{2(ef-dg)}{5e^2(d+ex)^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-5beg+4cdg+6cef)\left(\frac{4c\int \frac{1}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3(2cd-be)}-\frac{2}{3e(d+ex)(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{5e(2cd-be)}-\frac{2(ef-dg)}{5e^2(d+ex)^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{\left(\frac{8c(b+2cx)}{3(2cd-be)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2}{3e(d+ex)(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-5beg+4cdg+6cef)}{5e(2cd-be)}-\frac{2(ef-dg)}{5e^2(d+ex)^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input
```
Int[(f + g*x)/((d + e*x)^2*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2)),x]
```

output
```
(-2*(e*f - d*g))/(5*e^2*(2*c*d - b*e)*(d + e*x)^2*Sqrt[d*(c*d - b*e) - b*e
^2*x - c*e^2*x^2]) + ((6*c*e*f + 4*c*d*g - 5*b*e*g)*((8*c*(b + 2*c*x))/(3*
(2*c*d - b*e)^3*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) - 2/(3*e*(2*c*d
 - b*e)*(d + e*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])))/(5*e*(2*c*d
 - b*e))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.82

method	result
trager	$-\frac{2\left(-40b\,c^2e^4g\,x^3+32c^3d\,e^3g\,x^3+48c^3e^4f\,x^3-20b^2c\,e^4g\,x^2-64b\,c^2d\,e^3g\,x^2+24b\,c^2e^4f\,x^2+64c^3d^2e^2g\,x^2+96c^3d\,e^3f\,x^2+5b^3e^4gx\right.}{}$
gosper	$-\frac{2(cex+be-cd)\left(-40b\,c^2e^4g\,x^3+32c^3d\,e^3g\,x^3+48c^3e^4f\,x^3-20b^2c\,e^4g\,x^2-64b\,c^2d\,e^3g\,x^2+24b\,c^2e^4f\,x^2+64c^3d^2e^2g\,x^2+96c^3d\,e^3f\right.}{15(ex+d)e^2(b^4e}$
orering	$-\frac{2(cex+be-cd)\left(-40b\,c^2e^4g\,x^3+32c^3d\,e^3g\,x^3+48c^3e^4f\,x^3-20b^2c\,e^4g\,x^2-64b\,c^2d\,e^3g\,x^2+24b\,c^2e^4f\,x^2+64c^3d^2e^2g\,x^2+96c^3d\,e^3f\right.}{15(ex+d)e^2(b^4e}$
default	$\frac{g\left(-\frac{2}{3\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)\sqrt{-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)}}-\frac{8c\,e^2\left(-2c\,e^2\left(x+\frac{d}{e}\right)-b\,e^2+2dec\right)}{3\left(-b\,e^2+2dec\right)^3\sqrt{-c\,e^2\left(x+\frac{d}{e}\right)^2+\left(-b\,e^2+2dec\right)\left(x+\frac{d}{e}\right)}}\right)}{e^2}-\frac{(dg-ef}{}$

input
```
int((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/15*(-40*b*c^2*e^4*g*x^3+32*c^3*d*e^3*g*x^3+48*c^3*e^4*f*x^3-20*b^2*c*e^
4*g*x^2-64*b*c^2*d*e^3*g*x^2+24*b*c^2*e^4*f*x^2+64*c^3*d^2*e^2*g*x^2+96*c^
3*d*e^3*f*x^2+5*b^3*e^4*g*x-64*b^2*c*d*e^3*g*x-6*b^2*c*e^4*f*x+28*b*c^2*d^
2*e^2*g*x+72*b*c^2*d*e^3*f*x+16*c^3*d^3*e*g*x+24*c^3*d^2*e^2*f*x+2*b^3*d*e
^3*g+3*b^3*e^4*f-26*b^2*c*d^2*e^2*g-24*b^2*c*d*e^3*f+16*b*c^2*d^3*e*g+84*b
*c^2*d^2*e^2*f+8*c^3*d^4*g-48*c^3*d^3*e*f)/(c*e*x+b*e-c*d)/(b*e-2*c*d)/(b^
3*e^3-6*b^2*c*d*e^2+12*b*c^2*d^2*e-8*c^3*d^3)/e^2/(e*x+d)^3*(-c*e^2*x^2-b*
e^2*x-b*d*e+c*d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(197) = 394$.

Time = 15.75 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.11

$$\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(8}{15\left(16\,c^5d^8e^2-48\,bc^4d^7e^3+56\,b^2c^3d^6e^4-32\,b^3c^2d^5e^5+9\right.}$$

input

```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="fricas")
```

output

```
2/15*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(8*(6*c^3*e^4*f + (4*c^3*d
*e^3 - 5*b*c^2*e^4)*g)*x^3 + 4*(6*(4*c^3*d*e^3 + b*c^2*e^4)*f + (16*c^3*d^
2*e^2 - 16*b*c^2*d*e^3 - 5*b^2*c*e^4)*g)*x^2 - 3*(16*c^3*d^3*e - 28*b*c^2*
d^2*e^2 + 8*b^2*c*d*e^3 - b^3*e^4)*f + 2*(4*c^3*d^4 + 8*b*c^2*d^3*e - 13*b
^2*c*d^2*e^2 + b^3*d*e^3)*g + (6*(4*c^3*d^2*e^2 + 12*b*c^2*d*e^3 - b^2*c*e
^4)*f + (16*c^3*d^3*e + 28*b*c^2*d^2*e^2 - 64*b^2*c*d*e^3 + 5*b^3*e^4)*g)*
x)/(16*c^5*d^8*e^2 - 48*b*c^4*d^7*e^3 + 56*b^2*c^3*d^6*e^4 - 32*b^3*c^2*d^
5*e^5 + 9*b^4*c*d^4*e^6 - b^5*d^3*e^7 - (16*c^5*d^4*e^6 - 32*b*c^4*d^3*e^7
 + 24*b^2*c^3*d^2*e^8 - 8*b^3*c^2*d*e^9 + b^4*c*e^10)*x^4 - (32*c^5*d^5*e^
5 - 48*b*c^4*d^4*e^6 + 16*b^2*c^3*d^3*e^7 + 8*b^3*c^2*d^2*e^8 - 6*b^4*c*d*
e^9 + b^5*e^10)*x^3 - 3*(16*b*c^4*d^5*e^5 - 32*b^2*c^3*d^4*e^6 + 24*b^3*c^
2*d^3*e^7 - 8*b^4*c*d^2*e^8 + b^5*d*e^9)*x^2 + (32*c^5*d^7*e^3 - 112*b*c^4
*d^6*e^4 + 144*b^2*c^3*d^5*e^5 - 88*b^3*c^2*d^4*e^6 + 26*b^4*c*d^3*e^7 - 3
*b^5*d^2*e^8)*x)
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \int \frac{f+gx}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

input

```
integrate((g*x+f)/(e*x+d)**2/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x
)
```

output
```
Integral((f + g*x)/((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(d + e*x)**2),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7557 vs. $2(197) = 394$.

Time = 0.51 (sec) , antiderivative size = 7557, normalized size of antiderivative = 36.16

$$\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="giac")
```

output

```
-2/15*(8*(6*c^3*e*f + 4*c^3*d*g - 5*b*c^2*e*g)*sgn(1/(e*x + d))*sgn(e)/(16
*sqrt(-c)*c^4*d^4*e - 32*b*sqrt(-c)*c^3*d^3*e^2 + 24*b^2*sqrt(-c)*c^2*d^2*
e^3 - 8*b^3*sqrt(-c)*c*d*e^4 + b^4*sqrt(-c)*e^5) + (196608*(c - 2*c*d/(e*x
 + d) + b*e/(e*x + d))^2*c^16*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d
^16*e^5*f*sgn(1/(e*x + d))^4*sgn(e)^4 + 2949120*c^18*sqrt(-c + 2*c*d/(e*x
+ d) - b*e/(e*x + d))*d^16*e^5*f*sgn(1/(e*x + d))^4*sgn(e)^4 + 983040*c^17
*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*d^16*e^5*f*sgn(1/(e*x + d))^
4*sgn(e)^4 - 1572864*b*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^2*c^15*sqrt(-
c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^15*e^6*f*sgn(1/(e*x + d))^4*sgn(e)^
4 - 23592960*b*c^17*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^15*e^6*f*
sgn(1/(e*x + d))^4*sgn(e)^4 - 7864320*b*c^16*(-c + 2*c*d/(e*x + d) - b*e/(
e*x + d))^(3/2)*d^15*e^6*f*sgn(1/(e*x + d))^4*sgn(e)^4 + 5898240*b^2*(c -
2*c*d/(e*x + d) + b*e/(e*x + d))^2*c^14*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e
*x + d))*d^14*e^7*f*sgn(1/(e*x + d))^4*sgn(e)^4 + 88473600*b^2*c^16*sqrt(-
c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^14*e^7*f*sgn(1/(e*x + d))^4*sgn(e)^
4 + 29491200*b^2*c^15*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*d^14*e^
7*f*sgn(1/(e*x + d))^4*sgn(e)^4 - 13762560*b^3*(c - 2*c*d/(e*x + d) + b*e/
(e*x + d))^2*c^13*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^13*e^8*f*sg
n(1/(e*x + d))^4*sgn(e)^4 - 206438400*b^3*c^15*sqrt(-c + 2*c*d/(e*x + d) -
 b*e/(e*x + d))*d^13*e^8*f*sgn(1/(e*x + d))^4*sgn(e)^4 - 68812800*b^3*c...
```

Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 2126, normalized size of antiderivative = 10.17

$$\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((f + g*x)/((d + e*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2)),x)
```

output

```
(((d*((16*c^3*f - 16*b*c^2*g)/(15*(b*e - 2*c*d)^5) + (8*c^3*d*g)/(15*e*(b*
e - 2*c*d)^5)))/e + (2*b*c*(3*b*g - 4*c*f))/(15*(b*e - 2*c*d)^5))*(c*d^2 -
 c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((4*b*c*g)/(15*e*(b*e -
2*c*d)^4) - (8*c^2*d*g)/(15*e^2*(b*e - 2*c*d)^4))*(c*d^2 - c*e^2*x^2 - b*d
*e - b*e^2*x)^(1/2))/(d + e*x) + (((2*b*g)/(5*(3*b*e^2 - 6*c*d*e)*(b*e - 2
*c*d)^2) - (4*c*d*g)/(5*e*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)^2))*(c*d^2 - c
*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2 + (((4*c*g*(3*b*e - 4*c*d))
/(15*e^2*(b*e - 2*c*d)^4) - (8*c^2*d*g)/(15*e^2*(b*e - 2*c*d)^4))*(c*d^2 -
 c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + (((2*d*g)/(5*b^2*e^4 + 20
*c^2*d^2*e^2 - 20*b*c*d*e^3) - (2*e*f)/(5*b^2*e^4 + 20*c^2*d^2*e^2 - 20*b*
c*d*e^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^3 + (((d*
((2*c*e*(3*b*e*g + 2*c*d*g - 4*c*e*f))/(5*(b*e - 2*c*d)^2*(3*b^2*e^4 + 12*
c^2*d^2*e^2 - 12*b*c*d*e^3)) - (4*c^2*d*e*g)/(5*(b*e - 2*c*d)^2*(3*b^2*e^4
 + 12*c^2*d^2*e^2 - 12*b*c*d*e^3))))/e - (12*b^2*e^2*g + 12*c^2*d^2*g - 18
*b*c*e^2*f + 28*c^2*d*e*f - 24*b*c*d*e*g)/(5*(b*e - 2*c*d)^2*(3*b^2*e^4 +
12*c^2*d^2*e^2 - 12*b*c*d*e^3)))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/
2))/(d + e*x)^2 - ((x*((d*(b*e - c*d)*((16*c^5*g*(e*(b*e - c*d) + c*d*e))/
(15*(b*e - 2*c*d)^4*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e)) + (16*c^5*e*(c*
d*g - 3*b*e*g + 2*c*e*f))/(15*(b*e - 2*c*d)^4*(4*c^3*d^2 + b^2*c*e^2 - 4*b
*c^2*d*e)) - (8*b*c^5*e^2*g)/(15*(b*e - 2*c*d)^4*(4*c^3*d^2 + b^2*c*e^2...
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1782, normalized size of antiderivative = 8.53

$$\int \frac{f+gx}{(d+ex)^2\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(2*i*(40*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c*d**3*e**2*g + 120*sqrt(
c)*sqrt( - b*e + c*d - c*e*x)*b**2*c*d**2*e**3*g*x + 120*sqrt(c)*sqrt( - b
*e + c*d - c*e*x)*b**2*c*d*e**4*g*x**2 + 40*sqrt(c)*sqrt( - b*e + c*d - c*
e*x)*b**2*c*e**5*g*x**3 - 112*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d*
*4*e*g - 48*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**3*e**2*f - 336*sq
rt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**3*e**2*g*x - 144*sqrt(c)*sqrt(
- b*e + c*d - c*e*x)*b*c**2*d**2*e**3*f*x - 336*sqrt(c)*sqrt( - b*e + c*d
- c*e*x)*b*c**2*d**2*e**3*g*x**2 - 144*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*
b*c**2*d*e**4*f*x**2 - 112*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d*e**
4*g*x**3 - 48*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*e**5*f*x**3 + 64*s
qrt(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d**5*g + 96*sqrt(c)*sqrt( - b*e + c
*d - c*e*x)*c**3*d**4*e*f + 192*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d*
*4*e*g*x + 288*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d**3*e**2*f*x + 192
*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d**3*e**2*g*x**2 + 288*sqrt(c)*sq
rt( - b*e + c*d - c*e*x)*c**3*d**2*e**3*f*x**2 + 64*sqrt(c)*sqrt( - b*e +
c*d - c*e*x)*c**3*d**2*e**3*g*x**3 + 96*sqrt(c)*sqrt( - b*e + c*d - c*e*x)
*c**3*d*e**4*f*x**3 + 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*
d)*b**3*d*e**3*g + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*
b**3*e**4*f + 5*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b**3*
e**4*g*x - 26*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b**2...
```

3.186 $\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 284

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{16c^2(8cef+6cdg-7beg)(b+2cx)}{35e(2cd-be)^5\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$
$$-\frac{2(ef-dg)}{7e^2(2cd-be)(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$
$$-\frac{2(8cef+6cdg-7beg)}{35e^2(2cd-be)^2(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$
$$-\frac{4c(8cef+6cdg-7beg)}{35e^2(2cd-be)^3(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

output

```
16/35*c^2*(-7*b*e*g+6*c*d*g+8*c*e*f)*(2*c*x+b)/e/(-b*e+2*c*d)^5/(d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(1/2)-2/7*(-d*g+e*f)/e^2/(-b*e+2*c*d)/(e*x+d)^3/(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-2/35*(-7*b*e*g+6*c*d*g+8*c*e*f)/e^2/(-
b*e+2*c*d)^2/(e*x+d)^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-4/35*c*(-7*b
*e*g+6*c*d*g+8*c*e*f)/e^2/(-b*e+2*c*d)^3/(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e
^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.17

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{2(b^4e^4(5ef+2dg+7egx)+16c^4(d^5g-8e^5fx^4+d^3e^2x(4}{}$$

input
```
Integrate[(f + g*x)/((d + e*x)^3*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/
2)),x]
```

output
```
(2*(b^4*e^4*(5*e*f + 2*d*g + 7*e*g*x) + 16*c^4*(d^5*g - 8*e^5*f*x^4 + d^3*
e^2*x*(4*f - 15*g*x) - 6*d*e^4*x^3*(4*f + g*x) + d^4*e*(13*f + 3*g*x) - 2*
d^2*e^3*x^2*(10*f + 9*g*x)) - 2*b^3*c*e^3*(11*d^2*g + e^2*x*(4*f + 7*g*x)
+ d*e*(24*f + 38*g*x)) - 8*b*c^3*e*(15*d^4*g + d^2*e^2*x*(52*f - 11*g*x) +
 4*d*e^3*x^2*(8*f - 9*g*x) + 2*e^4*x^3*(4*f - 7*g*x) + d^3*e*(48*f + 46*g*
x)) + 4*b^2*c^2*e^2*(31*d^3*g + 2*e^3*x^2*(2*f + 7*g*x) + 2*d^2*e*(23*f +
53*g*x) + d*e^2*x*(20*f + 59*g*x))))/(35*e^2*(-2*c*d + b*e)^5*(d + e*x)^3*
Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1220, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^3\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(-7beg+6cdg+8cef)\int \frac{1}{(d+ex)^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{7e(2cd-be)}-\frac{2(ef-dg)}{7e^2(d+ex)^3(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \; 1129$$

$$\frac{(-7beg+6cdg+8cef)\left(\frac{6c\int \frac{1}{(d+ex)\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{5(2cd-be)}-\frac{2}{5e(d+ex)^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{7e(2cd-be)}-\frac{2(ef-dg)}{7e^2(d+ex)^3(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(-7beg+6cdg+8cef)\left(\frac{6c\left(\frac{4c\int \frac{1}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3(2cd-be)}-\frac{2}{3e(d+ex)(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{5(2cd-be)}-\frac{2}{5e(d+ex)^2(2cd-be)\sqrt{}}\right.}{7e(2cd-be)}$$

$$\frac{2(ef-dg)}{7e^2(d+ex)^3(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{\left(\frac{6c\left(\frac{8c(b+2cx)}{3(2cd-be)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2}{3e(d+ex)(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{5(2cd-be)}-\frac{2}{5e(d+ex)^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-7b}{7e(2cd-be)}$$

$$\frac{2(ef-dg)}{7e^2(d+ex)^3(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input

```
Int[(f + g*x)/((d + e*x)^3*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2)),x]
```

output

```
(-2*(e*f - d*g))/(7*e^2*(2*c*d - b*e)*(d + e*x)^3*Sqrt[d*(c*d - b*e) - b*e
^2*x - c*e^2*x^2]) + ((8*c*e*f + 6*c*d*g - 7*b*e*g)*(-2/(5*e*(2*c*d - b*e)
*(d + e*x)^2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) + (6*c*((8*c*(b +
2*c*x))/(3*(2*c*d - b*e)^3*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) - 2/
(3*e*(2*c*d - b*e)*(d + e*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])))/
(5*(2*c*d - b*e))))/(7*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(268) = 536$.

Time = 2.88 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.98

method	result
trager	$-\frac{2\left(112b\,c^{3}e^{5}g\,x^{4}-96c^{4}d\,e^{4}g\,x^{4}-128c^{4}e^{5}f\,x^{4}+56b^{2}c^{2}e^{5}g\,x^{3}+288b\,c^{3}d\,e^{4}g\,x^{3}-64b\,c^{3}e^{5}f\,x^{3}-288c^{4}d^{2}e^{3}g\,x^{3}-384c^{4}d\,e^{4}f\,x^{3}-14b}{}$
gosper	$-\frac{2(cex+be-cd)\left(112b\,c^{3}e^{5}g\,x^{4}-96c^{4}d\,e^{4}g\,x^{4}-128c^{4}e^{5}f\,x^{4}+56b^{2}c^{2}e^{5}g\,x^{3}+288b\,c^{3}d\,e^{4}g\,x^{3}-64b\,c^{3}e^{5}f\,x^{3}-288c^{4}d^{2}e^{3}g\,x^{3}-384c^{4}}{}$
orering	$-\frac{2(cex+be-cd)\left(112b\,c^{3}e^{5}g\,x^{4}-96c^{4}d\,e^{4}g\,x^{4}-128c^{4}e^{5}f\,x^{4}+56b^{2}c^{2}e^{5}g\,x^{3}+288b\,c^{3}d\,e^{4}g\,x^{3}-64b\,c^{3}e^{5}f\,x^{3}-288c^{4}d^{2}e^{3}g\,x^{3}-384c^{4}}{}$
default	$\frac{g\left(-\frac{2}{5\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{2}\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)}}+\frac{6c\,e^{2}\left(-\frac{2}{3\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\sqrt{-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)}}}{5\left(-b\,e^{2}+2de}\right.}{e^{3}}$

input

```
int((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=_RET
URNVERBOSE)
```

output

```
-2/35*(112*b*c^3*e^5*g*x^4-96*c^4*d*e^4*g*x^4-128*c^4*e^5*f*x^4+56*b^2*c^2
*e^5*g*x^3+288*b*c^3*d*e^4*g*x^3-64*b*c^3*e^5*f*x^3-288*c^4*d^2*e^3*g*x^3-
384*c^4*d*e^4*f*x^3-14*b^3*c*e^5*g*x^2+236*b^2*c^2*d*e^4*g*x^2+16*b^2*c^2*
e^5*f*x^2+88*b*c^3*d^2*e^3*g*x^2-256*b*c^3*d*e^4*f*x^2-240*c^4*d^3*e^2*g*x
^2-320*c^4*d^2*e^3*f*x^2+7*b^4*e^5*g*x-76*b^3*c*d*e^4*g*x-8*b^3*c*e^5*f*x+
424*b^2*c^2*d^2*e^3*g*x+80*b^2*c^2*d*e^4*f*x-368*b*c^3*d^3*e^2*g*x-416*b*c
^3*d^2*e^3*f*x+48*c^4*d^4*e*g*x+64*c^4*d^3*e^2*f*x+2*b^4*d*e^4*g+5*b^4*e^5
*f-22*b^3*c*d^2*e^3*g-48*b^3*c*d*e^4*f+124*b^2*c^2*d^3*e^2*g+184*b^2*c^2*d
^2*e^3*f-120*b*c^3*d^4*e*g-384*b*c^3*d^3*e^2*f+16*c^4*d^5*g+208*c^4*d^4*e*
f)/(c*e*x+b*e-c*d)/(b*e-2*c*d)/(b^4*e^4-8*b^3*c*d*e^3+24*b^2*c^2*d^2*e^2-3
2*b*c^3*d^3*e+16*c^4*d^4)/e^2/(e*x+d)^4*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(
1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(268) = 536$.

Time = 74.11 (sec) , antiderivative size = 974, normalized size of antiderivative = 3.43

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="fricas")
```

output

```
2/35*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(16*(8*c^4*e^5*f + (6*c^4*
d*e^4 - 7*b*c^3*e^5)*g)*x^4 + 8*(8*(6*c^4*d*e^4 + b*c^3*e^5)*f + (36*c^4*d
^2*e^3 - 36*b*c^3*d*e^4 - 7*b^2*c^2*e^5)*g)*x^3 + 2*(8*(20*c^4*d^2*e^3 + 1
6*b*c^3*d*e^4 - b^2*c^2*e^5)*f + (120*c^4*d^3*e^2 - 44*b*c^3*d^2*e^3 - 118
*b^2*c^2*d*e^4 + 7*b^3*c*e^5)*g)*x^2 - (208*c^4*d^4*e - 384*b*c^3*d^3*e^2
+ 184*b^2*c^2*d^2*e^3 - 48*b^3*c*d*e^4 + 5*b^4*e^5)*f - 2*(8*c^4*d^5 - 60*
b*c^3*d^4*e + 62*b^2*c^2*d^3*e^2 - 11*b^3*c*d^2*e^3 + b^4*d*e^4)*g - (8*(8
*c^4*d^3*e^2 - 52*b*c^3*d^2*e^3 + 10*b^2*c^2*d*e^4 - b^3*c*e^5)*f + (48*c^
4*d^4*e - 368*b*c^3*d^3*e^2 + 424*b^2*c^2*d^2*e^3 - 76*b^3*c*d*e^4 + 7*b^4
*e^5)*g)*x)/(32*c^6*d^10*e^2 - 112*b*c^5*d^9*e^3 + 160*b^2*c^4*d^8*e^4 - 1
20*b^3*c^3*d^7*e^5 + 50*b^4*c^2*d^6*e^6 - 11*b^5*c*d^5*e^7 + b^6*d^4*e^8 -
 (32*c^6*d^5*e^7 - 80*b*c^5*d^4*e^8 + 80*b^2*c^4*d^3*e^9 - 40*b^3*c^3*d^2*
e^10 + 10*b^4*c^2*d*e^11 - b^5*c*e^12)*x^5 - (96*c^6*d^6*e^6 - 208*b*c^5*d
^5*e^7 + 160*b^2*c^4*d^4*e^8 - 40*b^3*c^3*d^3*e^9 - 10*b^4*c^2*d^2*e^10 +
7*b^5*c*d*e^11 - b^6*e^12)*x^4 - 2*(32*c^6*d^7*e^5 - 16*b*c^5*d^6*e^6 - 80
*b^2*c^4*d^5*e^7 + 120*b^3*c^3*d^4*e^8 - 70*b^4*c^2*d^3*e^9 + 19*b^5*c*d^2
*e^10 - 2*b^6*d*e^11)*x^3 + 2*(32*c^6*d^8*e^4 - 176*b*c^5*d^7*e^5 + 320*b^
2*c^4*d^6*e^6 - 280*b^3*c^3*d^5*e^7 + 130*b^4*c^2*d^4*e^8 - 31*b^5*c*d^3*e
^9 + 3*b^6*d^2*e^10)*x^2 + (96*c^6*d^9*e^3 - 368*b*c^5*d^8*e^4 + 560*b^2*c
^4*d^7*e^5 - 440*b^3*c^3*d^6*e^6 + 190*b^4*c^2*d^5*e^7 - 43*b^5*c*d^4*e...
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^3\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{f+gx}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^3}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**3/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x
)
```

output
```
Integral((f + g*x)/((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(d + e*x)**3),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^3\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{f+gx}{(d+ex)^3\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{gx+f}{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}\left(ex+d\right)^3}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((g*x + f)/((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(e*x + d
)^3), x)
```

Mupad [B] (verification not implemented)

Time = 16.42 (sec) , antiderivative size = 4339, normalized size of antiderivative = 15.28

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((f + g*x)/((d + e*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2)),x)
```

output
```
(((8*c*g*(2*b*e - 3*c*d))/(35*e*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)^3) - (8*
c^2*d*g)/(35*e*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)^3))*(c*d^2 - c*e^2*x^2 -
b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2 - (((d*((d*((8*c^4*(2*c*d*g - 7*b*e*g
+ 6*c*e*f))/(105*(b*e - 2*c*d)^7) + (16*c^5*d*g)/(105*(b*e - 2*c*d)^7)))/e
 + (64*c^5*d^2*g - 20*b^2*c^3*e^2*g - 320*c^5*d*e*f + 112*b*c^4*e^2*f + 80
*b*c^4*d*e*g)/(105*e*(b*e - 2*c*d)^7)))/e - (2*b*c^2*(16*c^2*d^2*g - 11*b^
2*e^2*g + 34*b*c*e^2*f - 80*c^2*d*e*f + 22*b*c*d*e*g))/(105*e*(b*e - 2*c*d
)^7))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((4*b*c*g)
/(35*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)^3) - (8*c^2*d*g)/(35*e*(3*b*e^2 - 6
*c*d*e)*(b*e - 2*c*d)^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d
+ e*x)^2 - (((8*c^2*g*(3*b*e - 4*c*d))/(105*e^2*(b*e - 2*c*d)^5) - (16*c^3
*d*g)/(105*e^2*(b*e - 2*c*d)^5))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/
2))/(d + e*x) - (((d*((48*c^4*f - 40*b*c^3*g)/(105*(b*e - 2*c*d)^6) + (16*
c^4*d*g)/(105*e*(b*e - 2*c*d)^6)))/e + (8*b*c^2*(2*b*g - 3*c*f))/(105*(b*e
 - 2*c*d)^6))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + (((
d*((48*c^4*d*g + 48*c^4*e*f - 64*b*c^3*e*g)/(105*e*(b*e - 2*c*d)^6) + (16*
c^4*d*g)/(105*e*(b*e - 2*c*d)^6)))/e + (96*c^4*d*f - 72*b*c^3*d*g - 72*b*c
^3*e*f + 52*b^2*c^2*e*g)/(105*e*(b*e - 2*c*d)^6))*(c*d^2 - c*e^2*x^2 - b*d
*e - b*e^2*x)^(1/2))/(d + e*x) + (((2*b*g)/(7*(5*b*e^2 - 10*c*d*e)*(b*e -
2*c*d)^2) - (4*c*d*g)/(7*e*(5*b*e^2 - 10*c*d*e)*(b*e - 2*c*d)^2))*(c*d^...
```

Reduce [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 2565, normalized size of antiderivative = 9.03

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(2*i*( - 112*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**4*e**2*g - 44
8*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**3*e**3*g*x - 672*sqrt(c)
*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**2*e**4*g*x**2 - 448*sqrt(c)*sqrt(
 - b*e + c*d - c*e*x)*b**2*c**2*d*e**5*g*x**3 - 112*sqrt(c)*sqrt( - b*e +
c*d - c*e*x)*b**2*c**2*e**6*g*x**4 + 320*sqrt(c)*sqrt( - b*e + c*d - c*e*x
)*b*c**3*d**5*e*g + 128*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*d**4*e**
2*f + 1280*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*d**4*e**2*g*x + 512*s
qrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*d**3*e**3*f*x + 1920*sqrt(c)*sqrt
( - b*e + c*d - c*e*x)*b*c**3*d**3*e**3*g*x**2 + 768*sqrt(c)*sqrt( - b*e +
 c*d - c*e*x)*b*c**3*d**2*e**4*f*x**2 + 1280*sqrt(c)*sqrt( - b*e + c*d - c
*e*x)*b*c**3*d**2*e**4*g*x**3 + 512*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c
**3*d*e**5*f*x**3 + 320*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*d*e**5*g
*x**4 + 128*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*e**6*f*x**4 - 192*sq
rt(c)*sqrt( - b*e + c*d - c*e*x)*c**4*d**6*g - 256*sqrt(c)*sqrt( - b*e + c
*d - c*e*x)*c**4*d**5*e*f - 768*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**4*d*
*5*e*g*x - 1024*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**4*d**4*e**2*f*x - 11
52*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**4*d**4*e**2*g*x**2 - 1536*sqrt(c)
*sqrt( - b*e + c*d - c*e*x)*c**4*d**3*e**3*f*x**2 - 768*sqrt(c)*sqrt( - b*
e + c*d - c*e*x)*c**4*d**3*e**3*g*x**3 - 1024*sqrt(c)*sqrt( - b*e + c*d -
c*e*x)*c**4*d**2*e**4*f*x**3 - 192*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c...
```

3.187 $\int \frac{(d+ex)^5(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 311

$$\begin{aligned}
\int \frac{(d+ex)^5(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx &= \frac{2(cef+cdg-beg)(d+ex)^4}{3c^2e^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} \\
&- \frac{2(5cef+11cdg-8beg)(d+ex)^2}{3c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} \\
&- \frac{5(4cef+10cdg-7beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c^4e^2} \\
&- \frac{g(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2c^3e^2} \\
&+ \frac{5(2cd-be)(4cef+10cdg-7beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4c^{9/2}e^2}
\end{aligned}$$

output
```
2/3*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^4/c^2/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(3/2)-2/3*(-8*b*e*g+11*c*d*g+5*c*e*f)*(e*x+d)^2/c^3/e^2/(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(1/2)-5/4*(-7*b*e*g+10*c*d*g+4*c*e*f)*(d*(-b*e+c*d)-b*e^2
*x-c*e^2*x^2)^(1/2)/c^4/e^2-1/2*g*(e*x+d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)
^(1/2)/c^3/e^2+5/4*(-b*e+2*c*d)*(-7*b*e*g+10*c*d*g+4*c*e*f)*arctan(c^(1/2)
*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(9/2)/e^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.45

$$\int \frac{(d+ex)^5(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{2(d+ex)^5\left(-7(cef+cdg-beg)+(4cef+10cdg-7beg)\left(\frac{-cd+be+}{-2cd+}\right.\right.}{21ce^2(-2cd+be)((d+ex)(-be+}$$

input
```
Integrate[((d + e*x)^5*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5
/2),x]
```

output
```
(2*(d + e*x)^5*(-7*(c*e*f + c*d*g - b*e*g) + (4*c*e*f + 10*c*d*g - 7*b*e*g
)*((-(c*d) + b*e + c*e*x)/(-2*c*d + b*e))^(3/2)*Hypergeometric2F1[3/2, 7/2
, 9/2, (c*(d + e*x))/(2*c*d - b*e)]))/(21*c*e^2*(-2*c*d + b*e)*((d + e*x)*
(-(b*e) + c*(d - e*x)))^(3/2))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1218, 1124, 2192, 27, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{2(d+ex)^5(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-7beg+10cdg+4cef)\int \frac{(d+ex)^4}{(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{3ce(2cd-be)}$$

$$\Big\downarrow \text{1124}$$

$$\frac{2(d+ex)^5(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-7beg+10cdg+4cef)\left(\frac{2(d+ex)(2cd-be)^2}{c^3e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\int\frac{c^2x^2e^6+c(4cd-be)xe^5+\left(7c^2d^2-5bced+b^2e^2\right)e^4}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{c^3e^4}\right)}{3ce(2cd-be)}$$

$$\Big\downarrow \; 2192$$

$$\frac{2(d+ex)^5(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-7beg+10cdg+4cef)\Bigg(\frac{2(d+ex)(2cd-be)^2}{c^3e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{-\frac{\int-\frac{ce^6(2(5cd-2be)(3cd-be)+ce(16cd-7be)x)}{2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2ce^2}-\frac{1}{2}ce^4x\sqrt{d(cd-be)-be^2x-ce}}{c^3e^4}}{3ce(2cd-be)}$$

$$\Big\downarrow \; 27$$

$$\frac{2(d+ex)^5(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-7beg+10cdg+4cef)\Bigg(\frac{2(d+ex)(2cd-be)^2}{c^3e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{1}{4}e^4\int\frac{2(5cd-2be)(3cd-be)+ce(16cd-7be)x}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{1}{2}ce^4x\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^3e^4}}{3ce(2cd-be)}$$

$$\Big\downarrow \; 1160$$

$$\frac{2(d+ex)^5(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-7beg+10cdg+4cef)\Bigg(\frac{2(d+ex)(2cd-be)^2}{c^3e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{1}{4}e^4\Bigg(\frac{15}{2}(2cd-be)^2\int\frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{(16cd-7be)\sqrt{d(cd-be)-be^2}}{e}}{c^3e^4}}{3ce(2cd-be)}$$

$$\Big\downarrow \; 1092$$

$$\frac{2(d+ex)^5(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-7beg+10cdg+4cef)\Bigg(\frac{2(d+ex)(2cd-be)^2}{c^3e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{1}{4}e^4\Bigg(15(2cd-be)^2\int\frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\Bigg(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(}}}{c}}{3ce(2cd-be)}$$

$$\Big\downarrow 217$$

$$\frac{2(d+ex)^5(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{\left(\frac{2(d+ex)(2cd-be)^2}{c^3e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{1}{4}e^4\left(\frac{15(2cd-be)^2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2\sqrt{c}e}-\frac{(16cd-7be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}\right)-\frac{1}{2}ce^4x\sqrt{d(cd-}{c^3e^4}\right.}{3ce(2cd-be)}$$

input
```
Int[((d + e*x)^5*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^5)/(3*c*e^2*(2*c*d - b*e)*(d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2)^(3/2)) - ((4*c*e*f + 10*c*d*g - 7*b*e*g)*((2*(2
*c*d - b*e)^2*(d + e*x))/(c^3*e*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])
 - (-1/2*(c*e^4*x*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) + (e^4*(-(((1
6*c*d - 7*b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/e) + (15*(2*c*d
- b*e)^2*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x -
c*e^2*x^2])])/(2*Sqrt[c]*e)))/4)/(c^3*e^4)))/(3*c*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1218

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2192

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6818 vs. $2(287) = 574$.

Time = 5.94 (sec) , antiderivative size = 6819, normalized size of antiderivative = 21.93

method	result	size
default	Expression too large to display	6819

input
```
int((e*x+d)^5*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(287) = 574.

Time = 3.21 (sec) , antiderivative size = 1233, normalized size of antiderivative = 3.96

$$\int \frac{(d+ex)^5(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^5*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="fricas")
```

output
```
[-1/48*(15*((4*(2*c^4*d*e^3 - b*c^3*e^4)*f + (20*c^4*d^2*e^2 - 24*b*c^3*d*
e^3 + 7*b^2*c^2*e^4)*g)*x^2 + 4*(2*c^4*d^3*e - 5*b*c^3*d^2*e^2 + 4*b^2*c^2
*d*e^3 - b^3*c*e^4)*f + (20*c^4*d^4 - 64*b*c^3*d^3*e + 75*b^2*c^2*d^2*e^2
- 38*b^3*c*d*e^3 + 7*b^4*e^4)*g - 2*(4*(2*c^4*d^2*e^2 - 3*b*c^3*d*e^3 + b^
2*c^2*e^4)*f + (20*c^4*d^3*e - 44*b*c^3*d^2*e^2 + 31*b^2*c^2*d*e^3 - 7*b^3
*c*e^4)*g)*x)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c
*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b
*e)*sqrt(-c)) + 4*(6*c^4*e^3*g*x^3 + 3*(4*c^4*e^3*f + (16*c^4*d*e^2 - 7*b*
c^3*e^3)*g)*x^2 + 4*(23*c^4*d^2*e - 40*b*c^3*d*e^2 + 15*b^2*c^2*e^3)*f + (
236*c^4*d^3 - 561*b*c^3*d^2*e + 430*b^2*c^2*d*e^2 - 105*b^3*c*e^3)*g - 2*(
4*(17*c^4*d*e^2 - 10*b*c^3*e^3)*f + (161*c^4*d^2*e - 219*b*c^3*d*e^2 + 70*
b^2*c^2*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^7*e^4*x^
2 + c^7*d^2*e^2 - 2*b*c^6*d*e^3 + b^2*c^5*e^4 - 2*(c^7*d*e^3 - b*c^6*e^4)*
x), -1/24*(15*((4*(2*c^4*d*e^3 - b*c^3*e^4)*f + (20*c^4*d^2*e^2 - 24*b*c^3
*d*e^3 + 7*b^2*c^2*e^4)*g)*x^2 + 4*(2*c^4*d^3*e - 5*b*c^3*d^2*e^2 + 4*b^2*
c^2*d*e^3 - b^3*c*e^4)*f + (20*c^4*d^4 - 64*b*c^3*d^3*e + 75*b^2*c^2*d^2*e
^2 - 38*b^3*c*d*e^3 + 7*b^4*e^4)*g - 2*(4*(2*c^4*d^2*e^2 - 3*b*c^3*d*e^3 +
 b^2*c^2*e^4)*f + (20*c^4*d^3*e - 44*b*c^3*d^2*e^2 + 31*b^2*c^2*d*e^3 - 7*
b^3*c*e^4)*g)*x)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*
d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d...
```

Sympy [F]

$$\int \frac{(d+ex)^5(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^5\,(f+gx)}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**5*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x
)
```

output
```
Integral((d + e*x)**5*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(5/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^5(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^5*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(287) = 574$.

Time = 2.93 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.38

$$\int \frac{(d+ex)^5(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^5*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="giac")
```

output
```
-1/4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*g*x/(c^3*e) + (4*c^8*e^
3*f + 20*c^8*d*e^2*g - 11*b*c^7*e^3*g)/(c^11*e^4)) - 1/8*(8*c^2*d*e*f - 4*
b*c*e^2*f + 20*c^2*d^2*g - 24*b*c*d*e*g + 7*b^2*e^2*g)*log(abs(-b*c^4*d^4*
e^2 + 4*b^2*c^3*d^3*e^3 - 6*b^3*c^2*d^2*e^4 + 4*b^4*c*d*e^5 - b^5*e^6 + 2*
(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*c^4
*d^4*abs(e) - 12*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d
*e))*b*sqrt(-c)*c^3*d^3*e*abs(e) + 24*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 -
b*e^2*x + c*d^2 - b*d*e))*b^2*sqrt(-c)*c^2*d^2*e^2*abs(e) - 20*(sqrt(-c*e^
2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b^3*sqrt(-c)*c*d*e^3*ab
s(e) + 6*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b^4
*sqrt(-c)*e^4*abs(e) - 8*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d
^2 - b*d*e))^2*c^4*d^3*e + 30*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x
+ c*d^2 - b*d*e))^2*b*c^3*d^2*e^2 - 36*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 -
 b*e^2*x + c*d^2 - b*d*e))^2*b^2*c^2*d*e^3 + 14*(sqrt(-c*e^2)*x - sqrt(-c*
e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*b^3*c*e^4 - 12*(sqrt(-c*e^2)*x - sqr
t(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*sqrt(-c)*c^3*d^2*abs(e) + 28*(s
qrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*b*sqrt(-c)*c
^2*d*e*abs(e) - 16*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b
*d*e))^3*b^2*sqrt(-c)*c*e^2*abs(e) + 8*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 -
 b*e^2*x + c*d^2 - b*d*e))^4*c^3*d*e - 9*(sqrt(-c*e^2)*x - sqrt(-c*e^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^5(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \int \frac{(f+g\,x)\,\left(d+e\,x\right)^5}{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^5)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^5)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 2003, normalized size of antiderivative = 6.44

$$\int \frac{(d+ex)^5(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^5*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(i*(840*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*b**4*e**4*g - 5400*sqrt(c)*sqrt( - b*e + c*d -
 c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c*
d*e**3*g - 480*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d
 - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c*e**4*f + 840*sqrt(c)*sqrt( - b*e
 + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*b**3*c*e**4*g*x + 12720*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( -
 b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d**2*e**2*g + 2400*
sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sq
rt( - b*e + 2*c*d))*b**2*c**2*d*e**3*f - 4560*sqrt(c)*sqrt( - b*e + c*d -
c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**
2*d*e**3*g*x - 480*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e +
 c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*e**4*f*x - 12960*sqrt(c)*
sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*
e + 2*c*d))*b*c**3*d**3*e*g - 3840*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asin
h((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**3*d**2*e**2*f
+ 8160*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x
)*i)/sqrt( - b*e + 2*c*d))*b*c**3*d**2*e**2*g*x + 1920*sqrt(c)*sqrt( - b*e
 + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*b*c**3*d*e**3*f*x + 4800*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqr...
```

3.188 $\int \frac{(d+ex)^4(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 230

$$\int \frac{(d+ex)^4(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2(cef+cdg-beg)(d+ex)^3}{3c^2e^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} - \frac{2(cef+3cdg-2beg)(d+ex)}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} - \frac{g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^3e^2} + \frac{(2cef+8cdg-5beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{c^{7/2}e^2}$$

output

```
2/3*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^3/c^2/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(3/2)-2*(-2*b*e*g+3*c*d*g+c*e*f)*(e*x+d)/c^3/e^2/(d*(-b*e+c*d)-b*e^2*x-c
*e^2*x^2)^(1/2)-g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^3/e^2+(-5*b*e*g
+8*c*d*g+2*c*e*f)*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(1/2))/c^(7/2)/e^2
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^4(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{-\sqrt{c}(d+ex)^3(-be+c(d-ex))\left(15b^2e^2g-2bce(3ef+17dg-10e\right.}{}$$

input

```
Integrate[((d + e*x)^4*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5
/2),x]
```

output

```
(-(Sqrt[c]*(d + e*x)^3*(-(b*e) + c*(d - e*x))*(15*b^2*e^2*g - 2*b*c*e*(3*e
*f + 17*d*g - 10*e*g*x) + c^2*(19*d^2*g + d*e*(4*f - 26*g*x) + e^2*x*(-8*f
 + 3*g*x)))) - 3*(2*c*e*f + 8*c*d*g - 5*b*e*g)*(d + e*x)^(5/2)*(-(b*e) + c
*(d - e*x))^(5/2)*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])
/(3*c^(7/2)*e^2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {1218, 1124, 25, 27, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{2(d+ex)^4(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-5beg+8cdg+2cef)\int \frac{(d+ex)^3}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3ce(2cd-be)}$$

$$\Big\downarrow \text{1124}$$

$$\frac{2(d+ex)^4(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-5beg+8cdg+2cef)\left(\frac{\int -\frac{e^2(3cd-be+cex)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{c^2e^2}+\frac{2(d+ex)(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3ce(2cd-be)}$$

$$\Big\downarrow \; 25$$

$$\frac{2(d+ex)^4(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-5beg+8cdg+2cef)\left(\frac{2(d+ex)(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\int \frac{e^2(3cd-be+cex)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{c^2e^2}\right)}{3ce(2cd-be)}$$

$$\Big\downarrow \; 27$$

$$\frac{2(d+ex)^4(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-5beg+8cdg+2cef)\left(\frac{2(d+ex)(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\int \frac{3cd-be+cex}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{c^2}\right)}{3ce(2cd-be)}$$

$$\Big\downarrow \; 1160$$

$$\frac{2(d+ex)^4(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-5beg+8cdg+2cef)\left(\frac{2(d+ex)(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{3}{2}(2cd-be)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}}{c^2}\right)}{3ce(2cd-be)}$$

$$\Big\downarrow \; 1092$$

$$\frac{2(d+ex)^4(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-5beg+8cdg+2cef)\left(\frac{2(d+ex)(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{3(2cd-be)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)-}{c^2}\right.}{3ce(2cd-be)}$$

$$\Big\downarrow \; 217$$

$$\frac{2(d+ex)^4(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{\left(\frac{2(d+ex)(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\frac{3(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2\sqrt{c}e}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e}}{c^2}\right)(-5beg+8cdg+2cef)}{3ce(2cd-be)}$$

input
```
Int[((d + e*x)^4*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^4)/(3*c*e^2*(2*c*d - b*e)*(d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2)^(3/2)) - ((2*c*e*f + 8*c*d*g - 5*b*e*g)*((2*(2*
c*d - b*e)*(d + e*x))/(c^2*e*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) -
(-(Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/e) + (3*(2*c*d - b*e)*ArcTan[
(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(2
*Sqrt[c]*e))/c^2))/(3*c*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4115 vs. $2(216) = 432$.

Time = 4.58 (sec) , antiderivative size = 4116, normalized size of antiderivative = 17.90

method	result	size
default	Expression too large to display	4116

input
```
int((e*x+d)^4*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
f*d^4*(2/3*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^
2-b*e^2*x-b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)^
2*(-2*c*e^2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))+e^3*(4*d*g+e*
f)*(1/3*x^3/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-1/2*b/c*(x^2/c/e^
2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)+1/2*b/c*(1/2*x/c/e^2/(-c*e^2*x^2-
b*e^2*x-b*d*e+c*d^2)^(3/2)-1/4*b/c*(1/3/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*
d^2)^(3/2)-1/2*b/c*(2/3*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^
4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-4*c*e^2*(-b*d*e+c*d
^2)-b^2*e^4)^2*(-2*c*e^2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))
-1/2*(-b*d*e+c*d^2)/c/e^2*(2/3*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)
-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-4*c*e^2*(-b*
d*e+c*d^2)-b^2*e^4)^2*(-2*c*e^2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^
(1/2)))-2*(-b*d*e+c*d^2)/c/e^2*(1/3/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)
^(3/2)-1/2*b/c*(2/3*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(
-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-4*c*e^2*(-b*d*e+c*d^2)-
b^2*e^4)^2*(-2*c*e^2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))-1/
c/e^2*(x/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c*(1/c/e^2/(-c
*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-b/c*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d
*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))-1/c/e^2/(c*e^2)
^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(216) = 432$.

Time = 1.95 (sec) , antiderivative size = 881, normalized size of antiderivative = 3.83

$$\int \frac{(d+ex)^4(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="fricas")
```

output
```
[1/12*(3*((2*c^3*e^3*f + (8*c^3*d*e^2 - 5*b*c^2*e^3)*g)*x^2 + 2*(c^3*d^2*e
 - 2*b*c^2*d*e^2 + b^2*c*e^3)*f + (8*c^3*d^3 - 21*b*c^2*d^2*e + 18*b^2*c*d
*e^2 - 5*b^3*e^3)*g - 2*(2*(c^3*d*e^2 - b*c^2*e^3)*f + (8*c^3*d^2*e - 13*b
*c^2*d*e^2 + 5*b^2*c*e^3)*g)*x)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x -
 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b
*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*(3*c^3*e^2*g*x^2 + 2*(2*c^3*d*e - 3*b*
c^2*e^2)*f + (19*c^3*d^2 - 34*b*c^2*d*e + 15*b^2*c*e^2)*g - 2*(4*c^3*e^2*f
 + (13*c^3*d*e - 10*b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b
*d*e))/(c^6*e^4*x^2 + c^6*d^2*e^2 - 2*b*c^5*d*e^3 + b^2*c^4*e^4 - 2*(c^6*d
*e^3 - b*c^5*e^4)*x), -1/6*(3*((2*c^3*e^3*f + (8*c^3*d*e^2 - 5*b*c^2*e^3)*
g)*x^2 + 2*(c^3*d^2*e - 2*b*c^2*d*e^2 + b^2*c*e^3)*f + (8*c^3*d^3 - 21*b*c
^2*d^2*e + 18*b^2*c*d*e^2 - 5*b^3*e^3)*g - 2*(2*(c^3*d*e^2 - b*c^2*e^3)*f
+ (8*c^3*d^2*e - 13*b*c^2*d*e^2 + 5*b^2*c*e^3)*g)*x)*sqrt(c)*arctan(1/2*sq
rt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*
x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) + 2*(3*c^3*e^2*g*x^2 + 2*(2*c^3*d*e
- 3*b*c^2*e^2)*f + (19*c^3*d^2 - 34*b*c^2*d*e + 15*b^2*c*e^2)*g - 2*(4*c^3
*e^2*f + (13*c^3*d*e - 10*b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d
^2 - b*d*e))/(c^6*e^4*x^2 + c^6*d^2*e^2 - 2*b*c^5*d*e^3 + b^2*c^4*e^4 - 2*
(c^6*d*e^3 - b*c^5*e^4)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^4(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(d+ex)^4\,(f+gx)}{(-(d+ex)\,(be-cd+cex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**4*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x
)
```

output
```
Integral((d + e*x)**4*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(5/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^4(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^4*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(216) = 432.

Time = 2.02 (sec) , antiderivative size = 997, normalized size of antiderivative = 4.33

$$\int \frac{(d+ex)^4(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="giac")
```

output
```
-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*g/(c^3*e^2) - 1/10*(2*c*e*f +
8*c*d*g - 5*b*e*g)*log(abs(b*sqrt(-c)*c^4*d^4*e^2 - 4*b^2*sqrt(-c)*c^3*d^3
*e^3 + 6*b^3*sqrt(-c)*c^2*d^2*e^4 - 4*b^4*sqrt(-c)*c*d*e^5 + b^5*sqrt(-c)*
e^6 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*c^5*
d^4*abs(e) - 12*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*
e))*b*c^4*d^3*e*abs(e) + 24*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x +
c*d^2 - b*d*e))*b^2*c^3*d^2*e^2*abs(e) - 20*(sqrt(-c*e^2)*x - sqrt(-c*e^2*
x^2 - b*e^2*x + c*d^2 - b*d*e))*b^3*c^2*d*e^3*abs(e) + 6*(sqrt(-c*e^2)*x -
 sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b^4*c*e^4*abs(e) + 8*(sqrt(-c
*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*sqrt(-c)*c^4*d^3*e
 - 30*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*b*sq
rt(-c)*c^3*d^2*e^2 + 36*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^
2 - b*d*e))^2*b^2*sqrt(-c)*c^2*d*e^3 - 14*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^
2 - b*e^2*x + c*d^2 - b*d*e))^2*b^3*sqrt(-c)*c*e^4 - 12*(sqrt(-c*e^2)*x -
sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*c^4*d^2*abs(e) + 28*(sqrt(-c
*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*b*c^3*d*e*abs(e) -
 16*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*b^2*c^
2*e^2*abs(e) - 8*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d
*e))^4*sqrt(-c)*c^3*d*e + 9*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x +
c*d^2 - b*d*e))^4*b*sqrt(-c)*c^2*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(f+g\,x)\,(d+e\,x)^4}{(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)^{5/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^4)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^4)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1231, normalized size of antiderivative = 5.35

$$\int \frac{(d+ex)^4(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^4*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(i*( - 30*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*
e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*e**3*g + 138*sqrt(c)*sqrt( - b*e + c*d
- c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c
*d*e**2*g + 12*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d
 - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*e**3*f - 30*sqrt(c)*sqrt( - b*e
+ c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b**2*c*e**3*g*x - 204*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*
e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e*g - 36*sqrt(c)*sqr
t( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
 2*c*d))*b*c**2*d*e**2*f + 108*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((s
qrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**2*g*x + 12*
sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sq
rt( - b*e + 2*c*d))*b*c**2*e**3*f*x + 96*sqrt(c)*sqrt( - b*e + c*d - c*e*x
)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**3*g +
 24*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i
)/sqrt( - b*e + 2*c*d))*c**3*d**2*e*f - 96*sqrt(c)*sqrt( - b*e + c*d - c*e
*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**2*e
*g*x - 24*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*
e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d*e**2*f*x - 5*sqrt(c)*sqrt( - b*e + c*
d - c*e*x)*b**3*e**3*g + 25*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c*d...
```

3.189 $\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 173

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\frac{2(cef+cdg-beg)(d+ex)^3}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{2g(d+ex)}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}+\frac{2g\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{c^{5/2}e^2}$$

output
```
2/3*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^3/c/e^2/(-b*e+2*c*d)/(d*(-b*e+c*d)-b*e^2*
x-c*e^2*x^2)^(3/2)-2*g*(e*x+d)/c^2/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1
/2)+2*g*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(
5/2)/e^2
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\frac{2\left(-\frac{\sqrt{c}(d+ex)^3(-be+c(d-ex))\left(3b^2e^2g+4bceg(-2d+ex)+c^2\left(5d^2g-e^2fx-de(f+7gx)\right)\right)}{2cd-be}\right.}{3c^{5/2}e^2((d+ex)(-be+}$$

input
```
Integrate[((d + e*x)^3*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5
/2),x]
```

output
```
(2*(-((Sqrt[c]*(d + e*x)^3*(-(b*e) + c*(d - e*x))*(3*b^2*e^2*g + 4*b*c*e*g
*(-2*d + e*x) + c^2*(5*d^2*g - e^2*f*x - d*e*(f + 7*g*x))))/(2*c*d - b*e))
 - 3*g*(d + e*x)^(5/2)*(-(b*e) + c*(d - e*x))^(5/2)*ArcTan[Sqrt[c*d - b*e
- c*e*x]/(Sqrt[c]*Sqrt[d + e*x])]))/(3*c^(5/2)*e^2*((d + e*x)*(-(b*e) + c*
(d - e*x)))^(5/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1218, 1124, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1218}$$

$$\frac{2(d+ex)^3(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{g\int \frac{(d+ex)^2}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{ce}$$

$$\Big\downarrow \text{ 1124}$$

$$\frac{2(d+ex)^3(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{g\left(\frac{2(d+ex)}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{c}\right)}{ce}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{2(d+ex)^3(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{g\left(\frac{2(d+ex)}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2\int\frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{c}\right)}{ce}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{2(d+ex)^3(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{g\left(\frac{2(d+ex)}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{c^{3/2}e}\right)}{ce}$$

input
```
Int[((d + e*x)^3*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^3)/(3*c*e^2*(2*c*d - b*e)*(d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2)^(3/2)) - (g*((2*(d + e*x))/(c*e*Sqrt[d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2]) - ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*
d - b*e) - b*e^2*x - c*e^2*x^2])]/(c^(3/2)*e)))/(c*e)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2416 vs. $2(161) = 322$.

Time = 3.49 (sec) , antiderivative size = 2417, normalized size of antiderivative = 13.97

method	result	size
default	Expression too large to display	2417

input
```
int((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RET
URNVERBOSE)
```

output

```
d^3*f*(2/3*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^
2-b*e^2*x-b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)^
2*(-2*c*e^2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))+e^2*(3*d*g+e*
f)*(x^2/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)+1/2*b/c*(1/2*x/c/e^2/
(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-1/4*b/c*(1/3/c/e^2/(-c*e^2*x^2-b*e^
2*x-b*d*e+c*d^2)^(3/2)-1/2*b/c*(2/3*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c
*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-4*c*e^2
*(-b*d*e+c*d^2)-b^2*e^4)^2*(-2*c*e^2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*
d^2)^(1/2)))-1/2*(-b*d*e+c*d^2)/c/e^2*(2/3*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-
b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-
4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)^2*(-2*c*e^2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b
*d*e+c*d^2)^(1/2)))-2*(-b*d*e+c*d^2)/c/e^2*(1/3/c/e^2/(-c*e^2*x^2-b*e^2*x-
b*d*e+c*d^2)^(3/2)-1/2*b/c*(2/3*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2
)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-4*c*e^2*(-b
*d*e+c*d^2)-b^2*e^4)^2*(-2*c*e^2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)
^(1/2))))+3*d*e*(d*g+e*f)*(1/2*x/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3
/2)-1/4*b/c*(1/3/c/e^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)-1/2*b/c*(2/3
*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-b*e^2*x-
b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)^2*(-2*c*e^
2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))-1/2*(-b*d*e+c*d^2)/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(161) = 322.

Time = 1.22 (sec) , antiderivative size = 785, normalized size of antiderivative = 4.54

$$\int \frac{(d+ex)^3(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="fricas")
```

output

```
[-1/6*(3*((2*c^3*d*e^2 - b*c^2*e^3)*g*x^2 - 2*(2*c^3*d^2*e - 3*b*c^2*d*e^2
 + b^2*c*e^3)*g*x + (2*c^3*d^3 - 5*b*c^2*d^2*e + 4*b^2*c*d*e^2 - b^3*e^3)*
g)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*
e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c
)) - 4*(c^3*d*e*f - (5*c^3*d^2 - 8*b*c^2*d*e + 3*b^2*c*e^2)*g + (c^3*e^2*f
 + (7*c^3*d*e - 4*b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d
*e))/(2*c^6*d^3*e^2 - 5*b*c^5*d^2*e^3 + 4*b^2*c^4*d*e^4 - b^3*c^3*e^5 + (2
*c^6*d*e^4 - b*c^5*e^5)*x^2 - 2*(2*c^6*d^2*e^3 - 3*b*c^5*d*e^4 + b^2*c^4*e
^5)*x), -1/3*(3*((2*c^3*d*e^2 - b*c^2*e^3)*g*x^2 - 2*(2*c^3*d^2*e - 3*b*c^
2*d*e^2 + b^2*c*e^3)*g*x + (2*c^3*d^3 - 5*b*c^2*d^2*e + 4*b^2*c*d*e^2 - b^
3*e^3)*g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2
*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) - 2*(
c^3*d*e*f - (5*c^3*d^2 - 8*b*c^2*d*e + 3*b^2*c*e^2)*g + (c^3*e^2*f + (7*c^
3*d*e - 4*b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(2*
c^6*d^3*e^2 - 5*b*c^5*d^2*e^3 + 4*b^2*c^4*d*e^4 - b^3*c^3*e^5 + (2*c^6*d*e
^4 - b*c^5*e^5)*x^2 - 2*(2*c^6*d^2*e^3 - 3*b*c^5*d*e^4 + b^2*c^4*e^5)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(d+ex)^3\,(f+gx)}{(-\,(d+ex)\,(be-cd+cex))^{\frac{5}{2}}}\,dx$$

input

```
integrate((e*x+d)**3*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x
)
```

output

```
Integral((d + e*x)**3*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(5/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 943 vs. 2(161) = 322.

Time = 1.52 (sec) , antiderivative size = 943, normalized size of antiderivative = 5.45

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="giac")
```

output

```
-1/5*g*log(abs(b*sqrt(-c)*c^4*d^4*e^2 - 4*b^2*sqrt(-c)*c^3*d^3*e^3 + 6*b^3
*sqrt(-c)*c^2*d^2*e^4 - 4*b^4*sqrt(-c)*c*d*e^5 + b^5*sqrt(-c)*e^6 + 2*(sqr
t(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*c^5*d^4*abs(e) -
 12*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*b*c^4*d^
3*e*abs(e) + 24*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*
e))*b^2*c^3*d^2*e^2*abs(e) - 20*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*
x + c*d^2 - b*d*e))*b^3*c^2*d*e^3*abs(e) + 6*(sqrt(-c*e^2)*x - sqrt(-c*e^2
*x^2 - b*e^2*x + c*d^2 - b*d*e))*b^4*c*e^4*abs(e) + 8*(sqrt(-c*e^2)*x - sq
rt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*sqrt(-c)*c^4*d^3*e - 30*(sqrt(
-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^2*b*sqrt(-c)*c^3*d
^2*e^2 + 36*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^
2*b^2*sqrt(-c)*c^2*d*e^3 - 14*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x
+ c*d^2 - b*d*e))^2*b^3*sqrt(-c)*c*e^4 - 12*(sqrt(-c*e^2)*x - sqrt(-c*e^2*
x^2 - b*e^2*x + c*d^2 - b*d*e))^3*c^4*d^2*abs(e) + 28*(sqrt(-c*e^2)*x - sq
rt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*b*c^3*d*e*abs(e) - 16*(sqrt(-c
*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^3*b^2*c^2*e^2*abs(e)
 - 8*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))^4*sqrt(
-c)*c^3*d*e + 9*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*
e))^4*b*sqrt(-c)*c^2*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x +
 c*d^2 - b*d*e))^5*c^3*abs(e)))/(sqrt(-c)*c^2*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(f+g\,x)\,{(d+e\,x)}^3}{{(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)}^{5/2}} \,d x$$

input

```
int(((f + g*x)*(d + e*x)^3)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),x)
```

output

```
int(((f + g*x)*(d + e*x)^3)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 987, normalized size of antiderivative = 5.71

$$\int \frac{(d+ex)^3(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(2*i*(3*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*b**3*e**3*g - 15*sqrt(c)*sqrt( - b*e + c*d - c
*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d*
e**2*g + 3*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*e**3*g*x + 24*sqrt(c)*sqrt( - b*e +
c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*
c**2*d**2*e*g - 12*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e +
 c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d*e**2*g*x - 12*sqrt(c)*sqrt
( - b*e + c*d - c*e*x)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
2*c*d))*c**3*d**3*g + 12*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*asinh((sqrt( -
 b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**2*e*g*x - sqrt(c)*sqr
t( - b*e + c*d - c*e*x)*b**2*c*d*e**2*g + sqrt(c)*sqrt( - b*e + c*d - c*e*
x)*b**2*c*e**3*f + 3*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**2*e*g -
3*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d*e**2*f - sqrt(c)*sqrt( - b*e
 + c*d - c*e*x)*b*c**2*d*e**2*g*x + sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c
**2*e**3*f*x - 2*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d**3*g + 2*sqrt(c
)*sqrt( - b*e + c*d - c*e*x)*c**3*d**2*e*f + 2*sqrt(c)*sqrt( - b*e + c*d -
 c*e*x)*c**3*d**2*e*g*x - 2*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d*e**2
*f*x - 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b**2*c*e**2*
g + 8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*b*c**2*d*e*g...
```

3.190 $\int \frac{(d+ex)^2(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 134

$$\int \frac{(d+ex)^2(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2(ef-dg)(d+ex)^2}{e^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} - \frac{2(2cef-4cdg+beg)(d+ex)^3}{3e^2(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

output
```
2*(-d*g+e*f)*(e*x+d)^2/e^2/(-b*e+2*c*d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
3/2)-2/3*(b*e*g-4*c*d*g+2*c*e*f)*(e*x+d)^3/e^2/(-b*e+2*c*d)^2/(d*(-b*e+c*d
)-b*e^2*x-c*e^2*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^2(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2(d+ex)\left(be(3ef-2dg+egx)+2c\left(d^2g+e^2fx-2de(f+gx)\right)\right)}{3e^2(-2cd+be)^2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[((d + e*x)^2*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5
/2),x]
```

output
```
(2*(d + e*x)*(b*e*(3*e*f - 2*d*g + e*g*x) + 2*c*(d^2*g + e^2*f*x - 2*d*e*(
f + g*x))))/(3*e^2*(-2*c*d + b*e)^2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*
(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1218, 1124, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(beg-4cdg+2cef)\int \frac{d+ex}{(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{3ce(2cd-be)}+\frac{2(d+ex)^2(-beg+cdg+cef)}{3ce^2(2cd-be)\,(d(cd-be)-be^2x-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{1124}$$

$$\frac{(beg-4cdg+2cef)\left(e^2\int 0dx+\frac{2(d+ex)}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3ce(2cd-be)}+\frac{2(d+ex)^2(-beg+cdg+cef)}{3ce^2(2cd-be)\,(d(cd-be)-be^2x-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{24}$$

$$\frac{2(d+ex)^2(-beg+cdg+cef)}{3ce^2(2cd-be)\,(d(cd-be)-be^2x-ce^2x^2)^{3/2}}+\frac{2(d+ex)(beg-4cdg+2cef)}{3ce^2(2cd-be)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input
```
Int[((d + e*x)^2*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output

```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^2)/(3*c*e^2*(2*c*d - b*e)*(d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2)^(3/2)) + (2*(2*c*e*f - 4*c*d*g + b*e*g)*(d + e*
x))/(3*c*e^2*(2*c*d - b*e)^2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 1124

```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1218

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

method	result	size
trager	$\frac{2\left(-b\,e^{2} g x +4 c d e g x -2 c\,e^{2} f x +2 b d e g -3 b\,e^{2} f -2 c\,d^{2} g +4 c d e f \right) \sqrt{-x^{2} c\,e^{2}-x b\,e^{2}-b d e +c\,d^{2}}}{3 \left(b^{2} e^{2}-4 b c d e +4 c^{2} d^{2}\right) \left(c e x +b e -c d \right)^{2} e^{2}}$	123
gosper	$-\frac{2 \left(e x +d \right)^{3} \left(c e x +b e -c d \right) \left(-b\,e^{2} g x +4 c d e g x -2 c\,e^{2} f x +2 b d e g -3 b\,e^{2} f -2 c\,d^{2} g +4 c d e f \right)}{3 e^{2} \left(b^{2} e^{2}-4 b c d e +4 c^{2} d^{2}\right) \left(-x^{2} c\,e^{2}-x b\,e^{2}-b d e +c\,d^{2}\right)^{\frac{5}{2}}}$	128
orering	$-\frac{2 \left(e x +d \right)^{3} \left(c e x +b e -c d \right) \left(-b\,e^{2} g x +4 c d e g x -2 c\,e^{2} f x +2 b d e g -3 b\,e^{2} f -2 c\,d^{2} g +4 c d e f \right)}{3 e^{2} \left(b^{2} e^{2}-4 b c d e +4 c^{2} d^{2}\right) \left(-x^{2} c\,e^{2}-x b\,e^{2}-b d e +c\,d^{2}\right)^{\frac{5}{2}}}$	128
default	Expression too large to display	1458

input
```
int((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*(-b*e^2*g*x+4*c*d*e*g*x-2*c*e^2*f*x+2*b*d*e*g-3*b*e^2*f-2*c*d^2*g+4*c*
d*e*f)/(b^2*e^2-4*b*c*d*e+4*c^2*d^2)/(c*e*x+b*e-c*d)^2/e^2*(-c*e^2*x^2-b*e
^2*x-b*d*e+c*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.69

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}((4\,cde-3\,be^2)}{3\,(4\,c^4d^4e^2-12\,bc^3d^3e^3+13\,b^2c^2d^2e^4-6\,b^3cde^5+b^4e^6+(4\,c^4d^2e^4}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="fricas")
```

output
```
2/3*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((4*c*d*e - 3*b*e^2)*f - 2*
(c*d^2 - b*d*e)*g - (2*c*e^2*f - (4*c*d*e - b*e^2)*g)*x)/(4*c^4*d^4*e^2 -
12*b*c^3*d^3*e^3 + 13*b^2*c^2*d^2*e^4 - 6*b^3*c*d*e^5 + b^4*e^6 + (4*c^4*d
^2*e^4 - 4*b*c^3*d*e^5 + b^2*c^2*e^6)*x^2 - 2*(4*c^4*d^3*e^3 - 8*b*c^3*d^2
*e^4 + 5*b^2*c^2*d*e^5 - b^3*c*e^6)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^2\,(f+gx)}{(-(d+ex)\,(be-cd+cex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x
)
```

output
```
Integral((d + e*x)**2*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(5/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[6,0,10,2]%%%}+%%%{-10,[5,1,9,3]%%%}+%%%{41,[4,2,8,4
]%%%}+%%%
```

Mupad [B] (verification not implemented)

Time = 11.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx =$$
$$-\frac{2\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,(3\,b\,e^2\,f+2\,c\,d^2\,g+b\,e^2\,g\,x+2\,c\,e^2\,f\,x-2\,b\,d\,e\,g-4\,c\,d\,e\,f-4\,c\,d\,e\,g\,x}{3\,e^2\,(b\,e-2\,c\,d)^2\,(b\,e-c\,d+c\,e\,x)^2}$$

input
```
int(((f + g*x)*(d + e*x)^2)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),x)
```

output
```
-(2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*(3*b*e^2*f + 2*c*d^2*g + b
*e^2*g*x + 2*c*e^2*f*x - 2*b*d*e*g - 4*c*d*e*f - 4*c*d*e*g*x))/(3*e^2*(b*e
 - 2*c*d)^2*(b*e - c*d + c*e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 628, normalized size of antiderivative = 4.69

$$\int \frac{(d+ex)^2(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{2i(\sqrt{c}\sqrt{-cex-be+cd}\,b^3e^3g-3\sqrt{c}\sqrt{-cex-be+cd}\,b^2cd\,e^2g-2}{}$$

input
```
int((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(2*i*(sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**3*e**3*g - 3*sqrt(c)*sqrt( - b
*e + c*d - c*e*x)*b**2*c*d*e**2*g - 2*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b
**2*c*e**3*f + sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**3*g*x + 2*sqrt
(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**2*e*g + 6*sqrt(c)*sqrt( - b*e + c
*d - c*e*x)*b*c**2*d*e**2*f - 2*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*
d*e**2*g*x - 2*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*e**3*f*x - 4*sqrt
(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d**2*e*f + 4*sqrt(c)*sqrt( - b*e + c*d
 - c*e*x)*c**3*d*e**2*f*x - 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e
+ 2*c*d)*b*c**2*d*e*g + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*
c*d)*b*c**2*e**2*f + sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*
b*c**2*e**2*g*x + 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*c
**3*d**2*g - 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*c**3*d
*e*f - 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*c**3*d*e*g*x
 + 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*c**3*e**2*f*x))/
(3*sqrt( - b*e + c*d - c*e*x)*c**2*e**2*(b**4*e**4 - 7*b**3*c*d*e**3 + b**
3*c*e**4*x + 18*b**2*c**2*d**2*e**2 - 6*b**2*c**2*d*e**3*x - 20*b*c**3*d**
3*e + 12*b*c**3*d**2*e**2*x + 8*c**4*d**4 - 8*c**4*d**3*e*x))
```

3.191 $\int \frac{(d+ex)(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 146

$$\int \frac{(d+ex)(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\frac{2(cef+cdg-beg)(d+ex)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}+\frac{2(4cef-2cdg-beg)(b+2cx)}{3ce(2cd-be)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

output
```
2/3*(-b*e*g+c*d*g+c*e*f)*(e*x+d)/c/e^2/(-b*e+2*c*d)/(d*(-b*e+c*d)-b*e^2*x-
c*e^2*x^2)^(3/2)+2/3*(-b*e*g-2*c*d*g+4*c*e*f)*(2*c*x+b)/c/e/(-b*e+2*c*d)^3
/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\frac{6b^2e^2(-ef+2dg+egx)-4bce(5d^2g-2degx+e^2x(6f-gx))+8}{3e^2(-2cd+be)^3(-cd+be+cex)\sqrt{(d+}}$$

input
```
Integrate[((d + e*x)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2
),x]
```

output

```
(6*b^2*e^2*(-(e*f) + 2*d*g + e*g*x) - 4*b*c*e*(5*d^2*g - 2*d*e*g*x + e^2*x
*(6*f - g*x)) + 8*c^2*(d^3*g - 2*e^3*f*x^2 + d^2*e*(f - g*x) + d*e^2*x*(2*
f + g*x)))/(3*e^2*(-2*c*d + b*e)^3*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(
-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1218, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \; 1218$$

$$\frac{(-beg-2cdg+4cef)\int \frac{1}{(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{3ce(2cd-be)}+\frac{2(d+ex)(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 1088$$

$$\frac{2(b+2cx)(-beg-2cdg+4cef)}{3ce(2cd-be)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}+\frac{2(d+ex)(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input

```
Int[((d + e*x)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output

```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x))/(3*c*e^2*(2*c*d - b*e)*(d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2)^(3/2)) + (2*(4*c*e*f - 2*c*d*g - b*e*g)*(b + 2*c*
x))/(3*c*e*(2*c*d - b*e)^3*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.54

method	result
trager	$-\frac{2(2bce^3gx^2+4c^2de^2gx^2-8fc^2e^3x^2+3b^2e^3gx+4bcde^2gx-12bce^3fx-4c^2d^2egx+8c^2de^2fx+6b^2de^2g-3b^2e^3f-10bcd^2eg+4c}{3(ex+d)(b^2e^2-4bcde+4c^2d^2)(be-2cd)(cex+be-cd)^2e^2}$
gosper	$\frac{2(ex+d)^2(cex+be-cd)(2bce^3gx^2+4c^2de^2gx^2-8fc^2e^3x^2+3b^2e^3gx+4bcde^2gx-12bce^3fx-4c^2d^2egx+8c^2de^2fx+6b^2de^2g-3b^2}{3(b^3e^3-6de^2b^2c+12d^2ebc^2-8d^3c^3)e^2(-x^2ce^2-xbe^2-bde+cd^2)^{\frac{5}{2}}}$
orering	$\frac{2(ex+d)^2(cex+be-cd)(2bce^3gx^2+4c^2de^2gx^2-8fc^2e^3x^2+3b^2e^3gx+4bcde^2gx-12bce^3fx-4c^2d^2egx+8c^2de^2fx+6b^2de^2g-3b^2}{3(b^3e^3-6de^2b^2c+12d^2ebc^2-8d^3c^3)e^2(-x^2ce^2-xbe^2-bde+cd^2)^{\frac{5}{2}}}$
default	$df\left(\frac{-\frac{4}{3}ce^2x-\frac{2}{3}be^2}{(-4ce^2(-bde+cd^2)-b^2e^4)(-x^2ce^2-xbe^2-bde+cd^2)^{\frac{3}{2}}}-\frac{16ce^2(-2ce^2x-be^2)}{3(-4ce^2(-bde+cd^2)-b^2e^4)^2\sqrt{-x^2ce^2-xbe^2-bde+cd^2}}\right)+$

input
```
int((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
-2/3*(2*b*c*e^3*g*x^2+4*c^2*d*e^2*g*x^2-8*c^2*e^3*f*x^2+3*b^2*e^3*g*x+4*b*
c*d*e^2*g*x-12*b*c*e^3*f*x-4*c^2*d^2*e*g*x+8*c^2*d*e^2*f*x+6*b^2*d*e^2*g-3
*b^2*e^3*f-10*b*c*d^2*e*g+4*c^2*d^3*g+4*c^2*d^2*e*f)/(e*x+d)/(b^2*e^2-4*b*
c*d*e+4*c^2*d^2)/(b*e-2*c*d)/(c*e*x+b*e-c*d)^2/e^2*(-c*e^2*x^2-b*e^2*x-b*d
*e+c*d^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(138) = 276$.

Time = 5.19 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.95

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx =$$
$$-\frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,(4\,c^2e^3f-(2\,c^2de^2+bce^3)g)x^2}{3\,(8\,c^5d^6e^2-28\,bc^4d^5e^3+38\,b^2c^3d^4e^4-25\,b^3c^2d^3e^5+8\,b^4cd^2e^6-b^5de^7+(8\,c^5d^3e^5-12\,bc^4d^2e^6+6\,b^2c^3}$$

input
```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algori
thm="fricas")
```

output
```
-2/3*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*c^2*e^3*f - (2*c^2*d
*e^2 + b*c*e^3)*g)*x^2 - (4*c^2*d^2*e - 3*b^2*e^3)*f - 2*(2*c^2*d^3 - 5*b*
c*d^2*e + 3*b^2*d*e^2)*g - (4*(2*c^2*d*e^2 - 3*b*c*e^3)*f - (4*c^2*d^2*e -
 4*b*c*d*e^2 - 3*b^2*e^3)*g)*x)/(8*c^5*d^6*e^2 - 28*b*c^4*d^5*e^3 + 38*b^2
*c^3*d^4*e^4 - 25*b^3*c^2*d^3*e^5 + 8*b^4*c*d^2*e^6 - b^5*d*e^7 + (8*c^5*d
^3*e^5 - 12*b*c^4*d^2*e^6 + 6*b^2*c^3*d*e^7 - b^3*c^2*e^8)*x^3 - (8*c^5*d^
4*e^4 - 28*b*c^4*d^3*e^5 + 30*b^2*c^3*d^2*e^6 - 13*b^3*c^2*d*e^7 + 2*b^4*c
*e^8)*x^2 - (8*c^5*d^5*e^3 - 12*b*c^4*d^4*e^4 - 2*b^2*c^3*d^3*e^5 + 11*b^3
*c^2*d^2*e^6 - 6*b^4*c*d*e^7 + b^5*e^8)*x)
```

Sympy [F]

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(d+ex)\,(f+gx)}{(-\,(d+ex)\,(be-cd+cex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x)
```

output
```
Integral((d + e*x)*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [F]

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(ex+d)(gx+f)}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algori
thm="giac")
```

output
```
integrate((e*x + d)*(g*x + f)/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)
, x)
```

Mupad [B] (verification not implemented)

Time = 11.88 (sec) , antiderivative size = 795, normalized size of antiderivative = 5.45

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx =$$

$$-\frac{8\,c^2\,d^3\,g\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}-6\,b^2\,e^3\,f\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}-16\,c^2\,e^3\,f\,x^2\,\sqrt{c\,d^2-b}}{}$$

input
```
int(((f + g*x)*(d + e*x))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),x)
```

output
```
-(8*c^2*d^3*g*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) - 6*b^2*e^3*f*(c
*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) - 16*c^2*e^3*f*x^2*(c*d^2 - c*e^
2*x^2 - b*d*e - b*e^2*x)^(1/2) + 12*b^2*d*e^2*g*(c*d^2 - c*e^2*x^2 - b*d*e
 - b*e^2*x)^(1/2) + 8*c^2*d^2*e*f*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1
/2) + 6*b^2*e^3*g*x*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) + 4*b*c*e^
3*g*x^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) + 16*c^2*d*e^2*f*x*(c*
d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) - 8*c^2*d^2*e*g*x*(c*d^2 - c*e^2*
x^2 - b*d*e - b*e^2*x)^(1/2) + 8*c^2*d*e^2*g*x^2*(c*d^2 - c*e^2*x^2 - b*d*
e - b*e^2*x)^(1/2) - 20*b*c*d^2*e*g*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^
(1/2) - 24*b*c*e^3*f*x*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) + 8*b*c
*d*e^2*g*x*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(3*b^5*d*e^7 + 3*b
^5*e^8*x - 24*c^5*d^6*e^2 + 84*b*c^4*d^5*e^3 - 24*b^4*c*d^2*e^6 + 6*b^4*c*
e^8*x^2 + 24*c^5*d^5*e^3*x - 114*b^2*c^3*d^4*e^4 + 75*b^3*c^2*d^3*e^5 + 3*
b^3*c^2*e^8*x^3 + 24*c^5*d^4*e^4*x^2 - 24*c^5*d^3*e^5*x^3 - 18*b^4*c*d*e^7
*x + 90*b^2*c^3*d^2*e^6*x^2 - 36*b*c^4*d^4*e^4*x - 6*b^2*c^3*d^3*e^5*x + 3
3*b^3*c^2*d^2*e^6*x - 84*b*c^4*d^3*e^5*x^2 - 39*b^3*c^2*d*e^7*x^2 + 36*b*c
^4*d^2*e^6*x^3 - 18*b^2*c^3*d*e^7*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1117, normalized size of antiderivative = 7.65

$$\int \frac{(d+ex)(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(2*i*( - 2*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**3*d*e**3*g - 2*sqrt(c)*sq
rt( - b*e + c*d - c*e*x)*b**3*e**4*g*x + 2*sqrt(c)*sqrt( - b*e + c*d - c*e
*x)*b**2*c*d**2*e**2*g + 8*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**
3*f + 8*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**4*f*x - 2*sqrt(c)*sqr
t( - b*e + c*d - c*e*x)*b**2*c*e**4*g*x**2 + 8*sqrt(c)*sqrt( - b*e + c*d -
 c*e*x)*b*c**2*d**3*e*g - 24*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**
2*e**2*f + 8*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**2*e**2*g*x - 16*
sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d*e**3*f*x + 8*sqrt(c)*sqrt( - b
*e + c*d - c*e*x)*b*c**2*e**4*f*x**2 - 8*sqrt(c)*sqrt( - b*e + c*d - c*e*x
)*c**3*d**4*g + 16*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d**3*e*f + 8*sq
rt(c)*sqrt( - b*e + c*d - c*e*x)*c**3*d**2*e**2*g*x**2 - 16*sqrt(c)*sqrt(
- b*e + c*d - c*e*x)*c**3*d*e**3*f*x**2 + 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d
)*sqrt( - b*e + 2*c*d)*b**2*c*d*e**2*g - 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)
*sqrt( - b*e + 2*c*d)*b**2*c*e**3*f + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*b**2*c*e**3*g*x - 10*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*b*c**2*d**2*e*g + 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqr
t( - b*e + 2*c*d)*b*c**2*d*e**2*g*x - 12*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*s
qrt( - b*e + 2*c*d)*b*c**2*e**3*f*x + 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*b*c**2*e**3*g*x**2 + 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*
sqrt( - b*e + 2*c*d)*c**3*d**3*g + 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq...
```

3.192 $\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 208

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2(8cef+2cdg-5beg)(b+2cx)}{15e(2cd-be)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} - \frac{2(ef-dg)}{5e^2(2cd-be)(d+ex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} + \frac{16c(8cef+2cdg-5beg)(b+2cx)}{15e(2cd-be)^5\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

output
```
2/15*(-5*b*e*g+2*c*d*g+8*c*e*f)*(2*c*x+b)/e/(-b*e+2*c*d)^3/(d*(-b*e+c*d)-b
*e^2*x-c*e^2*x^2)^(3/2)-2/5*(-d*g+e*f)/e^2/(-b*e+2*c*d)/(e*x+d)/(d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(3/2)+16/15*c*(-5*b*e*g+2*c*d*g+8*c*e*f)*(2*c*x+b)/
e/(-b*e+2*c*d)^5/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.66

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{2(b^4e^4(3ef+2dg+5egx)+12b^2c^2e^2(d^3g+de^2x(12f-19gx)+2e^3x^2(2f-5gx)+2d^2e(7f-gx))-1}{}$$

input
```
Integrate[(f + g*x)/((d + e*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2)
),x]
```

output
```
(-2*(b^4*e^4*(3*e*f + 2*d*g + 5*e*g*x) + 12*b^2*c^2*e^2*(d^3*g + d*e^2*x*(
12*f - 19*g*x) + 2*e^3*x^2*(2*f - 5*g*x) + 2*d^2*e*(7*f - g*x)) - 16*c^4*(
3*d^5*g - 8*e^5*f*x^4 - 3*d^4*e*(f - g*x) - 2*d^2*e^3*x^2*(-6*f + g*x) + 3
*d^3*e^2*x*(4*f + g*x) - 2*d*e^4*x^3*(4*f + g*x)) + 8*b*c^3*e*(9*d^4*g + 2
*e^4*x^3*(12*f - 5*g*x) - 6*d^3*e*(4*f - 3*g*x) - 4*d*e^3*x^2*(-12*f + g*x
) + 3*d^2*e^2*x*(4*f + 9*g*x)) - 2*b^3*c*e^3*(19*d^2*g + e^2*x*(4*f + 15*g
*x) + 2*d*e*(8*f + 23*g*x))))/(15*e^2*(-2*c*d + b*e)^5*(d + e*x)^2*(-(c*d)
 + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {1220, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}\,dx$$
$$\Big\downarrow \text{1220}$$

$$\frac{(-5beg+2cdg+8cef)\int \frac{1}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{5e(2cd-be)}-\frac{2(ef-dg)}{5e^2(d+ex)(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1089$$

$$\frac{(-5beg+2cdg+8cef)\left(\frac{8c\int \frac{1}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3(2cd-be)^2}+\frac{2(b+2cx)}{3(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)}{5e(2cd-be)}-\frac{2(ef-dg)}{5e^2(d+ex)(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1088$$

$$\frac{\left(\frac{16c(b+2cx)}{3(2cd-be)^4\sqrt{d(cd-be)-be^2x-ce^2x^2}}+\frac{2(b+2cx)}{3(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)(-5beg+2cdg+8cef)}{5e(2cd-be)}-\frac{2(ef-dg)}{5e^2(d+ex)(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input

```
Int[(f + g*x)/((d + e*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2)),x]
```

output

```
(-2*(e*f - d*g))/(5*e^2*(2*c*d - b*e)*(d + e*x)*(d*(c*d - b*e) - b*e^2*x -
 c*e^2*x^2)^(3/2)) + ((8*c*e*f + 2*c*d*g - 5*b*e*g)*((2*(b + 2*c*x))/(3*(2
*c*d - b*e)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) + (16*c*(b + 2*
c*x))/(3*(2*c*d - b*e)^4*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])))/(5*e
*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(196) = 392$.

Time = 2.40 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.01

method	result
default	$\frac{g\left(\frac{-\frac{4}{3}c\,e^2x-\frac{2}{3}b\,e^2}{\left(-4c\,e^2\left(-bde+c\,d^2\right)-b^2e^4\right)\left(-x^2c\,e^2-xb\,e^2-bde+c\,d^2\right)^{\frac{3}{2}}}-\frac{16c\,e^2\left(-2c\,e^2x-b\,e^2\right)}{3\left(-4c\,e^2\left(-bde+c\,d^2\right)-b^2e^4\right)^2\sqrt{-x^2c\,e^2-xb\,e^2-bde+c\,d^2}}\right)}{e}-\frac{(dg-ef)}{}$
gosper	$-\frac{2(cex+be-cd)\left(-80b\,c^3e^5g\,x^4+32c^4d\,e^4g\,x^4+128c^4e^5f\,x^4-120b^2c^2e^5g\,x^3-32b\,c^3d\,e^4g\,x^3+192b\,c^3e^5f\,x^3+32c^4d^2e^3g\,x^3+128c^4\right.}{}$
orering	$-\frac{2(cex+be-cd)\left(-80b\,c^3e^5g\,x^4+32c^4d\,e^4g\,x^4+128c^4e^5f\,x^4-120b^2c^2e^5g\,x^3-32b\,c^3d\,e^4g\,x^3+192b\,c^3e^5f\,x^3+32c^4d^2e^3g\,x^3+128c^4\right.}{}$
trager	$\frac{2\left(-80b\,c^3e^5g\,x^4+32c^4d\,e^4g\,x^4+128c^4e^5f\,x^4-120b^2c^2e^5g\,x^3-32b\,c^3d\,e^4g\,x^3+192b\,c^3e^5f\,x^3+32c^4d^2e^3g\,x^3+128c^4d\,e^4f\,x^3-30b^3c\right.}{}$

input

```
int((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RETUR
NVERBOSE)
```

output

```
g/e*(2/3*(-2*c*e^2*x-b*e^2)/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)/(-c*e^2*x^2-
b*e^2*x-b*d*e+c*d^2)^(3/2)-16/3*c*e^2/(-4*c*e^2*(-b*d*e+c*d^2)-b^2*e^4)^2*
(-2*c*e^2*x-b*e^2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))-(d*g-e*f)/e^2*(
-2/5/(-b*e^2+2*c*d*e)/(x+d/e)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^
(3/2)+8/5*c*e^2/(-b*e^2+2*c*d*e)*(-2/3*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/(-
b*e^2+2*c*d*e)^2/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)-16/3*c*
e^2/(-b*e^2+2*c*d*e)^4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/(-c*e^2*(x+d/e)^2+
(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(196) = 392$.

Time = 52.10 (sec) , antiderivative size = 1028, normalized size of antiderivative = 4.94

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algori
thm="fricas")
```

output
```
-2/15*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(16*(8*c^4*e^5*f + (2*c^4
*d*e^4 - 5*b*c^3*e^5)*g)*x^4 + 8*(8*(2*c^4*d*e^4 + 3*b*c^3*e^5)*f + (4*c^4
*d^2*e^3 - 4*b*c^3*d*e^4 - 15*b^2*c^2*e^5)*g)*x^3 - 6*(8*(4*c^4*d^2*e^3 -
8*b*c^3*d*e^4 - b^2*c^2*e^5)*f + (8*c^4*d^3*e^2 - 36*b*c^3*d^2*e^3 + 38*b^
2*c^2*d*e^4 + 5*b^3*c*e^5)*g)*x^2 + (48*c^4*d^4*e - 192*b*c^3*d^3*e^2 + 16
8*b^2*c^2*d^2*e^3 - 32*b^3*c*d*e^4 + 3*b^4*e^5)*f - 2*(24*c^4*d^5 - 36*b*c
^3*d^4*e - 6*b^2*c^2*d^3*e^2 + 19*b^3*c*d^2*e^3 - b^4*d*e^4)*g - (8*(24*c^
4*d^3*e^2 - 12*b*c^3*d^2*e^3 - 18*b^2*c^2*d*e^4 + b^3*c*e^5)*f + (48*c^4*d
^4*e - 144*b*c^3*d^3*e^2 + 24*b^2*c^2*d^2*e^3 + 92*b^3*c*d*e^4 - 5*b^4*e^5
)*g)*x)/(32*c^7*d^10*e^2 - 144*b*c^6*d^9*e^3 + 272*b^2*c^5*d^8*e^4 - 280*b
^3*c^4*d^7*e^5 + 170*b^4*c^3*d^6*e^6 - 61*b^5*c^2*d^5*e^7 + 12*b^6*c*d^4*e
^8 - b^7*d^3*e^9 + (32*c^7*d^5*e^7 - 80*b*c^6*d^4*e^8 + 80*b^2*c^5*d^3*e^9
 - 40*b^3*c^4*d^2*e^10 + 10*b^4*c^3*d*e^11 - b^5*c^2*e^12)*x^5 + (32*c^7*d
^6*e^6 - 16*b*c^6*d^5*e^7 - 80*b^2*c^5*d^4*e^8 + 120*b^3*c^4*d^3*e^9 - 70*
b^4*c^3*d^2*e^10 + 19*b^5*c^2*d*e^11 - 2*b^6*c*e^12)*x^4 - (64*c^7*d^7*e^5
 - 288*b*c^6*d^6*e^6 + 448*b^2*c^5*d^5*e^7 - 320*b^3*c^4*d^4*e^8 + 100*b^4
*c^3*d^3*e^9 - 2*b^5*c^2*d^2*e^10 - 6*b^6*c*d*e^11 + b^7*e^12)*x^3 - (64*c
^7*d^8*e^4 - 160*b*c^6*d^7*e^5 + 64*b^2*c^5*d^6*e^6 + 160*b^3*c^4*d^5*e^7
- 220*b^4*c^3*d^4*e^8 + 118*b^5*c^2*d^3*e^9 - 30*b^6*c*d^2*e^10 + 3*b^7*d*
e^11)*x^2 + (32*c^7*d^9*e^3 - 208*b*c^6*d^8*e^4 + 496*b^2*c^5*d^7*e^5 -...
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\int \frac{f+gx}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x)
```

output
```
Integral((f + g*x)/((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \int \frac{gx+f}{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algori
thm="giac")
```

output
```
integrate((g*x + f)/((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*(e*x + d
)), x)
```

Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 3326, normalized size of antiderivative = 15.99

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((f + g*x)/((d + e*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2)),x)
```

output
```
(x*((16*c^2*(b*g - c*f))/(15*(b*e - 2*c*d)^5) - (8*b*c^2*g)/(15*(b*e - 2*c
*d)^5)) + (72*c^3*d*e*f - 56*c^3*d^2*g - 44*b*c^2*e^2*f + 10*b^2*c*e^2*g +
 20*b*c^2*d*e*g)/(15*e^2*(b*e - 2*c*d)^5) + (8*c^2*g*(c*d^2 - b*d*e))/(15*
e^2*(b*e - 2*c*d)^5))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) + (((4*b
*c*g)/(15*e*(b*e - 2*c*d)^5) - (8*c^2*d*g)/(15*e^2*(b*e - 2*c*d)^5))*(c*d^
2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + (((2*e^2*f)/(5*b^3*e^6
 - 40*c^3*d^3*e^3 + 60*b*c^2*d^2*e^4 - 30*b^2*c*d*e^5) - (2*d*e*g)/(5*b^3*
e^6 - 40*c^3*d^3*e^3 + 60*b*c^2*d^2*e^4 - 30*b^2*c*d*e^5))*(c*d^2 - c*e^2*
x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^3 - (((2*b*g)/(5*(3*b*e^2 - 6*c*d*
e)*(b*e - 2*c*d)^3) - (4*c*d*g)/(5*e*(3*b*e^2 - 6*c*d*e)*(b*e - 2*c*d)^3))
*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^2 - (((4*c*g*(3*b*
e - 4*c*d))/(15*e^2*(b*e - 2*c*d)^5) - (8*c^2*d*g)/(15*e^2*(b*e - 2*c*d)^5
))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) + ((x*(((e*(b*e
- c*d) + c*d*e)*(((e*(b*e - c*d) + c*d*e)*((4*c^4*e^2*(5*b*g - 4*c*f))/(15
*(b*e - 2*c*d)^3*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e)) - (8*c^4*g*(e*(b*e
 - c*d) + c*d*e))/(15*(b*e - 2*c*d)^3*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*d*e
)) + (4*b*c^4*e^2*g)/(15*(b*e - 2*c*d)^3*(4*c^3*d^2 + b^2*c*e^2 - 4*b*c^2*
d*e))))/(c*e^2) - (2*c^2*(8*b^2*c*e^3*g - 26*b*c^2*e^3*f + 36*c^3*d*e^2*f
- 32*c^3*d^2*e*g + 14*b*c^2*d*e^2*g))/(15*e*(b*e - 2*c*d)^3*(4*c^3*d^2 + b
^2*c*e^2 - 4*b*c^2*d*e)) - (2*b*c^3*e^2*(5*b*g - 4*c*f))/(15*(b*e - 2*c...
```

Reduce [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 2763, normalized size of antiderivative = 13.28

$$\int \frac{f+gx}{(d+ex)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(2*i*( - 80*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**3*c*d**3*e**3*g - 240*sq
rt(c)*sqrt( - b*e + c*d - c*e*x)*b**3*c*d**2*e**4*g*x - 240*sqrt(c)*sqrt(
- b*e + c*d - c*e*x)*b**3*c*d*e**5*g*x**2 - 80*sqrt(c)*sqrt( - b*e + c*d -
 c*e*x)*b**3*c*e**6*g*x**3 + 272*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c
**2*d**4*e**2*g + 128*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**3*e*
*3*f + 736*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**3*e**3*g*x + 38
4*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**2*e**4*f*x + 576*sqrt(c)
*sqrt( - b*e + c*d - c*e*x)*b**2*c**2*d**2*e**4*g*x**2 + 384*sqrt(c)*sqrt(
 - b*e + c*d - c*e*x)*b**2*c**2*d*e**5*f*x**2 + 32*sqrt(c)*sqrt( - b*e + c
*d - c*e*x)*b**2*c**2*d*e**5*g*x**3 + 128*sqrt(c)*sqrt( - b*e + c*d - c*e*
x)*b**2*c**2*e**6*f*x**3 - 80*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**2
*e**6*g*x**4 - 256*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*d**5*e*g - 38
4*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*d**4*e**2*f - 576*sqrt(c)*sqrt
( - b*e + c*d - c*e*x)*b*c**3*d**4*e**2*g*x - 1024*sqrt(c)*sqrt( - b*e + c
*d - c*e*x)*b*c**3*d**3*e**3*f*x - 192*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*
b*c**3*d**3*e**3*g*x**2 - 768*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*d*
*2*e**4*f*x**2 + 320*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*d**2*e**4*g
*x**3 + 192*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*d*e**5*g*x**4 + 128*
sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**3*e**6*f*x**4 + 64*sqrt(c)*sqrt( -
 b*e + c*d - c*e*x)*c**4*d**6*g + 256*sqrt(c)*sqrt( - b*e + c*d - c*e*x...
```

3.193 $\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 283

$$\begin{aligned}
\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx &= \frac{16c(10cef+4cdg-7beg)(b+2cx)}{105e(2cd-be)^4\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} \\
&- \frac{2(ef-dg)}{7e^2(2cd-be)(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} \\
&- \frac{2(10cef+4cdg-7beg)}{35e^2(2cd-be)^2(d+ex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} \\
&+ \frac{128c^2(10cef+4cdg-7beg)(b+2cx)}{105e(2cd-be)^6\sqrt{d(cd-be)-be^2x-ce^2x^2}}
\end{aligned}$$

output
```
16/105*c*(-7*b*e*g+4*c*d*g+10*c*e*f)*(2*c*x+b)/e/(-b*e+2*c*d)^4/(d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(3/2)-2/7*(-d*g+e*f)/e^2/(-b*e+2*c*d)/(e*x+d)^2/(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)-2/35*(-7*b*e*g+4*c*d*g+10*c*e*f)/e^2/(
-b*e+2*c*d)^2/(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)+128/105*c^2*(
-7*b*e*g+4*c*d*g+10*c*e*f)*(2*c*x+b)/e/(-b*e+2*c*d)^6/(d*(-b*e+c*d)-b*e^2*
x-c*e^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.65

$$\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{-6b^5e^5(5ef+2dg+7egx)+96b^2c^3e^2(17d^4g+d^2e^2x(65f}{}$$

input
```
Integrate[(f + g*x)/((d + e*x)^2*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/
2)),x]
```

output
```
(-6*b^5*e^5*(5*e*f + 2*d*g + 7*e*g*x) + 96*b^2*c^3*e^2*(17*d^4*g + d^2*e^2
*x*(65*f - 54*g*x) + 2*e^4*x^3*(5*f - 14*g*x) + 40*d*e^3*x^2*(f - 2*g*x) +
 20*d^3*e*(3*f + 2*g*x)) - 64*c^5*(9*d^6*g - 40*e^6*f*x^5 + 4*d^2*e^4*x^3*
(5*f - 8*g*x) - 6*d^5*e*(5*f - 3*g*x) - 16*d*e^5*x^4*(5*f + g*x) + 8*d^3*e
^3*x^2*(15*f + g*x) + 3*d^4*e^2*x*(15*f + 16*g*x)) + 32*b*c^4*e*(6*d^5*g +
 8*d*e^4*x^3*(45*f - 8*g*x) + 8*e^5*x^4*(15*f - 7*g*x) - 39*d^4*e*(5*f - g
*x) + 12*d^3*e^2*x*(-5*f + 24*g*x) + 4*d^2*e^3*x^2*(75*f + 43*g*x)) + 4*b^
4*c*e^4*(43*d^2*g + e^2*x*(15*f + 28*g*x) + 2*d*e*(45*f + 73*g*x)) - 16*b^
3*c^2*e^3*(88*d^3*g + 2*e^3*x^2*(5*f + 21*g*x) + 2*d*e^2*x*(25*f + 86*g*x)
 + d^2*e*(115*f + 293*g*x)))/(105*e^2*(-2*c*d + b*e)^6*(d + e*x)^3*(-(c*d)
 + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1220, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^2\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(-7beg+4cdg+10cef)\int \frac{1}{(d+ex)\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{7e(2cd-be)}-$$
$$\frac{2(ef-dg)}{7e^2(d+ex)^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(-7beg+4cdg+10cef)\left(\frac{8c\int \frac{1}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{5(2cd-be)}-\frac{2}{5e(d+ex)(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)}{7e(2cd-be)}-$$
$$\frac{2(ef-dg)}{7e^2(d+ex)^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{(-7beg+4cdg+10cef)\left(\frac{8c\left(\frac{8c\int \frac{1}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3(2cd-be)^2}+\frac{2(b+2cx)}{3(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)}{5(2cd-be)}-\frac{2}{5e(d+ex)(2cd-be)(d(cd}\right.}{7e(2cd-be)}$$
$$\frac{2(ef-dg)}{7e^2(d+ex)^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{\left(\frac{8c\left(\frac{16c(b+2cx)}{3(2cd-be)^4\sqrt{d(cd-be)-be^2x-ce^2x^2}}+\frac{2(b+2cx)}{3(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)}{5(2cd-be)}-\frac{2}{5e(d+ex)(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)(-7be}{7e(2cd-be)}$$
$$\frac{2(ef-dg)}{7e^2(d+ex)^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input

```
Int[(f + g*x)/((d + e*x)^2*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2)),x]
```

output
```
(-2*(e*f - d*g))/(7*e^2*(2*c*d - b*e)*(d + e*x)^2*(d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2)^(3/2)) + ((10*c*e*f + 4*c*d*g - 7*b*e*g)*(-2/(5*e*(2*c*d - b
*e)*(d + e*x)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) + (8*c*((2*(b +
 2*c*x))/(3*(2*c*d - b*e)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) +
 (16*c*(b + 2*c*x))/(3*(2*c*d - b*e)^4*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^
2*x^2])))/(5*(2*c*d - b*e))))/(7*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(267) = 534$.

Time = 2.50 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.11

method	result
default	$\frac{g\left(-\frac{2}{5\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{8c\,e^{2}\left(-\frac{2\left(-2c\,e^{2}\left(x+\frac{d}{e}\right)-b\,e^{2}+2dec\right)}{3\left(-b\,e^{2}+2dec\right)^{2}\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}\right.}{5\left(-b\,e^{2}+2d\right.}\right.}{e^{2}}$
trager	$2(896b\,c^{4}e^{6}g\,x^{5}-512c^{5}d\,e^{5}g\,x^{5}-1280c^{5}e^{6}f\,x^{5}+1344b^{2}c^{3}e^{6}g\,x^{4}+1024b\,c^{4}d\,e^{5}g\,x^{4}-1920b\,c^{4}e^{6}f\,x^{4}-1024c^{5}d^{2}e^{4}g\,x^{4}-2560c^{5}d\,e^{5}f$
gosper	$-\frac{2(cex+be-cd)(896b\,c^{4}e^{6}g\,x^{5}-512c^{5}d\,e^{5}g\,x^{5}-1280c^{5}e^{6}f\,x^{5}+1344b^{2}c^{3}e^{6}g\,x^{4}+1024b\,c^{4}d\,e^{5}g\,x^{4}-1920b\,c^{4}e^{6}f\,x^{4}-1024c^{5}d^{2}e^{4}g\,x}{}$
orering	$-\frac{2(cex+be-cd)(896b\,c^{4}e^{6}g\,x^{5}-512c^{5}d\,e^{5}g\,x^{5}-1280c^{5}e^{6}f\,x^{5}+1344b^{2}c^{3}e^{6}g\,x^{4}+1024b\,c^{4}d\,e^{5}g\,x^{4}-1920b\,c^{4}e^{6}f\,x^{4}-1024c^{5}d^{2}e^{4}g\,x}{}$

input
```
int((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
g/e^2*(-2/5/(-b*e^2+2*c*d*e)/(x+d/e)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x
+d/e))^(3/2)+8/5*c*e^2/(-b*e^2+2*c*d*e)*(-2/3*(-2*c*e^2*(x+d/e)-b*e^2+2*d*
e*c)/(-b*e^2+2*c*d*e)^2/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)-
16/3*c*e^2/(-b*e^2+2*c*d*e)^4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/(-c*e^2*(x+
d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)))-(d*g-e*f)/e^3*(-2/7/(-b*e^2+2*c*d
*e)/(x+d/e)^2/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+10/7*c*e^2
/(-b*e^2+2*c*d*e)*(-2/5/(-b*e^2+2*c*d*e)/(x+d/e)/(-c*e^2*(x+d/e)^2+(-b*e^2
+2*c*d*e)*(x+d/e))^(3/2)+8/5*c*e^2/(-b*e^2+2*c*d*e)*(-2/3*(-2*c*e^2*(x+d/e
)-b*e^2+2*d*e*c)/(-b*e^2+2*c*d*e)^2/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+
d/e))^(3/2)-16/3*c*e^2/(-b*e^2+2*c*d*e)^4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)
/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)^2\,(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)^2\,(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)/(e*x+d)**2/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x
)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^2\,(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22347 vs. 2(267) = 534.

Time = 0.79 (sec) , antiderivative size = 22347, normalized size of antiderivative = 78.96

$$\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="giac")
```

output
```
-2/105*(128*(10*c^4*e*f + 4*c^4*d*g - 7*b*c^3*e*g)*sgn(1/(e*x + d))*sgn(e)
/(64*sqrt(-c)*c^6*d^6*e - 192*b*sqrt(-c)*c^5*d^5*e^2 + 240*b^2*sqrt(-c)*c^
4*d^4*e^3 - 160*b^3*sqrt(-c)*c^3*d^3*e^4 + 60*b^4*sqrt(-c)*c^2*d^2*e^5 - 1
2*b^5*sqrt(-c)*c*d*e^6 + b^6*sqrt(-c)*e^7) - (1030792151040*(c - 2*c*d/(e*
x + d) + b*e/(e*x + d))^3*c^36*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*
d^36*e^7*f*sgn(1/(e*x + d))^6*sgn(e)^6 - 7215545057280*(c - 2*c*d/(e*x + d
) + b*e/(e*x + d))^2*c^37*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^36*
e^7*f*sgn(1/(e*x + d))^6*sgn(e)^6 - 72155450572800*c^39*sqrt(-c + 2*c*d/(e
*x + d) - b*e/(e*x + d))*d^36*e^7*f*sgn(1/(e*x + d))^6*sgn(e)^6 - 24051816
857600*c^38*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*d^36*e^7*f*sgn(1/
(e*x + d))^6*sgn(e)^6 - 18554258718720*b*(c - 2*c*d/(e*x + d) + b*e/(e*x +
 d))^3*c^35*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^35*e^8*f*sgn(1/(e
*x + d))^6*sgn(e)^6 + 129879811031040*b*(c - 2*c*d/(e*x + d) + b*e/(e*x +
d))^2*c^36*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^35*e^8*f*sgn(1/(e*
x + d))^6*sgn(e)^6 + 1298798110310400*b*c^38*sqrt(-c + 2*c*d/(e*x + d) - b
*e/(e*x + d))*d^35*e^8*f*sgn(1/(e*x + d))^6*sgn(e)^6 + 432932703436800*b*c
^37*(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))^(3/2)*d^35*e^8*f*sgn(1/(e*x + d
))^6*sgn(e)^6 + 162349763788800*b^2*(c - 2*c*d/(e*x + d) + b*e/(e*x + d))^
3*c^34*sqrt(-c + 2*c*d/(e*x + d) - b*e/(e*x + d))*d^34*e^9*f*sgn(1/(e*x +
d))^6*sgn(e)^6 - 1136448346521600*b^2*(c - 2*c*d/(e*x + d) + b*e/(e*x +...
```

Mupad [B] (verification not implemented)

Time = 18.94 (sec) , antiderivative size = 11539, normalized size of antiderivative = 40.77

$$\int \frac{f+gx}{(d+ex)^2\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input

```
int((f + g*x)/((d + e*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2)),x)
```

output

```
((800*c^6*d^4*g + 558*b^3*c^3*e^4*f - 222*b^4*c^2*e^4*g - 4192*c^6*d^3*e*f
 + 384*b*c^5*d^3*e*g + 6624*b*c^5*d^2*e^2*f - 3392*b^2*c^4*d*e^3*f + 1248*
b^3*c^3*d*e^3*g - 1984*b^2*c^4*d^2*e^2*g)/(105*e^2*(b*e - 2*c*d)^8) - x*((
b*((b*((8*c^5*e*(4*c*d*g - 9*b*e*g + 6*c*e*f))/(105*(b*e - 2*c*d)^8) + (16
*b*c^5*e^2*g)/(105*(b*e - 2*c*d)^8)))/c - (4*c^4*(40*c^2*d^2*g - 33*b^2*e^
2*g + 62*b*c*e^2*f - 88*c^2*d*e*f + 16*b*c*d*e*g))/(105*(b*e - 2*c*d)^8) +
 (16*c^5*g*(c*d^2 - b*d*e))/(105*(b*e - 2*c*d)^8)))/c + (44*b^2*c^4*e^4*f
- 30*b^3*c^3*e^4*g - 672*c^6*d^2*e^2*f + 224*c^6*d^3*e*g + 320*b*c^5*d*e^3
*f + 160*b*c^5*d^2*e^2*g - 128*b^2*c^4*d*e^3*g)/(105*e^2*(b*e - 2*c*d)^8)
+ ((c*d^2 - b*d*e)*((8*c^5*e*(4*c*d*g - 9*b*e*g + 6*c*e*f))/(105*(b*e - 2*
c*d)^8) + (16*b*c^5*e^2*g)/(105*(b*e - 2*c*d)^8)))/(c*e^2)) + ((c*d^2 - b*
d*e)*((b*((8*c^5*e*(4*c*d*g - 9*b*e*g + 6*c*e*f))/(105*(b*e - 2*c*d)^8) +
(16*b*c^5*e^2*g)/(105*(b*e - 2*c*d)^8)))/c - (4*c^4*(40*c^2*d^2*g - 33*b^2
*e^2*g + 62*b*c*e^2*f - 88*c^2*d*e*f + 16*b*c*d*e*g))/(105*(b*e - 2*c*d)^8
) + (16*c^5*g*(c*d^2 - b*d*e))/(105*(b*e - 2*c*d)^8)))/(c*e^2))/(c*d^2 - c
*e^2*x^2 - b*d*e - b*e^2*x)^(1/2) + (((8*c^2*g*(3*b*e - 4*c*d))/(105*e^2*(
b*e - 2*c*d)^6) - (16*c^3*d*g)/(105*e^2*(b*e - 2*c*d)^6))*(c*d^2 - c*e^2*x
^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x) - (((d*((d*((8*c^4*(2*c*d*g - 7*b*e
*g + 6*c*e*f))/(105*(b*e - 2*c*d)^8) + (16*c^5*d*g)/(105*(b*e - 2*c*d)^8))
)/e + (76*b^2*c^3*e^2*g - 176*c^5*d^2*g + 304*c^5*d*e*f - 200*b*c^4*e^2...
```

Reduce [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 3857, normalized size of antiderivative = 13.63

$$\int \frac{f+gx}{(d+ex)^2 (cd^2-bde-be^2x-ce^2x^2)^{5/2}}\, dx = \text{Too large to display}$$

input

```
int((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(2*i*(896*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d**4*e**3*g + 3584*
sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d**3*e**4*g*x + 5376*sqrt(c)*
sqrt( - b*e + c*d - c*e*x)*b**3*c**2*d**2*e**5*g*x**2 + 3584*sqrt(c)*sqrt(
 - b*e + c*d - c*e*x)*b**3*c**2*d*e**6*g*x**3 + 896*sqrt(c)*sqrt( - b*e +
c*d - c*e*x)*b**3*c**2*e**7*g*x**4 - 3200*sqrt(c)*sqrt( - b*e + c*d - c*e*
x)*b**2*c**3*d**5*e**2*g - 1280*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c*
*3*d**4*e**3*f - 11904*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**3*d**4*e
**3*g*x - 5120*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**3*d**3*e**4*f*x
- 15616*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**3*d**3*e**4*g*x**2 - 76
80*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**3*d**2*e**5*f*x**2 - 7424*sq
rt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**3*d**2*e**5*g*x**3 - 5120*sqrt(c)
*sqrt( - b*e + c*d - c*e*x)*b**2*c**3*d*e**6*f*x**3 + 384*sqrt(c)*sqrt( -
b*e + c*d - c*e*x)*b**2*c**3*d*e**6*g*x**4 - 1280*sqrt(c)*sqrt( - b*e + c*
d - c*e*x)*b**2*c**3*e**7*f*x**4 + 896*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*
b**2*c**3*e**7*g*x**5 + 3328*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**4*d**
6*e*g + 3840*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**4*d**5*e**2*f + 11008
*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**4*d**5*e**2*g*x + 14080*sqrt(c)*s
qrt( - b*e + c*d - c*e*x)*b*c**4*d**4*e**3*f*x + 10752*sqrt(c)*sqrt( - b*e
 + c*d - c*e*x)*b*c**4*d**4*e**3*g*x**2 + 17920*sqrt(c)*sqrt( - b*e + c*d
- c*e*x)*b*c**4*d**3*e**4*f*x**2 - 512*sqrt(c)*sqrt( - b*e + c*d - c*e*...
```

3.194 $\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 358

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\frac{32c^2(4cef+2cdg-3beg)(b+2cx)}{63e(2cd-be)^5\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$
$$-\frac{2(ef-dg)}{9e^2(2cd-be)(d+ex)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$
$$-\frac{2(4cef+2cdg-3beg)}{21e^2(2cd-be)^2(d+ex)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$
$$-\frac{4c(4cef+2cdg-3beg)}{21e^2(2cd-be)^3(d+ex)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$
$$+\frac{256c^3(4cef+2cdg-3beg)(b+2cx)}{63e(2cd-be)^7\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

output

```
32/63*c^2*(-3*b*e*g+2*c*d*g+4*c*e*f)*(2*c*x+b)/e/(-b*e+2*c*d)^5/(d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(3/2)-2/9*(-d*g+e*f)/e^2/(-b*e+2*c*d)/(e*x+d)^3/(d*
(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)-2/21*(-3*b*e*g+2*c*d*g+4*c*e*f)/e^2/(-
b*e+2*c*d)^2/(e*x+d)^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)-4/21*c*(-3*b
*e*g+2*c*d*g+4*c*e*f)/e^2/(-b*e+2*c*d)^3/(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e
^2*x^2)^(3/2)+256/63*c^3*(-3*b*e*g+2*c*d*g+4*c*e*f)*(2*c*x+b)/e/(-b*e+2*c*
d)^7/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.41

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{2(21c^6ef(d+ex)^6+21c^6dg(d+ex)^6-21bc^5eg(d+ex)^6-378c^5ef(d+ex)^5(-cd+be+cex)+315bc^4}{}$$

input

```
Integrate[(f + g*x)/((d + e*x)^3*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/
2)),x]
```

output

```
(-2*(21*c^6*e*f*(d + e*x)^6 + 21*c^6*d*g*(d + e*x)^6 - 21*b*c^5*e*g*(d + e
*x)^6 - 378*c^5*e*f*(d + e*x)^5*(-(c*d) + b*e + c*e*x) + 315*b*c^4*e*g*(d
+ e*x)^5*(-(c*d) + b*e + c*e*x) - 945*c^4*e*f*(d + e*x)^4*(-(c*d) + b*e +
c*e*x)^2 - 315*c^4*d*g*(d + e*x)^4*(-(c*d) + b*e + c*e*x)^2 + 630*b*c^3*e*
g*(d + e*x)^4*(-(c*d) + b*e + c*e*x)^2 + 420*c^3*e*f*(d + e*x)^3*(-(c*d) +
 b*e + c*e*x)^3 - 210*b*c^2*e*g*(d + e*x)^3*(-(c*d) + b*e + c*e*x)^3 - 189
*c^2*e*f*(d + e*x)^2*(-(c*d) + b*e + c*e*x)^4 + 63*c^2*d*g*(d + e*x)^2*(-(
c*d) + b*e + c*e*x)^4 + 63*b*c*e*g*(d + e*x)^2*(-(c*d) + b*e + c*e*x)^4 +
54*c*e*f*(d + e*x)*(-(c*d) + b*e + c*e*x)^5 - 9*b*e*g*(d + e*x)*(-(c*d) +
b*e + c*e*x)^5 - 7*e*f*(-(c*d) + b*e + c*e*x)^6 + 7*d*g*(-(c*d) + b*e + c*
e*x)^6 + 252*c^5*d*g*(d + e*x)^5*(-(b*e) + c*(d - e*x)) + 36*c*d*g*(d + e*
x)*(-(b*e) + c*(d - e*x))^5))/(63*e^2*(-2*c*d + b*e)^7*(d + e*x)^3*((d + e
*x)*(-(b*e) + c*(d - e*x)))^(3/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1220, 1129, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^3\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}\,dx$$

$$\Big\downarrow 1220$$

$$\frac{(-3beg+2cdg+4cef)\int \frac{1}{(d+ex)^2(-cx^2e^2-bxe^2+d(cd-be))^{5/2}}dx}{3e(2cd-be)}-\frac{2(ef-dg)}{9e^2(d+ex)^3(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1129$$

$$\frac{(-3beg+2cdg+4cef)\left(\frac{10c\int \frac{1}{(d+ex)\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{7(2cd-be)}-\frac{2}{7e(d+ex)^2(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{3/2}}\right)}{3e(2cd-be)}-\frac{2(ef-dg)}{9e^2(d+ex)^3(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1129$$

$$\frac{(-3beg+2cdg+4cef)\left(\frac{10c\left(\frac{8c\int \frac{1}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{5(2cd-be)}-\frac{2}{5e(d+ex)(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)}{7(2cd-be)}-\frac{}{7e(d+ex)^2(2cd-b}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)}{9e^2(d+ex)^3(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1089$$

$$\frac{(-3beg+2cdg+4cef)\left(\frac{10c\left(\frac{8c\left(\frac{8c\int \frac{1}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3(2cd-be)^2}+\frac{2(b+2cx)}{3(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)}{5(2cd-be)}-\frac{2}{5e(d+ex)(2cd-be)\left(d(cd-\right.}\right.}{7(2cd-be)}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)}{9e^2(d+ex)^3(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1088$$

$$\frac{\left(\frac{10c\left(\frac{8c\left(\frac{16c(b+2cx)}{3(2cd-be)^4\sqrt{d(cd-be)-be^2x-ce^2x^2}}+\frac{2(b+2cx)}{3(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)}{5(2cd-be)}-\frac{2}{5e(d+ex)(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)}{7(2cd-be)}-\frac{}{7e(d-}\right.}{3e(2cd-be)}$$

$$\frac{2(ef-dg)}{9e^2(d+ex)^3(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input
```
Int[(f + g*x)/((d + e*x)^3*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2)),x]
```

output
```
(-2*(e*f - d*g))/(9*e^2*(2*c*d - b*e)*(d + e*x)^3*(d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2)^(3/2)) + ((4*c*e*f + 2*c*d*g - 3*b*e*g)*(-2/(7*e*(2*c*d - b*
e)*(d + e*x)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) + (10*c*(-2/(5
*e*(2*c*d - b*e)*(d + e*x)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) +
(8*c*((2*(b + 2*c*x))/(3*(2*c*d - b*e)^2*(d*(c*d - b*e) - b*e^2*x - c*e^2*
x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*(2*c*d - b*e)^4*Sqrt[d*(c*d - b*e) - b
*e^2*x - c*e^2*x^2])))/(5*(2*c*d - b*e))))/(7*(2*c*d - b*e))))/(3*e*(2*c*d
 - b*e))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(338) = 676$.

Time = 2.86 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.14

method	result
default	$\frac{g\left(-\frac{2}{7\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)^{2}\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{10c\,e^{2}\left(-\frac{2}{5\left(-b\,e^{2}+2dec\right)\left(x+\frac{d}{e}\right)\left(-c\,e^{2}\left(x+\frac{d}{e}\right)^{2}+\left(-b\,e^{2}+2dec\right)\left(x+\right.\right.}\right.}{}\right.}{e}$
trager	Expression too large to display
gosper	Expression too large to display
orering	Expression too large to display

input
```
int((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
g/e^3*(-2/7/(-b*e^2+2*c*d*e)/(x+d/e)^2/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*
(x+d/e))^(3/2)+10/7*c*e^2/(-b*e^2+2*c*d*e)*(-2/5/(-b*e^2+2*c*d*e)/(x+d/e)/
(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+8/5*c*e^2/(-b*e^2+2*c*d*
e)*(-2/3*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)/(-b*e^2+2*c*d*e)^2/(-c*e^2*(x+d/
e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)-16/3*c*e^2/(-b*e^2+2*c*d*e)^4*(-2*c*e
^2*(x+d/e)-b*e^2+2*d*e*c)/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2
))))-(d*g-e*f)/e^4*(-2/9/(-b*e^2+2*c*d*e)/(x+d/e)^3/(-c*e^2*(x+d/e)^2+(-b*
e^2+2*c*d*e)*(x+d/e))^(3/2)+4/3*c*e^2/(-b*e^2+2*c*d*e)*(-2/7/(-b*e^2+2*c*d
*e)/(x+d/e)^2/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(3/2)+10/7*c*e^2
/(-b*e^2+2*c*d*e)*(-2/5/(-b*e^2+2*c*d*e)/(x+d/e)/(-c*e^2*(x+d/e)^2+(-b*e^2
+2*c*d*e)*(x+d/e))^(3/2)+8/5*c*e^2/(-b*e^2+2*c*d*e)*(-2/3*(-2*c*e^2*(x+d/e
)-b*e^2+2*d*e*c)/(-b*e^2+2*c*d*e)^2/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+
d/e))^(3/2)-16/3*c*e^2/(-b*e^2+2*c*d*e)^4*(-2*c*e^2*(x+d/e)-b*e^2+2*d*e*c)
/(-c*e^2*(x+d/e)^2+(-b*e^2+2*c*d*e)*(x+d/e))^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)/(e*x+d)**3/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x
)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-2*c*d>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \int \frac{gx+f}{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{5}{2}}(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((g*x + f)/((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*(e*x + d
)^3), x)
```

Mupad [B] (verification not implemented)

Time = 32.17 (sec) , antiderivative size = 33819, normalized size of antiderivative = 94.47

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int((f + g*x)/((d + e*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2)),x)
```

output

```
((35968*c^9*d^6*g - 10062*b^5*c^4*e^6*f + 5714*b^6*c^3*e^6*g + 279680*c^9*
d^5*e*f - 248960*b*c^8*d^5*e*g - 729600*b*c^8*d^4*e^2*f + 98950*b^4*c^5*d*
e^5*f - 57260*b^5*c^4*d*e^5*g + 755040*b^2*c^7*d^3*e^3*f - 387748*b^3*c^6*
d^2*e^4*f + 504032*b^2*c^7*d^4*e^2*g - 473132*b^3*c^6*d^3*e^3*g + 231104*b
^4*c^5*d^2*e^4*g)/(945*e^2*(b*e - 2*c*d)^11) - x*((b*((b*((b*((b*((32*c^8*
e^3*(4*c*d*g - 7*b*e*g + 4*c*e*f))/(945*(b*e - 2*c*d)^11) + (32*b*c^8*e^4*
g)/(945*(b*e - 2*c*d)^11)))/c - (16*c^7*e^2*(20*c^2*d^2*g - 43*b^2*e^2*g +
 61*b*c*e^2*f - 82*c^2*d*e*f + 41*b*c*d*e*g))/(945*(b*e - 2*c*d)^11) + (32
*c^8*e^2*g*(c*d^2 - b*d*e))/(945*(b*e - 2*c*d)^11)))/c + (1608*b^2*c^7*e^6
*f - 912*b^3*c^6*e^6*g - 96*c^9*d^2*e^4*f + 224*c^9*d^3*e^3*g - 2528*b*c^8
*d*e^5*f + 352*b*c^8*d^2*e^4*g + 1112*b^2*c^7*d*e^5*g)/(945*e^2*(b*e - 2*c
*d)^11) + (((32*c^8*e^3*(4*c*d*g - 7*b*e*g + 4*c*e*f))/(945*(b*e - 2*c*d)^
11) + (32*b*c^8*e^4*g)/(945*(b*e - 2*c*d)^11))*(c*d^2 - b*d*e))/(c*e^2)))/
c - (326*b^4*c^5*e^6*g - 1372*b^3*c^6*e^6*f + 19840*c^9*d^3*e^3*f - 5248*c
^9*d^4*e^2*g - 29904*b*c^8*d^2*e^4*f + 13056*b^2*c^7*d*e^5*f + 912*b*c^8*d
^3*e^3*g - 3972*b^3*c^6*d*e^5*g + 7056*b^2*c^7*d^2*e^4*g)/(945*e^2*(b*e -
2*c*d)^11) + ((c*d^2 - b*d*e)*((b*((32*c^8*e^3*(4*c*d*g - 7*b*e*g + 4*c*e*
f))/(945*(b*e - 2*c*d)^11) + (32*b*c^8*e^4*g)/(945*(b*e - 2*c*d)^11)))/c -
 (16*c^7*e^2*(20*c^2*d^2*g - 43*b^2*e^2*g + 61*b*c*e^2*f - 82*c^2*d*e*f +
41*b*c*d*e*g))/(945*(b*e - 2*c*d)^11) + (32*c^8*e^2*g*(c*d^2 - b*d*e))/...
```

Reduce [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 4847, normalized size of antiderivative = 13.54

$$\int \frac{f+gx}{(d+ex)^3\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output
```
(2*i*( - 768*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**3*c**3*d**5*e**3*g - 38
40*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**3*c**3*d**4*e**4*g*x - 7680*sqrt(
c)*sqrt( - b*e + c*d - c*e*x)*b**3*c**3*d**3*e**5*g*x**2 - 7680*sqrt(c)*sq
rt( - b*e + c*d - c*e*x)*b**3*c**3*d**2*e**6*g*x**3 - 3840*sqrt(c)*sqrt( -
 b*e + c*d - c*e*x)*b**3*c**3*d*e**7*g*x**4 - 768*sqrt(c)*sqrt( - b*e + c*
d - c*e*x)*b**3*c**3*e**8*g*x**5 + 2816*sqrt(c)*sqrt( - b*e + c*d - c*e*x)
*b**2*c**4*d**6*e**2*g + 1024*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**4
*d**5*e**3*f + 13312*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**4*d**5*e**
3*g*x + 5120*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**4*d**4*e**4*f*x +
24320*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**4*d**4*e**4*g*x**2 + 1024
0*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**4*d**3*e**5*f*x**2 + 20480*sq
rt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**4*d**3*e**5*g*x**3 + 10240*sqrt(c
)*sqrt( - b*e + c*d - c*e*x)*b**2*c**4*d**2*e**6*f*x**3 + 6400*sqrt(c)*sqr
t( - b*e + c*d - c*e*x)*b**2*c**4*d**2*e**6*g*x**4 + 5120*sqrt(c)*sqrt( -
b*e + c*d - c*e*x)*b**2*c**4*d*e**7*f*x**4 - 1024*sqrt(c)*sqrt( - b*e + c*
d - c*e*x)*b**2*c**4*d*e**7*g*x**5 + 1024*sqrt(c)*sqrt( - b*e + c*d - c*e*
x)*b**2*c**4*e**8*f*x**5 - 768*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b**2*c**
4*e**8*g*x**6 - 3072*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**5*d**7*e*g -
3072*sqrt(c)*sqrt( - b*e + c*d - c*e*x)*b*c**5*d**6*e**2*f - 13312*sqrt(c)
*sqrt( - b*e + c*d - c*e*x)*b*c**5*d**6*e**2*g*x - 14336*sqrt(c)*sqrt( ...
```

3.195 $\int (d+ex)^{5/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 343

$$\begin{aligned}\int (d+ex)^{5/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx= & \\ -\frac{2(2cd-be)^3(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^5e^2(d+ex)^{3/2}} & \\ +\frac{2(2cd-be)^2(3cef+5cdg-4beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^5e^2(d+ex)^{5/2}} & \\ -\frac{6(2cd-be)(cef+3cdg-2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^5e^2(d+ex)^{7/2}} & \\ +\frac{2(cef+7cdg-4beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^5e^2(d+ex)^{9/2}} & \\ -\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}{11c^5e^2(d+ex)^{11/2}} &\end{aligned}$$

output

```
-2/3*(-b*e+2*c*d)^3*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(3/2)/c^5/e^2/(e*x+d)^(3/2)+2/5*(-b*e+2*c*d)^2*(-4*b*e*g+5*c*d*g+3*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^5/e^2/(e*x+d)^(5/2)-6/7*(-b*e+2*c
*d)*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^5/e^
2/(e*x+d)^(7/2)+2/9*(-4*b*e*g+7*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(9/2)/c^5/e^2/(e*x+d)^(9/2)-2/11*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1
1/2)/c^5/e^2/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.76

$$\int (d+ex)^{5/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}(128b^4e^4g-16b^3ce^3(}{}$$

input
```
Integrate[(d + e*x)^(5/2)*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x
^2],x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(128*b^4*
e^4*g - 16*b^3*c*e^3*(11*e*f + 65*d*g + 12*e*g*x) + 24*b^2*c^2*e^2*(131*d^
2*g + e^2*x*(11*f + 10*g*x) + d*e*(55*f + 57*g*x)) - 2*b*c^3*e*(2071*d^3*g
 + 5*e^3*x^2*(33*f + 28*g*x) + 3*d*e^2*x*(286*f + 245*g*x) + 3*d^2*e*(583*
f + 558*g*x)) + c^4*(1910*d^4*g + 35*e^4*x^3*(11*f + 9*g*x) + 5*d*e^3*x^2*
(363*f + 287*g*x) + 3*d^2*e^2*x*(1177*f + 905*g*x) + d^3*e*(3509*f + 2865*
g*x))))/(3465*c^5*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1221, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{5/2}(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-8beg+5cdg+11cef)\int (d+ex)^{5/2}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{11ce}-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{11ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-8beg+5cdg+11cef)\left(\frac{2(2cd-be)\int (d+ex)^{3/2}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{3c}-\frac{2(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{9ce}\right)}{11ce}-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{11ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-8beg+5cdg+11cef)\left(\frac{2(2cd-be)\left(\frac{4(2cd-be)\int \sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{7c}-\frac{2\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{7ce}\right)}{3c}-2(d+ex)^{3}\right.}{11ce}$$

$$-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{11ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-8beg+5cdg+11cef)\left(\frac{2(2cd-be)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}dx}{5c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}\right)}{7c}-2\sqrt{d+ex}\left(d(cd\right.\right.}{3c}\right.}{11ce}$$

$$-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{11ce^2}$$

$$\Big\downarrow \text{1122}$$

$$\frac{\left(\frac{2(2cd-be)\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{15c^2e(d+ex)^{3/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}\right)}{7c}-\frac{2\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{7ce}\right)}{3c}-2(d+\right.}{11ce}$$

$$\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{11ce^2}$$

input

```
Int[(d + e*x)^(5/2)*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output

```
(-2*g*(d + e*x)^(5/2)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(11*c*e
^2) + ((11*c*e*f + 5*c*d*g - 8*b*e*g)*((-2*(d + e*x)^(3/2)*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(3/2))/(9*c*e) + (2*(2*c*d - b*e)*((-2*Sqrt[d + e*x
]*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(7*c*e) + (4*(2*c*d - b*e)*
((-4*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(15*c^2*e*
(d + e*x)^(3/2)) - (2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(5*c*e*
Sqrt[d + e*x])))/(7*c)))/(3*c)))/(11*c*e)
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.05

method	result
default	$\frac{2(cex+be-cd)(315g\,e^{4}x^{4}c^{4}-280b\,c^{3}e^{4}g\,x^{3}+1435c^{4}d\,e^{3}g\,x^{3}+385c^{4}e^{4}f\,x^{3}+240b^{2}c^{2}e^{4}g\,x^{2}-1470b\,c^{3}d\,e^{3}g\,x^{2}-330b\,c^{3}e^{4}f\,x^{2}+2715c}{}$
gosper	$\frac{2(cex+be-cd)(315g\,e^{4}x^{4}c^{4}-280b\,c^{3}e^{4}g\,x^{3}+1435c^{4}d\,e^{3}g\,x^{3}+385c^{4}e^{4}f\,x^{3}+240b^{2}c^{2}e^{4}g\,x^{2}-1470b\,c^{3}d\,e^{3}g\,x^{2}-330b\,c^{3}e^{4}f\,x^{2}+2715c}{}$
orering	$\frac{2(cex+be-cd)(315g\,e^{4}x^{4}c^{4}-280b\,c^{3}e^{4}g\,x^{3}+1435c^{4}d\,e^{3}g\,x^{3}+385c^{4}e^{4}f\,x^{3}+240b^{2}c^{2}e^{4}g\,x^{2}-1470b\,c^{3}d\,e^{3}g\,x^{2}-330b\,c^{3}e^{4}f\,x^{2}+2715c}{}$

input
```
int((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
2/3465*(c*e*x+b*e-c*d)*(315*c^4*e^4*g*x^4-280*b*c^3*e^4*g*x^3+1435*c^4*d*e
^3*g*x^3+385*c^4*e^4*f*x^3+240*b^2*c^2*e^4*g*x^2-1470*b*c^3*d*e^3*g*x^2-33
0*b*c^3*e^4*f*x^2+2715*c^4*d^2*e^2*g*x^2+1815*c^4*d*e^3*f*x^2-192*b^3*c*e^
4*g*x+1368*b^2*c^2*d*e^3*g*x+264*b^2*c^2*e^4*f*x-3348*b*c^3*d^2*e^2*g*x-17
16*b*c^3*d*e^3*f*x+2865*c^4*d^3*e*g*x+3531*c^4*d^2*e^2*f*x+128*b^4*e^4*g-1
040*b^3*c*d*e^3*g-176*b^3*c*e^4*f+3144*b^2*c^2*d^2*e^2*g+1320*b^2*c^2*d*e^
3*f-4142*b*c^3*d^3*e*g-3498*b*c^3*d^2*e^2*f+1910*c^4*d^4*g+3509*c^4*d^3*e*
f)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/c^5/e^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.45

$$\int (d+ex)^{5/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{2\,(315\,c^5e^5gx^5+35\,(11\,c^5e^5f+(32\,c^5de^4+bc^4e^5)g)x^4+5\,(11\,(26\,c^5}{}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="fricas")
```

output
```
2/3465*(315*c^5*e^5*g*x^5 + 35*(11*c^5*e^5*f + (32*c^5*d*e^4 + b*c^4*e^5)*
g)*x^4 + 5*(11*(26*c^5*d*e^4 + b*c^4*e^5)*f + (256*c^5*d^2*e^3 + 49*b*c^4*
d*e^4 - 8*b^2*c^3*e^5)*g)*x^3 + 3*(11*(52*c^5*d^2*e^3 + 13*b*c^4*d*e^4 - 2
*b^2*c^3*e^5)*f + (50*c^5*d^3*e^2 + 279*b*c^4*d^2*e^3 - 114*b^2*c^3*d*e^4
+ 16*b^3*c^2*e^5)*g)*x^2 - 11*(319*c^5*d^4*e - 637*b*c^4*d^3*e^2 + 438*b^2
*c^3*d^2*e^3 - 136*b^3*c^2*d*e^4 + 16*b^4*c*e^5)*f - 2*(955*c^5*d^5 - 3026
*b*c^4*d^4*e + 3643*b^2*c^3*d^3*e^2 - 2092*b^3*c^2*d^2*e^3 + 584*b^4*c*d*e
^4 - 64*b^5*e^5)*g - (11*(2*c^5*d^3*e^2 - 159*b*c^4*d^2*e^3 + 60*b^2*c^3*d
*e^4 - 8*b^3*c^2*e^5)*f + (955*c^5*d^4*e - 2071*b*c^4*d^3*e^2 + 1572*b^2*c
^3*d^2*e^3 - 520*b^3*c^2*d*e^4 + 64*b^4*c*e^5)*g)*x)*sqrt(-c*e^2*x^2 - b*e
^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^5*e^3*x + c^5*d*e^2)
```

Sympy [F]

$$\int (d+ex)^{5/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \int \sqrt{-(d+ex)\,(be-cd+cex)}(d+ex)^{\frac{5}{2}}\,(f+gx)\,dx$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/
2),x)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)**(5/2)*(f + g*x),
x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.46

$$\int (d+ex)^{5/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{2\left(35c^4e^4x^4-319c^4d^4+637bc^3d^3e-438b^2c^2d^2e^2+136b^3cde^3-1\right.}{}$$

$$+\frac{2\left(315c^5e^5x^5-1910c^5d^5+6052bc^4d^4e-7286b^2c^3d^3e^2+4184b^3c^2d^2e^3-1168b^4cde^4+128b^5e^5+35(3\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="maxima")
```

output
```
2/315*(35*c^4*e^4*x^4 - 319*c^4*d^4 + 637*b*c^3*d^3*e - 438*b^2*c^2*d^2*e^
2 + 136*b^3*c*d*e^3 - 16*b^4*e^4 + 5*(26*c^4*d*e^3 + b*c^3*e^4)*x^3 + 3*(5
2*c^4*d^2*e^2 + 13*b*c^3*d*e^3 - 2*b^2*c^2*e^4)*x^2 - (2*c^4*d^3*e - 159*b
*c^3*d^2*e^2 + 60*b^2*c^2*d*e^3 - 8*b^3*c*e^4)*x)*sqrt(-c*e*x + c*d - b*e)
*(e*x + d)*f/(c^4*e^2*x + c^4*d*e) + 2/3465*(315*c^5*e^5*x^5 - 1910*c^5*d^
5 + 6052*b*c^4*d^4*e - 7286*b^2*c^3*d^3*e^2 + 4184*b^3*c^2*d^2*e^3 - 1168*
b^4*c*d*e^4 + 128*b^5*e^5 + 35*(32*c^5*d*e^4 + b*c^4*e^5)*x^4 + 5*(256*c^5
*d^2*e^3 + 49*b*c^4*d*e^4 - 8*b^2*c^3*e^5)*x^3 + 3*(50*c^5*d^3*e^2 + 279*b
*c^4*d^2*e^3 - 114*b^2*c^3*d*e^4 + 16*b^3*c^2*e^5)*x^2 - (955*c^5*d^4*e -
2071*b*c^4*d^3*e^2 + 1572*b^2*c^3*d^2*e^3 - 520*b^3*c^2*d*e^4 + 64*b^4*c*e
^5)*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d)*g/(c^5*e^3*x + c^5*d*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3520 vs. $2(313) = 626$.

Time = 0.37 (sec) , antiderivative size = 3520, normalized size of antiderivative = 10.26

$$\int (d+ex)^{5/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="giac")
```

output

```
-2/3465*(3465*sqrt(-c*e*x + c*d - b*e)*c*d^4*e*f - 3465*sqrt(-c*e*x + c*d
- b*e)*b*d^3*e^2*f + 2310*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x
+ c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*d^3*e*f - 3465*(3*sqrt(-c*e
*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e
)^(3/2))*b*d^2*e^2*f/c + 1155*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*
e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*d^4*g - 1155*(3*sqrt(-c
*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b
*e)^(3/2))*b*d^3*e*g/c - 693*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqr
t(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(
-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e
*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))*b*d*e^2*f/c^2 + 462*(15*sqrt(-
c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt
(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*
e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b
*e))*d^3*g/c - 693*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x +
 c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c
*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d +
 b*e)^2*sqrt(-c*e*x + c*d - b*e))*b*d^2*e*g/c^2 - 198*(35*sqrt(-c*e*x + c*
d - b*e)*c^3*d^3 - 105*sqrt(-c*e*x + c*d - b*e)*b*c^2*d^2*e + 105*sqrt(-c*
e*x + c*d - b*e)*b^2*c*d*e^2 - 35*sqrt(-c*e*x + c*d - b*e)*b^3*e^3 - 35...
```

Mupad [B] (verification not implemented)

Time = 11.57 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.46

$$\int (d+ex)^{5/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\left(\frac{2\,x^3\,\sqrt{d+e\,x}\,\left(-8\,g\,b^2\,e^2+49\,g\,b\,c\,d\,e+11\,f\,b\,c\,e^2+2\right.}{693\,c^2}\right.}{}$$

input
```
int((f + g*x)*(d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),
x)
```

output
```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*x^3*(d + e*x)^(1/2)*(256*
c^2*d^2*g - 8*b^2*e^2*g + 11*b*c*e^2*f + 286*c^2*d*e*f + 49*b*c*d*e*g))/(6
93*c^2) + (2*e^2*g*x^5*(d + e*x)^(1/2))/11 + (2*(b*e - c*d)*(d + e*x)^(1/2
)*(128*b^4*e^4*g + 1910*c^4*d^4*g - 176*b^3*c*e^4*f + 3509*c^4*d^3*e*f - 4
142*b*c^3*d^3*e*g - 1040*b^3*c*d*e^3*g - 3498*b*c^3*d^2*e^2*f + 1320*b^2*c
^2*d*e^3*f + 3144*b^2*c^2*d^2*e^2*g))/(3465*c^5*e^3) - (x*(d + e*x)^(1/2)*
(44*c^5*d^3*e^2*f - 176*b^3*c^2*e^5*f + 128*b^4*c*e^5*g + 1910*c^5*d^4*e*g
 - 3498*b*c^4*d^2*e^3*f + 1320*b^2*c^3*d*e^4*f - 4142*b*c^4*d^3*e^2*g - 10
40*b^3*c^2*d*e^4*g + 3144*b^2*c^3*d^2*e^3*g))/(3465*c^5*e^3) + (x^2*(d + e
*x)^(1/2)*(96*b^3*c^2*e^5*g - 132*b^2*c^3*e^5*f + 3432*c^5*d^2*e^3*f + 300
*c^5*d^3*e^2*g + 858*b*c^4*d*e^4*f + 1674*b*c^4*d^2*e^3*g - 684*b^2*c^3*d*
e^4*g))/(3465*c^5*e^3) + (2*e*x^4*(d + e*x)^(1/2)*(b*e*g + 32*c*d*g + 11*c
*e*f))/(99*c)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.46

$$\int (d+ex)^{5/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{2\sqrt{-cex-be+cd}\,(315c^5e^5g\,x^5+35b\,c^4e^5g\,x^4+1120c^5d\,e^4g\,x^4+38}{}$$

input
```
int((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(2*sqrt( - b*e + c*d - c*e*x)*(128*b**5*e**5*g - 1168*b**4*c*d*e**4*g - 17
6*b**4*c*e**5*f - 64*b**4*c*e**5*g*x + 4184*b**3*c**2*d**2*e**3*g + 1496*b
**3*c**2*d*e**4*f + 520*b**3*c**2*d*e**4*g*x + 88*b**3*c**2*e**5*f*x + 48*
b**3*c**2*e**5*g*x**2 - 7286*b**2*c**3*d**3*e**2*g - 4818*b**2*c**3*d**2*e
**3*f - 1572*b**2*c**3*d**2*e**3*g*x - 660*b**2*c**3*d*e**4*f*x - 342*b**2
*c**3*d*e**4*g*x**2 - 66*b**2*c**3*e**5*f*x**2 - 40*b**2*c**3*e**5*g*x**3
+ 6052*b*c**4*d**4*e*g + 7007*b*c**4*d**3*e**2*f + 2071*b*c**4*d**3*e**2*g
*x + 1749*b*c**4*d**2*e**3*f*x + 837*b*c**4*d**2*e**3*g*x**2 + 429*b*c**4*
d*e**4*f*x**2 + 245*b*c**4*d*e**4*g*x**3 + 55*b*c**4*e**5*f*x**3 + 35*b*c*
*4*e**5*g*x**4 - 1910*c**5*d**5*g - 3509*c**5*d**4*e*f - 955*c**5*d**4*e*g
*x - 22*c**5*d**3*e**2*f*x + 150*c**5*d**3*e**2*g*x**2 + 1716*c**5*d**2*e*
*3*f*x**2 + 1280*c**5*d**2*e**3*g*x**3 + 1430*c**5*d*e**4*f*x**3 + 1120*c*
*5*d*e**4*g*x**4 + 385*c**5*e**5*f*x**4 + 315*c**5*e**5*g*x**5))/(3465*c**
5*e**2)
```

3.196 $\int (d+ex)^{3/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 267

$$\begin{aligned}
&\int (d+ex)^{3/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \\
&-\frac{2(2cd-be)^2(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^4e^2(d+ex)^{3/2}} \\
&+\frac{2(2cd-be)(2cef+4cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^4e^2(d+ex)^{5/2}} \\
&-\frac{2(cef+5cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^4e^2(d+ex)^{7/2}} \\
&+\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^4e^2(d+ex)^{9/2}}
\end{aligned}$$

output

```
-2/3*(-b*e+2*c*d)^2*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(3/2)/c^4/e^2/(e*x+d)^(3/2)+2/5*(-b*e+2*c*d)*(-3*b*e*g+4*c*d*g+2*c*e*f)*(d
*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^4/e^2/(e*x+d)^(5/2)-2/7*(-3*b*e*g+5
*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^4/e^2/(e*x+d)^(7/2)
+2/9*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(9/2)/c^4/e^2/(e*x+d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int (d+ex)^{3/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}(-16b^3e^3g+24b^2ce^2}{}$$

input

```
Integrate[(d + e*x)^(3/2)*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x
^2],x]
```

output

```
(2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-16*b^3*
e^3*g + 24*b^2*c*e^2*(4*d*g + e*(f + g*x)) - 6*b*c^2*e*(31*d^2*g + e^2*x*(
6*f + 5*g*x) + d*e*(22*f + 20*g*x)) + c^3*(106*d^3*g + 5*e^3*x^2*(9*f + 7*
g*x) + 6*d*e^2*x*(27*f + 20*g*x) + 3*d^2*e*(71*f + 53*g*x))))/(315*c^4*e^2
*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1221, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2}(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-2beg+cdg+3cef)\int (d+ex)^{3/2}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{3ce}-\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{9ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{4(2cd-be)\int \sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{7c}-\frac{2\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{7ce}\right)}{3ce}-$$
$$\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{9ce^2}$$

$$\Big\downarrow \textcolor{blue}{1128}$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}dx}{5c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}\right)}{7c}-\frac{2\sqrt{d+ex}(d(cd-be)-be^2}{7ce}\right.}{3ce}$$
$$\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{9ce^2}$$

$$\Big\downarrow \textcolor{blue}{1122}$$

$$\frac{\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{15c^2e(d+ex)^{3/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}\right)}{7c}-\frac{2\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{7ce}\right)(-2beg+c}{3ce}$$
$$\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{9ce^2}$$

input
```
Int[(d + e*x)^(3/2)*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
(-2*g*(d + e*x)^(3/2)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(9*c*e^
2) + ((3*c*e*f + c*d*g - 2*b*e*g)*((-2*Sqrt[d + e*x]*(d*(c*d - b*e) - b*e^
2*x - c*e^2*x^2)^(3/2))/(7*c*e) + (4*(2*c*d - b*e)*((-4*(2*c*d - b*e)*(d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(15*c^2*e*(d + e*x)^(3/2)) - (2*(
d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(5*c*e*Sqrt[d + e*x])))/(7*c))
)/(3*c*e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.85

method	result
default	$-\frac{2(cex+be-cd)\left(-35e^3g\,x^3c^3+30b\,c^2e^3g\,x^2-120c^3d\,e^2g\,x^2-45c^3e^3f\,x^2-24b^2c\,e^3gx+120b\,c^2d\,e^2gx+36b\,c^2e^3fx-159c^3d^2egx-1\right.}{315c^4e^2\sqrt{}}$
gosper	$-\frac{2(cex+be-cd)\left(-35e^3g\,x^3c^3+30b\,c^2e^3g\,x^2-120c^3d\,e^2g\,x^2-45c^3e^3f\,x^2-24b^2c\,e^3gx+120b\,c^2d\,e^2gx+36b\,c^2e^3fx-159c^3d^2egx-1\right.}{315c^4e^2\sqrt{}}$
orering	$-\frac{2(cex+be-cd)\left(-35e^3g\,x^3c^3+30b\,c^2e^3g\,x^2-120c^3d\,e^2g\,x^2-45c^3e^3f\,x^2-24b^2c\,e^3gx+120b\,c^2d\,e^2gx+36b\,c^2e^3fx-159c^3d^2egx-1\right.}{315c^4e^2\sqrt{}}$

input
```
int((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
-2/315*(c*e*x+b*e-c*d)*(-35*c^3*e^3*g*x^3+30*b*c^2*e^3*g*x^2-120*c^3*d*e^2
*g*x^2-45*c^3*e^3*f*x^2-24*b^2*c*e^3*g*x+120*b*c^2*d*e^2*g*x+36*b*c^2*e^3*
f*x-159*c^3*d^2*e*g*x-162*c^3*d*e^2*f*x+16*b^3*e^3*g-96*b^2*c*d*e^2*g-24*b
^2*c*e^3*f+186*b*c^2*d^2*e*g+132*b*c^2*d*e^2*f-106*c^3*d^3*g-213*c^3*d^2*e
*f)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/c^4/e^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.32

$$\int (d+ex)^{3/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{2\,(35\,c^4e^4gx^4+5\,(9\,c^4e^4f+(17\,c^4de^3+bc^3e^4)g)x^3+3\,(3\,(13\,c^4de^3+}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="fricas")
```

output
```
2/315*(35*c^4*e^4*g*x^4 + 5*(9*c^4*e^4*f + (17*c^4*d*e^3 + b*c^3*e^4)*g)*x
^3 + 3*(3*(13*c^4*d*e^3 + b*c^3*e^4)*f + (13*c^4*d^2*e^2 + 10*b*c^3*d*e^3
- 2*b^2*c^2*e^4)*g)*x^2 - 3*(71*c^4*d^3*e - 115*b*c^3*d^2*e^2 + 52*b^2*c^2
*d*e^3 - 8*b^3*c*e^4)*f - 2*(53*c^4*d^4 - 146*b*c^3*d^3*e + 141*b^2*c^2*d^
2*e^2 - 56*b^3*c*d*e^3 + 8*b^4*e^4)*g + (3*(17*c^4*d^2*e^2 + 22*b*c^3*d*e^
3 - 4*b^2*c^2*e^4)*f - (53*c^4*d^3*e - 93*b*c^3*d^2*e^2 + 48*b^2*c^2*d*e^3
 - 8*b^3*c*e^4)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x
+ d)/(c^4*e^3*x + c^4*d*e^2)
```

Sympy [F]

$$\int (d+ex)^{3/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \int \sqrt{-(d+ex)\,(be-cd+cex)}(d+ex)^{\frac{3}{2}}\,(f+gx)\;dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/
2),x)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)**(3/2)*(f + g*x),
x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.33

$$\int (d+ex)^{3/2}(f +gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{2\,(15\,c^3e^3x^3-71\,c^3d^3+115\,bc^2d^2e-52\,b^2cde^2+8\,b^3e^3+3\,(13\,c^3de^2}{105\,(c} + \frac{2\,(35\,c^4e^4x^4-106\,c^4d^4+292\,bc^3d^3e-282\,b^2c^2d^2e^2+112\,b^3cde^3-16\,b^4e^4+5\,(17\,c^4de^3+bc^3e^4)x^3+3\,(1}{315\,(c}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="maxima")
```

output
```
2/105*(15*c^3*e^3*x^3 - 71*c^3*d^3 + 115*b*c^2*d^2*e - 52*b^2*c*d*e^2 + 8*
b^3*e^3 + 3*(13*c^3*d*e^2 + b*c^2*e^3)*x^2 + (17*c^3*d^2*e + 22*b*c^2*d*e^
2 - 4*b^2*c*e^3)*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d)*f/(c^3*e^2*x + c^3*
d*e) + 2/315*(35*c^4*e^4*x^4 - 106*c^4*d^4 + 292*b*c^3*d^3*e - 282*b^2*c^2
*d^2*e^2 + 112*b^3*c*d*e^3 - 16*b^4*e^4 + 5*(17*c^4*d*e^3 + b*c^3*e^4)*x^3
 + 3*(13*c^4*d^2*e^2 + 10*b*c^3*d*e^3 - 2*b^2*c^2*e^4)*x^2 - (53*c^4*d^3*e
 - 93*b*c^3*d^2*e^2 + 48*b^2*c^2*d*e^3 - 8*b^3*c*e^4)*x)*sqrt(-c*e*x + c*d
 - b*e)*(e*x + d)*g/(c^4*e^3*x + c^4*d*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2139 vs. $2(243) = 486$.

Time = 0.30 (sec) , antiderivative size = 2139, normalized size of antiderivative = 8.01

$$\int (d+ex)^{3/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="giac")
```

output

```
-2/315*(315*sqrt(-c*e*x + c*d - b*e)*c*d^3*e*f - 315*sqrt(-c*e*x + c*d - b
*e)*b*d^2*e^2*f + 105*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*
d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*d^2*e*f - 210*(3*sqrt(-c*e*x +
c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/
2))*b*d*e^2*f/c + 105*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*
d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*d^3*g - 105*(3*sqrt(-c*e*x + c*
d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2)
)*b*d^2*e*g/c - 21*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x +
 c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c
*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d +
 b*e)^2*sqrt(-c*e*x + c*d - b*e))*d*e*f/c - 21*(15*sqrt(-c*e*x + c*d - b*e
)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b
*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^
(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))*b*e^2*f/c^2
+ 21*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*
c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2
)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-
c*e*x + c*d - b*e))*d^2*g/c - 42*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30
*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 -
10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e +...
```

Mupad [B] (verification not implemented)

Time = 11.21 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.26

$$\int (d+ex)^{3/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{2\,x^3\,\sqrt{d+e\,x}\,(b\,e\,g+17\,c\,d\,g+9\,c\,e\,f)}{63\,c}+\frac{2\,e\,g\,x^4\,\sqrt{}}{9}\right.}{}$$

input
```
int((f + g*x)*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),
x)
```

output
```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*x^3*(d + e*x)^(1/2)*(b*e*
g + 17*c*d*g + 9*c*e*f))/(63*c) + (2*e*g*x^4*(d + e*x)^(1/2))/9 + (2*x^2*(
d + e*x)^(1/2)*(13*c^2*d^2*g - 2*b^2*e^2*g + 3*b*c*e^2*f + 39*c^2*d*e*f +
10*b*c*d*e*g))/(105*c^2*e) + (2*(b*e - c*d)*(d + e*x)^(1/2)*(106*c^3*d^3*g
 - 16*b^3*e^3*g + 24*b^2*c*e^3*f + 213*c^3*d^2*e*f - 132*b*c^2*d*e^2*f - 1
86*b*c^2*d^2*e*g + 96*b^2*c*d*e^2*g))/(315*c^4*e^3) + (x*(d + e*x)^(1/2)*(
102*c^4*d^2*e^2*f - 24*b^2*c^2*e^4*f + 16*b^3*c*e^4*g - 106*c^4*d^3*e*g +
132*b*c^3*d*e^3*f + 186*b*c^3*d^2*e^2*g - 96*b^2*c^2*d*e^3*g))/(315*c^4*e^
3)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.24

$$\int (d+ex)^{3/2}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{2\sqrt{-cex-be+cd}\,(35c^4e^4g\,x^4+5b\,c^3e^4g\,x^3+85c^4d\,e^3g\,x^3+45c^4e^4}{}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(2*sqrt( - b*e + c*d - c*e*x)*( - 16*b**4*e**4*g + 112*b**3*c*d*e**3*g + 2
4*b**3*c*e**4*f + 8*b**3*c*e**4*g*x - 282*b**2*c**2*d**2*e**2*g - 156*b**2
*c**2*d*e**3*f - 48*b**2*c**2*d*e**3*g*x - 12*b**2*c**2*e**4*f*x - 6*b**2*
c**2*e**4*g*x**2 + 292*b*c**3*d**3*e*g + 345*b*c**3*d**2*e**2*f + 93*b*c**
3*d**2*e**2*g*x + 66*b*c**3*d*e**3*f*x + 30*b*c**3*d*e**3*g*x**2 + 9*b*c**
3*e**4*f*x**2 + 5*b*c**3*e**4*g*x**3 - 106*c**4*d**4*g - 213*c**4*d**3*e*f
 - 53*c**4*d**3*e*g*x + 51*c**4*d**2*e**2*f*x + 39*c**4*d**2*e**2*g*x**2 +
 117*c**4*d*e**3*f*x**2 + 85*c**4*d*e**3*g*x**3 + 45*c**4*e**4*f*x**3 + 35
*c**4*e**4*g*x**4))/(315*c**4*e**2)
```

3.197 $\int \sqrt{d+ex}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 190

$$\begin{aligned}\int \sqrt{d+ex}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx &= -\frac{2(2cd-be)(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^3e^2(d+ex)^{3/2}}\\ &\quad+\frac{2(cef+3cdg-2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^3e^2(d+ex)^{5/2}}\\ &\quad-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^3e^2(d+ex)^{7/2}}\end{aligned}$$

output

```
-2/3*(-b*e+2*c*d)*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3
/2)/c^3/e^2/(e*x+d)^(3/2)+2/5*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2
*x-c*e^2*x^2)^(5/2)/c^3/e^2/(e*x+d)^(5/2)-2/7*g*(d*(-b*e+c*d)-b*e^2*x-c*e^
2*x^2)^(7/2)/c^3/e^2/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

$$\int \sqrt{d+ex}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$=\frac{2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}(8b^2e^2g-2bce(7ef+15dg+6egx)+c^2(22d^2g+3e^2x(7}{105c^3e^2\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
],x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(8*b^2*e^
2*g - 2*b*c*e*(7*e*f + 15*d*g + 6*e*g*x) + c^2*(22*d^2*g + 3*e^2*x*(7*f +
5*g*x) + d*e*(49*f + 33*g*x))))/(105*c^3*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-4beg+cdg+7cef)\int \sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{7ce}-\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{7ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-4beg+cdg+7cef)\left(\frac{2(2cd-be)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}dx}{5c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}\right)}{7ce}-\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{7ce^2}$$

$$\Big\downarrow \; 1122$$

$$\frac{\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{15c^2e(d+ex)^{3/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}\right)(-4beg+cdg+7cef)}{7ce}-\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{7ce^2}$$

input
```
Int[Sqrt[d + e*x]*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
(-2*g*Sqrt[d + e*x]*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(7*c*e^2)
 + ((7*c*e*f + c*d*g - 4*b*e*g)*((-4*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*
x - c*e^2*x^2)^(3/2))/(15*c^2*e*(d + e*x)^(3/2)) - (2*(d*(c*d - b*e) - b*e
^2*x - c*e^2*x^2)^(3/2))/(5*c*e*Sqrt[d + e*x])))/(7*c*e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.69

method	result
default	$\frac{2(cex+be-cd)\left(15g\,x^2c^2e^2-12bc\,e^2gx+33c^2degx+21c^2e^2fx+8b^2e^2g-30bcdeg-14bc\,e^2f+22c^2d^2g+49c^2def\right)\sqrt{-(ex+d)(cex+be}}{105c^3e^2\sqrt{ex+d}}$
gosper	$\frac{2(cex+be-cd)\left(15g\,x^2c^2e^2-12bc\,e^2gx+33c^2degx+21c^2e^2fx+8b^2e^2g-30bcdeg-14bc\,e^2f+22c^2d^2g+49c^2def\right)\sqrt{-x^2c\,e^2-xb\,e^2-b}}{105c^3e^2\sqrt{ex+d}}$
orering	$\frac{2(cex+be-cd)\left(15g\,x^2c^2e^2-12bc\,e^2gx+33c^2degx+21c^2e^2fx+8b^2e^2g-30bcdeg-14bc\,e^2f+22c^2d^2g+49c^2def\right)\sqrt{-x^2c\,e^2-xb\,e^2-b}}{105c^3e^2\sqrt{ex+d}}$

input
```
int((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
2/105*(c*e*x+b*e-c*d)*(15*c^2*e^2*g*x^2-12*b*c*e^2*g*x+33*c^2*d*e*g*x+21*c
^2*e^2*f*x+8*b^2*e^2*g-30*b*c*d*e*g-14*b*c*e^2*f+22*c^2*d^2*g+49*c^2*d*e*f
)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/c^3/e^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.23

$$\int \sqrt{d+ex}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$

$$= \frac{2\left(15\,c^3e^3gx^3+3\left(7\,c^3e^3f+\left(6\,c^3de^2+bc^2e^3\right)g\right)x^2-7\left(7\,c^3d^2e-9\,bc^2de^2+2\,b^2ce^3\right)f-2\left(11\,c^3d^3-26\,b\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="fricas")
```

output

```
2/105*(15*c^3*e^3*g*x^3 + 3*(7*c^3*e^3*f + (6*c^3*d*e^2 + b*c^2*e^3)*g)*x^
2 - 7*(7*c^3*d^2*e - 9*b*c^2*d*e^2 + 2*b^2*c*e^3)*f - 2*(11*c^3*d^3 - 26*b
*c^2*d^2*e + 19*b^2*c*d*e^2 - 4*b^3*e^3)*g + (7*(4*c^3*d*e^2 + b*c^2*e^3)*
f - (11*c^3*d^2*e - 15*b*c^2*d*e^2 + 4*b^2*c*e^3)*g)*x)*sqrt(-c*e^2*x^2 -
b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^3*e^3*x + c^3*d*e^2)
```

Sympy [F]

$$\int \sqrt{d+ex}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \int \sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\sqrt{d+ex}\left(f+gx\right)\,dx$$

input

```
integrate((e*x+d)**(1/2)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/
2),x)
```

output

```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*sqrt(d + e*x)*(f + g*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.24

$$\int \sqrt{d+ex}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx = \frac{2\left(3\,c^2e^2x^2-7\,c^2d^2+9\,bcde-2\,b^2e^2+\left(4\,c^2de+bce^2\right)x\right)\sqrt{-cex+cd-be}(ex+d)f}{15\left(c^2e^2x+c^2de\right)} + \frac{2\left(15\,c^3e^3x^3-22\,c^3d^3+52\,bc^2d^2e-38\,b^2cde^2+8\,b^3e^3+3\left(6\,c^3de^2+bc^2e^3\right)x^2-\left(11\,c^3d^2e-15\,bc^2de^2\right.}{105\left(c^3e^3x+c^3de^2\right)}$$

input

```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="maxima")
```

output
```
2/15*(3*c^2*e^2*x^2 - 7*c^2*d^2 + 9*b*c*d*e - 2*b^2*e^2 + (4*c^2*d*e + b*c
*e^2)*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d)*f/(c^2*e^2*x + c^2*d*e) + 2/10
5*(15*c^3*e^3*x^3 - 22*c^3*d^3 + 52*b*c^2*d^2*e - 38*b^2*c*d*e^2 + 8*b^3*e
^3 + 3*(6*c^3*d*e^2 + b*c^2*e^3)*x^2 - (11*c^3*d^2*e - 15*b*c^2*d*e^2 + 4*
b^2*c*e^3)*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d)*g/(c^3*e^3*x + c^3*d*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(172) = 344$.

Time = 0.33 (sec) , antiderivative size = 804, normalized size of antiderivative = 4.23

$$\int \sqrt{d+ex}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx =$$

$$-\frac{2\left(105\sqrt{-cex+cd-be}cd^2ef-105\sqrt{-cex+cd-be}bde^2f-\frac{35\left(3\sqrt{-cex+cd-be}cd-3\sqrt{-cex+cd-be}be-(-cex}{c}}{}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="giac")
```

output
```
-2/105*(105*sqrt(-c*e*x + c*d - b*e)*c*d^2*e*f - 105*sqrt(-c*e*x + c*d - b
*e)*b*d*e^2*f - 35*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d -
 b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b*e^2*f/c + 35*(3*sqrt(-c*e*x + c*
d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2)
)*d^2*g - 35*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*
b*e - (-c*e*x + c*d - b*e)^(3/2))*b*d*e*g/c - 7*(15*sqrt(-c*e*x + c*d - b*
e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d -
b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)
^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))*e*f/c - 7*(
15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e
+ 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d
+ 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x
+ c*d - b*e))*b*e*g/c^2 - 3*(35*sqrt(-c*e*x + c*d - b*e)*c^3*d^3 - 105*sqr
t(-c*e*x + c*d - b*e)*b*c^2*d^2*e + 105*sqrt(-c*e*x + c*d - b*e)*b^2*c*d*e
^2 - 35*sqrt(-c*e*x + c*d - b*e)*b^3*e^3 - 35*(-c*e*x + c*d - b*e)^(3/2)*c
^2*d^2 + 70*(-c*e*x + c*d - b*e)^(3/2)*b*c*d*e - 35*(-c*e*x + c*d - b*e)^(
3/2)*b^2*e^2 + 21*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e)*c*d - 21*
(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e)*b*e + 5*(c*e*x - c*d + b*e)
^3*sqrt(-c*e*x + c*d - b*e))*g/c^2)/(c*e^2)
```

Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.15

$$\int \sqrt{d+ex}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$

$$= \frac{\bigg(\frac{2\,g\,x^3\,\sqrt{d+e\,x}}{7}+\frac{2\,x^2\,\sqrt{d+e\,x}\,\left(b\,e\,g+6\,c\,d\,g+7\,c\,e\,f\right)}{35\,c\,e}+\frac{2\,\left(b\,e-c\,d\right)\,\sqrt{d+e\,x}\,\left(8\,g\,b^2\,e^2-30\,g\,b\,c\,d\,e-14\,f\,b\,c\,e^2+22\,g\,c^2\,d^2+49\,f\,c^2\,d\,e\right)}{105\,c^3\,e^3}+}{x+\frac{d}{e}}$$

input
```
int((f + g*x)*(d + e*x)^(1/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),
x)
```

output
```
(((2*g*x^3*(d + e*x)^(1/2))/7 + (2*x^2*(d + e*x)^(1/2)*(b*e*g + 6*c*d*g +
7*c*e*f))/(35*c*e) + (2*(b*e - c*d)*(d + e*x)^(1/2)*(8*b^2*e^2*g + 22*c^2*
d^2*g - 14*b*c*e^2*f + 49*c^2*d*e*f - 30*b*c*d*e*g))/(105*c^3*e^3) + (x*(d
 + e*x)^(1/2)*(14*b*c^2*e^3*f - 8*b^2*c*e^3*g + 56*c^3*d*e^2*f - 22*c^3*d^
2*e*g + 30*b*c^2*d*e^2*g))/(105*c^3*e^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e
^2*x)^(1/2))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.05

$$\int \sqrt{d+ex}(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$= \frac{2\sqrt{-cex-be+cd}\,(15c^3e^3g\,x^3+3b\,c^2e^3g\,x^2+18c^3d\,e^2g\,x^2+21c^3e^3f\,x^2-4b^2c\,e^3gx+15b\,c^2d\,e^2gx+7b}{}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*(8*b**3*e**3*g - 38*b**2*c*d*e**2*g - 14*b**
2*c*e**3*f - 4*b**2*c*e**3*g*x + 52*b*c**2*d**2*e*g + 63*b*c**2*d*e**2*f +
 15*b*c**2*d*e**2*g*x + 7*b*c**2*e**3*f*x + 3*b*c**2*e**3*g*x**2 - 22*c**3
*d**3*g - 49*c**3*d**2*e*f - 11*c**3*d**2*e*g*x + 28*c**3*d*e**2*f*x + 18*
c**3*d*e**2*g*x**2 + 21*c**3*e**3*f*x**2 + 15*c**3*e**3*g*x**3))/(105*c**3
*e**2)
```

3.198 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 116

$$\begin{aligned}\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{\sqrt{d+ex}}\,dx &= -\frac{2(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^2e^2(d+ex)^{3/2}} \\ &\quad+\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^2e^2(d+ex)^{5/2}}\end{aligned}$$

output

```
-2/3*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^2/e^2/(
e*x+d)^(3/2)+2/5*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^2/e^2/(e*x+d)^
(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\begin{aligned}\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{\sqrt{d+ex}}\,dx \\ = \frac{2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}(-2beg+c(5ef+2dg+3egx))}{15c^2e^2\sqrt{d+ex}}\end{aligned}$$

input
```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/Sqrt[d + e
*x],x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-2*b*e*g
 + c*(5*e*f + 2*d*g + 3*e*g*x)))/(15*c^2*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{1221, 1122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-2beg-cdg+5cef)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}dx}{5ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}(-2beg-cdg+5cef)}{15c^2e^2(d+ex)^{3/2}}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{5ce^2\sqrt{d+ex}}$$

input
```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/Sqrt[d + e*x],x]
```

output
```
(-2*(5*c*e*f - c*d*g - 2*b*e*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2
))/(15*c^2*e^2*(d + e*x)^(3/2)) - (2*g*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^
2)^(3/2))/(5*c*e^2*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{2(cex+be-cd)(-3cegx+2beg-2cdg-5fce)\sqrt{-(ex+d)(cex+be-cd)}}{15c^{2}e^{2}\sqrt{ex+d}}$	71
gosper	$-\frac{2(cex+be-cd)(-3cegx+2beg-2cdg-5fce)\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}{15c^{2}e^{2}\sqrt{ex+d}}$	79
orering	$-\frac{2(cex+be-cd)(-3cegx+2beg-2cdg-5fce)\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}{15c^{2}e^{2}\sqrt{ex+d}}$	79

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
-2/15*(c*e*x+b*e-c*d)*(-3*c*e*g*x+2*b*e*g-2*c*d*g-5*c*e*f)*(-(e*x+d)*(c*e*
x+b*e-c*d))^(1/2)/c^2/e^2/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{\sqrt{d+ex}}\,dx = \frac{2\,(3\,c^2e^2gx^2-5\,(c^2de-bce^2)f-2\,(c^2d^2-2\,bcde+b^2e^2)g+(5\,c^2e^2f-(c^2de-bce^2)g)x)\sqrt{-ce^2x^2-be}}{15\,(c^2e^3x+c^2de^2)}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

output
```
2/15*(3*c^2*e^2*g*x^2 - 5*(c^2*d*e - b*c*e^2)*f - 2*(c^2*d^2 - 2*b*c*d*e +
 b^2*e^2)*g + (5*c^2*e^2*f - (c^2*d*e - b*c*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b
*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^2*e^3*x + c^2*d*e^2)
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**(1/
2),x)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{\sqrt{d+ex}}\,dx &= \frac{2\,(cex-cd+be)\sqrt{-cex+cd-be}f}{3\,ce}\\ &\quad+\frac{2\,(3\,c^2e^2x^2-2\,c^2d^2+4\,bcde-2\,b^2e^2-(c^2de-bce^2)x)\sqrt{-cex+cd-be}g}{15\,c^2e^2}\end{aligned}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

output
```
2/3*(c*e*x - c*d + b*e)*sqrt(-c*e*x + c*d - b*e)*f/(c*e) + 2/15*(3*c^2*e^2
*x^2 - 2*c^2*d^2 + 4*b*c*d*e - 2*b^2*e^2 - (c^2*d*e - b*c*e^2)*x)*sqrt(-c*
e*x + c*d - b*e)*g/(c^2*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(104) = 208.

Time = 0.33 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.27

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{\sqrt{d+ex}}\,dx =$$

$$-\frac{2\Bigl(15\sqrt{-cex+cd-be}cdef-15\sqrt{-cex+cd-be}be^2f-5\Bigl(3\sqrt{-cex+cd-be}cd-3\sqrt{-cex+cd}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="giac")
```

output

```
-2/15*(15*sqrt(-c*e*x + c*d - b*e)*c*d*e*f - 15*sqrt(-c*e*x + c*d - b*e)*b
*e^2*f - 5*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*
e - (-c*e*x + c*d - b*e)^(3/2))*e*f + 5*(3*sqrt(-c*e*x + c*d - b*e)*c*d -
3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*d*g - 5*(3*sq
rt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*
d - b*e)^(3/2))*b*e*g/c - (15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-
c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*
e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x
- c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))*g/c)/(c*e^2)
```

Mupad [B] (verification not implemented)

Time = 10.95 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{\left(\frac{2\,g\,x^2}{5}+\frac{2\,x\,\left(b\,e\,g-c\,d\,g+5\,c\,e\,f\right)}{15\,c\,e}+\frac{2\,\left(b\,e-c\,d\right)\,\left(2\,c\,d\,g-2\,b\,e\,g+5\,c\,e\,f\right)}{15\,c^2\,e^2}\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{\sqrt{d+e\,x}}$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(1/2
),x)
```

output

```
(((2*g*x^2)/5 + (2*x*(b*e*g - c*d*g + 5*c*e*f))/(15*c*e) + (2*(b*e - c*d)*
(2*c*d*g - 2*b*e*g + 5*c*e*f))/(15*c^2*e^2))*(c*d^2 - c*e^2*x^2 - b*d*e -
b*e^2*x)^(1/2))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\sqrt{-cex-be+cd}\,(3c^2e^2g\,x^2+bc\,e^2gx-c^2degx+5c^2e^2fx-2b^2e^2g+4bcdeg+5bc\,e^2f-2c^2d^2g-5c}{15c^2e^2}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*( - 2*b**2*e**2*g + 4*b*c*d*e*g + 5*b*c*e**2
*f + b*c*e**2*g*x - 2*c**2*d**2*g - 5*c**2*d*e*f - c**2*d*e*g*x + 5*c**2*e
**2*f*x + 3*c**2*e**2*g*x**2))/(15*c**2*e**2)
```

3.199 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 186

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{3/2}}\,dx = \frac{2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2\sqrt{d+ex}} - \frac{2g(d(cd-be)-be^2x-ce^2x^2)^{3/2}}{3ce^2(d+ex)^{3/2}} - \frac{2\sqrt{2cd-be}(ef-dg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{e^2}$$

output
```
2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)^(1/2)-2/3*
g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c/e^2/(e*x+d)^(3/2)-2*(-b*e+2*c*d
)^(1/2)*(-d*g+e*f)*arctanh((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*
c*d)^(1/2)/(e*x+d)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{d+ex}\sqrt{cd-be-cex}\Big(\sqrt{-be+c(d-ex)}(beg+c(3ef-}{3ce^2\sqrt{(d+ex)(}}$$

input
```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
(3/2),x]
```

output
```
(2*Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*x]*(Sqrt[-(b*e) + c*(d - e*x)]*(b*e*
g + c*(3*e*f - 4*d*g + e*g*x)) + 3*c*Sqrt[-2*c*d + b*e]*(-(e*f) + d*g)*Arc
Tan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]]))/(3*c*e^2*Sqrt[(d + e*x)*
(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1221, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(ef-dg)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{3/2}}dx}{e}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3ce^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(ef-dg)\left((2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}\right)}{e}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3ce^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(ef-dg)\left(2e(2cd-be)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}+\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}\right)}{e}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3ce^2(d+ex)^{3/2}}$$

$$\Big\downarrow 221$$

$$\frac{(ef-dg)\left(\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2cd-be}\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e}\right)}{e}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3ce^2(d+ex)^{3/2}}$$

input

```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^(3/2),
x]
```

output

```
(-2*g*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(3*c*e^2*(d + e*x)^(3/2
)) + ((e*f - d*g)*((2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e*Sqrt[d
 + e*x]) - (2*Sqrt[2*c*d - b*e]*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e
^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/e))/e
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1131

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.73

method	result
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,\left(3\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bcdeg-3\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bc\,e^{2}f-6\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c^{2}d^{2}g+6\operatorname{arc}\right.}{3\sqrt{ex+}}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2/3*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e
-2*c*d)^(1/2))*b*c*d*e*g-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2)
)*b*c*e^2*f-6*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^2*g+6
*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d*e*f+c*e*g*x*(-c*e*
x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+b*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d
)^(1/2)-4*c*d*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+3*c*e*f*(-c*e*x-b
*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2))/(e*x+d)^(1/2)/(-c*e*x-b*e+c*d)^(1/2)/c/e^
2/(b*e-2*c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.26

$$\int\frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{3/2}}\,dx=\left[-\frac{3\,(cdef-cd^2g+(ce^2f-cdeg)x)\sqrt{2\,cd-be}\log\left(-\frac{ce^2x^2-}{}\right.}{}\right.$$

$$-\frac{2\left(3\,(cdef-cd^2g+(ce^2f-cdeg)x)\sqrt{-2\,cd+be}\arctan\left(-\frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}\sqrt{-2\,cd+be}\sqrt{ex+d}}{2\,cd^2-bde+(2\,cde-be^2)x}\right)-\sqrt{-ce}\right.}{3\,(ce^3x+cde^2)}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(3/2),x,
algorithm="fricas")
```

output
```
[-1/3*(3*(c*d*e*f - c*d^2*g + (c*e^2*f - c*d*e*g)*x)*sqrt(2*c*d - b*e)*log
(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*sqrt(-c*e^2*x^2
 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2 + 2*
d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(c*e*g*x + 3*
c*e*f - (4*c*d - b*e)*g)*sqrt(e*x + d))/(c*e^3*x + c*d*e^2), -2/3*(3*(c*d*
e*f - c*d^2*g + (c*e^2*f - c*d*e*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqrt(-c*
e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d
^2 - b*d*e + (2*c*d*e - b*e^2)*x)) - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b
*d*e)*(c*e*g*x + 3*c*e*f - (4*c*d - b*e)*g)*sqrt(e*x + d))/(c*e^3*x + c*d*
e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{3/2}}\,dx=\int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**(3/
2),x)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**(3/2),
x)
```

Maxima [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{3/2}}\,dx=\int \frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(gx+f)}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(3/2),x,
algorithm="maxima")
```

output
```
integrate(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)/(e*x + d)^(
3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{3\,(2\,cdef-be^2f-2\,cd^2g+bdeg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{\sqrt{-2\,cd+be}}+\frac{3\sqrt{-(ex+d)}}{}\right.}{3\,e^2}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(3/2),x,
algorithm="giac")
```

output
```
2/3*(3*(2*c*d*e*f - b*e^2*f - 2*c*d^2*g + b*d*e*g)*arctan(sqrt(-(e*x + d)*
c + 2*c*d - b*e)/sqrt(-2*c*d + b*e))/sqrt(-2*c*d + b*e) + (3*sqrt(-(e*x +
d)*c + 2*c*d - b*e)*c^3*e*f - 3*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^3*d*g -
 (-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*g)/c^3)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{3/2}}\,dx = \int \frac{(f+g\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{{(d+e\,x)}^{3/2}} \,d x$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(3/2
),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(3/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)cdg-2\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-ce}}{\sqrt{be}}\right.}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(3/2),x)
```

output
```
(2*(3*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))
*c*d*g - 3*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*
c*d))*c*e*f + sqrt( - b*e + c*d - c*e*x)*b*e*g - 4*sqrt( - b*e + c*d - c*e
*x)*c*d*g + 3*sqrt( - b*e + c*d - c*e*x)*c*e*f + sqrt( - b*e + c*d - c*e*x
)*c*e*g*x))/(3*c*e**2)
```

3.200 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 187

$$\begin{aligned}\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{5/2}}\,dx = {} & -\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)^{3/2}}+\frac{2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2\sqrt{d+ex}} \\ & +\frac{(cef-5cdg+2beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{e^2\sqrt{2cd-be}}\end{aligned}$$

output

```
-(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)^(3/2)+2*g*(
d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)^(1/2)+(2*b*e*g-5*c*d*g+c
*e*f)*arctanh((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e
*x+d)^(1/2))/e^2/(-b*e+2*c*d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.73

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{(d+ex)(-be+c(d-ex))}\left(\frac{-ef+3dg+2egx}{d+ex}+\frac{(-cef+5cdg-2beg)}{\sqrt{-2cd+be}\sqrt{}}\right.}{e^2\sqrt{d+ex}}$$

input
```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
(5/2),x]
```

output
```
(Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*((-(e*f) + 3*d*g + 2*e*g*x)/(d + e
*x) + ((-(c*e*f) + 5*c*d*g - 2*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[
-2*c*d + b*e]])/(Sqrt[-2*c*d + b*e]*Sqrt[-(b*e) + c*(d - e*x)])))/(e^2*Sqr
t[d + e*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1220, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(2beg-5cdg+cef)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{3/2}}dx}{2e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{e^2(d+ex)^{5/2}(2cd-be)}$$

$$\Big\downarrow \text{1131}$$

$$-\frac{(2beg-5cdg+cef)\left((2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}\right)}{2e(2cd-be)}-$$
$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{e^2(d+ex)^{5/2}(2cd-be)}$$

$$\Big\downarrow \text{1136}$$

$$-\frac{(2beg-5cdg+cef)\left(2e(2cd-be)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}+\frac{2\sqrt{d(cd-be)-be^2x}}{e\sqrt{d+ex}}\right.}{2e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{e^2(d+ex)^{5/2}(2cd-be)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\left(\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2cd-be}\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e}\right)(2beg-5cdg+cef)}{2e(2cd-be)}-$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{e^2(d+ex)^{5/2}(2cd-be)}$$

input

```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^(5/2),
x]
```

output

```
-(((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(e^2*(2*c*d -
b*e)*(d + e*x)^(5/2))) - ((c*e*f - 5*c*d*g + 2*b*e*g)*((2*Sqrt[d*(c*d - b*
e) - b*e^2*x - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2*c*d - b*e]*ArcTan
h[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*
x])])/e))/(2*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(171) = 342$.

Time = 2.13 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.88

method	result
default	$\sqrt{-(ex+d)(cex+be-cd)}\,\left(-2\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)b\,e^{2}gx+5\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)cdegx-\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c\,e^{2}fx+2\sqrt{-}\right.$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(5/2),x,method=
_RETURNVERBOSE)
```

output

```
(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/(e*x+d)^(3/2)*(-2*arctan((-c*e*x-b*e+c*d)
^(1/2)/(b*e-2*c*d)^(1/2))*b*e^2*g*x+5*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2
*c*d)^(1/2))*c*d*e*g*x-arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c*
e^2*f*x+2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)*e*g*x-2*arctan((-c*e*x-
b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*d*e*g+5*arctan((-c*e*x-b*e+c*d)^(1/2)/
(b*e-2*c*d)^(1/2))*c*d^2*g-arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2)
)*c*d*e*f+3*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)*d*g-(-c*e*x-b*e+c*d)^
(1/2)*(b*e-2*c*d)^(1/2)*e*f)/(-c*e*x-b*e+c*d)^(1/2)/e^2/(b*e-2*c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.53

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{5/2}}\,dx = \Bigg[\frac{(cd^2ef+(ce^3f-(5\,cde^2-2\,be^3)g)x^2-(5\,cd^3-2\,bd^2e)g+}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(5/2),x,
algorithm="fricas")
```

output

```
[1/2*((c*d^2*e*f + (c*e^3*f - (5*c*d*e^2 - 2*b*e^3)*g)*x^2 - (5*c*d^3 - 2*
b*d^2*e)*g + 2*(c*d*e^2*f - (5*c*d^2*e - 2*b*d*e^2)*g)*x)*sqrt(2*c*d - b*e
)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*sqrt(-c*e^
2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2
 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(2*c*
d*e - b*e^2)*g*x - (2*c*d*e - b*e^2)*f + 3*(2*c*d^2 - b*d*e)*g)*sqrt(e*x +
 d))/(2*c*d^3*e^2 - b*d^2*e^3 + (2*c*d*e^4 - b*e^5)*x^2 + 2*(2*c*d^2*e^3 -
 b*d*e^4)*x), ((c*d^2*e*f + (c*e^3*f - (5*c*d*e^2 - 2*b*e^3)*g)*x^2 - (5*c
*d^3 - 2*b*d^2*e)*g + 2*(c*d*e^2*f - (5*c*d^2*e - 2*b*d*e^2)*g)*x)*sqrt(-2
*c*d + b*e)*arctan(-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d
 + b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)) + sqrt(-c*e
^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(2*c*d*e - b*e^2)*g*x - (2*c*d*e - b*
e^2)*f + 3*(2*c*d^2 - b*d*e)*g)*sqrt(e*x + d))/(2*c*d^3*e^2 - b*d^2*e^3 +
(2*c*d*e^4 - b*e^5)*x^2 + 2*(2*c*d^2*e^3 - b*d*e^4)*x)]
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{5/2}}\,dx=\int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**(5/
2),x)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**(5/2),
x)
```

Maxima [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{5/2}}\,dx=\int \frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(gx+f)}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(5/2),x,
algorithm="maxima")
```

output
```
integrate(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)/(e*x + d)^(
5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.84

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{5/2}}\,dx=\frac{2\sqrt{-(ex+d)c+2\,cd-be}cg-\frac{(c^2ef-5\,c^2dg+2\,bceg)\arctan\left(\frac{\sqrt{-(ex+}}{\sqrt{-}}\right.}{\sqrt{-2\,cd+be}}}{ce^2}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(5/2),x,
algorithm="giac")
```

output
```
(2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c*g - (c^2*e*f - 5*c^2*d*g + 2*b*c*e*g
)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(-2*c*d + b*e))/sqrt(-2*c*d
+ b*e) - (sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^2*e*f - sqrt(-(e*x + d)*c + 2
*c*d - b*e)*c^2*d*g)/((e*x + d)*c))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{5/2}}\,dx=\int \frac{(f+g\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{{(d+e\,x)}^{5/2}} \,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(5/2
),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(5/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.21

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{5/2}}\,dx=\frac{-2\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bdeg-2\sqrt{be-2cd}\,atan(}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(5/2),x)
```

output
```
( - 2*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))
*b*d*e*g - 2*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e -
2*c*d))*b*e**2*g*x + 5*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/s
qrt(b*e - 2*c*d))*c*d**2*g - sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*
e*x)/sqrt(b*e - 2*c*d))*c*d*e*f + 5*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c
*d - c*e*x)/sqrt(b*e - 2*c*d))*c*d*e*g*x - sqrt(b*e - 2*c*d)*atan(sqrt( -
b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c*e**2*f*x + 3*sqrt( - b*e + c*d - c
*e*x)*b*d*e*g - sqrt( - b*e + c*d - c*e*x)*b*e**2*f + 2*sqrt( - b*e + c*d
- c*e*x)*b*e**2*g*x - 6*sqrt( - b*e + c*d - c*e*x)*c*d**2*g + 2*sqrt( - b*
e + c*d - c*e*x)*c*d*e*f - 4*sqrt( - b*e + c*d - c*e*x)*c*d*e*g*x)/(e**2*(
b*d*e + b*e**2*x - 2*c*d**2 - 2*c*d*e*x))
```

3.201 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 220

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}}\,dx = -\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e^2(d+ex)^{5/2}} + \frac{(cef-9cdg+4beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4e^2(2cd-be)(d+ex)^{3/2}} + \frac{c(cef+7cdg-4beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{4e^2(2cd-be)^{3/2}}$$

output
```
-1/2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)^(5/2)+1
/4*(4*b*e*g-9*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*
e+2*c*d)/(e*x+d)^(3/2)+1/4*c*(-4*b*e*g+7*c*d*g+c*e*f)*arctanh((d*(-b*e+c*d
)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2/(-b*e+2*c
*d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}}\,dx=\frac{c\sqrt{(d+ex)(-be+c(d-ex))}\left(\frac{2be(dg+e(f+2gx))+c(-5d^2g+e^2fx-3}{c(2cd-be)(d+ex)^2}\right.}{4e^2\sqrt{d+ex}}$$

input

```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
(7/2),x]
```

output

```
(c*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*((2*b*e*(d*g + e*(f + 2*g*x)) +
c*(-5*d^2*g + e^2*f*x - 3*d*e*(f + 3*g*x)))/(c*(2*c*d - b*e)*(d + e*x)^2)
+ ((c*e*f + 7*c*d*g - 4*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d
+ b*e]])/((-2*c*d + b*e)^(3/2)*Sqrt[-(b*e) + c*(d - e*x)])))/(4*e^2*Sqrt[d
 + e*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1220, 1130, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-4beg+7cdg+cef)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{5/2}}dx}{4e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e^2(d+ex)^{7/2}(2cd-be)}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(-4beg+7cdg+cef)\left(-\frac{1}{2}c\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}}\right)}{4e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e^2(d+ex)^{7/2}(2cd-be)}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{(-4beg+7cdg+cef)\left(-ce\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}}\right)}{4e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e^2(d+ex)^{7/2}(2cd-be)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e\sqrt{2cd-be}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}}\right)(-4beg+7cdg+cef)}{4e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e^2(d+ex)^{7/2}(2cd-be)}$$

input

```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^(7/2),
x]
```

output

```
-1/2*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(7/2)) + ((c*e*f + 7*c*d*g - 4*b*e*g)*(-(Sqrt[d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2]/(e*(d + e*x)^(3/2))) + (c*ArcTanh[Sqrt[d*(c*d -
 b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/(e*Sqrt[2
*c*d - b*e])))/(4*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(198) = 396$.

Time = 2.21 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.83

method	result
default	$-\frac{\left(4\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bce^{3}g\,x^{2}-7\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c^{2}d\,e^{2}g\,x^{2}-\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c^{2}e^{3}f\,x^{2}+8\arctan\left(\frac{\sqrt{-cex-be}}{\sqrt{be-2c}}\right.\right.}{}$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(7/2),x,method=
_RETURNVERBOSE)
```

output

```
-1/4*(4*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c*e^3*g*x^2-7*a
rctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d*e^2*g*x^2-arctan((-c
*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*e^3*f*x^2+8*arctan((-c*e*x-b*e+
c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c*d*e^2*g*x-14*arctan((-c*e*x-b*e+c*d)^(1/
2)/(b*e-2*c*d)^(1/2))*c^2*d^2*e*g*x-2*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2
*c*d)^(1/2))*c^2*d*e^2*f*x+4*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/
2))*b*c*d^2*e*g-7*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^3
*g-arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^2*e*f+4*b*e^2*g*
x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-9*c*d*e*g*x*(-c*e*x-b*e+c*d)^(1
/2)*(b*e-2*c*d)^(1/2)+c*e^2*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+2
*b*d*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+2*b*e^2*f*(-c*e*x-b*e+c*
d)^(1/2)*(b*e-2*c*d)^(1/2)-5*c*d^2*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1
/2)-3*c*d*e*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2))*(-(e*x+d)*(c*e*x+b
*e-c*d))^(1/2)/(b*e-2*c*d)^(3/2)/e^2/(-c*e*x-b*e+c*d)^(1/2)/(e*x+d)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(198) = 396$.

Time = 0.11 (sec) , antiderivative size = 1045, normalized size of antiderivative = 4.75

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(7/2),x,
algorithm="fricas")
```

output
```
[1/8*((c^2*d^3*e*f + (c^2*e^4*f + (7*c^2*d*e^3 - 4*b*c*e^4)*g)*x^3 + 3*(c^
2*d*e^3*f + (7*c^2*d^2*e^2 - 4*b*c*d*e^3)*g)*x^2 + (7*c^2*d^4 - 4*b*c*d^3*
e)*g + 3*(c^2*d^2*e^2*f + (7*c^2*d^3*e - 4*b*c*d^2*e^2)*g)*x)*sqrt(2*c*d -
 b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*sqrt(-
c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2
*x^2 + 2*d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((6*
c^2*d^2*e - 7*b*c*d*e^2 + 2*b^2*e^3)*f + (10*c^2*d^3 - 9*b*c*d^2*e + 2*b^2
*d*e^2)*g - ((2*c^2*d*e^2 - b*c*e^3)*f - (18*c^2*d^2*e - 17*b*c*d*e^2 + 4*
b^2*e^3)*g)*x)*sqrt(e*x + d))/(4*c^2*d^5*e^2 - 4*b*c*d^4*e^3 + b^2*d^3*e^4
 + (4*c^2*d^2*e^5 - 4*b*c*d*e^6 + b^2*e^7)*x^3 + 3*(4*c^2*d^3*e^4 - 4*b*c*
d^2*e^5 + b^2*d*e^6)*x^2 + 3*(4*c^2*d^4*e^3 - 4*b*c*d^3*e^4 + b^2*d^2*e^5)
*x), 1/4*((c^2*d^3*e*f + (c^2*e^4*f + (7*c^2*d*e^3 - 4*b*c*e^4)*g)*x^3 + 3
*(c^2*d*e^3*f + (7*c^2*d^2*e^2 - 4*b*c*d*e^3)*g)*x^2 + (7*c^2*d^4 - 4*b*c*
d^3*e)*g + 3*(c^2*d^2*e^2*f + (7*c^2*d^3*e - 4*b*c*d^2*e^2)*g)*x)*sqrt(-2*
c*d + b*e)*arctan(-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d
+ b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)) - sqrt(-c*e^
2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((6*c^2*d^2*e - 7*b*c*d*e^2 + 2*b^2*e^3)*
f + (10*c^2*d^3 - 9*b*c*d^2*e + 2*b^2*d*e^2)*g - ((2*c^2*d*e^2 - b*c*e^3)*
f - (18*c^2*d^2*e - 17*b*c*d*e^2 + 4*b^2*e^3)*g)*x)*sqrt(e*x + d))/(4*c^2*
d^5*e^2 - 4*b*c*d^4*e^3 + b^2*d^3*e^4 + (4*c^2*d^2*e^5 - 4*b*c*d*e^6 + ...
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}}\,dx=\int \frac{\sqrt{-(d+ex)\,(be-cd+cex)}(f+gx)}{(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**(7/
2),x)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**(7/2),
x)
```

Maxima [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}}\,dx=\int \frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(gx+f)}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(7/2),x,
algorithm="maxima")
```

output
```
integrate(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)/(e*x + d)^(
7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.49

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}}\,dx=$$

$$-\frac{\frac{(c^3ef+7\,c^3dg-4\,bc^2eg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{(2\,cd-be)\sqrt{-2\,cd+be}}+\frac{2\,\sqrt{-(ex+d)c+2\,cd-be}c^4def-\sqrt{-(ex+d)c+2\,cd-be}bc^3e^2f+14\,\sqrt{-(ex+d)c+2\,cd-}}{}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(7/2),x,
algorithm="giac")
```

output
```
-1/4*((c^3*e*f + 7*c^3*d*g - 4*b*c^2*e*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d
 - b*e)/sqrt(-2*c*d + b*e))/((2*c*d - b*e)*sqrt(-2*c*d + b*e)) + (2*sqrt(-
(e*x + d)*c + 2*c*d - b*e)*c^4*d*e*f - sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*
c^3*e^2*f + 14*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4*d^2*g - 15*sqrt(-(e*x
+ d)*c + 2*c*d - b*e)*b*c^3*d*e*g + 4*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2
*c^2*e^2*g + (-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^3*e*f - 9*(-(e*x + d)*c
+ 2*c*d - b*e)^(3/2)*c^3*d*g + 4*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^2*
e*g)/((2*c*d - b*e)*(e*x + d)^2*c^2))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}}\,dx=\int \frac{(f+g\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{{(d+e\,x)}^{7/2}} \,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(7/2
),x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(7/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 780, normalized size of antiderivative = 3.55

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}}\,dx=\frac{-4\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bc\,d^2eg-8\sqrt{be-2cd}\,atan\left(\right.}{}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(7/2),x)
```

output

```
( - 4*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))
*b*c*d**2*e*g - 8*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b
*e - 2*c*d))*b*c*d*e**2*g*x - 4*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d -
 c*e*x)/sqrt(b*e - 2*c*d))*b*c*e**3*g*x**2 + 7*sqrt(b*e - 2*c*d)*atan(sqrt
( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**3*g + sqrt(b*e - 2*c*d)*
atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*e*f + 14*sqrt
(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2
*e*g*x + 2*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*
c*d))*c**2*d*e**2*f*x + 7*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x
)/sqrt(b*e - 2*c*d))*c**2*d*e**2*g*x**2 + sqrt(b*e - 2*c*d)*atan(sqrt( - b
*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*e**3*f*x**2 - 2*sqrt( - b*e + c*
d - c*e*x)*b**2*d*e**2*g - 2*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*f - 4*sq
rt( - b*e + c*d - c*e*x)*b**2*e**3*g*x + 9*sqrt( - b*e + c*d - c*e*x)*b*c*
d**2*e*g + 7*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*f + 17*sqrt( - b*e + c*
d - c*e*x)*b*c*d*e**2*g*x - sqrt( - b*e + c*d - c*e*x)*b*c*e**3*f*x - 10*s
qrt( - b*e + c*d - c*e*x)*c**2*d**3*g - 6*sqrt( - b*e + c*d - c*e*x)*c**2*
d**2*e*f - 18*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*e*g*x + 2*sqrt( - b*e +
 c*d - c*e*x)*c**2*d*e**2*f*x)/(4*e**2*(b**2*d**2*e**2 + 2*b**2*d*e**3*x +
 b**2*e**4*x**2 - 4*b*c*d**3*e - 8*b*c*d**2*e**2*x - 4*b*c*d*e**3*x**2 + 4
*c**2*d**4 + 8*c**2*d**3*e*x + 4*c**2*d**2*e**2*x**2))
```

3.202 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 296

$$\begin{aligned}\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{9/2}}\,dx = & -\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e^2(d+ex)^{7/2}} \\ & +\frac{(cef-13cdg+6beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{12e^2(2cd-be)(d+ex)^{5/2}} \\ & +\frac{c(cef+3cdg-2beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{8e^2(2cd-be)^2(d+ex)^{3/2}} \\ & +\frac{c^2(cef+3cdg-2beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{8e^2(2cd-be)^{5/2}}\end{aligned}$$

output
```
-1/3*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)^(7/2)+1
/12*(6*b*e*g-13*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-
b*e+2*c*d)/(e*x+d)^(5/2)+1/8*c*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^
2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^(3/2)+1/8*c^2*(-2*b*e*g+3*
c*d*g+c*e*f)*arctanh((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(
1/2)/(e*x+d)^(1/2))/e^2/(-b*e+2*c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{9/2}}\,dx=\frac{c^2\sqrt{(d+ex)(-be+c(d-ex))}\Bigg(\frac{-4b^2e^2(2ef+dg+3egx)+2bce\left(6d^2g-e\right.}{}}{}$$

input
```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
(9/2),x]
```

output
```
(c^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*((-4*b^2*e^2*(2*e*f + d*g + 3*
e*g*x) + 2*b*c*e*(6*d^2*g - e^2*x*(f + 3*g*x) + d*e*(15*f + 19*g*x)) + c^2
*(-11*d^3*g + 3*e^3*f*x^2 + d*e^2*x*(10*f + 9*g*x) - d^2*e*(25*f + 34*g*x)
))/(c^2*(-2*c*d + b*e)^2*(d + e*x)^3) - (3*(c*e*f + 3*c*d*g - 2*b*e*g)*Arc
Tan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/((-2*c*d + b*e)^(5/2)*Sqr
t[-(b*e) + c*(d - e*x)])))/(24*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1220, 1130, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-2beg+3cdg+cef)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{7/2}}dx}{2e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(-2beg+3cdg+cef)\left(-\frac{1}{4}c\int\frac{1}{(d+ex)^{3/2}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}}\right)}{2e(2cd-be)}-$$
$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(-2beg+3cdg+cef)\left(-\frac{1}{4}c\left(\frac{c\int\frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2(2cd-be)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}}\right)}{2e(2cd-be)}-$$
$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(-2beg+3cdg+cef)\left(-\frac{1}{4}c\left(\frac{ce\int\frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)\right.}{2e(2cd-be)}$$
$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(-\frac{1}{4}c\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}}\right)(-2beg+3cdg+ce}{2e(2cd-be)}$$
$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{9/2}(2cd-be)}$$

input
```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^(9/2),
x]
```

output
```
-1/3*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(9/2)) + ((c*e*f + 3*c*d*g - 2*b*e*g)*(-1/2*Sqrt[d*(c*d
- b*e) - b*e^2*x - c*e^2*x^2]/(e*(d + e*x)^(5/2)) - (c*(-(Sqrt[d*(c*d - b*
e) - b*e^2*x - c*e^2*x^2]/(e*(2*c*d - b*e)*(d + e*x)^(3/2))) - (c*ArcTanh[
Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x]
)])/(e*(2*c*d - b*e)^(3/2))))/4))/(2*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. $2(268) = 536$.

Time $= 2.20$ (sec) , antiderivative size $= 1025$, normalized size of antiderivative $= 3.46$

method	result	size
default	Expression too large to display	1025

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(9/2),x,method=
_RETURNVERBOSE)
```

output
```
1/24*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-6*b*c*e^3*g*x^2*(-c*e*x-b*e+c*d)^(
1/2)*(b*e-2*c*d)^(1/2)+9*c^2*d*e^2*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d
)^(1/2)+38*b*c*d*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-11*c^2*d
^3*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-3*arctan((-c*e*x-b*e+c*d)^(1
/2)/(b*e-2*c*d)^(1/2))*c^3*e^4*f*x^3-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-
2*c*d)^(1/2))*c^3*d^3*e*f+18*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/
2))*b*c^2*d*e^3*g*x^2+18*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*
b*c^2*d^2*e^2*g*x+12*b*c*d^2*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+
30*b*c*d*e^2*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+10*c^2*d*e^2*f*x*(
-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-34*c^2*d^2*e*g*x*(-c*e*x-b*e+c*d)^
(1/2)*(b*e-2*c*d)^(1/2)-2*b*c*e^3*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(
1/2)-9*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^4*g+6*arctan
((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*e^4*g*x^3-9*arctan((-c*e*
x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d*e^3*g*x^3-8*b^2*e^3*f*(-c*e*x-b*
e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-9*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)
^(1/2))*c^3*d*e^3*f*x^2-27*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2)
)*c^3*d^3*e*g*x-9*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^2
*e^2*f*x-12*b^2*e^3*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-4*b^2*d*e
^2*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-25*c^2*d^2*e*f*(-c*e*x-b*e+c
*d)^(1/2)*(b*e-2*c*d)^(1/2)+3*c^2*e^3*f*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(268) = 536$.

Time = 0.16 (sec) , antiderivative size = 1606, normalized size of antiderivative = 5.43

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(9/2),x,
algorithm="fricas")
```

output

```
[-1/48*(3*(c^3*d^4*e*f + (c^3*e^5*f + (3*c^3*d*e^4 - 2*b*c^2*e^5)*g)*x^4 +
 4*(c^3*d*e^4*f + (3*c^3*d^2*e^3 - 2*b*c^2*d*e^4)*g)*x^3 + 6*(c^3*d^2*e^3*
f + (3*c^3*d^3*e^2 - 2*b*c^2*d^2*e^3)*g)*x^2 + (3*c^3*d^5 - 2*b*c^2*d^4*e)
*g + 4*(c^3*d^3*e^2*f + (3*c^3*d^4*e - 2*b*c^2*d^3*e^2)*g)*x)*sqrt(2*c*d -
 b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x + 2*sqrt(-
c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2
*x^2 + 2*d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(3*(
(2*c^3*d*e^3 - b*c^2*e^4)*f + (6*c^3*d^2*e^2 - 7*b*c^2*d*e^3 + 2*b^2*c*e^4
)*g)*x^2 - (50*c^3*d^3*e - 85*b*c^2*d^2*e^2 + 46*b^2*c*d*e^3 - 8*b^3*e^4)*
f - (22*c^3*d^4 - 35*b*c^2*d^3*e + 20*b^2*c*d^2*e^2 - 4*b^3*d*e^3)*g + 2*(
(10*c^3*d^2*e^2 - 7*b*c^2*d*e^3 + b^2*c*e^4)*f - (34*c^3*d^3*e - 55*b*c^2*
d^2*e^2 + 31*b^2*c*d*e^3 - 6*b^3*e^4)*g)*x)*sqrt(e*x + d))/(8*c^3*d^7*e^2
- 12*b*c^2*d^6*e^3 + 6*b^2*c*d^5*e^4 - b^3*d^4*e^5 + (8*c^3*d^3*e^6 - 12*b
*c^2*d^2*e^7 + 6*b^2*c*d*e^8 - b^3*e^9)*x^4 + 4*(8*c^3*d^4*e^5 - 12*b*c^2*
d^3*e^6 + 6*b^2*c*d^2*e^7 - b^3*d*e^8)*x^3 + 6*(8*c^3*d^5*e^4 - 12*b*c^2*d
^4*e^5 + 6*b^2*c*d^3*e^6 - b^3*d^2*e^7)*x^2 + 4*(8*c^3*d^6*e^3 - 12*b*c^2*
d^5*e^4 + 6*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x), 1/24*(3*(c^3*d^4*e*f + (c^3*e
^5*f + (3*c^3*d*e^4 - 2*b*c^2*e^5)*g)*x^4 + 4*(c^3*d*e^4*f + (3*c^3*d^2*e^
3 - 2*b*c^2*d*e^4)*g)*x^3 + 6*(c^3*d^2*e^3*f + (3*c^3*d^3*e^2 - 2*b*c^2*d^
2*e^3)*g)*x^2 + (3*c^3*d^5 - 2*b*c^2*d^4*e)*g + 4*(c^3*d^3*e^2*f + (3*c...
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{9/2}}\,dx=\int \frac{\sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\left(f+gx\right)}{\left(d+ex\right)^{\frac{9}{2}}}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**(9/
2),x)
```

output

```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**(9/2),
x)
```

Maxima [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{9/2}}\,dx = \int \frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(gx+f)}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(9/2),x,
algorithm="maxima")
```

output
```
integrate(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)/(e*x + d)^(
9/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(268) = 536$.

Time = 0.38 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.98

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{9/2}}\,dx =$$

$$-\frac{\frac{3\,(c^4ef+3\,c^4dg-2\,bc^3eg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{(4\,c^2d^2-4\,bcde+b^2e^2)\sqrt{-2\,cd+be}} + 12\,\sqrt{-(ex+d)c+2\,cd-be}c^6d^2ef-12\,\sqrt{-(ex+d)c+2\,cd-be}bc^5de^2f+3\,\sqrt{-(ex+d)c}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(9/2),x,
algorithm="giac")
```

output

```
-1/24*(3*(c^4*e*f + 3*c^4*d*g - 2*b*c^3*e*g)*arctan(sqrt(-(e*x + d)*c + 2*
c*d - b*e)/sqrt(-2*c*d + b*e))/((4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*sqrt(-2*
c*d + b*e)) + (12*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d^2*e*f - 12*sqrt(-
(e*x + d)*c + 2*c*d - b*e)*b*c^5*d*e^2*f + 3*sqrt(-(e*x + d)*c + 2*c*d - b
*e)*b^2*c^4*e^3*f + 36*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d^3*g - 60*sqr
t(-(e*x + d)*c + 2*c*d - b*e)*b*c^5*d^2*e*g + 33*sqrt(-(e*x + d)*c + 2*c*d
 - b*e)*b^2*c^4*d*e^2*g - 6*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^3*e^3*g
 + 16*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^5*d*e*f - 8*(-(e*x + d)*c + 2*c
*d - b*e)^(3/2)*b*c^4*e^2*f - 16*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^5*d^
2*g + 8*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^4*d*e*g - 3*((e*x + d)*c -
2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4*e*f - 9*((e*x + d)*c -
 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4*d*g + 6*((e*x + d)*c
- 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^3*e*g)/((4*c^2*d^2 -
 4*b*c*d*e + b^2*e^2)*(e*x + d)^3*c^3))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{9/2}}\,dx=\int \frac{(f+g\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{{(d+e\,x)}^{9/2}}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(9/2
),x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(9/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1302, normalized size of antiderivative = 4.40

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{9/2}}\,dx=\text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(9/2),x)
```

output

```
(6*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*
c**2*d**3*e*g + 18*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(
b*e - 2*c*d))*b*c**2*d**2*e**2*g*x + 18*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e
 + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d*e**3*g*x**2 + 6*sqrt(b*e - 2*c
*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*e**4*g*x**3
- 9*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c
**3*d**4*g - 3*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e
- 2*c*d))*c**3*d**3*e*f - 27*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*
e*x)/sqrt(b*e - 2*c*d))*c**3*d**3*e*g*x - 9*sqrt(b*e - 2*c*d)*atan(sqrt( -
 b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**2*e**2*f*x - 27*sqrt(b*e -
2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**2*e**2*g
*x**2 - 9*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c
*d))*c**3*d*e**3*f*x**2 - 9*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e
*x)/sqrt(b*e - 2*c*d))*c**3*d*e**3*g*x**3 - 3*sqrt(b*e - 2*c*d)*atan(sqrt(
 - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*e**4*f*x**3 - 4*sqrt( - b*e
+ c*d - c*e*x)*b**3*d*e**3*g - 8*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*f -
12*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*g*x + 20*sqrt( - b*e + c*d - c*e*x
)*b**2*c*d**2*e**2*g + 46*sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**3*f + 62*
sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**3*g*x - 2*sqrt( - b*e + c*d - c*e*x
)*b**2*c*e**4*f*x - 6*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**4*g*x**2 - 3...
```

3.203 $\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 375

$$
\begin{aligned}
\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{11/2}}\,dx = \\
&-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4e^2(d+ex)^{9/2}} \\
&+\frac{(cef-17cdg+8beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{24e^2(2cd-be)(d+ex)^{7/2}} \\
&+\frac{c(5cef+11cdg-8beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{96e^2(2cd-be)^2(d+ex)^{5/2}} \\
&+\frac{c^2(5cef+11cdg-8beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{64e^2(2cd-be)^3(d+ex)^{3/2}} \\
&+\frac{c^3(5cef+11cdg-8beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{64e^2(2cd-be)^{7/2}}
\end{aligned}
$$

output
```
-1/4*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)^(9/2)+1
/24*(8*b*e*g-17*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-
b*e+2*c*d)/(e*x+d)^(7/2)+1/96*c*(-8*b*e*g+11*c*d*g+5*c*e*f)*(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^(5/2)+1/64*c^2*(-8*b*e
*g+11*c*d*g+5*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*
d)^3/(e*x+d)^(3/2)+1/64*c^3*(-8*b*e*g+11*c*d*g+5*c*e*f)*arctanh((d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2/(-b*e+2
*c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{11/2}}\,dx = \frac{c^3\sqrt{(d+ex)(-be+c(d-ex))}\left(\frac{16b^3e^3(3ef+dg+4egx)-8b^2ce^2(11d^2g}{}\right.}{}$$

input
```
Integrate[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^
(11/2),x]
```

output
```
(c^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*((16*b^3*e^3*(3*e*f + d*g + 4*
e*g*x) - 8*b^2*c*e^2*(11*d^2*g - e^2*x*(f + 2*g*x) + 5*d*e*(7*f + 9*g*x))
+ 2*b*c^2*e*(73*d^3*g - e^3*x^2*(5*f + 12*g*x) - d*e^2*x*(26*f + 63*g*x) +
 d^2*e*(267*f + 310*g*x)) + c^3*(-83*d^4*g + 15*e^4*f*x^3 + 13*d^2*e^2*x*(
9*f + 11*g*x) + d*e^3*x^2*(65*f + 33*g*x) - d^3*e*(317*f + 357*g*x)))/(c^3
*(2*c*d - b*e)^3*(d + e*x)^4) + (3*(5*c*e*f + 11*c*d*g - 8*b*e*g)*ArcTan[S
qrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/((-2*c*d + b*e)^(7/2)*Sqrt[-(b
*e) + c*(d - e*x)])))/(192*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1220, 1130, 1135, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}{(d+ex)^{11/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-8beg+11cdg+5cef)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{9/2}}dx}{8e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{4e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(-8beg+11cdg+5cef)\left(-\frac{1}{6}c\int \frac{1}{(d+ex)^{5/2}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e(d+ex)^{7/2}}\right)}{8e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{4e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(-8beg+11cdg+5cef)\left(-\frac{1}{6}c\left(\frac{3c\int \frac{1}{(d+ex)^{3/2}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4(2cd-be)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}(2cd-be)}\right)-\frac{\sqrt{d(cd-be)-be^2x-ce^2}}{3e(d+ex)^{7/2}}}{8e(2cd-be)}$$
$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{4e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(-8beg+11cdg+5cef)\left(-\frac{1}{6}c\left(\frac{3c\left(\frac{c\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2(2cd-be)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)}{4(2cd-be)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}(2cd-be)}\right)\right.}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{4e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \ 1136$$

$$\frac{(-8beg+11cdg+5cef)\left(-\frac{1}{6}c\left(\frac{3c\left(\frac{ce\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)}{4(2cd-be)}\right.\right.}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{4e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \ 221$$

$$\frac{\left(-\frac{1}{6}c\left(\frac{3c\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)}{4(2cd-be)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}(2cd-be)}\right)-\frac{\sqrt{d(cd-be)-be^2x-ce^2x}}{3e(d+ex)^{7/2}}\right.}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{4e^2(d+ex)^{11/2}(2cd-be)}$$

input

```
Int[((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2])/(d + e*x)^(11/2)
,x]
```

output

```
-1/4*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(11/2)) + ((5*c*e*f + 11*c*d*g - 8*b*e*g)*(-1/3*Sqrt[d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2]/(e*(d + e*x)^(7/2)) - (c*(-1/2*Sqrt[d*(c
*d - b*e) - b*e^2*x - c*e^2*x^2]/(e*(2*c*d - b*e)*(d + e*x)^(5/2)) + (3*c*
(-(Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(e*(2*c*d - b*e)*(d + e*x)^(3
/2))) - (c*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d -
 b*e]*Sqrt[d + e*x])])/(e*(2*c*d - b*e)^(3/2))))/(4*(2*c*d - b*e))))/6))/(
8*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1532 vs. $2(341) = 682$.

Time = 2.27 (sec) , antiderivative size = 1533, normalized size of antiderivative = 4.09

method	result	size
default	Expression too large to display	1533

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(11/2),x,method
=_RETURNVERBOSE)
```

output
```
-1/192*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(146*b*c^2*d^3*e*g*(-c*e*x-b*e+c*d
)^(1/2)*(b*e-2*c*d)^(1/2)-126*b*c^2*d*e^3*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*
e-2*c*d)^(1/2)+65*c^3*d*e^3*f*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)
+143*c^3*d^2*e^2*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+620*b*c^2*
d^2*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+16*b^2*c*e^4*g*x^2*(-
c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+96*arctan((-c*e*x-b*e+c*d)^(1/2)/(b
*e-2*c*d)^(1/2))*b*c^3*d^3*e^2*g*x+534*b*c^2*d^2*e^2*f*(-c*e*x-b*e+c*d)^(1
/2)*(b*e-2*c*d)^(1/2)-33*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*
c^4*d^5*g+16*b^3*d*e^3*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-317*c^3*
d^3*e*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+48*b^3*e^4*f*(-c*e*x-b*e+
c*d)^(1/2)*(b*e-2*c*d)^(1/2)-83*c^3*d^4*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*
d)^(1/2)+117*c^3*d^2*e^2*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+144*
arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^3*d^2*e^3*g*x^2-357*c
^3*d^3*e*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-280*b^2*c*d*e^3*f*(-
c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-10*b*c^2*e^4*f*x^2*(-c*e*x-b*e+c*d)
^(1/2)*(b*e-2*c*d)^(1/2)+33*c^3*d*e^3*g*x^3*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*
c*d)^(1/2)-88*b^2*c*d^2*e^2*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+8*b
^2*c*e^4*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-52*b*c^2*d*e^3*f*x*(
-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-24*b*c^2*e^4*g*x^3*(-c*e*x-b*e+c*d
)^(1/2)*(b*e-2*c*d)^(1/2)-360*b^2*c*d*e^3*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(341) = 682$.

Time = 0.24 (sec) , antiderivative size = 2258, normalized size of antiderivative = 6.02

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(11/2),x,
 algorithm="fricas")
```

output

```
[1/384*(3*(5*c^4*d^5*e*f + (5*c^4*e^6*f + (11*c^4*d*e^5 - 8*b*c^3*e^6)*g)*
x^5 + 5*(5*c^4*d*e^5*f + (11*c^4*d^2*e^4 - 8*b*c^3*d*e^5)*g)*x^4 + 10*(5*c
^4*d^2*e^4*f + (11*c^4*d^3*e^3 - 8*b*c^3*d^2*e^4)*g)*x^3 + 10*(5*c^4*d^3*e
^3*f + (11*c^4*d^4*e^2 - 8*b*c^3*d^3*e^3)*g)*x^2 + (11*c^4*d^6 - 8*b*c^3*d
^5*e)*g + 5*(5*c^4*d^4*e^2*f + (11*c^4*d^5*e - 8*b*c^3*d^4*e^2)*g)*x)*sqrt
(2*c*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x -
2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x +
d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d
*e)*(3*(5*(2*c^4*d*e^4 - b*c^3*e^5)*f + (22*c^4*d^2*e^3 - 27*b*c^3*d*e^4 +
 8*b^2*c^2*e^5)*g)*x^3 + (5*(26*c^4*d^2*e^3 - 17*b*c^3*d*e^4 + 2*b^2*c^2*e
^5)*f + (286*c^4*d^3*e^2 - 395*b*c^3*d^2*e^3 + 158*b^2*c^2*d*e^4 - 16*b^3*
c*e^5)*g)*x^2 - (634*c^4*d^4*e - 1385*b*c^3*d^3*e^2 + 1094*b^2*c^2*d^2*e^3
 - 376*b^3*c*d*e^4 + 48*b^4*e^5)*f - (166*c^4*d^5 - 375*b*c^3*d^4*e + 322*
b^2*c^2*d^3*e^2 - 120*b^3*c*d^2*e^3 + 16*b^4*d*e^4)*g + ((234*c^4*d^3*e^2
- 221*b*c^3*d^2*e^3 + 68*b^2*c^2*d*e^4 - 8*b^3*c*e^5)*f - (714*c^4*d^4*e -
 1597*b*c^3*d^3*e^2 + 1340*b^2*c^2*d^2*e^3 - 488*b^3*c*d*e^4 + 64*b^4*e^5)
*g)*x)*sqrt(e*x + d))/(16*c^4*d^9*e^2 - 32*b*c^3*d^8*e^3 + 24*b^2*c^2*d^7*
e^4 - 8*b^3*c*d^6*e^5 + b^4*d^5*e^6 + (16*c^4*d^4*e^7 - 32*b*c^3*d^3*e^8 +
 24*b^2*c^2*d^2*e^9 - 8*b^3*c*d*e^10 + b^4*e^11)*x^5 + 5*(16*c^4*d^5*e^6 -
 32*b*c^3*d^4*e^7 + 24*b^2*c^2*d^3*e^8 - 8*b^3*c*d^2*e^9 + b^4*d*e^10)*...
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{11/2}}\,dx = \int \frac{\sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\left(f+gx\right)}{\left(d+ex\right)^{\frac{11}{2}}}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2)/(e*x+d)**(11
/2),x)
```

output

```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)/(d + e*x)**(11/2),
 x)
```

Maxima [F]

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{11/2}}\,dx=\int \frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(gx+f)}{(ex+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(11/2),x,
 algorithm="maxima")
```

output
```
integrate(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)/(e*x + d)^(
11/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(341) = 682$.

Time = 0.37 (sec) , antiderivative size = 1013, normalized size of antiderivative = 2.70

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{11/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(11/2),x,
 algorithm="giac")
```

output

```
-1/192*(3*(5*c^5*e*f + 11*c^5*d*g - 8*b*c^4*e*g)*arctan(sqrt(-(e*x + d)*c
+ 2*c*d - b*e)/sqrt(-2*c*d + b*e))/((8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*
d*e^2 - b^3*e^3)*sqrt(-2*c*d + b*e)) + (120*sqrt(-(e*x + d)*c + 2*c*d - b*
e)*c^8*d^3*e*f - 180*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^7*d^2*e^2*f + 90
*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^6*d*e^3*f - 15*sqrt(-(e*x + d)*c +
 2*c*d - b*e)*b^3*c^5*e^4*f + 264*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^8*d^4
*g - 588*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^7*d^3*e*g + 486*sqrt(-(e*x +
 d)*c + 2*c*d - b*e)*b^2*c^6*d^2*e^2*g - 177*sqrt(-(e*x + d)*c + 2*c*d - b
*e)*b^3*c^5*d*e^3*g + 24*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^4*c^4*e^4*g +
292*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^7*d^2*e*f - 292*(-(e*x + d)*c + 2
*c*d - b*e)^(3/2)*b*c^6*d*e^2*f + 73*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^
2*c^5*e^3*f + 28*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^7*d^3*g - 188*(-(e*x
 + d)*c + 2*c*d - b*e)^(3/2)*b*c^6*d^2*e*g + 167*(-(e*x + d)*c + 2*c*d - b
*e)^(3/2)*b^2*c^5*d*e^2*g - 40*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^3*c^4*
e^3*g - 110*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)
*c^6*d*e*f + 55*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d -
b*e)*b*c^5*e^2*f - 242*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2
*c*d - b*e)*c^6*d^2*g + 297*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*
c + 2*c*d - b*e)*b*c^5*d*e*g - 88*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x
 + d)*c + 2*c*d - b*e)*b^2*c^4*e^2*g - 15*((e*x + d)*c - 2*c*d + b*e)^3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{11/2}}\,dx=\int \frac{(f+g\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{{(d+e\,x)}^{11/2}} \,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(11/
2),x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(d + e*x)^(11/
2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1916, normalized size of antiderivative = 5.11

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{11/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(11/2),x)
```

output

```
( - 24*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*b*c**3*d**4*e*g - 96*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/s
qrt(b*e - 2*c*d))*b*c**3*d**3*e**2*g*x - 144*sqrt(b*e - 2*c*d)*atan(sqrt(
- b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*d**2*e**3*g*x**2 - 96*sqrt(
b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*d*e
**4*g*x**3 - 24*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e
 - 2*c*d))*b*c**3*e**5*g*x**4 + 33*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*
d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**5*g + 15*sqrt(b*e - 2*c*d)*atan(sqrt
( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**4*e*f + 132*sqrt(b*e - 2
*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**4*e*g*x +
 60*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c
**4*d**3*e**2*f*x + 198*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/
sqrt(b*e - 2*c*d))*c**4*d**3*e**2*g*x**2 + 90*sqrt(b*e - 2*c*d)*atan(sqrt(
 - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**2*e**3*f*x**2 + 132*sqrt(
b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**2*
e**3*g*x**3 + 60*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*
e - 2*c*d))*c**4*d*e**4*f*x**3 + 33*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c
*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d*e**4*g*x**4 + 15*sqrt(b*e - 2*c*d)*a
tan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*e**5*f*x**4 - 16*sq
rt( - b*e + c*d - c*e*x)*b**4*d*e**4*g - 48*sqrt( - b*e + c*d - c*e*x)*...
```

3.204 $\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}$ d

Optimal result

Integrand size = 46, antiderivative size = 421

$$\begin{aligned}
&\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \\
&-\frac{2(2cd-be)^4(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^6e^2(d+ex)^{5/2}} \\
&+\frac{2(2cd-be)^3(4cef+6cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^6e^2(d+ex)^{7/2}} \\
&-\frac{4(2cd-be)^2(3cef+7cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^6e^2(d+ex)^{9/2}} \\
&+\frac{4(2cd-be)(2cef+8cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}{11c^6e^2(d+ex)^{11/2}} \\
&-\frac{2(cef+9cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}{13c^6e^2(d+ex)^{13/2}} \\
&+\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{15/2}}{15c^6e^2(d+ex)^{15/2}}
\end{aligned}$$

output
```
-2/5*(-b*e+2*c*d)^4*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(5/2)/c^6/e^2/(e*x+d)^(5/2)+2/7*(-b*e+2*c*d)^3*(-5*b*e*g+6*c*d*g+4*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^6/e^2/(e*x+d)^(7/2)-4/9*(-b*e+2*c
*d)^2*(-5*b*e*g+7*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(9/2)/c^
6/e^2/(e*x+d)^(9/2)+4/11*(-b*e+2*c*d)*(-5*b*e*g+8*c*d*g+2*c*e*f)*(d*(-b*e+
c*d)-b*e^2*x-c*e^2*x^2)^(11/2)/c^6/e^2/(e*x+d)^(11/2)-2/13*(-5*b*e*g+9*c*d
*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(13/2)/c^6/e^2/(e*x+d)^(13/2)+2
/15*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(15/2)/c^6/e^2/(e*x+d)^(15/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.86

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{2(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}(-256b^5e^5g+128b^4ce^4(3ef+22dg+5egx)-32b^3c^2e^3}{}$$

input
```
Integrate[(d + e*x)^(5/2)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^
(3/2),x]
```

output
```
(-2*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-256*
b^5*e^5*g + 128*b^4*c*e^4*(3*e*f + 22*d*g + 5*e*g*x) - 32*b^3*c^2*e^3*(389
*d^2*g + 5*e^2*x*(6*f + 7*g*x) + 2*d*e*(63*f + 100*g*x)) + 16*b^2*c^3*e^2*
(1724*d^3*g + 105*e^3*x^2*(f + g*x) + 30*d*e^2*x*(19*f + 21*g*x) + 3*d^2*e
*(347*f + 515*g*x)) - 2*b*c^4*e*(15191*d^4*g + 105*e^4*x^3*(12*f + 11*g*x)
 + 420*d*e^3*x^2*(17*f + 16*g*x) + 30*d^2*e^2*x*(542*f + 553*g*x) + 4*d^3*
e*(4131*f + 5530*g*x)) + c^5*(12686*d^5*g + 231*e^5*x^4*(15*f + 13*g*x) +
210*d*e^4*x^3*(90*f + 77*g*x) + 210*d^2*e^3*x^2*(203*f + 173*g*x) + 20*d^3
*e^2*x*(2505*f + 2212*g*x) + d^4*e*(29049*f + 31715*g*x))))/(45045*c^6*e^2
*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1221, 1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{5/2}(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}\,dx$$

$$\Big\downarrow \; 1221$$

$$\frac{(-2beg+cdg+3cef)\int (d+ex)^{5/2}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{3ce}-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{15ce^2}$$

$$\Big\downarrow \; 1128$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{8(2cd-be)\int (d+ex)^{3/2}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{13c}-\frac{2(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13ce}\right)}{3ce}-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{15ce^2}$$

$$\Big\downarrow \; 1128$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\int \sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{11c}-\frac{2\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11ce}\right)}{13c}-2(d+ex)\right.}{3ce}-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{15ce^2}$$

$$\Big\downarrow \; 1128$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{\sqrt{d+ex}}dx}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)}{11c}-\frac{2\sqrt{d+ex}\left(d(cd\right.}{}\right.}{13c}\right.}{3ce}$$

$$\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{15ce^2}$$

$$\Big\downarrow \text{1128}$$

$$(-2beg+cdg+3cef)\frac{\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{3/2}}dx}{7c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)}{9c}-2\right.}{11c}\right.}{13c}\right.}{3ce}$$

$$\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{15ce^2}$$

$$\Big\downarrow \; 1122$$

$$\frac{\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{35c^2e(d+ex)^{5/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)}{11c}-\frac{2\sqrt{}}{}\right.}{13c}\right.}{3ce}$$

$$\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{15ce^2}$$

input
```
Int[(d + e*x)^(5/2)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),
x]
```

output
```
(-2*g*(d + e*x)^(5/2)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(15*c*e
^2) + ((3*c*e*f + c*d*g - 2*b*e*g)*((-2*(d + e*x)^(3/2)*(d*(c*d - b*e) - b
*e^2*x - c*e^2*x^2)^(5/2))/(13*c*e) + (8*(2*c*d - b*e)*((-2*Sqrt[d + e*x]*
(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(11*c*e) + (6*(2*c*d - b*e)*(
(-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(9*c*e*Sqrt[d + e*x]) + (
4*(2*c*d - b*e)*((-4*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(
5/2))/(35*c^2*e*(d + e*x)^(5/2)) - (2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2
)^(5/2))/(7*c*e*(d + e*x)^(3/2))))/(9*c)))/(11*c)))/(13*c)))/(3*c*e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.26

method	result
default	$2\sqrt{-(ex+d)(cex+be-cd)}\,(cex+be-cd)^2\big(-3003g\,e^5x^5c^5+2310b\,c^4e^5g\,x^4-16170c^5d\,e^4g\,x^4-3465c^5e^5f\,x^4-1680b^2c^3e^5g\,x^3+1344$
gosper	$-\,2(cex+be-cd)\big(-3003g\,e^5x^5c^5+2310b\,c^4e^5g\,x^4-16170c^5d\,e^4g\,x^4-3465c^5e^5f\,x^4-1680b^2c^3e^5g\,x^3+13440b\,c^4d\,e^4g\,x^3+2520b\,c^4e$
orering	$-\,2(cex+be-cd)\big(-3003g\,e^5x^5c^5+2310b\,c^4e^5g\,x^4-16170c^5d\,e^4g\,x^4-3465c^5e^5f\,x^4-1680b^2c^3e^5g\,x^3+13440b\,c^4d\,e^4g\,x^3+2520b\,c^4e$

input
```
int((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2/45045/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(c*e*x+b*e-c*d)^2*(
-3003*c^5*e^5*g*x^5+2310*b*c^4*e^5*g*x^4-16170*c^5*d*e^4*g*x^4-3465*c^5*e^
5*f*x^4-1680*b^2*c^3*e^5*g*x^3+13440*b*c^4*d*e^4*g*x^3+2520*b*c^4*e^5*f*x^
3-36330*c^5*d^2*e^3*g*x^3-18900*c^5*d*e^4*f*x^3+1120*b^3*c^2*e^5*g*x^2-100
80*b^2*c^3*d*e^4*g*x^2-1680*b^2*c^3*e^5*f*x^2+33180*b*c^4*d^2*e^3*g*x^2+14
280*b*c^4*d*e^4*f*x^2-44240*c^5*d^3*e^2*g*x^2-42630*c^5*d^2*e^3*f*x^2-640*
b^4*c*e^5*g*x+6400*b^3*c^2*d*e^4*g*x+960*b^3*c^2*e^5*f*x-24720*b^2*c^3*d^2
*e^3*g*x-9120*b^2*c^3*d*e^4*f*x+44240*b*c^4*d^3*e^2*g*x+32520*b*c^4*d^2*e^
3*f*x-31715*c^5*d^4*e*g*x-50100*c^5*d^3*e^2*f*x+256*b^5*e^5*g-2816*b^4*c*d
*e^4*g-384*b^4*c*e^5*f+12448*b^3*c^2*d^2*e^3*g+4032*b^3*c^2*d*e^4*f-27584*
b^2*c^3*d^3*e^2*g-16656*b^2*c^3*d^2*e^3*f+30382*b*c^4*d^4*e*g+33048*b*c^4*
d^3*e^2*f-12686*c^5*d^5*g-29049*c^5*d^4*e*f)/c^6/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(385) = 770$.

Time = 0.13 (sec) , antiderivative size = 880, normalized size of antiderivative = 2.09

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output

```
-2/45045*(3003*c^7*e^7*g*x^7 + 231*(15*c^7*e^7*f + 4*(11*c^7*d*e^6 + 4*b*c
^6*e^7)*g)*x^6 + 63*(10*(19*c^7*d*e^6 + 7*b*c^6*e^7)*f + (111*c^7*d^2*e^5
+ 278*b*c^6*d*e^6 + b^2*c^5*e^7)*g)*x^5 + 35*(3*(79*c^7*d^2*e^5 + 206*b*c^
6*d*e^6 + b^2*c^5*e^7)*f - 2*(175*c^7*d^3*e^4 - 453*b*c^6*d^2*e^5 - 9*b^2*
c^5*d*e^6 + b^3*c^4*e^7)*g)*x^4 - 5*(12*(271*c^7*d^3*e^4 - 683*b*c^6*d^2*e
^5 - 19*b^2*c^5*d*e^6 + 2*b^3*c^4*e^7)*f + (4087*c^7*d^4*e^3 - 4900*b*c^6*
d^3*e^4 - 618*b^2*c^5*d^2*e^5 + 160*b^3*c^4*d*e^6 - 16*b^4*c^3*e^7)*g)*x^3
 - 3*(3*(3169*c^7*d^4*e^3 - 3628*b*c^6*d^3*e^4 - 694*b^2*c^5*d^2*e^5 + 168
*b^3*c^4*d*e^6 - 16*b^4*c^3*e^7)*f + 4*(542*c^7*d^5*e^2 + 11*b*c^6*d^4*e^3
 - 862*b^2*c^5*d^3*e^4 + 389*b^3*c^4*d^2*e^5 - 88*b^4*c^3*d*e^6 + 8*b^5*c^
2*e^7)*g)*x^2 + 3*(9683*c^7*d^6*e - 30382*b*c^6*d^5*e^2 + 37267*b^2*c^5*d^
4*e^3 - 23464*b^3*c^4*d^3*e^4 + 8368*b^4*c^3*d^2*e^5 - 1600*b^5*c^2*d*e^6
+ 128*b^6*c*e^7)*f + 2*(6343*c^7*d^7 - 27877*b*c^6*d^6*e + 50517*b^2*c^5*d
^5*e^2 - 48999*b^3*c^4*d^4*e^3 + 27648*b^4*c^3*d^3*e^4 - 9168*b^5*c^2*d^2*
e^5 + 1664*b^6*c*d*e^6 - 128*b^7*e^7)*g - (6*(1333*c^7*d^5*e^2 + 1421*b*c^
6*d^4*e^3 - 4142*b^2*c^5*d^3*e^4 + 1724*b^3*c^4*d^2*e^5 - 368*b^4*c^3*d*e^
6 + 32*b^5*c^2*e^7)*f - (6343*c^7*d^6*e - 21534*b*c^6*d^5*e^2 + 28983*b^2*
c^5*d^4*e^3 - 20016*b^3*c^4*d^3*e^4 + 7632*b^4*c^3*d^2*e^5 - 1536*b^5*c^2*
d*e^6 + 128*b^6*c*e^7)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sq
rt(e*x + d)/(c^6*e^3*x + c^6*d*e^2)
```

Sympy [F]

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\int\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}(d+ex)^{\frac{5}{2}}\,(f+gx)\,dx$$

input

```
integrate((e*x+d)**(5/2)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(d + e*x)**(5/2)*(f + g*x
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(385) = 770$.

Time = 0.11 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.08

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output

```
-2/15015*(1155*c^6*e^6*x^6 + 9683*c^6*d^6 - 30382*b*c^5*d^5*e + 37267*b^2*
c^4*d^4*e^2 - 23464*b^3*c^3*d^3*e^3 + 8368*b^4*c^2*d^2*e^4 - 1600*b^5*c*d*
e^5 + 128*b^6*e^6 + 210*(19*c^6*d*e^5 + 7*b*c^5*e^6)*x^5 + 35*(79*c^6*d^2*
e^4 + 206*b*c^5*d*e^5 + b^2*c^4*e^6)*x^4 - 20*(271*c^6*d^3*e^3 - 683*b*c^5
*d^2*e^4 - 19*b^2*c^4*d*e^5 + 2*b^3*c^3*e^6)*x^3 - 3*(3169*c^6*d^4*e^2 - 3
628*b*c^5*d^3*e^3 - 694*b^2*c^4*d^2*e^4 + 168*b^3*c^3*d*e^5 - 16*b^4*c^2*e
^6)*x^2 - 2*(1333*c^6*d^5*e + 1421*b*c^5*d^4*e^2 - 4142*b^2*c^4*d^3*e^3 +
1724*b^3*c^3*d^2*e^4 - 368*b^4*c^2*d*e^5 + 32*b^5*c*e^6)*x)*sqrt(-c*e*x +
c*d - b*e)*(e*x + d)*f/(c^5*e^2*x + c^5*d*e) - 2/45045*(3003*c^7*e^7*x^7 +
 12686*c^7*d^7 - 55754*b*c^6*d^6*e + 101034*b^2*c^5*d^5*e^2 - 97998*b^3*c^
4*d^4*e^3 + 55296*b^4*c^3*d^3*e^4 - 18336*b^5*c^2*d^2*e^5 + 3328*b^6*c*d*e
^6 - 256*b^7*e^7 + 924*(11*c^7*d*e^6 + 4*b*c^6*e^7)*x^6 + 63*(111*c^7*d^2*
e^5 + 278*b*c^6*d*e^6 + b^2*c^5*e^7)*x^5 - 70*(175*c^7*d^3*e^4 - 453*b*c^6
*d^2*e^5 - 9*b^2*c^5*d*e^6 + b^3*c^4*e^7)*x^4 - 5*(4087*c^7*d^4*e^3 - 4900
*b*c^6*d^3*e^4 - 618*b^2*c^5*d^2*e^5 + 160*b^3*c^4*d*e^6 - 16*b^4*c^3*e^7)
*x^3 - 12*(542*c^7*d^5*e^2 + 11*b*c^6*d^4*e^3 - 862*b^2*c^5*d^3*e^4 + 389*
b^3*c^4*d^2*e^5 - 88*b^4*c^3*d*e^6 + 8*b^5*c^2*e^7)*x^2 + (6343*c^7*d^6*e
- 21534*b*c^6*d^5*e^2 + 28983*b^2*c^5*d^4*e^3 - 20016*b^3*c^4*d^3*e^4 + 76
32*b^4*c^3*d^2*e^5 - 1536*b^5*c^2*d*e^6 + 128*b^6*c*e^7)*x)*sqrt(-c*e*x +
c*d - b*e)*(e*x + d)*g/(c^6*e^3*x + c^6*d*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12772 vs. 2(385) = 770.

Time = 0.48 (sec) , antiderivative size = 12772, normalized size of antiderivative = 30.34

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output

```
-2/45045*(45045*sqrt(-c*e*x + c*d - b*e)*c^2*d^6*e*f - 90090*sqrt(-c*e*x +
 c*d - b*e)*b*c*d^5*e^2*f + 45045*sqrt(-c*e*x + c*d - b*e)*b^2*d^4*e^3*f +
 30030*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e -
(-c*e*x + c*d - b*e)^(3/2))*c*d^5*e*f - 90090*(3*sqrt(-c*e*x + c*d - b*e)*
c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b*d^4*e
^2*f + 60060*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*
b*e - (-c*e*x + c*d - b*e)^(3/2))*b^2*d^3*e^3*f/c + 15015*(3*sqrt(-c*e*x +
 c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3
/2))*c*d^6*g - 30030*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d
 - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b*d^5*e*g + 15015*(3*sqrt(-c*e*x
 + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^
(3/2))*b^2*d^4*e^2*g/c - 3003*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sq
rt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*
(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*
e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))*d^4*e*f - 12012*(15*sqrt(-c*e
*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c
*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x
 + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e)
)*b*d^3*e^2*f/c + 18018*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*
e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c...
```

Mupad [B] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.05

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\left(\frac{2\,e^3\,x^6\,\sqrt{d+e\,x}\,(16\,b\,e\,g+44\,c\,d\,g+15\,c\,e\,f)}{195}+\frac{2\,e^2\,x^5\,\sqrt{d+e\,x}\,\left(g\,b^2\,e^2+278\,g\,b\,c\,d\,e+70\,f\,b\,c\,e^2+111\right.}{715\,c}\right.}{}$$

input

```
int((f + g*x)*(d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),
x)
```

output

```
-((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*e^3*x^6*(d + e*x)^(1/2)*
(16*b*e*g + 44*c*d*g + 15*c*e*f))/195 + (2*e^2*x^5*(d + e*x)^(1/2)*(b^2*e^
2*g + 111*c^2*d^2*g + 70*b*c*e^2*f + 190*c^2*d*e*f + 278*b*c*d*e*g))/(715*
c) + (2*c*e^4*g*x^7*(d + e*x)^(1/2))/15 + (2*(b*e - c*d)^2*(d + e*x)^(1/2)
*(12686*c^5*d^5*g - 256*b^5*e^5*g + 384*b^4*c*e^5*f + 29049*c^5*d^4*e*f -
30382*b*c^4*d^4*e*g + 2816*b^4*c*d*e^4*g - 33048*b*c^4*d^3*e^2*f - 4032*b^
3*c^2*d*e^4*f + 16656*b^2*c^3*d^2*e^3*f + 27584*b^2*c^3*d^3*e^2*g - 12448*
b^3*c^2*d^2*e^3*g))/(45045*c^6*e^3) + (x^3*(d + e*x)^(1/2)*(160*b^4*c^3*e^
7*g - 240*b^3*c^4*e^7*f - 32520*c^7*d^3*e^4*f - 40870*c^7*d^4*e^3*g + 8196
0*b*c^6*d^2*e^5*f + 2280*b^2*c^5*d*e^6*f + 49000*b*c^6*d^3*e^4*g - 1600*b^
3*c^4*d*e^6*g + 6180*b^2*c^5*d^2*e^5*g))/(45045*c^6*e^3) + (x^4*(d + e*x)^
(1/2)*(210*b^2*c^5*e^7*f - 140*b^3*c^4*e^7*g + 16590*c^7*d^2*e^5*f - 24500
*c^7*d^3*e^4*g + 43260*b*c^6*d*e^6*f + 63420*b*c^6*d^2*e^5*g + 1260*b^2*c^
5*d*e^6*g))/(45045*c^6*e^3) - (x^2*(d + e*x)^(1/2)*(192*b^5*c^2*e^7*g - 28
8*b^4*c^3*e^7*f + 57042*c^7*d^4*e^3*f + 13008*c^7*d^5*e^2*g - 65304*b*c^6*
d^3*e^4*f + 3024*b^3*c^4*d*e^6*f + 264*b*c^6*d^4*e^3*g - 2112*b^4*c^3*d*e^
6*g - 12492*b^2*c^5*d^2*e^5*f - 20688*b^2*c^5*d^3*e^4*g + 9336*b^3*c^4*d^2
*e^5*g))/(45045*c^6*e^3) + (2*x*(b*e - c*d)*(d + e*x)^(1/2)*(128*b^5*e^5*g
 - 6343*c^5*d^5*g - 192*b^4*c*e^5*f + 7998*c^5*d^4*e*f + 15191*b*c^4*d^4*e
*g - 1408*b^4*c*d*e^4*g + 16524*b*c^4*d^3*e^2*f + 2016*b^3*c^2*d*e^4*f ...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 944, normalized size of antiderivative = 2.24

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*(256*b**7*e**7*g - 3328*b**6*c*d*e**6*g - 38
4*b**6*c*e**7*f - 128*b**6*c*e**7*g*x + 18336*b**5*c**2*d**2*e**5*g + 4800
*b**5*c**2*d*e**6*f + 1536*b**5*c**2*d*e**6*g*x + 192*b**5*c**2*e**7*f*x +
 96*b**5*c**2*e**7*g*x**2 - 55296*b**4*c**3*d**3*e**4*g - 25104*b**4*c**3*
d**2*e**5*f - 7632*b**4*c**3*d**2*e**5*g*x - 2208*b**4*c**3*d*e**6*f*x - 1
056*b**4*c**3*d*e**6*g*x**2 - 144*b**4*c**3*e**7*f*x**2 - 80*b**4*c**3*e**
7*g*x**3 + 97998*b**3*c**4*d**4*e**3*g + 70392*b**3*c**4*d**3*e**4*f + 200
16*b**3*c**4*d**3*e**4*g*x + 10344*b**3*c**4*d**2*e**5*f*x + 4668*b**3*c**
4*d**2*e**5*g*x**2 + 1512*b**3*c**4*d*e**6*f*x**2 + 800*b**3*c**4*d*e**6*g
*x**3 + 120*b**3*c**4*e**7*f*x**3 + 70*b**3*c**4*e**7*g*x**4 - 101034*b**2
*c**5*d**5*e**2*g - 111801*b**2*c**5*d**4*e**3*f - 28983*b**2*c**5*d**4*e*
*3*g*x - 24852*b**2*c**5*d**3*e**4*f*x - 10344*b**2*c**5*d**3*e**4*g*x**2
- 6246*b**2*c**5*d**2*e**5*f*x**2 - 3090*b**2*c**5*d**2*e**5*g*x**3 - 1140
*b**2*c**5*d*e**6*f*x**3 - 630*b**2*c**5*d*e**6*g*x**4 - 105*b**2*c**5*e**
7*f*x**4 - 63*b**2*c**5*e**7*g*x**5 + 55754*b*c**6*d**6*e*g + 91146*b*c**6
*d**5*e**2*f + 21534*b*c**6*d**5*e**2*g*x + 8526*b*c**6*d**4*e**3*f*x + 13
2*b*c**6*d**4*e**3*g*x**2 - 32652*b*c**6*d**3*e**4*f*x**2 - 24500*b*c**6*d
**3*e**4*g*x**3 - 40980*b*c**6*d**2*e**5*f*x**3 - 31710*b*c**6*d**2*e**5*g
*x**4 - 21630*b*c**6*d*e**6*f*x**4 - 17514*b*c**6*d*e**6*g*x**5 - 4410*b*c
**6*e**7*f*x**5 - 3696*b*c**6*e**7*g*x**6 - 12686*c**7*d**7*g - 29049*c...
```

3.205 $\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2} d$

Optimal result

Integrand size = 46, antiderivative size = 343

$$\begin{aligned}
&\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx= \\
&-\frac{2(2cd-be)^3(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^5e^2(d+ex)^{5/2}} \\
&+\frac{2(2cd-be)^2(3cef+5cdg-4beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^5e^2(d+ex)^{7/2}} \\
&-\frac{2(2cd-be)(cef+3cdg-2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{3c^5e^2(d+ex)^{9/2}} \\
&+\frac{2(cef+7cdg-4beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}{11c^5e^2(d+ex)^{11/2}} \\
&-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}{13c^5e^2(d+ex)^{13/2}}
\end{aligned}$$

output

```
-2/5*(-b*e+2*c*d)^3*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(5/2)/c^5/e^2/(e*x+d)^(5/2)+2/7*(-b*e+2*c*d)^2*(-4*b*e*g+5*c*d*g+3*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^5/e^2/(e*x+d)^(7/2)-2/3*(-b*e+2*c
*d)*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(9/2)/c^5/e^
2/(e*x+d)^(9/2)+2/11*(-4*b*e*g+7*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*
x^2)^(11/2)/c^5/e^2/(e*x+d)^(11/2)-2/13*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)
^(13/2)/c^5/e^2/(e*x+d)^(13/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.77

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$
$$-\frac{2(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}(128b^4e^4g-16b^3ce^3(13ef+71dg+20egx)+8b^2c^2e^2(4}{}$$

input
```
Integrate[(d + e*x)^(3/2)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^
(3/2),x]
```

output
```
(-2*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(128*b
^4*e^4*g - 16*b^3*c*e^3*(13*e*f + 71*d*g + 20*e*g*x) + 8*b^2*c^2*e^2*(473*
d^2*g + 5*e^2*x*(13*f + 14*g*x) + d*e*(221*f + 315*g*x)) - 2*b*c^3*e*(2765
*d^3*g + 35*e^3*x^2*(13*f + 12*g*x) + 25*d*e^2*x*(78*f + 77*g*x) + d^2*e*(
2743*f + 3470*g*x)) + c^4*(2754*d^4*g + 105*e^4*x^3*(13*f + 11*g*x) + 35*d
*e^3*x^2*(169*f + 141*g*x) + 5*d^2*e^2*x*(1963*f + 1659*g*x) + d^3*e*(6929
*f + 6885*g*x))))/(15015*c^5*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1221, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2}(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}\,dx$$
$$\Big\downarrow \; 1221$$
$$\frac{(-8beg+3cdg+13cef)\int (d+ex)^{3/2}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{13ce}-\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-8beg+3cdg+13cef)\left(\frac{6(2cd-be)\int \sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{11c}-\frac{2\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11ce}\right)}{13ce}-\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-8beg+3cdg+13cef)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{\sqrt{d+ex}}dx}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)}{11c}-\frac{2\sqrt{d+ex}(d(cd-be}{1}\right.}{13ce}-\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-8beg+3cdg+13cef)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{3/2}}dx}{7c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)}{9c}-\frac{2\left(d(cd-be\right.}{}\right.}{11c}\right.}{13ce}-\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13ce^2}$$

$$\Big\downarrow \text{1122}$$

$$\frac{\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{35c^2e(d+ex)^{5/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)}{11c}-\frac{2\sqrt{d+ex}(d(}{}\right.}{13ce}$$

$$\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{13ce^2}$$

input
```
Int[(d + e*x)^(3/2)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),
x]
```

output
```
(-2*g*(d + e*x)^(3/2)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(13*c*e
^2) + ((13*c*e*f + 3*c*d*g - 8*b*e*g)*((-2*Sqrt[d + e*x]*(d*(c*d - b*e) -
b*e^2*x - c*e^2*x^2)^(5/2))/(11*c*e) + (6*(2*c*d - b*e)*((-2*(d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2)^(5/2))/(9*c*e*Sqrt[d + e*x]) + (4*(2*c*d - b*e)*(
(-4*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(35*c^2*e*(
d + e*x)^(5/2)) - (2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(7*c*e*(
d + e*x)^(3/2))))/(9*c)))/(11*c)))/(13*c*e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.05

method	result
default	$-\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(cex+be-cd)^2\left(1155g\,e^4x^4c^4-840b\,c^3e^4g\,x^3+4935c^4d\,e^3g\,x^3+1365c^4e^4f\,x^3+560b^2c^2e^4g\,x^2-3850b\,c\right.}{}$
gosper	$\frac{2(cex+be-cd)\left(1155g\,e^4x^4c^4-840b\,c^3e^4g\,x^3+4935c^4d\,e^3g\,x^3+1365c^4e^4f\,x^3+560b^2c^2e^4g\,x^2-3850b\,c^3d\,e^3g\,x^2-910b\,c^3e^4f\,x^2+829\right.}{}$
orering	$\frac{2(cex+be-cd)\left(1155g\,e^4x^4c^4-840b\,c^3e^4g\,x^3+4935c^4d\,e^3g\,x^3+1365c^4e^4f\,x^3+560b^2c^2e^4g\,x^2-3850b\,c^3d\,e^3g\,x^2-910b\,c^3e^4f\,x^2+829\right.}{}$

input
```
int((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-2/15015/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(c*e*x+b*e-c*d)^2*
(1155*c^4*e^4*g*x^4-840*b*c^3*e^4*g*x^3+4935*c^4*d*e^3*g*x^3+1365*c^4*e^4*
f*x^3+560*b^2*c^2*e^4*g*x^2-3850*b*c^3*d*e^3*g*x^2-910*b*c^3*e^4*f*x^2+829
5*c^4*d^2*e^2*g*x^2+5915*c^4*d*e^3*f*x^2-320*b^3*c*e^4*g*x+2520*b^2*c^2*d*
e^3*g*x+520*b^2*c^2*e^4*f*x-6940*b*c^3*d^2*e^2*g*x-3900*b*c^3*d*e^3*f*x+68
85*c^4*d^3*e*g*x+9815*c^4*d^2*e^2*f*x+128*b^4*e^4*g-1136*b^3*c*d*e^3*g-208
*b^3*c*e^4*f+3784*b^2*c^2*d^2*e^2*g+1768*b^2*c^2*d*e^3*f-5530*b*c^3*d^3*e*
g-5486*b*c^3*d^2*e^2*f+2754*c^4*d^4*g+6929*c^4*d^3*e*f)/c^5/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(313) = 626$.

Time = 0.11 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.98

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{2\,(1155\,c^6e^6gx^6+105\,(13\,c^6e^6f+(25\,c^6de^5+14\,bc^5e^6)g)x^5+35\,(13\,(7\,c^6de^5+4\,bc^5e^6)f-(12\,c^6d^2e^4-}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output
```
-2/15015*(1155*c^6*e^6*g*x^6 + 105*(13*c^6*e^6*f + (25*c^6*d*e^5 + 14*b*c^
5*e^6)*g)*x^5 + 35*(13*(7*c^6*d*e^5 + 4*b*c^5*e^6)*f - (12*c^6*d^2*e^4 - 1
54*b*c^5*d*e^5 - b^2*c^4*e^6)*g)*x^4 - 5*(13*(10*c^6*d^2*e^4 - 108*b*c^5*d
*e^5 - b^2*c^4*e^6)*f + (954*c^6*d^3*e^3 - 1328*b*c^5*d^2*e^4 - 63*b^2*c^4
*d*e^5 + 8*b^3*c^3*e^6)*g)*x^3 - 3*(13*(174*c^6*d^3*e^3 - 236*b*c^5*d^2*e^
4 - 17*b^2*c^4*d*e^5 + 2*b^3*c^3*e^6)*f + (907*c^6*d^4*e^2 - 560*b*c^5*d^3
*e^3 - 473*b^2*c^4*d^2*e^4 + 142*b^3*c^3*d*e^5 - 16*b^4*c^2*e^6)*g)*x^2 +
13*(533*c^6*d^5*e - 1488*b*c^5*d^4*e^2 + 1513*b^2*c^4*d^3*e^3 - 710*b^3*c^
3*d^2*e^4 + 168*b^4*c^2*d*e^5 - 16*b^5*c*e^6)*f + 2*(1377*c^6*d^6 - 5519*b
*c^5*d^5*e + 8799*b^2*c^4*d^4*e^2 - 7117*b^3*c^3*d^3*e^3 + 3092*b^4*c^2*d^
2*e^4 - 696*b^5*c*d*e^5 + 64*b^6*e^6)*g - (13*(311*c^6*d^4*e^2 - 100*b*c^5
*d^3*e^3 - 279*b^2*c^4*d^2*e^4 + 76*b^3*c^3*d*e^5 - 8*b^4*c^2*e^6)*f - (13
77*c^6*d^5*e - 4142*b*c^5*d^4*e^2 + 4657*b^2*c^4*d^3*e^3 - 2460*b^3*c^3*d^
2*e^4 + 632*b^4*c^2*d*e^5 - 64*b^5*c*e^6)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x
+ c*d^2 - b*d*e)*sqrt(e*x + d)/(c^5*e^3*x + c^5*d*e^2)
```

Sympy [F]

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x\right.$$
$$\left.-ce^2x^2\right)^{3/2}\,dx = \int \left(-(d+ex)\,(be-cd+cex)\right)^{\frac{3}{2}}(d+ex)^{\frac{3}{2}}\,(f+gx)\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(d + e*x)**(3/2)*(f + g*x
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(313) = 626$.

Time = 0.11 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.97

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{2\,(105\,c^5e^5x^5+533\,c^5d^5-1488\,bc^4d^4e+1513\,b^2c^3d^3e^2-710\,b^3c^2d^2e^3+168\,b^4cde^4-16\,b^5e^5+35\,(7\,c^5d}{}$$

$$-\frac{2\,(1155\,c^6e^6x^6+2754\,c^6d^6-11038\,bc^5d^5e+17598\,b^2c^4d^4e^2-14234\,b^3c^3d^3e^3+6184\,b^4c^2d^2e^4-1392\,b^5c}{}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output
```
-2/1155*(105*c^5*e^5*x^5 + 533*c^5*d^5 - 1488*b*c^4*d^4*e + 1513*b^2*c^3*d
^3*e^2 - 710*b^3*c^2*d^2*e^3 + 168*b^4*c*d*e^4 - 16*b^5*e^5 + 35*(7*c^5*d*
e^4 + 4*b*c^4*e^5)*x^4 - 5*(10*c^5*d^2*e^3 - 108*b*c^4*d*e^4 - b^2*c^3*e^5
)*x^3 - 3*(174*c^5*d^3*e^2 - 236*b*c^4*d^2*e^3 - 17*b^2*c^3*d*e^4 + 2*b^3*
c^2*e^5)*x^2 - (311*c^5*d^4*e - 100*b*c^4*d^3*e^2 - 279*b^2*c^3*d^2*e^3 +
76*b^3*c^2*d*e^4 - 8*b^4*c*e^5)*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d)*f/(c
^4*e^2*x + c^4*d*e) - 2/15015*(1155*c^6*e^6*x^6 + 2754*c^6*d^6 - 11038*b*c
^5*d^5*e + 17598*b^2*c^4*d^4*e^2 - 14234*b^3*c^3*d^3*e^3 + 6184*b^4*c^2*d^
2*e^4 - 1392*b^5*c*d*e^5 + 128*b^6*e^6 + 105*(25*c^6*d*e^5 + 14*b*c^5*e^6)
*x^5 - 35*(12*c^6*d^2*e^4 - 154*b*c^5*d*e^5 - b^2*c^4*e^6)*x^4 - 5*(954*c^
6*d^3*e^3 - 1328*b*c^5*d^2*e^4 - 63*b^2*c^4*d*e^5 + 8*b^3*c^3*e^6)*x^3 - 3
*(907*c^6*d^4*e^2 - 560*b*c^5*d^3*e^3 - 473*b^2*c^4*d^2*e^4 + 142*b^3*c^3*
d*e^5 - 16*b^4*c^2*e^6)*x^2 + (1377*c^6*d^5*e - 4142*b*c^5*d^4*e^2 + 4657*
b^2*c^4*d^3*e^3 - 2460*b^3*c^3*d^2*e^4 + 632*b^4*c^2*d*e^5 - 64*b^5*c*e^6)
*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d)*g/(c^5*e^3*x + c^5*d*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7604 vs. 2(313) = 626.

Time = 0.41 (sec) , antiderivative size = 7604, normalized size of antiderivative = 22.17

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output

```
-2/45045*(45045*sqrt(-c*e*x + c*d - b*e)*c^2*d^5*e*f - 90090*sqrt(-c*e*x +
 c*d - b*e)*b*c*d^4*e^2*f + 45045*sqrt(-c*e*x + c*d - b*e)*b^2*d^3*e^3*f +
 15015*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e -
(-c*e*x + c*d - b*e)^(3/2))*c*d^4*e*f - 60060*(3*sqrt(-c*e*x + c*d - b*e)*
c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b*d^3*e
^2*f + 45045*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*
b*e - (-c*e*x + c*d - b*e)^(3/2))*b^2*d^2*e^3*f/c + 15015*(3*sqrt(-c*e*x +
 c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3
/2))*c*d^5*g - 30030*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d
 - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b*d^4*e*g + 15015*(3*sqrt(-c*e*x
 + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^
(3/2))*b^2*d^3*e^2*g/c - 6006*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sq
rt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*
(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*
e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))*d^3*e*f + 9009*(15*sqrt(-c*e*
x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*
e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x
+ c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))
*b^2*d*e^3*f/c^2 + 3003*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*
e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c...
```

Mupad [B] (verification not implemented)

Time = 12.64 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.86

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = -\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{2\,e^2\,x^5\,\sqrt{d+e\,x}\,(14\,b\,e\,g+25\,c\,d\,g+13\,c\,e\,f)}{143}+\frac{2\,c\,e^3\,g\,x^6\,\sqrt{d+e\,x}}{13}+\frac{x^2\,\sqrt{d+e\,x}\,(96\,g\,b^4\,c^2\,e^6-852\,g}{}\right.}{}$$

input

```
int((f + g*x)*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),
x)
```

output

```
-((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*e^2*x^5*(d + e*x)^(1/2)*
(14*b*e*g + 25*c*d*g + 13*c*e*f))/143 + (2*c*e^3*g*x^6*(d + e*x)^(1/2))/13
 + (x^2*(d + e*x)^(1/2)*(96*b^4*c^2*e^6*g - 156*b^3*c^3*e^6*f - 13572*c^6*
d^3*e^3*f - 5442*c^6*d^4*e^2*g + 18408*b*c^5*d^2*e^4*f + 1326*b^2*c^4*d*e^
5*f + 3360*b*c^5*d^3*e^3*g - 852*b^3*c^3*d*e^5*g + 2838*b^2*c^4*d^2*e^4*g)
)/(15015*c^5*e^3) + (2*(b*e - c*d)^2*(d + e*x)^(1/2)*(128*b^4*e^4*g + 2754
*c^4*d^4*g - 208*b^3*c*e^4*f + 6929*c^4*d^3*e*f - 5530*b*c^3*d^3*e*g - 113
6*b^3*c*d*e^3*g - 5486*b*c^3*d^2*e^2*f + 1768*b^2*c^2*d*e^3*f + 3784*b^2*c
^2*d^2*e^2*g))/(15015*c^5*e^3) + (2*e*x^4*(d + e*x)^(1/2)*(b^2*e^2*g - 12*
c^2*d^2*g + 52*b*c*e^2*f + 91*c^2*d*e*f + 154*b*c*d*e*g))/(429*c) + (x^3*(
d + e*x)^(1/2)*(130*b^2*c^4*e^6*f - 80*b^3*c^3*e^6*g - 1300*c^6*d^2*e^4*f
- 9540*c^6*d^3*e^3*g + 14040*b*c^5*d*e^5*f + 13280*b*c^5*d^2*e^4*g + 630*b
^2*c^4*d*e^5*g))/(15015*c^5*e^3) + (2*x*(b*e - c*d)*(d + e*x)^(1/2)*(104*b
^3*c*e^4*f - 1377*c^4*d^4*g - 64*b^4*e^4*g + 4043*c^4*d^3*e*f + 2765*b*c^3
*d^3*e*g + 568*b^3*c*d*e^3*g + 2743*b*c^3*d^2*e^2*f - 884*b^2*c^2*d*e^3*f
- 1892*b^2*c^2*d^2*e^2*g))/(15015*c^4*e^2)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.05

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \frac{2\sqrt{-cex-be+cd}\,(-1155c^6e^6g\,x^6-1470b\,c^5e^6g\,x^5-2625c^6d\,e^5g\,x^5-1365c^6e^6f\,x^5-3}{}$$

input

```
int((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(2*sqrt( - b*e + c*d - c*e*x)*( - 128*b**6*e**6*g + 1392*b**5*c*d*e**5*g +
 208*b**5*c*e**6*f + 64*b**5*c*e**6*g*x - 6184*b**4*c**2*d**2*e**4*g - 218
4*b**4*c**2*d*e**5*f - 632*b**4*c**2*d*e**5*g*x - 104*b**4*c**2*e**6*f*x -
 48*b**4*c**2*e**6*g*x**2 + 14234*b**3*c**3*d**3*e**3*g + 9230*b**3*c**3*d
**2*e**4*f + 2460*b**3*c**3*d**2*e**4*g*x + 988*b**3*c**3*d*e**5*f*x + 426
*b**3*c**3*d*e**5*g*x**2 + 78*b**3*c**3*e**6*f*x**2 + 40*b**3*c**3*e**6*g*
x**3 - 17598*b**2*c**4*d**4*e**2*g - 19669*b**2*c**4*d**3*e**3*f - 4657*b*
*2*c**4*d**3*e**3*g*x - 3627*b**2*c**4*d**2*e**4*f*x - 1419*b**2*c**4*d**2
*e**4*g*x**2 - 663*b**2*c**4*d*e**5*f*x**2 - 315*b**2*c**4*d*e**5*g*x**3 -
 65*b**2*c**4*e**6*f*x**3 - 35*b**2*c**4*e**6*g*x**4 + 11038*b*c**5*d**5*e
*g + 19344*b*c**5*d**4*e**2*f + 4142*b*c**5*d**4*e**2*g*x - 1300*b*c**5*d*
*3*e**3*f*x - 1680*b*c**5*d**3*e**3*g*x**2 - 9204*b*c**5*d**2*e**4*f*x**2
- 6640*b*c**5*d**2*e**4*g*x**3 - 7020*b*c**5*d*e**5*f*x**3 - 5390*b*c**5*d
*e**5*g*x**4 - 1820*b*c**5*e**6*f*x**4 - 1470*b*c**5*e**6*g*x**5 - 2754*c*
*6*d**6*g - 6929*c**6*d**5*e*f - 1377*c**6*d**5*e*g*x + 4043*c**6*d**4*e**
2*f*x + 2721*c**6*d**4*e**2*g*x**2 + 6786*c**6*d**3*e**3*f*x**2 + 4770*c**
6*d**3*e**3*g*x**3 + 650*c**6*d**2*e**4*f*x**3 + 420*c**6*d**2*e**4*g*x**4
 - 3185*c**6*d*e**5*f*x**4 - 2625*c**6*d*e**5*g*x**5 - 1365*c**6*e**6*f*x*
*5 - 1155*c**6*e**6*g*x**6))/(15015*c**5*e**2)
```

3.206 $\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,d$

Optimal result

Integrand size = 46, antiderivative size = 267

$$\begin{aligned}
&\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \\
&-\frac{2(2cd-be)^2(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^4e^2(d+ex)^{5/2}} \\
&+\frac{2(2cd-be)(2cef+4cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^4e^2(d+ex)^{7/2}} \\
&-\frac{2(cef+5cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^4e^2(d+ex)^{9/2}} \\
&+\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}{11c^4e^2(d+ex)^{11/2}}
\end{aligned}$$

output
```
-2/5*(-b*e+2*c*d)^2*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(5/2)/c^4/e^2/(e*x+d)^(5/2)+2/7*(-b*e+2*c*d)*(-3*b*e*g+4*c*d*g+2*c*e*f)*(d
*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^4/e^2/(e*x+d)^(7/2)-2/9*(-3*b*e*g+5
*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(9/2)/c^4/e^2/(e*x+d)^(9/2)
+2/11*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(11/2)/c^4/e^2/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{2(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}(-48b^3e^3g+8b^2ce^2(11ef+40dg+15egx)-2bc^2e(347}{}$$

input
```
Integrate[Sqrt[d + e*x]*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3
/2),x]
```

output
```
(-2*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-48*b
^3*e^3*g + 8*b^2*c*e^2*(11*e*f + 40*d*g + 15*e*g*x) - 2*b*c^2*e*(347*d^2*g
 + 5*e^2*x*(22*f + 21*g*x) + d*e*(286*f + 340*g*x)) + c^3*(422*d^3*g + 35*
e^3*x^2*(11*f + 9*g*x) + 10*d*e^2*x*(121*f + 98*g*x) + d^2*e*(1177*f + 105
5*g*x))))/(3465*c^4*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1221, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-6beg+cdg+11cef)\int \sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{11ce}-\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-6beg+cdg+11cef)\left(\frac{4(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{\sqrt{d+ex}}dx}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)}{11ce}-$$

$$\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11ce^2}$$

$$\Big\downarrow 1128$$

$$\frac{(-6beg+cdg+11cef)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{3/2}}dx}{7c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)}{9c}-\frac{2\left(d(cd-be)-be^2x-\right.}{9ce\sqrt{d+e}}\right.}{11ce}$$

$$\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11ce^2}$$

$$\Big\downarrow 1122$$

$$\frac{\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{35c^2e(d+ex)^{5/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)(-6beg+cdg+}{11ce}$$

$$\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{11ce^2}$$

input
```
Int[Sqrt[d + e*x]*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*g*Sqrt[d + e*x]*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(11*c*e^2
) + ((11*c*e*f + c*d*g - 6*b*e*g)*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^
2)^(5/2))/(9*c*e*Sqrt[d + e*x]) + (4*(2*c*d - b*e)*((-4*(2*c*d - b*e)*(d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(35*c^2*e*(d + e*x)^(5/2)) - (2*(
d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(7*c*e*(d + e*x)^(3/2))))/(9*c
)))/(11*c*e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.86

method	result
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(cex+be-cd)^2\left(-315e^3g\,x^3c^3+210b\,c^2e^3g\,x^2-980c^3d\,e^2g\,x^2-385c^3e^3f\,x^2-120b^2c\,e^3gx+680b\,c^2d\,e^2gx\right.}{3465\sqrt{}}$
gosper	$-\frac{2(cex+be-cd)\left(-315e^3g\,x^3c^3+210b\,c^2e^3g\,x^2-980c^3d\,e^2g\,x^2-385c^3e^3f\,x^2-120b^2c\,e^3gx+680b\,c^2d\,e^2gx+220b\,c^2e^3fx-1055c^3d\right.}{346}$
orering	$-\frac{2(cex+be-cd)\left(-315e^3g\,x^3c^3+210b\,c^2e^3g\,x^2-980c^3d\,e^2g\,x^2-385c^3e^3f\,x^2-120b^2c\,e^3gx+680b\,c^2d\,e^2gx+220b\,c^2e^3fx-1055c^3d\right.}{346}$

input
```
int((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output

```
2/3465/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(c*e*x+b*e-c*d)^2*(-
315*c^3*e^3*g*x^3+210*b*c^2*e^3*g*x^2-980*c^3*d*e^2*g*x^2-385*c^3*e^3*f*x^
2-120*b^2*c*e^3*g*x+680*b*c^2*d*e^2*g*x+220*b*c^2*e^3*f*x-1055*c^3*d^2*e*g
*x-1210*c^3*d*e^2*f*x+48*b^3*e^3*g-320*b^2*c*d*e^2*g-88*b^2*c*e^3*f+694*b*
c^2*d^2*e*g+572*b*c^2*d*e^2*f-422*c^3*d^3*g-1177*c^3*d^2*e*f)/c^4/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(243) = 486$.

Time = 0.09 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.89

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{2\,(315\,c^5e^5gx^5+35\,(11\,c^5e^5f+2\,(5\,c^5de^4+6\,bc^4e^5)g)x^4+5\,(22\,(4\,c^5de^4+5\,bc^4e^5)f-(118\,c^5d^2e^3-214}{}$$

input

```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output

```
-2/3465*(315*c^5*e^5*g*x^5 + 35*(11*c^5*e^5*f + 2*(5*c^5*d*e^4 + 6*b*c^4*e
^5)*g)*x^4 + 5*(22*(4*c^5*d*e^4 + 5*b*c^4*e^5)*f - (118*c^5*d^2*e^3 - 214*
b*c^4*d*e^4 - 3*b^2*c^3*e^5)*g)*x^3 - 3*(11*(26*c^5*d^2*e^3 - 46*b*c^4*d*e
^4 - b^2*c^3*e^5)*f + 2*(118*c^5*d^3*e^2 - 101*b*c^4*d^2*e^3 - 20*b^2*c^3*
d*e^4 + 3*b^3*c^2*e^5)*g)*x^2 + 11*(107*c^5*d^4*e - 266*b*c^4*d^3*e^2 + 21
9*b^2*c^3*d^2*e^3 - 68*b^3*c^2*d*e^4 + 8*b^4*c*e^5)*f + 2*(211*c^5*d^5 - 7
69*b*c^4*d^4*e + 1065*b^2*c^3*d^3*e^2 - 691*b^3*c^2*d^2*e^3 + 208*b^4*c*d*
e^4 - 24*b^5*e^5)*g - (22*(52*c^5*d^3*e^2 - 39*b*c^4*d^2*e^3 - 15*b^2*c^3*
d*e^4 + 2*b^3*c^2*e^5)*f - (211*c^5*d^4*e - 558*b*c^4*d^3*e^2 + 507*b^2*c^
3*d^2*e^3 - 184*b^3*c^2*d*e^4 + 24*b^4*c*e^5)*g)*x)*sqrt(-c*e^2*x^2 - b*e^
2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^4*e^3*x + c^4*d*e^2)
```

Sympy [F]

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\int\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}\sqrt{d+ex}(f+gx)\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*sqrt(d + e*x)*(f + g*x),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(243) = 486.

Time = 0.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.88

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=$$
$$-\frac{2\left(35\,c^4e^4x^4+107\,c^4d^4-266\,bc^3d^3e+219\,b^2c^2d^2e^2-68\,b^3cde^3+8\,b^4e^4+10\,(4\,c^4de^3+5\,bc^3e^4)x^3-3\,(2\right.}{315\,(c}$$
$$-\frac{2\left(315\,c^5e^5x^5+422\,c^5d^5-1538\,bc^4d^4e+2130\,b^2c^3d^3e^2-1382\,b^3c^2d^2e^3+416\,b^4cde^4-48\,b^5e^5+70\,(5\,c^5\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output

```
-2/315*(35*c^4*e^4*x^4 + 107*c^4*d^4 - 266*b*c^3*d^3*e + 219*b^2*c^2*d^2*e
^2 - 68*b^3*c*d*e^3 + 8*b^4*e^4 + 10*(4*c^4*d*e^3 + 5*b*c^3*e^4)*x^3 - 3*(
26*c^4*d^2*e^2 - 46*b*c^3*d*e^3 - b^2*c^2*e^4)*x^2 - 2*(52*c^4*d^3*e - 39*
b*c^3*d^2*e^2 - 15*b^2*c^2*d*e^3 + 2*b^3*c*e^4)*x)*sqrt(-c*e*x + c*d - b*e
)*(e*x + d)*f/(c^3*e^2*x + c^3*d*e) - 2/3465*(315*c^5*e^5*x^5 + 422*c^5*d^
5 - 1538*b*c^4*d^4*e + 2130*b^2*c^3*d^3*e^2 - 1382*b^3*c^2*d^2*e^3 + 416*b
^4*c*d*e^4 - 48*b^5*e^5 + 70*(5*c^5*d*e^4 + 6*b*c^4*e^5)*x^4 - 5*(118*c^5*
d^2*e^3 - 214*b*c^4*d*e^4 - 3*b^2*c^3*e^5)*x^3 - 6*(118*c^5*d^3*e^2 - 101*
b*c^4*d^2*e^3 - 20*b^2*c^3*d*e^4 + 3*b^3*c^2*e^5)*x^2 + (211*c^5*d^4*e - 5
58*b*c^4*d^3*e^2 + 507*b^2*c^3*d^2*e^3 - 184*b^3*c^2*d*e^4 + 24*b^4*c*e^5)
*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d)*g/(c^4*e^3*x + c^4*d*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3758 vs. $2(243) = 486$.

Time = 0.37 (sec) , antiderivative size = 3758, normalized size of antiderivative = 14.07

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output
```
-2/3465*(3465*sqrt(-c*e*x + c*d - b*e)*c^2*d^4*e*f - 6930*sqrt(-c*e*x + c*
d - b*e)*b*c*d^3*e^2*f + 3465*sqrt(-c*e*x + c*d - b*e)*b^2*d^2*e^3*f - 231
0*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e
*x + c*d - b*e)^(3/2))*b*d^2*e^2*f + 2310*(3*sqrt(-c*e*x + c*d - b*e)*c*d
- 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b^2*d*e^3*f
/c + 1155*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e
 - (-c*e*x + c*d - b*e)^(3/2))*c*d^4*g - 2310*(3*sqrt(-c*e*x + c*d - b*e)*
c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b*d^3*e
*g + 1155*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e
 - (-c*e*x + c*d - b*e)^(3/2))*b^2*d^2*e^2*g/c - 462*(15*sqrt(-c*e*x + c*d
 - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c
*d - b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d -
 b*e)^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))*d^2*e*
f + 462*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)
*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(
3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqr
t(-c*e*x + c*d - b*e))*b*d*e^2*f/c + 231*(15*sqrt(-c*e*x + c*d - b*e)*c^2*
d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^
2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*
b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))*b^2*e^3*f/c^2 -...
```

Mupad [B] (verification not implemented)

Time = 12.11 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.65

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\Bigg(\frac{2\,e\,x^4\,\sqrt{d+e\,x}\,\left(12\,b\,e\,g+10\,c\,d\,g+11\,c\,e\,f\right)}{99}+\frac{2\,x^3\,\sqrt{d+e\,x}\,\left(3\,g\,b^2\,e^2+214\,g\,b\,c\,d\,e+110\,f\,b\,c\,e^2-118\,g}{693\,c}}{}$$

input
```
int((f + g*x)*(d + e*x)^(1/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),
x)
```

output
```
-((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*e*x^4*(d + e*x)^(1/2)*(1
2*b*e*g + 10*c*d*g + 11*c*e*f))/99 + (2*x^3*(d + e*x)^(1/2)*(3*b^2*e^2*g -
 118*c^2*d^2*g + 110*b*c*e^2*f + 88*c^2*d*e*f + 214*b*c*d*e*g))/(693*c) +
(2*c*e^2*g*x^5*(d + e*x)^(1/2))/11 + (x^2*(d + e*x)^(1/2)*(66*b^2*c^3*e^5*
f - 36*b^3*c^2*e^5*g - 1716*c^5*d^2*e^3*f - 1416*c^5*d^3*e^2*g + 3036*b*c^
4*d*e^4*f + 1212*b*c^4*d^2*e^3*g + 240*b^2*c^3*d*e^4*g))/(3465*c^4*e^3) +
(2*(b*e - c*d)^2*(d + e*x)^(1/2)*(422*c^3*d^3*g - 48*b^3*e^3*g + 88*b^2*c*
e^3*f + 1177*c^3*d^2*e*f - 572*b*c^2*d*e^2*f - 694*b*c^2*d^2*e*g + 320*b^2
*c*d*e^2*g))/(3465*c^4*e^3) + (2*x*(b*e - c*d)*(d + e*x)^(1/2)*(24*b^3*e^3
*g - 211*c^3*d^3*g - 44*b^2*c*e^3*f + 1144*c^3*d^2*e*f + 286*b*c^2*d*e^2*f
 + 347*b*c^2*d^2*e*g - 160*b^2*c*d*e^2*g))/(3465*c^3*e^2)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.87

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx = \frac{2\sqrt{-cex-be+cd}\,\left(-315c^5e^5g\,x^5-420b\,c^4e^5g\,x^4-350c^5d\,e^4g\,x^4-385c^5e^5f\,x^4-15b^2c\right.}{}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*(48*b**5*e**5*g - 416*b**4*c*d*e**4*g - 88*b
**4*c*e**5*f - 24*b**4*c*e**5*g*x + 1382*b**3*c**2*d**2*e**3*g + 748*b**3*
c**2*d*e**4*f + 184*b**3*c**2*d*e**4*g*x + 44*b**3*c**2*e**5*f*x + 18*b**3
*c**2*e**5*g*x**2 - 2130*b**2*c**3*d**3*e**2*g - 2409*b**2*c**3*d**2*e**3*
f - 507*b**2*c**3*d**2*e**3*g*x - 330*b**2*c**3*d*e**4*f*x - 120*b**2*c**3
*d*e**4*g*x**2 - 33*b**2*c**3*e**5*f*x**2 - 15*b**2*c**3*e**5*g*x**3 + 153
8*b*c**4*d**4*e*g + 2926*b*c**4*d**3*e**2*f + 558*b*c**4*d**3*e**2*g*x - 8
58*b*c**4*d**2*e**3*f*x - 606*b*c**4*d**2*e**3*g*x**2 - 1518*b*c**4*d*e**4
*f*x**2 - 1070*b*c**4*d*e**4*g*x**3 - 550*b*c**4*e**5*f*x**3 - 420*b*c**4*
e**5*g*x**4 - 422*c**5*d**5*g - 1177*c**5*d**4*e*f - 211*c**5*d**4*e*g*x +
 1144*c**5*d**3*e**2*f*x + 708*c**5*d**3*e**2*g*x**2 + 858*c**5*d**2*e**3*
f*x**2 + 590*c**5*d**2*e**3*g*x**3 - 440*c**5*d*e**4*f*x**3 - 350*c**5*d*e
**4*g*x**4 - 385*c**5*e**5*f*x**4 - 315*c**5*e**5*g*x**5))/(3465*c**4*e**2
)
```

3.207 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 190

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = &-\frac{2(2cd-be)(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^3e^2(d+ex)^{5/2}}\\ &+\frac{2(cef+3cdg-2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^3e^2(d+ex)^{7/2}}\\ &-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^3e^2(d+ex)^{9/2}}\end{aligned}$$

output
```
-2/5*(-b*e+2*c*d)*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5
/2)/c^3/e^2/(e*x+d)^(5/2)+2/7*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2
*x-c*e^2*x^2)^(7/2)/c^3/e^2/(e*x+d)^(7/2)-2/9*g*(d*(-b*e+c*d)-b*e^2*x-c*e^
2*x^2)^(9/2)/c^3/e^2/(e*x+d)^(9/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.64

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx =$$
$$-\frac{2(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}(8b^2e^2g-2bce(9ef+17dg+10egx)+c^2(26d^2g+5e^2x}{315c^3e^2\sqrt{d+ex}}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/Sqrt[d +
 e*x],x]
```

output
```
(-2*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(8*b^2
*e^2*g - 2*b*c*e*(9*e*f + 17*d*g + 10*e*g*x) + c^2*(26*d^2*g + 5*e^2*x*(9*
f + 7*g*x) + d*e*(81*f + 65*g*x))))/(315*c^3*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}dx$$
$$\Big\downarrow \text{1221}$$
$$\frac{(-4beg-cdg+9cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{\sqrt{d+ex}}dx}{9ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9ce^2\sqrt{d+ex}}$$
$$\Big\downarrow \text{1128}$$

$$\frac{(-4beg-cdg+9cef)\left(\frac{2(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{3/2}}dx}{7c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)}{9ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9ce^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{35c^2e(d+ex)^{5/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)(-4beg-cdg+9cef)}{9ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{9ce^2\sqrt{d+ex}}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/Sqrt[d + e*x],
x]
```

output

```
(-2*g*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(9*c*e^2*Sqrt[d + e*x])
 + ((9*c*e*f - c*d*g - 4*b*e*g)*((-4*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*
x - c*e^2*x^2)^(5/2))/(35*c^2*e*(d + e*x)^(5/2)) - (2*(d*(c*d - b*e) - b*e
^2*x - c*e^2*x^2)^(5/2))/(7*c*e*(d + e*x)^(3/2))))/(9*c*e)
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(cex+be-cd)^2\left(35g\,x^2c^2e^2-20bc\,e^2gx+65c^2degx+45c^2e^2fx+8b^2e^2g-34bcdeg-18bc\,e^2f+26c^2d^2g+\right.}{315\sqrt{ex+d}\,c^3e^2}$
gosper	$\frac{2(cex+be-cd)\left(35g\,x^2c^2e^2-20bc\,e^2gx+65c^2degx+45c^2e^2fx+8b^2e^2g-34bcdeg-18bc\,e^2f+26c^2d^2g+81c^2def\right)\left(-x^2c\,e^2-xb\,e^2-bd\right.}{315c^3e^2(ex+d)^{\frac{3}{2}}}$
orering	$\frac{2(cex+be-cd)\left(35g\,x^2c^2e^2-20bc\,e^2gx+65c^2degx+45c^2e^2fx+8b^2e^2g-34bcdeg-18bc\,e^2f+26c^2d^2g+81c^2def\right)\left(-x^2c\,e^2-xb\,e^2-bd\right.}{315c^3e^2(ex+d)^{\frac{3}{2}}}$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(1/2),x,method=
_RETURNVERBOSE)
```

output

```
-2/315*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/(e*x+d)^(1/2)*(c*e*x+b*e-c*d)^2*(3
5*c^2*e^2*g*x^2-20*b*c*e^2*g*x+65*c^2*d*e*g*x+45*c^2*e^2*f*x+8*b^2*e^2*g-3
4*b*c*d*e*g-18*b*c*e^2*f+26*c^2*d^2*g+81*c^2*d*e*f)/c^3/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(172) = 344.

Time = 0.10 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.86

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx =$$

$$-\frac{2\left(35\,c^4e^4gx^4+5\left(9\,c^4e^4f-(c^4de^3-10\,bc^3e^4)g\right)x^3-3\left(3\,(c^4de^3-8\,bc^3e^4)f+(23\,c^4d^2e^2-22\,bc^3de^3-\right.\right.}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

output

```
-2/315*(35*c^4*e^4*g*x^4 + 5*(9*c^4*e^4*f - (c^4*d*e^3 - 10*b*c^3*e^4)*g)*
x^3 - 3*(3*(c^4*d*e^3 - 8*b*c^3*e^4)*f + (23*c^4*d^2*e^2 - 22*b*c^3*d*e^3
- b^2*c^2*e^4)*g)*x^2 + 9*(9*c^4*d^3*e - 20*b*c^3*d^2*e^2 + 13*b^2*c^2*d*e
^3 - 2*b^3*c*e^4)*f + 2*(13*c^4*d^4 - 43*b*c^3*d^3*e + 51*b^2*c^2*d^2*e^2
- 25*b^3*c*d*e^3 + 4*b^4*e^4)*g - (9*(13*c^4*d^2*e^2 - 12*b*c^3*d*e^3 - b^
2*c^2*e^4)*f - (13*c^4*d^3*e - 30*b*c^3*d^2*e^2 + 21*b^2*c^2*d*e^3 - 4*b^3
*c*e^4)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^
3*e^3*x + c^3*d*e^2)
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=\int \frac{\left(-(d+ex)\,(be-cd+cex)\right)^{\frac{3}{2}}(f+gx)}{\sqrt{d+ex}}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**(1/
2),x)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/sqrt(d + e*x),
x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.68

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=$$

$$-\frac{2\,(5\,c^3e^3x^3+9\,c^3d^3-20\,bc^2d^2e+13\,b^2cde^2-2\,b^3e^3-(c^3de^2-8\,bc^2e^3)x^2-(13\,c^3d^2e-12\,bc^2de^2-b^2ce}{35\,c^2e}$$

$$-\frac{2\,(35\,c^4e^4x^4+26\,c^4d^4-86\,bc^3d^3e+102\,b^2c^2d^2e^2-50\,b^3cde^3+8\,b^4e^4-5\,(c^4de^3-10\,bc^3e^4)x^3-3\,(23\,c^4}{315\,c^3e}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

output

```
-2/35*(5*c^3*e^3*x^3 + 9*c^3*d^3 - 20*b*c^2*d^2*e + 13*b^2*c*d*e^2 - 2*b^3
*e^3 - (c^3*d*e^2 - 8*b*c^2*e^3)*x^2 - (13*c^3*d^2*e - 12*b*c^2*d*e^2 - b^
2*c*e^3)*x)*sqrt(-c*e*x + c*d - b*e)*f/(c^2*e) - 2/315*(35*c^4*e^4*x^4 + 2
6*c^4*d^4 - 86*b*c^3*d^3*e + 102*b^2*c^2*d^2*e^2 - 50*b^3*c*d*e^3 + 8*b^4*
e^4 - 5*(c^4*d*e^3 - 10*b*c^3*e^4)*x^3 - 3*(23*c^4*d^2*e^2 - 22*b*c^3*d*e^
3 - b^2*c^2*e^4)*x^2 + (13*c^4*d^3*e - 30*b*c^3*d^2*e^2 + 21*b^2*c^2*d*e^3
 - 4*b^3*c*e^4)*x)*sqrt(-c*e*x + c*d - b*e)*g/(c^3*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1123 vs. $2(172) = 344$.

Time = 0.38 (sec) , antiderivative size = 1123, normalized size of antiderivative = 5.91

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(1/2),x,
algorithm="giac")
```

output

```
-2/315*(105*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*d^2*e*f - 105*(-(e*x + d)*c
 + 2*c*d - b*e)^(3/2)*b*d*e^2*f/c - 21*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/
2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d
 + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*b*e^2*f/c^2 + 21*(5*(-(e*x + d
)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e -
3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*d^2*g/c
- 21*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d -
 b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*
d - b*e))*b*d*e*g/c^2 - 3*(35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*d^2 -
 70*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c*d*e + 35*(-(e*x + d)*c + 2*c*d
- b*e)^(3/2)*b^2*e^2 - 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c
+ 2*c*d - b*e)*c*d + 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c +
2*c*d - b*e)*b*e - 15*((e*x + d)*c - 2*c*d + b*e)^3*sqrt(-(e*x + d)*c + 2*
c*d - b*e))*e*f/c^2 - 3*(35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*d^2 - 7
0*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c*d*e + 35*(-(e*x + d)*c + 2*c*d -
b*e)^(3/2)*b^2*e^2 - 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c +
2*c*d - b*e)*c*d + 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*
c*d - b*e)*b*e - 15*((e*x + d)*c - 2*c*d + b*e)^3*sqrt(-(e*x + d)*c + 2*c*
d - b*e))*b*e*g/c^3 - (105*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^3*d^3 - 31
5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^2*d^2*e + 315*(-(e*x + d)*c + ...
```

Mupad [B] (verification not implemented)

Time = 11.75 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.26

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx =$$

$$-\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{2\,e\,x^3\,\left(10\,b\,e\,g-c\,d\,g+9\,c\,e\,f\right)}{63}+\frac{2\,x^2\,\left(g\,b^2\,e^2+22\,g\,b\,c\,d\,e+24\,f\,b\,c\,e^2-23\,g\,c^2\,d^2-3\,f\,c^2\,d\,e\right)}{105\,c}+2\,c\,e}{}$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(1/2
),x)
```

output
```
-((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*e*x^3*(10*b*e*g - c*d*g
+ 9*c*e*f))/63 + (2*x^2*(b^2*e^2*g - 23*c^2*d^2*g + 24*b*c*e^2*f - 3*c^2*d
*e*f + 22*b*c*d*e*g))/(105*c) + (2*c*e^2*g*x^4)/9 + (2*(b*e - c*d)^2*(8*b^
2*e^2*g + 26*c^2*d^2*g - 18*b*c*e^2*f + 81*c^2*d*e*f - 34*b*c*d*e*g))/(315
*c^3*e^2) + (2*x*(b*e - c*d)*(9*b*c*e^2*f - 13*c^2*d^2*g - 4*b^2*e^2*g + 1
17*c^2*d*e*f + 17*b*c*d*e*g))/(315*c^2*e)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.75

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{-cex-be+cd}\,\left(-35c^4e^4g\,x^4-50b\,c^3e^4g\,x^3+5c^4d\,e^3g\right.}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*( - 8*b**4*e**4*g + 50*b**3*c*d*e**3*g + 18*
b**3*c*e**4*f + 4*b**3*c*e**4*g*x - 102*b**2*c**2*d**2*e**2*g - 117*b**2*c
**2*d*e**3*f - 21*b**2*c**2*d*e**3*g*x - 9*b**2*c**2*e**4*f*x - 3*b**2*c**
2*e**4*g*x**2 + 86*b*c**3*d**3*e*g + 180*b*c**3*d**2*e**2*f + 30*b*c**3*d*
*2*e**2*g*x - 108*b*c**3*d*e**3*f*x - 66*b*c**3*d*e**3*g*x**2 - 72*b*c**3*
e**4*f*x**2 - 50*b*c**3*e**4*g*x**3 - 26*c**4*d**4*g - 81*c**4*d**3*e*f -
13*c**4*d**3*e*g*x + 117*c**4*d**2*e**2*f*x + 69*c**4*d**2*e**2*g*x**2 + 9
*c**4*d*e**3*f*x**2 + 5*c**4*d*e**3*g*x**3 - 45*c**4*e**4*f*x**3 - 35*c**4
*e**4*g*x**4))/(315*c**3*e**2)
```

3.208 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 116

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{2(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^2e^2(d+ex)^{5/2}} + \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^2e^2(d+ex)^{7/2}}$$

output
```
-2/5*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^2/e^2/(
e*x+d)^(5/2)+2/7*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^2/e^2/(e*x+d)^
(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{2(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}(-2beg+c(7ef+2dg+5egx))}{35c^2e^2\sqrt{d+ex}}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^(3/2),x]
```

output
```
(-2*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-2*b*
e*g + c*(7*e*f + 2*d*g + 5*e*g*x)))/(35*c^2*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-2beg-3cdg+7cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{3/2}}dx}{7ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}(-2beg-3cdg+7cef)}{35c^2e^2(d+ex)^{5/2}}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{7ce^2(d+ex)^{3/2}}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^(3/2
),x]
```

output

```
(-2*(7*c*e*f - 3*c*d*g - 2*b*e*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5
/2))/(35*c^2*e^2*(d + e*x)^(5/2)) - (2*g*(d*(c*d - b*e) - b*e^2*x - c*e^2*
x^2)^(5/2))/(7*c*e^2*(d + e*x)^(3/2))
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2\sqrt{-\left(e x +d \right) \left(c e x +b e -c d \right)}\, \left(c e x +b e -c d \right)^{2} \left(-5 c e g x +2 b e g -2 c d g -7 f c e \right)}{35 \sqrt{e x +d}\, c^{2} e^{2}}$	73
gosper	$-\frac{2 \left(c e x +b e -c d \right) \left(-5 c e g x +2 b e g -2 c d g -7 f c e \right) \left(-x^{2} c \,e^{2}-x b \,e^{2}-b d e +c \,d^{2}\right)^{\frac{3}{2}}}{35 c^{2} e^{2} \left(e x +d \right)^{\frac{3}{2}}}$	79
orering	$-\frac{2 \left(c e x +b e -c d \right) \left(-5 c e g x +2 b e g -2 c d g -7 f c e \right) \left(-x^{2} c \,e^{2}-x b \,e^{2}-b d e +c \,d^{2}\right)^{\frac{3}{2}}}{35 c^{2} e^{2} \left(e x +d \right)^{\frac{3}{2}}}$	79

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2/35*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/(e*x+d)^(1/2)*(c*e*x+b*e-c*d)^2*(-5*
c*e*g*x+2*b*e*g-2*c*d*g-7*c*e*f)/c^2/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(104) = 208$.

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.97

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{2\left(5\,c^3e^3gx^3+\left(7\,c^3e^3f-8\left(c^3de^2-bc^2e^3\right)g\right)x^2+7\left(c^3d^2e-2\,bc^2de^2+b^2ce^3\right)f+2\left(c^3d^3-3\,bc^2d^2e+3\,b\right.\right.}{35\left(c^2e\right.}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="fricas")
```

output
```
-2/35*(5*c^3*e^3*g*x^3 + (7*c^3*e^3*f - 8*(c^3*d*e^2 - b*c^2*e^3)*g)*x^2 +
 7*(c^3*d^2*e - 2*b*c^2*d*e^2 + b^2*c*e^3)*f + 2*(c^3*d^3 - 3*b*c^2*d^2*e
+ 3*b^2*c*d*e^2 - b^3*e^3)*g - (14*(c^3*d*e^2 - b*c^2*e^3)*f - (c^3*d^2*e
- 2*b*c^2*d*e^2 + b^2*c*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d
*e)*sqrt(e*x + d)/(c^2*e^3*x + c^2*d*e^2)
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left(-(d+ex)\,(be-cd+cex)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**(3/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**(3/2
), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx =$$
$$-\frac{2\,(c^2e^2x^2+c^2d^2-2\,bcde+b^2e^2-2\,(c^2de-bce^2)x)\sqrt{-cex+cd-be}f}{5\,ce}$$
$$-\frac{2\,(5\,c^3e^3x^3+2\,c^3d^3-6\,bc^2d^2e+6\,b^2cde^2-2\,b^3e^3-8\,(c^3de^2-bc^2e^3)x^2+(c^3d^2e-2\,bc^2de^2+b^2ce^3)x)\sqrt{}}{35\,c^2e^2}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="maxima")
```

output
```
-2/5*(c^2*e^2*x^2 + c^2*d^2 - 2*b*c*d*e + b^2*e^2 - 2*(c^2*d*e - b*c*e^2)*
x)*sqrt(-c*e*x + c*d - b*e)*f/(c*e) - 2/35*(5*c^3*e^3*x^3 + 2*c^3*d^3 - 6*
b*c^2*d^2*e + 6*b^2*c*d*e^2 - 2*b^3*e^3 - 8*(c^3*d*e^2 - b*c^2*e^3)*x^2 +
(c^3*d^2*e - 2*b*c^2*d*e^2 + b^2*c*e^3)*x)*sqrt(-c*e*x + c*d - b*e)*g/(c^2
*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(104) = 208.

Time = 0.35 (sec) , antiderivative size = 540, normalized size of antiderivative = 4.66

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx =$$
$$-\frac{2\left(35\,(-(ex+d)c+2\,cd-be)^{\frac{3}{2}}def-\frac{35\,(-(ex+d)c+2\,cd-be)^{\frac{3}{2}}be^2f}{c}-\frac{7\left(5\,(-(ex+d)c+2\,cd-be)^{\frac{3}{2}}cd-5\,(-(ex+d)c+2\,cd-b\right.}{}\right.}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="giac")
```

output
```
-2/105*(35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*d*e*f - 35*(-(e*x + d)*c + 2
*c*d - b*e)^(3/2)*b*e^2*f/c - 7*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d
- 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)
^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*e*f/c + 7*(5*(-(e*x + d)*c + 2*c*d -
b*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c
 - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*d*g/c - 7*(5*(-(e*x +
d)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e -
 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*b*e*g/c
^2 - (35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*d^2 - 70*(-(e*x + d)*c + 2
*c*d - b*e)^(3/2)*b*c*d*e + 35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*e^2
- 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c*d +
42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*e - 15
*((e*x + d)*c - 2*c*d + b*e)^3*sqrt(-(e*x + d)*c + 2*c*d - b*e))*g/c^2)/e^
2
```

Mupad [B] (verification not implemented)

Time = 11.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx =$$

$$-\frac{\left(x^2\left(\frac{16\,b\,e\,g}{35}-\frac{16\,c\,d\,g}{35}+\frac{2\,c\,e\,f}{5}\right)+\frac{2\,c\,e\,g\,x^3}{7}+\frac{2\,(b\,e-c\,d)^2\,(2\,c\,d\,g-2\,b\,e\,g+7\,c\,e\,f)}{35\,c^2\,e^2}+\frac{2\,x\,(b\,e-c\,d)\,(b\,e\,g-c\,d\,g+14\,c\,e\,f)}{35\,c\,e}\right)\,\sqrt{c\,d^2-}}{\sqrt{d+e\,x}}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(3/2
),x)
```

output
```
-((x^2*((16*b*e*g)/35 - (16*c*d*g)/35 + (2*c*e*f)/5) + (2*c*e*g*x^3)/7 + (
2*(b*e - c*d)^2*(2*c*d*g - 2*b*e*g + 7*c*e*f))/(35*c^2*e^2) + (2*x*(b*e -
c*d)*(b*e*g - c*d*g + 14*c*e*f))/(35*c*e))*(c*d^2 - c*e^2*x^2 - b*d*e - b*
e^2*x)^(1/2))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.72

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = 2\sqrt{-cex-be+cd}\,(-5c^3e^3g\,x^3-8b\,c^2e^3g\,x^2+8c^3d\,e^2g\,x^2$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*(2*b**3*e**3*g - 6*b**2*c*d*e**2*g - 7*b**2*
c*e**3*f - b**2*c*e**3*g*x + 6*b*c**2*d**2*e*g + 14*b*c**2*d*e**2*f + 2*b*
c**2*d*e**2*g*x - 14*b*c**2*e**3*f*x - 8*b*c**2*e**3*g*x**2 - 2*c**3*d**3*
g - 7*c**3*d**2*e*f - c**3*d**2*e*g*x + 14*c**3*d*e**2*f*x + 8*c**3*d*e**2
*g*x**2 - 7*c**3*e**3*f*x**2 - 5*c**3*e**3*g*x**3))/(35*c**2*e**2)
```

3.209 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 250

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx=\frac{2(2cd-be)(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2\sqrt{d+ex}}$$
$$+\frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{3/2}}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5ce^2(d+ex)^{5/2}}$$
$$-\frac{2(2cd-be)^{3/2}(ef-dg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{e^2}$$

output

```
2*(-b*e+2*c*d)*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+
d)^(1/2)+2/3*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)
^(3/2)-2/5*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c/e^2/(e*x+d)^(5/2)-2*
(-b*e+2*c*d)^(3/2)*(-d*g+e*f)*arctanh((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/
2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.78

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx=\frac{2((d+ex)(-be+c(d-ex)))^{3/2}\left(\frac{-3b^2e^2g-2bce(10ef-13dg+3eg}{c}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^(5/2),x]
```

output
```
(2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*((-3*b^2*e^2*g - 2*b*c*e*(10*e
*f - 13*d*g + 3*e*g*x) + c^2*(-38*d^2*g - e^2*x*(5*f + 3*g*x) + d*e*(35*f
+ 11*g*x)))/(c*(-(b*e) + c*(d - e*x))) - (15*(-2*c*d + b*e)^(3/2)*(-(e*f)
+ d*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/(-(b*e) + c*(d
- e*x))^(3/2)))/(15*e^2*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1221, 1131, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(ef-dg)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{5/2}}dx}{e}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5ce^2(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(ef-dg)\left((2cd-be)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{3/2}}dx+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}}\right)}{e}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5ce^2(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(ef-dg)\left((2cd-be)\left((2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}\right)+\frac{2(d(cd-be)-be^2x-ce}{3e(d+ex)^{3/2}}\right.}{e}$$
$$\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5ce^2(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(ef-dg)\left((2cd-be)\left(2e(2cd-be)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}+\frac{2\sqrt{d(cd-be)-be^2}}{e\sqrt{d+ex}}\right.\right.}{e}$$
$$\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5ce^2(d+ex)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{(ef-dg)\left((2cd-be)\left(\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2cd-be}\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e}\right)+\frac{2(d(cd-be)-be^2x-ce^2x^2)}{3e(d+ex)^{3/2}}\right.}{e}$$
$$\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5ce^2(d+ex)^{5/2}}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^(5/2
),x]
```

output
```
(-2*g*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(5*c*e^2*(d + e*x)^(5/2
)) + ((e*f - d*g)*((2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(3*e*(d
 + e*x)^(3/2)) + (2*c*d - b*e)*((2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^
2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2*c*d - b*e]*ArcTanh[Sqrt[d*(c*d - b*e) -
b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/e)))/e
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(226) = 452$.

Time = 1.72 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.37

method	result
default	$-\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,\left(3c^2e^2g\,x^2\sqrt{-cex-be+cd}\,\sqrt{be-2cd}+15\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)b^2cd\,e^2g-15\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)b\right.}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(5/2),x,method=
_RETURNVERBOSE)
```

output

```
-2/15*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(3*c^2*e^2*g*x^2*(-c*e*x-b*e+c*d)^(
1/2)*(b*e-2*c*d)^(1/2)+15*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))
*b^2*c*d*e^2*g-15*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c*e
^3*f-60*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d^2*e*g+60*
arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d*e^2*f+60*arctan((
-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^3*g-60*arctan((-c*e*x-b*e+c
*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^2*e*f+6*b*c*e^2*g*x*(-c*e*x-b*e+c*d)^(1
/2)*(b*e-2*c*d)^(1/2)-11*c^2*d*e*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1
/2)+5*c^2*e^2*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+3*b^2*e^2*g*(-c
*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-26*b*c*d*e*g*(-c*e*x-b*e+c*d)^(1/2)*
(b*e-2*c*d)^(1/2)+20*b*c*e^2*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+38
*c^2*d^2*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-35*c^2*d*e*f*(-c*e*x-b
*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2))/(e*x+d)^(1/2)/(-c*e*x-b*e+c*d)^(1/2)/e^2/
c/(b*e-2*c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.63

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \left[\frac{15\sqrt{2\,cd-be}\left((2\,c^2d^2e-bcde^2)f-(2\,c^2d^3-bcd^2e)g+(\right.}{}\right.$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(5/2),x,
algorithm="fricas")
```

output
```
[1/15*(15*sqrt(2*c*d - b*e)*((2*c^2*d^2*e - b*c*d*e^2)*f - (2*c^2*d^3 - b*
c*d^2*e)*g + ((2*c^2*d*e^2 - b*c*e^3)*f - (2*c^2*d^2*e - b*c*d*e^2)*g)*x)*
log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x + 2*sqrt(-c*e^2*
x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2 +
 2*d*e*x + d^2)) - 2*(3*c^2*e^2*g*x^2 - 5*(7*c^2*d*e - 4*b*c*e^2)*f + (38*
c^2*d^2 - 26*b*c*d*e + 3*b^2*e^2)*g + (5*c^2*e^2*f - (11*c^2*d*e - 6*b*c*e
^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d))/(c*e^3
*x + c*d*e^2), -2/15*(15*sqrt(-2*c*d + b*e)*((2*c^2*d^2*e - b*c*d*e^2)*f -
 (2*c^2*d^3 - b*c*d^2*e)*g + ((2*c^2*d*e^2 - b*c*e^3)*f - (2*c^2*d^2*e - b
*c*d*e^2)*g)*x)*arctan(-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(-2
*c*d + b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)) + (3*c^
2*e^2*g*x^2 - 5*(7*c^2*d*e - 4*b*c*e^2)*f + (38*c^2*d^2 - 26*b*c*d*e + 3*b
^2*e^2)*g + (5*c^2*e^2*f - (11*c^2*d*e - 6*b*c*e^2)*g)*x)*sqrt(-c*e^2*x^2
- b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d))/(c*e^3*x + c*d*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**(5/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**(5/2
), x)
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \int \frac{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(gx+f)}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(5/2),x,
algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(g*x + f)/(e*x + d)
^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.26

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{15\left(4c^2d^2ef-4bcde^2f+b^2e^3f-4c^2d^3g+4bcd^2eg-b^2de^2g\right)\arctan\left(\frac{\sqrt{-(ex}}{\sqrt{-}}\right.}{\sqrt{-2cd+be}}\right.}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(5/2),x,
algorithm="giac")
```

output
```
2/15*(15*(4*c^2*d^2*e*f - 4*b*c*d*e^2*f + b^2*e^3*f - 4*c^2*d^3*g + 4*b*c*
d^2*e*g - b^2*d*e^2*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(-2*c*d
 + b*e))/sqrt(-2*c*d + b*e) + (30*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d*e
*f - 15*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^5*e^2*f - 30*sqrt(-(e*x + d)*
c + 2*c*d - b*e)*c^6*d^2*g + 15*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^5*d*e
*g + 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^5*e*f - 5*(-(e*x + d)*c + 2*c*
d - b*e)^(3/2)*c^5*d*g - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c
 + 2*c*d - b*e)*c^4*g)/c^5)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{(d+e\,x)^{5/2}}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(5/2
),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(5/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.57

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{-2\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bcdeg+2\sqrt{be-2cd}\,atan}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(5/2),x)
```

output
```
(2*( - 15*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c
*d))*b*c*d*e*g + 15*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt
(b*e - 2*c*d))*b*c*e**2*f + 30*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d -
c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*g - 30*sqrt(b*e - 2*c*d)*atan(sqrt( -
b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d*e*f - 3*sqrt( - b*e + c*d - c
*e*x)*b**2*e**2*g + 26*sqrt( - b*e + c*d - c*e*x)*b*c*d*e*g - 20*sqrt( - b
*e + c*d - c*e*x)*b*c*e**2*f - 6*sqrt( - b*e + c*d - c*e*x)*b*c*e**2*g*x -
 38*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*g + 35*sqrt( - b*e + c*d - c*e*x)
*c**2*d*e*f + 11*sqrt( - b*e + c*d - c*e*x)*c**2*d*e*g*x - 5*sqrt( - b*e +
 c*d - c*e*x)*c**2*e**2*f*x - 3*sqrt( - b*e + c*d - c*e*x)*c**2*e**2*g*x**
2))/(15*c*e**2)
```

3.210 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 258

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx =&\\ &-\frac{(2cd-be)(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)^{3/2}}\\ &-\frac{2(cef-3cdg+beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2\sqrt{d+ex}}\\ &+\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{3/2}}\\ &+\frac{\sqrt{2cd-be}(3cef-7cdg+2beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{e^2}\end{aligned}$$

output

```
-(-b*e+2*c*d)*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d
)^(3/2)-2*(b*e*g-3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2
/(e*x+d)^(1/2)+2/3*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^(3
/2)+(-b*e+2*c*d)^(1/2)*(2*b*e*g-7*c*d*g+3*c*e*f)*arctanh((d*(-b*e+c*d)-b*e
^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.74

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx =$$

$$-\frac{\sqrt{(d+ex)(-be+c(d-ex))}\Big(\sqrt{cd-be-cex}(be(-3ef+11dg+8egx)+2c(-13d^2g+de(6f-9gx)+}{3e^2(d+ex)^{3/2}\sqrt{-be+c(}}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^(7/2),x]
```

output

```
-1/3*(Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(Sqrt[c*d - b*e - c*e*x]*(b*e
*(-3*e*f + 11*d*g + 8*e*g*x) + 2*c*(-13*d^2*g + d*e*(6*f - 9*g*x) + e^2*x*
(3*f + g*x))) + 3*Sqrt[-2*c*d + b*e]*(-3*c*e*f + 7*c*d*g - 2*b*e*g)*(d + e
*x)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]]))/(e^2*(d + e*x)^(3
/2)*Sqrt[-(b*e) + c*(d - e*x)])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1220, 1131, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(2beg-7cdg+3cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{5/2}}dx}{2e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^{7/2}(2cd-be)}$$

$$\Big\downarrow \; 1131$$

$$-\frac{(2beg-7cdg+3cef)\left((2cd-be)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{3/2}}dx+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}}\right)}{2e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^{7/2}(2cd-be)}$$

$$\Big\downarrow \; 1131$$

$$-\frac{(2beg-7cdg+3cef)\left((2cd-be)\left((2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}\right)+\frac{2(d(}{}\right.}{2e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^{7/2}(2cd-be)}$$

$$\Big\downarrow \; 1136$$

$$-\frac{(2beg-7cdg+3cef)\left((2cd-be)\left(2e(2cd-be)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}+\frac{2}{}\right.\right.}{2e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^{7/2}(2cd-be)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left((2cd-be)\left(\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2cd-be}\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e}\right)+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}}\right)(2b}{2e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e^2(d+ex)^{7/2}(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^(7/2
),x]
```

output

```
-(((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(e^2*(2*c*d -
b*e)*(d + e*x)^(7/2))) - ((3*c*e*f - 7*c*d*g + 2*b*e*g)*((2*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) + (2*c*d - b*e)*((2*S
qrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2*c*
d - b*e]*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b
*e]*Sqrt[d + e*x])])/e)))/(2*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1131

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. 2(236) = 472.

Time = 1.62 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.66

method	result
default	$\sqrt{-(ex+d)(cex+be-cd)}\left(6\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)b^2e^3gx-33\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bcde^2gx+9\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bce^3fx+\right.$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(7/2),x,method=
_RETURNVERBOSE)
```

output
```
1/3*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(6*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e
-2*c*d)^(1/2))*b^2*e^3*g*x-33*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1
/2))*b*c*d*e^2*g*x+9*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c*
e^3*f*x+42*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^2*e*g*x-
18*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d*e^2*f*x-2*c*e^2*
g*x^2*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)+6*arctan((-c*e*x-b*e+c*d)^(
1/2)/(b*e-2*c*d)^(1/2))*b^2*d*e^2*g-33*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-
2*c*d)^(1/2))*b*c*d^2*e*g+9*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2
))*b*c*d*e^2*f+42*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^3
*g-18*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^2*e*f-8*b*e^2
*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+18*c*d*e*g*x*(-c*e*x-b*e+c*d
)^(1/2)*(b*e-2*c*d)^(1/2)-6*c*e^2*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(
1/2)-11*b*d*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+3*b*e^2*f*(-c*e*x
-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+26*c*d^2*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2
*c*d)^(1/2)-12*c*d*e*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2))/(e*x+d)^(
3/2)/(-c*e*x-b*e+c*d)^(1/2)/e^2/(b*e-2*c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.48

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx = \left[\frac{3\left(3cd^2ef+\left(3ce^3f-\left(7cde^2-2be^3\right)g\right)x^2-\left(7cd^3-2b\right.\right.}{}\right.$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(7/2),x,
algorithm="fricas")
```

output

```
[1/6*(3*(3*c*d^2*e*f + (3*c*e^3*f - (7*c*d*e^2 - 2*b*e^3)*g)*x^2 - (7*c*d^
3 - 2*b*d^2*e)*g + 2*(3*c*d*e^2*f - (7*c*d^2*e - 2*b*d*e^2)*g)*x)*sqrt(2*c
*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*sq
rt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/
(e^2*x^2 + 2*d*e*x + d^2)) - 2*(2*c*e^2*g*x^2 + 3*(4*c*d*e - b*e^2)*f - (2
6*c*d^2 - 11*b*d*e)*g + 2*(3*c*e^2*f - (9*c*d*e - 4*b*e^2)*g)*x)*sqrt(-c*e
^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d))/(e^4*x^2 + 2*d*e^3*x + d^
2*e^2), 1/3*(3*(3*c*d^2*e*f + (3*c*e^3*f - (7*c*d*e^2 - 2*b*e^3)*g)*x^2 -
(7*c*d^3 - 2*b*d^2*e)*g + 2*(3*c*d*e^2*f - (7*c*d^2*e - 2*b*d*e^2)*g)*x)*s
qrt(-2*c*d + b*e)*arctan(-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(
-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)) - (2*
c*e^2*g*x^2 + 3*(4*c*d*e - b*e^2)*f - (26*c*d^2 - 11*b*d*e)*g + 2*(3*c*e^2
*f - (9*c*d*e - 4*b*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*
sqrt(e*x + d))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)]
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^{\frac{7}{2}}}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**(7/
2),x)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**(7/2
), x)
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=\int \frac{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(gx+f)}{(ex+d)^{\frac{7}{2}}}\,dx$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(7/2),x,
algorithm="maxima")
```

output

```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(g*x + f)/(e*x + d)
^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.25

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=$$

$$-\frac{6\,\sqrt{-(ex+d)c+2\,cd-be}c^2ef-18\,\sqrt{-(ex+d)c+2\,cd-be}c^2dg+6\,\sqrt{-(ex+d)c+2\,cd-be}bceg-}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(7/2),x,
algorithm="giac")
```

output

```
-1/3*(6*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^2*e*f - 18*sqrt(-(e*x + d)*c +
2*c*d - b*e)*c^2*d*g + 6*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c*e*g - 2*(-(e
*x + d)*c + 2*c*d - b*e)^(3/2)*c*g + 3*(6*c^3*d*e*f - 3*b*c^2*e^2*f - 14*c
^3*d^2*g + 11*b*c^2*d*e*g - 2*b^2*c*e^2*g)*arctan(sqrt(-(e*x + d)*c + 2*c*
d - b*e)/sqrt(-2*c*d + b*e))/sqrt(-2*c*d + b*e) + 3*(2*sqrt(-(e*x + d)*c +
 2*c*d - b*e)*c^3*d*e*f - sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^2*e^2*f - 2
*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^3*d^2*g + sqrt(-(e*x + d)*c + 2*c*d -
b*e)*b*c^2*d*e*g)/((e*x + d)*c))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{(d+e\,x)^{7/2}}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(7/2
),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(7/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.71

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=6\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bdeg+6\sqrt{be-2cd}\,atan\left(\right.$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(7/2),x)
```

output
```
(6*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*
d*e*g + 6*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c
*d))*b*e**2*g*x - 21*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqr
t(b*e - 2*c*d))*c*d**2*g + 9*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*
e*x)/sqrt(b*e - 2*c*d))*c*d*e*f - 21*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e +
c*d - c*e*x)/sqrt(b*e - 2*c*d))*c*d*e*g*x + 9*sqrt(b*e - 2*c*d)*atan(sqrt(
 - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c*e**2*f*x - 11*sqrt( - b*e + c*d
 - c*e*x)*b*d*e*g + 3*sqrt( - b*e + c*d - c*e*x)*b*e**2*f - 8*sqrt( - b*e
+ c*d - c*e*x)*b*e**2*g*x + 26*sqrt( - b*e + c*d - c*e*x)*c*d**2*g - 12*sq
rt( - b*e + c*d - c*e*x)*c*d*e*f + 18*sqrt( - b*e + c*d - c*e*x)*c*d*e*g*x
 - 6*sqrt( - b*e + c*d - c*e*x)*c*e**2*f*x - 2*sqrt( - b*e + c*d - c*e*x)*
c*e**2*g*x**2)/(3*e**2*(d + e*x))
```

3.211 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 257

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx=\frac{(3cef-11cdg+4beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4e^2(d+ex)^{3/2}}-\frac{2cg\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2\sqrt{d+ex}}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e^2(d+ex)^{7/2}}-\frac{3c(cef-9cdg+4beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{4e^2\sqrt{2cd-be}}$$

output

```
1/4*(4*b*e*g-11*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/
(e*x+d)^(3/2)-2*c*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)^(1/
2)-1/2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^(7/2)
-3/4*c*(4*b*e*g-9*c*d*g+c*e*f)*arctanh((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1
/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2/(-b*e+2*c*d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.75

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx=\frac{c((d+ex)(-be+c(d-ex)))^{3/2}\Bigg(\frac{c(-17d^2g+de(f-29gx)+e^2x(5f}{c(d+ex)^2(-be}}{4e^2(d+}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^(9/2),x]
```

output
```
(c*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*((c*(-17*d^2*g + d*e*(f - 29*g
*x) + e^2*x*(5*f - 8*g*x)) + 2*b*e*(d*g + e*(f + 2*g*x)))/(c*(d + e*x)^2*(
-(b*e) + c*(d - e*x))) + (3*(c*e*f - 9*c*d*g + 4*b*e*g)*ArcTan[Sqrt[c*d -
b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/(Sqrt[-2*c*d + b*e]*(-(b*e) + c*(d - e*x
))^(3/2))))/(4*e^2*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1220, 1130, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{(4beg-9cdg+cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{7/2}}dx}{4e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{2e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \text{1130}$$

$$-\frac{(4beg-9cdg+cef)\left(-\frac{3}{2}c\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{3/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}\right)}{4e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{2e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \; 1131$$

$$-\frac{(4beg-9cdg+cef)\left(-\frac{3}{2}c\left((2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}\right)-\frac{(d(cd-be)-be}{e(d+}\right.}{4e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{2e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \; 1136$$

$$-\frac{(4beg-9cdg+cef)\left(-\frac{3}{2}c\left(2e(2cd-be)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}+\frac{2\sqrt{d(cd-b}}{e}\right.\right.}{4e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{2e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left(-\frac{3}{2}c\left(\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2cd-be}\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e}\right)-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}\right)(4beg-9}{4e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{2e^2(d+ex)^{9/2}(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^(9/2
),x]
```

output
```
-1/2*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(9/2)) - ((c*e*f - 9*c*d*g + 4*b*e*g)*(-((d*(c*d - b*e)
- b*e^2*x - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(5/2))) - (3*c*((2*Sqrt[d*(c*d -
 b*e) - b*e^2*x - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2*c*d - b*e]*Arc
Tanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d +
 e*x])])/e))/2))/(4*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(231) = 462$.

Time = 1.54 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.56

method	result
default	$\sqrt{-(ex+d)(cex+be-cd)}\,\left(12\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bc\,e^{3}g\,x^{2}-27\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c^{2}d\,e^{2}g\,x^{2}+3\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c^{2}e$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(9/2),x,method=
_RETURNVERBOSE)
```

output
```
1/4*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(12*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*
e-2*c*d)^(1/2))*b*c*e^3*g*x^2-27*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)
^(1/2))*c^2*d*e^2*g*x^2+3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))
*c^2*e^3*f*x^2+24*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c*d*e
^2*g*x-54*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^2*e*g*x+6
*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d*e^2*f*x-8*c*e^2*g*
x^2*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)+12*arctan((-c*e*x-b*e+c*d)^(1
/2)/(b*e-2*c*d)^(1/2))*b*c*d^2*e*g-27*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2
*c*d)^(1/2))*c^2*d^3*g+3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*
c^2*d^2*e*f+4*b*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-29*c*d*e*
g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+5*c*e^2*f*x*(-c*e*x-b*e+c*d)^
(1/2)*(b*e-2*c*d)^(1/2)+2*b*d*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)
+2*b*e^2*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-17*c*d^2*g*(-c*e*x-b*e
+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+c*d*e*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(
1/2))/(e*x+d)^(5/2)/(-c*e*x-b*e+c*d)^(1/2)/e^2/(b*e-2*c*d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(231) = 462$.

Time = 0.12 (sec) , antiderivative size = 1000, normalized size of antiderivative = 3.89

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(9/2),x,
algorithm="fricas")
```

output

```
[1/8*(3*(c^2*d^3*e*f + (c^2*e^4*f - (9*c^2*d*e^3 - 4*b*c*e^4)*g)*x^3 + 3*(
c^2*d*e^3*f - (9*c^2*d^2*e^2 - 4*b*c*d*e^3)*g)*x^2 - (9*c^2*d^4 - 4*b*c*d^
3*e)*g + 3*(c^2*d^2*e^2*f - (9*c^2*d^3*e - 4*b*c*d^2*e^2)*g)*x)*sqrt(2*c*d
 - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x + 2*sqrt
(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e
^2*x^2 + 2*d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(8
*(2*c^2*d*e^2 - b*c*e^3)*g*x^2 - (2*c^2*d^2*e + 3*b*c*d*e^2 - 2*b^2*e^3)*f
 + (34*c^2*d^3 - 21*b*c*d^2*e + 2*b^2*d*e^2)*g - (5*(2*c^2*d*e^2 - b*c*e^3
)*f - (58*c^2*d^2*e - 37*b*c*d*e^2 + 4*b^2*e^3)*g)*x)*sqrt(e*x + d))/(2*c*
d^4*e^2 - b*d^3*e^3 + (2*c*d*e^5 - b*e^6)*x^3 + 3*(2*c*d^2*e^4 - b*d*e^5)*
x^2 + 3*(2*c*d^3*e^3 - b*d^2*e^4)*x), -1/4*(3*(c^2*d^3*e*f + (c^2*e^4*f -
(9*c^2*d*e^3 - 4*b*c*e^4)*g)*x^3 + 3*(c^2*d*e^3*f - (9*c^2*d^2*e^2 - 4*b*c
*d*e^3)*g)*x^2 - (9*c^2*d^4 - 4*b*c*d^3*e)*g + 3*(c^2*d^2*e^2*f - (9*c^2*d
^3*e - 4*b*c*d^2*e^2)*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqrt(-c*e^2*x^2 - b
*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e
+ (2*c*d*e - b*e^2)*x)) + sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(8*(2
*c^2*d*e^2 - b*c*e^3)*g*x^2 - (2*c^2*d^2*e + 3*b*c*d*e^2 - 2*b^2*e^3)*f +
(34*c^2*d^3 - 21*b*c*d^2*e + 2*b^2*d*e^2)*g - (5*(2*c^2*d*e^2 - b*c*e^3)*f
 - (58*c^2*d^2*e - 37*b*c*d*e^2 + 4*b^2*e^3)*g)*x)*sqrt(e*x + d))/(2*c*d^4
*e^2 - b*d^3*e^3 + (2*c*d*e^5 - b*e^6)*x^3 + 3*(2*c*d^2*e^4 - b*d*e^5)*...
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**(9/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**(9/2
), x)
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx=\int \frac{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(gx+f)}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(9/2),x,
algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(g*x + f)/(e*x + d)
^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.30

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx=$$

$$-\frac{8\sqrt{-(ex+d)c+2\,cd-be}\,c^2g-\frac{3\left(c^3ef-9\,c^3dg+4\,bc^2eg\right)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{\sqrt{-2\,cd+be}}-\frac{6\sqrt{-(ex+d)c+2\,cd-be}\,c^4def-3\sqrt{-}}{}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(9/2),x,
algorithm="giac")
```

output
```
-1/4*(8*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^2*g - 3*(c^3*e*f - 9*c^3*d*g +
4*b*c^2*e*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(-2*c*d + b*e))/s
qrt(-2*c*d + b*e) - (6*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4*d*e*f - 3*sqrt
(-(e*x + d)*c + 2*c*d - b*e)*b*c^3*e^2*f - 22*sqrt(-(e*x + d)*c + 2*c*d -
b*e)*c^4*d^2*g + 19*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^3*d*e*g - 4*sqrt(
-(e*x + d)*c + 2*c*d - b*e)*b^2*c^2*e^2*g - 5*(-(e*x + d)*c + 2*c*d - b*e)
^(3/2)*c^3*e*f + 13*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^3*d*g - 4*(-(e*x
+ d)*c + 2*c*d - b*e)^(3/2)*b*c^2*e*g)/((e*x + d)^2*c^2))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{(d+e\,x)^{9/2}}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(9/2
),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(9/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 788, normalized size of antiderivative = 3.07

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx=\frac{12\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bc\,d^2eg+24\sqrt{be-2cd}\,at}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(9/2),x)
```

output

```
(12*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b
*c*d**2*e*g + 24*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*
e - 2*c*d))*b*c*d*e**2*g*x + 12*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d -
 c*e*x)/sqrt(b*e - 2*c*d))*b*c*e**3*g*x**2 - 27*sqrt(b*e - 2*c*d)*atan(sqr
t( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**3*g + 3*sqrt(b*e - 2*c*
d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*e*f - 54*s
qrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d
**2*e*g*x + 6*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e -
 2*c*d))*c**2*d*e**2*f*x - 27*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c
*e*x)/sqrt(b*e - 2*c*d))*c**2*d*e**2*g*x**2 + 3*sqrt(b*e - 2*c*d)*atan(sqr
t( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*e**3*f*x**2 + 2*sqrt( - b*
e + c*d - c*e*x)*b**2*d*e**2*g + 2*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*f
+ 4*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*g*x - 21*sqrt( - b*e + c*d - c*e*
x)*b*c*d**2*e*g - 3*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*f - 37*sqrt( - b
*e + c*d - c*e*x)*b*c*d*e**2*g*x + 5*sqrt( - b*e + c*d - c*e*x)*b*c*e**3*f
*x - 8*sqrt( - b*e + c*d - c*e*x)*b*c*e**3*g*x**2 + 34*sqrt( - b*e + c*d -
 c*e*x)*c**2*d**3*g - 2*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*e*f + 58*sqrt
( - b*e + c*d - c*e*x)*c**2*d**2*e*g*x - 10*sqrt( - b*e + c*d - c*e*x)*c**
2*d*e**2*f*x + 16*sqrt( - b*e + c*d - c*e*x)*c**2*d*e**2*g*x**2)/(4*e**2*(
b*d**2*e + 2*b*d*e**2*x + b*e**3*x**2 - 2*c*d**3 - 4*c*d**2*e*x - 2*c*d...
```

3.212 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 285

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx &= \frac{(cef-5cdg+2beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4e^2(d+ex)^{5/2}}\\ &-\frac{c(cef-21cdg+10beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{8e^2(2cd-be)(d+ex)^{3/2}}\\ &-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{9/2}}\\ &-\frac{c^2(cef+11cdg-6beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{8e^2(2cd-be)^{3/2}}\end{aligned}$$

output

```
1/4*(2*b*e*g-5*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*
x+d)^(5/2)-1/8*c*(10*b*e*g-21*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(1/2)/e^2/(-b*e+2*c*d)/(e*x+d)^(3/2)-1/3*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*
x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^(9/2)-1/8*c^2*(-6*b*e*g+11*c*d*g+c*e*f)*arc
tanh((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/
2))/e^2/(-b*e+2*c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx=\frac{c^2((d+ex)(-be+c(d-ex)))^{3/2}\left(\frac{-4b^2e^2(2ef+dg+3egx)+2bce^2}{}\right.}{}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^(11/2),x]
```

output

```
(c^2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*((-4*b^2*e^2*(2*e*f + d*g +
3*e*g*x) + 2*b*c*e^2*(d*(9*f + g*x) - e*x*(7*f + 15*g*x)) + c^2*(19*d^3*g
- 3*e^3*f*x^2 + d^2*e*(-7*f + 50*g*x) + d*e^2*x*(22*f + 63*g*x)))/(c^2*(2*
c*d - b*e)*(d + e*x)^3*(-(b*e) + c*(d - e*x))) - (3*(c*e*f + 11*c*d*g - 6*
b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/((-2*c*d + b*e)
^(3/2)*(-(b*e) + c*(d - e*x))^(3/2))))/(24*e^2*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1220, 1130, 1130, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(-6beg+11cdg+cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{9/2}}dx}{6e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(-6beg+11cdg+cef)\left(-\frac{3}{4}c\int\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{5/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}\right)}{6e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(-6beg+11cdg+cef)\left(-\frac{3}{4}c\left(-\frac{1}{2}c\int\frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}}\right)-\frac{\left(d(cd-be)-be^2x-ce^2\right.}{2e(d+ex)^{7/2}}\right.}{6e(2cd-be)}$$
$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(-6beg+11cdg+cef)\left(-\frac{3}{4}c\left(-ce\int\frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}-\frac{\sqrt{d(cd-be)-be^2x-}}{e(d+ex)^{3/2}}\right.\right.}{6e(2cd-be)}$$
$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(-\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e\sqrt{2cd-be}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}}\right)-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}\right)(-6beg+11cdg+c}{6e(2cd-be)}$$
$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^{11/2}(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^(11/
2),x]
```

output

```
-1/3*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(11/2)) + ((c*e*f + 11*c*d*g - 6*b*e*g)*(-1/2*(d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(7/2)) - (3*c*(-(Sqrt[d*(c*
d - b*e) - b*e^2*x - c*e^2*x^2]/(e*(d + e*x)^(3/2))) + (c*ArcTanh[Sqrt[d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/(e*S
qrt[2*c*d - b*e])))/4))/(6*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. 2(257) = 514.

Time = 1.56 (sec) , antiderivative size = 991, normalized size of antiderivative = 3.48

method	result
default	$\frac{\left(30bc\,e^3 g\,x^2\sqrt{-cex-be+cd}\,\sqrt{be-2cd}-63c^2 d\,e^2 g\,x^2\sqrt{-cex-be+cd}\,\sqrt{be-2cd}-2bcd\,e^2 gx\sqrt{-cex-be+cd}\,\sqrt{be-2cd}-19c^2 d^3 g\sqrt{-cex}\right.}{}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(11/2),x,method
=_RETURNVERBOSE)
```

output
```
1/24*(30*b*c*e^3*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-63*c^2*d*e
^2*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-2*b*c*d*e^2*g*x*(-c*e*x-
b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-19*c^2*d^3*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-
2*c*d)^(1/2)-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*e^4*f*
x^3-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^3*e*f+54*arct
an((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d*e^3*g*x^2+54*arctan((
-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d^2*e^2*g*x-18*b*c*d*e^2*f*
(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-22*c^2*d*e^2*f*x*(-c*e*x-b*e+c*d)
^(1/2)*(b*e-2*c*d)^(1/2)-50*c^2*d^2*e*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*
d)^(1/2)+14*b*c*e^3*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-33*arctan
((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^4*g+18*arctan((-c*e*x-b*e
+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*e^4*g*x^3-33*arctan((-c*e*x-b*e+c*d)^
(1/2)/(b*e-2*c*d)^(1/2))*c^3*d*e^3*g*x^3+8*b^2*e^3*f*(-c*e*x-b*e+c*d)^(1/2
)*(b*e-2*c*d)^(1/2)-9*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3
*d*e^3*f*x^2-99*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^3*e
*g*x-9*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^2*e^2*f*x+12
*b^2*e^3*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+4*b^2*d*e^2*g*(-c*e*
x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+7*c^2*d^2*e*f*(-c*e*x-b*e+c*d)^(1/2)*(b
*e-2*c*d)^(1/2)+3*c^2*e^3*f*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+1
8*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d^3*e*g-99*arc...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(257) = 514$.

Time = 0.14 (sec) , antiderivative size = 1456, normalized size of antiderivative = 5.11

$$\int \frac{(f+gx)\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}{(d+ex)^{11/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(11/2),x,
 algorithm="fricas")
```

output

```
[1/48*(3*(c^3*d^4*e*f + (c^3*e^5*f + (11*c^3*d*e^4 - 6*b*c^2*e^5)*g)*x^4 +
 4*(c^3*d*e^4*f + (11*c^3*d^2*e^3 - 6*b*c^2*d*e^4)*g)*x^3 + 6*(c^3*d^2*e^3
*f + (11*c^3*d^3*e^2 - 6*b*c^2*d^2*e^3)*g)*x^2 + (11*c^3*d^5 - 6*b*c^2*d^4
*e)*g + 4*(c^3*d^3*e^2*f + (11*c^3*d^4*e - 6*b*c^2*d^3*e^2)*g)*x)*sqrt(2*c
*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x + 2*sq
rt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/
(e^2*x^2 + 2*d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*
(3*((2*c^3*d*e^3 - b*c^2*e^4)*f - (42*c^3*d^2*e^2 - 41*b*c^2*d*e^3 + 10*b^
2*c*e^4)*g)*x^2 + (14*c^3*d^3*e - 43*b*c^2*d^2*e^2 + 34*b^2*c*d*e^3 - 8*b^
3*e^4)*f - (38*c^3*d^4 - 19*b*c^2*d^3*e - 8*b^2*c*d^2*e^2 + 4*b^3*d*e^3)*g
 - 2*((22*c^3*d^2*e^2 - 25*b*c^2*d*e^3 + 7*b^2*c*e^4)*f + (50*c^3*d^3*e -
23*b*c^2*d^2*e^2 - 13*b^2*c*d*e^3 + 6*b^3*e^4)*g)*x)*sqrt(e*x + d))/(4*c^2
*d^6*e^2 - 4*b*c*d^5*e^3 + b^2*d^4*e^4 + (4*c^2*d^2*e^6 - 4*b*c*d*e^7 + b^
2*e^8)*x^4 + 4*(4*c^2*d^3*e^5 - 4*b*c*d^2*e^6 + b^2*d*e^7)*x^3 + 6*(4*c^2*
d^4*e^4 - 4*b*c*d^3*e^5 + b^2*d^2*e^6)*x^2 + 4*(4*c^2*d^5*e^3 - 4*b*c*d^4*
e^4 + b^2*d^3*e^5)*x), -1/24*(3*(c^3*d^4*e*f + (c^3*e^5*f + (11*c^3*d*e^4
- 6*b*c^2*e^5)*g)*x^4 + 4*(c^3*d*e^4*f + (11*c^3*d^2*e^3 - 6*b*c^2*d*e^4)*
g)*x^3 + 6*(c^3*d^2*e^3*f + (11*c^3*d^3*e^2 - 6*b*c^2*d^2*e^3)*g)*x^2 + (1
1*c^3*d^5 - 6*b*c^2*d^4*e)*g + 4*(c^3*d^3*e^2*f + (11*c^3*d^4*e - 6*b*c^2*
d^3*e^2)*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqrt(-c*e^2*x^2 - b*e^2*x + c...
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx=\int \frac{\left(-(d+ex)\,(be-cd+cex)\right)^{\frac{3}{2}}(f+gx)}{(d+ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**(11
/2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(f + g*x)/(d + e*x)**(11/
2), x)
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx=\int \frac{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(gx+f)}{(ex+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(11/2),x,
 algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(g*x + f)/(e*x + d)
^(11/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(257) = 514$.

Time = 0.44 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.08

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx=\frac{\frac{3\,(c^4ef+11\,c^4dg-6\,bc^3eg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{(2\,cd-be)\sqrt{-2\,cd+be}}+\frac{12\,\sqrt{-(ex+d)c+2}}{}}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(11/2),x,
 algorithm="giac")
```

output

```
1/24*(3*(c^4*e*f + 11*c^4*d*g - 6*b*c^3*e*g)*arctan(sqrt(-(e*x + d)*c + 2*
c*d - b*e)/sqrt(-2*c*d + b*e))/((2*c*d - b*e)*sqrt(-2*c*d + b*e)) + (12*sq
rt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d^2*e*f - 12*sqrt(-(e*x + d)*c + 2*c*d
- b*e)*b*c^5*d*e^2*f + 3*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^4*e^3*f +
132*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d^3*g - 204*sqrt(-(e*x + d)*c + 2
*c*d - b*e)*b*c^5*d^2*e*g + 105*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^4*d
*e^2*g - 18*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^3*e^3*g - 16*(-(e*x + d
)*c + 2*c*d - b*e)^(3/2)*c^5*d*e*f + 8*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*
b*c^4*e^2*f - 176*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^5*d^2*g + 184*(-(e*
x + d)*c + 2*c*d - b*e)^(3/2)*b*c^4*d*e*g - 48*(-(e*x + d)*c + 2*c*d - b*e
)^(3/2)*b^2*c^3*e^2*g - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c
+ 2*c*d - b*e)*c^4*e*f + 63*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*
c + 2*c*d - b*e)*c^4*d*g - 30*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d
)*c + 2*c*d - b*e)*b*c^3*e*g)/((2*c*d - b*e)*(e*x + d)^3*c^3))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{(d+e\,x)^{11/2}}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(11/
2),x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(11/
2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1239, normalized size of antiderivative = 4.35

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(11/2),x)
```

output

```
(18*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b
*c**2*d**3*e*g + 54*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt
(b*e - 2*c*d))*b*c**2*d**2*e**2*g*x + 54*sqrt(b*e - 2*c*d)*atan(sqrt( - b*
e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d*e**3*g*x**2 + 18*sqrt(b*e - 2
*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*e**4*g*x**
3 - 33*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*c**3*d**4*g - 3*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b
*e - 2*c*d))*c**3*d**3*e*f - 99*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d -
 c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**3*e*g*x - 9*sqrt(b*e - 2*c*d)*atan(sqrt
( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**2*e**2*f*x - 99*sqrt(b*e
 - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**2*e**
2*g*x**2 - 9*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e -
2*c*d))*c**3*d*e**3*f*x**2 - 33*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d -
 c*e*x)/sqrt(b*e - 2*c*d))*c**3*d*e**3*g*x**3 - 3*sqrt(b*e - 2*c*d)*atan(s
qrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*e**4*f*x**3 + 4*sqrt( -
b*e + c*d - c*e*x)*b**3*d*e**3*g + 8*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*
f + 12*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*g*x - 8*sqrt( - b*e + c*d - c*
e*x)*b**2*c*d**2*e**2*g - 34*sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**3*f -
26*sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**3*g*x + 14*sqrt( - b*e + c*d - c
*e*x)*b**2*c*e**4*f*x + 30*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**4*g*x**...
```

3.213 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 365

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx &= \frac{(3cef-19cdg+8beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{24e^2(d+ex)^{7/2}} \\
&- \frac{c(3cef-115cdg+56beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{96e^2(2cd-be)(d+ex)^{5/2}} \\
&- \frac{c^2(3cef+13cdg-8beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{64e^2(2cd-be)^2(d+ex)^{3/2}} \\
&- \frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{4e^2(d+ex)^{11/2}} \\
&- \frac{c^3(3cef+13cdg-8beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{64e^2(2cd-be)^{5/2}}\end{aligned}$$

output

```
1/24*(8*b*e*g-19*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2
/(e*x+d)^(7/2)-1/96*c*(56*b*e*g-115*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c
*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e*x+d)^(5/2)-1/64*c^2*(-8*b*e*g+13*c*d*g
+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^2/(e*x+d
)^(3/2)-1/4*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^
(11/2)-1/64*c^3*(-8*b*e*g+13*c*d*g+3*c*e*f)*arctanh((d*(-b*e+c*d)-b*e^2*x-
c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2/(-b*e+2*c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.93

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx=\frac{c^3((d+ex)(-be+c(d-ex)))^{3/2}\left(\frac{16b^3e^3(3ef+dg+4egx)-8b^2ce^2}{}\right.}{}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x
)^(13/2),x]
```

output

```
(c^3*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*((16*b^3*e^3*(3*e*f + d*g +
4*e*g*x) - 8*b^2*c*e^2*(7*d^2*g - e^2*x*(9*f + 14*g*x) + d*e*(27*f + 29*g*
x)) + 2*b*c^2*e*(25*d^3*g + 3*e^3*x^2*(f + 4*g*x) + d^2*e*(147*f + 110*g*x
) - d*e^2*x*(138*f + 191*g*x)) + c^3*(5*d^4*g - 9*e^4*f*x^3 + 3*d^3*e*(-39
*f + g*x) - 39*d*e^3*x^2*(f + g*x) + d^2*e^2*x*(237*f + 343*g*x)))/(c^3*(-
2*c*d + b*e)^2*(d + e*x)^4*(-(b*e) + c*(d - e*x))) + (3*(3*c*e*f + 13*c*d*
g - 8*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/((-2*c*d
+ b*e)^(5/2)*(-(b*e) + c*(d - e*x))^(3/2))))/(192*e^2*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1220, 1130, 1130, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx$$

$$\Big\downarrow \ 1220$$

$$\frac{(-8beg+13cdg+3cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{11/2}}dx}{8e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{4e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{(-8beg+13cdg+3cef)\left(-\frac{1}{2}c\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{7/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}\right)}{8e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{4e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{(-8beg+13cdg+3cef)\left(-\frac{1}{2}c\left(-\frac{1}{4}c\int \frac{1}{(d+ex)^{3/2}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}}\right)-\frac{(d(cd-be)-be^2x}{3e(d+ex}\right.}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{4e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(-8beg+13cdg+3cef)\left(-\frac{1}{2}c\left(-\frac{1}{4}c\left(\frac{c\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2(2cd-be)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)-\frac{\sqrt{d(cd-be)-be^2x-}}{2e(d+ex)^{5/2}}\right.\right.}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{4e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{(-8beg+13cdg+3cef)\left(-\frac{1}{2}c\left(-\frac{1}{4}c\left(\frac{ce\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}-\frac{\sqrt{d(cd-be)-be^2x}}{e(d+ex)^{3/2}(2cd}\right.\right.\right.}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{4e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\left(-\frac{1}{2}c\left(-\frac{1}{4}c\left(-\frac{c\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}}\right)-\frac{(d(cd-be)-be}{3e(d+e}}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{4e^2(d+ex)^{13/2}(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2))/(d + e*x)^(13/
2),x]
```

output
```
-1/4*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(13/2)) + ((3*c*e*f + 13*c*d*g - 8*b*e*g)*(-1/3*(d*(c*d
- b*e) - b*e^2*x - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(9/2)) - (c*(-1/2*Sqrt[d*
(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(e*(d + e*x)^(5/2)) - (c*(-(Sqrt[d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2]/(e*(2*c*d - b*e)*(d + e*x)^(3/2))) - (c*Arc
Tanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d +
 e*x])])/(e*(2*c*d - b*e)^(3/2))))/4))/2))/(8*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1508 vs. 2(331) = 662.

Time = 1.50 (sec) , antiderivative size = 1509, normalized size of antiderivative = 4.13

method	result	size
default	Expression too large to display	1509

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(13/2),x,method
=_RETURNVERBOSE)
```

output

```
-1/192*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-50*b*c^2*d^3*e*g*(-c*e*x-b*e+c*d
)^(1/2)*(b*e-2*c*d)^(1/2)+382*b*c^2*d*e^3*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*
e-2*c*d)^(1/2)+39*c^3*d*e^3*f*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)
-343*c^3*d^2*e^2*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-220*b*c^2*
d^2*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-112*b^2*c*e^4*g*x^2*(
-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+96*arctan((-c*e*x-b*e+c*d)^(1/2)/(
b*e-2*c*d)^(1/2))*b*c^3*d^3*e^2*g*x-294*b*c^2*d^2*e^2*f*(-c*e*x-b*e+c*d)^(
1/2)*(b*e-2*c*d)^(1/2)-39*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))
*c^4*d^5*g-16*b^3*d*e^3*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+117*c^3
*d^3*e*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-48*b^3*e^4*f*(-c*e*x-b*e
+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-5*c^3*d^4*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*
d)^(1/2)-237*c^3*d^2*e^2*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+144*
arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^3*d^2*e^3*g*x^2-3*c^3
*d^3*e*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+216*b^2*c*d*e^3*f*(-c*
e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-6*b*c^2*e^4*f*x^2*(-c*e*x-b*e+c*d)^(1
/2)*(b*e-2*c*d)^(1/2)+39*c^3*d*e^3*g*x^3*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d
)^(1/2)+56*b^2*c*d^2*e^2*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-72*b^2
*c*e^4*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+276*b*c^2*d*e^3*f*x*(-
c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-24*b*c^2*e^4*g*x^3*(-c*e*x-b*e+c*d)
^(1/2)*(b*e-2*c*d)^(1/2)+232*b^2*c*d*e^3*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(331) = 662$.

Time = 0.21 (sec) , antiderivative size = 2106, normalized size of antiderivative = 5.77

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(13/2),x,
 algorithm="fricas")
```

output

```
[-1/384*(3*(3*c^4*d^5*e*f + (3*c^4*e^6*f + (13*c^4*d*e^5 - 8*b*c^3*e^6)*g)
*x^5 + 5*(3*c^4*d*e^5*f + (13*c^4*d^2*e^4 - 8*b*c^3*d*e^5)*g)*x^4 + 10*(3*
c^4*d^2*e^4*f + (13*c^4*d^3*e^3 - 8*b*c^3*d^2*e^4)*g)*x^3 + 10*(3*c^4*d^3*
e^3*f + (13*c^4*d^4*e^2 - 8*b*c^3*d^3*e^3)*g)*x^2 + (13*c^4*d^6 - 8*b*c^3*
d^5*e)*g + 5*(3*c^4*d^4*e^2*f + (13*c^4*d^5*e - 8*b*c^3*d^4*e^2)*g)*x)*sqr
t(2*c*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x -
 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x +
 d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*
d*e)*(3*(3*(2*c^4*d*e^4 - b*c^3*e^5)*f + (26*c^4*d^2*e^3 - 29*b*c^3*d*e^4
+ 8*b^2*c^2*e^5)*g)*x^3 + (3*(26*c^4*d^2*e^3 - 17*b*c^3*d*e^4 + 2*b^2*c^2*
e^5)*f - (686*c^4*d^3*e^2 - 1107*b*c^3*d^2*e^3 + 606*b^2*c^2*d*e^4 - 112*b
^3*c*e^5)*g)*x^2 + 3*(78*c^4*d^4*e - 235*b*c^3*d^3*e^2 + 242*b^2*c^2*d^2*e
^3 - 104*b^3*c*d*e^4 + 16*b^4*e^5)*f - (10*c^4*d^5 + 95*b*c^3*d^4*e - 162*
b^2*c^2*d^3*e^2 + 88*b^3*c*d^2*e^3 - 16*b^4*d*e^4)*g - (3*(158*c^4*d^3*e^2
 - 263*b*c^3*d^2*e^3 + 140*b^2*c^2*d*e^4 - 24*b^3*c*e^5)*f + (6*c^4*d^4*e
+ 437*b*c^3*d^3*e^2 - 684*b^2*c^2*d^2*e^3 + 360*b^3*c*d*e^4 - 64*b^4*e^5)*
g)*x)*sqrt(e*x + d))/(8*c^3*d^8*e^2 - 12*b*c^2*d^7*e^3 + 6*b^2*c*d^6*e^4 -
 b^3*d^5*e^5 + (8*c^3*d^3*e^7 - 12*b*c^2*d^2*e^8 + 6*b^2*c*d*e^9 - b^3*e^1
0)*x^5 + 5*(8*c^3*d^4*e^6 - 12*b*c^2*d^3*e^7 + 6*b^2*c*d^2*e^8 - b^3*d*e^9
)*x^4 + 10*(8*c^3*d^5*e^5 - 12*b*c^2*d^4*e^6 + 6*b^2*c*d^3*e^7 - b^3*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx=\text{Timed out}$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2)/(e*x+d)**(13
/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx=\int \frac{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(gx+f)}{(ex+d)^{\frac{13}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(13/2),x,
 algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(g*x + f)/(e*x + d)
^(13/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(331) = 662$.

Time = 0.39 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.69

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(13/2),x,
 algorithm="giac")
```

output

```
1/192*(3*(3*c^5*e*f + 13*c^5*d*g - 8*b*c^4*e*g)*arctan(sqrt(-(e*x + d)*c +
 2*c*d - b*e)/sqrt(-2*c*d + b*e))/((4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*sqrt(
-2*c*d + b*e)) + (72*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^8*d^3*e*f - 108*sq
rt(-(e*x + d)*c + 2*c*d - b*e)*b*c^7*d^2*e^2*f + 54*sqrt(-(e*x + d)*c + 2*
c*d - b*e)*b^2*c^6*d*e^3*f - 9*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^5*e^
4*f + 312*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^8*d^4*g - 660*sqrt(-(e*x + d)
*c + 2*c*d - b*e)*b*c^7*d^3*e*g + 522*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2
*c^6*d^2*e^2*g - 183*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^5*d*e^3*g + 24
*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^4*c^4*e^4*g - 132*(-(e*x + d)*c + 2*c*
d - b*e)^(3/2)*c^7*d^2*e*f + 132*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^6*
d*e^2*f - 33*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c^5*e^3*f - 572*(-(e*x
 + d)*c + 2*c*d - b*e)^(3/2)*c^7*d^3*g + 924*(-(e*x + d)*c + 2*c*d - b*e)^
(3/2)*b*c^6*d^2*e*g - 495*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c^5*d*e^2
*g + 88*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^3*c^4*e^3*g - 66*((e*x + d)*c
 - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d*e*f + 33*((e*x +
d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^5*e^2*f + 226*(
(e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d^2*g -
193*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^5*d
*e*g + 40*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b
^2*c^4*e^2*g - 9*((e*x + d)*c - 2*c*d + b*e)^3*sqrt(-(e*x + d)*c + 2*c*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{3/2}}{{(d+e\,x)}^{13/2}}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(13/
2),x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2))/(d + e*x)^(13/
2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1836, normalized size of antiderivative = 5.03

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(13/2),x)
```

output

```
( - 24*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*b*c**3*d**4*e*g - 96*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/s
qrt(b*e - 2*c*d))*b*c**3*d**3*e**2*g*x - 144*sqrt(b*e - 2*c*d)*atan(sqrt(
- b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*d**2*e**3*g*x**2 - 96*sqrt(
b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*d*e
**4*g*x**3 - 24*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e
 - 2*c*d))*b*c**3*e**5*g*x**4 + 39*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*
d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**5*g + 9*sqrt(b*e - 2*c*d)*atan(sqrt(
 - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**4*e*f + 156*sqrt(b*e - 2*
c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**4*e*g*x +
36*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c*
*4*d**3*e**2*f*x + 234*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/s
qrt(b*e - 2*c*d))*c**4*d**3*e**2*g*x**2 + 54*sqrt(b*e - 2*c*d)*atan(sqrt(
- b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**2*e**3*f*x**2 + 156*sqrt(b
*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**2*e
**3*g*x**3 + 36*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e
 - 2*c*d))*c**4*d*e**4*f*x**3 + 39*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*
d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d*e**4*g*x**4 + 9*sqrt(b*e - 2*c*d)*ata
n(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*e**5*f*x**4 + 16*sqrt
( - b*e + c*d - c*e*x)*b**4*d*e**4*g + 48*sqrt( - b*e + c*d - c*e*x)*b*...
```

3.214 $\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,d$

Optimal result

Integrand size = 46, antiderivative size = 496

$$
\begin{aligned}
&\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \\
&-\frac{2(2cd-be)^5(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^7e^2(d+ex)^{7/2}} \\
&+\frac{2(2cd-be)^4(5cef+7cdg-6beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^7e^2(d+ex)^{9/2}} \\
&-\frac{10(2cd-be)^3(2cef+4cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}{11c^7e^2(d+ex)^{11/2}} \\
&+\frac{20(2cd-be)^2(cef+3cdg-2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}{13c^7e^2(d+ex)^{13/2}} \\
&-\frac{2(2cd-be)(cef+5cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{15/2}}{3c^7e^2(d+ex)^{15/2}} \\
&+\frac{2(cef+11cdg-6beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{17/2}}{17c^7e^2(d+ex)^{17/2}} \\
&-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{19/2}}{19c^7e^2(d+ex)^{19/2}}
\end{aligned}
$$

output

```
-2/7*(-b*e+2*c*d)^5*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(7/2)/c^7/e^2/(e*x+d)^(7/2)+2/9*(-b*e+2*c*d)^4*(-6*b*e*g+7*c*d*g+5*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(9/2)/c^7/e^2/(e*x+d)^(9/2)-10/11*(-b*e+2
*c*d)^3*(-3*b*e*g+4*c*d*g+2*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(11/2)
/c^7/e^2/(e*x+d)^(11/2)+20/13*(-b*e+2*c*d)^2*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(
-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(13/2)/c^7/e^2/(e*x+d)^(13/2)-2/3*(-b*e+2*c*d
)*(-3*b*e*g+5*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(15/2)/c^7/e^2
/(e*x+d)^(15/2)+2/17*(-6*b*e*g+11*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2
*x^2)^(17/2)/c^7/e^2/(e*x+d)^(17/2)-2/19*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(19/2)/c^7/e^2/(e*x+d)^(19/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.98

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(3072b^6e^6g-256b^5ce^5(19ef+167dg+}{}$$

input

```
Integrate[(d + e*x)^(5/2)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^
(5/2),x]
```

output

```
(2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(3072*b
^6*e^6*g - 256*b^5*c*e^5*(19*e*f + 167*d*g + 42*e*g*x) + 128*b^4*c^2*e^4*(
1956*d^2*g + 7*e^2*x*(19*f + 27*g*x) + d*e*(513*f + 1085*g*x)) - 32*b^3*c^
3*e^3*(24701*d^3*g + 63*e^3*x^2*(19*f + 22*g*x) + 7*d*e^2*x*(950*f + 1287*
g*x) + d^2*e*(11533*f + 23044*g*x)) + 8*b^2*c^4*e^2*(177311*d^4*g + 231*e^
4*x^3*(38*f + 39*g*x) + 42*d*e^3*x^2*(1311*f + 1441*g*x) + 42*d^2*e^2*x*(3
211*f + 4080*g*x) + 2*d^3*e*(68609*f + 126819*g*x)) - 2*b*c^5*e*(682101*d^
5*g + 3003*e^5*x^4*(19*f + 18*g*x) + 1617*d*e^4*x^3*(228*f + 221*g*x) + 12
6*d^2*e^3*x^2*(7885*f + 8052*g*x) + 98*d^3*e^2*x*(14098*f + 16299*g*x) + d
^4*e*(894273*f + 1467802*g*x)) + c^6*(525458*d^6*g + 9009*e^6*x^5*(19*f +
17*g*x) + 3003*d*e^5*x^4*(361*f + 321*g*x) + 462*d^2*e^4*x^3*(6289*f + 559
0*g*x) + 42*d^3*e^3*x^2*(100719*f + 91135*g*x) + 7*d^4*e^2*x*(499529*f + 4
87215*g*x) + d^5*e*(1414759*f + 1839103*g*x))))/(2909907*c^7*e^2*Sqrt[d +
e*x])
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1221, 1128, 1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{5/2}(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}dx$$

$$\Big\downarrow \; 1221$$

$$\frac{(-12beg+5cdg+19cef)\int (d+ex)^{5/2}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{19ce}-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{19ce^2}$$

$$\Big\downarrow \; 1128$$

$$\frac{(-12beg+5cdg+19cef)\left(\frac{10(2cd-be)\int (d+ex)^{3/2}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{17c}-\frac{2(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{17ce}\right)}{19ce}-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{19ce^2}$$

$$\Big\downarrow \; 1128$$

$$\frac{(-12beg+5cdg+19cef)\left(\frac{10(2cd-be)\left(\frac{8(2cd-be)\int \sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{15c}-\frac{2\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15ce}\right)}{17c}-2(\right.}{19ce}-\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{19ce^2}$$

$$\Big\downarrow \; 1128$$

$$\frac{(-12beg+5cdg+19cef)\left(\frac{10(2cd-be)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{\sqrt{d+ex}}dx}{13c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13ce\sqrt{d+ex}}\right)}{15c}-\frac{2\sqrt{d+e}}{}\right.}{17c}\right.}{19ce}$$

$$\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{19ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-12beg+5cdg+19cef)\left(\frac{10(2cd-be)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{3/2}}dx}{11c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce(d+ex)^{3/2}}\right.}{13c}\right.}{15c}\right.}{17c}\right.}{19ce}$$

$$\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{19ce^2}$$

$$\Big\downarrow \; 1128$$

$$(-12beg+5cdg+19cef)\left(\frac{10(2cd-be)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{5/2}}dx}{9c}-\frac{2\left(d(cd-be)-be^2x\right.}{9ce(d+ex}\right.}{11c}\right.}{13c}\right.}{15c}\right.}{}\right.$$

$$\Big\downarrow \text{1122}$$

$$\frac{\left(\frac{10(2cd-be)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{63c^2e(d+ex)^{7/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce(d+ex)^{5/2}}\right)}{11c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right.}{11ce(d+ex)^{3/2}}\right.}{13c}\right.}{15c}\right.}{17c}\right.}{}$$

$$\frac{2g(d+ex)^{5/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{19ce^2}$$

input
```
Int[(d + e*x)^(5/2)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),
x]
```

output
```
(-2*g*(d + e*x)^(5/2)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(19*c*e
^2) + ((19*c*e*f + 5*c*d*g - 12*b*e*g)*((-2*(d + e*x)^(3/2)*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(7/2))/(17*c*e) + (10*(2*c*d - b*e)*((-2*Sqrt[d +
e*x]*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(15*c*e) + (8*(2*c*d - b
*e)*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(13*c*e*Sqrt[d + e*x
]) + (6*(2*c*d - b*e)*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(1
1*c*e*(d + e*x)^(3/2)) + (4*(2*c*d - b*e)*((-4*(2*c*d - b*e)*(d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2)^(7/2))/(63*c^2*e*(d + e*x)^(7/2)) - (2*(d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(9*c*e*(d + e*x)^(5/2))))/(11*c)))/(13*
c)))/(15*c)))/(17*c)))/(19*c*e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.48

method	result
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(cex+be-cd)^3\left(153153g\,e^6x^6c^6-108108b\,c^5e^6g\,x^5+963963c^6d\,e^5g\,x^5+171171c^6e^6f\,x^5+72072b^2c^4e^6g\,x\right.}{}$
gosper	$\frac{2(cex+be-cd)\left(153153g\,e^6x^6c^6-108108b\,c^5e^6g\,x^5+963963c^6d\,e^5g\,x^5+171171c^6e^6f\,x^5+72072b^2c^4e^6g\,x^4-714714b\,c^5d\,e^5g\,x^4-11\right.}{}$
orering	$\frac{2(cex+be-cd)\left(153153g\,e^6x^6c^6-108108b\,c^5e^6g\,x^5+963963c^6d\,e^5g\,x^5+171171c^6e^6f\,x^5+72072b^2c^4e^6g\,x^4-714714b\,c^5d\,e^5g\,x^4-11\right.}{}$

input
```
int((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
2/2909907/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(c*e*x+b*e-c*d)^3
*(153153*c^6*e^6*g*x^6-108108*b*c^5*e^6*g*x^5+963963*c^6*d*e^5*g*x^5+17117
1*c^6*e^6*f*x^5+72072*b^2*c^4*e^6*g*x^4-714714*b*c^5*d*e^5*g*x^4-114114*b*
c^5*e^6*f*x^4+2582580*c^6*d^2*e^4*g*x^4+1084083*c^6*d*e^5*f*x^4-44352*b^3*
c^3*e^6*g*x^3+484176*b^2*c^4*d*e^5*g*x^3+70224*b^2*c^4*e^6*f*x^3-2029104*b
*c^5*d^2*e^4*g*x^3-737352*b*c^5*d*e^5*f*x^3+3827670*c^6*d^3*e^3*g*x^3+2905
518*c^6*d^2*e^4*f*x^3+24192*b^4*c^2*e^6*g*x^2-288288*b^3*c^3*d*e^5*g*x^2-3
8304*b^3*c^3*e^6*f*x^2+1370880*b^2*c^4*d^2*e^4*g*x^2+440496*b^2*c^4*d*e^5*
f*x^2-3194604*b*c^5*d^3*e^3*g*x^2-1987020*b*c^5*d^2*e^4*f*x^2+3410505*c^6*
d^4*e^2*g*x^2+4230198*c^6*d^3*e^3*f*x^2-10752*b^5*c*e^6*g*x+138880*b^4*c^2
*d*e^5*g*x+17024*b^4*c^2*e^6*f*x-737408*b^3*c^3*d^2*e^4*g*x-212800*b^3*c^3
*d*e^5*f*x+2029104*b^2*c^4*d^3*e^3*g*x+1078896*b^2*c^4*d^2*e^4*f*x-2935604
*b*c^5*d^4*e^2*g*x-2763208*b*c^5*d^3*e^3*f*x+1839103*c^6*d^5*e*g*x+3496703
*c^6*d^4*e^2*f*x+3072*b^6*e^6*g-42752*b^5*c*d*e^5*g-4864*b^5*c*e^6*f+25036
8*b^4*c^2*d^2*e^4*g+65664*b^4*c^2*d*e^5*f-790432*b^3*c^3*d^3*e^3*g-369056*
b^3*c^3*d^2*e^4*f+1418488*b^2*c^4*d^4*e^2*g+1097744*b^2*c^4*d^3*e^3*f-1364
202*b*c^5*d^5*e*g-1788546*b*c^5*d^4*e^2*f+525458*c^6*d^6*g+1414759*c^6*d^5
*e*f)/c^7/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1370 vs. $2(454) = 908$.

Time = 0.21 (sec) , antiderivative size = 1370, normalized size of antiderivative = 2.76

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output

```
2/2909907*(153153*c^9*e^9*g*x^9 + 9009*(19*c^9*e^9*f + (56*c^9*d*e^8 + 39*
b*c^8*e^9)*g)*x^8 + 3003*(19*(10*c^9*d*e^8 + 7*b*c^8*e^9)*f + (50*c^9*d^2*
e^7 + 527*b*c^8*d*e^8 + 69*b^2*c^7*e^9)*g)*x^7 + 231*(19*(38*c^9*d^2*e^7 +
 417*b*c^8*d*e^8 + 55*b^2*c^7*e^9)*f - (5114*c^9*d^3*e^6 - 9585*b*c^8*d^2*
e^7 - 5216*b^2*c^7*d*e^8 - 3*b^3*c^6*e^9)*g)*x^6 - 63*(19*(1174*c^9*d^3*e^
6 - 2179*b*c^8*d^2*e^7 - 1204*b^2*c^7*d*e^8 - b^3*c^6*e^9)*f + (20456*c^9*
d^4*e^5 + 4189*b*c^8*d^3*e^6 - 45509*b^2*c^7*d^2*e^7 - 143*b^3*c^6*d*e^8 +
 12*b^4*c^5*e^9)*g)*x^5 - 7*(95*(2348*c^9*d^4*e^5 + 587*b*c^8*d^3*e^6 - 53
43*b^2*c^7*d^2*e^7 - 25*b^3*c^6*d*e^8 + 2*b^4*c^5*e^9)*f - (72574*c^9*d^5*
e^4 - 530165*b*c^8*d^4*e^5 + 496980*b^2*c^7*d^3*e^6 + 8230*b^3*c^6*d^2*e^7
 - 1550*b^4*c^5*d*e^8 + 120*b^5*c^4*e^9)*g)*x^4 + (19*(37354*c^9*d^5*e^4 -
 257745*b*c^8*d^4*e^5 + 237200*b^2*c^7*d^3*e^6 + 6070*b^3*c^6*d^2*e^7 - 10
80*b^4*c^5*d*e^8 + 80*b^5*c^4*e^9)*f + (1411994*c^9*d^6*e^3 - 3574809*b*c^
8*d^5*e^4 + 1981645*b^2*c^7*d^4*e^5 + 247010*b^3*c^6*d^3*e^6 - 78240*b^4*c
^5*d^2*e^7 + 13360*b^5*c^4*d*e^8 - 960*b^6*c^3*e^9)*g)*x^3 + 3*(19*(35362*
c^9*d^6*e^3 - 87409*b*c^8*d^5*e^4 + 44825*b^2*c^7*d^4*e^5 + 9650*b^3*c^6*d
^3*e^6 - 2860*b^4*c^5*d^2*e^7 + 464*b^5*c^4*d*e^8 - 32*b^6*c^3*e^9)*f + (1
76810*c^9*d^7*e^2 - 248777*b*c^8*d^6*e^3 - 105344*b^2*c^7*d^5*e^4 + 276115
*b^3*c^6*d^4*e^5 - 130100*b^4*c^5*d^3*e^6 + 36640*b^5*c^4*d^2*e^7 - 5728*b
^6*c^3*d*e^8 + 384*b^7*c^2*e^9)*g)*x^2 - 19*(74461*c^9*d^8*e - 317517*b...
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1364 vs. 2(454) = 908.

Time = 0.13 (sec) , antiderivative size = 1364, normalized size of antiderivative = 2.75

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output

```
2/153153*(9009*c^8*e^8*x^8 - 74461*c^8*d^8 + 317517*b*c^7*d^7*e - 563561*b
^2*c^6*d^6*e^2 + 549615*b^3*c^5*d^5*e^3 - 329190*b^4*c^4*d^4*e^4 + 126672*
b^5*c^3*d^3*e^5 - 30560*b^6*c^2*d^2*e^6 + 4224*b^7*c*d*e^7 - 256*b^8*e^8 +
 3003*(10*c^8*d*e^7 + 7*b*c^7*e^8)*x^7 + 231*(38*c^8*d^2*e^6 + 417*b*c^7*d
*e^7 + 55*b^2*c^6*e^8)*x^6 - 63*(1174*c^8*d^3*e^5 - 2179*b*c^7*d^2*e^6 - 1
204*b^2*c^6*d*e^7 - b^3*c^5*e^8)*x^5 - 35*(2348*c^8*d^4*e^4 + 587*b*c^7*d^
3*e^5 - 5343*b^2*c^6*d^2*e^6 - 25*b^3*c^5*d*e^7 + 2*b^4*c^4*e^8)*x^4 + (37
354*c^8*d^5*e^3 - 257745*b*c^7*d^4*e^4 + 237200*b^2*c^6*d^3*e^5 + 6070*b^3
*c^5*d^2*e^6 - 1080*b^4*c^4*d*e^7 + 80*b^5*c^3*e^8)*x^3 + 3*(35362*c^8*d^6
*e^2 - 87409*b*c^7*d^5*e^3 + 44825*b^2*c^6*d^4*e^4 + 9650*b^3*c^5*d^3*e^5
- 2860*b^4*c^4*d^2*e^6 + 464*b^5*c^3*d*e^7 - 32*b^6*c^2*e^8)*x^2 + (39346*
c^8*d^7*e - 31625*b*c^7*d^6*e^2 - 83676*b^2*c^6*d^5*e^3 + 114555*b^3*c^5*d
^4*e^4 - 50040*b^4*c^4*d^3*e^5 + 13296*b^5*c^3*d^2*e^6 - 1984*b^6*c^2*d*e^
7 + 128*b^7*c*e^8)*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d)*f/(c^6*e^2*x + c^
6*d*e) + 2/2909907*(153153*c^9*e^9*x^9 - 525458*c^9*d^9 + 2940576*b*c^8*d^
8*e - 7087468*b^2*c^7*d^7*e^2 + 9663960*b^3*c^6*d^6*e^3 - 8241330*b^4*c^5*
d^5*e^4 + 4583640*b^5*c^4*d^4*e^5 - 1672864*b^6*c^3*d^3*e^6 + 387840*b^7*c
^2*d^2*e^7 - 51968*b^8*c*d*e^8 + 3072*b^9*e^9 + 9009*(56*c^9*d*e^8 + 39*b*
c^8*e^9)*x^8 + 3003*(50*c^9*d^2*e^7 + 527*b*c^8*d*e^8 + 69*b^2*c^7*e^9)*x^
7 - 231*(5114*c^9*d^3*e^6 - 9585*b*c^8*d^2*e^7 - 5216*b^2*c^7*d*e^8 - 3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29765 vs. $2(454) = 908$.

Time = 0.63 (sec) , antiderivative size = 29765, normalized size of antiderivative = 60.01

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output

```
-2/14549535*(14549535*sqrt(-c*e*x + c*d - b*e)*c^3*d^8*e*f - 43648605*sqrt
(-c*e*x + c*d - b*e)*b*c^2*d^7*e^2*f + 43648605*sqrt(-c*e*x + c*d - b*e)*b
^2*c*d^6*e^3*f - 14549535*sqrt(-c*e*x + c*d - b*e)*b^3*d^5*e^4*f + 9699690
*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*
x + c*d - b*e)^(3/2))*c^2*d^7*e*f - 43648605*(3*sqrt(-c*e*x + c*d - b*e)*c
*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b*c*d^6*
e^2*f + 58198140*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b
*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b^2*d^5*e^3*f - 24249225*(3*sqrt(-c*
e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*
e)^(3/2))*b^3*d^4*e^4*f/c + 4849845*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sq
rt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*c^2*d^8*g - 14549
535*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c
*e*x + c*d - b*e)^(3/2))*b*c*d^7*e*g + 14549535*(3*sqrt(-c*e*x + c*d - b*e
)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b^2*d
^6*e^2*g - 4849845*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d -
 b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b^3*d^5*e^3*g/c - 1939938*(15*sqrt
(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sq
rt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-
c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d -
 b*e))*c*d^6*e*f - 2909907*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sq...
```

Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 1307, normalized size of antiderivative = 2.64

$$\int (d+ex)^{5/2}(f+gx) \left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2} \, dx=\text{Too large to display}$$

input

```
int((f + g*x)*(d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),
x)
```

output

```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*e^4*x^7*(d + e*x)^(1/2)*(
69*b^2*e^2*g + 50*c^2*d^2*g + 133*b*c*e^2*f + 190*c^2*d*e*f + 527*b*c*d*e*
g))/969 + (x^5*(d + e*x)^(1/2)*(2394*b^3*c^6*e^9*f - 1512*b^4*c^5*e^9*g -
2810556*c^9*d^3*e^6*f - 2577456*c^9*d^4*e^5*g + 5216526*b*c^8*d^2*e^7*f +
2882376*b^2*c^7*d*e^8*f - 527814*b*c^8*d^3*e^6*g + 18018*b^3*c^6*d*e^8*g +
 5734134*b^2*c^7*d^2*e^7*g))/(2909907*c^7*e^3) + (2*c^2*e^6*g*x^9*(d + e*x
)^(1/2))/19 + (x^3*(d + e*x)^(1/2)*(3040*b^5*c^4*e^9*f - 1920*b^6*c^3*e^9*
g + 1419452*c^9*d^5*e^4*f + 2823988*c^9*d^6*e^3*g - 9794310*b*c^8*d^4*e^5*
f - 41040*b^4*c^5*d*e^8*f - 7149618*b*c^8*d^5*e^4*g + 26720*b^5*c^4*d*e^8*
g + 9013600*b^2*c^7*d^3*e^6*f + 230660*b^3*c^6*d^2*e^7*f + 3963290*b^2*c^7
*d^4*e^5*g + 494020*b^3*c^6*d^3*e^6*g - 156480*b^4*c^5*d^2*e^7*g))/(290990
7*c^7*e^3) + (x^6*(d + e*x)^(1/2)*(482790*b^2*c^7*e^9*f + 1386*b^3*c^6*e^9
*g + 333564*c^9*d^2*e^7*f - 2362668*c^9*d^3*e^6*g + 3660426*b*c^8*d*e^8*f
+ 4428270*b*c^8*d^2*e^7*g + 2409792*b^2*c^7*d*e^8*g))/(2909907*c^7*e^3) +
(2*c*e^5*x^8*(d + e*x)^(1/2)*(39*b*e*g + 56*c*d*g + 19*c*e*f))/323 + (2*(b
*e - c*d)^3*(d + e*x)^(1/2)*(3072*b^6*e^6*g + 525458*c^6*d^6*g - 4864*b^5*
c*e^6*f + 1414759*c^6*d^5*e*f - 1364202*b*c^5*d^5*e*g - 42752*b^5*c*d*e^5*
g - 1788546*b*c^5*d^4*e^2*f + 65664*b^4*c^2*d*e^5*f + 1097744*b^2*c^4*d^3*
e^3*f - 369056*b^3*c^3*d^2*e^4*f + 1418488*b^2*c^4*d^4*e^2*g - 790432*b^3*
c^3*d^3*e^3*g + 250368*b^4*c^2*d^2*e^4*g))/(2909907*c^7*e^3) + (x^4*(d ...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1532, normalized size of antiderivative = 3.09

$$\int (d+ex)^{5/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(2*sqrt( - b*e + c*d - c*e*x)*(3072*b**9*e**9*g - 51968*b**8*c*d*e**8*g -
4864*b**8*c*e**9*f - 1536*b**8*c*e**9*g*x + 387840*b**7*c**2*d**2*e**7*g +
 80256*b**7*c**2*d*e**8*f + 24448*b**7*c**2*d*e**8*g*x + 2432*b**7*c**2*e*
*9*f*x + 1152*b**7*c**2*e**9*g*x**2 - 1672864*b**6*c**3*d**3*e**6*g - 5806
40*b**6*c**3*d**2*e**7*f - 169472*b**6*c**3*d**2*e**7*g*x - 37696*b**6*c**
3*d*e**8*f*x - 17184*b**6*c**3*d*e**8*g*x**2 - 1824*b**6*c**3*e**9*f*x**2
- 960*b**6*c**3*e**9*g*x**3 + 4583640*b**5*c**4*d**4*e**5*g + 2406768*b**5
*c**4*d**3*e**6*f + 666960*b**5*c**4*d**3*e**6*g*x + 252624*b**5*c**4*d**2
*e**7*f*x + 109920*b**5*c**4*d**2*e**7*g*x**2 + 26448*b**5*c**4*d*e**8*f*x
**2 + 13360*b**5*c**4*d*e**8*g*x**3 + 1520*b**5*c**4*e**9*f*x**3 + 840*b**
5*c**4*e**9*g*x**4 - 8241330*b**4*c**5*d**5*e**4*g - 6254610*b**4*c**5*d**
4*e**5*f - 1624860*b**4*c**5*d**4*e**5*g*x - 950760*b**4*c**5*d**3*e**6*f*
x - 390300*b**4*c**5*d**3*e**6*g*x**2 - 163020*b**4*c**5*d**2*e**7*f*x**2
- 78240*b**4*c**5*d**2*e**7*g*x**3 - 20520*b**4*c**5*d*e**8*f*x**3 - 10850
*b**4*c**5*d*e**8*g*x**4 - 1330*b**4*c**5*e**9*f*x**4 - 756*b**4*c**5*e**9
*g*x**5 + 9663960*b**3*c**6*d**6*e**3*g + 10442685*b**3*c**6*d**5*e**4*f +
 2495805*b**3*c**6*d**5*e**4*g*x + 2176545*b**3*c**6*d**4*e**5*f*x + 82834
5*b**3*c**6*d**4*e**5*g*x**2 + 550050*b**3*c**6*d**3*e**6*f*x**2 + 247010*
b**3*c**6*d**3*e**6*g*x**3 + 115330*b**3*c**6*d**2*e**7*f*x**3 + 57610*b**
3*c**6*d**2*e**7*g*x**4 + 16625*b**3*c**6*d*e**8*f*x**4 + 9009*b**3*c**...
```

3.215 $\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}$

Optimal result

Integrand size = 46, antiderivative size = 421

$$\begin{aligned}
&\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx= \\
&-\frac{2(2cd-be)^4(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^6e^2(d+ex)^{7/2}} \\
&+\frac{2(2cd-be)^3(4cef+6cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^6e^2(d+ex)^{9/2}} \\
&-\frac{4(2cd-be)^2(3cef+7cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}{11c^6e^2(d+ex)^{11/2}} \\
&+\frac{4(2cd-be)(2cef+8cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}{13c^6e^2(d+ex)^{13/2}} \\
&-\frac{2(cef+9cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{15/2}}{15c^6e^2(d+ex)^{15/2}} \\
&+\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{17/2}}{17c^6e^2(d+ex)^{17/2}}
\end{aligned}$$

output
```
-2/7*(-b*e+2*c*d)^4*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(7/2)/c^6/e^2/(e*x+d)^(7/2)+2/9*(-b*e+2*c*d)^3*(-5*b*e*g+6*c*d*g+4*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(9/2)/c^6/e^2/(e*x+d)^(9/2)-4/11*(-b*e+2*
c*d)^2*(-5*b*e*g+7*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(11/2)/
c^6/e^2/(e*x+d)^(11/2)+4/13*(-b*e+2*c*d)*(-5*b*e*g+8*c*d*g+2*c*e*f)*(d*(-b
*e+c*d)-b*e^2*x-c*e^2*x^2)^(13/2)/c^6/e^2/(e*x+d)^(13/2)-2/15*(-5*b*e*g+9*
c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(15/2)/c^6/e^2/(e*x+d)^(15/2
)+2/17*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(17/2)/c^6/e^2/(e*x+d)^(17/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.87

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(-1280b^5e^5g+128b^4ce^4(17ef+118dg}{}$$

input
```
Integrate[(d + e*x)^(3/2)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^
(5/2),x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-1280*
b^5*e^5*g + 128*b^4*c*e^4*(17*e*f + 118*d*g + 35*e*g*x) - 32*b^3*c^2*e^3*(
2253*d^2*g + 7*e^2*x*(34*f + 45*g*x) + 2*d*e*(391*f + 756*g*x)) + 16*b^2*c
^3*e^2*(10864*d^3*g + 294*d*e^2*x*(17*f + 21*g*x) + 21*e^3*x^2*(51*f + 55*
g*x) + 3*d^2*e*(2397*f + 4249*g*x)) - 2*b*c^4*e*(104843*d^4*g + 231*e^4*x^
3*(68*f + 65*g*x) + 84*d*e^3*x^2*(969*f + 968*g*x) + 42*d^2*e^2*x*(3842*f
+ 4287*g*x) + 4*d^3*e*(32623*f + 50554*g*x)) + c^5*(94134*d^5*g + 3003*e^5
*x^4*(17*f + 15*g*x) + 462*d*e^4*x^3*(578*f + 507*g*x) + 126*d^2*e^3*x^2*(
4471*f + 3949*g*x) + 28*d^3*e^2*x*(21097*f + 19638*g*x) + d^4*e*(278171*f
+ 329469*g*x))))/(765765*c^6*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1221, 1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2}(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}\,dx$$

$$\Big\downarrow \; 1221$$

$$\frac{(-10beg+3cdg+17cef)\int (d+ex)^{3/2}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{17ce}-\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{17ce^2}$$

$$\Big\downarrow \; 1128$$

$$\frac{(-10beg+3cdg+17cef)\left(\frac{8(2cd-be)\int \sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{15c}-\frac{2\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15ce}\right)}{17ce}-\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{17ce^2}$$

$$\Big\downarrow \; 1128$$

$$\frac{(-10beg+3cdg+17cef)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{\sqrt{d+ex}}dx}{13c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13ce\sqrt{d+ex}}\right)}{15c}-2\sqrt{d+ex}(d(cd-\right.}{17ce}-\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{17ce^2}$$

$$\Big\downarrow \; 1128$$

$$\frac{(-10beg+3cdg+17cef)\Bigg(\frac{8(2cd-be)\Bigg(\frac{6(2cd-be)\left(\frac{4(2cd-be)\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{3/2}}dx}{11c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce(d+ex)^{3/2}}\right)}{13c}-\frac{2\big(d(cd-}{}}{15c}}{17ce}$$

$$\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{17ce^2}$$

$$\Big\downarrow 1128$$

$$\frac{(-10beg+3cdg+17cef)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{5/2}}dx}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce(d+ex)^{5/2}}\right)}{11c}\right.}{13c}\right.}{15c}\right.}{17ce}$$

$$\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{17ce^2}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{\left(8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{63c^2e(d+ex)^{7/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce(d+ex)^{5/2}}\right)}{11c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce(d+ex)^{3/2}}\right)}{13c}-\frac{2\left(d\right.}{}\right)}{15c}\right.}{17ce}$$

$$\frac{2g(d+ex)^{3/2}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{17ce^2}$$

input

```
Int[(d + e*x)^(3/2)*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),
x]
```

output

```
(-2*g*(d + e*x)^(3/2)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(17*c*e
^2) + ((17*c*e*f + 3*c*d*g - 10*b*e*g)*((-2*Sqrt[d + e*x]*(d*(c*d - b*e) -
 b*e^2*x - c*e^2*x^2)^(7/2))/(15*c*e) + (8*(2*c*d - b*e)*((-2*(d*(c*d - b*
e) - b*e^2*x - c*e^2*x^2)^(7/2))/(13*c*e*Sqrt[d + e*x]) + (6*(2*c*d - b*e)
*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(11*c*e*(d + e*x)^(3/2)
) + (4*(2*c*d - b*e)*((-4*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x
^2)^(7/2))/(63*c^2*e*(d + e*x)^(7/2)) - (2*(d*(c*d - b*e) - b*e^2*x - c*e^
2*x^2)^(7/2))/(9*c*e*(d + e*x)^(5/2))))/(11*c)))/(13*c)))/(15*c)))/(17*c*e
)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.26

method	result
default	$-\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(cex+be-cd)^{3}\left(-45045g\,e^{5}x^{5}c^{5}+30030b\,c^{4}e^{5}g\,x^{4}-234234c^{5}d\,e^{4}g\,x^{4}-51051c^{5}e^{5}f\,x^{4}-18480b^{2}c^{3}e^{5}g\right.}{}$
gosper	$-\frac{2(cex+be-cd)\left(-45045g\,e^{5}x^{5}c^{5}+30030b\,c^{4}e^{5}g\,x^{4}-234234c^{5}d\,e^{4}g\,x^{4}-51051c^{5}e^{5}f\,x^{4}-18480b^{2}c^{3}e^{5}g\,x^{3}+162624b\,c^{4}d\,e^{4}g\,x^{3}+31\right.}{}$
orering	$-\frac{2(cex+be-cd)\left(-45045g\,e^{5}x^{5}c^{5}+30030b\,c^{4}e^{5}g\,x^{4}-234234c^{5}d\,e^{4}g\,x^{4}-51051c^{5}e^{5}f\,x^{4}-18480b^{2}c^{3}e^{5}g\,x^{3}+162624b\,c^{4}d\,e^{4}g\,x^{3}+31\right.}{}$

input
```
int((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output

```
-2/765765/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(c*e*x+b*e-c*d)^3
*(-45045*c^5*e^5*g*x^5+30030*b*c^4*e^5*g*x^4-234234*c^5*d*e^4*g*x^4-51051*
c^5*e^5*f*x^4-18480*b^2*c^3*e^5*g*x^3+162624*b*c^4*d*e^4*g*x^3+31416*b*c^4
*e^5*f*x^3-497574*c^5*d^2*e^3*g*x^3-267036*c^5*d*e^4*f*x^3+10080*b^3*c^2*e
^5*g*x^2-98784*b^2*c^3*d*e^4*g*x^2-17136*b^2*c^3*e^5*f*x^2+360108*b*c^4*d^
2*e^3*g*x^2+162792*b*c^4*d*e^4*f*x^2-549864*c^5*d^3*e^2*g*x^2-563346*c^5*d
^2*e^3*f*x^2-4480*b^4*c*e^5*g*x+48384*b^3*c^2*d*e^4*g*x+7616*b^3*c^2*e^5*f
*x-203952*b^2*c^3*d^2*e^3*g*x-79968*b^2*c^3*d*e^4*f*x+404432*b*c^4*d^3*e^2
*g*x+322728*b*c^4*d^2*e^3*f*x-329469*c^5*d^4*e*g*x-590716*c^5*d^3*e^2*f*x+
1280*b^5*e^5*g-15104*b^4*c*d*e^4*g-2176*b^4*c*e^5*f+72096*b^3*c^2*d^2*e^3*
g+25024*b^3*c^2*d*e^4*f-173824*b^2*c^3*d^3*e^2*g-115056*b^2*c^3*d^2*e^3*f+
209686*b*c^4*d^4*e*g+260984*b*c^4*d^3*e^2*f-94134*c^5*d^5*g-278171*c^5*d^4
*e*f)/c^6/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(385) = 770$.

Time = 0.16 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.64

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output

```
2/765765*(45045*c^8*e^8*g*x^8 + 3003*(17*c^8*e^8*f + (33*c^8*d*e^7 + 35*b*
c^7*e^8)*g)*x^7 + 231*(17*(29*c^8*d*e^7 + 31*b*c^7*e^8)*f - (303*c^8*d^2*e
^6 - 1558*b*c^7*d*e^7 - 275*b^2*c^6*e^8)*g)*x^6 - 63*(17*(79*c^8*d^2*e^6 -
 398*b*c^7*d*e^7 - 71*b^2*c^6*e^8)*f + (4527*c^8*d^3*e^5 - 4129*b*c^7*d^2*
e^6 - 4813*b^2*c^6*d*e^7 - 5*b^3*c^5*e^8)*g)*x^5 - 35*(17*(587*c^8*d^3*e^5
 - 525*b*c^7*d^2*e^6 - 633*b^2*c^6*d*e^7 - b^3*c^5*e^8)*f + (1761*c^8*d^4*
e^4 + 11860*b*c^7*d^3*e^5 - 15954*b^2*c^6*d^2*e^6 - 108*b^3*c^5*d*e^7 + 10
*b^4*c^4*e^8)*g)*x^4 - 5*(17*(835*c^8*d^4*e^4 + 6548*b*c^7*d^3*e^5 - 8586*
b^2*c^6*d^2*e^6 - 92*b^3*c^5*d*e^7 + 8*b^4*c^4*e^8)*f - (51549*c^8*d^5*e^3
 - 146429*b*c^7*d^4*e^4 + 91238*b^2*c^6*d^3*e^5 + 4506*b^3*c^5*d^2*e^6 - 9
44*b^4*c^4*d*e^7 + 80*b^5*c^3*e^8)*g)*x^3 + 3*(17*(7339*c^8*d^5*e^3 - 2043
5*b*c^7*d^4*e^4 + 12250*b^2*c^6*d^3*e^5 + 1030*b^3*c^5*d^2*e^6 - 200*b^4*c
^4*d*e^7 + 16*b^5*c^3*e^8)*f + (52047*c^8*d^6*e^2 - 89650*b*c^7*d^5*e^3 +
15875*b^2*c^6*d^4*e^4 + 30740*b^3*c^5*d^3*e^5 - 10900*b^4*c^4*d^2*e^6 + 20
48*b^5*c^3*d*e^7 - 160*b^6*c^2*e^8)*g)*x^2 - 17*(16363*c^8*d^7*e - 64441*b
*c^7*d^6*e^2 + 101913*b^2*c^6*d^5*e^3 - 84195*b^3*c^5*d^4*e^4 + 40200*b^4*
c^4*d^3*e^5 - 11568*b^5*c^3*d^2*e^6 + 1856*b^6*c^2*d*e^7 - 128*b^7*c*e^8)*
f - 2*(47067*c^8*d^8 - 246044*b*c^7*d^7*e + 542642*b^2*c^6*d^6*e^2 - 65838
0*b^3*c^5*d^5*e^3 + 481275*b^4*c^4*d^4*e^4 - 218352*b^5*c^3*d^3*e^5 + 6062
4*b^6*c^2*d^2*e^6 - 9472*b^7*c*d*e^7 + 640*b^8*e^8)*g + (17*(14341*c^8*...
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Timed out}$$

input

```
integrate((e*x+d)**(3/2)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. 2(385) = 770.

Time = 0.11 (sec) , antiderivative size = 1108, normalized size of antiderivative = 2.63

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output

```
2/45045*(3003*c^7*e^7*x^7 - 16363*c^7*d^7 + 64441*b*c^6*d^6*e - 101913*b^2
*c^5*d^5*e^2 + 84195*b^3*c^4*d^4*e^3 - 40200*b^4*c^3*d^3*e^4 + 11568*b^5*c
^2*d^2*e^5 - 1856*b^6*c*d*e^6 + 128*b^7*e^7 + 231*(29*c^7*d*e^6 + 31*b*c^6
*e^7)*x^6 - 63*(79*c^7*d^2*e^5 - 398*b*c^6*d*e^6 - 71*b^2*c^5*e^7)*x^5 - 3
5*(587*c^7*d^3*e^4 - 525*b*c^6*d^2*e^5 - 633*b^2*c^5*d*e^6 - b^3*c^4*e^7)*
x^4 - 5*(835*c^7*d^4*e^3 + 6548*b*c^6*d^3*e^4 - 8586*b^2*c^5*d^2*e^5 - 92*
b^3*c^4*d*e^6 + 8*b^4*c^3*e^7)*x^3 + 3*(7339*c^7*d^5*e^2 - 20435*b*c^6*d^4
*e^3 + 12250*b^2*c^5*d^3*e^4 + 1030*b^3*c^4*d^2*e^5 - 200*b^4*c^3*d*e^6 +
16*b^5*c^2*e^7)*x^2 + (14341*c^7*d^6*e - 21006*b*c^6*d^5*e^2 - 4395*b^2*c^
5*d^4*e^3 + 15180*b^3*c^4*d^3*e^4 - 4920*b^4*c^3*d^2*e^5 + 864*b^5*c^2*d*e
^6 - 64*b^6*c*e^7)*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d)*f/(c^5*e^2*x + c^
5*d*e) + 2/765765*(45045*c^8*e^8*x^8 - 94134*c^8*d^8 + 492088*b*c^7*d^7*e
- 1085284*b^2*c^6*d^6*e^2 + 1316760*b^3*c^5*d^5*e^3 - 962550*b^4*c^4*d^4*e
^4 + 436704*b^5*c^3*d^3*e^5 - 121248*b^6*c^2*d^2*e^6 + 18944*b^7*c*d*e^7 -
 1280*b^8*e^8 + 3003*(33*c^8*d*e^7 + 35*b*c^7*e^8)*x^7 - 231*(303*c^8*d^2*
e^6 - 1558*b*c^7*d*e^7 - 275*b^2*c^6*e^8)*x^6 - 63*(4527*c^8*d^3*e^5 - 412
9*b*c^7*d^2*e^6 - 4813*b^2*c^6*d*e^7 - 5*b^3*c^5*e^8)*x^5 - 35*(1761*c^8*d
^4*e^4 + 11860*b*c^7*d^3*e^5 - 15954*b^2*c^6*d^2*e^6 - 108*b^3*c^5*d*e^7 +
 10*b^4*c^4*e^8)*x^4 + 5*(51549*c^8*d^5*e^3 - 146429*b*c^7*d^4*e^4 + 91238
*b^2*c^6*d^3*e^5 + 4506*b^3*c^5*d^2*e^6 - 944*b^4*c^4*d*e^7 + 80*b^5*c^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21712 vs. $2(385) = 770$.

Time = 0.52 (sec) , antiderivative size = 21712, normalized size of antiderivative = 51.57

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output

```
-2/765765*(765765*sqrt(-c*e*x + c*d - b*e)*c^3*d^7*e*f - 2297295*sqrt(-c*e
*x + c*d - b*e)*b*c^2*d^6*e^2*f + 2297295*sqrt(-c*e*x + c*d - b*e)*b^2*c*d
^5*e^3*f - 765765*sqrt(-c*e*x + c*d - b*e)*b^3*d^4*e^4*f + 255255*(3*sqrt(
-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d -
 b*e)^(3/2))*c^2*d^6*e*f - 1531530*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqr
t(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b*c*d^5*e^2*f + 22
97295*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (
-c*e*x + c*d - b*e)^(3/2))*b^2*d^4*e^3*f - 1021020*(3*sqrt(-c*e*x + c*d -
b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b^
3*d^3*e^4*f/c + 255255*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c
*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*c^2*d^7*g - 765765*(3*sqrt(-c*
e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*
e)^(3/2))*b*c*d^6*e*g + 765765*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c
*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b^2*d^5*e^2*g - 255255
*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*
x + c*d - b*e)^(3/2))*b^3*d^4*e^3*g/c - 153153*(15*sqrt(-c*e*x + c*d - b*e
)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b
*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^
(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e))*c*d^5*e*f +
153153*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*...
```

Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 1023, normalized size of antiderivative = 2.43

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
int((f + g*x)*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),
x)
```

output
```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*e^3*x^6*(d + e*x)^(1/2)*(
275*b^2*e^2*g - 303*c^2*d^2*g + 527*b*c*e^2*f + 493*c^2*d*e*f + 1558*b*c*d
*e*g))/3315 + (2*(b*e - c*d)^3*(d + e*x)^(1/2)*(94134*c^5*d^5*g - 1280*b^5
*e^5*g + 2176*b^4*c*e^5*f + 278171*c^5*d^4*e*f - 209686*b*c^4*d^4*e*g + 15
104*b^4*c*d*e^4*g - 260984*b*c^4*d^3*e^2*f - 25024*b^3*c^2*d*e^4*f + 11505
6*b^2*c^3*d^2*e^3*f + 173824*b^2*c^3*d^3*e^2*g - 72096*b^3*c^2*d^2*e^3*g))
/(765765*c^6*e^3) + (x^4*(d + e*x)^(1/2)*(1190*b^3*c^5*e^8*f - 700*b^4*c^4
*e^8*g - 698530*c^8*d^3*e^5*f - 123270*c^8*d^4*e^4*g + 624750*b*c^7*d^2*e^
6*f + 753270*b^2*c^6*d*e^7*f - 830200*b*c^7*d^3*e^5*g + 7560*b^3*c^5*d*e^7
*g + 1116780*b^2*c^6*d^2*e^6*g))/(765765*c^6*e^3) + (2*c^2*e^5*g*x^8*(d +
e*x)^(1/2))/17 + (x^5*(d + e*x)^(1/2)*(152082*b^2*c^6*e^8*f + 630*b^3*c^5*
e^8*g - 169218*c^8*d^2*e^6*f - 570402*c^8*d^3*e^5*g + 852516*b*c^7*d*e^7*f
 + 520254*b*c^7*d^2*e^6*g + 606438*b^2*c^6*d*e^7*g))/(765765*c^6*e^3) + (2
*c*e^4*x^7*(d + e*x)^(1/2)*(35*b*e*g + 33*c*d*g + 17*c*e*f))/255 + (x^3*(d
 + e*x)^(1/2)*(800*b^5*c^3*e^8*g - 1360*b^4*c^4*e^8*f - 141950*c^8*d^4*e^4
*f + 515490*c^8*d^5*e^3*g - 1113160*b*c^7*d^3*e^5*f + 15640*b^3*c^5*d*e^7*
f - 1464290*b*c^7*d^4*e^4*g - 9440*b^4*c^4*d*e^7*g + 1459620*b^2*c^6*d^2*e
^6*f + 912380*b^2*c^6*d^3*e^5*g + 45060*b^3*c^5*d^2*e^6*g))/(765765*c^6*e^
3) + (2*x^2*(b*e - c*d)*(d + e*x)^(1/2)*(272*b^4*c*e^5*f - 52047*c^5*d^5*g
 - 160*b^5*e^5*g - 124763*c^5*d^4*e*f + 37603*b*c^4*d^4*e*g + 1888*b^4*...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1220, normalized size of antiderivative = 2.90

$$\int (d+ex)^{3/2}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(2*sqrt( - b*e + c*d - c*e*x)*( - 1280*b**8*e**8*g + 18944*b**7*c*d*e**7*g
 + 2176*b**7*c*e**8*f + 640*b**7*c*e**8*g*x - 121248*b**6*c**2*d**2*e**6*g
 - 31552*b**6*c**2*d*e**7*f - 8832*b**6*c**2*d*e**7*g*x - 1088*b**6*c**2*e
**8*f*x - 480*b**6*c**2*e**8*g*x**2 + 436704*b**5*c**3*d**3*e**5*g + 19665
6*b**5*c**3*d**2*e**6*f + 51792*b**5*c**3*d**2*e**6*g*x + 14688*b**5*c**3*
d*e**7*f*x + 6144*b**5*c**3*d*e**7*g*x**2 + 816*b**5*c**3*e**8*f*x**2 + 40
0*b**5*c**3*e**8*g*x**3 - 962550*b**4*c**4*d**4*e**4*g - 683400*b**4*c**4*
d**3*e**5*f - 166560*b**4*c**4*d**3*e**5*g*x - 83640*b**4*c**4*d**2*e**6*f
*x - 32700*b**4*c**4*d**2*e**6*g*x**2 - 10200*b**4*c**4*d*e**7*f*x**2 - 47
20*b**4*c**4*d*e**7*g*x**3 - 680*b**4*c**4*e**8*f*x**3 - 350*b**4*c**4*e**
8*g*x**4 + 1316760*b**3*c**5*d**5*e**3*g + 1431315*b**3*c**5*d**4*e**4*f +
 314715*b**3*c**5*d**4*e**4*g*x + 258060*b**3*c**5*d**3*e**5*f*x + 92220*b
**3*c**5*d**3*e**5*g*x**2 + 52530*b**3*c**5*d**2*e**6*f*x**2 + 22530*b**3*
c**5*d**2*e**6*g*x**3 + 7820*b**3*c**5*d*e**7*f*x**3 + 3780*b**3*c**5*d*e*
*7*g*x**4 + 595*b**3*c**5*e**8*f*x**4 + 315*b**3*c**5*e**8*g*x**5 - 108528
4*b**2*c**6*d**6*e**2*g - 1732521*b**2*c**6*d**5*e**3*f - 343665*b**2*c**6
*d**5*e**3*g*x - 74715*b**2*c**6*d**4*e**4*f*x + 47625*b**2*c**6*d**4*e**4
*g*x**2 + 624750*b**2*c**6*d**3*e**5*f*x**2 + 456190*b**2*c**6*d**3*e**5*g
*x**3 + 729810*b**2*c**6*d**2*e**6*f*x**3 + 558390*b**2*c**6*d**2*e**6*g*x
**4 + 376635*b**2*c**6*d*e**7*f*x**4 + 303219*b**2*c**6*d*e**7*g*x**5 +...
```

3.216 $\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2} d$

Optimal result

Integrand size = 46, antiderivative size = 343

$$\begin{aligned}\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx =\\ -\frac{2(2cd-be)^3(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^5e^2(d+ex)^{7/2}}\\ +\frac{2(2cd-be)^2(3cef+5cdg-4beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^5e^2(d+ex)^{9/2}}\\ -\frac{6(2cd-be)(cef+3cdg-2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}{11c^5e^2(d+ex)^{11/2}}\\ +\frac{2(cef+7cdg-4beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}{13c^5e^2(d+ex)^{13/2}}\\ -\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{15/2}}{15c^5e^2(d+ex)^{15/2}}\end{aligned}$$

output
```
-2/7*(-b*e+2*c*d)^3*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(7/2)/c^5/e^2/(e*x+d)^(7/2)+2/9*(-b*e+2*c*d)^2*(-4*b*e*g+5*c*d*g+3*c*e*f)*
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(9/2)/c^5/e^2/(e*x+d)^(9/2)-6/11*(-b*e+2*
c*d)*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(11/2)/c^5/
e^2/(e*x+d)^(11/2)+2/13*(-4*b*e*g+7*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e
^2*x^2)^(13/2)/c^5/e^2/(e*x+d)^(13/2)-2/15*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(15/2)/c^5/e^2/(e*x+d)^(15/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.77

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(128b^4e^4g-16b^3ce^3(15ef+77dg+28e}{}$$

input
```
Integrate[Sqrt[d + e*x]*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5
/2),x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(128*b^
4*e^4*g - 16*b^3*c*e^3*(15*e*f + 77*d*g + 28*e*g*x) + 24*b^2*c^2*e^2*(187*
d^2*g + 7*e^2*x*(5*f + 6*g*x) + d*e*(95*f + 161*g*x)) - 2*b*c^3*e*(3611*d^
3*g + 21*e^3*x^2*(45*f + 44*g*x) + 21*d*e^2*x*(170*f + 183*g*x) + d^2*e*(4
065*f + 5922*g*x)) + c^4*(3838*d^4*g + 231*e^4*x^3*(15*f + 13*g*x) + 147*d
^2*e^2*x*(145*f + 129*g*x) + 21*d*e^3*x^2*(675*f + 583*g*x) + d^3*e*(12525
*f + 13433*g*x))))/(45045*c^5*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1221, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}\,dx$$

$$\Big\downarrow \ 1221$$

$$\frac{(-8beg+cdg+15cef)\int \sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}dx}{15ce}-\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-8beg+cdg+15cef)\left(\frac{6(2cd-be)\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{\sqrt{d+ex}}dx}{13c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13ce\sqrt{d+ex}}\right)}{15ce}-\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-8beg+cdg+15cef)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{3/2}}dx}{11c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce(d+ex)^{3/2}}\right)}{13c}-\frac{2(d(cd-be)-be^2x-}{13ce\sqrt{d+e}}\right.}{15ce}$$

$$\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-8beg+cdg+15cef)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{5/2}}dx}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce(d+ex)^{5/2}}\right)}{11c}-\frac{2(d(cd-be)}{11ce}\right.}{13c}\right.}{15ce}$$

$$\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15ce^2}$$

$$\Big\downarrow \text{1122}$$

$$\frac{\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{63c^2e(d+ex)^{7/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce(d+ex)^{5/2}}\right)}{11c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce(d+ex)^{3/2}}\right)}{13c}-\frac{2(d(cd-be)}{13}\right.}{15ce}$$

$$\frac{2g\sqrt{d+ex}\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{15ce^2}$$

input

```
Int[Sqrt[d + e*x]*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output

```
(-2*g*Sqrt[d + e*x]*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(15*c*e^2
) + ((15*c*e*f + c*d*g - 8*b*e*g)*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^
2)^(7/2))/(13*c*e*Sqrt[d + e*x]) + (6*(2*c*d - b*e)*((-2*(d*(c*d - b*e) -
b*e^2*x - c*e^2*x^2)^(7/2))/(11*c*e*(d + e*x)^(3/2)) + (4*(2*c*d - b*e)*((
-4*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(63*c^2*e*(d
 + e*x)^(7/2)) - (2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(9*c*e*(d
 + e*x)^(5/2))))/(11*c)))/(13*c)))/(15*c*e)
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.05

method	result
default	$2\sqrt{-(ex+d)(cex+be-cd)}\,(cex+be-cd)^3\left(3003g\,e^4x^4c^4-1848b\,c^3e^4g\,x^3+12243c^4d\,e^3g\,x^3+3465c^4e^4f\,x^3+1008b^2c^2e^4g\,x^2-7686b\right.$
gosper	$2(cex+be-cd)\left(3003g\,e^4x^4c^4-1848b\,c^3e^4g\,x^3+12243c^4d\,e^3g\,x^3+3465c^4e^4f\,x^3+1008b^2c^2e^4g\,x^2-7686b\,c^3d\,e^3g\,x^2-1890b\,c^3e^4f\,x^2\right.$
orering	$2(cex+be-cd)\left(3003g\,e^4x^4c^4-1848b\,c^3e^4g\,x^3+12243c^4d\,e^3g\,x^3+3465c^4e^4f\,x^3+1008b^2c^2e^4g\,x^2-7686b\,c^3d\,e^3g\,x^2-1890b\,c^3e^4f\,x^2\right.$

input
```
int((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
2/45045/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(c*e*x+b*e-c*d)^3*(
3003*c^4*e^4*g*x^4-1848*b*c^3*e^4*g*x^3+12243*c^4*d*e^3*g*x^3+3465*c^4*e^4
*f*x^3+1008*b^2*c^2*e^4*g*x^2-7686*b*c^3*d*e^3*g*x^2-1890*b*c^3*e^4*f*x^2+
18963*c^4*d^2*e^2*g*x^2+14175*c^4*d*e^3*f*x^2-448*b^3*c*e^4*g*x+3864*b^2*c
^2*d*e^3*g*x+840*b^2*c^2*e^4*f*x-11844*b*c^3*d^2*e^2*g*x-7140*b*c^3*d*e^3*
f*x+13433*c^4*d^3*e*g*x+21315*c^4*d^2*e^2*f*x+128*b^4*e^4*g-1232*b^3*c*d*e
^3*g-240*b^3*c*e^4*f+4488*b^2*c^2*d^2*e^2*g+2280*b^2*c^2*d*e^3*f-7222*b*c^
3*d^3*e*g-8130*b*c^3*d^2*e^2*f+3838*c^4*d^4*g+12525*c^4*d^3*e*f)/c^5/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(313) = 626$.

Time = 0.12 (sec) , antiderivative size = 881, normalized size of antiderivative = 2.57

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output
```
2/45045*(3003*c^7*e^7*g*x^7 + 231*(15*c^7*e^7*f + (14*c^7*d*e^6 + 31*b*c^6
*e^7)*g)*x^6 + 63*(15*(4*c^7*d*e^6 + 9*b*c^6*e^7)*f - (139*c^7*d^2*e^5 - 2
63*b*c^6*d*e^6 - 71*b^2*c^5*e^7)*g)*x^5 - 35*(3*(103*c^7*d^2*e^5 - 193*b*c
^6*d*e^6 - 53*b^2*c^5*e^7)*f + (278*c^7*d^3*e^4 + 54*b*c^6*d^2*e^5 - 474*b
^2*c^5*d*e^6 - b^3*c^4*e^7)*g)*x^4 - 5*(3*(824*c^7*d^3*e^4 + 206*b*c^6*d^2
*e^5 - 1454*b^2*c^5*d*e^6 - 5*b^3*c^4*e^7)*f - (1637*c^7*d^4*e^3 - 5930*b*
c^6*d^3*e^4 + 4224*b^2*c^5*d^2*e^5 + 77*b^3*c^4*d*e^6 - 8*b^4*c^3*e^7)*g)*
x^3 + 3*(15*(271*c^7*d^4*e^3 - 954*b*c^6*d^3*e^4 + 664*b^2*c^5*d^2*e^5 + 2
1*b^3*c^4*d*e^6 - 2*b^4*c^3*e^7)*f + (3274*c^7*d^5*e^2 - 6125*b*c^6*d^4*e^
3 + 2290*b^2*c^5*d^3*e^4 + 715*b^3*c^4*d^2*e^5 - 170*b^4*c^3*d*e^6 + 16*b^
5*c^2*e^7)*g)*x^2 - 15*(835*c^7*d^6*e - 3047*b*c^6*d^5*e^2 + 4283*b^2*c^5*
d^4*e^3 - 2933*b^3*c^4*d^3*e^4 + 1046*b^4*c^3*d^2*e^5 - 200*b^5*c^2*d*e^6
+ 16*b^6*c*e^7)*f - 2*(1919*c^7*d^7 - 9368*b*c^6*d^6*e + 18834*b^2*c^5*d^5
*e^2 - 20100*b^3*c^4*d^4*e^3 + 12255*b^4*c^3*d^3*e^4 - 4284*b^5*c^2*d^2*e^
5 + 808*b^6*c*d*e^6 - 64*b^7*e^7)*g + (15*(1084*c^7*d^5*e^2 - 1897*b*c^6*d
^4*e^3 + 466*b^2*c^5*d^3*e^4 + 431*b^3*c^4*d^2*e^5 - 92*b^4*c^3*d*e^6 + 8*
b^5*c^2*e^7)*f - (1919*c^7*d^6*e - 7449*b*c^6*d^5*e^2 + 11385*b^2*c^5*d^4*
e^3 - 8715*b^3*c^4*d^3*e^4 + 3540*b^4*c^3*d^2*e^5 - 744*b^5*c^2*d*e^6 + 64
*b^6*c*e^7)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)
/(c^5*e^3*x + c^5*d*e^2)
```

Sympy [F]

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\int \left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{5}{2}}\sqrt{d+ex}(f+gx)\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*sqrt(d + e*x)*(f + g*x),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(313) = 626$.

Time = 0.12 (sec) , antiderivative size = 878, normalized size of antiderivative = 2.56

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output

```
2/3003*(231*c^6*e^6*x^6 - 835*c^6*d^6 + 3047*b*c^5*d^5*e - 4283*b^2*c^4*d^
4*e^2 + 2933*b^3*c^3*d^3*e^3 - 1046*b^4*c^2*d^2*e^4 + 200*b^5*c*d*e^5 - 16
*b^6*e^6 + 63*(4*c^6*d*e^5 + 9*b*c^5*e^6)*x^5 - 7*(103*c^6*d^2*e^4 - 193*b
*c^5*d*e^5 - 53*b^2*c^4*e^6)*x^4 - (824*c^6*d^3*e^3 + 206*b*c^5*d^2*e^4 -
1454*b^2*c^4*d*e^5 - 5*b^3*c^3*e^6)*x^3 + 3*(271*c^6*d^4*e^2 - 954*b*c^5*d
^3*e^3 + 664*b^2*c^4*d^2*e^4 + 21*b^3*c^3*d*e^5 - 2*b^4*c^2*e^6)*x^2 + (10
84*c^6*d^5*e - 1897*b*c^5*d^4*e^2 + 466*b^2*c^4*d^3*e^3 + 431*b^3*c^3*d^2*
e^4 - 92*b^4*c^2*d*e^5 + 8*b^5*c*e^6)*x)*sqrt(-c*e*x + c*d - b*e)*(e*x + d
)*f/(c^4*e^2*x + c^4*d*e) + 2/45045*(3003*c^7*e^7*x^7 - 3838*c^7*d^7 + 187
36*b*c^6*d^6*e - 37668*b^2*c^5*d^5*e^2 + 40200*b^3*c^4*d^4*e^3 - 24510*b^4
*c^3*d^3*e^4 + 8568*b^5*c^2*d^2*e^5 - 1616*b^6*c*d*e^6 + 128*b^7*e^7 + 231
*(14*c^7*d*e^6 + 31*b*c^6*e^7)*x^6 - 63*(139*c^7*d^2*e^5 - 263*b*c^6*d*e^6
 - 71*b^2*c^5*e^7)*x^5 - 35*(278*c^7*d^3*e^4 + 54*b*c^6*d^2*e^5 - 474*b^2*
c^5*d*e^6 - b^3*c^4*e^7)*x^4 + 5*(1637*c^7*d^4*e^3 - 5930*b*c^6*d^3*e^4 +
4224*b^2*c^5*d^2*e^5 + 77*b^3*c^4*d*e^6 - 8*b^4*c^3*e^7)*x^3 + 3*(3274*c^7
*d^5*e^2 - 6125*b*c^6*d^4*e^3 + 2290*b^2*c^5*d^3*e^4 + 715*b^3*c^4*d^2*e^5
 - 170*b^4*c^3*d*e^6 + 16*b^5*c^2*e^7)*x^2 - (1919*c^7*d^6*e - 7449*b*c^6*
d^5*e^2 + 11385*b^2*c^5*d^4*e^3 - 8715*b^3*c^4*d^3*e^4 + 3540*b^4*c^3*d^2*
e^5 - 744*b^5*c^2*d*e^6 + 64*b^6*c*e^7)*x)*sqrt(-c*e*x + c*d - b*e)*(e*x +
 d)*g/(c^5*e^3*x + c^5*d*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11336 vs. $2(313) = 626$.

Time = 0.43 (sec) , antiderivative size = 11336, normalized size of antiderivative = 33.05

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output
```
-2/45045*(45045*sqrt(-c*e*x + c*d - b*e)*c^3*d^6*e*f - 135135*sqrt(-c*e*x
+ c*d - b*e)*b*c^2*d^5*e^2*f + 135135*sqrt(-c*e*x + c*d - b*e)*b^2*c*d^4*e
^3*f - 45045*sqrt(-c*e*x + c*d - b*e)*b^3*d^3*e^4*f - 45045*(3*sqrt(-c*e*x
 + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^
(3/2))*b*c*d^4*e^2*f + 90090*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e
*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b^2*d^3*e^3*f - 45045*(3
*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x +
 c*d - b*e)^(3/2))*b^3*d^2*e^4*f/c + 15015*(3*sqrt(-c*e*x + c*d - b*e)*c*d
 - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*c^2*d^6*g
- 45045*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e -
 (-c*e*x + c*d - b*e)^(3/2))*b*c*d^5*e*g + 45045*(3*sqrt(-c*e*x + c*d - b*
e)*c*d - 3*sqrt(-c*e*x + c*d - b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b^2*
d^4*e^2*g - 15015*(3*sqrt(-c*e*x + c*d - b*e)*c*d - 3*sqrt(-c*e*x + c*d -
b*e)*b*e - (-c*e*x + c*d - b*e)^(3/2))*b^3*d^3*e^3*g/c - 9009*(15*sqrt(-c*
e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x + c*d - b*e)*b*c*d*e + 15*sqrt(-
c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x + c*d - b*e)^(3/2)*c*d + 10*(-c*e*
x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - c*d + b*e)^2*sqrt(-c*e*x + c*d - b*e
))*c*d^4*e*f + 18018*(15*sqrt(-c*e*x + c*d - b*e)*c^2*d^2 - 30*sqrt(-c*e*x
 + c*d - b*e)*b*c*d*e + 15*sqrt(-c*e*x + c*d - b*e)*b^2*e^2 - 10*(-c*e*x +
 c*d - b*e)^(3/2)*c*d + 10*(-c*e*x + c*d - b*e)^(3/2)*b*e + 3*(c*e*x - ...
```

Mupad [B] (verification not implemented)

Time = 12.23 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.24

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{2\,e^2\,x^5\,\sqrt{d+e\,x}\,\left(71\,g\,b^2\,e^2+263\,g\,b\,c\,d\,e+135\,f\,b\,c\,e^2-139\,g\,c^2\,d^2+60\,f\,c^2\,d\,e\right)}{715}-$$

input
```
int((f + g*x)*(d + e*x)^(1/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),
x)
```

output
```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*e^2*x^5*(d + e*x)^(1/2)*(
71*b^2*e^2*g - 139*c^2*d^2*g + 135*b*c*e^2*f + 60*c^2*d*e*f + 263*b*c*d*e*
g))/715 + (x^3*(d + e*x)^(1/2)*(150*b^3*c^4*e^7*f - 80*b^4*c^3*e^7*g - 247
20*c^7*d^3*e^4*f + 16370*c^7*d^4*e^3*g - 6180*b*c^6*d^2*e^5*f + 43620*b^2*
c^5*d*e^6*f - 59300*b*c^6*d^3*e^4*g + 770*b^3*c^4*d*e^6*g + 42240*b^2*c^5*
d^2*e^5*g))/(45045*c^5*e^3) + (2*c^2*e^4*g*x^7*(d + e*x)^(1/2))/15 + (2*(b
*e - c*d)^3*(d + e*x)^(1/2)*(128*b^4*e^4*g + 3838*c^4*d^4*g - 240*b^3*c*e^
4*f + 12525*c^4*d^3*e*f - 7222*b*c^3*d^3*e*g - 1232*b^3*c*d*e^3*g - 8130*b
*c^3*d^2*e^2*f + 2280*b^2*c^2*d*e^3*f + 4488*b^2*c^2*d^2*e^2*g))/(45045*c^
5*e^3) + (x^4*(d + e*x)^(1/2)*(11130*b^2*c^5*e^7*f + 70*b^3*c^4*e^7*g - 21
630*c^7*d^2*e^5*f - 19460*c^7*d^3*e^4*g + 40530*b*c^6*d*e^6*f - 3780*b*c^6
*d^2*e^5*g + 33180*b^2*c^5*d*e^6*g))/(45045*c^5*e^3) + (2*c*e^3*x^6*(d + e
*x)^(1/2)*(31*b*e*g + 14*c*d*g + 15*c*e*f))/195 + (2*x^2*(b*e - c*d)*(d +
e*x)^(1/2)*(16*b^4*e^4*g - 3274*c^4*d^4*g - 30*b^3*c*e^4*f - 4065*c^4*d^3*
e*f + 2851*b*c^3*d^3*e*g - 154*b^3*c*d*e^3*g + 10245*b*c^3*d^2*e^2*f + 285
*b^2*c^2*d*e^3*f + 561*b^2*c^2*d^2*e^2*g))/(15015*c^3*e) + (2*x*(b*e - c*d
)^2*(d + e*x)^(1/2)*(120*b^3*c*e^4*f - 1919*c^4*d^4*g - 64*b^4*e^4*g + 162
60*c^4*d^3*e*f + 3611*b*c^3*d^3*e*g + 616*b^3*c*d*e^3*g + 4065*b*c^3*d^2*e
^2*f - 1140*b^2*c^2*d*e^3*f - 2244*b^2*c^2*d^2*e^2*g))/(45045*c^4*e^2)))/(
x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 944, normalized size of antiderivative = 2.75

$$\int \sqrt{d+ex}(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(2*sqrt( - b*e + c*d - c*e*x)*(128*b**7*e**7*g - 1616*b**6*c*d*e**6*g - 24
0*b**6*c*e**7*f - 64*b**6*c*e**7*g*x + 8568*b**5*c**2*d**2*e**5*g + 3000*b
**5*c**2*d*e**6*f + 744*b**5*c**2*d*e**6*g*x + 120*b**5*c**2*e**7*f*x + 48
*b**5*c**2*e**7*g*x**2 - 24510*b**4*c**3*d**3*e**4*g - 15690*b**4*c**3*d**
2*e**5*f - 3540*b**4*c**3*d**2*e**5*g*x - 1380*b**4*c**3*d*e**6*f*x - 510*
b**4*c**3*d*e**6*g*x**2 - 90*b**4*c**3*e**7*f*x**2 - 40*b**4*c**3*e**7*g*x
**3 + 40200*b**3*c**4*d**4*e**3*g + 43995*b**3*c**4*d**3*e**4*f + 8715*b**
3*c**4*d**3*e**4*g*x + 6465*b**3*c**4*d**2*e**5*f*x + 2145*b**3*c**4*d**2*
e**5*g*x**2 + 945*b**3*c**4*d*e**6*f*x**2 + 385*b**3*c**4*d*e**6*g*x**3 +
75*b**3*c**4*e**7*f*x**3 + 35*b**3*c**4*e**7*g*x**4 - 37668*b**2*c**5*d**5
*e**2*g - 64245*b**2*c**5*d**4*e**3*f - 11385*b**2*c**5*d**4*e**3*g*x + 69
90*b**2*c**5*d**3*e**4*f*x + 6870*b**2*c**5*d**3*e**4*g*x**2 + 29880*b**2*
c**5*d**2*e**5*f*x**2 + 21120*b**2*c**5*d**2*e**5*g*x**3 + 21810*b**2*c**5
*d*e**6*f*x**3 + 16590*b**2*c**5*d*e**6*g*x**4 + 5565*b**2*c**5*e**7*f*x**
4 + 4473*b**2*c**5*e**7*g*x**5 + 18736*b*c**6*d**6*e*g + 45705*b*c**6*d**5
*e**2*f + 7449*b*c**6*d**5*e**2*g*x - 28455*b*c**6*d**4*e**3*f*x - 18375*b
*c**6*d**4*e**3*g*x**2 - 42930*b*c**6*d**3*e**4*f*x**2 - 29650*b*c**6*d**3
*e**4*g*x**3 - 3090*b*c**6*d**2*e**5*f*x**3 - 1890*b*c**6*d**2*e**5*g*x**4
 + 20265*b*c**6*d*e**6*f*x**4 + 16569*b*c**6*d*e**6*g*x**5 + 8505*b*c**6*e
**7*f*x**5 + 7161*b*c**6*e**7*g*x**6 - 3838*c**7*d**7*g - 12525*c**7*d*...
```

3.217 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 267

$$
\begin{aligned}
\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx &= \\
&-\frac{2(2cd-be)^2(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^4e^2(d+ex)^{7/2}} \\
&+\frac{2(2cd-be)(2cef+4cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^4e^2(d+ex)^{9/2}} \\
&-\frac{2(cef+5cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}{11c^4e^2(d+ex)^{11/2}} \\
&+\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{13/2}}{13c^4e^2(d+ex)^{13/2}}
\end{aligned}
$$

output

```
-2/7*(-b*e+2*c*d)^2*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(7/2)/c^4/e^2/(e*x+d)^(7/2)+2/9*(-b*e+2*c*d)*(-3*b*e*g+4*c*d*g+2*c*e*f)*(d
*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(9/2)/c^4/e^2/(e*x+d)^(9/2)-2/11*(-3*b*e*g+
5*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(11/2)/c^4/e^2/(e*x+d)^(11
/2)+2/13*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(13/2)/c^4/e^2/(e*x+d)^(13/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(-48b^3e^3}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/Sqrt[d +
 e*x],x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-48*b^
3*e^3*g + 8*b^2*c*e^2*(13*e*f + 44*d*g + 21*e*g*x) - 2*b*c^2*e*(423*d^2*g
+ 7*e^2*x*(26*f + 27*g*x) + d*e*(390*f + 532*g*x)) + c^3*(542*d^3*g + 63*e
^3*x^2*(13*f + 11*g*x) + 14*d*e^2*x*(169*f + 144*g*x) + d^2*e*(1963*f + 18
97*g*x))))/(9009*c^4*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1221, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-6beg-cdg+13cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{\sqrt{d+ex}}dx}{13ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13ce^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-6beg-cdg+13cef)\left(\frac{4(2cd-be)\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{3/2}}dx}{11c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce(d+ex)^{3/2}}\right)}{13ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13ce^2\sqrt{d+ex}}$$

$$\Big\downarrow 1128$$

$$\frac{(-6beg-cdg+13cef)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int\frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{5/2}}dx}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce(d+ex)^{5/2}}\right)}{11c}-\frac{2(d(cd-be)-be^2x-}{11ce(d+ex)}\right.}{13ce}$$

$$\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13ce^2\sqrt{d+ex}}$$

$$\Big\downarrow 1122$$

$$\frac{\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{63c^2e(d+ex)^{7/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce(d+ex)^{5/2}}\right)}{11c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce(d+ex)^{3/2}}\right)(-6beg-cdg+}{13ce}$$

$$\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{13ce^2\sqrt{d+ex}}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/Sqrt[d + e*x],
x]
```

output

```
(-2*g*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(13*c*e^2*Sqrt[d + e*x]
) + ((13*c*e*f - c*d*g - 6*b*e*g)*((-2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^
2)^(7/2))/(11*c*e*(d + e*x)^(3/2)) + (4*(2*c*d - b*e)*((-4*(2*c*d - b*e)*(
d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(63*c^2*e*(d + e*x)^(7/2)) - (
2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(9*c*e*(d + e*x)^(5/2))))/(
11*c)))/(13*c*e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.86

method	result
default	$-\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(cex+be-cd)^3\left(-693e^3g\,x^3c^3+378b\,c^2e^3g\,x^2-2016c^3d\,e^2g\,x^2-819c^3e^3f\,x^2-168b^2c\,e^3gx+1064b\,c^2d}{900}$
gosper	$-\frac{2(cex+be-cd)\left(-693e^3g\,x^3c^3+378b\,c^2e^3g\,x^2-2016c^3d\,e^2g\,x^2-819c^3e^3f\,x^2-168b^2c\,e^3gx+1064b\,c^2d\,e^2gx+364b\,c^2e^3fx-1897c}{90}$
orering	$-\frac{2(cex+be-cd)\left(-693e^3g\,x^3c^3+378b\,c^2e^3g\,x^2-2016c^3d\,e^2g\,x^2-819c^3e^3f\,x^2-168b^2c\,e^3gx+1064b\,c^2d\,e^2gx+364b\,c^2e^3fx-1897c}{90}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
-2/9009*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/(e*x+d)^(1/2)*(c*e*x+b*e-c*d)^3*(
-693*c^3*e^3*g*x^3+378*b*c^2*e^3*g*x^2-2016*c^3*d*e^2*g*x^2-819*c^3*e^3*f*
x^2-168*b^2*c*e^3*g*x+1064*b*c^2*d*e^2*g*x+364*b*c^2*e^3*f*x-1897*c^3*d^2*
e*g*x-2366*c^3*d*e^2*f*x+48*b^3*e^3*g-352*b^2*c*d*e^2*g-104*b^2*c*e^3*f+84
6*b*c^2*d^2*e*g+780*b*c^2*d*e^2*f-542*c^3*d^3*g-1963*c^3*d^2*e*f)/c^4/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(243) = 486.

Time = 0.11 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.53

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2\left(693\,c^6e^6gx^6+63\,(13\,c^6e^6f-(c^6de^5-27\,bc^5e^6)g)x^5-7\right.}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

output
```
2/9009*(693*c^6*e^6*g*x^6 + 63*(13*c^6*e^6*f - (c^6*d*e^5 - 27*b*c^5*e^6)*
g)*x^5 - 7*(13*(c^6*d*e^5 - 23*b*c^5*e^6)*f + (296*c^6*d^2*e^4 - 280*b*c^5
*d*e^5 - 159*b^2*c^4*e^6)*g)*x^4 - (13*(206*c^6*d^2*e^4 - 192*b*c^5*d*e^5
- 113*b^2*c^4*e^6)*f - (206*c^6*d^3*e^3 - 3114*b*c^5*d^2*e^4 + 2893*b^2*c^
4*d*e^5 + 15*b^3*c^3*e^6)*g)*x^3 + 3*(13*(10*c^6*d^3*e^3 - 118*b*c^5*d^2*e
^4 + 107*b^2*c^4*d*e^5 + b^3*c^3*e^6)*f + (683*c^6*d^4*e^2 - 1328*b*c^5*d^
3*e^3 + 601*b^2*c^4*d^2*e^4 + 50*b^3*c^3*d*e^5 - 6*b^4*c^2*e^6)*g)*x^2 - 1
3*(151*c^6*d^5*e - 513*b*c^5*d^4*e^2 + 641*b^2*c^4*d^3*e^3 - 355*b^3*c^3*d
^2*e^4 + 84*b^4*c^2*d*e^5 - 8*b^5*c*e^6)*f - 2*(271*c^6*d^6 - 1236*b*c^5*d
^5*e + 2258*b^2*c^4*d^4*e^2 - 2092*b^3*c^3*d^3*e^3 + 1023*b^4*c^2*d^2*e^4
- 248*b^5*c*d*e^5 + 24*b^6*e^6)*g + (13*(271*c^6*d^4*e^2 - 512*b*c^5*d^3*e
^3 + 207*b^2*c^4*d^2*e^4 + 38*b^3*c^3*d*e^5 - 4*b^4*c^2*e^6)*f - (271*c^6*
d^5*e - 965*b*c^5*d^4*e^2 + 1293*b^2*c^4*d^3*e^3 - 799*b^3*c^3*d^2*e^4 + 2
24*b^4*c^2*d*e^5 - 24*b^5*c*e^6)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 -
 b*d*e)*sqrt(e*x + d)/(c^4*e^3*x + c^4*d*e^2)
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx=\int \frac{(-(d+ex)\,(be-cd+cex))^{\frac{5}{2}}\,(f+gx)}{\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**(1/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/sqrt(d + e*x),
x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(243) = 486$.

Time = 0.07 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.39

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx=\frac{2\,(63\,c^5e^5x^5-151\,c^5d^5+513\,bc^4d^4e-641\,b^2c^3d^3e^2+355}{}$$
$$+\frac{2\,(693\,c^6e^6x^6-542\,c^6d^6+2472\,bc^5d^5e-4516\,b^2c^4d^4e^2+4184\,b^3c^3d^3e^3-2046\,b^4c^2d^2e^4+496\,b^5cde^5-}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

output

```
2/693*(63*c^5*e^5*x^5 - 151*c^5*d^5 + 513*b*c^4*d^4*e - 641*b^2*c^3*d^3*e^
2 + 355*b^3*c^2*d^2*e^3 - 84*b^4*c*d*e^4 + 8*b^5*e^5 - 7*(c^5*d*e^4 - 23*b
*c^4*e^5)*x^4 - (206*c^5*d^2*e^3 - 192*b*c^4*d*e^4 - 113*b^2*c^3*e^5)*x^3
+ 3*(10*c^5*d^3*e^2 - 118*b*c^4*d^2*e^3 + 107*b^2*c^3*d*e^4 + b^3*c^2*e^5)
*x^2 + (271*c^5*d^4*e - 512*b*c^4*d^3*e^2 + 207*b^2*c^3*d^2*e^3 + 38*b^3*c
^2*d*e^4 - 4*b^4*c*e^5)*x)*sqrt(-c*e*x + c*d - b*e)*f/(c^3*e) + 2/9009*(69
3*c^6*e^6*x^6 - 542*c^6*d^6 + 2472*b*c^5*d^5*e - 4516*b^2*c^4*d^4*e^2 + 41
84*b^3*c^3*d^3*e^3 - 2046*b^4*c^2*d^2*e^4 + 496*b^5*c*d*e^5 - 48*b^6*e^6 -
 63*(c^6*d*e^5 - 27*b*c^5*e^6)*x^5 - 7*(296*c^6*d^2*e^4 - 280*b*c^5*d*e^5
- 159*b^2*c^4*e^6)*x^4 + (206*c^6*d^3*e^3 - 3114*b*c^5*d^2*e^4 + 2893*b^2*
c^4*d*e^5 + 15*b^3*c^3*e^6)*x^3 + 3*(683*c^6*d^4*e^2 - 1328*b*c^5*d^3*e^3
+ 601*b^2*c^4*d^2*e^4 + 50*b^3*c^3*d*e^5 - 6*b^4*c^2*e^6)*x^2 - (271*c^6*d
^5*e - 965*b*c^5*d^4*e^2 + 1293*b^2*c^4*d^3*e^3 - 799*b^3*c^3*d^2*e^4 + 22
4*b^4*c^2*d*e^5 - 24*b^5*c*e^6)*x)*sqrt(-c*e*x + c*d - b*e)*g/(c^4*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5116 vs. $2(243) = 486$.

Time = 0.46 (sec) , antiderivative size = 5116, normalized size of antiderivative = 19.16

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(1/2),x,
algorithm="giac")
```

output
```
-2/45045*(15015*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d^4*e*f - 30030*(-(e*
x + d)*c + 2*c*d - b*e)^(3/2)*b*d^3*e^2*f + 15015*(-(e*x + d)*c + 2*c*d -
b*e)^(3/2)*b^2*d^2*e^3*f/c - 6006*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*
d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*
e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*b*d^2*e^2*f/c + 6006*(5*(-(e*x + d)
*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3
*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*b^2*d*e^3
*f/c^2 + 3003*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c
+ 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)
*c + 2*c*d - b*e))*d^4*g - 6006*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d
- 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)
^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*b*d^3*e*g/c + 3003*(5*(-(e*x + d)*c +
 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e
*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*b^2*d^2*e^2*g
/c^2 - 858*(35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*d^2 - 70*(-(e*x + d)
*c + 2*c*d - b*e)^(3/2)*b*c*d*e + 35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^
2*e^2 - 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*
c*d + 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*
e - 15*((e*x + d)*c - 2*c*d + b*e)^3*sqrt(-(e*x + d)*c + 2*c*d - b*e))*d^2
*e*f/c + 858*(35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*d^2 - 70*(-(e*x...
```

Mupad [B] (verification not implemented)

Time = 11.70 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.84

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx=\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{2\,e^2\,x^4\,\left(159\,g\,b^2\,e^2+280\,g\,b\,c\,d\,e+29}{128}\right.}{}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(1/2
),x)
```

output
```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*e^2*x^4*(159*b^2*e^2*g -
296*c^2*d^2*g + 299*b*c*e^2*f - 13*c^2*d*e*f + 280*b*c*d*e*g))/1287 + (2*c
^2*e^4*g*x^6)/13 + (2*x^2*(b*e - c*d)*(13*b^2*c*e^3*f - 683*c^3*d^3*g - 6*
b^3*e^3*g - 130*c^3*d^2*e*f + 1404*b*c^2*d*e^2*f + 645*b*c^2*d^2*e*g + 44*
b^2*c*d*e^2*g))/(3003*c^2) + (x^3*(2938*b^2*c^4*e^6*f + 30*b^3*c^3*e^6*g -
 5356*c^6*d^2*e^4*f + 412*c^6*d^3*e^3*g + 4992*b*c^5*d*e^5*f - 6228*b*c^5*
d^2*e^4*g + 5786*b^2*c^4*d*e^5*g))/(9009*c^4*e^2) + (2*c*e^3*x^5*(27*b*e*g
 - c*d*g + 13*c*e*f))/143 + (2*(b*e - c*d)^3*(542*c^3*d^3*g - 48*b^3*e^3*g
 + 104*b^2*c*e^3*f + 1963*c^3*d^2*e*f - 780*b*c^2*d*e^2*f - 846*b*c^2*d^2*
e*g + 352*b^2*c*d*e^2*g))/(9009*c^4*e^2) + (2*x*(b*e - c*d)^2*(24*b^3*e^3*
g - 271*c^3*d^3*g - 52*b^2*c*e^3*f + 3523*c^3*d^2*e*f + 390*b*c^2*d*e^2*f
+ 423*b*c^2*d^2*e*g - 176*b^2*c*d*e^2*g))/(9009*c^3*e)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.64

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{-cex-be+cd}\,\left(693c^6e^6g\,x^6+1701b\,c^5e^6g\,x^5-63c^6d\,e\right.}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(1/2),x)
```

output

```
(2*sqrt( - b*e + c*d - c*e*x)*( - 48*b**6*e**6*g + 496*b**5*c*d*e**5*g + 1
04*b**5*c*e**6*f + 24*b**5*c*e**6*g*x - 2046*b**4*c**2*d**2*e**4*g - 1092*
b**4*c**2*d*e**5*f - 224*b**4*c**2*d*e**5*g*x - 52*b**4*c**2*e**6*f*x - 18
*b**4*c**2*e**6*g*x**2 + 4184*b**3*c**3*d**3*e**3*g + 4615*b**3*c**3*d**2*
e**4*f + 799*b**3*c**3*d**2*e**4*g*x + 494*b**3*c**3*d*e**5*f*x + 150*b**3
*c**3*d*e**5*g*x**2 + 39*b**3*c**3*e**6*f*x**2 + 15*b**3*c**3*e**6*g*x**3
- 4516*b**2*c**4*d**4*e**2*g - 8333*b**2*c**4*d**3*e**3*f - 1293*b**2*c**4
*d**3*e**3*g*x + 2691*b**2*c**4*d**2*e**4*f*x + 1803*b**2*c**4*d**2*e**4*g
*x**2 + 4173*b**2*c**4*d*e**5*f*x**2 + 2893*b**2*c**4*d*e**5*g*x**3 + 1469
*b**2*c**4*e**6*f*x**3 + 1113*b**2*c**4*e**6*g*x**4 + 2472*b*c**5*d**5*e*g
 + 6669*b*c**5*d**4*e**2*f + 965*b*c**5*d**4*e**2*g*x - 6656*b*c**5*d**3*e
**3*f*x - 3984*b*c**5*d**3*e**3*g*x**2 - 4602*b*c**5*d**2*e**4*f*x**2 - 31
14*b*c**5*d**2*e**4*g*x**3 + 2496*b*c**5*d*e**5*f*x**3 + 1960*b*c**5*d*e**
5*g*x**4 + 2093*b*c**5*e**6*f*x**4 + 1701*b*c**5*e**6*g*x**5 - 542*c**6*d*
*6*g - 1963*c**6*d**5*e*f - 271*c**6*d**5*e*g*x + 3523*c**6*d**4*e**2*f*x
+ 2049*c**6*d**4*e**2*g*x**2 + 390*c**6*d**3*e**3*f*x**2 + 206*c**6*d**3*e
**3*g*x**3 - 2678*c**6*d**2*e**4*f*x**3 - 2072*c**6*d**2*e**4*g*x**4 - 91*
c**6*d*e**5*f*x**4 - 63*c**6*d*e**5*g*x**5 + 819*c**6*e**6*f*x**5 + 693*c*
*6*e**6*g*x**6))/(9009*c**4*e**2)
```

3.218 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 190

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx = -\frac{2(2cd-be)(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^3e^2(d+ex)^{7/2}} + \frac{2(cef+3cdg-2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^3e^2(d+ex)^{9/2}} - \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{11/2}}{11c^3e^2(d+ex)^{11/2}}$$

output
```
-2/7*(-b*e+2*c*d)*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7
/2)/c^3/e^2/(e*x+d)^(7/2)+2/9*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2
*x-c*e^2*x^2)^(9/2)/c^3/e^2/(e*x+d)^(9/2)-2/11*g*(d*(-b*e+c*d)-b*e^2*x-c*e
^2*x^2)^(11/2)/c^3/e^2/(e*x+d)^(11/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.64

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx=\frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(8b^2e^2g-}{}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^(3/2),x]
```

output

```
(2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(8*b^2*
e^2*g - 2*b*c*e*(11*e*f + 19*d*g + 14*e*g*x) + c^2*(30*d^2*g + 7*e^2*x*(11
*f + 9*g*x) + d*e*(121*f + 105*g*x))))/(693*c^3*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-4beg-3cdg+11cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{3/2}}dx}{11ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-4beg-3cdg+11cef)\left(\frac{2(2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{5/2}}dx}{9c}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce(d+ex)^{5/2}}\right)}{11ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{\left(-\frac{4(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{63c^2e(d+ex)^{7/2}}-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce(d+ex)^{5/2}}\right)(-4beg-3cdg+11cef)}{11ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{11ce^2(d+ex)^{3/2}}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^(3/2
),x]
```

output
```
(-2*g*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(11*c*e^2*(d + e*x)^(3/
2)) + ((11*c*e*f - 3*c*d*g - 4*b*e*g)*((-4*(2*c*d - b*e)*(d*(c*d - b*e) -
b*e^2*x - c*e^2*x^2)^(7/2))/(63*c^2*e*(d + e*x)^(7/2)) - (2*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(7/2))/(9*c*e*(d + e*x)^(5/2))))/(11*c*e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.70

method	result
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(cex+be-cd)^3\,(63g\,x^2c^2e^2-28bc\,e^2gx+105c^2degx+77c^2e^2fx+8b^2e^2g-38bcdeg-22bc\,e^2f+30c^2d^2g+12}{693\sqrt{ex+d}\,c^3e^2}$
gosper	$\frac{2(cex+be-cd)\,(63g\,x^2c^2e^2-28bc\,e^2gx+105c^2degx+77c^2e^2fx+8b^2e^2g-38bcdeg-22bc\,e^2f+30c^2d^2g+121c^2def)\,(-x^2c\,e^2-xb\,e^2-}{693c^3e^2(ex+d)^{\frac{5}{2}}}$
orering	$\frac{2(cex+be-cd)\,(63g\,x^2c^2e^2-28bc\,e^2gx+105c^2degx+77c^2e^2fx+8b^2e^2g-38bcdeg-22bc\,e^2f+30c^2d^2g+121c^2def)\,(-x^2c\,e^2-xb\,e^2-}{693c^3e^2(ex+d)^{\frac{5}{2}}}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2/693*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/(e*x+d)^(1/2)*(c*e*x+b*e-c*d)^3*(63
*c^2*e^2*g*x^2-28*b*c*e^2*g*x+105*c^2*d*e*g*x+77*c^2*e^2*f*x+8*b^2*e^2*g-3
8*b*c*d*e*g-22*b*c*e^2*f+30*c^2*d^2*g+121*c^2*d*e*f)/c^3/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(172) = 344$.

Time = 0.09 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.63

$$\int \frac{(f+gx)\,(cd^2-bde-be^2x-ce^2x^2)^{5/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(63\,c^5e^5gx^5+7\,(11\,c^5e^5f-(12\,c^5de^4-23\,bc^4e^5)g)x^4-(}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(3/2),x,
algorithm="fricas")
```

output

```
2/693*(63*c^5*e^5*g*x^5 + 7*(11*c^5*e^5*f - (12*c^5*d*e^4 - 23*b*c^4*e^5)*
g)*x^4 - (11*(10*c^5*d*e^4 - 19*b*c^4*e^5)*f + (96*c^5*d^2*e^3 + 17*b*c^4*
d*e^4 - 113*b^2*c^3*e^5)*g)*x^3 - 3*(11*(4*c^5*d^2*e^3 + b*c^4*d*e^4 - 5*b
^2*c^3*e^5)*f - (54*c^5*d^3*e^2 - 107*b*c^4*d^2*e^3 + 52*b^2*c^3*d*e^4 + b
^3*c^2*e^5)*g)*x^2 - 11*(11*c^5*d^4*e - 35*b*c^4*d^3*e^2 + 39*b^2*c^3*d^2*
e^3 - 17*b^3*c^2*d*e^4 + 2*b^4*c*e^5)*f - 2*(15*c^5*d^5 - 64*b*c^4*d^4*e +
 106*b^2*c^3*d^3*e^2 - 84*b^3*c^2*d^2*e^3 + 31*b^4*c*d*e^4 - 4*b^5*e^5)*g
+ (11*(26*c^5*d^3*e^2 - 51*b*c^4*d^2*e^3 + 24*b^2*c^3*d*e^4 + b^3*c^2*e^5)
*f - (15*c^5*d^4*e - 49*b*c^4*d^3*e^2 + 57*b^2*c^3*d^2*e^3 - 27*b^3*c^2*d*
e^4 + 4*b^4*c*e^5)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e
*x + d)/(c^3*e^3*x + c^3*d*e^2)
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left(-(d+ex)\,(be-cd+cex)\right)^{\frac{5}{2}}(f+gx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**(3/
2),x)
```

output

```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**(3/2
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(172) = 344.

Time = 0.08 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.45

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx = \frac{2\,(7\,c^4e^4x^4-11\,c^4d^4+35\,bc^3d^3e-39\,b^2c^2d^2e^2+17\,b^3cde^3}{} \\ + \frac{2\,(63\,c^5e^5x^5-30\,c^5d^5+128\,bc^4d^4e-212\,b^2c^3d^3e^2+168\,b^3c^2d^2e^3-62\,b^4cde^4+8\,b^5e^5-7\,(12\,c^5de^4-23}{}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(3/2),x,
algorithm="maxima")
```

output

```
2/63*(7*c^4*e^4*x^4 - 11*c^4*d^4 + 35*b*c^3*d^3*e - 39*b^2*c^2*d^2*e^2 + 1
7*b^3*c*d*e^3 - 2*b^4*e^4 - (10*c^4*d*e^3 - 19*b*c^3*e^4)*x^3 - 3*(4*c^4*d
^2*e^2 + b*c^3*d*e^3 - 5*b^2*c^2*e^4)*x^2 + (26*c^4*d^3*e - 51*b*c^3*d^2*e
^2 + 24*b^2*c^2*d*e^3 + b^3*c*e^4)*x)*sqrt(-c*e*x + c*d - b*e)*f/(c^2*e) +
 2/693*(63*c^5*e^5*x^5 - 30*c^5*d^5 + 128*b*c^4*d^4*e - 212*b^2*c^3*d^3*e^
2 + 168*b^3*c^2*d^2*e^3 - 62*b^4*c*d*e^4 + 8*b^5*e^5 - 7*(12*c^5*d*e^4 - 2
3*b*c^4*e^5)*x^4 - (96*c^5*d^2*e^3 + 17*b*c^4*d*e^4 - 113*b^2*c^3*e^5)*x^3
 + 3*(54*c^5*d^3*e^2 - 107*b*c^4*d^2*e^3 + 52*b^2*c^3*d*e^4 + b^3*c^2*e^5)
*x^2 - (15*c^5*d^4*e - 49*b*c^4*d^3*e^2 + 57*b^2*c^3*d^2*e^3 - 27*b^3*c^2*
d*e^4 + 4*b^4*c*e^5)*x)*sqrt(-c*e*x + c*d - b*e)*g/(c^3*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3083 vs. 2(172) = 344.

Time = 0.41 (sec) , antiderivative size = 3083, normalized size of antiderivative = 16.23

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(3/2),x,
algorithm="giac")
```

output
```
-2/3465*(1155*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d^3*e*f - 2310*(-(e*x +
 d)*c + 2*c*d - b*e)^(3/2)*b*d^2*e^2*f + 1155*(-(e*x + d)*c + 2*c*d - b*e)
^(3/2)*b^2*d*e^3*f/c - 231*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(
-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)^2*sq
rt(-(e*x + d)*c + 2*c*d - b*e))*d^2*e*f + 231*(5*(-(e*x + d)*c + 2*c*d - b
*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c
- 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*b^2*e^3*f/c^2 + 231*(5*
(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3
/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)
)*d^3*g - 462*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c
+ 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)
*c + 2*c*d - b*e))*b*d^2*e*g/c + 231*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)
*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d +
 b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*b^2*d*e^2*g/c^2 - 33*(35*(-(e*x
+ d)*c + 2*c*d - b*e)^(3/2)*c^2*d^2 - 70*(-(e*x + d)*c + 2*c*d - b*e)^(3/2
)*b*c*d*e + 35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*e^2 - 42*((e*x + d)*
c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c*d + 42*((e*x + d)*c
- 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*e - 15*((e*x + d)*c -
2*c*d + b*e)^3*sqrt(-(e*x + d)*c + 2*c*d - b*e))*d*e*f/c + 66*(35*(-(e*x +
 d)*c + 2*c*d - b*e)^(3/2)*c^2*d^2 - 70*(-(e*x + d)*c + 2*c*d - b*e)^(3...
```

Mupad [B] (verification not implemented)

Time = 11.36 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.68

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{2\,c^2\,e^3\,g\,x^5}{11}+\frac{2\,c\,e^2\,x^4\,(23\,b\,e\,g-12}{99}\right.}{}$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(3/2
),x)
```

output
```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*c^2*e^3*g*x^5)/11 + (2*c*
e^2*x^4*(23*b*e*g - 12*c*d*g + 11*c*e*f))/99 + (2*x^2*(b*e - c*d)*(b^2*e^2
*g - 54*c^2*d^2*g + 55*b*c*e^2*f + 44*c^2*d*e*f + 53*b*c*d*e*g))/(231*c) -
 (x^3*(192*c^5*d^2*e^3*g - 226*b^2*c^3*e^5*g - 418*b*c^4*e^5*f + 220*c^5*d
*e^4*f + 34*b*c^4*d*e^4*g))/(693*c^3*e^2) + (2*(b*e - c*d)^3*(8*b^2*e^2*g
+ 30*c^2*d^2*g - 22*b*c*e^2*f + 121*c^2*d*e*f - 38*b*c*d*e*g))/(693*c^3*e^
2) + (2*x*(b*e - c*d)^2*(11*b*c*e^2*f - 15*c^2*d^2*g - 4*b^2*e^2*g + 286*c
^2*d*e*f + 19*b*c*d*e*g))/(693*c^2*e)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.63

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{-cex-be+cd}\,(63c^5e^5g\,x^5+161b\,c^4e^5g\,x^4-84c^5d\,e^4g}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*(8*b**5*e**5*g - 62*b**4*c*d*e**4*g - 22*b**
4*c*e**5*f - 4*b**4*c*e**5*g*x + 168*b**3*c**2*d**2*e**3*g + 187*b**3*c**2
*d*e**4*f + 27*b**3*c**2*d*e**4*g*x + 11*b**3*c**2*e**5*f*x + 3*b**3*c**2*
e**5*g*x**2 - 212*b**2*c**3*d**3*e**2*g - 429*b**2*c**3*d**2*e**3*f - 57*b
**2*c**3*d**2*e**3*g*x + 264*b**2*c**3*d*e**4*f*x + 156*b**2*c**3*d*e**4*g
*x**2 + 165*b**2*c**3*e**5*f*x**2 + 113*b**2*c**3*e**5*g*x**3 + 128*b*c**4
*d**4*e*g + 385*b*c**4*d**3*e**2*f + 49*b*c**4*d**3*e**2*g*x - 561*b*c**4*
d**2*e**3*f*x - 321*b*c**4*d**2*e**3*g*x**2 - 33*b*c**4*d*e**4*f*x**2 - 17
*b*c**4*d*e**4*g*x**3 + 209*b*c**4*e**5*f*x**3 + 161*b*c**4*e**5*g*x**4 -
30*c**5*d**5*g - 121*c**5*d**4*e*f - 15*c**5*d**4*e*g*x + 286*c**5*d**3*e*
*2*f*x + 162*c**5*d**3*e**2*g*x**2 - 132*c**5*d**2*e**3*f*x**2 - 96*c**5*d
**2*e**3*g*x**3 - 110*c**5*d*e**4*f*x**3 - 84*c**5*d*e**4*g*x**4 + 77*c**5
*e**5*f*x**4 + 63*c**5*e**5*g*x**5))/(693*c**3*e**2)
```

3.219 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 116

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{2(cef+cdg-beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^2e^2(d+ex)^{7/2}} + \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{9/2}}{9c^2e^2(d+ex)^{9/2}}$$

output

```
-2/7*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^2/e^2/(
e*x+d)^(7/2)+2/9*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(9/2)/c^2/e^2/(e*x+d)^
(9/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{2(-cd+be+cex)^3\sqrt{(d+ex)(-be+c(d-ex))}(-2beg+}{63c^2e^2\sqrt{d+ex}}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^(5/2),x]
```

output
```
(2*(-(c*d) + b*e + c*e*x)^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-2*b*e
*g + c*(9*e*f + 2*d*g + 7*e*g*x)))/(63*c^2*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-2beg-5cdg+9cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{5/2}}dx}{9ce}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce^2(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1122}$$

$$-\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}(-2beg-5cdg+9cef)}{63c^2e^2(d+ex)^{7/2}}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{9ce^2(d+ex)^{5/2}}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^(5/2
),x]
```

output
```
(-2*(9*c*e*f - 5*c*d*g - 2*b*e*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7
/2))/(63*c^2*e^2*(d + e*x)^(7/2)) - (2*g*(d*(c*d - b*e) - b*e^2*x - c*e^2*
x^2)^(7/2))/(9*c*e^2*(d + e*x)^(5/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{2 \sqrt{-\left(e x +d \right) \left(c e x +b e -c d \right)}\, \left(c e x +b e -c d \right)^{3} \left(-7 c e g x +2 b e g -2 c d g -9 f c e \right)}{63 \sqrt{e x +d}\, c^{2} e^{2}}$	73
gosper	$-\frac{2 \left(c e x +b e -c d \right) \left(-7 c e g x +2 b e g -2 c d g -9 f c e \right) \left(-x^{2} c \,e^{2}-x b \,e^{2}-b d e +c \,d^{2}\right)^{\frac{5}{2}}}{63 c^{2} e^{2} \left(e x +d \right)^{\frac{5}{2}}}$	79
orering	$-\frac{2 \left(c e x +b e -c d \right) \left(-7 c e g x +2 b e g -2 c d g -9 f c e \right) \left(-x^{2} c \,e^{2}-x b \,e^{2}-b d e +c \,d^{2}\right)^{\frac{5}{2}}}{63 c^{2} e^{2} \left(e x +d \right)^{\frac{5}{2}}}$	79

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
-2/63*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/(e*x+d)^(1/2)*(c*e*x+b*e-c*d)^3*(-7
*c*e*g*x+2*b*e*g-2*c*d*g-9*c*e*f)/c^2/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(104) = 208$.

Time = 0.09 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.97

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(7\,c^4e^4gx^4+(9\,c^4e^4f-19\,(c^4de^3-bc^3e^4)g)x^3-3\,(9\,(c^4}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="fricas")
```

output
```
2/63*(7*c^4*e^4*g*x^4 + (9*c^4*e^4*f - 19*(c^4*d*e^3 - b*c^3*e^4)*g)*x^3 -
 3*(9*(c^4*d*e^3 - b*c^3*e^4)*f - 5*(c^4*d^2*e^2 - 2*b*c^3*d*e^3 + b^2*c^2
*e^4)*g)*x^2 - 9*(c^4*d^3*e - 3*b*c^3*d^2*e^2 + 3*b^2*c^2*d*e^3 - b^3*c*e^
4)*f - 2*(c^4*d^4 - 4*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^
4*e^4)*g + (27*(c^4*d^2*e^2 - 2*b*c^3*d*e^3 + b^2*c^2*e^4)*f - (c^4*d^3*e
- 3*b*c^3*d^2*e^2 + 3*b^2*c^2*d*e^3 - b^3*c*e^4)*g)*x)*sqrt(-c*e^2*x^2 - b
*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^2*e^3*x + c^2*d*e^2)
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{\left(-(d+ex)\,(be-cd+cex)\right)^{\frac{5}{2}}\,(f+gx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**(5/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**(5/2
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(104) = 208$.

Time = 0.08 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.71

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\,(c^3e^3x^3-c^3d^3+3\,bc^2d^2e-3\,b^2cde^2+b^3e^3-3\,(c^3de^2-}{7}$$
$$+\frac{2\,(7\,c^4e^4x^4-2\,c^4d^4+8\,bc^3d^3e-12\,b^2c^2d^2e^2+8\,b^3cde^3-2\,b^4e^4-19\,(c^4de^3-bc^3e^4)x^3+15\,(c^4d^2e^2-2\,b}{63\,c^2e^2}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="maxima")
```

output
```
2/7*(c^3*e^3*x^3 - c^3*d^3 + 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + b^3*e^3 - 3*(
c^3*d*e^2 - b*c^2*e^3)*x^2 + 3*(c^3*d^2*e - 2*b*c^2*d*e^2 + b^2*c*e^3)*x)*
sqrt(-c*e*x + c*d - b*e)*f/(c*e) + 2/63*(7*c^4*e^4*x^4 - 2*c^4*d^4 + 8*b*c
^3*d^3*e - 12*b^2*c^2*d^2*e^2 + 8*b^3*c*d*e^3 - 2*b^4*e^4 - 19*(c^4*d*e^3
- b*c^3*e^4)*x^3 + 15*(c^4*d^2*e^2 - 2*b*c^3*d*e^3 + b^2*c^2*e^4)*x^2 - (c
^4*d^3*e - 3*b*c^3*d^2*e^2 + 3*b^2*c^2*d*e^3 - b^3*c*e^4)*x)*sqrt(-c*e*x +
 c*d - b*e)*g/(c^2*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(104) = 208$.

Time = 0.43 (sec) , antiderivative size = 1540, normalized size of antiderivative = 13.28

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="giac")
```

output

```
-2/315*(105*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d^2*e*f - 210*(-(e*x + d)
*c + 2*c*d - b*e)^(3/2)*b*d*e^2*f + 105*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)
*b^2*e^3*f/c - 42*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d
)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x
+ d)*c + 2*c*d - b*e))*d*e*f + 42*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*
d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*
e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*b*e^2*f/c + 21*(5*(-(e*x + d)*c + 2
*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x
 + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))*d^2*g - 42*(5*(
-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/
2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e))
*b*d*e*g/c + 21*(5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d - 5*(-(e*x + d)*
c + 2*c*d - b*e)^(3/2)*b*e - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x +
d)*c + 2*c*d - b*e))*b^2*e^2*g/c^2 + 3*(35*(-(e*x + d)*c + 2*c*d - b*e)^(3
/2)*c^2*d^2 - 70*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c*d*e + 35*(-(e*x +
d)*c + 2*c*d - b*e)^(3/2)*b^2*e^2 - 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(
-(e*x + d)*c + 2*c*d - b*e)*c*d + 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(
e*x + d)*c + 2*c*d - b*e)*b*e - 15*((e*x + d)*c - 2*c*d + b*e)^3*sqrt(-(e*
x + d)*c + 2*c*d - b*e))*e*f/c - 6*(35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*
c^2*d^2 - 70*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c*d*e + 35*(-(e*x + d...
```

Mupad [B] (verification not implemented)

Time = 11.02 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.47

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx=\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{2\,x^2\,(b\,e-c\,d)\,(5\,b\,e\,g-5\,c\,d\,g+9\,c\,e\,f)}{21}\right.}{}$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(5/2
),x)
```

output

```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*x^2*(b*e - c*d)*(5*b*e*g
- 5*c*d*g + 9*c*e*f))/21 + (2*c*e*x^3*(19*b*e*g - 19*c*d*g + 9*c*e*f))/63
+ (2*c^2*e^2*g*x^4)/9 + (2*(b*e - c*d)^3*(2*c*d*g - 2*b*e*g + 9*c*e*f))/(6
3*c^2*e^2) + (2*x*(b*e - c*d)^2*(b*e*g - c*d*g + 27*c*e*f))/(63*c*e)))/(d
+ e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.85

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{-cex-be+cd}\,(7c^4e^4g\,x^4+19b\,c^3e^4g\,x^3-19c^4d\,e^3g\,x^3}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*( - 2*b**4*e**4*g + 8*b**3*c*d*e**3*g + 9*b*
*3*c*e**4*f + b**3*c*e**4*g*x - 12*b**2*c**2*d**2*e**2*g - 27*b**2*c**2*d*
e**3*f - 3*b**2*c**2*d*e**3*g*x + 27*b**2*c**2*e**4*f*x + 15*b**2*c**2*e**
4*g*x**2 + 8*b*c**3*d**3*e*g + 27*b*c**3*d**2*e**2*f + 3*b*c**3*d**2*e**2*
g*x - 54*b*c**3*d*e**3*f*x - 30*b*c**3*d*e**3*g*x**2 + 27*b*c**3*e**4*f*x*
*2 + 19*b*c**3*e**4*g*x**3 - 2*c**4*d**4*g - 9*c**4*d**3*e*f - c**4*d**3*e
*g*x + 27*c**4*d**2*e**2*f*x + 15*c**4*d**2*e**2*g*x**2 - 27*c**4*d*e**3*f
*x**2 - 19*c**4*d*e**3*g*x**3 + 9*c**4*e**4*f*x**3 + 7*c**4*e**4*g*x**4))/
(63*c**2*e**2)
```

3.220 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 316

$$\begin{aligned}
\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx &= \frac{2(2cd-be)^2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2\sqrt{d+ex}} \\
&+ \frac{2(2cd-be)(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{3/2}} \\
&+ \frac{2(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(d+ex)^{5/2}} - \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7ce^2(d+ex)^{7/2}} \\
&- \frac{2(2cd-be)^{5/2}(ef-dg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{e^2}
\end{aligned}$$

output

```
2*(-b*e+2*c*d)^2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*
x+d)^(1/2)+2/3*(-b*e+2*c*d)*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3
/2)/e^2/(e*x+d)^(3/2)+2/5*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2
)/e^2/(e*x+d)^(5/2)-2/7*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c/e^2/(e*
x+d)^(7/2)-2*(-b*e+2*c*d)^(5/2)*(-d*g+e*f)*arctanh((d*(-b*e+c*d)-b*e^2*x-c
*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.80

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx=\frac{2((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{15b^3e^3g+b^2ce^2(161ef-206dg+}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^(7/2),x]
```

output
```
(2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((15*b^3*e^3*g + b^2*c*e^2*(16
1*e*f - 206*d*g + 45*e*g*x) + c^3*(-526*d^3*g + 3*e^3*x^2*(7*f + 5*g*x) -
2*d*e^2*x*(56*f + 33*g*x) + d^2*e*(511*f + 157*g*x)) + b*c^2*e*(612*d^2*g
+ e^2*x*(77*f + 45*g*x) - d*e*(567*f + 167*g*x)))/(c*(-(c*d) + b*e + c*e*x
)^2) + (105*(-2*c*d + b*e)^(5/2)*(-(e*f) + d*g)*ArcTan[Sqrt[c*d - b*e - c*
e*x]/Sqrt[-2*c*d + b*e]])/(-(b*e) + c*(d - e*x))^(5/2)))/(105*e^2*(d + e*x
)^(5/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1221, 1131, 1131, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \color{blue}{1221}$$

$$\frac{(ef-dg)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{7/2}}dx}{e}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7ce^2(d+ex)^{7/2}}$$

$$\Big\downarrow \color{blue}{1131}$$

$$\frac{(ef-dg)\left((2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{5/2}}dx+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e(d+ex)^{5/2}}\right)}{e}-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7ce^2(d+ex)^{7/2}}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(ef-dg)\left((2cd-be)\left((2cd-be)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{3/2}}dx+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}}\right)+\frac{2(d(cd-be)-be^2x-ce^2x^2}{5e(d+ex)^{5/2}}\right.}{e}$$

$$\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7ce^2(d+ex)^{7/2}}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(ef-dg)\left((2cd-be)\left((2cd-be)\left((2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}\right)+2(d(c\right.\right.}{e}$$

$$\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7ce^2(d+ex)^{7/2}}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{(ef-dg)\left((2cd-be)\left((2cd-be)\left(2e(2cd-be)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}+2\surd\right.\right.\right.}{e}$$

$$\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7ce^2(d+ex)^{7/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{(ef-dg)\left((2cd-be)\left((2cd-be)\left(\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2cd-be}\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e}\right)+\frac{2(d(cd-b}{3}\right.\right.}{e}$$

$$\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7ce^2(d+ex)^{7/2}}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^(7/2
),x]
```

output

```
(-2*g*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(7*c*e^2*(d + e*x)^(7/2
)) + ((e*f - d*g)*((2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(5*e*(d
 + e*x)^(5/2)) + (2*c*d - b*e)*((2*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(
3/2))/(3*e*(d + e*x)^(3/2)) + (2*c*d - b*e)*((2*Sqrt[d*(c*d - b*e) - b*e^2
*x - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2*c*d - b*e]*ArcTanh[Sqrt[d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/e)))
)/e
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1131

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. 2(286) = 572.

Time = 1.58 (sec) , antiderivative size = 948, normalized size of antiderivative = 3.00

method	result
default	$2\sqrt{-(ex+d)(cex+be-cd)}\,\Big(15c^3e^3g\,x^3\sqrt{-cex-be+cd}\,\sqrt{be-2cd}+45b\,c^2e^3g\,x^2\sqrt{-cex-be+cd}\,\sqrt{be-2cd}-66c^3d\,e^2g\,x^2\sqrt{-cex-be+}$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(7/2),x,method=
_RETURNVERBOSE)
```

output
```
2/105*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(15*c^3*e^3*g*x^3*(-c*e*x-b*e+c*d)^
(1/2)*(b*e-2*c*d)^(1/2)+45*b*c^2*e^3*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c
*d)^(1/2)-66*c^3*d*e^2*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+21*c
^3*e^3*f*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+105*arctan((-c*e*x-b
*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^3*c*d*e^3*g-105*arctan((-c*e*x-b*e+c*d)
^(1/2)/(b*e-2*c*d)^(1/2))*b^3*c*e^4*f-630*arctan((-c*e*x-b*e+c*d)^(1/2)/(b
*e-2*c*d)^(1/2))*b^2*c^2*d^2*e^2*g+630*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-
2*c*d)^(1/2))*b^2*c^2*d*e^3*f+1260*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*
d)^(1/2))*b*c^3*d^3*e*g-1260*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/
2))*b*c^3*d^2*e^2*f-840*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c
^4*d^4*g+840*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^4*d^3*e*f+
45*b^2*c*e^3*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-167*b*c^2*d*e^2*
g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+77*b*c^2*e^3*f*x*(-c*e*x-b*e+
c*d)^(1/2)*(b*e-2*c*d)^(1/2)+157*c^3*d^2*e*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e
-2*c*d)^(1/2)-112*c^3*d*e^2*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+1
5*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)*b^3*e^3*g-206*(b*e-2*c*d)^(1/2)
*(-c*e*x-b*e+c*d)^(1/2)*b^2*c*d*e^2*g+161*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*
d)^(1/2)*b^2*c*e^3*f+612*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)*b*c^2*d^
2*e*g-567*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)*b*c^2*d*e^2*f-526*(b*e-
2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)*c^3*d^3*g+511*(b*e-2*c*d)^(1/2)*(-c...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 951, normalized size of antiderivative = 3.01

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(7/2),x,
algorithm="fricas")
```

output

```
[-1/105*(105*sqrt(2*c*d - b*e)*((4*c^3*d^3*e - 4*b*c^2*d^2*e^2 + b^2*c*d*e
^3)*f - (4*c^3*d^4 - 4*b*c^2*d^3*e + b^2*c*d^2*e^2)*g + ((4*c^3*d^2*e^2 -
4*b*c^2*d*e^3 + b^2*c*e^4)*f - (4*c^3*d^3*e - 4*b*c^2*d^2*e^2 + b^2*c*d*e^
3)*g)*x)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*sqr
t(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(
e^2*x^2 + 2*d*e*x + d^2)) - 2*(15*c^3*e^3*g*x^3 + 3*(7*c^3*e^3*f - (22*c^3
*d*e^2 - 15*b*c^2*e^3)*g)*x^2 + 7*(73*c^3*d^2*e - 81*b*c^2*d*e^2 + 23*b^2*
c*e^3)*f - (526*c^3*d^3 - 612*b*c^2*d^2*e + 206*b^2*c*d*e^2 - 15*b^3*e^3)*
g - (7*(16*c^3*d*e^2 - 11*b*c^2*e^3)*f - (157*c^3*d^2*e - 167*b*c^2*d*e^2
+ 45*b^2*c*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x
+ d))/(c*e^3*x + c*d*e^2), -2/105*(105*sqrt(-2*c*d + b*e)*((4*c^3*d^3*e -
4*b*c^2*d^2*e^2 + b^2*c*d*e^3)*f - (4*c^3*d^4 - 4*b*c^2*d^3*e + b^2*c*d^2*
e^2)*g + ((4*c^3*d^2*e^2 - 4*b*c^2*d*e^3 + b^2*c*e^4)*f - (4*c^3*d^3*e - 4
*b*c^2*d^2*e^2 + b^2*c*d*e^3)*g)*x)*arctan(-sqrt(-c*e^2*x^2 - b*e^2*x + c*
d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e + (2*c*d*e
- b*e^2)*x)) - (15*c^3*e^3*g*x^3 + 3*(7*c^3*e^3*f - (22*c^3*d*e^2 - 15*b*c
^2*e^3)*g)*x^2 + 7*(73*c^3*d^2*e - 81*b*c^2*d*e^2 + 23*b^2*c*e^3)*f - (526
*c^3*d^3 - 612*b*c^2*d^2*e + 206*b^2*c*d*e^2 - 15*b^3*e^3)*g - (7*(16*c^3*
d*e^2 - 11*b*c^2*e^3)*f - (157*c^3*d^2*e - 167*b*c^2*d*e^2 + 45*b^2*c*e^3)
*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d))/(c*e^3...
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{5}{2}}(f+gx)}{(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**(7/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**(7/2
), x)
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx=\int \frac{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{5}{2}}(gx+f)}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(7/2),x,
algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*(g*x + f)/(e*x + d)
^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.77

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx=\frac{2\left(\frac{105\left(8\,c^3d^3ef-12\,bc^2d^2e^2f+6\,b^2cde^3f-b^3e^4f-8\,c^3d^4g+12\,bc^2d^3eg-6\,b^2c}{\sqrt{-2\,cd+be}}\right.}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(7/2),x,
algorithm="giac")
```

output
```
2/105*(105*(8*c^3*d^3*e*f - 12*b*c^2*d^2*e^2*f + 6*b^2*c*d*e^3*f - b^3*e^4
*f - 8*c^3*d^4*g + 12*b*c^2*d^3*e*g - 6*b^2*c*d^2*e^2*g + b^3*d*e^3*g)*arc
tan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(-2*c*d + b*e))/sqrt(-2*c*d + b*e
) + (420*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^9*d^2*e*f - 420*sqrt(-(e*x + d
)*c + 2*c*d - b*e)*b*c^8*d*e^2*f + 105*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^
2*c^7*e^3*f - 420*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^9*d^3*g + 420*sqrt(-(
e*x + d)*c + 2*c*d - b*e)*b*c^8*d^2*e*g - 105*sqrt(-(e*x + d)*c + 2*c*d -
b*e)*b^2*c^7*d*e^2*g + 70*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^8*d*e*f - 3
5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^7*e^2*f - 70*(-(e*x + d)*c + 2*c*
d - b*e)^(3/2)*c^8*d^2*g + 35*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^7*d*e
*g + 21*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^7
*e*f - 21*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c
^7*d*g + 15*((e*x + d)*c - 2*c*d + b*e)^3*sqrt(-(e*x + d)*c + 2*c*d - b*e)
*c^6*g)/c^7)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx=\int \frac{\left(f+g\,x\right)\,{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)}^{5/2}}{{\left(d+e\,x\right)}^{7/2}} \,d x$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(7/2
),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(7/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 688, normalized size of antiderivative = 2.18

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx=\frac{2\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)b^2cd\,e^2g-2\sqrt{be-2cd}\,ata}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(7/2),x)
```

output

```
(2*(105*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d
))*b**2*c*d*e**2*g - 105*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)
/sqrt(b*e - 2*c*d))*b**2*c*e**3*f - 420*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e
 + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d**2*e*g + 420*sqrt(b*e - 2*c*d)
*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d*e**2*f + 420*
sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*
d**3*g - 420*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e -
2*c*d))*c**3*d**2*e*f + 15*sqrt( - b*e + c*d - c*e*x)*b**3*e**3*g - 206*sq
rt( - b*e + c*d - c*e*x)*b**2*c*d*e**2*g + 161*sqrt( - b*e + c*d - c*e*x)*
b**2*c*e**3*f + 45*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**3*g*x + 612*sqrt(
- b*e + c*d - c*e*x)*b*c**2*d**2*e*g - 567*sqrt( - b*e + c*d - c*e*x)*b*c*
*2*d*e**2*f - 167*sqrt( - b*e + c*d - c*e*x)*b*c**2*d*e**2*g*x + 77*sqrt(
- b*e + c*d - c*e*x)*b*c**2*e**3*f*x + 45*sqrt( - b*e + c*d - c*e*x)*b*c**
2*e**3*g*x**2 - 526*sqrt( - b*e + c*d - c*e*x)*c**3*d**3*g + 511*sqrt( - b
*e + c*d - c*e*x)*c**3*d**2*e*f + 157*sqrt( - b*e + c*d - c*e*x)*c**3*d**2
*e*g*x - 112*sqrt( - b*e + c*d - c*e*x)*c**3*d*e**2*f*x - 66*sqrt( - b*e +
 c*d - c*e*x)*c**3*d*e**2*g*x**2 + 21*sqrt( - b*e + c*d - c*e*x)*c**3*e**3
*f*x**2 + 15*sqrt( - b*e + c*d - c*e*x)*c**3*e**3*g*x**3))/(105*c*e**2)
```

3.221 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 331

$$
\begin{aligned}
\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx &= \\
&-\frac{(2cd-be)^2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)^{3/2}} \\
&-\frac{2(2cd-be)(2cef-4cdg+beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2\sqrt{d+ex}} \\
&-\frac{2(cef-3cdg+beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{3/2}} \\
&+\frac{2g(d(cd-be)-be^2x-ce^2x^2)^{5/2}}{5e^2(d+ex)^{5/2}} \\
&+\frac{(2cd-be)^{3/2}(5cef-9cdg+2beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{e^2}
\end{aligned}
$$

output

```
-(-b*e+2*c*d)^2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x
+d)^(3/2)-2*(-b*e+2*c*d)*(b*e*g-4*c*d*g+2*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e
^2*x^2)^(1/2)/e^2/(e*x+d)^(1/2)-2/3*(b*e*g-3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^(3/2)+2/5*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2
*x^2)^(5/2)/e^2/(e*x+d)^(5/2)+(-b*e+2*c*d)^(3/2)*(2*b*e*g-9*c*d*g+5*c*e*f)
*arctanh((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)
^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.77

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx=\frac{((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{b^2e^2(-15ef+61dg+46egx)+2bce(}{}\right.}{}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^(9/2),x]
```

output

```
(((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((b^2*e^2*(-15*e*f + 61*d*g + 46
*e*g*x) + 2*b*c*e*(-146*d^2*g + 5*d*e*(13*f - 21*g*x) + e^2*x*(35*f + 11*g
*x)) + 2*c^2*(168*d^3*g + e^3*x^2*(5*f + 3*g*x) - 6*d*e^2*x*(10*f + 3*g*x)
 + d^2*e*(-95*f + 117*g*x)))/((d + e*x)*(-(c*d) + b*e + c*e*x)^2) + (15*(-
2*c*d + b*e)^(3/2)*(-5*c*e*f + 9*c*d*g - 2*b*e*g)*ArcTan[Sqrt[c*d - b*e -
c*e*x]/Sqrt[-2*c*d + b*e]])/(-(b*e) + c*(d - e*x))^(5/2)))/(15*e^2*(d + e*
x)^(5/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1220, 1131, 1131, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx$$

$$\Big\downarrow \; 1220$$

$$-\frac{(2beg-9cdg+5cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{7/2}}dx}{2e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \; 1131$$

$$-\frac{(2beg-9cdg+5cef)\left((2cd-be)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{5/2}}dx+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e(d+ex)^{5/2}}\right)}{2e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \; 1131$$

$$-\frac{(2beg-9cdg+5cef)\left((2cd-be)\left((2cd-be)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{3/2}}dx+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}}\right)+2\big(d(cd-}{2e(2cd-be)}$$

$$-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \; 1131$$

$$-\frac{(2beg-9cdg+5cef)\left((2cd-be)\left((2cd-be)\left((2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{2\sqrt{d(cd-be)-be^2x-}}{e\sqrt{d+ex}}\right.\right.\right.}{2e(2cd-be)}$$

$$-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \; 1136$$

$$-\frac{(2beg-9cdg+5cef)\left((2cd-be)\left((2cd-be)\left(2e(2cd-be)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2}}{\sqrt{d+e}}\right.\right.\right.}{2e(2cd-be)}$$

$$-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^{9/2}(2cd-be)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\left((2cd-be)\left((2cd-be)\left(\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2cd-be}\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e}\right)+\frac{2\left(d(cd-be)-be^2x-c\right.}{3e(d+ex)^{3/}}\right.\right.}{2e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{e^2(d+ex)^{9/2}(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^(9/2
),x]
```

output
```
-(((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(e^2*(2*c*d -
b*e)*(d + e*x)^(9/2))) - ((5*c*e*f - 9*c*d*g + 2*b*e*g)*((2*(d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2)^(5/2))/(5*e*(d + e*x)^(5/2)) + (2*c*d - b*e)*((2*(
d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) + (2*c*d
 - b*e)*((2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e*Sqrt[d + e*x]) -
 (2*Sqrt[2*c*d - b*e]*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(S
qrt[2*c*d - b*e]*Sqrt[d + e*x])])/e))))/(2*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(303) = 606$.

Time = 1.50 (sec) , antiderivative size = 1128, normalized size of antiderivative = 3.41

method	result	size
default	Expression too large to display	1128

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(9/2),x,method=
_RETURNVERBOSE)
```

output

```
-1/15*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-22*b*c*e^3*g*x^2*(-c*e*x-b*e+c*d)
^(1/2)*(b*e-2*c*d)^(1/2)+30*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2
))*b^3*d*e^3*g+36*c^2*d*e^2*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)
-255*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c*d*e^3*g*x+210*
b*c*d*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+75*arctan((-c*e*x-b
*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c*d*e^3*f-300*arctan((-c*e*x-b*e+c*d)
^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d^2*e^2*f-336*c^2*d^3*g*(-c*e*x-b*e+c*d)^(
1/2)*(b*e-2*c*d)^(1/2)+300*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2)
)*c^3*d^3*e*f-300*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d
*e^3*f*x+660*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d^2*e^
2*g*x+292*b*c*d^2*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-130*b*c*d*e
^2*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+30*arctan((-c*e*x-b*e+c*d)^(
1/2)/(b*e-2*c*d)^(1/2))*b^3*e^4*g*x+120*c^2*d*e^2*f*x*(-c*e*x-b*e+c*d)^(1/
2)*(b*e-2*c*d)^(1/2)-234*c^2*d^2*e*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^
(1/2)-70*b*c*e^3*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-540*arctan((
-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^4*g-255*arctan((-c*e*x-b*e+
c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c*d^2*e^2*g+75*arctan((-c*e*x-b*e+c*d)^(
1/2)/(b*e-2*c*d)^(1/2))*b^2*c*e^4*f*x+15*b^2*e^3*f*(-c*e*x-b*e+c*d)^(1/2)*
(b*e-2*c*d)^(1/2)-6*c^2*e^3*g*x^3*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)
-540*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^3*e*g*x+300...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 992, normalized size of antiderivative = 3.00

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(9/2),x,
algorithm="fricas")
```

output
```
[-1/30*(15*((5*(2*c^2*d*e^3 - b*c*e^4)*f - (18*c^2*d^2*e^2 - 13*b*c*d*e^3
+ 2*b^2*e^4)*g)*x^2 + 5*(2*c^2*d^3*e - b*c*d^2*e^2)*f - (18*c^2*d^4 - 13*b
*c*d^3*e + 2*b^2*d^2*e^2)*g + 2*(5*(2*c^2*d^2*e^2 - b*c*d*e^3)*f - (18*c^2
*d^3*e - 13*b*c*d^2*e^2 + 2*b^2*d*e^3)*g)*x)*sqrt(2*c*d - b*e)*log(-(c*e^2
*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x + 2*sqrt(-c*e^2*x^2 - b*e^2
*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x +
d^2)) - 2*(6*c^2*e^3*g*x^3 + 2*(5*c^2*e^3*f - (18*c^2*d*e^2 - 11*b*c*e^3)*
g)*x^2 - 5*(38*c^2*d^2*e - 26*b*c*d*e^2 + 3*b^2*e^3)*f + (336*c^2*d^3 - 29
2*b*c*d^2*e + 61*b^2*d*e^2)*g - 2*(5*(12*c^2*d*e^2 - 7*b*c*e^3)*f - (117*c
^2*d^2*e - 105*b*c*d*e^2 + 23*b^2*e^3)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c
*d^2 - b*d*e)*sqrt(e*x + d))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2), 1/15*(15*((5
*(2*c^2*d*e^3 - b*c*e^4)*f - (18*c^2*d^2*e^2 - 13*b*c*d*e^3 + 2*b^2*e^4)*g
)*x^2 + 5*(2*c^2*d^3*e - b*c*d^2*e^2)*f - (18*c^2*d^4 - 13*b*c*d^3*e + 2*b
^2*d^2*e^2)*g + 2*(5*(2*c^2*d^2*e^2 - b*c*d*e^3)*f - (18*c^2*d^3*e - 13*b*
c*d^2*e^2 + 2*b^2*d*e^3)*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqrt(-c*e^2*x^2
- b*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d
*e + (2*c*d*e - b*e^2)*x)) + (6*c^2*e^3*g*x^3 + 2*(5*c^2*e^3*f - (18*c^2*d
*e^2 - 11*b*c*e^3)*g)*x^2 - 5*(38*c^2*d^2*e - 26*b*c*d*e^2 + 3*b^2*e^3)*f
+ (336*c^2*d^3 - 292*b*c*d^2*e + 61*b^2*d*e^2)*g - 2*(5*(12*c^2*d*e^2 - 7*
b*c*e^3)*f - (117*c^2*d^2*e - 105*b*c*d*e^2 + 23*b^2*e^3)*g)*x)*sqrt(-c...
```

Sympy [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx=\int \frac{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{5}{2}}(f+gx)}{(d+ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**(9/
2),x)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(f + g*x)/(d + e*x)**(9/2
), x)
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx=\int \frac{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{5}{2}}(gx+f)}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(9/2),x,
algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*(g*x + f)/(e*x + d)
^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.75

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx=$$

$$-\frac{120\,\sqrt{-(ex+d)c+2\,cd-be}\,c^{3}def-60\,\sqrt{-(ex+d)c+2\,cd-be}\,bc^{2}e^{2}f-240\,\sqrt{-(ex+d)c+2\,cd-b}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(9/2),x,
algorithm="giac")
```

output
```
-1/15*(120*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^3*d*e*f - 60*sqrt(-(e*x + d)
*c + 2*c*d - b*e)*b*c^2*e^2*f - 240*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^3*d
^2*g + 180*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^2*d*e*g - 30*sqrt(-(e*x +
d)*c + 2*c*d - b*e)*b^2*c*e^2*g + 10*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^
2*e*f - 30*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*d*g + 10*(-(e*x + d)*c +
 2*c*d - b*e)^(3/2)*b*c*e*g - 6*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x +
 d)*c + 2*c*d - b*e)*c*g + 15*(20*c^4*d^2*e*f - 20*b*c^3*d*e^2*f + 5*b^2*c
^2*e^3*f - 36*c^4*d^3*g + 44*b*c^3*d^2*e*g - 17*b^2*c^2*d*e^2*g + 2*b^3*c*
e^3*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(-2*c*d + b*e))/sqrt(-2
*c*d + b*e) + 15*(4*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4*d^2*e*f - 4*sqrt(
-(e*x + d)*c + 2*c*d - b*e)*b*c^3*d*e^2*f + sqrt(-(e*x + d)*c + 2*c*d - b*
e)*b^2*c^2*e^3*f - 4*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4*d^3*g + 4*sqrt(-
(e*x + d)*c + 2*c*d - b*e)*b*c^3*d^2*e*g - sqrt(-(e*x + d)*c + 2*c*d - b*e
)*b^2*c^2*d*e^2*g)/((e*x + d)*c))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{{\left(d+e\,x\right)}^{9/2}}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(9/2
),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(9/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 836, normalized size of antiderivative = 2.53

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx=\frac{22\sqrt{-cex-be+cd}\,bc\,e^3g\,x^2-36\sqrt{-cex-be+cd}\,c^2d\,e^2}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(9/2),x)
```

output

```
( - 30*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*b**2*d*e**2*g - 30*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqr
t(b*e - 2*c*d))*b**2*e**3*g*x + 195*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c
*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*d**2*e*g - 75*sqrt(b*e - 2*c*d)*atan(sq
rt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*d*e**2*f + 195*sqrt(b*e -
2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*d*e**2*g*x -
 75*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b
*c*e**3*f*x - 270*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b
*e - 2*c*d))*c**2*d**3*g + 150*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d -
c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*e*f - 270*sqrt(b*e - 2*c*d)*atan(sqrt(
 - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*e*g*x + 150*sqrt(b*e -
2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d*e**2*f*x
+ 61*sqrt( - b*e + c*d - c*e*x)*b**2*d*e**2*g - 15*sqrt( - b*e + c*d - c*e
*x)*b**2*e**3*f + 46*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*g*x - 292*sqrt(
- b*e + c*d - c*e*x)*b*c*d**2*e*g + 130*sqrt( - b*e + c*d - c*e*x)*b*c*d*e
**2*f - 210*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*g*x + 70*sqrt( - b*e + c
*d - c*e*x)*b*c*e**3*f*x + 22*sqrt( - b*e + c*d - c*e*x)*b*c*e**3*g*x**2 +
 336*sqrt( - b*e + c*d - c*e*x)*c**2*d**3*g - 190*sqrt( - b*e + c*d - c*e*
x)*c**2*d**2*e*f + 234*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*e*g*x - 120*sq
rt( - b*e + c*d - c*e*x)*c**2*d*e**2*f*x - 36*sqrt( - b*e + c*d - c*e*x...
```

3.222 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 341

$$\begin{aligned}
&\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx= \\
&-\frac{(2cd-be)^2(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e^2(d+ex)^{5/2}} \\
&+\frac{(2cd-be)(9cef-17cdg+4beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4e^2(d+ex)^{3/2}} \\
&+\frac{2c(cef-5cdg+2beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2\sqrt{d+ex}} \\
&-\frac{2cg\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e^2(d+ex)^{3/2}} \\
&-\frac{5c\sqrt{2cd-be}(3cef-11cdg+4beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{4e^2}
\end{aligned}$$

output
```
-1/2*(-b*e+2*c*d)^2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/
(e*x+d)^(5/2)+1/4*(-b*e+2*c*d)*(4*b*e*g-17*c*d*g+9*c*e*f)*(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)^(3/2)+2*c*(2*b*e*g-5*c*d*g+c*e*f)*(d*(-
b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+d)^(1/2)-2/3*c*g*(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^(3/2)-5/4*c*(-b*e+2*c*d)^(1/2)*(4*b*e
*g-11*c*d*g+3*c*e*f)*arctanh((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+
2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.74

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx = \frac{c((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{c^2\left(-206d^3g+d^2e(54f-350gx)+\right.}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^(11/2),x]
```

output
```
(c*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((c^2*(-206*d^3*g + d^2*e*(54*
f - 350*g*x) + 2*d*e^2*x*(51*f - 56*g*x) + 8*e^3*x^2*(3*f + g*x)) - 6*b^2*
e^2*(d*g + e*(f + 2*g*x)) + b*c*e*(107*d^2*g + e^2*x*(-27*f + 56*g*x) + d*
e*(-3*f + 187*g*x)))/(c*(d + e*x)^2*(-(c*d) + b*e + c*e*x)^2) + (15*Sqrt[-
2*c*d + b*e]*(-3*c*e*f + 11*c*d*g - 4*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x
]/Sqrt[-2*c*d + b*e]])/(-(b*e) + c*(d - e*x))^(5/2)))/(12*e^2*(d + e*x)^(5
/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1220, 1130, 1131, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}dx$$

$$\Big\downarrow \ 1220$$

$$-\frac{(4beg-11cdg+3cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{9/2}}dx}{4e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{2e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \ 1130$$

$$-\frac{(4beg-11cdg+3cef)\left(-\frac{5}{2}c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{5/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{e(d+ex)^{7/2}}\right)}{4e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{2e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \ 1131$$

$$-\frac{(4beg-11cdg+3cef)\left(-\frac{5}{2}c\left((2cd-be)\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{3/2}}dx+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^{3/2}}\right)-\frac{(d(cd-be)-be}{e(d+e}}{4e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{2e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \ 1131$$

$$-\frac{(4beg-11cdg+3cef)\left(-\frac{5}{2}c\left((2cd-be)\left((2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}\right.\right.\right.}{4e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{2e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \ 1136$$

$$-\frac{(4beg-11cdg+3cef)\left(-\frac{5}{2}c\left((2cd-be)\left(2e(2cd-be)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-}}{\sqrt{d+ex}}\right.\right.\right.}{4e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{2e^2(d+ex)^{11/2}(2cd-be)}$$

$$\Big\downarrow \ 221$$

$$-\frac{\left(-\frac{5}{2}c\left((2cd-be)\left(\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2cd-be}\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e}\right)+\frac{2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/}}{3e(d+ex)^{3/2}}\right.\right.}{4e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{2e^2(d+ex)^{11/2}(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^(11/
2),x]
```

output
```
-1/2*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(11/2)) - ((3*c*e*f - 11*c*d*g + 4*b*e*g)*(-((d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2)^(5/2)/(e*(d + e*x)^(7/2))) - (5*c*((2*(d*(c*d -
 b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) + (2*c*d - b*e)*
((2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt
[2*c*d - b*e]*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*
d - b*e]*Sqrt[d + e*x])])/e)))/2))/(4*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. 2(309) = 618.

Time = 1.54 (sec) , antiderivative size = 1181, normalized size of antiderivative = 3.46

method	result	size
default	Expression too large to display	1181

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(11/2),x,method
=_RETURNVERBOSE)
```

output

```
-1/12*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-56*b*c*e^3*g*x^2*(-c*e*x-b*e+c*d)
^(1/2)*(b*e-2*c*d)^(1/2)+112*c^2*d*e^2*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2
*c*d)^(1/2)+120*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c*d*e
^3*g*x-187*b*c*d*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+45*arcta
n((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d^2*e^2*f+206*c^2*d^3*g*
(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-90*arctan((-c*e*x-b*e+c*d)^(1/2)/
(b*e-2*c*d)^(1/2))*c^3*d^3*e*f+90*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d
)^(1/2))*b*c^2*d*e^3*f*x-285*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/
2))*b*c^2*d*e^3*g*x^2-570*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))
*b*c^2*d^2*e^2*g*x-107*b*c*d^2*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2
)+3*b*c*d*e^2*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-102*c^2*d*e^2*f*x
*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+350*c^2*d^2*e*g*x*(-c*e*x-b*e+c*
d)^(1/2)*(b*e-2*c*d)^(1/2)+27*b*c*e^3*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*
d)^(1/2)+330*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^4*g+60
*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c*d^2*e^2*g+6*b^2*e^
3*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-90*arctan((-c*e*x-b*e+c*d)^(1
/2)/(b*e-2*c*d)^(1/2))*c^3*d*e^3*f*x^2-8*c^2*e^3*g*x^3*(-c*e*x-b*e+c*d)^(1
/2)*(b*e-2*c*d)^(1/2)+60*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*
b^2*c*e^4*g*x^2+45*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*
e^4*f*x^2+660*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^3*...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.82

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(11/2),x,
 algorithm="fricas")
```

output
```
[1/24*(15*(3*c^2*d^3*e*f + (3*c^2*e^4*f - (11*c^2*d*e^3 - 4*b*c*e^4)*g)*x^
3 + 3*(3*c^2*d*e^3*f - (11*c^2*d^2*e^2 - 4*b*c*d*e^3)*g)*x^2 - (11*c^2*d^4
 - 4*b*c*d^3*e)*g + 3*(3*c^2*d^2*e^2*f - (11*c^2*d^3*e - 4*b*c*d^2*e^2)*g)
*x)*sqrt(2*c*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e
^2)*x + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqr
t(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(8*c^2*e^3*g*x^3 + 8*(3*c^2*e^3
*f - 7*(2*c^2*d*e^2 - b*c*e^3)*g)*x^2 + 3*(18*c^2*d^2*e - b*c*d*e^2 - 2*b^
2*e^3)*f - (206*c^2*d^3 - 107*b*c*d^2*e + 6*b^2*d*e^2)*g + (3*(34*c^2*d*e^
2 - 9*b*c*e^3)*f - (350*c^2*d^2*e - 187*b*c*d*e^2 + 12*b^2*e^3)*g)*x)*sqrt
(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d))/(e^5*x^3 + 3*d*e^4*x
^2 + 3*d^2*e^3*x + d^3*e^2), -1/12*(15*(3*c^2*d^3*e*f + (3*c^2*e^4*f - (11
*c^2*d*e^3 - 4*b*c*e^4)*g)*x^3 + 3*(3*c^2*d*e^3*f - (11*c^2*d^2*e^2 - 4*b*
c*d*e^3)*g)*x^2 - (11*c^2*d^4 - 4*b*c*d^3*e)*g + 3*(3*c^2*d^2*e^2*f - (11*
c^2*d^3*e - 4*b*c*d^2*e^2)*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqrt(-c*e^2*x^
2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d^2 - b
*d*e + (2*c*d*e - b*e^2)*x)) - (8*c^2*e^3*g*x^3 + 8*(3*c^2*e^3*f - 7*(2*c^
2*d*e^2 - b*c*e^3)*g)*x^2 + 3*(18*c^2*d^2*e - b*c*d*e^2 - 2*b^2*e^3)*f - (
206*c^2*d^3 - 107*b*c*d^2*e + 6*b^2*d*e^2)*g + (3*(34*c^2*d*e^2 - 9*b*c*e^
3)*f - (350*c^2*d^2*e - 187*b*c*d*e^2 + 12*b^2*e^3)*g)*x)*sqrt(-c*e^2*x^2
- b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d))/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**(11
/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx=\int \frac{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{5}{2}}(gx+f)}{(ex+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(11/2),x,
 algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*(g*x + f)/(e*x + d)
^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.68

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx=\frac{24\sqrt{-(ex+d)c+2\,cd-be}c^3ef-120\sqrt{-(ex+d)c+2}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(11/2),x,
 algorithm="giac")
```

output
```
1/12*(24*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^3*e*f - 120*sqrt(-(e*x + d)*c
+ 2*c*d - b*e)*c^3*d*g + 48*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^2*e*g - 8
*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*g + 15*(6*c^4*d*e*f - 3*b*c^3*e^2*
f - 22*c^4*d^2*g + 19*b*c^3*d*e*g - 4*b^2*c^2*e^2*g)*arctan(sqrt(-(e*x + d
)*c + 2*c*d - b*e)/sqrt(-2*c*d + b*e))/sqrt(-2*c*d + b*e) + 3*(28*sqrt(-(e
*x + d)*c + 2*c*d - b*e)*c^5*d^2*e*f - 28*sqrt(-(e*x + d)*c + 2*c*d - b*e)
*b*c^4*d*e^2*f + 7*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^3*e^3*f - 60*sqr
t(-(e*x + d)*c + 2*c*d - b*e)*c^5*d^3*g + 76*sqrt(-(e*x + d)*c + 2*c*d - b
*e)*b*c^4*d^2*e*g - 31*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^3*d*e^2*g +
4*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^2*e^3*g - 18*(-(e*x + d)*c + 2*c*
d - b*e)^(3/2)*c^4*d*e*f + 9*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^3*e^2*
f + 34*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^4*d^2*g - 25*(-(e*x + d)*c + 2
*c*d - b*e)^(3/2)*b*c^3*d*e*g + 4*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c
^2*e^2*g)/((e*x + d)^2*c^2))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{(d+e\,x)^{11/2}}\,dx$$

input
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(11/
2),x)
```

output
```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(11/
2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 809, normalized size of antiderivative = 2.37

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx=\frac{-60\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bc\,d^2eg-120\sqrt{be-2cd}}{}$$

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(11/2),x)
```

output

```
( - 60*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*b*c*d**2*e*g - 120*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqr
t(b*e - 2*c*d))*b*c*d*e**2*g*x - 60*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c
*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*e**3*g*x**2 + 165*sqrt(b*e - 2*c*d)*ata
n(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**3*g - 45*sqrt(b*e
- 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*e*f
+ 330*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))
*c**2*d**2*e*g*x - 90*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sq
rt(b*e - 2*c*d))*c**2*d*e**2*f*x + 165*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e
+ c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d*e**2*g*x**2 - 45*sqrt(b*e - 2*c*d
)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*e**3*f*x**2 - 6*
sqrt( - b*e + c*d - c*e*x)*b**2*d*e**2*g - 6*sqrt( - b*e + c*d - c*e*x)*b*
*2*e**3*f - 12*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*g*x + 107*sqrt( - b*e
+ c*d - c*e*x)*b*c*d**2*e*g - 3*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*f +
187*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*g*x - 27*sqrt( - b*e + c*d - c*e
*x)*b*c*e**3*f*x + 56*sqrt( - b*e + c*d - c*e*x)*b*c*e**3*g*x**2 - 206*sqr
t( - b*e + c*d - c*e*x)*c**2*d**3*g + 54*sqrt( - b*e + c*d - c*e*x)*c**2*d
**2*e*f - 350*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*e*g*x + 102*sqrt( - b*e
 + c*d - c*e*x)*c**2*d*e**2*f*x - 112*sqrt( - b*e + c*d - c*e*x)*c**2*d*e*
*2*g*x**2 + 24*sqrt( - b*e + c*d - c*e*x)*c**2*e**3*f*x**2 + 8*sqrt( - ...
```

3.223 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 325

$$\begin{aligned}\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx = & -\frac{c(5cef-33cdg+14beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{8e^2(d+ex)^{3/2}} \\ & +\frac{2c^2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2\sqrt{d+ex}} \\ & +\frac{(5cef-17cdg+6beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{12e^2(d+ex)^{7/2}} \\ & -\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e^2(d+ex)^{11/2}} \\ & +\frac{5c^2(cef-13cdg+6beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{8e^2\sqrt{2cd-be}}\end{aligned}$$

output
```
-1/8*c*(14*b*e*g-33*c*d*g+5*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/
e^2/(e*x+d)^(3/2)+2*c^2*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(e*x+
d)^(1/2)+1/12*(6*b*e*g-17*c*d*g+5*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(3/2)/e^2/(e*x+d)^(7/2)-1/3*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5
/2)/e^2/(e*x+d)^(11/2)+5/8*c^2*(6*b*e*g-13*c*d*g+c*e*f)*arctanh((d*(-b*e+c
*d)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2/(-b*e+2
*c*d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.79

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx = \frac{c^2((d+ex)(-be+c(d-ex)))^{5/2}\Bigg(\frac{-4b^2e^2(2ef+dg+3egx)-2bce(}{}}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^(13/2),x]
```

output
```
(c^2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((-4*b^2*e^2*(2*e*f + d*g +
3*e*g*x) - 2*b*c*e*(6*d^2*g + d*e*(-3*f + 17*g*x) + e^2*x*(13*f + 27*g*x))
 + c^2*(121*d^3*g + 3*e^3*x^2*(-11*f + 16*g*x) + d*e^2*x*(-14*f + 285*g*x)
 + d^2*e*(-13*f + 326*g*x)))/(c^2*(d + e*x)^3*(-(c*d) + b*e + c*e*x)^2) -
(15*(c*e*f - 13*c*d*g + 6*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*
d + b*e]])/(Sqrt[-2*c*d + b*e]*(-(b*e) + c*(d - e*x))^(5/2))))/(24*e^2*(d
+ e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1220, 1130, 1130, 1131, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{13/2}}dx$$

$$\Big\downarrow \; 1220$$

$$-\frac{(6beg-13cdg+cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{11/2}}dx}{6e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \; 1130$$

$$-\frac{(6beg-13cdg+cef)\left(-\frac{5}{4}c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{7/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{2e(d+ex)^{9/2}}\right)}{6e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \; 1130$$

$$-\frac{(6beg-13cdg+cef)\Bigg(-\frac{5}{4}c\left(-\frac{3}{2}c\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{3/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}\right)-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right.}{2e(d+ex)^{9/2}}}{6e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \; 1131$$

$$-\frac{(6beg-13cdg+cef)\Bigg(-\frac{5}{4}c\Bigg(-\frac{3}{2}c\left((2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx+\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}\right)-\frac{(d(c}{}}{6e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \; 1136$$

$$-\frac{(6beg-13cdg+cef)\Bigg(-\frac{5}{4}c\Bigg(-\frac{3}{2}c\Bigg(2e(2cd-be)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}+\frac{2}{}}{6e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^{13/2}(2cd-be)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left(-\frac{5}{4}c\left(-\frac{3}{2}c\left(\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2cd-be}\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e}\right)-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}\right)-\left(\right.\right.}{6e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{3e^2(d+ex)^{13/2}(2cd-be)}$$

input

```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^(13/
2),x]
```

output

```
-1/3*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(13/2)) - ((c*e*f - 13*c*d*g + 6*b*e*g)*(-1/2*(d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2)^(5/2)/(e*(d + e*x)^(9/2)) - (5*c*(-((d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(5/2))) - (3*c*((2*Sqrt[d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2*c*d - b*e
]*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqr
t[d + e*x])])/e))/2))/4))/(6*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(293) = 586$.

Time = 1.50 (sec) , antiderivative size = 1062, normalized size of antiderivative = 3.27

method	result	size
default	Expression too large to display	1062

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(13/2),x,method
=_RETURNVERBOSE)
```

output

```
-1/24*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(54*b*c*e^3*g*x^2*(-c*e*x-b*e+c*d)^
(1/2)*(b*e-2*c*d)^(1/2)-285*c^2*d*e^2*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*
c*d)^(1/2)+34*b*c*d*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-121*c
^2*d^3*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+15*arctan((-c*e*x-b*e+c*
d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*e^4*f*x^3+15*arctan((-c*e*x-b*e+c*d)^(1/2)
/(b*e-2*c*d)^(1/2))*c^3*d^3*e*f+270*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c
*d)^(1/2))*b*c^2*d*e^3*g*x^2+270*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)
^(1/2))*b*c^2*d^2*e^2*g*x+12*b*c*d^2*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d
)^(1/2)-6*b*c*d*e^2*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+14*c^2*d*e^
2*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-326*c^2*d^2*e*g*x*(-c*e*x-b
*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+26*b*c*e^3*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e
-2*c*d)^(1/2)-195*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^4
*g+90*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*e^4*g*x^3-195
*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d*e^3*g*x^3+8*b^2*e^
3*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+45*arctan((-c*e*x-b*e+c*d)^(1
/2)/(b*e-2*c*d)^(1/2))*c^3*d*e^3*f*x^2-48*c^2*e^3*g*x^3*(-c*e*x-b*e+c*d)^(
1/2)*(b*e-2*c*d)^(1/2)-585*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2)
)*c^3*d^3*e*g*x+45*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^
2*e^2*f*x+12*b^2*e^3*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+4*b^2*d*
e^2*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+13*c^2*d^2*e*f*(-c*e*x-b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. 2(293) = 586.

Time = 0.17 (sec) , antiderivative size = 1390, normalized size of antiderivative = 4.28

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(13/2),x,
 algorithm="fricas")
```

output

```
[1/48*(15*(c^3*d^4*e*f + (c^3*e^5*f - (13*c^3*d*e^4 - 6*b*c^2*e^5)*g)*x^4
+ 4*(c^3*d*e^4*f - (13*c^3*d^2*e^3 - 6*b*c^2*d*e^4)*g)*x^3 + 6*(c^3*d^2*e^
3*f - (13*c^3*d^3*e^2 - 6*b*c^2*d^2*e^3)*g)*x^2 - (13*c^3*d^5 - 6*b*c^2*d^
4*e)*g + 4*(c^3*d^3*e^2*f - (13*c^3*d^4*e - 6*b*c^2*d^3*e^2)*g)*x)*sqrt(2*
c*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*s
qrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))
/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)
*(48*(2*c^3*d*e^3 - b*c^2*e^4)*g*x^3 - 3*(11*(2*c^3*d*e^3 - b*c^2*e^4)*f -
 (190*c^3*d^2*e^2 - 131*b*c^2*d*e^3 + 18*b^2*c*e^4)*g)*x^2 - (26*c^3*d^3*e
 - 25*b*c^2*d^2*e^2 + 22*b^2*c*d*e^3 - 8*b^3*e^4)*f + (242*c^3*d^4 - 145*b
*c^2*d^3*e + 4*b^2*c*d^2*e^2 + 4*b^3*d*e^3)*g - 2*((14*c^3*d^2*e^2 + 19*b*
c^2*d*e^3 - 13*b^2*c*e^4)*f - (326*c^3*d^3*e - 197*b*c^2*d^2*e^2 + 5*b^2*c
*d*e^3 + 6*b^3*e^4)*g)*x)*sqrt(e*x + d))/(2*c*d^5*e^2 - b*d^4*e^3 + (2*c*d
*e^6 - b*e^7)*x^4 + 4*(2*c*d^2*e^5 - b*d*e^6)*x^3 + 6*(2*c*d^3*e^4 - b*d^2
*e^5)*x^2 + 4*(2*c*d^4*e^3 - b*d^3*e^4)*x), 1/24*(15*(c^3*d^4*e*f + (c^3*e
^5*f - (13*c^3*d*e^4 - 6*b*c^2*e^5)*g)*x^4 + 4*(c^3*d*e^4*f - (13*c^3*d^2*
e^3 - 6*b*c^2*d*e^4)*g)*x^3 + 6*(c^3*d^2*e^3*f - (13*c^3*d^3*e^2 - 6*b*c^2
*d^2*e^3)*g)*x^2 - (13*c^3*d^5 - 6*b*c^2*d^4*e)*g + 4*(c^3*d^3*e^2*f - (13
*c^3*d^4*e - 6*b*c^2*d^3*e^2)*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqrt(-c*e^2
*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx = \text{Timed out}$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**(13
/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx=\int \frac{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{5}{2}}(gx+f)}{(ex+d)^{\frac{13}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(13/2),x,
 algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*(g*x + f)/(e*x + d)
^(13/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(293) = 586$.

Time = 0.41 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.83

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx=\frac{48\sqrt{-(ex+d)c+2\,cd-be}c^3g-\frac{15\left(c^4ef-13\,c^4dg+6\,bc^3eg\right)\mathrm{arcta}}{\sqrt{-2\,cd+}}}{}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(13/2),x,
 algorithm="giac")
```

output

```
1/24*(48*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^3*g - 15*(c^4*e*f - 13*c^4*d*g
 + 6*b*c^3*e*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(-2*c*d + b*e)
)/sqrt(-2*c*d + b*e) - (60*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d^2*e*f -
60*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^5*d*e^2*f + 15*sqrt(-(e*x + d)*c +
 2*c*d - b*e)*b^2*c^4*e^3*f - 396*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d^3
*g + 564*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^5*d^2*e*g - 267*sqrt(-(e*x +
 d)*c + 2*c*d - b*e)*b^2*c^4*d*e^2*g + 42*sqrt(-(e*x + d)*c + 2*c*d - b*e)
*b^3*c^3*e^3*g - 80*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^5*d*e*f + 40*(-(e
*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^4*e^2*f + 464*(-(e*x + d)*c + 2*c*d - b
*e)^(3/2)*c^5*d^2*g - 424*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^4*d*e*g +
 96*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c^3*e^2*g + 33*((e*x + d)*c - 2
*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4*e*f - 141*((e*x + d)*c
- 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4*d*g + 54*((e*x + d)*
c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^3*e*g)/((e*x + d)^
3*c^3))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\,(cd^2-bde-be^2x-ce^2x^2)^{5/2}}{(d+ex)^{13/2}}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{(d+e\,x)^{13/2}}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(13/
2),x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(13/
2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1230, normalized size of antiderivative = 3.78

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{13/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(13/2),x)
```

output

```
( - 90*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*b*c**2*d**3*e*g - 270*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/
sqrt(b*e - 2*c*d))*b*c**2*d**2*e**2*g*x - 270*sqrt(b*e - 2*c*d)*atan(sqrt(
 - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d*e**3*g*x**2 - 90*sqrt(b*
e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*e**4*
g*x**3 + 195*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e -
2*c*d))*c**3*d**4*g - 15*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)
/sqrt(b*e - 2*c*d))*c**3*d**3*e*f + 585*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e
 + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**3*e*g*x - 45*sqrt(b*e - 2*c*d)*
atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**2*e**2*f*x + 58
5*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**
3*d**2*e**2*g*x**2 - 45*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/
sqrt(b*e - 2*c*d))*c**3*d*e**3*f*x**2 + 195*sqrt(b*e - 2*c*d)*atan(sqrt( -
 b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d*e**3*g*x**3 - 15*sqrt(b*e -
2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*e**4*f*x**3
 - 4*sqrt( - b*e + c*d - c*e*x)*b**3*d*e**3*g - 8*sqrt( - b*e + c*d - c*e*
x)*b**3*e**4*f - 12*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*g*x - 4*sqrt( - b
*e + c*d - c*e*x)*b**2*c*d**2*e**2*g + 22*sqrt( - b*e + c*d - c*e*x)*b**2*
c*d*e**3*f - 10*sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**3*g*x - 26*sqrt( -
b*e + c*d - c*e*x)*b**2*c*e**4*f*x - 54*sqrt( - b*e + c*d - c*e*x)*b**2...
```

3.224 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 353

$$\begin{aligned}
\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx &= \\
&-\frac{c(5cef-53cdg+24beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{32e^2(d+ex)^{5/2}} \\
&+\frac{c^2(5cef-181cdg+88beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{64e^2(2cd-be)(d+ex)^{3/2}} \\
&+\frac{(5cef-21cdg+8beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{24e^2(d+ex)^{9/2}} \\
&-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{4e^2(d+ex)^{13/2}} \\
&+\frac{5c^3(cef+15cdg-8beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{64e^2(2cd-be)^{3/2}}
\end{aligned}$$

output
```
-1/32*c*(24*b*e*g-53*c*d*g+5*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)
/e^2/(e*x+d)^(5/2)+1/64*c^2*(88*b*e*g-181*c*d*g+5*c*e*f)*(d*(-b*e+c*d)-b*e
^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e*x+d)^(3/2)+1/24*(8*b*e*g-21*c*d*
g+5*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/e^2/(e*x+d)^(9/2)-1/4*(-
d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/e^2/(e*x+d)^(13/2)+5/64*c^
3*(-8*b*e*g+15*c*d*g+c*e*f)*arctanh((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)
/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))/e^2/(-b*e+2*c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx=\frac{c^3((d+ex)(-be+c(d-ex)))^{5/2}\left(\frac{16b^3e^3(3ef+dg+4egx)-8b^2ce^2}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^(15/2),x]
```

output
```
(c^3*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((16*b^3*e^3*(3*e*f + d*g +
4*e*g*x) - 8*b^2*c*e^2*(3*d^2*g + d*e*(19*f + 13*g*x) - e^2*x*(17*f + 26*g
*x)) + 2*b*c^2*e*(25*d^3*g + 3*d^2*e*(25*f + 34*g*x) - d*e^2*x*(154*f + 79
*g*x) + e^3*x^2*(59*f + 132*g*x)) - c^3*(147*d^4*g - 15*e^4*f*x^3 + 61*d^3
*e*(f + 9*g*x) + 3*d^2*e^2*x*(-39*f + 187*g*x) + d*e^3*x^2*(191*f + 543*g*
x)))/(c^3*(2*c*d - b*e)*(d + e*x)^4*(-(c*d) + b*e + c*e*x)^2) + (15*(c*e*f
 + 15*c*d*g - 8*b*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])
/((-2*c*d + b*e)^(3/2)*(-(b*e) + c*(d - e*x))^(5/2))))/(192*e^2*(d + e*x)^
(5/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1220, 1130, 1130, 1130, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(-8beg+15cdg+cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{13/2}}dx}{8e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{4e^2(d+ex)^{15/2}(2cd-be)}$$

$$\Big\downarrow \; 1130$$

$$\frac{(-8beg+15cdg+cef)\left(-\frac{5}{6}c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{9/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{3e(d+ex)^{11/2}}\right)}{8e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{4e^2(d+ex)^{15/2}(2cd-be)}$$

$$\Big\downarrow \; 1130$$

$$\frac{(-8beg+15cdg+cef)\left(-\frac{5}{6}c\left(-\frac{3}{4}c\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{5/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}\right)-\frac{(d(cd-be)-be^2x-ce^2x^2}{3e(d+ex)^{11/2}}\right.}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{4e^2(d+ex)^{15/2}(2cd-be)}$$

$$\Big\downarrow \; 1130$$

$$\frac{(-8beg+15cdg+cef)\left(-\frac{5}{6}c\left(-\frac{3}{4}c\left(-\frac{1}{2}c\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}}\right)-\frac{(d(cd-be)-b}{2e(d+}\right.\right.}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{4e^2(d+ex)^{15/2}(2cd-be)}$$

$$\Big\downarrow 1136$$

$$\frac{(-8beg+15cdg+cef)\Bigg(-\frac{5}{6}c\Bigg(-\frac{3}{4}c\Bigg(-ce\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}-\frac{\sqrt{d(cd-be)-}}{e(d+}}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{4e^2(d+ex)^{15/2}(2cd-be)}$$

$$\Big\downarrow 221$$

$$\frac{\Bigg(-\frac{5}{6}c\Bigg(-\frac{3}{4}c\Bigg(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e\sqrt{2cd-be}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}}\Bigg)-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}\Bigg)-\frac{(d(cd-be)-be}{3e(d+}}{8e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{4e^2(d+ex)^{15/2}(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^(15/
2),x]
```

output
```
-1/4*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(15/2)) + ((c*e*f + 15*c*d*g - 8*b*e*g)*(-1/3*(d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2)^(5/2)/(e*(d + e*x)^(11/2)) - (5*c*(-1/2*(d*(c*
d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(7/2)) - (3*c*(-(Sqrt[d
*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(e*(d + e*x)^(3/2))) + (c*ArcTanh[Sqrt
[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/
(e*Sqrt[2*c*d - b*e])))/4))/6))/(8*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1508 vs. $2(319) = 638$.

Time = 1.49 (sec) , antiderivative size = 1509, normalized size of antiderivative = 4.27

method	result	size
default	Expression too large to display	1509

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(15/2),x,method
=_RETURNVERBOSE)
```

output
```
-1/192*(50*b*c^2*d^3*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-158*b*c^
2*d*e^3*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-191*c^3*d*e^3*f*x^2
*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-561*c^3*d^2*e^2*g*x^2*(-c*e*x-b*
e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+204*b*c^2*d^2*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2
)*(b*e-2*c*d)^(1/2)+208*b^2*c*e^4*g*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)
^(1/2)+480*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^3*d^3*e^2*
g*x+150*b*c^2*d^2*e^2*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-225*arcta
n((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^4*d^5*g+16*b^3*d*e^3*g*(-c*e
*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-61*c^3*d^3*e*f*(-c*e*x-b*e+c*d)^(1/2)*
(b*e-2*c*d)^(1/2)+48*b^3*e^4*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-14
7*c^3*d^4*g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+117*c^3*d^2*e^2*f*x*(
-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+720*arctan((-c*e*x-b*e+c*d)^(1/2)/
(b*e-2*c*d)^(1/2))*b*c^3*d^2*e^3*g*x^2-549*c^3*d^3*e*g*x*(-c*e*x-b*e+c*d)^
(1/2)*(b*e-2*c*d)^(1/2)-152*b^2*c*d*e^3*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*
d)^(1/2)+118*b*c^2*e^4*f*x^2*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-543*
c^3*d*e^3*g*x^3*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-24*b^2*c*d^2*e^2*
g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+136*b^2*c*e^4*f*x*(-c*e*x-b*e+c
*d)^(1/2)*(b*e-2*c*d)^(1/2)-308*b*c^2*d*e^3*f*x*(-c*e*x-b*e+c*d)^(1/2)*(b*
e-2*c*d)^(1/2)+264*b*c^2*e^4*g*x^3*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2
)-104*b^2*c*d*e^3*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+480*arct...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 941 vs. 2(319) = 638.

Time = 0.18 (sec) , antiderivative size = 1912, normalized size of antiderivative = 5.42

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(15/2),x,
 algorithm="fricas")
```

output

```
[1/384*(15*(c^4*d^5*e*f + (c^4*e^6*f + (15*c^4*d*e^5 - 8*b*c^3*e^6)*g)*x^5
 + 5*(c^4*d*e^5*f + (15*c^4*d^2*e^4 - 8*b*c^3*d*e^5)*g)*x^4 + 10*(c^4*d^2*
e^4*f + (15*c^4*d^3*e^3 - 8*b*c^3*d^2*e^4)*g)*x^3 + 10*(c^4*d^3*e^3*f + (1
5*c^4*d^4*e^2 - 8*b*c^3*d^3*e^3)*g)*x^2 + (15*c^4*d^6 - 8*b*c^3*d^5*e)*g +
 5*(c^4*d^4*e^2*f + (15*c^4*d^5*e - 8*b*c^3*d^4*e^2)*g)*x)*sqrt(2*c*d - b*
e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*sqrt(-c*e
^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^
2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(3*(5*(
2*c^4*d*e^4 - b*c^3*e^5)*f - (362*c^4*d^2*e^3 - 357*b*c^3*d*e^4 + 88*b^2*c
^2*e^5)*g)*x^3 - ((382*c^4*d^2*e^3 - 427*b*c^3*d*e^4 + 118*b^2*c^2*e^5)*f
+ (1122*c^4*d^3*e^2 - 245*b*c^3*d^2*e^3 - 574*b^2*c^2*d*e^4 + 208*b^3*c*e^
5)*g)*x^2 - (122*c^4*d^4*e - 361*b*c^3*d^3*e^2 + 454*b^2*c^2*d^2*e^3 - 248
*b^3*c*d*e^4 + 48*b^4*e^5)*f - (294*c^4*d^5 - 247*b*c^3*d^4*e + 98*b^2*c^2
*d^3*e^2 - 56*b^3*c*d^2*e^3 + 16*b^4*d*e^4)*g + ((234*c^4*d^3*e^2 - 733*b*
c^3*d^2*e^3 + 580*b^2*c^2*d*e^4 - 136*b^3*c*e^5)*f - (1098*c^4*d^4*e - 957
*b*c^3*d^3*e^2 + 412*b^2*c^2*d^2*e^3 - 232*b^3*c*d*e^4 + 64*b^4*e^5)*g)*x)
*sqrt(e*x + d))/(4*c^2*d^7*e^2 - 4*b*c*d^6*e^3 + b^2*d^5*e^4 + (4*c^2*d^2*
e^7 - 4*b*c*d*e^8 + b^2*e^9)*x^5 + 5*(4*c^2*d^3*e^6 - 4*b*c*d^2*e^7 + b^2*
d*e^8)*x^4 + 10*(4*c^2*d^4*e^5 - 4*b*c*d^3*e^6 + b^2*d^2*e^7)*x^3 + 10*(4*
c^2*d^5*e^4 - 4*b*c*d^4*e^5 + b^2*d^3*e^6)*x^2 + 5*(4*c^2*d^6*e^3 - 4*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx=\text{Timed out}$$

input

```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**(15
/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx=\int \frac{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{5}{2}}(gx+f)}{(ex+d)^{\frac{15}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(15/2),x,
 algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*(g*x + f)/(e*x + d)
^(15/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 956 vs. $2(319) = 638$.

Time = 0.42 (sec) , antiderivative size = 956, normalized size of antiderivative = 2.71

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(15/2),x,
 algorithm="giac")
```

output

```
-1/192*(15*(c^5*e*f + 15*c^5*d*g - 8*b*c^4*e*g)*arctan(sqrt(-(e*x + d)*c +
 2*c*d - b*e)/sqrt(-2*c*d + b*e))/((2*c*d - b*e)*sqrt(-2*c*d + b*e)) + (12
0*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^8*d^3*e*f - 180*sqrt(-(e*x + d)*c + 2
*c*d - b*e)*b*c^7*d^2*e^2*f + 90*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^6*
d*e^3*f - 15*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^5*e^4*f + 1800*sqrt(-(
e*x + d)*c + 2*c*d - b*e)*c^8*d^4*g - 3660*sqrt(-(e*x + d)*c + 2*c*d - b*e
)*b*c^7*d^3*e*g + 2790*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^6*d^2*e^2*g
 - 945*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^5*d*e^3*g + 120*sqrt(-(e*x +
d)*c + 2*c*d - b*e)*b^4*c^4*e^4*g - 220*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)
*c^7*d^2*e*f + 220*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^6*d*e^2*f - 55*(
-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c^5*e^3*f - 3300*(-(e*x + d)*c + 2*c
*d - b*e)^(3/2)*c^7*d^3*g + 5060*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^6*
d^2*e*g - 2585*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c^5*d*e^2*g + 440*(-
(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^3*c^4*e^3*g + 146*((e*x + d)*c - 2*c*d
 + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d*e*f - 73*((e*x + d)*c - 2*
c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^5*e^2*f + 2190*((e*x + d
)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*d^2*g - 2263*((e
*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^5*d*e*g +
584*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^4
*e^2*g - 15*((e*x + d)*c - 2*c*d + b*e)^3*sqrt(-(e*x + d)*c + 2*c*d - b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{{(d+e\,x)}^{15/2}}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(15/
2),x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(15/
2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1756, normalized size of antiderivative = 4.97

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{15/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(15/2),x)
```

output

```
( - 120*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d
))*b*c**3*d**4*e*g - 480*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)
/sqrt(b*e - 2*c*d))*b*c**3*d**3*e**2*g*x - 720*sqrt(b*e - 2*c*d)*atan(sqrt
( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*d**2*e**3*g*x**2 - 480*sq
rt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*
d*e**4*g*x**3 - 120*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt
(b*e - 2*c*d))*b*c**3*e**5*g*x**4 + 225*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e
 + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**5*g + 15*sqrt(b*e - 2*c*d)*atan
(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**4*e*f + 900*sqrt(b*
e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**4*e*
g*x + 60*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*
d))*c**4*d**3*e**2*f*x + 1350*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c
*e*x)/sqrt(b*e - 2*c*d))*c**4*d**3*e**2*g*x**2 + 90*sqrt(b*e - 2*c*d)*atan
(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d**2*e**3*f*x**2 + 900
*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4
*d**2*e**3*g*x**3 + 60*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/s
qrt(b*e - 2*c*d))*c**4*d*e**4*f*x**3 + 225*sqrt(b*e - 2*c*d)*atan(sqrt( -
b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*d*e**4*g*x**4 + 15*sqrt(b*e - 2
*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**4*e**5*f*x**4
- 16*sqrt( - b*e + c*d - c*e*x)*b**4*d*e**4*g - 48*sqrt( - b*e + c*d - ...
```

3.225 $\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 430

$$\begin{aligned}
\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx &= \\
&-\frac{c(3cef-47cdg+22beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{48e^2(d+ex)^{7/2}} \\
&+\frac{c^2(3cef-239cdg+118beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{192e^2(2cd-be)(d+ex)^{5/2}} \\
&+\frac{c^3(3cef+17cdg-10beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{128e^2(2cd-be)^2(d+ex)^{3/2}} \\
&+\frac{(cef-5cdg+2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{8e^2(d+ex)^{11/2}} \\
&-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5e^2(d+ex)^{15/2}} \\
&+\frac{c^4(3cef+17cdg-10beg)\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{2cd-be}\sqrt{d+ex}}\right)}{128e^2(2cd-be)^{5/2}}
\end{aligned}$$

output

```
-1/48*c*(22*b*e*g-47*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)
/e^2/(e*x+d)^(7/2)+1/192*c^2*(118*b*e*g-239*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b
*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e*x+d)^(5/2)+1/128*c^3*(-10*b*e*
g+17*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d
)^2/(e*x+d)^(3/2)+1/8*(2*b*e*g-5*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*
x^2)^(3/2)/e^2/(e*x+d)^(11/2)-1/5*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(5/2)/e^2/(e*x+d)^(15/2)+1/128*c^4*(-10*b*e*g+17*c*d*g+3*c*e*f)*arctan
h((d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/(-b*e+2*c*d)^(1/2)/(e*x+d)^(1/2))
/e^2/(-b*e+2*c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 3.33 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx = \frac{c^4((d+ex)(-be+c(d-ex)))^{5/2}\Bigg(\frac{-96b^4e^4(4ef+dg+5egx)+16b^3}{}}{}$$

input

```
Integrate[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x
)^(17/2),x]
```

output

```
(c^4*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2)*((-96*b^4*e^4*(4*e*f + d*g +
 5*e*g*x) + 16*b^3*c*e^3*(26*d^2*g - e^2*x*(63*f + 85*g*x) + d*e*(129*f +
133*g*x)) - 4*b^2*c^2*e^2*(157*d^3*g + e^3*x^2*(186*f + 295*g*x) - 3*d*e^2
*x*(380*f + 447*g*x) + d^2*e*(978*f + 825*g*x)) + c^4*(-269*d^5*g + 45*e^5
*f*x^4 + 15*d*e^4*x^3*(16*f + 17*g*x) + 2*d^3*e^2*x*(1236*f + 523*g*x) - d
^4*e*(951*f + 1352*g*x) - 2*d^2*e^3*x^2*(1263*f + 1880*g*x)) + 2*b*c^3*e*(
236*d^4*g - 15*e^4*x^3*(f + 5*g*x) + 3*d^3*e*(523*f + 419*g*x) - 3*d^2*e^2
*x*(1039*f + 991*g*x) + d*e^3*x^2*(1443*f + 2075*g*x)))/(c^4*(-2*c*d + b*e
)^2*(d + e*x)^5*(-(c*d) + b*e + c*e*x)^2) - (15*(3*c*e*f + 17*c*d*g - 10*b
*e*g)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/((-2*c*d + b*e)^
(5/2)*(-(b*e) + c*(d - e*x))^(5/2))))/(1920*e^2*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1220, 1130, 1130, 1130, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}dx$$

$$\Big\downarrow 1220$$

$$\frac{(-10beg+17cdg+3cef)\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}{(d+ex)^{15/2}}dx}{10e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^{17/2}(2cd-be)}$$

$$\Big\downarrow 1130$$

$$\frac{(-10beg+17cdg+3cef)\left(-\frac{5}{8}c\int \frac{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}{(d+ex)^{11/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{4e(d+ex)^{13/2}}\right)}{10e(2cd-be)}-\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^{17/2}(2cd-be)}$$

$$\Big\downarrow 1130$$

$$\frac{(-10beg+17cdg+3cef)\Bigg(-\frac{5}{8}c\Bigg(-\frac{1}{2}c\int \frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{(d+ex)^{7/2}}dx-\frac{\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}\Bigg)-\frac{(d(cd-be)-be^2x-ce^2}{4e(d+ex)^{13/2}}}{10e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^{17/2}(2cd-be)}$$

$$\Big\downarrow 1130$$

$$\frac{(-10beg+17cdg+3cef)\Bigg(-\frac{5}{8}c\Bigg(-\frac{1}{2}c\Bigg(-\frac{1}{4}c\int \frac{1}{(d+ex)^{3/2}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}}\Bigg)-\frac{(d(cd-}{}}{10e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^{17/2}(2cd-be)}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(-10beg+17cdg+3cef)\left(-\frac{5}{8}c\left(-\frac{1}{2}c\left(-\frac{1}{4}c\left(\frac{c\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2(2cd-be)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)-\frac{\sqrt{d(cd-be}}{2e(}\right.\right.\right.}{10e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^{17/2}(2cd-be)}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(-10beg+17cdg+3cef)\left(-\frac{5}{8}c\left(-\frac{1}{2}c\left(-\frac{1}{4}c\left(\frac{ce\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}-\frac{\sqrt{d(cd-}}{e(d+e}\right.\right.\right.\right.}{10e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^{17/2}(2cd-be)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(-\frac{5}{8}c\left(-\frac{1}{2}c\left(-\frac{1}{4}c\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e(d+ex)^{5/2}}\right)-\frac{(d(cd-}{}\right.\right.}{10e(2cd-be)}$$

$$\frac{(ef-dg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{5e^2(d+ex)^{17/2}(2cd-be)}$$

input
```
Int[((f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2))/(d + e*x)^(17/
2),x]
```

output

```
-1/5*((e*f - d*g)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(7/2))/(e^2*(2*c*d
 - b*e)*(d + e*x)^(17/2)) + ((3*c*e*f + 17*c*d*g - 10*b*e*g)*(-1/4*(d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2)^(5/2)/(e*(d + e*x)^(13/2)) - (5*c*(-1/3*(d*
(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(9/2)) - (c*(-1/2*Sq
rt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(e*(d + e*x)^(5/2)) - (c*(-(Sqrt[d
*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(e*(2*c*d - b*e)*(d + e*x)^(3/2))) - (
c*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqr
t[d + e*x])])/(e*(2*c*d - b*e)^(3/2))))/4))/2))/8))/(10*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1130

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2078 vs. $2(390) = 780$.

Time = 1.53 (sec) , antiderivative size = 2079, normalized size of antiderivative = 4.83

method	result	size
default	Expression too large to display	2079

input
```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(17/2),x,method
=_RETURNVERBOSE)
```

output

```
1/1920*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(4560*b^2*c^2*d*e^4*f*x*(b*e-2*c*d
)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)+2514*b*c^3*d^3*e^2*g*x*(b*e-2*c*d)^(1/2)*(-
c*e*x-b*e+c*d)^(1/2)+4150*b*c^3*d*e^4*g*x^3*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+
c*d)^(1/2)-45*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^5*e^6*f*x
^5-45*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^5*d^5*e*f-3760*c^
4*d^2*e^3*g*x^3*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)+750*arctan((-c*e*
x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^4*d*e^5*g*x^4+1500*arctan((-c*e*x-
b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^4*d^2*e^4*g*x^3+1500*arctan((-c*e*x-
b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^4*d^3*e^3*g*x^2+750*arctan((-c*e*x-b
*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^4*d^4*e^2*g*x+1046*c^4*d^3*e^2*g*x^2*
(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)+416*b^3*c*d^2*e^3*g*(b*e-2*c*d)^(
1/2)*(-c*e*x-b*e+c*d)^(1/2)+2064*b^3*c*d*e^4*f*(b*e-2*c*d)^(1/2)*(-c*e*x-b
*e+c*d)^(1/2)-3912*b^2*c^2*d^2*e^3*f*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1
/2)+472*b*c^3*d^4*e*g*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)-6234*b*c^3*
d^2*e^3*f*x*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)+3138*b*c^3*d^3*e^2*f*
(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)-5946*b*c^3*d^2*e^3*g*x^2*(b*e-2*c
*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)+2886*b*c^3*d*e^4*f*x^2*(b*e-2*c*d)^(1/2)*
(-c*e*x-b*e+c*d)^(1/2)+240*c^4*d*e^4*f*x^3*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c
*d)^(1/2)-1352*c^4*d^4*e*g*x*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)+2472
*c^4*d^3*e^2*f*x*(b*e-2*c*d)^(1/2)*(-c*e*x-b*e+c*d)^(1/2)-255*arctan((-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1311 vs. 2(390) = 780.

Time = 0.31 (sec) , antiderivative size = 2652, normalized size of antiderivative = 6.17

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(17/2),x,
 algorithm="fricas")
```

output
```
[-1/3840*(15*(3*c^5*d^6*e*f + (3*c^5*e^7*f + (17*c^5*d*e^6 - 10*b*c^4*e^7)
*g)*x^6 + 6*(3*c^5*d*e^6*f + (17*c^5*d^2*e^5 - 10*b*c^4*d*e^6)*g)*x^5 + 15
*(3*c^5*d^2*e^5*f + (17*c^5*d^3*e^4 - 10*b*c^4*d^2*e^5)*g)*x^4 + 20*(3*c^5
*d^3*e^4*f + (17*c^5*d^4*e^3 - 10*b*c^4*d^3*e^4)*g)*x^3 + 15*(3*c^5*d^4*e^
3*f + (17*c^5*d^5*e^2 - 10*b*c^4*d^4*e^3)*g)*x^2 + (17*c^5*d^7 - 10*b*c^4*
d^6*e)*g + 6*(3*c^5*d^5*e^2*f + (17*c^5*d^6*e - 10*b*c^4*d^5*e^2)*g)*x)*sq
rt(2*c*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x
+ 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x
+ d))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b
*d*e)*(15*(3*(2*c^5*d*e^5 - b*c^4*e^6)*f + (34*c^5*d^2*e^4 - 37*b*c^4*d*e^
5 + 10*b^2*c^3*e^6)*g)*x^4 + 10*(3*(16*c^5*d^2*e^4 - 10*b*c^4*d*e^5 + b^2*
c^3*e^6)*f - (752*c^5*d^3*e^3 - 1206*b*c^4*d^2*e^4 + 651*b^2*c^3*d*e^5 - 1
18*b^3*c^2*e^6)*g)*x^3 - 2*(3*(842*c^5*d^3*e^3 - 1383*b*c^4*d^2*e^4 + 729*
b^2*c^3*d*e^5 - 124*b^3*c^2*e^6)*f - (1046*c^5*d^4*e^2 - 6469*b*c^4*d^3*e^
3 + 8337*b^2*c^3*d^2*e^4 - 4042*b^3*c^2*d*e^5 + 680*b^4*c*e^6)*g)*x^2 - 3*
(634*c^5*d^5*e - 2409*b*c^4*d^4*e^2 + 3654*b^2*c^3*d^3*e^3 - 2680*b^3*c^2*
d^2*e^4 + 944*b^4*c*d*e^5 - 128*b^5*e^6)*f - (538*c^5*d^6 - 1213*b*c^4*d^5
*e + 1728*b^2*c^3*d^4*e^2 - 1460*b^3*c^2*d^3*e^3 + 608*b^4*c*d^2*e^4 - 96*
b^5*d*e^5)*g + 2*(3*(824*c^5*d^4*e^2 - 2490*b*c^4*d^3*e^3 + 2559*b^2*c^3*d
^2*e^4 - 1096*b^3*c^2*d*e^5 + 168*b^4*c*e^6)*f - (1352*c^5*d^5*e - 3190...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2)/(e*x+d)**(17
/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx=\int \frac{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{5}{2}}(gx+f)}{(ex+d)^{\frac{17}{2}}}\,dx$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(17/2),x,
 algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*(g*x + f)/(e*x + d)
^(17/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1449 vs. $2(390) = 780$.

Time = 0.49 (sec) , antiderivative size = 1449, normalized size of antiderivative = 3.37

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(17/2),x,
 algorithm="giac")
```

output

```
-1/1920*(15*(3*c^6*e*f + 17*c^6*d*g - 10*b*c^5*e*g)*arctan(sqrt(-(e*x + d)
*c + 2*c*d - b*e)/sqrt(-2*c*d + b*e))/((4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*s
qrt(-2*c*d + b*e)) + (720*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^10*d^4*e*f -
1440*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^9*d^3*e^2*f + 1080*sqrt(-(e*x +
d)*c + 2*c*d - b*e)*b^2*c^8*d^2*e^3*f - 360*sqrt(-(e*x + d)*c + 2*c*d - b*
e)*b^3*c^7*d*e^4*f + 45*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^4*c^6*e^5*f + 4
080*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^10*d^5*g - 10560*sqrt(-(e*x + d)*c
+ 2*c*d - b*e)*b*c^9*d^4*e*g + 10920*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*
c^8*d^3*e^2*g - 5640*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^7*d^2*e^3*g +
1455*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^4*c^6*d*e^4*g - 150*sqrt(-(e*x + d
)*c + 2*c*d - b*e)*b^5*c^5*e^5*g - 1680*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)
*c^9*d^3*e*f + 2520*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^8*d^2*e^2*f - 1
260*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c^7*d*e^3*f + 210*(-(e*x + d)*c
 + 2*c*d - b*e)^(3/2)*b^3*c^6*e^4*f - 9520*(-(e*x + d)*c + 2*c*d - b*e)^(3
/2)*c^9*d^4*g + 19880*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^8*d^3*e*g - 1
5540*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c^7*d^2*e^2*g + 5390*(-(e*x +
d)*c + 2*c*d - b*e)^(3/2)*b^3*c^6*d*e^3*g - 700*(-(e*x + d)*c + 2*c*d - b*
e)^(3/2)*b^4*c^5*e^4*g + 1536*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d
)*c + 2*c*d - b*e)*c^8*d^2*e*f - 1536*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-
(e*x + d)*c + 2*c*d - b*e)*b*c^7*d*e^2*f + 384*((e*x + d)*c - 2*c*d + b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx=\int \frac{(f+g\,x)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}{{\left(d+e\,x\right)}^{17/2}}\,dx$$

input

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(17/
2),x)
```

output

```
int(((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2))/(d + e*x)^(17/
2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2434, normalized size of antiderivative = 5.66

$$\int \frac{(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}{(d+ex)^{17/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(17/2),x)
```

output

```
(150*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*
b*c**4*d**5*e*g + 750*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sq
rt(b*e - 2*c*d))*b*c**4*d**4*e**2*g*x + 1500*sqrt(b*e - 2*c*d)*atan(sqrt(
- b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**4*d**3*e**3*g*x**2 + 1500*sqr
t(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**4*d
**2*e**4*g*x**3 + 750*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sq
rt(b*e - 2*c*d))*b*c**4*d*e**5*g*x**4 + 150*sqrt(b*e - 2*c*d)*atan(sqrt( -
 b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**4*e**6*g*x**5 - 255*sqrt(b*e -
 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**5*d**6*g - 4
5*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**
5*d**5*e*f - 1275*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b
*e - 2*c*d))*c**5*d**5*e*g*x - 225*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*
d - c*e*x)/sqrt(b*e - 2*c*d))*c**5*d**4*e**2*f*x - 2550*sqrt(b*e - 2*c*d)*
atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**5*d**4*e**2*g*x**2 -
 450*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*
c**5*d**3*e**3*f*x**2 - 2550*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*
e*x)/sqrt(b*e - 2*c*d))*c**5*d**3*e**3*g*x**3 - 450*sqrt(b*e - 2*c*d)*atan
(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**5*d**2*e**4*f*x**3 - 127
5*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**
5*d**2*e**4*g*x**4 - 225*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e...
```

3.226 $\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 265

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{2(2cd-be)^2(cef+cdg-beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^4e^2\sqrt{d+ex}} + \frac{2(2cd-be)(2cef+4cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^4e^2(d+ex)^{3/2}} - \frac{2(cef+5cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^4e^2(d+ex)^{5/2}} + \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^4e^2(d+ex)^{7/2}}$$

output

```
-2*(-b*e+2*c*d)^2*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1
/2)/c^4/e^2/(e*x+d)^(1/2)+2/3*(-b*e+2*c*d)*(-3*b*e*g+4*c*d*g+2*c*e*f)*(d*(
-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^4/e^2/(e*x+d)^(3/2)-2/5*(-3*b*e*g+5*c
*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^4/e^2/(e*x+d)^(5/2)+2
/7*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^4/e^2/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = 2\sqrt{d+ex}(-cd+be+cex)\left(-48b^3e^3g+8b^2ce^2(7ef+32dg+3egx)-\right.$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2
*x^2],x]
```

output
```
(2*Sqrt[d + e*x]*(-(c*d) + b*e + c*e*x)*(-48*b^3*e^3*g + 8*b^2*c*e^2*(7*e*
f + 32*d*g + 3*e*g*x) - 2*b*c^2*e*(219*d^2*g + e^2*x*(14*f + 9*g*x) + 2*d*
e*(63*f + 26*g*x)) + c^3*(230*d^3*g + 3*e^3*x^2*(7*f + 5*g*x) + 2*d*e^2*x*
(49*f + 30*g*x) + d^2*e*(301*f + 115*g*x))))/(105*c^4*e^2*Sqrt[(d + e*x)*(
-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1221, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-6beg+5cdg+7cef)\int \frac{(d+ex)^{5/2}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{7ce}-\frac{2g(d+ex)^{5/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-6beg+5cdg+7cef)\left(\frac{4(2cd-be)\int \frac{(d+ex)^{3/2}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{5c}-\frac{2(d+ex)^{3/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5ce}\right)}{7ce}-\frac{2g(d+ex)^{5/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7ce^2}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{(-6beg+5cdg+7cef)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3c}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)}{5c}-\frac{2(d+ex)^{3/2}\sqrt{d(cd-}{5ce}\right.}{7ce}$$

$$\frac{2g(d+ex)^{5/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7ce^2}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3c^2e\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)}{5c}-\frac{2(d+ex)^{3/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5ce}\right)(-6beg+5}{7ce}$$

$$\frac{2g(d+ex)^{5/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7ce^2}$$

input

```
Int[((d + e*x)^(5/2)*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],
x]
```

output

```
(-2*g*(d + e*x)^(5/2)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(7*c*e^2)
 + ((7*c*e*f + 5*c*d*g - 6*b*e*g)*((-2*(d + e*x)^(3/2)*Sqrt[d*(c*d - b*e)
- b*e^2*x - c*e^2*x^2])/(5*c*e) + (4*(2*c*d - b*e)*((-4*(2*c*d - b*e)*Sqrt
[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*c^2*e*Sqrt[d + e*x]) - (2*Sqrt[d
 + e*x]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*c*e)))/(5*c)))/(7*c*
e)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.81

method	result
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(-15e^3g\,x^3c^3+18b\,c^2e^3g\,x^2-60c^3d\,e^2g\,x^2-21c^3e^3f\,x^2-24b^2c\,e^3gx+104b\,c^2d\,e^2gx+28b\,c^2e^3fx-115c^3}{105\sqrt{ex+d}\,c^4e^2}$
gosper	$-\frac{2(cex+be-cd)(-15e^3g\,x^3c^3+18b\,c^2e^3g\,x^2-60c^3d\,e^2g\,x^2-21c^3e^3f\,x^2-24b^2c\,e^3gx+104b\,c^2d\,e^2gx+28b\,c^2e^3fx-115c^3d^2egx-98}{105c^4e^2\sqrt{-x^2c\,e^2-xb\,e^2-bde}}$
orering	$-\frac{2(cex+be-cd)(-15e^3g\,x^3c^3+18b\,c^2e^3g\,x^2-60c^3d\,e^2g\,x^2-21c^3e^3f\,x^2-24b^2c\,e^3gx+104b\,c^2d\,e^2gx+28b\,c^2e^3fx-115c^3d^2egx-98}{105c^4e^2\sqrt{-x^2c\,e^2-xb\,e^2-bde}}$

input
```
int((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output

```
2/105/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-15*c^3*e^3*g*x^3+18
*b*c^2*e^3*g*x^2-60*c^3*d*e^2*g*x^2-21*c^3*e^3*f*x^2-24*b^2*c*e^3*g*x+104*
b*c^2*d*e^2*g*x+28*b*c^2*e^3*f*x-115*c^3*d^2*e*g*x-98*c^3*d*e^2*f*x+48*b^3
*e^3*g-256*b^2*c*d*e^2*g-56*b^2*c*e^3*f+438*b*c^2*d^2*e*g+252*b*c^2*d*e^2*
f-230*c^3*d^3*g-301*c^3*d^2*e*f)/c^4/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$
$$-\frac{2\,(15\,c^3e^3gx^3+3\,(7\,c^3e^3f+2\,(10\,c^3de^2-3\,bc^2e^3)g)x^2+7\,(43\,c^3d^2e-36\,bc^2de^2+8\,b^2ce^3)f+2\,(115\,c^3d}{}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="fricas")
```

output

```
-2/105*(15*c^3*e^3*g*x^3 + 3*(7*c^3*e^3*f + 2*(10*c^3*d*e^2 - 3*b*c^2*e^3)
*g)*x^2 + 7*(43*c^3*d^2*e - 36*b*c^2*d*e^2 + 8*b^2*c*e^3)*f + 2*(115*c^3*d
^3 - 219*b*c^2*d^2*e + 128*b^2*c*d*e^2 - 24*b^3*e^3)*g + (14*(7*c^3*d*e^2
- 2*b*c^2*e^3)*f + (115*c^3*d^2*e - 104*b*c^2*d*e^2 + 24*b^2*c*e^3)*g)*x)*
sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^4*e^3*x + c^4*
d*e^2)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}\,(f+gx)}{\sqrt{-\,(d+ex)\,(be-cd+cex)}}\,dx$$

input

```
integrate((e*x+d)**(5/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/
2),x)
```

output

```
Integral((d + e*x)**(5/2)*(f + g*x)/sqrt(-(d + e*x)*(b*e - c*d + c*e*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2\,(3\,c^3e^3x^3-43\,c^3d^3+79\,bc^2d^2e-44\,b^2cde^2+8\,b^3e^3+(11\,c^3de^2-bc}{15\,\sqrt{-cex+cd-be}c^3e}$$
$$+\frac{2\,(15\,c^4e^4x^4-230\,c^4d^4+668\,bc^3d^3e-694\,b^2c^2d^2e^2+304\,b^3cde^3-48\,b^4e^4+3\,(15\,c^4de^3-bc^3e^4)x^3+(55}{105\,\sqrt{-cex+cd-b}}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="maxima")
```

output

```
2/15*(3*c^3*e^3*x^3 - 43*c^3*d^3 + 79*b*c^2*d^2*e - 44*b^2*c*d*e^2 + 8*b^3
*e^3 + (11*c^3*d*e^2 - b*c^2*e^3)*x^2 + (29*c^3*d^2*e - 18*b*c^2*d*e^2 + 4
*b^2*c*e^3)*x)*f/(sqrt(-c*e*x + c*d - b*e)*c^3*e) + 2/105*(15*c^4*e^4*x^4
- 230*c^4*d^4 + 668*b*c^3*d^3*e - 694*b^2*c^2*d^2*e^2 + 304*b^3*c*d*e^3 -
48*b^4*e^4 + 3*(15*c^4*d*e^3 - b*c^3*e^4)*x^3 + (55*c^4*d^2*e^2 - 26*b*c^3
*d*e^3 + 6*b^2*c^2*e^4)*x^2 + (115*c^4*d^3*e - 219*b*c^3*d^2*e^2 + 128*b^2
*c^2*d*e^3 - 24*b^3*c*e^4)*x)*g/(sqrt(-c*e*x + c*d - b*e)*c^4*e^2)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$
$$-\frac{2\left(\frac{105\,(4\,c^3d^2ef-4\,bc^2de^2f+b^2ce^3f+4\,c^3d^3g-8\,bc^2d^2eg+5\,b^2cde^2g-b^3e^3g)\sqrt{-(ex+d)c+2\,cd-be}}{c^4}-\frac{140\,(-(ex+d)c+2\,cd-be)^{\frac{3}{2}}c^2def-70}{}\right.}{}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="giac")
```

output
```
-2/105*(105*(4*c^3*d^2*e*f - 4*b*c^2*d*e^2*f + b^2*c*e^3*f + 4*c^3*d^3*g -
 8*b*c^2*d^2*e*g + 5*b^2*c*d*e^2*g - b^3*e^3*g)*sqrt(-(e*x + d)*c + 2*c*d
- b*e)/c^4 - (140*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*d*e*f - 70*(-(e*x
 + d)*c + 2*c*d - b*e)^(3/2)*b*c*e^2*f + 280*(-(e*x + d)*c + 2*c*d - b*e)^
(3/2)*c^2*d^2*g - 350*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c*d*e*g + 105*(
-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*e^2*g - 21*((e*x + d)*c - 2*c*d + b*
e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c*e*f - 105*((e*x + d)*c - 2*c*d + b
*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c*d*g + 63*((e*x + d)*c - 2*c*d + b
*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*e*g - 15*((e*x + d)*c - 2*c*d + b
*e)^3*sqrt(-(e*x + d)*c + 2*c*d - b*e)*g)/c^4)/e^2
```

Mupad [B] (verification not implemented)

Time = 11.42 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$

$$-\frac{\left(\frac{2\,g\,x^3\,\sqrt{d+e\,x}}{7\,c}+\frac{\sqrt{d+e\,x}\,\left(-96\,g\,b^3\,e^3+512\,g\,b^2\,c\,d\,e^2+112\,f\,b^2\,c\,e^3-876\,g\,b\,c^2\,d^2\,e-504\,f\,b\,c^2\,d\,e^2+460\,g\,c^3\,d^3+602\,f\,c^3\,d^2\,e\right)}{105\,c^4\,e^3}+\frac{2\,x^2\,\sqrt{d+e}}{}\right.}{}$$

input
```
int(((f + g*x)*(d + e*x)^(5/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2
),x)
```

output
```
-(((2*g*x^3*(d + e*x)^(1/2))/(7*c) + ((d + e*x)^(1/2)*(460*c^3*d^3*g - 96*
b^3*e^3*g + 112*b^2*c*e^3*f + 602*c^3*d^2*e*f - 504*b*c^2*d*e^2*f - 876*b*
c^2*d^2*e*g + 512*b^2*c*d*e^2*g))/(105*c^4*e^3) + (2*x^2*(d + e*x)^(1/2)*(
20*c*d*g - 6*b*e*g + 7*c*e*f))/(35*c^2*e) + (x*(d + e*x)^(1/2)*(48*b^2*c*e
^3*g - 56*b*c^2*e^3*f + 196*c^3*d*e^2*f + 230*c^3*d^2*e*g - 208*b*c^2*d*e^
2*g))/(105*c^4*e^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(x + d/e
)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2\sqrt{-cex-be+cd}\,(-15c^3e^3g\,x^3+18b\,c^2e^3g\,x^2-60c^3d\,e^2g\,x^2-21c^3e}{}$$

input
```
int((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*(48*b**3*e**3*g - 256*b**2*c*d*e**2*g - 56*b
**2*c*e**3*f - 24*b**2*c*e**3*g*x + 438*b*c**2*d**2*e*g + 252*b*c**2*d*e**
2*f + 104*b*c**2*d*e**2*g*x + 28*b*c**2*e**3*f*x + 18*b*c**2*e**3*g*x**2 -
 230*c**3*d**3*g - 301*c**3*d**2*e*f - 115*c**3*d**2*e*g*x - 98*c**3*d*e**
2*f*x - 60*c**3*d*e**2*g*x**2 - 21*c**3*e**3*f*x**2 - 15*c**3*e**3*g*x**3)
)/(105*c**4*e**2)
```

3.227 $\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 188

$$\begin{aligned}
&\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \\
&-\frac{2(2cd-be)(cef+cdg-beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^3e^2\sqrt{d+ex}} \\
&+\frac{2(cef+3cdg-2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^3e^2(d+ex)^{3/2}} \\
&-\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^3e^2(d+ex)^{5/2}}
\end{aligned}$$

output

```
-2*(-b*e+2*c*d)*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2
)/c^3/e^2/(e*x+d)^(1/2)+2/3*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x
-c*e^2*x^2)^(3/2)/c^3/e^2/(e*x+d)^(3/2)-2/5*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*
x^2)^(5/2)/c^3/e^2/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2\sqrt{d+ex}(-cd+be+cex)\left(8b^2e^2g-2bce(5ef+13dg+2egx)+c^2(1\right.}{15c^3e^2\sqrt{(d+ex)(-be+c(d-e}}$$

input

```
Integrate[((d + e*x)^(3/2)*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2
*x^2],x]
```

output

```
(2*Sqrt[d + e*x]*(-(c*d) + b*e + c*e*x)*(8*b^2*e^2*g - 2*b*c*e*(5*e*f + 13
*d*g + 2*e*g*x) + c^2*(18*d^2*g + e^2*x*(5*f + 3*g*x) + d*e*(25*f + 9*g*x)
)))/(15*c^3*e^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-4beg+3cdg+5cef)\int \frac{(d+ex)^{3/2}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{5ce}-\frac{2g(d+ex)^{3/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5ce^2}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(-4beg+3cdg+5cef)\left(\frac{2(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3c}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)}{5ce}-\frac{2g(d+ex)^{3/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5ce^2}$$

$$\Big\downarrow \text{1122}$$

$$\frac{\left(-\frac{4(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3c^2e\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)(-4beg+3cdg+5cef)}{5ce}-\frac{2g(d+ex)^{3/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5ce^2}$$

input

```
Int[((d + e*x)^(3/2)*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],
x]
```

output

```
(-2*g*(d + e*x)^(3/2)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(5*c*e^2)
 + ((5*c*e*f + 3*c*d*g - 4*b*e*g)*((-4*(2*c*d - b*e)*Sqrt[d*(c*d - b*e) -
b*e^2*x - c*e^2*x^2])/(3*c^2*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*Sqrt[d*(c
*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*c*e)))/(5*c*e)
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(3g\,x^{2}c^{2}e^{2}-4bc\,e^{2}gx+9c^{2}degx+5c^{2}e^{2}fx+8b^{2}e^{2}g-26bcdeg-10bc\,e^{2}f+18c^{2}d^{2}g+25c^{2}def)}{15\sqrt{ex+d}\,c^{3}e^{2}}$	119
gosper	$\frac{2(cex+be-cd)\left(3g\,x^{2}c^{2}e^{2}-4bc\,e^{2}gx+9c^{2}degx+5c^{2}e^{2}fx+8b^{2}e^{2}g-26bcdeg-10bc\,e^{2}f+18c^{2}d^{2}g+25c^{2}def\right)\sqrt{ex+d}}{15c^{3}e^{2}\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}$	139
orering	$\frac{2(cex+be-cd)\left(3g\,x^{2}c^{2}e^{2}-4bc\,e^{2}gx+9c^{2}degx+5c^{2}e^{2}fx+8b^{2}e^{2}g-26bcdeg-10bc\,e^{2}f+18c^{2}d^{2}g+25c^{2}def\right)\sqrt{ex+d}}{15c^{3}e^{2}\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}$	139

input

```
int((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output

```
-2/15/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(3*c^2*e^2*g*x^2-4*b*
c*e^2*g*x+9*c^2*d*e*g*x+5*c^2*e^2*f*x+8*b^2*e^2*g-26*b*c*d*e*g-10*b*c*e^2*
f+18*c^2*d^2*g+25*c^2*d*e*f)/c^3/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$

$$-\frac{2\left(3\,c^{2}e^{2}gx^{2}+5\,(5\,c^{2}de-2\,bce^{2})f+2\,(9\,c^{2}d^{2}-13\,bcde+4\,b^{2}e^{2})g+(5\,c^{2}e^{2}f+(9\,c^{2}de-4\,bce^{2})g)x\right)\sqrt{-}}{15\,(c^{3}e^{3}x+c^{3}de^{2})}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="fricas")
```

output

```
-2/15*(3*c^2*e^2*g*x^2 + 5*(5*c^2*d*e - 2*b*c*e^2)*f + 2*(9*c^2*d^2 - 13*b
*c*d*e + 4*b^2*e^2)*g + (5*c^2*e^2*f + (9*c^2*d*e - 4*b*c*e^2)*g)*x)*sqrt(
-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^3*e^3*x + c^3*d*e^2
)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)}{\sqrt{-\,(d+ex)\,(be-cd+cex)}}\,dx$$

input

```
integrate((e*x+d)**(3/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/
2),x)
```

output

```
Integral((d + e*x)**(3/2)*(f + g*x)/sqrt(-(d + e*x)*(b*e - c*d + c*e*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2\,(c^2e^2x^2-5\,c^2d^2+7\,bcde-2\,b^2e^2+(4\,c^2de-bce^2)x)f}{3\,\sqrt{-cex+cd-be}c^2e}$$
$$+\frac{2\,(3\,c^3e^3x^3-18\,c^3d^3+44\,bc^2d^2e-34\,b^2cde^2+8\,b^3e^3+(6\,c^3de^2-bc^2e^3)x^2+(9\,c^3d^2e-13\,bc^2de^2+4\,b^2c}{15\,\sqrt{-cex+cd-be}c^3e^2}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="maxima")
```

output

```
2/3*(c^2*e^2*x^2 - 5*c^2*d^2 + 7*b*c*d*e - 2*b^2*e^2 + (4*c^2*d*e - b*c*e^
2)*x)*f/(sqrt(-c*e*x + c*d - b*e)*c^2*e) + 2/15*(3*c^3*e^3*x^3 - 18*c^3*d^
3 + 44*b*c^2*d^2*e - 34*b^2*c*d*e^2 + 8*b^3*e^3 + (6*c^3*d*e^2 - b*c^2*e^3
)*x^2 + (9*c^3*d^2*e - 13*b*c^2*d*e^2 + 4*b^2*c*e^3)*x)*g/(sqrt(-c*e*x + c
*d - b*e)*c^3*e^2)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$
$$-\frac{2\left(\frac{15\,(2\,c^2def-bce^2f+2\,c^2d^2g-3\,bcdeg+b^2e^2g)\,\sqrt{-(ex+d)c+2\,cd-be}}{c^3}-\frac{5\,(-(ex+d)c+2\,cd-be)^{\frac{3}{2}}\,cef+15\,(-(ex+d)c+2\,cd-be)^{\frac{3}{2}}\,cdg-10\,(-}{}\right.}{15\,e^2}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="giac")
```

output
```
-2/15*(15*(2*c^2*d*e*f - b*c*e^2*f + 2*c^2*d^2*g - 3*b*c*d*e*g + b^2*e^2*g
)*sqrt(-(e*x + d)*c + 2*c*d - b*e)/c^3 - (5*(-(e*x + d)*c + 2*c*d - b*e)^(
3/2)*c*e*f + 15*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c*d*g - 10*(-(e*x + d)*
c + 2*c*d - b*e)^(3/2)*b*e*g - 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x
+ d)*c + 2*c*d - b*e)*g)/c^3)/e^2
```

Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$
$$-\frac{\left(\frac{\sqrt{d+e\,x}\,\left(16\,g\,b^2\,e^2-52\,g\,b\,c\,d\,e-20\,f\,b\,c\,e^2+36\,g\,c^2\,d^2+50\,f\,c^2\,d\,e\right)}{15\,c^3\,e^3}+\frac{2\,g\,x^2\,\sqrt{d+e\,x}}{5\,c\,e}+\frac{2\,x\,\sqrt{d+e\,x}\,\left(9\,c\,d\,g-4\,b\,e\,g+5\,c\,e\,f\right)}{15\,c^2\,e^2}\right)\,\sqrt{c\,d^2-b\,c}}{x+\frac{d}{e}}$$

input
```
int(((f + g*x)*(d + e*x)^(3/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2
),x)
```

output
```
-((((d + e*x)^(1/2)*(16*b^2*e^2*g + 36*c^2*d^2*g - 20*b*c*e^2*f + 50*c^2*d
*e*f - 52*b*c*d*e*g))/(15*c^3*e^3) + (2*g*x^2*(d + e*x)^(1/2))/(5*c*e) + (
2*x*(d + e*x)^(1/2)*(9*c*d*g - 4*b*e*g + 5*c*e*f))/(15*c^2*e^2))*(c*d^2 -
c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2\sqrt{-cex-be+cd}\,(-3c^2e^2g\,x^2+4bc\,e^2gx-9c^2degx-5c^2e^2fx-8b^2}{15c^3e^2}$$

input

```
int((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(2*sqrt( - b*e + c*d - c*e*x)*( - 8*b**2*e**2*g + 26*b*c*d*e*g + 10*b*c*e*
*2*f + 4*b*c*e**2*g*x - 18*c**2*d**2*g - 25*c**2*d*e*f - 9*c**2*d*e*g*x -
5*c**2*e**2*f*x - 3*c**2*e**2*g*x**2))/(15*c**3*e**2)
```

3.228 $\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 114

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{2(cef+cdg-beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^2e^2\sqrt{d+ex}} + \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^2e^2(d+ex)^{3/2}}$$

output

```
-2*(-b*e*g+c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^2/e^2/(e*
x+d)^(1/2)+2/3*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^2/e^2/(e*x+d)^(3
/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{2\sqrt{(d+ex)(-be+c(d-ex))}(-2beg+c(3ef+2dg+egx))}{3c^2e^2\sqrt{d+ex}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x
^2],x]
```

output
```
(-2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(-2*b*e*g + c*(3*e*f + 2*d*g +
e*g*x)))/(3*c^2*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-2beg+cdg+3cef)\int \frac{\sqrt{d+ex}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3ce}-\frac{2g\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$\Big\downarrow \text{1122}$$

$$-\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}(-2beg+cdg+3cef)}{3c^2e^2\sqrt{d+ex}}-\frac{2g\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
(-2*(3*c*e*f + c*d*g - 2*b*e*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])
/(3*c^2*e^2*Sqrt[d + e*x]) - (2*g*Sqrt[d + e*x]*Sqrt[d*(c*d - b*e) - b*e^2
*x - c*e^2*x^2])/(3*c*e^2)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(-cegx+2beg-2cdg-3fce)}{3\sqrt{ex+d}\,c^{2}e^{2}}$	59
gosper	$-\frac{2(cex+be-cd)(-cegx+2beg-2cdg-3fce)\sqrt{ex+d}}{3c^{2}e^{2}\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}$	79
orering	$-\frac{2(cex+be-cd)(-cegx+2beg-2cdg-3fce)\sqrt{ex+d}}{3c^{2}e^{2}\sqrt{-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}}}$	79

input
```
int((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
2/3/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-c*e*g*x+2*b*e*g-2*c*d
*g-3*c*e*f)/c^2/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{2\,\sqrt{-ce^2x^2-be^2x+cd^2-bde}(cegx+3\,cef+2\,(cd-be)g)\sqrt{ex+d}}{3\,(c^2e^3x+c^2de^2)}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="fricas")
```

output
```
-2/3*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(c*e*g*x + 3*c*e*f + 2*(c*
d - b*e)*g)*sqrt(e*x + d)/(c^2*e^3*x + c^2*d*e^2)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{-(d+ex)\,(be-cd+cex)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/
2),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)/sqrt(-(d + e*x)*(b*e - c*d + c*e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2\,(cex-cd+be)f}{\sqrt{-cex+cd-be}ce} + \frac{2\,(c^2e^2x^2-2\,c^2d^2+4\,bcde-2\,b^2e^2+(c^2de-bce^2)x)g}{3\,\sqrt{-cex+cd-be}c^2e^2}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="maxima")
```

output
```
2*(c*e*x - c*d + b*e)*f/(sqrt(-c*e*x + c*d - b*e)*c*e) + 2/3*(c^2*e^2*x^2
- 2*c^2*d^2 + 4*b*c*d*e - 2*b^2*e^2 + (c^2*d*e - b*c*e^2)*x)*g/(sqrt(-c*e*
x + c*d - b*e)*c^2*e^2)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2\left(\frac{(-(ex+d)c+2\,cd-be)^{\frac{3}{2}}g}{c^2}-\frac{3\,(cef+cdg-beg)\sqrt{-(ex+d)c+2\,cd-be}}{c^2}\right)}{3\,e^2}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="giac")
```

output
```
2/3*((-(e*x + d)*c + 2*c*d - b*e)^(3/2)*g/c^2 - 3*(c*e*f + c*d*g - b*e*g)*
sqrt(-(e*x + d)*c + 2*c*d - b*e)/c^2)/e^2
```

Mupad [B] (verification not implemented)

Time = 10.91 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{\left(\frac{\sqrt{d+e\,x}\,(4\,c\,d\,g-4\,b\,e\,g+6\,c\,e\,f)}{3\,c^2\,e^3}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{3\,c\,e^2}\right)\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{x+\frac{d}{e}}$$

input
```
int(((f + g*x)*(d + e*x)^(1/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2
),x)
```

output
```
-((((d + e*x)^(1/2)*(4*c*d*g - 4*b*e*g + 6*c*e*f))/(3*c^2*e^3) + (2*g*x*(d
 + e*x)^(1/2))/(3*c*e^2))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(x
+ d/e)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\frac{2\sqrt{-cex-be+cd}\,(-cegx+2beg-2cdg-3cef)}{3c^2e^2}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
(2*sqrt( - b*e + c*d - c*e*x)*(2*b*e*g - 2*c*d*g - 3*c*e*f - c*e*g*x))/(3*
c**2*e**2)
```

3.229 $\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 131

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=-\frac{2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2\sqrt{d+ex}}-\frac{2(ef-dg)\text{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e^2\sqrt{2cd-be}}$$

output

```
-2*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c/e^2/(e*x+d)^(1/2)-2*(-d*g+e*
f)*arctanh((-b*e+2*c*d)^(1/2)*(e*x+d)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^
2)^(1/2))/e^2/(-b*e+2*c*d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =\frac{2\sqrt{d+ex}\left(\sqrt{-2cd+be}g(-cd+be+cex)+c(ef-dg)\sqrt{-be+c(d-ex)}\arctan\left(\frac{\sqrt{cd-be-cex}}{\sqrt{-2cd+be}}\right)\right)}{ce^2\sqrt{-2cd+be}\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[(f + g*x)/(Sqrt[d + e*x]*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^
2]),x]
```

output
```
(2*Sqrt[d + e*x]*(Sqrt[-2*c*d + b*e]*g*(-(c*d) + b*e + c*e*x) + c*(e*f - d
*g)*Sqrt[-(b*e) + c*(d - e*x)]*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d
+ b*e]]))/(c*e^2*Sqrt[-2*c*d + b*e]*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]
)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1221, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(ef-dg)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{e}-\frac{2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{1136}$$

$$2(ef-dg)\int \frac{1}{\frac{e^2(-cx^2e^2-bxe^2+d(cd-be))}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}-\frac{2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2(ef-dg)\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e^2\sqrt{2cd-be}}-\frac{2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2\sqrt{d+ex}}$$

input
```
Int[(f + g*x)/(Sqrt[d + e*x]*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output

```
(-2*g*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(c*e^2*Sqrt[d + e*x]) - (
2*(e*f - d*g)*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*
d - b*e]*Sqrt[d + e*x])])/(e^2*Sqrt[2*c*d - b*e])
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{2 \sqrt{-\left(e x +d \right) \left(c e x +b e -c d \right)}\, \left(\arctan \left(\frac{\sqrt{-c e x -b e +c d}}{\sqrt{b e -2 c d}}\right) c d g -\arctan \left(\frac{\sqrt{-c e x -b e +c d}}{\sqrt{b e -2 c d}}\right) c e f +g \sqrt{-c e x -b e +c d}\, \sqrt{b e -2 c d}\right)}{\sqrt{e x +d}\, \sqrt{-c e x -b e +c d}\, e^{2} c \sqrt{b e -2 c d}}$	153

input

```
int((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(arctan((-c*e*x-b*e+c*d)
^(1/2)/(b*e-2*c*d)^(1/2))*c*d*g-arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^
(1/2))*c*e*f+g*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2))/(-c*e*x-b*e+c*d)^
(1/2)/e^2/c/(b*e-2*c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.39

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \left[-\frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,cd-be)\sqrt{ex+d}g+(cdef-cd^2g+(ce^2f-cdeg)x)\sqrt{2\,cd-be}\log}{2\,c^2d^2e^2-bcde^3+(2\,c^2de^3-bce^4)x}\right.$$

$$-\frac{2\left(\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,cd-be)\sqrt{ex+d}g+(cdef-cd^2g+(ce^2f-cdeg)x)\sqrt{-2\,cd+be}\right.}{2\,c^2d^2e^2-bcde^3+(2\,c^2de^3-bce^4)x}$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="fricas")
```

output
```
[-(2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*d - b*e)*sqrt(e*x + d
)*g + (c*d*e*f - c*d^2*g + (c*e^2*f - c*d*e*g)*x)*sqrt(2*c*d - b*e)*log(-(
c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*sqrt(-c*e^2*x^2 -
b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e
*x + d^2)))/(2*c^2*d^2*e^2 - b*c*d*e^3 + (2*c^2*d*e^3 - b*c*e^4)*x), -2*(s
qrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*d - b*e)*sqrt(e*x + d)*g +
(c*d*e*f - c*d^2*g + (c*e^2*f - c*d*e*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqr
t(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(
2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)))/(2*c^2*d^2*e^2 - b*c*d*e^3 + (2*c
^2*d*e^3 - b*c*e^4)*x)]
```

Sympy [F]

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{f+gx}{\sqrt{-(d+ex)\,(be-cd+cex)}\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**(1/2)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/
2),x)
```

output
```
Integral((f + g*x)/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*sqrt(d + e*x)), x
)
```

Maxima [F]

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{gx+f}{\sqrt{-ce^2x^2-be^2x+cd^2-bde}\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((g*x + f)/(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x +
 d)), x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\frac{2\left(\frac{(ef-dg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{\sqrt{-2\,cd+be}}-\frac{\sqrt{-(ex+d)c+2\,cd-be}g}{c}\right)}{e^2}$$

input

```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="giac")
```

output

```
2*((e*f - d*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(-2*c*d + b*e))
/sqrt(-2*c*d + b*e) - sqrt(-(e*x + d)*c + 2*c*d - b*e)*g/c)/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{f+g\,x}{\sqrt{d+e\,x}\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input

```
int((f + g*x)/((d + e*x)^(1/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)
),x)
```

output

```
int((f + g*x)/((d + e*x)^(1/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{-2\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)cdg+2\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)cef-2\sqrt{-cex-be+cd}\,beg+}{c\,e^2\,(be-2cd)}$$

input

```
int((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
(2*( - sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*c*d*g + sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c
*d))*c*e*f - sqrt( - b*e + c*d - c*e*x)*b*e*g + 2*sqrt( - b*e + c*d - c*e*
x)*c*d*g))/(c*e**2*(b*e - 2*c*d))
```

3.230 $\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 153

$$\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(2cd-be)(d+ex)^{3/2}} - \frac{(cef+3cdg-2beg)\operatorname{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e^2(2cd-be)^{3/2}}$$

output
```
-(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e*x+d
)^(3/2)-(-2*b*e*g+3*c*d*g+c*e*f)*arctanh((-b*e+2*c*d)^(1/2)*(e*x+d)^(1/2)/
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2/(-b*e+2*c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00

$$\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\frac{\sqrt{d+ex}\Big(\frac{(ef-dg)(-cd+be+cex)}{(2cd-be)(d+ex)}-\frac{(cef+3cdg-2beg)\sqrt{cd-be-cex}\,\mathrm{arctan}}{(-2cd+be)^{3/2}}}{e^2\sqrt{(d+ex)(-be+c(d-ex))}}$$

input

```
Integrate[(f + g*x)/((d + e*x)^(3/2)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*
x^2]),x]
```

output

```
(Sqrt[d + e*x]*(((e*f - d*g)*(-(c*d) + b*e + c*e*x))/((2*c*d - b*e)*(d + e
*x)) - ((c*e*f + 3*c*d*g - 2*b*e*g)*Sqrt[c*d - b*e - c*e*x]*ArcTan[Sqrt[c*
d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/(-2*c*d + b*e)^(3/2)))/(e^2*Sqrt[(d
+ e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1220, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \textcolor{blue}{1220}$$

$$\frac{(-2beg+3cdg+cef)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2e(2cd-be)}-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)^{3/2}(2cd-be)}$$

$$\Big\downarrow \textcolor{blue}{1136}$$

$$\frac{(-2beg+3cdg+cef)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)^{3/2}(2cd-be)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)(-2beg+3cdg+cef)}{e^2(2cd-be)^{3/2}}-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(d+ex)^{3/2}(2cd-be)}$$

input
```
Int[(f + g*x)/((d + e*x)^(3/2)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),
x]
```

output
```
-(((e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e^2*(2*c*d - b*
e)*(d + e*x)^(3/2))) - ((c*e*f + 3*c*d*g - 2*b*e*g)*ArcTanh[Sqrt[d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/(e^2*(2*c*
d - b*e)^(3/2))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(141) = 282$.

Time = 1.68 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.09

method	result
default	$\frac{\left(2\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)b\,e^{2}gx-3\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)cdegx-\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c\,e^{2}fx+2\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bdeg-\right.}{(be-2cd)^{\frac{3}{2}}}$

input

```
int((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output

```
(2*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*e^2*g*x-3*arctan((-c
*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c*d*e*g*x-arctan((-c*e*x-b*e+c*d)^(
1/2)/(b*e-2*c*d)^(1/2))*c*e^2*f*x+2*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c
*d)^(1/2))*b*d*e*g-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c*d^
2*g-arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c*d*e*f-(-c*e*x-b*e+c
*d)^(1/2)*(b*e-2*c*d)^(1/2)*d*g+(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)*e
*f)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/(b*e-2*c*d)^(3/2)/e^2/(-c*e*x-b*e+c*d
)^(1/2)/(e*x+d)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(141) = 282$.

Time = 0.10 (sec) , antiderivative size = 700, normalized size of antiderivative = 4.58

$$\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \left[\frac{(cd^2ef+(ce^3f+(3\,cde^2-2\,be^3)g)x^2+(3\,cd^3-2\,bd^2e)g}{}\right.$$

$$-\frac{(cd^2ef+(ce^3f+(3\,cde^2-2\,be^3)g)x^2+(3\,cd^3-2\,bd^2e)g+2\,(cde^2f+(3\,cd^2e-2\,bde^2)g)x)\sqrt{-2\,cd+b}}{4\,c^2d^4e^2-4\,bcd^3e^3+b^2d^2e^4+(4\,c^2d^2e}$$

input
```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="fricas")
```

output
```
[1/2*((c*d^2*e*f + (c*e^3*f + (3*c*d*e^2 - 2*b*e^3)*g)*x^2 + (3*c*d^3 - 2*
b*d^2*e)*g + 2*(c*d*e^2*f + (3*c*d^2*e - 2*b*d*e^2)*g)*x)*sqrt(2*c*d - b*e
)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x + 2*sqrt(-c*e^
2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2
 + 2*d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((2*c*d*
e - b*e^2)*f - (2*c*d^2 - b*d*e)*g)*sqrt(e*x + d))/(4*c^2*d^4*e^2 - 4*b*c*
d^3*e^3 + b^2*d^2*e^4 + (4*c^2*d^2*e^4 - 4*b*c*d*e^5 + b^2*e^6)*x^2 + 2*(4
*c^2*d^3*e^3 - 4*b*c*d^2*e^4 + b^2*d*e^5)*x), -((c*d^2*e*f + (c*e^3*f + (3
*c*d*e^2 - 2*b*e^3)*g)*x^2 + (3*c*d^3 - 2*b*d^2*e)*g + 2*(c*d*e^2*f + (3*c
*d^2*e - 2*b*d*e^2)*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqrt(-c*e^2*x^2 - b*e
^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e +
(2*c*d*e - b*e^2)*x)) + sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((2*c*d
*e - b*e^2)*f - (2*c*d^2 - b*d*e)*g)*sqrt(e*x + d))/(4*c^2*d^4*e^2 - 4*b*c
*d^3*e^3 + b^2*d^2*e^4 + (4*c^2*d^2*e^4 - 4*b*c*d*e^5 + b^2*e^6)*x^2 + 2*(
4*c^2*d^3*e^3 - 4*b*c*d^2*e^4 + b^2*d*e^5)*x)]
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{f+gx}{\sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**(3/2)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/
2),x)
```

output
```
Integral((f + g*x)/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)**(3/2))
, x)
```

Maxima [F]

$$\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{gx+f}{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((g*x + f)/(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(e*x + d)^
(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.02

$$\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\frac{\frac{\left(c^2ef+3\,c^2dg-2\,bceg\right)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{(2\,cd-be)\sqrt{-2\,cd+be}}-\frac{\sqrt{-(ex+d)c+2\,cd-bec}}{(2\,cd}}{ce^2}$$

input
```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="giac")
```

output

```
((c^2*e*f + 3*c^2*d*g - 2*b*c*e*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)
/sqrt(-2*c*d + b*e))/((2*c*d - b*e)*sqrt(-2*c*d + b*e)) - (sqrt(-(e*x + d)
*c + 2*c*d - b*e)*c^2*e*f - sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^2*d*g)/((2*
c*d - b*e)*(e*x + d)*c))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{f+g\,x}{(d+e\,x)^{3/2}\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input

```
int((f + g*x)/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)
),x)
```

output

```
int((f + g*x)/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.60

$$\int \frac{f+gx}{(d+ex)^{3/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{2\sqrt{be-2cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bdeg+2\sqrt{be-2cd}\,atan\Bigg(}{}$$

input

```
int((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(2*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*
d*e*g + 2*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c
*d))*b*e**2*g*x - 3*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt
(b*e - 2*c*d))*c*d**2*g - sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x
)/sqrt(b*e - 2*c*d))*c*d*e*f - 3*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d
- c*e*x)/sqrt(b*e - 2*c*d))*c*d*e*g*x - sqrt(b*e - 2*c*d)*atan(sqrt( - b*e
 + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c*e**2*f*x - sqrt( - b*e + c*d - c*e*x)
*b*d*e*g + sqrt( - b*e + c*d - c*e*x)*b*e**2*f + 2*sqrt( - b*e + c*d - c*e
*x)*c*d**2*g - 2*sqrt( - b*e + c*d - c*e*x)*c*d*e*f)/(e**2*(b**2*d*e**2 +
b**2*e**3*x - 4*b*c*d**2*e - 4*b*c*d*e**2*x + 4*c**2*d**3 + 4*c**2*d**2*e*
x))
```

3.231 $\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 233

$$\begin{aligned}\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = &-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e^2(2cd-be)(d+ex)^{5/2}}\\ &-\frac{(3cef+5cdg-4beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4e^2(2cd-be)^2(d+ex)^{3/2}}\\ &-\frac{c(3cef+5cdg-4beg)\operatorname{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4e^2(2cd-be)^{5/2}}\end{aligned}$$

output

```
-1/2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)/(e
*x+d)^(5/2)-1/4*(-4*b*e*g+5*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(1/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^(3/2)-1/4*c*(-4*b*e*g+5*c*d*g+3*c*e*f)*
arctanh((-b*e+2*c*d)^(1/2)*(e*x+d)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^
(1/2))/e^2/(-b*e+2*c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.84

$$\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\frac{c\sqrt{d+ex}\left(-\frac{(-be+c(d-ex))(-2be(dg+e(f+2gx))+c(d^2g+3e^2fx+de(7f+}{c(-2cd+be)^2(d+ex)^2}\right.}{4e^2\sqrt{(d+ex)(-be-}}$$

input
```
Integrate[(f + g*x)/((d + e*x)^(5/2)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*
x^2]),x]
```

output
```
(c*Sqrt[d + e*x]*(-(((-(b*e) + c*(d - e*x))*(-2*b*e*(d*g + e*(f + 2*g*x))
+ c*(d^2*g + 3*e^2*f*x + d*e*(7*f + 5*g*x))))/(c*(-2*c*d + b*e)^2*(d + e*x
)^2)) + ((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[c*d - b*e - c*e*x]*ArcTan[Sqrt
[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/(-2*c*d + b*e)^(5/2)))/(4*e^2*Sqr
t[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1220, 1135, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(-4beg+5cdg+3cef)\int \frac{1}{(d+ex)^{3/2}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4e(2cd-be)}-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e^2(d+ex)^{5/2}(2cd-be)}$$

$$\Big\downarrow \; 1135$$

$$\frac{(-4beg+5cdg+3cef)\left(\frac{c\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2(2cd-be)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)}{4e(2cd-be)}-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e^2(d+ex)^{5/2}(2cd-be)}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(-4beg+5cdg+3cef)\left(\frac{ce\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)}{4e(2cd-be)}-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e^2(d+ex)^{5/2}(2cd-be)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e(d+ex)^{3/2}(2cd-be)}\right)(-4beg+5cdg+3cef)}{4e(2cd-be)}-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e^2(d+ex)^{5/2}(2cd-be)}$$

input

```
Int[(f + g*x)/((d + e*x)^(5/2)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),
x]
```

output

```
-1/2*((e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(e^2*(2*c*d -
 b*e)*(d + e*x)^(5/2)) + ((3*c*e*f + 5*c*d*g - 4*b*e*g)*(-(Sqrt[d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2]/(e*(2*c*d - b*e)*(d + e*x)^(3/2))) - (c*ArcTanh
[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x
])])/(e*(2*c*d - b*e)^(3/2))))/(4*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1135

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(211) = 422$.

Time = 1.56 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.67

method	result
default	$-\frac{\sqrt{-(ex+d)(cex+be-cd)}\,(4\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bce^3gx^2-5\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c^2de^2gx^2-3\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c^2e}{}$

input
```
int((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
-1/4*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(4*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*
e-2*c*d)^(1/2))*b*c*e^3*g*x^2-5*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^
(1/2))*c^2*d*e^2*g*x^2-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*
c^2*e^3*f*x^2+8*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c*d*e^2
*g*x-10*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^2*e*g*x-6*a
rctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d*e^2*f*x+4*arctan((-c
*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c*d^2*e*g-5*arctan((-c*e*x-b*e+c*
d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^3*g-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e
-2*c*d)^(1/2))*c^2*d^2*e*f-4*b*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^
(1/2)+5*c*d*e*g*x*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+3*c*e^2*f*x*(-c
*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)-2*b*d*e*g*(-c*e*x-b*e+c*d)^(1/2)*(b*
e-2*c*d)^(1/2)-2*b*e^2*f*(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+c*d^2*g*
(-c*e*x-b*e+c*d)^(1/2)*(b*e-2*c*d)^(1/2)+7*c*d*e*f*(-c*e*x-b*e+c*d)^(1/2)*
(b*e-2*c*d)^(1/2))/(e*x+d)^(5/2)/(b*e-2*c*d)^(5/2)/e^2/(-c*e*x-b*e+c*d)^(1
/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(211) = 422$.

Time = 0.13 (sec) , antiderivative size = 1170, normalized size of antiderivative = 5.02

$$\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="fricas")
```

output
```
[-1/8*((3*c^2*d^3*e*f + (3*c^2*e^4*f + (5*c^2*d*e^3 - 4*b*c*e^4)*g)*x^3 +
3*(3*c^2*d*e^3*f + (5*c^2*d^2*e^2 - 4*b*c*d*e^3)*g)*x^2 + (5*c^2*d^4 - 4*b
*c*d^3*e)*g + 3*(3*c^2*d^2*e^2*f + (5*c^2*d^3*e - 4*b*c*d^2*e^2)*g)*x)*sqr
t(2*c*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x -
 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x +
 d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*
d*e)*((14*c^2*d^2*e - 11*b*c*d*e^2 + 2*b^2*e^3)*f + (2*c^2*d^3 - 5*b*c*d^2
*e + 2*b^2*d*e^2)*g + (3*(2*c^2*d*e^2 - b*c*e^3)*f + (10*c^2*d^2*e - 13*b*
c*d*e^2 + 4*b^2*e^3)*g)*x)*sqrt(e*x + d))/(8*c^3*d^6*e^2 - 12*b*c^2*d^5*e^
3 + 6*b^2*c*d^4*e^4 - b^3*d^3*e^5 + (8*c^3*d^3*e^5 - 12*b*c^2*d^2*e^6 + 6*
b^2*c*d*e^7 - b^3*e^8)*x^3 + 3*(8*c^3*d^4*e^4 - 12*b*c^2*d^3*e^5 + 6*b^2*c
*d^2*e^6 - b^3*d*e^7)*x^2 + 3*(8*c^3*d^5*e^3 - 12*b*c^2*d^4*e^4 + 6*b^2*c*
d^3*e^5 - b^3*d^2*e^6)*x), -1/4*((3*c^2*d^3*e*f + (3*c^2*e^4*f + (5*c^2*d*
e^3 - 4*b*c*e^4)*g)*x^3 + 3*(3*c^2*d*e^3*f + (5*c^2*d^2*e^2 - 4*b*c*d*e^3)
*g)*x^2 + (5*c^2*d^4 - 4*b*c*d^3*e)*g + 3*(3*c^2*d^2*e^2*f + (5*c^2*d^3*e
- 4*b*c*d^2*e^2)*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqrt(-c*e^2*x^2 - b*e^2*
x + c*d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e + (2*
c*d*e - b*e^2)*x)) + sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((14*c^2*d
^2*e - 11*b*c*d*e^2 + 2*b^2*e^3)*f + (2*c^2*d^3 - 5*b*c*d^2*e + 2*b^2*d*e^
2)*g + (3*(2*c^2*d*e^2 - b*c*e^3)*f + (10*c^2*d^2*e - 13*b*c*d*e^2 + 4*...
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{f+gx}{\sqrt{-(d+ex)\,(be-cd+cex)}\,(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**(5/2)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/
2),x)
```

output
```
Integral((f + g*x)/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)**(5/2))
, x)
```

Maxima [F]

$$\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{gx+f}{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((g*x + f)/(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(e*x + d)^
(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.53

$$\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{\frac{(3\,c^3ef+5\,c^3dg-4\,bc^2eg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{(4\,c^2d^2-4\,bcde+b^2e^2)\sqrt{-2\,cd+be}} - \frac{10\,\sqrt{-(ex+d)c+2\,c}}{}}{}$$

input
```
integrate((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,
algorithm="giac")
```

output
```
1/4*((3*c^3*e*f + 5*c^3*d*g - 4*b*c^2*e*g)*arctan(sqrt(-(e*x + d)*c + 2*c*
d - b*e)/sqrt(-2*c*d + b*e))/((4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*sqrt(-2*c*
d + b*e)) - (10*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4*d*e*f - 5*sqrt(-(e*x
+ d)*c + 2*c*d - b*e)*b*c^3*e^2*f + 6*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^4
*d^2*g - 11*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^3*d*e*g + 4*sqrt(-(e*x +
d)*c + 2*c*d - b*e)*b^2*c^2*e^2*g - 3*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c
^3*e*f - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^3*d*g + 4*(-(e*x + d)*c +
2*c*d - b*e)^(3/2)*b*c^2*e*g)/((4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*(e*x + d)
^2*c^2))/(c*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{f+g\,x}{{\left(d+e\,x\right)}^{5/2}\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}} \,d x$$

input
```
int((f + g*x)/((d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)
),x)
```

output
```
int((f + g*x)/((d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 828, normalized size of antiderivative = 3.55

$$\int \frac{f+gx}{(d+ex)^{5/2}\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\frac{-4\sqrt{be-2cd}\,\mathit{atan}\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right) bc\,d^2eg-8\sqrt{be-2cd}\,\mathit{ata}}{}$$

input
```
int((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
( - 4*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))
*b*c*d**2*e*g - 8*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b
*e - 2*c*d))*b*c*d*e**2*g*x - 4*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d -
 c*e*x)/sqrt(b*e - 2*c*d))*b*c*e**3*g*x**2 + 5*sqrt(b*e - 2*c*d)*atan(sqrt
( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**3*g + 3*sqrt(b*e - 2*c*d
)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*e*f + 10*sq
rt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d*
*2*e*g*x + 6*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e -
2*c*d))*c**2*d*e**2*f*x + 5*sqrt(b*e - 2*c*d)*atan(sqrt( - b*e + c*d - c*e
*x)/sqrt(b*e - 2*c*d))*c**2*d*e**2*g*x**2 + 3*sqrt(b*e - 2*c*d)*atan(sqrt(
 - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*e**3*f*x**2 + 2*sqrt( - b*e
+ c*d - c*e*x)*b**2*d*e**2*g + 2*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*f +
4*sqrt( - b*e + c*d - c*e*x)*b**2*e**3*g*x - 5*sqrt( - b*e + c*d - c*e*x)*
b*c*d**2*e*g - 11*sqrt( - b*e + c*d - c*e*x)*b*c*d*e**2*f - 13*sqrt( - b*e
 + c*d - c*e*x)*b*c*d*e**2*g*x - 3*sqrt( - b*e + c*d - c*e*x)*b*c*e**3*f*x
 + 2*sqrt( - b*e + c*d - c*e*x)*c**2*d**3*g + 14*sqrt( - b*e + c*d - c*e*x
)*c**2*d**2*e*f + 10*sqrt( - b*e + c*d - c*e*x)*c**2*d**2*e*g*x + 6*sqrt(
- b*e + c*d - c*e*x)*c**2*d*e**2*f*x)/(4*e**2*(b**3*d**2*e**3 + 2*b**3*d*e
**4*x + b**3*e**5*x**2 - 6*b**2*c*d**3*e**2 - 12*b**2*c*d**2*e**3*x - 6*b*
*2*c*d*e**4*x**2 + 12*b*c**2*d**4*e + 24*b*c**2*d**3*e**2*x + 12*b*c**2...
```

3.232 $\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 337

$$\begin{aligned}
\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx &= \frac{2(2cd-be)^3(cef+cdg-beg)\sqrt{d+ex}}{c^5e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} \\
&+ \frac{2(2cd-be)^2(3cef+5cdg-4beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^5e^2\sqrt{d+ex}} \\
&- \frac{2(2cd-be)(cef+3cdg-2beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{c^5e^2(d+ex)^{3/2}} \\
&+ \frac{2(cef+7cdg-4beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^5e^2(d+ex)^{5/2}} \\
&- \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^5e^2(d+ex)^{7/2}}
\end{aligned}$$

output

```
2*(-b*e+2*c*d)^3*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1/2)/c^5/e^2/(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(1/2)+2*(-b*e+2*c*d)^2*(-4*b*e*g+5*c*d*g+3*c*e*f)*(d*(-
b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^5/e^2/(e*x+d)^(1/2)-2*(-b*e+2*c*d)*(-2
*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^5/e^2/(e*x+
d)^(3/2)+2/5*(-4*b*e*g+7*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/
2)/c^5/e^2/(e*x+d)^(5/2)-2/7*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^5/
e^2/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{d+ex}(-128b^4e^4g+16b^3ce^3(7ef+53dg-4egx)-8b^2c^2e^2(257d^2g+de(77f-45gx)-e^2x(7f+2g}{}$$

input
```
Integrate[((d + e*x)^(9/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(-128*b^4*e^4*g + 16*b^3*c*e^3*(7*e*f + 53*d*g - 4*e*g*x
) - 8*b^2*c^2*e^2*(257*d^2*g + d*e*(77*f - 45*g*x) - e^2*x*(7*f + 2*g*x))
- 2*b*c^3*e*(-1075*d^3*g + e^3*x^2*(7*f + 4*g*x) + d*e^2*x*(126*f + 37*g*x
) + d^2*e*(-553*f + 334*g*x)) + c^4*(-814*d^4*g + e^4*x^3*(7*f + 5*g*x) +
d*e^3*x^2*(49*f + 29*g*x) + d^2*e^2*x*(301*f + 93*g*x) + d^3*e*(-637*f + 4
07*g*x))))/(35*c^5*e^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1218, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1218}$$

$$\frac{2(d+ex)^{9/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-8beg+9cdg+7cef)\int \frac{(d+ex)^{7/2}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{ce(2cd-be)}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2(d+ex)^{9/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-8beg+9cdg+7cef)\left(\frac{6(2cd-be)\int \frac{(d+ex)^{5/2}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{7c}-\frac{2(d+ex)^{5/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{7ce}\right)}{ce(2cd-be)}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2(d+ex)^{9/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-8beg+9cdg+7cef)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\int \frac{(d+ex)^{3/2}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{5c}-\frac{2(d+ex)^{3/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5ce}\right)}{7c}-\frac{2(d+ex)^{5/2}\sqrt{d(c}{}\right.}{ce(2cd-be)}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2(d+ex)^{9/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-8beg+9cdg+7cef)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3c}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)}{5c}-\frac{2(d+ex)^{3/2}}{}\right.}{7c}\right.}{ce(2cd-be)}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(d+ex)^{9/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-$$

$$\frac{\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3c^2e\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)}{5c}-\frac{2(d+ex)^{3/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5ce}\right)}{7c}-2(d+e\right.}{ce(2cd-be)}$$

input
```
Int[((d + e*x)^(9/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2
),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(9/2))/(c*e^2*(2*c*d - b*e)*Sqrt[d*(c
*d - b*e) - b*e^2*x - c*e^2*x^2]) - ((7*c*e*f + 9*c*d*g - 8*b*e*g)*((-2*(d
 + e*x)^(5/2)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(7*c*e) + (6*(2*c
*d - b*e)*((-2*(d + e*x)^(3/2)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/
(5*c*e) + (4*(2*c*d - b*e)*((-4*(2*c*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2])/(3*c^2*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*Sqrt[d*(c*d - b*
e) - b*e^2*x - c*e^2*x^2])/(3*c*e)))/(5*c)))/(7*c)))/(c*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.07

method	result
default	$-2\sqrt{-(ex+d)(cex+be-cd)}\,(-5g\,e^4x^4c^4+8b\,c^3e^4g\,x^3-29c^4d\,e^3g\,x^3-7c^4e^4f\,x^3-16b^2c^2e^4g\,x^2+74b\,c^3d\,e^3g\,x^2+14b\,c^3e^4f\,x^2-93$
gosper	$-2(cex+be-cd)(-5g\,e^4x^4c^4+8b\,c^3e^4g\,x^3-29c^4d\,e^3g\,x^3-7c^4e^4f\,x^3-16b^2c^2e^4g\,x^2+74b\,c^3d\,e^3g\,x^2+14b\,c^3e^4f\,x^2-93c^4d^2e^2g\,x^2-$
orering	$-2(cex+be-cd)(-5g\,e^4x^4c^4+8b\,c^3e^4g\,x^3-29c^4d\,e^3g\,x^3-7c^4e^4f\,x^3-16b^2c^2e^4g\,x^2+74b\,c^3d\,e^3g\,x^2+14b\,c^3e^4f\,x^2-93c^4d^2e^2g\,x^2-$

input
```
int((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-2/35/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-5*c^4*e^4*g*x^4+8*b
*c^3*e^4*g*x^3-29*c^4*d*e^3*g*x^3-7*c^4*e^4*f*x^3-16*b^2*c^2*e^4*g*x^2+74*
b*c^3*d*e^3*g*x^2+14*b*c^3*e^4*f*x^2-93*c^4*d^2*e^2*g*x^2-49*c^4*d*e^3*f*x
^2+64*b^3*c*e^4*g*x-360*b^2*c^2*d*e^3*g*x-56*b^2*c^2*e^4*f*x+668*b*c^3*d^2
*e^2*g*x+252*b*c^3*d*e^3*f*x-407*c^4*d^3*e*g*x-301*c^4*d^2*e^2*f*x+128*b^4
*e^4*g-848*b^3*c*d*e^3*g-112*b^3*c*e^4*f+2056*b^2*c^2*d^2*e^2*g+616*b^2*c^
2*d*e^3*f-2150*b*c^3*d^3*e*g-1106*b*c^3*d^2*e^2*f+814*c^4*d^4*g+637*c^4*d^
3*e*f)/(c*e*x+b*e-c*d)/c^5/e^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{2\,(5\,c^4e^4gx^4+(7\,c^4e^4f+(29\,c^4de^3-8\,bc^3e^4)g)x^3+(7\,(7\,c^4de^3-2}{}$$

input
```
integrate((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output
```
2/35*(5*c^4*e^4*g*x^4 + (7*c^4*e^4*f + (29*c^4*d*e^3 - 8*b*c^3*e^4)*g)*x^3
 + (7*(7*c^4*d*e^3 - 2*b*c^3*e^4)*f + (93*c^4*d^2*e^2 - 74*b*c^3*d*e^3 + 1
6*b^2*c^2*e^4)*g)*x^2 - 7*(91*c^4*d^3*e - 158*b*c^3*d^2*e^2 + 88*b^2*c^2*d
*e^3 - 16*b^3*c*e^4)*f - 2*(407*c^4*d^4 - 1075*b*c^3*d^3*e + 1028*b^2*c^2*
d^2*e^2 - 424*b^3*c*d*e^3 + 64*b^4*e^4)*g + (7*(43*c^4*d^2*e^2 - 36*b*c^3*
d*e^3 + 8*b^2*c^2*e^4)*f + (407*c^4*d^3*e - 668*b*c^3*d^2*e^2 + 360*b^2*c^
2*d*e^3 - 64*b^3*c*e^4)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*s
qrt(e*x + d)/(c^6*e^4*x^2 + b*c^5*e^4*x - c^6*d^2*e^2 + b*c^5*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(9/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = -\frac{2\,(c^3e^3x^3-91\,c^3d^3+158\,bc^2d^2e-88\,b^2cde^2+16\,b^3e^3+(7\,c^3de^2-2\,bc^2e^3)x^2+(43\,c^3d^2e-36\,bc^2de^2+}{5\,\sqrt{-cex+cd-be}c^4e} -\frac{2\,(5\,c^4e^4x^4-814\,c^4d^4+2150\,bc^3d^3e-2056\,b^2c^2d^2e^2+848\,b^3cde^3-128\,b^4e^4+(29\,c^4de^3-8\,bc^3e^4)x^3+}{35\,\sqrt{-cex+cd-}}$$

input
```
integrate((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output
```
-2/5*(c^3*e^3*x^3 - 91*c^3*d^3 + 158*b*c^2*d^2*e - 88*b^2*c*d*e^2 + 16*b^3
*e^3 + (7*c^3*d*e^2 - 2*b*c^2*e^3)*x^2 + (43*c^3*d^2*e - 36*b*c^2*d*e^2 +
8*b^2*c*e^3)*x)*f/(sqrt(-c*e*x + c*d - b*e)*c^4*e) - 2/35*(5*c^4*e^4*x^4 -
 814*c^4*d^4 + 2150*b*c^3*d^3*e - 2056*b^2*c^2*d^2*e^2 + 848*b^3*c*d*e^3 -
 128*b^4*e^4 + (29*c^4*d*e^3 - 8*b*c^3*e^4)*x^3 + (93*c^4*d^2*e^2 - 74*b*c
^3*d*e^3 + 16*b^2*c^2*e^4)*x^2 + (407*c^4*d^3*e - 668*b*c^3*d^2*e^2 + 360*
b^2*c^2*d*e^3 - 64*b^3*c*e^4)*x)*g/(sqrt(-c*e*x + c*d - b*e)*c^5*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. 2(313) = 626.

Time = 0.36 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{2\left(\frac{35\,(8\,c^4d^3ef-12\,bc^3d^2e^2f+6\,b^2c^2de^3f-b^3ce^4f+8\,c^4d^4g-20\,bc^3d^3eg+18\,b^2c^2d^2e^2g-7}{\sqrt{-(ex+d)c+2\,cd-be}c^5e}\right.}{}$$

input
```
integrate((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output

```
2/35*(35*(8*c^4*d^3*e*f - 12*b*c^3*d^2*e^2*f + 6*b^2*c^2*d*e^3*f - b^3*c*e
^4*f + 8*c^4*d^4*g - 20*b*c^3*d^3*e*g + 18*b^2*c^2*d^2*e^2*g - 7*b^3*c*d*e
^3*g + b^4*e^4*g)/(sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^5*e) + (420*sqrt(-(e
*x + d)*c + 2*c*d - b*e)*c^33*d^2*e^7*f - 420*sqrt(-(e*x + d)*c + 2*c*d -
b*e)*b*c^32*d*e^8*f + 105*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^31*e^9*f
+ 700*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^33*d^3*e^6*g - 1260*sqrt(-(e*x +
d)*c + 2*c*d - b*e)*b*c^32*d^2*e^7*g + 735*sqrt(-(e*x + d)*c + 2*c*d - b*e
)*b^2*c^31*d*e^8*g - 140*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^30*e^9*g -
 70*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^32*d*e^7*f + 35*(-(e*x + d)*c + 2
*c*d - b*e)^(3/2)*b*c^31*e^8*f - 210*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^
32*d^2*e^6*g + 245*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^31*d*e^7*g - 70*
(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c^30*e^8*g + 7*((e*x + d)*c - 2*c*d
 + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^31*e^7*f + 49*((e*x + d)*c -
2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^31*d*e^6*g - 28*((e*x +
d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^30*e^7*g + 5*((
e*x + d)*c - 2*c*d + b*e)^3*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^30*e^6*g)/(
c^35*e^7))/e
```

Mupad [B] (verification not implemented)

Time = 11.35 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{2\,g\,x^4\,\sqrt{d+e\,x}}{7\,c^2}-\frac{\sqrt{d+e\,x}\,(256\,g\,b^4\,e^4-1696\,g\,b^3}{}\right.}{}$$

input

```
int(((f + g*x)*(d + e*x)^(9/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2
),x)
```

output
```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*g*x^4*(d + e*x)^(1/2))/(7
*c^2) - ((d + e*x)^(1/2)*(256*b^4*e^4*g + 1628*c^4*d^4*g - 224*b^3*c*e^4*f
 + 1274*c^4*d^3*e*f - 4300*b*c^3*d^3*e*g - 1696*b^3*c*d*e^3*g - 2212*b*c^3
*d^2*e^2*f + 1232*b^2*c^2*d*e^3*f + 4112*b^2*c^2*d^2*e^2*g))/(35*c^6*e^4)
+ (x^2*(d + e*x)^(1/2)*(32*b^2*c^2*e^4*g + 186*c^4*d^2*e^2*g - 28*b*c^3*e^
4*f + 98*c^4*d*e^3*f - 148*b*c^3*d*e^3*g))/(35*c^6*e^4) + (2*x^3*(d + e*x)
^(1/2)*(29*c*d*g - 8*b*e*g + 7*c*e*f))/(35*c^3*e) + (x*(d + e*x)^(1/2)*(11
2*b^2*c^2*e^4*f + 602*c^4*d^2*e^2*f - 128*b^3*c*e^4*g + 814*c^4*d^3*e*g -
504*b*c^3*d*e^3*f - 1336*b*c^3*d^2*e^2*g + 720*b^2*c^2*d*e^3*g))/(35*c^6*e
^4)))/(x^2 + (b*x)/c + (d*(b*e - c*d))/(c*e^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^{9/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{\frac{1628}{35}c^4d^4g - \frac{1696}{35}b^3cde^3g + \frac{4112}{35}b^2c^2d^2e^2g + \frac{176}{5}b^2c^2de^3f - \frac{16}{5}b^2c^2e^4f}{}$$

input
```
int((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
(2*(128*b**4*e**4*g - 848*b**3*c*d*e**3*g - 112*b**3*c*e**4*f + 64*b**3*c*
e**4*g*x + 2056*b**2*c**2*d**2*e**2*g + 616*b**2*c**2*d*e**3*f - 360*b**2*
c**2*d*e**3*g*x - 56*b**2*c**2*e**4*f*x - 16*b**2*c**2*e**4*g*x**2 - 2150*
b*c**3*d**3*e*g - 1106*b*c**3*d**2*e**2*f + 668*b*c**3*d**2*e**2*g*x + 252
*b*c**3*d*e**3*f*x + 74*b*c**3*d*e**3*g*x**2 + 14*b*c**3*e**4*f*x**2 + 8*b
*c**3*e**4*g*x**3 + 814*c**4*d**4*g + 637*c**4*d**3*e*f - 407*c**4*d**3*e*
g*x - 301*c**4*d**2*e**2*f*x - 93*c**4*d**2*e**2*g*x**2 - 49*c**4*d*e**3*f
*x**2 - 29*c**4*d*e**3*g*x**3 - 7*c**4*e**4*f*x**3 - 5*c**4*e**4*g*x**4))/
(35*sqrt( - b*e + c*d - c*e*x)*c**5*e**2)
```

3.233 $\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 263

$$\begin{aligned}
\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx &= \frac{2(2cd-be)^2(cef+cdg-beg)\sqrt{d+ex}}{c^4e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} \\
&+ \frac{2(2cd-be)(2cef+4cdg-3beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^4e^2\sqrt{d+ex}} \\
&- \frac{2(cef+5cdg-3beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^4e^2(d+ex)^{3/2}} \\
&+ \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^4e^2(d+ex)^{5/2}}
\end{aligned}$$

output

```
2*(-b*e+2*c*d)^2*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1/2)/c^4/e^2/(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(1/2)+2*(-b*e+2*c*d)*(-3*b*e*g+4*c*d*g+2*c*e*f)*(d*(-b*
e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^4/e^2/(e*x+d)^(1/2)-2/3*(-3*b*e*g+5*c*d*
g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^4/e^2/(e*x+d)^(3/2)+2/5*
g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^4/e^2/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{d+ex}(48b^3e^3g-8b^2ce^2(5ef+28dg-3egx)+2bc^2e(167d^2g+de(70f-44gx)-e^2x(10f+3gx))}{15c^4e^2\sqrt{(d+ex)(-be+c(d-}}$$

input
```
Integrate[((d + e*x)^(7/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(48*b^3*e^3*g - 8*b^2*c*e^2*(5*e*f + 28*d*g - 3*e*g*x) +
 2*b*c^2*e*(167*d^2*g + d*e*(70*f - 44*g*x) - e^2*x*(10*f + 3*g*x)) + c^3*
(-158*d^3*g + e^3*x^2*(5*f + 3*g*x) + 2*d*e^2*x*(25*f + 8*g*x) + d^2*e*(-1
15*f + 79*g*x))))/(15*c^4*e^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1218, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{2(d+ex)^{7/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-6beg+7cdg+5cef)\int \frac{(d+ex)^{5/2}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{ce(2cd-be)}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2(d+ex)^{7/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-6beg+7cdg+5cef)\left(\frac{4(2cd-be)\int\frac{(d+ex)^{3/2}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{5c}-\frac{2(d+ex)^{3/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5ce}\right)}{ce(2cd-be)}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2(d+ex)^{7/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-6beg+7cdg+5cef)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int\frac{\sqrt{d+ex}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3c}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)}{5c}-\frac{2(d+ex)^{3/2}\sqrt{d(cd-}}{5ce}\right.}{ce(2cd-be)}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2(d+ex)^{7/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\left(\frac{4(2cd-be)\left(-\frac{4(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3c^2e\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)}{5c}-\frac{2(d+ex)^{3/2}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{5ce}\right)(-6beg+7}{ce(2cd-be)}$$

input
```
Int[((d + e*x)^(7/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2
),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(7/2))/(c*e^2*(2*c*d - b*e)*Sqrt[d*(c
*d - b*e) - b*e^2*x - c*e^2*x^2]) - ((5*c*e*f + 7*c*d*g - 6*b*e*g)*((-2*(d
 + e*x)^(3/2)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(5*c*e) + (4*(2*c
*d - b*e)*((-4*(2*c*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3
*c^2*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*
e^2*x^2])/(3*c*e)))/(5*c)))/(c*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.87

method	result
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(3e^3g\,x^3c^3-6b\,c^2e^3g\,x^2+16c^3d\,e^2g\,x^2+5c^3e^3f\,x^2+24b^2c\,e^3gx-88b\,c^2d\,e^2gx-20b\,c^2e^3fx+79c^3d^2egx+}{15\sqrt{ex+d}\,(cex+be-cd)c^4e^2}$
gosper	$\frac{2(cex+be-cd)(3e^3g\,x^3c^3-6b\,c^2e^3g\,x^2+16c^3d\,e^2g\,x^2+5c^3e^3f\,x^2+24b^2c\,e^3gx-88b\,c^2d\,e^2gx-20b\,c^2e^3fx+79c^3d^2egx+50c^3d\,e^2fx+}{15c^4e^2(-x^2c\,e^2-xb\,e^2-bde+c\,d^2)^{\frac{3}{2}}}$
orering	$\frac{2(cex+be-cd)(3e^3g\,x^3c^3-6b\,c^2e^3g\,x^2+16c^3d\,e^2g\,x^2+5c^3e^3f\,x^2+24b^2c\,e^3gx-88b\,c^2d\,e^2gx-20b\,c^2e^3fx+79c^3d^2egx+50c^3d\,e^2fx+}{15c^4e^2(-x^2c\,e^2-xb\,e^2-bde+c\,d^2)^{\frac{3}{2}}}$

input
```
int((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2/15/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(3*c^3*e^3*g*x^3-6*b*c
^2*e^3*g*x^2+16*c^3*d*e^2*g*x^2+5*c^3*e^3*f*x^2+24*b^2*c*e^3*g*x-88*b*c^2*
d*e^2*g*x-20*b*c^2*e^3*f*x+79*c^3*d^2*e*g*x+50*c^3*d*e^2*f*x+48*b^3*e^3*g-
224*b^2*c*d*e^2*g-40*b^2*c*e^3*f+334*b*c^2*d^2*e*g+140*b*c^2*d*e^2*f-158*c
^3*d^3*g-115*c^3*d^2*e*f)/(c*e*x+b*e-c*d)/c^4/e^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{2\,(3\,c^3e^3gx^3+(5\,c^3e^3f+2\,(8\,c^3de^2-3\,bc^2e^3)g)x^2-5\,(23\,c^3d^2e-2}{}$$

input
```
integrate((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output
```
2/15*(3*c^3*e^3*g*x^3 + (5*c^3*e^3*f + 2*(8*c^3*d*e^2 - 3*b*c^2*e^3)*g)*x^
2 - 5*(23*c^3*d^2*e - 28*b*c^2*d*e^2 + 8*b^2*c*e^3)*f - 2*(79*c^3*d^3 - 16
7*b*c^2*d^2*e + 112*b^2*c*d*e^2 - 24*b^3*e^3)*g + (10*(5*c^3*d*e^2 - 2*b*c
^2*e^3)*f + (79*c^3*d^2*e - 88*b*c^2*d*e^2 + 24*b^2*c*e^3)*g)*x)*sqrt(-c*e
^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^5*e^4*x^2 + b*c^4*e^4*x
 - c^5*d^2*e^2 + b*c^4*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = -\frac{2\,(c^2e^2x^2-23\,c^2d^2+28\,bcde-8\,b^2e^2+2\,(5\,c^2de-2\,bce^2)x)f}{3\,\sqrt{-cex+cd-be}c^3e} - \frac{2\,(3\,c^3e^3x^3-158\,c^3d^3+334\,bc^2d^2e-224\,b^2cde^2+48\,b^3e^3+2\,(8\,c^3de^2-3\,bc^2e^3)x^2+(79\,c^3d^2e-88\,bc^2}{15\,\sqrt{-cex+cd-be}c^4e^2}$$

input
```
integrate((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output
```
-2/3*(c^2*e^2*x^2 - 23*c^2*d^2 + 28*b*c*d*e - 8*b^2*e^2 + 2*(5*c^2*d*e - 2
*b*c*e^2)*x)*f/(sqrt(-c*e*x + c*d - b*e)*c^3*e) - 2/15*(3*c^3*e^3*x^3 - 15
8*c^3*d^3 + 334*b*c^2*d^2*e - 224*b^2*c*d*e^2 + 48*b^3*e^3 + 2*(8*c^3*d*e^
2 - 3*b*c^2*e^3)*x^2 + (79*c^3*d^2*e - 88*b*c^2*d*e^2 + 24*b^2*c*e^3)*x)*g
/(sqrt(-c*e*x + c*d - b*e)*c^4*e^2)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{2\left(\frac{15\,(4\,c^3d^2ef-4\,bc^2de^2f+b^2ce^3f+4\,c^3d^3g-8\,bc^2d^2eg+5\,b^2cde^2g-b^3e^3g)}{\sqrt{-(ex+d)c+2\,cd-be}c^4e}+\frac{60\,\sqrt{-(ex}{}\right.}{}$$

input
```
integrate((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output
```
2/15*(15*(4*c^3*d^2*e*f - 4*b*c^2*d*e^2*f + b^2*c*e^3*f + 4*c^3*d^3*g - 8*
b*c^2*d^2*e*g + 5*b^2*c*d*e^2*g - b^3*e^3*g)/(sqrt(-(e*x + d)*c + 2*c*d -
b*e)*c^4*e) + (60*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^18*d*e^5*f - 30*sqrt(
-(e*x + d)*c + 2*c*d - b*e)*b*c^17*e^6*f + 120*sqrt(-(e*x + d)*c + 2*c*d -
 b*e)*c^18*d^2*e^4*g - 150*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^17*d*e^5*g
 + 45*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^16*e^6*g - 5*(-(e*x + d)*c +
2*c*d - b*e)^(3/2)*c^17*e^5*f - 25*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^17
*d*e^4*g + 15*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^16*e^5*g + 3*((e*x +
d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^16*e^4*g)/(c^20*e
^5))/e
```

Mupad [B] (verification not implemented)

Time = 11.05 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{2\,x^2\,\sqrt{d+e\,x}\,\left(16\,c\,d\,g-6\,b\,e\,g+5\,c\,e\,f\right)}{15\,c^3\,e^2}-\frac{\sqrt{d+e\,x}}{}\right.}{}$$

input
```
int(((f + g*x)*(d + e*x)^(7/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2
),x)
```

output
```
((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*((2*x^2*(d + e*x)^(1/2)*(16*c
*d*g - 6*b*e*g + 5*c*e*f))/(15*c^3*e^2) - ((d + e*x)^(1/2)*(316*c^3*d^3*g
- 96*b^3*e^3*g + 80*b^2*c*e^3*f + 230*c^3*d^2*e*f - 280*b*c^2*d*e^2*f - 66
8*b*c^2*d^2*e*g + 448*b^2*c*d*e^2*g))/(15*c^5*e^4) + (2*g*x^3*(d + e*x)^(1
/2))/(5*c^2*e) + (x*(d + e*x)^(1/2)*(48*b^2*c*e^3*g - 40*b*c^2*e^3*f + 100
*c^3*d*e^2*f + 158*c^3*d^2*e*g - 176*b*c^2*d*e^2*g))/(15*c^5*e^4)))/(x^2 +
 (b*x)/c + (d*(b*e - c*d))/(c*e^2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{-\frac{2}{5}c^3e^3g\,x^3+\frac{4}{5}b\,c^2e^3g\,x^2-\frac{32}{15}c^3d\,e^2g\,x^2-\frac{2}{3}c^3e^3f\,x^2-\frac{16}{5}b^2c\,e^3gx+}{}$$

input
```
int((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
(2*( - 48*b**3*e**3*g + 224*b**2*c*d*e**2*g + 40*b**2*c*e**3*f - 24*b**2*c
*e**3*g*x - 334*b*c**2*d**2*e*g - 140*b*c**2*d*e**2*f + 88*b*c**2*d*e**2*g
*x + 20*b*c**2*e**3*f*x + 6*b*c**2*e**3*g*x**2 + 158*c**3*d**3*g + 115*c**
3*d**2*e*f - 79*c**3*d**2*e*g*x - 50*c**3*d*e**2*f*x - 16*c**3*d*e**2*g*x*
*2 - 5*c**3*e**3*f*x**2 - 3*c**3*e**3*g*x**3))/(15*sqrt( - b*e + c*d - c*e
*x)*c**4*e**2)
```

3.234 $\int \frac{(d+ex)^{5/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 186

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{2(2cd-be)(cef+cdg-beg)\sqrt{d+ex}}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} + \frac{2(cef+3cdg-2beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^3e^2\sqrt{d+ex}} - \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^3e^2(d+ex)^{3/2}}$$

output
```
2*(-b*e+2*c*d)*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1/2)/c^3/e^2/(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(1/2)+2*(-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*
e^2*x^2)^(1/2)/c^3/e^2/(e*x+d)^(1/2)-2/3*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(3/2)/c^3/e^2/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(-8b^2e^2g+2bce(3ef+11dg-2egx)+c^2(-14d^2g+e^2x(3f+gx)+de(-9f+7gx)))}{3c^3e^2\sqrt{(d+ex)(-be+c(d-ex))}}$$

input

```
Integrate[((d + e*x)^(5/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
)^(3/2),x]
```

output

```
(-2*Sqrt[d + e*x]*(-8*b^2*e^2*g + 2*b*c*e*(3*e*f + 11*d*g - 2*e*g*x) + c^2
*(-14*d^2*g + e^2*x*(3*f + g*x) + d*e*(-9*f + 7*g*x))))/(3*c^3*e^2*Sqrt[(d
 + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1218, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{2(d+ex)^{5/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-4beg+5cdg+3cef)\int \frac{(d+ex)^{3/2}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{ce(2cd-be)}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2(d+ex)^{5/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-4beg+5cdg+3cef)\left(\frac{2(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3c}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)}{ce(2cd-be)}$$

$$\Big\downarrow \textcolor{blue}{1122}$$

$$\frac{2(d+ex)^{5/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{\left(-\frac{4(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3c^2e\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce}\right)(-4beg+5cdg+3cef)}{ce(2cd-be)}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2
),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(5/2))/(c*e^2*(2*c*d - b*e)*Sqrt[d*(c
*d - b*e) - b*e^2*x - c*e^2*x^2]) - ((3*c*e*f + 5*c*d*g - 4*b*e*g)*((-4*(2
*c*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*c^2*e*Sqrt[d + e
*x]) - (2*Sqrt[d + e*x]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*c*e)
))/(c*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,\left(-g\,x^{2}c^{2}e^{2}+4bc\,e^{2}gx-7c^{2}degx-3c^{2}e^{2}fx+8b^{2}e^{2}g-22bcdeg-6bc\,e^{2}f+14c^{2}d^{2}g+9c^{2}def\right)}{3\sqrt{ex+d}\,(cex+be-cd)c^{3}e^{2}}$	133
gosper	$-\frac{2(cex+be-cd)\left(-g\,x^{2}c^{2}e^{2}+4bc\,e^{2}gx-7c^{2}degx-3c^{2}e^{2}fx+8b^{2}e^{2}g-22bcdeg-6bc\,e^{2}f+14c^{2}d^{2}g+9c^{2}def\right)(ex+d)^{\frac{3}{2}}}{3c^{3}e^{2}\left(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}\right)^{\frac{3}{2}}}$	139
orering	$-\frac{2(cex+be-cd)\left(-g\,x^{2}c^{2}e^{2}+4bc\,e^{2}gx-7c^{2}degx-3c^{2}e^{2}fx+8b^{2}e^{2}g-22bcdeg-6bc\,e^{2}f+14c^{2}d^{2}g+9c^{2}def\right)(ex+d)^{\frac{3}{2}}}{3c^{3}e^{2}\left(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}\right)^{\frac{3}{2}}}$	139

input
```
int((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-c^2*e^2*g*x^2+4*b*c*
e^2*g*x-7*c^2*d*e*g*x-3*c^2*e^2*f*x+8*b^2*e^2*g-22*b*c*d*e*g-6*b*c*e^2*f+1
4*c^2*d^2*g+9*c^2*d*e*f)/(c*e*x+b*e-c*d)/c^3/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{2\,(c^2e^2gx^2-3\,(3\,c^2de-2\,bce^2)f-2\,(7\,c^2d^2-11\,bcde+4\,b^2e^2)g+}{3\,(c^4e^4x^2+bc^3e^4x}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output
```
2/3*(c^2*e^2*g*x^2 - 3*(3*c^2*d*e - 2*b*c*e^2)*f - 2*(7*c^2*d^2 - 11*b*c*d
*e + 4*b^2*e^2)*g + (3*c^2*e^2*f + (7*c^2*d*e - 4*b*c*e^2)*g)*x)*sqrt(-c*e
^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^4*e^4*x^2 + b*c^3*e^4*x
 - c^4*d^2*e^2 + b*c^3*d*e^3)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{(d+ex)^{\frac{5}{2}}\,(f+gx)}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output
```
Integral((d + e*x)**(5/2)*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(3/2
), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=-\frac{2\,(cex-3\,cd+2\,be)f}{\sqrt{-cex+cd-be}c^2e}-\frac{2\,(c^2e^2x^2-14\,c^2d^2+22\,bcde-8\,b^2e^2+(7\,c^2de-4\,bce^2)x)g}{3\,\sqrt{-cex+cd-be}c^3e^2}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output
```
-2*(c*e*x - 3*c*d + 2*b*e)*f/(sqrt(-c*e*x + c*d - b*e)*c^2*e) - 2/3*(c^2*e
^2*x^2 - 14*c^2*d^2 + 22*b*c*d*e - 8*b^2*e^2 + (7*c^2*d*e - 4*b*c*e^2)*x)*
g/(sqrt(-c*e*x + c*d - b*e)*c^3*e^2)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{2\Bigg(\frac{3\left(2c^2def-bce^2f+2c^2d^2g-3bcdeg+b^2e^2g\right)}{\sqrt{-(ex+d)c+2cd-be}c^3e}+\frac{3\sqrt{-(ex+d)c+2cd-be}c^7e^3f+9\sqrt{-(e}{}}{3e}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output
```
2/3*(3*(2*c^2*d*e*f - b*c*e^2*f + 2*c^2*d^2*g - 3*b*c*d*e*g + b^2*e^2*g)/(
sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^3*e) + (3*sqrt(-(e*x + d)*c + 2*c*d - b
*e)*c^7*e^3*f + 9*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^7*d*e^2*g - 6*sqrt(-(
e*x + d)*c + 2*c*d - b*e)*b*c^6*e^3*g - (-(e*x + d)*c + 2*c*d - b*e)^(3/2)
*c^6*e^2*g)/(c^9*e^3))/e
```

Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{\Bigg(\frac{2gx^2\sqrt{d+ex}}{3c^2e^2}-\frac{\sqrt{d+ex}\left(16gb^2e^2-44gbcde-12fbce^2+28gc^2d^2+18fc^2de\right)}{3c^4e^4}+2}{x^2+\frac{bx}{c}+\frac{d(be-}{ce}}$$

input
```
int(((f + g*x)*(d + e*x)^(5/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2
),x)
```

output
```
(((2*g*x^2*(d + e*x)^(1/2))/(3*c^2*e^2) - ((d + e*x)^(1/2)*(16*b^2*e^2*g +
 28*c^2*d^2*g - 12*b*c*e^2*f + 18*c^2*d*e*f - 44*b*c*d*e*g))/(3*c^4*e^4) +
 (2*x*(d + e*x)^(1/2)*(7*c*d*g - 4*b*e*g + 3*c*e*f))/(3*c^3*e^3))*(c*d^2 -
 c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(x^2 + (b*x)/c + (d*(b*e - c*d))/(c*e
^2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{-\frac{2}{3}c^2e^2g\,x^2+\frac{8}{3}bc\,e^2gx-\frac{14}{3}c^2degx-2c^2e^2fx+\frac{16}{3}b^2e^2g-\frac{44}{3}bcdeg}{\sqrt{-cex-be+cd}\,c^3e^2}$$

input

```
int((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(2*(8*b**2*e**2*g - 22*b*c*d*e*g - 6*b*c*e**2*f + 4*b*c*e**2*g*x + 14*c**2
*d**2*g + 9*c**2*d*e*f - 7*c**2*d*e*g*x - 3*c**2*e**2*f*x - c**2*e**2*g*x*
*2))/(3*sqrt( - b*e + c*d - c*e*x)*c**3*e**2)
```

3.235 $\int \frac{(d+ex)^{3/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 112

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{2(cef+cdg-beg)\sqrt{d+ex}}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} + \frac{2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^2e^2\sqrt{d+ex}}$$

output

```
2*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1/2)/c^2/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(1/2)+2*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^2/e^2/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{2\sqrt{d+ex}(2cdg-2beg+ce(f-gx))}{c^2e^2\sqrt{(d+ex)(-be+c(d-ex))}}$$

input

```
Integrate[((d + e*x)^(3/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
)^(3/2),x]
```

output
```
(2*Sqrt[d + e*x]*(2*c*d*g - 2*b*e*g + c*e*(f - g*x)))/(c^2*e^2*Sqrt[(d + e
*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1218, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{2(d+ex)^{3/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(-2beg+3cdg+cef)\int \frac{\sqrt{d+ex}}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{ce(2cd-be)}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2\sqrt{d(cd-be)-be^2x-ce^2x^2}(-2beg+3cdg+cef)}{c^2e^2\sqrt{d+ex}(2cd-be)}+\frac{2(d+ex)^{3/2}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input
```
Int[((d + e*x)^(3/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2
),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(3/2))/(c*e^2*(2*c*d - b*e)*Sqrt[d*(c
*d - b*e) - b*e^2*x - c*e^2*x^2]) + (2*(c*e*f + 3*c*d*g - 2*b*e*g)*Sqrt[d*
(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(c^2*e^2*(2*c*d - b*e)*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(cegx+2beg-2cdg-fce)}{\sqrt{ex+d}\,(cex+be-cd)c^2e^2}$	72
gosper	$\frac{2(cex+be-cd)(cegx+2beg-2cdg-fce)(ex+d)^{\frac{3}{2}}}{c^2e^2(-x^2c\,e^2-xb\,e^2-bde+c\,d^2)^{\frac{3}{2}}}$	78
orering	$\frac{2(cex+be-cd)(cegx+2beg-2cdg-fce)(ex+d)^{\frac{3}{2}}}{c^2e^2(-x^2c\,e^2-xb\,e^2-bde+c\,d^2)^{\frac{3}{2}}}$	78

input
```
int((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(c*e*g*x+2*b*e*g-2*c*d*g-
c*e*f)/(c*e*x+b*e-c*d)/c^2/e^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(cegx-cef-2\,(cd-be)g)\sqrt{ex+d}}{c^3e^4x^2+bc^2e^4x-c^3d^2e^2+bc^2de^3}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output

```
2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(c*e*g*x - c*e*f - 2*(c*d - b
*e)*g)*sqrt(e*x + d)/(c^3*e^4*x^2 + b*c^2*e^4*x - c^3*d^2*e^2 + b*c^2*d*e^
3)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)**(3/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output

```
Integral((d + e*x)**(3/2)*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(3/2
), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{2\,f}{\sqrt{-cex+cd-be}ce}-\frac{2\,(cex-2\,cd+2\,be)g}{\sqrt{-cex+cd-be}c^2e^2}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output
```
2*f/(sqrt(-c*e*x + c*d - b*e)*c*e) - 2*(c*e*x - 2*c*d + 2*b*e)*g/(sqrt(-c*
e*x + c*d - b*e)*c^2*e^2)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{2\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}g}{c^2e}+\frac{cef+cdg-beg}{\sqrt{-(ex+d)c+2\,cd-be}c^2e}\right)}{e}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output
```
2*(sqrt(-(e*x + d)*c + 2*c*d - b*e)*g/(c^2*e) + (c*e*f + c*d*g - b*e*g)/(s
qrt(-(e*x + d)*c + 2*c*d - b*e)*c^2*e))/e
```

Mupad [B] (verification not implemented)

Time = 10.87 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=-\frac{\left(\frac{\sqrt{d+e\,x}\,\left(4\,c\,d\,g-4\,b\,e\,g+2\,c\,e\,f\right)}{c^3\,e^4}-\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^2\,e^3}\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}{x^2+\frac{b\,x}{c}+\frac{d\,\left(b\,e-c\,d\right)}{c\,e^2}}$$

input
```
int(((f + g*x)*(d + e*x)^(3/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2
),x)
```

output
```
-((((d + e*x)^(1/2)*(4*c*d*g - 4*b*e*g + 2*c*e*f))/(c^3*e^4) - (2*g*x*(d +
 e*x)^(1/2))/(c^2*e^3))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(x^2
+ (b*x)/c + (d*(b*e - c*d))/(c*e^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{-2cegx-4beg+4cdg+2cef}{\sqrt{-cex-be+cd}\,c^2e^2}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
(2*( - 2*b*e*g + 2*c*d*g + c*e*f - c*e*g*x))/(sqrt( - b*e + c*d - c*e*x)*c
**2*e**2)
```

3.236 $\int \frac{\sqrt{d+ex}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 155

$$\int \frac{\sqrt{d+ex}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{2(cef+cdg-beg)\sqrt{d+ex}}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}} - \frac{2(ef-dg)\operatorname{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e^2(2cd-be)^{3/2}}$$

output
```
2*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1/2)/c/e^2/(-b*e+2*c*d)/(d*(-b*e+c*d)-b*e^
2*x-c*e^2*x^2)^(1/2)-2*(-d*g+e*f)*arctanh((-b*e+2*c*d)^(1/2)*(e*x+d)^(1/2)
/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2/(-b*e+2*c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{d+ex}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\left(\sqrt{-2cd+be}(cef+cdg-beg)+c(ef-dg)\sqrt{-be+c(d-ex)}\arctan\left(\frac{\sqrt{cd-be-cex}}{\sqrt{-2cd+be}}\right)\right)}{ce^2(-2cd+be)^{3/2}\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^
(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(Sqrt[-2*c*d + b*e]*(c*e*f + c*d*g - b*e*g) + c*(e*f - d
*g)*Sqrt[-(b*e) + c*(d - e*x)]*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d
+ b*e]]))/(c*e^2*(-2*c*d + b*e)^(3/2)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x)
)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1218, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(ef-dg)\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{e(2cd-be)}+\frac{2\sqrt{d+ex}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{2(ef-dg)\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}+\frac{2\sqrt{d+ex}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{d+ex}(-beg+cdg+cef)}{ce^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2(ef-dg)\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e^2(2cd-be)^{3/2}}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),
x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*Sqrt[d + e*x])/(c*e^2*(2*c*d - b*e)*Sqrt[d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2]) - (2*(e*f - d*g)*ArcTanh[Sqrt[d*(c*d - b*e
) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/(e^2*(2*c*d -
 b*e)^(3/2))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28

method	result
default	$-\frac{2\left(\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)cdg\sqrt{-cex-be+cd}-\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)cef\sqrt{-cex-be+cd}+\sqrt{be-2cd}\,beg-\sqrt{be-2cd}\,cdg-\sqrt{be-2}\right.}{(be-2cd)^{\frac{3}{2}}\,c\,e^{2}(cex+be-cd)\sqrt{ex+d}}$

input
```
int((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-2*(arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c*d*g*(-c*e*x-b*e+c*d
)^(1/2)-arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c*e*f*(-c*e*x-b*e
+c*d)^(1/2)+(b*e-2*c*d)^(1/2)*b*e*g-(b*e-2*c*d)^(1/2)*c*d*g-(b*e-2*c*d)^(1
/2)*c*e*f)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/(b*e-2*c*d)^(3/2)/c/e^2/(c*e*x
+b*e-c*d)/(e*x+d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(143) = 286.

Time = 0.10 (sec) , antiderivative size = 778, normalized size of antiderivative = 5.02

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \left[-\frac{((c^2e^3f-c^2de^2g)x^2-(c^2d^2e-bcde^2)f+(c^2d^3-bcd^2e)g+(bc}{}\right.$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output

```
[-(((c^2*e^3*f - c^2*d*e^2*g)*x^2 - (c^2*d^2*e - b*c*d*e^2)*f + (c^2*d^3 -
 b*c*d^2*e)*g + (b*c*e^3*f - b*c*d*e^2*g)*x)*sqrt(2*c*d - b*e)*log(-(c*e^2
*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x + 2*sqrt(-c*e^2*x^2 - b*e^2
*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x +
d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((2*c^2*d*e - b*c*e^2
)*f + (2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*g)*sqrt(e*x + d))/(4*c^4*d^4*e^2 -
 8*b*c^3*d^3*e^3 + 5*b^2*c^2*d^2*e^4 - b^3*c*d*e^5 - (4*c^4*d^2*e^4 - 4*b*
c^3*d*e^5 + b^2*c^2*e^6)*x^2 - (4*b*c^3*d^2*e^4 - 4*b^2*c^2*d*e^5 + b^3*c*
e^6)*x), 2*(((c^2*e^3*f - c^2*d*e^2*g)*x^2 - (c^2*d^2*e - b*c*d*e^2)*f + (
c^2*d^3 - b*c*d^2*e)*g + (b*c*e^3*f - b*c*d*e^2*g)*x)*sqrt(-2*c*d + b*e)*a
rctan(-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(
e*x + d)/(2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)) + sqrt(-c*e^2*x^2 - b*e^
2*x + c*d^2 - b*d*e)*((2*c^2*d*e - b*c*e^2)*f + (2*c^2*d^2 - 3*b*c*d*e + b
^2*e^2)*g)*sqrt(e*x + d))/(4*c^4*d^4*e^2 - 8*b*c^3*d^3*e^3 + 5*b^2*c^2*d^2
*e^4 - b^3*c*d*e^5 - (4*c^4*d^2*e^4 - 4*b*c^3*d*e^5 + b^2*c^2*e^6)*x^2 - (
4*b*c^3*d^2*e^4 - 4*b^2*c^2*d*e^5 + b^3*c*e^6)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{\sqrt{d+ex}(f+gx)}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)**(1/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output

```
Integral(sqrt(d + e*x)*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(3/2),
x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}(gx+f)}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(
3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{2\left(\frac{(ef-dg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{(2\,cde-be^2)\sqrt{-2\,cd+be}}+\frac{cef+cdg-beg}{(2\,c^2de-bce^2)\sqrt{-(ex+d)c+2\,cd-be}}\right)}{e}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output
```
2*((e*f - d*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(-2*c*d + b*e))
/((2*c*d*e - b*e^2)*sqrt(-2*c*d + b*e)) + (c*e*f + c*d*g - b*e*g)/((2*c^2*
d*e - b*c*e^2)*sqrt(-(e*x + d)*c + 2*c*d - b*e)))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \int \frac{(f+g\,x)\,\sqrt{d+e\,x}}{(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)^{3/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^(1/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2
),x)
```

output
```
int(((f + g*x)*(d + e*x)^(1/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{2\sqrt{be-2cd}\,\sqrt{-cex-be+cd}\,\mathrm{atan}\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)cdg-2\sqrt{be-2cd}}{\sqrt{-cex-b}}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
(2*(sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d -
c*e*x)/sqrt(b*e - 2*c*d))*c*d*g - sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*
e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c*e*f + b**2*e**2*
g - 3*b*c*d*e*g - b*c*e**2*f + 2*c**2*d**2*g + 2*c**2*d*e*f))/(sqrt( - b*e
 + c*d - c*e*x)*c*e**2*(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))
```

3.237 $\int \frac{f+gx}{\sqrt{d+ex}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 223

$$\begin{aligned}\int \frac{f+gx}{\sqrt{d+ex}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx &= \frac{2(cef+cdg-beg)\sqrt{d+ex}}{e^2(2cd-be)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}\\ &- \frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(2cd-be)^2(d+ex)^{3/2}}\\ &- \frac{(3cef+cdg-2beg)\operatorname{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e^2(2cd-be)^{5/2}}\end{aligned}$$

output

```
2*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1/2)/e^2/(-b*e+2*c*d)^2/(d*(-b*e+c*d)-b*e^
2*x-c*e^2*x^2)^(1/2)-(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2
/(-b*e+2*c*d)^2/(e*x+d)^(3/2)-(-2*b*e*g+c*d*g+3*c*e*f)*arctanh((-b*e+2*c*d
)^(1/2)*(e*x+d)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2/(-b*e+2*
c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.77

$$\int \frac{f+gx}{\sqrt{d+ex}\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{\sqrt{-2cd+be}(be(-3dg+e(f-2gx))+c(3d^2g+3e^2fx+}{e^2(-2cd+be)^{5/}}$$

input

```
Integrate[(f + g*x)/(Sqrt[d + e*x]*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(
3/2)),x]
```

output

```
(Sqrt[-2*c*d + b*e]*(b*e*(-3*d*g + e*(f - 2*g*x)) + c*(3*d^2*g + 3*e^2*f*x
 + d*e*(f + g*x))) + (3*c*e*f + c*d*g - 2*b*e*g)*(d + e*x)*Sqrt[c*d - b*e
- c*e*x]*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/(e^2*(-2*c*d
+ b*e)^(5/2)*Sqrt[d + e*x]*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1220, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\sqrt{d+ex}\,(-bde-be^2x+cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \, 1220$$

$$\frac{(-2beg+cdg+3cef)\int \frac{\sqrt{d+ex}}{(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{2e(2cd-be)}-\frac{ef-dg}{e^2\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \, 1132$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2e(2cd-be)}-\frac{ef-dg}{e^2\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow 1136$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{2e\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2e(2cd-be)}$$

$$\frac{ef-dg}{e^2\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow 221$$

$$\frac{\left(\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}\right)(-2beg+cdg+3cef)}{2e(2cd-be)}-\frac{ef-dg}{e^2\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input

```
Int[(f + g*x)/(Sqrt[d + e*x]*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2)),
x]
```

output

```
-((e*f - d*g)/(e^2*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[d*(c*d - b*e) - b*e^2*
x - c*e^2*x^2])) + ((3*c*e*f + c*d*g - 2*b*e*g)*((2*Sqrt[d + e*x])/(e*(2*c
*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) - (2*ArcTanh[Sqrt[d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/(e*(
2*c*d - b*e)^(3/2))))/(2*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1132

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(207) = 414$.

Time = 1.74 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.11

method	result
default	$\sqrt{-(ex+d)(cex+be-cd)}\,\left(2\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)\sqrt{-cex-be+cd}\,b\,e^2gx-\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)\sqrt{-cex-be+cd}\,cdegx-3\arctan\right.$

input

```
int((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output

```
1/(e*x+d)^(3/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(2*arctan((-c*e*x-b*e+c*d
)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*b*e^2*g*x-arctan((-c*e*x
-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c*d*e*g*x-3*arct
an((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c*e^2*
f*x+2*(b*e-2*c*d)^(1/2)*b*e^2*g*x-(b*e-2*c*d)^(1/2)*c*d*e*g*x-3*(b*e-2*c*d
)^(1/2)*c*e^2*f*x+2*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e
*x-b*e+c*d)^(1/2)*b*d*e*g-arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))
*(-c*e*x-b*e+c*d)^(1/2)*c*d^2*g-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d
)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c*d*e*f+3*(b*e-2*c*d)^(1/2)*b*d*e*g-(b*e-2
*c*d)^(1/2)*b*e^2*f-3*(b*e-2*c*d)^(1/2)*c*d^2*g-(b*e-2*c*d)^(1/2)*c*d*e*f)
/(c*e*x+b*e-c*d)/e^2/(b*e-2*c*d)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(207) = 414$.

Time = 0.17 (sec) , antiderivative size = 1353, normalized size of antiderivative = 6.07

$$\int \frac{f + gx}{\sqrt{d + ex}\,(cd^2 - bde - be^2x - ce^2x^2)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output

```
[1/2*(((3*c^2*e^4*f + (c^2*d*e^3 - 2*b*c*e^4)*g)*x^3 + (3*(c^2*d*e^3 + b*c
*e^4)*f + (c^2*d^2*e^2 - b*c*d*e^3 - 2*b^2*e^4)*g)*x^2 - 3*(c^2*d^3*e - b*
c*d^2*e^2)*f - (c^2*d^4 - 3*b*c*d^3*e + 2*b^2*d^2*e^2)*g - (3*(c^2*d^2*e^2
 - 2*b*c*d*e^3)*f + (c^2*d^3*e - 4*b*c*d^2*e^2 + 4*b^2*d*e^3)*g)*x)*sqrt(2
*c*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*
sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d)
)/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e
)*((2*c^2*d^2*e + b*c*d*e^2 - b^2*e^3)*f + 3*(2*c^2*d^3 - 3*b*c*d^2*e + b^
2*d*e^2)*g + (3*(2*c^2*d*e^2 - b*c*e^3)*f + (2*c^2*d^2*e - 5*b*c*d*e^2 + 2
*b^2*e^3)*g)*x)*sqrt(e*x + d))/(8*c^4*d^6*e^2 - 20*b*c^3*d^5*e^3 + 18*b^2*
c^2*d^4*e^4 - 7*b^3*c*d^3*e^5 + b^4*d^2*e^6 - (8*c^4*d^3*e^5 - 12*b*c^3*d^
2*e^6 + 6*b^2*c^2*d*e^7 - b^3*c*e^8)*x^3 - (8*c^4*d^4*e^4 - 4*b*c^3*d^3*e^
5 - 6*b^2*c^2*d^2*e^6 + 5*b^3*c*d*e^7 - b^4*e^8)*x^2 + (8*c^4*d^5*e^3 - 28
*b*c^3*d^4*e^4 + 30*b^2*c^2*d^3*e^5 - 13*b^3*c*d^2*e^6 + 2*b^4*d*e^7)*x),
(((3*c^2*e^4*f + (c^2*d*e^3 - 2*b*c*e^4)*g)*x^3 + (3*(c^2*d*e^3 + b*c*e^4)
*f + (c^2*d^2*e^2 - b*c*d*e^3 - 2*b^2*e^4)*g)*x^2 - 3*(c^2*d^3*e - b*c*d^2
*e^2)*f - (c^2*d^4 - 3*b*c*d^3*e + 2*b^2*d^2*e^2)*g - (3*(c^2*d^2*e^2 - 2*
b*c*d*e^3)*f + (c^2*d^3*e - 4*b*c*d^2*e^2 + 4*b^2*d*e^3)*g)*x)*sqrt(-2*c*d
 + b*e)*arctan(-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(-2*c*d + b
*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)) + sqrt(-c*e^...
```

Sympy [F]

$$\int \frac{f+gx}{\sqrt{d+ex}\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{f+gx}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}\sqrt{d+ex}}\,dx$$

input

```
integrate((g*x+f)/(e*x+d)**(1/2)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output

```
Integral((f + g*x)/((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*sqrt(d + e*x))
, x)
```

Maxima [F]

$$\int \frac{f+gx}{\sqrt{d+ex}\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \int \frac{gx+f}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output
```
integrate((g*x + f)/((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*sqrt(e*x
 + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.29

$$\int \frac{f+gx}{\sqrt{d+ex}\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{\frac{(3\,cef+cdg-2\,beg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-2\,cd+be}} + \frac{4\,c^2def-2\,bce^2f+4\,c^2d^2g-}{(4\,c^2d^2e-4\,bcde^2-}}{}$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output
```
((3*c*e*f + c*d*g - 2*b*e*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(
-2*c*d + b*e))/((4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-2*c*d + b*e))
+ (4*c^2*d*e*f - 2*b*c*e^2*f + 4*c^2*d^2*g - 6*b*c*d*e*g + 2*b^2*e^2*g + 3
*((e*x + d)*c - 2*c*d + b*e)*c*e*f + ((e*x + d)*c - 2*c*d + b*e)*c*d*g - 2
*((e*x + d)*c - 2*c*d + b*e)*b*e*g)/((4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)
*(2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c*d - sqrt(-(e*x + d)*c + 2*c*d - b*e
)*b*e - (-(e*x + d)*c + 2*c*d - b*e)^(3/2))))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\sqrt{d+ex}\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \int \frac{f+g\,x}{\sqrt{d+e\,x}\,(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)^{3/2}}\,dx$$

input
```
int((f + g*x)/((d + e*x)^(1/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2)
),x)
```

output
```
int((f + g*x)/((d + e*x)^(1/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.47

$$\int \frac{f+gx}{\sqrt{d+ex}\,(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{-2\sqrt{be-2cd}\,\sqrt{-cex-be+cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bdeg -}{}$$

input
```
int((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
( - 2*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d
- c*e*x)/sqrt(b*e - 2*c*d))*b*d*e*g - 2*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*
d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*e**2*g*x +
 sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e
*x)/sqrt(b*e - 2*c*d))*c*d**2*g + 3*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d -
c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c*d*e*f + sqrt(b
*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqr
t(b*e - 2*c*d))*c*d*e*g*x + 3*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)
*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c*e**2*f*x - 3*b**2*d*
e**2*g + b**2*e**3*f - 2*b**2*e**3*g*x + 9*b*c*d**2*e*g - b*c*d*e**2*f + 5
*b*c*d*e**2*g*x + 3*b*c*e**3*f*x - 6*c**2*d**3*g - 2*c**2*d**2*e*f - 2*c**
2*d**2*e*g*x - 6*c**2*d*e**2*f*x)/(sqrt( - b*e + c*d - c*e*x)*e**2*(b**3*d
*e**3 + b**3*e**4*x - 6*b**2*c*d**2*e**2 - 6*b**2*c*d*e**3*x + 12*b*c**2*d
**3*e + 12*b*c**2*d**2*e**2*x - 8*c**3*d**4 - 8*c**3*d**3*e*x))
```

3.238 $\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 303

$$\begin{aligned}\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx &= \frac{2c(cef+cdg-beg)\sqrt{d+ex}}{e^2(2cd-be)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}} \\ &- \frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e^2(2cd-be)^2(d+ex)^{5/2}} \\ &- \frac{(7cef+cdg-4beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4e^2(2cd-be)^3(d+ex)^{3/2}} \\ &- \frac{3c(5cef+3cdg-4beg)\operatorname{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4e^2(2cd-be)^{7/2}}\end{aligned}$$

output
```
2*c*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1/2)/e^2/(-b*e+2*c*d)^3/(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(1/2)-1/2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/
2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^(5/2)-1/4*(-4*b*e*g+c*d*g+7*c*e*f)*(d*(-b*e+
c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^(3/2)-3/4*c*(-4*b
*e*g+3*c*d*g+5*c*e*f)*arctanh((-b*e+2*c*d)^(1/2)*(e*x+d)^(1/2)/(d*(-b*e+c*
d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2/(-b*e+2*c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.84

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{c(d+ex)^{3/2}\left(\frac{(-be+c(d-ex))(bce(-9d^2g+e^2x(5f-12gx)+13de(f-gx}{}\right.}{}$$

input
```
Integrate[(f + g*x)/((d + e*x)^(3/2)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)
^(3/2)),x]
```

output
```
(c*(d + e*x)^(3/2)*(((-(b*e) + c*(d - e*x))*(b*c*e*(-9*d^2*g + e^2*x*(5*f
- 12*g*x) + 13*d*e*(f - g*x)) - 2*b^2*e^2*(d*g + e*(f + 2*g*x)) + c^2*(11*
d^3*g + 15*e^3*f*x^2 - 3*d^2*e*(f - 4*g*x) + d*e^2*x*(20*f + 9*g*x))))/(c*
(2*c*d - b*e)^3*(d + e*x)^2) - (3*(5*c*e*f + 3*c*d*g - 4*b*e*g)*(-(b*e) +
c*(d - e*x))^(3/2)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/(-2
*c*d + b*e)^(7/2)))/(4*e^2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {1220, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(-4beg+3cdg+5cef)\int \frac{1}{\sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{4e(2cd-be)}-\frac{ef-dg}{2e^2(d+ex)^{3/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \; 1135$$

$$\frac{(-4beg+3cdg+5cef)\left(\frac{3c\int \frac{\sqrt{d+ex}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4e(2cd-be)}-\frac{ef-dg}{2e^2(d+ex)^{3/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{1132}$$

$$\frac{(-4beg+3cdg+5cef)\left(\frac{3c\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)}}\right.}{4e(2cd-be)}$$

$$\frac{ef-dg}{2e^2(d+ex)^{3/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(-4beg+3cdg+5cef)\left(\frac{3c\left(\frac{2e\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2(2cd-be)}\right.}{4e(2cd-be)}$$

$$\frac{ef-dg}{2e^2(d+ex)^{3/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(\frac{3c\left(\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-4beg}{4e(2cd-be)}$$

$$\frac{ef-dg}{2e^2(d+ex)^{3/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input
```
Int[(f + g*x)/((d + e*x)^(3/2)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2)
),x]
```

output
```
-1/2*(e*f - d*g)/(e^2*(2*c*d - b*e)*(d + e*x)^(3/2)*Sqrt[d*(c*d - b*e) - b
*e^2*x - c*e^2*x^2]) + ((5*c*e*f + 3*c*d*g - 4*b*e*g)*(-(1/(e*(2*c*d - b*e
)*Sqrt[d + e*x]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])) + (3*c*((2*Sqr
t[d + e*x])/(e*(2*c*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) -
(2*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sq
rt[d + e*x])])/(e*(2*c*d - b*e)^(3/2))))/(2*(2*c*d - b*e))))/(4*e*(2*c*d -
 b*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(277) = 554$.

Time = 1.63 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.69

method	result
default	$-\frac{\sqrt{-(ex+d)(cex+be-cd)}\left(12\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)\sqrt{-cex-be+cd}\,bc\,e^{3}g\,x^{2}-9\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)\sqrt{-cex-be+cd}\,c^{2}d\,e^{2}g}{}$

input
```
int((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-1/4/(e*x+d)^(5/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(12*arctan((-c*e*x-b*e
+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*b*c*e^3*g*x^2-9*arct
an((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c^2*d*
e^2*g*x^2-15*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+
c*d)^(1/2)*c^2*e^3*f*x^2+12*(b*e-2*c*d)^(1/2)*b*c*e^3*g*x^2-9*(b*e-2*c*d)^
(1/2)*c^2*d*e^2*g*x^2-15*(b*e-2*c*d)^(1/2)*c^2*e^3*f*x^2+24*arctan((-c*e*x
-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*b*c*d*e^2*g*x-18
*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c
^2*d^2*e*g*x-30*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b
*e+c*d)^(1/2)*c^2*d*e^2*f*x+4*(b*e-2*c*d)^(1/2)*b^2*e^3*g*x+13*(b*e-2*c*d)
^(1/2)*b*c*d*e^2*g*x-5*(b*e-2*c*d)^(1/2)*b*c*e^3*f*x-12*(b*e-2*c*d)^(1/2)*
c^2*d^2*e*g*x-20*(b*e-2*c*d)^(1/2)*c^2*d*e^2*f*x+12*arctan((-c*e*x-b*e+c*d
)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*b*c*d^2*e*g-9*arctan((-c
*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c^2*d^3*g-15
*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c
^2*d^2*e*f+2*(b*e-2*c*d)^(1/2)*b^2*d*e^2*g+2*(b*e-2*c*d)^(1/2)*b^2*e^3*f+9
*(b*e-2*c*d)^(1/2)*b*c*d^2*e*g-13*(b*e-2*c*d)^(1/2)*b*c*d*e^2*f-11*(b*e-2*
c*d)^(1/2)*c^2*d^3*g+3*(b*e-2*c*d)^(1/2)*c^2*d^2*e*f)/(c*e*x+b*e-c*d)/e^2/
(b*e-2*c*d)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(277) = 554$.

Time = 0.32 (sec) , antiderivative size = 1978, normalized size of antiderivative = 6.53

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output

```
[-1/8*(3*((5*c^3*e^5*f + (3*c^3*d*e^4 - 4*b*c^2*e^5)*g)*x^4 + (5*(2*c^3*d*
e^4 + b*c^2*e^5)*f + (6*c^3*d^2*e^3 - 5*b*c^2*d*e^4 - 4*b^2*c*e^5)*g)*x^3
+ 3*(5*b*c^2*d*e^4*f + (3*b*c^2*d^2*e^3 - 4*b^2*c*d*e^4)*g)*x^2 - 5*(c^3*d
^4*e - b*c^2*d^3*e^2)*f - (3*c^3*d^5 - 7*b*c^2*d^4*e + 4*b^2*c*d^3*e^2)*g
- (5*(2*c^3*d^3*e^2 - 3*b*c^2*d^2*e^3)*f + (6*c^3*d^4*e - 17*b*c^2*d^3*e^2
 + 12*b^2*c*d^2*e^3)*g)*x)*sqrt(2*c*d - b*e)*log(-(c*e^2*x^2 - 3*c*d^2 + 2
*b*d*e - 2*(c*d*e - b*e^2)*x + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e
)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*sqrt(-c*
e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(3*(5*(2*c^3*d*e^3 - b*c^2*e^4)*f + (6*
c^3*d^2*e^2 - 11*b*c^2*d*e^3 + 4*b^2*c*e^4)*g)*x^2 - (6*c^3*d^3*e - 29*b*c
^2*d^2*e^2 + 17*b^2*c*d*e^3 - 2*b^3*e^4)*f + (22*c^3*d^4 - 29*b*c^2*d^3*e
+ 5*b^2*c*d^2*e^2 + 2*b^3*d*e^3)*g + (5*(8*c^3*d^2*e^2 - 2*b*c^2*d*e^3 - b
^2*c*e^4)*f + (24*c^3*d^3*e - 38*b*c^2*d^2*e^2 + 5*b^2*c*d*e^3 + 4*b^3*e^4
)*g)*x)*sqrt(e*x + d))/(16*c^5*d^8*e^2 - 48*b*c^4*d^7*e^3 + 56*b^2*c^3*d^6
*e^4 - 32*b^3*c^2*d^5*e^5 + 9*b^4*c*d^4*e^6 - b^5*d^3*e^7 - (16*c^5*d^4*e^
6 - 32*b*c^4*d^3*e^7 + 24*b^2*c^3*d^2*e^8 - 8*b^3*c^2*d*e^9 + b^4*c*e^10)*
x^4 - (32*c^5*d^5*e^5 - 48*b*c^4*d^4*e^6 + 16*b^2*c^3*d^3*e^7 + 8*b^3*c^2*
d^2*e^8 - 6*b^4*c*d*e^9 + b^5*e^10)*x^3 - 3*(16*b*c^4*d^5*e^5 - 32*b^2*c^3
*d^4*e^6 + 24*b^3*c^2*d^3*e^7 - 8*b^4*c*d^2*e^8 + b^5*d*e^9)*x^2 + (32*c^5
*d^7*e^3 - 112*b*c^4*d^6*e^4 + 144*b^2*c^3*d^5*e^5 - 88*b^3*c^2*d^4*e^6...
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\int \frac{f+gx}{\left(-(d+ex)\left(be-cd+cex\right)\right)^{\frac{3}{2}}(d+ex)^{\frac{3}{2}}}\,dx$$

input

```
integrate((g*x+f)/(e*x+d)**(3/2)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output

```
Integral((f + g*x)/((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(d + e*x)**(3/
2)), x)
```

Maxima [F]

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\int \frac{gx+f}{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output
```
integrate((g*x + f)/((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(e*x + d
)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.54

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\frac{\frac{3\,(5\,c^2ef+3\,c^2dg-4\,bceg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{(8\,c^3d^3e-12\,bc^2d^2e^2+6\,b^2cde^3-b^3e^4)\sqrt{-2\,cd+be}}+\frac{}{(8\,c^3d^3e-12\,bc^2}}{}$$

input
```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output
```
1/4*(3*(5*c^2*e*f + 3*c^2*d*g - 4*b*c*e*g)*arctan(sqrt(-(e*x + d)*c + 2*c*
d - b*e)/sqrt(-2*c*d + b*e))/((8*c^3*d^3*e - 12*b*c^2*d^2*e^2 + 6*b^2*c*d*
e^3 - b^3*e^4)*sqrt(-2*c*d + b*e)) + 8*(c^2*e*f + c^2*d*g - b*c*e*g)/((8*c
^3*d^3*e - 12*b*c^2*d^2*e^2 + 6*b^2*c*d*e^3 - b^3*e^4)*sqrt(-(e*x + d)*c +
 2*c*d - b*e)) - (18*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^3*d*e*f - 9*sqrt(-
(e*x + d)*c + 2*c*d - b*e)*b*c^2*e^2*f - 2*sqrt(-(e*x + d)*c + 2*c*d - b*e
)*c^3*d^2*g - 7*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^2*d*e*g + 4*sqrt(-(e*
x + d)*c + 2*c*d - b*e)*b^2*c*e^2*g - 7*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)
*c^2*e*f - (-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*d*g + 4*(-(e*x + d)*c +
2*c*d - b*e)^(3/2)*b*c*e*g)/((8*c^3*d^3*e - 12*b*c^2*d^2*e^2 + 6*b^2*c*d*e
^3 - b^3*e^4)*(e*x + d)^2*c^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\int \frac{f+g\,x}{{\left(d+e\,x\right)}^{3/2}\,{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)}^{3/2}} \,d x$$

input
```
int((f + g*x)/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2)
),x)
```

output
```
int((f + g*x)/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 992, normalized size of antiderivative = 3.27

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
(12*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d -
c*e*x)/sqrt(b*e - 2*c*d))*b*c*d**2*e*g + 24*sqrt(b*e - 2*c*d)*sqrt( - b*e
+ c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*d*e*
*2*g*x + 12*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e
+ c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*e**3*g*x**2 - 9*sqrt(b*e - 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*c**2*d**3*g - 15*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt(
 - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*e*f - 18*sqrt(b*e - 2*c
*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e -
2*c*d))*c**2*d**2*e*g*x - 30*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d*e**2*f*x - 9*sqr
t(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/
sqrt(b*e - 2*c*d))*c**2*d*e**2*g*x**2 - 15*sqrt(b*e - 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*e**3
*f*x**2 + 2*b**3*d*e**3*g + 2*b**3*e**4*f + 4*b**3*e**4*g*x + 5*b**2*c*d**
2*e**2*g - 17*b**2*c*d*e**3*f + 5*b**2*c*d*e**3*g*x - 5*b**2*c*e**4*f*x +
12*b**2*c*e**4*g*x**2 - 29*b*c**2*d**3*e*g + 29*b*c**2*d**2*e**2*f - 38*b*
c**2*d**2*e**2*g*x - 10*b*c**2*d*e**3*f*x - 33*b*c**2*d*e**3*g*x**2 - 15*b
*c**2*e**4*f*x**2 + 22*c**3*d**4*g - 6*c**3*d**3*e*f + 24*c**3*d**3*e*g*x
+ 40*c**3*d**2*e**2*f*x + 18*c**3*d**2*e**2*g*x**2 + 30*c**3*d*e**3*f*x...
```

3.239 $\int \frac{f+gx}{(d+ex)^{5/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 382

$$
\begin{aligned}
\int \frac{f+gx}{(d+ex)^{5/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx &= \frac{2c^2(cef+cdg-beg)\sqrt{d+ex}}{e^2(2cd-be)^4\sqrt{d(cd-be)-be^2x-ce^2x^2}} \\
&- \frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e^2(2cd-be)^2(d+ex)^{7/2}} \\
&- \frac{(11cef+cdg-6beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{12e^2(2cd-be)^3(d+ex)^{5/2}} \\
&- \frac{c(19cef+9cdg-14beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{8e^2(2cd-be)^4(d+ex)^{3/2}} \\
&- \frac{5c^2(7cef+5cdg-6beg)\operatorname{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8e^2(2cd-be)^{9/2}}
\end{aligned}
$$

output

```
2*c^2*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1/2)/e^2/(-b*e+2*c*d)^4/(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(1/2)-1/3*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
1/2)/e^2/(-b*e+2*c*d)^2/(e*x+d)^(7/2)-1/12*(-6*b*e*g+c*d*g+11*c*e*f)*(d*(-
b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^(5/2)-1/8*c*(
-14*b*e*g+9*c*d*g+19*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b
*e+2*c*d)^4/(e*x+d)^(3/2)-5/8*c^2*(-6*b*e*g+5*c*d*g+7*c*e*f)*arctanh((-b*e
+2*c*d)^(1/2)*(e*x+d)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2/(-
b*e+2*c*d)^(9/2)
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.89

$$\int \frac{f+gx}{(d+ex)^{5/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \frac{c^2(d+ex)^{3/2}\left(\frac{(-be+c(d-ex))\left(4b^3e^3(2ef+dg+3egx)+bc^2e\left(-13d^3g+\right.\right.}{}\right.}{}$$

input
```
Integrate[(f + g*x)/((d + e*x)^(5/2)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)
^(3/2)),x]
```

output
```
(c^2*(d + e*x)^(3/2)*(((-(b*e) + c*(d - e*x))*(4*b^3*e^3*(2*e*f + d*g + 3*
e*g*x) + b*c^2*e*(-13*d^3*g + d*e^2*x*(126*f - 185*g*x) + 5*e^3*x^2*(7*f -
 18*g*x) + d^2*e*(187*f - 12*g*x)) + c^3*(49*d^4*g + 105*e^4*f*x^3 - 85*d^
3*e*(f - g*x) + 5*d*e^3*x^2*(49*f + 15*g*x) + 7*d^2*e^2*x*(17*f + 25*g*x))
 - 2*b^2*c*e^2*(20*d^2*g + e^2*x*(7*f + 15*g*x) + d*e*(31*f + 59*g*x))))/(
c^2*(-2*c*d + b*e)^4*(d + e*x)^3) + (15*(7*c*e*f + 5*c*d*g - 6*b*e*g)*(-(b
*e) + c*(d - e*x))^(3/2)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]
])/(-2*c*d + b*e)^(9/2)))/(24*e^2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)
)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1220, 1135, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^{5/2}\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(-6beg+5cdg+7cef)\int \frac{1}{(d+ex)^{3/2}(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{6e(2cd-be)}-\frac{ef-dg}{3e^2(d+ex)^{5/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(-6beg+5cdg+7cef)\left(\frac{5c\int \frac{1}{\sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{4(2cd-be)}-\frac{1}{2e(d+ex)^{3/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{6e(2cd-be)}-\frac{ef-dg}{3e^2(d+ex)^{5/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(-6beg+5cdg+7cef)\left(\frac{5c\left(\frac{3c\int \frac{\sqrt{d+ex}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4(2cd-be)}-\frac{1}{2e(d+ex)^{3/2}(2cd-be)\sqrt{}}\right.}{6e(2cd-be)}$$

$$\frac{ef-dg}{3e^2(d+ex)^{5/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{ 1132}$$

$$\frac{(-6beg+5cdg+7cef)\left(\frac{5c\left(\frac{3c\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-b}}\right.}{4(2cd-be)}\right.}{6e(2cd-be)}$$

$$\frac{ef-dg}{3e^2(d+ex)^{5/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{(-6beg+5cdg+7cef)\left(\frac{5c\left(\frac{3c\left(\frac{2e\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2}}\right.}{2(2cd-be)}\right.}{4(2cd-be)}\right.}{6e(2cd-be)}$$

$$\frac{ef-dg}{3e^2(d+ex)^{5/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{5c\left(\frac{3c\left(\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4(2cd-be)}-\frac{}{2e(d+ex)^{3/}}\right.}{6e(2cd-be)}$$

$$\frac{ef-dg}{3e^2(d+ex)^{5/2}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input
```
Int[(f + g*x)/((d + e*x)^(5/2)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2)
),x]
```

output
```
-1/3*(e*f - d*g)/(e^2*(2*c*d - b*e)*(d + e*x)^(5/2)*Sqrt[d*(c*d - b*e) - b
*e^2*x - c*e^2*x^2]) + ((7*c*e*f + 5*c*d*g - 6*b*e*g)*(-1/2*1/(e*(2*c*d -
b*e)*(d + e*x)^(3/2)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) + (5*c*(-(
1/(e*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]
)) + (3*c*((2*Sqrt[d + e*x])/(e*(2*c*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2]) - (2*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqr
t[2*c*d - b*e]*Sqrt[d + e*x])])/(e*(2*c*d - b*e)^(3/2))))/(2*(2*c*d - b*e)
)))/(4*(2*c*d - b*e))))/(6*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. $2(350) = 700$.

Time = 1.66 (sec) , antiderivative size = 1216, normalized size of antiderivative = 3.18

method	result	size
default	Expression too large to display	1216

input
```
int((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
1/24/(e*x+d)^(7/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-126*(b*e-2*c*d)^(1/2
)*b*c^2*d*e^3*f*x+270*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)
/(b*e-2*c*d)^(1/2))*b*c^2*d^2*e^2*g*x-105*(-c*e*x-b*e+c*d)^(1/2)*arctan((-
c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*e^4*f*x^3+90*(b*e-2*c*d)^(1/2)
*b*c^2*e^4*g*x^3-75*(b*e-2*c*d)^(1/2)*c^3*d*e^3*g*x^3+30*(b*e-2*c*d)^(1/2)
*b^2*c*e^4*g*x^2-35*(b*e-2*c*d)^(1/2)*b*c^2*e^4*f*x^2-175*(b*e-2*c*d)^(1/2
)*c^3*d^2*e^2*g*x^2+270*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/
2)/(b*e-2*c*d)^(1/2))*b*c^2*d*e^3*g*x^2-105*(b*e-2*c*d)^(1/2)*c^3*e^4*f*x^
3-75*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2
))*c^3*d^4*g-12*(b*e-2*c*d)^(1/2)*b^3*e^4*g*x-4*(b*e-2*c*d)^(1/2)*b^3*d*e^
3*g+85*(b*e-2*c*d)^(1/2)*c^3*d^3*e*f+90*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*
e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d^3*e*g+118*(b*e-2*c*d)^(1/2)*
b^2*c*d*e^3*g*x+12*(b*e-2*c*d)^(1/2)*b*c^2*d^2*e^2*g*x+90*(-c*e*x-b*e+c*d)
^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*e^4*g*x^3-75
*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c
^3*d*e^3*g*x^3-225*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)/(b
*e-2*c*d)^(1/2))*c^3*d^2*e^2*g*x^2-315*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e
*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d*e^3*f*x^2-225*(-c*e*x-b*e+c*d)^
(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*d^3*e*g*x-315*(
-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(350) = 700$.

Time = 0.67 (sec) , antiderivative size = 2834, normalized size of antiderivative = 7.42

$$\int \frac{f+gx}{(d+ex)^{5/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="fricas")
```

output
```
[1/48*(15*((7*c^4*e^6*f + (5*c^4*d*e^5 - 6*b*c^3*e^6)*g)*x^5 + (7*(3*c^4*d
*e^5 + b*c^3*e^6)*f + (15*c^4*d^2*e^4 - 13*b*c^3*d*e^5 - 6*b^2*c^2*e^6)*g)
*x^4 + 2*(7*(c^4*d^2*e^4 + 2*b*c^3*d*e^5)*f + (5*c^4*d^3*e^3 + 4*b*c^3*d^2
*e^4 - 12*b^2*c^2*d*e^5)*g)*x^3 - 2*(7*(c^4*d^3*e^3 - 3*b*c^3*d^2*e^4)*f +
 (5*c^4*d^4*e^2 - 21*b*c^3*d^3*e^3 + 18*b^2*c^2*d^2*e^4)*g)*x^2 - 7*(c^4*d
^5*e - b*c^3*d^4*e^2)*f - (5*c^4*d^6 - 11*b*c^3*d^5*e + 6*b^2*c^2*d^4*e^2)
*g - (7*(3*c^4*d^4*e^2 - 4*b*c^3*d^3*e^3)*f + (15*c^4*d^5*e - 38*b*c^3*d^4
*e^2 + 24*b^2*c^2*d^3*e^3)*g)*x)*sqrt(2*c*d - b*e)*log(-(c*e^2*x^2 - 3*c*d
^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 -
 b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sq
rt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(15*(7*(2*c^4*d*e^4 - b*c^3*e^5)*
f + (10*c^4*d^2*e^3 - 17*b*c^3*d*e^4 + 6*b^2*c^2*e^5)*g)*x^3 + 5*(7*(14*c^
4*d^2*e^3 - 5*b*c^3*d*e^4 - b^2*c^2*e^5)*f + (70*c^4*d^3*e^2 - 109*b*c^3*d
^2*e^3 + 25*b^2*c^2*d*e^4 + 6*b^3*c*e^5)*g)*x^2 - (170*c^4*d^4*e - 459*b*c
^3*d^3*e^2 + 311*b^2*c^2*d^2*e^3 - 78*b^3*c*d*e^4 + 8*b^4*e^5)*f + (98*c^4
*d^5 - 75*b*c^3*d^4*e - 67*b^2*c^2*d^3*e^2 + 48*b^3*c*d^2*e^3 - 4*b^4*d*e^
4)*g + (7*(34*c^4*d^3*e^2 + 19*b*c^3*d^2*e^3 - 22*b^2*c^2*d*e^4 + 2*b^3*c*
e^5)*f + (170*c^4*d^4*e - 109*b*c^3*d^3*e^2 - 224*b^2*c^2*d^2*e^3 + 142*b^
3*c*d*e^4 - 12*b^4*e^5)*g)*x)*sqrt(e*x + d))/(32*c^6*d^10*e^2 - 112*b*c^5*
d^9*e^3 + 160*b^2*c^4*d^8*e^4 - 120*b^3*c^3*d^7*e^5 + 50*b^4*c^2*d^6*e^...
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^{5/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\int \frac{f+gx}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**(5/2)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/
2),x)
```

output
```
Integral((f + g*x)/((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(d + e*x)**(5/
2)), x)
```

Maxima [F]

$$\int \frac{f+gx}{(d+ex)^{5/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\int \frac{gx+f}{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="maxima")
```

output
```
integrate((g*x + f)/((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(e*x + d
)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. 2(350) = 700.

Time = 0.35 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.03

$$\int \frac{f+gx}{(d+ex)^{5/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x,
algorithm="giac")
```

output

```
1/24*(15*(7*c^3*e*f + 5*c^3*d*g - 6*b*c^2*e*g)*arctan(sqrt(-(e*x + d)*c +
2*c*d - b*e)/sqrt(-2*c*d + b*e))/((16*c^4*d^4*e - 32*b*c^3*d^3*e^2 + 24*b^
2*c^2*d^2*e^3 - 8*b^3*c*d*e^4 + b^4*e^5)*sqrt(-2*c*d + b*e)) + 48*(c^3*e*f
 + c^3*d*g - b*c^2*e*g)/((16*c^4*d^4*e - 32*b*c^3*d^3*e^2 + 24*b^2*c^2*d^2
*e^3 - 8*b^3*c*d*e^4 + b^4*e^5)*sqrt(-(e*x + d)*c + 2*c*d - b*e)) - (348*s
qrt(-(e*x + d)*c + 2*c*d - b*e)*c^5*d^2*e*f - 348*sqrt(-(e*x + d)*c + 2*c*
d - b*e)*b*c^4*d*e^2*f + 87*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^3*e^3*f
 + 84*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^5*d^3*g - 300*sqrt(-(e*x + d)*c +
 2*c*d - b*e)*b*c^4*d^2*e*g + 237*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^3
*d*e^2*g - 54*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^2*e^3*g - 272*(-(e*x
+ d)*c + 2*c*d - b*e)^(3/2)*c^4*d*e*f + 136*(-(e*x + d)*c + 2*c*d - b*e)^(
3/2)*b*c^3*e^2*f - 112*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^4*d^2*g + 248*
(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^3*d*e*g - 96*(-(e*x + d)*c + 2*c*d
- b*e)^(3/2)*b^2*c^2*e^2*g + 57*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x +
 d)*c + 2*c*d - b*e)*c^3*e*f + 27*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x
 + d)*c + 2*c*d - b*e)*c^3*d*g - 42*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e
*x + d)*c + 2*c*d - b*e)*b*c^2*e*g)/((16*c^4*d^4*e - 32*b*c^3*d^3*e^2 + 24
*b^2*c^2*d^2*e^3 - 8*b^3*c*d*e^4 + b^4*e^5)*(e*x + d)^3*c^3))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)^{5/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx=\int \frac{f+g\,x}{{\left(d+e\,x\right)}^{5/2}\,{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)}^{3/2}}\,dx$$

input

```
int((f + g*x)/((d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2)
),x)
```

output

```
int((f + g*x)/((d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1500, normalized size of antiderivative = 3.93

$$\int \frac{f+gx}{(d+ex)^{5/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
( - 90*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d
 - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d**3*e*g - 270*sqrt(b*e - 2*c*d)*sqrt(
 - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b
*c**2*d**2*e**2*g*x - 270*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*ata
n(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d*e**3*g*x**2 - 90*
sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*
x)/sqrt(b*e - 2*c*d))*b*c**2*e**4*g*x**3 + 75*sqrt(b*e - 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d
**4*g + 105*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e
+ c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**3*e*f + 225*sqrt(b*e - 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*c**3*d**3*e*g*x + 315*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(
sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**2*e**2*f*x + 225*sqr
t(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/
sqrt(b*e - 2*c*d))*c**3*d**2*e**2*g*x**2 + 315*sqrt(b*e - 2*c*d)*sqrt( - b
*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*
d*e**3*f*x**2 + 75*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt(
 - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d*e**3*g*x**3 + 105*sqrt(b*e
 - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(
b*e - 2*c*d))*c**3*e**4*f*x**3 + 4*b**4*d*e**4*g + 8*b**4*e**5*f + 12*b...
```

3.240 $\int \frac{(d+ex)^{13/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 46, antiderivative size = 417

$$
\begin{aligned}
\int \frac{(d+ex)^{13/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx &= \frac{2(2cd-be)^4(cef+cdg-beg)(d+ex)^{3/2}}{3c^6e^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} \\
&- \frac{2(2cd-be)^3(4cef+6cdg-5beg)\sqrt{d+ex}}{c^6e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} \\
&- \frac{4(2cd-be)^2(3cef+7cdg-5beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^6e^2\sqrt{d+ex}} \\
&+ \frac{4(2cd-be)(2cef+8cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^6e^2(d+ex)^{3/2}} \\
&- \frac{2(cef+9cdg-5beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^6e^2(d+ex)^{5/2}} \\
&+ \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{7/2}}{7c^6e^2(d+ex)^{7/2}}
\end{aligned}
$$

output
```
2/3*(-b*e+2*c*d)^4*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(3/2)/c^6/e^2/(d*(-b*e+c*d
)-b*e^2*x-c*e^2*x^2)^(3/2)-2*(-b*e+2*c*d)^3*(-5*b*e*g+6*c*d*g+4*c*e*f)*(e*
x+d)^(1/2)/c^6/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-4*(-b*e+2*c*d)^2
*(-5*b*e*g+7*c*d*g+3*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^6/e^2
/(e*x+d)^(1/2)+4/3*(-b*e+2*c*d)*(-5*b*e*g+8*c*d*g+2*c*e*f)*(d*(-b*e+c*d)-b
*e^2*x-c*e^2*x^2)^(3/2)/c^6/e^2/(e*x+d)^(3/2)-2/5*(-5*b*e*g+9*c*d*g+c*e*f)
*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^6/e^2/(e*x+d)^(5/2)+2/7*g*(d*(-b
*e+c*d)-b*e^2*x-c*e^2*x^2)^(7/2)/c^6/e^2/(e*x+d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^{13/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2\sqrt{d+ex}(-1280b^5e^5g+128b^4ce^4(7ef+78dg-15egx)-32b^3c^2e^3}{}$$

input
```
Integrate[((d + e*x)^(13/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^
2)^(5/2),x]
```

output
```
(2*Sqrt[d + e*x]*(-1280*b^5*e^5*g + 128*b^4*c*e^4*(7*e*f + 78*d*g - 15*e*g
*x) - 32*b^3*c^2*e^3*(953*d^2*g + 2*d*e*(91*f - 204*g*x) + 3*e^2*x*(-14*f
+ 5*g*x)) + 16*b^2*c^3*e^2*(2844*d^3*g + 3*d^2*e*(287*f - 681*g*x) + e^3*x
^2*(21*f + 5*g*x) + 6*d*e^2*x*(-77*f + 29*g*x)) + c^5*(9414*d^5*g + 3*d^4*
e*(1687*f - 4707*g*x) + 3*e^5*x^4*(7*f + 5*g*x) + 2*d*e^4*x^3*(98*f + 57*g
*x) + 2*d^2*e^3*x^2*(903*f + 257*g*x) + 12*d^3*e^2*x*(-637*f + 292*g*x)) -
 2*b*c^4*e*(16563*d^4*g + 12*d^3*e*(581*f - 1482*g*x) + e^4*x^3*(28*f + 15
*g*x) + 12*d*e^3*x^2*(63*f + 16*g*x) + 6*d^2*e^2*x*(-1106*f + 449*g*x))))/
(105*c^6*e^2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]
)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1218, 1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{13/2}(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 1218$$

$$\frac{2(d+ex)^{13/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-10beg+13cdg+7cef)\int \frac{(d+ex)^{11/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3ce(2cd-be)}$$

$$\Big\downarrow \; 1128$$

$$\frac{2(d+ex)^{13/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-10beg+13cdg+7cef)\left(\frac{8(2cd-be)\int \frac{(d+ex)^{9/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{7c}-\frac{2(d+ex)^{9/2}}{7ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3ce(2cd-be)}$$

$$\Big\downarrow \; 1128$$

$$\frac{2(d+ex)^{13/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-10beg+13cdg+7cef)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\int \frac{(d+ex)^{7/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{5c}-\frac{2(d+ex)^{7/2}}{5ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{7c}-\frac{2(d+ex)^9}{7ce\sqrt{d(cd-be)-b}}\right.}{3ce(2cd-be)}$$

$$\Big\downarrow \; 1128$$

$$\frac{2(d+ex)^{13/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-$$

$$\frac{(-10beg+13cdg+7cef)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\int\frac{(d+ex)^{5/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3c}-\frac{2(d+ex)^{5/2}}{3ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{5c}-\frac{2(}{5ce\sqrt{d(cd-}}\right.}{7c}\right.}{3ce(2cd-be)}$$

$\downarrow$ 1128

$$\frac{2(d+ex)^{13/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-$$

$$\frac{(-10beg+13cdg+7cef)\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int\frac{(d+ex)^{3/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{c}-\frac{2(d+ex)^{3/2}}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3c}-\right.}{5c}\right.}{7c}\right.}{3ce(2cd-be)}$$

$$\Big\downarrow 1122$$

$$\frac{2(d+ex)^{13/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{\left(\frac{8(2cd-be)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{4\sqrt{d+ex}(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3c}-\frac{2(d+ex)^{5/2}}{3ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{5c}-\frac{2(d+ex)^{7/2}}{5ce\sqrt{d(cd-be)-be^2x}}\right)}{7c}\right.}{3ce(2cd-be)}$$

input
```
Int[((d + e*x)^(13/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/
2),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(13/2))/(3*c*e^2*(2*c*d - b*e)*(d*(c*
d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) - ((7*c*e*f + 13*c*d*g - 10*b*e*g)*
((-2*(d + e*x)^(9/2))/(7*c*e*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) +
(8*(2*c*d - b*e)*((-2*(d + e*x)^(7/2))/(5*c*e*Sqrt[d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2]) + (6*(2*c*d - b*e)*((-2*(d + e*x)^(5/2))/(3*c*e*Sqrt[d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2]) + (4*(2*c*d - b*e)*((4*(2*c*d - b*e)*Sqrt[
d + e*x])/(c^2*e*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) - (2*(d + e*x)
^(3/2))/(c*e*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])))/(3*c)))/(5*c)))/
(7*c)))/(3*c*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.27

method	result
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(-15g\,e^5x^5c^5+30b\,c^4e^5g\,x^4-114c^5d\,e^4g\,x^4-21c^5e^5f\,x^4-80b^2c^3e^5g\,x^3+384b\,c^4d\,e^4g\,x^3+56b\,c^4e^5f\,x^3-}{}$
gosper	$-\frac{2(cex+be-cd)(-15g\,e^5x^5c^5+30b\,c^4e^5g\,x^4-114c^5d\,e^4g\,x^4-21c^5e^5f\,x^4-80b^2c^3e^5g\,x^3+384b\,c^4d\,e^4g\,x^3+56b\,c^4e^5f\,x^3-514c^5d^2e}{}$
orering	$-\frac{2(cex+be-cd)(-15g\,e^5x^5c^5+30b\,c^4e^5g\,x^4-114c^5d\,e^4g\,x^4-21c^5e^5f\,x^4-80b^2c^3e^5g\,x^3+384b\,c^4d\,e^4g\,x^3+56b\,c^4e^5f\,x^3-514c^5d^2e}{}$

input
```
int((e*x+d)^(13/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method
=_RETURNVERBOSE)
```

output
```
2/105/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-15*c^5*e^5*g*x^5+30
*b*c^4*e^5*g*x^4-114*c^5*d*e^4*g*x^4-21*c^5*e^5*f*x^4-80*b^2*c^3*e^5*g*x^3
+384*b*c^4*d*e^4*g*x^3+56*b*c^4*e^5*f*x^3-514*c^5*d^2*e^3*g*x^3-196*c^5*d*
e^4*f*x^3+480*b^3*c^2*e^5*g*x^2-2784*b^2*c^3*d*e^4*g*x^2-336*b^2*c^3*e^5*f
*x^2+5388*b*c^4*d^2*e^3*g*x^2+1512*b*c^4*d*e^4*f*x^2-3504*c^5*d^3*e^2*g*x^
2-1806*c^5*d^2*e^3*f*x^2+1920*b^4*c*e^5*g*x-13056*b^3*c^2*d*e^4*g*x-1344*b
^3*c^2*e^5*f*x+32688*b^2*c^3*d^2*e^3*g*x+7392*b^2*c^3*d*e^4*f*x-35568*b*c^
4*d^3*e^2*g*x-13272*b*c^4*d^2*e^3*f*x+14121*c^5*d^4*e*g*x+7644*c^5*d^3*e^2
*f*x+1280*b^5*e^5*g-9984*b^4*c*d*e^4*g-896*b^4*c*e^5*f+30496*b^3*c^2*d^2*e
^3*g+5824*b^3*c^2*d*e^4*f-45504*b^2*c^3*d^3*e^2*g-13776*b^2*c^3*d^2*e^3*f+
33126*b*c^4*d^4*e*g+13944*b*c^4*d^3*e^2*f-9414*c^5*d^5*g-5061*c^5*d^4*e*f)
/(c*e*x+b*e-c*d)^2/c^6/e^2
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^{13/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx =$$
$$-\frac{2\,(15\,c^5e^5gx^5+3\,(7\,c^5e^5f+2\,(19\,c^5de^4-5\,bc^4e^5)g)x^4+2\,(14\,(7\,c^5de^4-2\,bc^4e^5)f+(257\,c^5d^2e^3-192\,bc$$

input
```
integrate((e*x+d)^(13/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
 algorithm="fricas")
```

output
```
-2/105*(15*c^5*e^5*g*x^5 + 3*(7*c^5*e^5*f + 2*(19*c^5*d*e^4 - 5*b*c^4*e^5)
*g)*x^4 + 2*(14*(7*c^5*d*e^4 - 2*b*c^4*e^5)*f + (257*c^5*d^2*e^3 - 192*b*c
^4*d*e^4 + 40*b^2*c^3*e^5)*g)*x^3 + 6*(7*(43*c^5*d^2*e^3 - 36*b*c^4*d*e^4
+ 8*b^2*c^3*e^5)*f + 2*(292*c^5*d^3*e^2 - 449*b*c^4*d^2*e^3 + 232*b^2*c^3*
d*e^4 - 40*b^3*c^2*e^5)*g)*x^2 + 7*(723*c^5*d^4*e - 1992*b*c^4*d^3*e^2 + 1
968*b^2*c^3*d^2*e^3 - 832*b^3*c^2*d*e^4 + 128*b^4*c*e^5)*f + 2*(4707*c^5*d
^5 - 16563*b*c^4*d^4*e + 22752*b^2*c^3*d^3*e^2 - 15248*b^3*c^2*d^2*e^3 + 4
992*b^4*c*d*e^4 - 640*b^5*e^5)*g - 3*(28*(91*c^5*d^3*e^2 - 158*b*c^4*d^2*e
^3 + 88*b^2*c^3*d*e^4 - 16*b^3*c^2*e^5)*f + (4707*c^5*d^4*e - 11856*b*c^4*
d^3*e^2 + 10896*b^2*c^3*d^2*e^3 - 4352*b^3*c^2*d*e^4 + 640*b^4*c*e^5)*g)*x
)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^8*e^5*x^3 +
c^8*d^3*e^2 - 2*b*c^7*d^2*e^3 + b^2*c^6*d*e^4 - (c^8*d*e^4 - 2*b*c^7*e^5)*
x^2 - (c^8*d^2*e^3 - b^2*c^6*e^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{13/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(13/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5
/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^{13/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{2\,(3\,c^4e^4x^4+723\,c^4d^4-1992\,bc^3d^3e+1968\,b^2c^2d^2e^2-832\,b^3cde^3+}{} + \frac{2\,(15\,c^5e^5x^5+9414\,c^5d^5-33126\,bc^4d^4e+45504\,b^2c^3d^3e^2-30496\,b^3c^2d^2e^3+9984\,b^4cde^4-1280\,b^5e^5+6}{}$$

input
```
integrate((e*x+d)^(13/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
 algorithm="maxima")
```

output

```
2/15*(3*c^4*e^4*x^4 + 723*c^4*d^4 - 1992*b*c^3*d^3*e + 1968*b^2*c^2*d^2*e^
2 - 832*b^3*c*d*e^3 + 128*b^4*e^4 + 4*(7*c^4*d*e^3 - 2*b*c^3*e^4)*x^3 + 6*
(43*c^4*d^2*e^2 - 36*b*c^3*d*e^3 + 8*b^2*c^2*e^4)*x^2 - 12*(91*c^4*d^3*e -
 158*b*c^3*d^2*e^2 + 88*b^2*c^2*d*e^3 - 16*b^3*c*e^4)*x)*f/((c^6*e^2*x - c
^6*d*e + b*c^5*e^2)*sqrt(-c*e*x + c*d - b*e)) + 2/105*(15*c^5*e^5*x^5 + 94
14*c^5*d^5 - 33126*b*c^4*d^4*e + 45504*b^2*c^3*d^3*e^2 - 30496*b^3*c^2*d^2
*e^3 + 9984*b^4*c*d*e^4 - 1280*b^5*e^5 + 6*(19*c^5*d*e^4 - 5*b*c^4*e^5)*x^
4 + 2*(257*c^5*d^2*e^3 - 192*b*c^4*d*e^4 + 40*b^2*c^3*e^5)*x^3 + 12*(292*c
^5*d^3*e^2 - 449*b*c^4*d^2*e^3 + 232*b^2*c^3*d*e^4 - 40*b^3*c^2*e^5)*x^2 -
 3*(4707*c^5*d^4*e - 11856*b*c^4*d^3*e^2 + 10896*b^2*c^3*d^2*e^3 - 4352*b^
3*c^2*d*e^4 + 640*b^4*c*e^5)*x)*g/((c^7*e^3*x - c^7*d*e^2 + b*c^6*e^3)*sqr
t(-c*e*x + c*d - b*e))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(385) = 770$.

Time = 0.39 (sec) , antiderivative size = 975, normalized size of antiderivative = 2.34

$$\int \frac{(d+ex)^{13/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(13/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
 algorithm="giac")
```

output
```
-2/105*(35*(16*c^5*d^4*e*f - 32*b*c^4*d^3*e^2*f + 24*b^2*c^3*d^2*e^3*f - 8
*b^3*c^2*d*e^4*f + b^4*c*e^5*f + 16*c^5*d^5*g - 48*b*c^4*d^4*e*g + 56*b^2*
c^3*d^3*e^2*g - 32*b^3*c^2*d^2*e^3*g + 9*b^4*c*d*e^4*g - b^5*e^5*g + 96*((
e*x + d)*c - 2*c*d + b*e)*c^4*d^3*e*f - 144*((e*x + d)*c - 2*c*d + b*e)*b*
c^3*d^2*e^2*f + 72*((e*x + d)*c - 2*c*d + b*e)*b^2*c^2*d*e^3*f - 12*((e*x
+ d)*c - 2*c*d + b*e)*b^3*c*e^4*f + 144*((e*x + d)*c - 2*c*d + b*e)*c^4*d^
4*g - 336*((e*x + d)*c - 2*c*d + b*e)*b*c^3*d^3*e*g + 288*((e*x + d)*c - 2
*c*d + b*e)*b^2*c^2*d^2*e^2*g - 108*((e*x + d)*c - 2*c*d + b*e)*b^3*c*d*e^
3*g + 15*((e*x + d)*c - 2*c*d + b*e)*b^4*e^4*g)/((((e*x + d)*c - 2*c*d + b*
e)*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^6*e) + (2520*sqrt(-(e*x + d)*c + 2*c
*d - b*e)*c^39*d^2*e^7*f - 2520*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^38*d*
e^8*f + 630*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^37*e^9*f + 5880*sqrt(-(
e*x + d)*c + 2*c*d - b*e)*c^39*d^3*e^6*g - 10080*sqrt(-(e*x + d)*c + 2*c*d
 - b*e)*b*c^38*d^2*e^7*g + 5670*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^37*
d*e^8*g - 1050*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^3*c^36*e^9*g - 280*(-(e*
x + d)*c + 2*c*d - b*e)^(3/2)*c^38*d*e^7*f + 140*(-(e*x + d)*c + 2*c*d - b
*e)^(3/2)*b*c^37*e^8*f - 1120*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^38*d^2*
e^6*g + 1260*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c^37*d*e^7*g - 350*(-(e*
x + d)*c + 2*c*d - b*e)^(3/2)*b^2*c^36*e^8*g + 21*((e*x + d)*c - 2*c*d + b
*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^37*e^7*f + 189*((e*x + d)*c - ...
```

Mupad [B] (verification not implemented)

Time = 12.57 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)^{13/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = -\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{\sqrt{d+e\,x}\,\left(-2560\,g\,b^5\,e^5+19968\,g\,b^4\,c\,d\,e^4+1792\,f\,b^4\,c\,e^5-60992\,g\,b^3\,c^2\,d^2\,e^3-11648\,f\,b^3\,c^2\,d\,e^4+910\right.}{10}\right.}{}$$

input
```
int(((f + g*x)*(d + e*x)^(13/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/
2),x)
```

output

```
-((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*(((d + e*x)^(1/2)*(18828*c^5
*d^5*g - 2560*b^5*e^5*g + 1792*b^4*c*e^5*f + 10122*c^5*d^4*e*f - 66252*b*c
^4*d^4*e*g + 19968*b^4*c*d*e^4*g - 27888*b*c^4*d^3*e^2*f - 11648*b^3*c^2*d
*e^4*f + 27552*b^2*c^3*d^2*e^3*f + 91008*b^2*c^3*d^3*e^2*g - 60992*b^3*c^2
*d^2*e^3*g))/(105*c^8*e^5) + (2*g*x^5*(d + e*x)^(1/2))/(7*c^3) + (4*x^3*(d
 + e*x)^(1/2)*(40*b^2*e^2*g + 257*c^2*d^2*g - 28*b*c*e^2*f + 98*c^2*d*e*f
- 192*b*c*d*e*g))/(105*c^5*e^2) + (2*x^4*(d + e*x)^(1/2)*(38*c*d*g - 10*b*
e*g + 7*c*e*f))/(35*c^4*e) - (x*(d + e*x)^(1/2)*(15288*c^5*d^3*e^2*f - 268
8*b^3*c^2*e^5*f + 3840*b^4*c*e^5*g + 28242*c^5*d^4*e*g - 26544*b*c^4*d^2*e
^3*f + 14784*b^2*c^3*d*e^4*f - 71136*b*c^4*d^3*e^2*g - 26112*b^3*c^2*d*e^4
*g + 65376*b^2*c^3*d^2*e^3*g))/(105*c^8*e^5) + (x^2*(d + e*x)^(1/2)*(672*b
^2*c^3*e^5*f - 960*b^3*c^2*e^5*g + 3612*c^5*d^2*e^3*f + 7008*c^5*d^3*e^2*g
 - 3024*b*c^4*d*e^4*f - 10776*b*c^4*d^2*e^3*g + 5568*b^2*c^3*d*e^4*g))/(10
5*c^8*e^5)))/(x^3 + (x*(105*b^2*c^6*e^5 - 105*c^8*d^2*e^3))/(105*c^8*e^5)
+ (d*(b*e - c*d)^2)/(c^2*e^3) + (x^2*(2*b*e - c*d))/(c*e))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^{13/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{\frac{256}{15}b^4ce^5f+\frac{2}{5}c^5e^5fx^4+\frac{2}{7}c^5e^5gx^5+\frac{128}{5}b^3c^2e^5fx-\frac{64}{7}b^3c^2e^5gx^2+}{}$$

input

```
int((e*x+d)^(13/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(2*( - 1280*b**5*e**5*g + 9984*b**4*c*d*e**4*g + 896*b**4*c*e**5*f - 1920*
b**4*c*e**5*g*x - 30496*b**3*c**2*d**2*e**3*g - 5824*b**3*c**2*d*e**4*f +
13056*b**3*c**2*d*e**4*g*x + 1344*b**3*c**2*e**5*f*x - 480*b**3*c**2*e**5*
g*x**2 + 45504*b**2*c**3*d**3*e**2*g + 13776*b**2*c**3*d**2*e**3*f - 32688
*b**2*c**3*d**2*e**3*g*x - 7392*b**2*c**3*d*e**4*f*x + 2784*b**2*c**3*d*e*
*4*g*x**2 + 336*b**2*c**3*e**5*f*x**2 + 80*b**2*c**3*e**5*g*x**3 - 33126*b
*c**4*d**4*e*g - 13944*b*c**4*d**3*e**2*f + 35568*b*c**4*d**3*e**2*g*x + 1
3272*b*c**4*d**2*e**3*f*x - 5388*b*c**4*d**2*e**3*g*x**2 - 1512*b*c**4*d*e
**4*f*x**2 - 384*b*c**4*d*e**4*g*x**3 - 56*b*c**4*e**5*f*x**3 - 30*b*c**4*
e**5*g*x**4 + 9414*c**5*d**5*g + 5061*c**5*d**4*e*f - 14121*c**5*d**4*e*g*
x - 7644*c**5*d**3*e**2*f*x + 3504*c**5*d**3*e**2*g*x**2 + 1806*c**5*d**2*
e**3*f*x**2 + 514*c**5*d**2*e**3*g*x**3 + 196*c**5*d*e**4*f*x**3 + 114*c**
5*d*e**4*g*x**4 + 21*c**5*e**5*f*x**4 + 15*c**5*e**5*g*x**5))/(105*sqrt( -
 b*e + c*d - c*e*x)*c**6*e**2*(b*e - c*d + c*e*x))
```

3.241 $\int \frac{(d+ex)^{11/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 339

$$\begin{aligned}\int \frac{(d+ex)^{11/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx &= \frac{2(2cd-be)^3(cef+cdg-beg)(d+ex)^{3/2}}{3c^5e^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} \\ &\quad - \frac{2(2cd-be)^2(3cef+5cdg-4beg)\sqrt{d+ex}}{c^5e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} \\ &\quad - \frac{6(2cd-be)(cef+3cdg-2beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^5e^2\sqrt{d+ex}} \\ &\quad + \frac{2(cef+7cdg-4beg)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^5e^2(d+ex)^{3/2}} \\ &\quad - \frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{5c^5e^2(d+ex)^{5/2}}\end{aligned}$$

output

```
2/3*(-b*e+2*c*d)^3*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(3/2)/c^5/e^2/(d*(-b*e+c*d
)-b*e^2*x-c*e^2*x^2)^(3/2)-2*(-b*e+2*c*d)^2*(-4*b*e*g+5*c*d*g+3*c*e*f)*(e*
x+d)^(1/2)/c^5/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-6*(-b*e+2*c*d)*(
-2*b*e*g+3*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^5/e^2/(e*
x+d)^(1/2)+2/3*(-4*b*e*g+7*c*d*g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
3/2)/c^5/e^2/(e*x+d)^(3/2)-2/5*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c^
5/e^2/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^{11/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{d+ex}(128b^4e^4g-16b^3ce^3(5ef+47dg-12egx)+24b^2c^2e^2(67}{}$$

input
```
Integrate[((d + e*x)^(11/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^
2)^(5/2),x]
```

output
```
(2*Sqrt[d + e*x]*(128*b^4*e^4*g - 16*b^3*c*e^3*(5*e*f + 47*d*g - 12*e*g*x)
 + 24*b^2*c^2*e^2*(67*d^2*g + 3*d*e*(5*f - 13*g*x) + e^2*x*(-5*f + 2*g*x))
 - 2*b*c^3*e*(741*d^3*g + 3*d^2*e*(85*f - 246*g*x) + e^3*x^2*(15*f + 4*g*x
) + 3*d*e^2*x*(-70*f + 31*g*x)) + c^4*(498*d^4*g + 9*d^3*e*(25*f - 83*g*x)
 + e^4*x^3*(5*f + 3*g*x) + d*e^3*x^2*(75*f + 23*g*x) + 3*d^2*e^2*x*(-115*f
 + 61*g*x))))/(15*c^5*e^2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) +
c*(d - e*x))])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1218, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{11/2}(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1218$$

$$\frac{2(d+ex)^{11/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} - \frac{(-8beg+11cdg+5cef)\int \frac{(d+ex)^{9/2}}{(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{3ce(2cd-be)}$$

$$\Big\downarrow \; 1128$$

$$\frac{2(d+ex)^{11/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-8beg+11cdg+5cef)\left(\frac{6(2cd-be)\int\frac{(d+ex)^{7/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{5c}-\frac{2(d+ex)^{7/2}}{5ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3ce(2cd-be)}$$

$$\Big\downarrow 1128$$

$$\frac{2(d+ex)^{11/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-8beg+11cdg+5cef)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\int\frac{(d+ex)^{5/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3c}-\frac{2(d+ex)^{5/2}}{3ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{5c}-\frac{2(d+ex)^{7/}}{5ce\sqrt{d(cd-be)-be^2}}\right.}{3ce(2cd-be)}$$

$$\Big\downarrow 1128$$

$$\frac{2(d+ex)^{11/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-8beg+11cdg+5cef)\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int\frac{(d+ex)^{3/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{c}-\frac{2(d+ex)^{3/2}}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3c}-\frac{2(d+}{3ce\sqrt{d(cd-be}}\right.}{5c}\right.}{3ce(2cd-be)}$$

$$\Big\downarrow 1122$$

$$\frac{2(d+ex)^{11/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-$$

$$\frac{\left(\frac{6(2cd-be)\left(\frac{4(2cd-be)\left(\frac{4\sqrt{d+ex}(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3c}-\frac{2(d+ex)^{5/2}}{3ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{5c}-\frac{2(d+ex)^{7/2}}{5ce\sqrt{d(cd-be)-be^2x-ce^2x}}\right.}{3ce(2cd-be)}$$

input
```
Int[((d + e*x)^(11/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/
2),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(11/2))/(3*c*e^2*(2*c*d - b*e)*(d*(c*
d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) - ((5*c*e*f + 11*c*d*g - 8*b*e*g)*(
(-2*(d + e*x)^(7/2))/(5*c*e*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) + (
6*(2*c*d - b*e)*((-2*(d + e*x)^(5/2))/(3*c*e*Sqrt[d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2]) + (4*(2*c*d - b*e)*((4*(2*c*d - b*e)*Sqrt[d + e*x])/(c^2*e*S
qrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) - (2*(d + e*x)^(3/2))/(c*e*Sqrt[
d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])))/(3*c)))/(5*c)))/(3*c*e*(2*c*d - b*
e))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.06

method	result
default	$-\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(3g\,e^4x^4c^4-8b\,c^3e^4g\,x^3+23c^4d\,e^3g\,x^3+5c^4e^4f\,x^3+48b^2c^2e^4g\,x^2-186b\,c^3d\,e^3g\,x^2-30b\,c^3e^4f\,x^2+183$
gosper	$\frac{2(cex+be-cd)\,(3g\,e^4x^4c^4-8b\,c^3e^4g\,x^3+23c^4d\,e^3g\,x^3+5c^4e^4f\,x^3+48b^2c^2e^4g\,x^2-186b\,c^3d\,e^3g\,x^2-30b\,c^3e^4f\,x^2+183c^4d^2e^2g\,x^2+75$
orering	$\frac{2(cex+be-cd)\,(3g\,e^4x^4c^4-8b\,c^3e^4g\,x^3+23c^4d\,e^3g\,x^3+5c^4e^4f\,x^3+48b^2c^2e^4g\,x^2-186b\,c^3d\,e^3g\,x^2-30b\,c^3e^4f\,x^2+183c^4d^2e^2g\,x^2+75$

input
```
int((e*x+d)^(11/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method
=_RETURNVERBOSE)
```

output
```
-2/15/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(3*c^4*e^4*g*x^4-8*b*
c^3*e^4*g*x^3+23*c^4*d*e^3*g*x^3+5*c^4*e^4*f*x^3+48*b^2*c^2*e^4*g*x^2-186*
b*c^3*d*e^3*g*x^2-30*b*c^3*e^4*f*x^2+183*c^4*d^2*e^2*g*x^2+75*c^4*d*e^3*f*
x^2+192*b^3*c*e^4*g*x-936*b^2*c^2*d*e^3*g*x-120*b^2*c^2*e^4*f*x+1476*b*c^3
*d^2*e^2*g*x+420*b*c^3*d*e^3*f*x-747*c^4*d^3*e*g*x-345*c^4*d^2*e^2*f*x+128
*b^4*e^4*g-752*b^3*c*d*e^3*g-80*b^3*c*e^4*f+1608*b^2*c^2*d^2*e^2*g+360*b^2
*c^2*d*e^3*f-1482*b*c^3*d^3*e*g-510*b*c^3*d^2*e^2*f+498*c^4*d^4*g+225*c^4*
d^3*e*f)/(c*e*x+b*e-c*d)^2/c^5/e^2
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^{11/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx =$$

$$-\frac{2\,(3\,c^4e^4gx^4+(5\,c^4e^4f+(23\,c^4de^3-8\,bc^3e^4)g)x^3+3\,(5\,(5\,c^4de^3-2\,bc^3e^4)f+(61\,c^4d^2e^2-62\,bc^3de^3+}{}$$

input
```
integrate((e*x+d)^(11/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
 algorithm="fricas")
```

output
```
-2/15*(3*c^4*e^4*g*x^4 + (5*c^4*e^4*f + (23*c^4*d*e^3 - 8*b*c^3*e^4)*g)*x^
3 + 3*(5*(5*c^4*d*e^3 - 2*b*c^3*e^4)*f + (61*c^4*d^2*e^2 - 62*b*c^3*d*e^3
+ 16*b^2*c^2*e^4)*g)*x^2 + 5*(45*c^4*d^3*e - 102*b*c^3*d^2*e^2 + 72*b^2*c^
2*d*e^3 - 16*b^3*c*e^4)*f + 2*(249*c^4*d^4 - 741*b*c^3*d^3*e + 804*b^2*c^2
*d^2*e^2 - 376*b^3*c*d*e^3 + 64*b^4*e^4)*g - 3*(5*(23*c^4*d^2*e^2 - 28*b*c
^3*d*e^3 + 8*b^2*c^2*e^4)*f + (249*c^4*d^3*e - 492*b*c^3*d^2*e^2 + 312*b^2
*c^2*d*e^3 - 64*b^3*c*e^4)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e
)*sqrt(e*x + d)/(c^7*e^5*x^3 + c^7*d^3*e^2 - 2*b*c^6*d^2*e^3 + b^2*c^5*d*e
^4 - (c^7*d*e^4 - 2*b*c^6*e^5)*x^2 - (c^7*d^2*e^3 - b^2*c^5*e^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{11/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(11/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5
/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{11/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2\,(c^3e^3x^3+45\,c^3d^3-102\,bc^2d^2e+72\,b^2cde^2-16\,b^3e^3+3\,(5\,c^3de^2-}{3\,(c^5e^2x-c^5de+bc^4e^2)\sqrt{-c}} + \frac{2\,(3\,c^4e^4x^4+498\,c^4d^4-1482\,bc^3d^3e+1608\,b^2c^2d^2e^2-752\,b^3cde^3+128\,b^4e^4+(23\,c^4de^3-8\,bc^3e^4)x^3+3}{15\,(c^6e^3x-c^6de^2+bc^5e^3)}$$

input
```
integrate((e*x+d)^(11/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
 algorithm="maxima")
```

output
```
2/3*(c^3*e^3*x^3 + 45*c^3*d^3 - 102*b*c^2*d^2*e + 72*b^2*c*d*e^2 - 16*b^3*
e^3 + 3*(5*c^3*d*e^2 - 2*b*c^2*e^3)*x^2 - 3*(23*c^3*d^2*e - 28*b*c^2*d*e^2
 + 8*b^2*c*e^3)*x)*f/((c^5*e^2*x - c^5*d*e + b*c^4*e^2)*sqrt(-c*e*x + c*d
- b*e)) + 2/15*(3*c^4*e^4*x^4 + 498*c^4*d^4 - 1482*b*c^3*d^3*e + 1608*b^2*
c^2*d^2*e^2 - 752*b^3*c*d*e^3 + 128*b^4*e^4 + (23*c^4*d*e^3 - 8*b*c^3*e^4)
*x^3 + 3*(61*c^4*d^2*e^2 - 62*b*c^3*d*e^3 + 16*b^2*c^2*e^4)*x^2 - 3*(249*c
^4*d^3*e - 492*b*c^3*d^2*e^2 + 312*b^2*c^2*d*e^3 - 64*b^3*c*e^4)*x)*g/((c^
6*e^3*x - c^6*d*e^2 + b*c^5*e^3)*sqrt(-c*e*x + c*d - b*e))
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)^{11/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{2\,\Big(\frac{5\,(8\,c^4d^3ef-12\,bc^3d^2e^2f+6\,b^2c^2de^3f-b^3ce^4f+8\,c^4d^4g-20\,bc^3d^3eg+18\,b^2c^2d^2e^2g-7\,b^3cde^3g+b^4e^4g+36\,((ex+d)c-2\,cd+be)c^3d^2ef-36\,(}{}}{}$$

input
```
integrate((e*x+d)^(11/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
 algorithm="giac")
```

output
```
-2/15*(5*(8*c^4*d^3*e*f - 12*b*c^3*d^2*e^2*f + 6*b^2*c^2*d*e^3*f - b^3*c*e
^4*f + 8*c^4*d^4*g - 20*b*c^3*d^3*e*g + 18*b^2*c^2*d^2*e^2*g - 7*b^3*c*d*e
^3*g + b^4*e^4*g + 36*((e*x + d)*c - 2*c*d + b*e)*c^3*d^2*e*f - 36*((e*x +
 d)*c - 2*c*d + b*e)*b*c^2*d*e^2*f + 9*((e*x + d)*c - 2*c*d + b*e)*b^2*c*e
^3*f + 60*((e*x + d)*c - 2*c*d + b*e)*c^3*d^3*g - 108*((e*x + d)*c - 2*c*d
 + b*e)*b*c^2*d^2*e*g + 63*((e*x + d)*c - 2*c*d + b*e)*b^2*c*d*e^2*g - 12*
((e*x + d)*c - 2*c*d + b*e)*b^3*e^3*g)/(((e*x + d)*c - 2*c*d + b*e)*sqrt(-
(e*x + d)*c + 2*c*d - b*e)*c^5*e) + (90*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c
^22*d*e^5*f - 45*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^21*e^6*f + 270*sqrt(
-(e*x + d)*c + 2*c*d - b*e)*c^22*d^2*e^4*g - 315*sqrt(-(e*x + d)*c + 2*c*d
 - b*e)*b*c^21*d*e^5*g + 90*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c^20*e^6*
g - 5*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^21*e^5*f - 35*(-(e*x + d)*c + 2
*c*d - b*e)^(3/2)*c^21*d*e^4*g + 20*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c
^20*e^5*g + 3*((e*x + d)*c - 2*c*d + b*e)^2*sqrt(-(e*x + d)*c + 2*c*d - b*
e)*c^20*e^4*g)/(c^25*e^5))/e
```

Mupad [B] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^{11/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,\left(\frac{\sqrt{d+e\,x}\,\left(256\,g\,b^4\,e^4-1504\,g\,b^3\,c\,d\,e^3-160\,f\,b^3\,c\,e^4+3216\,g\,b^2\,c^2\,d^2\,e^2+720\,f\,b^2\,c^2\,d\,e^3-2964\,g\,b\,c^3\,d^3}{15\,c^7\,e^5}}{}$$

input
```
int(((f + g*x)*(d + e*x)^(11/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/
2),x)
```

output
```
-((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*(((d + e*x)^(1/2)*(256*b^4*e
^4*g + 996*c^4*d^4*g - 160*b^3*c*e^4*f + 450*c^4*d^3*e*f - 2964*b*c^3*d^3*
e*g - 1504*b^3*c*d*e^3*g - 1020*b*c^3*d^2*e^2*f + 720*b^2*c^2*d*e^3*f + 32
16*b^2*c^2*d^2*e^2*g))/(15*c^7*e^5) + (2*x^2*(d + e*x)^(1/2)*(16*b^2*e^2*g
 + 61*c^2*d^2*g - 10*b*c*e^2*f + 25*c^2*d*e*f - 62*b*c*d*e*g))/(5*c^5*e^3)
 + (2*x^3*(d + e*x)^(1/2)*(23*c*d*g - 8*b*e*g + 5*c*e*f))/(15*c^4*e^2) + (
2*g*x^4*(d + e*x)^(1/2))/(5*c^3*e) - (x*(d + e*x)^(1/2)*(240*b^2*c^2*e^4*f
 + 690*c^4*d^2*e^2*f - 384*b^3*c*e^4*g + 1494*c^4*d^3*e*g - 840*b*c^3*d*e^
3*f - 2952*b*c^3*d^2*e^2*g + 1872*b^2*c^2*d*e^3*g))/(15*c^7*e^5)))/(x^3 +
(x*(15*b^2*c^5*e^5 - 15*c^7*d^2*e^3))/(15*c^7*e^5) + (d*(b*e - c*d)^2)/(c^
2*e^3) + (x^2*(2*b*e - c*d))/(c*e))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^{11/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{\frac{332}{5}c^4d^4g-\frac{1504}{15}b^3cd\,e^3g+\frac{1072}{5}b^2c^2d^2e^2g+48b^2c^2d\,e^3f-16b^2c^2e^4fx}{}$$

input
```
int((e*x+d)^(11/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output
```
(2*(128*b**4*e**4*g - 752*b**3*c*d*e**3*g - 80*b**3*c*e**4*f + 192*b**3*c*
e**4*g*x + 1608*b**2*c**2*d**2*e**2*g + 360*b**2*c**2*d*e**3*f - 936*b**2*
c**2*d*e**3*g*x - 120*b**2*c**2*e**4*f*x + 48*b**2*c**2*e**4*g*x**2 - 1482
*b*c**3*d**3*e*g - 510*b*c**3*d**2*e**2*f + 1476*b*c**3*d**2*e**2*g*x + 42
0*b*c**3*d*e**3*f*x - 186*b*c**3*d*e**3*g*x**2 - 30*b*c**3*e**4*f*x**2 - 8
*b*c**3*e**4*g*x**3 + 498*c**4*d**4*g + 225*c**4*d**3*e*f - 747*c**4*d**3*
e*g*x - 345*c**4*d**2*e**2*f*x + 183*c**4*d**2*e**2*g*x**2 + 75*c**4*d*e**
3*f*x**2 + 23*c**4*d*e**3*g*x**3 + 5*c**4*e**4*f*x**3 + 3*c**4*e**4*g*x**4
))/(15*sqrt( - b*e + c*d - c*e*x)*c**5*e**2*(b*e - c*d + c*e*x))
```

3.242 $\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 263

$$\begin{aligned}\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx &= \frac{2(2cd-be)^2(cef+cdg-beg)(d+ex)^{3/2}}{3c^4e^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\\ &-\frac{2(2cd-be)(2cef+4cdg-3beg)\sqrt{d+ex}}{c^4e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}\\ &-\frac{2(cef+5cdg-3beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^4e^2\sqrt{d+ex}}\\ &+\frac{2g\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{3c^4e^2(d+ex)^{3/2}}\end{aligned}$$

output
```
2/3*(-b*e+2*c*d)^2*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(3/2)/c^4/e^2/(d*(-b*e+c*d
)-b*e^2*x-c*e^2*x^2)^(3/2)-2*(-b*e+2*c*d)*(-3*b*e*g+4*c*d*g+2*c*e*f)*(e*x+
d)^(1/2)/c^4/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-2*(-3*b*e*g+5*c*d*
g+c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^4/e^2/(e*x+d)^(1/2)+2/3*
g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c^4/e^2/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{9/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{d+ex}(-16b^3e^3g+8b^2ce^2(8dg+e(f-3gx))-2bc^2e(41d^2g+2}{3c^4e^2(}$$

input

```
Integrate[((d + e*x)^(9/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
)^(5/2),x]
```

output

```
(2*Sqrt[d + e*x]*(-16*b^3*e^3*g + 8*b^2*c*e^2*(8*d*g + e*(f - 3*g*x)) - 2*
b*c^2*e*(41*d^2*g + 2*d*e*(5*f - 18*g*x) + 3*e^2*x*(-2*f + g*x)) + c^3*(34
*d^3*g + d^2*e*(11*f - 51*g*x) + e^3*x^2*(3*f + g*x) + 6*d*e^2*x*(-3*f + 2
*g*x))))/(3*c^4*e^2*(-(c*d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d -
 e*x))])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1218, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{2(d+ex)^{9/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-2beg+3cdg+cef)\int \frac{(d+ex)^{7/2}}{(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{ce(2cd-be)}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2(d+ex)^{9/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-2beg+3cdg+cef)\left(\frac{4(2cd-be)\int\frac{(d+ex)^{5/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3c}-\frac{2(d+ex)^{5/2}}{3ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{ce(2cd-be)}$$

$$\Big\downarrow \; 1128$$

$$\frac{2(d+ex)^{9/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-2beg+3cdg+cef)\left(\frac{4(2cd-be)\left(\frac{2(2cd-be)\int\frac{(d+ex)^{3/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{c}-\frac{2(d+ex)^{3/2}}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3c}-\frac{2(d+ex)^{5/2}}{3ce\sqrt{d(cd-be)-be^2x-c}}\right.}{ce(2cd-be)}$$

$$\Big\downarrow \; 1122$$

$$\frac{2(d+ex)^{9/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{\left(\frac{4(2cd-be)\left(\frac{4\sqrt{d+ex}(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3c}-\frac{2(d+ex)^{5/2}}{3ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-2beg+3cdg+cef)}{ce(2cd-be)}$$

input
```
Int[((d + e*x)^(9/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2
),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(9/2))/(3*c*e^2*(2*c*d - b*e)*(d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) - ((c*e*f + 3*c*d*g - 2*b*e*g)*((-2*
(d + e*x)^(5/2))/(3*c*e*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) + (4*(2
*c*d - b*e)*((4*(2*c*d - b*e)*Sqrt[d + e*x])/(c^2*e*Sqrt[d*(c*d - b*e) - b
*e^2*x - c*e^2*x^2]) - (2*(d + e*x)^(3/2))/(c*e*Sqrt[d*(c*d - b*e) - b*e^2
*x - c*e^2*x^2])))/(3*c)))/(c*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.87

method	result
default	$\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(-e^3 g\,x^3 c^3+6b\,c^2 e^3 g\,x^2-12c^3 d\,e^2 g\,x^2-3c^3 e^3 f\,x^2+24b^2 c\,e^3 gx-72b\,c^2 d\,e^2 gx-12b\,c^2 e^3 fx+51c^3 d^2 egx+}{3\sqrt{ex+d}\,(cex+be-cd)^2 c^4 e^2}$
gosper	$-\frac{2(cex+be-cd)(-e^3 g\,x^3 c^3+6b\,c^2 e^3 g\,x^2-12c^3 d\,e^2 g\,x^2-3c^3 e^3 f\,x^2+24b^2 c\,e^3 gx-72b\,c^2 d\,e^2 gx-12b\,c^2 e^3 fx+51c^3 d^2 egx+18c^3 d\,e^2}{3c^4 e^2(-x^2 c\,e^2-xb\,e^2-bde+c\,d^2)^{\frac{5}{2}}}$
orering	$-\frac{2(cex+be-cd)(-e^3 g\,x^3 c^3+6b\,c^2 e^3 g\,x^2-12c^3 d\,e^2 g\,x^2-3c^3 e^3 f\,x^2+24b^2 c\,e^3 gx-72b\,c^2 d\,e^2 gx-12b\,c^2 e^3 fx+51c^3 d^2 egx+18c^3 d\,e^2}{3c^4 e^2(-x^2 c\,e^2-xb\,e^2-bde+c\,d^2)^{\frac{5}{2}}}$

input
```
int((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
2/3/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-c^3*e^3*g*x^3+6*b*c^2
*e^3*g*x^2-12*c^3*d*e^2*g*x^2-3*c^3*e^3*f*x^2+24*b^2*c*e^3*g*x-72*b*c^2*d*
e^2*g*x-12*b*c^2*e^3*f*x+51*c^3*d^2*e*g*x+18*c^3*d*e^2*f*x+16*b^3*e^3*g-64
*b^2*c*d*e^2*g-8*b^2*c*e^3*f+82*b*c^2*d^2*e*g+20*b*c^2*d*e^2*f-34*c^3*d^3*
g-11*c^3*d^2*e*f)/(c*e*x+b*e-c*d)^2/c^4/e^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx =$$

$$-\frac{2\,(c^3e^3gx^3+3\,(c^3e^3f+2\,(2\,c^3de^2-bc^2e^3)g)x^2+(11\,c^3d^2e-20\,bc^2de^2+8\,b^2ce^3)f+2\,(17\,c^3d^3-41\,bc^2}{3\,(c^6e^5x^3+c^6d^3e^2-2\,bc^5d^2e^3}$$

input
```
integrate((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output
```
-2/3*(c^3*e^3*g*x^3 + 3*(c^3*e^3*f + 2*(2*c^3*d*e^2 - b*c^2*e^3)*g)*x^2 +
(11*c^3*d^2*e - 20*b*c^2*d*e^2 + 8*b^2*c*e^3)*f + 2*(17*c^3*d^3 - 41*b*c^2
*d^2*e + 32*b^2*c*d*e^2 - 8*b^3*e^3)*g - 3*(2*(3*c^3*d*e^2 - 2*b*c^2*e^3)*
f + (17*c^3*d^2*e - 24*b*c^2*d*e^2 + 8*b^2*c*e^3)*g)*x)*sqrt(-c*e^2*x^2 -
b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^6*e^5*x^3 + c^6*d^3*e^2 - 2*b*c^
5*d^2*e^3 + b^2*c^4*d*e^4 - (c^6*d*e^4 - 2*b*c^5*e^5)*x^2 - (c^6*d^2*e^3 -
 b^2*c^4*e^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(9/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^{9/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{2\,(3\,c^2e^2x^2+11\,c^2d^2-20\,bcde+8\,b^2e^2-6\,(3\,c^2de-2\,bce^2)x)f}{3\,(c^4e^2x-c^4de+bc^3e^2)\sqrt{-cex+cd-be}} + \frac{2\,(c^3e^3x^3+34\,c^3d^3-82\,bc^2d^2e+64\,b^2cde^2-16\,b^3e^3+6\,(2\,c^3de^2-bc^2e^3)x^2-3\,(17\,c^3d^2e-24\,bc^2de^2+}{3\,(c^5e^3x-c^5de^2+bc^4e^3)\sqrt{-cex+cd-be}}$$

input
```
integrate((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output
```
2/3*(3*c^2*e^2*x^2 + 11*c^2*d^2 - 20*b*c*d*e + 8*b^2*e^2 - 6*(3*c^2*d*e -
2*b*c*e^2)*x)*f/((c^4*e^2*x - c^4*d*e + b*c^3*e^2)*sqrt(-c*e*x + c*d - b*e
)) + 2/3*(c^3*e^3*x^3 + 34*c^3*d^3 - 82*b*c^2*d^2*e + 64*b^2*c*d*e^2 - 16*
b^3*e^3 + 6*(2*c^3*d*e^2 - b*c^2*e^3)*x^2 - 3*(17*c^3*d^2*e - 24*b*c^2*d*e
^2 + 8*b^2*c*e^3)*x)*g/((c^5*e^3*x - c^5*d*e^2 + b*c^4*e^3)*sqrt(-c*e*x +
c*d - b*e))
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^{9/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = -\frac{2\left(\frac{4\,c^3d^2ef-4\,bc^2de^2f+b^2ce^3f+4\,c^3d^3g-8\,bc^2d^2eg+5\,b^2cde^2g-b^3e^3g+12\,((ex+d)c-2\,cd+be)c^2def-6\,((ex+d)c-2\,cd+be)bce^2f+24\,((ex+d}{((ex+d)c-2\,cd+be)\sqrt{-(ex+d)c+2\,cd-be}c^4e}\right.}{}$$

input
```
integrate((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output
```
-2/3*((4*c^3*d^2*e*f - 4*b*c^2*d*e^2*f + b^2*c*e^3*f + 4*c^3*d^3*g - 8*b*c
^2*d^2*e*g + 5*b^2*c*d*e^2*g - b^3*e^3*g + 12*((e*x + d)*c - 2*c*d + b*e)*
c^2*d*e*f - 6*((e*x + d)*c - 2*c*d + b*e)*b*c*e^2*f + 24*((e*x + d)*c - 2*
c*d + b*e)*c^2*d^2*g - 30*((e*x + d)*c - 2*c*d + b*e)*b*c*d*e*g + 9*((e*x
+ d)*c - 2*c*d + b*e)*b^2*e^2*g)/(((e*x + d)*c - 2*c*d + b*e)*sqrt(-(e*x +
 d)*c + 2*c*d - b*e)*c^4*e) + (3*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^9*e^3*
f + 15*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^9*d*e^2*g - 9*sqrt(-(e*x + d)*c
+ 2*c*d - b*e)*b*c^8*e^3*g - (-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^8*e^2*g)
/(c^12*e^3))/e
```

Mupad [B] (verification not implemented)

Time = 12.17 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^{9/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = -\frac{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\left(\frac{\sqrt{d+e\,x}\,(-32\,g\,b^3\,e^3+128\,g\,b^2\,c\,d\,e^2+16\,f\,b^2\,c\,e^3-164\,g\,b\,c^2\,d^2\,e-40\,f\,b\,c^2\,d\,e^2+68\,g\,c^3\,d^3+22\,f\,c^3\,d^2}{3\,c^6\,e^5}\right.}{x^3+\frac{x\,(3\,b^2\,c^4\,e^5-3\,c^6\,d^2\,e^3)}{3\,c^6\,e^5}}$$

input
```
int(((f + g*x)*(d + e*x)^(9/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2
),x)
```

output
```
-((c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)*(((d + e*x)^(1/2)*(68*c^3*d^
3*g - 32*b^3*e^3*g + 16*b^2*c*e^3*f + 22*c^3*d^2*e*f - 40*b*c^2*d*e^2*f -
164*b*c^2*d^2*e*g + 128*b^2*c*d*e^2*g))/(3*c^6*e^5) + (2*x^2*(d + e*x)^(1/
2)*(4*c*d*g - 2*b*e*g + c*e*f))/(c^4*e^3) + (2*g*x^3*(d + e*x)^(1/2))/(3*c
^3*e^2) - (x*(d + e*x)^(1/2)*(48*b^2*c*e^3*g - 24*b*c^2*e^3*f + 36*c^3*d*e
^2*f + 102*c^3*d^2*e*g - 144*b*c^2*d*e^2*g))/(3*c^6*e^5)))/(x^3 + (x*(3*b^
2*c^4*e^5 - 3*c^6*d^2*e^3))/(3*c^6*e^5) + (d*(b*e - c*d)^2)/(c^2*e^3) + (x
^2*(2*b*e - c*d))/(c*e))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{9/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{\frac{2}{3}c^3e^3g\,x^3-4b\,c^2e^3g\,x^2+8c^3d\,e^2g\,x^2+2c^3e^3f\,x^2-16b^2c\,e^3gx+48}{}$$

input
```
int((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output
```
(2*( - 16*b**3*e**3*g + 64*b**2*c*d*e**2*g + 8*b**2*c*e**3*f - 24*b**2*c*e
**3*g*x - 82*b*c**2*d**2*e*g - 20*b*c**2*d*e**2*f + 72*b*c**2*d*e**2*g*x +
 12*b*c**2*e**3*f*x - 6*b*c**2*e**3*g*x**2 + 34*c**3*d**3*g + 11*c**3*d**2
*e*f - 51*c**3*d**2*e*g*x - 18*c**3*d*e**2*f*x + 12*c**3*d*e**2*g*x**2 + 3
*c**3*e**3*f*x**2 + c**3*e**3*g*x**3))/(3*sqrt( - b*e + c*d - c*e*x)*c**4*
e**2*(b*e - c*d + c*e*x))
```

3.243 $\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 186

$$\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2(2cd-be)(cef+cdg-beg)(d+ex)^{3/2}}{3c^3e^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} - \frac{2(cef+3cdg-2beg)\sqrt{d+ex}}{c^3e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}} - \frac{2g\sqrt{d(cd-be)-be^2x-ce^2x^2}}{c^3e^2\sqrt{d+ex}}$$

output
```
2/3*(-b*e+2*c*d)*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(3/2)/c^3/e^2/(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(3/2)-2*(-2*b*e*g+3*c*d*g+c*e*f)*(e*x+d)^(1/2)/c^3/e^2/
(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-2*g*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(1/2)/c^3/e^2/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2\sqrt{d+ex}(8b^2e^2g-2bce(9dg+e(f-6gx))+c^2(10d^2g+de(f-15}{3c^3e^2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-}$$

input
```
Integrate[((d + e*x)^(7/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
)^(5/2),x]
```

output

```
(2*Sqrt[d + e*x]*(8*b^2*e^2*g - 2*b*c*e*(9*d*g + e*(f - 6*g*x)) + c^2*(10*
d^2*g + d*e*(f - 15*g*x) + 3*e^2*x*(-f + g*x))))/(3*c^3*e^2*(-(c*d) + b*e
+ c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1218, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{2(d+ex)^{7/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-4beg+7cdg+cef)\int \frac{(d+ex)^{5/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3ce(2cd-be)}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2(d+ex)^{7/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(-4beg+7cdg+cef)\left(\frac{2(2cd-be)\int \frac{(d+ex)^{3/2}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{c}-\frac{2(d+ex)^{3/2}}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3ce(2cd-be)}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2(d+ex)^{7/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{\left(\frac{4\sqrt{d+ex}(2cd-be)}{c^2e\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-4beg+7cdg+cef)}{3ce(2cd-be)}$$

input
```
Int[((d + e*x)^(7/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2
),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(7/2))/(3*c*e^2*(2*c*d - b*e)*(d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) - ((c*e*f + 7*c*d*g - 4*b*e*g)*((4*(
2*c*d - b*e)*Sqrt[d + e*x])/(c^2*e*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^
2]) - (2*(d + e*x)^(3/2))/(c*e*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]))
)/(3*c*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{2\sqrt{-(ex+d)(cex+be-cd)}\,(3g\,x^{2}c^{2}e^{2}+12bc\,e^{2}gx-15c^{2}degx-3c^{2}e^{2}fx+8b^{2}e^{2}g-18bcdeg-2bc\,e^{2}f+10c^{2}d^{2}g+c^{2}def)}{3\sqrt{ex+d}\,(cex+be-cd)^{2}c^{3}e^{2}}$	132
gosper	$\frac{2(cex+be-cd)\left(3g\,x^{2}c^{2}e^{2}+12bc\,e^{2}gx-15c^{2}degx-3c^{2}e^{2}fx+8b^{2}e^{2}g-18bcdeg-2bc\,e^{2}f+10c^{2}d^{2}g+c^{2}def\right)(ex+d)^{\frac{5}{2}}}{3c^{3}e^{2}(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2})^{\frac{5}{2}}}$	138
orering	$\frac{2(cex+be-cd)\left(3g\,x^{2}c^{2}e^{2}+12bc\,e^{2}gx-15c^{2}degx-3c^{2}e^{2}fx+8b^{2}e^{2}g-18bcdeg-2bc\,e^{2}f+10c^{2}d^{2}g+c^{2}def\right)(ex+d)^{\frac{5}{2}}}{3c^{3}e^{2}(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2})^{\frac{5}{2}}}$	138

input

```
int((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output

```
-2/3/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(3*c^2*e^2*g*x^2+12*b*
c*e^2*g*x-15*c^2*d*e*g*x-3*c^2*e^2*f*x+8*b^2*e^2*g-18*b*c*d*e*g-2*b*c*e^2*
f+10*c^2*d^2*g+c^2*d*e*f)/(c*e*x+b*e-c*d)^2/c^3/e^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{2\left(3\,c^{2}e^{2}gx^{2}+(c^{2}de-2\,bce^{2})f+2\,(5\,c^{2}d^{2}-9\,bcde+4\,b^{2}e^{2})g-3\,(c^{2}e^{2}f+(5\,c^{2}de-4\,bce^{2})g)x\right)\sqrt{-ce^{2}x}}{3\,(c^{5}e^{5}x^{3}+c^{5}d^{3}e^{2}-2\,bc^{4}d^{2}e^{3}+b^{2}c^{3}de^{4}-(c^{5}de^{4}-2\,bc^{4}e^{5})x^{2}-(c^{5}d^{2}e^{3}-b^{2}c^{3}e}$$

input

```
integrate((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output

```
-2/3*(3*c^2*e^2*g*x^2 + (c^2*d*e - 2*b*c*e^2)*f + 2*(5*c^2*d^2 - 9*b*c*d*e
 + 4*b^2*e^2)*g - 3*(c^2*e^2*f + (5*c^2*d*e - 4*b*c*e^2)*g)*x)*sqrt(-c*e^2
*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(e*x + d)/(c^5*e^5*x^3 + c^5*d^3*e^2 -
 2*b*c^4*d^2*e^3 + b^2*c^3*d*e^4 - (c^5*d*e^4 - 2*b*c^4*e^5)*x^2 - (c^5*d^
2*e^3 - b^2*c^3*e^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = -\frac{2\,(3\,cex-cd+2\,be)f}{3\,(c^3e^2x-c^3de+bc^2e^2)\sqrt{-cex+cd-be}} + \frac{2\,(3\,c^2e^2x^2+10\,c^2d^2-18\,bcde+8\,b^2e^2-3\,(5\,c^2de-4\,bce^2)x)g}{3\,(c^4e^3x-c^4de^2+bc^3e^3)\sqrt{-cex+cd-be}}$$

input
```
integrate((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output
```
-2/3*(3*c*e*x - c*d + 2*b*e)*f/((c^3*e^2*x - c^3*d*e + b*c^2*e^2)*sqrt(-c*
e*x + c*d - b*e)) + 2/3*(3*c^2*e^2*x^2 + 10*c^2*d^2 - 18*b*c*d*e + 8*b^2*e
^2 - 3*(5*c^2*d*e - 4*b*c*e^2)*x)*g/((c^4*e^3*x - c^4*d*e^2 + b*c^3*e^3)*s
qrt(-c*e*x + c*d - b*e))
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{2\Bigg(\frac{3\sqrt{-(ex+d)c+2\,cd-be}g}{c^3e}+\frac{2\,c^2def-bce^2f+2\,c^2d^2g-3\,bcdeg+b^2e^2g+3\,((ex+d)c-2\,cd+be)cef+9\,((ex+d)c-2\,cd+be)cdg-6\,((ex+d)c-2}{((ex+d)c-2\,cd+be)\sqrt{-(ex+d)c+2\,cd-be}c^3e}}{3\,e}$$

input
```
integrate((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output
```
-2/3*(3*sqrt(-(e*x + d)*c + 2*c*d - b*e)*g/(c^3*e) + (2*c^2*d*e*f - b*c*e^
2*f + 2*c^2*d^2*g - 3*b*c*d*e*g + b^2*e^2*g + 3*((e*x + d)*c - 2*c*d + b*e
)*c*e*f + 9*((e*x + d)*c - 2*c*d + b*e)*c*d*g - 6*((e*x + d)*c - 2*c*d + b
*e)*b*e*g)/(((e*x + d)*c - 2*c*d + b*e)*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c
^3*e))/e
```

Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{\left(\frac{\sqrt{d+e\,x}\,\left(16\,g\,b^2\,e^2-36\,g\,b\,c\,d\,e-4\,f\,b\,c\,e^2+20\,g\,c^2\,d^2+2\,f\,c^2\,d\,e\right)}{3\,c^5\,e^5}+\frac{2\,g\,x^2\,\sqrt{d+e\,x}}{c^3\,e^3}-\frac{2\,x\,\sqrt{d+e\,x}\,\left(5\,c\,d\,g-4\,b\,e\,g+c\,e\,f\right)}{c^4\,e^4}\right)\,\sqrt{c\,d^2-b\,d\,e}}{x^3+\frac{x\,\left(3\,b^2\,c^3\,e^5-3\,c^5\,d^2\,e^3\right)}{3\,c^5\,e^5}+\frac{d\,{\left(b\,e-c\,d\right)}^2}{c^2\,e^3}+\frac{x^2\,\left(2\,b\,e-c\,d\right)}{c\,e}}$$

input
```
int(((f + g*x)*(d + e*x)^(7/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2
),x)
```

output
```
-((((d + e*x)^(1/2)*(16*b^2*e^2*g + 20*c^2*d^2*g - 4*b*c*e^2*f + 2*c^2*d*e
*f - 36*b*c*d*e*g))/(3*c^5*e^5) + (2*g*x^2*(d + e*x)^(1/2))/(c^3*e^3) - (2
*x*(d + e*x)^(1/2)*(5*c*d*g - 4*b*e*g + c*e*f))/(c^4*e^4))*(c*d^2 - c*e^2*
x^2 - b*d*e - b*e^2*x)^(1/2))/(x^3 + (x*(3*b^2*c^3*e^5 - 3*c^5*d^2*e^3))/(
3*c^5*e^5) + (d*(b*e - c*d)^2)/(c^2*e^3) + (x^2*(2*b*e - c*d))/(c*e))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{7/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\frac{2c^2e^2g\,x^2+8bc\,e^2gx-10c^2degx-2c^2e^2fx+\frac{16}{3}b^2e^2g-12bcdeg-}{\sqrt{-cex-be+cd}\,c^3e^2\left(cex+be-cd\right)}$$

input
```
int((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output
```
(2*(8*b**2*e**2*g - 18*b*c*d*e*g - 2*b*c*e**2*f + 12*b*c*e**2*g*x + 10*c**
2*d**2*g + c**2*d*e*f - 15*c**2*d*e*g*x - 3*c**2*e**2*f*x + 3*c**2*e**2*g*
x**2))/(3*sqrt( - b*e + c*d - c*e*x)*c**3*e**2*(b*e - c*d + c*e*x))
```

3.244 $\int \frac{(d+ex)^{5/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 114

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\frac{2(cef+cdg-beg)(d+ex)^{3/2}}{3c^2e^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}}{c^2e^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

output
```
2/3*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(3/2)/c^2/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2
*x^2)^(3/2)-2*g*(e*x+d)^(1/2)/c^2/e^2/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=-\frac{2\sqrt{d+ex}(-2cdg+2beg+ce(f+3gx))}{3c^2e^2(-cd+be+cex)\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[((d + e*x)^(5/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
)^(5/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(-2*c*d*g + 2*b*e*g + c*e*(f + 3*g*x)))/(3*c^2*e^2*(-(c*
d) + b*e + c*e*x)*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1218, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(-bde-be^2x+cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 1218}$$

$$\frac{(2beg-5cdg+cef)\int \frac{(d+ex)^{3/2}}{(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{3ce(2cd-be)}+\frac{2(d+ex)^{5/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2\sqrt{d+ex}(2beg-5cdg+cef)}{3c^2e^2(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}+\frac{2(d+ex)^{5/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input
```
Int[((d + e*x)^(5/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2
),x]
```

output
```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(5/2))/(3*c*e^2*(2*c*d - b*e)*(d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) + (2*(c*e*f - 5*c*d*g + 2*b*e*g)*Sqr
t[d + e*x])/(3*c^2*e^2*(2*c*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*
x^2])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2\sqrt{-\left(ex+d\right)\left(cex+be-cd\right)}\,\left(3cegx+2beg-2cdg+fce\right)}{3\sqrt{ex+d}\,\left(cex+be-cd\right)^{2}c^{2}e^{2}}$	72
gosper	$-\frac{2\left(cex+be-cd\right)\left(3cegx+2beg-2cdg+fce\right)\left(ex+d\right)^{\frac{5}{2}}}{3c^{2}e^{2}\left(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}\right)^{\frac{5}{2}}}$	78
orering	$-\frac{2\left(cex+be-cd\right)\left(3cegx+2beg-2cdg+fce\right)\left(ex+d\right)^{\frac{5}{2}}}{3c^{2}e^{2}\left(-x^{2}c\,e^{2}-xb\,e^{2}-bde+c\,d^{2}\right)^{\frac{5}{2}}}$	78

input
```
int((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
2/3/(e*x+d)^(1/2)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(3*c*e*g*x+2*b*e*g-2*c*
d*g+c*e*f)/(c*e*x+b*e-c*d)^2/c^2/e^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{-ce^2x^2-be^2x+cd^2-bde}(3\,cegx+cef-2\,(cd-be)g)}{3\,(c^4e^5x^3+c^4d^3e^2-2\,bc^3d^2e^3+b^2c^2de^4-(c^4de^4-2\,bc^3e^5)x^2-(c^4}$$

input
```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output
```
2/3*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(3*c*e*g*x + c*e*f - 2*(c*d
 - b*e)*g)*sqrt(e*x + d)/(c^4*e^5*x^3 + c^4*d^3*e^2 - 2*b*c^3*d^2*e^3 + b^
2*c^2*d*e^4 - (c^4*d*e^4 - 2*b*c^3*e^5)*x^2 - (c^4*d^2*e^3 - b^2*c^2*e^5)*
x)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}\,(f+gx)}{(-\,(d+ex)\,(be-cd+cex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(5/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output
```
Integral((d + e*x)**(5/2)*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(5/2
), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = -\frac{2\,(3\,cex-2\,cd+2\,be)g}{3\,(c^3e^3x-c^3de^2+bc^2e^3)\sqrt{-cex+cd-be}} - \frac{2\,f}{3\,(c^2e^2x-c^2de+bce^2)\sqrt{-cex+cd-be}}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output

```
-2/3*(3*c*e*x - 2*c*d + 2*b*e)*g/((c^3*e^3*x - c^3*d*e^2 + b*c^2*e^3)*sqrt
(-c*e*x + c*d - b*e)) - 2/3*f/((c^2*e^2*x - c^2*d*e + b*c*e^2)*sqrt(-c*e*x
 + c*d - b*e))
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = -\frac{2\,(cef+cdg-beg+3\,((ex+d)c-2\,cd+be)g)}{3\,((ex+d)c-2\,cd+be)\sqrt{-(ex+d)c+2\,cd-be}c^2e^2}$$

input

```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output

```
-2/3*(c*e*f + c*d*g - b*e*g + 3*((e*x + d)*c - 2*c*d + b*e)*g)/(((e*x + d)
*c - 2*c*d + b*e)*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 11.92 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\frac{\left(\frac{\sqrt{d+e\,x}\,\left(4\,b\,e\,g-4\,c\,d\,g+2\,c\,e\,f\right)}{3\,c^4\,e^5}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^3\,e^4}\right)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2}}{x^3+\frac{x\,\left(3\,b^2\,c^2\,e^5-3\,c^4\,d^2\,e^3\right)}{3\,c^4\,e^5}+\frac{d\,{\left(b\,e-c\,d\right)}^2}{c^2\,e^3}+\frac{x^2\,\left(2\,b\,e-c\,d\right)}{c\,e}}$$

input
```
int(((f + g*x)*(d + e*x)^(5/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2
),x)
```

output
```
((((d + e*x)^(1/2)*(4*b*e*g - 4*c*d*g + 2*c*e*f))/(3*c^4*e^5) + (2*g*x*(d
+ e*x)^(1/2))/(c^3*e^4))*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2))/(x^3
 + (x*(3*b^2*c^2*e^5 - 3*c^4*d^2*e^3))/(3*c^4*e^5) + (d*(b*e - c*d)^2)/(c^
2*e^3) + (x^2*(2*b*e - c*d))/(c*e))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\frac{-2cegx-\frac{4}{3}beg+\frac{4}{3}cdg-\frac{2}{3}cef}{\sqrt{-cex-be+cd}\,c^2e^2\left(cex+be-cd\right)}$$

input
```
int((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output
```
(2*( - 2*b*e*g + 2*c*d*g - c*e*f - 3*c*e*g*x))/(3*sqrt( - b*e + c*d - c*e*
x)*c**2*e**2*(b*e - c*d + c*e*x))
```

3.245 $\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 221

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\frac{2(cef+cdg-beg)(d+ex)^{3/2}}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}+\frac{2(ef-dg)\sqrt{d+ex}}{e^2(2cd-be)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2(ef-dg)\text{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e^2(2cd-be)^{5/2}}$$

output

```
2/3*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(3/2)/c/e^2/(-b*e+2*c*d)/(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(3/2)+2*(-d*g+e*f)*(e*x+d)^(1/2)/e^2/(-b*e+2*c*d)^2/(d*(-
b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-2*(-d*g+e*f)*arctanh((-b*e+2*c*d)^(1/2)*
(e*x+d)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2/(-b*e+2*c*d)^(5/
2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2(d+ex)^{5/2}\left(-\frac{(-be+c(d-ex))\left(4bce^2f-b^2e^2g+c^2\left(d^2g+3e^2fx-de(5f+3gx)\right)\right)}{c(-2cd+be)^2}+3\right.}{3e^2((d+ex)(-be+c(d-ex)))}$$

input
```
Integrate[((d + e*x)^(3/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
)^(5/2),x]
```

output
```
(2*(d + e*x)^(5/2)*(-(((-(b*e) + c*(d - e*x))*(4*b*c*e^2*f - b^2*e^2*g + c
^2*(d^2*g + 3*e^2*f*x - d*e*(5*f + 3*g*x))))/(c*(-2*c*d + b*e)^2)) + (3*(e
*f - d*g)*(-(b*e) + c*(d - e*x))^(5/2)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt
[-2*c*d + b*e]])/(-2*c*d + b*e)^(5/2)))/(3*e^2*((d + e*x)*(-(b*e) + c*(d -
 e*x)))^(5/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1218, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(ef-dg)\int \frac{\sqrt{d+ex}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{e(2cd-be)}+\frac{2(d+ex)^{3/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{1132}$$

$$\frac{(ef-dg)\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e(2cd-be)}+\frac{2(d+ex)^{3/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{1136}$$

$$\frac{(ef-dg)\left(\frac{2e\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e(2cd-be)}+\frac{2(d+ex)^{3/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{(ef-dg)\left(\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}\right)}{e(2cd-be)}+\frac{2(d+ex)^{3/2}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input

```
Int[((d + e*x)^(3/2)*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2
),x]
```

output

```
(2*(c*e*f + c*d*g - b*e*g)*(d + e*x)^(3/2))/(3*c*e^2*(2*c*d - b*e)*(d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) + ((e*f - d*g)*((2*Sqrt[d + e*x])/(e
*(2*c*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]) - (2*ArcTanh[Sqr
t[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])
/(e*(2*c*d - b*e)^(3/2))))/(e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1132

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1218

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(203) = 406$.

Time = 1.54 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.16

method	result
default	$\frac{2\left(3\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c^{2}degx\sqrt{-cex-be+cd}-3\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)c^{2}e^{2}fx\sqrt{-cex-be+cd}+3\sqrt{-cex-be+cd}\,\arctan\left(\right.\right.}{}$

input

```
int((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output

```
2/3*(3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d*e*g*x*(-c*e*
x-b*e+c*d)^(1/2)-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*e^
2*f*x*(-c*e*x-b*e+c*d)^(1/2)+3*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c
*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c*d*e*g-3*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c
*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c*e^2*f-3*(-c*e*x-b*e+c*d)^(1/2)*
arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d^2*g+3*(-c*e*x-b*e+c
*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^2*d*e*f+3*(b*
e-2*c*d)^(1/2)*c^2*d*e*g*x-3*(b*e-2*c*d)^(1/2)*c^2*e^2*f*x+(b*e-2*c*d)^(1/
2)*b^2*e^2*g-4*(b*e-2*c*d)^(1/2)*b*c*e^2*f-(b*e-2*c*d)^(1/2)*c^2*d^2*g+5*(
b*e-2*c*d)^(1/2)*c^2*d*e*f)*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)/(b*e-2*c*d)^(
5/2)/c/e^2/(c*e*x+b*e-c*d)^2/(e*x+d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(203) = 406$.

Time = 0.21 (sec) , antiderivative size = 1439, normalized size of antiderivative = 6.51

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output
```
[-1/3*(3*((c^3*e^4*f - c^3*d*e^3*g)*x^3 - ((c^3*d*e^3 - 2*b*c^2*e^4)*f - (
c^3*d^2*e^2 - 2*b*c^2*d*e^3)*g)*x^2 + (c^3*d^3*e - 2*b*c^2*d^2*e^2 + b^2*c
*d*e^3)*f - (c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2)*g - ((c^3*d^2*e^2 -
b^2*c*e^4)*f - (c^3*d^3*e - b^2*c*d*e^3)*g)*x)*sqrt(2*c*d - b*e)*log(-(c*e
^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x - 2*sqrt(-c*e^2*x^2 - b*e
^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x
+ d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*((10*c^3*d^2*e - 13
*b*c^2*d*e^2 + 4*b^2*c*e^3)*f - (2*c^3*d^3 - b*c^2*d^2*e - 2*b^2*c*d*e^2 +
 b^3*e^3)*g - 3*((2*c^3*d*e^2 - b*c^2*e^3)*f - (2*c^3*d^2*e - b*c^2*d*e^2)
*g)*x)*sqrt(e*x + d))/(8*c^6*d^6*e^2 - 28*b*c^5*d^5*e^3 + 38*b^2*c^4*d^4*e
^4 - 25*b^3*c^3*d^3*e^5 + 8*b^4*c^2*d^2*e^6 - b^5*c*d*e^7 + (8*c^6*d^3*e^5
 - 12*b*c^5*d^2*e^6 + 6*b^2*c^4*d*e^7 - b^3*c^3*e^8)*x^3 - (8*c^6*d^4*e^4
- 28*b*c^5*d^3*e^5 + 30*b^2*c^4*d^2*e^6 - 13*b^3*c^3*d*e^7 + 2*b^4*c^2*e^8
)*x^2 - (8*c^6*d^5*e^3 - 12*b*c^5*d^4*e^4 - 2*b^2*c^4*d^3*e^5 + 11*b^3*c^3
*d^2*e^6 - 6*b^4*c^2*d*e^7 + b^5*c*e^8)*x), -2/3*(3*((c^3*e^4*f - c^3*d*e^
3*g)*x^3 - ((c^3*d*e^3 - 2*b*c^2*e^4)*f - (c^3*d^2*e^2 - 2*b*c^2*d*e^3)*g)
*x^2 + (c^3*d^3*e - 2*b*c^2*d^2*e^2 + b^2*c*d*e^3)*f - (c^3*d^4 - 2*b*c^2*
d^3*e + b^2*c*d^2*e^2)*g - ((c^3*d^2*e^2 - b^2*c*e^4)*f - (c^3*d^3*e - b^2
*c*d*e^3)*g)*x)*sqrt(-2*c*d + b*e)*arctan(-sqrt(-c*e^2*x^2 - b*e^2*x + c*d
^2 - b*d*e)*sqrt(-2*c*d + b*e)*sqrt(e*x + d)/(2*c*d^2 - b*d*e + (2*c*d*...
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output
```
Integral((d + e*x)**(3/2)*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(5/2
), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{5}{2}}}\,dx$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output

```
integrate((e*x + d)^(3/2)*(g*x + f)/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)
^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\frac{2\left(\frac{3\,(ef-dg)\arctan\left(\frac{\sqrt{-(ex+d)c+2\,cd-be}}{\sqrt{-2\,cd+be}}\right)}{(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-2\,cd+be}}-\frac{2\,c^2def-bce^2f+2\,c^2d^2g-3\,bcdeg+b^2e^2g-}{(4\,c^3d^2e-4\,bc^2de^2+b^2ce^3)((e}\right.}{3\,e}$$

input

```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output

```
2/3*(3*(e*f - d*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)/sqrt(-2*c*d + b
*e))/((4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*e^3)*sqrt(-2*c*d + b*e)) - (2*c^2*d
*e*f - b*c*e^2*f + 2*c^2*d^2*g - 3*b*c*d*e*g + b^2*e^2*g - 3*((e*x + d)*c
- 2*c*d + b*e)*c*e*f + 3*((e*x + d)*c - 2*c*d + b*e)*c*d*g)/((4*c^3*d^2*e
- 4*b*c^2*d*e^2 + b^2*c*e^3)*((e*x + d)*c - 2*c*d + b*e)*sqrt(-(e*x + d)*c
 + 2*c*d - b*e)))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(f+g\,x)\,(d+e\,x)^{3/2}}{(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)^{5/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^(3/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2
),x)
```

output
```
int(((f + g*x)*(d + e*x)^(3/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.66

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\frac{-2\sqrt{be-2cd}\,\sqrt{-cex-be+cd}\,atan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)bcdeg+2\sqrt{be-}}{}$$

input
```
int((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output
```
(2*( - 3*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c
*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*d*e*g + 3*sqrt(b*e - 2*c*d)*sqrt( - b*e
 + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*e**
2*f + 3*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*
d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*g - 3*sqrt(b*e - 2*c*d)*sqrt( - b*
e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d
*e*f - 3*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c
*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d*e*g*x + 3*sqrt(b*e - 2*c*d)*sqrt( -
b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2
*e**2*f*x - b**3*e**3*g + 2*b**2*c*d*e**2*g + 4*b**2*c*e**3*f + b*c**2*d**
2*e*g - 13*b*c**2*d*e**2*f - 3*b*c**2*d*e**2*g*x + 3*b*c**2*e**3*f*x - 2*c
**3*d**3*g + 10*c**3*d**2*e*f + 6*c**3*d**2*e*g*x - 6*c**3*d*e**2*f*x))/(3
*sqrt( - b*e + c*d - c*e*x)*c*e**2*(b**4*e**4 - 7*b**3*c*d*e**3 + b**3*c*e
**4*x + 18*b**2*c**2*d**2*e**2 - 6*b**2*c**2*d*e**3*x - 20*b*c**3*d**3*e +
 12*b*c**3*d**2*e**2*x + 8*c**4*d**4 - 8*c**4*d**3*e*x))
```

3.246 $\int \frac{\sqrt{d+ex}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 291

$$\begin{aligned}\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx &= \frac{2(cef+cdg-beg)(d+ex)^{3/2}}{3e^2(2cd-be)^2\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} \\ &+ \frac{2(2cf-bg)\sqrt{d+ex}}{e(2cd-be)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}} - \frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{e^2(2cd-be)^3(d+ex)^{3/2}} \\ &- \frac{(5cef-cdg-2beg)\operatorname{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{e^2(2cd-be)^{7/2}}\end{aligned}$$

output
```
2/3*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(3/2)/e^2/(-b*e+2*c*d)^2/(d*(-b*e+c*d)-b*
e^2*x-c*e^2*x^2)^(3/2)+2*(-b*g+2*c*f)*(e*x+d)^(1/2)/e/(-b*e+2*c*d)^3/(d*(-
b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(1/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^(3/2)-(-2*b*e*g-c*d*g+5*c*e*f)*arctan
h((-b*e+2*c*d)^(1/2)*(e*x+d)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))
/e^2/(-b*e+2*c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{d+ex}(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{(d+ex)^{5/2}\left(\frac{(-be+c(d-ex))\left(b^2e^2(-3ef+11dg+8egx)-2bce\left(4def+9d^2g+e^2x(10f-3g\right.\right.}{(2cd-be)^3(d+e}\right.}{3e^2}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^
(5/2),x]
```

output
```
((d + e*x)^(5/2)*(((-(b*e) + c*(d - e*x))*(b^2*e^2*(-3*e*f + 11*d*g + 8*e*
g*x) - 2*b*c*e*(4*d*e*f + 9*d^2*g + e^2*x*(10*f - 3*g*x)) + c^2*(7*d^3*g -
 15*e^3*f*x^2 + d^2*e*(13*f - 2*g*x) + d*e^2*x*(10*f + 3*g*x))))/((2*c*d -
 b*e)^3*(d + e*x)) + (3*(-5*c*e*f + c*d*g + 2*b*e*g)*(-(b*e) + c*(d - e*x)
)^(5/2)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/(-2*c*d + b*e)
^(7/2)))/(3*e^2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1218, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(-2beg-cdg+5cef)\int \frac{1}{\sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3ce(2cd-be)}+\frac{2\sqrt{d+ex}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(-2beg-cdg+5cef)\left(\frac{3c\int \frac{\sqrt{d+ex}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3ce(2cd-be)}+\frac{2\sqrt{d+ex}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1132$$

$$\frac{(-2beg-cdg+5cef)\left(\frac{3c\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-}}\right.}{3ce(2cd-be)}$$

$$\frac{2\sqrt{d+ex}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1136$$

$$\frac{(-2beg-cdg+5cef)\left(\frac{3c\left(\frac{2e\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2(2cd-be)}\right.}{3ce(2cd-be)}$$

$$\frac{2\sqrt{d+ex}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 221$$

$$\frac{\left(\frac{3c\left(\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-2beg}{3ce(2cd-be)}$$

$$\frac{2\sqrt{d+ex}(-beg+cdg+cef)}{3ce^2(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input

```
Int[(Sqrt[d + e*x]*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),
x]
```

output

```
(2*(c*e*f + c*d*g - b*e*g)*Sqrt[d + e*x])/(3*c*e^2*(2*c*d - b*e)*(d*(c*d -
 b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) + ((5*c*e*f - c*d*g - 2*b*e*g)*(-(1/(e
*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])) +
 (3*c*((2*Sqrt[d + e*x])/(e*(2*c*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x - c
*e^2*x^2]) - (2*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*
c*d - b*e]*Sqrt[d + e*x])])/(e*(2*c*d - b*e)^(3/2))))/(2*(2*c*d - b*e))))/
(3*c*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1132

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(269) = 538$.

Time = 1.61 (sec) , antiderivative size = 896, normalized size of antiderivative = 3.08

method	result
default	$-\frac{\sqrt{-(ex+d)(cex+be-cd)}\,\left(6\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)\sqrt{-cex-be+cd}\,bc\,e^3 g\,x^2+3\arctan\left(\frac{\sqrt{-cex-be+cd}}{\sqrt{be-2cd}}\right)\sqrt{-cex-be+cd}\,c^2 d\,e^2 g\,x}{}$

input
```
int((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output

```
-1/3*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(6*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*
e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*b*c*e^3*g*x^2+3*arctan((-c*e*x-b*e+
c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c^2*d*e^2*g*x^2-15*ar
ctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c^2*
e^3*f*x^2+6*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*e^3*g*x*(
-c*e*x-b*e+c*d)^(1/2)+3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(
-c*e*x-b*e+c*d)^(1/2)*b*c*d*e^2*g*x-15*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-
2*c*d)^(1/2))*b*c*e^3*f*x*(-c*e*x-b*e+c*d)^(1/2)+6*(b*e-2*c*d)^(1/2)*b*c*e
^3*g*x^2+3*(b*e-2*c*d)^(1/2)*c^2*d*e^2*g*x^2-15*(b*e-2*c*d)^(1/2)*c^2*e^3*
f*x^2+6*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*d*e^2*g*(-c*e
*x-b*e+c*d)^(1/2)-3*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e
*x-b*e+c*d)^(1/2)*b*c*d^2*e*g-15*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)
^(1/2))*b*c*d*e^2*f*(-c*e*x-b*e+c*d)^(1/2)-3*arctan((-c*e*x-b*e+c*d)^(1/2)
/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c^2*d^3*g+15*arctan((-c*e*x-b*e
+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*(-c*e*x-b*e+c*d)^(1/2)*c^2*d^2*e*f+8*(b*e-2
*c*d)^(1/2)*b^2*e^3*g*x-20*(b*e-2*c*d)^(1/2)*b*c*e^3*f*x-2*(b*e-2*c*d)^(1/
2)*c^2*d^2*e*g*x+10*(b*e-2*c*d)^(1/2)*c^2*d*e^2*f*x+11*(b*e-2*c*d)^(1/2)*b
^2*d*e^2*g-3*(b*e-2*c*d)^(1/2)*b^2*e^3*f-18*(b*e-2*c*d)^(1/2)*b*c*d^2*e*g-
8*(b*e-2*c*d)^(1/2)*b*c*d*e^2*f+7*(b*e-2*c*d)^(1/2)*c^2*d^3*g+13*(b*e-2*c*
d)^(1/2)*c^2*d^2*e*f)/(e*x+d)^(3/2)/(c*e*x+b*e-c*d)^2/e^2/(b*e-2*c*d)^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1039 vs. 2(269) = 538.

Time = 0.44 (sec) , antiderivative size = 2108, normalized size of antiderivative = 7.24

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output

```
[1/6*(3*((5*c^3*e^5*f - (c^3*d*e^4 + 2*b*c^2*e^5)*g)*x^4 + 2*(5*b*c^2*e^5*
f - (b*c^2*d*e^4 + 2*b^2*c*e^5)*g)*x^3 - (5*(2*c^3*d^2*e^3 - 2*b*c^2*d*e^4
 - b^2*c*e^5)*f - (2*c^3*d^3*e^2 + 2*b*c^2*d^2*e^3 - 5*b^2*c*d*e^4 - 2*b^3
*e^5)*g)*x^2 + 5*(c^3*d^4*e - 2*b*c^2*d^3*e^2 + b^2*c*d^2*e^3)*f - (c^3*d^
5 - 3*b^2*c*d^3*e^2 + 2*b^3*d^2*e^3)*g - 2*(5*(b*c^2*d^2*e^3 - b^2*c*d*e^4
)*f - (b*c^2*d^3*e^2 + b^2*c*d^2*e^3 - 2*b^3*d*e^4)*g)*x)*sqrt(2*c*d - b*e
)*log(-(c*e^2*x^2 - 3*c*d^2 + 2*b*d*e - 2*(c*d*e - b*e^2)*x + 2*sqrt(-c*e^
2*x^2 - b*e^2*x + c*d^2 - b*d*e)*sqrt(2*c*d - b*e)*sqrt(e*x + d))/(e^2*x^2
 + 2*d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(3*(5*(2
*c^3*d*e^3 - b*c^2*e^4)*f - (2*c^3*d^2*e^2 + 3*b*c^2*d*e^3 - 2*b^2*c*e^4)*
g)*x^2 - (26*c^3*d^3*e - 29*b*c^2*d^2*e^2 + 2*b^2*c*d*e^3 + 3*b^3*e^4)*f -
 (14*c^3*d^4 - 43*b*c^2*d^3*e + 40*b^2*c*d^2*e^2 - 11*b^3*d*e^3)*g - 2*(5*
(2*c^3*d^2*e^2 - 5*b*c^2*d*e^3 + 2*b^2*c*e^4)*f - (2*c^3*d^3*e - b*c^2*d^2
*e^2 - 8*b^2*c*d*e^3 + 4*b^3*e^4)*g)*x)*sqrt(e*x + d))/(16*c^6*d^8*e^2 - 6
4*b*c^5*d^7*e^3 + 104*b^2*c^4*d^6*e^4 - 88*b^3*c^3*d^5*e^5 + 41*b^4*c^2*d^
4*e^6 - 10*b^5*c*d^3*e^7 + b^6*d^2*e^8 + (16*c^6*d^4*e^6 - 32*b*c^5*d^3*e^
7 + 24*b^2*c^4*d^2*e^8 - 8*b^3*c^3*d*e^9 + b^4*c^2*e^10)*x^4 + 2*(16*b*c^5
*d^4*e^6 - 32*b^2*c^4*d^3*e^7 + 24*b^3*c^3*d^2*e^8 - 8*b^4*c^2*d*e^9 + b^5
*c*e^10)*x^3 - (32*c^6*d^6*e^4 - 96*b*c^5*d^5*e^5 + 96*b^2*c^4*d^4*e^6 - 3
2*b^3*c^3*d^3*e^7 - 6*b^4*c^2*d^2*e^8 + 6*b^5*c*d*e^9 - b^6*e^10)*x^2 -...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{\sqrt{d+ex}(f+gx)}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate((e*x+d)**(1/2)*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output

```
Integral(sqrt(d + e*x)*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(5/2),
x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \int \frac{\sqrt{ex+d}(gx+f)}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(
5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \frac{\frac{3(5cef-cdg-2beg)\arctan\left(\frac{\sqrt{-(ex+d)c+2cd-be}}{\sqrt{-2cd+be}}\right)}{(8c^3d^3e-12bc^2d^2e^2+6b^2cde^3-b^3e^4)\sqrt{-2cd+be}} - \frac{2(2c^2def-bce^2f+2c^2d^2g-3bcdeg+}{(8c^3d^3e-12bc^2d^2e^2+6b^2cd}}{}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output
```
1/3*(3*(5*c*e*f - c*d*g - 2*b*e*g)*arctan(sqrt(-(e*x + d)*c + 2*c*d - b*e)
/sqrt(-2*c*d + b*e))/((8*c^3*d^3*e - 12*b*c^2*d^2*e^2 + 6*b^2*c*d*e^3 - b^
3*e^4)*sqrt(-2*c*d + b*e)) - 2*(2*c^2*d*e*f - b*c*e^2*f + 2*c^2*d^2*g - 3*
b*c*d*e*g + b^2*e^2*g - 6*((e*x + d)*c - 2*c*d + b*e)*c*e*f + 3*((e*x + d)
*c - 2*c*d + b*e)*b*e*g)/((8*c^3*d^3*e - 12*b*c^2*d^2*e^2 + 6*b^2*c*d*e^3
- b^3*e^4)*((e*x + d)*c - 2*c*d + b*e)*sqrt(-(e*x + d)*c + 2*c*d - b*e)) -
 3*(sqrt(-(e*x + d)*c + 2*c*d - b*e)*c*e*f - sqrt(-(e*x + d)*c + 2*c*d - b
*e)*c*d*g)/((8*c^3*d^3*e - 12*b*c^2*d^2*e^2 + 6*b^2*c*d*e^3 - b^3*e^4)*(e*
x + d)*c))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(f+g\,x)\,\sqrt{d+e\,x}}{(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)^{5/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^(1/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2
),x)
```

output
```
int(((f + g*x)*(d + e*x)^(1/2))/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1124, normalized size of antiderivative = 3.86

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(6*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c
*e*x)/sqrt(b*e - 2*c*d))*b**2*d*e**2*g + 6*sqrt(b*e - 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b**2*e**3
*g*x - 3*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c
*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*d**2*e*g - 15*sqrt(b*e - 2*c*d)*sqrt( -
 b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c
*d*e**2*f + 3*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*
e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*d*e**2*g*x - 15*sqrt(b*e - 2*c*d)*
sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*
d))*b*c*e**3*f*x + 6*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqr
t( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c*e**3*g*x**2 - 3*sqrt(b*e -
2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e
 - 2*c*d))*c**2*d**3*g + 15*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*a
tan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*d**2*e*f + 3*sqrt(b
*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqr
t(b*e - 2*c*d))*c**2*d*e**2*g*x**2 - 15*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*
d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**2*e**3*f*
x**2 + 11*b**3*d*e**3*g - 3*b**3*e**4*f + 8*b**3*e**4*g*x - 40*b**2*c*d**2
*e**2*g - 2*b**2*c*d*e**3*f - 16*b**2*c*d*e**3*g*x - 20*b**2*c*e**4*f*x +
6*b**2*c*e**4*g*x**2 + 43*b*c**2*d**3*e*g + 29*b*c**2*d**2*e**2*f - 2*b...
```

3.247 $\int \frac{f+gx}{\sqrt{d+ex}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 377

$$
\begin{aligned}
\int \frac{f+gx}{\sqrt{d+ex}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx &= \frac{2c(cef+cdg-beg)(d+ex)^{3/2}}{3e^2(2cd-be)^3\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}} \\
&+ \frac{2c(3cef+cdg-2beg)\sqrt{d+ex}}{e^2(2cd-be)^4\sqrt{d(cd-be)-be^2x-ce^2x^2}} - \frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2e^2(2cd-be)^3(d+ex)^{5/2}} \\
&- \frac{(11cef-3cdg-4beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4e^2(2cd-be)^4(d+ex)^{3/2}} \\
&- \frac{5c(7cef+cdg-4beg)\operatorname{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4e^2(2cd-be)^{9/2}}
\end{aligned}
$$

output

```
2/3*c*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(3/2)/e^2/(-b*e+2*c*d)^3/(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(3/2)+2*c*(-2*b*e*g+c*d*g+3*c*e*f)*(e*x+d)^(1/2)/e^2/(-
b*e+2*c*d)^4/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-1/2*(-d*g+e*f)*(d*(-b*
e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^(5/2)-1/4*(-4*b
*e*g-3*c*d*g+11*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*
c*d)^4/(e*x+d)^(3/2)-5/4*c*(-4*b*e*g+c*d*g+7*c*e*f)*arctanh((-b*e+2*c*d)^(
1/2)*(e*x+d)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2/(-b*e+2*c*d
)^(9/2)
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.87

$$\int \frac{f+gx}{\sqrt{d+ex}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{c(d+ex)^{5/2}\left(\frac{(-be+c(d-ex))\left(6b^3e^3(dg+e(f+2gx))+c^3\left(61d^4g-105e^4fx^3\right.\right.}{}\right.}{}$$

input
```
Integrate[(f + g*x)/(Sqrt[d + e*x]*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(
5/2)),x]
```

output
```
(c*(d + e*x)^(5/2)*(((-(b*e) + c*(d - e*x))*(6*b^3*e^3*(d*g + e*(f + 2*g*x
)) + c^3*(61*d^4*g - 105*e^4*f*x^3 + d^2*e^2*x*(161*f - 5*g*x) - 5*d*e^3*x
^2*(7*f + 3*g*x) + d^3*e*(43*f + 23*g*x)) - 4*b*c^2*e*(33*d^3*g + 49*d*e^2
*f*x + 5*e^3*x^2*(7*f - 3*g*x) + d^2*e*(-4*f + 30*g*x)) + b^2*c*e^2*(65*d^
2*g + e^2*x*(-21*f + 80*g*x) + d*e*(-57*f + 109*g*x))))/(c*(-2*c*d + b*e)^
4*(d + e*x)^2) + (15*(7*c*e*f + c*d*g - 4*b*e*g)*(-(b*e) + c*(d - e*x))^(5
/2)*ArcTan[Sqrt[c*d - b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/(-2*c*d + b*e)^(9/
2)))/(12*e^2*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1220, 1132, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\sqrt{d+ex}\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(-4beg+cdg+7cef)\int \frac{\sqrt{d+ex}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{4e(2cd-be)}-\frac{ef-dg}{2e^2\sqrt{d+ex}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1132$$

$$\frac{(-4beg+cdg+7cef)\left(\frac{5\int \frac{1}{\sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3(2cd-be)}+\frac{2\sqrt{d+ex}}{3e(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{3/2}}\right)}{4e(2cd-be)}-\frac{ef-dg}{2e^2\sqrt{d+ex}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1135$$

$$\frac{(-4beg+cdg+7cef)\left(\frac{5\left(\frac{3c\int \frac{\sqrt{d+ex}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3(2cd-be)}+\frac{2\sqrt{d+ex}}{3e(2cd-be)(d(cd-be)-be^2}\right.}{4e(2cd-be)}$$

$$\frac{ef-dg}{2e^2\sqrt{d+ex}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1132$$

$$\frac{(-4beg+cdg+7cef)\left(\frac{5\left(\frac{3c\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x}}\right.}{3(2cd-be)}\right.}{4e(2cd-be)}$$

$$\frac{ef-dg}{2e^2\sqrt{d+ex}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow 1136$$

$$\frac{(-4beg+cdg+7cef)\left(\frac{5\left(\frac{3c\left(\frac{2e\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right.}{2(2cd-be)}\right.}{3(2cd-be)}\right.}{4e(2cd-be)}$$

$$\frac{ef-dg}{2e^2\sqrt{d+ex}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(\frac{5\left(\frac{3c\left(\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3(2cd-be)}+\frac{}{3e(2cd-be)(}\right.}{4e(2cd-be)}$$

$$\frac{ef-dg}{2e^2\sqrt{d+ex}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input

```
Int[(f + g*x)/(Sqrt[d + e*x]*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2)),
x]
```

output

```
-1/2*(e*f - d*g)/(e^2*(2*c*d - b*e)*Sqrt[d + e*x]*(d*(c*d - b*e) - b*e^2*x
 - c*e^2*x^2)^(3/2)) + ((7*c*e*f + c*d*g - 4*b*e*g)*((2*Sqrt[d + e*x])/(3*
e*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) + (5*(-(1/(e*
(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])) +
(3*c*((2*Sqrt[d + e*x])/(e*(2*c*d - b*e)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*
e^2*x^2]) - (2*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(Sqrt[2*c
*d - b*e]*Sqrt[d + e*x])])/(e*(2*c*d - b*e)^(3/2))))/(2*(2*c*d - b*e))))/(
3*(2*c*d - b*e))))/(4*e*(2*c*d - b*e))
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1132

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1519 vs. $2(345) = 690$.

Time = 1.61 (sec) , antiderivative size = 1520, normalized size of antiderivative = 4.03

method	result	size
default	Expression too large to display	1520

input

```
int((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
1/12*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-196*(b*e-2*c*d)^(1/2)*b*c^2*d*e^3*
f*x-90*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1
/2))*b*c^2*d^2*e^2*g*x-210*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2)
)*b*c^2*d*e^3*f*x*(-c*e*x-b*e+c*d)^(1/2)-105*(-c*e*x-b*e+c*d)^(1/2)*arctan
((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*c^3*e^4*f*x^3+60*(b*e-2*c*d)^(1
/2)*b*c^2*e^4*g*x^3-15*(b*e-2*c*d)^(1/2)*c^3*d*e^3*g*x^3+80*(b*e-2*c*d)^(1
/2)*b^2*c*e^4*g*x^2-140*(b*e-2*c*d)^(1/2)*b*c^2*e^4*f*x^2-5*(b*e-2*c*d)^(1
/2)*c^3*d^2*e^2*g*x^2+45*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1
/2)/(b*e-2*c*d)^(1/2))*b*c^2*d*e^3*g*x^2-105*(b*e-2*c*d)^(1/2)*c^3*e^4*f*x
^3+15*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/
2))*c^3*d^4*g+12*(b*e-2*c*d)^(1/2)*b^3*e^4*g*x+6*(b*e-2*c*d)^(1/2)*b^3*d*e
^3*g+43*(b*e-2*c*d)^(1/2)*c^3*d^3*e*f-75*(-c*e*x-b*e+c*d)^(1/2)*arctan((-c
*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d^3*e*g-105*arctan((-c*e*x-b*
e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*d^2*e^2*f*(-c*e*x-b*e+c*d)^(1/2)+60*
arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c*e^4*g*x^2*(-c*e*x-b
*e+c*d)^(1/2)-105*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^2*e
^4*f*x^2*(-c*e*x-b*e+c*d)^(1/2)+120*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c
*d)^(1/2))*b^2*c*d*e^3*g*x*(-c*e*x-b*e+c*d)^(1/2)+109*(b*e-2*c*d)^(1/2)*b^
2*c*d*e^3*g*x-120*(b*e-2*c*d)^(1/2)*b*c^2*d^2*e^2*g*x+60*arctan((-c*e*x-b*
e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c*d^2*e^2*g*(-c*e*x-b*e+c*d)^(1/2)+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. 2(345) = 690.

Time = 1.58 (sec) , antiderivative size = 3098, normalized size of antiderivative = 8.22

$$\int \frac{f+gx}{\sqrt{d+ex}\,(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{f+gx}{\sqrt{d+ex}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \int \frac{f+gx}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{5}{2}}\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**(1/2)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output
```
Integral((f + g*x)/((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*sqrt(d + e*x))
, x)
```

Maxima [F]

$$\int \frac{f+gx}{\sqrt{d+ex}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \int \frac{gx+f}{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output
```
integrate((g*x + f)/((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*sqrt(e*x
 + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.63

$$\int \frac{f+gx}{\sqrt{d+ex}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{\frac{15\left(7c^2ef+c^2dg-4bceg\right)\arctan\left(\frac{\sqrt{-(ex+d)c+2cd-be}}{\sqrt{-2cd+be}}\right)}{\left(16c^4d^4e-32bc^3d^3e^2+24b^2c^2d^2e^3-8b^3cde^4+b^4e^5\right)\sqrt{-2cd+be}} - \frac{8\left(2c^3def-\right.}{}}{}$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output
```
1/12*(15*(7*c^2*e*f + c^2*d*g - 4*b*c*e*g)*arctan(sqrt(-(e*x + d)*c + 2*c*
d - b*e)/sqrt(-2*c*d + b*e))/((16*c^4*d^4*e - 32*b*c^3*d^3*e^2 + 24*b^2*c^
2*d^2*e^3 - 8*b^3*c*d*e^4 + b^4*e^5)*sqrt(-2*c*d + b*e)) - 8*(2*c^3*d*e*f
- b*c^2*e^2*f + 2*c^3*d^2*g - 3*b*c^2*d*e*g + b^2*c*e^2*g - 9*((e*x + d)*c
 - 2*c*d + b*e)*c^2*e*f - 3*((e*x + d)*c - 2*c*d + b*e)*c^2*d*g + 6*((e*x
+ d)*c - 2*c*d + b*e)*b*c*e*g)/((16*c^4*d^4*e - 32*b*c^3*d^3*e^2 + 24*b^2*
c^2*d^2*e^3 - 8*b^3*c*d*e^4 + b^4*e^5)*((e*x + d)*c - 2*c*d + b*e)*sqrt(-(
e*x + d)*c + 2*c*d - b*e)) - 3*(26*sqrt(-(e*x + d)*c + 2*c*d - b*e)*c^3*d*
e*f - 13*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^2*e^2*f - 10*sqrt(-(e*x + d)
*c + 2*c*d - b*e)*c^3*d^2*g - 3*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b*c^2*d*e
*g + 4*sqrt(-(e*x + d)*c + 2*c*d - b*e)*b^2*c*e^2*g - 11*(-(e*x + d)*c + 2
*c*d - b*e)^(3/2)*c^2*e*f + 3*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*c^2*d*g +
 4*(-(e*x + d)*c + 2*c*d - b*e)^(3/2)*b*c*e*g)/((16*c^4*d^4*e - 32*b*c^3*d
^3*e^2 + 24*b^2*c^2*d^2*e^3 - 8*b^3*c*d*e^4 + b^4*e^5)*(e*x + d)^2*c^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\sqrt{d+ex}\,(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{f+g\,x}{\sqrt{d+e\,x}\,(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)^{5/2}}\,dx$$

input
```
int((f + g*x)/((d + e*x)^(1/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2)
),x)
```

output
```
int((f + g*x)/((d + e*x)^(1/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1902, normalized size of antiderivative = 5.05

$$\int \frac{f+gx}{\sqrt{d+ex}\,(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
( - 60*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d
 - c*e*x)/sqrt(b*e - 2*c*d))*b**2*c*d**2*e**2*g - 120*sqrt(b*e - 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*b**2*c*d*e**3*g*x - 60*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan
(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b**2*c*e**4*g*x**2 + 75*sqr
t(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/
sqrt(b*e - 2*c*d))*b*c**2*d**3*e*g + 105*sqrt(b*e - 2*c*d)*sqrt( - b*e + c
*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d**2
*e**2*f + 90*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e
 + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d**2*e**2*g*x + 210*sqrt(b*e - 2
*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e
- 2*c*d))*b*c**2*d*e**3*f*x - 45*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e
*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*d*e**3*g*x**
2 + 105*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*
d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**2*e**4*f*x**2 - 60*sqrt(b*e - 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d)
)*b*c**2*e**4*g*x**3 - 15*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*ata
n(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*c**3*d**4*g - 105*sqrt(b*e
 - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(
b*e - 2*c*d))*c**3*d**3*e*f - 15*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - ...
```

3.248 $\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 459

$$\begin{aligned}
&\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{2c^2(cef+cdg-beg)(d+ex)^{3/2}}{3e^2(2cd-be)^4\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\\
&+\frac{2c^2(4cef+2cdg-3beg)\sqrt{d+ex}}{e^2(2cd-be)^5\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3e^2(2cd-be)^3(d+ex)^{7/2}}\\
&-\frac{(17cef-5cdg-6beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{12e^2(2cd-be)^4(d+ex)^{5/2}}\\
&-\frac{c(41cef+3cdg-22beg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{8e^2(2cd-be)^5(d+ex)^{3/2}}\\
&-\frac{35c^2(3cef+cdg-2beg)\operatorname{arctanh}\left(\frac{\sqrt{2cd-be}\sqrt{d+ex}}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8e^2(2cd-be)^{11/2}}
\end{aligned}$$

output
```
2/3*c^2*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(3/2)/e^2/(-b*e+2*c*d)^4/(d*(-b*e+c*d
)-b*e^2*x-c*e^2*x^2)^(3/2)+2*c^2*(-3*b*e*g+2*c*d*g+4*c*e*f)*(e*x+d)^(1/2)/
e^2/(-b*e+2*c*d)^5/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-1/3*(-d*g+e*f)*(
d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^3/(e*x+d)^(7/2)-1/1
2*(-6*b*e*g-5*c*d*g+17*c*e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(
-b*e+2*c*d)^4/(e*x+d)^(5/2)-1/8*c*(-22*b*e*g+3*c*d*g+41*c*e*f)*(d*(-b*e+c*
d)-b*e^2*x-c*e^2*x^2)^(1/2)/e^2/(-b*e+2*c*d)^5/(e*x+d)^(3/2)-35/8*c^2*(-2*
b*e*g+c*d*g+3*c*e*f)*arctanh((-b*e+2*c*d)^(1/2)*(e*x+d)^(1/2)/(d*(-b*e+c*d
)-b*e^2*x-c*e^2*x^2)^(1/2))/e^2/(-b*e+2*c*d)^(11/2)
```

Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.95

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \frac{c^2(d+ex)^{5/2}\left(\frac{(-be+c(d-ex))\left(-4b^4e^4(2ef+dg+3egx)+2b^3ce^3\left(28d^2g\right.\right.}{}\right.}{}$$

input
```
Integrate[(f + g*x)/((d + e*x)^(3/2)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)
^(5/2)),x]
```

output
```
(c^2*(d + e*x)^(5/2)*(((-(b*e) + c*(d - e*x))*(-4*b^4*e^4*(2*e*f + d*g + 3
*e*g*x) + 2*b^3*c*e^3*(28*d^2*g + 3*e^2*x*(3*f + 7*g*x) + d*e*(41*f + 81*g
*x)) + c^4*(171*d^5*g - 315*e^5*f*x^4 + 14*d^2*e^3*x^2*(27*f - 10*g*x) - 1
05*d*e^4*x^3*(4*f + g*x) + 18*d^3*e^2*x*(34*f + 7*g*x) + d^4*e*(f + 204*g*
x)) + b^2*c^2*e^2*(103*d^3*g + 7*e^3*x^2*(-9*f + 40*g*x) + 3*d*e^2*x*(-78*
f + 217*g*x) + d^2*e*(-363*f + 282*g*x)) - 2*b*c^3*e*(163*d^4*g + 14*d*e^3
*x^2*(36*f - 5*g*x) - 105*e^4*x^3*(-2*f + g*x) + 6*d^2*e^2*x*(45*f + 49*g*
x) + d^3*e*(-152*f + 294*g*x))))/(c^2*(2*c*d - b*e)^5*(d + e*x)^3) - (105*
(3*c*e*f + c*d*g - 2*b*e*g)*(-(b*e) + c*(d - e*x))^(5/2)*ArcTan[Sqrt[c*d -
 b*e - c*e*x]/Sqrt[-2*c*d + b*e]])/(-2*c*d + b*e)^(11/2)))/(24*e^2*((d + e
*x)*(-(b*e) + c*(d - e*x)))^(5/2))
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1220, 1135, 1132, 1135, 1132, 1136, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(-2beg+cdg+3cef)\int \frac{1}{\sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{2e(2cd-be)}-\frac{ef-dg}{3e^2(d+ex)^{3/2}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{7c\int \frac{\sqrt{d+ex}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{4(2cd-be)}-\frac{1}{2e\sqrt{d+ex}(2cd-be)(d(cd-be)-be^2x-ce^2x^2)^{3/2}}\right)}{2e(2cd-be)}-\frac{ef-dg}{3e^2(d+ex)^{3/2}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1132}$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{7c\left(\frac{5\int \frac{1}{\sqrt{d+ex}\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{3(2cd-be)}+\frac{2\sqrt{d+ex}}{3e(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}\right)}{4(2cd-be)}-\frac{}{2e\sqrt{d+ex}(2cd-be)(d(}\right.}{2e(2cd-be)}$$

$$\frac{ef-dg}{3e^2(d+ex)^{3/2}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{7c\left(\frac{5\left(\frac{3c\int \frac{\sqrt{d+ex}}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}}dx}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3(2cd-be)}+\frac{2\sqrt{d+ex}}{3e(2cd-be)\left(d(cd-be)-be^2x}}{4(2cd-be)}}{2e(2cd-be)}$$

$$\frac{ef-dg}{3e^2(d+ex)^{3/2}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{1132}$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{7c\left(\frac{5\left(\frac{3c\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-}}}{3(2cd-be)}}{4(2cd-be)}}{2e(2cd-be)}$$

$$\frac{ef-dg}{3e^2(d+ex)^{3/2}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(-2beg+cdg+3cef)\left(\frac{7c\left(\frac{5\left(\frac{3c\left(\frac{2e\int \frac{1}{\frac{e^2\left(-cx^2e^2-bxe^2+d(cd-be)\right)}{d+ex}-e^2(2cd-be)}d\frac{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}{\sqrt{d+ex}}}{2cd-be}+\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce}}\right.}{2(2cd-be)}\right.}{3(2cd-be)}\right.}{4(2cd-be)}\right.}{2e(2c}$$

$$\frac{ef-dg}{3e^2(d+ex)^{3/2}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(\frac{7c\left(\frac{5\left(\frac{3c\left(\frac{2\sqrt{d+ex}}{e(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{\sqrt{d+ex}\sqrt{2cd-be}}\right)}{e(2cd-be)^{3/2}}\right)}{2(2cd-be)}-\frac{1}{e\sqrt{d+ex}(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{3(2cd-be)}+\frac{}{3e(2cd-be)}\right)}{4(2cd-be)}\right.}{2e(2cd-be)}$$

$$\frac{ef-dg}{3e^2(d+ex)^{3/2}(2cd-be)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

input

```
Int[(f + g*x)/((d + e*x)^(3/2)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2)
),x]
```

output
```
-1/3*(e*f - d*g)/(e^2*(2*c*d - b*e)*(d + e*x)^(3/2)*(d*(c*d - b*e) - b*e^2
*x - c*e^2*x^2)^(3/2)) + ((3*c*e*f + c*d*g - 2*b*e*g)*(-1/2*1/(e*(2*c*d -
b*e)*Sqrt[d + e*x]*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)) + (7*c*((2
*Sqrt[d + e*x])/(3*e*(2*c*d - b*e)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(
3/2)) + (5*(-(1/(e*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[d*(c*d - b*e) - b*e^2*
x - c*e^2*x^2])) + (3*c*((2*Sqrt[d + e*x])/(e*(2*c*d - b*e)*Sqrt[d*(c*d -
b*e) - b*e^2*x - c*e^2*x^2]) - (2*ArcTanh[Sqrt[d*(c*d - b*e) - b*e^2*x - c
*e^2*x^2]/(Sqrt[2*c*d - b*e]*Sqrt[d + e*x])])/(e*(2*c*d - b*e)^(3/2))))/(2
*(2*c*d - b*e))))/(3*(2*c*d - b*e))))/(4*(2*c*d - b*e))))/(2*e*(2*c*d - b*
e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2002 vs. $2(421) = 842$.

Time = 1.58 (sec) , antiderivative size = 2003, normalized size of antiderivative = 4.36

method	result	size
default	Expression too large to display	2003

input

```
int((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
-1/24*(-(e*x+d)*(c*e*x+b*e-c*d))^(1/2)*(-945*arctan((-c*e*x-b*e+c*d)^(1/2)
/(b*e-2*c*d)^(1/2))*b*c^3*d*e^4*f*x^2*(-c*e*x-b*e+c*d)^(1/2)+162*(b*e-2*c*
d)^(1/2)*b^3*c*d*e^4*g*x+282*(b*e-2*c*d)^(1/2)*b^2*c^2*d^2*e^3*g*x-234*(b*
e-2*c*d)^(1/2)*b^2*c^2*d*e^4*f*x-588*(b*e-2*c*d)^(1/2)*b*c^3*d^3*e^2*g*x-5
40*(b*e-2*c*d)^(1/2)*b*c^3*d^2*e^3*f*x+140*(b*e-2*c*d)^(1/2)*b*c^3*d*e^4*g
*x^3-588*(b*e-2*c*d)^(1/2)*b*c^3*d^2*e^3*g*x^2-1008*(b*e-2*c*d)^(1/2)*b*c^
3*d*e^4*f*x^2-315*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^3*d
^3*e^2*f*(-c*e*x-b*e+c*d)^(1/2)-105*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c
*d)^(1/2))*c^4*d*e^4*g*x^4*(-c*e*x-b*e+c*d)^(1/2)+210*arctan((-c*e*x-b*e+c
*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c^2*e^5*g*x^3*(-c*e*x-b*e+c*d)^(1/2)+82*(
b*e-2*c*d)^(1/2)*b^3*c*d*e^4*f+103*(b*e-2*c*d)^(1/2)*b^2*c^2*d^3*e^2*g+210
*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^3*e^5*g*x^4*(-c*e*x-
b*e+c*d)^(1/2)-315*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^3*
d^4*e*g*(-c*e*x-b*e+c*d)^(1/2)+651*(b*e-2*c*d)^(1/2)*b^2*c^2*d*e^4*g*x^2+6
30*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c^2*d^2*e^3*g*x*(-
c*e*x-b*e+c*d)^(1/2)-735*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*
b*c^3*d^3*e^2*g*x*(-c*e*x-b*e+c*d)^(1/2)+315*arctan((-c*e*x-b*e+c*d)^(1/2)
/(b*e-2*c*d)^(1/2))*b*c^3*d*e^4*g*x^3*(-c*e*x-b*e+c*d)^(1/2)+630*arctan((-
c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b^2*c^2*d*e^4*g*x^2*(-c*e*x-b*e+c*
d)^(1/2)-315*arctan((-c*e*x-b*e+c*d)^(1/2)/(b*e-2*c*d)^(1/2))*b*c^3*d^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2028 vs. $2(421) = 842$.

Time = 3.37 (sec) , antiderivative size = 4086, normalized size of antiderivative = 8.90

$$\int \frac{f + gx}{(d + ex)^{3/2} \left(cd^2 - bde - be^2x - ce^2x^2\right)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\int \frac{f+gx}{\left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{5}{2}}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**(3/2)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/
2),x)
```

output
```
Integral((f + g*x)/((-(d + e*x)*(b*e - c*d + c*e*x))**(5/2)*(d + e*x)**(3/
2)), x)
```

Maxima [F]

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\int \frac{gx+f}{\left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{5}{2}}\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="maxima")
```

output
```
integrate((g*x + f)/((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/2)*(e*x + d
)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(421) = 842$.

Time = 0.39 (sec) , antiderivative size = 1067, normalized size of antiderivative = 2.32

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x,
algorithm="giac")
```

output

```
1/24*(105*(3*c^3*e*f + c^3*d*g - 2*b*c^2*e*g)*arctan(sqrt(-(e*x + d)*c + 2
*c*d - b*e)/sqrt(-2*c*d + b*e))/((32*c^5*d^5*e - 80*b*c^4*d^4*e^2 + 80*b^2
*c^3*d^3*e^3 - 40*b^3*c^2*d^2*e^4 + 10*b^4*c*d*e^5 - b^5*e^6)*sqrt(-2*c*d
+ b*e)) + (256*c^7*d^4*e*f - 512*b*c^6*d^3*e^2*f + 384*b^2*c^5*d^2*e^3*f -
 128*b^3*c^4*d*e^4*f + 16*b^4*c^3*e^5*f + 256*c^7*d^5*g - 768*b*c^6*d^4*e*
g + 896*b^2*c^5*d^3*e^2*g - 512*b^3*c^4*d^2*e^3*g + 144*b^4*c^3*d*e^4*g -
16*b^5*c^2*e^5*g - 1152*((e*x + d)*c - 2*c*d + b*e)*c^6*d^3*e*f + 1728*((e
*x + d)*c - 2*c*d + b*e)*b*c^5*d^2*e^2*f - 864*((e*x + d)*c - 2*c*d + b*e)
*b^2*c^4*d*e^3*f + 144*((e*x + d)*c - 2*c*d + b*e)*b^3*c^3*e^4*f - 384*((e
*x + d)*c - 2*c*d + b*e)*c^6*d^4*g + 1344*((e*x + d)*c - 2*c*d + b*e)*b*c^
5*d^3*e*g - 1440*((e*x + d)*c - 2*c*d + b*e)*b^2*c^4*d^2*e^2*g + 624*((e*x
 + d)*c - 2*c*d + b*e)*b^3*c^3*d*e^3*g - 96*((e*x + d)*c - 2*c*d + b*e)*b^
4*c^2*e^4*g - 2772*((e*x + d)*c - 2*c*d + b*e)^2*c^5*d^2*e*f + 2772*((e*x
+ d)*c - 2*c*d + b*e)^2*b*c^4*d*e^2*f - 693*((e*x + d)*c - 2*c*d + b*e)^2*
b^2*c^3*e^3*f - 924*((e*x + d)*c - 2*c*d + b*e)^2*c^5*d^3*g + 2772*((e*x +
 d)*c - 2*c*d + b*e)^2*b*c^4*d^2*e*g - 2079*((e*x + d)*c - 2*c*d + b*e)^2*
b^2*c^3*d*e^2*g + 462*((e*x + d)*c - 2*c*d + b*e)^2*b^3*c^2*e^3*g - 1680*(
(e*x + d)*c - 2*c*d + b*e)^3*c^4*d*e*f + 840*((e*x + d)*c - 2*c*d + b*e)^3
*b*c^3*e^2*f - 560*((e*x + d)*c - 2*c*d + b*e)^3*c^4*d^2*g + 1400*((e*x +
d)*c - 2*c*d + b*e)^3*b*c^3*d*e*g - 560*((e*x + d)*c - 2*c*d + b*e)^3*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx=\int \frac{f+g\,x}{\left(d+e\,x\right)^{3/2}\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^{5/2}}\,dx$$

input

```
int((f + g*x)/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2)
),x)
```

output

```
int((f + g*x)/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2504, normalized size of antiderivative = 5.46

$$\int \frac{f+gx}{(d+ex)^{3/2}\left(cd^2-bde-be^2x-ce^2x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output

```
(210*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d -
 c*e*x)/sqrt(b*e - 2*c*d))*b**2*c**2*d**3*e**2*g + 630*sqrt(b*e - 2*c*d)*s
qrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d
))*b**2*c**2*d**2*e**3*g*x + 630*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e
*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b**2*c**2*d*e**4*g*
x**2 + 210*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e +
 c*d - c*e*x)/sqrt(b*e - 2*c*d))*b**2*c**2*e**5*g*x**3 - 315*sqrt(b*e - 2*
c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e -
 2*c*d))*b*c**3*d**4*e*g - 315*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x
)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*d**3*e**2*f -
735*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d -
c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*d**3*e**2*g*x - 945*sqrt(b*e - 2*c*d)*sqr
t( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))
*b*c**3*d**2*e**3*f*x - 315*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*a
tan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*d**2*e**3*g*x**2
- 945*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d
- c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*d*e**4*f*x**2 + 315*sqrt(b*e - 2*c*d)*s
qrt( - b*e + c*d - c*e*x)*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d
))*b*c**3*d*e**4*g*x**3 - 315*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)
*atan(sqrt( - b*e + c*d - c*e*x)/sqrt(b*e - 2*c*d))*b*c**3*e**5*f*x**3 ...
```

3.249 $\int \frac{1+x}{\sqrt{5-2x}\sqrt{2+3x+x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 51

$$\int \frac{1+x}{\sqrt{5-2x}\sqrt{2+3x+x^2}}\,dx=-\frac{\sqrt{7}\sqrt{1+x}\sqrt{2+x}E\left(\arcsin\left(\frac{1}{3}\sqrt{5-2x}\right)|\frac{9}{7}\right)}{\sqrt{2+3x+x^2}}$$

output
```
-7^(1/2)*(1+x)^(1/2)*(2+x)^(1/2)*EllipticE(1/3*(5-2*x)^(1/2),3/7*7^(1/2))/
(x^2+3*x+2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(51) = 102.

Time = 31.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.92

$$\int \frac{1+x}{\sqrt{5-2x}\sqrt{2+3x+x^2}}\,dx$$
$$=\frac{12+18x+6x^2+9(5-2x)^{3/2}\sqrt{\frac{1+x}{-5+2x}}\sqrt{\frac{2+x}{-5+2x}}E\left(\arcsin\left(\frac{3}{\sqrt{5-2x}}\right)|\frac{7}{9}\right)-2(5-2x)^{3/2}\sqrt{\frac{1+x}{-5+2x}}\sqrt{\frac{2+x}{-5+2x}}\,\mathrm{El}}{3\sqrt{5-2x}\sqrt{2+3x+x^2}}$$

input
```
Integrate[(1 + x)/(Sqrt[5 - 2*x]*Sqrt[2 + 3*x + x^2]),x]
```

output
```
(12 + 18*x + 6*x^2 + 9*(5 - 2*x)^(3/2)*Sqrt[(1 + x)/(-5 + 2*x)]*Sqrt[(2 +
x)/(-5 + 2*x)]*EllipticE[ArcSin[3/Sqrt[5 - 2*x]], 7/9] - 2*(5 - 2*x)^(3/2)
*Sqrt[(1 + x)/(-5 + 2*x)]*Sqrt[(2 + x)/(-5 + 2*x)]*EllipticF[ArcSin[3/Sqrt
[5 - 2*x]], 7/9])/(3*Sqrt[5 - 2*x]*Sqrt[2 + 3*x + x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1268, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\sqrt{5-2x}\sqrt{x^2+3x+2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{x+1}\sqrt{x+2}\int \frac{\sqrt{x+1}}{\sqrt{5-2x}\sqrt{x+2}}dx}{\sqrt{x^2+3x+2}}$$

$$\Big\downarrow \text{123}$$

$$-\frac{\sqrt{7}\sqrt{x+1}\sqrt{x+2}E\left(\arcsin\left(\frac{1}{3}\sqrt{5-2x}\right)|\frac{9}{7}\right)}{\sqrt{x^2+3x+2}}$$

input
```
Int[(1 + x)/(Sqrt[5 - 2*x]*Sqrt[2 + 3*x + x^2]),x]
```

output
```
-((Sqrt[7]*Sqrt[1 + x]*Sqrt[2 + x]*EllipticE[ArcSin[Sqrt[5 - 2*x]/3], 9/7]
)/Sqrt[2 + 3*x + x^2])
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result
default	$-\frac{\operatorname{EllipticE}\left(\frac{\sqrt{5-2x}}{3}, \frac{3\sqrt{7}}{7}\right)\sqrt{2x+4}\,\sqrt{14x+14}}{2\sqrt{x^{2}+3x+2}}$
elliptic	$\frac{\sqrt{-\left(-5+2x\right)\left(x^{2}+3x+2\right)}\,\Bigg(-\frac{\sqrt{5-2x}\,\sqrt{14x+14}\,\sqrt{2x+4}\,\operatorname{EllipticF}\left(\frac{\sqrt{5-2x}}{3}, \frac{3\sqrt{7}}{7}\right)}{7\sqrt{-2x^{3}-x^{2}+11x+10}}-\frac{\sqrt{5-2x}\,\sqrt{14x+14}\,\sqrt{2x+4}\,\Bigg(\frac{7\operatorname{EllipticE}\left(\frac{\sqrt{5-2x}}{3}, \frac{3\sqrt{7}}{7}\right)}{2}-\mathrm{El}}{7\sqrt{-2x^{3}-x^{2}+11x+10}}}{\sqrt{5-2x}\,\sqrt{x^{2}+3x+2}}$

input
```
int((x+1)/(5-2*x)^(1/2)/(x^2+3*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*EllipticE(1/3*(5-2*x)^(1/2),3/7*7^(1/2))*(2*x+4)^(1/2)*(14*x+14)^(1/2
)/(x^2+3*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.49

$$\int \frac{1+x}{\sqrt{5-2x}\sqrt{2+3x+x^2}}\,dx$$
$$= -\frac{5}{6}\sqrt{-2}\text{weierstrassPInverse}\left(\frac{67}{3},\frac{440}{27},x+\frac{1}{6}\right)$$
$$+\sqrt{-2}\text{weierstrassZeta}\left(\frac{67}{3},\frac{440}{27},\text{weierstrassPInverse}\left(\frac{67}{3},\frac{440}{27},x+\frac{1}{6}\right)\right)$$

input
```
integrate((1+x)/(5-2*x)^(1/2)/(x^2+3*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-5/6*sqrt(-2)*weierstrassPInverse(67/3, 440/27, x + 1/6) + sqrt(-2)*weiers
trassZeta(67/3, 440/27, weierstrassPInverse(67/3, 440/27, x + 1/6))
```

Sympy [F]

$$\int \frac{1+x}{\sqrt{5-2x}\sqrt{2+3x+x^2}}\,dx = \int \frac{x+1}{\sqrt{(x+1)(x+2)}\sqrt{5-2x}}\,dx$$

input
```
integrate((1+x)/(5-2*x)**(1/2)/(x**2+3*x+2)**(1/2),x)
```

output
```
Integral((x + 1)/(sqrt((x + 1)*(x + 2))*sqrt(5 - 2*x)), x)
```

Maxima [F]

$$\int \frac{1+x}{\sqrt{5-2x}\sqrt{2+3x+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+3x+2}\sqrt{-2x+5}}\,dx$$

input
```
integrate((1+x)/(5-2*x)^(1/2)/(x^2+3*x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(sqrt(x^2 + 3*x + 2)*sqrt(-2*x + 5)), x)
```

Giac [F]

$$\int \frac{1+x}{\sqrt{5-2x}\sqrt{2+3x+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+3\,x+2}\sqrt{-2\,x+5}}\,dx$$

input
```
integrate((1+x)/(5-2*x)^(1/2)/(x^2+3*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + 1)/(sqrt(x^2 + 3*x + 2)*sqrt(-2*x + 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{\sqrt{5-2x}\sqrt{2+3x+x^2}}\,dx = \int \frac{x+1}{\sqrt{5-2\,x}\,\sqrt{x^2+3\,x+2}}\,dx$$

input
```
int((x + 1)/((5 - 2*x)^(1/2)*(3*x + x^2 + 2)^(1/2)),x)
```

output
```
int((x + 1)/((5 - 2*x)^(1/2)*(3*x + x^2 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+x}{\sqrt{5-2x}\sqrt{2+3x+x^2}}\,dx = -\left(\int \frac{\sqrt{-2x+5}\,\sqrt{x^2+3x+2}}{2x^2-x-10}dx\right)$$

input
```
int((1+x)/(5-2*x)^(1/2)/(x^2+3*x+2)^(1/2),x)
```

output
```
 - int((sqrt( - 2*x + 5)*sqrt(x**2 + 3*x + 2))/(2*x**2 - x - 10),x)
```

3.250 $\int (d+ex)^m(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 174

$$\int (d+ex)^m(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{g(d+ex)^m\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{ce^2(5+m)}$$

$$+\frac{(beg(5+2m)-2c(dgm+ef(5+m)))(d+ex)^m\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}\,\text{Hypergeometric2F1}\left(1,\right.}{5ce^2(2cd-be)(5+m)}$$

output
```
-g*(e*x+d)^m*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)/c/e^2/(5+m)+1/5*(b*e*g
*(5+2*m)-2*c*(d*g*m+e*f*(5+m)))*(e*x+d)^m*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)
^(5/2)*hypergeom([1, 5+m],[7/2],(-c*e*x-b*e+c*d)/(-b*e+2*c*d))/c/e^2/(-b*e
+2*c*d)/(5+m)
```

Mathematica [A] (warning: unable to verify)

Time = 1.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int (d+ex)^m(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\frac{(d+ex)^m(-cd+be+cex)^2\sqrt{(d+ex)(-be+c(d-ex))}\Big(-5c^2g(d+ex)^2+(-2cd+be}{}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2
),x]
```

output
```
((d + e*x)^m*(-(c*d) + b*e + c*e*x)^2*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x)
)]*(-5*c^2*g*(d + e*x)^2 + (-2*c*d + b*e)*(-(b*e*g*(5 + 2*m)) + 2*c*(d*g*m
 + e*f*(5 + m)))*((c*(d + e*x))/(2*c*d - b*e))^(-1/2 - m)*Hypergeometric2F
1[5/2, -3/2 - m, 7/2, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)]))/(5*c^3*e^2*
(5 + m))
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1221, 1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)(d+ex)^m\left(-bde-be^2x+cd^2-ce^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1221}$$

$$-\frac{(beg(2m+5)-2c(dgm+ef(m+5)))\int (d+ex)^m\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2}dx}{2ce(m+5)}-\frac{g(d+ex)^m\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{ce^2(m+5)}$$

$$\Big\downarrow \text{1139}$$

$$-\frac{(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m} (beg(2m+5)-2c(dgm+ef(m+5))) \int \left(\frac{ex}{d}+1\right)^m \left(-cx^2e^2-bxe^2+d(cd-be)\right)^{3/2} dx}{2ce(m+5)}$$

$$\frac{g(d+ex)^m \left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{ce^2(m+5)}$$

$$\Big\downarrow \textcolor{blue}{1138}$$

$$-\frac{(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m-\frac{1}{2}} \sqrt{d(cd-be)-be^2x-ce^2x^2}(beg(2m+5)-2c(dgm+ef(m+5))) \int \left(\frac{ex}{d}+1\right)^{m+\frac{3}{2}} (}{2ce(m+5)\sqrt{d(cd-be)-cdex}}$$

$$\frac{g(d+ex)^m \left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{ce^2(m+5)}$$

$$\Big\downarrow \textcolor{blue}{80}$$

$$-\frac{(2cd-be)(d+ex)^m\sqrt{d(cd-be)-be^2x-ce^2x^2}\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-\frac{1}{2}} (beg(2m+5)-2c(dgm+ef(m+5))) \int (d(c}{2c^2de(m+5)\sqrt{d(cd-be)-cdex}}$$

$$\frac{g(d+ex)^m \left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{ce^2(m+5)}$$

$$\Big\downarrow \textcolor{blue}{79}$$

$$\frac{(2cd-be)(d+ex)^m(d(cd-be)-cdex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-\frac{1}{2}} (beg(2m+5)-2c(dgm+}{5c^3d^2e^2(m+5)}$$

$$\frac{g(d+ex)^m \left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}{ce^2(m+5)}$$

input
```
Int[(d + e*x)^m*(f + g*x)*(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),x]
```

output
```
-((g*(d + e*x)^m*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2))/(c*e^2*(5 +
m))) + ((2*c*d - b*e)*(b*e*g*(5 + 2*m) - 2*c*(d*g*m + e*f*(5 + m)))*(d + e
*x)^m*((c*(d + e*x))/(2*c*d - b*e))^(-1/2 - m)*(d*(c*d - b*e) - c*d*e*x)^2
*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]*Hypergeometric2F1[5/2, -3/2 - m
, 7/2, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(5*c^3*d^2*e^2*(5 + m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [F]

$$\int (ex+d)^{m} (gx+f) \left(-x^{2} c\, e^{2}-x b\, e^{2}-b d e+c\, d^{2}\right)^{\frac{3}{2}}d x$$

input
```
int((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx) \left(cd^2-bde-be^2x -ce^2x^2\right)^{3/2}\,dx=\int \left(-ce^{2}x^{2}-be^{2}x+cd^{2}-bde\right)^{\frac{3}{2}}(gx+f)(ex+d)^{m}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(-(c*e^2*g*x^3 + (c*e^2*f + b*e^2*g)*x^2 - (c*d^2 - b*d*e)*f + (b*
e^2*f - (c*d^2 - b*d*e)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(
e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m(f+gx) \left(cd^2-bde-be^2x -ce^2x^2\right)^{3/2}\,dx=\int \left(-\left(d+ex\right)\left(be-cd+cex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{m}\left(f+gx\right)\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x
)
```

output
```
Integral((-(d + e*x)*(b*e - c*d + c*e*x))**(3/2)*(d + e*x)**m*(f + g*x), x
)
```

Maxima [F]

$$\int (d+ex)^m(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\int \left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(g*x + f)*(e*x + d)
^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\int \left(-ce^2x^2-be^2x+cd^2-bde\right)^{\frac{3}{2}}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/2)*(g*x + f)*(e*x + d)
^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\int (f+g\,x)\,\left(d+e\,x\right)^m\,{\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)}^{3/2}\,dx$$

input
```
int((f + g*x)*(d + e*x)^m*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),x)
```

output
```
int((f + g*x)*(d + e*x)^m*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2), x)
```

Reduce [F]

$$\int (d+ex)^m(f+gx)\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}\,dx=\int (ex+d)^m\,(gx+f)\left(-c\,e^2x^2-b\,e^2x-bde+c\,d^2\right)^{\frac{3}{2}}dx$$

input
```
int((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

3.251 $\int (d+ex)^m(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 174

$$\begin{aligned}&\int (d+ex)^m(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx\\&=-\frac{g(d+ex)^m\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{ce^2(3+m)}\\&\quad+\frac{(beg(3+2m)-2c(dgm+ef(3+m)))(d+ex)^m\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}\,\text{Hypergeometric2F1}}{3ce^2(2cd-be)(3+m)}\end{aligned}$$

output

```
-g*(e*x+d)^m*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)/c/e^2/(3+m)+1/3*(b*e*g
*(3+2*m)-2*c*(d*g*m+e*f*(3+m)))*(e*x+d)^m*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)
^(3/2)*hypergeom([1, 3+m],[5/2],(-c*e*x-b*e+c*d)/(-b*e+2*c*d))/c/e^2/(-b*e
+2*c*d)/(3+m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.86

$$\int (d+ex)^m(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$=\frac{(d+ex)^{-1+m}((d+ex)(-be+c(d-ex)))^{3/2}\left(-3ceg(d+ex)-e(-beg(3+2m)+2c(dgm+ef(3+m\right.}{3c^2e^3(3+m)}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],
x]
```

output
```
((d + e*x)^(-1 + m)*((d + e*x)*(-(b*e) + c*(d - e*x)))^(3/2)*(-3*c*e*g*(d
+ e*x) - e*(-(b*e*g*(3 + 2*m)) + 2*c*(d*g*m + e*f*(3 + m)))*((c*(d + e*x))
/(2*c*d - b*e))^(-1/2 - m)*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (-(c*d) +
 b*e + c*e*x)/(-2*c*d + b*e)]))/(3*c^2*e^3*(3 + m))
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1221, 1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)(d+ex)^m\sqrt{-bde-be^2x+cd^2-ce^2x^2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$-\frac{(beg(2m+3)-2c(dgm+ef(m+3)))\int (d+ex)^m\sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{2ce(m+3)}-\frac{g(d+ex)^m\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{ce^2(m+3)}$$

$$\Big\downarrow \text{1139}$$

$$-\frac{(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m} (beg(2m+3)-2c(dgm+ef(m+3)))\int \left(\frac{ex}{d}+1\right)^m \sqrt{-cx^2e^2-bxe^2+d(cd-be)}dx}{2ce(m+3)}-$$
$$\frac{g(d+ex)^m \left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{ce^2(m+3)}$$

$$\Big\downarrow \text{ 1138}$$

$$-\frac{(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m-\frac{1}{2}} \sqrt{d(cd-be)-be^2x-ce^2x^2}(beg(2m+3)-2c(dgm+ef(m+3)))\int \left(\frac{ex}{d}+1\right)^{m+\frac{1}{2}} \surd}{2ce(m+3)\sqrt{d(cd-be)-cdex}}$$
$$\frac{g(d+ex)^m \left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{ce^2(m+3)}$$

$$\Big\downarrow \text{ 80}$$

$$-\frac{(d+ex)^m \sqrt{d(cd-be)-be^2x-ce^2x^2}\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-\frac{1}{2}} (beg(2m+3)-2c(dgm+ef(m+3)))\int \sqrt{d(cd-be)}}{2ce(m+3)\sqrt{d(cd-be)-cdex}}$$
$$\frac{g(d+ex)^m \left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{ce^2(m+3)}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{(d+ex)^m(d(cd-be)-cdex)\sqrt{d(cd-be)-be^2x-ce^2x^2}\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-\frac{1}{2}} (beg(2m+3)-2c(dgm+ef(m+3)}{3c^2de^2(m+3)}$$
$$\frac{g(d+ex)^m \left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}{ce^2(m+3)}$$

input
```
Int[(d + e*x)^m*(f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
-((g*(d + e*x)^m*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2))/(c*e^2*(3 +
m))) + ((b*e*g*(3 + 2*m) - 2*c*(d*g*m + e*f*(3 + m)))*(d + e*x)^m*((c*(d +
 e*x))/(2*c*d - b*e))^(-1/2 - m)*(d*(c*d - b*e) - c*d*e*x)*Sqrt[d*(c*d - b
*e) - b*e^2*x - c*e^2*x^2]*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (c*d - b*
e - c*e*x)/(2*c*d - b*e)])/(3*c^2*d*e^2*(3 + m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [F]

$$\int (ex+d)^m (gx+f)\sqrt{-x^2 c\, e^2 - xb\, e^2 - bde + c\, d^2}dx$$

input
```
int((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$= \int \sqrt{-ce^2x^2-be^2x+cd^2-bde}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)*(e*x + d)^m,
 x)
```

Sympy [F]

$$\int (d+ex)^m(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$= \int \sqrt{-(d+ex)(be-cd+cex)}(d+ex)^m(f+gx)\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)*(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output
```
Integral(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(d + e*x)**m*(f + g*x), x)
```

Maxima [F]

$$\int (d+ex)^m(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$=\int \sqrt{-ce^2x^2-be^2x+cd^2-bde}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)*(e*x + d)^m
, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$=\int \sqrt{-ce^2x^2-be^2x+cd^2-bde}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)*(e*x + d)^m
, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$=\int (f+g\,x)\,\left(d+e\,x\right)^m\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}\,dx$$

input
```
int((f + g*x)*(d + e*x)^m*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output
```
int((f + g*x)*(d + e*x)^m*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2), x)
```

Reduce [F]

$$\int (d+ex)^m(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}\,dx$$
$$=\left(\int (ex+d)^{m+\frac{1}{2}}\sqrt{-cex-be+cd}\,xdx\right)g$$
$$+\left(\int (ex+d)^{m+\frac{1}{2}}\sqrt{-cex-be+cd}dx\right)f$$

input
```
int((e*x+d)^m*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
int((d + e*x)**((2*m + 1)/2)*sqrt( - b*e + c*d - c*e*x)*x,x)*g + int((d +
e*x)**((2*m + 1)/2)*sqrt( - b*e + c*d - c*e*x),x)*f
```

3.252 $\int \frac{(d+ex)^m(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 171

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{g(d+ex)^m\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2(1+m)} + \frac{(beg(1+2m)-2c(dgm+ef(1+m)))(d+ex)^m\sqrt{d(cd-be)-be^2x-ce^2x^2}\,\text{Hypergeometric2F1}\,(1,}{ce^2(2cd-be)(1+m)}$$

output
```
-g*(e*x+d)^m*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c/e^2/(1+m)+(b*e*g*(1+
2*m)-2*c*(d*g*m+e*f*(1+m)))*(e*x+d)^m*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/
2)*hypergeom([1, 1+m],[3/2],(-c*e*x-b*e+c*d)/(-b*e+2*c*d))/c/e^2/(-b*e+2*c
*d)/(1+m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{2(d+ex)^m\sqrt{(d+ex)(-be+c(d-ex))}\left(e(ef-dg)+\frac{e(beg(1+2m)-2c(dgm+ef(1+m)))\left(\frac{c(d+ex)}{2cd-be}\right)^{-\frac{1}{2}-m}\text{Hyperge}}{c}\right.}{e^3(-2cd+be)(1+2m)}$$

input

```
Integrate[((d + e*x)^m*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
],x]
```

output

```
(-2*(d + e*x)^m*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]*(e*(e*f - d*g) + (e
*(b*e*g*(1 + 2*m) - 2*c*(d*g*m + e*f*(1 + m)))*((c*(d + e*x))/(2*c*d - b*e
))^(-1/2 - m)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (-(c*d) + b*e + c*e*x)
/(-2*c*d + b*e)])/c))/(e^3*(-2*c*d + b*e)*(1 + 2*m))
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1221, 1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(d+ex)^m}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{ 1221}$$

$$-\frac{(beg(2m+1)-2c(dgm+ef(m+1)))\int \frac{(d+ex)^m}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2ce(m+1)}-\frac{g(d+ex)^m\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2(m+1)}$$

$$\Big\downarrow \text{ 1139}$$

$$-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}(beg(2m+1)-2c(dgm+ef(m+1)))\int \frac{\left(\frac{ex}{d}+1\right)^m}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2ce(m+1)}-\frac{g(d+ex)^m\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2(m+1)}$$

$$\Big\downarrow \text{ 1138}$$

$$-\frac{(d+ex)^m \left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m} \sqrt{d(cd-be)-cdex}\,(beg(2m+1)-2c(dgm+ef(m+1))) \int \frac{\left(\frac{ex}{d}+1\right)^{m-\frac{1}{2}}}{\sqrt{d(cd-be)-cdex}}dx}{2ce(m+1)\sqrt{d(cd-be)-be^2x-ce^2x^2}} - \frac{g(d+ex)^m\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2(m+1)}$$

$$\Big\downarrow \text{ 80}$$

$$-\frac{(d+ex)^m \sqrt{d(cd-be)-cdex}\left(\frac{c(d+ex)}{2cd-be}\right)^{\frac{1}{2}-m} (beg(2m+1)-2c(dgm+ef(m+1))) \int \frac{\left(\frac{cd}{2cd-be}+\frac{cex}{2cd-be}\right)^{m-\frac{1}{2}}}{\sqrt{d(cd-be)-cdex}}dx}{2ce(m+1)\sqrt{d(cd-be)-be^2x-ce^2x^2}} - \frac{g(d+ex)^m\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2(m+1)}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{(d+ex)^m(d(cd-be)-cdex)\left(\frac{c(d+ex)}{2cd-be}\right)^{\frac{1}{2}-m}(beg(2m+1)-2c(dgm+ef(m+1)))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\right.}{c^2de^2(m+1)\sqrt{d(cd-be)-be^2x-ce^2x^2}} \qquad \frac{g(d+ex)^m\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce^2(m+1)}$$

input

```
Int[((d + e*x)^m*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output

```
-((g*(d + e*x)^m*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(c*e^2*(1 + m)
)) + ((b*e*g*(1 + 2*m) - 2*c*(d*g*m + e*f*(1 + m)))*(d + e*x)^m*((c*(d + e
*x))/(2*c*d - b*e))^(1/2 - m)*(d*(c*d - b*e) - c*d*e*x)*Hypergeometric2F1[
1/2, 1/2 - m, 3/2, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(c^2*d*e^2*(1 + m)*
Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])
```

Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^m (gx+f)}{\sqrt{-x^2c\,e^2-xb\,e^2-bde+c\,d^2}}dx$$

input
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{\sqrt{-ce^2x^2-be^2x+cd^2-bde}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)*(e*x + d)^m
/(c*e^2*x^2 + b*e^2*x - c*d^2 + b*d*e), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{(d+ex)^m\,(f+gx)}{\sqrt{-\,(d+ex)\,(be-cd+cex)}}\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output
```
Integral((d + e*x)**m*(f + g*x)/sqrt(-(d + e*x)*(b*e - c*d + c*e*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{\sqrt{-ce^2x^2-be^2x+cd^2-bde}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((g*x + f)*(e*x + d)^m/sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)
, x)
```

Giac [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{\sqrt{-ce^2x^2-be^2x+cd^2-bde}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((g*x + f)*(e*x + d)^m/sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{(f+g\,x)\,(d+e\,x)^m}{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^m)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^m)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2), x
)
```

Reduce [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\left(\int \frac{(ex+d)^m}{\sqrt{ex+d}\,\sqrt{-cex-be+cd}}dx\right)f+\left(\int \frac{(ex+d)^m\,x}{\sqrt{ex+d}\,\sqrt{-cex-be+cd}}dx\right)g$$

input
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
int((d + e*x)**m/(sqrt(d + e*x)*sqrt( - b*e + c*d - c*e*x)),x)*f + int(((d
 + e*x)**m*x)/(sqrt(d + e*x)*sqrt( - b*e + c*d - c*e*x)),x)*g
```

3.253 $\int \frac{(d+ex)^m(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 178

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{g(d+ex)^m}{ce^2(1-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}} - \frac{(beg(1-2m)-2c(ef(1-m)-dgm))(d+ex)^m \operatorname{Hypergeometric2F1}\left(1,-1+m,\frac{1}{2},\frac{cd-be-cex}{2cd-be}\right)}{ce^2(2cd-be)(1-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

output
```
g*(e*x+d)^m/c/e^2/(1-m)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)-(b*e*g*(1-2
*m)-2*c*(e*f*(1-m)-d*g*m))*(e*x+d)^m*hypergeom([1, -1+m],[1/2],(-c*e*x-b*e
+c*d)/(-b*e+2*c*d))/c/e^2/(-b*e+2*c*d)/(1-m)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx = \frac{2(d+ex)^m \Big(e(2cd-be)(cef+cdg-beg)-e(beg(1-2m)+2c(e}{ce^3(-2}$$

input
```
Integrate[((d + e*x)^m*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3
/2),x]
```

output
```
(2*(d + e*x)^m*(e*(2*c*d - b*e)*(c*e*f + c*d*g - b*e*g) - e*(b*e*g*(1 - 2*
m) + 2*c*(e*f*(-1 + m) + d*g*m))*((c*(d + e*x))/(2*c*d - b*e))^(1/2 - m)*(
-(c*d) + b*e + c*e*x)*Hypergeometric2F1[1/2, 3/2 - m, 3/2, (-(c*d) + b*e +
 c*e*x)/(-2*c*d + b*e)]))/(c*e^3*(-2*c*d + b*e)^2*Sqrt[(d + e*x)*(-(b*e) +
 c*(d - e*x))])
```

Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1221, 1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(d+ex)^m}{(-bde-be^2x+cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{g(d+ex)^m}{ce^2(1-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(beg(1-2m)-2c(ef(1-m)-dgm))\int \frac{(d+ex)^m}{(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{2ce(1-m)}$$

$$\Big\downarrow \text{1139}$$

$$\frac{g(d+ex)^m}{ce^2(1-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}(beg(1-2m)-2c(ef(1-m)-dgm))\int \frac{\left(\frac{ex}{d}+1\right)^m}{(-cx^2e^2-bxe^2+d(cd-be))^{3/2}}dx}{2ce(1-m)}$$

$$\Big\downarrow \text{1138}$$

$$\frac{g(d+ex)^m}{ce^2(1-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m}\sqrt{d(cd-be)-cdex}(beg(1-2m)-2c(ef(1-m)-dgm))\int\frac{\left(\frac{ex}{d}+1\right)^{m-\frac{3}{2}}}{(d(cd-be)-cdex)^{3/2}}dx}{2ce(1-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow 80$$

$$\frac{g(d+ex)^m}{ce^2(1-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{d(d+ex)^m\sqrt{d(cd-be)-cdex}\left(\frac{c(d+ex)}{2cd-be}\right)^{\frac{1}{2}-m}(beg(1-2m)-2c(ef(1-m)-dgm))\int\frac{\left(\frac{cd}{2cd-be}+\frac{cex}{2cd-be}\right)^{m-\frac{3}{2}}}{(d(cd-be)-cdex)^{3/2}}dx}{2e(1-m)(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow 79$$

$$\frac{g(d+ex)^m}{ce^2(1-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}}-\frac{(d+ex)^m\left(\frac{c(d+ex)}{2cd-be}\right)^{\frac{1}{2}-m}(beg(1-2m)-2c(ef(1-m)-dgm))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-m,\frac{1}{2},\frac{cd-be-cex}{2cd-be}\right)}{ce^2(1-m)(2cd-be)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input

```
Int[((d + e*x)^m*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(3/2),x]
```

output

```
(g*(d + e*x)^m)/(c*e^2*(1 - m)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])
- ((b*e*g*(1 - 2*m) - 2*c*(e*f*(1 - m) - d*g*m))*(d + e*x)^m*((c*(d + e*x)
)/(2*c*d - b*e))^(1/2 - m)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (c*d - b*
e - c*e*x)/(2*c*d - b*e)])/(c*e^2*(2*c*d - b*e)*(1 - m)*Sqrt[d*(c*d - b*e)
 - b*e^2*x - c*e^2*x^2])
```

Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])    Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m])   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{m} (gx+f)}{(-x^{2} c\,e^{2}-xb\,e^{2}-bde+c\,d^{2})^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)*(e*x + d)^m/
(c^2*e^4*x^4 + 2*b*c*e^4*x^3 + c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 - (2*c^
2*d^2*e^2 - 2*b*c*d*e^3 - b^2*e^4)*x^2 - 2*(b*c*d^2*e^2 - b^2*d*e^3)*x), x
)
```

Sympy [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{(d+ex)^m\,(f+gx)}{(-\,(d+ex)\,(be-cd+cex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(3/2),x
)
```

output
```
Integral((d + e*x)**m*(f + g*x)/(-(d + e*x)*(b*e - c*d + c*e*x))**(3/2), x
)
```

Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((g*x + f)*(e*x + d)^m/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/
2), x)
```

Giac [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((g*x + f)*(e*x + d)^m/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx=\int \frac{(f+g\,x)\,(d+e\,x)^m}{(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)^{3/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^m)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^m)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(3/2), x
)
```

Reduce [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}}\,dx =$$

$$-\left(\int \frac{(ex+d)^m}{\sqrt{ex+d}\,\sqrt{-cex-be+cd}\,bde+\sqrt{ex+d}\,\sqrt{-cex-be+cd}\,b\,e^2x-\sqrt{ex+d}\,\sqrt{-cex-be+cd}\,c\,d^2}\right.$$

$$-\left(\int \frac{(ex+d)^m\,x}{\sqrt{ex+d}\,\sqrt{-cex-be+cd}\,bde+\sqrt{ex+d}\,\sqrt{-cex-be+cd}\,b\,e^2x-\sqrt{ex+d}\,\sqrt{-cex-be+cd}\,c\,d^2}\right.$$

input

```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x)
```

output

```
 - (int((d + e*x)**m/(sqrt(d + e*x)*sqrt( - b*e + c*d - c*e*x)*b*d*e + sqr
t(d + e*x)*sqrt( - b*e + c*d - c*e*x)*b*e**2*x - sqrt(d + e*x)*sqrt( - b*e
 + c*d - c*e*x)*c*d**2 + sqrt(d + e*x)*sqrt( - b*e + c*d - c*e*x)*c*e**2*x
**2),x)*f + int(((d + e*x)**m*x)/(sqrt(d + e*x)*sqrt( - b*e + c*d - c*e*x)
*b*d*e + sqrt(d + e*x)*sqrt( - b*e + c*d - c*e*x)*b*e**2*x - sqrt(d + e*x)
*sqrt( - b*e + c*d - c*e*x)*c*d**2 + sqrt(d + e*x)*sqrt( - b*e + c*d - c*e
*x)*c*e**2*x**2),x)*g)
```

3.254 $\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 180

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\frac{g(d+ex)^m}{ce^2(3-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(beg(3-2m)-2c(ef(3-m)-dgm))(d+ex)^m\,\text{Hypergeometric2F1}\left(1,-3+m,-\frac{1}{2},\frac{cd-be-cex}{2cd-be}\right)}{3ce^2(2cd-be)(3-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}$$

output
```
g*(e*x+d)^m/c/e^2/(3-m)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(3/2)-1/3*(b*e*g*
(3-2*m)-2*c*(e*f*(3-m)-d*g*m))*(e*x+d)^m*hypergeom([1, -3+m],[-1/2],(-c*e*
x-b*e+c*d)/(-b*e+2*c*d))/c/e^2/(-b*e+2*c*d)/(3-m)/(d*(-b*e+c*d)-b*e^2*x-c*
e^2*x^2)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\frac{2(d+ex)^m\Bigg(-\frac{e(-2cd+be)^2(cef+cdg-beg)}{c}-e(beg(3-2m)+2c(ef(-3}{3e^3(-}$$

input
```
Integrate[((d + e*x)^m*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5
/2),x]
```

output
```
(2*(d + e*x)^m*(-((e*(-2*c*d + b*e)^2*(c*e*f + c*d*g - b*e*g))/c) - e*(b*e
*g*(3 - 2*m) + 2*c*(e*f*(-3 + m) + d*g*m))*(d + e*x)*((c*(d + e*x))/(2*c*d
 - b*e))^(1/2 - m)*(-(c*d) + b*e + c*e*x)*Hypergeometric2F1[-1/2, 5/2 - m,
 1/2, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)]))/(3*e^3*(-2*c*d + b*e)^3*((d
 + e*x)*(-(b*e) + c*(d - e*x)))^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1221, 1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(d+ex)^m}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{g(d+ex)^m}{ce^2(3-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(beg(3-2m)-2c(ef(3-m)-dgm))\int \frac{(d+ex)^m}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{2ce(3-m)}$$

$$\Big\downarrow \text{1139}$$

$$\frac{g(d+ex)^m}{ce^2(3-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}(beg(3-2m)-2c(ef(3-m)-dgm))\int \frac{\left(\frac{ex}{d}+1\right)^m}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{5/2}}dx}{2ce(3-m)}$$

$$\Big\downarrow \text{1138}$$

$$\frac{g(d+ex)^m}{ce^2(3-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m}\sqrt{d(cd-be)-cdex}(beg(3-2m)-2c(ef(3-m)-dgm))\int\frac{\left(\frac{ex}{d}+1\right)^{m-\frac{5}{2}}}{(d(cd-be)-cdex)^{5/2}}dx}{2ce(3-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{g(d+ex)^m}{ce^2(3-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{cd^2(d+ex)^m\sqrt{d(cd-be)-cdex}\left(\frac{c(d+ex)}{2cd-be}\right)^{\frac{1}{2}-m}(beg(3-2m)-2c(ef(3-m)-dgm))\int\frac{\left(\frac{cd}{2cd-be}+\frac{cex}{2cd-be}\right)^{m-\frac{5}{2}}}{(d(cd-be)-cdex)^{5/2}}dx}{2e(3-m)(2cd-be)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{g(d+ex)^m}{ce^2(3-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{3/2}}-\frac{d(d+ex)^m\left(\frac{c(d+ex)}{2cd-be}\right)^{\frac{1}{2}-m}(beg(3-2m)-2c(ef(3-m)-dgm))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-m,-\frac{1}{2},\frac{cd-be-ce}{2cd-be}\right.}{3e^2(3-m)(2cd-be)^2(d(cd-be)-cdex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input
```
Int[((d + e*x)^m*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(5/2),x]
```

output
```
(g*(d + e*x)^m)/(c*e^2*(3 - m)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(3/2)
) - (d*(b*e*g*(3 - 2*m) - 2*c*(e*f*(3 - m) - d*g*m))*(d + e*x)^m*((c*(d +
e*x))/(2*c*d - b*e))^(1/2 - m)*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, (c*d
 - b*e - c*e*x)/(2*c*d - b*e)])/(3*e^2*(2*c*d - b*e)^2*(3 - m)*(d*(c*d - b
*e) - c*d*e*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^{m}(gx+f)}{\left(-x^{2} c\, e^{2}-x b\, e^{2}-bde+c\, d^{2}\right)^{\frac{5}{2}}}dx$$

input
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(-sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)*(e*x + d)^m
/(c^3*e^6*x^6 + 3*b*c^2*e^6*x^5 - c^3*d^6 + 3*b*c^2*d^5*e - 3*b^2*c*d^4*e^
2 + b^3*d^3*e^3 - 3*(c^3*d^2*e^4 - b*c^2*d*e^5 - b^2*c*e^6)*x^4 - (6*b*c^2
*d^2*e^4 - 6*b^2*c*d*e^5 - b^3*e^6)*x^3 + 3*(c^3*d^4*e^2 - 2*b*c^2*d^3*e^3
 + b^3*d*e^5)*x^2 + 3*(b*c^2*d^4*e^2 - 2*b^2*c*d^3*e^3 + b^3*d^2*e^4)*x),
x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(5/2),x
)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((g*x + f)*(e*x + d)^m/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/
2), x)
```

Giac [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algo
rithm="giac")
```

output
```
integrate((g*x + f)*(e*x + d)^m/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(f+g\,x)\,(d+e\,x)^m}{(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)^{5/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^m)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^m)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(5/2), x
)
```

Reduce [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{5/2}}\,dx=\int \frac{(ex+d)^m\,(gx+f)}{(-c\,e^2x^2-b\,e^2x-bde+c\,d^2)^{\frac{5}{2}}}dx$$

input
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x)
```

3.255 $\int \frac{(d+ex)^m(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 180

$$\int \frac{(d+ex)^m(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{7/2}}\,dx = \frac{g(d+ex)^m}{ce^2(5-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}} - \frac{(beg(5-2m)-2c(ef(5-m)-dgm))(d+ex)^m \operatorname{Hypergeometric2F1}\left(1,-5+m,-\frac{3}{2},\frac{cd-be-cex}{2cd-be}\right)}{5ce^2(2cd-be)(5-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}$$

output
```
g*(e*x+d)^m/c/e^2/(5-m)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(5/2)-1/5*(b*e*g*
(5-2*m)-2*c*(e*f*(5-m)-d*g*m))*(e*x+d)^m*hypergeom([1, -5+m],[-3/2],(-c*e*
x-b*e+c*d)/(-b*e+2*c*d))/c/e^2/(-b*e+2*c*d)/(5-m)/(d*(-b*e+c*d)-b*e^2*x-c*
e^2*x^2)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.42 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^m(f+gx)}{\left(cd^2-bde-be^2x-ce^2x^2\right)^{7/2}}\,dx = \frac{2(d+ex)^m \Big(-3e^3(-2cd+be)^3(cef+cdg-beg)+c^2e^3(beg(5-2}{}$$

input
```
Integrate[((d + e*x)^m*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(7
/2),x]
```

output
```
(2*(d + e*x)^m*(-3*e^3*(-2*c*d + b*e)^3*(c*e*f + c*d*g - b*e*g) + c^2*e^3*
(b*e*g*(5 - 2*m) + 2*c*(e*f*(-5 + m) + d*g*m))*(d + e*x)^2*((c*(d + e*x))/
(2*c*d - b*e))^(1/2 - m)*(-(c*d) + b*e + c*e*x)*Hypergeometric2F1[-3/2, 7/
2 - m, -1/2, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)]))/(15*c*e^5*(-2*c*d +
b*e)^4*((d + e*x)*(-(b*e) + c*(d - e*x)))^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1221, 1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(d+ex)^m}{\left(-bde-be^2x+cd^2-ce^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{g(d+ex)^m}{ce^2(5-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}-\frac{(beg(5-2m)-2c(ef(5-m)-dgm))\int \frac{(d+ex)^m}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{7/2}}dx}{2ce(5-m)}$$

$$\Big\downarrow \text{1139}$$

$$\frac{g(d+ex)^m}{ce^2(5-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}(beg(5-2m)-2c(ef(5-m)-dgm))\int \frac{\left(\frac{ex}{d}+1\right)^m}{\left(-cx^2e^2-bxe^2+d(cd-be)\right)^{7/2}}dx}{2ce(5-m)}$$

$$\Big\downarrow \text{1138}$$

$$\frac{g(d+ex)^m}{ce^2(5-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{\frac{1}{2}-m}\sqrt{d(cd-be)-cdex}\,(beg(5-2m)-2c(ef(5-m)-dgm))\int\frac{\left(\frac{ex}{d}+1\right)^{m-\frac{7}{2}}}{(d(cd-be)-cdex)^{7/2}}dx}{2ce(5-m)\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{g(d+ex)^m}{ce^2(5-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}-\frac{c^2d^3(d+ex)^m\sqrt{d(cd-be)-cdex}\left(\frac{c(d+ex)}{2cd-be}\right)^{\frac{1}{2}-m}(beg(5-2m)-2c(ef(5-m)-dgm))\int\frac{\left(\frac{cd}{2cd-be}+\frac{cex}{2cd-be}\right)^{m-\frac{7}{2}}}{(d(cd-be)-cdex)^{7/2}}dx}{2e(5-m)(2cd-be)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{g(d+ex)^m}{ce^2(5-m)\left(d(cd-be)-be^2x-ce^2x^2\right)^{5/2}}-\frac{cd^2(d+ex)^m\left(\frac{c(d+ex)}{2cd-be}\right)^{\frac{1}{2}-m}(beg(5-2m)-2c(ef(5-m)-dgm))\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-m,-\frac{3}{2},\frac{cd-be-}{2cd-}\right.}{5e^2(5-m)(2cd-be)^3(d(cd-be)-cdex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}$$

input

```
Int[((d + e*x)^m*(f + g*x))/(c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2)^(7/2),x]
```

output

```
(g*(d + e*x)^m)/(c*e^2*(5 - m)*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(5/2)
) - (c*d^2*(b*e*g*(5 - 2*m) - 2*c*(e*f*(5 - m) - d*g*m))*(d + e*x)^m*((c*(
d + e*x))/(2*c*d - b*e))^(1/2 - m)*Hypergeometric2F1[-5/2, 7/2 - m, -3/2,
(c*d - b*e - c*e*x)/(2*c*d - b*e)])/(5*e^2*(2*c*d - b*e)^3*(5 - m)*(d*(c*d
 - b*e) - c*d*e*x)^2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])
```

Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^m (gx+f)}{(-x^2c\,e^2-xb\,e^2-bde+c\,d^2)^{\frac{7}{2}}}dx$$

input
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{7/2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{7}{2}}}\,dx$$

input

```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2),x, algo
rithm="fricas")
```

output

```
integral(sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(g*x + f)*(e*x + d)^m/
(c^4*e^8*x^8 + 4*b*c^3*e^8*x^7 + c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e
^2 - 4*b^3*c*d^5*e^3 + b^4*d^4*e^4 - 2*(2*c^4*d^2*e^6 - 2*b*c^3*d*e^7 - 3*
b^2*c^2*e^8)*x^6 - 4*(3*b*c^3*d^2*e^6 - 3*b^2*c^2*d*e^7 - b^3*c*e^8)*x^5 +
 (6*c^4*d^4*e^4 - 12*b*c^3*d^3*e^5 - 6*b^2*c^2*d^2*e^6 + 12*b^3*c*d*e^7 +
b^4*e^8)*x^4 + 4*(3*b*c^3*d^4*e^4 - 6*b^2*c^2*d^3*e^5 + 2*b^3*c*d^2*e^6 +
b^4*d*e^7)*x^3 - 2*(2*c^4*d^6*e^2 - 6*b*c^3*d^5*e^3 + 3*b^2*c^2*d^4*e^4 +
4*b^3*c*d^3*e^5 - 3*b^4*d^2*e^6)*x^2 - 4*(b*c^3*d^6*e^2 - 3*b^2*c^2*d^5*e^
3 + 3*b^3*c*d^4*e^4 - b^4*d^3*e^5)*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{7/2}}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input

```
integrate((e*x+d)**m*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(7/2),x
)
```

output

```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{7/2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((g*x + f)*(e*x + d)^m/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(7/
2), x)
```

Giac [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{7/2}}\,dx=\int \frac{(gx+f)(ex+d)^m}{(-ce^2x^2-be^2x+cd^2-bde)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2),x, algo
rithm="giac")
```

output
```
integrate((g*x + f)*(e*x + d)^m/(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)^(7/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{7/2}}\,dx=\int \frac{(f+g\,x)\,(d+e\,x)^m}{(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x)^{7/2}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^m)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(7/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^m)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(7/2), x
)
```

Reduce [F]

$$\int \frac{(d+ex)^m(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^m\,(gx+f)}{(-c\,e^2x^2-b\,e^2x-bde+c\,d^2)^{\frac{7}{2}}}dx$$

input
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(7/2),x)
```

3.256 $\int (d+ex)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(cd^2-bde-be^2x-ce^2x^2\right)^p\,dx$

Optimal result

Integrand size = 61, antiderivative size = 42

$$\int (d+ex)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(cd^2-bde-be^2x-ce^2x^2\right)^p\,dx = \frac{(d+ex)^m\left(d(cd-be)-be^2x-ce^2x^2\right)^{1+p}}{e}$$

output
```
(e*x+d)^m*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(p+1)/e
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (d+ex)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(cd^2-bde-be^2x-ce^2x^2\right)^p\,dx = \frac{(d+ex)^m((d+ex)(-be+c(d-ex)))^{1+p}}{e}$$

input
```
Integrate[(d + e*x)^m*(c*d*m - b*e*(1 + m + p) - c*e*(2 + m + 2*p)*x)*(c*d
^2 - b*d*e - b*e^2*x - c*e^2*x^2)^p,x]
```

output
```
((d + e*x)^m*((d + e*x)*(-(b*e) + c*(d - e*x)))^(1 + p))/e
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.016$, Rules used = $\{1217\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(-bde-be^2x+cd^2-ce^2x^2\right)^p (-be(m+p+1)+cdm-cex(m+2p+2))\,dx$$

$$\Big\downarrow \; 1217$$

$$\frac{(d+ex)^m \left(d(cd-be)-be^2x-ce^2x^2\right)^{p+1}}{e}$$

input
```
Int[(d + e*x)^m*(c*d*m - b*e*(1 + m + p) - c*e*(2 + m + 2*p)*x)*(c*d^2 - b
*d*e - b*e^2*x - c*e^2*x^2)^p,x]
```

output
```
((d + e*x)^m*(d*(c*d - b*e) - b*e^2*x - c*e^2*x^2)^(1 + p))/e
```

Defintions of rubi rules used

rule 1217
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && EqQ[c*e*f*(m + 2*p + 2) + g*(c*d*m - b*e*(m + p
+ 1)), 0]
```

Maple [A] (verified)

Time = 5.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

method	result
gosper	$-\frac{(ex+d)^{1+m}(cex+be-cd)\left(-x^2 c\,e^2-xb\,e^2-bde+c\,d^2\right)^p}{e}$
orering	$\frac{(ex+d)(cex+be-cd)(ex+d)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(-x^2 c\,e^2-xb\,e^2-bde+c\,d^2\right)^p}{e(cexm+2cexp+bem+bep-cdm+2cex+be)}$
parallelrisch	$-\frac{x^2(ex+d)^m\left(-x^2 c\,e^2-xb\,e^2-bde+c\,d^2\right)^p bc\,e^3+x(ex+d)^m\left(-x^2 c\,e^2-xb\,e^2-bde+c\,d^2\right)^p b^2 e^3+(ex+d)^m\left(-x^2 c\,e^2-xb\,e^2-b\right.}{b\,e^2}$
risch	$-\frac{\left(x^2 c\,e^2+xb\,e^2+bde-c\,d^2\right)(ex+d)^m(be+c(ex-d))^p(ex+d)^p\mathrm{e}^{-\frac{ip\pi\left(-\operatorname{csgn}(i(ex+d)(be+c(ex-d)))^3-\operatorname{csgn}(i(ex+d)(be+c(ex-d)))\right.}{}}}{}$

input
```
int((e*x+d)^m*(c*d*m-b*e*(1+m+p)-c*e*(2+m+2*p)*x)*(-c*e^2*x^2-b*e^2*x-b*d*
e+c*d^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/e*(e*x+d)^(1+m)*(c*e*x+b*e-c*d)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int (d+ex)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(cd^2-bde-be^2x-ce^2x^2\right)^p\,dx$$
$$=-\frac{(ce^2x^2+be^2x-cd^2+bde)(-ce^2x^2-be^2x+cd^2-bde)^p(ex+d)^m}{e}$$

input
```
integrate((e*x+d)^m*(c*d*m-b*e*(1+m+p)-c*e*(2+m+2*p)*x)*(-c*e^2*x^2-b*e^2*
x-b*d*e+c*d^2)^p,x, algorithm="fricas")
```

output
```
-(c*e^2*x^2 + b*e^2*x - c*d^2 + b*d*e)*(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d
*e)^p*(e*x + d)^m/e
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(34) = 68$.

Time = 7.80 (sec) , antiderivative size = 173, normalized size of antiderivative = 4.12

$$\int (d+ex)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(cd^2-bde-be^2x-ce^2x^2\right)^p\,dx$$
$$= \begin{cases} -bd(d+ex)^m\left(-bde-be^2x+cd^2-ce^2x^2\right)^p - bex(d+ex)^m\left(-bde-be^2x+cd^2-ce^2x^2\right)^p + \frac{cd^2(d+ex)}{} \\ cdd^m mx\left(cd^2\right)^p \end{cases}$$

input
```
integrate((e*x+d)**m*(c*d*m-b*e*(1+m+p)-c*e*(2+m+2*p)*x)*(-c*e**2*x**2-b*e
**2*x-b*d*e+c*d**2)**p,x)
```

output
```
Piecewise((-b*d*(d + e*x)**m*(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)**p
 - b*e*x*(d + e*x)**m*(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)**p + c*d*
*2*(d + e*x)**m*(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)**p/e - c*e*x**2
*(d + e*x)**m*(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)**p, Ne(e, 0)), (c
*d*d**m*m*x*(c*d**2)**p, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

$$\int (d+ex)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(cd^2-bde-be^2x-ce^2x^2\right)^p\,dx$$
$$= -\frac{(ce^2x^2+be^2x-cd^2+bde)e^{(p\log(-cex+cd-be)+m\log(ex+d)+p\log(ex+d))}}{e}$$

input
```
integrate((e*x+d)^m*(c*d*m-b*e*(1+m+p)-c*e*(2+m+2*p)*x)*(-c*e^2*x^2-b*e^2*
x-b*d*e+c*d^2)^p,x, algorithm="maxima")
```

output
```
-(c*e^2*x^2 + b*e^2*x - c*d^2 + b*d*e)*e^(p*log(-c*e*x + c*d - b*e) + m*lo
g(e*x + d) + p*log(e*x + d))/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(42) = 84$.

Time = 0.37 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.86

$$\int (d+ex)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(cd^2-bde-be^2x-ce^2x^2\right)^p\,dx = -\frac{(ex+d)^m ce^2x^2e^{(p\log(-cex+cd-be)+p\log(ex+d))}+(ex+d)^m be^2xe^{(p\log(-cex+cd-be)+p\log(ex+d))}-(ex+d)^m c}{e}$$

input
```
integrate((e*x+d)^m*(c*d*m-b*e*(1+m+p)-c*e*(2+m+2*p)*x)*(-c*e^2*x^2-b*e^2*
x-b*d*e+c*d^2)^p,x, algorithm="giac")
```

output
```
-((e*x + d)^m*c*e^2*x^2*e^(p*log(-c*e*x + c*d - b*e) + p*log(e*x + d)) + (
e*x + d)^m*b*e^2*x*e^(p*log(-c*e*x + c*d - b*e) + p*log(e*x + d)) - (e*x +
 d)^m*c*d^2*e^(p*log(-c*e*x + c*d - b*e) + p*log(e*x + d)) + (e*x + d)^m*b
*d*e*e^(p*log(-c*e*x + c*d - b*e) + p*log(e*x + d)))/e
```

Mupad [B] (verification not implemented)

Time = 10.72 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int (d+ex)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(cd^2-bde-be^2x-ce^2x^2\right)^p\,dx$$

$$= -\left(b\,e\,x\,(d+e\,x)^m-\frac{(c\,d^2-b\,d\,e)\,(d+e\,x)^m}{e}+c\,e\,x^2\,(d+e\,x)^m\right)\,\left(c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x\right)^p$$

input
```
int(-(d + e*x)^m*(b*e*(m + p + 1) - c*d*m + c*e*x*(m + 2*p + 2))*(c*d^2 -
c*e^2*x^2 - b*d*e - b*e^2*x)^p,x)
```

output
```
-(b*e*x*(d + e*x)^m - ((c*d^2 - b*d*e)*(d + e*x)^m)/e + c*e*x^2*(d + e*x)^
m)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^p
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

$$\int (d+ex)^m(cdm-be(1+m+p)-ce(2+m+2p)x)\left(cd^2-bde-be^2x-ce^2x^2\right)^p\,dx$$
$$=\frac{(ex+d)^m\left(-c\,e^2x^2-b\,e^2x-bde+c\,d^2\right)^p\left(-c\,e^2x^2-b\,e^2x-bde+c\,d^2\right)}{e}$$

input
```
int((e*x+d)^m*(c*d*m-b*e*(1+m+p)-c*e*(2+m+2*p)*x)*(-c*e^2*x^2-b*e^2*x-b*d*
e+c*d^2)^p,x)
```

output
```
((d + e*x)**m*( - b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)**p*( - b*d*e -
b*e**2*x + c*d**2 - c*e**2*x**2))/e
```

3.257 $\int (d+ex)^{-3-2p}(f+gx)\,(d(ef+dg+dgp)+e(ef+$

Optimal result

Integrand size = 60, antiderivative size = 64

$$\int (d+ex)^{-3-2p}(f+gx)\left(d(ef+dg+dgp)+e(ef+3dg+2dgp)x+e^2g(2+p)x^2\right)^p\,dx = -\frac{(d+ex)^{-3-2p}\left(d(ef+dg(1+p))+e(ef+dg(3+2p))x+e^2g(2+p)x^2\right)^{1+p}}{e^2(2+p)}$$

output
```
-(e*x+d)^(-3-2*p)*(d*(e*f+d*g*(p+1))+e*(e*f+d*g*(3+2*p))*x+e^2*g*(2+p)*x^2
)^(p+1)/e^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int (d+ex)^{-3-2p}(f+gx)\left(d(ef+dg+dgp)+e(ef+3dg+2dgp)x+e^2g(2+p)x^2\right)^p\,dx = -\frac{(d+ex)^{-3-2p}((d+ex)(dg(1+p)+e(f+g(2+p)x)))^{1+p}}{e^2(2+p)}$$

input
```
Integrate[(d + e*x)^(-3 - 2*p)*(f + g*x)*(d*(e*f + d*g + d*g*p) + e*(e*f +
 3*d*g + 2*d*g*p)*x + e^2*g*(2 + p)*x^2)^p,x]
```

output
```
-(((d + e*x)^(-3 - 2*p)*((d + e*x)*(d*g*(1 + p) + e*(f + g*(2 + p)*x)))^(1
 + p))/(e^2*(2 + p)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.017$, Rules used = $\{1217\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)(d+ex)^{-2p-3} \left(ex(2dgp+3dg+ef)+d(dgp+dg+ef)+e^2g(p+2)x^2\right)^p \, dx$$

$$\Big\downarrow \; 1217$$

$$-\frac{(d+ex)^{-2p-3} \left(ex(dg(2p+3)+ef)+d(dg(p+1)+ef)+e^2g(p+2)x^2\right)^{p+1}}{e^2(p+2)}$$

input
```
Int[(d + e*x)^(-3 - 2*p)*(f + g*x)*(d*(e*f + d*g + d*g*p) + e*(e*f + 3*d*g
 + 2*d*g*p)*x + e^2*g*(2 + p)*x^2)^p,x]
```

output
```
-(((d + e*x)^(-3 - 2*p)*(d*(e*f + d*g*(1 + p)) + e*(e*f + d*g*(3 + 2*p))*x
 + e^2*g*(2 + p)*x^2)^(1 + p))/(e^2*(2 + p)))
```

Defintions of rubi rules used

rule 1217
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && EqQ[c*e*f*(m + 2*p + 2) + g*(c*d*m - b*e*(m + p
+ 1)), 0]
```

Maple [A] (verified)

Time = 5.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

method	result
orering	$-\frac{(ex+d)(egxp+dgp+2egx+dg+ef)(ex+d)^{-3-2p}\left(d(dgp+dg+ef)+e(2dgp+3dg+ef)x+e^{2}g(2+p)x^{2}\right)^{p}}{e^{2}(2+p)}$
gosper	$-\frac{(ex+d)^{-2p-2}(egxp+dgp+2egx+dg+ef)\left(e^{2}g\,x^{2}p+2degpx+2e^{2}g\,x^{2}+d^{2}gp+3degx+e^{2}fx+d^{2}g+def\right)^{p}}{e^{2}(2+p)}$
risch	$-\frac{(ex+d)^{-3-2p}\left(e^{2}g\,x^{2}p+2degpx+2e^{2}g\,x^{2}+d^{2}gp+3degx+e^{2}fx+d^{2}g+def\right)(dg(p+1)+(f+(xp+2x)g)e)^{p}(ex+d)^{p}\mathrm{e}^{-\frac{i\,\mathrm{csgn}(i}{}}}{}$
parallelrisch	$-\frac{x^{2}(ex+d)^{-3-2p}\left(d(dgp+dg+ef)+e(2dgp+3dg+ef)x+e^{2}g(2+p)x^{2}\right)^{p}e^{2}g^{2}p+2x^{2}(ex+d)^{-3-2p}\left(d(dgp+dg+ef)+e(2dgp+\right.}{}$

input
```
int((e*x+d)^(-3-2*p)*(g*x+f)*(d*(d*g*p+d*g+e*f)+e*(2*d*g*p+3*d*g+e*f)*x+e^
2*g*(2+p)*x^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-(e*x+d)*(e*g*p*x+d*g*p+2*e*g*x+d*g+e*f)/e^2/(2+p)*(e*x+d)^(-3-2*p)*(d*(d*
g*p+d*g+e*f)+e*(2*d*g*p+3*d*g+e*f)*x+e^2*g*(2+p)*x^2)^p
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.06

$$\int (d+ex)^{-3-2p}(f+gx)\left(d(ef+dg+dgp)+e(ef+3dg+2dgp)x+e^{2}g(2+p)x^{2}\right)^{p}\,dx =$$
$$-\frac{(d^{2}gp+def+d^{2}g+(e^{2}gp+2\,e^{2}g)x^{2}+(2\,degp+e^{2}f+3\,deg)x)(d^{2}gp+def+d^{2}g+(e^{2}gp+2\,e^{2}g)}{e^{2}p+2\,e^{2}}$$

input
```
integrate((e*x+d)^(-3-2*p)*(g*x+f)*(d*(d*g*p+d*g+e*f)+e*(2*d*g*p+3*d*g+e*f
)*x+e^2*g*(2+p)*x^2)^p,x, algorithm="fricas")
```

output
```
-(d^2*g*p + d*e*f + d^2*g + (e^2*g*p + 2*e^2*g)*x^2 + (2*d*e*g*p + e^2*f +
 3*d*e*g)*x)*(d^2*g*p + d*e*f + d^2*g + (e^2*g*p + 2*e^2*g)*x^2 + (2*d*e*g
*p + e^2*f + 3*d*e*g)*x)^p*(e*x + d)^(-2*p - 3)/(e^2*p + 2*e^2)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{-3-2p}(f+gx)\left(d(ef+dg+dgp)\right.$$
$$\left.+e(ef+3dg+2dgp)x+e^2g(2+p)x^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(-3-2*p)*(g*x+f)*(d*(d*g*p+d*g+e*f)+e*(2*d*g*p+3*d*g+e*
f)*x+e**2*g*(2+p)*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^{-3-2p}(f+gx)\left(d(ef+dg+dgp)+e(ef+3dg+2dgp)x+e^2g(2+p)x^2\right)^p\,dx$$
$$=\int (gx+f)\left(e^2g(p+2)x^2+(2\,dgp+ef+3\,dg)ex+(dgp+ef+dg)d\right)^p(ex+d)^{-2\,p-3}\,dx$$

input
```
integrate((e*x+d)^(-3-2*p)*(g*x+f)*(d*(d*g*p+d*g+e*f)+e*(2*d*g*p+3*d*g+e*f
)*x+e^2*g*(2+p)*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((g*x + f)*(e^2*g*(p + 2)*x^2 + (2*d*g*p + e*f + 3*d*g)*e*x + (d*
g*p + e*f + d*g)*d)^p*(e*x + d)^(-2*p - 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(64) = 128.

Time = 0.37 (sec) , antiderivative size = 412, normalized size of antiderivative = 6.44

$$\int (d+ex)^{-3-2p}(f+gx)\left(d(ef+dg+dgp)+e(ef+3dg+2dgp)x+e^2g(2+p)x^2\right)^p\,dx=$$
$$-\frac{e^2gpx^2e^{(p\log(egpx+dgp+2\,egx+ef+dg)-p\log(ex+d)-3\,\log(ex+d))}+2\,degpxe^{(p\log(egpx+dgp+2\,egx+ef+dg)-p\log(ex+d)-3}}{}$$

input

```
integrate((e*x+d)^(-3-2*p)*(g*x+f)*(d*(d*g*p+d*g+e*f)+e*(2*d*g*p+3*d*g+e*f
)*x+e^2*g*(2+p)*x^2)^p,x, algorithm="giac")
```

output

```
-(e^2*g*p*x^2*e^(p*log(e*g*p*x + d*g*p + 2*e*g*x + e*f + d*g) - p*log(e*x
+ d) - 3*log(e*x + d)) + 2*d*e*g*p*x*e^(p*log(e*g*p*x + d*g*p + 2*e*g*x +
e*f + d*g) - p*log(e*x + d) - 3*log(e*x + d)) + 2*e^2*g*x^2*e^(p*log(e*g*p
*x + d*g*p + 2*e*g*x + e*f + d*g) - p*log(e*x + d) - 3*log(e*x + d)) + d^2
*g*p*e^(p*log(e*g*p*x + d*g*p + 2*e*g*x + e*f + d*g) - p*log(e*x + d) - 3*
log(e*x + d)) + e^2*f*x*e^(p*log(e*g*p*x + d*g*p + 2*e*g*x + e*f + d*g) -
p*log(e*x + d) - 3*log(e*x + d)) + 3*d*e*g*x*e^(p*log(e*g*p*x + d*g*p + 2*
e*g*x + e*f + d*g) - p*log(e*x + d) - 3*log(e*x + d)) + d*e*f*e^(p*log(e*g
*p*x + d*g*p + 2*e*g*x + e*f + d*g) - p*log(e*x + d) - 3*log(e*x + d)) + d
^2*g*e^(p*log(e*g*p*x + d*g*p + 2*e*g*x + e*f + d*g) - p*log(e*x + d) - 3*
log(e*x + d)))/(e^2*p + 2*e^2)
```

Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.16

$$\int (d+ex)^{-3-2p}(f+gx)\left(d(ef+dg+dgp)+e(ef+3dg+2dgp)x+e^2g(2+p)x^2\right)^p\,dx = -\left(d\,(d\,g+e\,f+d\,g\,p)+e\,x\,(3\,d\,g+e\,f+2\,d\,g\,p)+e^2\,g\,x^2\,(p+2)\right)^p\left(\frac{g\,x^2}{{(d+e\,x)}^{2\,p+3}}+\frac{d^2\,g+d\,e\,f+d^2\,g\,p}{e^2\,(p+2)\,{(d+e\,x)}^{2\,p+3}}+\frac{x\,(3\,d\,g+e\,f+2\,d\,g\,p)}{e\,(p+2)\,{(d+e\,x)}^{2\,p+3}}\right)$$

input

```
int(((f + g*x)*(d*(d*g + e*f + d*g*p) + e*x*(3*d*g + e*f + 2*d*g*p) + e^2*
g*x^2*(p + 2))^p)/(d + e*x)^(2*p + 3),x)
```

output

```
-(d*(d*g + e*f + d*g*p) + e*x*(3*d*g + e*f + 2*d*g*p) + e^2*g*x^2*(p + 2))
^p*((g*x^2)/(d + e*x)^(2*p + 3) + (d^2*g + d*e*f + d^2*g*p)/(e^2*(p + 2)*(
d + e*x)^(2*p + 3)) + (x*(3*d*g + e*f + 2*d*g*p))/(e*(p + 2)*(d + e*x)^(2*
p + 3)))
```

Reduce [F]

$$\int (d+ex)^{-3-2p}(f+gx)\left(d(ef+dg+dgp)+e(ef+3dg+2dgp)x+e^2g(2+p)x^2\right)^p\,dx$$
$$=\left(\int \frac{\left(e^2gp\,x^2+2degpx+2e^2g\,x^2+d^2gp+3degx+e^2fx+d^2g+def\right)^p}{\left(ex+d\right)^{2p}d^3+3\left(ex+d\right)^{2p}d^2ex+3\left(ex+d\right)^{2p}d\,e^2x^2+\left(ex+d\right)^{2p}e^3x^3}dx\right)f$$
$$+\left(\int \frac{\left(e^2gp\,x^2+2degpx+2e^2g\,x^2+d^2gp+3degx+e^2fx+d^2g+def\right)^p x}{\left(ex+d\right)^{2p}d^3+3\left(ex+d\right)^{2p}d^2ex+3\left(ex+d\right)^{2p}d\,e^2x^2+\left(ex+d\right)^{2p}e^3x^3}dx\right)g$$

input

```
int((e*x+d)^(-3-2*p)*(g*x+f)*(d*(d*g*p+d*g+e*f)+e*(2*d*g*p+3*d*g+e*f)*x+e^
2*g*(2+p)*x^2)^p,x)
```

output

```
int((d**2*g*p + d**2*g + d*e*f + 2*d*e*g*p*x + 3*d*e*g*x + e**2*f*x + e**2
*g*p*x**2 + 2*e**2*g*x**2)**p/((d + e*x)**(2*p)*d**3 + 3*(d + e*x)**(2*p)*
d**2*e*x + 3*(d + e*x)**(2*p)*d*e**2*x**2 + (d + e*x)**(2*p)*e**3*x**3),x)
*f + int(((d**2*g*p + d**2*g + d*e*f + 2*d*e*g*p*x + 3*d*e*g*x + e**2*f*x
+ e**2*g*p*x**2 + 2*e**2*g*x**2)**p*x)/((d + e*x)**(2*p)*d**3 + 3*(d + e*x
)**(2*p)*d**2*e*x + 3*(d + e*x)**(2*p)*d*e**2*x**2 + (d + e*x)**(2*p)*e**3
*x**3),x)*g
```

3.258 $\int \frac{(d+ex)^2(f+gx)}{\left(cf^2-bfg-bg^2x-cg^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 143

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)}{\left(cf^2-bfg-bg^2x-cg^2x^2\right)^2}\,dx &= \frac{(cef+cdg-beg)^2}{c^2g^3(2cf-bg)(cf-bg-cgx)}+\frac{(ef-dg)^2\log(f+gx)}{g^3(2cf-bg)^2} \\ &\quad +\frac{(3cef-cdg-beg)(cef+cdg-beg)\log(cf-bg-cgx)}{c^2g^3(2cf-bg)^2}\end{aligned}$$

output
```
(-b*e*g+c*d*g+c*e*f)^2/c^2/g^3/(-b*g+2*c*f)/(-c*g*x-b*g+c*f)+(-d*g+e*f)^2*
ln(g*x+f)/g^3/(-b*g+2*c*f)^2+(-b*e*g-c*d*g+3*c*e*f)*(-b*e*g+c*d*g+c*e*f)*l
n(-c*g*x-b*g+c*f)/c^2/g^3/(-b*g+2*c*f)^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^2(f+gx)}{\left(cf^2-bfg-bg^2x-cg^2x^2\right)^2}\,dx = \frac{\frac{(cef+cdg-beg)^2}{c^2(2cf-bg)(-bg+c(f-gx))}+\frac{(ef-dg)^2\log(f+gx)}{(-2cf+bg)^2}+\frac{\left(-4bce^2fg+b^2e^2g^2+c^2\left(3e^2f^2+2defg-d^2g^2\right)\right)\log(cf-bg-cgx)}{c^2(-2cf+bg)^2}}{g^3}$$

input
```
Integrate[((d + e*x)^2*(f + g*x))/(c*f^2 - b*f*g - b*g^2*x - c*g^2*x^2)^2,
x]
```

output
```
((c*e*f + c*d*g - b*e*g)^2/(c^2*(2*c*f - b*g)*(-(b*g) + c*(f - g*x))) + ((
e*f - d*g)^2*Log[f + g*x])/(-2*c*f + b*g)^2 + ((-4*b*c*e^2*f*g + b^2*e^2*g
^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2))*Log[c*f - b*g - c*g*x])/(c^2*(
-2*c*f + b*g)^2))/g^3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1207, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)}{\left(-bfg-bg^2x+cf^2-cg^2x^2\right)^2}dx$$

$$\Big\downarrow \textcolor{blue}{1207}$$

$$c^2g^4\int \left(\frac{(ef-dg)^2}{c^2g^6(2cf-bg)^2(f+gx)}-\frac{(3cef-cdg-beg)(cef+cdg-beg)}{c^3g^6(2cf-bg)^2(cf-bg-cgx)}+\frac{(cef+cdg-beg)^2}{c^3g^6(2cf-bg)(cf-bg-cgx)^2}\right)$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$c^2g^4\left(\frac{(-beg+cdg+cef)^2}{c^4g^7(2cf-bg)(-bg+cf-cgx)}+\frac{(-beg-cdg+3cef)(-beg+cdg+cef)\log(-bg+cf-cgx)}{c^4g^7(2cf-bg)^2}+\right.$$

input
```
Int[((d + e*x)^2*(f + g*x))/(c*f^2 - b*f*g - b*g^2*x - c*g^2*x^2)^2,x]
```

output
```
c^2*g^4*((c*e*f + c*d*g - b*e*g)^2/(c^4*g^7*(2*c*f - b*g)*(c*f - b*g - c*g
*x)) + ((e*f - d*g)^2*Log[f + g*x])/(c^2*g^7*(2*c*f - b*g)^2) + ((3*c*e*f
- c*d*g - b*e*g)*(c*e*f + c*d*g - b*e*g)*Log[c*f - b*g - c*g*x])/(c^4*g^7*
(2*c*f - b*g)^2))
```

Defintions of rubi rules used

rule 1207
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1
/c^p   Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(b/2 - q/2 + c*x)^p*(b/2
 + q/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && ILtQ[p, -1] && IntegersQ[m, n] && NiceSqrtQ[b^2 - 4*
a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.52

method	result
default	$\frac{\left(b^{2} e^{2} g^{2}-4 b c\, e^{2} f g-c^{2} d^{2} g^{2}+2 c^{2} d e f g+3 c^{2} e^{2} f^{2}\right) \ln \left(x g c+b g-c f\right)}{g^{3} \left(b g-2 c f\right)^{2} c^{2}}-\frac{-b^{2} e^{2} g^{2}+2 b c d e\, g^{2}+2 b c\, e^{2} f g-c^{2} d^{2} g^{2}-2 c^{2} d e f g-c^{2} e^{2} f}{c^{2} g^{3} \left(b g-2 c f\right) \left(x g c+b g-c f\right)}$
norman	$\frac{\frac{\left(b^{2} e^{2} g^{2}-2 b c d e\, g^{2}-2 b c\, e^{2} f g+c^{2} d^{2} g^{2}+2 c^{2} d e f g+c^{2} e^{2} f^{2}\right) x}{c^{2} g^{2} \left(b g-2 c f\right)}+\frac{\left(b^{2} e^{2} g^{2}-2 b c d e\, g^{2}-2 b c\, e^{2} f g+c^{2} d^{2} g^{2}+2 c^{2} d e f g+c^{2} e^{2} f^{2}\right) f}{c^{2} g^{3} \left(b g-2 c f\right)}}{\left(g x+f\right) \left(x g c+b g-c f\right)}+\frac{(d^{2} g^{2}-2}{g^{3} (b^{2}}$
risch	$\frac{b^{2} e^{2}}{c^{2} g \left(b g-2 c f\right) \left(x g c+b g-c f\right)}-\frac{2 b d e}{c g \left(b g-2 c f\right) \left(x g c+b g-c f\right)}-\frac{2 b\, e^{2} f}{c\, g^{2} \left(b g-2 c f\right) \left(x g c+b g-c f\right)}+\frac{d^{2}}{g \left(b g-2 c f\right) \left(x g c+b g-c f\right)}+\frac{}{g^{2}}$
parallelrisch	$\frac{-2 c^{3} e^{2} f^{3}-2 b^{2} d e\, g^{3} c-4 b^{2} e^{2} f\, g^{2} c+\ln \left(x g c+b g-c f\right) b^{3} e^{2} g^{3}-3 \ln \left(x g c+b g-c f\right) c^{3} e^{2} f^{3}-\ln \left(g x+f\right) c^{3} e^{2} f^{3}-2 \ln \left(g x+f\right) x\, c^{3} d e f}{}$

input
```
int((e*x+d)^2*(g*x+f)/(-c*g^2*x^2-b*g^2*x-b*f*g+c*f^2)^2,x,method=_RETURNV
ERBOSE)
```

output
```
(b^2*e^2*g^2-4*b*c*e^2*f*g-c^2*d^2*g^2+2*c^2*d*e*f*g+3*c^2*e^2*f^2)/g^3/(b
*g-2*c*f)^2/c^2*ln(c*g*x+b*g-c*f)-(-b^2*e^2*g^2+2*b*c*d*e*g^2+2*b*c*e^2*f*
g-c^2*d^2*g^2-2*c^2*d*e*f*g-c^2*e^2*f^2)/c^2/g^3/(b*g-2*c*f)/(c*g*x+b*g-c*
f)+(d^2*g^2-2*d*e*f*g+e^2*f^2)/g^3/(b*g-2*c*f)^2*ln(g*x+f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(143) = 286$.

Time = 0.10 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.13

$$\int \frac{(d+ex)^2(f+gx)}{(cf^2-bfg-bg^2x-cg^2x^2)^2}\,dx$$
$$= \frac{2\,c^3e^2f^3+(4\,c^3de-5\,bc^2e^2)f^2g+2\,(c^3d^2-3\,bc^2de+2\,b^2ce^2)fg^2-(bc^2d^2-2\,b^2cde+b^3e^2)g^3+(3\,c^3e^2}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*g^2*x^2-b*g^2*x-b*f*g+c*f^2)^2,x, algorith
m="fricas")
```

output
```
(2*c^3*e^2*f^3 + (4*c^3*d*e - 5*b*c^2*e^2)*f^2*g + 2*(c^3*d^2 - 3*b*c^2*d*
e + 2*b^2*c*e^2)*f*g^2 - (b*c^2*d^2 - 2*b^2*c*d*e + b^3*e^2)*g^3 + (3*c^3*
e^2*f^3 + (2*c^3*d*e - 7*b*c^2*e^2)*f^2*g - (c^3*d^2 + 2*b*c^2*d*e - 5*b^2
*c*e^2)*f*g^2 + (b*c^2*d^2 - b^3*e^2)*g^3 - (3*c^3*e^2*f^2*g + 2*(c^3*d*e
- 2*b*c^2*e^2)*f*g^2 - (c^3*d^2 - b^2*c*e^2)*g^3)*x)*log(c*g*x - c*f + b*g
) + (c^3*e^2*f^3 - b*c^2*d^2*g^3 - (2*c^3*d*e + b*c^2*e^2)*f^2*g + (c^3*d^
2 + 2*b*c^2*d*e)*f*g^2 - (c^3*e^2*f^2*g - 2*c^3*d*e*f*g^2 + c^3*d^2*g^3)*x
)*log(g*x + f))/(4*c^5*f^3*g^3 - 8*b*c^4*f^2*g^4 + 5*b^2*c^3*f*g^5 - b^3*c
^2*g^6 - (4*c^5*f^2*g^4 - 4*b*c^4*f*g^5 + b^2*c^3*g^6)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)}{(cf^2-bfg-bg^2x-cg^2x^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2*(g*x+f)/(-c*g**2*x**2-b*g**2*x-b*f*g+c*f**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.87

$$\begin{aligned}&\int \frac{(d+ex)^2(f+gx)}{(cf^2-bfg-bg^2x-cg^2x^2)^2}\,dx\\&=\frac{(3\,c^2e^2f^2+2\,(c^2de-2\,bce^2)fg-(c^2d^2-b^2e^2)g^2)\log(cgx-cf+bg)}{4\,c^4f^2g^3-4\,bc^3fg^4+b^2c^2g^5}\\&\quad+\frac{(e^2f^2-2\,defg+d^2g^2)\log(gx+f)}{4\,c^2f^2g^3-4\,bcfg^4+b^2g^5}\\&\quad+\frac{c^2e^2f^2+2\,(c^2de-bce^2)fg+(c^2d^2-2\,bcde+b^2e^2)g^2}{2\,c^4f^2g^3-3\,bc^3fg^4+b^2c^2g^5-(2\,c^4fg^4-bc^3g^5)x}\end{aligned}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*g^2*x^2-b*g^2*x-b*f*g+c*f^2)^2,x, algorith
m="maxima")
```

output
```
(3*c^2*e^2*f^2 + 2*(c^2*d*e - 2*b*c*e^2)*f*g - (c^2*d^2 - b^2*e^2)*g^2)*lo
g(c*g*x - c*f + b*g)/(4*c^4*f^2*g^3 - 4*b*c^3*f*g^4 + b^2*c^2*g^5) + (e^2*
f^2 - 2*d*e*f*g + d^2*g^2)*log(g*x + f)/(4*c^2*f^2*g^3 - 4*b*c*f*g^4 + b^2
*g^5) + (c^2*e^2*f^2 + 2*(c^2*d*e - b*c*e^2)*f*g + (c^2*d^2 - 2*b*c*d*e +
b^2*e^2)*g^2)/(2*c^4*f^2*g^3 - 3*b*c^3*f*g^4 + b^2*c^2*g^5 - (2*c^4*f*g^4
- b*c^3*g^5)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(143) = 286$.

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.09

$$\begin{aligned}&\int \frac{(d+ex)^2(f+gx)}{(cf^2-bfg-bg^2x-cg^2x^2)^2}\,dx\\&=\frac{(3\,c^2e^2f^2+2\,c^2defg-4\,bce^2fg-c^2d^2g^2+b^2e^2g^2)\log(|cgx-cf+bg|)}{4\,c^4f^2g^3-4\,bc^3fg^4+b^2c^2g^5}\\&\quad+\frac{(e^2f^2-2\,defg+d^2g^2)\log(|gx+f|)}{4\,c^2f^2g^3-4\,bcfg^4+b^2g^5}\\&\quad-\frac{2\,c^3e^2f^3+4\,c^3def^2g-5\,bc^2e^2f^2g+2\,c^3d^2fg^2-6\,bc^2defg^2+4\,b^2ce^2fg^2-bc^2d^2g^3+2\,b^2cdeg^3-b^3e^2}{(cgx-cf+bg)(2\,cf-bg)^2c^2g^3}\end{aligned}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*g^2*x^2-b*g^2*x-b*f*g+c*f^2)^2,x, algorith
m="giac")
```

output
```
(3*c^2*e^2*f^2 + 2*c^2*d*e*f*g - 4*b*c*e^2*f*g - c^2*d^2*g^2 + b^2*e^2*g^2
)*log(abs(c*g*x - c*f + b*g))/(4*c^4*f^2*g^3 - 4*b*c^3*f*g^4 + b^2*c^2*g^5
) + (e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(abs(g*x + f))/(4*c^2*f^2*g^3 - 4*b
*c*f*g^4 + b^2*g^5) - (2*c^3*e^2*f^3 + 4*c^3*d*e*f^2*g - 5*b*c^2*e^2*f^2*g
 + 2*c^3*d^2*f*g^2 - 6*b*c^2*d*e*f*g^2 + 4*b^2*c*e^2*f*g^2 - b*c^2*d^2*g^3
 + 2*b^2*c*d*e*g^3 - b^3*e^2*g^3)/((c*g*x - c*f + b*g)*(2*c*f - b*g)^2*c^2
*g^3)
```

Mupad [B] (verification not implemented)

Time = 11.39 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.57

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)}{(cf^2-bfg-bg^2x-cg^2x^2)^2}\,dx &= \frac{\ln(f+g\,x)\,(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2)}{b^2\,g^5-4\,b\,c\,f\,g^4+4\,c^2\,f^2\,g^3}\\ &\quad+\frac{b^2\,e^2\,g^2-2\,b\,c\,d\,e\,g^2-2\,b\,c\,e^2\,f\,g+c^2\,d^2\,g^2+2\,c^2\,d\,e\,f\,g+c^2\,e^2\,f^2}{c^2\,g^3\,(b\,g-2\,c\,f)\,(b\,g-c\,f+c\,g\,x)}\\ &\quad+\frac{\ln(b\,g-c\,f+c\,g\,x)\,\left(c^2\,(-d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2)+b^2\,e^2\,g^2-4\,b\,c\,e^2\,f\,g\right)}{c^2\,g^3\,(b\,g-2\,c\,f)^2}\end{aligned}$$

input
```
int(((f + g*x)*(d + e*x)^2)/(c*g^2*x^2 - c*f^2 + b*f*g + b*g^2*x)^2,x)
```

output
```
(log(f + g*x)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))/(b^2*g^5 + 4*c^2*f^2*g^3 -
4*b*c*f*g^4) + (b^2*e^2*g^2 + c^2*d^2*g^2 + c^2*e^2*f^2 - 2*b*c*d*e*g^2 -
2*b*c*e^2*f*g + 2*c^2*d*e*f*g)/(c^2*g^3*(b*g - 2*c*f)*(b*g - c*f + c*g*x))
 + (log(b*g - c*f + c*g*x)*(c^2*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g) + b^2*e^
2*g^2 - 4*b*c*e^2*f*g))/(c^2*g^3*(b*g - 2*c*f)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1017, normalized size of antiderivative = 7.11

$$\int \frac{(d+ex)^2(f+gx)}{(cf^2-bfg-bg^2x-cg^2x^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(g*x+f)/(-c*g^2*x^2-b*g^2*x-b*f*g+c*f^2)^2,x)
```

output
```
(log(b*g - c*f + c*g*x)*b**4*e**2*g**4 - 6*log(b*g - c*f + c*g*x)*b**3*c*e
**2*f*g**3 + log(b*g - c*f + c*g*x)*b**3*c*e**2*g**4*x - log(b*g - c*f + c
*g*x)*b**2*c**2*d**2*g**4 + 2*log(b*g - c*f + c*g*x)*b**2*c**2*d*e*f*g**3
+ 12*log(b*g - c*f + c*g*x)*b**2*c**2*e**2*f**2*g**2 - 5*log(b*g - c*f + c
*g*x)*b**2*c**2*e**2*f*g**3*x + 2*log(b*g - c*f + c*g*x)*b*c**3*d**2*f*g**
3 - log(b*g - c*f + c*g*x)*b*c**3*d**2*g**4*x - 4*log(b*g - c*f + c*g*x)*b
*c**3*d*e*f**2*g**2 + 2*log(b*g - c*f + c*g*x)*b*c**3*d*e*f*g**3*x - 10*lo
g(b*g - c*f + c*g*x)*b*c**3*e**2*f**3*g + 7*log(b*g - c*f + c*g*x)*b*c**3*
e**2*f**2*g**2*x - log(b*g - c*f + c*g*x)*c**4*d**2*f**2*g**2 + log(b*g -
c*f + c*g*x)*c**4*d**2*f*g**3*x + 2*log(b*g - c*f + c*g*x)*c**4*d*e*f**3*g
 - 2*log(b*g - c*f + c*g*x)*c**4*d*e*f**2*g**2*x + 3*log(b*g - c*f + c*g*x
)*c**4*e**2*f**4 - 3*log(b*g - c*f + c*g*x)*c**4*e**2*f**3*g*x + log(f + g
*x)*b**2*c**2*d**2*g**4 - 2*log(f + g*x)*b**2*c**2*d*e*f*g**3 + log(f + g*
x)*b**2*c**2*e**2*f**2*g**2 - 2*log(f + g*x)*b*c**3*d**2*f*g**3 + log(f +
g*x)*b*c**3*d**2*g**4*x + 4*log(f + g*x)*b*c**3*d*e*f**2*g**2 - 2*log(f +
g*x)*b*c**3*d*e*f*g**3*x - 2*log(f + g*x)*b*c**3*e**2*f**3*g + log(f + g*x
)*b*c**3*e**2*f**2*g**2*x + log(f + g*x)*c**4*d**2*f**2*g**2 - log(f + g*x
)*c**4*d**2*f*g**3*x - 2*log(f + g*x)*c**4*d*e*f**3*g + 2*log(f + g*x)*c**
4*d*e*f**2*g**2*x + log(f + g*x)*c**4*e**2*f**4 - log(f + g*x)*c**4*e**2*f
**3*g*x - b**3*c*e**2*g**4*x + 2*b**2*c**2*d*e*g**4*x + 4*b**2*c**2*e**...
```

3.259 $\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 527

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$$

$$= \frac{1}{24}\left(\frac{ag}{cd}+\frac{6ef-7dg}{e^2}\right)(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}$$

$$+ \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e}$$

$$+ \frac{(15a^3e^6g^3-a^2cde^4g^2(72ef-17dg)+ac^2d^2e^2g(136e^2f^2-96defg+25d^2g^2)+c^3d^3(96e^3f^3-376de^2}{}$$

$$+ \frac{(cd^2-ae^2)(5a^3e^6g^3-3a^2cde^4g^2(8ef-3dg)+3ac^2d^2e^2g(16e^2f^2-16defg+5d^2g^2)-c^3d^3(64e^3f^3 -}{64c^{7/2}d^{7/2}e^{9/2}}$$

output
```
1/24*(a*g/c/d+(-7*d*g+6*e*f)/e^2)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+1/4*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e+1/192*(1
5*a^3*e^6*g^3-a^2*c*d*e^4*g^2*(-17*d*g+72*e*f)+a*c^2*d^2*e^2*g*(25*d^2*g^2
-96*d*e*f*g+136*e^2*f^2)+c^3*d^3*(-105*d^3*g^3+360*d^2*e*f*g^2-376*d*e^2*f
^2*g+96*e^3*f^3)-2*c*d*e*g*(5*a^2*e^4*g^2-2*a*c*d*e^2*g*(-3*d*g+8*e*f)-c^2
*d^2*(35*d^2*g^2-64*d*e*f*g+24*e^2*f^2))*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/c^3/d^3/e^4+1/64*(-a*e^2+c*d^2)*(5*a^3*e^6*g^3-3*a^2*c*d*e^4*g^2
*(-3*d*g+8*e*f)+3*a*c^2*d^2*e^2*g*(5*d^2*g^2-16*d*e*f*g+16*e^2*f^2)-c^3*d^
3*(-35*d^3*g^3+120*d^2*e*f*g^2-144*d*e^2*f^2*g+64*e^3*f^3))*arctanh(c^(1/2
)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)
/d^(7/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 13.95 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$

$$= \sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(15a^3e^6g^3+a^2cde^4g^2(17dg-2e(36f+5gx))+ac^2d^2e^2g(25d^2g^2-12\right.$$

input
```
Integrate[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e
*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(15*a^3*e^6*g^3 +
a^2*c*d*e^4*g^2*(17*d*g - 2*e*(36*f + 5*g*x)) + a*c^2*d^2*e^2*g*(25*d^2*g^
2 - 12*d*e*g*(8*f + g*x) + 8*e^2*(18*f^2 + 6*f*g*x + g^2*x^2)) + c^3*d^3*(
-105*d^3*g^3 + 10*d^2*e*g^2*(36*f + 7*g*x) - 8*d*e^2*g*(54*f^2 + 30*f*g*x
+ 7*g^2*x^2) + 48*e^3*(4*f^3 + 6*f^2*g*x + 4*f*g^2*x^2 + g^3*x^3))) + (3*S
qrt[c*d]*Sqrt[c*d^2 - a*e^2]*(5*a^3*e^6*g^3 + 3*a^2*c*d*e^4*g^2*(-8*e*f +
3*d*g) + 3*a*c^2*d^2*e^2*g*(16*e^2*f^2 - 16*d*e*f*g + 5*d^2*g^2) + c^3*d^3
*(-64*e^3*f^3 + 144*d*e^2*f^2*g - 120*d^2*e*f*g^2 + 35*d^3*g^3))*ArcSinh[(
Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])
])/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(192*c^(7/2
)*d^(7/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1215, 1236, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}dx$$

$$\Big\downarrow \ 1215$$

$$\int \frac{(f+gx)^3(ae+cdx)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1236$$

$$\frac{\int -\frac{cd(f+gx)^2\left(cfd^2-ae(7ef-6dg)-\left(age^2+cd(6ef-7dg)\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cde}+\frac{(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}$$

$$\Big\downarrow \ 27$$

$$\frac{(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\int \frac{(f+gx)^2\left(cfd^2-ae(7ef-6dg)-\left(age^2+cd(6ef-7dg)\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8e}$$

$$\Big\downarrow \ 1236$$

$$\frac{(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\frac{\int \frac{(f+gx)\left(c^2f(12ef-7dg)d^3-2ace\left(18e^2f^2-27degf+14d^2g^2\right)d+a^2e^3g(ef+4dg)+\left(5a^2g^2e^4-2acdg(8ef-3dg)e^2-c^2d^2\left(24e^2f^2-64degf+35d^2g^2\right)\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}-}{8e}$$

$$\Big\downarrow \ 27$$

$$\frac{(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\frac{\int \frac{(f+gx)\left(c^2f(12ef-7dg)d^3-2ace\left(18e^2f^2-27degf+14d^2g^2\right)d+a^2e^3g(ef+4dg)+\left(5a^2g^2e^4-2acdg(8ef-3dg)e^2-c^2d^2\left(24e^2f^2-64degf+35d^2g^2\right)\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}-}{8e}$$

$$\Big\downarrow \text{1225}$$

$$\frac{(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-$$

$$-\frac{3\left(cd^2-ae^2\right)\left(5a^3e^6g^3-3a^2cde^4g^2(8ef-3dg)+3ac^2d^2e^2g\left(5d^2g^2-16defg+16e^2f^2\right)-c^3d^3\left(-35d^3g^3+120d^2efg^2-144de^2f^2g+64e^3f^3\right)\right)\int\frac{1}{\sqrt{cdex^2+\left(cd^2+a\right.}}}{8c^2d^2e^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-$$

$$-\frac{3\left(cd^2-ae^2\right)\left(5a^3e^6g^3-3a^2cde^4g^2(8ef-3dg)+3ac^2d^2e^2g\left(5d^2g^2-16defg+16e^2f^2\right)-c^3d^3\left(-35d^3g^3+120d^2efg^2-144de^2f^2g+64e^3f^3\right)\right)\int\frac{1}{4cde-\frac{\left(cd^2+2c\right.}{cdex^2+\left(cd\right.}}}{4c^2d^2e^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-$$

$$-\frac{3\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)\left(5a^3e^6g^3-3a^2cde^4g^2(8ef-3dg)+3ac^2d^2e^2g\left(5d^2g^2-16defg+16e^2f^2\right)-c^3d^3\left(-35d^3g^3+1\right.\right.}{8c^{5/2}d^{5/2}e^{5/2}}$$

input

```
Int[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output

```
((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - (-1/3*((
6*f - (7*d*g)/e + (a*e*g)/(c*d))*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) + (-1/4*((15*a^3*e^6*g^3 - a^2*c*d*e^4*g^2*(72*e*f - 17*d*
g) + a*c^2*d^2*e^2*g*(136*e^2*f^2 - 96*d*e*f*g + 25*d^2*g^2) + c^3*d^3*(96
*e^3*f^3 - 376*d*e^2*f^2*g + 360*d^2*e*f*g^2 - 105*d^3*g^3) - 2*c*d*e*g*(5
*a^2*e^4*g^2 - 2*a*c*d*e^2*g*(8*e*f - 3*d*g) - c^2*d^2*(24*e^2*f^2 - 64*d*
e*f*g + 35*d^2*g^2))*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*
d^2*e^2) - (3*(c*d^2 - a*e^2)*(5*a^3*e^6*g^3 - 3*a^2*c*d*e^4*g^2*(8*e*f -
3*d*g) + 3*a*c^2*d^2*e^2*g*(16*e^2*f^2 - 16*d*e*f*g + 5*d^2*g^2) - c^3*d^3
*(64*e^3*f^3 - 144*d*e^2*f^2*g + 120*d^2*e*f*g^2 - 35*d^3*g^3))*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(5/2)))/(6*c*d*e))/(8*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1322 vs. $2(499) = 998$.

Time = 2.26 (sec) , antiderivative size = 1323, normalized size of antiderivative = 2.51

method	result	size
default	Expression too large to display	1323

input

```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d),x,method=_RE
TURNVERBOSE)
```

output

```
g/e^3*(d^2*g^2*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/
2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d
*e*c)^(1/2))-e*g*(d*g-3*e*f)*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/
d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*l
n((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2))/(d*e*c)^(1/2)))+e^2*g^2*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(3/2)/d/e/c-5/8*(a*e^2+c*d^2)/d/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+3*e^2*f^2*(1/4*(2*c*d*e*x+a
*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e
^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-3*d*e*f*g*(1/4*(2*c
*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(...
```

Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 1274, normalized size of antiderivative = 2.42

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, alg
orithm="fricas")
```

output

```
[1/768*(3*(64*(c^4*d^5*e^3 - a*c^3*d^3*e^5)*f^3 - 48*(3*c^4*d^6*e^2 - 2*a*
c^3*d^4*e^4 - a^2*c^2*d^2*e^6)*f^2*g + 24*(5*c^4*d^7*e - 3*a*c^3*d^5*e^3 -
 a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*f*g^2 - (35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6
*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*g^3)*sqrt(c*d*e)*log(8*c^2
*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^
3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*e^4*g^3*x^3 + 192*c^4*d^4*e^4*f^3 - 14
4*(3*c^4*d^5*e^3 - a*c^3*d^3*e^5)*f^2*g + 24*(15*c^4*d^6*e^2 - 4*a*c^3*d^4
*e^4 - 3*a^2*c^2*d^2*e^6)*f*g^2 - (105*c^4*d^7*e - 25*a*c^3*d^5*e^3 - 17*a
^2*c^2*d^3*e^5 - 15*a^3*c*d*e^7)*g^3 + 8*(24*c^4*d^4*e^4*f*g^2 - (7*c^4*d^
5*e^3 - a*c^3*d^3*e^5)*g^3)*x^2 + 2*(144*c^4*d^4*e^4*f^2*g - 24*(5*c^4*d^5
*e^3 - a*c^3*d^3*e^5)*f*g^2 + (35*c^4*d^6*e^2 - 6*a*c^3*d^4*e^4 - 5*a^2*c^
2*d^2*e^6)*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e
^5), 1/384*(3*(64*(c^4*d^5*e^3 - a*c^3*d^3*e^5)*f^3 - 48*(3*c^4*d^6*e^2 -
2*a*c^3*d^4*e^4 - a^2*c^2*d^2*e^6)*f^2*g + 24*(5*c^4*d^7*e - 3*a*c^3*d^5*e
^3 - a^2*c^2*d^3*e^5 - a^3*c*d*e^7)*f*g^2 - (35*c^4*d^8 - 20*a*c^3*d^6*e^2
 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*g^3)*sqrt(-c*d*e)*arct
an(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*
e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)
*x)) + 2*(48*c^4*d^4*e^4*g^3*x^3 + 192*c^4*d^4*e^4*f^3 - 144*(3*c^4*d^5...
```

Sympy [F]

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^3}{d+ex}\,dx$$

input
```
integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x
)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, alg
orithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.20

$$\begin{aligned}&\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx\\&=\frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(\frac{6\,g^3x}{e}+\frac{24\,c^3d^3e^3fg^2-7\,c^3d^4e^2g^3+ac^2d^2e^4g^3}{c^3d^3e^4}\right)x+\frac{144\,c^3d^3}{}\right.\right.\right.\\&\quad+\frac{(64\,c^4d^5e^3f^3-64\,ac^3d^3e^5f^3-144\,c^4d^6e^2f^2g+96\,ac^3d^4e^4f^2g+48\,a^2c^2d^2e^6f^2g+120\,c^4d^7efg^2-72\,a}{}\end{aligned}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, alg
orithm="giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*g^3*x/e + (24*c
^3*d^3*e^3*f*g^2 - 7*c^3*d^4*e^2*g^3 + a*c^2*d^2*e^4*g^3)/(c^3*d^3*e^4))*x
 + (144*c^3*d^3*e^3*f^2*g - 120*c^3*d^4*e^2*f*g^2 + 24*a*c^2*d^2*e^4*f*g^2
 + 35*c^3*d^5*e*g^3 - 6*a*c^2*d^3*e^3*g^3 - 5*a^2*c*d*e^5*g^3)/(c^3*d^3*e^
4))*x + (192*c^3*d^3*e^3*f^3 - 432*c^3*d^4*e^2*f^2*g + 144*a*c^2*d^2*e^4*f
^2*g + 360*c^3*d^5*e*f*g^2 - 96*a*c^2*d^3*e^3*f*g^2 - 72*a^2*c*d*e^5*f*g^2
 - 105*c^3*d^6*g^3 + 25*a*c^2*d^4*e^2*g^3 + 17*a^2*c*d^2*e^4*g^3 + 15*a^3*
e^6*g^3)/(c^3*d^3*e^4)) + 1/128*(64*c^4*d^5*e^3*f^3 - 64*a*c^3*d^3*e^5*f^3
 - 144*c^4*d^6*e^2*f^2*g + 96*a*c^3*d^4*e^4*f^2*g + 48*a^2*c^2*d^2*e^6*f^2
*g + 120*c^4*d^7*e*f*g^2 - 72*a*c^3*d^5*e^3*f*g^2 - 24*a^2*c^2*d^3*e^5*f*g
^2 - 24*a^3*c*d*e^7*f*g^2 - 35*c^4*d^8*g^3 + 20*a*c^3*d^6*e^2*g^3 + 6*a^2*
c^2*d^4*e^4*g^3 + 4*a^3*c*d^2*e^6*g^3 + 5*a^4*e^8*g^3)*log(abs(-c*d^2 - a*
e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \int \frac{(f+g\,x)^3\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),
x)
```

output
```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),
 x)
```

Reduce [F]

$$\int \frac{(f+gx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$$
$$=\int \frac{\left(gx+f\right)^3\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2}}{ex+d}dx$$

input

```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

output

```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

3.260 $\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 314

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e}$$

$$-\frac{(3a^2e^4g^2-4acde^2g(3ef-dg)-c^2d^2(16e^2f^2-36defg+15d^2g^2)-2cdeg(ae^2g+cd(4ef-5dg))\,x)}{24c^2d^2e^3}$$

$$-\frac{(cd^2-ae^2)\,(a^2e^4g^2-2acde^2g(2ef-dg)+c^2d^2(8e^2f^2-12defg+5d^2g^2))\,\text{arctanh}\Big(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x}}}{8c^{5/2}d^{5/2}e^{7/2}}$$

output

```
1/3*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e-1/24*(3*a^2*e^4*g^
2-4*a*c*d*e^2*g*(-d*g+3*e*f)-c^2*d^2*(15*d^2*g^2-36*d*e*f*g+16*e^2*f^2)-2*
c*d*e*g*(a*e^2*g+c*d*(-5*d*g+4*e*f))*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/c^2/d^2/e^3-1/8*(-a*e^2+c*d^2)*(a^2*e^4*g^2-2*a*c*d*e^2*g*(-d*g+2*e*
f)+c^2*d^2*(5*d^2*g^2-12*d*e*f*g+8*e^2*f^2))*arctanh(c^(1/2)*d^(1/2)*(e*x+
d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2
)
```

Mathematica [A] (verified)

Time = 11.33 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.17

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{g(3ae^2g+cd(-8ef+5dg))(ae+cdx)}{4cde}+g(ae+cdx)(f+gx)+\frac{3\left(a^2e^4g^2+2acde^2g(-2ef+dg)+c^2\right.}{}\right.}{3cde}$$

input
```
Integrate[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e
*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-1/4*(g*(3*a*e^2*g + c*d*(-8*e*f + 5*d*g))
*(a*e + c*d*x))/(c*d*e) + g*(a*e + c*d*x)*(f + g*x) + (3*(a^2*e^4*g^2 + 2*
a*c*d*e^2*g*(-2*e*f + d*g) + c^2*d^2*(8*e^2*f^2 - 12*d*e*f*g + 5*d^2*g^2))
*(Sqrt[c*d]*Sqrt[e]*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x)*Sqrt[(c*d*(d + e*x))
/(c*d^2 - a*e^2)] - Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)*Sqrt[a*e + c*d*x]*ArcS
inh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*
e^2])]))/(8*(c*d)^(3/2)*e^(5/2)*Sqrt[c*d^2 - a*e^2]*(a*e + c*d*x)*Sqrt[(c*
d*(d + e*x))/(c*d^2 - a*e^2)])))/(3*c*d*e)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1215, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}\,dx$$
$$\Big\downarrow \ 1215$$
$$\int \frac{(f+gx)^2(ae+cdx)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{\int -\frac{cd(f+gx)\left(cfd^2-ae(5ef-4dg)-\left(age^2+cd(4ef-5dg)\right)x\right)}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3cde}+\frac{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}-\frac{\int \frac{(f+gx)\left(cfd^2-ae(5ef-4dg)-\left(age^2+cd(4ef-5dg)\right)x\right)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6e}$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}-\frac{\frac{3\left(cd^2-ae^2\right)\left(a^2e^4g^2-2acde^2g(2ef-dg)+c^2d^2\left(5d^2g^2-12defg+8e^2f^2\right)\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8c^2d^2e^2}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\Big(3a^2e^4g^2}{}}{6e}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}-\frac{\frac{3\left(cd^2-ae^2\right)\left(a^2e^4g^2-2acde^2g(2ef-dg)+c^2d^2\left(5d^2g^2-12defg+8e^2f^2\right)\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4c^2d^2e^2}+\frac{\sqrt{x(ae^2}{}}{6e}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3e}-\frac{\frac{3(cd^2-ae^2)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)\left(a^2e^4g^2-2acde^2g(2ef-dg)+c^2d^2\left(5d^2g^2-12defg+8e^2f^2\right)\right)}{8c^{5/2}d^{5/2}e^{5/2}}+\frac{\sqrt{x(ae^2+cd^2)+ad}{}}{6e}$$

input

```
Int[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*e) - (((3*a^2
*e^4*g^2 - 4*a*c*d*e^2*g*(3*e*f - d*g) - 2*c^2*(8*d^2*e^2*f^2 - 18*d^3*e*f
*g + (15*d^4*g^2)/2) - 2*c*d*e*g*(a*e^2*g + c*d*(4*e*f - 5*d*g))*x)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^2*d^2*e^2) + (3*(c*d^2 - a*e^2
)*(a^2*e^4*g^2 - 2*a*c*d*e^2*g*(2*e*f - d*g) + c^2*d^2*(8*e^2*f^2 - 12*d*e
*f*g + 5*d^2*g^2))*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*
e^(5/2)))/(6*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(290) = 580$.

Time = 2.02 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.20

method	result
default	$\frac{\left(d^{2} g^{2}-2 d e f g+e^{2} f^{2}\right) \left(\sqrt{d e c \left(x+\frac{d}{e}\right)^{2}+\left(a e^{2}-c d^{2}\right) \left(x+\frac{d}{e}\right)}+\frac{\left(a e^{2}-c d^{2}\right) \ln \left(\frac{\frac{a e^{2}}{2}-\frac{c d^{2}}{2}+d e c \left(x+\frac{d}{e}\right)}{\sqrt{d e c}}+\sqrt{d e c \left(x+\frac{d}{e}\right)^{2}+\left(a e^{2}-c d^{2}\right) \left(x+\frac{d}{e}\right.}}{2 \sqrt{d e c}}}{e^{3}}$

input

```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d),x,method=_RE
TURNVERBOSE)
```

output
```
(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2
)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))-g/e^2*(d*g
*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/
e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*
e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2
*e*f*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/
c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*
d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2
))-e*g*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2
)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2
+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(
1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 812, normalized size of antiderivative = 2.59

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, alg
orithm="fricas")
```

output
```
[-1/96*(3*sqrt(c*d*e)*(8*(c^3*d^4*e^2 - a*c^2*d^2*e^4)*f^2 - 4*(3*c^3*d^5*
e - 2*a*c^2*d^3*e^3 - a^2*c*d*e^5)*f*g + (5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^
2*c*d^2*e^4 - a^3*e^6)*g^2)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^
3*g^2*x^2 + 24*c^3*d^3*e^3*f^2 - 12*(3*c^3*d^4*e^2 - a*c^2*d^2*e^4)*f*g +
(15*c^3*d^5*e - 4*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5)*g^2 + 2*(12*c^3*d^3*e^3*f
*g - (5*c^3*d^4*e^2 - a*c^2*d^2*e^4)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/48*(3*sqrt(-c*d*e)*(8*(c^3*d^4*e^2 - a*c^
2*d^2*e^4)*f^2 - 4*(3*c^3*d^5*e - 2*a*c^2*d^3*e^3 - a^2*c*d*e^5)*f*g + (5*
c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*g^2)*arctan(1/2*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c
*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*
c^3*d^3*e^3*g^2*x^2 + 24*c^3*d^3*e^3*f^2 - 12*(3*c^3*d^4*e^2 - a*c^2*d^2*e
^4)*f*g + (15*c^3*d^5*e - 4*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5)*g^2 + 2*(12*c^3
*d^3*e^3*f*g - (5*c^3*d^4*e^2 - a*c^2*d^2*e^4)*g^2)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

Sympy [F]

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)^2}{d+ex}\,dx$$

input
```
integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x
)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, alg
orithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.17

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$

$$= \frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(\frac{4g^2x}{e}+\frac{12c^2d^2e^2fg-5c^2d^3eg^2+acde^3g^2}{c^2d^2e^3}\right)x+\frac{24c^2d^2e^2f^2-36}{}\right.$$

$$+\frac{(8c^3d^4e^2f^2-8ac^2d^2e^4f^2-12c^3d^5efg+8ac^2d^3e^3fg+4a^2cde^5fg+5c^3d^6g^2-3ac^2d^4e^2g^2-a^2cd^2e}{16\sqrt{cde}c^2d^2e^3}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, alg
orithm="giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*g^2*x/e + (12*c^2*d
^2*e^2*f*g - 5*c^2*d^3*e*g^2 + a*c*d*e^3*g^2)/(c^2*d^2*e^3))*x + (24*c^2*d
^2*e^2*f^2 - 36*c^2*d^3*e*f*g + 12*a*c*d*e^3*f*g + 15*c^2*d^4*g^2 - 4*a*c*
d^2*e^2*g^2 - 3*a^2*e^4*g^2)/(c^2*d^2*e^3)) + 1/16*(8*c^3*d^4*e^2*f^2 - 8*
a*c^2*d^2*e^4*f^2 - 12*c^3*d^5*e*f*g + 8*a*c^2*d^3*e^3*f*g + 4*a^2*c*d*e^5
*f*g + 5*c^3*d^6*g^2 - 3*a*c^2*d^4*e^2*g^2 - a^2*c*d^2*e^4*g^2 - a^3*e^6*g
^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned}\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx\\ =\int \frac{(f+g\,x)^2\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{d+e\,x}\,dx\end{aligned}$$

input
```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),
x)
```

output
```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),
 x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.69

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

output

```
( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5*g**2 - 4*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a*c**2*d**3*e**3*g**2 + 12*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*a*c**2*d**2*e**4*f*g + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e
**4*g**2*x + 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e*g**2 - 36*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*f*g - 10*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*c**3*d**4*e**2*g**2*x + 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*
d**3*e**3*f**2 + 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**3*e**3*f*g*x +
 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**3*e**3*g**2*x**2 + 3*sqrt(e)*sq
rt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*
x))/sqrt(a*e**2 - c*d**2))*a**3*e**6*g**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
c*d**2))*a**2*c*d**2*e**4*g**2 - 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*s
qrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a
**2*c*d*e**5*f*g + 9*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**2
*g**2 - 24*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d
)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**3*e**3*f*g + 24*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**2*e**4*f**2 - 15*sqrt(e)*sq
rt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d +...
```

3.261 $\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 171

$$\begin{aligned}\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx &= \frac{\left(ae^2g+cd(4ef-3dg)+2cdegx\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4cde^2} \\ &\quad+\frac{\left(cd^2-ae^2\right)\left(ae^2g-cd(4ef-3dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{4c^{3/2}d^{3/2}e^{5/2}}\end{aligned}$$

output

```
1/4*(a*e^2*g+c*d*(-3*d*g+4*e*f)+2*c*d*e*g*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/c/d/e^2+1/4*(-a*e^2+c*d^2)*(a*e^2*g-c*d*(-3*d*g+4*e*f))*arctanh
(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
c^(3/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.13

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$

$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(ae^2g+cd(4ef-3dg+2egx)) - \frac{2(cd^2-ae^2)\,(ae^2g+cd(-4ef+3dg))\text{arctanh}\left(\right.}{\sqrt{ae+cdx}\sqrt{d+ex}}\right.}{4c^{3/2}d^{3/2}e^{5/2}}$$

input
```
Integrate[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x
),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e^2*g + c*d*(4*
e*f - 3*d*g + 2*e*g*x)) - (2*(c*d^2 - a*e^2)*(a*e^2*g + c*d*(-4*e*f + 3*d*
g))*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*(Sqrt[d - (a*e^2)
/(c*d)] - Sqrt[d + e*x]))])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(4*c^(3/2)
*d^(3/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1215, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{d+ex}\,dx$$

$$\Big\downarrow \; 1215$$

$$\int \frac{(f+gx)(ae+cdx)}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{\left(cd^2-ae^2\right)\left(ae^2g-cd(4ef-3dg)\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2g+cd(4ef-3dg)+2cdegx\right)}{4cde^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(cd^2-ae^2\right)\left(ae^2g-cd(4ef-3dg)\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4cde^2}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2g+cd(4ef-3dg)+2cdegx\right)}{4cde^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)\left(ae^2g-cd(4ef-3dg)\right)}{8c^{3/2}d^{3/2}e^{5/2}}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2g+cd(4ef-3dg)+2cdegx\right)}{4cde^2}$$

input

```
Int[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output

```
((a*e^2*g + c*d*(4*e*f - 3*d*g) + 2*c*d*e*g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(4*c*d*e^2) + ((c*d^2 - a*e^2)*(a*e^2*g - c*d*(4*e*f - 3
*d*g))*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(5/2))
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.75

method	result
default	$\frac{g\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{4cde}+\frac{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}\right)}{8dec\sqrt{dec}}\right)}{e}-$

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d),x,method=_RETU
RNVERBOSE)
```

output
```
g/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c
/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d
*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)
)-(d*g-e*f)/e^2*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-
c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.94

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\left[\frac{\sqrt{cde}(4\,(c^2d^3e-acde^3)f-(3\,c^2d^4-2\,acd^2e^2-a^2e^4)g)\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{}\right.}{}\right.$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algor
ithm="fricas")
```

output
```
[1/16*(sqrt(c*d*e)*(4*(c^2*d^3*e - a*c*d*e^3)*f - (3*c^2*d^4 - 2*a*c*d^2*e
^2 - a^2*e^4)*g)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^
2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^2*d^2*e^2*g*x + 4*c
^2*d^2*e^2*f - (3*c^2*d^3*e - a*c*d*e^3)*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c^2*d^2*e^3), 1/8*(sqrt(-c*d*e)*(4*(c^2*d^3*e - a*c*d*e^3)
*f - (3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*g)*arctan(1/2*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d
^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*
g*x + 4*c^2*d^2*e^2*f - (3*c^2*d^3*e - a*c*d*e^3)*g)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]
```

Sympy [F]

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(f+gx)}{d+ex}\,dx$$

input
```
integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,gx}{e}+\frac{4\,cdef-3\,cd^2g+ae^2g}{cde^2}\right)$$
$$+ \frac{(4\,c^2d^3ef-4\,acde^3f-3\,c^2d^4g+2\,acd^2e^2g+a^2e^4g)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+c}\right.\right.\right.}{8\sqrt{cde}cde^2}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algor
ithm="giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*g*x/e + (4*c*d*e*f - 3*
c*d^2*g + a*e^2*g)/(c*d*e^2)) + 1/8*(4*c^2*d^3*e*f - 4*a*c*d*e^3*f - 3*c^2
*d^4*g + 2*a*c*d^2*e^2*g + a^2*e^4*g)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*
e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c
*d*e)*c*d*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$$
$$=\int \frac{(f+g\,x)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

input

```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

output

```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x
)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.33

$$\int \frac{(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$$
$$=\frac{\sqrt{ex+d}\,\sqrt{cdx+ae}\,acde^3g-3\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^3eg+4\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^2e^2f+2\sqrt{ex+d}}{}$$

input

```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

output

```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**3*g - 3*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c**2*d**3*e*g + 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**2*f
 + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**2*g*x - sqrt(e)*sqrt(d)*
sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sq
rt(a*e**2 - c*d**2))*a**2*e**4*g - 2*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*
sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*
a*c*d**2*e**2*g + 4*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d*e**3*f + 3*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4*g - 4*sqrt(e)*sqrt(d)*sqrt(
c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*
e**2 - c*d**2))*c**2*d**3*e*f)/(4*c**2*d**2*e**3)
```

3.262 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 114

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e} - \frac{(cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e-(-a*e^2+c*d^2)*arctanh(c^(1/2)*d
^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(1/2)/d^
(1/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}+\frac{(-cd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{e^{3/2}}$$

input

```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

output

```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] + ((-(c*d^2) + a*e^2)*ArcTanh[(Sqr
t[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]
*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/e^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1131, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

output

```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(
c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+(a\,e^{2}-c\,d^{2})\left(x+\frac{d}{e}\right)}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\ln\left(\frac{\frac{a\,e^{2}}{2}-\frac{c\,d^{2}}{2}+dec\left(x+\frac{d}{e}\right)}{\sqrt{dec}}+\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{dec}}}{e}$	131

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output

```
1/e*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1
/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$

$$=\left[\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}cde-(cd^2-ae^2)\sqrt{cde}\log\left(8c^2d^2e^2x^2+c^2d^4+6acd^2e^2+a^2e^4+4\sqrt{}\right.}{4cde^2}\right.$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fr
icas")
```

output

```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2
)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 +
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c*d*e^2), 1/2*(2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e + (c*d^2 - a*e^2)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e
^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*
x))/(c*d*e^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output

```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{d + ex}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{d + ex}\, dx = \frac{(cd^2 - ae^2)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{2\sqrt{cde}e} + \frac{\sqrt{cdex^2 + cd^2x + ae^2x + ade}}{e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/2*(c*d^2 - a*e^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e) + sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{\sqrt{ex+d}\,\sqrt{cdx+ae}\,cde+\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2-\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}}{\sqrt{a\,e^2-c\,d^2}}\right.}{cd\,e^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

output
```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d*e + sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a*e**2 - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2)/(c*d*e**2)
```

3.263 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)(f+gx)}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 165

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)}\,dx = \frac{2\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}g} - \frac{2\sqrt{cdf-aeg}\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{g\sqrt{ef-dg}}$$

output
```
2*c^(1/2)*d^(1/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2))/e^(1/2)/g-2*(-a*e*g+c*d*f)^(1/2)*arctanh((-a*e*g+
c*d*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/g/(-d*g+e*f)^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)}\,dx$$
$$=\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{e}\sqrt{cdf-aeg}\arctan\left(\frac{\sqrt{c}\sqrt{d}\left(-\sqrt{\frac{e}{cd}}g\sqrt{ae+cdx}\sqrt{d+ex}+e(f+gx)\right)}{\sqrt{e}\sqrt{-ef+dg}\sqrt{cdf-aeg}}\right)-\sqrt{c}\sqrt{d}\sqrt{-ef+dg}\,\mathrm{lo}}{\sqrt{c}\sqrt{d}\sqrt{\frac{e}{cd}}g\sqrt{-ef+dg}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((d + e*x)*(f + g*x)
),x]
```

output
```
(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[e]*Sqrt[c*d*f - a*e*g]*ArcTan[(Sq
rt[c]*Sqrt[d]*(-(Sqrt[e/(c*d)]*g*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]) + e*(f +
 g*x)))/(Sqrt[e]*Sqrt[-(e*f) + d*g]*Sqrt[c*d*f - a*e*g])] - Sqrt[c]*Sqrt[d
]*Sqrt[-(e*f) + d*g]*Log[-(Sqrt[e/(c*d)]*Sqrt[a*e + c*d*x]) + Sqrt[d + e*x
]]))/(Sqrt[c]*Sqrt[d]*Sqrt[e/(c*d)]*g*Sqrt[-(e*f) + d*g]*Sqrt[(a*e + c*d*x
)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1215, 1268, 140, 27, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)(f+gx)}\,dx$$
$$\Big\downarrow \text{1215}$$
$$\int \frac{ae+cdx}{(f+gx)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$
$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{\sqrt{ae+cdx}}{\sqrt{d+ex}(f+gx)}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 140$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\int \frac{ae-\frac{cdf}{g}}{\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)}dx+\frac{cd\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx}{g}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\left(ae-\frac{cdf}{g}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)}dx+\frac{cd\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx}{g}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 66$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\left(ae-\frac{cdf}{g}\right)\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}(f+gx)}dx+\frac{2cd\int \frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}}{g}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 104$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(2\left(ae-\frac{cdf}{g}\right)\int \frac{1}{ef-dg-\frac{(cdf-aeg)(d+ex)}{ae+cdx}}d\frac{\sqrt{d+ex}}{\sqrt{ae+cdx}}+\frac{2cd\int \frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}}{g}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2\left(ae-\frac{cdf}{g}\right)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{cdf-aeg}}{\sqrt{ef-dg}\sqrt{ae+cdx}}\right)}{\sqrt{ef-dg}\sqrt{cdf-aeg}}+\frac{2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{e}g}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((d + e*x)*(f + g*x)),x]
```

output

```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[e]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]])/(Sqrt[e]*g) + (2*(a*e - (
c*d*f)/g)*ArcTanh[(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])/(Sqrt[e*f - d*g]*Sqr
t[a*e + c*d*x])]])/(Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g])))/Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(141) = 282$.

Time = 2.35 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.00

method	result
default	$\sqrt{cd\left(x+\frac{f}{g}\right)^{2}e+\frac{\left(a\,e^{2}g+c\,d^{2}g-2cdef\right)\left(x+\frac{f}{g}\right)}{g}+\frac{ade\,g^{2}-a\,e^{2}fg-c\,d^{2}fg+cde\,f^{2}}{g^{2}}}+\left(a\,e^{2}g+c\,d^{2}g-2cdef\right)\ln\left(\frac{\frac{a\,e^{2}g+c\,d^{2}g-2cdef}{2g}+dec\left(x+\frac{f}{g}\right)}{\sqrt{dec}}\right.$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)/(g*x+f),x,method=_RETU
RNVERBOSE)
```

output
```
1/(d*g-e*f)*((c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e
*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)+1/2*(a*e^2*g+c*d^2*g-2*c*d*
e*f)/g*ln((1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)/g+d*e*c*(x+f/g))/(d*e*c)^(1/2)+
(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*
g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2))/(d*e*c)^(1/2)-(a*d*e*g^2-a*e^2*f*g-c*d^
2*f*g+c*d*e*f^2)/g^2/((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)
*ln((2*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2+(a*e^2*g+c*d^2*g-2*c*
d*e*f)/g*(x+f/g)+2*((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*(
c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g
-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2))/(x+f/g)))-1/(d*g-e*f)*((d*e*c*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e
*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(
d*e*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(141) = 282$.

Time = 8.91 (sec) , antiderivative size = 1689, normalized size of antiderivative = 10.24

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)}\,dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f),x, algor
ithm="fricas")
```

output

```
[1/2*(sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^
4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + sqrt((c*d*f - a*e*g)/
(e*f - d*g))*log((8*a^2*d^2*e^2*g^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
f^2 - 8*(a*c*d^3*e + a^2*d*e^3)*f*g + (8*c^2*d^2*e^2*f^2 - 8*(c^2*d^3*e +
a*c*d*e^3)*f*g + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*g^2)*x^2 + 2*(4*(c^2*
d^3*e + a*c*d*e^3)*f^2 - (3*c^2*d^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*f*g + 4*
(a*c*d^3*e + a^2*d*e^3)*g^2)*x - 4*(2*a*d^2*e*g^2 + (c*d^2*e + a*e^3)*f^2
- (c*d^3 + 3*a*d*e^2)*f*g + (2*c*d*e^2*f^2 - (3*c*d^2*e + a*e^3)*f*g + (c*
d^3 + a*d*e^2)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt((c
*d*f - a*e*g)/(e*f - d*g)))/(g^2*x^2 + 2*f*g*x + f^2)))/g, -1/2*(2*sqrt(-c
*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*
e^2)*x)) - sqrt((c*d*f - a*e*g)/(e*f - d*g))*log((8*a^2*d^2*e^2*g^2 + (c^2
*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*f^2 - 8*(a*c*d^3*e + a^2*d*e^3)*f*g + (8*c
^2*d^2*e^2*f^2 - 8*(c^2*d^3*e + a*c*d*e^3)*f*g + (c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4)*g^2)*x^2 + 2*(4*(c^2*d^3*e + a*c*d*e^3)*f^2 - (3*c^2*d^4 + 10*a
*c*d^2*e^2 + 3*a^2*e^4)*f*g + 4*(a*c*d^3*e + a^2*d*e^3)*g^2)*x - 4*(2*a*d^
2*e*g^2 + (c*d^2*e + a*e^3)*f^2 - (c*d^3 + 3*a*d*e^2)*f*g + (2*c*d*e^2*f^2
 - (3*c*d^2*e + a*e^3)*f*g + (c*d^3 + a*d*e^2)*g^2)*x)*sqrt(c*d*e*x^2 +...
```

Sympy [F]

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)}\,dx=\int \frac{\sqrt{(d+ex)\left(ae+cdx\right)}}{(d+ex)\left(f+gx\right)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)/(g*x+f),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/((d + e*x)*(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.31

$$\begin{aligned}&\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)}\,dx\\&=-\frac{2\left(cdf-aeg\right)\arctan\left(-\frac{\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)g+\sqrt{cde}f}{\sqrt{-cdef^2+cd^2fg+ae^2fg-adeg^2}}\right)}{\sqrt{-cdef^2+cd^2fg+ae^2fg-adeg^2}\,g}\\&\quad-\frac{\sqrt{cde}\log\left(\left|-2\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)cde-\sqrt{cde}cd^2-\sqrt{cde}ae^2\right|\right)}{eg}\end{aligned}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f),x, algor
ithm="giac")
```

output
```
-2*(c*d*f - a*e*g)*arctan(-((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))*g + sqrt(c*d*e)*f)/sqrt(-c*d*e*f^2 + c*d^2*f*g + a*e^2*f*g
 - a*d*e*g^2))/(sqrt(-c*d*e*f^2 + c*d^2*f*g + a*e^2*f*g - a*d*e*g^2)*g) -
sqrt(c*d*e)*log(abs(-2*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))*c*d*e - sqrt(c*d*e)*c*d^2 - sqrt(c*d*e)*a*e^2))/(e*g)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)(f + gx)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{(f + g\,x)\,(d + e\,x)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)), x
)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)(f + gx)}\,dx$$

$$= \frac{\sqrt{dg - ef}\,\sqrt{aeg - cdf}\,\log\left(\sqrt{g}\,\sqrt{e}\,\sqrt{cdx + ae} - \sqrt{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{dg - ef}\,\sqrt{aeg - cdf} + a\,e^2g + c\,d^2g -}\right.}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f),x)
```

output

```
(sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*
e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*e
+ sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a
*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*e
 - sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
d*g - e*f)*sqrt(a*e*g - c*d*f) + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*g + 2*c*d*e*f + 2*c*d*e*g*x)*e + 2*sqrt(e)*sqrt(d)*sqrt(c
)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e
**2 - c*d**2))*d*g - 2*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*e*f)/(e*g*(d*g
 - e*f))
```

3.264 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 150

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)^2}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(ef-dg)(f+gx)} - \frac{\left(cd^2-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{(ef-dg)^{3/2}\sqrt{cdf-aeg}}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-d*g+e*f)/(g*x+f)-(-a*e^2+c*d^2)*
arctanh((-a*e*g+c*d*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/(-d*g+e*f)^(3/2)/(-a*e*g+c*d*f)^(1/2)
```

Mathematica [A] (verified)

Time = 10.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)^2}\,dx = \sqrt{(ae+cdx)(d+ex)}\left(\frac{1}{(ef-dg)(f+gx)} + \frac{\left(-cd^2+ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ef-dg}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(ef-dg)^{3/2}\sqrt{cdf-aeg}\sqrt{ae+cdx}\sqrt{d+ex}}\right)$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((d + e*x)*(f + g*x)
^2),x]
```

output
```
Sqrt[(a*e + c*d*x)*(d + e*x)]*(1/((e*f - d*g)*(f + g*x)) + ((-(c*d^2) + a*
e^2)*ArcTanh[(Sqrt[e*f - d*g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d*f - a*e*g]*Sqrt
[d + e*x]])]/((e*f - d*g)^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x]*Sqrt
[d + e*x]))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1215, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)(f+gx)^2}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{ae+cdx}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1228}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(f+gx)(ef-dg)}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2(ef-dg)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\left(cd^2-ae^2\right)\int \frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae(ef-2dg)-(ae^2g-cd(2ef-dg))x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}\right)}{ef-dg}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(f+gx)(ef-dg)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(f+gx)(ef-dg)}-\frac{\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{2(ef-dg)^{3/2}\sqrt{cdf-aeg}}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((d + e*x)*(f + g*x)^2),x]
```

output

```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((e*f - d*g)*(f + g*x)) - ((c*
d^2 - a*e^2)*ArcTanh[(c*d^2*f + a*e*(e*f - 2*d*g) - (a*e^2*g - c*d*(2*e*f
- d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])])/(2*(e*f - d*g)^(3/2)*Sqrt[c*d*f - a*e*g])
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215

```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1747 vs. $2(138) = 276$.

Time = 2.51 (sec) , antiderivative size = 1748, normalized size of antiderivative = 11.65

method	result	size
default	Expression too large to display	1748

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)/(g*x+f)^2,x,method=_RE
TURNVERBOSE)
```

output

```
e/(d*g-e*f)^2*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*
d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))+1/g/(d*g-e*f)*(-1/(a*d*e*g^2
-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2/(x+f/g)*(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^
2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(3
/2)+1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e
*f^2)*((c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a
*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)+1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)/g
*ln((1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)/g+d*e*c*(x+f/g))/(d*e*c)^(1/2)+(c*d*(
x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^
2*f*g+c*d*e*f^2)/g^2)^(1/2))/(d*e*c)^(1/2)-(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+
c*d*e*f^2)/g^2/((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*ln((2
*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2+(a*e^2*g+c*d^2*g-2*c*d*e*f)
/g*(x+f/g)+2*((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*(c*d*(x
+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2
*f*g+c*d*e*f^2)/g^2)^(1/2))/(x+f/g)))+2*d*e*c/(a*d*e*g^2-a*e^2*f*g-c*d^2*f
*g+c*d*e*f^2)*g^2*(1/4*(2*d*e*c*(x+f/g)+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g)/d/e
/c*(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2
*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)+1/8*(4*d*e*c*(a*d*e*g^2-a*e^2*f*g-c*d
^2*f*g+c*d*e*f^2)/g^2-(a*e^2*g+c*d^2*g-2*c*d*e*f)^2/g^2)/d/e/c*ln((1/2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(138) = 276$.

Time = 0.97 (sec) , antiderivative size = 1113, normalized size of antiderivative = 7.42

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{(d + ex)(f + gx)^2}\, dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^2,x, alg
orithm="fricas")
```

output

```
[1/4*(sqrt(c*d*e*f^2 + a*d*e*g^2 - (c*d^2 + a*e^2)*f*g)*((c*d^2 - a*e^2)*g
*x + (c*d^2 - a*e^2)*f)*log((8*a^2*d^2*e^2*g^2 + (c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4)*f^2 - 8*(a*c*d^3*e + a^2*d*e^3)*f*g + (8*c^2*d^2*e^2*f^2 - 8*(c
^2*d^3*e + a*c*d*e^3)*f*g + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*g^2)*x^2 +
 4*sqrt(c*d*e*f^2 + a*d*e*g^2 - (c*d^2 + a*e^2)*f*g)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*a*d*e*g - (c*d^2 + a*e^2)*f - (2*c*d*e*f - (c*d^
2 + a*e^2)*g)*x) + 2*(4*(c^2*d^3*e + a*c*d*e^3)*f^2 - (3*c^2*d^4 + 10*a*c*
d^2*e^2 + 3*a^2*e^4)*f*g + 4*(a*c*d^3*e + a^2*d*e^3)*g^2)*x)/(g^2*x^2 + 2*
f*g*x + f^2)) + 4*(c*d*e*f^2 + a*d*e*g^2 - (c*d^2 + a*e^2)*f*g)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^2*f^4 - a*d^2*e*f*g^3 - (2*c*d^2
*e + a*e^3)*f^3*g + (c*d^3 + 2*a*d*e^2)*f^2*g^2 + (c*d*e^2*f^3*g - a*d^2*e
*g^4 - (2*c*d^2*e + a*e^3)*f^2*g^2 + (c*d^3 + 2*a*d*e^2)*f*g^3)*x), -1/2*(
sqrt(-c*d*e*f^2 - a*d*e*g^2 + (c*d^2 + a*e^2)*f*g)*((c*d^2 - a*e^2)*g*x +
(c*d^2 - a*e^2)*f)*arctan(1/2*sqrt(-c*d*e*f^2 - a*d*e*g^2 + (c*d^2 + a*e^2
)*f*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e*g - (c*d^2 + a
*e^2)*f - (2*c*d*e*f - (c*d^2 + a*e^2)*g)*x)/(a*c*d^2*e^2*f^2 + a^2*d^2*e^
2*g^2 - (a*c*d^3*e + a^2*d*e^3)*f*g + (c^2*d^2*e^2*f^2 + a*c*d^2*e^2*g^2 -
 (c^2*d^3*e + a*c*d*e^3)*f*g)*x^2 + ((c^2*d^3*e + a*c*d*e^3)*f^2 - (c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4)*f*g + (a*c*d^3*e + a^2*d*e^3)*g^2)*x)) - 2*(c
*d*e*f^2 + a*d*e*g^2 - (c*d^2 + a*e^2)*f*g)*sqrt(c*d*e*x^2 + a*d*e + (c...
```

Sympy [F]

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)^2}\,dx=\int \frac{\sqrt{(d+ex)\left(ae+cdx\right)}}{\left(d+ex\right)\left(f+gx\right)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)/(g*x+f)**2,x
)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/((d + e*x)*(f + g*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)^2}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{(ex+d)(gx+f)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^2,x, alg
orithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*(g*x + f)
^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(138)=276$.

Time = 0.73 (sec) , antiderivative size = 744, normalized size of antiderivative = 4.96

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)^2}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^2,x, alg
orithm="giac")
```

output

```
1/2*(2*sqrt(c*d*e - 2*c*d*e*f/(g*x + f) + c*d*e*f^2/(g*x + f)^2 + c*d^2*g/
(g*x + f) + a*e^2*g/(g*x + f) - c*d^2*f*g/(g*x + f)^2 - a*e^2*f*g/(g*x + f
)^2 + a*d*e*g^2/(g*x + f)^2)*g^2*sgn(1/(g*x + f))*sgn(g)/(e*f*g^5 - d*g^6)
 - (c*d^2*g^2*log(abs(2*c*d*e*f*g - c*d^2*g^2 - a*e^2*g^2 - 2*sqrt(c*d*e*f
^2 - c*d^2*f*g - a*e^2*f*g + a*d*e*g^2)*sqrt(c*d*e)*abs(g))) - a*e^2*g^2*l
og(abs(2*c*d*e*f*g - c*d^2*g^2 - a*e^2*g^2 - 2*sqrt(c*d*e*f^2 - c*d^2*f*g
- a*e^2*f*g + a*d*e*g^2)*sqrt(c*d*e)*abs(g))) + 2*sqrt(c*d*e*f^2 - c*d^2*f
*g - a*e^2*f*g + a*d*e*g^2)*sqrt(c*d*e)*abs(g))*sgn(1/(g*x + f))*sgn(g)/(s
qrt(c*d*e*f^2 - c*d^2*f*g - a*e^2*f*g + a*d*e*g^2)*e*f*g^3*abs(g) - sqrt(c
*d*e*f^2 - c*d^2*f*g - a*e^2*f*g + a*d*e*g^2)*d*g^4*abs(g)) + (c*d^2*sgn(1
/(g*x + f))*sgn(g) - a*e^2*sgn(1/(g*x + f))*sgn(g))*log(abs(2*c*d*e*f*g -
c*d^2*g^2 - a*e^2*g^2 - 2*sqrt(c*d*e*f^2 - c*d^2*f*g - a*e^2*f*g + a*d*e*g
^2)*(sqrt(c*d*e - 2*c*d*e*f/(g*x + f) + c*d*e*f^2/(g*x + f)^2 + c*d^2*g/(g
*x + f) + a*e^2*g/(g*x + f) - c*d^2*f*g/(g*x + f)^2 - a*e^2*f*g/(g*x + f)^
2 + a*d*e*g^2/(g*x + f)^2) + sqrt(c*d*e*f^2*g^2 - c*d^2*f*g^3 - a*e^2*f*g^
3 + a*d*e*g^4)/((g*x + f)*g))*abs(g)))/(sqrt(c*d*e*f^2 - c*d^2*f*g - a*e^2
*f*g + a*d*e*g^2)*(e*f*g - d*g^2)*abs(g)))*g^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^2}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^2\,\left(d+e\,x\right)}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)),
x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)),
 x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1453, normalized size of antiderivative = 9.69

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(d+ex)(f+gx)^2}\,dx = \text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^2,x)
```

output

```
( - sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d
*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) +
 a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))
*a*e**2*f - sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a
*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g -
c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*a*e**2*g*x + sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqr
t(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt
(c)*sqrt(d + e*x))*c*d**2*f + sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt
(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g -
e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt
(d)*sqrt(c)*sqrt(d + e*x))*c*d**2*g*x - sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f
)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sq
rt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*e**2*f - sqrt(d*g - e*f)*sqrt(a*e*g
 - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(e)*sqrt(d)*s
qrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e
*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*e**2*g*x + sqrt(d*g - e*f)*
sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt...
```

3.265 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)(f+gx)^3}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 258

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^3}\,dx &= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2(ef-dg)(f+gx)^2} \\ &\quad - \frac{(3ae^2g-cd(2ef+dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4(ef-dg)^2(cdf-aeg)(f+gx)} \\ &\quad + \frac{(cd^2-ae^2)\,(3ae^2g-cd(4ef-dg))\,\mathrm{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4(ef-dg)^{5/2}(cdf-aeg)^{3/2}}\end{aligned}$$

output

```
1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-d*g+e*f)/(g*x+f)^2-1/4*(3*a*
e^2*g-c*d*(d*g+2*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-d*g+e*f)^
2/(-a*e*g+c*d*f)/(g*x+f)+1/4*(-a*e^2+c*d^2)*(3*a*e^2*g-c*d*(-d*g+4*e*f))*a
rctanh((-a*e*g+c*d*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/(-d*g+e*f)^(5/2)/(-a*e*g+c*d*f)^(3/2)
```

Mathematica [A] (verified)

Time = 10.48 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^3}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{g(ae+cdx)}{(f+gx)^2}-\frac{(3ae^2g+cd(-4ef+dg))\left(\frac{\sqrt{ae+cdx}\sqrt{d+ex}}{(ef-dg)(f+gx)}+\frac{(-cd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{ef-dg}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(ef-dg)^{3/2}\sqrt{cdf-aeg}}\right)}{2\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{2(-ef+dg)(-cdf+aeg)}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((d + e*x)*(f + g*x)
^3),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((g*(a*e + c*d*x))/(f + g*x)^2) - ((3*a*e
^2*g + c*d*(-4*e*f + d*g))*((Sqrt[a*e + c*d*x]*Sqrt[d + e*x])/((e*f - d*g)
*(f + g*x)) + ((-(c*d^2) + a*e^2)*ArcTanh[(Sqrt[e*f - d*g]*Sqrt[a*e + c*d*
x])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]))/((e*f - d*g)^(3/2)*Sqrt[c*d*f -
a*e*g]))/(2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(2*(-(e*f) + d*g)*(-(c*d*f
) + a*e*g))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1215, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)(f+gx)^3}\,dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int \frac{ae+cdx}{(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1237}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(f+gx)^2(ef-dg)}-\frac{\int \frac{(cdf-aeg)\left(cd^2-2cexd-3ae^2\right)}{2(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(ef-dg)(cdf-aeg)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(f+gx)^2(ef-dg)}-\frac{\int \frac{cd^2-2cexd-3ae^2}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(ef-dg)}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(f+gx)^2(ef-dg)}-\frac{\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(3ae^2g-cd(dg+2ef)\right)}{(f+gx)(ef-dg)(cdf-aeg)}-\frac{\left(cd^2-ae^2\right)\left(3ae^2g-cd(4ef-dg)\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(ef-dg)(cdf-aeg)}}{4(ef-dg)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(f+gx)^2(ef-dg)}-\frac{\frac{\left(cd^2-ae^2\right)\left(3ae^2g-cd(4ef-dg)\right)\int \frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right)}{(ef-dg)(cdf-aeg)}}{4(ef-dg)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2(f+gx)^2(ef-dg)}-\frac{\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(3ae^2g-cd(dg+2ef)\right)}{(f+gx)(ef-dg)(cdf-aeg)}-\frac{\left(cd^2-ae^2\right)\left(3ae^2g-cd(4ef-dg)\right)\text{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{2(ef-dg)^{3/2}(cdf-aeg)^{3/2}}}{4(ef-dg)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((d + e*x)*(f + g*x)^3),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(e*f - d*g)*(f + g*x)^2) -
(((3*a*e^2*g - c*d*(2*e*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/((e*f - d*g)*(c*d*f - a*e*g)*(f + g*x)) - ((c*d^2 - a*e^2)*(3*a*e^2*g
 - c*d*(4*e*f - d*g))*ArcTanh[(c*d^2*f + a*e*(e*f - 2*d*g) - (a*e^2*g - c*
d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*(e*f - d*g)^(3/2)*(c*d*f - a*e*g)^(3/2)
))/(4*(e*f - d*g))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3573 vs. $2(238) = 476$.

Time = 3.05 (sec) , antiderivative size = 3574, normalized size of antiderivative = 13.85

method	result	size
default	Expression too large to display	3574

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)/(g*x+f)^3,x,method=_RE
TURNVERBOSE)
```

output

```
1/g^2/(d*g-e*f)*(-1/2/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2/(x+f/g
)^2*(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^
2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(3/2)-1/4*(a*e^2*g+c*d^2*g-2*c*d*e*f)*g/(a
*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*(-1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g
+c*d*e*f^2)*g^2/(x+f/g)*(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+
f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(3/2)+1/2*(a*e^2*g+c*d
^2*g-2*c*d*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*((c*d*(x+f/g)^
2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c
*d*e*f^2)/g^2)^(1/2)+1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*ln((1/2*(a*e^2*g+c*
d^2*g-2*c*d*e*f)/g+d*e*c*(x+f/g))/(d*e*c)^(1/2)+(c*d*(x+f/g)^2*e+(a*e^2*g+
c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2
)^(1/2))/(d*e*c)^(1/2)-(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2/((a*d
*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*ln((2*(a*d*e*g^2-a*e^2*f*
g-c*d^2*f*g+c*d*e*f^2)/g^2+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+2*((a*d*e
*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*(c*d*(x+f/g)^2*e+(a*e^2*g+c
*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)
^(1/2))/(x+f/g)))+2*d*e*c/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2*(1
/4*(2*d*e*c*(x+f/g)+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g)/d/e/c*(c*d*(x+f/g)^2*e+
(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e
*f^2)/g^2)^(1/2)+1/8*(4*d*e*c*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(238) = 476$.

Time = 14.73 (sec) , antiderivative size = 2441, normalized size of antiderivative = 9.46

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^3}\,dx=\text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^3,x, alg
orithm="fricas")
```

output

```
[-1/16*(sqrt(c*d*e*f^2 + a*d*e*g^2 - (c*d^2 + a*e^2)*f*g)*(4*(c^2*d^3*e -
a*c*d*e^3)*f^3 - (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*f^2*g + (4*(c^2*d^3
*e - a*c*d*e^3)*f*g^2 - (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*g^3)*x^2 + 2
*(4*(c^2*d^3*e - a*c*d*e^3)*f^2*g - (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*
f*g^2)*x)*log((8*a^2*d^2*e^2*g^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*f^2
 - 8*(a*c*d^3*e + a^2*d*e^3)*f*g + (8*c^2*d^2*e^2*f^2 - 8*(c^2*d^3*e + a*c
*d*e^3)*f*g + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*g^2)*x^2 - 4*sqrt(c*d*e*
f^2 + a*d*e*g^2 - (c*d^2 + a*e^2)*f*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*a*d*e*g - (c*d^2 + a*e^2)*f - (2*c*d*e*f - (c*d^2 + a*e^2)*g)*
x) + 2*(4*(c^2*d^3*e + a*c*d*e^3)*f^2 - (3*c^2*d^4 + 10*a*c*d^2*e^2 + 3*a^
2*e^4)*f*g + 4*(a*c*d^3*e + a^2*d*e^3)*g^2)*x)/(g^2*x^2 + 2*f*g*x + f^2))
- 4*(4*c^2*d^2*e^2*f^4 + 2*a^2*d^2*e^2*g^4 - (5*c^2*d^3*e + 9*a*c*d*e^3)*f
^3*g + (c^2*d^4 + 12*a*c*d^2*e^2 + 5*a^2*e^4)*f^2*g^2 - (3*a*c*d^3*e + 7*a
^2*d*e^3)*f*g^3 + (2*c^2*d^2*e^2*f^3*g - (c^2*d^3*e + 5*a*c*d*e^3)*f^2*g^2
 - (c^2*d^4 - 4*a*c*d^2*e^2 - 3*a^2*e^4)*f*g^3 + (a*c*d^3*e - 3*a^2*d*e^3)
*g^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3*f^7 - a
^2*d^3*e^2*f^2*g^5 - (3*c^2*d^3*e^2 + 2*a*c*d*e^4)*f^6*g + (3*c^2*d^4*e +
6*a*c*d^2*e^3 + a^2*e^5)*f^5*g^2 - (c^2*d^5 + 6*a*c*d^3*e^2 + 3*a^2*d*e^4)
*f^4*g^3 + (2*a*c*d^4*e + 3*a^2*d^2*e^3)*f^3*g^4 + (c^2*d^2*e^3*f^5*g^2 -
a^2*d^3*e^2*g^7 - (3*c^2*d^3*e^2 + 2*a*c*d*e^4)*f^4*g^3 + (3*c^2*d^4*e ...
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)(f + gx)^3}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{(d + ex)\,(f + gx)^3}\,dx$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)/(g*x+f)**3,x
)
```

output

```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/((d + e*x)*(f + g*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^3}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{(ex+d)(gx+f)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^3,x, alg
orithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*(g*x + f)
^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1848 vs. 2(238) = 476.

Time = 0.39 (sec) , antiderivative size = 1848, normalized size of antiderivative = 7.16

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^3}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^3,x, alg
orithm="giac")
```

output
```
-1/4*(4*c^2*d^3*e*f - 4*a*c*d*e^3*f - c^2*d^4*g - 2*a*c*d^2*e^2*g + 3*a^2*
e^4*g)*arctan(-((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))*g + sqrt(c*d*e)*f)/sqrt(-c*d*e*f^2 + c*d^2*f*g + a*e^2*f*g - a*d*e*g^2
))/((c*d*e^2*f^3 - 2*c*d^2*e*f^2*g - a*e^3*f^2*g + c*d^3*f*g^2 + 2*a*d*e^2
*f*g^2 - a*d^2*e*g^3)*sqrt(-c*d*e*f^2 + c*d^2*f*g + a*e^2*f*g - a*d*e*g^2)
) + 1/4*(8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*c
^3*d^4*e^2*f^3 + 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))*a*c^2*d^2*e^4*f^3 - 32*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))*a*c^2*d^3*e^3*f^2*g - 16*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c*d*e^5*f^2*g + (sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*c^3*d^6*f*g^2 + 7*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c^2*d^4*e^2*f*g^2 + 35*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c*d^2*e^4*f*
g^2 + 5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*
e^6*f*g^2 - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*
a*c^2*d^5*e*g^3 - 10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))*a^2*c*d^3*e^3*g^3 - 5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))*a^3*d*e^5*g^3 + 2*sqrt(c*d*e)*c^3*d^5*e*f^3 + 4*sqrt(c*
d*e)*a*c^2*d^3*e^3*f^3 + 2*sqrt(c*d*e)*a^2*c*d*e^5*f^3 + sqrt(c*d*e)*c^3*d
^6*f^2*g - 9*sqrt(c*d*e)*a*c^2*d^4*e^2*f^2*g - 13*sqrt(c*d*e)*a^2*c*d^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^3}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^3\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)),
x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)),
 x)
```

Reduce [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 11755, normalized size of antiderivative = 45.56

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^3}\,dx = \text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^3,x)
```

output

```
(3*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*
x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) +
a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*
a**3*e**6*f**2*g**3 + 6*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sq
rt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*s
qrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sq
rt(c)*sqrt(d + e*x))*a**3*e**6*f*g**4*x + 3*sqrt(d*g - e*f)*sqrt(a*e*g - c
*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(
c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f)
+ sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*g**5*x**2 + sqrt(d*g -
e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sq
rt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d
**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e*
*4*f**2*g**3 + 2*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*s
qrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e
*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*s
qrt(d + e*x))*a**2*c*d**2*e**4*f*g**4*x + sqrt(d*g - e*f)*sqrt(a*e*g - c*d
*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)
*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) +
sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**4*g**5*x**2 - 10*...
```

3.266 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)(f+gx)^4}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 400

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)(f+gx)^4}\,dx &= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(ef-dg)(f+gx)^3}\\ &- \frac{(5ae^2g-cd(4ef+dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12(ef-dg)^2(cdf-aeg)(f+gx)^2}\\ &+ \frac{(3ae^2g-cd(4ef-dg))(5ae^2g-cd(2ef+3dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24(ef-dg)^3(cdf-aeg)^2(f+gx)}\\ &- \frac{(cd^2-ae^2)(5a^2e^4g^2-2acde^2g(6ef-dg)+c^2d^2(8e^2f^2-4defg+d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+e}{\sqrt{ef-dg}\sqrt{ade+(cd^2+a}}\right.}{8(ef-dg)^{7/2}(cdf-aeg)^{5/2}}\end{aligned}$$

output
```
1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-d*g+e*f)/(g*x+f)^3-1/12*(5*a
*e^2*g-c*d*(d*g+4*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-d*g+e*f)
^2/(-a*e*g+c*d*f)/(g*x+f)^2+1/24*(3*a*e^2*g-c*d*(-d*g+4*e*f))*(5*a*e^2*g-c
*d*(3*d*g+2*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-d*g+e*f)^3/(-a
*e*g+c*d*f)^2/(g*x+f)-1/8*(-a*e^2+c*d^2)*(5*a^2*e^4*g^2-2*a*c*d*e^2*g*(-d*
g+6*e*f)+c^2*d^2*(d^2*g^2-4*d*e*f*g+8*e^2*f^2))*arctanh((-a*e*g+c*d*f)^(1/
2)*(e*x+d)/(-d*g+e*f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(-d*g
+e*f)^(7/2)/(-a*e*g+c*d*f)^(5/2)
```

Mathematica [A] (verified)

Time = 11.22 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^4}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-8g(ae+cdx)-\frac{2g(5ae^2g+cd(-8ef+3dg))(ae+cdx)(f+gx)}{(ef-dg)(-cdf+aeg)}+\frac{3(5a^2e^4g^2+2acde^2g(-6ef+dg)+c^2}{}\right.}{24(-ef+dg)(-cdf+a}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((d + e*x)*(f + g*x)
^4),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-8*g*(a*e + c*d*x) - (2*g*(5*a*e^2*g + c*d
*(-8*e*f + 3*d*g))*(a*e + c*d*x)*(f + g*x))/((e*f - d*g)*(-(c*d*f) + a*e*g
)) + (3*(5*a^2*e^4*g^2 + 2*a*c*d*e^2*g*(-6*e*f + d*g) + c^2*d^2*(8*e^2*f^2
 - 4*d*e*f*g + d^2*g^2))*(f + g*x)^2*(Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*
Sqrt[a*e + c*d*x]*Sqrt[d + e*x] - (c*d^2 - a*e^2)*(f + g*x)*ArcTanh[(Sqrt[
e*f - d*g]*Sqrt[a*e + c*d*x])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]))/((e*f
 - d*g)^(5/2)*(c*d*f - a*e*g)^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24
*(-(e*f) + d*g)*(-(c*d*f) + a*e*g)*(f + g*x)^3)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1215, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)(f+gx)^4}\,dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int \frac{ae+cdx}{(f+gx)^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1237}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(f+gx)^3(ef-dg)}-\frac{\int \frac{(cdf-aeg)\left(cd^2-4cexd-5ae^2\right)}{2(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3(ef-dg)(cdf-aeg)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(f+gx)^3(ef-dg)}-\frac{\int \frac{cd^2-4cexd-5ae^2}{(f+gx)^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6(ef-dg)}$$

$$\Big\downarrow \text{1237}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(f+gx)^3(ef-dg)}-\frac{\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(5ae^2g-cd(dg+4ef))}{2(f+gx)^2(ef-dg)(cdf-aeg)}-\frac{\int -\frac{15a^2ge^4-4acd(4ef+dg)e^2+2cd\left(5ae^2g-cd(4ef+dg)\right)xe+c^2d^3(8ef-3dg)}{2(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(ef-dg)(cdf-aeg)}}{6(ef-dg)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(f+gx)^3(ef-dg)}-\frac{\frac{\int \frac{15a^2ge^4-4acd(4ef+dg)e^2+2cd\left(5ae^2g-cd(4ef+dg)\right)xe+c^2d^3(8ef-3dg)}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(ef-dg)(cdf-aeg)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(5ae^2g-cd(dg+4ef))}{2(f+gx)^2(ef-dg)(cdf-aeg)}}{6(ef-dg)}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(f+gx)^3(ef-dg)}-\frac{\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4g^2-2acde^2g(6ef-dg)+c^2d^2\left(d^2g^2-4defg+8e^2f^2\right)\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(ef-dg)(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-5ae^2g+3cd^2g}{(f+gx)(ef-dg)(c}}{4(ef-dg)(cdf-aeg)}}{6(ef-dg)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(f+gx)^3(ef-dg)}-\frac{-\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4g^2-2acde^2g(6ef-dg)+c^2d^2\left(d^2g^2-4defg+8e^2f^2\right)\right)\int \frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae}{\sqrt{}}\right.}{(ef-dg)(cdf-aeg)}}{4(ef-dg)(cdf-aeg)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3(f+gx)^3(ef-dg)}-$$

$$\frac{\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4g^2-2acde^2g(6ef-dg)+c^2d^2\left(d^2g^2-4defg+8e^2f^2\right)\right)\operatorname{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{2(ef-dg)^{3/2}(cdf-aeg)^{3/2}}-\sqrt{x\left(ae^2+cd^2\right)+ade+}}{4(ef-dg)(cdf-aeg)}}{6(ef-dg)}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((d + e*x)*(f + g*x)^4),x]
```

output

```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(e*f - d*g)*(f + g*x)^3) -
(((5*a*e^2*g - c*d*(4*e*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(2*(e*f - d*g)*(c*d*f - a*e*g)*(f + g*x)^2) + (-(((2*c*d*e*f + 3*c*d^
2*g - 5*a*e^2*g)*(4*c*d*e*f - c*d^2*g - 3*a*e^2*g)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/((e*f - d*g)*(c*d*f - a*e*g)*(f + g*x))) + (3*(c*d^2
 - a*e^2)*(5*a^2*e^4*g^2 - 2*a*c*d*e^2*g*(6*e*f - d*g) + c^2*d^2*(8*e^2*f^
2 - 4*d*e*f*g + d^2*g^2))*ArcTanh[(c*d^2*f + a*e*(e*f - 2*d*g) - (a*e^2*g
- c*d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*(e*f - d*g)^(3/2)*(c*d*f - a*e*g)^(
3/2)))/(4*(e*f - d*g)*(c*d*f - a*e*g)))/(6*(e*f - d*g))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5580 vs. $2(376) = 752$.

Time = 3.74 (sec) , antiderivative size = 5581, normalized size of antiderivative = 13.95

method	result	size
default	Expression too large to display	5581

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)/(g*x+f)^4,x,method=_RE
TURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^4,x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^4}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)/(g*x+f)**4,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)(f+gx)^4}\,dx=\int \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{(ex+d)(gx+f)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^4,x, alg
orithm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*(g*x + f)
^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6024 vs. $2(376) = 752$.

Time = 0.55 (sec) , antiderivative size = 6024, normalized size of antiderivative = 15.06

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)(f + gx)^4}\,dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^4,x, alg
orithm="giac")
```

output

```
-1/8*(8*c^3*d^4*e^2*f^2 - 8*a*c^2*d^2*e^4*f^2 - 4*c^3*d^5*e*f*g - 8*a*c^2*
d^3*e^3*f*g + 12*a^2*c*d*e^5*f*g + c^3*d^6*g^2 + a*c^2*d^4*e^2*g^2 + 3*a^2
*c*d^2*e^4*g^2 - 5*a^3*e^6*g^2)*arctan(-((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e))*g + sqrt(c*d*e)*f)/sqrt(-c*d*e*f^2 + c*d^2*f*
g + a*e^2*f*g - a*d*e*g^2))/((c^2*d^2*e^3*f^5 - 3*c^2*d^3*e^2*f^4*g - 2*a*
c*d*e^4*f^4*g + 3*c^2*d^4*e*f^3*g^2 + 6*a*c*d^2*e^3*f^3*g^2 + a^2*e^5*f^3*
g^2 - c^2*d^5*f^2*g^3 - 6*a*c*d^3*e^2*f^2*g^3 - 3*a^2*d*e^4*f^2*g^3 + 2*a*
c*d^4*e*f*g^4 + 3*a^2*d^2*e^3*f*g^4 - a^2*d^3*e^2*g^5)*sqrt(-c*d*e*f^2 + c
*d^2*f*g + a*e^2*f*g - a*d*e*g^2)) + 1/24*(48*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))*c^5*d^7*e^3*f^5 + 96*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c^4*d^5*e^5*f^5 + 48*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c^3*d^3*e^7*f^5
+ 36*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*c^5*d^8
*e^2*f^4*g - 300*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))*a*c^4*d^6*e^4*f^4*g - 516*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))*a^2*c^3*d^4*e^6*f^4*g - 180*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^2*e^8*f^4*g - 6*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*c^5*d^9*e*f^3*g^2 - 12*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c^4*d^7*e^3*
f^3*g^2 + 864*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d+ex)(f+gx)^4}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\ x + a\,d\,e}}{(f+g\,x)^4\ (d+e\,x)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)),
x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)),
 x)
```

Reduce [B] (verification not implemented)

Time = 17.70 (sec) , antiderivative size = 25163, normalized size of antiderivative = 62.91

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d+ex)(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)/(g*x+f)^4,x)
```

output

```
( - 15*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e +
c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f
) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*
x))*a**4*e**8*f**3*g**4 - 45*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(
g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e
*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(
d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*f**2*g**5*x - 45*sqrt(d*g - e*f)*sqrt(
a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*
c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*f*g**6*x**2 -
15*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*
x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) +
a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*
a**4*e**8*g**7*x**3 - 6*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sq
rt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*s
qrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sq
rt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*f**3*g**4 - 18*sqrt(d*g - e*f)*sqrt(
a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*
c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*f**2...
```

3.267 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 716

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\frac{\left(cd^2-ae^2\right)\left(7a^3e^6g^3-3a^2cde^4g^2(12ef-5dg)+3ac^2d\right.}{}$$

$$+\frac{1}{20}\left(\frac{ag}{cd}+\frac{2ef-3dg}{e^2}\right)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}$$

$$+\frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e}$$

$$+\frac{\left(35a^3e^6g^3-3a^2cde^4g^2(60ef-11dg)+3ac^2d^2e^2g\left(104e^2f^2-48defg+7d^2g^2\right)+c^3d^3\left(64e^3f^3-456de^2f\right.\right.}{}$$

$$-\frac{\left(cd^2-ae^2\right)^3\left(7a^3e^6g^3-3a^2cde^4g^2(12ef-5dg)+3ac^2d^2e^2g\left(24e^2f^2-24defg+7d^2g^2\right)-c^3d^3\left(64e^3f^3-\right.\right.}{512c^{9/2}d^{9/2}e^{11/2}}$$

output
```
1/512*(-a*e^2+c*d^2)*(7*a^3*e^6*g^3-3*a^2*c*d*e^4*g^2*(-5*d*g+12*e*f)+3*a*
c^2*d^2*e^2*g*(7*d^2*g^2-24*d*e*f*g+24*e^2*f^2)-c^3*d^3*(-21*d^3*g^3+84*d^
2*e*f*g^2-120*d*e^2*f^2*g+64*e^3*f^3))*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^5+1/20*(a*g/c/d+(-3*d*g+2*e*f)/e^2)
*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/6*(g*x+f)^3*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/960*(35*a^3*e^6*g^3-3*a^2*c*d*e^4*g^2*(
-11*d*g+60*e*f)+3*a*c^2*d^2*e^2*g*(7*d^2*g^2-48*d*e*f*g+104*e^2*f^2)+c^3*d
^3*(-105*d^3*g^3+420*d^2*e*f*g^2-456*d*e^2*f^2*g+64*e^3*f^3)-6*c*d*e*g*(7*
a^2*e^4*g^2-2*a*c*d*e^2*g*(-3*d*g+10*e*f)-c^2*d^2*(21*d^2*g^2-36*d*e*f*g+8
*e^2*f^2))*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4-1/512*(-
a*e^2+c*d^2)^3*(7*a^3*e^6*g^3-3*a^2*c*d*e^4*g^2*(-5*d*g+12*e*f)+3*a*c^2*d^
2*e^2*g*(7*d^2*g^2-24*d*e*f*g+24*e^2*f^2)-c^3*d^3*(-21*d^3*g^3+84*d^2*e*f*
g^2-120*d*e^2*f^2*g+64*e^3*f^3))*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(11/2)
```

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 753, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-105a^5e^{10}g^3+5a^4c\right.}{}$$

input
```
Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x),x]
```

output

```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10*g^3
 + 5*a^4*c*d*e^8*g^2*(11*d*g + 2*e*(54*f + 7*g*x)) + 2*a^3*c^2*d^2*e^6*g*(
27*d^2*g^2 - 4*d*e*g*(45*f + 4*g*x) - 4*e^2*(135*f^2 + 45*f*g*x + 7*g^2*x^
2)) + 6*a^2*c^3*d^3*e^4*(13*d^3*g^3 - 6*d^2*e*g^2*(12*f + g*x) + 4*d*e^2*g
*(45*f^2 + 9*f*g*x + g^2*x^2) + 8*e^3*(20*f^3 + 15*f^2*g*x + 6*f*g^2*x^2 +
 g^3*x^3)) + c^5*d^5*(315*d^5*g^3 - 210*d^4*e*g^2*(6*f + g*x) + 24*d^3*e^2
*g*(75*f^2 + 35*f*g*x + 7*g^2*x^2) + 64*d*e^4*x*(10*f^3 + 15*f^2*g*x + 9*f
*g^2*x^2 + 2*g^3*x^3) - 48*d^2*e^3*(20*f^3 + 25*f^2*g*x + 14*f*g^2*x^2 + 3
*g^3*x^3) + 128*e^5*x^2*(20*f^3 + 45*f^2*g*x + 36*f*g^2*x^2 + 10*g^3*x^3))
 + a*c^4*d^4*e^2*(-525*d^4*g^3 + 24*d^3*e*g^2*(95*f + 14*g*x) - 24*d^2*e^2
*g*(155*f^2 + 61*f*g*x + 11*g^2*x^2) + 32*d*e^3*(80*f^3 + 75*f^2*g*x + 36*
f*g^2*x^2 + 7*g^3*x^3) + 64*e^4*x*(70*f^3 + 135*f^2*g*x + 99*f*g^2*x^2 + 2
6*g^3*x^3))) - (15*(c*d^2 - a*e^2)^3*(7*a^3*e^6*g^3 + 3*a^2*c*d*e^4*g^2*(-
12*e*f + 5*d*g) + 3*a*c^2*d^2*e^2*g*(24*e^2*f^2 - 24*d*e*f*g + 7*d^2*g^2)
+ c^3*d^3*(-64*e^3*f^3 + 120*d*e^2*f^2*g - 84*d^2*e*f*g^2 + 21*d^3*g^3))*A
rcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt
[a*e + c*d*x]*Sqrt[d + e*x])))/(7680*c^(9/2)*d^(9/2)*e^(11/2))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 650, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {1215, 1236, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{1215}$$

$$\int (f+gx)^3(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{3}{2}cd(f+gx)^2\left(cfd^2-ae(3ef-2dg)-\left(age^2+cd(2ef-3dg)\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{6cde}+\frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow 27$$

$$\frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\int (f+gx)^2\left(cfd^2-ae(3ef-2dg)-\left(age^2+cd(2ef-3dg)\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{4e}$$

$$\Big\downarrow 1236$$

$$\frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-$$

$$\int \tfrac{1}{2}(f+gx)\left(c^2f(16ef-9dg)d^3-2ace\left(12e^2f^2-11degf+6d^2g^2\right)d+a^2e^3g(3ef+4dg)+\left(7a^2g^2e^4-2acdg(10ef-3dg)e^2-c^2d^2\left(8e^2f^2-36degf+21d\right.\right.\right.$$

$$5cde$$

$$4e$$

$$\Big\downarrow 27$$

$$\frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-$$

$$\int(f+gx)\left(c^2f(16ef-9dg)d^3-2ace\left(12e^2f^2-11degf+6d^2g^2\right)d+a^2e^3g(3ef+4dg)+\left(7a^2g^2e^4-2acdg(10ef-3dg)e^2-c^2d^2\left(8e^2f^2-36degf+21d^2g\right.\right.\right.$$

$$10cde$$

$$4e$$

$$\Big\downarrow 1225$$

$$\frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-$$

$$-\frac{5\left(cd^2-ae^2\right)\left(7a^3e^6g^3-3a^2cde^4g^2(12ef-5dg)+3ac^2d^2e^2g\left(7d^2g^2-24defg+24e^2f^2\right)-c^3d^3\left(-21d^3g^3+84d^2efg^2-120de^2f^2g+64e^3f^3\right)\right)\int\sqrt{cdex^2+\left(cd^2+a\right.}}{16c^2d^2e^2}$$

$$\Big\downarrow 1087$$

$$\frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-$$

$$-\frac{5\left(cd^2-ae^2\right)\left(7a^3e^6g^3-3a^2cde^4g^2(12ef-5dg)+3ac^2d^2e^2g\left(7d^2g^2-24defg+24e^2f^2\right)-c^3d^3\left(-21d^3g^3+84d^2efg^2-120de^2f^2g+64e^3f^3\right)\right)\left(\frac{\left(ae^2+cd^2+2cdex\right.}{}\right.}{16c^2d^2e^2}$$

$$\Big\downarrow 1092$$

$$\frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-$$

$$-\frac{\frac{5\left(cd^2-ae^2\right)\left(7a^3e^6g^3-3a^2cde^4g^2(12ef-5dg)+3ac^2d^2e^2g\left(7d^2g^2-24defg+24e^2f^2\right)-c^3d^3\left(-21d^3g^3+84d^2efg^2-120de^2f^2g+64e^3f^3\right)\right)\left(\frac{\left(ae^2+cd^2+2cdex\right.}{}\right.}{16c^2d^2e^2}}{}$$

$$\Big\downarrow 219$$

$$\frac{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-$$

$$-\frac{\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)\left(7a^3e^6g^3-3a^2cde^4g^2(12\right.}{16c^2d^2e^2}}{}$$

input
```
Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),
x]
```

output
```
((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*e) - (-1/5*
((2*f - (3*d*g)/e + (a*e*g)/(c*d))*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)) + (-1/24*((35*a^3*e^6*g^3 - 3*a^2*c*d*e^4*g^2*(60*e*f
- 11*d*g) + 3*a*c^2*d^2*e^2*g*(104*e^2*f^2 - 48*d*e*f*g + 7*d^2*g^2) + c^3
*d^3*(64*e^3*f^3 - 456*d*e^2*f^2*g + 420*d^2*e*f*g^2 - 105*d^3*g^3) - 6*c*
d*e*g*(7*a^2*e^4*g^2 - 2*a*c*d*e^2*g*(10*e*f - 3*d*g) - c^2*d^2*(8*e^2*f^2
 - 36*d*e*f*g + 21*d^2*g^2))*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2))/(c^2*d^2*e^2) - (5*(c*d^2 - a*e^2)*(7*a^3*e^6*g^3 - 3*a^2*c*d*e^4*g^2
*(12*e*f - 5*d*g) + 3*a*c^2*d^2*e^2*g*(24*e^2*f^2 - 24*d*e*f*g + 7*d^2*g^2
) - c^3*d^3*(64*e^3*f^3 - 120*d*e^2*f^2*g + 84*d^2*e*f*g^2 - 21*d^3*g^3))*
(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqr
t[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(
3/2)*d^(3/2)*e^(3/2))))/(16*c^2*d^2*e^2))/(10*c*d*e))/(4*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1979 vs. $2(684) = 1368$.

Time = 2.48 (sec) , antiderivative size = 1980, normalized size of antiderivative = 2.77

method	result	size
default	Expression too large to display	1980

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d),x,method=_RE
TURNVERBOSE)
```

output

```
g/e^3*(d^2*g^2*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x
+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))-e*g*(d*g-3*e*f)*
(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c
*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/
e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))))+e^2*g^2*(1/6*x*(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-7/12*(a*e^2+c*d^2)/d/e/c*(1/5*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/8*(2*c*d*e*x+
a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2))/(d*e*c)^(1/2))))-1/6*a/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/
d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1...
```

Fricas [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 2430, normalized size of antiderivative = 3.39

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, alg
orithm="fricas")
```

output
```
[1/30720*(15*(64*(c^6*d^9*e^3 - 3*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - a^3*
c^3*d^3*e^9)*f^3 - 24*(5*c^6*d^10*e^2 - 12*a*c^5*d^8*e^4 + 6*a^2*c^4*d^6*e
^6 + 4*a^3*c^3*d^4*e^8 - 3*a^4*c^2*d^2*e^10)*f^2*g + 12*(7*c^6*d^11*e - 15
*a*c^5*d^9*e^3 + 6*a^2*c^4*d^7*e^5 + 2*a^3*c^3*d^5*e^7 + 3*a^4*c^2*d^3*e^9
 - 3*a^5*c*d*e^11)*f*g^2 - (21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d
^8*e^4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*
e^12)*g^3)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a
^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
+ a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(1280*c^6*d^6*e^6*
g^3*x^5 + 128*(36*c^6*d^6*e^6*f*g^2 + (c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*g^3
)*x^4 - 320*(3*c^6*d^8*e^4 - 8*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*f^3 + 12
0*(15*c^6*d^9*e^3 - 31*a*c^5*d^7*e^5 + 9*a^2*c^4*d^5*e^7 - 9*a^3*c^3*d^3*e
^9)*f^2*g - 12*(105*c^6*d^10*e^2 - 190*a*c^5*d^8*e^4 + 36*a^2*c^4*d^6*e^6
+ 30*a^3*c^3*d^4*e^8 - 45*a^4*c^2*d^2*e^10)*f*g^2 + (315*c^6*d^11*e - 525*
a*c^5*d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*c^2*d^3*e
^9 - 105*a^5*c*d*e^11)*g^3 + 16*(360*c^6*d^6*e^6*f^2*g + 36*(c^6*d^7*e^5 +
 11*a*c^5*d^5*e^7)*f*g^2 - (9*c^6*d^8*e^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d
^4*e^8)*g^3)*x^3 + 8*(320*c^6*d^6*e^6*f^3 + 120*(c^6*d^7*e^5 + 9*a*c^5*d^5
*e^7)*f^2*g - 12*(7*c^6*d^8*e^4 - 12*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*f*
g^2 + (21*c^6*d^9*e^3 - 33*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{d+ex}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x
)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, alg
orithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 1337, normalized size of antiderivative = 1.87

$$\int \frac{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, alg
orithm="giac")
```

output

```
1/7680*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*c*d*g^3
*x + (36*c^6*d^6*e^5*f*g^2 + c^6*d^7*e^4*g^3 + 13*a*c^5*d^5*e^6*g^3)/(c^5*
d^5*e^5))*x + (360*c^6*d^6*e^5*f^2*g + 36*c^6*d^7*e^4*f*g^2 + 396*a*c^5*d^
5*e^6*f*g^2 - 9*c^6*d^8*e^3*g^3 + 14*a*c^5*d^6*e^5*g^3 + 3*a^2*c^4*d^4*e^7
*g^3)/(c^5*d^5*e^5))*x + (320*c^6*d^6*e^5*f^3 + 120*c^6*d^7*e^4*f^2*g + 10
80*a*c^5*d^5*e^6*f^2*g - 84*c^6*d^8*e^3*f*g^2 + 144*a*c^5*d^6*e^5*f*g^2 +
36*a^2*c^4*d^4*e^7*f*g^2 + 21*c^6*d^9*e^2*g^3 - 33*a*c^5*d^7*e^4*g^3 + 3*a
^2*c^4*d^5*e^6*g^3 - 7*a^3*c^3*d^3*e^8*g^3)/(c^5*d^5*e^5))*x + (320*c^6*d^
7*e^4*f^3 + 2240*a*c^5*d^5*e^6*f^3 - 600*c^6*d^8*e^3*f^2*g + 1200*a*c^5*d^
6*e^5*f^2*g + 360*a^2*c^4*d^4*e^7*f^2*g + 420*c^6*d^9*e^2*f*g^2 - 732*a*c^
5*d^7*e^4*f*g^2 + 108*a^2*c^4*d^5*e^6*f*g^2 - 180*a^3*c^3*d^3*e^8*f*g^2 -
105*c^6*d^10*e*g^3 + 168*a*c^5*d^8*e^3*g^3 - 18*a^2*c^4*d^6*e^5*g^3 - 16*a
^3*c^3*d^4*e^7*g^3 + 35*a^4*c^2*d^2*e^9*g^3)/(c^5*d^5*e^5))*x - (960*c^6*d
^8*e^3*f^3 - 2560*a*c^5*d^6*e^5*f^3 - 960*a^2*c^4*d^4*e^7*f^3 - 1800*c^6*d
^9*e^2*f^2*g + 3720*a*c^5*d^7*e^4*f^2*g - 1080*a^2*c^4*d^5*e^6*f^2*g + 108
0*a^3*c^3*d^3*e^8*f^2*g + 1260*c^6*d^10*e*f*g^2 - 2280*a*c^5*d^8*e^3*f*g^2
 + 432*a^2*c^4*d^6*e^5*f*g^2 + 360*a^3*c^3*d^4*e^7*f*g^2 - 540*a^4*c^2*d^2
*e^9*f*g^2 - 315*c^6*d^11*g^3 + 525*a*c^5*d^9*e^2*g^3 - 78*a^2*c^4*d^7*e^4
*g^3 - 54*a^3*c^3*d^5*e^6*g^3 - 55*a^4*c^2*d^3*e^8*g^3 + 105*a^5*c*d*e^10*
g^3)/(c^5*d^5*e^5)) - 1/1024*(64*c^6*d^9*e^3*f^3 - 192*a*c^5*d^7*e^5*f^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \int \frac{(f+g\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

input

```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),
x)
```

output

```
int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),
 x)
```

Reduce [F]

$$\int \frac{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int \frac{(gx+f)^3 \left(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2\right)^{\frac{3}{2}}}{ex+d}dx$$

input
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

output
```
int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

3.268 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 449

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx =$$
$$-\frac{\left(cd^2-ae^2\right)\left(3a^2e^4g^2-6acde^2g(2ef-dg)+c^2d^2(16e^2f^2-20defg+7d^2g^2)\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade}}{128c^3d^3e^4}$$
$$+\frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5e}$$
$$-\frac{\left(15a^2e^4g^2-12acde^2g(5ef-dg)-c^2d^2(32e^2f^2-100defg+35d^2g^2)-6cdeg(3ae^2g+cd(4ef-7dg))\right.}{240c^2d^2e^3}$$
$$+\frac{\left(cd^2-ae^2\right)^3\left(3a^2e^4g^2-6acde^2g(2ef-dg)+c^2d^2(16e^2f^2-20defg+7d^2g^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x}}\right.}{128c^{7/2}d^{7/2}e^{9/2}}$$

output
```
-1/128*(-a*e^2+c*d^2)*(3*a^2*e^4*g^2-6*a*c*d*e^2*g*(-d*g+2*e*f)+c^2*d^2*(7
*d^2*g^2-20*d*e*f*g+16*e^2*f^2))*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4+1/5*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(3/2)/e-1/240*(15*a^2*e^4*g^2-12*a*c*d*e^2*g*(-d*g+5*e*f)-c^2*d^2
*(35*d^2*g^2-100*d*e*f*g+32*e^2*f^2)-6*c*d*e*g*(3*a*e^2*g+c*d*(-7*d*g+4*e*
f))*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e^3+1/128*(-a*e^2+c
*d^2)^3*(3*a^2*e^4*g^2-6*a*c*d*e^2*g*(-d*g+2*e*f)+c^2*d^2*(7*d^2*g^2-20*d*
e*f*g+16*e^2*f^2))*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.10

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\frac{\left(cd^2-ae^2\right)^3\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-45a^4e^8g^2+}{}\right.}{}$$

input
```
Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
 e*x),x]
```

output
```
((c*d^2 - a*e^2)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[c]*Sqrt[d]*Sqrt[
e]*(-45*a^4*e^8*g^2 + 30*a^3*c*d*e^6*g*(6*e*f + d*g + e*g*x) + 6*a^2*c^2*d
^2*e^4*(6*d^2*g^2 - 3*d*e*g*(10*f + g*x) - 4*e^2*(10*f^2 + 5*f*g*x + g^2*x
^2)) + c^4*d^4*(105*d^4*g^2 - 10*d^3*e*g*(30*f + 7*g*x) - 16*d*e^3*x*(10*f
^2 + 10*f*g*x + 3*g^2*x^2) - 64*e^4*x^2*(10*f^2 + 15*f*g*x + 6*g^2*x^2) +
8*d^2*e^2*(30*f^2 + 25*f*g*x + 7*g^2*x^2)) - 2*a*c^3*d^3*e^2*(95*d^3*g^2 -
 d^2*e*g*(310*f + 61*g*x) + 8*d*e^2*(40*f^2 + 25*f*g*x + 6*g^2*x^2) + 8*e^
3*x*(70*f^2 + 90*f*g*x + 33*g^2*x^2))))/(c*d^2 - a*e^2)^3) + (15*(3*a^2*e^
4*g^2 + 6*a*c*d*e^2*g*(-2*e*f + d*g) + c^2*d^2*(16*e^2*f^2 - 20*d*e*f*g +
7*d^2*g^2))*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*
d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(1920*c^(7/2)*d^(7/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1215, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{1215}$$

$$\int (f+gx)^2(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}cd(f+gx)\left(3cfd^2-ae(7ef-4dg)-\left(3age^2+cd(4ef-7dg)\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{5cde}+\frac{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{27}$$

$$\frac{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\int (f+gx)\left(3cfd^2-ae(7ef-4dg)-\left(3age^2+cd(4ef-7dg)\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{10e}$$

$$\Big\downarrow \text{1225}$$

$$\frac{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-$$

$$\frac{\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4g^2-6acde^2g(2ef-dg)+c^2d^2\left(7d^2g^2-20defg+16e^2f^2\right)\right)\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16c^2d^2e^2}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(}{}}{10e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-$$

$$\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4g^2-6acde^2g(2ef-dg)+c^2d^2\left(7d^2g^2-20defg+16e^2f^2\right)\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{}{\sqrt{cdex^2+}}}{8cde}\right.}{16c^2d^2e^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-$$

$$\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4g^2-6acde^2g(2ef-dg)+c^2d^2\left(7d^2g^2-20defg+16e^2f^2\right)\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{}{4cde-\frac{(}{cde}}}{}\right.}{16c^2d^2e^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-$$

$$\frac{\frac{5\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)(3a^2e^4g^2-6acde^2g(2ef}{16c^2d^2e^2}}{}$$

input

```
Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),
x]
```

output

```
((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) - (((15*
a^2*e^4*g^2 - 12*a*c*d*e^2*g*(5*e*f - d*g) - 2*c^2*(16*d^2*e^2*f^2 - 50*d^
3*e*f*g + (35*d^4*g^2)/2) - 6*c*d*e*g*(3*a*e^2*g + c*d*(4*e*f - 7*d*g))*x)
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*c^2*d^2*e^2) + (5*(c*d
^2 - a*e^2)*(3*a^2*e^4*g^2 - 6*a*c*d*e^2*g*(2*e*f - d*g) + c^2*d^2*(16*e^2
*f^2 - 20*d*e*f*g + 7*d^2*g^2))*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c^2*d^2*e^2))/
(10*e)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(421) = 842$.

Time = 2.20 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.38

method	result	size
default	Expression too large to display	1069

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d),x,method=_RE
TURNVERBOSE)
```

output

```
(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c
*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*
a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))-g/e^2*(d*g*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/
2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d
*e*c)^(1/2)))-2*e*f*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*
d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^
(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))-e*g*(1/5*(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/8*(
2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)
/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 1592, normalized size of antiderivative = 3.55

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, alg
orithm="fricas")
```

output

```
[-1/7680*(15*sqrt(c*d*e)*(16*(c^5*d^8*e^2 - 3*a*c^4*d^6*e^4 + 3*a^2*c^3*d^
4*e^6 - a^3*c^2*d^2*e^8)*f^2 - 4*(5*c^5*d^9*e - 12*a*c^4*d^7*e^3 + 6*a^2*c
^3*d^5*e^5 + 4*a^3*c^2*d^3*e^7 - 3*a^4*c*d*e^9)*f*g + (7*c^5*d^10 - 15*a*c
^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a
^5*e^10)*g^2)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 -
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(384*c^5*d^5*e^5*g^2*x^4 +
48*(20*c^5*d^5*e^5*f*g + (c^5*d^6*e^4 + 11*a*c^4*d^4*e^6)*g^2)*x^3 - 80*(3
*c^5*d^7*e^3 - 8*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*f^2 + 20*(15*c^5*d^8*e
^2 - 31*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 9*a^3*c^2*d^2*e^8)*f*g - (105*
c^5*d^9*e - 190*a*c^4*d^7*e^3 + 36*a^2*c^3*d^5*e^5 + 30*a^3*c^2*d^3*e^7 -
45*a^4*c*d*e^9)*g^2 + 8*(80*c^5*d^5*e^5*f^2 + 20*(c^5*d^6*e^4 + 9*a*c^4*d^
4*e^6)*f*g - (7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*g^2)*x
^2 + 2*(80*(c^5*d^6*e^4 + 7*a*c^4*d^4*e^6)*f^2 - 20*(5*c^5*d^7*e^3 - 10*a*
c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*f*g + (35*c^5*d^8*e^2 - 61*a*c^4*d^6*e^4
+ 9*a^2*c^3*d^4*e^6 - 15*a^3*c^2*d^2*e^8)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5), -1/3840*(15*sqrt(-c*d*e)*(16*(c^5*d^8*
e^2 - 3*a*c^4*d^6*e^4 + 3*a^2*c^3*d^4*e^6 - a^3*c^2*d^2*e^8)*f^2 - 4*(5*c^
5*d^9*e - 12*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 + 4*a^3*c^2*d^3*e^7 - 3*a^4
*c*d*e^9)*f*g + (7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*...
```

Sympy [A] (verification not implemented)

Time = 66.02 (sec) , antiderivative size = 3509, normalized size of antiderivative = 7.82

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x
)
```

output

```
a*e*f**2*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*d
*e*x**2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2
+ c*d**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c
*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*
e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e
))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2
)/(2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))*
*(3/2)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), Tru
e)) + 2*a*e*f*g*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d
**2)*(a*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))
/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqr
t(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e*
*2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x
- (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*
e))**2), True)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 -
 (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(
a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*
e + x*(a*e**2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5
)/(a*e**2 + c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True
)) + a*e*g**2*Piecewise(((-a*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, alg
orithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 828, normalized size of antiderivative = 1.84

$$\int \frac{(f+gx)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{1}{1920}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(6\left(8\,cdg^2x\right.\right.\right.\right.$$

$$-\frac{(16\,c^5d^8e^2f^2-48\,ac^4d^6e^4f^2+48\,a^2c^3d^4e^6f^2-16\,a^3c^2d^2e^8f^2-20\,c^5d^9efg+48\,ac^4d^7e^3fg-24\,a^2c^3d^5e^5}{}$$

input
```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, alg
orithm="giac")
```

output
```
1/1920*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*(8*c*d*g^2*x +
 (20*c^5*d^5*e^4*f*g + c^5*d^6*e^3*g^2 + 11*a*c^4*d^4*e^5*g^2)/(c^4*d^4*e^
4))*x + (80*c^5*d^5*e^4*f^2 + 20*c^5*d^6*e^3*f*g + 180*a*c^4*d^4*e^5*f*g -
 7*c^5*d^7*e^2*g^2 + 12*a*c^4*d^5*e^4*g^2 + 3*a^2*c^3*d^3*e^6*g^2)/(c^4*d^
4*e^4))*x + (80*c^5*d^6*e^3*f^2 + 560*a*c^4*d^4*e^5*f^2 - 100*c^5*d^7*e^2*
f*g + 200*a*c^4*d^5*e^4*f*g + 60*a^2*c^3*d^3*e^6*f*g + 35*c^5*d^8*e*g^2 -
61*a*c^4*d^6*e^3*g^2 + 9*a^2*c^3*d^4*e^5*g^2 - 15*a^3*c^2*d^2*e^7*g^2)/(c^
4*d^4*e^4))*x - (240*c^5*d^7*e^2*f^2 - 640*a*c^4*d^5*e^4*f^2 - 240*a^2*c^3
*d^3*e^6*f^2 - 300*c^5*d^8*e*f*g + 620*a*c^4*d^6*e^3*f*g - 180*a^2*c^3*d^4
*e^5*f*g + 180*a^3*c^2*d^2*e^7*f*g + 105*c^5*d^9*g^2 - 190*a*c^4*d^7*e^2*g
^2 + 36*a^2*c^3*d^5*e^4*g^2 + 30*a^3*c^2*d^3*e^6*g^2 - 45*a^4*c*d*e^8*g^2)
/(c^4*d^4*e^4)) - 1/256*(16*c^5*d^8*e^2*f^2 - 48*a*c^4*d^6*e^4*f^2 + 48*a^
2*c^3*d^4*e^6*f^2 - 16*a^3*c^2*d^2*e^8*f^2 - 20*c^5*d^9*e*f*g + 48*a*c^4*d
^7*e^3*f*g - 24*a^2*c^3*d^5*e^5*f*g - 16*a^3*c^2*d^3*e^7*f*g + 12*a^4*c*d*
e^9*f*g + 7*c^5*d^10*g^2 - 15*a*c^4*d^8*e^2*g^2 + 6*a^2*c^3*d^6*e^4*g^2 +
2*a^3*c^2*d^4*e^6*g^2 + 3*a^4*c*d^2*e^8*g^2 - 3*a^5*e^10*g^2)*log(abs(-c*d
^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int \frac{{\left(f+g\,x\right)}^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),
x)
```

output
```
int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),
 x)
```

Reduce [F]

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int \frac{(gx+f)^2\left(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2\right)^{\frac{3}{2}}}{ex+d}dx$$

input
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

output
```
int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

3.269 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 267

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(cd^2-ae^2\right)\left(3ae^2g-cd(8ef-5dg)\right)\left(cd^2+ae^2+2cde}{64c^2d^2e^3}$$

$$+\frac{\left(3ae^2g+cd(8ef-5dg)+6cdegx\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24cde^2}$$

$$-\frac{\left(cd^2-ae^2\right)^3\left(3ae^2g-cd(8ef-5dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{5/2}d^{5/2}e^{7/2}}$$

output

```
1/64*(-a*e^2+c*d^2)*(3*a*e^2*g-c*d*(-5*d*g+8*e*f))*(2*c*d*e*x+a*e^2+c*d^2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3+1/24*(3*a*e^2*g+c*d*(
-5*d*g+8*e*f)+6*c*d*e*g*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e^2
-1/64*(-a*e^2+c*d^2)^3*(3*a*e^2*g-c*d*(-5*d*g+8*e*f))*arctanh(c^(1/2)*d^(1
/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/
2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.12

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\frac{\left(cd^2-ae^2\right)^3\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-9a^3e^6g+3a^2c}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e
*x),x]
```

output
```
((c*d^2 - a*e^2)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[c]*Sqrt[d]*Sqrt[e]
*(-9*a^3*e^6*g + 3*a^2*c*d*e^4*(8*e*f + 3*d*g + 2*e*g*x) + c^3*d^3*(15*d^3
*g + 8*d*e^2*x*(2*f + g*x) + 16*e^3*x^2*(4*f + 3*g*x) - 2*d^2*e*(12*f + 5*
g*x)) + a*c^2*d^2*e^2*(-31*d^2*g + 4*d*e*(16*f + 5*g*x) + 8*e^2*x*(14*f +
9*g*x))))/(c*d^2 - a*e^2)^3 - (3*(3*a*e^2*g + c*d*(-8*e*f + 5*d*g))*ArcTan
h[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e
+ c*d*x]*Sqrt[d + e*x])))/(192*c^(5/2)*d^(5/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {1215, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int (f+gx)(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{\left(cd^2-ae^2\right)\left(3ae^2g-cd(8ef-5dg)\right)\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16cde^2}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(3ae^2g+cd(8ef-5dg)+6cdegx\right)}{24cde^2}$$

$$\Big\downarrow 1087$$

$$\frac{\left(cd^2-ae^2\right)\left(3ae^2g-cd(8ef-5dg)\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right.}{16cde^2}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(3ae^2g+cd(8ef-5dg)+6cdegx\right)}{24cde^2}$$

$$\Big\downarrow 1092$$

$$\frac{\left(cd^2-ae^2\right)\left(3ae^2g-cd(8ef-5dg)\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ad}}}{4cde}\right.}{16cde^2}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(3ae^2g+cd(8ef-5dg)+6cdegx\right)}{24cde^2}$$

$$\Big\downarrow 219$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)\left(3ae^2g-\right.}{16cde^2}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}\left(3ae^2g+cd(8ef-5dg)+6cdegx\right)}{24cde^2}$$

input
```
Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
((3*a*e^2*g + c*d*(8*e*f - 5*d*g) + 6*c*d*e*g*x)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2))/(24*c*d*e^2) + ((c*d^2 - a*e^2)*(3*a*e^2*g - c*d*(8*
e*f - 5*d*g))*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c
*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(243) = 486.

Time = 1.81 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.84

method	result
default	$\frac{g\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{4cde}+\frac{\left(4ac\right.}{}\right.}{16dec}\right.}{e}$

input
```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d),x,method=_RETU
RNVERBOSE)
```

output
```
g/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c
/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))-(d*g-e*f)/e^2*(1/3*(d*e*c*(
x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d
/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(
a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+
(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 960, normalized size of antiderivative = 3.60

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algor
ithm="fricas")
```

output

```
[1/768*(3*sqrt(c*d*e)*(8*(c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5
- a^3*c*d*e^7)*f - (5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a
^3*c*d^2*e^6 - 3*a^4*e^8)*g)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4*
e^4*g*x^3 + 8*(8*c^4*d^4*e^4*f + (c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*g)*x^2 -
8*(3*c^4*d^6*e^2 - 8*a*c^3*d^4*e^4 - 3*a^2*c^2*d^2*e^6)*f + (15*c^4*d^7*e
- 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7)*g + 2*(8*(c^4*d^5*
e^3 + 7*a*c^3*d^3*e^5)*f - (5*c^4*d^6*e^2 - 10*a*c^3*d^4*e^4 - 3*a^2*c^2*d
^2*e^6)*g)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4),
-1/384*(3*sqrt(-c*d*e)*(8*(c^4*d^7*e - 3*a*c^3*d^5*e^3 + 3*a^2*c^2*d^3*e^5
 - a^3*c*d*e^7)*f - (5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*
a^3*c*d^2*e^6 - 3*a^4*e^8)*g)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c
*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4*d^4*e^4*g*x^3 + 8*(8*c^
4*d^4*e^4*f + (c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*g)*x^2 - 8*(3*c^4*d^6*e^2 -
8*a*c^3*d^4*e^4 - 3*a^2*c^2*d^2*e^6)*f + (15*c^4*d^7*e - 31*a*c^3*d^5*e^3
+ 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e^7)*g + 2*(8*(c^4*d^5*e^3 + 7*a*c^3*d^3*e
^5)*f - (5*c^4*d^6*e^2 - 10*a*c^3*d^4*e^4 - 3*a^2*c^2*d^2*e^6)*g)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

Sympy [A] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 1853, normalized size of antiderivative = 6.94

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\text{Too large to display}$$

input

```
integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output

```
a*e*f*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*d*e*
x**2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2 + c
*d**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*
e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e -
 (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*
log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(
2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))**(3
/2)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), True))
 + a*e*g*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a
*d*e/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d
*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e
 + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*
d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e
**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2)
, True)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e**
2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e +
 c*d*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*(
a*e**2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e*
*2 + c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True)) + c*
d*f*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a*d...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algor
ithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.68

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\Bigg(2\Bigg(4\Bigg(6\,cdgx+\frac{8\,c^4}{}$$

$$-\frac{(8\,c^4d^7ef-24\,ac^3d^5e^3f+24\,a^2c^2d^3e^5f-8\,a^3cde^7f-5\,c^4d^8g+12\,ac^3d^6e^2g-6\,a^2c^2d^4e^4g-4\,a^3cd^2e^6g}{128\,\sqrt{cde}c^2d^2e^3}$$

input

```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algor
ithm="giac")
```

output

```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*c*d*g*x + (8*c^
4*d^4*e^3*f + c^4*d^5*e^2*g + 9*a*c^3*d^3*e^4*g)/(c^3*d^3*e^3))*x + (8*c^4
*d^5*e^2*f + 56*a*c^3*d^3*e^4*f - 5*c^4*d^6*e*g + 10*a*c^3*d^4*e^3*g + 3*a
^2*c^2*d^2*e^5*g)/(c^3*d^3*e^3))*x - (24*c^4*d^6*e*f - 64*a*c^3*d^4*e^3*f
- 24*a^2*c^2*d^2*e^5*f - 15*c^4*d^7*g + 31*a*c^3*d^5*e^2*g - 9*a^2*c^2*d^3
*e^4*g + 9*a^3*c*d*e^6*g)/(c^3*d^3*e^3)) - 1/128*(8*c^4*d^7*e*f - 24*a*c^3
*d^5*e^3*f + 24*a^2*c^2*d^3*e^5*f - 8*a^3*c*d*e^7*f - 5*c^4*d^8*g + 12*a*c
^3*d^6*e^2*g - 6*a^2*c^2*d^4*e^4*g - 4*a^3*c*d^2*e^6*g + 3*a^4*e^8*g)*log(
abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \int \frac{(f+g\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

input

```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

output

```
int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x
)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.76

$$\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\text{Too large to display}$$

input

```
int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

output

```
( - 9*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7*g + 9*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**2*c**2*d**3*e**5*g + 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**2*c**2*d**2*e**6*f + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**2*
e**6*g*x - 31*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**3*g + 64*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*f + 20*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*a*c**3*d**4*e**4*g*x + 112*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**
3*d**3*e**5*f*x + 72*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e**5*g*x*
*2 + 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**7*e*g - 24*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*c**4*d**6*e**2*f - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**
4*d**6*e**2*g*x + 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*f*x +
8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*g*x**2 + 64*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*c**4*d**4*e**4*f*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*c**4*d**4*e**4*g*x**3 + 9*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*
e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*e*
*8*g - 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)
*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2*e**6*g - 24*sqr
t(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt
(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d*e**7*f - 18*sqrt(e)*sqrt(d)*sqr
t(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(
a*e**2 - c*d**2))*a**2*c**2*d**4*e**4*g + 72*sqrt(e)*sqrt(d)*sqrt(c)*lo...
```

3.270 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 186

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e}+\frac{\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}$$

output
```
1/8*(a/c/d-d/e^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/8*(-a*e^2+c*d^2)^3
*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(3a^2e^4+2acde^2(4d+7ex)+c^2\right.}{24c^{3/2}d^{3/}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^2*e^4 + 2*a*c
*d*e^2*(4*d + 7*e*x) + c^2*d^2*(-3*d^2 + 2*d*e*x + 8*e^2*x^2)) + (3*(c*d^2
 - a*e^2)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*
d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*c^(3/2)*d^(3/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1131, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3e}-\frac{(cd^2-ae^2)\int \sqrt{cdex^2+(cd^2+ae^2)x+ade}dx}{2e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3e}-\frac{(cd^2-ae^2)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8cde}\right)}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4cde}\right)}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)}{2e}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

output

```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) - ((c*d^2 - a*e^2)*(((
c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4
*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2
)*d^(3/2)*e^(3/2))))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.24

method	result
default	$\frac{\frac{\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{4dec}-\frac{\left(a\,e^{2}-c\,d^{2}\right)^{2}\ln\left(\frac{\frac{a\,e^{2}}{2}-}{}\right.}{}\right.}{2}}{e}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(
1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e
))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.86

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\left[-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+\right.}{}\right.$$

$$-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\left(2\,cdex+cd^2+ae^2\right)\sqrt{-cde}}{2\left(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x\right)}\right)-2\left(8\,c^3\right.}{48\,c^2d^2e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d
*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e
) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*
a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/48*(3*(c^3*d^6 -
 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-
c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8
*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*
d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^2*d^2*e^3)]
```

Sympy [A] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 751, normalized size of antiderivative = 4.04

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
a*e*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*d*e*x*
*2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2 + c*d
**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)
*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (
a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*lo
g(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*
c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))**(3/2
)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), True)) +
 c*d*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a*d*e
/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d*e))
*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c
*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2
)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2
- c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), Tr
ue)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e**2/6
+ c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e + c*d
*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*(a*e*
*2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e**2 +
 c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.23

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\,cdx+\frac{c^3d^4e+7\,ac^2d^2e^3}{c^2d^2e^2}\right)\right.$$

$$-\frac{\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ad}\right.\right.\right.}{16\,\sqrt{cde}cde^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c*d*x + (c^3*d^4*e
+ 7*a*c^2*d^2*e^3)/(c^2*d^2*e^2))*x - (3*c^3*d^5 - 8*a*c^2*d^3*e^2 - 3*a^2
*c*d*e^4)/(c^2*d^2*e^2)) - 1/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e
^4 - a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c*d*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.15

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{3\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^5+8\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^3e^3+1}{}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

output

```
(3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5 + 8*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c**2*d**3*e**3 + 14*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d*
*2*e**4*x - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e + 2*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c
**3*d**3*e**3*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 + 9*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 - 9*sqrt(e)*sqrt(d)*
sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sq
rt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqr
t(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d*
*2))*c**3*d**6)/(24*c**2*d**2*e**3)
```

3.271 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 294

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx = \frac{\left(5ae^2g-cd(4ef-dg)+2cdegx\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4eg^2}$$

$$+\frac{\left(3a^2e^4g^2-6acde^2g(2ef-dg)+c^2d^2\left(8e^2f^2-4defg-d^2g^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4\sqrt{c}\sqrt{d}e^{3/2}g^3}$$

$$-\frac{2\sqrt{ef-dg}(cdf-aeg)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{g^3}$$

output
```
1/4*(5*a*e^2*g-c*d*(-d*g+4*e*f)+2*c*d*e*g*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/e/g^2+1/4*(3*a^2*e^4*g^2-6*a*c*d*e^2*g*(-d*g+2*e*f)+c^2*d^2*(-d
^2*g^2-4*d*e*f*g+8*e^2*f^2))*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(1/2)/d^(1/2)/e^(3/2)/g^3-2*(-d*g+e*
f)^(1/2)*(-a*e*g+c*d*f)^(3/2)*arctanh((-a*e*g+c*d*f)^(1/2)*(e*x+d)/(-d*g+e
*f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/g^3
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.03

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\Big(\sqrt{c}\sqrt{d}\sqrt{e}\Big(g\sqrt{ae+cdx}\sqrt{d+ex}(5ae^2g+cd}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)*(f + g*
x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(g*Sqrt[a*e + c*
d*x]*Sqrt[d + e*x]*(5*a*e^2*g + c*d*(-4*e*f + d*g + 2*e*g*x)) - 8*e*Sqrt[-
(e*f) + d*g]*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*
x])/(Sqrt[-(e*f) + d*g]*Sqrt[a*e + c*d*x])]) + (3*a^2*e^4*g^2 + 6*a*c*d*e^
2*g*(-2*e*f + d*g) - c^2*d^2*(-8*e^2*f^2 + 4*d*e*f*g + d^2*g^2))*ArcTanh[(
Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])]))/(4*Sqrt[c]*S
qrt[d]*e^(3/2)*g^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1215, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{f+gx}dx$$

$$\Big\downarrow \text{1231}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2g-cd(4ef-dg)+2cdegx\right)}{4eg^2}-\frac{\int\frac{cd\left(-c^2f(4ef-dg)d^3-2ace^2f(2ef-5dg)d+a^2e^3g(5ef-8dg)-\left(3a^2g^2e^4-6acdg(2ef-dg)e^2+c^2d^2\left(8e^2f^2-4degf-d^2g^2\right)\right)x\right)}{2(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4cdeg^2}$$

$$\Big\downarrow\ 27$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2g-cd(4ef-dg)+2cdegx\right)}{4eg^2}-\frac{\int\frac{-c^2f(4ef-dg)d^3-2ace^2f(2ef-5dg)d+a^2e^3g(5ef-8dg)-\left(3a^2g^2e^4-6acdg(2ef-dg)e^2+c^2d^2\left(8e^2f^2-4degf-d^2g^2\right)\right)x}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8eg^2}$$

$$\Big\downarrow\ 1269$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2g-cd(4ef-dg)+2cdegx\right)}{4eg^2}-\frac{\frac{8e(ef-dg)(cdf-aeg)^2\int\frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{\left(3a^2e^4g^2-6acde^2g(2ef-dg)+c^2d^2\left(-d^2g^2-4defg+8e^2f^2\right)\right)\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae\right.}}}{g}}{8eg^2}$$

$$\Big\downarrow\ 1092$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2g-cd(4ef-dg)+2cdegx\right)}{4eg^2}-\frac{\frac{8e(ef-dg)(cdf-aeg)^2\int\frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{2\left(3a^2e^4g^2-6acde^2g(2ef-dg)+c^2d^2\left(-d^2g^2-4defg+8e^2f^2\right)\right)\int\frac{1}{4cde-\frac{\left(cd^2+2\right.}{cdex^2+\left(c\right.}}}{g}}{8eg^2}$$

$$\Big\downarrow\ 219$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2g-cd(4ef-dg)+2cdegx\right)}{4eg^2}-\frac{\frac{8e(ef-dg)(cdf-aeg)^2\int\frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)\left(3a^2e^4g^2-6acde^2g(2ef-dg)+c\right.}{\sqrt{c}\sqrt{d}\sqrt{e}g}}{8eg^2}$$

$$\Big\downarrow\ 1154$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2g-cd(4ef-dg)+2cdegx\right)}{4eg^2}-\frac{-\frac{16e(ef-dg)(cdf-aeg)^2\int\frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right)}{g}-\frac{\text{ar}}{}}{8eg^2}$$

$$\Big\downarrow 219$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(5ae^2g-cd(4ef-dg)+2cdegx\right)}{4eg^2}-\frac{\frac{8e\sqrt{ef-dg}(cdf-aeg)^{3/2}\text{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{g}-\frac{\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)(3a^2e^4g}{\sqrt{c}\sqrt{d}}}{8eg^2}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

output

```
((5*a*e^2*g - c*d*(4*e*f - d*g) + 2*c*d*e*g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(4*e*g^2) - (-(((3*a^2*e^4*g^2 - 6*a*c*d*e^2*g*(2*e*f -
d*g) + c^2*d^2*(8*e^2*f^2 - 4*d*e*f*g - d^2*g^2))*ArcTanh[(c*d^2 + a*e^2 +
 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[e]*g)) + (8*e*Sqrt[e*f - d*g]*(c*d*f - a
*e*g)^(3/2)*ArcTanh[(c*d^2*f + a*e*(e*f - 2*d*g) - (a*e^2*g - c*d*(2*e*f -
 d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])])/g)/(8*e*g^2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. $2(264) = 528$.

Time = 2.29 (sec) , antiderivative size = 1240, normalized size of antiderivative = 4.22

method	result	size
default	Expression too large to display	1240

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)/(g*x+f),x,method=_RETU
RNVERBOSE)
```

output
```
1/(d*g-e*f)*(1/3*(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a
*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(3/2)+1/2*(a*e^2*g+c*d^2*g-2*
c*d*e*f)/g*(1/4*(2*d*e*c*(x+f/g)+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g)/d/e/c*(c*d
*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*
d^2*f*g+c*d*e*f^2)/g^2)^(1/2)+1/8*(4*d*e*c*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+
c*d*e*f^2)/g^2-(a*e^2*g+c*d^2*g-2*c*d*e*f)^2/g^2)/d/e/c*ln((1/2*(a*e^2*g+c
*d^2*g-2*c*d*e*f)/g+d*e*c*(x+f/g))/(d*e*c)^(1/2)+(c*d*(x+f/g)^2*e+(a*e^2*g
+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^
2)^(1/2))/(d*e*c)^(1/2))+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2*((c
*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-
c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)+1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*ln((1/2*
(a*e^2*g+c*d^2*g-2*c*d*e*f)/g+d*e*c*(x+f/g))/(d*e*c)^(1/2)+(c*d*(x+f/g)^2*
e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d
*e*f^2)/g^2)^(1/2))/(d*e*c)^(1/2)-(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2
)/g^2/((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*ln((2*(a*d*e*g
^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g
)+2*((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*(c*d*(x+f/g)^2*e
+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*
e*f^2)/g^2)^(1/2))/(x+f/g)))-1/(d*g-e*f)*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx=\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{\left(d+ex\right)\left(f+gx\right)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)/(g*x+f),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)*(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f),x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f),x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)), x
)
```

Reduce [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 1106, normalized size of antiderivative = 3.76

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f),x)
```

output

```
(4*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*
x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) +
a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*
a*c*d*e**3*g - 4*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*s
qrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e
*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*s
qrt(d + e*x))*c**2*d**2*e**2*f + 4*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log
(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d
*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)
*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c*d*e**3*g - 4*sqrt(d*g - e*f)*sqrt(a*e*
g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(e)*sqrt(d)*
sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*
e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**2*e**2*f - 4*sqrt(d*
g - e*f)*sqrt(a*e*g - c*d*f)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)
*sqrt(a*e*g - c*d*f) + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*g + 2*c*d*e*f + 2*c*d*e*g*x)*a*c*d*e**3*g + 4*sqrt(d*g - e*f)*sqrt(
a*e*g - c*d*f)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g -
c*d*f) + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x)*g + 2*c
*d*e*f + 2*c*d*e*g*x)*c**2*d**2*e**2*f + 5*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a*c*d*e**3*g**2 + sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**3*e*g**2 - 4...
```

3.272 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 261

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx &= \frac{(2cdf-aeg+cdgx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2(f+gx)} \\ &+ \frac{\sqrt{c}\sqrt{d}\left(3ae^2g-cd(4ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}g^3} \\ &- \frac{\sqrt{cdf-aeg}\left(ae^2g-cd(4ef-3dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{g^3\sqrt{ef-dg}}\end{aligned}$$

output
```
(c*d*g*x-a*e*g+2*c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f
)+c^(1/2)*d^(1/2)*(3*a*e^2*g-c*d*(-d*g+4*e*f))*arctanh(c^(1/2)*d^(1/2)*(e*
x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(1/2)/g^3-(-a*e*g+
c*d*f)^(1/2)*(a*e^2*g-c*d*(-3*d*g+4*e*f))*arctanh((-a*e*g+c*d*f)^(1/2)*(e*
x+d)/(-d*g+e*f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/g^3/(-d*g+e
*f)^(1/2)
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.99

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{g(-aeg+cd(2f+gx))}{f+gx}+\frac{\sqrt{cdf-aeg}(ae^2g+cd(-4ef+3}{\sqrt{-ef+dg}}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)*(f + g*
x)^2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((g*(-(a*e*g) + c*d*(2*f + g*x)))/(f + g*x)
 + (Sqrt[c*d*f - a*e*g]*(a*e^2*g + c*d*(-4*e*f + 3*d*g))*ArcTan[(Sqrt[c*d*
f - a*e*g]*Sqrt[d + e*x])/(Sqrt[-(e*f) + d*g]*Sqrt[a*e + c*d*x])])/(Sqrt[-
(e*f) + d*g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]) + (Sqrt[c]*Sqrt[d]*(3*a*e^2*
g + c*d*(-4*e*f + d*g))*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*S
qrt[a*e + c*d*x])])/(Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/g^3
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {1215, 1230, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^2}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(f+gx)^2}dx$$

$$\Big\downarrow \text{1230}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(-aeg+2cdf+cdgx)}{g^2(f+gx)}-\frac{\int \frac{2c^2fd^3+ace(2ef-3dg)d-c(3ae^2g-cd(4ef-dg))xd-a^2e^3g}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2g^2}$$

$$\Big\downarrow \ 1269$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(-aeg+2cdf+cdgx)}{g^2(f+gx)}-\frac{-\frac{cd\left(3ae^2g-cd(4ef-dg)\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{(cdf-aeg)\left(-ae^2g-3cd^2g+4cdef\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}}{2g^2}$$

$$\Big\downarrow \ 1092$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(-aeg+2cdf+cdgx)}{g^2(f+gx)}-\frac{-\frac{(cdf-aeg)\left(-ae^2g-3cd^2g+4cdef\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{2cd\left(3ae^2g-cd(4ef-dg)\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd}{\sqrt{cdex^2}}}{g}}{2g^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(-aeg+2cdf+cdgx)}{g^2(f+gx)}-\frac{-\frac{(cdf-aeg)\left(-ae^2g-3cd^2g+4cdef\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}-\frac{\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)\left(3ae^2g-cd\right.}{\sqrt{e}g}}{2g^2}$$

$$\Big\downarrow \ 1154$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(-aeg+2cdf+cdgx)}{g^2(f+gx)}-\frac{\frac{2(cdf-aeg)\left(-ae^2g-3cd^2g+4cdef\right)\int \frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right.}{g}}{2g^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(-aeg+2cdf+cdgx)}{g^2(f+gx)}-\frac{-\frac{\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)(3ae^2g-cd(4ef-dg))}{\sqrt{e}g}-\frac{\sqrt{cdf-aeg}(-ae^2g-3cd^2g+4cdef)\text{arctanh}\left(\frac{-x(ae^2g-cd}{2\sqrt{ef-dg}\sqrt{x(c}}\right.}{g\sqrt{ef-dg}}}{2g^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),
x]
```

output
```
((2*c*d*f - a*e*g + c*d*g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(g^2*(f + g*x)) - (-((Sqrt[c]*Sqrt[d]*(3*a*e^2*g - c*d*(4*e*f - d*g))*ArcT
anh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[e]*g)) - (Sqrt[c*d*f - a*e*g]*(4*c*d
*e*f - 3*c*d^2*g - a*e^2*g)*ArcTanh[(c*d^2*f + a*e*(e*f - 2*d*g) - (a*e^2*
g - c*d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(g*Sqrt[e*f - d*g]))/(2*g^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3014 vs. $2(235) = 470$.

Time = 2.67 (sec) , antiderivative size = 3015, normalized size of antiderivative = 11.55

method	result	size
default	Expression too large to display	3015

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)/(g*x+f)^2,x,method=_RE
TURNVERBOSE)
```

output

```
e/(d*g-e*f)^2*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^
2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-
c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*
e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/
(d*e*c)^(1/2)))+1/g/(d*g-e*f)*(-1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2
)*g^2/(x+f/g)*(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*
e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(5/2)+3/2*(a*e^2*g+c*d^2*g-2*c*d
*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*(1/3*(c*d*(x+f/g)^2*e+(a
*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f
^2)/g^2)^(3/2)+1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(1/4*(2*d*e*c*(x+f/g)+(a*
e^2*g+c*d^2*g-2*c*d*e*f)/g)/d/e/c*(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*
e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)+1/8*(4
*d*e*c*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2-(a*e^2*g+c*d^2*g-2*c*
d*e*f)^2/g^2)/d/e/c*ln((1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)/g+d*e*c*(x+f/g))/(
d*e*c)^(1/2)+(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e
*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2))/(d*e*c)^(1/2))+(a*d*e*g^2-
a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2*((c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*
d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)+1/2*
(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*ln((1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)/g+d*e*c*
(x+f/g))/(d*e*c)^(1/2)+(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\text{Timed out}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^2,x, alg
orithm="fricas")
```

output

```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)/(g*x+f)**2,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)(gx+f)^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^2,x, alg
orithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*(g*x +
f)^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^2,x, alg
orithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)),
x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)),
 x)
```

Reduce [B] (verification not implemented)

Time = 19.72 (sec) , antiderivative size = 2635, normalized size of antiderivative = 10.10

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^2,x)
```

output

```
(sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*
e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*
e**3*f*g + sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*
e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c
*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a*e**3*g**2*x + 3*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*
sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)
*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*
sqrt(c)*sqrt(d + e*x))*c*d**2*e*f*g + 3*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f
)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sq
rt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*d**2*e*g**2*x - 4*sqrt(d*g - e*f)*s
qrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*
sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g
- 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*d*e**2*f**2 - 4*sq
rt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) -
sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**
2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*d*e
**2*f*g*x + sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqr...
```

3.273 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 364

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx = & -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2(f+gx)} \\ & -\frac{\left(cd^2f+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g(ef-dg)(f+gx)^2} \\ & +\frac{2c^{3/2}d^{3/2}\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{g^3} \\ & +\frac{\left(a^2e^4g^2+2acde^2g(2ef-3dg)-c^2d^2\left(8e^2f^2-12defg+3d^2g^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4g^3(ef-dg)^{3/2}\sqrt{cdf-aeg}}\end{aligned}$$

output

```
-c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)-1/4*(c*d^2*f+a*e*
(-2*d*g+e*f)-(a*e^2*g-c*d*(-d*g+2*e*f))*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/g/(-d*g+e*f)/(g*x+f)^2+2*c^(3/2)*d^(3/2)*e^(1/2)*arctanh(c^(1/2)*
d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/g^3+1/4*(
a^2*e^4*g^2+2*a*c*d*e^2*g*(-3*d*g+2*e*f)-c^2*d^2*(3*d^2*g^2-12*d*e*f*g+8*e
^2*f^2))*arctanh((-a*e*g+c*d*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/g^3/(-d*g+e*f)^(3/2)/(-a*e*g+c*d*f)^(1/2)
```

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.88

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{g(aeg(-ef+2dg+egx)+cd(-2ef(2f+3gx)+dg(3f+5gx}{(ef-dg)(f+gx)^2}\right.}{}$$

input

```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)*(f + g*
x)^3),x]
```

output

```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((g*(a*e*g*(-(e*f) + 2*d*g + e*g*x) + c*d*(
-2*e*f*(2*f + 3*g*x) + d*g*(3*f + 5*g*x))))/((e*f - d*g)*(f + g*x)^2) + ((
a^2*e^4*g^2 + 2*a*c*d*e^2*g*(2*e*f - 3*d*g) + c^2*d^2*(-8*e^2*f^2 + 12*d*e
*f*g - 3*d^2*g^2))*ArcTan[(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])/(Sqrt[-(e*f)
 + d*g]*Sqrt[a*e + c*d*x])])/((-(e*f) + d*g)^(3/2)*Sqrt[c*d*f - a*e*g]*Sqr
t[a*e + c*d*x]*Sqrt[d + e*x]) + (8*c^(3/2)*d^(3/2)*Sqrt[e]*ArcTanh[(Sqrt[c
]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*
Sqrt[d + e*x])))/(4*g^3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1215, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^3}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(f+gx)^3}dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{\int -\frac{(cdf-aeg)\left(a^2ge^4+2acd(2ef-3dg)e^2+8c^2d^2(ef-dg)xe+c^2d^3(4ef-3dg)\right)}{2(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4g^2(ef-dg)(cdf-aeg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left((cdf-aeg)\left(-2adeg^2+ae^2fg-3cd^2fg+4cdef^2\right)+gx(cdf-aeg)\left(-ae^2g-5\right.\right.}{4g^2(f+gx)^2(ef-dg)(cdf-aeg)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a^2ge^4+2acd(2ef-3dg)e^2+8c^2d^2(ef-dg)xe+c^2d^3(4ef-3dg)}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8g^2(ef-dg)}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left((cdf-aeg)\left(-2adeg^2+ae^2fg-3cd^2fg+4cdef^2\right)+gx(cdf-aeg)\left(-ae^2g-5\right.\right.}{4g^2(f+gx)^2(ef-dg)(cdf-aeg)}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{\left(a^2e^4g^2+2acde^2g(2ef-3dg)-c^2d^2\left(3d^2g^2-12defg+8e^2f^2\right)\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}+\frac{8c^2d^2e(ef-dg)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+a}}}{g}}{8g^2(ef-dg)}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left((cdf-aeg)\left(-2adeg^2+ae^2fg-3cd^2fg+4cdef^2\right)+gx(cdf-aeg)\left(-ae^2g-5\right.\right.}{4g^2(f+gx)^2(ef-dg)(cdf-aeg)}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{\left(a^2e^4g^2+2acde^2g(2ef-3dg)-c^2d^2\left(3d^2g^2-12defg+8e^2f^2\right)\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}+\frac{16c^2d^2e(ef-dg)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+a\right.}{cdex^2+\left(cd^2+ae^2\right.}}}{g}}{8g^2(ef-dg)}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left((cdf-aeg)\left(-2adeg^2+ae^2fg-3cd^2fg+4cdef^2\right)+gx(cdf-aeg)\left(-ae^2g-5\right.\right.}{4g^2(f+gx)^2(ef-dg)(cdf-aeg)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(a^2e^4g^2+2acde^2g(2ef-3dg)-c^2d^2\left(3d^2g^2-12defg+8e^2f^2\right)\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{g}+\frac{8c^{3/2}d^{3/2}\sqrt{e}(ef-dg)\text{arctanh}\left(\frac{}{2\sqrt{c}\sqrt{d}}\right.}{g}}{8g^2(ef-dg)}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left((cdf-aeg)\left(-2adeg^2+ae^2fg-3cd^2fg+4cdef^2\right)+gx(cdf-aeg)\left(-ae^2g-5\right.\right.}{4g^2(f+gx)^2(ef-dg)(cdf-aeg)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{8c^{3/2}d^{3/2}\sqrt{e}(ef-dg)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{g}-\frac{2\left(a^2e^4g^2+2acde^2g(2ef-3dg)-c^2d^2\left(3d^2g^2-12defg+8e^2f^2\right)\right)\int \frac{}{4(ef}}{}}{8g^2(ef-dg)}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left((cdf-aeg)\left(-2adeg^2+ae^2fg-3cd^2fg+4cdef^2\right)+gx(cdf-aeg)\left(-ae^2g-5\right.\right.}{4g^2(f+gx)^2(ef-dg)(cdf-aeg)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(a^2e^4g^2+2acde^2g(2ef-3dg)-c^2d^2\left(3d^2g^2-12defg+8e^2f^2\right)\right)\text{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{g\sqrt{ef-dg}\sqrt{cdf-aeg}}+\frac{8c^{3/2}d^{3/2}\sqrt{e}(ef-dg}{}}{8g^2(ef-dg)}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left((cdf-aeg)\left(-2adeg^2+ae^2fg-3cd^2fg+4cdef^2\right)+gx(cdf-aeg)\left(-ae^2g-5\right.\right.}{4g^2(f+gx)^2(ef-dg)(cdf-aeg)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),
x]
```

output

```
-1/4*(((c*d*f - a*e*g)*(4*c*d*e*f^2 - 3*c*d^2*f*g + a*e^2*f*g - 2*a*d*e*g^
2) + g*(c*d*f - a*e*g)*(6*c*d*e*f - 5*c*d^2*g - a*e^2*g)*x)*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*(e*f - d*g)*(c*d*f - a*e*g)*(f + g*x)^
2) + ((8*c^(3/2)*d^(3/2)*Sqrt[e]*(e*f - d*g)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])])/g + ((a^2*e^4*g^2 + 2*a*c*d*e^2*g*(2*e*f - 3*d*g) - c^2*d^2*(8*e^2*
f^2 - 12*d*e*f*g + 3*d^2*g^2))*ArcTanh[(c*d^2*f + a*e*(e*f - 2*d*g) - (a*e
^2*g - c*d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(g*Sqrt[e*f - d*g]*Sqrt[c*d*f - a
*e*g]))/(8*g^2*(e*f - d*g))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6008 vs. $2(332) = 664$.

Time = 3.05 (sec) , antiderivative size = 6009, normalized size of antiderivative = 16.51

method	result	size
default	Expression too large to display	6009

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)/(g*x+f)^3,x,method=_RE
TURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)(f + gx)^3} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^3,x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)/(g*x+f)**3,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)(gx+f)^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^3,x, alg
orithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*(g*x +
f)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2052 vs. $2(332)=664$.

Time $=1.18$ (sec) , antiderivative size $=2052$, normalized size of antiderivative $=5.64$

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^3,x, alg
orithm="giac")
```

output

```
-c^2*d^2*e*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*g^3) - 1/4*(8*c^2*d^2
*e^2*f^2 - 12*c^2*d^3*e*f*g - 4*a*c*d*e^3*f*g + 3*c^2*d^4*g^2 + 6*a*c*d^2*
e^2*g^2 - a^2*e^4*g^2)*arctan(-((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
+ a*e^2*x + a*d*e))*g + sqrt(c*d*e)*f)/sqrt(-c*d*e*f^2 + c*d^2*f*g + a*e^2
*f*g - a*d*e*g^2))/(sqrt(-c*d*e*f^2 + c*d^2*f*g + a*e^2*f*g - a*d*e*g^2)*(
e*f*g^3 - d*g^4)) - 1/4*(6*c^4*d^6*e^2*f^3 + 12*a*c^3*d^4*e^4*f^3 + 6*a^2*
c^2*d^2*e^6*f^3 - 5*c^4*d^7*e*f^2*g - 27*a*c^3*d^5*e^3*f^2*g - 23*a^2*c^2*
d^3*e^5*f^2*g - a^3*c*d*e^7*f^2*g + 13*a*c^3*d^6*e^2*f*g^2 + 26*a^2*c^2*d^
4*e^4*f*g^2 + a^3*c*d^2*e^6*f*g^2 - 8*a^2*c^2*d^5*e^3*g^3 + 24*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*c^3*d^3*e^3*f^3 - 20*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*c^3*d^4*e^
2*f^2*g - 4*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^
2*a*c^2*d^2*e^4*f^2*g - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^2*c^3*d^5*e*f*g^2 - 18*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^2*a*c^2*d^3*e^3*f*g^2 - 5*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*c*d*e^5*f*g^2 + 16*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a*c^2*d^4*e^2*g^3 + 8*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*c*d^2*
e^4*g^3 + 24*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^3\,\left(d+e\,x\right)}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)),
x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)),
 x)
```

Reduce [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 14401, normalized size of antiderivative = 39.56

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^3}\,dx=\text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^3,x)
```

output

```
( - sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d
*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) +
 a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))
*a**3*e**6*f**2*g**3 - 2*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*s
qrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*
sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*s
qrt(c)*sqrt(d + e*x))*a**3*e**6*f*g**4*x - sqrt(d*g - e*f)*sqrt(a*e*g - c*
d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c
)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) +
 sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*g**5*x**2 + 5*sqrt(d*g -
 e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*s
qrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*
d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e
**4*f**2*g**3 + 10*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)
*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a
*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)
*sqrt(d + e*x))*a**2*c*d**2*e**4*f*g**4*x + 5*sqrt(d*g - e*f)*sqrt(a*e*g -
 c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqr
t(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f
) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**4*g**5*x**2 -...
```

3.274 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^4}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 272

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^4}\,dx = -\frac{\left(cd^2-ae^2\right)\left(cd^2f+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8(ef-dg)^2(cdf-aeg)(f+gx)^2} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3(ef-dg)(f+gx)^3} + \frac{\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8(ef-dg)^{5/2}(cdf-aeg)^{3/2}}$$

output
```
-1/8*(-a*e^2+c*d^2)*(c*d^2*f+a*e*(-2*d*g+e*f)-(a*e^2*g-c*d*(-d*g+2*e*f))*x
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-d*g+e*f)^2/(-a*e*g+c*d*f)/(g*x
+f)^2+1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-d*g+e*f)/(g*x+f)^3+1/8
*(-a*e^2+c*d^2)^3*arctanh((-a*e*g+c*d*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(-d*g+e*f)^(5/2)/(-a*e*g+c*d*f)^(3/
2)
```

Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.15

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^4}\,dx=\frac{1}{24}\sqrt{(ae+cdx)(d+ex)}\left(\frac{2acde(d^2g(f-7gx)-4de(f-gx)^2+\right.$$

$$\left.-\frac{3\left(cd^2-ae^2\right)^3\arctan\left(\frac{\sqrt{cdf-aeg}\sqrt{d+ex}}{\sqrt{-ef+dg}\sqrt{ae+cdx}}\right)}{(-ef+dg)^{5/2}(cdf-aeg)^{3/2}\sqrt{ae+cdx}\sqrt{d+ex}}\right)$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)*(f + g*
x)^4),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((2*a*c*d*e*(d^2*g*(f - 7*g*x) - 4*d*e*(f -
 g*x)^2 + e^2*f*x*(-7*f + g*x)) + c^2*d^2*(-8*e^2*f^2*x^2 - 2*d*e*f*x*(f -
 7*g*x) + d^2*(3*f^2 + 8*f*g*x - 3*g^2*x^2)) + a^2*e^2*(-8*d^2*g^2 - 2*d*e
*g*(-7*f + g*x) + e^2*(-3*f^2 + 8*f*g*x + 3*g^2*x^2)))/((e*f - d*g)^2*(-(c
*d*f) + a*e*g)*(f + g*x)^3) - (3*(c*d^2 - a*e^2)^3*ArcTan[(Sqrt[c*d*f - a*
e*g]*Sqrt[d + e*x])/(Sqrt[-(e*f) + d*g]*Sqrt[a*e + c*d*x])])/((-(e*f) + d*
g)^(5/2)*(c*d*f - a*e*g)^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/24
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1215, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)(f+gx)^4}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(f+gx)^4}dx$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(f+gx)^3(ef-dg)}-\frac{\left(cd^2-ae^2\right)\int\frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{(f+gx)^3}dx}{2(ef-dg)}$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(f+gx)^3(ef-dg)}-$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f\right)}{4(f+gx)^2(ef-dg)(cdf-aeg)}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}d}{8(ef-dg)(cdf-aeg)}\right.}{2(ef-dg)}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(f+gx)^3(ef-dg)}-$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right)}{4(ef-dg)(cdf-aeg)}+\right.}{2(ef-dg)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3(f+gx)^3(ef-dg)}-$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f\right)}{4(f+gx)^2(ef-dg)(cdf-aeg)}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)}{2\sqrt{ef-dg}\sqrt{x\left(ae^2+cd^2\right)+}}\right.}{8(ef-dg)^{3/2}(cdf-aeg)^{3/}}\right.}{2(ef-dg)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)*(f + g*x)^4),
x]
```

output

```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*(e*f - d*g)*(f + g*x)^3)
- ((c*d^2 - a*e^2)*(((c*d^2*f + a*e*(e*f - 2*d*g) - (a*e^2*g - c*d*(2*e*f
- d*g))*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(e*f - d*g)*(c*
d*f - a*e*g)*(f + g*x)^2) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2*f + a*e*(e*f
 - 2*d*g) - (a*e^2*g - c*d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f
 - a*e*g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*(e*f - d*g)^(3
/2)*(c*d*f - a*e*g)^(3/2))))/(2*(e*f - d*g))
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215

```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10983 vs. $2(252) = 504$.

Time = 3.49 (sec) , antiderivative size = 10984, normalized size of antiderivative = 40.38

method	result	size
default	Expression too large to display	10984

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)/(g*x+f)^4,x,method=_RE
TURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1684 vs. $2(252) = 504$.

Time = 90.33 (sec) , antiderivative size = 3425, normalized size of antiderivative = 12.59

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)(f + gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^4,x, alg
orithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)(f+gx)^4}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)/(g*x+f)**4,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)(f+gx)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex+d)(gx+f)^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^4,x, alg
orithm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*(g*x +
f)^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5573 vs. $2(252) = 504$.

Time = 2.40 (sec) , antiderivative size = 5573, normalized size of antiderivative = 20.49

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d+ex)(f+gx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^4,x, alg
orithm="giac")
```

output

```
1/8*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-((sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*g + sqrt(c*d*e)*f
)/sqrt(-c*d*e*f^2 + c*d^2*f*g + a*e^2*f*g - a*d*e*g^2))/((c*d*e^2*f^3 - 2*
c*d^2*e*f^2*g - a*e^3*f^2*g + c*d^3*f*g^2 + 2*a*d*e^2*f*g^2 - a*d^2*e*g^3)
*sqrt(-c*d*e*f^2 + c*d^2*f*g + a*e^2*f*g - a*d*e*g^2)) + 1/24*(48*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*c^5*d^7*e^3*f^5 + 96
*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c^4*d^5*e
^5*f^5 + 48*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*
a^2*c^3*d^3*e^7*f^5 - 84*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))*c^5*d^8*e^2*f^4*g - 276*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))*a*c^4*d^6*e^4*f^4*g - 204*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c^3*d^4*e^6*f^4*g - 12*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^2*e^8*f^4*
g + 18*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*c^5*d
^9*e*f^3*g^2 + 252*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))*a*c^4*d^7*e^3*f^3*g^2 + 336*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))*a^2*c^3*d^5*e^5*f^3*g^2 + 36*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^3*e^7*f^3*g^2 - 18*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c*d*e^9*f^3*g
^2 + 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*c^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)(f+gx)^4}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^4\,\left(d+e\,x\right)}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)),
x)
```

output

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)),
 x)
```

Reduce [F]

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)(f+gx)^4}\,dx=\int \frac{(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2)^{\frac{3}{2}}}{(ex+d)\,(gx+f)^4}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^4,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)/(g*x+f)^4,x)
```

3.275 $\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 348

$$\begin{aligned}
&\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx\\
&= -\frac{g(16cef-2cdg-7beg)(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{24c^2e^3}\\
&\quad -\frac{g^2(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^3}\\
&\quad -\frac{(35b^2e^2g^2-20bceg(4ef+3dg)+4c^2(12e^2f^2+16defg+7d^2g^2))\,(8cd-3be+2cex)\sqrt{d(cd-be)-b}}{192c^4e^3}\\
&\quad +\frac{(2cd-be)^2\,(35b^2e^2g^2-20bceg(4ef+3dg)+4c^2(12e^2f^2+16defg+7d^2g^2))\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-c}}\right.}{64c^{9/2}e^3}
\end{aligned}$$

output
```
-1/24*g*(-7*b*e*g-2*c*d*g+16*c*e*f)*(e*x+d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*
x^2)^(1/2)/c^2/e^3-1/4*g^2*(e*x+d)^3*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2
)/c/e^3-1/192*(35*b^2*e^2*g^2-20*b*c*e*g*(3*d*g+4*e*f)+4*c^2*(7*d^2*g^2+16
*d*e*f*g+12*e^2*f^2))*(2*c*e*x-3*b*e+8*c*d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^
2)^(1/2)/c^4/e^3+1/64*(-b*e+2*c*d)^2*(35*b^2*e^2*g^2-20*b*c*e*g*(3*d*g+4*e
*f)+4*c^2*(7*d^2*g^2+16*d*e*f*g+12*e^2*f^2))*arctan(c^(1/2)*(e*x+d)/(d*(-b
*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(9/2)/e^3
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\frac{-\sqrt{c}(d+ex)(-be+c(d-ex))\left(-105b^3e^3g^2+10b^2ce^2g(24ef+46dg+7egx)+8c^3(32d^3g^2+d^2eg(80f\right.}{}$$

input
```
Integrate[((d + e*x)^2*(f + g*x)^2)/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x
^2],x]
```

output
```
(-(Sqrt[c]*(d + e*x)*(-(b*e) + c*(d - e*x))*(-105*b^3*e^3*g^2 + 10*b^2*c*e
^2*g*(24*e*f + 46*d*g + 7*e*g*x) + 8*c^3*(32*d^3*g^2 + d^2*e*g*(80*f + 21*
g*x) + 16*d*e^2*(3*f^2 + 3*f*g*x + g^2*x^2) + 2*e^3*x*(6*f^2 + 8*f*g*x + 3
*g^2*x^2)) - 4*b*c^2*e*(155*d^2*g^2 + 2*d*e*g*(104*f + 29*g*x) + 2*e^2*(18
*f^2 + 20*f*g*x + 7*g^2*x^2)))) - 3*(-2*c*d + b*e)^2*(35*b^2*e^2*g^2 - 20*
b*c*e*g*(4*e*f + 3*d*g) + 4*c^2*(12*e^2*f^2 + 16*d*e*f*g + 7*d^2*g^2))*Sqr
t[d + e*x]*Sqrt[c*d - b*e - c*e*x]*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]
*Sqrt[d + e*x])])/(192*c^(9/2)*e^3*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1262, 27, 1221, 1134, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}\,dx$$
$$\Big\downarrow \; 1262$$

$$-\frac{\int -\frac{e^2(d+ex)^2\left(8ce^2f^2+6cd^2g^2-7bdeg^2+eg(16cef-2cdg-7beg)x\right)}{2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4ce^4}-\frac{g^2(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d+ex)^2\left(8ce^2f^2+6cd^2g^2-7bdeg^2+eg(16cef-2cdg-7beg)x\right)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{8ce^2}-\frac{g^2(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^3}$$

$$\Big\downarrow \; 1221$$

$$\frac{\frac{\left(35b^2e^2g^2-20bceg(3dg+4ef)+4c^2\left(7d^2g^2+16defg+12e^2f^2\right)\right)\int \frac{(d+ex)^2}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{6c}-\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}(-7beg-2cd}{3ce}}{8ce^2}$$
$$\frac{g^2(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^3}$$

$$\Big\downarrow \; 1134$$

$$\frac{\frac{\left(35b^2e^2g^2-20bceg(3dg+4ef)+4c^2\left(7d^2g^2+16defg+12e^2f^2\right)\right)\left(\frac{3(2cd-be)\int \frac{d+ex}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}\right)}{6c}-\frac{g(}{}}{8ce^2}$$
$$\frac{g^2(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^3}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{\left(35b^2e^2g^2-20bceg(3dg+4ef)+4c^2\left(7d^2g^2+16defg+12e^2f^2\right)\right)\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2c}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(}{}\right.}{6c}}{8ce^2}$$
$$\frac{g^2(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^3}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{\left(35b^2e^2g^2-20bceg(3dg+4ef)+4c^2\left(7d^2g^2+16defg+12e^2f^2\right)\right)\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-b}}\right.}{c}\right.}{4c}\right.}{6c}}{8ce^2}$$

$$\frac{g^2(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^3}$$

$$\Big\downarrow 217$$

$$\frac{\frac{\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2c^{3/2}e}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}\right)\left(35b^2e^2g^2-20bceg(3dg+\right.}{6c}}{8ce^2}$$

$$\frac{g^2(d+ex)^3\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4ce^3}$$

input
```
Int[((d + e*x)^2*(f + g*x)^2)/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
-1/4*(g^2*(d + e*x)^3*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(c*e^3) +
 (-1/3*(g*(16*c*e*f - 2*c*d*g - 7*b*e*g)*(d + e*x)^2*Sqrt[d*(c*d - b*e) -
b*e^2*x - c*e^2*x^2])/(c*e) + ((35*b^2*e^2*g^2 - 20*b*c*e*g*(4*e*f + 3*d*g
) + 4*c^2*(12*e^2*f^2 + 16*d*e*f*g + 7*d^2*g^2))*(-1/2*((d + e*x)*Sqrt[d*(
c*d - b*e) - b*e^2*x - c*e^2*x^2])/(c*e) + (3*(2*c*d - b*e)*(-(Sqrt[d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2]/(c*e)) + ((2*c*d - b*e)*ArcTan[(e*(b + 2*c*
x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(2*c^(3/2)*e))
)/(4*c)))/(6*c))/(8*c*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 1262
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n + e*g^n*(
m + p + n)*(d + e*x)^(n - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x], x] /;
 FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(328) = 656$.

Time = 2.76 (sec) , antiderivative size = 1458, normalized size of antiderivative = 4.19

method	result	size
default	Expression too large to display	1458

input
```
int((e*x+d)^2*(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_R
ETURNVERBOSE)
```

output

```
d^2*f^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x
-b*d*e+c*d^2)^(1/2))+2*e*g*(d*g+e*f)*(-1/3*x^2/c/e^2*(-c*e^2*x^2-b*e^2*x-b
*d*e+c*d^2)^(1/2)-5/6*b/c*(-1/2*x/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(
1/2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e
^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2
)^(1/2)))+1/2*(-b*d*e+c*d^2)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1
/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+2/3*(-b*d*e+c*d^2)/c/e^2*
(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arc
tan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))+2*
d*f*(d*g+e*f)*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*
e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^
2)^(1/2)))+(d^2*g^2+4*d*e*f*g+e^2*f^2)*(-1/2*x/c/e^2*(-c*e^2*x^2-b*e^2*x-b
*d*e+c*d^2)^(1/2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)
-1/2*b/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*
x-b*d*e+c*d^2)^(1/2)))+1/2*(-b*d*e+c*d^2)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^
2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+e^2*g^2*(-1/
4*x^3/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-7/8*b/c*(-1/3*x^2/c/e^2
*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-5/6*b/c*(-1/2*x/c/e^2*(-c*e^2*x^2-
b*e^2*x-b*d*e+c*d^2)^(1/2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d
^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^...
```

Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 997, normalized size of antiderivative = 2.86

$$\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\text{Too large to display}$$

input

```
integrate((e*x+d)^2*(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="fricas")
```

output
```
[-1/768*(3*(48*(4*c^4*d^2*e^2 - 4*b*c^3*d*e^3 + b^2*c^2*e^4)*f^2 + 16*(16*
c^4*d^3*e - 36*b*c^3*d^2*e^2 + 24*b^2*c^2*d*e^3 - 5*b^3*c*e^4)*f*g + (112*
c^4*d^4 - 352*b*c^3*d^3*e + 408*b^2*c^2*d^2*e^2 - 200*b^3*c*d*e^3 + 35*b^4
*e^4)*g^2)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*
e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)
*sqrt(-c)) + 4*(48*c^4*e^3*g^2*x^3 + 48*(8*c^4*d*e^2 - 3*b*c^3*e^3)*f^2 +
16*(40*c^4*d^2*e - 52*b*c^3*d*e^2 + 15*b^2*c^2*e^3)*f*g + (256*c^4*d^3 - 6
20*b*c^3*d^2*e + 460*b^2*c^2*d*e^2 - 105*b^3*c*e^3)*g^2 + 8*(16*c^4*e^3*f*
g + (16*c^4*d*e^2 - 7*b*c^3*e^3)*g^2)*x^2 + 2*(48*c^4*e^3*f^2 + 16*(12*c^4
*d*e^2 - 5*b*c^3*e^3)*f*g + (84*c^4*d^2*e - 116*b*c^3*d*e^2 + 35*b^2*c^2*e
^3)*g^2)*x)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^5*e^3), -1/384*
(3*(48*(4*c^4*d^2*e^2 - 4*b*c^3*d*e^3 + b^2*c^2*e^4)*f^2 + 16*(16*c^4*d^3*
e - 36*b*c^3*d^2*e^2 + 24*b^2*c^2*d*e^3 - 5*b^3*c*e^4)*f*g + (112*c^4*d^4
- 352*b*c^3*d^3*e + 408*b^2*c^2*d^2*e^2 - 200*b^3*c*d*e^3 + 35*b^4*e^4)*g^
2)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x
+ b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) + 2*(48*c^4*
e^3*g^2*x^3 + 48*(8*c^4*d*e^2 - 3*b*c^3*e^3)*f^2 + 16*(40*c^4*d^2*e - 52*b
*c^3*d*e^2 + 15*b^2*c^2*e^3)*f*g + (256*c^4*d^3 - 620*b*c^3*d^2*e + 460*b^
2*c^2*d*e^2 - 105*b^3*c*e^3)*g^2 + 8*(16*c^4*e^3*f*g + (16*c^4*d*e^2 - 7*b
*c^3*e^3)*g^2)*x^2 + 2*(48*c^4*e^3*f^2 + 16*(12*c^4*d*e^2 - 5*b*c^3*e^3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. 2(343) = 686.

Time = 1.29 (sec) , antiderivative size = 1282, normalized size of antiderivative = 3.68

$$\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2*(g*x+f)**2/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2
),x)
```

output

```
Piecewise(((-b*(-3*b*(-5*b*(-7*b*e**2*g**2/(8*c) + 2*d*e*g**2 + 2*e**2*f*g
)/(6*c) + d**2*g**2 + 4*d*e*f*g + e**2*f**2 + g**2*(-3*b*d*e + 3*c*d**2)/(
4*c))/(4*c) + 2*d**2*f*g + 2*d*e*f**2 + (-2*b*d*e + 2*c*d**2)*(-7*b*e**2*g
**2/(8*c) + 2*d*e*g**2 + 2*e**2*f*g)/(3*c*e**2))/(2*c) + d**2*f**2 + (-b*d
*e + c*d**2)*(-5*b*(-7*b*e**2*g**2/(8*c) + 2*d*e*g**2 + 2*e**2*f*g)/(6*c)
+ d**2*g**2 + 4*d*e*f*g + e**2*f**2 + g**2*(-3*b*d*e + 3*c*d**2)/(4*c))/(2
*c*e**2))*Piecewise((log(-b*e**2 - 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*
e - b*e**2*x + c*d**2 - c*e**2*x**2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) -
b*d*e + c*d**2, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c)
 + x)**2), True)) + sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)*(-g**2*
x**3/(4*c) - x**2*(-7*b*e**2*g**2/(8*c) + 2*d*e*g**2 + 2*e**2*f*g)/(3*c*e*
*2) - x*(-5*b*(-7*b*e**2*g**2/(8*c) + 2*d*e*g**2 + 2*e**2*f*g)/(6*c) + d**
2*g**2 + 4*d*e*f*g + e**2*f**2 + g**2*(-3*b*d*e + 3*c*d**2)/(4*c))/(2*c*e*
*2) - (-3*b*(-5*b*(-7*b*e**2*g**2/(8*c) + 2*d*e*g**2 + 2*e**2*f*g)/(6*c) +
 d**2*g**2 + 4*d*e*f*g + e**2*f**2 + g**2*(-3*b*d*e + 3*c*d**2)/(4*c))/(4*
c) + 2*d**2*f*g + 2*d*e*f**2 + (-2*b*d*e + 2*c*d**2)*(-7*b*e**2*g**2/(8*c)
 + 2*d*e*g**2 + 2*e**2*f*g)/(3*c*e**2))/(c*e**2)), Ne(c*e**2, 0)), (-2*(g*
*2*(-b*d*e - b*e**2*x + c*d**2)**(9/2)/(9*b**4*e**6) + (-b*d*e - b*e**2*x
+ c*d**2)**(7/2)*(2*b*d*e*g**2 - 2*b*e**2*f*g - 4*c*d**2*g**2)/(7*b**4*e**
6) + (-b*d*e - b*e**2*x + c*d**2)**(5/2)*(b**2*d**2*e**2*g**2 - 2*b**2*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^2*(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$

$$-\frac{1}{192}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\,\Bigg(2\,\Bigg(4\,\Bigg(\frac{6\,g^2x}{c}+\frac{16\,c^3e^5fg+16\,c^3de^4g^2-7\,bc^2e^5g^2}{c^4e^5}\Bigg)x+\ 48\,c^3e^5f^2$$

$$-\ (192\,c^4d^2e^2f^2-192\,bc^3de^3f^2+48\,b^2c^2e^4f^2+256\,c^4d^3efg-576\,bc^3d^2e^2fg+384\,b^2c^2de^3fg-80\,b^3ce$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="giac")
```

output
```
-1/192*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*(6*g^2*x/c + (16*c
^3*e^5*f*g + 16*c^3*d*e^4*g^2 - 7*b*c^2*e^5*g^2)/(c^4*e^5))*x + (48*c^3*e^
5*f^2 + 192*c^3*d*e^4*f*g - 80*b*c^2*e^5*f*g + 84*c^3*d^2*e^3*g^2 - 116*b*
c^2*d*e^4*g^2 + 35*b^2*c*e^5*g^2)/(c^4*e^5))*x + (384*c^3*d*e^4*f^2 - 144*
b*c^2*e^5*f^2 + 640*c^3*d^2*e^3*f*g - 832*b*c^2*d*e^4*f*g + 240*b^2*c*e^5*
f*g + 256*c^3*d^3*e^2*g^2 - 620*b*c^2*d^2*e^3*g^2 + 460*b^2*c*d*e^4*g^2 -
105*b^3*e^5*g^2)/(c^4*e^5)) - 1/128*(192*c^4*d^2*e^2*f^2 - 192*b*c^3*d*e^3
*f^2 + 48*b^2*c^2*e^4*f^2 + 256*c^4*d^3*e*f*g - 576*b*c^3*d^2*e^2*f*g + 38
4*b^2*c^2*d*e^3*f*g - 80*b^3*c*e^4*f*g + 112*c^4*d^4*g^2 - 352*b*c^3*d^3*e
*g^2 + 408*b^2*c^2*d^2*e^2*g^2 - 200*b^3*c*d*e^3*g^2 + 35*b^4*e^4*g^2)*log
(abs(-b*e^2 + 2*(sqrt(-c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*
e))*sqrt(-c)*abs(e)))/(sqrt(-c)*c^4*e^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^2}{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input
```
int(((f + g*x)^2*(d + e*x)^2)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),
x)
```

output

```
int(((f + g*x)^2*(d + e*x)^2)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),
 x)
```

Reduce [B] (verification not implemented)

Time = 16.02 (sec) , antiderivative size = 1678, normalized size of antiderivative = 4.82

$$\int \frac{(d+ex)^2(f+gx)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^2*(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*(105*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))
*b**5*e**5*g**2 - 810*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
 b*e + 2*c*d))*b**4*c*d*e**4*g**2 - 240*sqrt(c)*asinh((sqrt( - b*e + c*d -
 c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**4*c*e**5*f*g + 2424*sqrt(c)*asinh((sqr
t( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**3*c**2*d**2*e**3*g**2
+ 1632*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b**3*c**2*d*e**4*f*g + 144*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sq
rt( - b*e + 2*c*d))*b**3*c**2*e**5*f**2 - 3504*sqrt(c)*asinh((sqrt( - b*e
+ c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**3*d**3*e**2*g**2 - 4032*sq
rt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**3
*d**2*e**3*f*g - 864*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
b*e + 2*c*d))*b**2*c**3*d*e**4*f**2 + 2448*sqrt(c)*asinh((sqrt( - b*e + c*
d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*d**4*e*g**2 + 4224*sqrt(c)*asin
h((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**4*d**3*e**2*f*
g + 1728*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d)
)*b*c**4*d**2*e**3*f**2 - 672*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)
/sqrt( - b*e + 2*c*d))*c**5*d**5*g**2 - 1536*sqrt(c)*asinh((sqrt( - b*e +
c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**5*d**4*e*f*g - 1152*sqrt(c)*asinh
((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**5*d**3*e**2*f**2
+ 105*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e ...
```

3.276 $\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 203

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx &= -\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2} \\ &\quad -\frac{(6cef+4cdg-5beg)(8cd-3be+2cex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{24c^3e^2} \\ &\quad +\frac{(2cd-be)^2(6cef+4cdg-5beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8c^{7/2}e^2}\end{aligned}$$

output

```
-1/3*g*(e*x+d)^2*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c/e^2-1/24*(-5*b*e
*g+4*c*d*g+6*c*e*f)*(2*c*e*x-3*b*e+8*c*d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)
^(1/2)/c^3/e^2+1/8*(-b*e+2*c*d)^2*(-5*b*e*g+4*c*d*g+6*c*e*f)*arctan(c^(1/2
)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(7/2)/e^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \frac{(-2cd+be)^2 \Bigl(-\frac{\sqrt{c}(d+ex)(-be+c(d-ex))\left(15b^2e^2g-2bce(9ef+26dg+5egx)+4c^2\left(10d^2g+6de(2f+gx)+e^2x(3f+2gx)\right)\right)}{(-2cd+be)^2} - 3(6cef -}{24c^{7/2}e^2\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[((d + e*x)^2*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2
],x]
```

output
```
((-2*c*d + b*e)^2*(-((Sqrt[c]*(d + e*x)*(-(b*e) + c*(d - e*x))*(15*b^2*e^2
*g - 2*b*c*e*(9*e*f + 26*d*g + 5*e*g*x) + 4*c^2*(10*d^2*g + 6*d*e*(2*f + g
*x) + e^2*x*(3*f + 2*g*x))))/(-2*c*d + b*e)^2) - 3*(6*c*e*f + 4*c*d*g - 5*
b*e*g)*Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*x]*ArcTan[Sqrt[c*d - b*e - c*e*x
]/(Sqrt[c]*Sqrt[d + e*x])]))/(24*c^(7/2)*e^2*Sqrt[(d + e*x)*(-(b*e) + c*(d
 - e*x))])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1221, 1134, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{(-5beg+4cdg+6cef)\int \frac{(d+ex)^2}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{6ce}-\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$\Big\downarrow \text{1134}$$

$$\frac{(-5beg+4cdg+6cef)\left(\frac{3(2cd-be)\int \frac{d+ex}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}\right)}{6ce}-\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$\Big\downarrow \textcolor{blue}{1160}$$

$$\frac{(-5beg+4cdg+6cef)\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2c}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce}}{2ce}\right.}{6ce}$$

$$\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$\Big\downarrow \textcolor{blue}{1092}$$

$$\frac{(-5beg+4cdg+6cef)\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{c}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right.}{4c}\right.}{6ce}$$

$$\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{\left(\frac{3(2cd-be)\left(\frac{(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2c^{3/2}e}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}\right)(-5beg+4c}{6ce}$$

$$\frac{g(d+ex)^2\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3ce^2}$$

input

```
Int[((d + e*x)^2*(f + g*x))/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output

```
-1/3*(g*(d + e*x)^2*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(c*e^2) + (
(6*c*e*f + 4*c*d*g - 5*b*e*g)*(-1/2*((d + e*x)*Sqrt[d*(c*d - b*e) - b*e^2*
x - c*e^2*x^2])/(c*e) + (3*(2*c*d - b*e)*(-(Sqrt[d*(c*d - b*e) - b*e^2*x -
 c*e^2*x^2]/(c*e)) + ((2*c*d - b*e)*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt
[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(2*c^(3/2)*e)))/(4*c)))/(6*c*e)
```

Defintions of rubi rules used

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(187) = 374$.

Time = 2.02 (sec) , antiderivative size = 773, normalized size of antiderivative = 3.81

method	result
default	$\frac{d^2 f \arctan\left(\frac{\sqrt{c e^2}\left(x+\frac{b}{2c}\right)}{\sqrt{-x^2 c e^2-x b e^2-b d e+c d^2}}\right)}{\sqrt{c e^2}}+e(2dg+ef)\left(-\frac{x\sqrt{-x^2 c e^2-x b e^2-b d e+c d^2}}{2c e^2}-\frac{3b\left(-\frac{\sqrt{-x^2 c e^2-x b e^2-b d e+c}}{c e^2}\right.}{}\right.$

input
```
int((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
d^2*f/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b
*d*e+c*d^2)^(1/2))+e*(2*d*g+e*f)*(-1/2*x/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c
*d^2)^(1/2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b
/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*
e+c*d^2)^(1/2)))+1/2*(-b*d*e+c*d^2)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/
2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+d*(d*g+2*e*f)*(-1/
c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arctan(
(c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+e^2*g*(
-1/3*x^2/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-5/6*b/c*(-1/2*x/c/e^
2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e
^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*
b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+1/2*(-b*d*e+c*d^2)/c/e^2/(c*
e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^
2)^(1/2)))+2/3*(-b*d*e+c*d^2)/c/e^2*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*
d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*
x^2-b*e^2*x-b*d*e+c*d^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.88

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \left[\frac{3\,(6\,(4\,c^3d^2e-4\,bc^2de^2+b^2ce^3)f+(16\,c^3d^3-36\,bc^2d^2e+24\,b^2cde^2-5\,b^3e^3)g)\sqrt{-c}\log\left(8\,c^2e^2x^2+8\right.}{}\right.$$

$$-\frac{3\,(6\,(4\,c^3d^2e-4\,bc^2de^2+b^2ce^3)f+(16\,c^3d^3-36\,bc^2d^2e+24\,b^2cde^2-5\,b^3e^3)g)\sqrt{c}\arctan\left(\frac{\sqrt{-ce^2x^2-be}}{2\,(c^2e^2x}\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output
```
[1/96*(3*(6*(4*c^3*d^2*e - 4*b*c^2*d*e^2 + b^2*c*e^3)*f + (16*c^3*d^3 - 36
*b*c^2*d^2*e + 24*b^2*c*d*e^2 - 5*b^3*e^3)*g)*sqrt(-c)*log(8*c^2*e^2*x^2 +
 8*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 4*sqrt(-c*e^2*x^2 - b*e^2
*x + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) - 4*(8*c^3*e^2*g*x^2 + 6*(8*
c^3*d*e - 3*b*c^2*e^2)*f + (40*c^3*d^2 - 52*b*c^2*d*e + 15*b^2*c*e^2)*g +
2*(6*c^3*e^2*f + (12*c^3*d*e - 5*b*c^2*e^2)*g)*x)*sqrt(-c*e^2*x^2 - b*e^2*
x + c*d^2 - b*d*e))/(c^4*e^2), -1/48*(3*(6*(4*c^3*d^2*e - 4*b*c^2*d*e^2 +
b^2*c*e^3)*f + (16*c^3*d^3 - 36*b*c^2*d^2*e + 24*b^2*c*d*e^2 - 5*b^3*e^3)*
g)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c*e*x
+ b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2 + b*c*d*e)) + 2*(8*c^3*e
^2*g*x^2 + 6*(8*c^3*d*e - 3*b*c^2*e^2)*f + (40*c^3*d^2 - 52*b*c^2*d*e + 15
*b^2*c*e^2)*g + 2*(6*c^3*e^2*f + (12*c^3*d*e - 5*b*c^2*e^2)*g)*x)*sqrt(-c*
e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))/(c^4*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(194) = 388$.

Time = 1.37 (sec) , antiderivative size = 660, normalized size of antiderivative = 3.25

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \begin{cases} \left(-\frac{b\left(-\frac{3b\left(-\frac{5be^2g}{6c}+2deg+e^2f\right)}{4c}+d^2g+2def+\frac{g\left(-2bde+2cd^2\right)}{3c}\right)}{2c}+d^2f+\frac{\left(-bde+cd^2\right)\left(-\frac{5be^2g}{6c}+2deg+e^2f\right)}{2ce^2}\right)\left(\left\{\begin{array}{l}\log\left(-be^2-2ce^2\right. \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}\right.}{\sqrt{-ce^2\left(\frac{b}{2c}+\right.}}\end{array}\right.\right. \\ -\frac{2\left(-\frac{g\left(-bde-be^2x+cd^2\right)^{\frac{7}{2}}}{7b^3e^4}-\frac{\left(-bde-be^2x+cd^2\right)^{\frac{5}{2}}\left(bdeg-be^2f-3cd^2g\right)}{5b^3e^4}-\frac{\left(-bde-be^2x+cd^2\right)^{\frac{3}{2}}\left(-2bcd^3eg+2bcd^2e^2f+3c^2d^4g\right)}{3b^3e^4}-\frac{\sqrt{-bde-be^2x+cd^2}\left(\right.}{}\right.}{be^2} \\ \frac{d^2fx+\frac{e^2gx^4}{4}+\frac{x^3\cdot\left(2deg+e^2f\right)}{3}+\frac{x^2\left(d^2g+2def\right)}{2}}{\sqrt{-bde+cd^2}} \end{cases}$$

input
```
integrate((e*x+d)**2*(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output

```
Piecewise(((-b*(-3*b*(-5*b*e**2*g/(6*c) + 2*d*e*g + e**2*f)/(4*c) + d**2*g
 + 2*d*e*f + g*(-2*b*d*e + 2*c*d**2)/(3*c))/(2*c) + d**2*f + (-b*d*e + c*d
**2)*(-5*b*e**2*g/(6*c) + 2*d*e*g + e**2*f)/(2*c*e**2))*Piecewise((log(-b*
e**2 - 2*c*e**2*x + 2*sqrt(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e*
*2*x**2))/sqrt(-c*e**2), Ne(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*
c) + x)*log(b/(2*c) + x)/sqrt(-c*e**2*(b/(2*c) + x)**2), True)) + (-g*x**2
/(3*c) - x*(-5*b*e**2*g/(6*c) + 2*d*e*g + e**2*f)/(2*c*e**2) - (-3*b*(-5*b
*e**2*g/(6*c) + 2*d*e*g + e**2*f)/(4*c) + d**2*g + 2*d*e*f + g*(-2*b*d*e +
 2*c*d**2)/(3*c))/(c*e**2))*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2)
, Ne(c*e**2, 0)), (-2*(-g*(-b*d*e - b*e**2*x + c*d**2)**(7/2)/(7*b**3*e**4
) - (-b*d*e - b*e**2*x + c*d**2)**(5/2)*(b*d*e*g - b*e**2*f - 3*c*d**2*g)/
(5*b**3*e**4) - (-b*d*e - b*e**2*x + c*d**2)**(3/2)*(-2*b*c*d**3*e*g + 2*b
*c*d**2*e**2*f + 3*c**2*d**4*g)/(3*b**3*e**4) - sqrt(-b*d*e - b*e**2*x + c
*d**2)*(b*c**2*d**5*e*g - b*c**2*d**4*e**2*f - c**3*d**6*g)/(b**3*e**4))/(
b*e**2), Ne(b*e**2, 0)), ((d**2*f*x + e**2*g*x**4/4 + x**3*(2*d*e*g + e**2
*f)/3 + x**2*(d**2*g + 2*d*e*f)/2)/sqrt(-b*d*e + c*d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx =$$
$$-\frac{1}{24}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\Bigg(2\left(\frac{4\,gx}{c}+\frac{6\,c^2e^3f+12\,c^2de^2g-5\,bce^3g}{c^3e^3}\right)x+\frac{48\,c^2de^2f-18\,bce^3f}{}$$
$$-\frac{(24\,c^3d^2ef-24\,bc^2de^2f+6\,b^2ce^3f+16\,c^3d^3g-36\,bc^2d^2eg+24\,b^2cde^2g-5\,b^3e^3g)\log\left(\left|-be^2+2\,(\sqrt{}\right.\right.}{16\,\sqrt{-c}c^3e|e|}$$

input

```
integrate((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output

```
-1/24*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*(4*g*x/c + (6*c^2*e^3*
f + 12*c^2*d*e^2*g - 5*b*c*e^3*g)/(c^3*e^3))*x + (48*c^2*d*e^2*f - 18*b*c*
e^3*f + 40*c^2*d^2*e*g - 52*b*c*d*e^2*g + 15*b^2*e^3*g)/(c^3*e^3)) - 1/16*
(24*c^3*d^2*e*f - 24*b*c^2*d*e^2*f + 6*b^2*c*e^3*f + 16*c^3*d^3*g - 36*b*c
^2*d^2*e*g + 24*b^2*c*d*e^2*g - 5*b^3*e^3*g)*log(abs(-b*e^2 + 2*(sqrt(-c*e
^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(sqr
t(-c)*c^3*e*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{(f+g\,x)\,{(d+e\,x)}^2}{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input

```
int(((f + g*x)*(d + e*x)^2)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output

```
int(((f + g*x)*(d + e*x)^2)/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 839, normalized size of antiderivative = 4.13

$$\int \frac{(d+ex)^2(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^2*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*( - 15*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d
))*b**4*e**4*g + 102*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
b*e + 2*c*d))*b**3*c*d*e**3*g + 18*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*b**3*c*e**4*f - 252*sqrt(c)*asinh((sqrt( - b*e
 + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d**2*e**2*g - 108*sqrt(
c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c**2*d*
e**3*f + 264*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*
c*d))*b*c**3*d**3*e*g + 216*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/s
qrt( - b*e + 2*c*d))*b*c**3*d**2*e**2*f - 96*sqrt(c)*asinh((sqrt( - b*e +
c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**4*g - 144*sqrt(c)*asinh((sqr
t( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**4*d**3*e*f - 15*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
b**2*c*e**2*g + 52*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*b*c**2*d*e*g + 18*sqrt(d + e*x)*sqrt(b*e - 2*c*d)
*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*e**2*f + 10*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
b*c**2*e**2*g*x - 40*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*
sqrt( - b*e + c*d - c*e*x)*c**3*d**2*g - 48*sqrt(d + e*x)*sqrt(b*e - 2*c*d
)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**3*d*e*f - 24*sqrt(d +
 e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)...
```

3.277 $\int \frac{(d+ex)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 122

$$\int \frac{(d+ex)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=-\frac{(8cd-3be+2cex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4c^2e}+\frac{3(2cd-be)^2\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4c^{5/2}e}$$

output
```
-1/4*(2*c*e*x-3*b*e+8*c*d)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c^2/e+3/
4*(-b*e+2*c*d)^2*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(
1/2))/c^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\frac{-\sqrt{c}(d+ex)\left(3b^2e^2+bce(-11d+ex)+c^2(8d^2-6dex-2e^2x^2)\right)-3(-2cd+be)^2\sqrt{d+ex}\sqrt{cd-be-}}{4c^{5/2}e\sqrt{(d+ex)(-be+c(d-ex))}}$$

input
```
Integrate[(d + e*x)^2/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output
```
(-(Sqrt[c]*(d + e*x)*(3*b^2*e^2 + b*c*e*(-11*d + e*x) + c^2*(8*d^2 - 6*d*e
*x - 2*e^2*x^2))) - 3*(-2*c*d + b*e)^2*Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*
x]*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/(4*c^(5/2)*e*S
qrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1134, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{1134}$$

$$\frac{3(2cd-be)\int \frac{d+ex}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}$$

$$\Big\downarrow \text{1160}$$

$$\frac{3(2cd-be)\left(\frac{(2cd-be)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2c}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}$$

$$\Big\downarrow \text{1092}$$

$$\frac{3(2cd-be)\left(\frac{(2cd-be)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{c}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}$$

$$\Big\downarrow 217$$

$$\frac{3(2cd-be)\left(\frac{(2cd-be)\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{2c^{3/2}e}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{ce}\right)}{4c}-\frac{(d+ex)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2ce}$$

input

```
Int[(d + e*x)^2/Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2],x]
```

output

```
-1/2*((d + e*x)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(c*e) + (3*(2*c
*d - b*e)*(-(Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(c*e)) + ((2*c*d -
b*e)*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^
2*x^2])])/(2*c^(3/2)*e)))/(4*c)
```

Defintions of rubi rules used

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1134

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(110) = 220$.

Time = 1.74 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.10

method	result
default	$\frac{d^{2} \arctan\left(\frac{\sqrt{c\,e^{2}}\,\left(x+\frac{b}{2c}\right)}{\sqrt{-x^{2} c\,e^{2}-x b\,e^{2}-b d e+c\,d^{2}}}\right)}{\sqrt{c\,e^{2}}}+e^{2} \left(-\frac{x \sqrt{-x^{2} c\,e^{2}-x b\,e^{2}-b d e+c\,d^{2}}}{2 c\,e^{2}}-\frac{3 b \left(-\frac{\sqrt{-x^{2} c\,e^{2}-x b\,e^{2}-b d e+c\,d^{2}}}{c\,e^{2}}-\frac{b \arctan\left(\right.}{}\right.}{4 c}\right.$

input

```
int((e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RETURNVERBO
SE)
```

output

```
d^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d
*e+c*d^2)^(1/2))+e^2*(-1/2*x/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-
3/4*b/c*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(
1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/
2)))+1/2*(-b*d*e+c*d^2)/c/e^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/
c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))+2*d*e*(-1/c/e^2*(-c*e^2*x^2-b*
e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2
*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \Bigg[-\frac{3\,(4\,c^2d^2-4\,bcde+b^2e^2)\sqrt{-c}\log\big(8\,c^2e^2x^2+8\,bce^2x-4\,c^2d^2+4\,bcde+b^2e^2-4\,\sqrt{-ce^2x^2-be^2x+}}{16\,c^3e}$$

$$-\frac{3\,(4\,c^2d^2-4\,bcde+b^2e^2)\sqrt{c}\arctan\Big(\frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(2\,cex+be)\sqrt{c}}{2\,(c^2e^2x^2+bce^2x-c^2d^2+bcde)}\Big)+2\,\sqrt{-ce^2x^2-be^2x+cd^2-bde}}{8\,c^3e}$$

input
```
integrate((e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="f
ricas")
```

output
```
[-1/16*(3*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*sqrt(-c)*log(8*c^2*e^2*x^2 + 8
*b*c*e^2*x - 4*c^2*d^2 + 4*b*c*d*e + b^2*e^2 - 4*sqrt(-c*e^2*x^2 - b*e^2*x
 + c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(-c)) + 4*sqrt(-c*e^2*x^2 - b*e^2*x
+ c*d^2 - b*d*e)*(2*c^2*e*x + 8*c^2*d - 3*b*c*e))/(c^3*e), -1/8*(3*(4*c^2*
d^2 - 4*b*c*d*e + b^2*e^2)*sqrt(c)*arctan(1/2*sqrt(-c*e^2*x^2 - b*e^2*x +
c*d^2 - b*d*e)*(2*c*e*x + b*e)*sqrt(c)/(c^2*e^2*x^2 + b*c*e^2*x - c^2*d^2
+ b*c*d*e)) + 2*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*c^2*e*x + 8*
c^2*d - 3*b*c*e))/(c^3*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(109) = 218$.

Time = 1.06 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.90

$$\int \frac{(d+ex)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$

$$= \begin{cases} \left(-\frac{x}{2c}-\frac{-\frac{3be^2}{4c}+2de}{ce^2}\right)\sqrt{-bde-be^2x+cd^2-ce^2x^2}+\left(-\frac{b\left(-\frac{3be^2}{4c}+2de\right)}{2c}+d^2+\frac{-bde+cd^2}{2c}\right)\left(\left(\begin{cases}\frac{\log\left(-be^2-2ce^2\right.}{} \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}\right.}{\sqrt{-ce^2\left(\frac{b}{2c}+x\right.}}\end{cases}\right.\right. \\ -\frac{2\left(\frac{c^2d^4\sqrt{-bde-be^2x+cd^2}}{b^2e^2}-\frac{2cd^2\left(-bde-be^2x+cd^2\right)^{\frac{3}{2}}}{3b^2e^2}+\frac{\left(-bde-be^2x+cd^2\right)^{\frac{5}{2}}}{5b^2e^2}\right)}{be^2} \\ \frac{\begin{cases} d^2x & \text{for } e=0 \\ \frac{(d+ex)^3}{3e} & \text{otherwise} \end{cases}}{\sqrt{-bde+cd^2}} \end{cases}$$

input

```
integrate((e*x+d)**2/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x)
```

output

```
Piecewise(((-x/(2*c) - (-3*b*e**2/(4*c) + 2*d*e)/(c*e**2))*sqrt(-b*d*e - b
*e**2*x + c*d**2 - c*e**2*x**2) + (-b*(-3*b*e**2/(4*c) + 2*d*e)/(2*c) + d*
*2 + (-b*d*e + c*d**2)/(2*c))*Piecewise((log(-b*e**2 - 2*c*e**2*x + 2*sqrt
(-c*e**2)*sqrt(-b*d*e - b*e**2*x + c*d**2 - c*e**2*x**2))/sqrt(-c*e**2), N
e(b**2*e**2/(4*c) - b*d*e + c*d**2, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/s
qrt(-c*e**2*(b/(2*c) + x)**2), True)), Ne(c*e**2, 0)), (-2*(c**2*d**4*sqrt
(-b*d*e - b*e**2*x + c*d**2)/(b**2*e**2) - 2*c*d**2*(-b*d*e - b*e**2*x + c
*d**2)**(3/2)/(3*b**2*e**2) + (-b*d*e - b*e**2*x + c*d**2)**(5/2)/(5*b**2*
e**2))/(b*e**2), Ne(b*e**2, 0)), (Piecewise((d**2*x, Eq(e, 0)), ((d + e*x)
**3/(3*e), True))/sqrt(-b*d*e + c*d**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(b*e-2*c*d)>0)', see `assume?`
 for more
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.24

$$\begin{aligned}&\int \frac{(d+ex)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx\\&=-\frac{1}{4}\sqrt{-ce^2x^2-be^2x+cd^2-bde}\left(\frac{2x}{c}+\frac{8cd-3be}{c^2e}\right)\\&\quad-\frac{3(4c^2d^2-4bcde+b^2e^2)\log\left(\left|-be^2+2\left(\sqrt{-ce^2}x-\sqrt{-ce^2x^2-be^2x+cd^2-bde}\right)\sqrt{-c}|e|\right|\right)}{8\sqrt{-c}c^2|e|}\end{aligned}$$

input
```
integrate((e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="g
iac")
```

output
```
-1/4*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)*(2*x/c + (8*c*d - 3*b*e)/(
c^2*e)) - 3/8*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*log(abs(-b*e^2 + 2*(sqrt(-
c*e^2)*x - sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e))*sqrt(-c)*abs(e)))/(
sqrt(-c)*c^2*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\int \frac{{\left(d+e\,x\right)}^2}{\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}} \,d x$$

input
```
int((d + e*x)^2/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2),x)
```

output
```
int((d + e*x)^2/(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.58

$$\int \frac{(d+ex)^2}{\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{i\left(3\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^3e^3-18\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b^2cd\,e^2+36\sqrt{c}\,asinh\left(\frac{\sqrt{-cex-be+cd}\,i}{\sqrt{-be+2cd}}\right)b\,c^2d}{}$$

input
```
int((e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output
```
(i*(3*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b
**3*e**3 - 18*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2
*c*d))*b**2*c*d*e**2 + 36*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqr
t( - b*e + 2*c*d))*b*c**2*d**2*e - 24*sqrt(c)*asinh((sqrt( - b*e + c*d - c
*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**3 + 3*sqrt(d + e*x)*sqrt(b*e - 2*c*
d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*e - 8*sqrt(d + e*x)
*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d
 - 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d
 - c*e*x)*c**2*e*x))/(4*c**3*e*(b*e - 2*c*d))
```

3.278 $\int \frac{(d+ex)^2}{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 211

$$\int \frac{(d+ex)^2}{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = -\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{cg} - \frac{(2cef-4cdg+beg)\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{c^{3/2}g^2} + \frac{2(ef-dg)^{3/2}\arctan\left(\frac{\sqrt{cef+cdg-beg}(d+ex)}{\sqrt{ef-dg}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{g^2\sqrt{cef+cdg-beg}}$$

output

```
-(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/c/g-(b*e*g-4*c*d*g+2*c*e*f)*arctan
(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/c^(3/2)/g^2+2*(-d
*g+e*f)^(3/2)*arctan((-b*e*g+c*d*g+c*e*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(
d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/g^2/(-b*e*g+c*d*g+c*e*f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^2}{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{\sqrt{cd-be-cex}\left(\sqrt{cef+cdg-beg}(2cef-4cdg+beg)\sqrt{d+ex}\arctan\left(\frac{\sqrt{cd-be-cex}}{\sqrt{c}\sqrt{d+ex}}\right)-\sqrt{c}\left(g\sqrt{cef+cd}\right.\right.}{c^{3/2}g^2\sqrt{cef+cdg-beg}\sqrt{(d+e}}$$

input
```
Integrate[(d + e*x)^2/((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]
),x]
```

output
```
(Sqrt[c*d - b*e - c*e*x]*(Sqrt[c*e*f + c*d*g - b*e*g]*(2*c*e*f - 4*c*d*g +
 b*e*g)*Sqrt[d + e*x]*ArcTan[Sqrt[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x
])] - Sqrt[c]*(g*Sqrt[c*e*f + c*d*g - b*e*g]*(d + e*x)*Sqrt[-(b*e) + c*(d
- e*x)] + 2*c*(e*f - d*g)^(3/2)*Sqrt[d + e*x]*ArcTan[(Sqrt[e*f - d*g]*Sqrt
[c*d - b*e - c*e*x])/(Sqrt[c*e*f + c*d*g - b*e*g]*Sqrt[d + e*x])])))/(c^(3
/2)*g^2*Sqrt[c*e*f + c*d*g - b*e*g]*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))]
)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1267, 27, 1269, 1092, 217, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(f+gx)\sqrt{-bde-be^2x+cd^2-ce^2x^2}}\,dx$$

$$\Big\downarrow \; 1267$$

$$-\frac{\int \frac{e^2g(-2cgd^2+be^2f+e(2cef-4cdg+beg)x)}{2(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{ce^2g^2}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{cg}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{-2cgd^2+be^2f+e(2cef-4cdg+beg)x}{(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2cg}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{cg}$$

$$\Big\downarrow \ 1269$$

$$-\frac{\frac{e(beg-4cdg+2cef)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{g}-\frac{2c(ef-dg)^2\int \frac{1}{(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{g}}{2cg}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{cg}$$

$$\Big\downarrow \ 1092$$

$$-\frac{\frac{2e(beg-4cdg+2cef)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{g}-\frac{2c(ef-dg)^2\int \frac{1}{(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{g}}{2cg}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{cg}$$

$$\Big\downarrow \ 217$$

$$-\frac{\frac{\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(beg-4cdg+2cef)}{\sqrt{cg}}-\frac{2c(ef-dg)^2\int \frac{1}{(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{g}}{2cg}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{cg}$$

$$\Big\downarrow \ 1154$$

$$-\frac{\frac{4c(ef-dg)^2\int \frac{1}{-\frac{\left(2cgd^2+be(ef-2dg)+e^2(2cf-bg)x\right)^2}{-cx^2e^2-bxe^2+d(cd-be)}-4(ef-dg)(cef+cdg-beg)}d\frac{2cgd^2+be(ef-2dg)+e^2(2cf-bg)x}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}}{g}+\frac{\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-}}\right.}{\sqrt{cg}}}{2cg}$$

$$\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{cg}$$

$$\Big\downarrow \ 217$$

$$-\frac{\frac{\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(beg-4cdg+2cef)}{\sqrt{cg}}-\frac{2c(ef-dg)^{3/2}\arctan\left(\frac{e^2x(2cf-bg)+be(ef-2dg)+2cd^2g}{2\sqrt{ef-dg}\sqrt{d(cd-be)-be^2x-ce^2x^2}\sqrt{-beg+cdg+cef}}\right)}{g\sqrt{-beg+cdg+cef}}}{2cg}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}}{cg}$$

input

```
Int[(d + e*x)^2/((f + g*x)*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output

```
-(Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2]/(c*g)) - (((2*c*e*f - 4*c*d*g
+ b*e*g)*ArcTan[(e*(b + 2*c*x))/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x -
c*e^2*x^2])])/(Sqrt[c]*g) - (2*c*(e*f - d*g)^(3/2)*ArcTan[(2*c*d^2*g + b*e
*(e*f - 2*d*g) + e^2*(2*c*f - b*g)*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*e*f + c*d*
g - b*e*g]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(g*Sqrt[c*e*f + c*
d*g - b*e*g]))/(2*c*g)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(193) = 386.

Time = 2.14 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.28

method	result
default	$\frac{e\Bigg(eg\Bigg(-\frac{\sqrt{-x^2ce^2-xbe^2-bde+cd^2}}{ce^2}-\frac{b\arctan\left(\frac{\sqrt{ce^2}\left(x+\frac{b}{2c}\right)}{\sqrt{-x^2ce^2-xbe^2-bde+cd^2}}\right)}{2c\sqrt{ce^2}}\Bigg)+\frac{2dg\arctan\left(\frac{\sqrt{ce^2}\left(x+\frac{b}{2c}\right)}{\sqrt{-x^2ce^2-xbe^2-bde+cd^2}}\right)}{\sqrt{ce^2}}-}{g^2}$ $ef\arctan\Bigg($

input
```
int((e*x+d)^2/(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
e/g^2*(e*g*(-1/c/e^2*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)-1/2*b/c/(c*e^2
)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^
(1/2)))+2*d*g/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b
*e^2*x-b*d*e+c*d^2)^(1/2))-e*f/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b
/c)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)))-(d^2*g^2-2*d*e*f*g+e^2*f^2)/g
^3/(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*ln((-2*(b*d*e*g^
2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2-e^2*(b*g-2*c*f)/g*(x+f/g)+2*(-(b*d*e*
g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2*c*e^2-e^2*(b*g-2
*c*f)/g*(x+f/g)-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2))/(x+f
/g))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^2/(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{(d+ex)^2}{\sqrt{-(d+ex)\,(be-cd+cex)}\,(f+gx)}\,dx$$

input
```
integrate((e*x+d)**2/(g*x+f)/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2),x
)
```

output
```
Integral((d + e*x)**2/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((4*c*e^2>0)', see `assume?` for
more detai
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x+d)^2/(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\int \frac{(d+e\,x)^2}{(f+g\,x)\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),x)
```

output
```
int((d + e*x)^2/((f + g*x)*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)), x
)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 2483, normalized size of antiderivative = 11.77

$$\int \frac{(d+ex)^2}{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*( - sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*
b**3*e**3*g**2 + 7*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*
e + 2*c*d))*b**2*c*d*e**2*g**2 - sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)
*i)/sqrt( - b*e + 2*c*d))*b**2*c*e**3*f*g - 14*sqrt(c)*asinh((sqrt( - b*e
+ c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e*g**2 + 2*sqrt(c)*asi
nh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*e**3*f**2 +
 8*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3
*d**3*g**2 + 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
2*c*d))*c**3*d**2*e*f*g - 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/s
qrt( - b*e + 2*c*d))*c**3*d*e**2*f**2 - sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c*e*g**2 + sqrt(d + e*x)*s
qrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*c**2*d*g*
*2 + sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*
d - c*e*x)*c**2*e*f*g - sqrt(d*g - e*f)*sqrt(b*e*g - c*d*g - c*e*f)*log((s
qrt(g)*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*i - sqrt(2*sqrt(c)*sqr
t(d*g - e*f)*sqrt(b*e*g - c*d*g - c*e*f) - b*e*g + 2*c*e*f)*sqrt( - b*e +
2*c*d) + sqrt(g)*sqrt(c)*sqrt(d + e*x)*sqrt( - b*e + 2*c*d)*i)/sqrt( - b*e
 + 2*c*d))*b*c**2*d*e*g + sqrt(d*g - e*f)*sqrt(b*e*g - c*d*g - c*e*f)*log(
(sqrt(g)*sqrt(b*e - 2*c*d)*sqrt( - b*e + c*d - c*e*x)*i - sqrt(2*sqrt(c)*s
qrt(d*g - e*f)*sqrt(b*e*g - c*d*g - c*e*f) - b*e*g + 2*c*e*f)*sqrt( - b...
```

3.279 $\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 241

$$\begin{aligned}\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx &= \frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{g(cef+cdg-beg)(f+gx)}+\frac{2e\arctan\left(\frac{\sqrt{c}(d+ex)}{\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{\sqrt{c}g^2}\\ &\quad-\frac{e\sqrt{ef-dg}(2cef+4cdg-3beg)\arctan\left(\frac{\sqrt{cef+cdg-beg}(d+ex)}{\sqrt{ef-dg}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{g^2(cef+cdg-beg)^{3/2}}\end{aligned}$$

output

```
(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/g/(-b*e*g+c*d*g+c*e*f)/(
g*x+f)+2*e*arctan(c^(1/2)*(e*x+d)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/
c^(1/2)/g^2-e*(-d*g+e*f)^(1/2)*(-3*b*e*g+4*c*d*g+2*c*e*f)*arctan((-b*e*g+c
*d*g+c*e*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2
)^(1/2))/g^2/(-b*e*g+c*d*g+c*e*f)^(3/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \frac{e\left(\frac{g(-ef+dg)(d+ex)(-cd+be+cex)}{e(cef+cdg-beg)(f+gx)}-\frac{2\sqrt{d+ex}\sqrt{cd-be-cex}\arctan\left(\frac{\sqrt{cd-be-cex}}{\sqrt{c}\sqrt{d+ex}}\right)}{\sqrt{c}}+\frac{\sqrt{ef-dg}(2cef+4cdg-3beg)\sqrt{d+ex}\sqrt{cd-be-cex}\,\mathrm{ar}}{(cef+cdg-beg)^{3/2}}\right.}{g^2\sqrt{(d+ex)(-be+c(d-ex))}}$$

input

```
Integrate[(d + e*x)^2/((f + g*x)^2*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^
2]),x]
```

output

```
(e*((g*(-(e*f) + d*g)*(d + e*x)*(-(c*d) + b*e + c*e*x))/(e*(c*e*f + c*d*g
 - b*e*g)*(f + g*x)) - (2*Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*x]*ArcTan[Sqrt
[c*d - b*e - c*e*x]/(Sqrt[c]*Sqrt[d + e*x])])/Sqrt[c] + (Sqrt[e*f - d*g]*(
2*c*e*f + 4*c*d*g - 3*b*e*g)*Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*x]*ArcTan[
(Sqrt[e*f - d*g]*Sqrt[c*d - b*e - c*e*x])/(Sqrt[c*e*f + c*d*g - b*e*g]*Sqr
t[d + e*x])])/(c*e*f + c*d*g - b*e*g)^(3/2)))/(g^2*Sqrt[(d + e*x)*(-(b*e)
 + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1266, 27, 1269, 1092, 217, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{-bde-be^2x+cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{1266}$$

$$\frac{\int \frac{e(ef-dg)\left(4cgd^2+be(ef-3dg)+2e(cef+cdg-beg)x\right)}{2g(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{(ef-dg)(-beg+cdg+cef)}+\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{g(f+gx)(-beg+cdg+cef)}$$

$$\Big\downarrow \; 27$$

$$\frac{e\int \frac{4cgd^2+be(ef-3dg)+2e(cef+cdg-beg)x}{(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2g(-beg+cdg+cef)}+\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{g(f+gx)(-beg+cdg+cef)}$$

$$\Big\downarrow \; 1269$$

$$\frac{e\left(\frac{2e(-beg+cdg+cef)\int \frac{1}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{g}-\frac{(ef-dg)(-3beg+4cdg+2cef)\int \frac{1}{(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{g}\right)}{2g(-beg+cdg+cef)}+\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{g(f+gx)(-beg+cdg+cef)}$$

$$\Big\downarrow \; 1092$$

$$\frac{e\left(\frac{4e(-beg+cdg+cef)\int \frac{1}{-\frac{(b+2cx)^2e^4}{-cx^2e^2-bxe^2+d(cd-be)}-4ce^2}d\left(-\frac{e^2(b+2cx)}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}\right)}{g}-\frac{(ef-dg)(-3beg+4cdg+2cef)\int \frac{1}{(f+gx)\sqrt{-cx^2e^2-b}}}{g}\right.}{2g(-beg+cdg+cef)}$$

$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{g(f+gx)(-beg+cdg+cef)}$$

$$\Big\downarrow \; 217$$

$$\frac{e\left(\frac{2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-beg+cdg+cef)}{\sqrt{c}g}-\frac{(ef-dg)(-3beg+4cdg+2cef)\int \frac{1}{(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{g}\right)}{2g(-beg+cdg+cef)}+\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{g(f+gx)(-beg+cdg+cef)}$$

$$\Big\downarrow \; 1154$$

$$\frac{e\left(\frac{2(ef-dg)(-3beg+4cdg+2cef)\int \frac{1}{-\frac{\left(2cgd^2+be(ef-2dg)+e^2(2cf-bg)x\right)^2}{-cx^2e^2-bxe^2+d(cd-be)}-4(ef-dg)(cef+cdg-beg)}d\frac{2cgd^2+be(ef-2dg)+e^2(2cf-bg)x}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}}{g}+\frac{2\arctan\left(\right.}{}\right.}{2g(-beg+cdg+cef)}$$

$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{g(f+gx)(-beg+cdg+cef)}$$

$$\Big\downarrow 217$$

$$\frac{e\left(\frac{2\arctan\left(\frac{e(b+2cx)}{2\sqrt{c}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)(-beg+cdg+cef)}{\sqrt{c}g}-\frac{\sqrt{ef-dg}(-3beg+4cdg+2cef)\arctan\left(\frac{e^2x(2cf-bg)+be(ef-2dg)+2cd^2g}{2\sqrt{ef-dg}\sqrt{d(cd-be)-be^2x-ce^2x^2}\sqrt{-beg+c}}\right.}{g\sqrt{-beg+cdg+cef}}\right.}{2g(-beg+cdg+cef)}$$

$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{g(f+gx)(-beg+cdg+cef)}$$

input
```
Int[(d + e*x)^2/((f + g*x)^2*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output
```
((e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(g*(c*e*f + c*d*g
- b*e*g)*(f + g*x)) + (e*((2*(c*e*f + c*d*g - b*e*g)*ArcTan[(e*(b + 2*c*x)
)/(2*Sqrt[c]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(Sqrt[c]*g) - (S
qrt[e*f - d*g]*(2*c*e*f + 4*c*d*g - 3*b*e*g)*ArcTan[(2*c*d^2*g + b*e*(e*f
- 2*d*g) + e^2*(2*c*f - b*g)*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*e*f + c*d*g - b*
e*g]*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])])/(g*Sqrt[c*e*f + c*d*g -
b*e*g])))/(2*g*(c*e*f + c*d*g - b*e*g))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1266
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^
n, d + e*x, x], R = PolynomialRemainder[(f + g*x)^n, d + e*x, x]}, Simp[(e*
R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*
e^2)), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1
)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R
*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 1] && ILtQ[m, -1] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && (NeQ[m + n, 0] || EqQ[p, -2^(-1)])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(223) = 446$.

Time = 2.17 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.10

method	result
default	$\frac{e^2\arctan\left(\frac{\sqrt{c\,e^2}\left(x+\frac{b}{2c}\right)}{\sqrt{-x^2c\,e^2-xb\,e^2-bde+c\,d^2}}\right)}{g^2\sqrt{c\,e^2}}+\frac{\left(d^2g^2-2defg+e^2f^2\right)\left(\frac{g^2\sqrt{-\left(x+\frac{f}{g}\right)^2c\,e^2-\frac{e^2(bg-2cf)\left(x+\frac{f}{g}\right)}{g}-\frac{bde\,g^2-b\,e^2fg-c\,d^2g^2+c\,e}{g^2}}}{\left(bde\,g^2-b\,e^2fg-c\,d^2g^2+c\,e^2f^2\right)\left(x+\frac{f}{g}\right)}\right.}{}$

input
```
int((e*x+d)^2/(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
e^2/g^2/(c*e^2)^(1/2)*arctan((c*e^2)^(1/2)*(x+1/2*b/c)/(-c*e^2*x^2-b*e^2*x
-b*d*e+c*d^2)^(1/2))+1/g^4*(d^2*g^2-2*d*e*f*g+e^2*f^2)*(1/(b*d*e*g^2-b*e^2
*f*g-c*d^2*g^2+c*e^2*f^2)*g^2/(x+f/g)*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)/g*
(x+f/g)-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)+1/2*e^2*(b*g-
2*c*f)*g/(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/(-(b*d*e*g^2-b*e^2*f*g-
c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*ln((-2*(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^
2*f^2)/g^2-e^2*(b*g-2*c*f)/g*(x+f/g)+2*(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*
e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-(b*d*e*g^2
-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2))/(x+f/g)))-2*e/g^3*(d*g-e*f)/(-
(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*ln((-2*(b*d*e*g^2-b*e
^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2-e^2*(b*g-2*c*f)/g*(x+f/g)+2*(-(b*d*e*g^2-b
*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)
/g*(x+f/g)-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2))/(x+f/g))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^2/(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$
\begin{aligned}
&\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx\\
&=\int \frac{(d+ex)^2}{\sqrt{-\left(d+ex\right)\left(be-cd+cex\right)}\left(f+gx\right)^2}\,dx
\end{aligned}
$$

input

```
integrate((e*x+d)**2/(g*x+f)**2/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2
),x)
```

output

```
Integral((d + e*x)**2/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)**2),
 x)
```

Maxima [F(-2)]

Exception generated.

$$
\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx=\text{Exception raised: ValueError}
$$

input

```
integrate((e*x+d)^2/(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((4*c*e^2>0)', see `assume?` for
more detai
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{(f+g\,x)^2\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),
x)
```

output
```
int((d + e*x)^2/((f + g*x)^2*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),
 x)
```

Reduce [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 6230, normalized size of antiderivative = 25.85

$$\int \frac{(d+ex)^2}{(f+gx)^2\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^2/(g*x+f)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*(4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b
**3*e**4*f*g**2 + 4*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b
*e + 2*c*d))*b**3*e**4*g**3*x - 16*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*
x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d*e**3*f*g**2 - 16*sqrt(c)*asinh((sqrt(
 - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*d*e**3*g**3*x - 8*sq
rt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b**2*c*e*
*4*f**2*g - 8*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2
*c*d))*b**2*c*e**4*f*g**2*x + 20*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)
*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e**2*f*g**2 + 20*sqrt(c)*asinh((sqrt
( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*d**2*e**2*g**3*x +
24*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c*
*2*d*e**3*f**2*g + 24*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( -
 b*e + 2*c*d))*b*c**2*d*e**3*f*g**2*x + 4*sqrt(c)*asinh((sqrt( - b*e + c*d
 - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*e**4*f**3 + 4*sqrt(c)*asinh((sqr
t( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*b*c**2*e**4*f**2*g*x - 8*
sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d*
*3*e*f*g**2 - 8*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)*i)/sqrt( - b*e +
 2*c*d))*c**3*d**3*e*g**3*x - 16*sqrt(c)*asinh((sqrt( - b*e + c*d - c*e*x)
*i)/sqrt( - b*e + 2*c*d))*c**3*d**2*e**2*f**2*g - 16*sqrt(c)*asinh((sqrt(
- b*e + c*d - c*e*x)*i)/sqrt( - b*e + 2*c*d))*c**3*d**2*e**2*f*g**2*x -...
```

3.280 $\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 265

$$
\begin{aligned}
\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx
&= \frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2g(cef+cdg-beg)(f+gx)^2} \\
&\quad + \frac{e(5beg-2c(ef+4dg))\sqrt{d(cd-be)-be^2x-ce^2x^2}}{4g(cef+cdg-beg)^2(f+gx)} \\
&\quad + \frac{3e^2(2cd-be)^2\arctan\left(\frac{\sqrt{cef+cdg-beg}(d+ex)}{\sqrt{ef-dg}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{4\sqrt{ef-dg}(cef+cdg-beg)^{5/2}}
\end{aligned}
$$

output

```
1/2*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/g/(-b*e*g+c*d*g+c*e*
f)/(g*x+f)^2+1/4*e*(5*b*e*g-2*c*(4*d*g+e*f))*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x
^2)^(1/2)/g/(-b*e*g+c*d*g+c*e*f)^2/(g*x+f)+3/4*e^2*(-b*e+2*c*d)^2*arctan((
-b*e*g+c*d*g+c*e*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c
*e^2*x^2)^(1/2))/(-d*g+e*f)^(1/2)/(-b*e*g+c*d*g+c*e*f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\frac{e^2(-2cd+be)^2\left(\frac{(d+ex)(-cd+be+cex)\left(-be(3ef+2dg+5egx)+2c\left(d^2g+e^2fx+4de(f+gx)\right)\right)}{e^2(-2cd+be)^2(cef+cdg-beg)^2(f+gx)^2}-\frac{3\sqrt{d+ex}\sqrt{cd-be-cex}\arctan\left(\frac{\sqrt{ef-dg}}{\sqrt{cef+c}}\right.}{\sqrt{ef-dg}(cef+cdg-beg)^{5}}\right.}{4\sqrt{(d+ex)(-be+c(d-ex))}}$$

input

```
Integrate[(d + e*x)^2/((f + g*x)^3*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^
2]),x]
```

output

```
(e^2*(-2*c*d + b*e)^2*(((d + e*x)*(-(c*d) + b*e + c*e*x)*(-(b*e*(3*e*f + 2
*d*g + 5*e*g*x)) + 2*c*(d^2*g + e^2*f*x + 4*d*e*(f + g*x))))/(e^2*(-2*c*d
+ b*e)^2*(c*e*f + c*d*g - b*e*g)^2*(f + g*x)^2) - (3*Sqrt[d + e*x]*Sqrt[c*
d - b*e - c*e*x]*ArcTan[(Sqrt[e*f - d*g]*Sqrt[c*d - b*e - c*e*x])/(Sqrt[c*
e*f + c*d*g - b*e*g]*Sqrt[d + e*x])])/(Sqrt[e*f - d*g]*(c*e*f + c*d*g - b*
e*g)^(5/2))))/(4*Sqrt[(d + e*x)*(-(b*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {1266, 27, 1228, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{-bde-be^2x+cd^2-ce^2x^2}}\,dx$$
$$\Big\downarrow \ 1266$$
$$\frac{\int \frac{e(ef-dg)\left(8cgd^2+be(ef-5dg)+2e(cef+3cdg-2beg)x\right)}{2g(f+gx)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2(ef-dg)(-beg+cdg+cef)}+\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2g(f+gx)^2(-beg+cdg+cef)}$$
$$\Big\downarrow \ 27$$

$$\frac{e\int \frac{8cgd^2+be(ef-5dg)+2e(cef+3cdg-2beg)x}{(f+gx)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4g(-beg+cdg+cef)}+\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2g(f+gx)^2(-beg+cdg+cef)}$$

$$\Big\downarrow 1228$$

$$\frac{e\left(\frac{3eg(2cd-be)^2\int \frac{1}{(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2(-beg+cdg+cef)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(-5beg+8cdg+2cef)}{(f+gx)(-beg+cdg+cef)}\right)}{4g(-beg+cdg+cef)}+$$
$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2g(f+gx)^2(-beg+cdg+cef)}$$

$$\Big\downarrow 1154$$

$$\frac{e\left(-\frac{3eg(2cd-be)^2\int \frac{1}{-\frac{\left(2cgd^2+be(ef-2dg)+e^2(2cf-bg)x\right)^2}{-cx^2e^2-bxe^2+d(cd-be)}-4(ef-dg)(cef+cdg-beg)}d\frac{2cgd^2+be(ef-2dg)+e^2(2cf-bg)x}{\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}}{-beg+cdg+cef}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x}}{(f+gx)(-beg}\right.}{4g(-beg+cdg+cef)}$$
$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2g(f+gx)^2(-beg+cdg+cef)}$$

$$\Big\downarrow 217$$

$$\frac{e\left(\frac{3eg(2cd-be)^2\arctan\left(\frac{e^2x(2cf-bg)+be(ef-2dg)+2cd^2g}{2\sqrt{ef-dg}\sqrt{d(cd-be)-be^2x-ce^2x^2}\sqrt{-beg+cdg+cef}}\right)}{2\sqrt{ef-dg}(-beg+cdg+cef)^{3/2}}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(-5beg+8cdg+2cef)}{(f+gx)(-beg+cdg+cef)}\right)}{4g(-beg+cdg+cef)}+$$
$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{2g(f+gx)^2(-beg+cdg+cef)}$$

input

```
Int[(d + e*x)^2/((f + g*x)^3*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output

```
((e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(2*g*(c*e*f + c*d*
g - b*e*g)*(f + g*x)^2) + (e*(-(((2*c*e*f + 8*c*d*g - 5*b*e*g)*Sqrt[d*(c*d
 - b*e) - b*e^2*x - c*e^2*x^2])/((c*e*f + c*d*g - b*e*g)*(f + g*x))) + (3*
e*(2*c*d - b*e)^2*g*ArcTan[(2*c*d^2*g + b*e*(e*f - 2*d*g) + e^2*(2*c*f - b
*g)*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*e*f + c*d*g - b*e*g]*Sqrt[d*(c*d - b*e) -
 b*e^2*x - c*e^2*x^2])])/(2*Sqrt[e*f - d*g]*(c*e*f + c*d*g - b*e*g)^(3/2))
))/(4*g*(c*e*f + c*d*g - b*e*g))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1266
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^
n, d + e*x, x], R = PolynomialRemainder[(f + g*x)^n, d + e*x, x]}, Simp[(e*
R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*
e^2)), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1
)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R
*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 1] && ILtQ[m, -1] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && (NeQ[m + n, 0] || EqQ[p, -2^(-1)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1560 vs. 2(245) = 490.

Time = 2.57 (sec) , antiderivative size = 1561, normalized size of antiderivative = 5.89

method	result	size
default	Expression too large to display	1561

input

```
int((e*x+d)^2/(g*x+f)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_R
ETURNVERBOSE)
```

output

```
-e^2/g^3/(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*ln((-2*(b*
d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2-e^2*(b*g-2*c*f)/g*(x+f/g)+2*(-(
b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2*c*e^2-e^2*
(b*g-2*c*f)/g*(x+f/g)-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)
)/(x+f/g))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/g^5*(1/2/(b*d*e*g^2-b*e^2*f*g-c*d^2
*g^2+c*e^2*f^2)*g^2/(x+f/g)^2*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-
(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)-3/4*e^2*(b*g-2*c*f)*g
/(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)*(1/(b*d*e*g^2-b*e^2*f*g-c*d^2*g
^2+c*e^2*f^2)*g^2/(x+f/g)*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-(b*d
*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)+1/2*e^2*(b*g-2*c*f)*g/(b*
d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*
e^2*f^2)/g^2)^(1/2)*ln((-2*(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2-e
^2*(b*g-2*c*f)/g*(x+f/g)+2*(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2
)^(1/2)*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-(b*d*e*g^2-b*e^2*f*g-c
*d^2*g^2+c*e^2*f^2)/g^2)^(1/2))/(x+f/g)))+1/2*c*e^2/(b*d*e*g^2-b*e^2*f*g-c
*d^2*g^2+c*e^2*f^2)*g^2/(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(
1/2)*ln((-2*(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2-e^2*(b*g-2*c*f)/
g*(x+f/g)+2*(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*(-(x+f/
g)^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*
f^2)/g^2)^(1/2))/(x+f/g)))+2*(d*g-e*f)*e/g^4*(1/(b*d*e*g^2-b*e^2*f*g-c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. $2(245) = 490$.

Time = 4.48 (sec) , antiderivative size = 2302, normalized size of antiderivative = 8.69

$$\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(g*x+f)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="fricas")
```

output
```
[-1/16*(3*sqrt(-c*e^2*f^2 + b*e^2*f*g + (c*d^2 - b*d*e)*g^2)*((4*c^2*d^2*e
^2 - 4*b*c*d*e^3 + b^2*e^4)*g^2*x^2 + 2*(4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2
*e^4)*f*g*x + (4*c^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*f^2)*log(-((4*c^2*d^
2*e^2 - 4*b*c*d*e^3 - b^2*e^4)*f^2 - 8*(b*c*d^2*e^2 - b^2*d*e^3)*f*g - 8*(
c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*g^2 - (8*c^2*e^4*f^2 - 8*b*c*e^4*f*g
- (4*c^2*d^2*e^2 - 4*b*c*d*e^3 - b^2*e^4)*g^2)*x^2 + 4*sqrt(-c*e^2*f^2 + b
*e^2*f*g + (c*d^2 - b*d*e)*g^2)*sqrt(-c*e^2*x^2 - b*e^2*x + c*d^2 - b*d*e)
*(b*e^2*f + 2*(c*d^2 - b*d*e)*g + (2*c*e^2*f - b*e^2*g)*x) - 2*(4*b*c*e^4*
f^2 + (4*c^2*d^2*e^2 - 4*b*c*d*e^3 - 3*b^2*e^4)*f*g - 4*(b*c*d^2*e^2 - b^2
*d*e^3)*g^2)*x)/(g^2*x^2 + 2*f*g*x + f^2)) + 4*sqrt(-c*e^2*x^2 - b*e^2*x +
 c*d^2 - b*d*e)*((8*c^2*d*e^3 - 3*b*c*e^4)*f^3 + (2*c^2*d^2*e^2 - 10*b*c*d
*e^3 + 3*b^2*e^4)*f^2*g - (8*c^2*d^3*e - 9*b*c*d^2*e^2 + b^2*d*e^3)*f*g^2
- 2*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*g^3 + (2*c^2*e^4*f^3 + (8*c^2*d*
e^3 - 7*b*c*e^4)*f^2*g - (2*c^2*d^2*e^2 + 6*b*c*d*e^3 - 5*b^2*e^4)*f*g^2 -
 (8*c^2*d^3*e - 13*b*c*d^2*e^2 + 5*b^2*d*e^3)*g^3)*x))/(c^3*e^4*f^6 + (2*c
^3*d*e^3 - 3*b*c^2*e^4)*f^5*g - 3*(b*c^2*d*e^3 - b^2*c*e^4)*f^4*g^2 - (2*c
^3*d^3*e - 3*b*c^2*d^2*e^2 + b^3*e^4)*f^3*g^3 - (c^3*d^4 - 3*b*c^2*d^3*e +
 3*b^2*c*d^2*e^2 - b^3*d*e^3)*f^2*g^4 + (c^3*e^4*f^4*g^2 + (2*c^3*d*e^3 -
3*b*c^2*e^4)*f^3*g^3 - 3*(b*c^2*d*e^3 - b^2*c*e^4)*f^2*g^4 - (2*c^3*d^3*e
- 3*b*c^2*d^2*e^2 + b^3*e^4)*f*g^5 - (c^3*d^4 - 3*b*c^2*d^3*e + 3*b^2*c...
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\int \frac{(d+ex)^2}{\sqrt{-(d+ex)\,(be-cd+cex)}\,(f+gx)^3}\,dx$$

input
```
integrate((e*x+d)**2/(g*x+f)**3/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2
),x)
```

output
```
Integral((d + e*x)**2/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)**3),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(g*x+f)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((d*g-e*f)>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4728 vs. $2(245) = 490$.

Time = 11.78 (sec) , antiderivative size = 4728, normalized size of antiderivative = 17.84

$$\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2/(g*x+f)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="giac")
```

output

```
1/4*(48*c^4*d^4*e^2*f^2*g^2*arctan((sqrt(-c*e^2)*f - sqrt(c*d^2 - b*d*e)*g
)/sqrt(c*e^2*f^2 - b*e^2*f*g - c*d^2*g^2 + b*d*e*g^2)) - 96*b*c^3*d^3*e^3*
f^2*g^2*arctan((sqrt(-c*e^2)*f - sqrt(c*d^2 - b*d*e)*g)/sqrt(c*e^2*f^2 - b
*e^2*f*g - c*d^2*g^2 + b*d*e*g^2)) + 48*b^2*c^2*d^2*e^4*f^2*g^2*arctan((sq
rt(-c*e^2)*f - sqrt(c*d^2 - b*d*e)*g)/sqrt(c*e^2*f^2 - b*e^2*f*g - c*d^2*g
^2 + b*d*e*g^2)) - 3*b^4*e^6*f^2*g^2*arctan((sqrt(-c*e^2)*f - sqrt(c*d^2 -
 b*d*e)*g)/sqrt(c*e^2*f^2 - b*e^2*f*g - c*d^2*g^2 + b*d*e*g^2)) - 48*sqrt(
c*d^2 - b*d*e)*sqrt(-c*e^2)*b*c^2*d^2*e^2*f^2*g^2*arctan((sqrt(-c*e^2)*f -
 sqrt(c*d^2 - b*d*e)*g)/sqrt(c*e^2*f^2 - b*e^2*f*g - c*d^2*g^2 + b*d*e*g^2
)) + 48*sqrt(c*d^2 - b*d*e)*sqrt(-c*e^2)*b^2*c*d*e^3*f^2*g^2*arctan((sqrt(
-c*e^2)*f - sqrt(c*d^2 - b*d*e)*g)/sqrt(c*e^2*f^2 - b*e^2*f*g - c*d^2*g^2
+ b*d*e*g^2)) - 12*sqrt(c*d^2 - b*d*e)*sqrt(-c*e^2)*b^3*e^4*f^2*g^2*arctan
((sqrt(-c*e^2)*f - sqrt(c*d^2 - b*d*e)*g)/sqrt(c*e^2*f^2 - b*e^2*f*g - c*d
^2*g^2 + b*d*e*g^2)) + 8*sqrt(c*e^2*f^2 - b*e^2*f*g - c*d^2*g^2 + b*d*e*g^
2)*sqrt(-c*e^2)*c^3*d^2*e^2*f^3 - 8*sqrt(c*e^2*f^2 - b*e^2*f*g - c*d^2*g^2
 + b*d*e*g^2)*sqrt(-c*e^2)*b*c^2*d*e^3*f^3 - 2*sqrt(c*e^2*f^2 - b*e^2*f*g
- c*d^2*g^2 + b*d*e*g^2)*sqrt(-c*e^2)*b^2*c*e^4*f^3 + 8*sqrt(c*e^2*f^2 - b
*e^2*f*g - c*d^2*g^2 + b*d*e*g^2)*sqrt(c*d^2 - b*d*e)*b*c^2*e^4*f^3 + 32*s
qrt(c*e^2*f^2 - b*e^2*f*g - c*d^2*g^2 + b*d*e*g^2)*sqrt(-c*e^2)*c^3*d^3*e*
f^2*g - 52*sqrt(c*e^2*f^2 - b*e^2*f*g - c*d^2*g^2 + b*d*e*g^2)*sqrt(-c*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\int \frac{(d+e\,x)^2}{(f+g\,x)^3\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),
x)
```

output
```
int((d + e*x)^2/((f + g*x)^3*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),
 x)
```

Reduce [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 15967, normalized size of antiderivative = 60.25

$$\int \frac{(d+ex)^2}{(f+gx)^3\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/(g*x+f)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*(4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c
*d - c*e*x)*b**3*d**2*e**2*g**5 + 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
- b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*d*e**3*f*g**4 + 10*sqrt(d +
 e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*
*3*d*e**3*g**5*x - 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*
sqrt( - b*e + c*d - c*e*x)*b**3*e**4*f**2*g**3 - 10*sqrt(d + e*x)*sqrt(b*e
 - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*e**4*f*g**4
*x - 8*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b**2*c*d**3*e*g**5 - 26*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt(
 - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*d**2*e**2*f*g**4 - 26*sq
rt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e
*x)*b**2*c*d**2*e**2*g**5*x + 16*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b
*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*d*e**3*f**2*g**3 - 8*sqrt(d
+ e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b
**2*c*d*e**3*f*g**4*x + 18*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2
*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*e**4*f**3*g**2 + 34*sqrt(d + e*x)*
sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**2*c*e
**4*f**2*g**3*x + 4*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*s
qrt( - b*e + c*d - c*e*x)*b*c**2*d**4*g**5 + 32*sqrt(d + e*x)*sqrt(b*e - 2
*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b*c**2*d**3*e*f*g...
```

3.281 $\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$

Optimal result

Integrand size $=46$, antiderivative size $=410$

$$\begin{aligned}\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx &= \frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3g(cef+cdg-beg)(f+gx)^3}\\ &+ \frac{e(7beg-2c(ef+6dg))\sqrt{d(cd-be)-be^2x-ce^2x^2}}{12g(cef+cdg-beg)^2(f+gx)^2}\\ &- \frac{e^2(3b^2e^2g^2+4bceg(4ef-7dg)-4c^2(e^2f^2+6defg-10d^2g^2))\sqrt{d(cd-be)-be^2x-ce^2x^2}}{24g(ef-dg)(beg-c(ef+dg))^3(f+gx)}\\ &+ \frac{e^3(2cd-be)^2(6cef-4cdg-beg)\arctan\left(\frac{\sqrt{cef+cdg-beg}\,(d+ex)}{\sqrt{ef-dg}\sqrt{d(cd-be)-be^2x-ce^2x^2}}\right)}{8(ef-dg)^{3/2}(cef+cdg-beg)^{7/2}}\end{aligned}$$

output
```
1/3*(-d*g+e*f)*(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2)/g/(-b*e*g+c*d*g+c*e*
f)/(g*x+f)^3+1/12*e*(7*b*e*g-2*c*(6*d*g+e*f))*(d*(-b*e+c*d)-b*e^2*x-c*e^2*
x^2)^(1/2)/g/(-b*e*g+c*d*g+c*e*f)^2/(g*x+f)^2-1/24*e^2*(3*b^2*e^2*g^2+4*b*
c*e*g*(-7*d*g+4*e*f)-4*c^2*(-10*d^2*g^2+6*d*e*f*g+e^2*f^2))*(d*(-b*e+c*d)-
b*e^2*x-c*e^2*x^2)^(1/2)/g/(-d*g+e*f)/(b*e*g-c*(d*g+e*f))^3/(g*x+f)+1/8*e^
3*(-b*e+2*c*d)^2*(-b*e*g-4*c*d*g+6*c*e*f)*arctan((-b*e*g+c*d*g+c*e*f)^(1/2
)*(e*x+d)/(-d*g+e*f)^(1/2)/(d*(-b*e+c*d)-b*e^2*x-c*e^2*x^2)^(1/2))/(-d*g+e
*f)^(3/2)/(-b*e*g+c*d*g+c*e*f)^(7/2)
```

Mathematica [A] (verified)

Time = 4.35 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$= \frac{e^3(-2cd+be)^2 \left(\frac{(d+ex)(-cd+be+cex)\left(b^2e^2g\left(-8d^2g^2+2deg(f-7gx)+e^2\left(3f^2+8fgx-3g^2x^2\right)\right)-2bce\left(-8d^3g^3-d^2eg^2(11f+19gx)-2d\right.\right.}{e^3(-}\right.}{}$$

input

```
Integrate[(d + e*x)^2/((f + g*x)^4*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^
2]),x]
```

output

```
(e^3*(-2*c*d + b*e)^2*(((d + e*x)*(-(c*d) + b*e + c*e*x)*(b^2*e^2*g*(-8*d^
2*g^2 + 2*d*e*g*(f - 7*g*x) + e^2*(3*f^2 + 8*f*g*x - 3*g^2*x^2)) - 2*b*c*e
*(-8*d^3*g^3 - d^2*e*g^2*(11*f + 19*g*x) - 2*d*e^2*g*(-2*f^2 + 9*f*g*x + 7
*g^2*x^2) + e^3*f*(9*f^2 + 25*f*g*x + 8*g^2*x^2)) - 4*c^2*(2*d^4*g^3 + 6*d
^3*e*g^2*(f + g*x) - e^4*f^2*x*(3*f + g*x) - 6*d*e^3*f*(2*f^2 + 3*f*g*x +
g^2*x^2) + d^2*e^2*g*(7*f^2 + 21*f*g*x + 10*g^2*x^2))))/(e^3*(-2*c*d + b*e
)^2*(e*f - d*g)*(c*e*f + c*d*g - b*e*g)^3*(f + g*x)^3) - (3*(6*c*e*f - 4*c
*d*g - b*e*g)*Sqrt[d + e*x]*Sqrt[c*d - b*e - c*e*x]*ArcTan[(Sqrt[e*f - d*g
]*Sqrt[c*d - b*e - c*e*x])/(Sqrt[c*e*f + c*d*g - b*e*g]*Sqrt[d + e*x])])/(
(e*f - d*g)^(3/2)*(c*e*f + c*d*g - b*e*g)^(7/2))))/(24*Sqrt[(d + e*x)*(-(b
*e) + c*(d - e*x))])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1266, 27, 1237, 27, 1228, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{-bde-be^2x+cd^2-ce^2x^2}}\,dx$$
$$\Big\downarrow \; 1266$$

$$\frac{\int \frac{e(ef-dg)(12cgd^2+be(ef-7dg)+2e(cef+5cdg-3beg)x)}{2g(f+gx)^3\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{3(ef-dg)(-beg+cdg+cef)}+\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3g(f+gx)^3(-beg+cdg+cef)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int \frac{12cgd^2+be(ef-7dg)+2e(cef+5cdg-3beg)x}{(f+gx)^3\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{6g(-beg+cdg+cef)}+\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3g(f+gx)^3(-beg+cdg+cef)}$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{e\left(\frac{\int \frac{e(ef-dg)\left(40c^2gd^2+3b^2e^2g+2bce(ef-14dg)+2ce(2cef+12cdg-7beg)x\right)}{2(f+gx)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2(ef-dg)(-beg+cdg+cef)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(-7beg+12cdg+2cef)}{2(f+gx)^2(-beg+cdg+cef)}\right)}{6g(-beg+cdg+cef)}+$$
$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3g(f+gx)^3(-beg+cdg+cef)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\left(\frac{e\int \frac{40c^2gd^2+3b^2e^2g+2bce(ef-14dg)+2ce(2cef+12cdg-7beg)x}{(f+gx)^2\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{4(-beg+cdg+cef)}-\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}(-7beg+12cdg+2cef)}{2(f+gx)^2(-beg+cdg+cef)}\right)}{6g(-beg+cdg+cef)}+$$
$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3g(f+gx)^3(-beg+cdg+cef)}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{e\Bigg(\frac{e\Big(\frac{3eg(2cd-be)^2(-beg-4cdg+6cef)\int \frac{1}{(f+gx)\sqrt{-cx^2e^2-bxe^2+d(cd-be)}}dx}{2(ef-dg)(-beg+cdg+cef)}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}\big(3b^2e^2g^2+4bceg(4ef-7dg)-4c^2\big(-10d^2g^2+6defg}{(f+gx)(ef-dg)(-beg+cdg+cef)}}{4(-beg+cdg+cef)}}{6g(-beg+cdg+cef)}$$
$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3g(f+gx)^3(-beg+cdg+cef)}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{e\left(\frac{e\left(\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}\left(3b^2e^2g^2+4bceg(4ef-7dg)-4c^2\left(-10d^2g^2+6defg+e^2f^2\right)\right)}{(f+gx)(ef-dg)(-beg+cdg+cef)}-\frac{3eg(2cd-be)^2(-beg-4cdg+6cef)\int\frac{-\frac{\left(2cgd^2+be(ef-2dg)+e^2(\right.}{-cx^2e^2-bxe^2+d(c}}{}}{(ef-dg}\right.}{4(-beg+cdg+cef)}\right.}{6g(-beg+cdg+c}$$

$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3g(f+gx)^3(-beg+cdg+cef)}$$

$$\Big\downarrow \text{217}$$

$$\frac{e\left(\frac{e\left(\frac{3eg(2cd-be)^2(-beg-4cdg+6cef)\arctan\left(\frac{e^2x(2cf-bg)+be(ef-2dg)+2cd^2g}{2\sqrt{ef-dg}\sqrt{d(cd-be)-be^2x-ce^2x^2}\sqrt{-beg+cdg+cef}}\right)}{2(ef-dg)^{3/2}(-beg+cdg+cef)^{3/2}}+\frac{\sqrt{d(cd-be)-be^2x-ce^2x^2}\left(3b^2e^2g^2+4bceg(4ef-7dg)\right.}{(f+gx)(ef-dg)(-beg+cdg}\right.}{4(-beg+cdg+cef)}\right.}{6g(-beg+cdg+cef)}$$

$$\frac{(ef-dg)\sqrt{d(cd-be)-be^2x-ce^2x^2}}{3g(f+gx)^3(-beg+cdg+cef)}$$

input

```
Int[(d + e*x)^2/((f + g*x)^4*Sqrt[c*d^2 - b*d*e - b*e^2*x - c*e^2*x^2]),x]
```

output

```
((e*f - d*g)*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/(3*g*(c*e*f + c*d*
g - b*e*g)*(f + g*x)^3) + (e*(-1/2*((2*c*e*f + 12*c*d*g - 7*b*e*g)*Sqrt[d*
(c*d - b*e) - b*e^2*x - c*e^2*x^2])/((c*e*f + c*d*g - b*e*g)*(f + g*x)^2)
 + (e*(((3*b^2*e^2*g^2 + 4*b*c*e*g*(4*e*f - 7*d*g) - 4*c^2*(e^2*f^2 + 6*d*e
*f*g - 10*d^2*g^2))*Sqrt[d*(c*d - b*e) - b*e^2*x - c*e^2*x^2])/((e*f - d*g
)*(c*e*f + c*d*g - b*e*g)*(f + g*x)) + (3*e*(2*c*d - b*e)^2*g*(6*c*e*f - 4
*c*d*g - b*e*g)*ArcTan[(2*c*d^2*g + b*e*(e*f - 2*d*g) + e^2*(2*c*f - b*g)*
x)/(2*Sqrt[e*f - d*g]*Sqrt[c*e*f + c*d*g - b*e*g]*Sqrt[d*(c*d - b*e) - b*e
^2*x - c*e^2*x^2])])/(2*(e*f - d*g)^(3/2)*(c*e*f + c*d*g - b*e*g)^(3/2)))
/(4*(c*e*f + c*d*g - b*e*g))))/(6*g*(c*e*f + c*d*g - b*e*g))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1266
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)^
n, d + e*x, x], R = PolynomialRemainder[(f + g*x)^n, d + e*x, x]}, Simp[(e*
R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*
e^2)), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1
)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R
*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 1] && ILtQ[m, -1] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && (NeQ[m + n, 0] || EqQ[p, -2^(-1)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2825 vs. $2(386) = 772$.

Time = 3.23 (sec) , antiderivative size = 2826, normalized size of antiderivative = 6.89

method	result	size
default	Expression too large to display	2826

input
```
int((e*x+d)^2/(g*x+f)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x,method=_R
ETURNVERBOSE)
```

output

```
e^2/g^4*(1/(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)*g^2/(x+f/g)*(-(x+f/g)
^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^
2)/g^2)^(1/2)+1/2*e^2*(b*g-2*c*f)*g/(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f
^2)/(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*ln((-2*(b*d*e*g
^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2-e^2*(b*g-2*c*f)/g*(x+f/g)+2*(-(b*d*e
*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2*c*e^2-e^2*(b*g-
2*c*f)/g*(x+f/g)-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2))/(x+
f/g)))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/g^6*(1/3/(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2
+c*e^2*f^2)*g^2/(x+f/g)^3*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-(b*d
*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)-5/6*e^2*(b*g-2*c*f)*g/(b*
d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)*(1/2/(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2
+c*e^2*f^2)*g^2/(x+f/g)^2*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-(b*d
*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)-3/4*e^2*(b*g-2*c*f)*g/(b*
d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)*(1/(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c
*e^2*f^2)*g^2/(x+f/g)*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-(b*d*e*g
^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1/2)+1/2*e^2*(b*g-2*c*f)*g/(b*d*e*
g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*
f^2)/g^2)^(1/2)*ln((-2*(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2-e^2*(
b*g-2*c*f)/g*(x+f/g)+2*(-(b*d*e*g^2-b*e^2*f*g-c*d^2*g^2+c*e^2*f^2)/g^2)^(1
/2)*(-(x+f/g)^2*c*e^2-e^2*(b*g-2*c*f)/g*(x+f/g)-(b*d*e*g^2-b*e^2*f*g-c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2501 vs. $2(386) = 772$.

Time $= 53.14$ (sec) , antiderivative size $= 5060$, normalized size of antiderivative $= 12.34$

$$\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2/(g*x+f)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="fricas")
```

output

```
Too large to include
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx$$
$$=\int \frac{(d+ex)^2}{\sqrt{-(d+ex)\,(be-cd+cex)}\,(f+gx)^4}\,dx$$

input
```
integrate((e*x+d)**2/(g*x+f)**4/(-c*e**2*x**2-b*e**2*x-b*d*e+c*d**2)**(1/2
),x)
```

output
```
Integral((d + e*x)**2/(sqrt(-(d + e*x)*(b*e - c*d + c*e*x))*(f + g*x)**4),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(g*x+f)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((d*g-e*f)>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^2/(g*x+f)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \int \frac{(d+e\,x)^2}{(f+g\,x)^4\,\sqrt{c\,d^2-b\,d\,e-c\,e^2\,x^2-b\,e^2\,x}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)^4*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),
x)
```

output
```
int((d + e*x)^2/((f + g*x)^4*(c*d^2 - c*e^2*x^2 - b*d*e - b*e^2*x)^(1/2)),
 x)
```

Reduce [B] (verification not implemented)

Time = 32.37 (sec) , antiderivative size = 38018, normalized size of antiderivative = 92.73

$$\int \frac{(d+ex)^2}{(f+gx)^4\sqrt{cd^2-bde-be^2x-ce^2x^2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/(g*x+f)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x)
```

output

```
(i*(16*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
c*d - c*e*x)*b**4*d**3*e**3*g**8 - 20*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt
( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*d**2*e**4*f*g**7 + 28*sqr
t(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*
x)*b**4*d**2*e**4*g**8*x - 2*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e +
 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*d*e**5*f**2*g**6 - 44*sqrt(d + e*x
)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*d
*e**5*f*g**7*x + 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sq
rt( - b*e + c*d - c*e*x)*b**4*d*e**5*g**8*x**2 + 6*sqrt(d + e*x)*sqrt(b*e
- 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**6*f**3*g*
*5 + 16*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e +
 c*d - c*e*x)*b**4*e**6*f**2*g**6*x - 6*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sq
rt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**4*e**6*f*g**7*x**2 - 48*s
qrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*
e*x)*b**3*c*d**4*e**2*g**8 - 40*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*
e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*d**3*e**3*f*g**7 - 104*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*x)*
b**3*c*d**3*e**3*g**8*x + 122*sqrt(d + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e
+ 2*c*d)*sqrt( - b*e + c*d - c*e*x)*b**3*c*d**2*e**4*f**2*g**6 - 36*sqrt(d
 + e*x)*sqrt(b*e - 2*c*d)*sqrt( - b*e + 2*c*d)*sqrt( - b*e + c*d - c*e*...
```

3.282 $\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 453

$$\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(cdf-aeg)^3(d+ex)}{c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{g\left(57a^2e^4g^2-2acde^2g(63ef+5dg)+3c^2d^2\left(24e^2f^2+6defg-d^2g^2\right)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24c^4d^4e^2}$$
$$-\frac{g^2\left(11ae^2g-cd(18ef+dg)\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12c^3d^3e}$$
$$+\frac{g^3x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2}$$
$$-\frac{(35a^3e^6g^3-15a^2cde^4g^2(6ef+dg)+3ac^2d^2e^2g\left(24e^2f^2+12defg-d^2g^2\right)-c^3d^3(16e^3f^3+24de^2f^2g-}{8c^{9/2}d^{9/2}e^{5/2}}$$

output
```
-2*(-a*e*g+c*d*f)^3*(e*x+d)/c^4/d^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)+1/24*g*(57*a^2*e^4*g^2-2*a*c*d*e^2*g*(5*d*g+63*e*f)+3*c^2*d^2*(-d^2*g^2+
6*d*e*f*g+24*e^2*f^2))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^2
-1/12*g^2*(11*a*e^2*g-c*d*(d*g+18*e*f))*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/c^3/d^3/e+1/3*g^3*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/
d^2-1/8*(35*a^3*e^6*g^3-15*a^2*c*d*e^4*g^2*(d*g+6*e*f)+3*a*c^2*d^2*e^2*g*(
-d^2*g^2+12*d*e*f*g+24*e^2*f^2)-c^3*d^3*(d^3*g^3-6*d^2*e*f*g^2+24*d*e^2*f^
2*g+16*e^3*f^3))*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)\left(-105a^3e^5g^3+5a^2cde^3g^2(54ef+2dg-7eg\right.}{}$$

input

```
Integrate[((d + e*x)^2*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2),x]
```

output

```
(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-105*a^3*e^5*g^3 + 5*a^2*c*d*e^3*g^2
*(54*e*f + 2*d*g - 7*e*g*x) + a*c^2*d^2*e*g*(3*d^2*g^2 + 2*d*e*g*(-9*f + 4
*g*x) + e^2*(-216*f^2 + 90*f*g*x + 14*g^2*x^2)) + c^3*d^3*(3*d^2*g^3*x - 2
*d*e*g^2*x*(9*f + g*x) + 4*e^2*(12*f^3 - 18*f^2*g*x - 9*f*g^2*x^2 - 2*g^3*
x^3)))) + 3*(-35*a^3*e^6*g^3 + 15*a^2*c*d*e^4*g^2*(6*e*f + d*g) + 3*a*c^2*
d^2*e^2*g*(-24*e^2*f^2 - 12*d*e*f*g + d^2*g^2) + c^3*d^3*(16*e^3*f^3 + 24*
d*e^2*f^2*g - 6*d^2*e*f*g^2 + d^3*g^3))*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Ar
cTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(24*c^
(9/2)*d^(9/2)*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.205, Rules used = {1211, 25, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)^3}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int -\frac{-c^3d^3g^3x^3e^6+c^2d^2g^2\left(ae^2g-cd(3ef+dg)\right)x^2e^5+\left(a^3g^3e^4-a^2cdg^2(3ef+dg)e^2+3ac^2d^2fg(ef+dg)e-c^3d^3f^2(ef+3dg)\right)e^5-cdg\left(a^2g^2e^3-ac}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{c^4d^4e^5}$$

$$\frac{2(d+ex)(cdf-aeg)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 25$$

$$-\frac{\int \frac{-c^3d^3g^3x^3e^6+c^2d^2g^2\left(ae^2g-cd(3ef+dg)\right)x^2e^5+\left(a^3g^3e^4-a^2cdg^2(3ef+dg)e^2+3ac^2d^2fg(ef+dg)e-c^3d^3f^2(ef+3dg)\right)e^5-cdg\left(a^2g^2e^3-ac}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{c^4d^4e^5}$$

$$\frac{2(d+ex)(cdf-aeg)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 2192$$

$$-\frac{\frac{\int \frac{c^3d^3g^2\left(11ae^2g-cd(18ef+dg)\right)x^2e^6+6cd\left(a^3g^3e^4-a^2cdg^2(3ef+dg)e^2+3ac^2d^2fg(ef+dg)e-c^3d^3f^2(ef+3dg)\right)e^6-2c^2d^2g\left(3a^2g^2e^3-acdg(9ef+5dg)e+9c}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{3cde}}{c^4d^4e^5}$$

$$\frac{2(d+ex)(cdf-aeg)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\int \frac{c^3d^3g^2\left(11ae^2g-cd(18ef+dg)\right)x^2e^6+6cd\left(a^3g^3e^4-a^2cdg^2(3ef+dg)e^2+3ac^2d^2fg(ef+dg)e-c^3d^3f^2(ef+3dg)\right)e^6-2c^2d^2g\left(3a^2g^2e^3-acdg(9ef+5dg)e+9c}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{6cde}}{c^4d^4e^5}$$

$$\frac{2(d+ex)(cdf-aeg)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 2192$$

$$-\frac{\frac{\frac{\int \frac{c^2d^2e^6\left(2e\left(12a^3g^3e^4-a^2cdg^2(36ef+23dg)e^2-12c^3d^3f^2(ef+3dg)+ac^2d^2g\left(36e^2f^2+54degf+d^2g^2\right)\right)-cdg\left(57a^2g^2e^4-2acdg(63ef+5dg)e^2+3c^2d^2\left(24e^2f}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{2cde}}{6cde}}{c}$$

$$\frac{2(d+ex)(cdf-aeg)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\frac{1}{4}cde^5\int \frac{2e\left(12a^3g^3e^4-a^2cdg^2(36ef+23dg)e^2-12c^3d^3f^2(ef+3dg)+ac^2d^2g\left(36e^2f^2+54degf+d^2g^2\right)\right)-cdg\left(57a^2g^2e^4-2acdg(63ef+5dg)e^2+3c^2d^2\left(24e^2f^2\right.\right.}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{6cde}}{c^4}$$

$$\frac{2(d+ex)(cdf-aeg)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1160$$

$$-\frac{\frac{1}{4}cde^5\left(\frac{3\left(35a^3e^6g^3-15a^2cde^4g^2(dg+6ef)+3ac^2d^2e^2g\left(-d^2g^2+12defg+24e^2f^2\right)-c^3d^3\left(d^3g^3-6d^2efg^2+24de^2f^2g+16e^3f^3\right)\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+}}}{2e}\right.}{}$$

$$\frac{2(d+ex)(cdf-aeg)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1092$$

$$-\frac{\frac{1}{4}cde^5\left(\frac{3\left(35a^3e^6g^3-15a^2cde^4g^2(dg+6ef)+3ac^2d^2e^2g\left(-d^2g^2+12defg+24e^2f^2\right)-c^3d^3\left(d^3g^3-6d^2efg^2+24de^2f^2g+16e^3f^3\right)\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+a\right.}{cdex^2+\left(cd^2+ae^2\right.}}}{e}\right.}{}$$

$$\frac{2(d+ex)(cdf-aeg)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{1}{4}cde^5\left(\frac{3\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)\left(35a^3e^6g^3-15a^2cde^4g^2(dg+6ef)+3ac^2d^2e^2g\left(-d^2g^2+12defg+24e^2f^2\right)-c^3d^3\left(d^3g^3-6d^2e\right.\right.}{2\sqrt{c}\sqrt{d}e^{3/2}}\right.}{}$$

$$\frac{2(d+ex)(cdf-aeg)^3}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^2*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
),x]
```

output

```
(-2*(c*d*f - a*e*g)^3*(d + e*x))/(c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) - (-1/3*(c^2*d^2*e^5*g^3*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((c^2*d^2*e^5*g^2*(11*a*e^2*g - c*d*(18*e*f + d*g))*x*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2 + (c*d*e^5*(-((g*(57*a^2*e^4*g^2
 - 2*a*c*d*e^2*g*(63*e*f + 5*d*g) + 3*c^2*d^2*(24*e^2*f^2 + 6*d*e*f*g - d^
2*g^2))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/e) + (3*(35*a^3*e^6*g
^3 - 15*a^2*c*d*e^4*g^2*(6*e*f + d*g) + 3*a*c^2*d^2*e^2*g*(24*e^2*f^2 + 12
*d*e*f*g - d^2*g^2) - c^3*d^3*(16*e^3*f^3 + 24*d*e^2*f^2*g - 6*d^2*e*f*g^2
 + d^3*g^3))*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2
))))/4)/(6*c*d*e))/(c^4*d^4*e^5)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3170 vs. $2(423) = 846$.

Time = 3.31 (sec) , antiderivative size = 3171, normalized size of antiderivative = 7.00

method	result	size
default	Expression too large to display	3171

input
```
int((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_
RETURNVERBOSE)
```

output

```
2*d^2*f^3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+e*g^2*(2*d*g+3*e*f)*(1/2*x^3/d/e/c/(a*d*e+
(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a*e^2+c*d^2)/d/e/c*(x^2/d/e/c/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d
^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))-3/2*a/c*
(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*
(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c
*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+d*f^2*(3*d*g+2*e*
f)*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))+g*(d^2*g^2+6*d*e*f*g+3*e^2*f^2)*(x^2/d/e/c/(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e...
```

Fricas [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 1402, normalized size of antiderivative = 3.09

$$\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="fricas")
```

output
```
[-1/96*(3*(16*a*c^3*d^3*e^4*f^3 + 24*(a*c^3*d^4*e^3 - 3*a^2*c^2*d^2*e^5)*f
^2*g - 6*(a*c^3*d^5*e^2 + 6*a^2*c^2*d^3*e^4 - 15*a^3*c*d*e^6)*f*g^2 + (a*c
^3*d^6*e + 3*a^2*c^2*d^4*e^3 + 15*a^3*c*d^2*e^5 - 35*a^4*e^7)*g^3 + (16*c^
4*d^4*e^3*f^3 + 24*(c^4*d^5*e^2 - 3*a*c^3*d^3*e^4)*f^2*g - 6*(c^4*d^6*e +
6*a*c^3*d^4*e^3 - 15*a^2*c^2*d^2*e^5)*f*g^2 + (c^4*d^7 + 3*a*c^3*d^5*e^2 +
 15*a^2*c^2*d^3*e^4 - 35*a^3*c*d*e^6)*g^3)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^
2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a
*c*d*e^3)*x) - 4*(8*c^4*d^4*e^3*g^3*x^3 - 48*c^4*d^4*e^3*f^3 + 216*a*c^3*d
^3*e^4*f^2*g + 18*(a*c^3*d^4*e^3 - 15*a^2*c^2*d^2*e^5)*f*g^2 - (3*a*c^3*d^
5*e^2 + 10*a^2*c^2*d^3*e^4 - 105*a^3*c*d*e^6)*g^3 + 2*(18*c^4*d^4*e^3*f*g^
2 + (c^4*d^5*e^2 - 7*a*c^3*d^3*e^4)*g^3)*x^2 + (72*c^4*d^4*e^3*f^2*g + 18*
(c^4*d^5*e^2 - 5*a*c^3*d^3*e^4)*f*g^2 - (3*c^4*d^6*e + 8*a*c^3*d^4*e^3 - 3
5*a^2*c^2*d^2*e^5)*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c
^6*d^6*e^3*x + a*c^5*d^5*e^4), -1/48*(3*(16*a*c^3*d^3*e^4*f^3 + 24*(a*c^3*
d^4*e^3 - 3*a^2*c^2*d^2*e^5)*f^2*g - 6*(a*c^3*d^5*e^2 + 6*a^2*c^2*d^3*e^4
- 15*a^3*c*d*e^6)*f*g^2 + (a*c^3*d^6*e + 3*a^2*c^2*d^4*e^3 + 15*a^3*c*d^2*
e^5 - 35*a^4*e^7)*g^3 + (16*c^4*d^4*e^3*f^3 + 24*(c^4*d^5*e^2 - 3*a*c^3*d^
3*e^4)*f^2*g - 6*(c^4*d^6*e + 6*a*c^3*d^4*e^3 - 15*a^2*c^2*d^2*e^5)*f*g^2
+ (c^4*d^7 + 3*a*c^3*d^5*e^2 + 15*a^2*c^2*d^3*e^4 - 35*a^3*c*d*e^6)*g^3...
```

Sympy [F]

$$\int \frac{(d+ex)^2(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+ex)^2\,(f+gx)^3}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

output
```
Integral((d + e*x)**2*(f + g*x)**3/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[5,5,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,0,
0,0]%%%}+
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{\left(f+g\,x\right)^3\,\left(d+e\,x\right)^2}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(((f + g*x)^3*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
),x)
```

output
```
int(((f + g*x)^3*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{\left(ex+d\right)^2\left(gx+f\right)^3}{\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.283 $\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 290

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx &= -\frac{2(cdf-aeg)^2(d+ex)}{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &- \frac{g(7ae^2g-cd(8ef+dg))\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^3d^3e} \\ &+ \frac{g^2x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2c^2d^2} \\ &+ \frac{\left(15a^2e^4g^2-6acde^2g(4ef+dg)+c^2d^2\left(8e^2f^2+8defg-d^2g^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{4c^{7/2}d^{7/2}e^{3/2}}\end{aligned}$$

output

```
-2*(-a*e*g+c*d*f)^2*(e*x+d)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)-1/4*g*(7*a*e^2*g-c*d*(d*g+8*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c^3/d^3/e+1/2*g^2*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+1/4*
(15*a^2*e^4*g^2-6*a*c*d*e^2*g*(d*g+4*e*f)+c^2*d^2*(-d^2*g^2+8*d*e*f*g+8*e^
2*f^2))*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)\left(-15a^2e^3g^2+acdeg(24ef+dg-5egx)+c^2d\right.}{}$$

input
```
Integrate[((d + e*x)^2*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2),x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-15*a^2*e^3*g^2 + a*c*d*e*g*(24*e*f +
d*g - 5*e*g*x) + c^2*d^2*(d*g^2*x + e*(-8*f^2 + 8*f*g*x + 2*g^2*x^2))) - (
-15*a^2*e^4*g^2 + 6*a*c*d*e^2*g*(4*e*f + d*g) + c^2*d^2*(-8*e^2*f^2 - 8*d*
e*f*g + d^2*g^2))*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]
*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(4*c^(7/2)*d^(7/2)*e^(3/2)*S
qrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1211, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{c^2d^2g^2x^2e^4+\left(a^2g^2e^3-acdg(2ef+dg)e+c^2d^2f(ef+2dg)\right)e^3-cdg\left(ae^2g-cd(2ef+dg)\right)xe^3}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^3d^3e^3}-\frac{2(d+ex)(cdf-aeg)^2}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{cde^4\left(2\left(2a^2g^2e^3-acdg(4ef+3dg)e+2c^2d^2f(ef+2dg)\right)-cdg\left(7ae^2g-cd(8ef+dg)\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}cde^3g^2x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^3}-$$

$$\frac{2(d+ex)(cdf-aeg)^2}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}e^3\int \frac{2\left(2a^2g^2e^3-acdg(4ef+3dg)e+2c^2d^2f(ef+2dg)\right)-cdg\left(7ae^2g-cd(8ef+dg)\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}cde^3g^2x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^3}$$

$$\frac{2(d+ex)(cdf-aeg)^2}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{1}{4}e^3\left(\frac{\left(15a^2e^4g^2-6acde^2g(dg+4ef)+c^2d^2\left(-d^2g^2+8defg+8e^2f^2\right)\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}-\frac{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(7ae^2g-cd\right.}{e}\right.}{c^3d^3e^3}$$

$$\frac{2(d+ex)(cdf-aeg)^2}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{1}{4}e^3\left(\frac{\left(15a^2e^4g^2-6acde^2g(dg+4ef)+c^2d^2\left(-d^2g^2+8defg+8e^2f^2\right)\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}-\frac{g\sqrt{x\left(ae^2+\right.}}{}\right.}{c^3d^3e^3}$$

$$\frac{2(d+ex)(cdf-aeg)^2}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{4}e^3\left(\frac{\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)\left(15a^2e^4g^2-6acde^2g(dg+4ef)+c^2d^2\left(-d^2g^2+8defg+8e^2f^2\right)\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}-\frac{g\sqrt{x\left(ae^2+cd^2\right)+ade-}}{}\right.}{c^3d^3e^3}$$

$$\frac{2(d+ex)(cdf-aeg)^2}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input

```
Int[((d + e*x)^2*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
),x]
```

output

```
(-2*(c*d*f - a*e*g)^2*(d + e*x))/(c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((c*d*e^3*g^2*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/2 + (e^3*(-((g*(7*a*e^2*g - c*d*(8*e*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/e) + ((15*a^2*e^4*g^2 - 6*a*c*d*e^2*g*(4*e*f + d*g) +
c^2*d^2*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2))*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*
e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))))/4)/(c^3*d^3*e^3)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1799 vs. $2(264) = 528$.

Time = 2.52 (sec) , antiderivative size = 1800, normalized size of antiderivative = 6.21

method	result	size
default	Expression too large to display	1800

input

```
int((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_
RETURNVERBOSE)
```

output
```
2*d^2*f^2*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+2*e*g*(d*g+e*f)*(x^2/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/
c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a
*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+2*d*f*(d*g+e*f)*(
-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*
d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))+(d^2*g^2+4*d*e*f*g+e^2*f^2)*(-x/d/e/c/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*l
n((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2))/(d*e*c)^(1/2))+e^2*g^2*(1/2*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)-5/4*(a*e^2+c*d^2)/d/e/c*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2...
```

Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.01

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="fricas")
```

output

```
[1/16*((8*a*c^2*d^2*e^3*f^2 + 8*(a*c^2*d^3*e^2 - 3*a^2*c*d*e^4)*f*g - (a*c
^2*d^4*e + 6*a^2*c*d^2*e^3 - 15*a^3*e^5)*g^2 + (8*c^3*d^3*e^2*f^2 + 8*(c^3
*d^4*e - 3*a*c^2*d^2*e^3)*f*g - (c^3*d^5 + 6*a*c^2*d^3*e^2 - 15*a^2*c*d*e^
4)*g^2)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a
^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
+ a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^3*d^3*e^2*g^2
*x^2 - 8*c^3*d^3*e^2*f^2 + 24*a*c^2*d^2*e^3*f*g + (a*c^2*d^3*e^2 - 15*a^2*
c*d*e^4)*g^2 + (8*c^3*d^3*e^2*f*g + (c^3*d^4*e - 5*a*c^2*d^2*e^3)*g^2)*x)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^2*x + a*c^4*d^4*e^
3), -1/8*((8*a*c^2*d^2*e^3*f^2 + 8*(a*c^2*d^3*e^2 - 3*a^2*c*d*e^4)*f*g - (
a*c^2*d^4*e + 6*a^2*c*d^2*e^3 - 15*a^3*e^5)*g^2 + (8*c^3*d^3*e^2*f^2 + 8*(
c^3*d^4*e - 3*a*c^2*d^2*e^3)*f*g - (c^3*d^5 + 6*a*c^2*d^3*e^2 - 15*a^2*c*d
*e^4)*g^2)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^
2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^3*d^3*e^2*g^2*x^2 - 8*c^3*d^3
*e^2*f^2 + 24*a*c^2*d^2*e^3*f*g + (a*c^2*d^3*e^2 - 15*a^2*c*d*e^4)*g^2 + (
8*c^3*d^3*e^2*f*g + (c^3*d^4*e - 5*a*c^2*d^2*e^3)*g^2)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^2*x + a*c^4*d^4*e^3)]
```

Sympy [F]

$$\int \frac{(d+ex)^2(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{(d+ex)^2\,(f+gx)^2}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*x+d)**2*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

output

```
Integral((d + e*x)**2*(f + g*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[4,4,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,0,
0,0]%%%}+
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^2}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,dx$$

input

```
int(((f + g*x)^2*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
),x)
```

output

```
int(((f + g*x)^2*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 725, normalized size of antiderivative = 2.50

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx= 15\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4g^2-6\surd$$

input

```
int((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(15*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4*g**2
- 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**2*e**2*g
**2 - 24*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
 c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d*e**3
*f*g - sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4*g*
*2 + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**3*e*
f*g + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**2*e
**2*f**2 - 10*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**4*g**2 + 2
*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**2*e**2*g**2 + 18*sqrt(e)
*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d*e**3*f*g - 2*sqrt(e)*sqrt(d)*sqrt
(c)*sqrt(a*e + c*d*x)*c**2*d**3*e*f*g - 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
 + c*d*x)*c**2*d**2*e**2*f**2 - 15*sqrt(d + e*x)*a**2*c*d*e**4*g**2 + sqrt
(d + e*x)*a*c**2*d**3*e**2*g**2 + 24*sqrt(d + e*x)*a*c**2*d**2*e**3*f*g -
5*sqrt(d + e*x)*a*c**2*d**2*e**3*g**2*x + sqrt(d + e*x)*c**3*d**4*e*g**2*x
 - 8*sqrt(d + e*x)*c**3*d**3*e**2*f**2 + 8*sqrt(d + e*x)*c**3*d**3*e**2...
```

3.284 $\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 178

$$\begin{aligned}
&\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \\
&-\frac{2(cdf-aeg)(d+ex)}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2} \\
&-\frac{\left(3ae^2g-cd(2ef+dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{c^{5/2}d^{5/2}\sqrt{e}}
\end{aligned}$$

output

```
-2*(-a*e*g+c*d*f)*(e*x+d)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2-(3*a*e^2*g-c*d*(d*g+2*e*
f))*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/c^(5/2)/d^(5/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)(3aeg+cd(-2f+gx))+(-3ae^2g+cd(2ef+}{c^{5/2}d^{5/2}\sqrt{e}\sqrt{(ae+cdx)(}}$$

input
```
Integrate[((d + e*x)^2*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2),x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(3*a*e*g + c*d*(-2*f + g*x)) + (-3*a*e^
2*g + c*d*(2*e*f + d*g))*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*
Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(c^(5/2)*d^(5/2)*Sqrt
[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1211, 25, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int -\frac{e\left(age^2-cdgxe-cd(ef+dg)\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2e}-\frac{2(d+ex)(cdf-aeg)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{e\left(age^2-cdgxe-cd(ef+dg)\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2e}-\frac{2(d+ex)(cdf-aeg)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{age^2-cdgxe-cd(ef+dg)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^2d^2}-\frac{2(d+ex)(cdf-aeg)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1160$$

$$-\frac{\frac{1}{2}\left(3ae^2g-cd(dg+2ef)\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2}-\frac{2(d+ex)(cdf-aeg)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1092$$

$$-\frac{\left(3ae^2g-cd(dg+2ef)\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2}-\frac{2(d+ex)(cdf-aeg)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)\left(3ae^2g-cd(dg+2ef)\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2}-\frac{2(d+ex)(cdf-aeg)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[((d + e*x)^2*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),
x]
```

output
```
(-2*(c*d*f - a*e*g)*(d + e*x))/(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) - (-(g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((3*a*e^2
*g - c*d*(2*e*f + d*g))*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqr
t[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqr
t[d]*Sqrt[e]))/(c^2*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(160) = 320$.

Time = 2.06 (sec) , antiderivative size = 970, normalized size of antiderivative = 5.45

method	result
default	$\frac{2d^{2}f\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}+e(2dg+ef)\left(-\frac{x}{dec\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}-\frac{\left(a\,e^{2}+c\,d^{2}\right)}{}\right.$

input

```
int((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RE
TURNVERBOSE)
```

output

```
2*d^2*f*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+e*(2*d*g+e*f)*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2))/(d*e*c)^(1/2))+d*(d*g+2*e*f)*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2
-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+e^2*g*(x^2/d/e/
c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/
c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/
c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a
*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.88

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[-\frac{(2\,acde^2f+(acd^2e-3\,a^2e^3)g+(2\,c^2d^2ef+(c^2d^3-3\,acde^2)}{}\right.$$

$$-\frac{(2\,acde^2f+(acd^2e-3\,a^2e^3)g+(2\,c^2d^2ef+(c^2d^3-3\,acde^2)g)x)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,c}{2\,(c^2d^2e^2x^2+acd^2e^2+(c}\right.}{2\,(c^4d^4ex+ac^3d^3e^2}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, alg
orithm="fricas")
```

output
```
[-1/4*((2*a*c*d*e^2*f + (a*c*d^2*e - 3*a^2*e^3)*g + (2*c^2*d^2*e*f + (c^2*
d^3 - 3*a*c*d*e^2)*g)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*
d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(c^2
*d^2*e*g*x - 2*c^2*d^2*e*f + 3*a*c*d*e^2*g)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(c^4*d^4*e*x + a*c^3*d^3*e^2), -1/2*((2*a*c*d*e^2*f + (a*c*
d^2*e - 3*a^2*e^3)*g + (2*c^2*d^2*e*f + (c^2*d^3 - 3*a*c*d*e^2)*g)*x)*sqrt
(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
 + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e
 + a*c*d*e^3)*x)) - 2*(c^2*d^2*e*g*x - 2*c^2*d^2*e*f + 3*a*c*d*e^2*g)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e*x + a*c^3*d^3*e^2)]
```

Sympy [F]

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{\left(d+ex\right)^2\left(f+gx\right)}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x
)
```

output
```
Integral((d + e*x)**2*(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, alg
orithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{2,[3,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{(f+g\,x)\,\left(d+e\,x\right)^2}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input

```
int(((f + g*x)*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),
x)
```

output

```
int(((f + g*x)*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),
 x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{-12\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2g+4}{}$$

input

```
int((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
( - 12*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*e**2*g + 4
*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2*g + 8*sqrt(
e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d*e*f + 9*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e**2*g - sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
+ c*d*x)*c*d**2*g - 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*e*f +
12*sqrt(d + e*x)*a*c*d*e**2*g - 8*sqrt(d + e*x)*c**2*d**2*e*f + 4*sqrt(d +
 e*x)*c**2*d**2*e*g*x)/(4*sqrt(a*e + c*d*x)*c**3*d**3*e)
```

3.285 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 111

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(d+ex)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}}$$

output

```
(-2*e*x-2*d)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*e^(1/2)*arctanh
(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
c^(3/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{-2\sqrt{c}\sqrt{d}(d+ex)+2\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{c^{3/2}d^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input

```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[c]*Sqrt[d]*(d + e*x) + 2*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*
ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(c^(
3/2)*d^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {1124, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1124$$

$$\frac{e\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2(d+ex)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1092$$

$$\frac{2e\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{cd}-\frac{2(d+ex)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{e}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}}-\frac{2(d+ex)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (Sqrt[e
]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(c^(3/2)*d^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(97) = 194$.

Time = 1.83 (sec) , antiderivative size = 485, normalized size of antiderivative = 4.37

method	result
default	$\frac{2d^2\left(2cdxe+a\,e^2+c\,d^2\right)}{\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}+e^2\left(-\frac{x}{dec\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\left(a\,e^2+c\,d^2\right)\left(-\frac{}{dec\sqrt{ade+(}}\right.}{}\right.$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output

```
2*d^2*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)+e^2*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1
/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(
d*e*c)^(1/2))+2*d*e*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e
^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.14

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[\frac{(cdx+ae)\sqrt{\frac{e}{cd}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+8\,(c^2\right.}{}\right.$$

$$\left.-\frac{(cdx+ae)\sqrt{-\frac{e}{cd}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{e}{cd}}}{2\,(cde^2x^2+ade^2+(cd^2e+ae^3)x)}\right)+2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{c^2d^2x+acde}\right]$$

input

```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output

```
[1/2*((c*d*x + a*e)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*
d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d
^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d)))
 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*x + a*c*d*e), -
((c*d*x + a*e)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e
^2 + (c*d^2*e + a*e^3)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(c^2*d^2*x + a*c*d*e)]
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^2}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[1,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^2}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

input

```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output

```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)-2\sqrt{e}\,\sqrt{d}}{\sqrt{cdx+ae}\,c^2 d^2}$$

input

```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(2*(sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)) - sqrt(e)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x) - sqrt(d + e*x)*c*d))/(sqrt(a*e + c*d*x)*c**2
*d**2)
```

3.286 $\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 139

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(d+ex)}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{(cdf-aeg)^{3/2}}$$

output

```
(-2*e*x-2*d)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*(-d*
g+e*f)^(1/2)*arctanh((-a*e*g+c*d*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(-a*e*g+c*d*f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{-2\sqrt{cdf-aeg}(d+ex)+2\sqrt{-ef+dg}\sqrt{ae+cdx}\sqrt{d+}}{(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+}}$$

input
```
Integrate[(d + e*x)^2/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2)),x]
```

output
```
(-2*Sqrt[c*d*f - a*e*g]*(d + e*x) + 2*Sqrt[-(e*f) + d*g]*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTan[(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])/(Sqrt[-(e*f) + d
*g]*Sqrt[a*e + c*d*x])])/((c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*
x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1264, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1264}$$

$$-\frac{2\int -\frac{\left(cd^2-ae^2\right)^2(ef-dg)}{2(cdf-aeg)(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{\left(cd^2-ae^2\right)^2}-\frac{2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ef-dg)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}-\frac{2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{2(ef-dg)\int \frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae(ef-2dg)-(ae^2g-cd(2ef-dg))x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}\right)}{cdf-aeg}-\frac{2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ef-dg}\operatorname{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}-\frac{2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}$$

input

```
Int[(d + e*x)^2/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output

```
(-2*(d + e*x))/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) + (Sqrt[e*f - d*g]*ArcTanh[(c*d^2*f + a*e*(e*f - 2*d*g) - (a*e^2*g - c*
d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])])/(c*d*f - a*e*g)^(3/2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(129) = 258$.

Time = 2.07 (sec) , antiderivative size = 953, normalized size of antiderivative = 6.86

method	result
default	$\frac{e\left(eg\left(-\frac{1}{dec\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{dec\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}\right)+\frac{4dg\left(2cdxe+a\,e^{2}+\right.}{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\sqrt{ade+}}\right.}{g^{2}}$

input
```
int((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RE
TURNVERBOSE)
```

output
```
e/g^2*(e*g*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)
/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2))+4*d*g*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2
-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-2*e*f*(2*c*d*e*x
+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/g^3*(1/(a*d*e*g^2-a*e^2*f*g-c*d^2
*f*g+c*d*e*f^2)*g^2/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)
+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)-(a*e^2*g+c*d^2*g-2*c
*d*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*(2*d*e*c*(x+f/g)+(a*e^
2*g+c*d^2*g-2*c*d*e*f)/g)/(4*d*e*c*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^
2)/g^2-(a*e^2*g+c*d^2*g-2*c*d*e*f)^2/g^2)/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*
g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2
)-1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2/((a*d*e*g^2-a*e^2*f*g-c*
d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*ln((2*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f
^2)/g^2+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+2*((a*d*e*g^2-a*e^2*f*g-c*d^
2*f*g+c*d*e*f^2)/g^2)^(1/2)*(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g
*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2))/(x+f/g)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(127) = 254$.

Time = 1.34 (sec) , antiderivative size = 758, normalized size of antiderivative = 5.45

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \left[-\frac{(cdx+ae)\sqrt{\frac{ef-dg}{cdf-aeg}}\log\left(\frac{8\,a^2d^2e^2g^2+(c^2d^4+6\,acd^2e^2+a^2e^4)}{}\right.}{}\right.$$

$$-\frac{(cdx+ae)\sqrt{-\frac{ef-dg}{cdf-aeg}}\arctan\left(-\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,adeg-(cd^2+ae^2)f-(2\,cdef-(cd^2+ae^2)g)x)\sqrt{-\frac{ef-dg}{cdf-aeg}}}{2\,(ade^2f-ad^2eg+(cde^2f-cd^2eg)x^2+((cd^2e+ae^3)f-(cd^3+ade^2)g)x)}\right)+2}{acdef-a^2e^2g+(c^2d^2f-acdeg)x}$$

input
```
integrate((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, alg
orithm="fricas")
```

output
```
[-1/2*((c*d*x + a*e)*sqrt((e*f - d*g)/(c*d*f - a*e*g))*log((8*a^2*d^2*e^2*
g^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*f^2 - 8*(a*c*d^3*e + a^2*d*e^3)*
f*g + (8*c^2*d^2*e^2*f^2 - 8*(c^2*d^3*e + a*c*d*e^3)*f*g + (c^2*d^4 + 6*a*
c*d^2*e^2 + a^2*e^4)*g^2)*x^2 + 2*(4*(c^2*d^3*e + a*c*d*e^3)*f^2 - (3*c^2*
d^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*f*g + 4*(a*c*d^3*e + a^2*d*e^3)*g^2)*x -
 4*(2*a^2*d*e^2*g^2 + (c^2*d^3 + a*c*d*e^2)*f^2 - (3*a*c*d^2*e + a^2*e^3)*
f*g + (2*c^2*d^2*e*f^2 - (c^2*d^3 + 3*a*c*d*e^2)*f*g + (a*c*d^2*e + a^2*e^
3)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt((e*f - d*g)/(c
*d*f - a*e*g)))/(g^2*x^2 + 2*f*g*x + f^2)) + 4*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(a*c*d*e*f - a^2*e^2*g + (c^2*d^2*f - a*c*d*e*g)*x), -((
c*d*x + a*e)*sqrt(-(e*f - d*g)/(c*d*f - a*e*g))*arctan(-1/2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e*g - (c*d^2 + a*e^2)*f - (2*c*d*e*f
- (c*d^2 + a*e^2)*g)*x)*sqrt(-(e*f - d*g)/(c*d*f - a*e*g))/(a*d*e^2*f - a*
d^2*e*g + (c*d*e^2*f - c*d^2*e*g)*x^2 + ((c*d^2*e + a*e^3)*f - (c*d^3 + a*
d*e^2)*g)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c*d*e*f
- a^2*e^2*g + (c^2*d^2*f - a*c*d*e*g)*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{(d+ex)^2}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(f+gx)}\,dx$$

input
```
integrate((e*x+d)**2/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x
)
```

output
```
Integral((d + e*x)**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, alg
orithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((a*e^2)/g>0)', see `assume?` fo
r more det
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-1,[1,1,5]%%%},[2,1,3,0]%%%}+%%%{%%%{2,[2,3,3]%%%},[2,
1,2,0]%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+e\,x)^2}{(f+g\,x)\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),
x)
```

output
```
int((d + e*x)^2/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),
 x)
```

Reduce [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.15

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{dg-ef}\,\sqrt{cdx+ae}\,\sqrt{aeg-cdf}\,\log\Bigg(\sqrt{g}\,\sqrt{e}\,\sqrt{cdx+a}}{}$$

input

```
int((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)
*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a
*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)
*sqrt(d + e*x))*c*d + sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f
)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sq
rt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*d - sqrt(d*g - e*f)*sqrt(a*e + c*d*
x)*sqrt(a*e*g - c*d*f)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(
a*e*g - c*d*f) + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*g + 2*c*d*e*f + 2*c*d*e*g*x)*c*d + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c
*d*x)*a*e*g - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*f + 2*sqrt(d
 + e*x)*a*c*d*e*g - 2*sqrt(d + e*x)*c**2*d**2*f)/(sqrt(a*e + c*d*x)*c*d*(a
**2*e**2*g**2 - 2*a*c*d*e*f*g + c**2*d**2*f**2))
```

3.287 $\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 210

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2cd(d+ex)}{(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)^2(f+gx)}+\frac{\left(ae^2g+cd(2ef-3dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{ef-dg}(cdf-aeg)^{5/2}}$$

output

```
-2*c*d*(e*x+d)/(-a*e*g+c*d*f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-g*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)+(a*e^2*g+
c*d*(-3*d*g+2*e*f))*arctanh((-a*e*g+c*d*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(-d*g+e*f)^(1/2)/(-a*e*g+c*d*f)^(
5/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{-\frac{(ae+cdx)(d+ex)^2(aeg+cd(2f+3gx))}{(cdf-aeg)^2(f+gx)}-\frac{\left(ae^2g+cd(2ef-3dg)\right)(ae+cdx}{\sqrt{-e}}}{((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)^2/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2)),x]
```

output
```
(-(((a*e + c*d*x)*(d + e*x)^2*(a*e*g + c*d*(2*f + 3*g*x)))/((c*d*f - a*e*g
)^2*(f + g*x))) - ((a*e^2*g + c*d*(2*e*f - 3*d*g))*(a*e + c*d*x)^(3/2)*(d
+ e*x)^(3/2)*ArcTan[(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])/(Sqrt[-(e*f) + d*g
]*Sqrt[a*e + c*d*x])])/(Sqrt[-(e*f) + d*g]*(c*d*f - a*e*g)^(5/2)))/((a*e +
 c*d*x)*(d + e*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1264, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \color{blue}{1264}$$

$$-\frac{2\int -\frac{\left(cd^2-ae^2\right)^2\left(d\left(aeg^2+cf(ef-2dg)\right)-\left(cd^2-ae^2\right)g^2x\right)}{2(cdf-aeg)^2(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{\left(cd^2-ae^2\right)^2}-\frac{2cd(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\int \frac{d(aeg^2+cf(ef-2dg))-(cd^2-ae^2)g^2x}{(f+gx)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{(cdf-aeg)^2}-\frac{2cd(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{\frac{1}{2}\left(ae^2g+cd(2ef-3dg)\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{f+gx}}{(cdf-aeg)^2}-\frac{2cd(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{-\left(ae^2g+cd(2ef-3dg)\right)\int \frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-\right.}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}\right.}{(cdf-aeg)^2}$$
$$\frac{2cd(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{(ae^2g+cd(2ef-3dg))\text{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{2\sqrt{ef-dg}\sqrt{cdf-aeg}}-\frac{g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{f+gx}}{(cdf-aeg)^2}-$$
$$\frac{2cd(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}$$

input
```
Int[(d + e*x)^2/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
),x]
```

output
```
(-2*c*d*(d + e*x))/((c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]) + (-((g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(f + g*x)) +
 ((a*e^2*g + c*d*(2*e*f - 3*d*g))*ArcTanh[(c*d^2*f + a*e*(e*f - 2*d*g) - (
a*e^2*g - c*d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f
- a*e*g]))/(c*d*f - a*e*g)^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(196) = 392$.

Time = 2.17 (sec) , antiderivative size = 1795, normalized size of antiderivative = 8.55

method	result	size
default	Expression too large to display	1795

input

```
int((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_
RETURNVERBOSE)
```

output

```
2*e^2/g^2*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/g^4*(d^2*g^2-2*d*e*f*g+e^2*f^2)*(-1/(a*d
*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2/(x+f/g)/(c*d*(x+f/g)^2*e+(a*e^2*
g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g
^2)^(1/2)-3/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g
+c*d*e*f^2)*(1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2/(c*d*(x+f/g)^
2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c
*d*e*f^2)/g^2)^(1/2)-(a*e^2*g+c*d^2*g-2*c*d*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*
d^2*f*g+c*d*e*f^2)*(2*d*e*c*(x+f/g)+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g)/(4*d*e*
c*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2-(a*e^2*g+c*d^2*g-2*c*d*e*f
)^2/g^2)/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2
-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)-1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*
g+c*d*e*f^2)*g^2/((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*ln(
(2*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2+(a*e^2*g+c*d^2*g-2*c*d*e*
f)/g*(x+f/g)+2*((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*(c*d*
(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d
^2*f*g+c*d*e*f^2)/g^2)^(1/2))/(x+f/g)))-4*d*e*c/(a*d*e*g^2-a*e^2*f*g-c*d^2
*f*g+c*d*e*f^2)*g^2*(2*d*e*c*(x+f/g)+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g)/(4*d*e
*c*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2-(a*e^2*g+c*d^2*g-2*c*d*e*
f)^2/g^2)/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(196) = 392$.

Time = 9.97 (sec) , antiderivative size = 1923, normalized size of antiderivative = 9.16

$$\int \frac{(d+ex)^2}{(f+gx)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="fricas")
```

output

```
[1/4*((2*a*c*d*e^2*f^2 - (3*a*c*d^2*e - a^2*e^3)*f*g + (2*c^2*d^2*e*f*g -
(3*c^2*d^3 - a*c*d*e^2)*g^2)*x^2 + (2*c^2*d^2*e*f^2 - 3*(c^2*d^3 - a*c*d*e
^2)*f*g - (3*a*c*d^2*e - a^2*e^3)*g^2)*x)*sqrt(c*d*e*f^2 + a*d*e*g^2 - (c*
d^2 + a*e^2)*f*g)*log((8*a^2*d^2*e^2*g^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*
e^4)*f^2 - 8*(a*c*d^3*e + a^2*d*e^3)*f*g + (8*c^2*d^2*e^2*f^2 - 8*(c^2*d^3
*e + a*c*d*e^3)*f*g + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*g^2)*x^2 - 4*sqr
t(c*d*e*f^2 + a*d*e*g^2 - (c*d^2 + a*e^2)*f*g)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*a*d*e*g - (c*d^2 + a*e^2)*f - (2*c*d*e*f - (c*d^2 + a*
e^2)*g)*x) + 2*(4*(c^2*d^3*e + a*c*d*e^3)*f^2 - (3*c^2*d^4 + 10*a*c*d^2*e^
2 + 3*a^2*e^4)*f*g + 4*(a*c*d^3*e + a^2*d*e^3)*g^2)*x)/(g^2*x^2 + 2*f*g*x
+ f^2)) - 4*(2*c^2*d^2*e*f^3 + a^2*d*e^2*g^3 - (2*c^2*d^3 + a*c*d*e^2)*f^2
*g + (a*c*d^2*e - a^2*e^3)*f*g^2 + 3*(c^2*d^2*e*f^2*g + a*c*d^2*e*g^3 - (c
^2*d^3 + a*c*d*e^2)*f*g^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(a*c^3*d^3*e^2*f^5 + a^4*d*e^4*f*g^4 - (a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*
f^4*g + 3*(a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f^3*g^2 - (3*a^3*c*d^2*e^3 + a^4
*e^5)*f^2*g^3 + (c^4*d^4*e*f^4*g + a^3*c*d^2*e^3*g^5 - (c^4*d^5 + 3*a*c^3*
d^3*e^2)*f^3*g^2 + 3*(a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f^2*g^3 - (3*a^2*c^2*
d^3*e^2 + a^3*c*d*e^4)*f*g^4)*x^2 + (c^4*d^4*e*f^5 + 2*a*c^3*d^4*e*f^3*g^2
 + 2*a^3*c*d*e^4*f^2*g^3 + a^4*d*e^4*g^5 - (c^4*d^5 + 2*a*c^3*d^3*e^2)*f^4
*g - (2*a^3*c*d^2*e^3 + a^4*e^5)*f*g^4)*x), 1/2*((2*a*c*d*e^2*f^2 - (3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**2/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((a*e^2)/g>0)', see `assume?` fo
r more det
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2888 vs. $2(196) = 392$.

Time = 8.49 (sec) , antiderivative size = 2888, normalized size of antiderivative = 13.75

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="giac")
```

output
```
1/6*((2*c*d*e*f*g^3*log(abs(-8*sqrt(c*d*e)*c^3*d^3*e^2*f^3*abs(g) + 12*sqr
t(c*d*e)*c^3*d^4*e*f^2*g*abs(g) + 12*sqrt(c*d*e)*a*c^2*d^2*e^3*f^2*g*abs(g
) - 4*sqrt(c*d*e)*c^3*d^5*f*g^2*abs(g) - 16*sqrt(c*d*e)*a*c^2*d^3*e^2*f*g^
2*abs(g) - 4*sqrt(c*d*e)*a^2*c*d*e^4*f*g^2*abs(g) + 4*sqrt(c*d*e)*a*c^2*d^
4*e*g^3*abs(g) + 4*sqrt(c*d*e)*a^2*c*d^2*e^3*g^3*abs(g) + 8*sqrt(c*d*e*f^2
 - c*d^2*f*g - a*e^2*f*g + a*d*e*g^2)*c^3*d^3*e^2*f^2*g - 8*sqrt(c*d*e*f^2
 - c*d^2*f*g - a*e^2*f*g + a*d*e*g^2)*c^3*d^4*e*f*g^2 - 8*sqrt(c*d*e*f^2 -
 c*d^2*f*g - a*e^2*f*g + a*d*e*g^2)*a*c^2*d^2*e^3*f*g^2 + sqrt(c*d*e*f^2 -
 c*d^2*f*g - a*e^2*f*g + a*d*e*g^2)*c^3*d^5*g^3 + 6*sqrt(c*d*e*f^2 - c*d^2
*f*g - a*e^2*f*g + a*d*e*g^2)*a*c^2*d^3*e^2*g^3 + sqrt(c*d*e*f^2 - c*d^2*f
*g - a*e^2*f*g + a*d*e*g^2)*a^2*c*d*e^4*g^3)) - 3*c*d^2*g^4*log(abs(-8*sqr
t(c*d*e)*c^3*d^3*e^2*f^3*abs(g) + 12*sqrt(c*d*e)*c^3*d^4*e*f^2*g*abs(g) +
12*sqrt(c*d*e)*a*c^2*d^2*e^3*f^2*g*abs(g) - 4*sqrt(c*d*e)*c^3*d^5*f*g^2*ab
s(g) - 16*sqrt(c*d*e)*a*c^2*d^3*e^2*f*g^2*abs(g) - 4*sqrt(c*d*e)*a^2*c*d*e
^4*f*g^2*abs(g) + 4*sqrt(c*d*e)*a*c^2*d^4*e*g^3*abs(g) + 4*sqrt(c*d*e)*a^2
*c*d^2*e^3*g^3*abs(g) + 8*sqrt(c*d*e*f^2 - c*d^2*f*g - a*e^2*f*g + a*d*e*g
^2)*c^3*d^3*e^2*f^2*g - 8*sqrt(c*d*e*f^2 - c*d^2*f*g - a*e^2*f*g + a*d*e*g
^2)*c^3*d^4*e*f*g^2 - 8*sqrt(c*d*e*f^2 - c*d^2*f*g - a*e^2*f*g + a*d*e*g^2
)*a*c^2*d^2*e^3*f*g^2 + sqrt(c*d*e*f^2 - c*d^2*f*g - a*e^2*f*g + a*d*e*g^2
)*c^3*d^5*g^3 + 6*sqrt(c*d*e*f^2 - c*d^2*f*g - a*e^2*f*g + a*d*e*g^2)*a...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^2}{{\left(f+g\,x\right)}^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)
),x)
```

output
```
int((d + e*x)^2/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 4829, normalized size of antiderivative = 23.00

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)
*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a
*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)
*sqrt(d + e*x))*a**2*e**4*f*g**2 + sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(
a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*
c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*e**4*g**3*x - 2*sqr
t(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqr
t(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g
 - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqr
t(d + e*x))*a*c*d*e**3*f**2*g - 2*sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a
*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(
d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c
*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c*d*e**3*f*g**2*x - 9*s
qrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*s
qrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e
*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*s
qrt(d + e*x))*c**2*d**4*f*g**2 - 9*sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(
a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g -...
```

3.288 $\int \frac{(d+ex)^2}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 342

$$\int \frac{(d+ex)^2}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2c^2d^2(d+ex)}{(cdf-aeg)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)^2(f+gx)^2}$$

$$-\frac{g\left(ae^2g+cd(6ef-7dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4(ef-dg)(cdf-aeg)^3(f+gx)}$$

$$-\frac{\left(a^2e^4g^2-2acde^2g(4ef-3dg)-c^2d^2\left(8e^2f^2-24defg+15d^2g^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{4(ef-dg)^{3/2}(cdf-aeg)^{7/2}}$$

output

```
-2*c^2*d^2*(e*x+d)/(-a*e*g+c*d*f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)-1/2*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2
-1/4*g*(a*e^2*g+c*d*(-7*d*g+6*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/(-d*g+e*f)/(-a*e*g+c*d*f)^3/(g*x+f)-1/4*(a^2*e^4*g^2-2*a*c*d*e^2*g*(-3*d
*g+4*e*f)-c^2*d^2*(15*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*arctanh((-a*e*g+c*d*f
)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
(-d*g+e*f)^(3/2)/(-a*e*g+c*d*f)^(7/2)
```

Mathematica [A] (verified)

Time = 3.36 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\frac{(ae+cdx)(d+ex)^2\left(a^2e^2g^2(-ef+2dg+egx)+acdeg\left(-dg(9f+5gx)+e\left(8f^2+\right.\right.\right.}{(ef-dg)(-cd}}{}$$

input
```
Integrate[(d + e*x)^2/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2)),x]
```

output
```
(((a*e + c*d*x)*(d + e*x)^2*(a^2*e^2*g^2*(-(e*f) + 2*d*g + e*g*x) + a*c*d*
e*g*(-(d*g*(9*f + 5*g*x)) + e*(8*f^2 + 5*f*g*x + g^2*x^2)) + c^2*d^2*(2*e*
f*(4*f^2 + 12*f*g*x + 7*g^2*x^2) - d*g*(8*f^2 + 25*f*g*x + 15*g^2*x^2))))/
((e*f - d*g)*(-(c*d*f) + a*e*g)^3*(f + g*x)^2) + ((-(a^2*e^4*g^2) + 2*a*c*
d*e^2*g*(4*e*f - 3*d*g) + c^2*d^2*(8*e^2*f^2 - 24*d*e*f*g + 15*d^2*g^2))*(
a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTan[(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*
x])/(Sqrt[-(e*f) + d*g]*Sqrt[a*e + c*d*x])])/((-(e*f) + d*g)^(3/2)*(c*d*f
- a*e*g)^(7/2)))/(4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {1264, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1264}$$

$$-\frac{2\int -\frac{-\frac{cdg^3x^2\left(cd^2-ae^2\right)^3}{(cdf-aeg)^3}-\frac{g^2(3cdf-aeg)x\left(cd^2-ae^2\right)^3}{(cdf-aeg)^3}+\frac{d\left(-a^2e^2g^3+3acdefg^2+c^2df^2(ef-3dg)\right)\left(cd^2-ae^2\right)^2}{(cdf-aeg)^3}}{2(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{\left(cd^2-ae^2\right)^2}-\frac{2c^2d^2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{-\frac{cdg^3x^2\left(cd^2-ae^2\right)^3}{(cdf-aeg)^3}-\frac{g^2(3cdf-aeg)x\left(cd^2-ae^2\right)^3}{(cdf-aeg)^3}+\frac{d\left(-a^2e^2g^3+3acdefg^2+c^2df^2(ef-3dg)\right)\left(cd^2-ae^2\right)^2}{(cdf-aeg)^3}}{(f+gx)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{\left(cd^2-ae^2\right)^2}-\frac{2c^2d^2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}$$

$$\Big\downarrow \ 2181$$

$$\frac{-\frac{\int \frac{\left(cd^2-ae^2\right)^2(ef-dg)\left(a^2g^2e^3-acdg(ef+7dg)e-c^2d^2f(4ef-11dg)-2cdg\left(3ae^2g-cd(ef+2dg)\right)x\right)}{2(cdf-aeg)^2(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(ef-dg)(cdf-aeg)}-\frac{g\left(cd^2-ae^2\right)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2(f+gx)^2(cdf-aeg)^2}}{\left(cd^2-ae^2\right)^2}-\frac{2c^2d^2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{(cd^2-ae^2)^2\int \frac{a^2g^2e^3-acdg(ef+7dg)e-c^2d^2f(4ef-11dg)-2cdg\left(3ae^2g-cd(ef+2dg)\right)x}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4(cdf-aeg)^3}-\frac{g\left(cd^2-ae^2\right)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2(f+gx)^2(cdf-aeg)^2}}{\left(cd^2-ae^2\right)^2}-\frac{2c^2d^2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}$$

$$\Big\downarrow \ 1228$$

$$\frac{-\frac{(cd^2-ae^2)^2\left(\frac{\left(a^2e^4g^2-2acde^2g(4ef-3dg)-c^2d^2\left(15d^2g^2-24defg+8e^2f^2\right)\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(ef-dg)}+\frac{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2g\right.}{(f+gx)(ef-dg)}\right.}{4(cdf-aeg)^3}}{\left(cd^2-ae^2\right)^2}$$

$$\frac{2c^2d^2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}$$

$$\Big\downarrow \ 1154$$

$$-\frac{\frac{(cd^2-ae^2)^2\left(\frac{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2g+cd(6ef-7dg)\right)}{(f+gx)(ef-dg)}-\frac{\left(a^2e^4g^2-2acde^2g(4ef-3dg)-c^2d^2\left(15d^2g^2-24defg+8e^2f^2\right)\right)\int\frac{}{4(ef-dg)(cdf-aeg)-}}{}\right.}{4(cdf-aeg)^3}}{(cd^2-ae^2}$$

$$\frac{2c^2d^2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}$$

$$\Big\downarrow \color{blue}{219}$$

$$-\frac{\frac{(cd^2-ae^2)^2\left(\frac{\left(a^2e^4g^2-2acde^2g(4ef-3dg)-c^2d^2\left(15d^2g^2-24defg+8e^2f^2\right)\right)\text{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{2(ef-dg)^{3/2}\sqrt{cdf-aeg}}+\frac{g\sqrt{x\left(ae^2+c\right.}}{}\right.}{4(cdf-aeg)^3}}{(cd^2-ae^2)^2}$$

$$\frac{2c^2d^2(d+ex)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^3}$$

input
```
Int[(d + e*x)^2/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
),x]
```

output
```
(-2*c^2*d^2*(d + e*x))/((c*d*f - a*e*g)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + (-1/2*((c*d^2 - a*e^2)^2*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/((c*d*f - a*e*g)^2*(f + g*x)^2) - ((c*d^2 - a*e^2)^2*((g*(a*e
^2*g + c*d*(6*e*f - 7*d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
(e*f - d*g)*(f + g*x)) + ((a^2*e^4*g^2 - 2*a*c*d*e^2*g*(4*e*f - 3*d*g) - c
^2*d^2*(8*e^2*f^2 - 24*d*e*f*g + 15*d^2*g^2))*ArcTanh[(c*d^2*f + a*e*(e*f
- 2*d*g) - (a*e^2*g - c*d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f
- a*e*g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*(e*f - d*g)^(3/
2)*Sqrt[c*d*f - a*e*g])))/(4*(c*d*f - a*e*g)^3))/(c*d^2 - a*e^2)^2
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1264

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3620 vs. $2(320) = 640$.

Time = 2.45 (sec) , antiderivative size = 3621, normalized size of antiderivative = 10.59

method	result	size
default	Expression too large to display	3621

input
```
int((e*x+d)^2/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_
RETURNVERBOSE)
```

output
```
e^2/g^3*(1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2/(c*d*(x+f/g)^2*e+
(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e
*f^2)/g^2)^(1/2)-(a*e^2*g+c*d^2*g-2*c*d*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*d^2*
f*g+c*d*e*f^2)*(2*d*e*c*(x+f/g)+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g)/(4*d*e*c*(a
*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2-(a*e^2*g+c*d^2*g-2*c*d*e*f)^2/
g^2)/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e
^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)-1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*
d*e*f^2)*g^2/((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*ln((2*(
a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g
*(x+f/g)+2*((a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)*(c*d*(x+f
/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f
*g+c*d*e*f^2)/g^2)^(1/2))/(x+f/g)))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/g^5*(-1/2/
(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2/(x+f/g)^2/(c*d*(x+f/g)^2*e+(
a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*
f^2)/g^2)^(1/2)-5/4*(a*e^2*g+c*d^2*g-2*c*d*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*d
^2*f*g+c*d*e*f^2)*(-1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2/(x+f/g
)/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*
f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)-3/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)*g/(a*d
*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*(1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*
d*e*f^2)*g^2/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2145 vs. $2(320) = 640$.

Time = 126.40 (sec) , antiderivative size = 4347, normalized size of antiderivative = 12.71

$$\int \frac{(d+ex)^2}{(f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((d*g-e*f)>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-1,[1,1,11]%%%},[2,3,7,0]%%%}+%%%{%%%{4,[2,3,9]%%%},[2
,3,6,0]%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{{(d+e\,x)}^2}{{(f+g\,x)}^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)
),x)
```

output
```
int((d + e*x)^2/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 14503, normalized size of antiderivative = 42.41

$$\int \frac{(d+ex)^2}{(f+gx)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^2/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
( - 3*sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sq
rt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*s
qrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sq
rt(c)*sqrt(d + e*x))*a**3*e**6*f**2*g**3 - 6*sqrt(d*g - e*f)*sqrt(a*e + c*
d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sq
rt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d
**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*f*g*
*4*x - 3*sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)
*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f
)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)
*sqrt(c)*sqrt(d + e*x))*a**3*e**6*g**5*x**2 - 23*sqrt(d*g - e*f)*sqrt(a*e
+ c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(
2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g +
 c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**
2*e**4*f**2*g**3 - 46*sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f
)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sq
rt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**4*f*g**4*x - 23*sqrt(d
*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a
*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g...
```

3.289 $\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 297

$$\begin{aligned}
&\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \\
&-\frac{2(cdf-aeg)^3(d+ex)^2}{3c^3d^3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\
&+\frac{2(cdf-aeg)^2\left(7ae^2g+cd(2ef-9dg)\right)(d+ex)}{3c^3d^3\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&+\frac{g^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^3d^3e} \\
&-\frac{g^2\left(5ae^2g-cd(6ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{7/2}d^{7/2}e^{3/2}}
\end{aligned}$$

output

```
-2/3*(-a*e*g+c*d*f)^3*(e*x+d)^2/c^3/d^3/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)+2/3*(-a*e*g+c*d*f)^2*(7*a*e^2*g+c*d*(-9*d*g+2*e*f))*(
e*x+d)/c^3/d^3/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+g^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e-g^2*(5*a*e^2*g-c*d*(-d
*g+6*e*f))*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^2(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)(15a^4e^6g^3-2a^3cde^4g^2(9ef+11dg-10egx)+3a^2c^2d^2e^2g^2(d^2g+10de(f-}{}$$

input
```
Integrate[((d + e*x)^2*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2),x]
```

output
```
((Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(15*a^4*e^6*g^3 - 2*a^3*c*d*e^4*g^2*(9
*e*f + 11*d*g - 10*e*g*x) + 3*a^2*c^2*d^2*e^2*g^2*(d^2*g + 10*d*e*(f - g*x
) + e^2*x*(-8*f + g*x)) + c^4*d^4*(4*e^2*f^3*x + 3*d^2*g^3*x^2 - 2*d*e*f^2
*(f + 9*g*x)) + 6*a*c^3*d^3*e*(d^2*g^3*x + e^2*f^2*(f + g*x) - d*e*g*(2*f^
2 - 6*f*g*x + g^2*x^2))))/((c*d^2 - a*e^2)^2*(a*e + c*d*x)) - 3*g^2*(5*a*e
^2*g + c*d*(-6*e*f + d*g))*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c
]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(3*c^(7/2)*d^(7/2)*
e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1242, 27, 2165, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)^3}{(x\,(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1242$$

$$-\frac{2\int \frac{(d+ex)\left(-\frac{3\left(cd^2-ae^2\right)^2x^2g^3}{cd}-\frac{3\left(cd^2-ae^2\right)^2(3cdf-aeg)xg^2}{c^2d^2}+\frac{\left(cd^2-ae^2\right)\left(a^3g^3e^4-3a^2cdg^2(ef+dg)e^2+3ac^2d^2fg(ef+3dg)e+c^3d^3f^2(2ef-9dg)\right)}{c^3d^3}\right)}{2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}}{3\left(cd^2-ae^2\right)^2}$$

$$\frac{2(d+ex)^2(cdf-aeg)^3}{3c^3d^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{\int \frac{(d+ex)\left(-\frac{3\left(cd^2-ae^2\right)^2x^2g^3}{cd}-\frac{3\left(cd^2-ae^2\right)^2(3cdf-aeg)xg^2}{c^2d^2}+\frac{\left(cd^2-ae^2\right)\left(a^3g^3e^4-3a^2cdg^2(ef+dg)e^2+3ac^2d^2fg(ef+3dg)e+c^3d^3f^2(2ef-9dg)\right)}{c^3d^3}\right)}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}d}{3\left(cd^2-ae^2\right)^2}$$

$$\frac{2(d+ex)^2(cdf-aeg)^3}{3c^3d^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{2165}$$

$$-\frac{-\frac{2\int \frac{3\left(cd^2-ae^2\right)^4g^2(3cdf-2aeg+cdgx)}{2c^3d^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{(cd^2-ae^2)^2}-\frac{2(d+ex)\left(7ae^2g+cd(2ef-9dg)\right)(cdf-aeg)^2}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3\left(cd^2-ae^2\right)^2}-$$

$$\frac{2(d+ex)^2(cdf-aeg)^3}{3c^3d^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{-\frac{3g^2\left(cd^2-ae^2\right)^2\int \frac{3cdf-2aeg+cdgx}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^3d^3}-\frac{2(d+ex)(cdf-aeg)^2\left(7ae^2g+cd(2ef-9dg)\right)}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3\left(cd^2-ae^2\right)^2}-$$

$$\frac{2(d+ex)^2(cdf-aeg)^3}{3c^3d^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1160}$$

$$-\frac{-\frac{3g^2\left(cd^2-ae^2\right)^2\left(\frac{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(5ae^2g-cd(6ef-dg)\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}\right)}{c^3d^3}-\frac{2(d+ex)(cdf-aeg)^2\left(7ae^2g+cd(2ef-\right.}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex}}}{3\left(cd^2-ae^2\right)^2}$$

$$\frac{2(d+ex)^2(cdf-aeg)^3}{3c^3d^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1092}$$

$$-\frac{-\frac{3g^2\left(cd^2-ae^2\right)^2\left(\frac{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(5ae^2g-cd(6ef-dg)\right)\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}\right)}{c^3d^3}-\frac{2(d+ex}{c^3}}{3\left(cd^2-ae^2\right)^2}$$

$$\frac{2(d+ex)^2(cdf-aeg)^3}{3c^3d^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow 219$$

$$-\frac{-\frac{3g^2\left(cd^2-ae^2\right)^2\left(\frac{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)\left(5ae^2g-cd(6ef-dg)\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}\right)}{c^3d^3}-\frac{2(d+ex)(cdf-aeg}{c^3d^3\sqrt{x(ae^2}}}{3\left(cd^2-ae^2\right)^2}$$

$$\frac{2(d+ex)^2(cdf-aeg)^3}{3c^3d^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[((d + e*x)^2*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
),x]
```

output

```
(-2*(c*d*f - a*e*g)^3*(d + e*x)^2)/(3*c^3*d^3*(c*d^2 - a*e^2)*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - ((-2*(c*d*f - a*e*g)^2*(7*a*e^2*g + c
*d*(2*e*f - 9*d*g))*(d + e*x))/(c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) - (3*(c*d^2 - a*e^2)^2*g^2*((g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/e - ((5*a*e^2*g - c*d*(6*e*f - d*g))*ArcTanh[(c*d^2 + a*e^2 +
 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))))/(c^3*d^3))/(3*(c*d^2 - a*e^2)^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1242
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)
^n, a*e + c*d*x, x], R = PolynomialRemainder[(f + g*x)^n, a*e + c*d*x, x]},
 Simp[R*(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p + 1)*(b^
2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(
a + b*x + c*x^2)^(p + 1)*ExpandToSum[d*e*(p + 1)*(b^2 - 4*a*c)*Q - R*(2*c*d
 - b*e)*(m + 2*p + 2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IG
tQ[n, 1] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 2165
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a*e + c*d*x, x], R = Pol
ynomialRemainder[Pq, a*e + c*d*x, x]}, Simp[R*(2*c*d - b*e)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(e*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b
^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[
d*e*(p + 1)*(b^2 - 4*a*c)*Qx - R*(2*c*d - b*e)*(m + 2*p + 2), x], x], x]] /
; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4500 vs. $2(273) = 546$.

Time = 3.34 (sec) , antiderivative size = 4501, normalized size of antiderivative = 15.15

method	result	size
default	Expression too large to display	4501

input

```
int((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_
RETURNVERBOSE)
```

output

```
d^2*f^3*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+e*g
^2*(2*d*g+3*e*f)*(-1/3*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1
/2*(a*e^2+c*d^2)/d/e/c*(-x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)
+1/2*(a*e^2+c*d^2)/d/e/c*(-1/2*x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
3/2)-1/4*(a*e^2+c*d^2)/d/e/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(3/2)-1/2*(a*e^2+c*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2
-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)))+1/2*a/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(
a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*
d^2*e^2-(a*e^2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)))+2*a/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/
2)-1/2*(a*e^2+c*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*
e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^
2*e^2-(a*e^2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))))+1/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(273) = 546$.

Time = 4.63 (sec) , antiderivative size = 1718, normalized size of antiderivative = 5.78

$$\int \frac{(d+ex)^2(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, a
lgorithm="fricas")
```

output

```
[-1/12*(3*(6*(a^2*c^3*d^5*e^3 - 2*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*f*g^2 - (
a^2*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 - 9*a^4*c*d^2*e^6 + 5*a^5*e^8)*g^3 + (
6*(c^5*d^7*e - 2*a*c^4*d^5*e^3 + a^2*c^3*d^3*e^5)*f*g^2 - (c^5*d^8 + 3*a*c
^4*d^6*e^2 - 9*a^2*c^3*d^4*e^4 + 5*a^3*c^2*d^2*e^6)*g^3)*x^2 + 2*(6*(a*c^4
*d^6*e^2 - 2*a^2*c^3*d^4*e^4 + a^3*c^2*d^2*e^6)*f*g^2 - (a*c^4*d^7*e + 3*a
^2*c^3*d^5*e^3 - 9*a^3*c^2*d^3*e^5 + 5*a^4*c*d*e^7)*g^3)*x)*sqrt(c*d*e)*lo
g(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*
(c^2*d^3*e + a*c*d*e^3)*x) + 4*(12*a*c^4*d^5*e^3*f^2*g - 3*(c^5*d^7*e - 2*
a*c^4*d^5*e^3 + a^2*c^3*d^3*e^5)*g^3*x^2 + 2*(c^5*d^6*e^2 - 3*a*c^4*d^4*e^
4)*f^3 - 6*(5*a^2*c^3*d^4*e^4 - 3*a^3*c^2*d^2*e^6)*f*g^2 - (3*a^2*c^3*d^5*
e^3 - 22*a^3*c^2*d^3*e^5 + 15*a^4*c*d*e^7)*g^3 - 2*(2*c^5*d^5*e^3*f^3 - 3*
(3*c^5*d^6*e^2 - a*c^4*d^4*e^4)*f^2*g + 6*(3*a*c^4*d^5*e^3 - 2*a^2*c^3*d^3
*e^5)*f*g^2 + (3*a*c^4*d^6*e^2 - 15*a^2*c^3*d^4*e^4 + 10*a^3*c^2*d^2*e^6)*
g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*c^6*d^8*e^4 - 2*
a^3*c^5*d^6*e^6 + a^4*c^4*d^4*e^8 + (c^8*d^10*e^2 - 2*a*c^7*d^8*e^4 + a^2*
c^6*d^6*e^6)*x^2 + 2*(a*c^7*d^9*e^3 - 2*a^2*c^6*d^7*e^5 + a^3*c^5*d^5*e^7)
*x), -1/6*(3*(6*(a^2*c^3*d^5*e^3 - 2*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*f*g^2
- (a^2*c^3*d^6*e^2 + 3*a^3*c^2*d^4*e^4 - 9*a^4*c*d^2*e^6 + 5*a^5*e^8)*g^3
+ (6*(c^5*d^7*e - 2*a*c^4*d^5*e^3 + a^2*c^3*d^3*e^5)*f*g^2 - (c^5*d^8 +...
```

Sympy [F]

$$\int \frac{(d+ex)^2(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(d+ex)^2\,(f+gx)^3}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

output
```
Integral((d + e*x)**2*(f + g*x)**3/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, a
lgorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[5,5,9]%%%},[4,4]%%%}+%%%{%%%{-8,[6,7,7]%%%},[4,3]%%
%}+%%%{%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{(f+g\,x)^3\,(d+e\,x)^2}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{5/2}}\,dx$$

input
```
int(((f + g*x)^3*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
),x)
```

output
```
int(((f + g*x)^3*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 2000, normalized size of antiderivative = 6.73

$$\int \frac{(d+ex)^2(f+gx)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^2*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
( - 30*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*e**7*g*
*3 + 54*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2
*e**5*g**3 + 36*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqr
t(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**
3*c*d*e**6*f*g**2 - 30*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a**3*c*d*e**6*g**3*x - 18*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*lo
g((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2
- c*d**2))*a**2*c**2*d**4*e**3*g**3 - 72*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
+ c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/s
qrt(a*e**2 - c*d**2))*a**2*c**2*d**3*e**4*f*g**2 + 54*sqrt(e)*sqrt(d)*sqrt
(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**3*e**4*g**3*x + 36*sqrt(e)
*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d
)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**2*e**5*f*g**2
*x - 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**3*d**6*
e*g**3 + 36*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt...
```

3.290 $\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 231

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = & -\frac{2(cdf-aeg)^2(d+ex)^2}{3c^2d^2\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\ & +\frac{4(cdf-aeg)\left(2ae^2g+cd(ef-3dg)\right)(d+ex)}{3c^2d^2\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ & +\frac{2g^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{c^{5/2}d^{5/2}\sqrt{e}}\end{aligned}$$

output

```
-2/3*(-a*e*g+c*d*f)^2*(e*x+d)^2/c^2/d^2/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)+4/3*(-a*e*g+c*d*f)*(2*a*e^2*g+c*d*(-3*d*g+e*f))*(e*x+
d)/c^2/d^2/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*g^2*
arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/c^(5/2)/d^(5/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^2(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\Bigg(-\frac{\sqrt{c}\sqrt{d}(cdf-aeg)(d+ex)(-3a^2e^3g+acde(-3ef+5dg-4egx)+c^2d^2(-2efx+d(f+6}{(cd^2-ae^2)^2(ae+cdx)}}{3c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+}}$$

input

```
Integrate[((d + e*x)^2*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2),x]
```

output

```
(2*(-((Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*(d + e*x)*(-3*a^2*e^3*g + a*c*d*e*(
-3*e*f + 5*d*g - 4*e*g*x) + c^2*d^2*(-2*e*f*x + d*(f + 6*g*x))))/((c*d^2 -
 a*e^2)^2*(a*e + c*d*x))) + (3*g^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh
[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/Sqrt[e]))/(
3*c^(5/2)*d^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1242, 27, 1211, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)^2}{(x\,(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \, 1242$$

$$-\frac{2\int -\frac{(cd^2-ae^2)(d+ex)\left(a^2g^2e^3-acdg(2ef+3dg)e-2c^2d^2f(ef-3dg)+3cd\left(cd^2-ae^2\right)g^2x\right)}{2c^2d^2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3\,(cd^2-ae^2)^2}-\frac{2(d+ex)^2(cdf-aeg)^2}{3c^2d^2\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \, 27$$

$$\frac{\int \frac{(d+ex)\left(a^2g^2e^3-acdg(2ef+3dg)e-2c^2d^2f(ef-3dg)+3cd\left(cd^2-ae^2\right)g^2x\right)}{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}dx}{3c^2d^2\left(cd^2-ae^2\right)}-\frac{2(d+ex)^2(cdf-aeg)^2}{3c^2d^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1211}$$

$$\frac{\frac{\int \frac{3cd\left(cd^2-ae^2\right)g^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cd}+\frac{4(d+ex)(cdf-aeg)\left(2ae^2g-3cd^2g+cdef\right)}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3c^2d^2\left(cd^2-ae^2\right)}-\frac{2(d+ex)^2(cdf-aeg)^2}{3c^2d^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3g^2\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{4(d+ex)(cdf-aeg)\left(2ae^2g-3cd^2g+cdef\right)}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3c^2d^2\left(cd^2-ae^2\right)}-\frac{2(d+ex)^2(cdf-aeg)^2}{3c^2d^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{6g^2\left(cd^2-ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\frac{4(d+ex)(cdf-aeg)\left(2ae^2g-3cd^2g+cdef\right)}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3c^2d^2\left(cd^2-ae^2\right)}-\frac{2(d+ex)^2(cdf-aeg)^2}{3c^2d^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3g^2\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}+\frac{4(d+ex)(cdf-aeg)\left(2ae^2g-3cd^2g+cdef\right)}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3c^2d^2\left(cd^2-ae^2\right)}-\frac{2(d+ex)^2(cdf-aeg)^2}{3c^2d^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[((d + e*x)^2*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
),x]
```

output

```
(-2*(c*d*f - a*e*g)^2*(d + e*x)^2)/(3*c^2*d^2*(c*d^2 - a*e^2)*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + ((4*(c*d*f - a*e*g)*(c*d*e*f - 3*c*d^
2*g + 2*a*e^2*g)*(d + e*x))/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) + (3*(c*d^2 - a*e^2)*g^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*
x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
])/(Sqrt[c]*Sqrt[d]*Sqrt[e]))/(3*c^2*d^2*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1211

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 1242
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(f + g*x)
^n, a*e + c*d*x, x], R = PolynomialRemainder[(f + g*x)^n, a*e + c*d*x, x]},
 Simp[R*(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p + 1)*(b^
2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(
a + b*x + c*x^2)^(p + 1)*ExpandToSum[d*e*(p + 1)*(b^2 - 4*a*c)*Q - R*(2*c*d
 - b*e)*(m + 2*p + 2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IG
tQ[n, 1] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2628 vs. $2(209) = 418$.

Time = 2.55 (sec) , antiderivative size = 2629, normalized size of antiderivative = 11.38

method	result	size
default	Expression too large to display	2629

input
```
int((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_
RETURNVERBOSE)
```

output

```
d^2*f^2*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+2*e
*g*(d*g+e*f)*(-x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^
2+c*d^2)/d/e/c*(-1/2*x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(
a*e^2+c*d^2)/d/e/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2
*(a*e^2+c*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)))+1/2*a/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a
*e^2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2)))+2*a/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*
e^2+c*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2))))+2*d*f*(d*g+e*f)*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2
)-1/2*(a*e^2+c*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(209) = 418.

Time = 4.09 (sec) , antiderivative size = 1058, normalized size of antiderivative = 4.58

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, a
lgorithm="fricas")
```

output

```
[1/6*(3*((c^4*d^6 - 2*a*c^3*d^4*e^2 + a^2*c^2*d^2*e^4)*g^2*x^2 + 2*(a*c^3*
d^5*e - 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*g^2*x + (a^2*c^2*d^4*e^2 - 2*a^3*
c*d^2*e^4 + a^4*e^6)*g^2)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c
*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(4*
a*c^3*d^4*e^2*f*g + (c^4*d^5*e - 3*a*c^3*d^3*e^3)*f^2 - (5*a^2*c^2*d^3*e^3
 - 3*a^3*c*d*e^5)*g^2 - 2*(c^4*d^4*e^2*f^2 - (3*c^4*d^5*e - a*c^3*d^3*e^3)
*f*g + (3*a*c^3*d^4*e^2 - 2*a^2*c^2*d^2*e^4)*g^2)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a^2*c^5*d^7*e^3 - 2*a^3*c^4*d^5*e^5 + a^4*c^3*d^3
*e^7 + (c^7*d^9*e - 2*a*c^6*d^7*e^3 + a^2*c^5*d^5*e^5)*x^2 + 2*(a*c^6*d^8*
e^2 - 2*a^2*c^5*d^6*e^4 + a^3*c^4*d^4*e^6)*x), -1/3*(3*((c^4*d^6 - 2*a*c^3
*d^4*e^2 + a^2*c^2*d^2*e^4)*g^2*x^2 + 2*(a*c^3*d^5*e - 2*a^2*c^2*d^3*e^3 +
 a^3*c*d*e^5)*g^2*x + (a^2*c^2*d^4*e^2 - 2*a^3*c*d^2*e^4 + a^4*e^6)*g^2)*s
qrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^
3*e + a*c*d*e^3)*x)) + 2*(4*a*c^3*d^4*e^2*f*g + (c^4*d^5*e - 3*a*c^3*d^3*e
^3)*f^2 - (5*a^2*c^2*d^3*e^3 - 3*a^3*c*d*e^5)*g^2 - 2*(c^4*d^4*e^2*f^2 - (
3*c^4*d^5*e - a*c^3*d^3*e^3)*f*g + (3*a*c^3*d^4*e^2 - 2*a^2*c^2*d^2*e^4)*g
^2)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*c^5*d^7*e^3 - 2*a
^3*c^4*d^5*e^5 + a^4*c^3*d^3*e^7 + (c^7*d^9*e - 2*a*c^6*d^7*e^3 + a^2*c...
```

Sympy [F]

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{\left(d+ex\right)^2\left(f+gx\right)^2}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate((e*x+d)**2*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

output

```
Integral((d + e*x)**2*(f + g*x)**2/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, a
lgorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, a
lgorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[2,2,8]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[4,4]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\int \frac{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^2}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

input
```
int(((f + g*x)^2*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
),x)
```

output
```
int(((f + g*x)^2*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 943, normalized size of antiderivative = 4.08

$$\int \frac{(d+ex)^2(f+gx)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^2*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(2*(3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**5*g**
2 - 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e
**3*g**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a
*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c
*d*e**4*g**2*x + 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*
sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*
a*c**2*d**4*e*g**2 - 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a*c**2*d**3*e**2*g**2*x + 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*
log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**
2 - c*d**2))*c**3*d**5*g**2*x - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x
)*a**2*c*d**2*e**3*g**2 + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2
*c*d*e**4*f*g + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**3*e*
*2*f*g - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**2*g**2
*x - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**2*e**3*f**2 + 2
*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**2*e**3*f*g*x + 2*sqrt
(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**4*e*f*g*x - 2*sqrt(e)*sqrt(d
)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**3*e**2*f**2*x - 3*sqrt(d + e*x)*a**...
```

3.291 $\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 140

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(ef-dg)(d+ex)^2}{e\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\left(ae^2g+cd(2ef-3dg)\right)(d+ex)^3}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

output

```
-2*(-d*g+e*f)*(e*x+d)^2/e/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)+2/3*(a*e^2*g+c*d*(-3*d*g+2*e*f))*(e*x+d)^3/e/(-a*e^2+c*d^2)^2/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^2(-ae(3ef-2dg+egx)+cd(-2efx+d(f+3gx)))}{3\left(cd^2-ae^2\right)^2\left((ae+cdx)(d+ex)\right)^{3/2}}$$

input
```
Integrate[((d + e*x)^2*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2),x]
```

output
```
(-2*(d + e*x)^2*(-(a*e*(3*e*f - 2*d*g + e*g*x)) + c*d*(-2*e*f*x + d*(f + 3
*g*x))))/(3*(c*d^2 - a*e^2)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1218, 1124, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx)}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{\left(ae^2g+cd(2ef-3dg)\right)\int \frac{d+ex}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3cd\left(cd^2-ae^2\right)}-\frac{2(d+ex)^2(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1124}$$

$$-\frac{\left(ae^2g+cd(2ef-3dg)\right)\left(e^2\int 0dx-\frac{2(d+ex)}{(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{3cd\left(cd^2-ae^2\right)}-\frac{2(d+ex)^2(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{24}$$

$$\frac{2(d+ex)\left(ae^2g+cd(2ef-3dg)\right)}{3cd\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^2(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[((d + e*x)^2*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),
x]
```

output
```
(-2*(c*d*f - a*e*g)*(d + e*x)^2)/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(a*e^2*g + c*d*(2*e*f - 3*d*g))*(d + e*x
))/(3*c*d*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

method	result	size
trager	$-\frac{2\left(-a\,e^{2} g x+3 c\,d^{2} g x-2 c d e f x+2 a d e g-3 a\,e^{2} f+c\,d^{2} f\right) \sqrt{c d\,x^{2} e+a\,e^{2} x+c\,d^{2} x+a d e}}{3 \left(a^{2} e^{4}-2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \left(c d x+a e\right)^{2}}$	109
gosper	$-\frac{2 \left(e x+d\right)^{3} \left(c d x+a e\right) \left(-a\,e^{2} g x+3 c\,d^{2} g x-2 c d e f x+2 a d e g-3 a\,e^{2} f+c\,d^{2} f\right)}{3 \left(a^{2} e^{4}-2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \left(c d\,x^{2} e+a\,e^{2} x+c\,d^{2} x+a d e\right)^{\frac{5}{2}}}$	114
orering	$-\frac{2\left(-a\,e^{2} g x+3 c\,d^{2} g x-2 c d e f x+2 a d e g-3 a\,e^{2} f+c\,d^{2} f\right) \left(e x+d\right)^{3} \left(c d x+a e\right)}{3 \left(a^{2} e^{4}-2 a c\,d^{2} e^{2}+c^{2} d^{4}\right) \left(a d e+\left(a\,e^{2}+c\,d^{2}\right) x+c d\,x^{2} e\right)^{\frac{5}{2}}}$	115
default	Expression too large to display	1546

input
```
int((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RE
TURNVERBOSE)
```

output
```
-2/3*(-a*e^2*g*x+3*c*d^2*g*x-2*c*d*e*f*x+2*a*d*e*g-3*a*e^2*f+c*d^2*f)/(a^2
*e^4-2*a*c*d^2*e^2+c^2*d^4)/(c*d*x+a*e)^2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,adeg+(cd^2-3\,ae^2)f-(2\,cdef-(3\,cd^2-ae^2)g)x)}{3\,(a^2c^2d^4e^2-2\,a^3cd^2e^4+a^4e^6+(c^4d^6-2\,ac^3d^4e^2+a^2c^2d^2e^4)x^2+2\,(ac^3d^5e-2\,a^2c^2d^3e^3+a^3cde^5)x)}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, alg
orithm="fricas")
```

output
```
-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e*g + (c*d^2 - 3*a
*e^2)*f - (2*c*d*e*f - (3*c*d^2 - a*e^2)*g)*x)/(a^2*c^2*d^4*e^2 - 2*a^3*c*
d^2*e^4 + a^4*e^6 + (c^4*d^6 - 2*a*c^3*d^4*e^2 + a^2*c^2*d^2*e^4)*x^2 + 2*
(a*c^3*d^5*e - 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^2(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(d+ex)^2\,(f+gx)}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x
)
```

output
```
Integral((d + e*x)**2*(f + g*x)/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, alg
orithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[2,2,0]%%%},[4,0]%%%}+%%%{%%{[%%%{-4,[1,1,1]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 10.93 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex)^2(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,(3\,a\,e^2\,f-c\,d^2\,f+a\,e^2\,g\,x-}{3\,{\left(a\,e+c\,d\,x\right)}^2\,{\left(a\,e^2-c\,d^2\right)}^2}$$

input
```
int(((f + g*x)*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),
x)
```

output
```
(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*a*e^2*f - c*d^2*f + a*
e^2*g*x - 3*c*d^2*g*x - 2*a*d*e*g + 2*c*d*e*f*x))/(3*(a*e + c*d*x)^2*(a*e^
2 - c*d^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex)^2(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{\frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a^2e^3g}{3}+\frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,ac\,d^2eg}{3}-\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a}{3}}{}$$

input
```
int((e*x+d)^2*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**3*g + sqrt(e)*sqrt(d
)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**2*e*g - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*a*c*d*e**2*f + sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*
d*e**2*g*x + sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**3*g*x - 2*s
qrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**2*e*f*x - 2*sqrt(d + e*x)
*a*c**2*d**3*e*g + 3*sqrt(d + e*x)*a*c**2*d**2*e**2*f + sqrt(d + e*x)*a*c*
*2*d**2*e**2*g*x - sqrt(d + e*x)*c**3*d**4*f - 3*sqrt(d + e*x)*c**3*d**4*g
*x + 2*sqrt(d + e*x)*c**3*d**3*e*f*x))/(3*sqrt(a*e + c*d*x)*c**2*d**2*(a**
3*e**5 - 2*a**2*c*d**2*e**3 + a**2*c*d*e**4*x + a*c**2*d**4*e - 2*a*c**2*d
**3*e**2*x + c**3*d**5*x))
```

3.292 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 116

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2e(cd^2+ae^2+2cdex)}{3cd\left(cd^2-ae^2\right)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
1/3*(-2*e*x-2*d)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+2/3*e*(2*c*d*
e*x+a*e^2+c*d^2)/c/d/(-a*e^2+c*d^2)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^2\left(-3ae^2+cd(d-2ex)\right)}{3\left(cd^2-ae^2\right)^2\left((ae+cdx)(d+ex)\right)^{3/2}}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^2*(-3*a*e^2 + c*d*(d - 2*e*x)))/(3*(c*d^2 - a*e^2)^2*((a*e +
 c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1126, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 1126$$

$$-\frac{e\int \frac{1}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd}-\frac{2(d+ex)}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 1088$$

$$\frac{2e\left(ae^2+cd^2+2cdex\right)}{3cd\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*
e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*c*d*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1126
```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[e*(d + e*x)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Simp[
e^2*((p + 2)/(c*(p + 1)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[
{a, b, c, d, e, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

method	result
trager	$\frac{2\left(2cdxe+3a\,e^{2}-c\,d^{2}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})\left(cdx+ae\right)^{2}}$
gosper	$\frac{2(ex+d)^{3}(cdx+ae)\left(2cdxe+3a\,e^{2}-c\,d^{2}\right)}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$
orering	$\frac{2\left(2cdxe+3a\,e^{2}-c\,d^{2}\right)(ex+d)^{3}(cdx+ae)}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{5}{2}}}$
default	$d^{2}\left(\frac{\frac{4}{3}cdxe+\frac{2}{3}a\,e^{2}+\frac{2}{3}c\,d^{2}}{\left(4ac\,d^{2}e^{2}-(a\,e^{2}+c\,d^{2})^{2}\right)(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}+\frac{16dec\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{3\left(4ac\,d^{2}e^{2}-(a\,e^{2}+c\,d^{2})^{2}\right)^{2}\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}\right)+e$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*(2*c*d*e*x+3*a*e^2-c*d^2)/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)/(c*d*x+a*e)^
2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdex -}{3\,(a^2c^2d^4e^2-2\,a^3cd^2e^4+a^4e^6+(c^4d^6-2\,ac^3d^4e^2+a^2c^2d^2e^4)x^2}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x - c*d^2 + 3*a*e
^2)/(a^2*c^2*d^4*e^2 - 2*a^3*c*d^2*e^4 + a^4*e^6 + (c^4*d^6 - 2*a*c^3*d^4*
e^2 + a^2*c^2*d^2*e^4)*x^2 + 2*(a*c^3*d^5*e - 2*a^2*c^2*d^3*e^3 + a^3*c*d*
e^5)*x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \int \frac{\left(d+ex\right)^2}{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[2,2,0]%%%},[4,0]%%%}+%%%{%%{[%%%{-4,[1,1,1]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2\left(-c\,d^2+2\,c\,x\,d\,e+3\,a\,e^2\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{3\,\left(a\,e+c\,d\,x\right)^2\,\left(a\,e^2-c\,d^2\right)^2}$$

input

```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output

```
(2*(3*a*e^2 - c*d^2 + 2*c*d*e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2))/(3*(a*e + c*d*x)^2*(a*e^2 - c*d^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{-\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a\,e^2}{3}-\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,cdex}{3}+2\sqrt{ex+d}\,acd\,e^2-}{\sqrt{cdx+ae}\,cd\,(a^2cd\,e^4x-2a\,c^2d^3e^2x+c^3d^5x+a^3e^5-2}$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*( - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e**2 - 2*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*e*x + 3*sqrt(d + e*x)*a*c*d*e**2 - sqrt(
d + e*x)*c**2*d**3 + 2*sqrt(d + e*x)*c**2*d**2*e*x))/(3*sqrt(a*e + c*d*x)*
c*d*(a**3*e**5 - 2*a**2*c*d**2*e**3 + a**2*c*d*e**4*x + a*c**2*d**4*e - 2*
a*c**2*d**3*e**2*x + c**3*d**5*x))
```

3.293 $\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 270

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(a^2e^4g-acde^2(ef-4dg)-c^2d^3(ef+3dg)+cde\left(5ae^2g-cd(2ef+3dg)\right)x\right)}{3\left(cd^2-ae^2\right)^2(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2g^2\operatorname{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{ef-dg}(cdf-aeg)^{5/2}}$$

output

```
1/3*(-2*e*x-2*d)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-2/
3*(a^2*e^4*g-a*c*d*e^2*(-4*d*g+e*f)-c^2*d^3*(3*d*g+e*f)+c*d*e*(5*a*e^2*g-c
*d*(3*d*g+2*e*f))*x)/(-a*e^2+c*d^2)^2/(-a*e*g+c*d*f)^2/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)+2*g^2*arctanh((-a*e*g+c*d*f)^(1/2)*(e*x+d)/(-d*g+e*f)
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(-d*g+e*f)^(1/2)/(-a*e*g+c
*d*f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2\left(\frac{cd(d+ex)\left(-6a^2e^3g+c^2d^2(-df+2efx+3dgx)+acde(3ef+4dg-5egx)\right)}{\left(cd^2-ae^2\right)^2(cdf-aeg)^2(ae+cdx)}\right.}{3\sqrt{(ae+cdx)(d+}}$$

input
```
Integrate[(d + e*x)^2/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)),x]
```

output
```
(2*((c*d*(d + e*x)*(-6*a^2*e^3*g + c^2*d^2*(-(d*f) + 2*e*f*x + 3*d*g*x) +
a*c*d*e*(3*e*f + 4*d*g - 5*e*g*x)))/((c*d^2 - a*e^2)^2*(c*d*f - a*e*g)^2*(
a*e + c*d*x)) - (3*g^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[c*d*f
- a*e*g]*Sqrt[d + e*x])/(Sqrt[-(e*f) + d*g]*Sqrt[a*e + c*d*x])])/(Sqrt[-(e
*f) + d*g]*(c*d*f - a*e*g)^(5/2))))/(3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1264, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1264}$$

$$-\frac{2\int \frac{\left(cd^2-ae^2\right)^2(ef+3dg+4egx)}{2(cdf-aeg)(f+gx)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)^2}-\frac{2(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{ef+3dg+4egx}{(f+gx)(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3(cdf-aeg)}-\frac{2(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1235}$$

$$-\frac{\frac{2\left(a^2e^4g+cdex\left(5ae^2g-cd(3dg+2ef)\right)-acde^2(ef-4dg)-c^2d^3(3dg+ef)\right)}{(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}-\frac{2\int \frac{3\left(cd^2-ae^2\right)^2g^2(ef-dg)}{2(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{(cd^2-ae^2)^2(ef-dg)(cdf-aeg)}}{3(cdf-aeg)}-\frac{2(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2\left(a^2e^4g+cdex\left(5ae^2g-cd(3dg+2ef)\right)-acde^2(ef-4dg)-c^2d^3(3dg+ef)\right)}{(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}-\frac{3g^2\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cdf-aeg}}{3(cdf-aeg)}-\frac{2(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{\frac{6g^2\int \frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right)}{cdf-aeg}+\frac{2(a^2e^4g+cdex(5ae^2g-}{(cd^2-ae^2)^2}}{3(cdf-aeg)}$$

$$\frac{2(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{2\left(a^2e^4g+cdex\left(5ae^2g-cd(3dg+2ef)\right)-acde^2(ef-4dg)-c^2d^3(3dg+ef)\right)}{(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}-\frac{3g^2\text{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{\sqrt{ef-dg}(cdf-aeg)^{3/2}}}{3(cdf-aeg)}-\frac{2(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

input
```
Int[(d + e*x)^2/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),
x]
```

output

```
(-2*(d + e*x))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)) - ((2*(a^2*e^4*g - a*c*d*e^2*(e*f - 4*d*g) - c^2*d^3*(e*f + 3*d*g)
+ c*d*e*(5*a*e^2*g - c*d*(2*e*f + 3*d*g))*x))/((c*d^2 - a*e^2)^2*(c*d*f -
a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*g^2*ArcTanh[(c*d^
2*f + a*e*(e*f - 2*d*g) - (a*e^2*g - c*d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d
*g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sq
rt[e*f - d*g]*(c*d*f - a*e*g)^(3/2)))/(3*(c*d*f - a*e*g))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1782 vs. $2(255) = 510$.

Time = 2.05 (sec) , antiderivative size = 1783, normalized size of antiderivative = 6.60

method	result	size
default	Expression too large to display	1783

input
```
int((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RE
TURNVERBOSE)
```

output

```
e/g^2*(e*g*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+
c*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
))+2*d*g*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2
)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))-e*
f*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2*
(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+(d^2*g^2
-2*d*e*f*g+e^2*f^2)/g^3*(1/3/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2
/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f
*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(3/2)-1/2*(a*e^2*g+c*d^2*g-2*c*d*e*f)*g/(a*d*
e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*(2/3*(2*d*e*c*(x+f/g)+(a*e^2*g+c*d^2*
g-2*c*d*e*f)/g)/(4*d*e*c*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2-(a*
e^2*g+c*d^2*g-2*c*d*e*f)^2/g^2)/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*
f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(3/2)+16/3*d*e
*c/(4*d*e*c*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2-(a*e^2*g+c*d^2*g
-2*c*d*e*f)^2/g^2)^2*(2*d*e*c*(x+f/g)+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g)/(c*d*
(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(252) = 504$.

Time = 12.55 (sec) , antiderivative size = 3059, normalized size of antiderivative = 11.33

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, alg
orithm="fricas")
```

output

```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**2/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x
)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, alg
orithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((a*e^2)/g>0)', see `assume?` fo
r more det
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[2,2,10]%%%},[4,2,6,0]%%%}+%%%{%%%{-4,[3,4,8]%%%},[4
,2,5,0]%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^2}{\left(f+g\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}} \,dx$$

input
```
int((d + e*x)^2/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),
x)
```

output
```
int((d + e*x)^2/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),
 x)
```

Reduce [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 3593, normalized size of antiderivative = 13.31

$$\int \frac{(d+ex)^2}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output

```
(3*sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(
e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt
(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(
c)*sqrt(d + e*x))*a**3*e**5*g**2 - 6*sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqr
t(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g -
2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**3*g**2
+ 3*sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqr
t(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt
(c)*sqrt(d + e*x))*a**2*c*d*e**4*g**2*x + 3*sqrt(d*g - e*f)*sqrt(a*e + c*d
*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqr
t(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d*
*2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e*g
**2 - 6*sqrt(d*g - e*f)*sqrt(a*e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*
sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)
*sqrt(a*e*g - c*d*f) + a*e**2*g + c*d**2*g - 2*c*d*e*f) + sqrt(g)*sqrt(d)*
sqrt(c)*sqrt(d + e*x))*a*c**2*d**3*e**2*g**2*x + 3*sqrt(d*g - e*f)*sqrt(a*
e + c*d*x)*sqrt(a*e*g - c*d*f)*log(sqrt(g)*sqrt(e)*sqrt(a*e + c*d*x) - sqr
t(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d*g - e*f)*sqrt(a*e*g - c*d*f) + a*e**...
```

3.294 $\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 424

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2cd(d+ex)}{3(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2cd(ae^2(ef-8dg)+cd^2(ef+6dg)-e(7ae^2g-cd(2ef+5dg))x)}{3\left(cd^2-ae^2\right)^2(cdf-aeg)^2(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{g(3a^2e^4g^2+2acde^2g(8ef-11dg)-c^2d^2(4e^2f^2+8defg-15d^2g^2))\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3\left(cd^2-ae^2\right)^2(ef-dg)(cdf-aeg)^3(f+gx)} - \frac{g^2(ae^2g-cd(6ef-5dg))\,\mathrm{arctanh}\left(\frac{\sqrt{cdf-aeg}(d+ex)}{\sqrt{ef-dg}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{(ef-dg)^{3/2}(cdf-aeg)^{7/2}}$$

output
```
-2/3*c*d*(e*x+d)/(-a*e*g+c*d*f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+
2/3*c*d*(a*e^2*(-8*d*g+e*f)+c*d^2*(6*d*g+e*f)-e*(7*a*e^2*g-c*d*(5*d*g+2*e*
f))*x)/(-a*e^2+c*d^2)^2/(-a*e*g+c*d*f)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)-1/3*g*(3*a^2*e^4*g^2+2*a*c*d*e^2*g*(-11*d*g+8*e*f)-c^2*d^2*
(-15*d^2*g^2+8*d*e*f*g+4*e^2*f^2))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/(-a*e^2+c*d^2)^2/(-d*g+e*f)/(-a*e*g+c*d*f)^3/(g*x+f)-g^2*(a*e^2*g-c*d*(-5
*d*g+6*e*f))*arctanh((-a*e*g+c*d*f)^(1/2)*(e*x+d)/(-d*g+e*f)^(1/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(-d*g+e*f)^(3/2)/(-a*e*g+c*d*f)^(7/2)
```

Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{\frac{(d+ex)(3a^4e^6g^3-6a^3cde^4g^3(d-ex)-c^4d^4(4e^2f^2x(f+gx)+d^2g(2f^2-10f}{}}{}$$

input
```
Integrate[(d + e*x)^2/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2)),x]
```

output
```
(((d + e*x)*(3*a^4*e^6*g^3 - 6*a^3*c*d*e^4*g^3*(d - e*x) - c^4*d^4*(4*e^2*
f^2*x*(f + g*x) + d^2*g*(2*f^2 - 10*f*g*x - 15*g^2*x^2) - 2*d*e*f*(f^2 - 3
*f*g*x - 4*g^2*x^2)) + 3*a^2*c^2*d^2*e^2*g*(d^2*g^2 - 2*d*e*g*(3*f + 5*g*x
) + e^2*(6*f^2 + 6*f*g*x + g^2*x^2)) + 2*a*c^3*d^3*e*(d^2*g^2*(7*f + 10*g*
x) + e^2*f*(-3*f^2 + 5*f*g*x + 8*g^2*x^2) - d*e*g*(4*f^2 + 12*f*g*x + 11*g
^2*x^2))))/((c*d^2 - a*e^2)^2*(-(e*f) + d*g)*(c*d*f - a*e*g)^3*(a*e + c*d*
x)*(f + g*x)) - (3*g^2*(a*e^2*g + c*d*(-6*e*f + 5*d*g))*Sqrt[a*e + c*d*x]*
Sqrt[d + e*x]*ArcTan[(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])/(Sqrt[-(e*f) + d*
g]*Sqrt[a*e + c*d*x])])/((-(e*f) + d*g)^(3/2)*(c*d*f - a*e*g)^(7/2)))/(3*S
qrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {1264, 27, 2177, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1264}$$

$$-\frac{2\int -\frac{-\frac{4cdeg^2x^2\left(cd^2-ae^2\right)^2}{(cdf-aeg)^2}+\frac{d\left(3aeg^2-cf(ef+6dg)\right)\left(cd^2-ae^2\right)^2}{(cdf-aeg)^2}+\frac{g\left(3ae^2g-cd(8ef+3dg)\right)x\left(cd^2-ae^2\right)^2}{(cdf-aeg)^2}}{2(f+gx)^2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3\left(cd^2-ae^2\right)^2}-\frac{2cd(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-\frac{4cdeg^2x^2\left(cd^2-ae^2\right)^2}{(cdf-aeg)^2}+\frac{d\left(3aeg^2-cf(ef+6dg)\right)\left(cd^2-ae^2\right)^2}{(cdf-aeg)^2}+\frac{g\left(3ae^2g-cd(8ef+3dg)\right)x\left(cd^2-ae^2\right)^2}{(cdf-aeg)^2}}{(f+gx)^2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3\left(cd^2-ae^2\right)^2}-\frac{2cd(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}$$

$$\Big\downarrow \; 2177$$

$$\frac{-\frac{2\int -\frac{3\left(cd^2-ae^2\right)^4g^2(3cdf-aeg+2cdgx)}{2(cdf-aeg)^3(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{(cd^2-ae^2)^2}-\frac{2cd(a^2e^4g+2cdex(4ae^2g-cd(3dg+ef))-acde^2(ef-7dg)-c^2d^3(6dg+ef))}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}}{3\left(cd^2-ae^2\right)^2}-\frac{2cd(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3g^2\left(cd^2-ae^2\right)^2\int \frac{3cdf-aeg+2cdgx}{(f+gx)^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{(cdf-aeg)^3}-\frac{2cd(a^2e^4g+2cdex(4ae^2g-cd(3dg+ef))-acde^2(ef-7dg)-c^2d^3(6dg+ef))}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}}{3\left(cd^2-ae^2\right)^2}-\frac{2cd(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}$$

$$\Big\downarrow \; 1228$$

$$\frac{\frac{3g^2\left(cd^2-ae^2\right)^2\left(-\frac{\left(ae^2g-cd(6ef-5dg)\right)\int \frac{1}{(f+gx)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2(ef-dg)}-\frac{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(f+gx)(ef-dg)}\right)}{(cdf-aeg)^3}-\frac{2cd(a^2e^4g+2cdex(4ae^2g-cd(3dg}{\sqrt{x(ae^2+cd^2)-}}}{3\left(cd^2-ae^2\right)^2}-\frac{2cd(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{3g^2\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2g-cd(6ef-5dg)\right)\int \frac{1}{4(ef-dg)(cdf-aeg)-\frac{\left(cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\left(-\frac{cfd^2+ae(ef-2dg)-\left(ae^2g-cd(2ef-dg)\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right.}{ef-dg}\right.}{(cdf-aeg)^3}}{3\left(cd^2-ae^2\right)^2}$$

$$\frac{2cd(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}$$

$$\Big\downarrow 219$$

$$\frac{\frac{3g^2\left(cd^2-ae^2\right)^2\left(-\frac{\left(ae^2g-cd(6ef-5dg)\right)\text{arctanh}\left(\frac{-x\left(ae^2g-cd(2ef-dg)\right)+ae(ef-2dg)+cd^2f}{2\sqrt{ef-dg}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\sqrt{cdf-aeg}}\right)}{2(ef-dg)^{3/2}\sqrt{cdf-aeg}}-\frac{g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(f+gx)(ef-dg)}\right)}{(cdf-aeg)^3}-\frac{2cd(a^2e^4g+}{}}{3\left(cd^2-ae^2\right)^2}$$

$$\frac{2cd(d+ex)}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}$$

input
```
Int[(d + e*x)^2/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)
),x]
```

output
```
(-2*c*d*(d + e*x))/(3*(c*d*f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)) + ((-2*c*d*(a^2*e^4*g - a*c*d*e^2*(e*f - 7*d*g) - c^2*d^3*(e*
f + 6*d*g) + 2*c*d*e*(4*a*e^2*g - c*d*(e*f + 3*d*g))*x))/((c*d*f - a*e*g)^
3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*(c*d^2 - a*e^2)^2*g^2*
(-((g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((e*f - d*g)*(f + g*x))
) - ((a*e^2*g - c*d*(6*e*f - 5*d*g))*ArcTanh[(c*d^2*f + a*e*(e*f - 2*d*g)
- (a*e^2*g - c*d*(2*e*f - d*g))*x)/(2*Sqrt[e*f - d*g]*Sqrt[c*d*f - a*e*g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*(e*f - d*g)^(3/2)*Sqrt[c
*d*f - a*e*g])))/(c*d*f - a*e*g)^3)/(3*(c*d^2 - a*e^2)^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1264
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)
^m*(f + g*x)^n, a + b*x + c*x^2, x], R = Coeff[PolynomialRemainder[(d + e*x
)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 0], S = Coeff[PolynomialRemainder[
(d + e*x)^m*(f + g*x)^n, a + b*x + c*x^2, x], x, 1]}, Simp[(b*R - 2*a*S + (
2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + S
imp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*E
xpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*R - b*S)
)/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1]
 && LtQ[p, -1] && ILtQ[m, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3246 vs. $2(402) = 804$.

Time = 2.03 (sec) , antiderivative size = 3247, normalized size of antiderivative = 7.66

method	result	size
default	Expression too large to display	3247

input
```
int((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_
RETURNVERBOSE)
```

output
```
e^2/g^2*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/g
^4*(d^2*g^2-2*d*e*f*g+e^2*f^2)*(-1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^
2)*g^2/(x+f/g)/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d
*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(3/2)-5/2*(a*e^2*g+c*d^2*g-2*c*
d*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*(1/3/(a*d*e*g^2-a*e^2*f
*g-c*d^2*f*g+c*d*e*f^2)*g^2/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g
*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(3/2)-1/2*(a*e^2*g
+c*d^2*g-2*c*d*e*f)*g/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)*(2/3*(2*d*
e*c*(x+f/g)+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g)/(4*d*e*c*(a*d*e*g^2-a*e^2*f*g-c
*d^2*f*g+c*d*e*f^2)/g^2-(a*e^2*g+c*d^2*g-2*c*d*e*f)^2/g^2)/(c*d*(x+f/g)^2*
e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d
*e*f^2)/g^2)^(3/2)+16/3*d*e*c/(4*d*e*c*(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*
e*f^2)/g^2-(a*e^2*g+c*d^2*g-2*c*d*e*f)^2/g^2)^2*(2*d*e*c*(x+f/g)+(a*e^2*g+
c*d^2*g-2*c*d*e*f)/g)/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/
g)+(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2))+1/(a*d*e*g^2-a*e^
2*f*g-c*d^2*f*g+c*d*e*f^2)*g^2*(1/(a*d*e*g^2-a*e^2*f*g-c*d^2*f*g+c*d*e*f^2
)*g^2/(c*d*(x+f/g)^2*e+(a*e^2*g+c*d^2*g-2*c*d*e*f)/g*(x+f/g)+(a*d*e*g^2-a*
e^2*f*g-c*d^2*f*g+c*d*e*f^2)/g^2)^(1/2)-(a*e^2*g+c*d^2*g-2*c*d*e*f)*g/(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3368 vs. $2(402) = 804$.

Time = 68.33 (sec) , antiderivative size = 6793, normalized size of antiderivative = 16.02

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, a
lgorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**2/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, a
lgorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((a*e^2)/g>0)', see `assume?` fo
r more det
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, a
lgorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^2}{{\left(f+g\,x\right)}^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}} \,dx$$

input
```
int((d + e*x)^2/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)
),x)
```

output
```
int((d + e*x)^2/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)
), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx=\int \frac{\left(ex+d\right)^{2}}{\left(gx+f\right)^{2}\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{5}{2}}}d x$$

input
```
int((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
int((e*x+d)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

3.295 $\int \sqrt{2+3x}(f+gx)^3\sqrt{1+\frac{5x}{6}-x^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 148

$$\begin{aligned}\int \sqrt{2+3x}(f+gx)^3\sqrt{1+\frac{5x}{6}-x^2}\,dx = &-\frac{13(2f+3g)^3(3-2x)^{3/2}}{48\sqrt{6}}\\ &+\frac{1}{40}\sqrt{\frac{3}{2}}(2f+3g)^2(f+8g)(3-2x)^{5/2}\\ &-\frac{1}{56}\sqrt{\frac{3}{2}}g(2f+3g)(3f+11g)(3-2x)^{7/2}\\ &+\frac{g^2(9f+20g)(3-2x)^{9/2}}{72\sqrt{6}}\\ &-\frac{1}{176}\sqrt{\frac{3}{2}}g^3(3-2x)^{11/2}\end{aligned}$$

output

```
-13/288*(2*f+3*g)^3*(3-2*x)^(3/2)*6^(1/2)+1/80*6^(1/2)*(2*f+3*g)^2*(f+8*g)
*(3-2*x)^(5/2)-1/112*6^(1/2)*g*(2*f+3*g)*(3*f+11*g)*(3-2*x)^(7/2)+1/432*g^
2*(9*f+20*g)*(3-2*x)^(9/2)*6^(1/2)-1/352*6^(1/2)*g^3*(3-2*x)^(11/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \sqrt{2+3x}(f+gx)^3\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{(-3+2x)\sqrt{6+5x-6x^2}(231f^3(19+9x)+297f^2g(32+32x+15x^2)+495fg^2(18+18x+15x^2+7x^3}{3465\sqrt{6}\sqrt{2+3x}}$$

input
```
Integrate[Sqrt[2 + 3*x]*(f + g*x)^3*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
((-3 + 2*x)*Sqrt[6 + 5*x - 6*x^2]*(231*f^3*(19 + 9*x) + 297*f^2*g*(32 + 32
*x + 15*x^2) + 495*f*g^2*(18 + 18*x + 15*x^2 + 7*x^3) + g^3*(3132 + 3132*x
 + 2610*x^2 + 2030*x^3 + 945*x^4)))/(3465*Sqrt[6]*Sqrt[2 + 3*x])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1245, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}(f+gx)^3\,dx$$
$$\Big\downarrow \text{1245}$$
$$\int \sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)(f+gx)^3dx$$
$$\Big\downarrow \text{86}$$
$$\int \left(\frac{27}{2}g\left(\frac{1}{2}-\frac{x}{3}\right)^{5/2}(6f^2+31fg+33g^2)-27g^2\left(\frac{1}{2}-\frac{x}{3}\right)^{7/2}(9f+20g)-\frac{9}{4}\left(\frac{1}{2}-\frac{x}{3}\right)^{3/2}(2f+3g)^2(f+8g)\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{g^2(3-2x)^{9/2}(9f+20g)}{72\sqrt{6}}-\frac{1}{56}\sqrt{\frac{3}{2}}g(3-2x)^{7/2}(2f+3g)(3f+11g)+\frac{1}{40}\sqrt{\frac{3}{2}}(3-2x)^{5/2}(2f+3g)^2(f+8g)-\frac{13(3-2x)^{3/2}(2f+3g)^3}{48\sqrt{6}}-\frac{1}{176}\sqrt{\frac{3}{2}}g^3(3-2x)^{11/2}$$

input
```
Int[Sqrt[2 + 3*x]*(f + g*x)^3*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
(-13*(2*f + 3*g)^3*(3 - 2*x)^(3/2))/(48*Sqrt[6]) + (Sqrt[3/2]*(2*f + 3*g)^
2*(f + 8*g)*(3 - 2*x)^(5/2))/40 - (Sqrt[3/2]*g*(2*f + 3*g)*(3*f + 11*g)*(3
 - 2*x)^(7/2))/56 + (g^2*(9*f + 20*g)*(3 - 2*x)^(9/2))/(72*Sqrt[6]) - (Sqr
t[3/2]*g^3*(3 - 2*x)^(11/2))/176
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{(2x-3)\left(945g^3x^4+3465x^3f\,g^2+2030x^3g^3+4455x^2f^2g+7425f\,g^2x^2+2610x^2g^3+2079x\,f^3+9504x\,f^2g+8910xf\,g^2+3132x\,g^3+4389}{20790\sqrt{3x+2}}$
default	$\frac{(2x-3)\left(945g^3x^4+3465x^3f\,g^2+2030x^3g^3+4455x^2f^2g+7425f\,g^2x^2+2610x^2g^3+2079x\,f^3+9504x\,f^2g+8910xf\,g^2+3132x\,g^3+4389}{20790\sqrt{3x+2}}$
orering	$\frac{(2x-3)\left(945g^3x^4+3465x^3f\,g^2+2030x^3g^3+4455x^2f^2g+7425f\,g^2x^2+2610x^2g^3+2079x\,f^3+9504x\,f^2g+8910xf\,g^2+3132x\,g^3+4389}{20790\sqrt{3x+2}}$
risch	$-\frac{\sqrt{\frac{-36x^2+30x+36}{3x+2}}\,\sqrt{3x+2}\,\left(1890g^3x^5+6930f\,g^2x^4+1225g^3x^4+8910f^2g\,x^3+4455x^3f\,g^2-870x^3g^3+4158f^3x^2+5643x^2f^2g-4455}{3465\sqrt{-36x^2+30x+36}}$

input

```
int(1/6*(3*x+2)^(1/2)*(g*x+f)^3*(-36*x^2+30*x+36)^(1/2),x,method=_RETURNVE
RBOSE)
```

output

```
1/20790*(2*x-3)*(945*g^3*x^4+3465*f*g^2*x^3+2030*g^3*x^3+4455*f^2*g*x^2+74
25*f*g^2*x^2+2610*g^3*x^2+2079*f^3*x+9504*f^2*g*x+8910*f*g^2*x+3132*g^3*x+
4389*f^3+9504*f^2*g+8910*f*g^2+3132*g^3)*(-36*x^2+30*x+36)^(1/2)/(3*x+2)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \sqrt{2+3x}(f+gx)^3\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{(1890\,g^3x^5+35\,(198\,fg^2+35\,g^3)x^4+15\,(594\,f^2g+297\,fg^2-58\,g^3)x^3-13167\,f^3-28512\,f^2g-2673}{}$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^3*(-36*x^2+30*x+36)^(1/2),x, algorithm
="fricas")
```

output
```
1/20790*(1890*g^3*x^5 + 35*(198*f*g^2 + 35*g^3)*x^4 + 15*(594*f^2*g + 297*
f*g^2 - 58*g^3)*x^3 - 13167*f^3 - 28512*f^2*g - 26730*f*g^2 - 9396*g^3 + 2
7*(154*f^3 + 209*f^2*g - 165*f*g^2 - 58*g^3)*x^2 + 3*(847*f^3 - 3168*f^2*g
 - 2970*f*g^2 - 1044*g^3)*x)*sqrt(-36*x^2 + 30*x + 36)/sqrt(3*x + 2)
```

Sympy [F]

$$\int \sqrt{2+3x}(f+gx)^3\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{\sqrt{6}\left(\int f^3\sqrt{3x+2}\sqrt{-6x^2+5x+6}\,dx+\int g^3x^3\sqrt{3x+2}\sqrt{-6x^2+5x+6}\,dx+\int 3fg^2x^2\sqrt{3x+2}\sqrt{-6x^2\right.}{6}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(g*x+f)**3*(-36*x**2+30*x+36)**(1/2),x)
```

output
```
sqrt(6)*(Integral(f**3*sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x) + Integra
l(g**3*x**3*sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x) + Integral(3*f*g**2*
x**2*sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x) + Integral(3*f**2*g*x*sqrt(
3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x))/6
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.57

$$\int \sqrt{2+3x}(f+gx)^3\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$\begin{aligned}&=\frac{1}{90}\left(18i\sqrt{3}\sqrt{2}x^2+11i\sqrt{3}\sqrt{2}x-57i\sqrt{3}\sqrt{2}\right)f^3\sqrt{2x-3}\\&\quad+\frac{1}{70}\left(30i\sqrt{3}\sqrt{2}x^3+19i\sqrt{3}\sqrt{2}x^2-32i\sqrt{3}\sqrt{2}x-96i\sqrt{3}\sqrt{2}\right)f^2g\sqrt{2x-3}\\&\quad+\frac{1}{42}\left(14i\sqrt{3}\sqrt{2}x^4+9i\sqrt{3}\sqrt{2}x^3-9i\sqrt{3}\sqrt{2}x^2-18i\sqrt{3}\sqrt{2}x-54i\sqrt{3}\sqrt{2}\right)fg^2\sqrt{2x-3}\\&\quad+\frac{1}{20790}\left(1890i\sqrt{3}\sqrt{2}x^5+1225i\sqrt{3}\sqrt{2}x^4-870i\sqrt{3}\sqrt{2}x^3-1566i\sqrt{3}\sqrt{2}x^2-3132i\sqrt{3}\sqrt{2}x-9396i\sqrt{\right.}\end{aligned}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^3*(-36*x^2+30*x+36)^(1/2),x, algorithm
="maxima")
```

output
```
1/90*(18*I*sqrt(3)*sqrt(2)*x^2 + 11*I*sqrt(3)*sqrt(2)*x - 57*I*sqrt(3)*sqr
t(2))*f^3*sqrt(2*x - 3) + 1/70*(30*I*sqrt(3)*sqrt(2)*x^3 + 19*I*sqrt(3)*sq
rt(2)*x^2 - 32*I*sqrt(3)*sqrt(2)*x - 96*I*sqrt(3)*sqrt(2))*f^2*g*sqrt(2*x
- 3) + 1/42*(14*I*sqrt(3)*sqrt(2)*x^4 + 9*I*sqrt(3)*sqrt(2)*x^3 - 9*I*sqrt
(3)*sqrt(2)*x^2 - 18*I*sqrt(3)*sqrt(2)*x - 54*I*sqrt(3)*sqrt(2))*f*g^2*sqr
t(2*x - 3) + 1/20790*(1890*I*sqrt(3)*sqrt(2)*x^5 + 1225*I*sqrt(3)*sqrt(2)*
x^4 - 870*I*sqrt(3)*sqrt(2)*x^3 - 1566*I*sqrt(3)*sqrt(2)*x^2 - 3132*I*sqrt
(3)*sqrt(2)*x - 9396*I*sqrt(3)*sqrt(2))*g^3*sqrt(2*x - 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(112) = 224$.

Time = 0.50 (sec) , antiderivative size = 585, normalized size of antiderivative = 3.95

$$\int \sqrt{2+3x}(f+gx)^3\sqrt{1+\frac{5x}{6}-x^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^3*(-36*x^2+30*x+36)^(1/2),x, algorithm
="giac")
```

output

```
1/332640*sqrt(6)*(16632*((2*x - 3)^2*sqrt(-2*x + 3) - 10*(-2*x + 3)^(3/2)
+ 45*sqrt(-2*x + 3))*f^3 + 46200*((-2*x + 3)^(3/2) - 9*sqrt(-2*x + 3))*f^3
 + 3564*(5*(2*x - 3)^3*sqrt(-2*x + 3) + 63*(2*x - 3)^2*sqrt(-2*x + 3) - 31
5*(-2*x + 3)^(3/2) + 945*sqrt(-2*x + 3))*f^2*g - 41580*((2*x - 3)^2*sqrt(-
2*x + 3) - 10*(-2*x + 3)^(3/2) + 45*sqrt(-2*x + 3))*f^2*g + 166320*((-2*x
+ 3)^(3/2) - 9*sqrt(-2*x + 3))*f^2*g + 198*(35*(2*x - 3)^4*sqrt(-2*x + 3)
+ 540*(2*x - 3)^3*sqrt(-2*x + 3) + 3402*(2*x - 3)^2*sqrt(-2*x + 3) - 11340
*(-2*x + 3)^(3/2) + 25515*sqrt(-2*x + 3))*f*g^2 - 2970*(5*(2*x - 3)^3*sqrt
(-2*x + 3) + 63*(2*x - 3)^2*sqrt(-2*x + 3) - 315*(-2*x + 3)^(3/2) + 945*sq
rt(-2*x + 3))*f*g^2 - 49896*((2*x - 3)^2*sqrt(-2*x + 3) - 10*(-2*x + 3)^(3
/2) + 45*sqrt(-2*x + 3))*f*g^2 + 45*(21*(2*x - 3)^5*sqrt(-2*x + 3) + 385*(
2*x - 3)^4*sqrt(-2*x + 3) + 2970*(2*x - 3)^3*sqrt(-2*x + 3) + 12474*(2*x -
 3)^2*sqrt(-2*x + 3) - 31185*(-2*x + 3)^(3/2) + 56133*sqrt(-2*x + 3))*g^3
- 55*(35*(2*x - 3)^4*sqrt(-2*x + 3) + 540*(2*x - 3)^3*sqrt(-2*x + 3) + 340
2*(2*x - 3)^2*sqrt(-2*x + 3) - 11340*(-2*x + 3)^(3/2) + 25515*sqrt(-2*x +
3))*g^3 - 1188*(5*(2*x - 3)^3*sqrt(-2*x + 3) + 63*(2*x - 3)^2*sqrt(-2*x +
3) - 315*(-2*x + 3)^(3/2) + 945*sqrt(-2*x + 3))*g^3 - 332640*f^3*sqrt(-2*x
 + 3))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.25

$$\int \sqrt{2+3x}(f+gx)^3\sqrt{1+\frac{5x}{6}-x^2}\,dx =$$

$$-\frac{\sqrt{6}\,\sqrt{-6\,x^2+5\,x+6}\,\left(\sqrt{3\,x+2}\,\left(\frac{19\,f^3}{90}+\frac{16\,f^2\,g}{35}+\frac{3\,f\,g^2}{7}+\frac{58\,g^3}{385}\right)+x\,\sqrt{3\,x+2}\,\left(-\frac{11\,f^3}{270}+\frac{16\,f^2\,g}{105}+\frac{f\,g^2}{7}+\right.\right.}{}$$

input

```
int(((f + g*x)^3*(3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/6,x)
```

output

```
-(6^(1/2)*(5*x - 6*x^2 + 6)^(1/2)*((3*x + 2)^(1/2)*((3*f*g^2)/7 + (16*f^2*
g)/35 + (19*f^3)/90 + (58*g^3)/385) + x*(3*x + 2)^(1/2)*((f*g^2)/7 + (16*f
^2*g)/105 - (11*f^3)/270 + (58*g^3)/1155) + x^2*(3*x + 2)^(1/2)*((f*g^2)/1
4 - (19*f^2*g)/210 - f^3/15 + (29*g^3)/1155) - (g^3*x^5*(3*x + 2)^(1/2))/3
3 - (g*x^3*(3*x + 2)^(1/2)*(297*f*g + 594*f^2 - 58*g^2))/4158 - g^2*x^4*(3
*x + 2)^(1/2)*(f/9 + (35*g)/1782)))/(x + 2/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\int \sqrt{2+3x}(f+gx)^3\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{\sqrt{-2x+3}\,\sqrt{6}\,(1890g^3x^5+6930f\,g^2x^4+1225g^3x^4+8910f^2g\,x^3+4455f\,g^2x^3-870g^3x^3+4158f^3x^2+}{}$$

input
```
int(1/6*(2+3*x)^(1/2)*(g*x+f)^3*(-36*x^2+30*x+36)^(1/2),x)
```

output
```
(sqrt( - 2*x + 3)*sqrt(6)*(4158*f**3*x**2 + 2541*f**3*x - 13167*f**3 + 891
0*f**2*g*x**3 + 5643*f**2*g*x**2 - 9504*f**2*g*x - 28512*f**2*g + 6930*f*g
**2*x**4 + 4455*f*g**2*x**3 - 4455*f*g**2*x**2 - 8910*f*g**2*x - 26730*f*g
**2 + 1890*g**3*x**5 + 1225*g**3*x**4 - 870*g**3*x**3 - 1566*g**3*x**2 - 3
132*g**3*x - 9396*g**3))/20790
```

3.296 $\int \sqrt{2+3x}(f+gx)^2\sqrt{1+\frac{5x}{6}-x^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \sqrt{2+3x}(f+gx)^2\sqrt{1+\frac{5x}{6}-x^2}\,dx = -\frac{13(2f+3g)^2(3-2x)^{3/2}}{24\sqrt{6}} + \frac{(2f+3g)(6f+35g)(3-2x)^{5/2}}{40\sqrt{6}} - \frac{g(12f+31g)(3-2x)^{7/2}}{56\sqrt{6}} + \frac{g^2(3-2x)^{9/2}}{24\sqrt{6}}$$

output

```
-13/144*(2*f+3*g)^2*(3-2*x)^(3/2)*6^(1/2)+1/240*(2*f+3*g)*(6*f+35*g)*(3-2*
x)^(5/2)*6^(1/2)-1/336*g*(12*f+31*g)*(3-2*x)^(7/2)*6^(1/2)+1/144*g^2*(3-2*
x)^(9/2)*6^(1/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \sqrt{2+3x}(f+gx)^2\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$= \frac{(-3+2x)\sqrt{6+5x-6x^2}\left(7f^2(19+9x)+6fg(32+32x+15x^2)+5g^2(18+18x+15x^2+7x^3)\right)}{105\sqrt{6}\sqrt{2+3x}}$$

input
```
Integrate[Sqrt[2 + 3*x]*(f + g*x)^2*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
((-3 + 2*x)*Sqrt[6 + 5*x - 6*x^2]*(7*f^2*(19 + 9*x) + 6*f*g*(32 + 32*x + 1
5*x^2) + 5*g^2*(18 + 18*x + 15*x^2 + 7*x^3)))/(105*Sqrt[6]*Sqrt[2 + 3*x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1245, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}(f+gx)^2\,dx$$

$$\Big\downarrow \text{ 1245}$$

$$\int \sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)(f+gx)^2dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \biggl(-\frac{3}{4}\left(\frac{1}{2}-\frac{x}{3}\right)^{3/2}\left(12f^2+88fg+105g^2\right)+\frac{9}{2}g\left(\frac{1}{2}-\frac{x}{3}\right)^{5/2}(12f+31g)+\frac{13}{8}\sqrt{\frac{1}{2}-\frac{x}{3}}(2f+3g)^2-81g^2\biggl(\frac{1}{2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{g(3-2x)^{7/2}(12f+31g)}{56\sqrt{6}}+\frac{(3-2x)^{5/2}(2f+3g)(6f+35g)}{40\sqrt{6}}-\frac{13(3-2x)^{3/2}(2f+3g)^2}{24\sqrt{6}}+\frac{g^2(3-2x)^{9/2}}{24\sqrt{6}}$$

input
```
Int[Sqrt[2 + 3*x]*(f + g*x)^2*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
(-13*(2*f + 3*g)^2*(3 - 2*x)^(3/2))/(24*Sqrt[6]) + ((2*f + 3*g)*(6*f + 35*
g)*(3 - 2*x)^(5/2))/(40*Sqrt[6]) - (g*(12*f + 31*g)*(3 - 2*x)^(7/2))/(56*S
qrt[6]) + (g^2*(3 - 2*x)^(9/2))/(24*Sqrt[6])
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{(2x-3)\left(35g^2x^3+90fg\,x^2+75g^2x^2+63f^2x+192fgx+90g^2x+133f^2+192fg+90g^2\right)\sqrt{-36x^2+30x+36}}{630\sqrt{3x+2}}$
default	$\frac{(2x-3)\left(35g^2x^3+90fg\,x^2+75g^2x^2+63f^2x+192fgx+90g^2x+133f^2+192fg+90g^2\right)\sqrt{-36x^2+30x+36}}{630\sqrt{3x+2}}$
orering	$\frac{(2x-3)\left(35g^2x^3+90fg\,x^2+75g^2x^2+63f^2x+192fgx+90g^2x+133f^2+192fg+90g^2\right)\sqrt{-36x^2+30x+36}}{630\sqrt{3x+2}}$
risch	$-\frac{\sqrt{\frac{-36x^2+30x+36}{3x+2}}\,\sqrt{3x+2}\,\left(70g^2x^4+180fg\,x^3+45g^2x^3+126f^2x^2+114fg\,x^2-45g^2x^2+77f^2x-192fgx-90g^2x-399f^2-576fg-}{105\sqrt{-36x^2+30x+36}\,\sqrt{-12x+18}}$

input
```
int(1/6*(3*x+2)^(1/2)*(g*x+f)^2*(-36*x^2+30*x+36)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/630*(2*x-3)*(35*g^2*x^3+90*f*g*x^2+75*g^2*x^2+63*f^2*x+192*f*g*x+90*g^2*
x+133*f^2+192*f*g+90*g^2)*(-36*x^2+30*x+36)^(1/2)/(3*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \sqrt{2+3x}(f+gx)^2\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{(70\,g^2x^4+45\,(4\,fg+g^2)x^3+3\,(42\,f^2+38\,fg-15\,g^2)x^2-399\,f^2-576\,fg-270\,g^2+(77\,f^2-192\,fg}{630\,\sqrt{3\,x+2}}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^2*(-36*x^2+30*x+36)^(1/2),x, algorithm
="fricas")
```

output
```
1/630*(70*g^2*x^4 + 45*(4*f*g + g^2)*x^3 + 3*(42*f^2 + 38*f*g - 15*g^2)*x^
2 - 399*f^2 - 576*f*g - 270*g^2 + (77*f^2 - 192*f*g - 90*g^2)*x)*sqrt(-36*
x^2 + 30*x + 36)/sqrt(3*x + 2)
```

Sympy [F]

$$\int \sqrt{2+3x}(f+gx)^2\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{\sqrt{6}\left(\int f^2\sqrt{3x+2}\sqrt{-6x^2+5x+6}\,dx+\int g^2x^2\sqrt{3x+2}\sqrt{-6x^2+5x+6}\,dx+\int 2fgx\sqrt{3x+2}\sqrt{-6x^2+}\right.}{6}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(g*x+f)**2*(-36*x**2+30*x+36)**(1/2),x)
```

output
```
sqrt(6)*(Integral(f**2*sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x) + Integra
l(g**2*x**2*sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x) + Integral(2*f*g*x*s
qrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x))/6
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.46

$$\int \sqrt{2+3x}(f+gx)^2\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$= \frac{1}{90}\left(18i\sqrt{3}\sqrt{2}x^2 + 11i\sqrt{3}\sqrt{2}x - 57i\sqrt{3}\sqrt{2}\right)f^2\sqrt{2x-3}$$
$$+ \frac{1}{105}\left(30i\sqrt{3}\sqrt{2}x^3 + 19i\sqrt{3}\sqrt{2}x^2 - 32i\sqrt{3}\sqrt{2}x - 96i\sqrt{3}\sqrt{2}\right)fg\sqrt{2x-3}$$
$$+ \frac{1}{126}\left(14i\sqrt{3}\sqrt{2}x^4 + 9i\sqrt{3}\sqrt{2}x^3 - 9i\sqrt{3}\sqrt{2}x^2 - 18i\sqrt{3}\sqrt{2}x - 54i\sqrt{3}\sqrt{2}\right)g^2\sqrt{2x-3}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^2*(-36*x^2+30*x+36)^(1/2),x, algorithm
="maxima")
```

output
```
1/90*(18*I*sqrt(3)*sqrt(2)*x^2 + 11*I*sqrt(3)*sqrt(2)*x - 57*I*sqrt(3)*sqr
t(2))*f^2*sqrt(2*x - 3) + 1/105*(30*I*sqrt(3)*sqrt(2)*x^3 + 19*I*sqrt(3)*s
qrt(2)*x^2 - 32*I*sqrt(3)*sqrt(2)*x - 96*I*sqrt(3)*sqrt(2))*f*g*sqrt(2*x -
 3) + 1/126*(14*I*sqrt(3)*sqrt(2)*x^4 + 9*I*sqrt(3)*sqrt(2)*x^3 - 9*I*sqrt
(3)*sqrt(2)*x^2 - 18*I*sqrt(3)*sqrt(2)*x - 54*I*sqrt(3)*sqrt(2))*g^2*sqrt(
2*x - 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.36

$$\int \sqrt{2+3x}(f+gx)^2\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$= \frac{1}{5040}\sqrt{6}\Big(252\left((2x-3)^2\sqrt{-2x+3} - 10(-2x+3)^{\frac{3}{2}} + 45\sqrt{-2x+3}\right)f^2 + 700\Big((-2x+3)^{\frac{3}{2}} - 9\sqrt{-2}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^2*(-36*x^2+30*x+36)^(1/2),x, algorithm
="giac")
```

output

```
1/5040*sqrt(6)*(252*((2*x - 3)^2*sqrt(-2*x + 3) - 10*(-2*x + 3)^(3/2) + 45
*sqrt(-2*x + 3))*f^2 + 700*((-2*x + 3)^(3/2) - 9*sqrt(-2*x + 3))*f^2 + 36*
(5*(2*x - 3)^3*sqrt(-2*x + 3) + 63*(2*x - 3)^2*sqrt(-2*x + 3) - 315*(-2*x
+ 3)^(3/2) + 945*sqrt(-2*x + 3))*f*g - 420*((2*x - 3)^2*sqrt(-2*x + 3) - 1
0*(-2*x + 3)^(3/2) + 45*sqrt(-2*x + 3))*f*g + 1680*((-2*x + 3)^(3/2) - 9*s
qrt(-2*x + 3))*f*g + (35*(2*x - 3)^4*sqrt(-2*x + 3) + 540*(2*x - 3)^3*sqrt
(-2*x + 3) + 3402*(2*x - 3)^2*sqrt(-2*x + 3) - 11340*(-2*x + 3)^(3/2) + 25
515*sqrt(-2*x + 3))*g^2 - 15*(5*(2*x - 3)^3*sqrt(-2*x + 3) + 63*(2*x - 3)^
2*sqrt(-2*x + 3) - 315*(-2*x + 3)^(3/2) + 945*sqrt(-2*x + 3))*g^2 - 252*((
2*x - 3)^2*sqrt(-2*x + 3) - 10*(-2*x + 3)^(3/2) + 45*sqrt(-2*x + 3))*g^2 -
 5040*f^2*sqrt(-2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int \sqrt{2+3x}(f+gx)^2\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{(2\,x-3)\,\sqrt{-36\,x^2+30\,x+36}\,(63\,f^2\,x+133\,f^2+90\,f\,g\,x^2+192\,f\,g\,x+192\,f\,g+35\,g^2\,x^3+75\,g^2\,x^2}{630\,\sqrt{3\,x+2}}$$

input

```
int(((f + g*x)^2*(3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/6,x)
```

output

```
((2*x - 3)*(30*x - 36*x^2 + 36)^(1/2)*(192*f*g + 63*f^2*x + 90*g^2*x + 133
*f^2 + 90*g^2 + 75*g^2*x^2 + 35*g^2*x^3 + 90*f*g*x^2 + 192*f*g*x))/(630*(3
*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82

$$\int \sqrt{2+3x}(f+gx)^2\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{\sqrt{-2x+3}\,\sqrt{6}\,(70g^2x^4+180fg\,x^3+45g^2x^3+126f^2x^2+114fg\,x^2-45g^2x^2+77f^2x-192fgx-90g^2}{630}$$

input
```
int(1/6*(2+3*x)^(1/2)*(g*x+f)^2*(-36*x^2+30*x+36)^(1/2),x)
```

output
```
(sqrt( - 2*x + 3)*sqrt(6)*(126*f**2*x**2 + 77*f**2*x - 399*f**2 + 180*f*g*
x**3 + 114*f*g*x**2 - 192*f*g*x - 576*f*g + 70*g**2*x**4 + 45*g**2*x**3 -
45*g**2*x**2 - 90*g**2*x - 270*g**2))/630
```

3.297 $\int \sqrt{2+3x}(f+gx)\sqrt{1+\frac{5x}{6}-x^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \sqrt{2+3x}(f+gx)\sqrt{1+\frac{5x}{6}-x^2}\,dx=-\frac{13(2f+3g)(3-2x)^{3/2}}{12\sqrt{6}}+\frac{(3f+11g)(3-2x)^{5/2}}{10\sqrt{6}}-\frac{1}{28}\sqrt{\frac{3}{2}}g(3-2x)^{7/2}$$

output
```
-13/72*(2*f+3*g)*(3-2*x)^(3/2)*6^(1/2)+1/60*(3*f+11*g)*(3-2*x)^(5/2)*6^(1/
2)-1/56*6^(1/2)*g*(3-2*x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \sqrt{2+3x}(f+gx)\sqrt{1+\frac{5x}{6}-x^2}\,dx=\frac{(-3+2x)\sqrt{6+5x-6x^2}(7f(19+9x)+3g(32+32x+15x^2))}{105\sqrt{6}\sqrt{2+3x}}$$

input
```
Integrate[Sqrt[2 + 3*x]*(f + g*x)*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
((-3 + 2*x)*Sqrt[6 + 5*x - 6*x^2]*(7*f*(19 + 9*x) + 3*g*(32 + 32*x + 15*x^
2)))/(105*Sqrt[6]*Sqrt[2 + 3*x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1221, 27, 1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}(f+gx)\,dx$$

$$\Big\downarrow \textcolor{blue}{1221}$$

$$\frac{1}{7}(7f+4g)\int \frac{\sqrt{3x+2}\sqrt{-6x^2+5x+6}}{\sqrt{6}}dx-\frac{g\sqrt{3x+2}\left(-6x^2+5x+6\right)^{3/2}}{21\sqrt{6}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{(7f+4g)\int \sqrt{3x+2}\sqrt{-6x^2+5x+6}dx}{7\sqrt{6}}-\frac{g\sqrt{3x+2}\left(-6x^2+5x+6\right)^{3/2}}{21\sqrt{6}}$$

$$\Big\downarrow \textcolor{blue}{1121}$$

$$\frac{(7f+4g)\int \left(\frac{13}{2}\sqrt{3-2x}-\frac{3}{2}(3-2x)^{3/2}\right)dx}{7\sqrt{6}}-\frac{g\sqrt{3x+2}\left(-6x^2+5x+6\right)^{3/2}}{21\sqrt{6}}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{\left(\frac{3}{10}(3-2x)^{5/2}-\frac{13}{6}(3-2x)^{3/2}\right)(7f+4g)}{7\sqrt{6}}-\frac{g\sqrt{3x+2}\left(-6x^2+5x+6\right)^{3/2}}{21\sqrt{6}}$$

input
```
Int[Sqrt[2 + 3*x]*(f + g*x)*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
((7*f + 4*g)*((-13*(3 - 2*x)^(3/2))/6 + (3*(3 - 2*x)^(5/2))/10))/(7*Sqrt[6
]) - (g*Sqrt[2 + 3*x]*(6 + 5*x - 6*x^2)^(3/2))/(21*Sqrt[6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{(2x-3)\left(45g\,x^{2}+63fx+96gx+133f+96g\right)\sqrt{-36x^{2}+30x+36}}{630\sqrt{3x+2}}$	48
default	$\frac{(2x-3)\left(45g\,x^{2}+63fx+96gx+133f+96g\right)\sqrt{-36x^{2}+30x+36}}{630\sqrt{3x+2}}$	48
orering	$\frac{(2x-3)\left(45g\,x^{2}+63fx+96gx+133f+96g\right)\sqrt{-36x^{2}+30x+36}}{630\sqrt{3x+2}}$	48
risch	$-\frac{\sqrt{\frac{-36x^{2}+30x+36}{3x+2}}\,\sqrt{3x+2}\,\left(90g\,x^{3}+126f\,x^{2}+57g\,x^{2}+77fx-96gx-399f-288g\right)(2x-3)}{105\sqrt{-36x^{2}+30x+36}\,\sqrt{-12x+18}}$	87

input
```
int(1/6*(3*x+2)^(1/2)*(g*x+f)*(-36*x^2+30*x+36)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/630*(2*x-3)*(45*g*x^2+63*f*x+96*g*x+133*f+96*g)*(-36*x^2+30*x+36)^(1/2)/
(3*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \sqrt{2+3x}(f+gx)\sqrt{1+\frac{5x}{6}-x^2}\,dx = \frac{(90\,gx^3+3\,(42\,f+19\,g)x^2+(77\,f-96\,g)x-399\,f-288\,g)\sqrt{-36\,x^2+30\,x+36}}{630\,\sqrt{3\,x+2}}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)*(-36*x^2+30*x+36)^(1/2),x, algorithm="
fricas")
```

output
```
1/630*(90*g*x^3 + 3*(42*f + 19*g)*x^2 + (77*f - 96*g)*x - 399*f - 288*g)*s
qrt(-36*x^2 + 30*x + 36)/sqrt(3*x + 2)
```

Sympy [F]

$$\int \sqrt{2+3x}(f+gx)\sqrt{1+\frac{5x}{6}-x^2}\,dx = \frac{\sqrt{6}\left(\int f\sqrt{3x+2}\sqrt{-6x^2+5x+6}\,dx+\int gx\sqrt{3x+2}\sqrt{-6x^2+5x+6}\,dx\right)}{6}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(g*x+f)*(-36*x**2+30*x+36)**(1/2),x)
```

output
```
sqrt(6)*(Integral(f*sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x) + Integral(g
*x*sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x))/6
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \sqrt{2+3x}(f+gx)\sqrt{1+\frac{5x}{6}-x^2}\,dx &= \frac{1}{90}\Big(18i\,\sqrt{3}\sqrt{2}x^2+11i\,\sqrt{3}\sqrt{2}x-57i\,\sqrt{3}\sqrt{2}\Big)f\sqrt{2\,x-3}\\ &\quad+\frac{1}{210}\Big(30i\,\sqrt{3}\sqrt{2}x^3+19i\,\sqrt{3}\sqrt{2}x^2-32i\,\sqrt{3}\sqrt{2}x-96i\,\sqrt{3}\sqrt{2}\Big)g\sqrt{2\,x-3}\end{aligned}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)*(-36*x^2+30*x+36)^(1/2),x, algorithm="
maxima")
```

output
```
1/90*(18*I*sqrt(3)*sqrt(2)*x^2 + 11*I*sqrt(3)*sqrt(2)*x - 57*I*sqrt(3)*sqr
t(2))*f*sqrt(2*x - 3) + 1/210*(30*I*sqrt(3)*sqrt(2)*x^3 + 19*I*sqrt(3)*sqr
t(2)*x^2 - 32*I*sqrt(3)*sqrt(2)*x - 96*I*sqrt(3)*sqrt(2))*g*sqrt(2*x - 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(52) = 104$.

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.56

$$\int \sqrt{2+3x}(f+gx)\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{1}{2520}\sqrt{6}\Big(126\left((2\,x-3)^2\sqrt{-2\,x+3}-10\,(-2\,x+3)^{\frac{3}{2}}+45\,\sqrt{-2\,x+3}\right)f+350\,\Big((-2\,x+3)^{\frac{3}{2}}-9\,\sqrt{-2}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)*(-36*x^2+30*x+36)^(1/2),x, algorithm="
giac")
```

output
```
1/2520*sqrt(6)*(126*((2*x - 3)^2*sqrt(-2*x + 3) - 10*(-2*x + 3)^(3/2) + 45
*sqrt(-2*x + 3))*f + 350*((-2*x + 3)^(3/2) - 9*sqrt(-2*x + 3))*f + 9*(5*(2
*x - 3)^3*sqrt(-2*x + 3) + 63*(2*x - 3)^2*sqrt(-2*x + 3) - 315*(-2*x + 3)^
(3/2) + 945*sqrt(-2*x + 3))*g - 105*((2*x - 3)^2*sqrt(-2*x + 3) - 10*(-2*x
 + 3)^(3/2) + 45*sqrt(-2*x + 3))*g + 420*((-2*x + 3)^(3/2) - 9*sqrt(-2*x +
 3))*g - 2520*f*sqrt(-2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 11.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

$$\int \sqrt{2+3x}(f+gx)\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{(2\,x-3)\,\sqrt{-36\,x^2+30\,x+36}\,(133\,f+96\,g+63\,f\,x+96\,g\,x+45\,g\,x^2)}{630\,\sqrt{3\,x+2}}$$

input
```
int(((f + g*x)*(3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/6,x)
```

output
```
((2*x - 3)*(30*x - 36*x^2 + 36)^(1/2)*(133*f + 96*g + 63*f*x + 96*g*x + 45
*g*x^2))/(630*(3*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \sqrt{2+3x}(f+gx)\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{\sqrt{-2x+3}\,\sqrt{6}\,(90g\,x^3+126f\,x^2+57g\,x^2+77fx-96gx-399f-288g)}{630}$$

input
```
int(1/6*(2+3*x)^(1/2)*(g*x+f)*(-36*x^2+30*x+36)^(1/2),x)
```

output
```
(sqrt( - 2*x + 3)*sqrt(6)*(126*f*x**2 + 77*f*x - 399*f + 90*g*x**3 + 57*g*
x**2 - 96*g*x - 288*g))/630
```

3.298 $\int \sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 39

$$\int \sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}\,dx=-\frac{13(3-2x)^{3/2}}{6\sqrt{6}}+\frac{1}{10}\sqrt{\frac{3}{2}}(3-2x)^{5/2}$$

output
```
-13/36*(3-2*x)^(3/2)*6^(1/2)+1/20*6^(1/2)*(3-2*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}\,dx=\frac{(-3+2x)(19+9x)\sqrt{6+5x-6x^2}}{15\sqrt{6}\sqrt{2+3x}}$$

input
```
Integrate[Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
((-3 + 2*x)*(19 + 9*x)*Sqrt[6 + 5*x - 6*x^2])/(15*Sqrt[6]*Sqrt[2 + 3*x])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{13}{2}\sqrt{\frac{1}{2}-\frac{x}{3}}-9\left(\frac{1}{2}-\frac{x}{3}\right)^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}\sqrt{\frac{3}{2}}(3-2x)^{5/2}-\frac{13(3-2x)^{3/2}}{6\sqrt{6}}$$

input
```
Int[Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
(-13*(3 - 2*x)^(3/2))/(6*Sqrt[6]) + (Sqrt[3/2]*(3 - 2*x)^(5/2))/10
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{(2x-3)(9x+19)\sqrt{-36x^2+30x+36}}{90\sqrt{3x+2}}$	32
default	$\frac{(2x-3)(9x+19)\sqrt{-36x^2+30x+36}}{90\sqrt{3x+2}}$	32
orering	$\frac{(2x-3)(9x+19)\sqrt{-36x^2+30x+36}}{90\sqrt{3x+2}}$	32
risch	$-\frac{\sqrt{\frac{-36x^2+30x+36}{3x+2}}\,\sqrt{3x+2}\,(18x^2+11x-57)(2x-3)}{15\sqrt{-36x^2+30x+36}\,\sqrt{-12x+18}}$	64

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/90*(2*x-3)*(9*x+19)*(-36*x^2+30*x+36)^(1/2)/(3*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}\,dx=\frac{(18\,x^2+11\,x-57)\sqrt{-36\,x^2+30\,x+36}}{90\,\sqrt{3\,x+2}}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2),x, algorithm="fricas")
```

output
```
1/90*(18*x^2 + 11*x - 57)*sqrt(-36*x^2 + 30*x + 36)/sqrt(3*x + 2)
```

Sympy [F]

$$\int \sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}\,dx=\frac{\sqrt{6}\int \sqrt{3x+2}\sqrt{-6x^2+5x+6}\,dx}{6}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2),x)
```

output
```
sqrt(6)*Integral(sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x)/6
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}\,dx=\frac{1}{90}\left(18i\sqrt{3}\sqrt{2}x^2+11i\sqrt{3}\sqrt{2}x-57i\sqrt{3}\sqrt{2}\right)\sqrt{2x-3}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2),x, algorithm="maxima")
```

output
```
1/90*(18*I*sqrt(3)*sqrt(2)*x^2 + 11*I*sqrt(3)*sqrt(2)*x - 57*I*sqrt(3)*sqr
t(2))*sqrt(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}\,dx=\frac{1}{180}\sqrt{6}\left(9(2x-3)^2\sqrt{-2x+3}-65(-2x+3)^{\frac{3}{2}}\right)$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2),x, algorithm="giac")
```

output
```
1/180*sqrt(6)*(9*(2*x - 3)^2*sqrt(-2*x + 3) - 65*(-2*x + 3)^(3/2))
```

Mupad [B] (verification not implemented)

Time = 10.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \frac{(18\,x^2+11\,x-57)\,\sqrt{-36\,x^2+30\,x+36}}{90\,\sqrt{3\,x+2}}$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/6,x)
```

output
```
((11*x + 18*x^2 - 57)*(30*x - 36*x^2 + 36)^(1/2))/(90*(3*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \frac{\sqrt{-2x+3}\,\sqrt{6}\,(18x^2+11x-57)}{90}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2),x)
```

output
```
(sqrt( - 2*x + 3)*sqrt(6)*(18*x**2 + 11*x - 57))/90
```

3.299 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{f+gx}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 106

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{f+gx}\,dx = -\frac{\sqrt{\frac{2}{3}}(3f-2g)\sqrt{3-2x}}{g^2}-\frac{(3-2x)^{3/2}}{\sqrt{6}g}+\frac{\sqrt{\frac{2}{3}}(3f-2g)\sqrt{2f+3g}\,\text{arctanh}\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2f+3g}}\right)}{g^{5/2}}$$

output
```
-1/3*6^(1/2)*(3*f-2*g)*(3-2*x)^(1/2)/g^2-1/6*(3-2*x)^(3/2)*6^(1/2)/g+1/3*6
^(1/2)*(3*f-2*g)*(2*f+3*g)^(1/2)*arctanh(g^(1/2)*(3-2*x)^(1/2)/(2*f+3*g)^(
1/2))/g^(5/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{f+gx}\,dx = \frac{\frac{3\sqrt{g}(-6f+g+2gx)\sqrt{6+5x-6x^2}}{\sqrt{2+3x}}+\frac{6(6f^2+5fg-6g^2)\text{arctanh}\left(\frac{\sqrt{2f+3g}\sqrt{6+5x-6x^2}}{\sqrt{g}(3-2x)\sqrt{2+3x}}\right)}{\sqrt{2f+3g}}}{3\sqrt{6}g^{5/2}}$$

input

```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x),x]
```

output

```
((3*Sqrt[g]*(-6*f + g + 2*g*x)*Sqrt[6 + 5*x - 6*x^2])/Sqrt[2 + 3*x] + (6*(
6*f^2 + 5*f*g - 6*g^2)*ArcTanh[(Sqrt[2*f + 3*g]*Sqrt[6 + 5*x - 6*x^2])/(Sq
rt[g]*(3 - 2*x)*Sqrt[2 + 3*x])])/Sqrt[2*f + 3*g])/(3*Sqrt[6]*g^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1245, 90, 27, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{f+gx}dx$$

$$\Big\downarrow \text{1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{f+gx}dx$$

$$\Big\downarrow \text{90}$$

$$-\frac{(3f-2g)\int \frac{\sqrt{3-2x}}{\sqrt{6}(f+gx)}dx}{g}-\frac{(3-2x)^{3/2}}{\sqrt{6}g}$$

$$\Big\downarrow \text{27}$$

$$-\frac{(3f-2g)\int \frac{\sqrt{3-2x}}{f+gx}dx}{\sqrt{6}g}-\frac{(3-2x)^{3/2}}{\sqrt{6}g}$$

$$\Big\downarrow \text{60}$$

$$-\frac{(3f-2g)\left(\frac{(2f+3g)\int \frac{1}{\sqrt{3-2x}(f+gx)}dx}{g}+\frac{2\sqrt{3-2x}}{g}\right)}{\sqrt{6}g}-\frac{(3-2x)^{3/2}}{\sqrt{6}g}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(3f-2g)\left(\frac{2\sqrt{3-2x}}{g}-\frac{(2f+3g)\int \frac{1}{\frac{1}{2}(2f+3g)-\frac{1}{2}g(3-2x)}d\sqrt{3-2x}}{g}\right)}{\sqrt{6}g}-\frac{(3-2x)^{3/2}}{\sqrt{6}g}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(3f-2g)\left(\frac{2\sqrt{3-2x}}{g}-\frac{2\sqrt{2f+3g}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2f+3g}}\right)}{g^{3/2}}\right)}{\sqrt{6}g}-\frac{(3-2x)^{3/2}}{\sqrt{6}g}$$

input
```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x),x]
```

output
```
-((3 - 2*x)^(3/2)/(Sqrt[6]*g)) - ((3*f - 2*g)*((2*Sqrt[3 - 2*x])/g - (2*Sq
rt[2*f + 3*g]*ArcTanh[(Sqrt[g]*Sqrt[3 - 2*x])/Sqrt[2*f + 3*g]])/g^(3/2)))/
(Sqrt[6]*g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.57

method	result
risch	$\frac{(-2gx+6f-g)(2x-3)\sqrt{\frac{-36x^2+30x+36}{3x+2}}\,\sqrt{3x+2}}{g^2\sqrt{-12x+18}\,\sqrt{-36x^2+30x+36}}+\frac{(6f^2+5fg-6g^2)\sqrt{6}\,\operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)\sqrt{\frac{-36x^2+30x+36}{3x+2}}\,\sqrt{3x+2}}{3g^2\sqrt{g(2f+3g)}\,\sqrt{-36x^2+30x+36}}$
default	$\sqrt{-6x^2+5x+6}\,\sqrt{6}\,\Big(2\sqrt{g(2f+3g)}\,\sqrt{3-2x}\,gx+12\operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)f^2+10\operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)fg-12\operatorname{arctanh}\Big($ $6\sqrt{3x+2}\,\sqrt{3-2x}\,\sqrt{g(2f+3g)}\,g^2$

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f),x,method=_RETURNVERB
OSE)
```

output
```
(-2*g*x+6*f-g)*(2*x-3)/g^2/(-12*x+18)^(1/2)*((-36*x^2+30*x+36)/(3*x+2))^(1
/2)*(3*x+2)^(1/2)/(-36*x^2+30*x+36)^(1/2)+1/3*(6*f^2+5*f*g-6*g^2)/g^2*6^(1
/2)/(g*(2*f+3*g))^(1/2)*arctanh(1/6*g*(-12*x+18)^(1/2)*6^(1/2)/(g*(2*f+3*g
))^(1/2))*((-36*x^2+30*x+36)/(3*x+2))^(1/2)*(3*x+2)^(1/2)/(-36*x^2+30*x+36
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.92

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{f+gx}\,dx$$
$$= \left[-\frac{3\sqrt{\frac{2}{3}}(3\,(3\,f-2\,g)x+6\,f-4\,g)\sqrt{\frac{2\,f+3\,g}{g}}\log\left(-\frac{6\,gx^2+\sqrt{\frac{2}{3}}\sqrt{-36\,x^2+30\,x+36}g\sqrt{3\,x+2}\sqrt{\frac{2\,f+3\,g}{g}}-2\,(3\,f+7\,g)x-4\,f-1}{3\,gx^2+(3\,f+2\,g)x+2\,f}\right.}{6\,(3\,g^2x+2\,g^2)}\right.$$
$$-\frac{6\sqrt{\frac{2}{3}}(3\,(3\,f-2\,g)x+6\,f-4\,g)\sqrt{-\frac{2\,f+3\,g}{g}}\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt{-36\,x^2+30\,x+36}g\sqrt{3\,x+2}\sqrt{-\frac{2\,f+3\,g}{g}}}{2\,(3\,(2\,f+3\,g)x+4\,f+6\,g)}\right)-(2\,gx-6\,f}{6\,(3\,g^2x+2\,g^2)}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f),x, algorithm="
fricas")
```

output
```
[-1/6*(3*sqrt(2/3)*(3*(3*f - 2*g)*x + 6*f - 4*g)*sqrt((2*f + 3*g)/g)*log(-
(6*g*x^2 + sqrt(2/3)*sqrt(-36*x^2 + 30*x + 36)*g*sqrt(3*x + 2)*sqrt((2*f +
 3*g)/g) - 2*(3*f + 7*g)*x - 4*f - 12*g)/(3*g*x^2 + (3*f + 2*g)*x + 2*f))
- (2*g*x - 6*f + g)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2))/(3*g^2*x + 2*
g^2), -1/6*(6*sqrt(2/3)*(3*(3*f - 2*g)*x + 6*f - 4*g)*sqrt(-(2*f + 3*g)/g)
*arctan(1/2*sqrt(2/3)*sqrt(-36*x^2 + 30*x + 36)*g*sqrt(3*x + 2)*sqrt(-(2*f
 + 3*g)/g)/(3*(2*f + 3*g)*x + 4*f + 6*g)) - (2*g*x - 6*f + g)*sqrt(-36*x^2
 + 30*x + 36)*sqrt(3*x + 2))/(3*g^2*x + 2*g^2)]
```

Sympy [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{f+gx}\,dx = \frac{\sqrt{6}\int \frac{\sqrt{3x+2}\sqrt{-6x^2+5x+6}}{f+gx}\,dx}{6}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f),x)
```

output
```
sqrt(6)*Integral(sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6)/(f + g*x), x)/6
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{f+gx}\,dx=\int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,(gx+f)}\,dx$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f),x, algorithm="
maxima")
```

output
```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(g*x + f), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{f+gx}\,dx= -\frac{1}{6}\sqrt{6}\left(\frac{2\,(6\,f^2+5\,fg-6\,g^2)\arctan\left(\frac{g\sqrt{-2\,x+3}}{\sqrt{-2\,fg-3\,g^2}}\right)}{\sqrt{-2\,fg-3\,g^2}g^2}+\frac{g^2(-2\,x+3)^{\frac{3}{2}}+6\,fg\sqrt{-2\,x+3}-4\,g^2\sqrt{-2\,x+}}{g^3}\right.$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f),x, algorithm="
giac")
```

output
```
-1/6*sqrt(6)*(2*(6*f^2 + 5*f*g - 6*g^2)*arctan(g*sqrt(-2*x + 3)/sqrt(-2*f*
g - 3*g^2))/(sqrt(-2*f*g - 3*g^2)*g^2) + (g^2*(-2*x + 3)^(3/2) + 6*f*g*sqr
t(-2*x + 3) - 4*g^2*sqrt(-2*x + 3))/g^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{f+gx}\,dx=\int \frac{\sqrt{3\,x+2}\,\sqrt{-36\,x^2+30\,x+36}}{6\,\left(f+g\,x\right)}\,dx$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)),x)
```

output
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{f+gx}\,dx$$
$$=\frac{\sqrt{6}\left(6\sqrt{g}\,\sqrt{-2f-3g}\,atan\left(\frac{\sqrt{-2x+3}\,g}{\sqrt{g}\,\sqrt{-2f-3g}}\right)f-4\sqrt{g}\,\sqrt{-2f-3g}\,atan\left(\frac{\sqrt{-2x+3}\,g}{\sqrt{g}\,\sqrt{-2f-3g}}\right)g-6\sqrt{-2x+3}\,fg+2}{6g^3}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f),x)
```

output
```
(sqrt(6)*(6*sqrt(g)*sqrt( - 2*f - 3*g)*atan((sqrt( - 2*x + 3)*g)/(sqrt(g)*
sqrt( - 2*f - 3*g)))*f - 4*sqrt(g)*sqrt( - 2*f - 3*g)*atan((sqrt( - 2*x +
3)*g)/(sqrt(g)*sqrt( - 2*f - 3*g)))*g - 6*sqrt( - 2*x + 3)*f*g + 2*sqrt( -
 2*x + 3)*g**2*x + sqrt( - 2*x + 3)*g**2))/(6*g**3)
```

3.300 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 110

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^2}\,dx=\frac{\sqrt{6}\sqrt{3-2x}}{g^2}+\frac{(3f-2g)\sqrt{3-2x}}{\sqrt{6}g^2(f+gx)}-\frac{\sqrt{\frac{2}{3}}(9f+7g)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2f+3g}}\right)}{g^{5/2}\sqrt{2f+3g}}$$

output
```
6^(1/2)*(3-2*x)^(1/2)/g^2+1/6*(3*f-2*g)*(3-2*x)^(1/2)*6^(1/2)/g^2/(g*x+f)-
1/3*6^(1/2)*(9*f+7*g)*arctanh(g^(1/2)*(3-2*x)^(1/2)/(2*f+3*g)^(1/2))/g^(5/
2)/(2*f+3*g)^(1/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^2}\,dx=\frac{\frac{\sqrt{g}(9f-2g+6gx)\sqrt{6+5x-6x^2}}{\sqrt{2+3x}(f+gx)}+\frac{2(9f+7g)\text{arctanh}\left(\frac{\sqrt{2f+3g}\sqrt{6+5x-6x^2}}{\sqrt{g}(-3+2x)\sqrt{2+3x}}\right)}{\sqrt{2f+3g}}}{\sqrt{6}g^{5/2}}$$

input

```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^2,x]
```

output

```
((Sqrt[g]*(9*f - 2*g + 6*g*x)*Sqrt[6 + 5*x - 6*x^2])/(Sqrt[2 + 3*x]*(f + g
*x)) + (2*(9*f + 7*g)*ArcTanh[(Sqrt[2*f + 3*g]*Sqrt[6 + 5*x - 6*x^2])/(Sqr
t[g]*(-3 + 2*x)*Sqrt[2 + 3*x])])/Sqrt[2*f + 3*g])/(Sqrt[6]*g^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1245, 87, 27, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{(f+gx)^2}dx$$

$$\Big\downarrow \text{ 1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{(f+gx)^2}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(9f+7g)\int \frac{\sqrt{3-2x}}{\sqrt{6}(f+gx)}dx}{g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{\sqrt{6}g(2f+3g)(f+gx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(9f+7g)\int \frac{\sqrt{3-2x}}{f+gx}dx}{\sqrt{6}g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{\sqrt{6}g(2f+3g)(f+gx)}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(9f+7g)\left(\frac{(2f+3g)\int \frac{1}{\sqrt{3-2x}(f+gx)}dx}{g}+\frac{2\sqrt{3-2x}}{g}\right)}{\sqrt{6}g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{\sqrt{6}g(2f+3g)(f+gx)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(9f+7g)\left(\frac{2\sqrt{3-2x}}{g}-\frac{(2f+3g)\int \frac{1}{\frac{1}{2}(2f+3g)-\frac{1}{2}g(3-2x)}d\sqrt{3-2x}}{g}\right)}{\sqrt{6}g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{\sqrt{6}g(2f+3g)(f+gx)}$$

$$\Big\downarrow 221$$

$$\frac{(9f+7g)\left(\frac{2\sqrt{3-2x}}{g}-\frac{2\sqrt{2f+3g}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2f+3g}}\right)}{g^{3/2}}\right)}{\sqrt{6}g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{\sqrt{6}g(2f+3g)(f+gx)}$$

input

```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^2,x]
```

output

```
((3*f - 2*g)*(3 - 2*x)^(3/2))/(Sqrt[6]*g*(2*f + 3*g)*(f + g*x)) + ((9*f +
7*g)*((2*Sqrt[3 - 2*x])/g - (2*Sqrt[2*f + 3*g]*ArcTanh[(Sqrt[g]*Sqrt[3 - 2
*x])/Sqrt[2*f + 3*g]])/g^(3/2)))/(Sqrt[6]*g*(2*f + 3*g))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(89) = 178$.

Time = 1.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{6(2x-3)\sqrt{\frac{-36x^{2}+30x+36}{3x+2}}\,\sqrt{3x+2}}{g^{2}\sqrt{-12x+18}\,\sqrt{-36x^{2}+30x+36}}+\frac{\left(\frac{2(-3f+2g)\sqrt{-12x+18}}{g(-12x+18)-12f-18g}-\frac{(9f+7g)\sqrt{6}\,\operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)}{3\sqrt{g(2f+3g)}}\right)\sqrt{\frac{-36x^{2}+30x+36}{3x+2}}\,\sqrt{3x+2}}{g^{2}\sqrt{-36x^{2}+30x+36}}$
default	$\frac{\Bigg(-18\operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)fgx-14\operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)g^{2}x+6\sqrt{g(2f+3g)}\,\sqrt{3-2x}\,gx-18\operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)f}{6\sqrt{3x+2}\,\sqrt{3-2x}\,\sqrt{g(2f+3g)}\,g^{2}(}$

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^2,x,method=_RETURNVE
RBOSE)
```

output
```
-6*(2*x-3)/g^2/(-12*x+18)^(1/2)*((-36*x^2+30*x+36)/(3*x+2))^(1/2)*(3*x+2)^
(1/2)/(-36*x^2+30*x+36)^(1/2)+1/g^2*(2*(-3*f+2*g)*(-12*x+18)^(1/2)/(g*(-12
*x+18)-12*f-18*g)-1/3*(9*f+7*g)*6^(1/2)/(g*(2*f+3*g))^(1/2)*arctanh(1/6*g*
(-12*x+18)^(1/2)*6^(1/2)/(g*(2*f+3*g))^(1/2)))*((-36*x^2+30*x+36)/(3*x+2))
^(1/2)*(3*x+2)^(1/2)/(-36*x^2+30*x+36)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(89) = 178$.

Time = 0.08 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.60

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^2}\,dx$$

$$= \left[\frac{(6\,gx+9\,f-2\,g)\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}+\frac{3\sqrt{\frac{2}{3}}(3\,(9\,fg+7\,g^2)x^2+18\,f^2+14\,fg+(27\,f^2+39\,fg+14\,g^2)x)\log\Bigl(-}{\sqrt{2\,fg-}}}{6\,(3\,g^3x^2+2\,fg^2+(3\,fg^2+2\,g^3)x)} \right.$$

$$-\frac{6\sqrt{\frac{2}{3}}(3\,(9\,fg+7\,g^2)x^2+18\,f^2+14\,fg+(27\,f^2+39\,fg+14\,g^2)x)\sqrt{-\frac{1}{2\,fg+3\,g^2}}\arctan\Biggl(\frac{\sqrt{\frac{2}{3}}\sqrt{-36\,x^2+30}}{}}{6\,(3\,g^3x^2+2\,fg^2+(3\,fg^2+}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^2,x, algorithm
="fricas")
```

output
```
[1/6*((6*g*x + 9*f - 2*g)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2) + 3*sqrt
(2/3)*(3*(9*f*g + 7*g^2)*x^2 + 18*f^2 + 14*f*g + (27*f^2 + 39*f*g + 14*g^2
)*x)*log(-(6*g*x^2 + sqrt(2/3)*sqrt(2*f*g + 3*g^2)*sqrt(-36*x^2 + 30*x + 3
6)*sqrt(3*x + 2) - 2*(3*f + 7*g)*x - 4*f - 12*g)/(3*g*x^2 + (3*f + 2*g)*x
+ 2*f))/sqrt(2*f*g + 3*g^2))/(3*g^3*x^2 + 2*f*g^2 + (3*f*g^2 + 2*g^3)*x),
-1/6*(6*sqrt(2/3)*(3*(9*f*g + 7*g^2)*x^2 + 18*f^2 + 14*f*g + (27*f^2 + 39*
f*g + 14*g^2)*x)*sqrt(-1/(2*f*g + 3*g^2))*arctan(1/2*sqrt(2/3)*sqrt(-36*x^
2 + 30*x + 36)*(2*f + 3*g)*sqrt(3*x + 2)*sqrt(-1/(2*f*g + 3*g^2))/(6*x^2 -
 5*x - 6)) - (6*g*x + 9*f - 2*g)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2))/
(3*g^3*x^2 + 2*f*g^2 + (3*f*g^2 + 2*g^3)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^2}\,dx=\frac{\sqrt{6}\int \frac{\sqrt{3x+2}\sqrt{-6x^2+5x+6}}{f^2+2fgx+g^2x^2}\,dx}{6}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f)**2,x)
```

output
```
sqrt(6)*Integral(sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6)/(f**2 + 2*f*g*x + g
**2*x**2), x)/6
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^2}\,dx=\int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,(gx+f)^2}\,dx$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^2,x, algorithm
="maxima")
```

output
```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(g*x + f)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^2}\,dx$$
$$=\frac{1}{3}\,\sqrt{6}\left(\frac{(9\,f+7\,g)\arctan\left(\frac{g\sqrt{-2\,x+3}}{\sqrt{-2\,fg-3\,g^2}}\right)}{\sqrt{-2\,fg-3\,g^2}g^2}+\frac{3\,\sqrt{-2\,x+3}}{g^2}+\frac{3\,f\sqrt{-2\,x+3}-2\,g\sqrt{-2\,x+3}}{(g(2\,x-3)+2\,f+3\,g)g^2}\right)$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^2,x, algorithm
="giac")
```

output
```
1/3*sqrt(6)*((9*f + 7*g)*arctan(g*sqrt(-2*x + 3)/sqrt(-2*f*g - 3*g^2))/(sq
rt(-2*f*g - 3*g^2)*g^2) + 3*sqrt(-2*x + 3)/g^2 + (3*f*sqrt(-2*x + 3) - 2*g
*sqrt(-2*x + 3))/((g*(2*x - 3) + 2*f + 3*g)*g^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^2}\,dx=\int \frac{\sqrt{3\,x+2}\,\sqrt{-36\,x^2+30\,x+36}}{6\,{\left(f+g\,x\right)}^2}\,dx$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^2),x)
```

output
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^2}\,dx$$

$$=\frac{\sqrt{6}\left(-18\sqrt{g}\,\sqrt{-2f-3g}\,atan\!\left(\frac{\sqrt{-2x+3}\,g}{\sqrt{g}\,\sqrt{-2f-3g}}\right)f^2-18\sqrt{g}\,\sqrt{-2f-3g}\,atan\!\left(\frac{\sqrt{-2x+3}\,g}{\sqrt{g}\,\sqrt{-2f-3g}}\right)fgx-14\sqrt{g}\,\sqrt{-2}\right.}{}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^2,x)
```

output
```
(sqrt(6)*( - 18*sqrt(g)*sqrt( - 2*f - 3*g)*atan((sqrt( - 2*x + 3)*g)/(sqrt
(g)*sqrt( - 2*f - 3*g)))*f**2 - 18*sqrt(g)*sqrt( - 2*f - 3*g)*atan((sqrt(
- 2*x + 3)*g)/(sqrt(g)*sqrt( - 2*f - 3*g)))*f*g*x - 14*sqrt(g)*sqrt( - 2*f
 - 3*g)*atan((sqrt( - 2*x + 3)*g)/(sqrt(g)*sqrt( - 2*f - 3*g)))*f*g - 14*s
qrt(g)*sqrt( - 2*f - 3*g)*atan((sqrt( - 2*x + 3)*g)/(sqrt(g)*sqrt( - 2*f -
 3*g)))*g**2*x + 18*sqrt( - 2*x + 3)*f**2*g + 12*sqrt( - 2*x + 3)*f*g**2*x
 + 23*sqrt( - 2*x + 3)*f*g**2 + 18*sqrt( - 2*x + 3)*g**3*x - 6*sqrt( - 2*x
 + 3)*g**3))/(6*g**3*(2*f**2 + 2*f*g*x + 3*f*g + 3*g**2*x))
```

3.301 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 136

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^3}\,dx=\frac{(3f-2g)\sqrt{3-2x}}{2\sqrt{6}g^2(f+gx)^2}-\frac{(15f+16g)\sqrt{3-2x}}{2\sqrt{6}g^2(2f+3g)(f+gx)}+\frac{(9f+20g)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2f+3g}}\right)}{\sqrt{6}g^{5/2}(2f+3g)^{3/2}}$$

output
```
1/12*(3*f-2*g)*(3-2*x)^(1/2)*6^(1/2)/g^2/(g*x+f)^2-1/12*(15*f+16*g)*(3-2*x
)^(1/2)*6^(1/2)/g^2/(2*f+3*g)/(g*x+f)+1/6*(9*f+20*g)*arctanh(g^(1/2)*(3-2*
x)^(1/2)/(2*f+3*g)^(1/2))*6^(1/2)/g^(5/2)/(2*f+3*g)^(3/2)
```

Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^3}\,dx=\frac{-\frac{\sqrt{g}\sqrt{6+5x-6x^2}(9f^2+2g^2(3+8x)+fg(11+15x))}{(2f+3g)\sqrt{2+3x}(f+gx)^2}+\frac{2(9f+20g)\text{arctanh}\left(\frac{\sqrt{2f+3g}\sqrt{6+5x-6x^2}}{\sqrt{g}(3-2x)\sqrt{2+3x}}\right)}{(2f+3g)^{3/2}}}{2\sqrt{6}g^{5/2}}$$

input

```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^3,x]
```

output

```
(-((Sqrt[g]*Sqrt[6 + 5*x - 6*x^2]*(9*f^2 + 2*g^2*(3 + 8*x) + f*g*(11 + 15*
x)))/((2*f + 3*g)*Sqrt[2 + 3*x]*(f + g*x)^2)) + (2*(9*f + 20*g)*ArcTanh[(S
qrt[2*f + 3*g]*Sqrt[6 + 5*x - 6*x^2])/(Sqrt[g]*(3 - 2*x)*Sqrt[2 + 3*x])])/
(2*f + 3*g)^(3/2))/(2*Sqrt[6]*g^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1245, 87, 27, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{(f+gx)^3}dx$$

$$\Big\downarrow \text{1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{(f+gx)^3}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(9f+20g)\int \frac{\sqrt{3-2x}}{\sqrt{6}(f+gx)^2}dx}{2g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{2\sqrt{6}g(2f+3g)(f+gx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(9f+20g)\int \frac{\sqrt{3-2x}}{(f+gx)^2}dx}{2\sqrt{6}g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{2\sqrt{6}g(2f+3g)(f+gx)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{(9f+20g)\left(-\frac{\int \frac{1}{\sqrt{3-2x}(f+gx)}dx}{g}-\frac{\sqrt{3-2x}}{g(f+gx)}\right)}{2\sqrt{6}g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{2\sqrt{6}g(2f+3g)(f+gx)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{(9f+20g)\left(\frac{\int \frac{1}{\frac{1}{2}(2f+3g)-\frac{1}{2}g(3-2x)}d\sqrt{3-2x}}{g}-\frac{\sqrt{3-2x}}{g(f+gx)}\right)}{2\sqrt{6}g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{2\sqrt{6}g(2f+3g)(f+gx)^2}$$

$$\Big\downarrow 221$$

$$\frac{(9f+20g)\left(\frac{2\text{arctanh}\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2f+3g}}\right)}{g^{3/2}\sqrt{2f+3g}}-\frac{\sqrt{3-2x}}{g(f+gx)}\right)}{2\sqrt{6}g(2f+3g)}+\frac{(3-2x)^{3/2}(3f-2g)}{2\sqrt{6}g(2f+3g)(f+gx)^2}$$

input

```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^3,x]
```

output

```
((3*f - 2*g)*(3 - 2*x)^(3/2))/(2*Sqrt[6]*g*(2*f + 3*g)*(f + g*x)^2) + ((9*
f + 20*g)*(-(Sqrt[3 - 2*x]/(g*(f + g*x))) + (2*ArcTanh[(Sqrt[g]*Sqrt[3 - 2
*x])/Sqrt[2*f + 3*g]])/(g^(3/2)*Sqrt[2*f + 3*g])))/(2*Sqrt[6]*g*(2*f + 3*g
))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(113) = 226$.

Time = 1.41 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.72

method	result
default	$\sqrt{-6x^2+5x+6}\,\sqrt{6}\left(18\ \operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)f\,g^2x^2+40\ \operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)g^3x^2+36\ \operatorname{arctanh}\left(\frac{g\sqrt{-12x+18}\,\sqrt{6}}{6\sqrt{g(2f+3g)}}\right)f^2gx+$

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^3,x,method=_RETURNVE
RBOSE)
```

output

```
1/12*(-6*x^2+5*x+6)^(1/2)*6^(1/2)*(18*arctanh(1/6*g*(-12*x+18)^(1/2)*6^(1/
2)/(g*(2*f+3*g))^(1/2))*f*g^2*x^2+40*arctanh(1/6*g*(-12*x+18)^(1/2)*6^(1/2
)/(g*(2*f+3*g))^(1/2))*g^3*x^2+36*arctanh(1/6*g*(-12*x+18)^(1/2)*6^(1/2)/(
g*(2*f+3*g))^(1/2))*f^2*g*x+80*arctanh(1/6*g*(-12*x+18)^(1/2)*6^(1/2)/(g*(
2*f+3*g))^(1/2))*f*g^2*x-15*(g*(2*f+3*g))^(1/2)*(3-2*x)^(1/2)*f*g*x-16*(g*
(2*f+3*g))^(1/2)*(3-2*x)^(1/2)*g^2*x+18*arctanh(1/6*g*(-12*x+18)^(1/2)*6^(
1/2)/(g*(2*f+3*g))^(1/2))*f^3+40*arctanh(1/6*g*(-12*x+18)^(1/2)*6^(1/2)/(g
*(2*f+3*g))^(1/2))*f^2*g-9*(g*(2*f+3*g))^(1/2)*(3-2*x)^(1/2)*f^2-11*(g*(2*
f+3*g))^(1/2)*(3-2*x)^(1/2)*f*g-6*(g*(2*f+3*g))^(1/2)*(3-2*x)^(1/2)*g^2)/(
3*x+2)^(1/2)/(3-2*x)^(1/2)/(g*(2*f+3*g))^(1/2)/g^2/(2*f+3*g)/(g*x+f)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(113) = 226$.

Time = 0.10 (sec) , antiderivative size = 684, normalized size of antiderivative = 5.03

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^3}\,dx = \text{Too large to display}$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^3,x, algorithm
="fricas")
```

output
```
[1/12*((3*(9*f*g^2 + 20*g^3)*x^3 + 18*f^3 + 40*f^2*g + 2*(27*f^2*g + 69*f*
g^2 + 20*g^3)*x^2 + (27*f^3 + 96*f^2*g + 80*f*g^2)*x)*sqrt(12*f*g + 18*g^2
)*log(-(18*g*x^2 - 6*(3*f + 7*g)*x - sqrt(12*f*g + 18*g^2)*sqrt(-36*x^2 +
30*x + 36)*sqrt(3*x + 2) - 12*f - 36*g)/(3*g*x^2 + (3*f + 2*g)*x + 2*f)) -
 (18*f^3*g + 49*f^2*g^2 + 45*f*g^3 + 18*g^4 + (30*f^2*g^2 + 77*f*g^3 + 48*
g^4)*x)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2))/(8*f^4*g^3 + 24*f^3*g^4 +
 18*f^2*g^5 + 3*(4*f^2*g^5 + 12*f*g^6 + 9*g^7)*x^3 + 2*(12*f^3*g^4 + 40*f^
2*g^5 + 39*f*g^6 + 9*g^7)*x^2 + (12*f^4*g^3 + 52*f^3*g^4 + 75*f^2*g^5 + 36
*f*g^6)*x), -1/12*(2*(3*(9*f*g^2 + 20*g^3)*x^3 + 18*f^3 + 40*f^2*g + 2*(27
*f^2*g + 69*f*g^2 + 20*g^3)*x^2 + (27*f^3 + 96*f^2*g + 80*f*g^2)*x)*sqrt(-
12*f*g - 18*g^2)*arctan(1/6*sqrt(-12*f*g - 18*g^2)*sqrt(-36*x^2 + 30*x + 3
6)*sqrt(3*x + 2)/(3*(2*f + 3*g)*x + 4*f + 6*g)) + (18*f^3*g + 49*f^2*g^2 +
 45*f*g^3 + 18*g^4 + (30*f^2*g^2 + 77*f*g^3 + 48*g^4)*x)*sqrt(-36*x^2 + 30
*x + 36)*sqrt(3*x + 2))/(8*f^4*g^3 + 24*f^3*g^4 + 18*f^2*g^5 + 3*(4*f^2*g^
5 + 12*f*g^6 + 9*g^7)*x^3 + 2*(12*f^3*g^4 + 40*f^2*g^5 + 39*f*g^6 + 9*g^7)
*x^2 + (12*f^4*g^3 + 52*f^3*g^4 + 75*f^2*g^5 + 36*f*g^6)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^3}\,dx = \frac{\sqrt{6}\int \frac{\sqrt{3x+2}\sqrt{-6x^2+5x+6}}{f^3+3f^2gx+3fg^2x^2+g^3x^3}\,dx}{6}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f)**3,x)
```

output
```
sqrt(6)*Integral(sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6)/(f**3 + 3*f**2*g*x
+ 3*f*g**2*x**2 + g**3*x**3), x)/6
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^3}\,dx = \int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,(gx+f)^3}\,dx$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^3,x, algorithm
="maxima")
```

output
```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(g*x + f)^3, x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^3}\,dx =$$

$$-\frac{1}{6}\sqrt{6}\left(\frac{(9f+20g)\arctan\left(\frac{g\sqrt{-2x+3}}{\sqrt{-2fg-3g^2}}\right)}{(2fg^2+3g^3)\sqrt{-2fg-3g^2}}-\frac{15fg(-2x+3)^{\frac{3}{2}}+16g^2(-2x+3)^{\frac{3}{2}}-18f^2\sqrt{-2x+3}-}{(2fg^2+3g^3)(g(2x-3)+2}\right.$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^3,x, algorithm
="giac")
```

output
```
-1/6*sqrt(6)*((9*f + 20*g)*arctan(g*sqrt(-2*x + 3)/sqrt(-2*f*g - 3*g^2))/(
(2*f*g^2 + 3*g^3)*sqrt(-2*f*g - 3*g^2)) - (15*f*g*(-2*x + 3)^(3/2) + 16*g^
2*(-2*x + 3)^(3/2) - 18*f^2*sqrt(-2*x + 3) - 67*f*g*sqrt(-2*x + 3) - 60*g^
2*sqrt(-2*x + 3))/((2*f*g^2 + 3*g^3)*(g*(2*x - 3) + 2*f + 3*g)^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^3}\,dx = \int \frac{\sqrt{3x+2}\,\sqrt{-36x^2+30x+36}}{6\,(f+g\,x)^3}\,dx$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^3),x)
```

output
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.01

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^3}\,dx$$
$$= \frac{\sqrt{6}\left(18\sqrt{g}\,\sqrt{-2f-3g}\,atan\left(\frac{\sqrt{-2x+3}\,g}{\sqrt{g}\,\sqrt{-2f-3g}}\right)f^3+36\sqrt{g}\,\sqrt{-2f-3g}\,atan\left(\frac{\sqrt{-2x+3}\,g}{\sqrt{g}\,\sqrt{-2f-3g}}\right)f^2gx+40\sqrt{g}\,\sqrt{-2\right.}{}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^3,x)
```

output
```
(sqrt(6)*(18*sqrt(g)*sqrt( - 2*f - 3*g)*atan((sqrt( - 2*x + 3)*g)/(sqrt(g)
*sqrt( - 2*f - 3*g)))*f**3 + 36*sqrt(g)*sqrt( - 2*f - 3*g)*atan((sqrt( - 2
*x + 3)*g)/(sqrt(g)*sqrt( - 2*f - 3*g)))*f**2*g*x + 40*sqrt(g)*sqrt( - 2*f
 - 3*g)*atan((sqrt( - 2*x + 3)*g)/(sqrt(g)*sqrt( - 2*f - 3*g)))*f**2*g + 1
8*sqrt(g)*sqrt( - 2*f - 3*g)*atan((sqrt( - 2*x + 3)*g)/(sqrt(g)*sqrt( - 2*
f - 3*g)))*f*g**2*x**2 + 80*sqrt(g)*sqrt( - 2*f - 3*g)*atan((sqrt( - 2*x +
 3)*g)/(sqrt(g)*sqrt( - 2*f - 3*g)))*f*g**2*x + 40*sqrt(g)*sqrt( - 2*f - 3
*g)*atan((sqrt( - 2*x + 3)*g)/(sqrt(g)*sqrt( - 2*f - 3*g)))*g**3*x**2 - 18
*sqrt( - 2*x + 3)*f**3*g - 30*sqrt( - 2*x + 3)*f**2*g**2*x - 49*sqrt( - 2*
x + 3)*f**2*g**2 - 77*sqrt( - 2*x + 3)*f*g**3*x - 45*sqrt( - 2*x + 3)*f*g*
*3 - 48*sqrt( - 2*x + 3)*g**4*x - 18*sqrt( - 2*x + 3)*g**4))/(12*g**3*(4*f
**4 + 8*f**3*g*x + 12*f**3*g + 4*f**2*g**2*x**2 + 24*f**2*g**2*x + 9*f**2*
g**2 + 12*f*g**3*x**2 + 18*f*g**3*x + 9*g**4*x**2))
```

3.302 $\int \sqrt{2+3x}(f+gx)^{3/2}\sqrt{1+\frac{5x}{6}-x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 220

$$\begin{aligned}
&\int \sqrt{2+3x}(f+gx)^{3/2}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \\
&-\frac{(18f-77g)(2f+3g)^2\sqrt{3-2x}\sqrt{f+gx}}{512\sqrt{6}g^2} \\
&+\frac{(18f-77g)(2f+3g)(3-2x)^{3/2}\sqrt{f+gx}}{256\sqrt{6}g}+\frac{(18f-77g)(3-2x)^{3/2}(f+gx)^{3/2}}{96\sqrt{6}g} \\
&-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}(f+gx)^{5/2}}{8g}+\frac{(18f-77g)(2f+3g)^3\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{1024\sqrt{3}g^{5/2}}
\end{aligned}$$

output

```
-1/3072*(18*f-77*g)*(2*f+3*g)^2*(3-2*x)^(1/2)*(g*x+f)^(1/2)*6^(1/2)/g^2+1/
1536*(18*f-77*g)*(2*f+3*g)*(3-2*x)^(3/2)*(g*x+f)^(1/2)*6^(1/2)/g+1/576*(18
*f-77*g)*(3-2*x)^(3/2)*(g*x+f)^(3/2)*6^(1/2)/g-1/16*6^(1/2)*(3-2*x)^(3/2)*
(g*x+f)^(5/2)/g+1/3072*(18*f-77*g)*(2*f+3*g)^3*arctan(1/2*g^(1/2)*(3-2*x)^
(1/2)*2^(1/2)/(g*x+f)^(1/2))*3^(1/2)/g^(5/2)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.15

$$\int \sqrt{2+3x}(f+gx)^{3/2}\sqrt{1+\frac{5x}{6}-x^2}\,dx =$$
$$-\frac{\sqrt{6+5x-6x^2}\left(-\sqrt{3}\sqrt{g}\sqrt{-3+2x}+\sqrt{6f+6gx}\right)\left(27\sqrt{2}\sqrt{g}\sqrt{-3+2x}\sqrt{f+gx}(-216f^3+12f^2g(5+1\right.}{41472g^{5/2}}$$

input
```
Integrate[Sqrt[2 + 3*x]*(f + g*x)^(3/2)*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
-1/41472*(Sqrt[6 + 5*x - 6*x^2]*(-(Sqrt[3]*Sqrt[g]*Sqrt[-3 + 2*x]) + Sqrt[
6*f + 6*g*x])*(27*Sqrt[2]*Sqrt[g]*Sqrt[-3 + 2*x]*Sqrt[f + g*x]*(-216*f^3 +
 12*f^2*g*(5 + 12*x) + 2*f*g^2*(-1605 + 536*x + 864*x^2) + g^3*(-2079 - 92
4*x + 736*x^2 + 1152*x^3)) + 81*(2*f + 3*g)^3*(-18*f + 77*g)*Log[-(Sqrt[3]
*Sqrt[g]*Sqrt[-3 + 2*x]) + Sqrt[6]*Sqrt[f + g*x]]))/(g^(5/2)*Sqrt[-9 + 6*x
]*Sqrt[4 + 6*x]*(Sqrt[6]*Sqrt[g]*Sqrt[-3 + 2*x] - 2*Sqrt[3]*Sqrt[f + g*x])
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1245, 90, 27, 60, 60, 60, 66, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}(f+gx)^{3/2}\,dx$$
$$\Big\downarrow \text{1245}$$
$$\int \sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)(f+gx)^{3/2}dx$$
$$\Big\downarrow \text{90}$$

$$-\frac{(18f-77g)\int \frac{\sqrt{3-2x}(f+gx)^{3/2}}{\sqrt{6}}dx}{16g}-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}(f+gx)^{5/2}}{8g}$$

$$\Big\downarrow \; 27$$

$$-\frac{(18f-77g)\int \sqrt{3-2x}(f+gx)^{3/2}dx}{16\sqrt{6}g}-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}(f+gx)^{5/2}}{8g}$$

$$\Big\downarrow \; 60$$

$$-\frac{(18f-77g)\left(\frac{1}{4}(2f+3g)\int \sqrt{3-2x}\sqrt{f+gx}dx-\frac{1}{6}(3-2x)^{3/2}(f+gx)^{3/2}\right)}{16\sqrt{6}g}-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}(f+gx)^{5/2}}{8g}$$

$$\Big\downarrow \; 60$$

$$-\frac{(18f-77g)\left(\frac{1}{4}(2f+3g)\left(\frac{1}{8}(2f+3g)\int \frac{\sqrt{3-2x}}{\sqrt{f+gx}}dx-\frac{1}{4}(3-2x)^{3/2}\sqrt{f+gx}\right)-\frac{1}{6}(3-2x)^{3/2}(f+gx)^{3/2}\right)}{16\sqrt{6}g}-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}(f+gx)^{5/2}}{8g}$$

$$\Big\downarrow \; 60$$

$$-\frac{(18f-77g)\left(\frac{1}{4}(2f+3g)\left(\frac{1}{8}(2f+3g)\left(\frac{(2f+3g)\int \frac{1}{\sqrt{3-2x}\sqrt{f+gx}}dx}{2g}+\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}\right)-\frac{1}{4}(3-2x)^{3/2}\sqrt{f+gx}\right)-\frac{1}{6}(\right.}{16\sqrt{6}g}$$

$$\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}(f+gx)^{5/2}}{8g}$$

$$\Big\downarrow \; 66$$

$$-\frac{(18f-77g)\left(\frac{1}{4}(2f+3g)\left(\frac{1}{8}(2f+3g)\left(\frac{(2f+3g)\int \frac{1}{-\frac{g(3-2x)}{f+gx}-2}d\frac{\sqrt{3-2x}}{\sqrt{f+gx}}}{g}+\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}\right)-\frac{1}{4}(3-2x)^{3/2}\sqrt{f+gx}\right)\right.}{16\sqrt{6}g}$$

$$\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}(f+gx)^{5/2}}{8g}$$

$$\Big\downarrow \; 217$$

$$-\frac{(18f-77g)\left(\frac{1}{4}(2f+3g)\left(\frac{1}{8}(2f+3g)\left(\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}-\frac{(2f+3g)\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{\sqrt{2}g^{3/2}}\right)-\frac{1}{4}(3-2x)^{3/2}\sqrt{f+gx}\right)-\right.}{16\sqrt{6}g}$$

$$\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}(f+gx)^{5/2}}{8g}$$

input

```
Int[Sqrt[2 + 3*x]*(f + g*x)^(3/2)*Sqrt[1 + (5*x)/6 - x^2],x]
```

output

```
-1/8*(Sqrt[3/2]*(3 - 2*x)^(3/2)*(f + g*x)^(5/2))/g - ((18*f - 77*g)*(-1/6*
((3 - 2*x)^(3/2)*(f + g*x)^(3/2)) + ((2*f + 3*g)*(-1/4*((3 - 2*x)^(3/2)*Sq
rt[f + g*x]) + ((2*f + 3*g)*((Sqrt[3 - 2*x]*Sqrt[f + g*x])/g - ((2*f + 3*g
)*ArcTan[(Sqrt[g]*Sqrt[3 - 2*x])/(Sqrt[2]*Sqrt[f + g*x])])/(Sqrt[2]*g^(3/2
))))/8))/4))/(16*Sqrt[6]*g)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(173) = 346$.

Time = 1.36 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.25

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{-6x^2+5x+6}\,\sqrt{3}\,\bigg(-2304\sqrt{2}\,g^{\frac{7}{2}}x^3\sqrt{-(2x-3)(gx+f)}-3456\sqrt{2}\,f\,g^{\frac{5}{2}}x^2\sqrt{-(2x-3)(gx+f)}-1472\sqrt{2}\,g^{\frac{7}{2}}x^2\sqrt{-(2x-3)(}}{}$

input
```
int(1/6*(3*x+2)^(1/2)*(g*x+f)^(3/2)*(-36*x^2+30*x+36)^(1/2),x,method=_RETU
RNVERBOSE)
```

output

```
-1/18432*(g*x+f)^(1/2)*(-6*x^2+5*x+6)^(1/2)*3^(1/2)/g^(5/2)*(-2304*2^(1/2)
*g^(7/2)*x^3*(-(2*x-3)*(g*x+f))^(1/2)-3456*2^(1/2)*f*g^(5/2)*x^2*(-(2*x-3)
*(g*x+f))^(1/2)-1472*2^(1/2)*g^(7/2)*x^2*(-(2*x-3)*(g*x+f))^(1/2)-288*g^(3
/2)*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*f^2*x-2144*g^(5/2)*2^(1/2)*(-(2*x-3)*
(g*x+f))^(1/2)*f*x+1848*g^(7/2)*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*x+432*g^(
1/2)*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*f^3-120*g^(3/2)*2^(1/2)*(-(2*x-3)*(g
*x+f))^(1/2)*f^2+6420*g^(5/2)*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*f+4158*g^(7
/2)*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)+432*arctan(1/4/g^(1/2)*(4*g*x+2*f-3*g
)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f^4+96*arctan(1/4/g^(1/2)*(4*g*x+2*f-3
*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f^3*g-5400*arctan(1/4/g^(1/2)*(4*g*x
+2*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f^2*g^2-11016*arctan(1/4/g^(1/
2)*(4*g*x+2*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f*g^3-6237*arctan(1/4
/g^(1/2)*(4*g*x+2*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*g^4)/(3*x+2)^(1
/2)/(-(2*x-3)*(g*x+f))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.67

$$\int \sqrt{2+3x}(f+gx)^{3/2}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \left[\frac{3\sqrt{3}(288\,f^4+64\,f^3g-3600\,f^2g^2-7344\,fg^3-4158\,g^4+3\,(144\,f^4+32\,f^3g\right.$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^(3/2)*(-36*x^2+30*x+36)^(1/2),x, algor
ithm="fricas")
```

output

```
[1/36864*(3*sqrt(3)*(288*f^4 + 64*f^3*g - 3600*f^2*g^2 - 7344*f*g^3 - 4158
*g^4 + 3*(144*f^4 + 32*f^3*g - 1800*f^2*g^2 - 3672*f*g^3 - 2079*g^4)*x)*sq
rt(-g)*log(-(288*g^2*x^3 - 4*sqrt(3)*(4*g*x + 2*f - 3*g)*sqrt(g*x + f)*sqr
t(-36*x^2 + 30*x + 36)*sqrt(-g)*sqrt(3*x + 2) + 48*(6*f*g - 5*g^2)*x^2 + 2
4*f^2 - 216*f*g + 54*g^2 + 3*(12*f^2 - 44*f*g - 69*g^2)*x)/(3*x + 2)) + 4*
(1152*g^4*x^3 - 216*f^3*g + 60*f^2*g^2 - 3210*f*g^3 - 2079*g^4 + 32*(54*f*
g^3 + 23*g^4)*x^2 + 4*(36*f^2*g^2 + 268*f*g^3 - 231*g^4)*x)*sqrt(g*x + f)*
sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2))/(3*g^3*x + 2*g^3), 1/18432*(3*sqr
t(3)*(288*f^4 + 64*f^3*g - 3600*f^2*g^2 - 7344*f*g^3 - 4158*g^4 + 3*(144*f
^4 + 32*f^3*g - 1800*f^2*g^2 - 3672*f*g^3 - 2079*g^4)*x)*sqrt(g)*arctan(1/
12*sqrt(3)*(4*g*x + 2*f - 3*g)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqr
t(g)*sqrt(3*x + 2)/(6*g^2*x^3 + (6*f*g - 5*g^2)*x^2 - 6*f*g - (5*f*g + 6*g
^2)*x)) + 2*(1152*g^4*x^3 - 216*f^3*g + 60*f^2*g^2 - 3210*f*g^3 - 2079*g^4
 + 32*(54*f*g^3 + 23*g^4)*x^2 + 4*(36*f^2*g^2 + 268*f*g^3 - 231*g^4)*x)*sq
rt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2))/(3*g^3*x + 2*g^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{2+3x}(f+gx)^{3/2}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \text{Timed out}$$

input

```
integrate(1/6*(2+3*x)**(1/2)*(g*x+f)**(3/2)*(-36*x**2+30*x+36)**(1/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \sqrt{2+3x}(f+gx)^{3/2}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \int \frac{1}{6}\,(gx+f)^{\frac{3}{2}}\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}\,dx$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^(3/2)*(-36*x^2+30*x+36)^(1/2),x, algor
ithm="maxima")
```

output

```
1/6*integrate((g*x + f)^(3/2)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2), x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{2+3x}(f+gx)^{3/2}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \text{Timed out}$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^(3/2)*(-36*x^2+30*x+36)^(1/2),x, algor
ithm="giac")
```

output

```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+3x}(f+gx)^{3/2}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \int \frac{(f+g\,x)^{3/2}\,\sqrt{3\,x+2}\,\sqrt{-36\,x^2+30\,x+36}}{6}\,dx$$

input

```
int(((f + g*x)^(3/2)*(3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/6,x)
```

output

```
int(((f + g*x)^(3/2)*(3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/6, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.24

$$\int \sqrt{2+3x}(f+gx)^{3/2}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \frac{\sqrt{3}\left(864\sqrt{g}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f^5+1488\sqrt{g}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f^4g-10512\sqrt{g}\,a\right.}{}$$

input

```
int(1/6*(2+3*x)^(1/2)*(g*x+f)^(3/2)*(-36*x^2+30*x+36)^(1/2),x)
```

output

```
(sqrt(3)*(864*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f**
5 + 1488*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f**4*g -
 10512*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f**3*g**2
- 38232*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f**2*g**3
 - 45522*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f*g**4 -
 18711*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*g**5 - 432
*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*f**4*g + 288*sqrt(f + g*x)*sqrt( -
 2*x + 3)*sqrt(2)*f**3*g**2*x - 528*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)
*f**3*g**2 + 3456*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*f**2*g**3*x**2 +
2576*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*f**2*g**3*x - 6240*sqrt(f + g*
x)*sqrt( - 2*x + 3)*sqrt(2)*f**2*g**3 + 2304*sqrt(f + g*x)*sqrt( - 2*x + 3
)*sqrt(2)*f*g**4*x**3 + 6656*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*f*g**4
*x**2 + 1368*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*f*g**4*x - 13788*sqrt(
f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*f*g**4 + 3456*sqrt(f + g*x)*sqrt( - 2*x
+ 3)*sqrt(2)*g**5*x**3 + 2208*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*g**5*
x**2 - 2772*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*g**5*x - 6237*sqrt(f +
g*x)*sqrt( - 2*x + 3)*sqrt(2)*g**5))/(9216*g**3*(2*f + 3*g))
```

3.303 $\int \sqrt{2+3x}\sqrt{f+gx}\sqrt{1+\frac{5x}{6}-x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 172

$$\begin{aligned}\int \sqrt{2+3x}\sqrt{f+gx}\sqrt{1+\frac{5x}{6}-x^2}\,dx = {}& -\frac{(6f-17g)(2f+3g)\sqrt{3-2x}\sqrt{f+gx}}{32\sqrt{6}g^2} \\ & +\frac{(6f-17g)(3-2x)^{3/2}\sqrt{f+gx}}{16\sqrt{6}g} \\ & -\frac{(3-2x)^{3/2}(f+gx)^{3/2}}{2\sqrt{6}g} \\ & +\frac{(6f-17g)(2f+3g)^2\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{64\sqrt{3}g^{5/2}}\end{aligned}$$

output

```
-1/192*(6*f-17*g)*(2*f+3*g)*(3-2*x)^(1/2)*(g*x+f)^(1/2)*6^(1/2)/g^2+1/96*(
6*f-17*g)*(3-2*x)^(3/2)*(g*x+f)^(1/2)*6^(1/2)/g-1/12*(3-2*x)^(3/2)*(g*x+f)
^(3/2)*6^(1/2)/g+1/192*(6*f-17*g)*(2*f+3*g)^2*arctan(1/2*g^(1/2)*(3-2*x)^(
1/2)*2^(1/2)/(g*x+f)^(1/2))*3^(1/2)/g^(5/2)
```

Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19

$$\int \sqrt{2+3x}\sqrt{f+gx}\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$=\frac{\sqrt{6+5x-6x^2}\Big(-2\sqrt{-2f-3g}\sqrt{g}(-3+2x)\left(12f^3+4f^2g(-1+x)+fg^2(51-24x-40x^2)+g^3x(51-\right.}{64\sqrt{6}\sqrt{-2f-3g}g^{5/2}(-3+2x)\sqrt{2+3}}$$

input
```
Integrate[Sqrt[2 + 3*x]*Sqrt[f + g*x]*Sqrt[1 + (5*x)/6 - x^2],x]
```

output
```
(Sqrt[6 + 5*x - 6*x^2]*(-2*Sqrt[-2*f - 3*g]*Sqrt[g]*(-3 + 2*x)*(12*f^3 + 4
*f^2*g*(-1 + x) + f*g^2*(51 - 24*x - 40*x^2) + g^3*x*(51 - 20*x - 32*x^2))
 + (6*f - 17*g)*(2*f + 3*g)^3*Sqrt[6 - 4*x]*Sqrt[(f + g*x)/(2*f + 3*g)]*Ar
cSinh[(Sqrt[g]*Sqrt[3 - 2*x])/Sqrt[-2*f - 3*g]]))/(64*Sqrt[6]*Sqrt[-2*f -
3*g]*g^(5/2)*(-3 + 2*x)*Sqrt[2 + 3*x]*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1245, 90, 27, 60, 60, 66, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}\sqrt{f+gx}\,dx$$
$$\Big\downarrow \text{1245}$$
$$\int \sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)\sqrt{f+gx}dx$$
$$\Big\downarrow \text{90}$$
$$-\frac{(6f-17g)\int \frac{\sqrt{3-2x}\sqrt{f+gx}}{\sqrt{6}}dx}{4g}-\frac{(3-2x)^{3/2}(f+gx)^{3/2}}{2\sqrt{6}g}$$

$$\Big\downarrow 27$$

$$-\frac{(6f-17g)\int \sqrt{3-2x}\sqrt{f+gx}dx}{4\sqrt{6}g}-\frac{(3-2x)^{3/2}(f+gx)^{3/2}}{2\sqrt{6}g}$$

$$\Big\downarrow 60$$

$$-\frac{(6f-17g)\left(\frac{1}{8}(2f+3g)\int \frac{\sqrt{3-2x}}{\sqrt{f+gx}}dx-\frac{1}{4}(3-2x)^{3/2}\sqrt{f+gx}\right)}{4\sqrt{6}g}-\frac{(3-2x)^{3/2}(f+gx)^{3/2}}{2\sqrt{6}g}$$

$$\Big\downarrow 60$$

$$-\frac{(6f-17g)\left(\frac{1}{8}(2f+3g)\left(\frac{(2f+3g)\int \frac{1}{\sqrt{3-2x}\sqrt{f+gx}}dx}{2g}+\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}\right)-\frac{1}{4}(3-2x)^{3/2}\sqrt{f+gx}\right)}{4\sqrt{6}g}-\frac{(3-2x)^{3/2}(f+gx)^{3/2}}{2\sqrt{6}g}$$

$$\Big\downarrow 66$$

$$-\frac{(6f-17g)\left(\frac{1}{8}(2f+3g)\left(\frac{(2f+3g)\int \frac{1}{-\frac{g(3-2x)}{f+gx}-2}d\frac{\sqrt{3-2x}}{\sqrt{f+gx}}}{g}+\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}\right)-\frac{1}{4}(3-2x)^{3/2}\sqrt{f+gx}\right)}{4\sqrt{6}g}-\frac{(3-2x)^{3/2}(f+gx)^{3/2}}{2\sqrt{6}g}$$

$$\Big\downarrow 217$$

$$-\frac{(6f-17g)\left(\frac{1}{8}(2f+3g)\left(\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}-\frac{(2f+3g)\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{\sqrt{2}g^{3/2}}\right)-\frac{1}{4}(3-2x)^{3/2}\sqrt{f+gx}\right)}{4\sqrt{6}g}-\frac{(3-2x)^{3/2}(f+gx)^{3/2}}{2\sqrt{6}g}$$

input

```
Int[Sqrt[2 + 3*x]*Sqrt[f + g*x]*Sqrt[1 + (5*x)/6 - x^2],x]
```

output

```
-1/2*((3 - 2*x)^(3/2)*(f + g*x)^(3/2))/(Sqrt[6]*g) - ((6*f - 17*g)*(-1/4*(
(3 - 2*x)^(3/2)*Sqrt[f + g*x]) + ((2*f + 3*g)*((Sqrt[3 - 2*x]*Sqrt[f + g*x
])/g - ((2*f + 3*g)*ArcTan[(Sqrt[g]*Sqrt[3 - 2*x])/(Sqrt[2]*Sqrt[f + g*x])
])/(Sqrt[2]*g^(3/2))))/8))/(4*Sqrt[6]*g)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(135) = 270$.

Time = 1.39 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{-6x^{2}+5x+6}\,\sqrt{3}\,\left(-64\sqrt{2}\,g^{\frac{5}{2}}x^{2}\sqrt{-(2x-3)(gx+f)}-16\sqrt{2}\,\sqrt{-(2x-3)(gx+f)}\,g^{\frac{3}{2}}fx-40\sqrt{2}\,\sqrt{-(2x-3)(gx+f)}\,g^{\frac{5}{2}}x+\right.}{}$

input
```
int(1/6*(3*x+2)^(1/2)*(g*x+f)^(1/2)*(-36*x^2+30*x+36)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
-1/384*(g*x+f)^(1/2)*(-6*x^2+5*x+6)^(1/2)*3^(1/2)/g^(5/2)*(-64*2^(1/2)*g^(
5/2)*x^2*(-(2*x-3)*(g*x+f))^(1/2)-16*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*g^(3
/2)*f*x-40*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*g^(5/2)*x+24*2^(1/2)*(-(2*x-3)
*(g*x+f))^(1/2)*g^(1/2)*f^2-8*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*g^(3/2)*f+1
02*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*g^(5/2)+24*arctan(1/4/g^(1/2)*(4*g*x+2
*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f^3+4*arctan(1/4/g^(1/2)*(4*g*x+
2*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f^2*g-150*arctan(1/4/g^(1/2)*(4
*g*x+2*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f*g^2-153*arctan(1/4/g^(1/
2)*(4*g*x+2*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*g^3)/(3*x+2)^(1/2)/(-
(2*x-3)*(g*x+f))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.84

$$\int \sqrt{2+3x}\sqrt{f+gx}\sqrt{1+\frac{5x}{6}-x^2}\,dx$$

$$= \left[\frac{\sqrt{3}(48\,f^3+8\,f^2g-300\,fg^2-306\,g^3+3\,(24\,f^3+4\,f^2g-150\,fg^2-153\,g^3)x)\sqrt{-g}\log\left(-\frac{288\,g^2x^3-4\,\sqrt{3}}{}\right.}{} \right.$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^(1/2)*(-36*x^2+30*x+36)^(1/2),x, algor
ithm="fricas")
```

output
```
[1/768*(sqrt(3)*(48*f^3 + 8*f^2*g - 300*f*g^2 - 306*g^3 + 3*(24*f^3 + 4*f^
2*g - 150*f*g^2 - 153*g^3)*x)*sqrt(-g)*log(-(288*g^2*x^3 - 4*sqrt(3)*(4*g*
x + 2*f - 3*g)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(-g)*sqrt(3*x +
 2) + 48*(6*f*g - 5*g^2)*x^2 + 24*f^2 - 216*f*g + 54*g^2 + 3*(12*f^2 - 44*
f*g - 69*g^2)*x)/(3*x + 2)) + 4*(32*g^3*x^2 - 12*f^2*g + 4*f*g^2 - 51*g^3
+ 4*(2*f*g^2 + 5*g^3)*x)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x
+ 2))/(3*g^3*x + 2*g^3), 1/384*(sqrt(3)*(48*f^3 + 8*f^2*g - 300*f*g^2 - 30
6*g^3 + 3*(24*f^3 + 4*f^2*g - 150*f*g^2 - 153*g^3)*x)*sqrt(g)*arctan(1/12*
sqrt(3)*(4*g*x + 2*f - 3*g)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(g
)*sqrt(3*x + 2)/(6*g^2*x^3 + (6*f*g - 5*g^2)*x^2 - 6*f*g - (5*f*g + 6*g^2)
*x)) + 2*(32*g^3*x^2 - 12*f^2*g + 4*f*g^2 - 51*g^3 + 4*(2*f*g^2 + 5*g^3)*x
)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2))/(3*g^3*x + 2*g^3)
]
```

Sympy [F]

$$\int \sqrt{2+3x}\sqrt{f+gx}\sqrt{1+\frac{5x}{6}-x^2}\,dx = \frac{\sqrt{6}\int \sqrt{f+gx}\sqrt{3x+2}\sqrt{-6x^2+5x+6}\,dx}{6}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(g*x+f)**(1/2)*(-36*x**2+30*x+36)**(1/2),x)
```

output
```
sqrt(6)*Integral(sqrt(f + g*x)*sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6), x)/6
```

Maxima [F]

$$\int \sqrt{2+3x}\sqrt{f+gx}\sqrt{1+\frac{5x}{6}-x^2}\,dx=\int \frac{1}{6}\sqrt{gx+f}\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}\,dx$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^(1/2)*(-36*x^2+30*x+36)^(1/2),x, algor
ithm="maxima")
```

output
```
1/6*integrate(sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2), x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{2+3x}\sqrt{f+gx}\sqrt{1+\frac{5x}{6}-x^2}\,dx=\text{Timed out}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(g*x+f)^(1/2)*(-36*x^2+30*x+36)^(1/2),x, algor
ithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+3x}\sqrt{f+gx}\sqrt{1+\frac{5x}{6}-x^2}\,dx=\int \frac{\sqrt{f+g\,x}\,\sqrt{3\,x+2}\,\sqrt{-36\,x^2+30\,x+36}}{6}\,dx$$

input
```
int(((f + g*x)^(1/2)*(3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/6,x)
```

output
```
int(((f + g*x)^(1/2)*(3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/6, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.02

$$\int \sqrt{2+3x}\sqrt{f+gx}\sqrt{1+\frac{5x}{6}-x^2}\,dx$$
$$= \frac{\sqrt{3}\left(48\sqrt{g}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f^4+80\sqrt{g}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f^3g-288\sqrt{g}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f^2g^2-756\sqrt{g}\right.}{}$$

input
```
int(1/6*(2+3*x)^(1/2)*(g*x+f)^(1/2)*(-36*x^2+30*x+36)^(1/2),x)
```

output
```
(sqrt(3)*(48*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f**4
 + 80*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f**3*g - 28
8*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f**2*g**2 - 756
*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f*g**3 - 459*sqr
t(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*g**4 - 24*sqrt(f + g
*x)*sqrt( - 2*x + 3)*sqrt(2)*f**3*g + 16*sqrt(f + g*x)*sqrt( - 2*x + 3)*sq
rt(2)*f**2*g**2*x - 28*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*f**2*g**2 +
64*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*f*g**3*x**2 + 64*sqrt(f + g*x)*s
qrt( - 2*x + 3)*sqrt(2)*f*g**3*x - 90*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(
2)*f*g**3 + 96*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*g**4*x**2 + 60*sqrt(
f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*g**4*x - 153*sqrt(f + g*x)*sqrt( - 2*x +
 3)*sqrt(2)*g**4))/(192*g**3*(2*f + 3*g))
```

3.304 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 128

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{\sqrt{f+gx}}\,dx = -\frac{(18f-25g)\sqrt{3-2x}\sqrt{f+gx}}{8\sqrt{6}g^2} - \frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}\sqrt{f+gx}}{4g} + \frac{(18f-25g)(2f+3g)\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{16\sqrt{3}g^{5/2}}$$

output

```
-1/48*(18*f-25*g)*(3-2*x)^(1/2)*(g*x+f)^(1/2)*6^(1/2)/g^2-1/8*6^(1/2)*(3-2
*x)^(3/2)*(g*x+f)^(1/2)/g+1/48*(18*f-25*g)*(2*f+3*g)*arctan(1/2*g^(1/2)*(3
-2*x)^(1/2)*2^(1/2)/(g*x+f)^(1/2))*3^(1/2)/g^(5/2)
```

Mathematica [A] (verified)

Time = 5.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{\sqrt{f+gx}}\,dx$$
$$=\frac{\sqrt{6+5x-6x^2}\Big(-2\sqrt{-2f-3g}\sqrt{g}(-3+2x)\left(18f^2+fg(-7+6x)-g^2x(7+12x)\right)+(18f-25g)(2f}{16\sqrt{6}\sqrt{-2f-3g}\,g^{5/2}(-3+2x)\sqrt{2+3x}\sqrt{f+gx}}$$

input
```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/Sqrt[f + g*x],x]
```

output
```
(Sqrt[6 + 5*x - 6*x^2]*(-2*Sqrt[-2*f - 3*g]*Sqrt[g]*(-3 + 2*x)*(18*f^2 + f
*g*(-7 + 6*x) - g^2*x*(7 + 12*x)) + (18*f - 25*g)*(2*f + 3*g)^2*Sqrt[6 - 4
*x]*Sqrt[(f + g*x)/(2*f + 3*g)]*ArcSinh[(Sqrt[g]*Sqrt[3 - 2*x])/Sqrt[-2*f
- 3*g]]))/(16*Sqrt[6]*Sqrt[-2*f - 3*g]*g^(5/2)*(-3 + 2*x)*Sqrt[2 + 3*x]*Sq
rt[f + g*x])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1245, 90, 27, 60, 66, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 90}$$

$$-\frac{(18f-25g)\int \frac{\sqrt{3-2x}}{\sqrt{6}\sqrt{f+gx}}dx}{8g}-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}\sqrt{f+gx}}{4g}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{(18f-25g)\int \frac{\sqrt{3-2x}}{\sqrt{f+gx}}dx}{8\sqrt{6}g}-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}\sqrt{f+gx}}{4g}$$

$$\Big\downarrow \textcolor{blue}{60}$$

$$-\frac{(18f-25g)\left(\frac{(2f+3g)\int \frac{1}{\sqrt{3-2x}\sqrt{f+gx}}dx}{2g}+\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}\right)}{8\sqrt{6}g}-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}\sqrt{f+gx}}{4g}$$

$$\Big\downarrow \textcolor{blue}{66}$$

$$-\frac{(18f-25g)\left(\frac{(2f+3g)\int \frac{1}{-\frac{g(3-2x)}{f+gx}-2}d\frac{\sqrt{3-2x}}{\sqrt{f+gx}}}{g}+\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}\right)}{8\sqrt{6}g}-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}\sqrt{f+gx}}{4g}$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$-\frac{(18f-25g)\left(\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}-\frac{(2f+3g)\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{\sqrt{2}g^{3/2}}\right)}{8\sqrt{6}g}-\frac{\sqrt{\frac{3}{2}}(3-2x)^{3/2}\sqrt{f+gx}}{4g}$$

input

```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/Sqrt[f + g*x],x]
```

output

```
-1/4*(Sqrt[3/2]*(3 - 2*x)^(3/2)*Sqrt[f + g*x])/g - ((18*f - 25*g)*((Sqrt[3
 - 2*x]*Sqrt[f + g*x])/g - ((2*f + 3*g)*ArcTan[(Sqrt[g]*Sqrt[3 - 2*x])/(Sq
rt[2]*Sqrt[f + g*x])])/(Sqrt[2]*g^(3/2))))/(8*Sqrt[6]*g)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1245

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(97) = 194$.

Time = 1.31 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\sqrt{-6x^2+5x+6}\,\sqrt{gx+f}\,\sqrt{3}\left(-24\sqrt{2}\,g^{\frac{3}{2}}x\sqrt{-(2x-3)(gx+f)}+36\sqrt{2}\,f\sqrt{-(2x-3)(gx+f)}\,\sqrt{g}-14\sqrt{2}\,g^{\frac{3}{2}}\sqrt{-(2x-3)(gx+f)}+36\right.}{96g^{\frac{5}{2}}\sqrt{3x+2}\,\sqrt{-(2x-3)}}$

input

```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(1/2),x,method=_RETU
RNVERBOSE)
```

output

```
-1/96*(-6*x^2+5*x+6)^(1/2)*(g*x+f)^(1/2)*3^(1/2)/g^(5/2)*(-24*2^(1/2)*g^(3
/2)*x*(-(2*x-3)*(g*x+f))^(1/2)+36*2^(1/2)*f*(-(2*x-3)*(g*x+f))^(1/2)*g^(1/
2)-14*2^(1/2)*g^(3/2)*(-(2*x-3)*(g*x+f))^(1/2)+36*arctan(1/4/g^(1/2)*(4*g*
x+2*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f^2+4*arctan(1/4/g^(1/2)*(4*g
*x+2*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f*g-75*arctan(1/4/g^(1/2)*(4
*g*x+2*f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*g^2)/(3*x+2)^(1/2)/(-(2*x-
3)*(g*x+f))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{\sqrt{f+gx}}\,dx$$

$$= \Bigg[\frac{\sqrt{3}(72\,f^2+8\,fg-150\,g^2+3\,(36\,f^2+4\,fg-75\,g^2)x)\sqrt{-g}\log\left(-\frac{288\,g^2x^3-4\,\sqrt{3}(4\,gx+2\,f-3\,g)\sqrt{gx+f}\sqrt{-36\,x^2+}}{}\right.}{}$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(1/2),x, algor
ithm="fricas")
```

output

```
[1/192*(sqrt(3)*(72*f^2 + 8*f*g - 150*g^2 + 3*(36*f^2 + 4*f*g - 75*g^2)*x)
*sqrt(-g)*log(-(288*g^2*x^3 - 4*sqrt(3)*(4*g*x + 2*f - 3*g)*sqrt(g*x + f)*
sqrt(-36*x^2 + 30*x + 36)*sqrt(-g)*sqrt(3*x + 2) + 48*(6*f*g - 5*g^2)*x^2
+ 24*f^2 - 216*f*g + 54*g^2 + 3*(12*f^2 - 44*f*g - 69*g^2)*x)/(3*x + 2)) +
 4*(12*g^2*x - 18*f*g + 7*g^2)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqr
t(3*x + 2))/(3*g^3*x + 2*g^3), 1/96*(sqrt(3)*(72*f^2 + 8*f*g - 150*g^2 + 3
*(36*f^2 + 4*f*g - 75*g^2)*x)*sqrt(g)*arctan(1/12*sqrt(3)*(4*g*x + 2*f - 3
*g)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(g)*sqrt(3*x + 2)/(6*g^2*x
^3 + (6*f*g - 5*g^2)*x^2 - 6*f*g - (5*f*g + 6*g^2)*x)) + 2*(12*g^2*x - 18*
f*g + 7*g^2)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2))/(3*g^3
*x + 2*g^3)]
```

Sympy [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{\sqrt{f+gx}}\,dx = \frac{\sqrt{6}\int \frac{\sqrt{3x+2}\sqrt{-6x^2+5x+6}}{\sqrt{f+gx}}\,dx}{6}$$

input

```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f)**(1/2),x)
```

output

```
sqrt(6)*Integral(sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6)/sqrt(f + g*x), x)/6
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,\sqrt{gx+f}}\,dx$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(1/2),x, algor
ithm="maxima")
```

output

```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/sqrt(g*x + f), x)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{1}{96}\sqrt{6}\sqrt{2}\left(\sqrt{g(2\,x-3)+2\,f+3\,g}\sqrt{-2\,x+3}\left(\frac{6\,(2\,x-3)}{g}-\frac{18\,fg-25\,g^2}{g^3}\right)-\frac{(36\,f^2+4\,fg-75\,g^2)}{}\right.$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(1/2),x, algor
ithm="giac")
```

output
```
1/96*sqrt(6)*sqrt(2)*(sqrt(g*(2*x - 3) + 2*f + 3*g)*sqrt(-2*x + 3)*(6*(2*x
 - 3)/g - (18*f*g - 25*g^2)/g^3) - (36*f^2 + 4*f*g - 75*g^2)*log(abs(-sqrt
(-g)*sqrt(-2*x + 3) + sqrt(g*(2*x - 3) + 2*f + 3*g)))/(sqrt(-g)*g^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{3\,x+2}\,\sqrt{-36\,x^2+30\,x+36}}{6\,\sqrt{f+g\,x}}\,dx$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(1/2)),x)
```

output
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{\sqrt{f+gx}}\,dx$$
$$=\frac{\sqrt{3}\,\Bigl(72\sqrt{g}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f^3+116\sqrt{g}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f^2g-138\sqrt{g}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f\,g^2-225\sqrt{g}}{}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(1/2),x)
```

output
```
(sqrt(3)*(72*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f**3
 + 116*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f**2*g - 1
38*sqrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*f*g**2 - 225*s
qrt(g)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*f + 3*g))*g**3 - 36*sqrt(f +
 g*x)*sqrt( - 2*x + 3)*sqrt(2)*f**2*g + 24*sqrt(f + g*x)*sqrt( - 2*x + 3)*
sqrt(2)*f*g**2*x - 40*sqrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*f*g**2 + 36*s
qrt(f + g*x)*sqrt( - 2*x + 3)*sqrt(2)*g**3*x + 21*sqrt(f + g*x)*sqrt( - 2*
x + 3)*sqrt(2)*g**3))/(48*g**3*(2*f + 3*g))
```

3.305 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 115

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{3/2}}\,dx = \frac{\sqrt{\frac{2}{3}}(3f-2g)\sqrt{3-2x}}{g^2\sqrt{f+gx}} + \frac{\sqrt{\frac{3}{2}}\sqrt{3-2x}\sqrt{f+gx}}{g^2} - \frac{(18f+g)\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{2\sqrt{3}g^{5/2}}$$

output
```
1/3*6^(1/2)*(3*f-2*g)*(3-2*x)^(1/2)/g^2/(g*x+f)^(1/2)+1/2*6^(1/2)*(3-2*x)^
(1/2)*(g*x+f)^(1/2)/g^2-1/6*(18*f+g)*arctan(1/2*g^(1/2)*(3-2*x)^(1/2)*2^(1
/2)/(g*x+f)^(1/2))*3^(1/2)/g^(5/2)
```

Mathematica [A] (verified)

Time = 10.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{3/2}}\,dx = \frac{\sqrt{6+5x-6x^2}\left(2\sqrt{-2f-3g}\sqrt{g}(-3+2x)(9f+g(-4+3x))-(36f^2+5\right.}{2\sqrt{6}\sqrt{-2f-3g}g^{5/2}(-3+2x)\sqrt{2+3}}$$

input
```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(3/2),x]
```

output
```
(Sqrt[6 + 5*x - 6*x^2]*(2*Sqrt[-2*f - 3*g]*Sqrt[g]*(-3 + 2*x)*(9*f + g*(-4
 + 3*x)) - (36*f^2 + 56*f*g + 3*g^2)*Sqrt[6 - 4*x]*Sqrt[(f + g*x)/(2*f + 3
*g)]*ArcSinh[(Sqrt[g]*Sqrt[3 - 2*x])/Sqrt[-2*f - 3*g]]))/(2*Sqrt[6]*Sqrt[-
2*f - 3*g]*g^(5/2)*(-3 + 2*x)*Sqrt[2 + 3*x]*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1245, 87, 27, 60, 66, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{ 1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(18f+g)\int \frac{\sqrt{3-2x}}{\sqrt{6}\sqrt{f+gx}}dx}{g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{g(2f+3g)\sqrt{f+gx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(18f+g)\int \frac{\sqrt{3-2x}}{\sqrt{f+gx}}dx}{\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{g(2f+3g)\sqrt{f+gx}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(18f+g)\left(\frac{(2f+3g)\int \frac{1}{\sqrt{3-2x}\sqrt{f+gx}}dx}{2g}+\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}\right)}{\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{g(2f+3g)\sqrt{f+gx}}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{(18f+g)\left(\frac{(2f+3g)\int \frac{1}{-\frac{g(3-2x)}{f+gx}-2}d\frac{\sqrt{3-2x}}{\sqrt{f+gx}}}{g}+\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}\right)}{\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{g(2f+3g)\sqrt{f+gx}}$$

$$\Big\downarrow \text{217}$$

$$\frac{(18f+g)\left(\frac{\sqrt{3-2x}\sqrt{f+gx}}{g}-\frac{(2f+3g)\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{\sqrt{2}g^{3/2}}\right)}{\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{g(2f+3g)\sqrt{f+gx}}$$

input

```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(3/2),x]
```

output

```
(Sqrt[2/3]*(3*f - 2*g)*(3 - 2*x)^(3/2))/(g*(2*f + 3*g)*Sqrt[f + g*x]) + ((
18*f + g)*((Sqrt[3 - 2*x]*Sqrt[f + g*x])/g - ((2*f + 3*g)*ArcTan[(Sqrt[g]*
Sqrt[3 - 2*x])/(Sqrt[2]*Sqrt[f + g*x])])/(Sqrt[2]*g^(3/2))))/(Sqrt[6]*g*(2
*f + 3*g))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(88) = 176$.

Time = 1.38 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.37

method	result
default	$\frac{\sqrt{-6x^2+5x+6}\,\sqrt{3}\left(6\sqrt{2}\,g^{\frac{3}{2}}x\sqrt{-(2x-3)(gx+f)}+18\sqrt{2}\,f\sqrt{-(2x-3)(gx+f)}\,\sqrt{g}-8\sqrt{2}\,g^{\frac{3}{2}}\sqrt{-(2x-3)(gx+f)}+18\arctan\left(\frac{(4gx+}{4\sqrt{g}\,\sqrt{-}}\right.\right.}{12\sqrt{-(2x-3)(gx+}}$

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/12*(-6*x^2+5*x+6)^(1/2)*3^(1/2)*(6*2^(1/2)*g^(3/2)*x*(-(2*x-3)*(g*x+f))^
(1/2)+18*2^(1/2)*f*(-(2*x-3)*(g*x+f))^(1/2)*g^(1/2)-8*2^(1/2)*g^(3/2)*(-(2
*x-3)*(g*x+f))^(1/2)+18*arctan(1/4/g^(1/2)*(4*g*x+2*f-3*g)*2^(1/2)/(-(2*x-
3)*(g*x+f))^(1/2))*f*g*x+arctan(1/4/g^(1/2)*(4*g*x+2*f-3*g)*2^(1/2)/(-(2*x
-3)*(g*x+f))^(1/2))*g^2*x+18*arctan(1/4/g^(1/2)*(4*g*x+2*f-3*g)*2^(1/2)/(-
(2*x-3)*(g*x+f))^(1/2))*f^2+arctan(1/4/g^(1/2)*(4*g*x+2*f-3*g)*2^(1/2)/(-(
2*x-3)*(g*x+f))^(1/2))*f*g)/(-(2*x-3)*(g*x+f))^(1/2)/g^(5/2)/(g*x+f)^(1/2)
/(3*x+2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.

Time = 0.11 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.95

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{3/2}}\,dx = \left[-\frac{\sqrt{3}(3\,(18\,fg+g^2)x^2+36\,f^2+2\,fg+(54\,f^2+39\,fg+2\,g^2)x)\sqrt{-g}\log}{}\right.$$

$$-\frac{\sqrt{3}(3\,(18\,fg+g^2)x^2+36\,f^2+2\,fg+(54\,f^2+39\,fg+2\,g^2)x)\sqrt{g}\arctan\left(\frac{\sqrt{3}(4\,gx+2\,f-3\,g)\sqrt{gx+f}\sqrt{-36\,x^2+30}}{12\,(6\,g^2x^3+(6\,fg-5\,g^2)x^2-6\,fg-(5}\right.}{12\,(3\,g^4x^2+2\,fg^3+(3\,fg^3+2}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(3/2),x, algor
ithm="fricas")
```

output
```
[-1/24*(sqrt(3)*(3*(18*f*g + g^2)*x^2 + 36*f^2 + 2*f*g + (54*f^2 + 39*f*g
+ 2*g^2)*x)*sqrt(-g)*log(-(288*g^2*x^3 - 4*sqrt(3)*(4*g*x + 2*f - 3*g)*sqr
t(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(-g)*sqrt(3*x + 2) + 48*(6*f*g -
5*g^2)*x^2 + 24*f^2 - 216*f*g + 54*g^2 + 3*(12*f^2 - 44*f*g - 69*g^2)*x)/(
3*x + 2)) - 4*(3*g^2*x + 9*f*g - 4*g^2)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x
+ 36)*sqrt(3*x + 2))/(3*g^4*x^2 + 2*f*g^3 + (3*f*g^3 + 2*g^4)*x), -1/12*(s
qrt(3)*(3*(18*f*g + g^2)*x^2 + 36*f^2 + 2*f*g + (54*f^2 + 39*f*g + 2*g^2)*
x)*sqrt(g)*arctan(1/12*sqrt(3)*(4*g*x + 2*f - 3*g)*sqrt(g*x + f)*sqrt(-36*
x^2 + 30*x + 36)*sqrt(g)*sqrt(3*x + 2)/(6*g^2*x^3 + (6*f*g - 5*g^2)*x^2 -
6*f*g - (5*f*g + 6*g^2)*x)) - 2*(3*g^2*x + 9*f*g - 4*g^2)*sqrt(g*x + f)*sq
rt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2))/(3*g^4*x^2 + 2*f*g^3 + (3*f*g^3 + 2
*g^4)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{3/2}}\,dx = \frac{\sqrt{6}\int \frac{\sqrt{3x+2}\sqrt{-6x^2+5x+6}}{f\sqrt{f+gx}+gx\sqrt{f+gx}}\,dx}{6}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f)**(3/2),x)
```

output
```
sqrt(6)*Integral(sqrt(3*x + 2)*sqrt(-6*x**2 + 5*x + 6)/(f*sqrt(f + g*x) +
g*x*sqrt(f + g*x)), x)/6
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(3/2),x, algor
ithm="maxima")
```

output
```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(g*x + f)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{3/2}}\,dx = \frac{1}{12}\,\sqrt{6}\Bigg(\frac{\sqrt{-2\,x+3}\left(\frac{3\,\sqrt{2}(2\,x-3)}{g}+\frac{18\,\sqrt{2}fg+\sqrt{2}g^2}{g^3}\right)}{\sqrt{g(2\,x-3)+2\,f+3\,g}}+\frac{(18\,\sqrt{2}f+\sqrt{2}g)\log\Big(\Big|-}{}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(3/2),x, algor
ithm="giac")
```

output
```
1/12*sqrt(6)*(sqrt(-2*x + 3)*(3*sqrt(2)*(2*x - 3)/g + (18*sqrt(2)*f*g + sq
rt(2)*g^2)/g^3)/sqrt(g*(2*x - 3) + 2*f + 3*g) + (18*sqrt(2)*f + sqrt(2)*g)
*log(abs(-sqrt(-g)*sqrt(-2*x + 3) + sqrt(g*(2*x - 3) + 2*f + 3*g)))/(sqrt(
-g)*g^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{3/2}}\,dx = \int \frac{\sqrt{3\,x+2}\,\sqrt{-36\,x^2+30\,x+36}}{6\,{\left(f+g\,x\right)}^{3/2}}\,dx$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(3/2)),x)
```

output
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{3/2}}\,dx = \frac{\sqrt{3}\left(-18\sqrt{g}\,\sqrt{gx+f}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f-\sqrt{g}\,\sqrt{gx+f}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)}{6\sqrt{gx+f}}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(3/2),x)
```

output
```
(sqrt(3)*( - 18*sqrt(g)*sqrt(f + g*x)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt
(2*f + 3*g))*f - sqrt(g)*sqrt(f + g*x)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqr
t(2*f + 3*g))*g + 9*sqrt( - 2*x + 3)*sqrt(2)*f*g + 3*sqrt( - 2*x + 3)*sqrt
(2)*g**2*x - 4*sqrt( - 2*x + 3)*sqrt(2)*g**2))/(6*sqrt(f + g*x)*g**3)
```

3.306 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 119

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{5/2}}\,dx=\frac{\sqrt{\frac{2}{3}}(3f-2g)(3-2x)^{3/2}}{3g(2f+3g)(f+gx)^{3/2}}-\frac{\sqrt{6}\sqrt{3-2x}}{g^2\sqrt{f+gx}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{g^{5/2}}$$

output
```
1/9*6^(1/2)*(3*f-2*g)*(3-2*x)^(3/2)/g/(2*f+3*g)/(g*x+f)^(3/2)-6^(1/2)*(3-2
*x)^(1/2)/g^2/(g*x+f)^(1/2)+2*3^(1/2)*arctan(1/2*g^(1/2)*(3-2*x)^(1/2)*2^(
1/2)/(g*x+f)^(1/2))/g^(5/2)
```

Mathematica [A] (verified)

Time = 10.93 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{5/2}}\,dx=\frac{\sqrt{6+5x-6x^2}\left(\frac{\sqrt{g}(18f^2+6fg(3+4x)+g^2(6+23x))}{-2f-3g}+\frac{9\sqrt{6-4x}(f+gx)^2\operatorname{arcsinh}\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{-2f-3g}}\right.}{\sqrt{-2f-3g}(-3+2x)\sqrt{\frac{f+gx}{2f+3g}}}\right.}{3g^{5/2}\sqrt{3+\frac{9x}{2}}(f+gx)^{3/2}}$$

input
```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(5/2),x]
```

output
```
(Sqrt[6 + 5*x - 6*x^2]*((Sqrt[g]*(18*f^2 + 6*f*g*(3 + 4*x) + g^2*(6 + 23*x
)))/(-2*f - 3*g) + (9*Sqrt[6 - 4*x]*(f + g*x)^2*ArcSinh[(Sqrt[g]*Sqrt[3 -
2*x])/Sqrt[-2*f - 3*g]])/(Sqrt[-2*f - 3*g]*(-3 + 2*x)*Sqrt[(f + g*x)/(2*f
+ 3*g)])))/(3*g^(5/2)*Sqrt[3 + (9*x)/2]*(f + g*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1245, 87, 27, 57, 66, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{(f+gx)^{5/2}}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{3\int \frac{\sqrt{3-2x}}{\sqrt{6}(f+gx)^{3/2}}dx}{g}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{3g(2f+3g)(f+gx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\frac{3}{2}}\int \frac{\sqrt{3-2x}}{(f+gx)^{3/2}}dx}{g}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{3g(2f+3g)(f+gx)^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$\frac{\sqrt{\frac{3}{2}}\left(-\frac{2\int \frac{1}{\sqrt{3-2x}\sqrt{f+gx}}dx}{g}-\frac{2\sqrt{3-2x}}{g\sqrt{f+gx}}\right)}{g}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{3g(2f+3g)(f+gx)^{3/2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{\sqrt{\frac{3}{2}}\left(-\frac{4\int \frac{1}{-\frac{g(3-2x)}{f+gx}-2}d\frac{\sqrt{3-2x}}{\sqrt{f+gx}}}{g}-\frac{2\sqrt{3-2x}}{g\sqrt{f+gx}}\right)}{g}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{3g(2f+3g)(f+gx)^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{\frac{3}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{g}\sqrt{3-2x}}{\sqrt{2}\sqrt{f+gx}}\right)}{g^{3/2}}-\frac{2\sqrt{3-2x}}{g\sqrt{f+gx}}\right)}{g}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{3g(2f+3g)(f+gx)^{3/2}}$$

input
```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(5/2),x]
```

output
```
(Sqrt[2/3]*(3*f - 2*g)*(3 - 2*x)^(3/2))/(3*g*(2*f + 3*g)*(f + g*x)^(3/2))
+ (Sqrt[3/2]*((-2*Sqrt[3 - 2*x])/(g*Sqrt[f + g*x]) + (2*Sqrt[2]*ArcTan[(Sq
rt[g]*Sqrt[3 - 2*x])/(Sqrt[2]*Sqrt[f + g*x])])/g^(3/2)))/g
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(92) = 184$.

Time = 1.50 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.55

method	result
default	$-\frac{\left(24\sqrt{2}\,\sqrt{-(2x-3)(gx+f)}\,g^{\frac{3}{2}}\,fx+23\sqrt{2}\,\sqrt{-(2x-3)(gx+f)}\,g^{\frac{5}{2}}\,x+18\arctan\left(\frac{(4gx+2f-3g)\sqrt{2}}{4\sqrt{g}\,\sqrt{-(2x-3)(gx+f)}}\right)f\,g^{2}x^{2}+27\arctan\left(\frac{(4gx}{4\sqrt{g}\,\sqrt{-}}\right.\right.}{}$

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(5/2),x,method=_RETU
RNVERBOSE)
```

output
```
-1/9*(24*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*g^(3/2)*f*x+23*2^(1/2)*(-(2*x-3)
*(g*x+f))^(1/2)*g^(5/2)*x+18*arctan(1/4/g^(1/2)*(4*g*x+2*f-3*g)*2^(1/2)/(-
(2*x-3)*(g*x+f))^(1/2))*f*g^2*x^2+27*arctan(1/4/g^(1/2)*(4*g*x+2*f-3*g)*2^
(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*g^3*x^2+18*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2
)*g^(1/2)*f^2+18*2^(1/2)*(-(2*x-3)*(g*x+f))^(1/2)*g^(3/2)*f+6*2^(1/2)*(-(2
*x-3)*(g*x+f))^(1/2)*g^(5/2)+36*arctan(1/4/g^(1/2)*(4*g*x+2*f-3*g)*2^(1/2)
/(-(2*x-3)*(g*x+f))^(1/2))*f^2*g*x+54*arctan(1/4/g^(1/2)*(4*g*x+2*f-3*g)*2
^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f*g^2*x+18*arctan(1/4/g^(1/2)*(4*g*x+2*f-
3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f^3+27*arctan(1/4/g^(1/2)*(4*g*x+2*
f-3*g)*2^(1/2)/(-(2*x-3)*(g*x+f))^(1/2))*f^2*g)*3^(1/2)*(-6*x^2+5*x+6)^(1/
2)/(2*f+3*g)/(-(2*x-3)*(g*x+f))^(1/2)/g^(5/2)/(g*x+f)^(3/2)/(3*x+2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(92) = 184$.

Time = 0.11 (sec) , antiderivative size = 631, normalized size of antiderivative = 5.30

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{5/2}}\,dx = \Bigg[\frac{9\sqrt{3}(3(2fg^2+3g^3)x^3+4f^3+6f^2g+2(6f^2g+11fg^2+3g^3)x^2+(6}{}$$

$$-\frac{(18f^2+18fg+6g^2+(24fg+23g^2)x)\sqrt{gx+f}\sqrt{-36x^2+30x+36}\sqrt{3x+2}-\frac{9\sqrt{3}(3(2fg^2+3g^3)x^3+4f}{}}{9(4f^3g^2+6f^2g^3+3(2fg^4+3g^5)x^3+2(6f^2g^3+11fg^4+}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(5/2),x, algor
ithm="fricas")
```

output

```
[1/18*(9*sqrt(3)*(3*(2*f*g^2 + 3*g^3)*x^3 + 4*f^3 + 6*f^2*g + 2*(6*f^2*g +
 11*f*g^2 + 3*g^3)*x^2 + (6*f^3 + 17*f^2*g + 12*f*g^2)*x)*sqrt(-1/g)*log(-
(288*g^2*x^3 - 4*sqrt(3)*(4*g^2*x + 2*f*g - 3*g^2)*sqrt(g*x + f)*sqrt(-36*
x^2 + 30*x + 36)*sqrt(3*x + 2)*sqrt(-1/g) + 48*(6*f*g - 5*g^2)*x^2 + 24*f^
2 - 216*f*g + 54*g^2 + 3*(12*f^2 - 44*f*g - 69*g^2)*x)/(3*x + 2)) - 2*(18*
f^2 + 18*f*g + 6*g^2 + (24*f*g + 23*g^2)*x)*sqrt(g*x + f)*sqrt(-36*x^2 + 3
0*x + 36)*sqrt(3*x + 2))/(4*f^3*g^2 + 6*f^2*g^3 + 3*(2*f*g^4 + 3*g^5)*x^3
+ 2*(6*f^2*g^3 + 11*f*g^4 + 3*g^5)*x^2 + (6*f^3*g^2 + 17*f^2*g^3 + 12*f*g^
4)*x), -1/9*((18*f^2 + 18*f*g + 6*g^2 + (24*f*g + 23*g^2)*x)*sqrt(g*x + f)
*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2) - 9*sqrt(3)*(3*(2*f*g^2 + 3*g^3)*
x^3 + 4*f^3 + 6*f^2*g + 2*(6*f^2*g + 11*f*g^2 + 3*g^3)*x^2 + (6*f^3 + 17*f
^2*g + 12*f*g^2)*x)*arctan(2/3*sqrt(3)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x +
 36)*sqrt(g)*sqrt(3*x + 2)/(12*g*x^2 + (6*f - g)*x + 4*f - 6*g))/sqrt(g))/
(4*f^3*g^2 + 6*f^2*g^3 + 3*(2*f*g^4 + 3*g^5)*x^3 + 2*(6*f^2*g^3 + 11*f*g^4
 + 3*g^5)*x^2 + (6*f^3*g^2 + 17*f^2*g^3 + 12*f*g^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f)**(5/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{5/2}}\,dx = \int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,(gx+f)^{\frac{5}{2}}}\,dx$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(5/2),x, algor
ithm="maxima")
```

output
```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(g*x + f)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{5/2}}\,dx =$$

$$-\frac{1}{9}\sqrt{6}\left(\frac{\left(\frac{\left(24\sqrt{2}fg^2+23\sqrt{2}g^3\right)(2x-3)}{2fg^3+3g^4}+\frac{9\left(4\sqrt{2}f^2g+12\sqrt{2}fg^2+9\sqrt{2}g^3\right)}{2fg^3+3g^4}\right)\sqrt{-2x+3}}{(g(2x-3)+2f+3g)^{\frac{3}{2}}}+\frac{9\sqrt{2}\log\left(\left|-\sqrt{-g}\sqrt{-2x+3}\right.\right.}{\sqrt{-}}\right.$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(5/2),x, algor
ithm="giac")
```

output
```
-1/9*sqrt(6)*(((24*sqrt(2)*f*g^2 + 23*sqrt(2)*g^3)*(2*x - 3)/(2*f*g^3 + 3*
g^4) + 9*(4*sqrt(2)*f^2*g + 12*sqrt(2)*f*g^2 + 9*sqrt(2)*g^3)/(2*f*g^3 + 3
*g^4))*sqrt(-2*x + 3)/(g*(2*x - 3) + 2*f + 3*g)^(3/2) + 9*sqrt(2)*log(abs(
-sqrt(-g)*sqrt(-2*x + 3) + sqrt(g*(2*x - 3) + 2*f + 3*g)))/(sqrt(-g)*g^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{5/2}}\,dx = \int \frac{\sqrt{3\,x+2}\,\sqrt{-36\,x^2+30\,x+36}}{6\,{\left(f+g\,x\right)}^{5/2}}\,dx$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(5/2)),x)
```

output
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.03

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{5/2}}\,dx = \frac{\sqrt{3}\left(36\sqrt{g}\,\sqrt{gx+f}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right)f^2+36\sqrt{g}\,\sqrt{gx+f}\,asin\left(\frac{\sqrt{g}\,\sqrt{-2x+3}}{\sqrt{2f+3g}}\right.\right.}{}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(5/2),x)
```

output
```
(sqrt(3)*(36*sqrt(g)*sqrt(f + g*x)*asin((sqrt(g)*sqrt( - 2*x + 3))/sqrt(2*
f + 3*g))*f**2 + 36*sqrt(g)*sqrt(f + g*x)*asin((sqrt(g)*sqrt( - 2*x + 3))/
sqrt(2*f + 3*g))*f*g*x + 54*sqrt(g)*sqrt(f + g*x)*asin((sqrt(g)*sqrt( - 2*
x + 3))/sqrt(2*f + 3*g))*f*g + 54*sqrt(g)*sqrt(f + g*x)*asin((sqrt(g)*sqrt
( - 2*x + 3))/sqrt(2*f + 3*g))*g**2*x - 18*sqrt( - 2*x + 3)*sqrt(2)*f**2*g
 - 24*sqrt( - 2*x + 3)*sqrt(2)*f*g**2*x - 18*sqrt( - 2*x + 3)*sqrt(2)*f*g*
*2 - 23*sqrt( - 2*x + 3)*sqrt(2)*g**3*x - 6*sqrt( - 2*x + 3)*sqrt(2)*g**3)
)/(9*sqrt(f + g*x)*g**3*(2*f**2 + 2*f*g*x + 3*f*g + 3*g**2*x))
```

3.307 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 97

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{7/2}}\,dx=\frac{\sqrt{\frac{2}{3}}(3f-2g)(3-2x)^{3/2}}{5g(2f+3g)(f+gx)^{5/2}}-\frac{\sqrt{\frac{2}{3}}(18f+53g)(3-2x)^{3/2}}{15g(2f+3g)^2(f+gx)^{3/2}}$$

output
```
1/15*6^(1/2)*(3*f-2*g)*(3-2*x)^(3/2)/g/(2*f+3*g)/(g*x+f)^(5/2)-1/45*6^(1/2
)*(18*f+53*g)*(3-2*x)^(3/2)/g/(2*f+3*g)^2/(g*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{7/2}}\,dx=\frac{(-3+2x)\sqrt{4+\frac{10x}{3}-4x^2}(2f(19+9x)+g(18+53x))}{15(2f+3g)^2\sqrt{2+3x}(f+gx)^{5/2}}$$

input
```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(7/2),x]
```

output
```
((-3 + 2*x)*Sqrt[4 + (10*x)/3 - 4*x^2]*(2*f*(19 + 9*x) + g*(18 + 53*x)))/(
15*(2*f + 3*g)^2*Sqrt[2 + 3*x]*(f + g*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1245, 87, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{(f+gx)^{7/2}}dx$$

$$\Big\downarrow \textit{1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{(f+gx)^{7/2}}dx$$

$$\Big\downarrow \textit{87}$$

$$\frac{(18f+53g)\int \frac{\sqrt{3-2x}}{\sqrt{6}(f+gx)^{5/2}}dx}{5g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{5g(2f+3g)(f+gx)^{5/2}}$$

$$\Big\downarrow \textit{27}$$

$$\frac{(18f+53g)\int \frac{\sqrt{3-2x}}{(f+gx)^{5/2}}dx}{5\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{5g(2f+3g)(f+gx)^{5/2}}$$

$$\Big\downarrow \textit{48}$$

$$\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{5g(2f+3g)(f+gx)^{5/2}}-\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(18f+53g)}{15g(2f+3g)^2(f+gx)^{3/2}}$$

input

```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(7/2),x]
```

output

```
(Sqrt[2/3]*(3*f - 2*g)*(3 - 2*x)^(3/2))/(5*g*(2*f + 3*g)*(f + g*x)^(5/2))
- (Sqrt[2/3]*(18*f + 53*g)*(3 - 2*x)^(3/2))/(15*g*(2*f + 3*g)^2*(f + g*x)^
(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\sqrt{-36x^{2}+30x+36}\,(2x-3)(18fx+53gx+38f+18g)}{45\sqrt{3x+2}\,(gx+f)^{\frac{5}{2}}(2f+3g)^{2}}$	58
gosper	$\frac{(2x-3)(18fx+53gx+38f+18g)\sqrt{-36x^{2}+30x+36}}{45(gx+f)^{\frac{5}{2}}(4f^{2}+12fg+9g^{2})\sqrt{3x+2}}$	66
orering	$\frac{(2x-3)(18fx+53gx+38f+18g)\sqrt{-36x^{2}+30x+36}}{45(gx+f)^{\frac{5}{2}}(4f^{2}+12fg+9g^{2})\sqrt{3x+2}}$	66

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(7/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/45/(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(5/2)*(2*x-3)*(18*f*x+5
3*g*x+38*f+18*g)/(2*f+3*g)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(77) = 154$.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{7/2}}\,dx = \frac{(2\,(18\,f+53\,g)x^2+(22\,f}{45\,(8\,f^5+24\,f^4g+18\,f^3g^2+3\,(4\,f^2g^3+12\,fg^4+9\,g^5)x^4+(36\,f^3g^2+11}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(7/2),x, algor
ithm="fricas")
```

output
```
1/45*(2*(18*f + 53*g)*x^2 + (22*f - 123*g)*x - 114*f - 54*g)*sqrt(g*x + f)
*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(8*f^5 + 24*f^4*g + 18*f^3*g^2 +
3*(4*f^2*g^3 + 12*f*g^4 + 9*g^5)*x^4 + (36*f^3*g^2 + 116*f^2*g^3 + 105*f*g
^4 + 18*g^5)*x^3 + 3*(12*f^4*g + 44*f^3*g^2 + 51*f^2*g^3 + 18*f*g^4)*x^2 +
 3*(4*f^5 + 20*f^4*g + 33*f^3*g^2 + 18*f^2*g^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{7/2}}\,dx=\int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,(gx+f)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(7/2),x, algor
ithm="maxima")
```

output
```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(g*x + f)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{7/2}}\,dx=\frac{2\,\sqrt{6}\left(\frac{\left(18\,\sqrt{2}fg^2+53\,\sqrt{2}g^3\right)(2\,x-3)}{4\,f^2g^2+12\,fg^3+9\,g^4}+\frac{65\,\left(2\,\sqrt{2}fg^2+3\,\sqrt{2}g^3\right)}{4\,f^2g^2+12\,fg^3+9\,g^4}\right)(2\,x-3)\sqrt{-2\,x+3}}{45\,(g(2\,x-3)+2\,f+3\,g)^{\frac{5}{2}}}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(7/2),x, algor
ithm="giac")
```

output
```
2/45*sqrt(6)*((18*sqrt(2)*f*g^2 + 53*sqrt(2)*g^3)*(2*x - 3)/(4*f^2*g^2 + 1
2*f*g^3 + 9*g^4) + 65*(2*sqrt(2)*f*g^2 + 3*sqrt(2)*g^3)/(4*f^2*g^2 + 12*f*
g^3 + 9*g^4))*(2*x - 3)*sqrt(-2*x + 3)/(g*(2*x - 3) + 2*f + 3*g)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 11.71 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{7/2}}\,dx=\frac{\sqrt{-36\,x^2+30\,x+36}\,\left(\frac{x\,\sqrt{3\,x+2}\,(22\,f-123\,g)}{135\,g^2\,{(2\,f+3\,g)}^2}-\frac{\sqrt{3\,x+2}\,(114\,f+54\,g)}{135\,g^2\,{(2\,f+3\,g)}^2}+\frac{x^2\,\sqrt{3\,x+2}\,(36\,f-}{135\,g^2\,(2\,f+3}\right.}{x^3\,\sqrt{f+g\,x}+\frac{2\,f^2\,\sqrt{f+g\,x}}{3\,g^2}+\frac{2\,x^2\,\sqrt{f+g\,x}\,(3\,f+g)}{3\,g}+\frac{f\,x\,\sqrt{f+g\,x}\,(3\,f+4\,g)}{3\,g^2}}$$

input

```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(7/2)),x)
```

output

```
((30*x - 36*x^2 + 36)^(1/2)*((x*(3*x + 2)^(1/2)*(22*f - 123*g))/(135*g^2*(
2*f + 3*g)^2) - ((3*x + 2)^(1/2)*(114*f + 54*g))/(135*g^2*(2*f + 3*g)^2) +
 (x^2*(3*x + 2)^(1/2)*(36*f + 106*g))/(135*g^2*(2*f + 3*g)^2)))/(x^3*(f +
g*x)^(1/2) + (2*f^2*(f + g*x)^(1/2))/(3*g^2) + (2*x^2*(f + g*x)^(1/2)*(3*f
 + g))/(3*g) + (f*x*(f + g*x)^(1/2)*(3*f + 4*g))/(3*g^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{7/2}}\,dx = \frac{\sqrt{-2x+3}\,\sqrt{6}\,(36f\,x^2+106g\,x^2+22fx-123gx-114f}{45\sqrt{gx+f}\,\left(4f^2g^2x^2+12f\,g^3x^2+9g^4x^2+8f^3gx+24f^2g^2x+18f\,g^3x+\right.}$$

input

```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(7/2),x)
```

output

```
(sqrt( - 2*x + 3)*sqrt(6)*(36*f*x**2 + 22*f*x - 114*f + 106*g*x**2 - 123*g
*x - 54*g))/(45*sqrt(f + g*x)*(4*f**4 + 8*f**3*g*x + 12*f**3*g + 4*f**2*g*
*2*x**2 + 24*f**2*g**2*x + 9*f**2*g**2 + 12*f*g**3*x**2 + 18*f*g**3*x + 9*
g**4*x**2))
```

3.308 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{9/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 145

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{9/2}}\,dx=\frac{\sqrt{\frac{2}{3}}(3f-2g)(3-2x)^{3/2}}{7g(2f+3g)(f+gx)^{7/2}}-\frac{\sqrt{\frac{2}{3}}(18f+79g)(3-2x)^{3/2}}{35g(2f+3g)^2(f+gx)^{5/2}}-\frac{4\sqrt{\frac{2}{3}}(18f+79g)(3-2x)^{3/2}}{105g(2f+3g)^3(f+gx)^{3/2}}$$

output

```
1/21*6^(1/2)*(3*f-2*g)*(3-2*x)^(3/2)/g/(2*f+3*g)/(g*x+f)^(7/2)-1/105*6^(1/
2)*(18*f+79*g)*(3-2*x)^(3/2)/g/(2*f+3*g)^2/(g*x+f)^(5/2)-4/315*6^(1/2)*(18
*f+79*g)*(3-2*x)^(3/2)/g/(2*f+3*g)^3/(g*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{9/2}}\,dx=\frac{(-3+2x)\sqrt{4+\frac{10x}{3}-4x^2}(28f^2(19+9x)+2fg(333+634x+36x^2)+g^2}{105(2f+3g)^3\sqrt{2+3x}(f+gx)^{7/2}}$$

input

```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(9/2),x]
```

output
```
((-3 + 2*x)*Sqrt[4 + (10*x)/3 - 4*x^2]*(28*f^2*(19 + 9*x) + 2*f*g*(333 + 6
34*x + 36*x^2) + g^2*(270 + 711*x + 316*x^2)))/(105*(2*f + 3*g)^3*Sqrt[2 +
 3*x]*(f + g*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1245, 87, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{(f+gx)^{9/2}}dx$$

$$\Big\downarrow \text{1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{(f+gx)^{9/2}}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(18f+79g)\int \frac{\sqrt{3-2x}}{\sqrt{6}(f+gx)^{7/2}}dx}{7g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{7g(2f+3g)(f+gx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(18f+79g)\int \frac{\sqrt{3-2x}}{(f+gx)^{7/2}}dx}{7\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{7g(2f+3g)(f+gx)^{7/2}}$$

$$\Big\downarrow \text{55}$$

$$\frac{(18f+79g)\left(\frac{4\int \frac{\sqrt{3-2x}}{(f+gx)^{5/2}}dx}{5(2f+3g)}-\frac{2(3-2x)^{3/2}}{5(2f+3g)(f+gx)^{5/2}}\right)}{7\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{7g(2f+3g)(f+gx)^{7/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{7g(2f+3g)(f+gx)^{7/2}}+\frac{(18f+79g)\left(-\frac{8(3-2x)^{3/2}}{15(2f+3g)^2(f+gx)^{3/2}}-\frac{2(3-2x)^{3/2}}{5(2f+3g)(f+gx)^{5/2}}\right)}{7\sqrt{6}g(2f+3g)}$$

input

```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(9/2),x]
```

output

```
(Sqrt[2/3]*(3*f - 2*g)*(3 - 2*x)^(3/2))/(7*g*(2*f + 3*g)*(f + g*x)^(7/2))
+ ((18*f + 79*g)*((-2*(3 - 2*x)^(3/2))/(5*(2*f + 3*g)*(f + g*x)^(5/2)) - (
8*(3 - 2*x)^(3/2))/(15*(2*f + 3*g)^2*(f + g*x)^(3/2))))/(7*Sqrt[6]*g*(2*f
+ 3*g))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{\sqrt{-36x^{2}+30x+36}\,(2x-3)\left(72fg\,x^{2}+316g^{2}x^{2}+252f^{2}x+1268fgx+711g^{2}x+532f^{2}+666fg+270g^{2}\right)}{315\sqrt{3x+2}\,(gx+f)^{\frac{7}{2}}\,(2f+3g)^{3}}$	90
gosper	$\frac{(2x-3)\left(72fg\,x^{2}+316g^{2}x^{2}+252f^{2}x+1268fgx+711g^{2}x+532f^{2}+666fg+270g^{2}\right)\sqrt{-36x^{2}+30x+36}}{315(gx+f)^{\frac{7}{2}}\,(8f^{3}+36f^{2}g+54f\,g^{2}+27g^{3})\sqrt{3x+2}}$	106
orering	$\frac{(2x-3)\left(72fg\,x^{2}+316g^{2}x^{2}+252f^{2}x+1268fgx+711g^{2}x+532f^{2}+666fg+270g^{2}\right)\sqrt{-36x^{2}+30x+36}}{315(gx+f)^{\frac{7}{2}}\,(8f^{3}+36f^{2}g+54f\,g^{2}+27g^{3})\sqrt{3x+2}}$	106

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(9/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/315/(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(7/2)*(2*x-3)*(72*f*g*
x^2+316*g^2*x^2+252*f^2*x+1268*f*g*x+711*g^2*x+532*f^2+666*f*g+270*g^2)/(2
*f+3*g)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(115) = 230.

Time = 0.09 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{9/2}}\,dx = \frac{}{315\,(16\,f^7+72\,f^6g+108\,f^5g^2+54\,f^4g^3+3\,(8\,f^3g^4+36\,f^2g^5+54\,fg^6+}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(9/2),x, algor
ithm="fricas")
```

output
```
1/315*(8*(18*f*g + 79*g^2)*x^3 + 2*(252*f^2 + 1160*f*g + 237*g^2)*x^2 - 15
96*f^2 - 1998*f*g - 810*g^2 + (308*f^2 - 2472*f*g - 1593*g^2)*x)*sqrt(g*x
+ f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(16*f^7 + 72*f^6*g + 108*f^5*
g^2 + 54*f^4*g^3 + 3*(8*f^3*g^4 + 36*f^2*g^5 + 54*f*g^6 + 27*g^7)*x^5 + 2*
(48*f^4*g^3 + 224*f^3*g^4 + 360*f^2*g^5 + 216*f*g^6 + 27*g^7)*x^4 + 2*(72*
f^5*g^2 + 356*f^4*g^3 + 630*f^3*g^4 + 459*f^2*g^5 + 108*f*g^6)*x^3 + 12*(8
*f^6*g + 44*f^5*g^2 + 90*f^4*g^3 + 81*f^3*g^4 + 27*f^2*g^5)*x^2 + (24*f^7
+ 172*f^6*g + 450*f^5*g^2 + 513*f^4*g^3 + 216*f^3*g^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{9/2}}\,dx=\int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,(gx+f)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(9/2),x, algor
ithm="maxima")
```

output
```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(g*x + f)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{9/2}}\,dx = \frac{4\sqrt{6}\Bigg(\bigg(\frac{2\left(18\sqrt{2}fg^4+79\sqrt{2}g^5\right)(2x-3)}{8f^3g^3+36f^2g^4+54fg^5+27g^6}+\frac{7\left(36\sqrt{2}f^2g^3+212\sqrt{2}fg^4+237\sqrt{2}g^5\right)}{8f^3g^3+36f^2g^4+54fg^5+27g^6}\bigg)(2x-3)}{315\,(g(2x-3)+2f+3}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(9/2),x, algor
ithm="giac")
```

output
```
4/315*sqrt(6)*((2*(18*sqrt(2)*f*g^4 + 79*sqrt(2)*g^5)*(2*x - 3)/(8*f^3*g^3
 + 36*f^2*g^4 + 54*f*g^5 + 27*g^6) + 7*(36*sqrt(2)*f^2*g^3 + 212*sqrt(2)*f
*g^4 + 237*sqrt(2)*g^5)/(8*f^3*g^3 + 36*f^2*g^4 + 54*f*g^5 + 27*g^6))*(2*x
 - 3) + 455*(4*sqrt(2)*f^2*g^3 + 12*sqrt(2)*f*g^4 + 9*sqrt(2)*g^5)/(8*f^3*
g^3 + 36*f^2*g^4 + 54*f*g^5 + 27*g^6))*(2*x - 3)*sqrt(-2*x + 3)/(g*(2*x -
3) + 2*f + 3*g)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 11.87 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{9/2}}\,dx =$$

$$-\frac{\sqrt{-36\,x^2+30\,x+36}\,\Bigg(\frac{\sqrt{3\,x+2}\,\left(1596\,f^2+1998\,f\,g+810\,g^2\right)}{945\,g^3\,{\left(2\,f+3\,g\right)}^3}+\frac{x\,\sqrt{3\,x+2}\,\left(-308\,f^2+2472\,f\,g+1593\,g^2\right)}{945\,g^3\,{\left(2\,f+3\,g\right)}^3}-\frac{x^2\,\sqrt{3\,x+2}\,\left(504\,f^2+2320\right.}{945\,g^3\,\left(2\,f+3\,g\right)}}{x^4\,\sqrt{f+g\,x}+\frac{2\,f^3\,\sqrt{f+g\,x}}{3\,g^3}+\frac{x^3\,\sqrt{f+g\,x}\,\left(9\,f+2\,g\right)}{3\,g}+\frac{f^2\,x\,\sqrt{f+g\,x}\,\left(f+2\,g\right)}{g^3}+\frac{f\,x^2\,\sqrt{f+g\,x}\,\left(3\,f\right.}{g^2}}$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(9/2)),x)
```

output
```
-((30*x - 36*x^2 + 36)^(1/2)*(((3*x + 2)^(1/2)*(1998*f*g + 1596*f^2 + 810*
g^2))/(945*g^3*(2*f + 3*g)^3) + (x*(3*x + 2)^(1/2)*(2472*f*g - 308*f^2 + 1
593*g^2))/(945*g^3*(2*f + 3*g)^3) - (x^2*(3*x + 2)^(1/2)*(2320*f*g + 504*f
^2 + 474*g^2))/(945*g^3*(2*f + 3*g)^3) - (8*x^3*(3*x + 2)^(1/2)*(18*f + 79
*g))/(945*g^2*(2*f + 3*g)^3)))/(x^4*(f + g*x)^(1/2) + (2*f^3*(f + g*x)^(1/
2))/(3*g^3) + (x^3*(f + g*x)^(1/2)*(9*f + 2*g))/(3*g) + (f^2*x*(f + g*x)^(
1/2)*(f + 2*g))/g^3 + (f*x^2*(f + g*x)^(1/2)*(3*f + 2*g))/g^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{9/2}}\,dx = \frac{\sqrt{-2x+3}\,\sqrt{6}\,(144fg\,x^3+632g^2x^3+504f^2x^2}{315\sqrt{gx+f}\,(8f^3g^3x^3+36f^2g^4x^3+54f\,g^5x^3+27g^6x^3+24f^4g^2x^2+108}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(9/2),x)
```

output
```
(sqrt( - 2*x + 3)*sqrt(6)*(504*f**2*x**2 + 308*f**2*x - 1596*f**2 + 144*f*
g*x**3 + 2320*f*g*x**2 - 2472*f*g*x - 1998*f*g + 632*g**2*x**3 + 474*g**2*
x**2 - 1593*g**2*x - 810*g**2))/(315*sqrt(f + g*x)*(8*f**6 + 24*f**5*g*x +
 36*f**5*g + 24*f**4*g**2*x**2 + 108*f**4*g**2*x + 54*f**4*g**2 + 8*f**3*g
**3*x**3 + 108*f**3*g**3*x**2 + 162*f**3*g**3*x + 27*f**3*g**3 + 36*f**2*g
**4*x**3 + 162*f**2*g**4*x**2 + 81*f**2*g**4*x + 54*f*g**5*x**3 + 81*f*g**
5*x**2 + 27*g**6*x**3))
```

3.309 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{11/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 193

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{11/2}}\,dx = \frac{\sqrt{\frac{2}{3}}(3f-2g)(3-2x)^{3/2}}{9g(2f+3g)(f+gx)^{9/2}} - \frac{\sqrt{\frac{2}{3}}(6f+35g)(3-2x)^{3/2}}{21g(2f+3g)^2(f+gx)^{7/2}} - \frac{8\sqrt{\frac{2}{3}}(6f+35g)(3-2x)^{3/2}}{105g(2f+3g)^3(f+gx)^{5/2}} - \frac{32\sqrt{\frac{2}{3}}(6f+35g)(3-2x)^{3/2}}{315g(2f+3g)^4(f+gx)^{3/2}}$$

output

```
1/27*6^(1/2)*(3*f-2*g)*(3-2*x)^(3/2)/g/(2*f+3*g)/(g*x+f)^(9/2)-1/63*6^(1/2
)*(6*f+35*g)*(3-2*x)^(3/2)/g/(2*f+3*g)^2/(g*x+f)^(7/2)-8/315*6^(1/2)*(6*f+
35*g)*(3-2*x)^(3/2)/g/(2*f+3*g)^3/(g*x+f)^(5/2)-32/945*6^(1/2)*(6*f+35*g)*
(3-2*x)^(3/2)/g/(2*f+3*g)^4/(g*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 7.93 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{11/2}}\,dx = \frac{(-3+2x)\sqrt{4+\frac{10x}{3}-4x^2}(168f^3(19+9x)+36f^2g(180+299x+24x^2)+}{315(2f+3g)^4\sqrt{2}}$$

input
```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(11/2),x]
```

output
```
((-3 + 2*x)*Sqrt[4 + (10*x)/3 - 4*x^2]*(168*f^3*(19 + 9*x) + 36*f^2*g*(180
 + 299*x + 24*x^2) + 35*g^3*(54 + 135*x + 72*x^2 + 32*x^3) + 6*f*g^2*(945
+ 2025*x + 912*x^2 + 32*x^3)))/(315*(2*f + 3*g)^4*Sqrt[2 + 3*x]*(f + g*x)^
(9/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1245, 87, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{(f+gx)^{11/2}}dx$$

$$\Big\downarrow \text{1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{(f+gx)^{11/2}}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(6f+35g)\int \frac{\sqrt{3-2x}}{\sqrt{6}(f+gx)^{9/2}}dx}{3g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{9g(2f+3g)(f+gx)^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(6f+35g)\int \frac{\sqrt{3-2x}}{(f+gx)^{9/2}}dx}{3\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{9g(2f+3g)(f+gx)^{9/2}}$$

$$\Big\downarrow \ 55$$

$$\frac{(6f+35g)\left(\frac{8\int \frac{\sqrt{3-2x}}{(f+gx)^{7/2}}dx}{7(2f+3g)}-\frac{2(3-2x)^{3/2}}{7(2f+3g)(f+gx)^{7/2}}\right)}{3\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{9g(2f+3g)(f+gx)^{9/2}}$$

$$\Big\downarrow \ 55$$

$$\frac{(6f+35g)\left(\frac{8\left(\frac{4\int \frac{\sqrt{3-2x}}{(f+gx)^{5/2}}dx}{5(2f+3g)}-\frac{2(3-2x)^{3/2}}{5(2f+3g)(f+gx)^{5/2}}\right)}{7(2f+3g)}-\frac{2(3-2x)^{3/2}}{7(2f+3g)(f+gx)^{7/2}}\right)}{3\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{9g(2f+3g)(f+gx)^{9/2}}$$

$$\Big\downarrow \ 48$$

$$\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{9g(2f+3g)(f+gx)^{9/2}}+\frac{(6f+35g)\left(\frac{8\left(-\frac{8(3-2x)^{3/2}}{15(2f+3g)^2(f+gx)^{3/2}}-\frac{2(3-2x)^{3/2}}{5(2f+3g)(f+gx)^{5/2}}\right)}{7(2f+3g)}-\frac{2(3-2x)^{3/2}}{7(2f+3g)(f+gx)^{7/2}}\right)}{3\sqrt{6}g(2f+3g)}$$

input

```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(11/2),x]
```

output

```
(Sqrt[2/3]*(3*f - 2*g)*(3 - 2*x)^(3/2))/(9*g*(2*f + 3*g)*(f + g*x)^(9/2))
+ ((6*f + 35*g)*((-2*(3 - 2*x)^(3/2))/(7*(2*f + 3*g)*(f + g*x)^(7/2)) + (8
*((-2*(3 - 2*x)^(3/2))/(5*(2*f + 3*g)*(f + g*x)^(5/2)) - (8*(3 - 2*x)^(3/2
))/(15*(2*f + 3*g)^2*(f + g*x)^(3/2))))/(7*(2*f + 3*g))))/(3*Sqrt[6]*g*(2*
f + 3*g))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1245

```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.70

method	result
default	$\frac{\sqrt{-36x^2+30x+36}\,(2x-3)\left(192x^3 f\,g^2+1120x^3 g^3+864x^2 f^2 g+5472f\,g^2 x^2+2520x^2 g^3+1512x\,f^3+10764x\,f^2 g+12150x f\,g^2+4725x}{945\sqrt{3x+2}\,(gx+f)^{\frac{9}{2}}\,(2f+3g)^4}$
gosper	$\frac{(2x-3)\left(192x^3 f\,g^2+1120x^3 g^3+864x^2 f^2 g+5472f\,g^2 x^2+2520x^2 g^3+1512x\,f^3+10764x\,f^2 g+12150x f\,g^2+4725x\,g^3+3192f^3+6480}{945(gx+f)^{\frac{9}{2}}\,(16f^4+96g\,f^3+216g^2 f^2+216f\,g^3+81g^4)\sqrt{3x+2}}$
orering	$\frac{(2x-3)\left(192x^3 f\,g^2+1120x^3 g^3+864x^2 f^2 g+5472f\,g^2 x^2+2520x^2 g^3+1512x\,f^3+10764x\,f^2 g+12150x f\,g^2+4725x\,g^3+3192f^3+6480}{945(gx+f)^{\frac{9}{2}}\,(16f^4+96g\,f^3+216g^2 f^2+216f\,g^3+81g^4)\sqrt{3x+2}}$

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(11/2),x,method=_RET
URNVERBOSE)
```

output
```
1/945/(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(9/2)*(2*x-3)*(192*f*g
^2*x^3+1120*g^3*x^3+864*f^2*g*x^2+5472*f*g^2*x^2+2520*g^3*x^2+1512*f^3*x+1
0764*f^2*g*x+12150*f*g^2*x+4725*g^3*x+3192*f^3+6480*f^2*g+5670*f*g^2+1890*
g^3)/(2*f+3*g)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(153) = 306.

Time = 0.10 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{11/2}}\,dx = \frac{}{945\,(32\,f^9+192\,f^8g+432\,f^7g^2+432\,f^6g^3+162\,f^5g^4+3\,(16\,f^4g^5+96\,f}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(11/2),x, algo
rithm="fricas")
```

output
```
1/945*(64*(6*f*g^2 + 35*g^3)*x^4 + 48*(36*f^2*g + 216*f*g^2 + 35*g^3)*x^3
- 9576*f^3 - 19440*f^2*g - 17010*f*g^2 - 5670*g^3 + 18*(168*f^3 + 1052*f^2
*g + 438*f*g^2 + 105*g^3)*x^2 + 3*(616*f^3 - 6444*f^2*g - 8370*f*g^2 - 346
5*g^3)*x)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(32*f^9 +
192*f^8*g + 432*f^7*g^2 + 432*f^6*g^3 + 162*f^5*g^4 + 3*(16*f^4*g^5 + 96*f
^3*g^6 + 216*f^2*g^7 + 216*f*g^8 + 81*g^9)*x^6 + (240*f^5*g^4 + 1472*f^4*g
^5 + 3432*f^3*g^6 + 3672*f^2*g^7 + 1647*f*g^8 + 162*g^9)*x^5 + 10*(48*f^6*
g^3 + 304*f^5*g^4 + 744*f^4*g^5 + 864*f^3*g^6 + 459*f^2*g^7 + 81*f*g^8)*x^
4 + 10*(48*f^7*g^2 + 320*f^6*g^3 + 840*f^5*g^4 + 1080*f^4*g^5 + 675*f^3*g^
6 + 162*f^2*g^7)*x^3 + 5*(48*f^8*g + 352*f^7*g^2 + 1032*f^6*g^3 + 1512*f^5
*g^4 + 1107*f^4*g^5 + 324*f^3*g^6)*x^2 + (48*f^9 + 448*f^8*g + 1608*f^7*g^
2 + 2808*f^6*g^3 + 2403*f^5*g^4 + 810*f^4*g^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{11/2}}\,dx = \int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,(gx+f)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(11/2),x, algo
rithm="maxima")
```

output
```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(g*x + f)^(11/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(153) = 306$.

Time = 0.77 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{11/2}}\,dx = \frac{8\sqrt{6}\Bigg(\Bigg(4\Bigg(\frac{2\left(6\sqrt{2}fg^6+35\sqrt{2}g^7\right)(2x-3)}{16f^4g^4+96f^3g^5+216f^2g^6+216fg^7+81g^8}+\frac{9\left(12\sqrt{2}f^2g^5+88\sqrt{2}fg^6+105\sqrt{2}}{16f^4g^4+96f^3g^5+216f^2g^6+216fg^7}}{}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(11/2),x, algo
rithm="giac")
```

output
```
8/945*sqrt(6)*((4*(2*(6*sqrt(2)*f*g^6 + 35*sqrt(2)*g^7)*(2*x - 3)/(16*f^4*
g^4 + 96*f^3*g^5 + 216*f^2*g^6 + 216*f*g^7 + 81*g^8) + 9*(12*sqrt(2)*f^2*g
^5 + 88*sqrt(2)*f*g^6 + 105*sqrt(2)*g^7)/(16*f^4*g^4 + 96*f^3*g^5 + 216*f^
2*g^6 + 216*f*g^7 + 81*g^8))*(2*x - 3) + 63*(24*sqrt(2)*f^3*g^4 + 212*sqrt
(2)*f^2*g^5 + 474*sqrt(2)*f*g^6 + 315*sqrt(2)*g^7)/(16*f^4*g^4 + 96*f^3*g^
5 + 216*f^2*g^6 + 216*f*g^7 + 81*g^8))*(2*x - 3) + 1365*(8*sqrt(2)*f^3*g^4
 + 36*sqrt(2)*f^2*g^5 + 54*sqrt(2)*f*g^6 + 27*sqrt(2)*g^7)/(16*f^4*g^4 + 9
6*f^3*g^5 + 216*f^2*g^6 + 216*f*g^7 + 81*g^8))*(2*x - 3)*sqrt(-2*x + 3)/(g
*(2*x - 3) + 2*f + 3*g)^(9/2)
```

Mupad [B] (verification not implemented)

Time = 11.91 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{11/2}}\,dx = \frac{\sqrt{-36\,x^2+30\,x+36}\,\Bigg(\frac{x^2\,\sqrt{3\,x+2}\,\left(3024\,f^3+18936\,f^2\,g+7884\,f\,g^2+1890\,g^3\right)}{2835\,g^4\,{\left(2\,f+3\,g\right)}^4}-\frac{x\,\sqrt{3\,x+2}\,(}{}}{x^5\,\sqrt{f+g\,x}+\frac{2\,f^4\,\sqrt{f+g\,x}}{3\,g^4}}$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(11/2)),x)
```

output
```
((30*x - 36*x^2 + 36)^(1/2)*((x^2*(3*x + 2)^(1/2)*(7884*f*g^2 + 18936*f^2*
g + 3024*f^3 + 1890*g^3))/(2835*g^4*(2*f + 3*g)^4) - (x*(3*x + 2)^(1/2)*(2
5110*f*g^2 + 19332*f^2*g - 1848*f^3 + 10395*g^3))/(2835*g^4*(2*f + 3*g)^4)
 - ((3*x + 2)^(1/2)*(17010*f*g^2 + 19440*f^2*g + 9576*f^3 + 5670*g^3))/(28
35*g^4*(2*f + 3*g)^4) + (16*x^3*(3*x + 2)^(1/2)*(216*f*g + 36*f^2 + 35*g^2
))/(945*g^3*(2*f + 3*g)^4) + (64*x^4*(3*x + 2)^(1/2)*(6*f + 35*g))/(2835*g
^2*(2*f + 3*g)^4)))/(x^5*(f + g*x)^(1/2) + (2*f^4*(f + g*x)^(1/2))/(3*g^4)
 + (2*x^4*(f + g*x)^(1/2)*(6*f + g))/(3*g) + (4*f^2*x^2*(f + g*x)^(1/2)*(f
 + g))/g^3 + (2*f*x^3*(f + g*x)^(1/2)*(9*f + 4*g))/(3*g^2) + (f^3*x*(f + g
*x)^(1/2)*(3*f + 8*g))/(3*g^4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{11/2}}\,dx = \frac{\sqrt{-2x+3}\,\sqrt{6}\,(384f\,g^2x^4+2240}{945\sqrt{gx+f}\,(16f^4g^4x^4+96f^3g^5x^4+216f^2g^6x^4+216f\,g^7x^4+81g^8x^4+}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(11/2),x)
```

output
```
(sqrt( - 2*x + 3)*sqrt(6)*(3024*f**3*x**2 + 1848*f**3*x - 9576*f**3 + 1728
*f**2*g*x**3 + 18936*f**2*g*x**2 - 19332*f**2*g*x - 19440*f**2*g + 384*f*g
**2*x**4 + 10368*f*g**2*x**3 + 7884*f*g**2*x**2 - 25110*f*g**2*x - 17010*f
*g**2 + 2240*g**3*x**4 + 1680*g**3*x**3 + 1890*g**3*x**2 - 10395*g**3*x -
5670*g**3))/(945*sqrt(f + g*x)*(16*f**8 + 64*f**7*g*x + 96*f**7*g + 96*f**
6*g**2*x**2 + 384*f**6*g**2*x + 216*f**6*g**2 + 64*f**5*g**3*x**3 + 576*f*
*5*g**3*x**2 + 864*f**5*g**3*x + 216*f**5*g**3 + 16*f**4*g**4*x**4 + 384*f
**4*g**4*x**3 + 1296*f**4*g**4*x**2 + 864*f**4*g**4*x + 81*f**4*g**4 + 96*
f**3*g**5*x**4 + 864*f**3*g**5*x**3 + 1296*f**3*g**5*x**2 + 324*f**3*g**5*
x + 216*f**2*g**6*x**4 + 864*f**2*g**6*x**3 + 486*f**2*g**6*x**2 + 216*f*g
**7*x**4 + 324*f*g**7*x**3 + 81*g**8*x**4))
```

3.310 $\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{13/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 241

$$\begin{aligned}\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{13/2}}\,dx &= \frac{\sqrt{\frac{2}{3}}(3f-2g)(3-2x)^{3/2}}{11g(2f+3g)(f+gx)^{11/2}} \\ &- \frac{\sqrt{\frac{2}{3}}(18f+131g)(3-2x)^{3/2}}{99g(2f+3g)^2(f+gx)^{9/2}} - \frac{4\sqrt{\frac{2}{3}}(18f+131g)(3-2x)^{3/2}}{231g(2f+3g)^3(f+gx)^{7/2}} \\ &- \frac{32\sqrt{\frac{2}{3}}(18f+131g)(3-2x)^{3/2}}{1155g(2f+3g)^4(f+gx)^{5/2}} - \frac{128\sqrt{\frac{2}{3}}(18f+131g)(3-2x)^{3/2}}{3465g(2f+3g)^5(f+gx)^{3/2}}\end{aligned}$$

output

```
1/33*6^(1/2)*(3*f-2*g)*(3-2*x)^(3/2)/g/(2*f+3*g)/(g*x+f)^(11/2)-1/297*6^(1
/2)*(18*f+131*g)*(3-2*x)^(3/2)/g/(2*f+3*g)^2/(g*x+f)^(9/2)-4/693*6^(1/2)*(
18*f+131*g)*(3-2*x)^(3/2)/g/(2*f+3*g)^3/(g*x+f)^(7/2)-32/3465*6^(1/2)*(18*
f+131*g)*(3-2*x)^(3/2)/g/(2*f+3*g)^4/(g*x+f)^(5/2)-128/10395*6^(1/2)*(18*f
+131*g)*(3-2*x)^(3/2)/g/(2*f+3*g)^5/(g*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 8.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{13/2}}\,dx = \frac{(-3+2x)\sqrt{4+\frac{10x}{3}-4x^2}\left(3696f^4(19+9x)+264f^3g(747+1160x+108\right.}{}$$

input
```
Integrate[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(13/2),x]
```

output
```
((-3 + 2*x)*Sqrt[4 + (10*x)/3 - 4*x^2]*(3696*f^4*(19 + 9*x) + 264*f^3*g*(7
47 + 1160*x + 108*x^2) + 396*f^2*g^2*(675 + 1314*x + 596*x^2 + 32*x^3) + 2
*f*g^3*(91665 + 203040*x + 108612*x^2 + 48704*x^3 + 1152*x^4) + g^4*(51030
 + 123795*x + 70740*x^2 + 37728*x^3 + 16768*x^4)))/(3465*(2*f + 3*g)^5*Sqr
t[2 + 3*x]*(f + g*x)^(11/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1245, 87, 27, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}\sqrt{-x^2+\frac{5x}{6}+1}}{(f+gx)^{13/2}}dx$$

$$\Big\downarrow \text{ 1245}$$

$$\int \frac{\sqrt{\frac{1}{2}-\frac{x}{3}}(3x+2)}{(f+gx)^{13/2}}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(18f+131g)\int \frac{\sqrt{3-2x}}{\sqrt{6}(f+gx)^{11/2}}dx}{11g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{11g(2f+3g)(f+gx)^{11/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(18f+131g)\int \frac{\sqrt{3-2x}}{(f+gx)^{11/2}}dx}{11\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{11g(2f+3g)(f+gx)^{11/2}}$$

$$\Big\downarrow 55$$

$$\frac{(18f+131g)\left(\frac{4\int \frac{\sqrt{3-2x}}{(f+gx)^{9/2}}dx}{3(2f+3g)}-\frac{2(3-2x)^{3/2}}{9(2f+3g)(f+gx)^{9/2}}\right)}{11\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{11g(2f+3g)(f+gx)^{11/2}}$$

$$\Big\downarrow 55$$

$$\frac{(18f+131g)\left(\frac{4\left(\frac{8\int \frac{\sqrt{3-2x}}{(f+gx)^{7/2}}dx}{7(2f+3g)}-\frac{2(3-2x)^{3/2}}{7(2f+3g)(f+gx)^{7/2}}\right)}{3(2f+3g)}-\frac{2(3-2x)^{3/2}}{9(2f+3g)(f+gx)^{9/2}}\right)}{11\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{11g(2f+3g)(f+gx)^{11/2}}$$

$$\Big\downarrow 55$$

$$\frac{(18f+131g)\left(\frac{4\left(\frac{8\left(\frac{4\int \frac{\sqrt{3-2x}}{(f+gx)^{5/2}}dx}{5(2f+3g)}-\frac{2(3-2x)^{3/2}}{5(2f+3g)(f+gx)^{5/2}}\right)}{7(2f+3g)}-\frac{2(3-2x)^{3/2}}{7(2f+3g)(f+gx)^{7/2}}\right)}{3(2f+3g)}-\frac{2(3-2x)^{3/2}}{9(2f+3g)(f+gx)^{9/2}}\right)}{11\sqrt{6}g(2f+3g)}+\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{11g(2f+3g)(f+gx)^{11/2}}$$

$$\Big\downarrow 48$$

$$\frac{\sqrt{\frac{2}{3}}(3-2x)^{3/2}(3f-2g)}{11g(2f+3g)(f+gx)^{11/2}}+\frac{(18f+131g)\left(\frac{4\left(\frac{8\left(-\frac{8(3-2x)^{3/2}}{15(2f+3g)^2(f+gx)^{3/2}}-\frac{2(3-2x)^{3/2}}{5(2f+3g)(f+gx)^{5/2}}\right)}{7(2f+3g)}-\frac{2(3-2x)^{3/2}}{7(2f+3g)(f+gx)^{7/2}}\right)}{3(2f+3g)}-\frac{2(3-2x)^{3/2}}{9(2f+3g)(f+gx)^{9/2}}\right)}{11\sqrt{6}g(2f+3g)}$$

input
```
Int[(Sqrt[2 + 3*x]*Sqrt[1 + (5*x)/6 - x^2])/(f + g*x)^(13/2),x]
```

output
```
(Sqrt[2/3]*(3*f - 2*g)*(3 - 2*x)^(3/2))/(11*g*(2*f + 3*g)*(f + g*x)^(11/2)
) + ((18*f + 131*g)*((-2*(3 - 2*x)^(3/2))/(9*(2*f + 3*g)*(f + g*x)^(9/2))
+ (4*((-2*(3 - 2*x)^(3/2))/(7*(2*f + 3*g)*(f + g*x)^(7/2)) + (8*((-2*(3 -
2*x)^(3/2))/(5*(2*f + 3*g)*(f + g*x)^(5/2)) - (8*(3 - 2*x)^(3/2))/(15*(2*f
 + 3*g)^2*(f + g*x)^(3/2))))/(7*(2*f + 3*g))))/(3*(2*f + 3*g))))/(11*Sqrt[
6]*g*(2*f + 3*g))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1245
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d
+ (c/e)*x)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.79

method	result
default	$\sqrt{-36x^2+30x+36}\,(2x-3)\left(2304g^3x^4f+16768g^4x^4+12672f^2g^2x^3+97408f\,g^3x^3+37728g^4x^3+28512f^3g\,x^2+236016f^2g^2x^2+2172\right.$ 10395
gosper	$(2x-3)\left(2304g^3x^4f+16768g^4x^4+12672f^2g^2x^3+97408f\,g^3x^3+37728g^4x^3+28512f^3g\,x^2+236016f^2g^2x^2+217224f\,g^3x^2+70740g^4\right.$ $10395(gx+f)^{\frac{11}{2}}\left(32f^5+240f\right.$
orering	$(2x-3)\left(2304g^3x^4f+16768g^4x^4+12672f^2g^2x^3+97408f\,g^3x^3+37728g^4x^3+28512f^3g\,x^2+236016f^2g^2x^2+217224f\,g^3x^2+70740g^4\right.$ $10395(gx+f)^{\frac{11}{2}}\left(32f^5+240f\right.$

input
```
int(1/6*(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(13/2),x,method=_RET
URNVERBOSE)
```

output
```
1/10395/(3*x+2)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(11/2)*(2*x-3)*(2304
*f*g^3*x^4+16768*g^4*x^4+12672*f^2*g^2*x^3+97408*f*g^3*x^3+37728*g^4*x^3+2
8512*f^3*g*x^2+236016*f^2*g^2*x^2+217224*f*g^3*x^2+70740*g^4*x^2+33264*f^4
*x+306240*f^3*g*x+520344*f^2*g^2*x+406080*f*g^3*x+123795*g^4*x+70224*f^4+1
97208*f^3*g+267300*f^2*g^2+183330*f*g^3+51030*g^4)/(2*f+3*g)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(191) = 382.

Time = 0.11 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{13/2}}\,dx = \text{Too large to display}$$

input

```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(13/2),x, algo
rithm="fricas")
```

output

```
1/10395*(256*(18*f*g^3 + 131*g^4)*x^5 + 64*(396*f^2*g^2 + 2936*f*g^3 + 393
*g^4)*x^4 - 210672*f^4 - 591624*f^3*g - 801900*f^2*g^2 - 549990*f*g^3 - 15
3090*g^4 + 24*(2376*f^3*g + 18084*f^2*g^2 + 5926*f*g^3 + 1179*g^4)*x^3 + 6
*(11088*f^4 + 87824*f^3*g + 55440*f^2*g^2 + 26748*f*g^3 + 5895*g^4)*x^2 +
3*(13552*f^4 - 174768*f^3*g - 342144*f^2*g^2 - 283860*f*g^3 - 89775*g^4)*x
)*sqrt(g*x + f)*sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(64*f^11 + 480*f^1
0*g + 1440*f^9*g^2 + 2160*f^8*g^3 + 1620*f^7*g^4 + 486*f^6*g^5 + 3*(32*f^5
*g^6 + 240*f^4*g^7 + 720*f^3*g^8 + 1080*f^2*g^9 + 810*f*g^10 + 243*g^11)*x
^7 + 2*(288*f^6*g^5 + 2192*f^5*g^6 + 6720*f^4*g^7 + 10440*f^3*g^8 + 8370*f
^2*g^9 + 2997*f*g^10 + 243*g^11)*x^6 + 3*(480*f^7*g^4 + 3728*f^6*g^5 + 117
60*f^5*g^6 + 19080*f^4*g^7 + 16470*f^3*g^8 + 6885*f^2*g^9 + 972*f*g^10)*x^
5 + 30*(64*f^8*g^3 + 512*f^7*g^4 + 1680*f^6*g^5 + 2880*f^5*g^6 + 2700*f^4*
g^7 + 1296*f^3*g^8 + 243*f^2*g^9)*x^4 + 5*(288*f^9*g^2 + 2416*f^8*g^3 + 84
00*f^7*g^4 + 15480*f^6*g^5 + 15930*f^5*g^6 + 8667*f^4*g^7 + 1944*f^3*g^8)*
x^3 + 6*(96*f^10*g + 880*f^9*g^2 + 3360*f^8*g^3 + 6840*f^7*g^4 + 7830*f^6*
g^5 + 4779*f^5*g^6 + 1215*f^4*g^7)*x^2 + 3*(32*f^11 + 368*f^10*g + 1680*f^
9*g^2 + 3960*f^8*g^3 + 5130*f^7*g^4 + 3483*f^6*g^5 + 972*f^5*g^6)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{13/2}}\,dx = \text{Timed out}$$

input

```
integrate(1/6*(2+3*x)**(1/2)*(-36*x**2+30*x+36)**(1/2)/(g*x+f)**(13/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{13/2}}\,dx = \int \frac{\sqrt{-36\,x^2+30\,x+36}\sqrt{3\,x+2}}{6\,(gx+f)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(13/2),x, algo
rithm="maxima")
```

output
```
1/6*integrate(sqrt(-36*x^2 + 30*x + 36)*sqrt(3*x + 2)/(g*x + f)^(13/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(191) = 382$.

Time = 0.41 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{13/2}}\,dx = \frac{16\,\sqrt{6}\Bigg(\Bigg(2\Bigg(4\Bigg(\frac{2\left(18\,\sqrt{2}fg^8+131\,\sqrt{2}g^9\right)(2\,x-3)}{32\,f^5g^5+240\,f^4g^6+720\,f^3g^7+1080\,f^2g^8+810\,fg^9+243\,g^{10}}+\frac{11\,\Big(36}{32\,f^5g^5+240\,f^4}}{}$$

input
```
integrate(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(13/2),x, algo
rithm="giac")
```

output
```
16/10395*sqrt(6)*((2*(4*(2*(18*sqrt(2)*f*g^8 + 131*sqrt(2)*g^9)*(2*x - 3)/
(32*f^5*g^5 + 240*f^4*g^6 + 720*f^3*g^7 + 1080*f^2*g^8 + 810*f*g^9 + 243*g
^10) + 11*(36*sqrt(2)*f^2*g^7 + 316*sqrt(2)*f*g^8 + 393*sqrt(2)*g^9)/(32*f
^5*g^5 + 240*f^4*g^6 + 720*f^3*g^7 + 1080*f^2*g^8 + 810*f*g^9 + 243*g^10))
*(2*x - 3) + 99*(72*sqrt(2)*f^3*g^6 + 740*sqrt(2)*f^2*g^7 + 1734*sqrt(2)*f
*g^8 + 1179*sqrt(2)*g^9)/(32*f^5*g^5 + 240*f^4*g^6 + 720*f^3*g^7 + 1080*f^
2*g^8 + 810*f*g^9 + 243*g^10))*(2*x - 3) + 231*(144*sqrt(2)*f^4*g^5 + 1696
*sqrt(2)*f^3*g^6 + 5688*sqrt(2)*f^2*g^7 + 7560*sqrt(2)*f*g^8 + 3537*sqrt(2
)*g^9)/(32*f^5*g^5 + 240*f^4*g^6 + 720*f^3*g^7 + 1080*f^2*g^8 + 810*f*g^9
+ 243*g^10))*(2*x - 3) + 15015*(16*sqrt(2)*f^4*g^5 + 96*sqrt(2)*f^3*g^6 +
216*sqrt(2)*f^2*g^7 + 216*sqrt(2)*f*g^8 + 81*sqrt(2)*g^9)/(32*f^5*g^5 + 24
0*f^4*g^6 + 720*f^3*g^7 + 1080*f^2*g^8 + 810*f*g^9 + 243*g^10))*(2*x - 3)*
sqrt(-2*x + 3)/(g*(2*x - 3) + 2*f + 3*g)^(11/2)
```

Mupad [B] (verification not implemented)

Time = 12.91 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{13/2}}\,dx = \frac{\sqrt{-36\,x^2+30\,x+36}\,\left(\frac{x^2\,\sqrt{3\,x+2}\,\left(66528\,f^4+526944\,f^3\,g+332640\,f^2\,g^2+160488\,f\,g^3+35370}{31185\,g^5\,\left(2\,f+3\,g\right)^5}\right.}{}$$

input
```
int(((3*x + 2)^(1/2)*(30*x - 36*x^2 + 36)^(1/2))/(6*(f + g*x)^(13/2)),x)
```

output
```
((30*x - 36*x^2 + 36)^(1/2)*((x^2*(3*x + 2)^(1/2)*(160488*f*g^3 + 526944*f
^3*g + 66528*f^4 + 35370*g^4 + 332640*f^2*g^2))/(31185*g^5*(2*f + 3*g)^5)
- (x*(3*x + 2)^(1/2)*(851580*f*g^3 + 524304*f^3*g - 40656*f^4 + 269325*g^4
 + 1026432*f^2*g^2))/(31185*g^5*(2*f + 3*g)^5) - ((3*x + 2)^(1/2)*(549990*
f*g^3 + 591624*f^3*g + 210672*f^4 + 153090*g^4 + 801900*f^2*g^2))/(31185*g
^5*(2*f + 3*g)^5) + (8*x^3*(3*x + 2)^(1/2)*(5926*f*g^2 + 18084*f^2*g + 237
6*f^3 + 1179*g^3))/(10395*g^4*(2*f + 3*g)^5) + (64*x^4*(3*x + 2)^(1/2)*(29
36*f*g + 396*f^2 + 393*g^2))/(31185*g^3*(2*f + 3*g)^5) + (256*x^5*(3*x + 2
)^(1/2)*(18*f + 131*g))/(31185*g^2*(2*f + 3*g)^5)))/(x^6*(f + g*x)^(1/2) +
 (2*f^5*(f + g*x)^(1/2))/(3*g^5) + (x^5*(f + g*x)^(1/2)*(15*f + 2*g))/(3*g
) + (10*f^2*x^3*(f + g*x)^(1/2)*(3*f + 2*g))/(3*g^3) + (5*f^3*x^2*(f + g*x
)^(1/2)*(3*f + 4*g))/(3*g^4) + (10*f*x^4*(f + g*x)^(1/2)*(3*f + g))/(3*g^2
) + (f^4*x*(f + g*x)^(1/2)*(3*f + 10*g))/(3*g^5))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{2+3x}\sqrt{1+\frac{5x}{6}-x^2}}{(f+gx)^{13/2}}\,dx = \frac{\sqrt{-2x+}}{10395\sqrt{gx+f}\,\left(32f^5g^5x^5+240f^4g^6x^5+720f^3g^7x^5+1080f^2g^8x^5+810f\right.}$$

input
```
int(1/6*(2+3*x)^(1/2)*(-36*x^2+30*x+36)^(1/2)/(g*x+f)^(13/2),x)
```

output

```
(sqrt( - 2*x + 3)*sqrt(6)*(66528*f**4*x**2 + 40656*f**4*x - 210672*f**4 +
57024*f**3*g*x**3 + 526944*f**3*g*x**2 - 524304*f**3*g*x - 591624*f**3*g +
 25344*f**2*g**2*x**4 + 434016*f**2*g**2*x**3 + 332640*f**2*g**2*x**2 - 10
26432*f**2*g**2*x - 801900*f**2*g**2 + 4608*f*g**3*x**5 + 187904*f*g**3*x*
*4 + 142224*f*g**3*x**3 + 160488*f*g**3*x**2 - 851580*f*g**3*x - 549990*f*
g**3 + 33536*g**4*x**5 + 25152*g**4*x**4 + 28296*g**4*x**3 + 35370*g**4*x*
*2 - 269325*g**4*x - 153090*g**4))/(10395*sqrt(f + g*x)*(32*f**10 + 160*f*
*9*g*x + 240*f**9*g + 320*f**8*g**2*x**2 + 1200*f**8*g**2*x + 720*f**8*g**
2 + 320*f**7*g**3*x**3 + 2400*f**7*g**3*x**2 + 3600*f**7*g**3*x + 1080*f**
7*g**3 + 160*f**6*g**4*x**4 + 2400*f**6*g**4*x**3 + 7200*f**6*g**4*x**2 +
5400*f**6*g**4*x + 810*f**6*g**4 + 32*f**5*g**5*x**5 + 1200*f**5*g**5*x**4
 + 7200*f**5*g**5*x**3 + 10800*f**5*g**5*x**2 + 4050*f**5*g**5*x + 243*f**
5*g**5 + 240*f**4*g**6*x**5 + 3600*f**4*g**6*x**4 + 10800*f**4*g**6*x**3 +
 8100*f**4*g**6*x**2 + 1215*f**4*g**6*x + 720*f**3*g**7*x**5 + 5400*f**3*g
**7*x**4 + 8100*f**3*g**7*x**3 + 2430*f**3*g**7*x**2 + 1080*f**2*g**8*x**5
 + 4050*f**2*g**8*x**4 + 2430*f**2*g**8*x**3 + 810*f*g**9*x**5 + 1215*f*g*
*9*x**4 + 243*g**10*x**5))
```

3.311 $\int (d+ex)^2(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int (d+ex)^2(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \,dx = \frac{(bd-ae)^2(a+bx)\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}(f+gx)^n\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\left(ad+(bd+ae)x+bex^2\right)^p \operatorname{AppellF1}\left(1+p,-2-\right.}{b^3(1+p)}$$

output
```
(-a*e+b*d)^2*(b*x+a)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(p+1,-2
-p,-n,2+p,-e*(b*x+a)/(-a*e+b*d),-g*(b*x+a)/(-a*g+b*f))/b^3/(p+1)/((b*(e*x+
d)/(-a*e+b*d))^p)/((b*(g*x+f)/(-a*g+b*f))^n)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int (d+ex)^2(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \,dx = \frac{(bd-ae)^2(a+bx)\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}((a+bx)(d+ex))^p(f+gx)^n\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\operatorname{AppellF1}\left(1+p,-2-p,-n,\right.}{b^3(1+p)}$$

input
```
Integrate[(d + e*x)^2*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
((b*d - a*e)^2*(a + b*x)*((a + b*x)*(d + e*x))^p*(f + g*x)^n*AppellF1[1 +
p, -2 - p, -n, 2 + p, (e*(a + b*x))/(-(b*d) + a*e), (g*(a + b*x))/(-(b*f)
+ a*g)])/(b^3*(1 + p)*((b*(d + e*x))/(b*d - a*e))^p*((b*(f + g*x))/(b*f -
a*g))^n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1268, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2(f+gx)^n \left(x(ae+bd)+ad+bex^2\right)^p \, dx$$

$$\Big\downarrow \textbf{1268}$$

$$(a+bx)^{-p}(d+ex)^{-p} \left(x(ae+bd)+ad+bex^2\right)^p \int (a+bx)^p(d+ex)^{p+2}(f+gx)^n dx$$

$$\Big\downarrow \textbf{157}$$

$$\frac{(bd-ae)^2(a+bx)^{-p} \left(\frac{b(d+ex)}{bd-ae}\right)^{-p} \left(x(ae+bd)+ad+bex^2\right)^p \int (a+bx)^p \left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)^{p+2}(f+gx)^n dx}{b^2}$$

$$\Big\downarrow \textbf{156}$$

$$\frac{(bd-ae)^2(a+bx)^{-p}(f+gx)^n \left(\frac{b(d+ex)}{bd-ae}\right)^{-p} \left(x(ae+bd)+ad+bex^2\right)^p \left(\frac{b(f+gx)}{bf-ag}\right)^{-n} \int (a+bx)^p \left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)}{b^2}$$

$$\Big\downarrow \textbf{155}$$

$$\frac{(a+bx)(bd-ae)^2(f+gx)^n \left(\frac{b(d+ex)}{bd-ae}\right)^{-p} \left(x(ae+bd)+ad+bex^2\right)^p \left(\frac{b(f+gx)}{bf-ag}\right)^{-n} \text{AppellF1}\left(p+1,-p-2,-n,\right.}{b^3(p+1)}$$

input
```
Int[(d + e*x)^2*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output

```
((b*d - a*e)^2*(a + b*x)*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p*App
ellF1[1 + p, -2 - p, -n, 2 + p, -((e*(a + b*x))/(b*d - a*e)), -((g*(a + b*
x))/(b*f - a*g))])/(b^3*(1 + p)*((b*(d + e*x))/(b*d - a*e))^p*((b*(f + g*x
))/(b*f - a*g))^n)
```

Defintions of rubi rules used

rule 155

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& !GtQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x] && !Si
mplerQ[e + f*x, a + b*x]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int (ex+d)^2 (gx+f)^n \left(ad+(ae+bd)\,x+be\,x^2\right)^p dx$$

input
```
int((e*x+d)^2*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((e*x+d)^2*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

Fricas [F]

$$\int (d+ex)^2(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \,dx$$
$$= \int (ex+d)^2 \left(bex^2+ad+(bd+ae)x\right)^p (gx+f)^n \,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="fr
icas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x
+ f)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^2(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2*(g*x+f)**n*(a*d+(a*e+b*d)*x+b*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^2(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$
$$= \int (ex+d)^2 \left(bex^2+ad+(bd+ae)x\right)^p (gx+f)^n \, dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="ma
xima")
```

output
```
integrate((e*x + d)^2*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n, x)
```

Giac [F]

$$\int (d+ex)^2(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$
$$= \int (ex+d)^2 \left(bex^2+ad+(bd+ae)x\right)^p (gx+f)^n \, dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="gi
ac")
```

output
```
integrate((e*x + d)^2*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p\,dx$$
$$=\int (f+g\,x)^n\,(d+e\,x)^2\,\left(b\,e\,x^2+(a\,e+b\,d)\,x+a\,d\right)^p\,dx$$

input
```
int((f + g*x)^n*(d + e*x)^2*(a*d + x*(a*e + b*d) + b*e*x^2)^p,x)
```

output
```
int((f + g*x)^n*(d + e*x)^2*(a*d + x*(a*e + b*d) + b*e*x^2)^p, x)
```

Reduce [F]

$$\int (d+ex)^2(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p\,dx$$
$$=\int (ex+d)^2\,(gx+f)^n\,\left(ad+(ae+bd)\,x+be\,x^2\right)^p dx$$

input
```
int((e*x+d)^2*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((e*x+d)^2*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

3.312 $\int (d+ex)(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$

Optimal result

Integrand size = 34, antiderivative size = 143

$$\int (d+ex)(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$

$$= \frac{(bd-ae)(a+bx)\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}(f+gx)^n\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\left(ad+(bd+ae)x+bex^2\right)^p \operatorname{AppellF1}\left(1+p,-1-\right.}{b^2(1+p)}$$

output
```
(-a*e+b*d)*(b*x+a)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(p+1,-1-p
,-n,2+p,-e*(b*x+a)/(-a*e+b*d),-g*(b*x+a)/(-a*g+b*f))/b^2/(p+1)/((b*(e*x+d)
/(-a*e+b*d))^p)/((b*(g*x+f)/(-a*g+b*f))^n)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

$$\int (d+ex)(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$

$$= \frac{\left(\frac{b(d+ex)}{bd-ae}\right)^{-1-p}((a+bx)(d+ex))^{1+p}(f+gx)^n\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\operatorname{AppellF1}\left(1+p,-1-p,-n,2+p,\frac{e(a+bx)}{-bd+ae},\right.}{b(1+p)}$$

input
```
Integrate[(d + e*x)*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
(((b*(d + e*x))/(b*d - a*e))^(-1 - p)*((a + b*x)*(d + e*x))^(1 + p)*(f + g
*x)^n*AppellF1[1 + p, -1 - p, -n, 2 + p, (e*(a + b*x))/(-(b*d) + a*e), (g*
(a + b*x))/(-(b*f) + a*g)])/(b*(1 + p)*((b*(f + g*x))/(b*f - a*g))^n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1268, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(f+gx)^n \left(x(ae+bd)+ad+bex^2\right)^p \, dx$$

$$\Big\downarrow \text{1268}$$

$$(a+bx)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (a+bx)^p(d+ex)^{p+1}(f+gx)^n dx$$

$$\Big\downarrow \text{157}$$

$$\frac{(bd-ae)(a+bx)^{-p}\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (a+bx)^p\left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)^{p+1}(f+gx)^n dx}{b}$$

$$\Big\downarrow \text{156}$$

$$\frac{(bd-ae)(a+bx)^{-p}(f+gx)^n\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\int (a+bx)^p\left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)^{p}}{b}$$

$$\Big\downarrow \text{155}$$

$$\frac{(a+bx)(bd-ae)(f+gx)^n\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\text{AppellF1}\,\Big(p+1,-p-1,-n,p}{b^2(p+1)}$$

input
```
Int[(d + e*x)*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
((b*d - a*e)*(a + b*x)*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p*Appel
lF1[1 + p, -1 - p, -n, 2 + p, -((e*(a + b*x))/(b*d - a*e)), -((g*(a + b*x)
)/(b*f - a*g))])/(b^2*(1 + p)*((b*(d + e*x))/(b*d - a*e))^p*((b*(f + g*x))
/(b*f - a*g))^n)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int (ex+d)\,(gx+f)^n\,\left(ad+(ae+bd)\,x+be\,x^2\right)^p\,dx$$

input
```
int((e*x+d)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((e*x+d)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

Fricas [F]

$$\begin{aligned}\int (d+ex)(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p\,dx \\ = \int (ex+d)\left(bex^2+ad+(bd+ae)x\right)^p(gx+f)^n\,dx\end{aligned}$$

input
```
integrate((e*x+d)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="fric
as")
```

output
```
integral((e*x + d)*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)*(g*x+f)**n*(a*d+(a*e+b*d)*x+b*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int (ex+d)\left(bex^2+ad+(bd+ae)x\right)^p(gx+f)^n \, dx$$

input
```
integrate((e*x+d)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="maxi
ma")
```

output
```
integrate((e*x + d)*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n, x)
```

Giac [F]

$$\int (d+ex)(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int (ex+d)\left(bex^2+ad+(bd+ae)x\right)^p(gx+f)^n \, dx$$

input
```
integrate((e*x+d)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="giac
")
```

output
```
integrate((e*x + d)*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int (f+g\,x)^n\,(d+e\,x)\,\left(b\,e\,x^2+(a\,e+b\,d)\,x+a\,d\right)^p \, dx$$

input
```
int((f + g*x)^n*(d + e*x)*(a*d + x*(a*e + b*d) + b*e*x^2)^p,x)
```

output
```
int((f + g*x)^n*(d + e*x)*(a*d + x*(a*e + b*d) + b*e*x^2)^p, x)
```

Reduce [F]

$$\int (d+ex)(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int (ex+d)\,(gx+f)^n\,\left(ad+(ae+bd)\,x+be\,x^2\right)^p dx$$

input
```
int((e*x+d)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((e*x+d)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

3.313 $\int (f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$

Optimal result

Integrand size = 29, antiderivative size = 134

$$\int (f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$
$$= \frac{(f+gx)^{1+n} \left(ad+(bd+ae)x+bex^2\right)^p \left(1-\frac{b(f+gx)}{bf-ag}\right)^{-p} \left(1-\frac{e(f+gx)}{ef-dg}\right)^{-p} \operatorname{AppellF1}\Bigg(1+n,-p,-p,2+n}{g(1+n)}$$

output
```
(g*x+f)^(1+n)*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(1+n,-p,-p,2+n,b*(g*x+f)
/(-a*g+b*f),e*(g*x+f)/(-d*g+e*f))/g/(1+n)/((1-b*(g*x+f)/(-a*g+b*f))^p)/((1
-e*(g*x+f)/(-d*g+e*f))^p)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int (f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$
$$= \frac{(a+bx)\left(\frac{b(d+ex)}{bd-ae}\right)^{-p} ((a+bx)(d+ex))^p (f+gx)^n \left(\frac{b(f+gx)}{bf-ag}\right)^{-n} \operatorname{AppellF1}\Bigg(1+p,-p,-n,2+p,\frac{e(a+bx)}{-bd+ae},}{b(1+p)}$$

input
```
Integrate[(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
((a + b*x)*((a + b*x)*(d + e*x))^p*(f + g*x)^n*AppellF1[1 + p, -p, -n, 2 +
 p, (e*(a + b*x))/(-(b*d) + a*e), (g*(a + b*x))/(-(b*f) + a*g)])/(b*(1 + p
)*((b*(d + e*x))/(b*d - a*e))^p*((b*(f + g*x))/(b*f - a*g))^n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^n \left(x(ae+bd)+ad+bex^2\right)^p dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(x(ae+bd)+ad+bex^2\right)^p \left(1-\frac{b(f+gx)}{bf-ag}\right)^{-p} \left(1-\frac{e(f+gx)}{ef-dg}\right)^{-p} \int (f+gx)^n \left(1-\frac{b(f+gx)}{bf-ag}\right)^p \left(1-\frac{e(f+gx)}{ef-dg}\right)^p d(f+gx}{g}$$

$$\Big\downarrow \text{150}$$

$$\frac{(f+gx)^{n+1} \left(x(ae+bd)+ad+bex^2\right)^p \left(1-\frac{b(f+gx)}{bf-ag}\right)^{-p} \left(1-\frac{e(f+gx)}{ef-dg}\right)^{-p} \text{AppellF1}\left(n+1,-p,-p,n+2,\frac{b(f+gx}{bf-ag}\right.}{g(n+1)}$$

input
```
Int[(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
((f + g*x)^(1 + n)*(a*d + (b*d + a*e)*x + b*e*x^2)^p*AppellF1[1 + n, -p, -
p, 2 + n, (b*(f + g*x))/(b*f - a*g), (e*(f + g*x))/(e*f - d*g)])/(g*(1 + n
)*(1 - (b*(f + g*x))/(b*f - a*g))^p*(1 - (e*(f + g*x))/(e*f - d*g))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (gx+f)^{n} \left(ad+(ae+bd)\,x+be\,x^{2}\right)^{p} dx$$

input
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

Fricas [F]

$$\int (f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int \left(bex^2+ad+(bd+ae)x\right)^p (gx+f)^n \, dx$$

input
```
integrate((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**n*(a*d+(a*e+b*d)*x+b*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p\,dx=\int \left(bex^2+ad+(bd+ae)x\right)^p(gx+f)^n\,dx$$

input
```
integrate((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n, x)
```

Giac [F]

$$\int (f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p\,dx=\int \left(bex^2+ad+(bd+ae)x\right)^p(gx+f)^n\,dx$$

input
```
integrate((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p\,dx=\int (f+g\,x)^n\,\left(b\,e\,x^2+(a\,e+b\,d)\,x+a\,d\right)^p\,dx$$

input
```
int((f + g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p,x)
```

output
```
int((f + g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p, x)
```

Reduce [F]

$$\int (f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p\,dx=\int (gx+f)^n\left(ad+(ae+bd)\,x+be\,x^2\right)^p dx$$

input
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

3.314 $\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{d+ex}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 142

$$\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{d+ex}\,dx = \frac{(a+bx)\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}(f+gx)^n\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\left(ad+(bd+ae)x+bex^2\right)^p\operatorname{AppellF1}\Big(1+p,1-p,-n,2+p}{(bd-ae)(1+p)}$$

output
```
(b*x+a)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(p+1,1-p,-n,2+p,-e*(
b*x+a)/(-a*e+b*d),-g*(b*x+a)/(-a*g+b*f))/(-a*e+b*d)/(p+1)/((b*(e*x+d)/(-a*
e+b*d))^p)/((b*(g*x+f)/(-a*g+b*f))^n)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

$$\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{d+ex}\,dx = \frac{(a+bx)\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}((a+bx)(d+ex))^p(f+gx)^n\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\operatorname{AppellF1}\Big(1+p,1-p,-n,2+p,\frac{e(a+bx)}{-bd+ae}}{(bd-ae)(1+p)}$$

input
```
Integrate[((f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p)/(d + e*x),x]
```

output
```
((a + b*x)*((a + b*x)*(d + e*x))^p*(f + g*x)^n*AppellF1[1 + p, 1 - p, -n,
2 + p, (e*(a + b*x))/(-(b*d) + a*e), (g*(a + b*x))/(-(b*f) + a*g)])/((b*d
- a*e)*(1 + p)*((b*(d + e*x))/(b*d - a*e))^p*((b*(f + g*x))/(b*f - a*g))^n
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1268, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n \left(x(ae+bd)+ad+bex^2\right)^p}{d+ex} \, dx$$

$$\Big\downarrow \text{ 1268}$$

$$(a+bx)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (a+bx)^p(d+ex)^{p-1}(f+gx)^n dx$$

$$\Big\downarrow \text{ 157}$$

$$\frac{b(a+bx)^{-p}\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (a+bx)^p\left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)^{p-1}(f+gx)^n dx}{bd-ae}$$

$$\Big\downarrow \text{ 156}$$

$$\frac{b(a+bx)^{-p}(f+gx)^n\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\int (a+bx)^p\left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)^{p-1}\left(\frac{bf}{bf-a}}{bd-ae}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{(a+bx)(f+gx)^n\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\text{AppellF1}\left(p+1,1-p,-n,p+2,-\frac{e(a+}{bd-}}{(p+1)(bd-ae)}$$

input

```
Int[((f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p)/(d + e*x),x]
```

output

```
((a + b*x)*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p*AppellF1[1 + p, 1
 - p, -n, 2 + p, -((e*(a + b*x))/(b*d - a*e)), -((g*(a + b*x))/(b*f - a*g)
)])/((b*d - a*e)*(1 + p)*((b*(d + e*x))/(b*d - a*e))^p*((b*(f + g*x))/(b*f
 - a*g))^n)
```

Defintions of rubi rules used

rule 155

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(gx+f)^n\,(ad+(ae+bd)\,x+be\,x^2)^p}{ex+d}dx$$

input
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x)
```

output
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(f+gx)^n\,(ad+(bd+ae)x+bex^2)^p}{d+ex}\,dx=\int \frac{(bex^2+ad+(bd+ae)x)^p(gx+f)^n}{ex+d}\,dx$$

input
```
integrate((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x, algorithm="fric
as")
```

output
```
integral((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\,(ad+(bd+ae)x+bex^2)^p}{d+ex}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**n*(a*d+(a*e+b*d)*x+b*e*x**2)**p/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p}{d+ex} \, dx = \int \frac{(bex^2+ad+(bd+ae)x)^p (gx+f)^n}{ex+d} \, dx$$

input
```
integrate((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x, algorithm="maxi
ma")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n/(e*x + d), x)
```

Giac [F]

$$\int \frac{(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p}{d+ex} \, dx = \int \frac{(bex^2+ad+(bd+ae)x)^p (gx+f)^n}{ex+d} \, dx$$

input
```
integrate((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x, algorithm="giac
")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p}{d+ex} \, dx = \int \frac{(f+g\,x)^n\,\left(b\,e\,x^2+(a\,e+b\,d)\,x+a\,d\right)^p}{d+e\,x} \, dx$$

input
```
int(((f + g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p)/(d + e*x),x)
```

output
```
int(((f + g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(f+gx)^n\,(ad+(bd+ae)x+bex^2)^p}{d+ex}\,dx=\int \frac{(gx+f)^n\,(ad+(ae+bd)\,x+be\,x^2)^p}{ex+d}dx$$

input
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x)
```

output
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x)
```

3.315 $\int \frac{(f+gx)^n(ad+(bd+ae)x+bex^2)^p}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 143

$$\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx = \frac{b(a+bx)\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}(f+gx)^n\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\left(ad+(bd+ae)x+bex^2\right)^p \operatorname{AppellF1}\left(1+p,2-p,-n,2+\right.}{(bd-ae)^2(1+p)}$$

output
```
b*(b*x+a)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(p+1,2-p,-n,2+p,-e
*(b*x+a)/(-a*e+b*d),-g*(b*x+a)/(-a*g+b*f))/(-a*e+b*d)^2/(p+1)/((b*(e*x+d)/
(-a*e+b*d))^p)/((b*(g*x+f)/(-a*g+b*f))^n)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93

$$\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx = \frac{b(a+bx)\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}((a+bx)(d+ex))^p(f+gx)^n\left(\frac{b(f+gx)}{bf-ag}\right)^{-n} \operatorname{AppellF1}\left(1+p,2-p,-n,2+p,\frac{e(a+b}{-bd+}\right.}{(bd-ae)^2(1+p)}$$

input
```
Integrate[((f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p)/(d + e*x)^2,x]
```

output
```
(b*(a + b*x)*((a + b*x)*(d + e*x))^p*(f + g*x)^n*AppellF1[1 + p, 2 - p, -n
, 2 + p, (e*(a + b*x))/(-(b*d) + a*e), (g*(a + b*x))/(-(b*f) + a*g)])/((b*
d - a*e)^2*(1 + p)*((b*(d + e*x))/(b*d - a*e))^p*((b*(f + g*x))/(b*f - a*g
))^n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1268, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^n\left(x(ae+bd)+ad+bex^2\right)^p}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 1268}$$

$$(a+bx)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\int (a+bx)^p(d+ex)^{p-2}(f+gx)^ndx$$

$$\Big\downarrow \text{ 157}$$

$$\frac{b^2(a+bx)^{-p}\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\int (a+bx)^p\left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)^{p-2}(f+gx)^ndx}{(bd-ae)^2}$$

$$\Big\downarrow \text{ 156}$$

$$\frac{b^2(a+bx)^{-p}(f+gx)^n\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\int (a+bx)^p\left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)^{p-2}}{(bd-ae)^2}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{b(a+bx)(f+gx)^n\left(\frac{b(d+ex)}{bd-ae}\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\left(\frac{b(f+gx)}{bf-ag}\right)^{-n}\text{AppellF1}\left(p+1,2-p,-n,p+2,-\right.}{(p+1)(bd-ae)^2}$$

input

```
Int[((f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p)/(d + e*x)^2,x]
```

output

```
(b*(a + b*x)*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p*AppellF1[1 + p,
 2 - p, -n, 2 + p, -((e*(a + b*x))/(b*d - a*e)), -((g*(a + b*x))/(b*f - a*
g))])/((b*d - a*e)^2*(1 + p)*((b*(d + e*x))/(b*d - a*e))^p*((b*(f + g*x))/
(b*f - a*g))^n)
```

Defintions of rubi rules used

rule 155

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(gx+f)^n\,(ad+(ae+bd)\,x+be\,x^2)^p}{(ex+d)^2}dx$$

input
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x)
```

output
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(f+gx)^n\,(ad+(bd+ae)x+bex^2)^p}{(d+ex)^2}\,dx = \int \frac{(bex^2+ad+(bd+ae)x)^p(gx+f)^n}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x, algorithm="fr
icas")
```

output
```
integral((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n/(e^2*x^2 + 2*d*e*x
+ d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**n*(a*d+(a*e+b*d)*x+b*e*x**2)**p/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx=\int \frac{(bex^2+ad+(bd+ae)x)^p(gx+f)^n}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x, algorithm="ma
xima")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx=\int \frac{(bex^2+ad+(bd+ae)x)^p(gx+f)^n}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x, algorithm="gi
ac")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x + f)^n/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx\\&=\int \frac{{\left(f+g\,x\right)}^n\,{\left(b\,e\,x^2+\left(a\,e+b\,d\right)\,x+a\,d\right)}^p}{{\left(d+e\,x\right)}^2}\,dx\end{aligned}$$

input
```
int(((f + g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p)/(d + e*x)^2,x)
```

output
```
int(((f + g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx=\int \frac{\left(gx+f\right)^n\left(ad+\left(ae+bd\right)x+be\,x^2\right)^p}{\left(ex+d\right)^2}dx$$

input
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x)
```

output
```
int((g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x)
```

3.316 $\int (d+ex)^m (f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$

Optimal result

Integrand size = 36, antiderivative size = 147

$$\int (d+ex)^m (f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$
$$= \frac{(d+ex)^{-1+m} \left(\frac{b(d+ex)}{bd-ae}\right)^{-m-p} (f+gx)^n \left(\frac{b(f+gx)}{bf-ag}\right)^{-n} \left(ad+(bd+ae)x+bex^2\right)^{1+p} \operatorname{AppellF1}\Big(1+p,-m}{b(1+p)}$$

output
```
(e*x+d)^(-1+m)*(b*(e*x+d)/(-a*e+b*d))^(-m-p)*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*
e*x^2)^(p+1)*AppellF1(p+1,-m-p,-n,2+p,-e*(b*x+a)/(-a*e+b*d),-g*(b*x+a)/(-a
*g+b*f))/b/(p+1)/((b*(g*x+f)/(-a*g+b*f))^n)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int (d+ex)^m (f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$
$$= \frac{(a+bx)(d+ex)^m \left(\frac{b(d+ex)}{bd-ae}\right)^{-m-p} ((a+bx)(d+ex))^p (f+gx)^n \left(\frac{b(f+gx)}{bf-ag}\right)^{-n} \operatorname{AppellF1}\Big(1+p,-m-p,}{b(1+p)}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output

```
((a + b*x)*(d + e*x)^m*((b*(d + e*x))/(b*d - a*e))^(-m - p)*((a + b*x)*(d
+ e*x))^p*(f + g*x)^n*AppellF1[1 + p, -m - p, -n, 2 + p, (e*(a + b*x))/(-(
b*d) + a*e), (g*(a + b*x))/(-(b*f) + a*g)])/(b*(1 + p)*((b*(f + g*x))/(b*f
 - a*g))^n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1268, 157, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m(f+gx)^n \left(x(ae+bd)+ad+bex^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1268}$$

$$(a+bx)^{-p}(d+ex)^{-p} \left(x(ae+bd)+ad+bex^2\right)^p \int (a+bx)^p(d+ex)^{m+p}(f+gx)^n dx$$

$$\Big\downarrow \text{ 157}$$

$$(a+bx)^{-p}(d+ex)^m \left(x(ae+bd)+ad+bex^2\right)^p \left(\frac{b(d+ex)}{bd-ae}\right)^{-m-p} \int (a+bx)^p \left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)^{m+p} (f+gx)^n dx$$

$$\Big\downarrow \text{ 156}$$

$$(a+bx)^{-p}(d+ex)^m(f+gx)^n \left(x(ae+bd)+ad+bex^2\right)^p \left(\frac{b(f+gx)}{bf-ag}\right)^{-n} \left(\frac{b(d+ex)}{bd-ae}\right)^{-m-p} \int (a+bx)^p \left(\frac{bd}{bd-ae}+\frac{bex}{bd-ae}\right)^{m+p} \left(\frac{bf}{bf-ag}+\frac{bgx}{bf-ag}\right)^n dx$$

$$\Big\downarrow \text{ 155}$$

$$\frac{(a+bx)(d+ex)^m(f+gx)^n \left(x(ae+bd)+ad+bex^2\right)^p \left(\frac{b(f+gx)}{bf-ag}\right)^{-n} \left(\frac{b(d+ex)}{bd-ae}\right)^{-m-p} \text{AppellF1}\left(p+1,-m-p,-\right.}{b(p+1)}$$

input

```
Int[(d + e*x)^m*(f + g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output

```
((a + b*x)*(d + e*x)^m*((b*(d + e*x))/(b*d - a*e))^(-m - p)*(f + g*x)^n*(a
*d + (b*d + a*e)*x + b*e*x^2)^p*AppellF1[1 + p, -m - p, -n, 2 + p, -((e*(a
 + b*x))/(b*d - a*e)), -((g*(a + b*x))/(b*f - a*g))])/(b*(1 + p)*((b*(f +
g*x))/(b*f - a*g))^n)
```

Defintions of rubi rules used

rule 155

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x] && !Si
mplerQ[e + f*x, a + b*x]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int (ex+d)^{m} (gx+f)^{n} \left(ad+(ae+bd)\, x+be\, x^{2}\right)^{p}dx$$

input
```
int((e*x+d)^m*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((e*x+d)^m*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p\,dx = \int (bex^2+ad+(bd+ae)x)^p(ex+d)^m(gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="fr
icas")
```

output
```
integral((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(e*x + d)^m*(g*x + f)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**n*(a*d+(a*e+b*d)*x+b*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p\,dx$$
$$=\int (bex^2+ad+(bd+ae)x)^p(ex+d)^m(gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="ma
xima")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(e*x + d)^m*(g*x + f)^n, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)^n\left(ad+(bd+ae)x+bex^2\right)^p\,dx$$
$$=\int (bex^2+ad+(bd+ae)x)^p(ex+d)^m(gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="gi
ac")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(e*x + d)^m*(g*x + f)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int {\left (f+g\,x\right)}^n\,{\left (d+e\,x\right)}^m\,{\left (b\,e\,x^2+\left (a\,e+b\,d\right)\,x+a\,d\right)}^p \,d x$$

input
```
int((f + g*x)^n*(d + e*x)^m*(a*d + x*(a*e + b*d) + b*e*x^2)^p,x)
```

output
```
int((f + g*x)^n*(d + e*x)^m*(a*d + x*(a*e + b*d) + b*e*x^2)^p, x)
```

Reduce [F]

$$\int (d+ex)^m(f+gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int \left(e x +d \right)^{m} \left(g x +f \right)^{n} \left(a d +\left(a e +b d \right) x +b e \,x^{2}\right)^{p}d x$$

input
```
int((e*x+d)^m*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((e*x+d)^m*(g*x+f)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file