Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.3/98-1.2.1.3-e1

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**246**]. This is test number [98].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (246)	0.00 (0)
Rubi	97.56 (240)	2.44 (6)
Maple	93.50 (230)	6.50 (16)
Fricas	93.50 (230)	6.50 (16)
Giac	69.51 (171)	30.49 (75)
Reduce	63.01 (155)	36.99 (91)
Mupad	50.81 (125)	49.19 (121)
Sympy	41.46 (102)	58.54 (144)
Maxima	28.46 (70)	71.54 (176)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

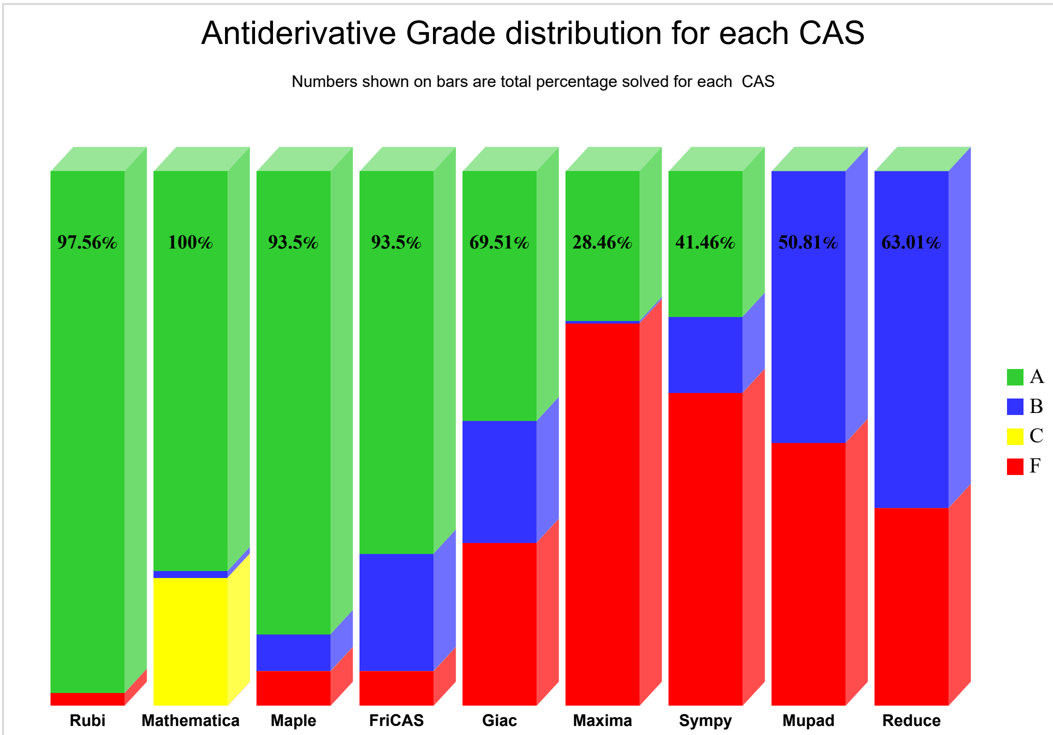
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

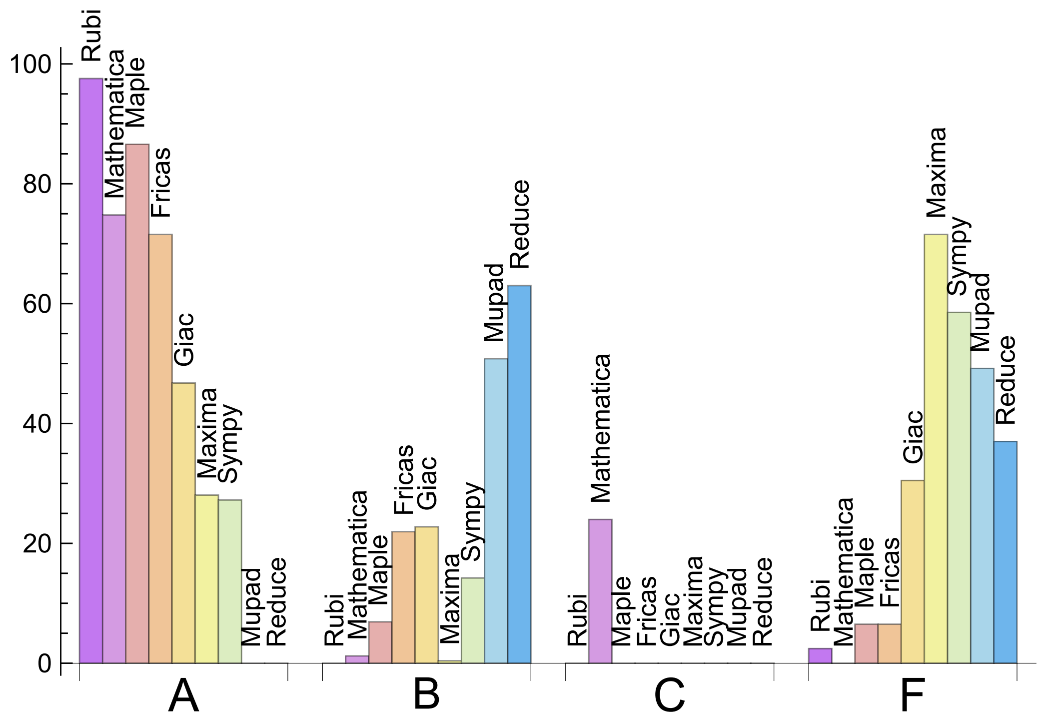
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.561	0.000	0.000	2.439
Maple	86.585	6.911	0.000	6.504
Mathematica	74.797	1.220	23.984	0.000
Fricas	71.545	21.951	0.000	6.504
Giac	46.748	22.764	0.000	30.488
Maxima	28.049	0.407	0.000	71.545
Sympy	27.236	14.228	0.000	58.537
Mupad	0.000	50.813	0.000	49.187
Reduce	0.000	63.008	0.000	36.992

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	6	100.00	0.00	0.00
Fricas	16	100.00	0.00	0.00
Maple	16	100.00	0.00	0.00
Giac	75	94.67	0.00	5.33
Reduce	91	100.00	0.00	0.00
Mupad	121	0.00	100.00	0.00
Sympy	144	72.22	27.78	0.00
Maxima	176	51.14	0.00	48.86

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Giac	0.35
Rubi	0.41
Maple	1.10
Reduce	1.71
Fricas	2.08
Sympy	4.81
Mathematica	6.09
Mupad	6.62

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	106.76	0.95	93.00	0.92
Rubi	199.75	1.02	186.50	1.00
Mathematica	208.87	1.02	168.00	0.98
Maple	242.33	1.13	164.50	1.02
Reduce	760.88	3.16	175.00	1.44
Giac	914.92	3.29	193.00	1.18
Fricas	947.55	3.42	170.00	1.08
Mupad	2610.26	7.67	164.00	1.04
Sympy	4633.41	21.42	193.00	1.42

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

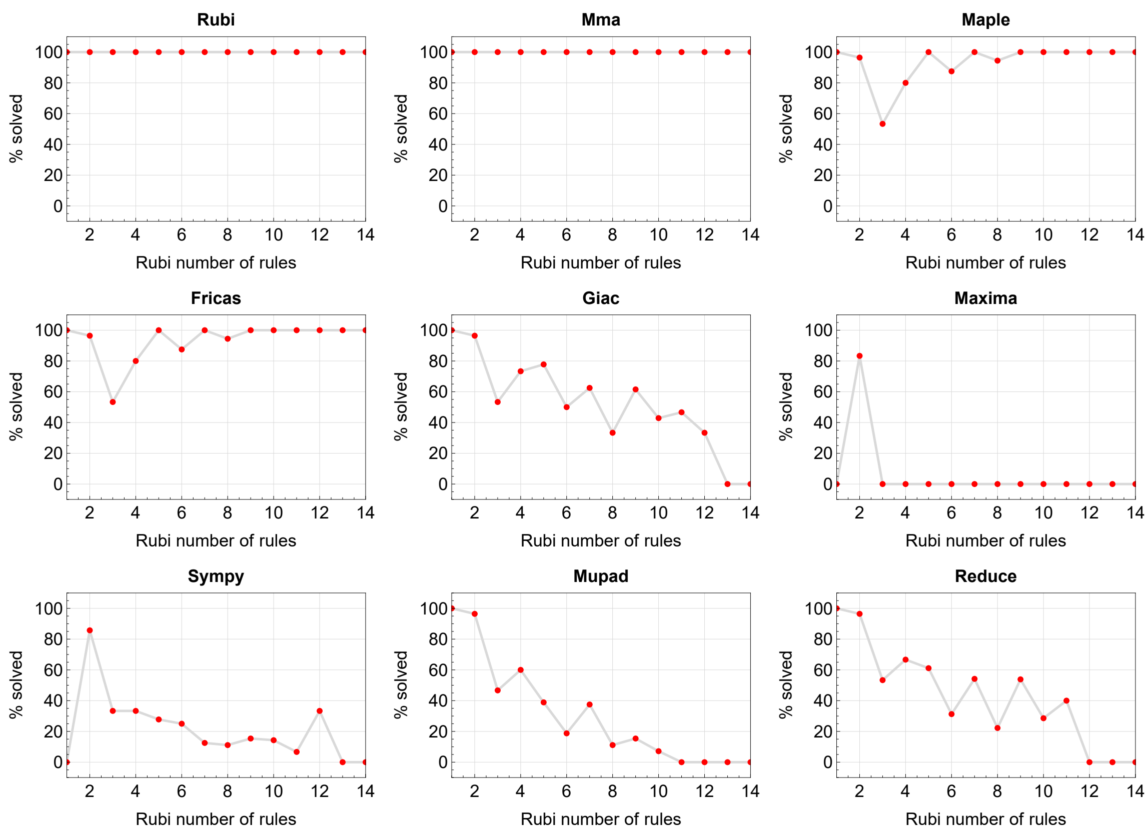


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

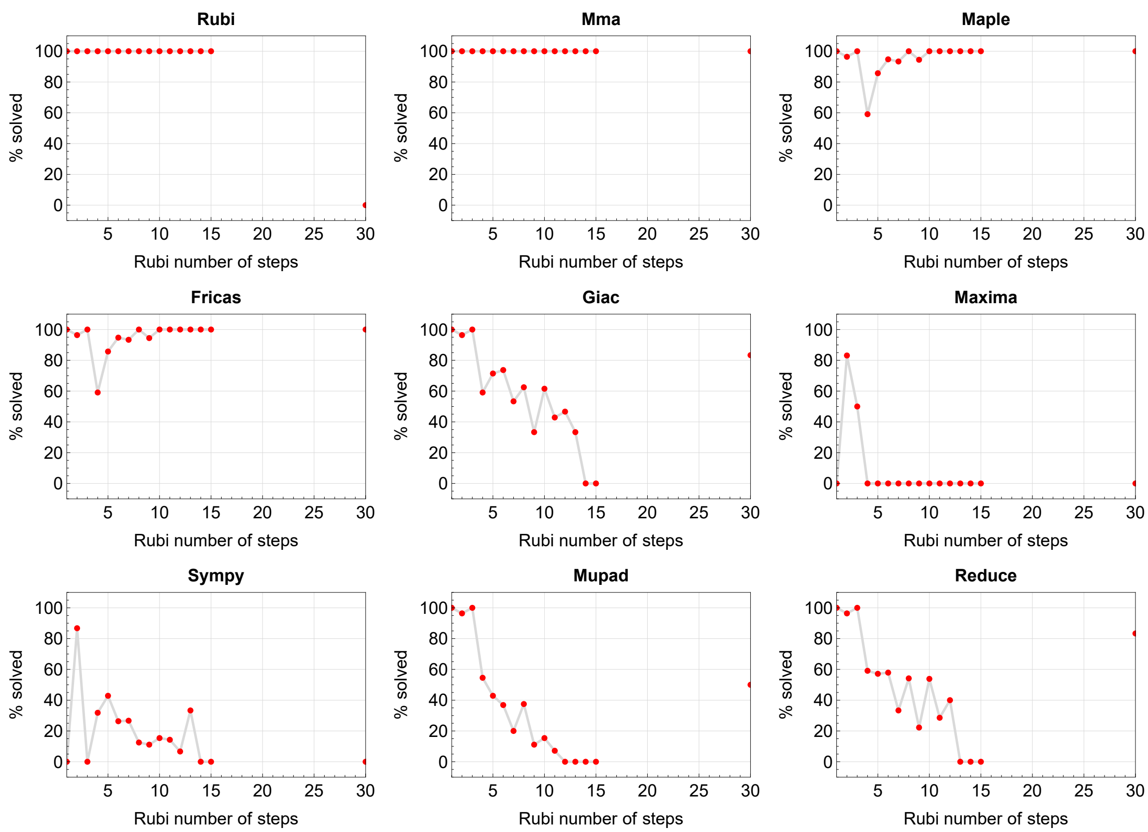


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

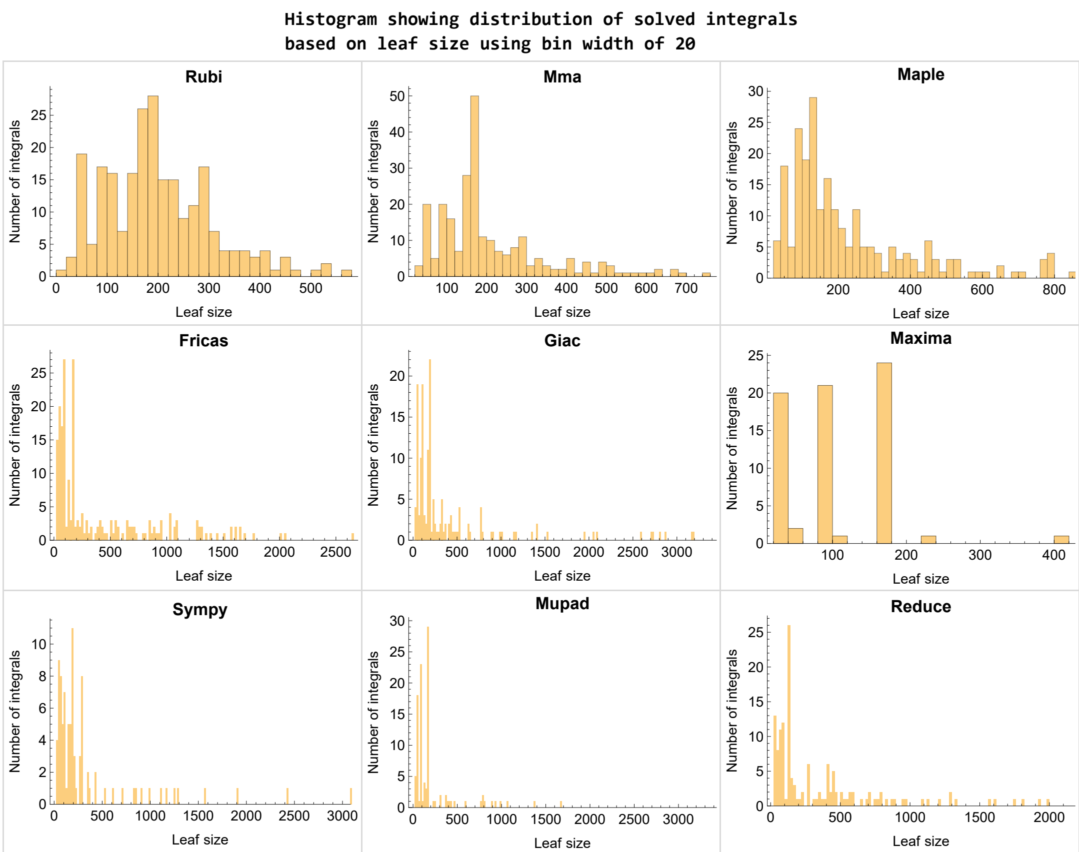


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

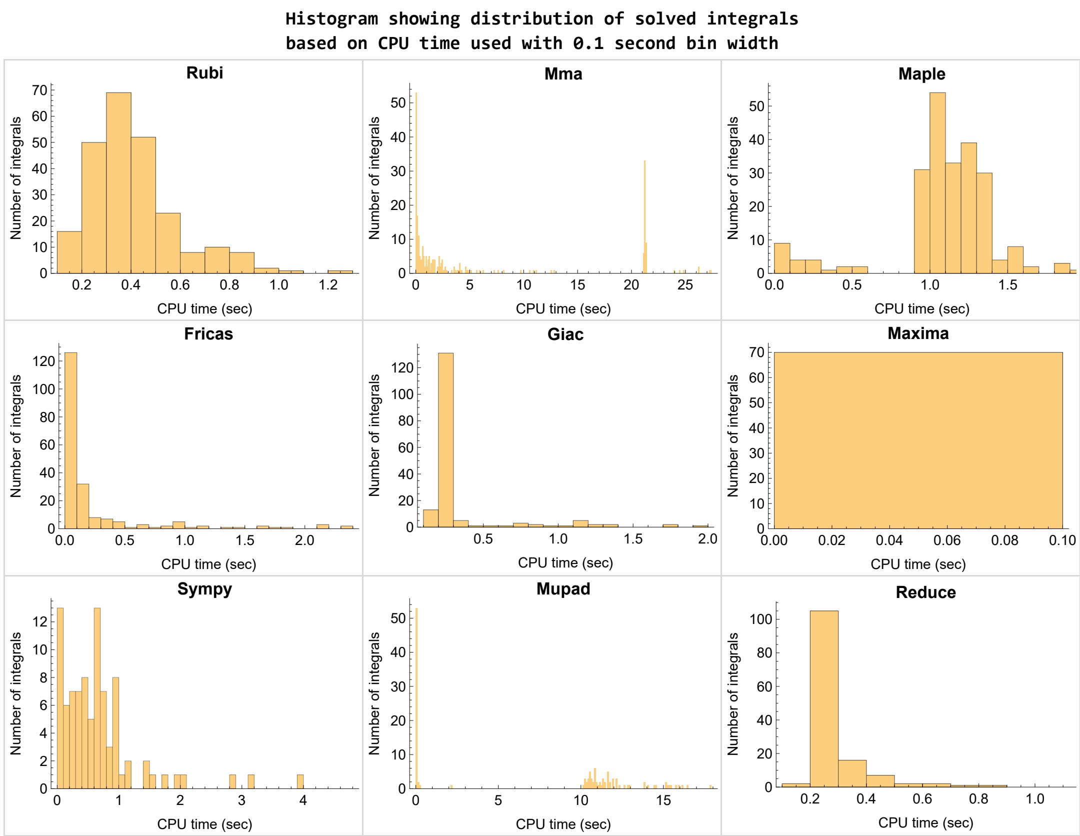


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

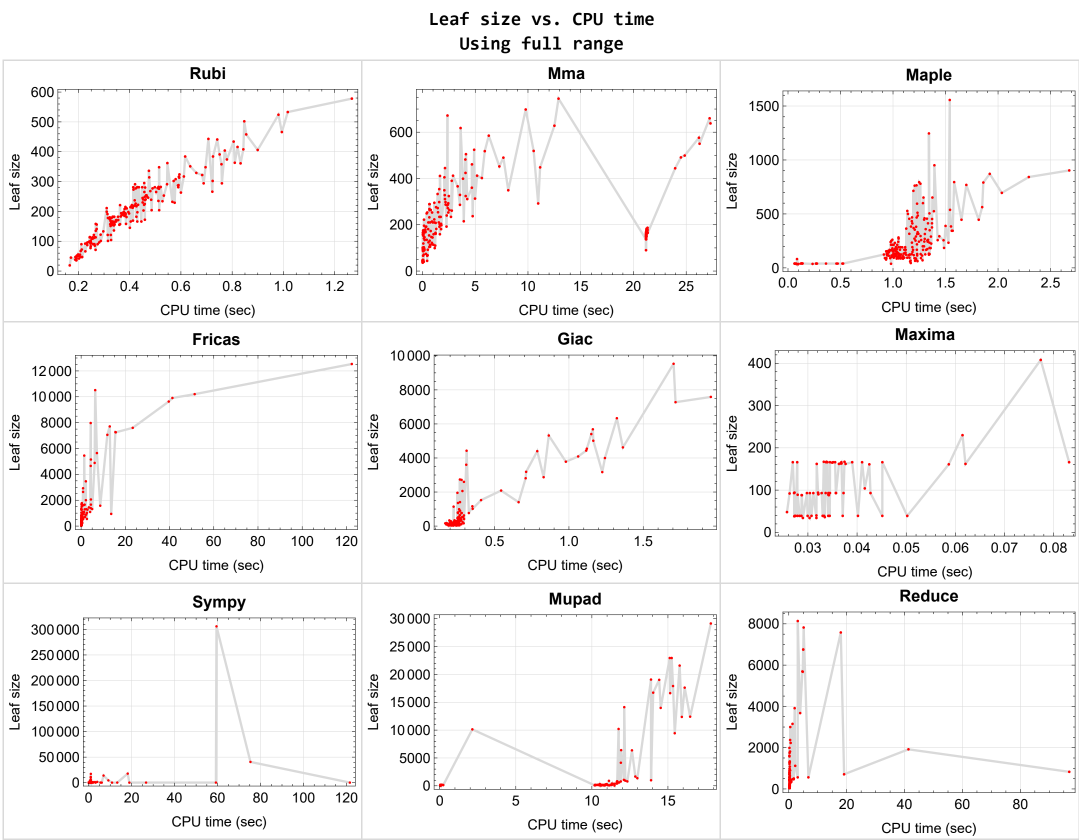


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {240, 241, 242, 244, 245, 246}

Mathematica {233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

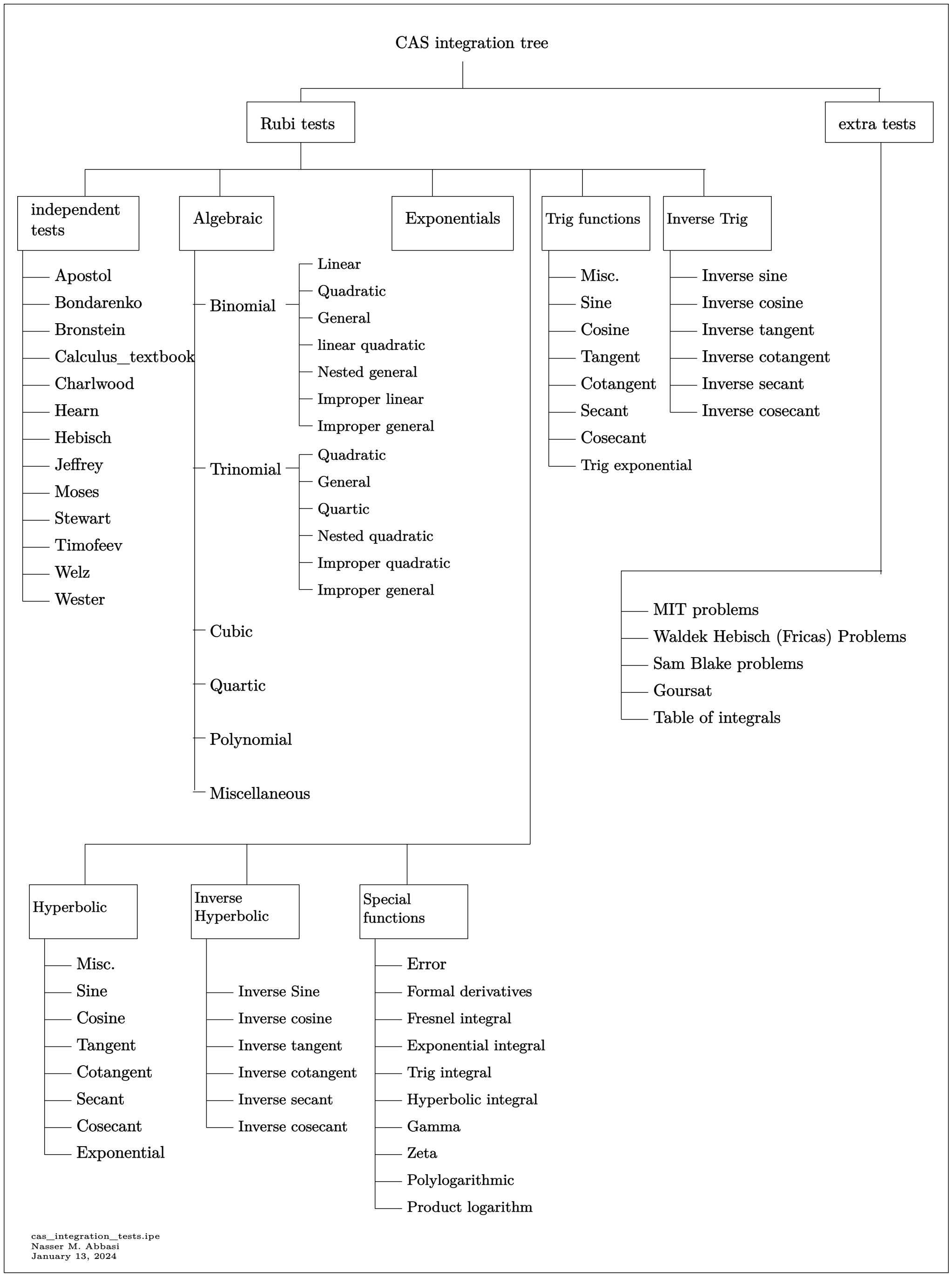
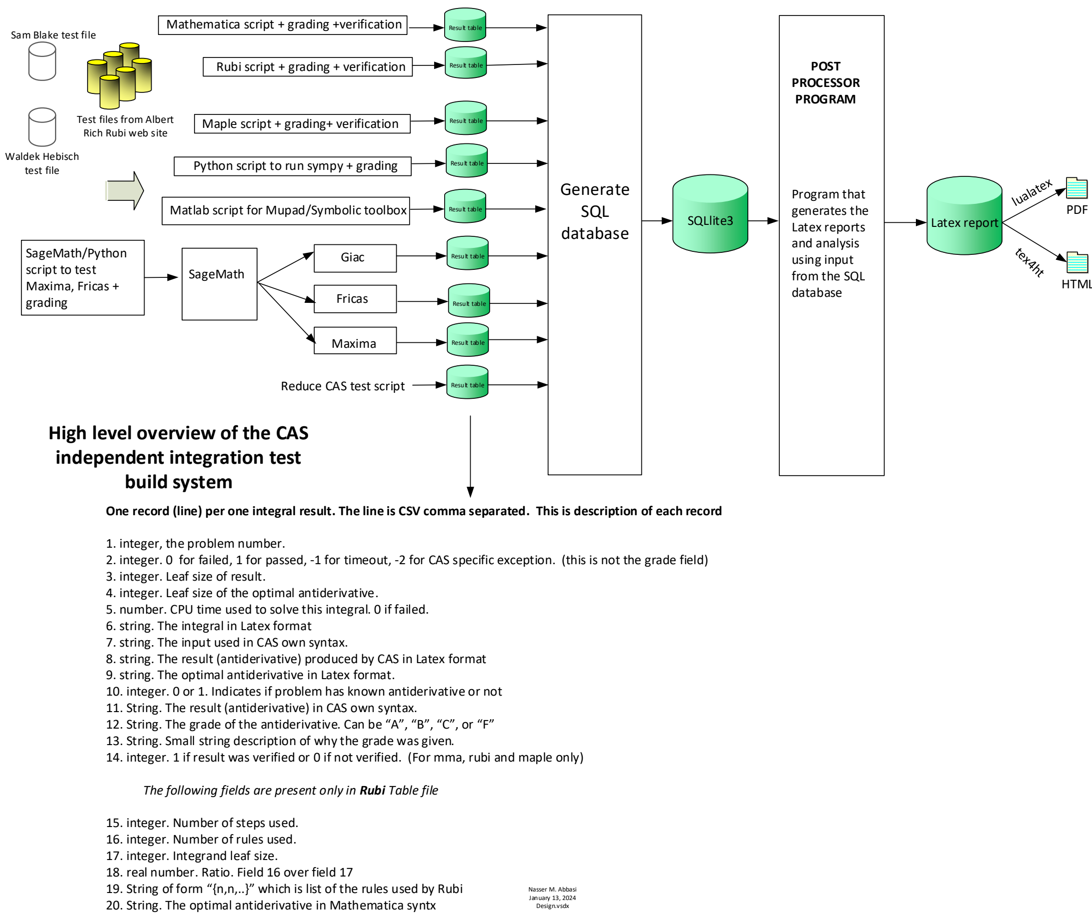


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

B grade { }

C grade { }

F normal fail { 89, 90, 91, 111, 124, 177 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 244, 245, 246 }

B grade { 172, 228, 233 }

C grade { 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 240, 241, 242, 243 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 138, 139, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 173, 174, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230 }

B grade { 102, 131, 132, 135, 136, 137, 141, 142, 143, 162, 163, 169, 170, 171, 172, 175, 176 }

C grade { }

F normal fail { 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 134, 135, 136, 137, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 157, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

B grade { 48, 49, 50, 51, 52, 53, 54, 55, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 131, 132, 133, 141, 142, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 228, 229, 230 }

C grade { }

F normal fail { 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 228, 229, 230 }

B grade { 172 }

C grade { }

F normal fail { 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

F(-1) timedout fail { }

F(-2) exception fail { 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 104, 105, 106, 107, 108, 110, 116, 117, 118, 119, 120, 122, 129, 130, 135, 144, 145, 146, 147, 149, 153, 154, 155, 156, 157, 158, 159, 162, 163, 165, 167, 168 }

B grade { 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 111, 112, 113, 114, 115, 123, 124, 125, 126, 127, 128, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 150, 151, 152, 160, 161, 164, 166, 169, 170, 171, 172, 228, 229, 230 }

C grade { }

F normal fail { 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

F(-1) timedout fail { }

F(-2) exception fail { 109, 121, 134, 148 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 120, 147, 148, 149, 156, 157, 164, 165, 166, 170, 171, 172, 228, 229, 230 }

C grade { }

F normal fail { }

F(-1) timedout fail { 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 167, 168, 169, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 144, 145 }

B grade { 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 84, 85, 86, 87, 88, 104, 105, 106, 107, 108, 116, 117, 118, 119, 120, 129, 130, 131, 132, 133, 146, 147, 228, 229, 230 }

C grade { }

F normal fail { 109, 110, 111, 112, 113, 114, 115, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 222, 223, 224, 225, 226, 231, 233, 234, 235, 236, 237, 241, 242, 243, 244, 245, 246 }

F(-1) timedout fail { 38, 44, 45, 46, 47, 53, 54, 55, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 161, 167, 168, 169, 170, 171, 199, 217, 218, 219, 220, 221, 227, 232, 238, 239, 240 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 114, 115, 122, 123, 124, 125, 126, 127, 128, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 228, 229, 230 }

C grade { }

F normal fail { 104, 105, 106, 107, 116, 117, 118, 119, 120, 121, 129, 130, 131, 132, 133, 134, 144, 154, 162, 163, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	40	39	39	42	43	38	41
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.89	0.91	0.81	0.87
time (sec)	N/A	0.214	0.013	0.276	0.031	0.063	0.018	0.201	0.213	0.049

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	40	39	39	42	43	38	41
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.89	0.91	0.81	0.87
time (sec)	N/A	0.215	0.015	0.267	0.027	0.063	0.017	0.211	0.210	0.043

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	40	39	39	42	43	38	41
N.S.	1	1.00	0.87	0.85	0.83	0.83	0.89	0.91	0.81	0.87
time (sec)	N/A	0.209	0.014	0.269	0.028	0.062	0.021	0.221	0.214	0.043

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	36	39	40	36	38
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.93	0.95	0.86	0.90
time (sec)	N/A	0.202	0.014	0.234	0.033	0.062	0.016	0.219	0.205	0.043

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	36	34	34	36	36	34	35
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.95	0.95	0.89	0.92
time (sec)	N/A	0.188	0.013	0.098	0.032	0.065	0.050	0.191	0.224	0.043

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	34	40	31	34	39	34
N.S.	1	1.00	1.00	0.94	0.94	1.11	0.86	0.94	1.08	0.94
time (sec)	N/A	0.192	0.019	0.096	0.029	0.064	0.078	0.221	0.293	0.040

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	34	41	36	35	42	34
N.S.	1	1.00	1.03	0.97	0.94	1.14	1.00	0.97	1.17	0.94
time (sec)	N/A	0.191	0.023	0.083	0.030	0.069	0.154	0.209	0.222	0.053

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	38	38	41	44	39	40	38
N.S.	1	1.00	1.00	0.93	0.93	1.00	1.07	0.95	0.98	0.93
time (sec)	N/A	0.192	0.030	0.069	0.027	0.064	0.320	0.198	0.234	0.056

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	40	39	39	46	41	38	40
N.S.	1	1.00	0.93	0.89	0.87	0.87	1.02	0.91	0.84	0.89
time (sec)	N/A	0.192	0.017	0.062	0.028	0.061	0.490	0.188	0.247	0.030

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	40	39	39	46	41	38	41
N.S.	1	1.00	0.94	0.85	0.83	0.83	0.98	0.87	0.81	0.87
time (sec)	N/A	0.191	0.017	0.070	0.034	0.065	0.884	0.213	0.230	0.030

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	40	39	39	46	41	38	41
N.S.	1	1.00	0.96	0.85	0.83	0.83	0.98	0.87	0.81	0.87
time (sec)	N/A	0.188	0.018	0.063	0.045	0.062	1.105	0.224	0.275	0.030

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	40	39	39	46	41	38	41
N.S.	1	1.00	0.98	0.85	0.83	0.83	0.98	0.87	0.81	0.87
time (sec)	N/A	0.190	0.019	0.079	0.030	0.064	1.562	0.242	0.238	0.031

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	94	93	93	105	103	79	93
N.S.	1	1.00	1.00	0.93	0.92	0.92	1.04	1.02	0.78	0.92
time (sec)	N/A	0.315	0.028	1.157	0.031	0.068	0.026	0.190	0.232	0.046

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	94	93	93	105	103	79	93
N.S.	1	1.00	1.00	0.93	0.92	0.92	1.04	1.02	0.78	0.92
time (sec)	N/A	0.287	0.023	1.081	0.036	0.081	0.025	0.228	0.231	0.032

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	90	90	90	100	99	77	89
N.S.	1	1.00	1.00	0.94	0.94	0.94	1.04	1.03	0.80	0.93
time (sec)	N/A	0.281	0.022	1.093	0.032	0.077	0.036	0.219	0.216	0.032

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	87	88	88	95	95	74	86
N.S.	1	1.00	1.00	0.95	0.96	0.96	1.03	1.03	0.80	0.93
time (sec)	N/A	0.248	0.037	0.931	0.029	0.072	0.095	0.204	0.216	0.037

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	90	88	95	88	92	81	86
N.S.	1	1.00	0.97	1.00	0.98	1.06	0.98	1.02	0.90	0.96
time (sec)	N/A	0.275	0.056	1.069	0.034	0.075	0.113	0.204	0.216	10.620

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	86	87	88	95	94	89	83	87
N.S.	1	1.00	0.96	0.97	0.98	1.06	1.04	0.99	0.92	0.97
time (sec)	N/A	0.263	0.053	1.073	0.029	0.073	0.227	0.213	0.232	10.581

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	86	89	95	99	89	83	87
N.S.	1	1.00	1.00	0.96	0.99	1.06	1.10	0.99	0.92	0.97
time (sec)	N/A	0.257	0.056	1.035	0.034	0.075	0.652	0.180	0.217	10.514

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	92	85	89	95	99	90	83	86
N.S.	1	1.00	1.02	0.94	0.99	1.06	1.10	1.00	0.92	0.96
time (sec)	N/A	0.258	0.055	1.095	0.028	0.070	1.424	0.247	0.226	10.181

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	92	88	92	95	105	93	81	89
N.S.	1	1.00	0.97	0.93	0.97	1.00	1.11	0.98	0.85	0.94
time (sec)	N/A	0.249	0.063	1.053	0.026	0.069	3.186	0.211	0.217	0.067

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	97	90	93	93	107	101	79	91
N.S.	1	1.00	0.98	0.91	0.94	0.94	1.08	1.02	0.80	0.92
time (sec)	N/A	0.242	0.048	0.937	0.034	0.068	6.037	0.236	0.234	10.229

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	100	90	93	93	107	101	79	93
N.S.	1	1.00	0.99	0.89	0.92	0.92	1.06	1.00	0.78	0.92
time (sec)	N/A	0.248	0.069	0.973	0.043	0.070	10.847	0.174	0.237	0.049

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	99	90	93	93	107	101	79	93
N.S.	1	1.00	0.98	0.89	0.92	0.92	1.06	1.00	0.78	0.92
time (sec)	N/A	0.247	0.042	0.923	0.036	0.071	18.917	0.233	0.231	0.048

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	169	166	166	201	192	134	168
N.S.	1	1.00	1.00	1.02	1.00	1.00	1.21	1.16	0.81	1.01
time (sec)	N/A	0.446	0.054	1.016	0.045	0.069	0.033	0.199	0.222	10.223

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	168	166	166	199	191	134	167
N.S.	1	1.00	1.00	1.01	1.00	1.00	1.20	1.15	0.81	1.01
time (sec)	N/A	0.404	0.041	1.107	0.027	0.072	0.033	0.225	0.282	0.048

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	158	164	162	162	190	187	132	163
N.S.	1	1.00	1.00	1.04	1.03	1.03	1.20	1.18	0.84	1.03
time (sec)	N/A	0.386	0.038	1.008	0.062	0.069	0.032	0.213	0.233	0.049

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	157	163	161	161	192	185	130	162
N.S.	1	1.00	1.00	1.04	1.03	1.03	1.22	1.18	0.83	1.03
time (sec)	N/A	0.324	0.070	0.980	0.059	0.078	0.137	0.218	0.273	10.205

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	156	168	162	168	184	183	136	163
N.S.	1	1.00	1.00	1.08	1.04	1.08	1.18	1.17	0.87	1.04
time (sec)	N/A	0.328	0.086	1.066	0.036	0.074	0.166	0.219	0.216	10.329

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	153	165	161	168	175	174	138	162
N.S.	1	1.00	1.00	1.08	1.05	1.10	1.14	1.14	0.90	1.06
time (sec)	N/A	0.338	0.077	0.987	0.042	0.074	0.300	0.219	0.217	0.058

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	160	162	162	168	187	169	138	164
N.S.	1	1.00	1.03	1.05	1.05	1.08	1.21	1.09	0.89	1.06
time (sec)	N/A	0.360	0.066	1.017	0.038	0.077	0.665	0.222	0.215	10.592

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	154	159	163	168	189	166	140	164
N.S.	1	1.00	0.99	1.02	1.04	1.08	1.21	1.06	0.90	1.05
time (sec)	N/A	0.341	0.067	1.022	0.037	0.071	2.085	0.176	0.231	10.398

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	161	151	163	168	182	162	138	162
N.S.	1	1.00	1.05	0.98	1.06	1.09	1.18	1.05	0.90	1.05
time (sec)	N/A	0.335	0.095	1.000	0.037	0.075	5.456	0.220	0.237	0.072

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	169	149	162	168	187	162	138	163
N.S.	1	1.00	1.09	0.96	1.05	1.08	1.21	1.05	0.89	1.05
time (sec)	N/A	0.333	0.096	1.038	0.032	0.070	13.282	0.168	0.226	10.425

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	175	152	165	168	194	165	136	165
N.S.	1	1.00	1.09	0.95	1.03	1.05	1.21	1.03	0.85	1.03
time (sec)	N/A	0.330	0.087	0.937	0.035	0.072	26.691	0.219	0.232	10.463

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	172	154	166	166	196	191	134	165
N.S.	1	1.00	1.06	0.95	1.02	1.02	1.21	1.18	0.83	1.02
time (sec)	N/A	0.340	0.069	0.951	0.037	0.070	59.224	0.217	0.218	0.075

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	175	154	166	166	196	191	134	167
N.S.	1	1.00	1.05	0.93	1.00	1.00	1.18	1.15	0.81	1.01
time (sec)	N/A	0.334	0.066	0.960	0.041	0.070	121.609	0.245	0.206	0.074

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	176	154	166	166	0	191	134	168
N.S.	1	1.00	1.06	0.93	1.00	1.00	0.00	1.15	0.81	1.01
time (sec)	N/A	0.331	0.071	0.964	0.083	0.069	0.000	0.210	0.296	0.074

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	222	260	0	730	1100	235	587	302
N.S.	1	1.00	0.97	1.14	0.00	3.19	4.80	1.03	2.56	1.32
time (sec)	N/A	0.576	0.143	1.334	0.000	0.102	1.998	0.229	0.227	10.531

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	165	183	0	563	840	169	447	221
N.S.	1	1.00	0.98	1.08	0.00	3.33	4.97	1.00	2.64	1.31
time (sec)	N/A	0.413	0.100	1.358	0.000	0.101	1.708	0.217	0.237	10.479

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	119	127	0	414	609	116	321	168
N.S.	1	1.00	0.98	1.05	0.00	3.42	5.03	0.96	2.65	1.39
time (sec)	N/A	0.327	0.079	1.348	0.000	0.090	1.125	0.175	0.240	0.181

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	86	90	0	289	423	84	215	127
N.S.	1	1.00	1.01	1.06	0.00	3.40	4.98	0.99	2.53	1.49
time (sec)	N/A	0.264	0.090	1.266	0.000	0.086	0.744	0.240	0.229	0.231

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	62	0	204	280	63	123	162
N.S.	1	1.00	1.00	0.94	0.00	3.09	4.24	0.95	1.86	2.45
time (sec)	N/A	0.227	0.056	1.233	0.000	0.085	0.413	0.226	0.230	0.082

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	70	0	228	0	71	137	375
N.S.	1	1.00	1.00	0.99	0.00	3.21	0.00	1.00	1.93	5.28
time (sec)	N/A	0.267	0.113	1.281	0.000	0.109	0.000	0.231	0.215	11.377

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	100	122	0	361	0	102	261	791
N.S.	1	1.00	0.96	1.17	0.00	3.47	0.00	0.98	2.51	7.61
time (sec)	N/A	0.328	0.087	1.342	0.000	0.156	0.000	0.205	0.232	12.332

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	141	174	0	517	0	147	407	814
N.S.	1	1.00	0.97	1.20	0.00	3.57	0.00	1.01	2.81	5.61
time (sec)	N/A	0.401	0.132	1.326	0.000	0.317	0.000	0.212	0.275	11.666

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	196	243	0	687	0	206	559	1063
N.S.	1	1.00	0.96	1.19	0.00	3.37	0.00	1.01	2.74	5.21
time (sec)	N/A	0.486	0.146	1.330	0.000	0.737	0.000	0.220	0.231	12.104

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	249	349	0	1696	1572	284	1610	427
N.S.	1	1.00	0.94	1.31	0.00	6.38	5.91	1.07	6.05	1.61
time (sec)	N/A	0.723	0.395	1.349	0.000	0.173	3.946	0.205	0.221	11.666

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	204	190	269	0	1283	1248	225	1294	360
N.S.	1	1.01	0.94	1.33	0.00	6.35	6.18	1.11	6.41	1.78
time (sec)	N/A	0.515	0.309	1.302	0.000	0.104	2.890	0.202	0.224	11.499

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	157	146	200	0	813	901	162	766	895
N.S.	1	1.19	1.11	1.52	0.00	6.16	6.83	1.23	5.80	6.78
time (sec)	N/A	0.363	0.189	1.295	0.000	0.087	1.493	0.217	0.287	11.121

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	99	99	118	0	521	379	109	406	177
N.S.	1	1.14	1.14	1.36	0.00	5.99	4.36	1.25	4.67	2.03
time (sec)	N/A	0.246	0.077	1.197	0.000	0.083	0.679	0.190	0.225	10.887

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	89	0	459	359	96	404	159
N.S.	1	1.00	1.01	1.02	0.00	5.28	4.13	1.10	4.64	1.83
time (sec)	N/A	0.235	0.078	1.204	0.000	0.083	0.514	0.214	0.211	0.118

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	166	134	200	0	955	0	157	873	920
N.S.	1	1.23	0.99	1.48	0.00	7.07	0.00	1.16	6.47	6.81
time (sec)	N/A	0.430	0.204	1.283	0.000	0.316	0.000	0.180	0.218	12.198

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	232	192	293	0	1615	0	238	1575	1366
N.S.	1	1.15	0.96	1.46	0.00	8.03	0.00	1.18	7.84	6.80
time (sec)	N/A	0.567	0.352	1.441	0.000	1.063	0.000	0.231	0.236	12.989

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	294	253	375	0	2003	0	334	1989	1661
N.S.	1	1.11	0.95	1.41	0.00	7.53	0.00	1.26	7.48	6.24
time (sec)	N/A	0.760	0.447	1.371	0.000	2.146	0.000	0.195	0.232	12.857

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	45	40	39	44	70	43	40	41
N.S.	1	1.00	0.82	0.73	0.71	0.80	1.27	0.78	0.73	0.75
time (sec)	N/A	0.200	0.056	0.525	0.033	0.083	0.572	0.251	0.299	0.052

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	40	39	44	70	43	40	41
N.S.	1	1.00	0.87	0.73	0.71	0.80	1.27	0.78	0.73	0.75
time (sec)	N/A	0.198	0.052	0.518	0.050	0.095	0.315	0.253	0.209	0.047

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	40	39	44	70	43	40	41
N.S.	1	1.00	0.87	0.73	0.71	0.80	1.27	0.78	0.73	0.75
time (sec)	N/A	0.206	0.052	0.473	0.034	0.081	0.206	0.259	0.227	0.045

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	47	40	39	42	53	43	38	41
N.S.	1	1.00	0.85	0.73	0.71	0.76	0.96	0.78	0.69	0.75
time (sec)	N/A	0.201	0.051	0.461	0.029	0.076	0.609	0.239	0.232	0.047

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	46	40	39	39	68	43	37	41
N.S.	1	1.00	0.87	0.75	0.74	0.74	1.28	0.81	0.70	0.77
time (sec)	N/A	0.199	0.054	0.362	0.028	0.074	0.126	0.263	0.228	0.046

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	45	42	39	39	65	43	39	41
N.S.	1	1.00	0.88	0.82	0.76	0.76	1.27	0.84	0.76	0.80
time (sec)	N/A	0.193	0.054	0.129	0.040	0.081	0.187	0.246	0.223	0.051

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	41	39	38	63	41	41	41
N.S.	1	1.00	0.86	0.80	0.76	0.75	1.24	0.80	0.80	0.80
time (sec)	N/A	0.197	0.068	0.139	0.037	0.076	0.204	0.243	0.223	0.047

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	42	40	40	39	65	42	42	42
N.S.	1	1.00	0.82	0.78	0.78	0.76	1.27	0.82	0.82	0.82
time (sec)	N/A	0.202	0.068	0.122	0.032	0.082	0.261	0.243	0.282	0.052

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	45	40	39	39	70	41	42	41
N.S.	1	1.00	0.85	0.75	0.74	0.74	1.32	0.77	0.79	0.77
time (sec)	N/A	0.196	0.078	0.135	0.033	0.076	0.406	0.221	0.234	0.041

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	94	93	98	162	103	81	93
N.S.	1	1.00	0.90	0.83	0.82	0.87	1.43	0.91	0.72	0.82
time (sec)	N/A	0.263	0.123	1.107	0.030	0.081	0.729	0.243	0.270	0.056

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	94	93	98	162	103	81	93
N.S.	1	1.00	0.90	0.83	0.82	0.87	1.43	0.91	0.72	0.82
time (sec)	N/A	0.255	0.113	1.046	0.033	0.078	0.484	0.231	0.228	0.037

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	94	93	98	162	103	81	93
N.S.	1	1.00	0.90	0.83	0.82	0.87	1.43	0.91	0.72	0.82
time (sec)	N/A	0.264	0.121	1.135	0.034	0.088	0.313	0.264	0.227	0.035

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	94	93	96	121	103	79	93
N.S.	1	1.00	0.90	0.83	0.82	0.85	1.07	0.91	0.70	0.82
time (sec)	N/A	0.267	0.115	1.007	0.027	0.088	0.871	0.261	0.249	0.035

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	100	94	93	93	160	103	78	93
N.S.	1	1.00	0.90	0.85	0.84	0.84	1.44	0.93	0.70	0.84
time (sec)	N/A	0.260	0.115	1.069	0.037	0.086	0.209	0.243	0.225	0.036

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	97	102	93	93	156	103	80	93
N.S.	1	1.00	0.89	0.94	0.85	0.85	1.43	0.94	0.73	0.85
time (sec)	N/A	0.256	0.153	0.985	0.030	0.084	0.279	0.261	0.220	0.038

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	101	93	93	153	101	83	94
N.S.	1	1.00	0.86	0.93	0.85	0.85	1.40	0.93	0.76	0.86
time (sec)	N/A	0.252	0.164	1.045	0.034	0.084	0.307	0.252	0.215	0.039

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	97	94	93	151	102	83	94
N.S.	1	1.00	0.87	0.89	0.86	0.85	1.39	0.94	0.76	0.86
time (sec)	N/A	0.253	0.167	1.075	0.032	0.097	0.370	0.231	0.220	0.060

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	93	94	93	153	102	83	94
N.S.	1	1.00	0.87	0.85	0.86	0.85	1.40	0.94	0.76	0.86
time (sec)	N/A	0.254	0.154	1.040	0.027	0.081	0.446	0.261	0.222	0.066

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	178	192	166	171	294	193	136	169
N.S.	1	1.00	0.98	1.05	0.91	0.94	1.62	1.06	0.75	0.93
time (sec)	N/A	0.339	0.241	1.184	0.034	0.081	1.037	0.217	0.213	10.917

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	178	192	166	171	294	193	136	169
N.S.	1	1.00	0.98	1.05	0.91	0.94	1.62	1.06	0.75	0.93
time (sec)	N/A	0.340	0.259	1.116	0.034	0.085	0.731	0.248	0.215	0.053

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	178	192	166	171	294	193	136	169
N.S.	1	1.00	0.98	1.05	0.91	0.94	1.62	1.06	0.75	0.93
time (sec)	N/A	0.334	0.231	1.029	0.035	0.086	0.638	0.240	0.230	0.053

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	178	192	166	169	216	193	134	169
N.S.	1	1.00	0.98	1.05	0.91	0.93	1.19	1.06	0.74	0.93
time (sec)	N/A	0.334	0.243	1.072	0.039	0.075	0.978	0.249	0.226	0.053

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	176	191	166	166	291	193	133	169
N.S.	1	1.00	0.98	1.06	0.92	0.92	1.62	1.07	0.74	0.94
time (sec)	N/A	0.323	0.237	1.088	0.034	0.082	0.329	0.239	0.250	0.053

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	173	192	166	166	284	193	135	169
N.S.	1	1.00	0.98	1.09	0.94	0.94	1.61	1.10	0.77	0.96
time (sec)	N/A	0.324	0.263	1.053	0.028	0.085	0.436	0.240	0.252	0.056

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	170	191	166	166	280	191	138	170
N.S.	1	1.00	0.96	1.07	0.93	0.93	1.57	1.07	0.78	0.96
time (sec)	N/A	0.324	0.272	1.027	0.036	0.080	0.471	0.227	0.247	0.055

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	169	181	167	166	275	192	138	170
N.S.	1	1.00	0.95	1.02	0.94	0.93	1.54	1.08	0.78	0.96
time (sec)	N/A	0.329	0.250	0.994	0.037	0.090	0.561	0.262	0.320	0.056

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	168	174	167	166	270	192	138	170
N.S.	1	1.00	0.97	1.00	0.96	0.95	1.55	1.10	0.79	0.98
time (sec)	N/A	0.322	0.285	1.037	0.034	0.126	0.656	0.236	0.311	10.653

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	172	167	167	166	275	192	138	170
N.S.	1	1.00	0.97	0.94	0.94	0.93	1.54	1.08	0.78	0.96
time (sec)	N/A	0.329	0.266	0.966	0.033	0.077	0.929	0.257	0.272	0.078

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	363	411	381	0	7707	40613	5319	1213	14120
N.S.	1	1.05	1.18	1.10	0.00	22.21	117.04	15.33	3.50	40.69
time (sec)	N/A	0.810	1.656	1.220	0.000	12.948	75.315	0.864	0.251	12.130

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	287	333	291	0	5148	17734	4399	975	10204
N.S.	1	1.04	1.21	1.06	0.00	18.72	64.49	16.00	3.55	37.11
time (sec)	N/A	0.589	1.432	1.218	0.000	4.420	18.140	0.787	0.269	11.756

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	225	264	218	0	2642	13942	3186	741	6401
N.S.	1	1.02	1.19	0.99	0.00	11.95	63.09	14.42	3.35	28.96
time (sec)	N/A	0.459	0.783	1.228	0.000	0.886	6.900	0.712	0.251	11.924

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	182	181	168	0	1577	4488	1404	533	4141
N.S.	1	1.01	1.01	0.93	0.00	8.76	24.93	7.80	2.96	23.01
time (sec)	N/A	0.355	0.539	1.121	0.000	0.411	9.139	0.661	0.244	11.905

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	197	214	175	0	2925	305978	2809	377	6367
N.S.	1	0.99	1.08	0.88	0.00	14.70	1537.58	14.12	1.89	31.99
time (sec)	N/A	0.430	0.901	1.019	0.000	0.884	59.492	0.709	0.285	12.635

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	F(-1)	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	0	276	233	0	5453	0	2870	403	10133
N.S.	1	0.00	0.97	0.82	0.00	19.20	0.00	10.11	1.42	35.68
time (sec)	N/A	0.000	1.284	1.528	0.000	1.418	0.000	0.830	0.292	2.143

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	F(-1)	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	0	354	314	0	7971	0	5013	653	13983
N.S.	1	0.00	1.15	1.02	0.00	25.96	0.00	16.33	2.13	45.55
time (sec)	N/A	0.000	1.589	1.303	0.000	4.322	0.000	1.164	0.362	14.524

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	B	F(-1)	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	0	445	412	0	10514	0	4090	896	17910
N.S.	1	0.00	1.17	1.08	0.00	27.60	0.00	10.73	2.35	47.01
time (sec)	N/A	0.000	2.135	1.184	0.000	6.373	0.000	1.062	0.464	15.335

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	404	461	455	0	7251	0	5685	3675	16631
N.S.	1	1.00	1.14	1.13	0.00	17.99	0.00	14.11	9.12	41.27
time (sec)	N/A	0.771	4.690	1.155	0.000	15.666	0.000	1.161	3.930	15.147

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	316	366	364	0	4653	0	4544	3001	12408
N.S.	1	0.98	1.14	1.13	0.00	14.50	0.00	14.16	9.35	38.65
time (sec)	N/A	0.592	3.332	1.126	0.000	4.331	0.000	1.121	0.512	16.459

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	268	298	295	0	3462	0	3781	1813	9434
N.S.	1	0.97	1.08	1.07	0.00	12.54	0.00	13.70	6.57	34.18
time (sec)	N/A	0.481	2.181	1.262	0.000	2.144	0.000	0.979	0.430	15.446

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	281	311	468	0	4884	0	4434	2372	12364
N.S.	1	0.96	1.07	1.60	0.00	16.73	0.00	15.18	8.12	42.34
time (sec)	N/A	0.590	2.131	1.181	0.000	6.118	0.000	1.116	0.464	15.913

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	384	399	383	0	7597	0	5405	3153	17623
N.S.	1	0.95	0.98	0.94	0.00	18.71	0.00	13.31	7.77	43.41
time (sec)	N/A	0.725	3.840	1.171	0.000	23.343	0.000	1.150	1.285	16.100

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	521	524	518	485	0	10203	0	6335	3914	21585
N.S.	1	1.01	0.99	0.93	0.00	19.58	0.00	12.16	7.51	41.43
time (sec)	N/A	0.981	5.903	1.236	0.000	51.362	0.000	1.323	2.024	15.768

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	533	745	714	0	9631	0	3997	7823	22943
N.S.	1	1.01	1.41	1.35	0.00	18.24	0.00	7.57	14.82	43.45
time (sec)	N/A	1.017	12.897	1.194	0.000	39.634	0.000	1.242	5.128	15.117

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	458	519	595	0	7056	0	7586	6753	19073
N.S.	1	1.00	1.13	1.30	0.00	15.37	0.00	16.53	14.71	41.55
time (sec)	N/A	0.855	10.539	1.229	0.000	11.865	0.000	1.955	5.017	13.887

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	416	448	500	0	5646	0	3170	5680	16720
N.S.	1	1.00	1.08	1.21	0.00	13.64	0.00	7.66	13.72	40.39
time (sec)	N/A	0.821	11.153	1.142	0.000	7.147	0.000	1.224	4.830	14.031

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	406	452	528	0	7267	0	7277	5692	19024
N.S.	1	0.91	1.01	1.18	0.00	16.29	0.00	16.32	12.76	42.65
time (sec)	N/A	0.900	7.287	1.136	0.000	15.519	0.000	1.718	4.695	14.425

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	466	524	1556	0	9907	0	4621	6756	22946
N.S.	1	1.00	1.12	3.32	0.00	21.17	0.00	9.87	14.44	49.03
time (sec)	N/A	0.994	4.900	1.540	0.000	41.298	0.000	1.363	4.977	15.246

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	664	578	628	762	0	12534	0	9534	8137	29137
N.S.	1	0.87	0.95	1.15	0.00	18.88	0.00	14.36	12.25	43.88
time (sec)	N/A	1.267	12.507	1.206	0.000	122.439	0.000	1.704	3.100	17.819

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	348	349	413	0	843	1292	412	23	992
N.S.	1	0.95	0.95	1.13	0.00	2.30	3.52	1.12	0.06	2.70
time (sec)	N/A	0.696	2.071	1.371	0.000	0.152	0.703	0.268	200.025	13.890

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	265	272	316	0	667	831	321	23	781
N.S.	1	0.95	0.97	1.13	0.00	2.38	2.97	1.15	0.08	2.79
time (sec)	N/A	0.503	1.443	1.242	0.000	0.150	0.675	0.271	200.032	11.901

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	194	210	234	0	517	539	243	23	463
N.S.	1	0.95	1.02	1.14	0.00	2.52	2.63	1.19	0.11	2.26
time (sec)	N/A	0.367	0.981	1.347	0.000	0.117	0.621	0.243	200.022	11.632

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	135	154	167	0	393	350	176	21	256
N.S.	1	0.94	1.07	1.16	0.00	2.73	2.43	1.22	0.15	1.78
time (sec)	N/A	0.268	0.628	1.191	0.000	0.111	0.762	0.255	200.023	10.898

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	115	114	0	291	224	121	233	145
N.S.	1	1.00	1.02	1.01	0.00	2.58	1.98	1.07	2.06	1.28
time (sec)	N/A	0.244	0.908	1.277	0.000	0.102	0.609	0.253	0.241	10.726

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	136	131	148	0	651	0	0	955	146
N.S.	1	1.05	1.02	1.15	0.00	5.05	0.00	0.00	7.40	1.13
time (sec)	N/A	0.339	0.681	1.185	0.000	0.637	0.000	0.000	0.326	10.801

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	113	167	0	648	0	161	142	166
N.S.	1	1.00	0.93	1.38	0.00	5.36	0.00	1.33	1.17	1.37
time (sec)	N/A	0.314	0.709	1.286	0.000	0.340	0.000	0.292	0.363	11.251

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	0	159	125	0	699	0	356	164	0
N.S.	1	0.00	1.20	0.94	0.00	5.26	0.00	2.68	1.23	0.00
time (sec)	N/A	0.000	1.048	1.307	0.000	0.388	0.000	0.275	0.254	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	165	128	0	317	0	524	175	0
N.S.	1	1.00	1.36	1.06	0.00	2.62	0.00	4.33	1.45	0.00
time (sec)	N/A	0.260	1.231	1.346	0.000	0.230	0.000	0.289	0.270	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	171	198	185	0	425	0	991	274	0
N.S.	1	0.99	1.15	1.08	0.00	2.47	0.00	5.76	1.59	0.00
time (sec)	N/A	0.347	1.791	1.486	0.000	0.408	0.000	0.261	0.406	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	230	264	257	0	553	0	1407	343	0
N.S.	1	0.98	1.12	1.09	0.00	2.35	0.00	5.99	1.46	0.00
time (sec)	N/A	0.451	2.544	1.425	0.000	0.927	0.000	0.267	0.669	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	302	329	344	0	709	0	1955	467	0
N.S.	1	0.97	1.06	1.11	0.00	2.29	0.00	6.31	1.51	0.00
time (sec)	N/A	0.572	3.554	1.559	0.000	1.709	0.000	0.249	1.283	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	391	505	652	0	1263	4983	637	23	0
N.S.	1	0.86	1.11	1.43	0.00	2.78	10.95	1.40	0.05	0.00
time (sec)	N/A	0.751	4.121	1.375	0.000	0.220	0.778	0.264	200.084	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	308	412	525	0	1037	3089	522	23	0
N.S.	1	0.87	1.16	1.47	0.00	2.91	8.68	1.47	0.06	0.00
time (sec)	N/A	0.579	2.894	1.389	0.000	0.191	0.763	0.259	200.049	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	237	328	413	0	845	1911	420	23	0
N.S.	1	0.88	1.22	1.54	0.00	3.14	7.10	1.56	0.09	0.00
time (sec)	N/A	0.420	2.111	1.333	0.000	0.154	0.698	0.263	200.029	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	178	254	316	0	669	1175	330	21	0
N.S.	1	0.90	1.28	1.60	0.00	3.38	5.93	1.67	0.11	0.00
time (sec)	N/A	0.315	1.473	1.155	0.000	0.130	0.902	0.254	200.030	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	156	192	234	0	515	711	249	20	305
N.S.	1	0.99	1.22	1.48	0.00	3.26	4.50	1.58	0.13	1.93
time (sec)	N/A	0.271	2.277	1.180	0.000	0.123	0.597	0.256	200.031	10.979

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	234	208	271	0	1023	0	0	23	0
N.S.	1	1.07	0.95	1.24	0.00	4.69	0.00	0.00	0.11	0.00
time (sec)	N/A	0.528	1.249	1.204	0.000	2.627	0.000	0.000	200.026	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	203	182	331	0	917	0	238	274	0
N.S.	1	1.05	0.94	1.72	0.00	4.75	0.00	1.23	1.42	0.00
time (sec)	N/A	0.454	1.309	1.234	0.000	1.173	0.000	0.284	0.355	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	182	159	290	0	921	0	412	267	0
N.S.	1	1.02	0.89	1.62	0.00	5.15	0.00	2.30	1.49	0.00
time (sec)	N/A	0.426	1.649	1.361	0.000	0.917	0.000	0.294	0.404	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	0	181	254	0	953	0	627	264	0
N.S.	1	0.00	0.88	1.23	0.00	4.63	0.00	3.04	1.28	0.00
time (sec)	N/A	0.000	1.762	1.327	0.000	0.986	0.000	0.293	0.440	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	235	240	225	0	1083	0	1016	310	0
N.S.	1	1.08	1.11	1.04	0.00	4.99	0.00	4.68	1.43	0.00
time (sec)	N/A	0.512	2.298	1.364	0.000	1.325	0.000	0.352	0.566	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	168	261	256	0	555	0	1357	343	0
N.S.	1	0.99	1.54	1.51	0.00	3.26	0.00	7.98	2.02	0.00
time (sec)	N/A	0.325	2.777	1.481	0.000	0.922	0.000	0.253	0.885	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	218	324	344	0	709	0	2059	467	0
N.S.	1	0.95	1.41	1.50	0.00	3.08	0.00	8.95	2.03	0.00
time (sec)	N/A	0.412	4.096	1.568	0.000	1.826	0.000	0.283	1.687	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	277	412	446	0	889	0	2713	561	0
N.S.	1	0.91	1.36	1.47	0.00	2.93	0.00	8.95	1.85	0.00
time (sec)	N/A	0.513	5.176	1.652	0.000	2.332	0.000	0.277	3.082	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	434	698	951	0	1775	17279	906	23	0
N.S.	1	0.80	1.29	1.75	0.00	3.27	31.82	1.67	0.04	0.00
time (sec)	N/A	0.806	9.785	1.394	0.000	0.326	0.980	0.267	200.019	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	351	585	794	0	1511	10654	767	23	0
N.S.	1	0.81	1.35	1.84	0.00	3.50	24.66	1.78	0.05	0.00
time (sec)	N/A	0.636	6.284	1.249	0.000	0.285	0.907	0.278	200.031	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	280	482	652	0	1263	6552	641	23	0
N.S.	1	0.84	1.45	1.96	0.00	3.79	19.68	1.92	0.07	0.00
time (sec)	N/A	0.461	4.064	1.290	0.000	0.223	0.945	0.267	200.060	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	221	390	511	0	1039	4009	526	21	0
N.S.	1	0.88	1.55	2.03	0.00	4.12	15.91	2.09	0.08	0.00
time (sec)	N/A	0.363	2.771	1.233	0.000	0.171	0.962	0.281	200.029	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	199	311	314	0	843	2428	423	20	0
N.S.	1	0.98	1.53	1.55	0.00	4.15	11.96	2.08	0.10	0.00
time (sec)	N/A	0.311	4.243	1.230	0.000	0.151	0.695	0.266	200.049	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	374	324	433	0	1575	0	0	23	0
N.S.	1	1.08	0.94	1.25	0.00	4.55	0.00	0.00	0.07	0.00
time (sec)	N/A	0.780	2.495	1.179	0.000	8.646	0.000	0.000	200.040	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	329	287	600	0	1393	0	368	449	0
N.S.	1	1.06	0.93	1.94	0.00	4.49	0.00	1.19	1.45	0.00
time (sec)	N/A	0.660	2.192	1.285	0.000	4.339	0.000	0.274	0.296	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	288	253	505	0	1269	0	527	433	0
N.S.	1	1.05	0.93	1.85	0.00	4.65	0.00	1.93	1.59	0.00
time (sec)	N/A	0.591	2.447	1.347	0.000	2.877	0.000	0.288	0.241	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	263	234	456	0	1293	0	765	396	0
N.S.	1	1.03	0.92	1.79	0.00	5.07	0.00	3.00	1.55	0.00
time (sec)	N/A	0.600	2.700	1.323	0.000	1.624	0.000	0.326	0.304	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	294	286	453	0	1305	0	1163	409	0
N.S.	1	1.04	1.01	1.60	0.00	4.60	0.00	4.10	1.44	0.00
time (sec)	N/A	0.689	3.552	1.372	0.000	2.314	0.000	0.350	0.490	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	361	313	389	0	1445	0	1525	441	0
N.S.	1	1.04	0.90	1.12	0.00	4.18	0.00	4.41	1.27	0.00
time (sec)	N/A	0.833	4.982	1.500	0.000	3.508	0.000	0.408	0.786	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	358	360	384	0	1659	0	2086	505	0
N.S.	1	1.08	1.08	1.16	0.00	5.00	0.00	6.28	1.52	0.00
time (sec)	N/A	0.758	4.680	1.552	0.000	4.427	0.000	0.543	1.727	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	215	402	446	0	887	0	2598	561	0
N.S.	1	0.98	1.84	2.04	0.00	4.05	0.00	11.86	2.56	0.00
time (sec)	N/A	0.384	5.628	1.816	0.000	2.199	0.000	0.293	6.773	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	265	490	563	0	1091	0	3603	710	0
N.S.	1	0.92	1.70	1.95	0.00	3.79	0.00	12.51	2.47	0.00
time (sec)	N/A	0.457	7.669	1.849	0.000	4.076	0.000	0.309	19.106	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	324	292	695	0	1315	0	4427	829	0
N.S.	1	0.86	0.78	1.85	0.00	3.51	0.00	11.81	2.21	0.00
time (sec)	N/A	0.594	10.969	2.036	0.000	5.002	0.000	0.311	96.887	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	222	173	167	0	395	423	181	23	0
N.S.	1	1.08	0.84	0.81	0.00	1.92	2.05	0.88	0.11	0.00
time (sec)	N/A	0.456	0.777	1.316	0.000	0.122	0.640	0.252	200.059	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	151	124	115	0	295	292	126	233	0
N.S.	1	1.06	0.87	0.80	0.00	2.06	2.04	0.88	1.63	0.00
time (sec)	N/A	0.326	0.565	1.230	0.000	0.110	0.578	0.247	0.248	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	80	0	213	204	88	144	0
N.S.	1	1.00	1.00	0.87	0.00	2.32	2.22	0.96	1.57	0.00
time (sec)	N/A	0.226	0.505	1.220	0.000	0.100	0.831	0.250	0.224	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	66	58	0	162	146	60	106	80
N.S.	1	1.00	0.99	0.87	0.00	2.42	2.18	0.90	1.58	1.19
time (sec)	N/A	0.209	0.449	1.163	0.000	0.091	0.425	0.253	0.216	11.159

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	74	67	0	468	0	0	794	66
N.S.	1	1.00	0.96	0.87	0.00	6.08	0.00	0.00	10.31	0.86
time (sec)	N/A	0.247	0.330	1.167	0.000	0.151	0.000	0.000	0.238	10.796

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	68	65	0	177	0	110	60	87
N.S.	1	1.00	0.94	0.90	0.00	2.46	0.00	1.53	0.83	1.21
time (sec)	N/A	0.213	0.487	1.210	0.000	0.118	0.000	0.222	0.263	10.851

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	122	122	88	0	235	0	303	130	0
N.S.	1	1.05	1.05	0.76	0.00	2.03	0.00	2.61	1.12	0.00
time (sec)	N/A	0.287	0.885	1.216	0.000	0.169	0.000	0.239	0.238	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	181	161	128	0	321	0	511	175	0
N.S.	1	1.08	0.96	0.77	0.00	1.92	0.00	3.06	1.05	0.00
time (sec)	N/A	0.388	1.251	1.263	0.000	0.234	0.000	0.253	0.314	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	253	201	185	0	425	0	884	274	0
N.S.	1	1.10	0.87	0.80	0.00	1.84	0.00	3.83	1.19	0.00
time (sec)	N/A	0.533	1.631	1.348	0.000	0.396	0.000	0.264	0.393	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	291	298	447	0	1035	0	365	1083	0
N.S.	1	1.04	1.06	1.60	0.00	3.70	0.00	1.30	3.87	0.00
time (sec)	N/A	0.546	1.584	1.296	0.000	0.288	0.000	0.257	0.369	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	210	223	359	0	793	0	266	23	0
N.S.	1	1.07	1.13	1.82	0.00	4.03	0.00	1.35	0.12	0.00
time (sec)	N/A	0.389	1.118	1.311	0.000	0.253	0.000	0.227	200.027	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	166	147	244	0	603	0	175	709	0
N.S.	1	1.08	0.96	1.59	0.00	3.94	0.00	1.14	4.63	0.00
time (sec)	N/A	0.347	0.842	1.298	0.000	0.192	0.000	0.247	0.208	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	104	168	0	405	0	108	456	111
N.S.	1	1.00	1.08	1.75	0.00	4.22	0.00	1.12	4.75	1.16
time (sec)	N/A	0.236	0.648	1.227	0.000	0.182	0.000	0.276	0.204	11.056

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	45	0	74	0	55	157	44
N.S.	1	1.00	0.98	1.00	0.00	1.64	0.00	1.22	3.49	0.98
time (sec)	N/A	0.171	0.440	1.169	0.000	0.128	0.000	0.236	0.194	10.595

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	105	124	0	412	0	125	2230	0
N.S.	1	1.00	1.09	1.29	0.00	4.29	0.00	1.30	23.23	0.00
time (sec)	N/A	0.244	0.718	1.158	0.000	0.203	0.000	0.261	0.649	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	171	145	243	0	657	0	220	451	0
N.S.	1	1.15	0.97	1.63	0.00	4.41	0.00	1.48	3.03	0.00
time (sec)	N/A	0.356	0.950	1.247	0.000	0.301	0.000	0.252	0.232	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	239	210	369	0	869	0	467	696	0
N.S.	1	1.09	0.96	1.68	0.00	3.97	0.00	2.13	3.18	0.00
time (sec)	N/A	0.484	1.548	1.196	0.000	0.463	0.000	0.250	0.199	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	322	293	460	0	1093	0	798	796	0
N.S.	1	1.10	1.00	1.58	0.00	3.74	0.00	2.73	2.73	0.00
time (sec)	N/A	0.683	2.443	1.300	0.000	0.969	0.000	0.252	0.241	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	313	287	1246	0	1601	0	456	23	0
N.S.	1	1.10	1.01	4.37	0.00	5.62	0.00	1.60	0.08	0.00
time (sec)	N/A	0.593	2.408	1.342	0.000	0.646	0.000	0.254	200.076	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	214	204	765	0	1061	0	312	23	0
N.S.	1	1.13	1.08	4.05	0.00	5.61	0.00	1.65	0.12	0.00
time (sec)	N/A	0.403	1.760	1.224	0.000	0.613	0.000	0.265	200.032	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	110	132	0	248	0	195	503	131
N.S.	1	1.00	1.17	1.40	0.00	2.64	0.00	2.07	5.35	1.39
time (sec)	N/A	0.233	1.209	1.341	0.000	0.460	0.000	0.259	0.301	10.898

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	114	114	129	0	244	0	196	401	128
N.S.	1	0.99	0.99	1.12	0.00	2.12	0.00	1.70	3.49	1.11
time (sec)	N/A	0.245	1.033	1.230	0.000	0.510	0.000	0.255	0.272	10.813

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	99	123	0	245	0	193	435	121
N.S.	1	1.00	1.10	1.37	0.00	2.72	0.00	2.14	4.83	1.34
time (sec)	N/A	0.212	1.160	1.161	0.000	0.480	0.000	0.247	0.360	10.697

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	208	191	253	0	1077	0	333	7582	0
N.S.	1	1.13	1.04	1.38	0.00	5.85	0.00	1.81	41.21	0.00
time (sec)	N/A	0.380	1.607	1.214	0.000	1.146	0.000	0.256	17.965	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	317	284	467	0	1655	0	483	1284	0
N.S.	1	1.14	1.03	1.69	0.00	5.97	0.00	1.74	4.64	0.00
time (sec)	N/A	0.612	2.332	1.225	0.000	1.606	0.000	0.265	0.340	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	408	424	780	0	2057	0	765	1748	0
N.S.	1	1.11	1.16	2.13	0.00	5.62	0.00	2.09	4.78	0.00
time (sec)	N/A	0.844	4.079	1.261	0.000	4.431	0.000	0.276	0.275	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	141	215	258	0	550	0	436	1120	395
N.S.	1	1.04	1.59	1.91	0.00	4.07	0.00	3.23	8.30	2.93
time (sec)	N/A	0.271	3.913	1.268	0.000	2.624	0.000	0.270	2.220	11.315

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	191	349	354	0	941	0	768	1920	599
N.S.	1	1.06	1.93	1.96	0.00	5.20	0.00	4.24	10.61	3.31
time (sec)	N/A	0.313	8.132	1.207	0.000	13.605	0.000	0.253	41.342	11.794

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	40	38	48	47	0	56	71	41
N.S.	1	1.00	2.11	2.00	2.53	2.47	0.00	2.95	3.74	2.16
time (sec)	N/A	0.166	0.122	0.981	0.026	0.074	0.000	0.259	0.268	11.485

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	441	638	871	0	265	0	0	292	0
N.S.	1	0.80	1.16	1.59	0.00	0.48	0.00	0.00	0.53	0.00
time (sec)	N/A	0.742	27.302	1.921	0.000	0.100	0.000	0.000	1.651	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	362	550	795	0	215	0	0	138	0
N.S.	1	0.76	1.16	1.67	0.00	0.45	0.00	0.00	0.29	0.00
time (sec)	N/A	0.547	26.269	1.582	0.000	0.097	0.000	0.000	1.429	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	336	491	769	0	190	0	0	142	0
N.S.	1	0.78	1.14	1.78	0.00	0.44	0.00	0.00	0.33	0.00
time (sec)	N/A	0.475	24.498	1.698	0.000	0.091	0.000	0.000	8.405	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	348	499	791	0	198	0	0	160	0
N.S.	1	0.76	1.09	1.73	0.00	0.43	0.00	0.00	0.35	0.00
time (sec)	N/A	0.515	24.859	1.860	0.000	0.101	0.000	0.000	3.109	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	0	576	842	0	255	0	0	254	0
N.S.	1	0.00	1.09	1.60	0.00	0.48	0.00	0.00	0.48	0.00
time (sec)	N/A	0.000	26.209	2.293	0.000	0.089	0.000	0.000	1.006	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	502	660	903	0	325	0	0	220	0
N.S.	1	0.79	1.04	1.42	0.00	0.51	0.00	0.00	0.35	0.00
time (sec)	N/A	0.847	27.221	2.678	0.000	0.100	0.000	0.000	3.846	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	277	178	137	0	68	0	0	169	0
N.S.	1	1.12	0.72	0.55	0.00	0.28	0.00	0.00	0.68	0.00
time (sec)	N/A	0.489	21.261	1.110	0.000	0.081	0.000	0.000	0.492	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	249	170	132	0	63	0	0	151	0
N.S.	1	1.11	0.76	0.59	0.00	0.28	0.00	0.00	0.67	0.00
time (sec)	N/A	0.459	21.204	1.102	0.000	0.087	0.000	0.000	0.623	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	221	165	127	0	58	0	0	133	0
N.S.	1	1.10	0.82	0.63	0.00	0.29	0.00	0.00	0.66	0.00
time (sec)	N/A	0.409	21.193	1.062	0.000	0.084	0.000	0.000	0.554	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	193	163	122	0	53	0	0	115	0
N.S.	1	1.08	0.92	0.69	0.00	0.30	0.00	0.00	0.65	0.00
time (sec)	N/A	0.372	21.209	1.124	0.000	0.080	0.000	0.000	0.553	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	165	158	117	0	48	0	0	97	0
N.S.	1	1.06	1.02	0.75	0.00	0.31	0.00	0.00	0.63	0.00
time (sec)	N/A	0.330	21.197	1.067	0.000	0.077	0.000	0.000	0.681	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	165	153	113	0	55	0	0	102	0
N.S.	1	1.06	0.99	0.73	0.00	0.35	0.00	0.00	0.66	0.00
time (sec)	N/A	0.337	21.210	1.032	0.000	0.088	0.000	0.000	0.676	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	163	153	115	0	59	0	0	111	0
N.S.	1	1.07	1.00	0.75	0.00	0.39	0.00	0.00	0.73	0.00
time (sec)	N/A	0.328	21.206	0.945	0.000	0.075	0.000	0.000	0.515	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	189	153	124	0	64	0	0	174	0
N.S.	1	1.07	0.87	0.70	0.00	0.36	0.00	0.00	0.99	0.00
time (sec)	N/A	0.381	21.211	1.078	0.000	0.086	0.000	0.000	0.478	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	217	155	129	0	69	0	0	168	0
N.S.	1	1.08	0.77	0.64	0.00	0.34	0.00	0.00	0.84	0.00
time (sec)	N/A	0.410	21.201	0.948	0.000	0.083	0.000	0.000	0.523	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	245	160	134	0	74	0	0	245	0
N.S.	1	1.09	0.71	0.60	0.00	0.33	0.00	0.00	1.09	0.00
time (sec)	N/A	0.450	21.205	1.019	0.000	0.120	0.000	0.000	0.711	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	282	183	142	0	73	0	0	187	0
N.S.	1	1.12	0.73	0.56	0.00	0.29	0.00	0.00	0.74	0.00
time (sec)	N/A	0.521	21.242	1.119	0.000	0.082	0.000	0.000	0.441	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	254	178	137	0	68	0	0	169	0
N.S.	1	1.11	0.78	0.60	0.00	0.30	0.00	0.00	0.74	0.00
time (sec)	N/A	0.465	21.230	1.049	0.000	0.086	0.000	0.000	0.441	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	226	173	132	0	63	0	0	151	0
N.S.	1	1.10	0.84	0.64	0.00	0.31	0.00	0.00	0.73	0.00
time (sec)	N/A	0.423	21.226	1.053	0.000	0.080	0.000	0.000	0.429	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	198	165	127	0	58	0	0	133	0
N.S.	1	1.08	0.90	0.69	0.00	0.32	0.00	0.00	0.73	0.00
time (sec)	N/A	0.377	21.214	1.178	0.000	0.087	0.000	0.000	0.415	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	198	163	123	0	65	0	0	134	0
N.S.	1	1.08	0.89	0.67	0.00	0.36	0.00	0.00	0.73	0.00
time (sec)	N/A	0.377	21.218	1.056	0.000	0.089	0.000	0.000	0.486	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	189	163	125	0	69	0	0	145	0
N.S.	1	1.10	0.95	0.73	0.00	0.40	0.00	0.00	0.84	0.00
time (sec)	N/A	0.374	21.250	1.014	0.000	0.081	0.000	0.000	0.403	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	194	163	129	0	69	0	0	190	0
N.S.	1	1.07	0.90	0.71	0.00	0.38	0.00	0.00	1.05	0.00
time (sec)	N/A	0.382	21.217	0.941	0.000	0.085	0.000	0.000	0.481	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	193	163	129	0	69	0	0	200	0
N.S.	1	1.05	0.89	0.70	0.00	0.38	0.00	0.00	1.09	0.00
time (sec)	N/A	0.398	21.219	1.003	0.000	0.081	0.000	0.000	0.550	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	222	160	134	0	74	0	0	261	0
N.S.	1	1.08	0.78	0.65	0.00	0.36	0.00	0.00	1.27	0.00
time (sec)	N/A	0.433	21.216	1.034	0.000	0.085	0.000	0.000	0.731	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	250	165	139	0	79	0	0	249	0
N.S.	1	1.09	0.72	0.61	0.00	0.34	0.00	0.00	1.09	0.00
time (sec)	N/A	0.467	21.218	1.043	0.000	0.076	0.000	0.000	0.830	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	278	170	144	0	84	0	0	249	0
N.S.	1	1.10	0.67	0.57	0.00	0.33	0.00	0.00	0.99	0.00
time (sec)	N/A	0.509	21.229	0.960	0.000	0.083	0.000	0.000	1.128	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	291	444	538	0	144	0	0	71	0
N.S.	1	0.73	1.11	1.34	0.00	0.36	0.00	0.00	0.18	0.00
time (sec)	N/A	0.421	23.955	1.543	0.000	0.098	0.000	0.000	1.119	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	240	168	127	0	58	0	0	133	0
N.S.	1	1.10	0.77	0.58	0.00	0.26	0.00	0.00	0.61	0.00
time (sec)	N/A	0.453	21.240	1.007	0.000	0.085	0.000	0.000	0.440	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	216	160	122	0	53	0	0	115	0
N.S.	1	1.10	0.82	0.62	0.00	0.27	0.00	0.00	0.59	0.00
time (sec)	N/A	0.405	21.212	1.018	0.000	0.083	0.000	0.000	0.461	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	188	158	117	0	48	0	0	97	0
N.S.	1	1.09	0.91	0.68	0.00	0.28	0.00	0.00	0.56	0.00
time (sec)	N/A	0.369	21.211	1.148	0.000	0.081	0.000	0.000	0.482	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	160	150	112	0	43	0	0	81	0
N.S.	1	1.07	1.00	0.75	0.00	0.29	0.00	0.00	0.54	0.00
time (sec)	N/A	0.329	21.239	1.056	0.000	0.081	0.000	0.000	0.436	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	150	77	0	26	0	0	65	0
N.S.	1	1.06	1.20	0.62	0.00	0.21	0.00	0.00	0.52	0.00
time (sec)	N/A	0.297	21.209	0.934	0.000	0.079	0.000	0.000	0.409	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	156	90	108	0	50	0	0	86	0
N.S.	1	1.10	0.63	0.76	0.00	0.35	0.00	0.00	0.61	0.00
time (sec)	N/A	0.331	21.191	1.021	0.000	0.077	0.000	0.000	0.443	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	183	148	115	0	59	0	0	90	0
N.S.	1	1.07	0.87	0.67	0.00	0.35	0.00	0.00	0.53	0.00
time (sec)	N/A	0.365	21.193	1.003	0.000	0.076	0.000	0.000	0.412	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	212	150	124	0	64	0	0	166	0
N.S.	1	1.10	0.78	0.65	0.00	0.33	0.00	0.00	0.86	0.00
time (sec)	N/A	0.411	21.205	0.913	0.000	0.084	0.000	0.000	0.526	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	217	156	117	0	69	0	0	349	0
N.S.	1	1.11	0.80	0.60	0.00	0.35	0.00	0.00	1.79	0.00
time (sec)	N/A	0.426	21.230	1.030	0.000	0.079	0.000	0.000	0.569	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	193	150	112	0	87	0	0	331	0
N.S.	1	1.08	0.84	0.63	0.00	0.49	0.00	0.00	1.86	0.00
time (sec)	N/A	0.380	21.210	0.961	0.000	0.077	0.000	0.000	0.543	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	165	145	107	0	82	0	0	331	0
N.S.	1	1.06	0.94	0.69	0.00	0.53	0.00	0.00	2.14	0.00
time (sec)	N/A	0.340	21.203	1.003	0.000	0.084	0.000	0.000	0.515	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	161	140	107	0	82	0	0	347	0
N.S.	1	1.07	0.93	0.71	0.00	0.54	0.00	0.00	2.30	0.00
time (sec)	N/A	0.331	21.191	0.989	0.000	0.078	0.000	0.000	0.520	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	161	137	107	0	82	0	0	90	0
N.S.	1	1.10	0.93	0.73	0.00	0.56	0.00	0.00	0.61	0.00
time (sec)	N/A	0.328	21.188	0.988	0.000	0.080	0.000	0.000	0.544	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	185	137	108	0	99	0	0	648	0
N.S.	1	1.10	0.82	0.64	0.00	0.59	0.00	0.00	3.86	0.00
time (sec)	N/A	0.380	21.212	1.095	0.000	0.084	0.000	0.000	0.496	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	213	145	115	0	110	0	0	726	0
N.S.	1	1.07	0.73	0.58	0.00	0.55	0.00	0.00	3.65	0.00
time (sec)	N/A	0.412	21.212	0.986	0.000	0.079	0.000	0.000	0.505	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	238	150	124	0	115	0	0	584	0
N.S.	1	1.07	0.68	0.56	0.00	0.52	0.00	0.00	2.63	0.00
time (sec)	N/A	0.460	21.217	1.030	0.000	0.079	0.000	0.000	0.513	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	282	187	272	0	137	0	0	736	0
N.S.	1	1.12	0.74	1.08	0.00	0.54	0.00	0.00	2.92	0.00
time (sec)	N/A	0.513	21.336	1.053	0.000	0.082	0.000	0.000	0.466	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	254	179	251	0	132	0	0	718	0
N.S.	1	1.11	0.78	1.10	0.00	0.58	0.00	0.00	3.14	0.00
time (sec)	N/A	0.466	21.324	0.984	0.000	0.083	0.000	0.000	0.457	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	219	177	232	0	127	0	0	700	0
N.S.	1	1.06	0.86	1.13	0.00	0.62	0.00	0.00	3.40	0.00
time (sec)	N/A	0.417	21.324	1.018	0.000	0.083	0.000	0.000	0.456	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	198	169	214	0	122	0	0	682	0
N.S.	1	1.08	0.92	1.17	0.00	0.67	0.00	0.00	3.73	0.00
time (sec)	N/A	0.395	21.318	0.992	0.000	0.081	0.000	0.000	0.442	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	194	167	214	0	122	0	0	664	0
N.S.	1	1.06	0.91	1.17	0.00	0.67	0.00	0.00	3.63	0.00
time (sec)	N/A	0.380	21.301	0.993	0.000	0.080	0.000	0.000	0.386	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	193	164	214	0	122	0	0	645	0
N.S.	1	1.05	0.90	1.17	0.00	0.67	0.00	0.00	3.52	0.00
time (sec)	N/A	0.383	21.298	0.981	0.000	0.081	0.000	0.000	0.448	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	189	165	214	0	122	0	0	630	0
N.S.	1	1.08	0.94	1.22	0.00	0.70	0.00	0.00	3.60	0.00
time (sec)	N/A	0.384	21.356	1.006	0.000	0.079	0.000	0.000	0.446	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	193	167	214	0	122	0	0	640	0
N.S.	1	1.07	0.92	1.18	0.00	0.67	0.00	0.00	3.54	0.00
time (sec)	N/A	0.384	21.288	1.040	0.000	0.087	0.000	0.000	0.520	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	220	167	240	0	139	0	0	743	0
N.S.	1	1.09	0.83	1.19	0.00	0.69	0.00	0.00	3.68	0.00
time (sec)	N/A	0.420	21.307	0.977	0.000	0.077	0.000	0.000	0.498	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	246	169	261	0	150	0	0	1119	0
N.S.	1	1.12	0.77	1.19	0.00	0.68	0.00	0.00	5.09	0.00
time (sec)	N/A	0.472	21.309	0.994	0.000	0.088	0.000	0.000	0.452	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	276	177	282	0	155	0	0	1119	0
N.S.	1	1.10	0.70	1.12	0.00	0.62	0.00	0.00	4.44	0.00
time (sec)	N/A	0.515	21.333	1.052	0.000	0.089	0.000	0.000	0.511	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	672	242	408	1350	11116	2736	1335	769
N.S.	1	1.00	2.80	1.01	1.70	5.62	46.32	11.40	5.56	3.20
time (sec)	N/A	0.462	2.369	1.050	0.077	0.107	0.938	0.266	0.344	11.648

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	289	154	230	573	4027	1142	590	405
N.S.	1	1.00	1.86	0.99	1.48	3.70	25.98	7.37	3.81	2.61
time (sec)	N/A	0.340	0.678	0.935	0.061	0.085	0.602	0.224	0.346	11.629

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	91	82	104	171	981	338	188	171
N.S.	1	1.00	1.10	0.99	1.25	2.06	11.82	4.07	2.27	2.06
time (sec)	N/A	0.248	0.187	0.086	0.042	0.081	0.332	0.230	0.332	11.319

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	172	135	0	0	0	0	0	92	0
N.S.	1	0.88	0.69	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.459	0.396	0.000	0.000	0.000	0.000	0.000	0.350	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	324	302	253	0	0	0	0	0	0	0
N.S.	1	0.93	0.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.724	0.911	0.000	0.000	0.000	0.000	0.000	0.380	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	618	0	0	0	0	0	25	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.498	3.610	0.000	0.000	0.000	0.000	0.000	200.040	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	405	0	0	0	0	0	25	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.430	2.077	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	234	0	0	0	0	0	25	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.414	1.598	0.000	0.000	0.000	0.000	0.000	200.033	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	234	0	0	0	0	0	48	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.449	1.724	0.000	0.000	0.000	0.000	0.000	0.716	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	272	0	0	0	0	0	114	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.436	2.393	0.000	0.000	0.000	0.000	0.000	8.948	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	237	0	0	0	0	0	234	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.431	4.756	0.000	0.000	0.000	0.000	0.000	156.556	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	232	0	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	0.651	0.000	0.000	0.000	0.000	0.000	0.374	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	443	210	0	0	0	0	0	0	0
N.S.	1	1.14	0.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.707	1.134	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	295	210	0	0	0	0	0	0	0
N.S.	1	1.19	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.457	0.721	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	211	210	0	0	0	0	0	1544	0
N.S.	1	1.21	1.20	0.00	0.00	0.00	0.00	0.00	8.82	0.00
time (sec)	N/A	0.310	0.554	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	158	268	0	0	0	0	0	650	0
N.S.	1	1.30	2.20	0.00	0.00	0.00	0.00	0.00	5.33	0.00
time (sec)	N/A	0.268	0.656	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	273	263	0	0	0	0	0	483	0
N.S.	1	1.16	1.11	0.00	0.00	0.00	0.00	0.00	2.05	0.00
time (sec)	N/A	0.415	0.676	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	314	289	0	0	0	0	0	892	0
N.S.	1	1.15	1.05	0.00	0.00	0.00	0.00	0.00	3.26	0.00
time (sec)	N/A	0.477	0.648	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	384	295	0	0	0	0	0	1048	0
N.S.	1	1.14	0.88	0.00	0.00	0.00	0.00	0.00	3.12	0.00
time (sec)	N/A	0.617	1.125	0.000	0.000	0.000	0.000	0.000	0.233	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [189] had the largest ratio of [.560000000000000053]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	19	0.105
2	A	2	2	1.00	19	0.105
3	A	2	2	1.00	17	0.118
4	A	2	2	1.00	16	0.125
5	A	2	2	1.00	19	0.105
6	A	2	2	1.00	19	0.105
7	A	2	2	1.00	19	0.105
8	A	2	2	1.00	19	0.105
9	A	2	2	1.00	19	0.105
10	A	2	2	1.00	19	0.105
11	A	2	2	1.00	19	0.105
12	A	2	2	1.00	19	0.105
13	A	2	2	1.00	21	0.095
14	A	2	2	1.00	19	0.105
15	A	2	2	1.00	18	0.111
16	A	2	2	1.00	21	0.095
17	A	2	2	1.00	21	0.095
18	A	2	2	1.00	21	0.095
19	A	2	2	1.00	21	0.095
20	A	2	2	1.00	21	0.095
21	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	21	0.095
23	A	2	2	1.00	21	0.095
24	A	2	2	1.00	21	0.095
25	A	2	2	1.00	21	0.095
26	A	2	2	1.00	19	0.105
27	A	2	2	1.00	18	0.111
28	A	2	2	1.00	21	0.095
29	A	2	2	1.00	21	0.095
30	A	2	2	1.00	21	0.095
31	A	2	2	1.00	21	0.095
32	A	2	2	1.00	21	0.095
33	A	2	2	1.00	21	0.095
34	A	2	2	1.00	21	0.095
35	A	2	2	1.00	21	0.095
36	A	2	2	1.00	21	0.095
37	A	2	2	1.00	21	0.095
38	A	2	2	1.00	21	0.095
39	A	2	2	1.00	21	0.095
40	A	2	2	1.00	21	0.095
41	A	2	2	1.00	21	0.095
42	A	2	2	1.00	19	0.105
43	A	5	4	1.00	18	0.222
44	A	2	2	1.00	21	0.095
45	A	2	2	1.00	21	0.095
46	A	2	2	1.00	21	0.095
47	A	2	2	1.00	21	0.095
48	A	4	4	1.00	21	0.190
49	A	4	4	1.01	21	0.190
50	A	7	6	1.19	21	0.286
51	A	4	3	1.14	19	0.158
52	A	4	3	1.00	18	0.167
53	A	4	4	1.23	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
54	A	4	4	1.15	21	0.190
55	A	4	4	1.11	21	0.190
56	A	2	2	1.00	21	0.095
57	A	2	2	1.00	21	0.095
58	A	2	2	1.00	21	0.095
59	A	2	2	1.00	21	0.095
60	A	2	2	1.00	21	0.095
61	A	2	2	1.00	21	0.095
62	A	2	2	1.00	21	0.095
63	A	2	2	1.00	21	0.095
64	A	2	2	1.00	21	0.095
65	A	2	2	1.00	23	0.087
66	A	2	2	1.00	23	0.087
67	A	2	2	1.00	23	0.087
68	A	2	2	1.00	23	0.087
69	A	2	2	1.00	23	0.087
70	A	2	2	1.00	23	0.087
71	A	2	2	1.00	23	0.087
72	A	2	2	1.00	23	0.087
73	A	2	2	1.00	23	0.087
74	A	2	2	1.00	23	0.087
75	A	2	2	1.00	23	0.087
76	A	2	2	1.00	23	0.087
77	A	2	2	1.00	23	0.087
78	A	2	2	1.00	23	0.087
79	A	2	2	1.00	23	0.087
80	A	2	2	1.00	23	0.087
81	A	2	2	1.00	23	0.087
82	A	2	2	1.00	23	0.087
83	A	2	2	1.00	23	0.087
84	A	10	9	1.05	23	0.391
85	A	8	7	1.04	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	6	5	1.02	23	0.217
87	A	4	3	1.01	23	0.130
88	A	6	5	0.99	23	0.217
89	F	0	0	N/A	0.000	N/A
90	F	0	0	N/A	0.000	N/A
91	F	0	0	N/A	0.000	N/A
92	A	8	7	1.00	23	0.304
93	A	6	5	0.98	23	0.217
94	A	6	5	0.97	23	0.217
95	A	6	5	0.96	23	0.217
96	A	7	6	0.95	23	0.261
97	A	9	8	1.01	23	0.348
98	A	8	7	1.01	23	0.304
99	A	8	7	1.00	23	0.304
100	A	8	7	1.00	23	0.304
101	A	8	7	0.91	23	0.304
102	A	8	7	1.00	23	0.304
103	A	10	9	0.87	23	0.391
104	A	11	10	0.95	23	0.435
105	A	9	8	0.95	23	0.348
106	A	7	6	0.95	23	0.261
107	A	5	4	0.94	21	0.190
108	A	5	4	1.00	20	0.200
109	A	8	7	1.05	23	0.304
110	A	8	7	1.00	23	0.304
111	F	0	0	N/A	0.000	N/A
112	A	5	4	1.00	23	0.174
113	A	7	6	0.99	23	0.261
114	A	9	8	0.98	23	0.348
115	A	11	10	0.97	23	0.435
116	A	12	11	0.86	23	0.478
117	A	10	9	0.87	23	0.391

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	8	7	0.88	23	0.304
119	A	6	5	0.90	21	0.238
120	A	6	5	0.99	20	0.250
121	A	10	9	1.07	23	0.391
122	A	10	9	1.05	23	0.391
123	A	10	9	1.02	23	0.391
124	F	0	0	N/A	0.000	N/A
125	A	10	9	1.08	23	0.391
126	A	6	5	0.99	23	0.217
127	A	8	7	0.95	23	0.304
128	A	10	9	0.91	23	0.391
129	A	13	12	0.80	23	0.522
130	A	11	10	0.81	23	0.435
131	A	9	8	0.84	23	0.348
132	A	7	6	0.88	21	0.286
133	A	7	6	0.98	20	0.300
134	A	12	11	1.08	23	0.478
135	A	12	11	1.06	23	0.478
136	A	12	11	1.05	23	0.478
137	A	12	11	1.03	23	0.478
138	A	12	11	1.04	23	0.478
139	A	12	11	1.04	23	0.478
140	A	12	11	1.08	23	0.478
141	A	7	6	0.98	23	0.261
142	A	9	8	0.92	23	0.348
143	A	11	10	0.86	23	0.435
144	A	8	7	1.08	23	0.304
145	A	6	5	1.06	23	0.217
146	A	4	3	1.00	21	0.143
147	A	4	3	1.00	20	0.150
148	A	6	5	1.00	23	0.217
149	A	4	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	6	5	1.05	23	0.217
151	A	8	7	1.08	23	0.304
152	A	11	10	1.10	23	0.435
153	A	8	7	1.04	23	0.304
154	A	6	5	1.07	23	0.217
155	A	6	5	1.08	23	0.217
156	A	4	3	1.00	21	0.143
157	A	1	1	1.00	20	0.050
158	A	5	4	1.00	23	0.174
159	A	6	5	1.15	23	0.217
160	A	9	8	1.09	23	0.348
161	A	11	10	1.10	23	0.435
162	A	8	7	1.10	23	0.304
163	A	6	5	1.13	23	0.217
164	A	2	2	1.00	23	0.087
165	A	2	2	0.99	21	0.095
166	A	2	2	1.00	20	0.100
167	A	7	6	1.13	23	0.261
168	A	8	7	1.14	23	0.304
169	A	10	9	1.11	23	0.391
170	A	3	3	1.04	20	0.150
171	A	4	4	1.06	20	0.200
172	A	3	2	1.00	21	0.095
173	A	10	9	0.80	25	0.360
174	A	8	7	0.76	25	0.280
175	A	8	7	0.78	25	0.280
176	A	8	7	0.76	25	0.280
177	F	0	0	N/A	0.000	N/A
178	A	12	11	0.79	25	0.440
179	A	13	12	1.12	25	0.480
180	A	12	11	1.11	25	0.440
181	A	11	10	1.10	25	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	8	7	1.08	25	0.280
183	A	7	6	1.06	25	0.240
184	A	7	6	1.06	25	0.240
185	A	7	6	1.07	25	0.240
186	A	8	7	1.07	25	0.280
187	A	11	10	1.08	25	0.400
188	A	12	11	1.09	25	0.440
189	A	15	14	1.12	25	0.560
190	A	12	11	1.11	25	0.440
191	A	10	9	1.10	25	0.360
192	A	9	8	1.08	25	0.320
193	A	9	8	1.08	25	0.320
194	A	9	8	1.10	25	0.320
195	A	8	7	1.07	25	0.280
196	A	9	8	1.05	25	0.320
197	A	10	9	1.08	25	0.360
198	A	11	10	1.09	25	0.400
199	A	14	13	1.10	25	0.520
200	A	7	6	0.73	27	0.222
201	A	12	11	1.10	25	0.440
202	A	10	9	1.10	25	0.360
203	A	9	8	1.09	25	0.320
204	A	6	5	1.07	25	0.200
205	A	5	4	1.06	25	0.160
206	A	6	5	1.10	25	0.200
207	A	8	7	1.07	25	0.280
208	A	11	10	1.10	25	0.400
209	A	11	10	1.11	25	0.400
210	A	9	8	1.08	25	0.320
211	A	7	6	1.06	25	0.240
212	A	7	6	1.07	25	0.240
213	A	6	5	1.10	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	9	8	1.10	25	0.320
215	A	11	10	1.07	25	0.400
216	A	12	11	1.07	25	0.440
217	A	15	14	1.12	25	0.560
218	A	13	12	1.11	25	0.480
219	A	11	10	1.06	25	0.400
220	A	9	8	1.08	25	0.320
221	A	9	8	1.06	25	0.320
222	A	9	8	1.05	25	0.320
223	A	8	7	1.08	25	0.280
224	A	9	8	1.07	25	0.320
225	A	11	10	1.09	25	0.400
226	A	12	11	1.12	25	0.440
227	A	14	13	1.10	25	0.520
228	A	2	2	1.00	23	0.087
229	A	2	2	1.00	23	0.087
230	A	2	2	1.00	21	0.095
231	A	2	2	0.88	23	0.087
232	A	4	4	0.93	23	0.174
233	A	4	3	1.00	25	0.120
234	A	4	3	1.00	25	0.120
235	A	4	3	1.00	25	0.120
236	A	4	3	1.00	25	0.120
237	A	4	3	1.00	25	0.120
238	A	4	3	1.00	25	0.120
239	A	4	3	1.00	23	0.130
240	A	6	6	1.14	21	0.286
241	A	4	4	1.19	21	0.190
242	A	2	2	1.21	19	0.105
243	A	2	2	1.30	18	0.111
244	A	5	4	1.16	21	0.190
245	A	7	6	1.15	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	9	8	1.14	21	0.381

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{4}aAx^4+\frac{1}{5}(Ab+aB)x^5+\frac{1}{6}(bB+Ac)x^6+\frac{1}{7}Bcx^7$$

output
```
1/4*a*A*x^4+1/5*(A*b+B*a)*x^5+1/6*(A*c+B*b)*x^6+1/7*B*c*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{4}aAx^4+\frac{1}{5}(Ab+aB)x^5+\frac{1}{6}(bB+Ac)x^6+\frac{1}{7}Bcx^7$$

input
```
Integrate[x^3*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(a*A*x^4)/4 + ((A*b + a*B)*x^5)/5 + ((b*B + A*c)*x^6)/6 + (B*c*x^7)/7
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(x^4(aB+Ab)+aAx^3+x^5(Ac+bB)+Bcx^6\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{4}aAx^4+\frac{1}{6}x^6(Ac+bB)+\frac{1}{7}Bcx^7$$

input
```
Int[x^3*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(a*A*x^4)/4 + ((A*b + a*B)*x^5)/5 + ((b*B + A*c)*x^6)/6 + (B*c*x^7)/7
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{4}}{4}+\frac{(Ab+Ba)x^{5}}{5}+\frac{(Ac+Bb)x^{6}}{6}+\frac{Bc\,x^{7}}{7}$	40
norman	$\frac{Bc\,x^{7}}{7}+\left(\frac{Ac}{6}+\frac{Bb}{6}\right)x^{6}+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^{5}+\frac{aA\,x^{4}}{4}$	42
orering	$\frac{x^{4}(60Bc\,x^{3}+70Ac\,x^{2}+70Bb\,x^{2}+84Abx+84Bax+105Aa)}{420}$	42
gosper	$\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}x^{6}Bb+\frac{1}{5}Ab\,x^{5}+\frac{1}{5}x^{5}Ba+\frac{1}{4}aA\,x^{4}$	44
risch	$\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}x^{6}Bb+\frac{1}{5}Ab\,x^{5}+\frac{1}{5}x^{5}Ba+\frac{1}{4}aA\,x^{4}$	44
parallelrisch	$\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}x^{6}Bb+\frac{1}{5}Ab\,x^{5}+\frac{1}{5}x^{5}Ba+\frac{1}{4}aA\,x^{4}$	44

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*A*x^4+1/5*(A*b+B*a)*x^5+1/6*(A*c+B*b)*x^6+1/7*B*c*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{7}\,Bcx^7+\frac{1}{6}\,(Bb+Ac)x^6+\frac{1}{4}\,Aax^4+\frac{1}{5}\,(Ba+Ab)x^5$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/7*B*c*x^7 + 1/6*(B*b + A*c)*x^6 + 1/4*A*a*x^4 + 1/5*(B*a + A*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{Aax^4}{4}+\frac{Bcx^7}{7}+x^6\left(\frac{Ac}{6}+\frac{Bb}{6}\right)+x^5\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

input
```
integrate(x**3*(B*x+A)*(c*x**2+b*x+a),x)
```

output
```
A*a*x**4/4 + B*c*x**7/7 + x**6*(A*c/6 + B*b/6) + x**5*(A*b/5 + B*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{7}\,Bcx^7+\frac{1}{6}\,(Bb+Ac)x^6+\frac{1}{4}\,Aax^4+\frac{1}{5}\,(Ba+Ab)x^5$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/7*B*c*x^7 + 1/6*(B*b + A*c)*x^6 + 1/4*A*a*x^4 + 1/5*(B*a + A*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{7}\,Bcx^7+\frac{1}{6}\,Bbx^6+\frac{1}{6}\,Acx^6+\frac{1}{5}\,Bax^5+\frac{1}{5}\,Abx^5+\frac{1}{4}\,Aax^4$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/7*B*c*x^7 + 1/6*B*b*x^6 + 1/6*A*c*x^6 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/4*
A*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{B\,c\,x^7}{7}+\left(\frac{A\,c}{6}+\frac{B\,b}{6}\right)\,x^6+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x^5+\frac{A\,a\,x^4}{4}$$

input
```
int(x^3*(A + B*x)*(a + b*x + c*x^2),x)
```

output
```
x^5*((A*b)/5 + (B*a)/5) + x^6*((A*c)/6 + (B*b)/6) + (A*a*x^4)/4 + (B*c*x^7
)/7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{x^4(60bc\,x^3+70ac\,x^2+70b^2x^2+168abx+105a^2)}{420}$$

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a),x)
```

output
```
(x**4*(105*a**2 + 168*a*b*x + 70*a*c*x**2 + 70*b**2*x**2 + 60*b*c*x**3))/4
20
```

3.2 $\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}(Ab+aB)x^4+\frac{1}{5}(bB+Ac)x^5+\frac{1}{6}Bcx^6$$

output
```
1/3*a*A*x^3+1/4*(A*b+B*a)*x^4+1/5*(A*c+B*b)*x^5+1/6*B*c*x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}(Ab+aB)x^4+\frac{1}{5}(bB+Ac)x^5+\frac{1}{6}Bcx^6$$

input
```
Integrate[x^2*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^4)/4 + ((b*B + A*c)*x^5)/5 + (B*c*x^6)/6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(x^3(aB+Ab)+aAx^2+x^4(Ac+bB)+Bcx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{5}x^5(Ac+bB)+\frac{1}{6}Bcx^6$$

input
```
Int[x^2*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^4)/4 + ((b*B + A*c)*x^5)/5 + (B*c*x^6)/6
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{(Ab+Ba)x^{4}}{4}+\frac{(Ac+Bb)x^{5}}{5}+\frac{Bc\,x^{6}}{6}$	40
norman	$\frac{Bc\,x^{6}}{6}+\left(\frac{Ac}{5}+\frac{Bb}{5}\right)x^{5}+\left(\frac{Ab}{4}+\frac{Ba}{4}\right)x^{4}+\frac{aA\,x^{3}}{3}$	42
orering	$\frac{x^{3}\left(10Bc\,x^{3}+12Ac\,x^{2}+12Bb\,x^{2}+15Abx+15Bax+20Aa\right)}{60}$	42
gosper	$\frac{1}{6}Bc\,x^{6}+\frac{1}{5}x^{5}Ac+\frac{1}{5}bB\,x^{5}+\frac{1}{4}Ab\,x^{4}+\frac{1}{4}Ba\,x^{4}+\frac{1}{3}aA\,x^{3}$	44
risch	$\frac{1}{6}Bc\,x^{6}+\frac{1}{5}x^{5}Ac+\frac{1}{5}bB\,x^{5}+\frac{1}{4}Ab\,x^{4}+\frac{1}{4}Ba\,x^{4}+\frac{1}{3}aA\,x^{3}$	44
parallelrisch	$\frac{1}{6}Bc\,x^{6}+\frac{1}{5}x^{5}Ac+\frac{1}{5}bB\,x^{5}+\frac{1}{4}Ab\,x^{4}+\frac{1}{4}Ba\,x^{4}+\frac{1}{3}aA\,x^{3}$	44

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/4*(A*b+B*a)*x^4+1/5*(A*c+B*b)*x^5+1/6*B*c*x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{6}\,Bcx^6+\frac{1}{5}\,(Bb+Ac)x^5+\frac{1}{3}\,Aax^3+\frac{1}{4}\,(Ba+Ab)x^4$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/6*B*c*x^6 + 1/5*(B*b + A*c)*x^5 + 1/3*A*a*x^3 + 1/4*(B*a + A*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{Aax^3}{3}+\frac{Bcx^6}{6}+x^5\left(\frac{Ac}{5}+\frac{Bb}{5}\right)+x^4\left(\frac{Ab}{4}+\frac{Ba}{4}\right)$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x+a),x)
```

output
```
A*a*x**3/3 + B*c*x**6/6 + x**5*(A*c/5 + B*b/5) + x**4*(A*b/4 + B*a/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{6}\,Bcx^6+\frac{1}{5}\,(Bb+Ac)x^5+\frac{1}{3}\,Aax^3+\frac{1}{4}\,(Ba+Ab)x^4$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/6*B*c*x^6 + 1/5*(B*b + A*c)*x^5 + 1/3*A*a*x^3 + 1/4*(B*a + A*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{6}\,Bcx^6+\frac{1}{5}\,Bbx^5+\frac{1}{5}\,Acx^5+\frac{1}{4}\,Bax^4+\frac{1}{4}\,Abx^4+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/6*B*c*x^6 + 1/5*B*b*x^5 + 1/5*A*c*x^5 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + 1/3*
A*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{B\,c\,x^6}{6}+\left(\frac{A\,c}{5}+\frac{B\,b}{5}\right)\,x^5+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,x^4+\frac{A\,a\,x^3}{3}$$

input
```
int(x^2*(A + B*x)*(a + b*x + c*x^2),x)
```

output
```
x^4*((A*b)/4 + (B*a)/4) + x^5*((A*c)/5 + (B*b)/5) + (A*a*x^3)/3 + (B*c*x^6
)/6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{x^3(5bc\,x^3+6ac\,x^2+6b^2x^2+15abx+10a^2)}{30}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a),x)
```

output
```
(x**3*(10*a**2 + 15*a*b*x + 6*a*c*x**2 + 6*b**2*x**2 + 5*b*c*x**3))/30
```

3.3 $\int x(A+Bx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int x(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}(Ab+aB)x^3+\frac{1}{4}(bB+Ac)x^4+\frac{1}{5}Bcx^5$$

output
```
1/2*a*A*x^2+1/3*(A*b+B*a)*x^3+1/4*(A*c+B*b)*x^4+1/5*B*c*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int x(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{60}x^2\left(30aA+20(Ab+aB)x+15(bB+Ac)x^2+12Bcx^3\right)$$

input
```
Integrate[x*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(x^2*(30*a*A + 20*(A*b + a*B)*x + 15*(b*B + A*c)*x^2 + 12*B*c*x^3))/60
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \ 1195$$

$$\int \left(x^2(aB+Ab)+aAx+x^3(Ac+bB)+Bcx^4\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}x^3(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{4}x^4(Ac+bB)+\frac{1}{5}Bcx^5$$

input
```
Int[x*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(a*A*x^2)/2 + ((A*b + a*B)*x^3)/3 + ((b*B + A*c)*x^4)/4 + (B*c*x^5)/5
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{2}}{2}+\frac{(Ab+Ba)x^{3}}{3}+\frac{(Ac+Bb)x^{4}}{4}+\frac{Bc\,x^{5}}{5}$	40
norman	$\frac{Bc\,x^{5}}{5}+\left(\frac{Ac}{4}+\frac{Bb}{4}\right)x^{4}+\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^{3}+\frac{aA\,x^{2}}{2}$	42
orering	$\frac{x^{2}\left(12Bc\,x^{3}+15Ac\,x^{2}+15Bb\,x^{2}+20Abx+20Bax+30Aa\right)}{60}$	42
gosper	$\frac{1}{5}Bc\,x^{5}+\frac{1}{4}Ac\,x^{4}+\frac{1}{4}Bb\,x^{4}+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}Ba\,x^{3}+\frac{1}{2}aA\,x^{2}$	44
risch	$\frac{1}{5}Bc\,x^{5}+\frac{1}{4}Ac\,x^{4}+\frac{1}{4}Bb\,x^{4}+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}Ba\,x^{3}+\frac{1}{2}aA\,x^{2}$	44
parallelrisch	$\frac{1}{5}Bc\,x^{5}+\frac{1}{4}Ac\,x^{4}+\frac{1}{4}Bb\,x^{4}+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}Ba\,x^{3}+\frac{1}{2}aA\,x^{2}$	44

input
```
int(x*(B*x+A)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*A*x^2+1/3*(A*b+B*a)*x^3+1/4*(A*c+B*b)*x^4+1/5*B*c*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x(A+Bx)\left(a+bx+cx^{2}\right)\,dx=\frac{1}{5}\,Bcx^{5}+\frac{1}{4}\,(Bb+Ac)x^{4}+\frac{1}{2}\,Aax^{2}+\frac{1}{3}\,(Ba+Ab)x^{3}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/5*B*c*x^5 + 1/4*(B*b + A*c)*x^4 + 1/2*A*a*x^2 + 1/3*(B*a + A*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int x(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{Aax^2}{2}+\frac{Bcx^5}{5}+x^4\left(\frac{Ac}{4}+\frac{Bb}{4}\right)+x^3\left(\frac{Ab}{3}+\frac{Ba}{3}\right)$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x+a),x)
```

output
```
A*a*x**2/2 + B*c*x**5/5 + x**4*(A*c/4 + B*b/4) + x**3*(A*b/3 + B*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{5}\,Bcx^5+\frac{1}{4}\,(Bb+Ac)x^4+\frac{1}{2}\,Aax^2+\frac{1}{3}\,(Ba+Ab)x^3$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/5*B*c*x^5 + 1/4*(B*b + A*c)*x^4 + 1/2*A*a*x^2 + 1/3*(B*a + A*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int x(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{5}\,Bcx^5+\frac{1}{4}\,Bbx^4+\frac{1}{4}\,Acx^4+\frac{1}{3}\,Bax^3+\frac{1}{3}\,Abx^3+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/5*B*c*x^5 + 1/4*B*b*x^4 + 1/4*A*c*x^4 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + 1/2*
A*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int x(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{B\,c\,x^5}{5}+\left(\frac{A\,c}{4}+\frac{B\,b}{4}\right)\,x^4+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x^3+\frac{A\,a\,x^2}{2}$$

input
```
int(x*(A + B*x)*(a + b*x + c*x^2),x)
```

output
```
x^3*((A*b)/3 + (B*a)/3) + x^4*((A*c)/4 + (B*b)/4) + (A*a*x^2)/2 + (B*c*x^5
)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int x(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{x^2(12bc\,x^3+15ac\,x^2+15b^2x^2+40abx+30a^2)}{60}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a),x)
```

output
```
(x**2*(30*a**2 + 40*a*b*x + 15*a*c*x**2 + 15*b**2*x**2 + 12*b*c*x**3))/60
```

3.4 $\int (A+Bx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int (A+Bx)\left(a+bx+cx^2\right)\,dx=aAx+\frac{1}{2}(Ab+aB)x^2+\frac{1}{3}(bB+Ac)x^3+\frac{1}{4}Bcx^4$$

output
```
a*A*x+1/2*(A*b+B*a)*x^2+1/3*(A*c+B*b)*x^3+1/4*B*c*x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(a+bx+cx^2\right)\,dx=aAx+\frac{1}{2}(Ab+aB)x^2+\frac{1}{3}(bB+Ac)x^3+\frac{1}{4}Bcx^4$$

input
```
Integrate[(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
a*A*x + ((A*b + a*B)*x^2)/2 + ((b*B + A*c)*x^3)/3 + (B*c*x^4)/4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(x(aB+Ab)+aA+x^2(Ac+bB)+Bcx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(aB+Ab)+aAx+\frac{1}{3}x^3(Ac+bB)+\frac{1}{4}Bcx^4$$

input
```
Int[(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
a*A*x + ((A*b + a*B)*x^2)/2 + ((b*B + A*c)*x^3)/3 + (B*c*x^4)/4
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
default	$aAx+\frac{(Ab+Ba)x^2}{2}+\frac{(Ac+Bb)x^3}{3}+\frac{Bc\,x^4}{4}$	37
norman	$\frac{Bc\,x^4}{4}+\left(\frac{Ac}{3}+\frac{Bb}{3}\right)x^3+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x^2+aAx$	39
orering	$\frac{x(3Bc\,x^3+4Ac\,x^2+4Bb\,x^2+6Abx+6Bax+12Aa)}{12}$	40
gosper	$\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}Bb\,x^3+\frac{1}{2}Ab\,x^2+\frac{1}{2}Ba\,x^2+aAx$	41
risch	$\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}Bb\,x^3+\frac{1}{2}Ab\,x^2+\frac{1}{2}Ba\,x^2+aAx$	41
parallelrisch	$\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}Bb\,x^3+\frac{1}{2}Ab\,x^2+\frac{1}{2}Ba\,x^2+aAx$	41

input
```
int((B*x+A)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*(A*b+B*a)*x^2+1/3*(A*c+B*b)*x^3+1/4*B*c*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{3}\,(Bb+Ac)x^3+Aax+\frac{1}{2}\,(Ba+Ab)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/4*B*c*x^4 + 1/3*(B*b + A*c)*x^3 + A*a*x + 1/2*(B*a + A*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int (A+Bx)\left(a+bx+cx^2\right)\,dx=Aax+\frac{Bcx^4}{4}+x^3\left(\frac{Ac}{3}+\frac{Bb}{3}\right)+x^2\left(\frac{Ab}{2}+\frac{Ba}{2}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a),x)
```

output
```
A*a*x + B*c*x**4/4 + x**3*(A*c/3 + B*b/3) + x**2*(A*b/2 + B*a/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{3}\,(Bb+Ac)x^3+Aax+\frac{1}{2}\,(Ba+Ab)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/4*B*c*x^4 + 1/3*(B*b + A*c)*x^3 + A*a*x + 1/2*(B*a + A*b)*x^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int (A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{3}\,Bbx^3+\frac{1}{3}\,Acx^3+\frac{1}{2}\,Bax^2+\frac{1}{2}\,Abx^2+Aax$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*B*c*x^4 + 1/3*B*b*x^3 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + 1/2*A*b*x^2 + A*a*
x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int (A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{B\,c\,x^4}{4}+\left(\frac{A\,c}{3}+\frac{B\,b}{3}\right)\,x^3+\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\,x^2+A\,a\,x$$

input
```
int((A + B*x)*(a + b*x + c*x^2),x)
```

output
```
x^2*((A*b)/2 + (B*a)/2) + x^3*((A*c)/3 + (B*b)/3) + A*a*x + (B*c*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{x(3bc\,x^3+4ac\,x^2+4b^2x^2+12abx+12a^2)}{12}$$

input
```
int((B*x+A)*(c*x^2+b*x+a),x)
```

output
```
(x*(12*a**2 + 12*a*b*x + 4*a*c*x**2 + 4*b**2*x**2 + 3*b*c*x**3))/12
```

3.5 $\int \frac{(A+Bx)(a+bx+cx^2)}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x}\,dx=(Ab+aB)x+\frac{1}{2}(bB+Ac)x^2+\frac{1}{3}Bcx^3+aA\log(x)$$

output
```
(A*b+B*a)*x+1/2*(A*c+B*b)*x^2+1/3*B*c*x^3+a*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x}\,dx=(Ab+aB)x+\frac{1}{2}(bB+Ac)x^2+\frac{1}{3}Bcx^3+aA\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x,x]
```

output
```
(A*b + a*B)*x + ((b*B + A*c)*x^2)/2 + (B*c*x^3)/3 + a*A*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x}\,dx$$

$$\Big\downarrow \ 1195$$

$$\int \left(\frac{aA}{x}+aB+x(Ac+bB)+Ab+Bcx^2\right)dx$$

$$\Big\downarrow \ 2009$$

$$x(aB+Ab)+aA\log(x)+\frac{1}{2}x^2(Ac+bB)+\frac{1}{3}Bcx^3$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x,x]
```

output
```
(A*b + a*B)*x + ((b*B + A*c)*x^2)/2 + (B*c*x^3)/3 + a*A*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{Bc\,x^{3}}{3}+\frac{Ac\,x^{2}}{2}+\frac{Bb\,x^{2}}{2}+Abx+Bax+aA\ln\left(x\right)$	36
norman	$\left(\frac{Ac}{2}+\frac{Bb}{2}\right)x^{2}+\left(Ab+Ba\right)x+\frac{Bc\,x^{3}}{3}+aA\ln\left(x\right)$	36
risch	$\frac{Bc\,x^{3}}{3}+\frac{Ac\,x^{2}}{2}+\frac{Bb\,x^{2}}{2}+Abx+Bax+aA\ln\left(x\right)$	36
parallelrisch	$\frac{Bc\,x^{3}}{3}+\frac{Ac\,x^{2}}{2}+\frac{Bb\,x^{2}}{2}+Abx+Bax+aA\ln\left(x\right)$	36

input
```
int((B*x+A)*(c*x^2+b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*c*x^3+1/2*A*c*x^2+1/2*B*b*x^2+A*b*x+B*a*x+a*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x}\,dx=\frac{1}{3}\,Bcx^{3}+\frac{1}{2}\,(Bb+Ac)x^{2}+Aa\log\left(x\right)+(Ba+Ab)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x,x, algorithm="fricas")
```

output
```
1/3*B*c*x^3 + 1/2*(B*b + A*c)*x^2 + A*a*log(x) + (B*a + A*b)*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x}\,dx=Aa\log\left(x\right)+\frac{Bcx^{3}}{3}+x^{2}\left(\frac{Ac}{2}+\frac{Bb}{2}\right)+x(Ab+Ba)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x,x)
```

output
```
A*a*log(x) + B*c*x**3/3 + x**2*(A*c/2 + B*b/2) + x*(A*b + B*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x}\,dx = \frac{1}{3}\,Bcx^3 + \frac{1}{2}\,(Bb+Ac)x^2 + Aa\log\left(x\right) + (Ba+Ab)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x,x, algorithm="maxima")
```

output
```
1/3*B*c*x^3 + 1/2*(B*b + A*c)*x^2 + A*a*log(x) + (B*a + A*b)*x
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x}\,dx = \frac{1}{3}\,Bcx^3 + \frac{1}{2}\,Bbx^2 + \frac{1}{2}\,Acx^2 + Bax + Abx + Aa\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x,x, algorithm="giac")
```

output
```
1/3*B*c*x^3 + 1/2*B*b*x^2 + 1/2*A*c*x^2 + B*a*x + A*b*x + A*a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x}\,dx = x\,(A\,b+B\,a) + x^2\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right) + \frac{B\,c\,x^3}{3} + A\,a\,\ln\left(x\right)$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x,x)
```

output
```
x*(A*b + B*a) + x^2*((A*c)/2 + (B*b)/2) + (B*c*x^3)/3 + A*a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x}\,dx = \log(x)\,a^2 + 2abx + \frac{ac\,x^2}{2} + \frac{b^2x^2}{2} + \frac{bc\,x^3}{3}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x,x)
```

output
```
(6*log(x)*a**2 + 12*a*b*x + 3*a*c*x**2 + 3*b**2*x**2 + 2*b*c*x**3)/6
```

3.6 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^2} \, dx = -\frac{aA}{x}+(bB+Ac)x+\frac{1}{2}Bcx^2+(Ab+aB)\log(x)$$

output
```
-a*A/x+(A*c+B*b)*x+1/2*B*c*x^2+(A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^2} \, dx = -\frac{aA}{x}+(bB+Ac)x+\frac{1}{2}Bcx^2+(Ab+aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^2,x]
```

output
```
-((a*A)/x) + (b*B + A*c)*x + (B*c*x^2)/2 + (A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^2}\,dx$$

$$\Big\downarrow \ 1195$$

$$\int \left(\frac{aB+Ab}{x}+\frac{aA}{x^2}+bB\left(\frac{Ac}{bB}+1\right)+Bcx\right)dx$$

$$\Big\downarrow \ 2009$$

$$\log(x)(aB+Ab)-\frac{aA}{x}+x(Ac+bB)+\frac{1}{2}Bcx^2$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^2,x]
```

output
```
-((a*A)/x) + (b*B + A*c)*x + (B*c*x^2)/2 + (A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{Bc\,x^{2}}{2}+Acx+Bbx+\left(Ab+Ba\right)\ln\left(x\right)-\frac{aA}{x}$	34
risch	$\frac{Bc\,x^{2}}{2}+Acx+Bbx+Ab\ln\left(x\right)+B\ln\left(x\right)a-\frac{aA}{x}$	34
norman	$\frac{\left(Ac+Bb\right)x^{2}-Aa+\frac{Bc\,x^{3}}{2}}{x}+\left(Ab+Ba\right)\ln\left(x\right)$	39
parallelrisch	$\frac{Bc\,x^{3}+2A\ln\left(x\right)xb+2Ac\,x^{2}+2B\ln\left(x\right)xa+2Bb\,x^{2}-2Aa}{2x}$	45

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*c*x^2+A*c*x+B*b*x+(A*b+B*a)*ln(x)-a*A/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^2}\,dx=\frac{Bcx^3+2\,(Bb+Ac)x^2+2\,(Ba+Ab)x\log\left(x\right)-2\,Aa}{2\,x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^2,x, algorithm="fricas")
```

output
```
1/2*(B*c*x^3 + 2*(B*b + A*c)*x^2 + 2*(B*a + A*b)*x*log(x) - 2*A*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^2}\,dx=-\frac{Aa}{x}+\frac{Bcx^2}{2}+x(Ac+Bb)+(Ab+Ba)\log\left(x\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**2,x)
```

output
```
-A*a/x + B*c*x**2/2 + x*(A*c + B*b) + (A*b + B*a)*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^2}\,dx = \frac{1}{2}\,Bcx^2 + (Bb+Ac)x + (Ba+Ab)\log{(x)} - \frac{Aa}{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^2,x, algorithm="maxima")
```

output
```
1/2*B*c*x^2 + (B*b + A*c)*x + (B*a + A*b)*log(x) - A*a/x
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^2}\,dx = \frac{1}{2}\,Bcx^2 + Bbx + Acx + (Ba+Ab)\log{(|x|)} - \frac{Aa}{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^2,x, algorithm="giac")
```

output
```
1/2*B*c*x^2 + B*b*x + A*c*x + (B*a + A*b)*log(abs(x)) - A*a/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^2}\,dx = x\,(A\,c+B\,b) + \ln{(x)}\,\left(A\,b+B\,a\right) - \frac{A\,a}{x} + \frac{B\,c\,x^2}{2}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^2,x)
```

output
```
x*(A*c + B*b) + log(x)*(A*b + B*a) - (A*a)/x + (B*c*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^2}\,dx = \frac{4\log(x)\,abx-2a^2+2ac\,x^2+2b^2x^2+bc\,x^3}{2x}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^2,x)
```

output
```
(4*log(x)*a*b*x - 2*a**2 + 2*a*c*x**2 + 2*b**2*x**2 + b*c*x**3)/(2*x)
```

3.7 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^3} dx = -\frac{aA}{2x^2} - \frac{Ab+aB}{x} + Bcx + (bB+Ac)\log(x)$$

output
```
-1/2*a*A/x^2-(A*b+B*a)/x+B*c*x+(A*c+B*b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^3} dx = -\frac{aA}{2x^2} + \frac{-Ab-aB}{x} + Bcx + (bB+Ac)\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 + (-(A*b) - a*B)/x + B*c*x + (b*B + A*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^3}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{aB+Ab}{x^2}+\frac{aA}{x^3}+\frac{Ac+bB}{x}+Bc\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aB+Ab}{x}-\frac{aA}{2x^2}+\log(x)(Ac+bB)+Bcx$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 - (A*b + a*B)/x + B*c*x + (b*B + A*c)*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{aA}{2x^2}-\frac{Ab+Ba}{x}+Bcx+(Ac+Bb)\ln(x)$	35
risch	$Bcx+\frac{(-Ab-Ba)x-\frac{Aa}{2}}{x^2}+A\ln(x)c+B\ln(x)b$	36
norman	$\frac{(-Ab-Ba)x+Bc\,x^3-\frac{Aa}{2}}{x^2}+(Ac+Bb)\ln(x)$	38
parallelrisch	$\frac{2A\ln(x)x^2c+2B\ln(x)x^2b+2Bc\,x^3-2Abx-2Bax-Aa}{2x^2}$	46

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*A/x^2-(A*b+B*a)/x+B*c*x+(A*c+B*b)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int\frac{(A+Bx)(a+bx+cx^2)}{x^3}\,dx=\frac{2\,Bcx^3+2\,(Bb+Ac)x^2\log(x)-Aa-2\,(Ba+Ab)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*B*c*x^3 + 2*(B*b + A*c)*x^2*log(x) - A*a - 2*(B*a + A*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int\frac{(A+Bx)(a+bx+cx^2)}{x^3}\,dx=Bcx+(Ac+Bb)\log(x)+\frac{-Aa+x(-2Ab-2Ba)}{2x^2}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**3,x)
```

output
```
B*c*x + (A*c + B*b)*log(x) + (-A*a + x*(-2*A*b - 2*B*a))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^3}\,dx = Bcx + (Bb+Ac)\log{(x)} - \frac{Aa+2\,(Ba+Ab)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^3,x, algorithm="maxima")
```

output
```
B*c*x + (B*b + A*c)*log(x) - 1/2*(A*a + 2*(B*a + A*b)*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^3}\,dx = Bcx + (Bb+Ac)\log{(|x|)} - \frac{Aa+2\,(Ba+Ab)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^3,x, algorithm="giac")
```

output
```
B*c*x + (B*b + A*c)*log(abs(x)) - 1/2*(A*a + 2*(B*a + A*b)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^3}\,dx = \ln{(x)}\ (A\,c+B\,b) - \frac{\frac{A\,a}{2}+x\,(A\,b+B\,a)}{x^2} + B\,c\,x$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^3,x)
```

output
```
log(x)*(A*c + B*b) - ((A*a)/2 + x*(A*b + B*a))/x^2 + B*c*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^3}\,dx = \frac{2\log(x)\,ac\,x^2+2\log(x)\,b^2x^2-a^2-4abx+2bc\,x^3}{2x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^3,x)
```

output
```
(2*log(x)*a*c*x**2 + 2*log(x)*b**2*x**2 - a**2 - 4*a*b*x + 2*b*c*x**3)/(2*
x**2)
```

3.8 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^4}\,dx=-\frac{aA}{3x^3}-\frac{Ab+aB}{2x^2}-\frac{bB+Ac}{x}+Bc\log(x)$$

output
```
-1/3*a*A/x^3-1/2*(A*b+B*a)/x^2-(A*c+B*b)/x+B*c*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^4}\,dx=-\frac{a(2A+3Bx)+3x(Ab+2bBx+2Acx)}{6x^3}+Bc\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^4,x]
```

output
```
-1/6*(a*(2*A + 3*B*x) + 3*x*(A*b + 2*b*B*x + 2*A*c*x))/x^3 + B*c*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^4}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{aB+Ab}{x^3}+\frac{aA}{x^4}+\frac{Ac+bB}{x^2}+\frac{Bc}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{3x^3}-\frac{Ac+bB}{x}+Bc\log(x)$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (A*b + a*B)/(2*x^2) - (b*B + A*c)/x + B*c*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{aA}{3x^3}-\frac{Ab+Ba}{2x^2}-\frac{Ac+Bb}{x}+Bc\ln\left(x\right)$	38
norman	$\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x+(-Ac-Bb)x^2-\frac{Aa}{3}}{x^3}+Bc\ln\left(x\right)$	40
risch	$\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x+(-Ac-Bb)x^2-\frac{Aa}{3}}{x^3}+Bc\ln\left(x\right)$	40
parallelrisch	$-\frac{-6Bc\ln(x)x^3+6Ac\,x^2+6Bb\,x^2+3Abx+3Bax+2Aa}{6x^3}$	44

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*A/x^3-1/2*(A*b+B*a)/x^2-(A*c+B*b)/x+B*c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int\frac{(A+Bx)\left(a+bx+cx^2\right)}{x^4}\,dx=\frac{6\,Bcx^3\log\left(x\right)-6\,(Bb+Ac)x^2-2\,Aa-3\,(Ba+Ab)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*B*c*x^3*log(x) - 6*(B*b + A*c)*x^2 - 2*A*a - 3*(B*a + A*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int\frac{(A+Bx)\left(a+bx+cx^2\right)}{x^4}\,dx$$
$$=Bc\log\left(x\right)+\frac{-2Aa+x^2(-6Ac-6Bb)+x(-3Ab-3Ba)}{6x^3}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**4,x)
```

output
```
B*c*log(x) + (-2*A*a + x**2*(-6*A*c - 6*B*b) + x*(-3*A*b - 3*B*a))/(6*x**3
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^4}\,dx = Bc\log\left(x\right) - \frac{6\,(Bb+Ac)x^2 + 2\,Aa + 3\,(Ba+Ab)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^4,x, algorithm="maxima")
```

output
```
B*c*log(x) - 1/6*(6*(B*b + A*c)*x^2 + 2*A*a + 3*(B*a + A*b)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^4}\,dx = Bc\log\left(|x|\right) - \frac{6\,(Bb+Ac)x^2 + 2\,Aa + 3\,(Ba+Ab)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^4,x, algorithm="giac")
```

output
```
B*c*log(abs(x)) - 1/6*(6*(B*b + A*c)*x^2 + 2*A*a + 3*(B*a + A*b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^4}\,dx = B\,c\,\ln(x) - \frac{(A\,c+B\,b)\;x^2+\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\;x+\frac{A\,a}{3}}{x^3}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^4,x)
```

output
```
B*c*log(x) - ((A*a)/3 + x*((A*b)/2 + (B*a)/2) + x^2*(A*c + B*b))/x^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^4}\,dx = \frac{3\log(x)\,bc\,x^3-a^2-3abx-3ac\,x^2-3b^2x^2}{3x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^4,x)
```

output
```
(3*log(x)*b*c*x**3 - a**2 - 3*a*b*x - 3*a*c*x**2 - 3*b**2*x**2)/(3*x**3)
```

3.9 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^5}\,dx=-\frac{aA}{4x^4}-\frac{Ab+aB}{3x^3}-\frac{bB+Ac}{2x^2}-\frac{Bc}{x}$$

output
```
-1/4*a*A/x^4-1/3*(A*b+B*a)/x^3-1/2*(A*c+B*b)/x^2-B*c/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^5}\,dx=-\frac{a(3A+4Bx)+2x(3Bx(b+2cx)+A(2b+3cx))}{12x^4}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^5,x]
```

output
```
-1/12*(a*(3*A + 4*B*x) + 2*x*(3*B*x*(b + 2*c*x) + A*(2*b + 3*c*x)))/x^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^5}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{aB+Ab}{x^4}+\frac{aA}{x^5}+\frac{Ac+bB}{x^3}+\frac{Bc}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{4x^4}-\frac{Ac+bB}{2x^2}-\frac{Bc}{x}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^5,x]
```

output
```
-1/4*(a*A)/x^4 - (A*b + a*B)/(3*x^3) - (b*B + A*c)/(2*x^2) - (B*c)/x
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{aA}{4x^4}-\frac{Ab+Ba}{3x^3}-\frac{Ac+Bb}{2x^2}-\frac{Bc}{x}$	40
norman	$\frac{-Bc\,x^3+\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x^2+\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x-\frac{Aa}{4}}{x^4}$	41
risch	$\frac{-Bc\,x^3+\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x^2+\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x-\frac{Aa}{4}}{x^4}$	41
gosper	$-\frac{12Bc\,x^3+6Ac\,x^2+6Bb\,x^2+4Abx+4Bax+3Aa}{12x^4}$	42
parallelrisch	$-\frac{12Bc\,x^3+6Ac\,x^2+6Bb\,x^2+4Abx+4Bax+3Aa}{12x^4}$	42
orering	$-\frac{12Bc\,x^3+6Ac\,x^2+6Bb\,x^2+4Abx+4Bax+3Aa}{12x^4}$	42

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*A/x^4-1/3*(A*b+B*a)/x^3-1/2*(A*c+B*b)/x^2-B*c/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^5}\,dx=-\frac{12\,Bcx^3+6\,(Bb+Ac)x^2+3\,Aa+4\,(Ba+Ab)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(12*B*c*x^3 + 6*(B*b + A*c)*x^2 + 3*A*a + 4*(B*a + A*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^5}\,dx = \frac{-3Aa-12Bcx^3+x^2(-6Ac-6Bb)+x(-4Ab-4Ba)}{12x^4}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**5,x)
```

output
```
(-3*A*a - 12*B*c*x**3 + x**2*(-6*A*c - 6*B*b) + x*(-4*A*b - 4*B*a))/(12*x*
*4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^5}\,dx = -\frac{12\,Bcx^3+6\,(Bb+Ac)x^2+3\,Aa+4\,(Ba+Ab)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(12*B*c*x^3 + 6*(B*b + A*c)*x^2 + 3*A*a + 4*(B*a + A*b)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^5}\,dx = -\frac{12\,Bcx^3+6\,Bbx^2+6\,Acx^2+4\,Bax+4\,Abx+3\,Aa}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^5,x, algorithm="giac")
```

output
```
-1/12*(12*B*c*x^3 + 6*B*b*x^2 + 6*A*c*x^2 + 4*B*a*x + 4*A*b*x + 3*A*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^5}\,dx=-\frac{B\,c\,x^3+\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right)\,x^2+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x+\frac{A\,a}{4}}{x^4}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^5,x)
```

output
```
-((A*a)/4 + x*((A*b)/3 + (B*a)/3) + x^2*((A*c)/2 + (B*b)/2) + B*c*x^3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^5}\,dx=\frac{-12bc\,x^3-6ac\,x^2-6b^2x^2-8abx-3a^2}{12x^4}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^5,x)
```

output
```
( - 3*a**2 - 8*a*b*x - 6*a*c*x**2 - 6*b**2*x**2 - 12*b*c*x**3)/(12*x**4)
```

3.10 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^6}\,dx = -\frac{aA}{5x^5} - \frac{Ab+aB}{4x^4} - \frac{bB+Ac}{3x^3} - \frac{Bc}{2x^2}$$

output
```
-1/5*a*A/x^5-1/4*(A*b+B*a)/x^4-1/3*(A*c+B*b)/x^3-1/2*B*c/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^6}\,dx = -\frac{3a(4A+5Bx)+5x(3Ab+4bBx+4Acx+6Bcx^2)}{60x^5}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^6,x]
```

output
```
-1/60*(3*a*(4*A + 5*B*x) + 5*x*(3*A*b + 4*b*B*x + 4*A*c*x + 6*B*c*x^2))/x^
5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^6}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{aB+Ab}{x^5}+\frac{aA}{x^6}+\frac{Ac+bB}{x^4}+\frac{Bc}{x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{aB+Ab}{4x^4}-\frac{aA}{5x^5}-\frac{Ac+bB}{3x^3}-\frac{Bc}{2x^2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 - (A*b + a*B)/(4*x^4) - (b*B + A*c)/(3*x^3) - (B*c)/(2*x^2)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{aA}{5x^5}-\frac{Ab+Ba}{4x^4}-\frac{Ac+Bb}{3x^3}-\frac{Bc}{2x^2}$	40
norman	$\frac{-\frac{Bc\,x^3}{2}+\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x^2+\left(-\frac{Ab}{4}-\frac{Ba}{4}\right)x-\frac{Aa}{5}}{x^5}$	41
risch	$\frac{-\frac{Bc\,x^3}{2}+\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x^2+\left(-\frac{Ab}{4}-\frac{Ba}{4}\right)x-\frac{Aa}{5}}{x^5}$	41
gosper	$-\frac{30Bc\,x^3+20Ac\,x^2+20Bb\,x^2+15Abx+15Bax+12Aa}{60x^5}$	42
parallelrisch	$-\frac{30Bc\,x^3+20Ac\,x^2+20Bb\,x^2+15Abx+15Bax+12Aa}{60x^5}$	42
orering	$-\frac{30Bc\,x^3+20Ac\,x^2+20Bb\,x^2+15Abx+15Bax+12Aa}{60x^5}$	42

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a*A/x^5-1/4*(A*b+B*a)/x^4-1/3*(A*c+B*b)/x^3-1/2*B*c/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^6}\,dx=-\frac{30\,Bcx^3+20\,(Bb+Ac)x^2+12\,Aa+15\,(Ba+Ab)x}{60\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^6,x, algorithm="fricas")
```

output
```
-1/60*(30*B*c*x^3 + 20*(B*b + A*c)*x^2 + 12*A*a + 15*(B*a + A*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^6}\,dx = \frac{-12Aa-30Bcx^3+x^2(-20Ac-20Bb)+x(-15Ab-15Ba)}{60x^5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**6,x)
```

output
```
(-12*A*a - 30*B*c*x**3 + x**2*(-20*A*c - 20*B*b) + x*(-15*A*b - 15*B*a))/(
60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^6}\,dx = -\frac{30\,Bcx^3+20\,(Bb+Ac)x^2+12\,Aa+15\,(Ba+Ab)x}{60\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^6,x, algorithm="maxima")
```

output
```
-1/60*(30*B*c*x^3 + 20*(B*b + A*c)*x^2 + 12*A*a + 15*(B*a + A*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^6}\,dx = -\frac{30\,Bcx^3+20\,Bbx^2+20\,Acx^2+15\,Bax+15\,Abx+12\,Aa}{60\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^6,x, algorithm="giac")
```

output
```
-1/60*(30*B*c*x^3 + 20*B*b*x^2 + 20*A*c*x^2 + 15*B*a*x + 15*A*b*x + 12*A*a
)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^6}\,dx=-\frac{\frac{B\,c\,x^3}{2}+\left(\frac{A\,c}{3}+\frac{B\,b}{3}\right)\,x^2+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,x+\frac{A\,a}{5}}{x^5}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^6,x)
```

output
```
-((A*a)/5 + x*((A*b)/4 + (B*a)/4) + x^2*((A*c)/3 + (B*b)/3) + (B*c*x^3)/2)
/x^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^6}\,dx=\frac{-15bc\,x^3-10ac\,x^2-10b^2x^2-15abx-6a^2}{30x^5}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^6,x)
```

output
```
( - 6*a**2 - 15*a*b*x - 10*a*c*x**2 - 10*b**2*x**2 - 15*b*c*x**3)/(30*x**5
)
```

3.11 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^7}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^7}\,dx=-\frac{aA}{6x^6}-\frac{Ab+aB}{5x^5}-\frac{bB+Ac}{4x^4}-\frac{Bc}{3x^3}$$

output
```
-1/6*a*A/x^6-1/5*(A*b+B*a)/x^5-1/4*(A*c+B*b)/x^4-1/3*B*c/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^7}\,dx=-\frac{2a(5A+6Bx)+x(5Bx(3b+4cx)+3A(4b+5cx))}{60x^6}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^7,x]
```

output
```
-1/60*(2*a*(5*A + 6*B*x) + x*(5*B*x*(3*b + 4*c*x) + 3*A*(4*b + 5*c*x)))/x^
6
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^7}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{aB+Ab}{x^6}+\frac{aA}{x^7}+\frac{Ac+bB}{x^5}+\frac{Bc}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aB+Ab}{5x^5}-\frac{aA}{6x^6}-\frac{Ac+bB}{4x^4}-\frac{Bc}{3x^3}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^7,x]
```

output
```
-1/6*(a*A)/x^6 - (A*b + a*B)/(5*x^5) - (b*B + A*c)/(4*x^4) - (B*c)/(3*x^3)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{aA}{6x^6}-\frac{Ab+Ba}{5x^5}-\frac{Ac+Bb}{4x^4}-\frac{Bc}{3x^3}$	40
norman	$\frac{-\frac{Bc\,x^3}{3}+\left(-\frac{Ac}{4}-\frac{Bb}{4}\right)x^2+\left(-\frac{Ab}{5}-\frac{Ba}{5}\right)x-\frac{Aa}{6}}{x^6}$	41
risch	$\frac{-\frac{Bc\,x^3}{3}+\left(-\frac{Ac}{4}-\frac{Bb}{4}\right)x^2+\left(-\frac{Ab}{5}-\frac{Ba}{5}\right)x-\frac{Aa}{6}}{x^6}$	41
gosper	$-\frac{20Bc\,x^3+15Ac\,x^2+15Bb\,x^2+12Abx+12Bax+10Aa}{60x^6}$	42
parallelrisch	$-\frac{20Bc\,x^3+15Ac\,x^2+15Bb\,x^2+12Abx+12Bax+10Aa}{60x^6}$	42
orering	$-\frac{20Bc\,x^3+15Ac\,x^2+15Bb\,x^2+12Abx+12Bax+10Aa}{60x^6}$	42

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*A/x^6-1/5*(A*b+B*a)/x^5-1/4*(A*c+B*b)/x^4-1/3*B*c/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^7}\,dx=-\frac{20\,Bcx^3+15\,(Bb+Ac)x^2+10\,Aa+12\,(Ba+Ab)x}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^7,x, algorithm="fricas")
```

output
```
-1/60*(20*B*c*x^3 + 15*(B*b + A*c)*x^2 + 10*A*a + 12*(B*a + A*b)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^7}\,dx = \frac{-10Aa-20Bcx^3+x^2(-15Ac-15Bb)+x(-12Ab-12Ba)}{60x^6}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**7,x)
```

output
```
(-10*A*a - 20*B*c*x**3 + x**2*(-15*A*c - 15*B*b) + x*(-12*A*b - 12*B*a))/(
60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^7}\,dx = -\frac{20\,Bcx^3+15\,(Bb+Ac)x^2+10\,Aa+12\,(Ba+Ab)x}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^7,x, algorithm="maxima")
```

output
```
-1/60*(20*B*c*x^3 + 15*(B*b + A*c)*x^2 + 10*A*a + 12*(B*a + A*b)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^7}\,dx = -\frac{20\,Bcx^3+15\,Bbx^2+15\,Acx^2+12\,Bax+12\,Abx+10\,Aa}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^7,x, algorithm="giac")
```

output
```
-1/60*(20*B*c*x^3 + 15*B*b*x^2 + 15*A*c*x^2 + 12*B*a*x + 12*A*b*x + 10*A*a
)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^7}\,dx=-\frac{\frac{B\,c\,x^3}{3}+\left(\frac{A\,c}{4}+\frac{B\,b}{4}\right)\,x^2+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x+\frac{A\,a}{6}}{x^6}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^7,x)
```

output
```
-((A*a)/6 + x*((A*b)/5 + (B*a)/5) + x^2*((A*c)/4 + (B*b)/4) + (B*c*x^3)/3)
/x^6
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^7}\,dx=\frac{-20bc\,x^3-15ac\,x^2-15b^2x^2-24abx-10a^2}{60x^6}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^7,x)
```

output
```
( - 10*a**2 - 24*a*b*x - 15*a*c*x**2 - 15*b**2*x**2 - 20*b*c*x**3)/(60*x**
6)
```

3.12 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^8} dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^8} dx = -\frac{aA}{7x^7} - \frac{Ab+aB}{6x^6} - \frac{bB+Ac}{5x^5} - \frac{Bc}{4x^4}$$

output
```
-1/7*a*A/x^7-1/6*(A*b+B*a)/x^6-1/5*(A*c+B*b)/x^5-1/4*B*c/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^8} dx = -\frac{10a(6A+7Bx)+7x(3Bx(4b+5cx)+2A(5b+6cx))}{420x^7}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^8,x]
```

output
```
-1/420*(10*a*(6*A + 7*B*x) + 7*x*(3*B*x*(4*b + 5*c*x) + 2*A*(5*b + 6*c*x))
)/x^7
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^8}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{aB+Ab}{x^7}+\frac{aA}{x^8}+\frac{Ac+bB}{x^6}+\frac{Bc}{x^5}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{aB+Ab}{6x^6}-\frac{aA}{7x^7}-\frac{Ac+bB}{5x^5}-\frac{Bc}{4x^4}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^8,x]
```

output
```
-1/7*(a*A)/x^7 - (A*b + a*B)/(6*x^6) - (b*B + A*c)/(5*x^5) - (B*c)/(4*x^4)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{aA}{7x^7}-\frac{Ab+Ba}{6x^6}-\frac{Ac+Bb}{5x^5}-\frac{Bc}{4x^4}$	40
norman	$\frac{-\frac{Bc\,x^3}{4}+\left(-\frac{Ac}{5}-\frac{Bb}{5}\right)x^2+\left(-\frac{Ab}{6}-\frac{Ba}{6}\right)x-\frac{Aa}{7}}{x^7}$	41
risch	$\frac{-\frac{Bc\,x^3}{4}+\left(-\frac{Ac}{5}-\frac{Bb}{5}\right)x^2+\left(-\frac{Ab}{6}-\frac{Ba}{6}\right)x-\frac{Aa}{7}}{x^7}$	41
gosper	$-\frac{105Bc\,x^3+84Ac\,x^2+84Bb\,x^2+70Abx+70Bax+60Aa}{420x^7}$	42
parallelrisch	$-\frac{105Bc\,x^3+84Ac\,x^2+84Bb\,x^2+70Abx+70Bax+60Aa}{420x^7}$	42
orering	$-\frac{105Bc\,x^3+84Ac\,x^2+84Bb\,x^2+70Abx+70Bax+60Aa}{420x^7}$	42

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a*A/x^7-1/6*(A*b+B*a)/x^6-1/5*(A*c+B*b)/x^5-1/4*B*c/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^8}\,dx = -\frac{105\,Bcx^3+84\,(Bb+Ac)x^2+60\,Aa+70\,(Ba+Ab)x}{420\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^8,x, algorithm="fricas")
```

output
```
-1/420*(105*B*c*x^3 + 84*(B*b + A*c)*x^2 + 60*A*a + 70*(B*a + A*b)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^8}\,dx = \frac{-60Aa-105Bcx^3+x^2(-84Ac-84Bb)+x(-70Ab-70Ba)}{420x^7}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**8,x)
```

output
```
(-60*A*a - 105*B*c*x**3 + x**2*(-84*A*c - 84*B*b) + x*(-70*A*b - 70*B*a))/
(420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^8}\,dx = -\frac{105\,Bcx^3+84\,(Bb+Ac)x^2+60\,Aa+70\,(Ba+Ab)x}{420\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^8,x, algorithm="maxima")
```

output
```
-1/420*(105*B*c*x^3 + 84*(B*b + A*c)*x^2 + 60*A*a + 70*(B*a + A*b)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^8}\,dx = -\frac{105\,Bcx^3+84\,Bbx^2+84\,Acx^2+70\,Bax+70\,Abx+60\,Aa}{420\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^8,x, algorithm="giac")
```

output
```
-1/420*(105*B*c*x^3 + 84*B*b*x^2 + 84*A*c*x^2 + 70*B*a*x + 70*A*b*x + 60*A
*a)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^8}\,dx = -\frac{\frac{B\,c\,x^3}{4}+\left(\frac{A\,c}{5}+\frac{B\,b}{5}\right)\,x^2+\left(\frac{A\,b}{6}+\frac{B\,a}{6}\right)\,x+\frac{A\,a}{7}}{x^7}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^8,x)
```

output
```
-((A*a)/7 + x*((A*b)/6 + (B*a)/6) + x^2*((A*c)/5 + (B*b)/5) + (B*c*x^3)/4)
/x^7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^8}\,dx = \frac{-105bc\,x^3-84ac\,x^2-84b^2x^2-140abx-60a^2}{420x^7}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^8,x)
```

output
```
( - 60*a**2 - 140*a*b*x - 84*a*c*x**2 - 84*b**2*x**2 - 105*b*c*x**3)/(420*
x**7)
```

3.13 $\int x^2(A+Bx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a(2Ab+aB)x^4+\frac{1}{5}\left(2abB+A\left(b^2+2ac\right)\right)x^5+\frac{1}{6}\left(b^2B+2Abc+2aBc\right)x^6+\frac{1}{7}c(2bB+Ac)x^7+\frac{1}{8}Bc^2x^8$$

output
```
1/3*a^2*A*x^3+1/4*a*(2*A*b+B*a)*x^4+1/5*(2*a*b*B+A*(2*a*c+b^2))*x^5+1/6*(2
*A*b*c+2*B*a*c+B*b^2)*x^6+1/7*c*(A*c+2*B*b)*x^7+1/8*B*c^2*x^8
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a(2Ab+aB)x^4+\frac{1}{5}\left(Ab^2+2abB+2aAc\right)x^5+\frac{1}{6}\left(b^2B+2Abc+2aBc\right)x^6+\frac{1}{7}c(2bB+Ac)x^7+\frac{1}{8}Bc^2x^8$$

input
```
Integrate[x^2*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^4)/4 + ((A*b^2 + 2*a*b*B + 2*a*A*c)*x^5
)/5 + ((b^2*B + 2*A*b*c + 2*a*B*c)*x^6)/6 + (c*(2*b*B + A*c)*x^7)/7 + (B*c
^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^2 dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^2Ax^2+x^5\left(2aBc+2Abc+b^2B\right)+x^4\left(A\left(2ac+b^2\right)+2abB\right)+ax^3(aB+2Ab)+cx^6(Ac+2bB)+Bc^2x^7\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{6}x^6\left(2aBc+2Abc+b^2B\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abB\right)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{7}cx^7(Ac+2bB)+\frac{1}{8}Bc^2x^8$$

input
```
Int[x^2*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^4)/4 + ((2*a*b*B + A*(b^2 + 2*a*c))*x^5
)/5 + ((b^2*B + 2*A*b*c + 2*a*B*c)*x^6)/6 + (c*(2*b*B + A*c)*x^7)/7 + (B*c
^2*x^8)/8
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

method	result
default	$\frac{B\,c^2 x^8}{8}+\frac{(A\,c^2+2Bbc)x^7}{7}+\frac{(2Abc+B(2ac+b^2))x^6}{6}+\frac{(2abB+A(2ac+b^2))x^5}{5}+\frac{(2abA+a^2B)x^4}{4}+\frac{a^2A\,x^3}{3}$
norman	$\frac{B\,c^2 x^8}{8}+\left(\frac{1}{7}A\,c^2+\frac{2}{7}Bbc\right)x^7+\left(\frac{1}{3}Abc+\frac{1}{3}aBc+\frac{1}{6}B\,b^2\right)x^6+\left(\frac{2}{5}Aac+\frac{1}{5}b^2A+\frac{2}{5}abB\right)x^5+($
orering	$\frac{x^3(105B\,c^2x^5+120x^4A\,c^2+240x^4Bbc+280x^3Abc+280Bac\,x^3+140x^3B\,b^2+336Aac\,x^2+168x^2b^2A+336Ba\,x^2b+420abAx+}{840}$
gosper	$\frac{1}{8}B\,c^2x^8+\frac{1}{7}x^7A\,c^2+\frac{2}{7}x^7Bbc+\frac{1}{3}Abc\,x^6+\frac{1}{3}x^6aBc+\frac{1}{6}B\,b^2x^6+\frac{2}{5}x^5Aac+\frac{1}{5}A\,b^2x^5+\frac{2}{5}Ba$
risch	$\frac{1}{8}B\,c^2x^8+\frac{1}{7}x^7A\,c^2+\frac{2}{7}x^7Bbc+\frac{1}{3}Abc\,x^6+\frac{1}{3}x^6aBc+\frac{1}{6}B\,b^2x^6+\frac{2}{5}x^5Aac+\frac{1}{5}A\,b^2x^5+\frac{2}{5}Ba$
parallelrisch	$\frac{1}{8}B\,c^2x^8+\frac{1}{7}x^7A\,c^2+\frac{2}{7}x^7Bbc+\frac{1}{3}Abc\,x^6+\frac{1}{3}x^6aBc+\frac{1}{6}B\,b^2x^6+\frac{2}{5}x^5Aac+\frac{1}{5}A\,b^2x^5+\frac{2}{5}Ba$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*B*c^2*x^8+1/7*(A*c^2+2*B*b*c)*x^7+1/6*(2*A*b*c+B*(2*a*c+b^2))*x^6+1/5*
(2*a*b*B+A*(2*a*c+b^2))*x^5+1/4*(2*A*a*b+B*a^2)*x^4+1/3*a^2*A*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{1}{8}Bc^2x^8 + \frac{1}{7}\left(2Bbc+Ac^2\right)x^7 + \frac{1}{6}\left(Bb^2+2(Ba+Ab)c\right)x^6 + \frac{1}{3}Aa^2x^3 + \frac{1}{5}\left(2Bab+Ab^2+2Aac\right)x^5 + \frac{1}{4}\left(Ba^2+2Aab\right)x^4$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*B*c^2*x^8 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/6*(B*b^2 + 2*(B*a + A*b)*c)*
x^6 + 1/3*A*a^2*x^3 + 1/5*(2*B*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/4*(B*a^2 + 2
*A*a*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{Aa^2x^3}{3} + \frac{Bc^2x^8}{8} + x^7\left(\frac{Ac^2}{7}+\frac{2Bbc}{7}\right) + x^6\left(\frac{Abc}{3}+\frac{Bac}{3}+\frac{Bb^2}{6}\right) + x^5 \cdot \left(\frac{2Aac}{5}+\frac{Ab^2}{5}+\frac{2Bab}{5}\right) + x^4\left(\frac{Aab}{2}+\frac{Ba^2}{4}\right)$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x+a)**2,x)
```

output
```
A*a**2*x**3/3 + B*c**2*x**8/8 + x**7*(A*c**2/7 + 2*B*b*c/7) + x**6*(A*b*c/
3 + B*a*c/3 + B*b**2/6) + x**5*(2*A*a*c/5 + A*b**2/5 + 2*B*a*b/5) + x**4*(
A*a*b/2 + B*a**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{1}{8}\,Bc^2x^8+\frac{1}{7}\left(2\,Bbc+Ac^2\right)x^7 + \frac{1}{6}\left(Bb^2+2\,(Ba+Ab)c\right)x^6+\frac{1}{3}\,Aa^2x^3 + \frac{1}{5}\left(2\,Bab+Ab^2+2\,Aac\right)x^5+\frac{1}{4}\left(Ba^2+2\,Aab\right)x^4$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/8*B*c^2*x^8 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/6*(B*b^2 + 2*(B*a + A*b)*c)*
x^6 + 1/3*A*a^2*x^3 + 1/5*(2*B*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/4*(B*a^2 + 2
*A*a*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{1}{8}\,Bc^2x^8+\frac{2}{7}\,Bbcx^7+\frac{1}{7}\,Ac^2x^7+\frac{1}{6}\,Bb^2x^6 + \frac{1}{3}\,Bacx^6+\frac{1}{3}\,Abcx^6+\frac{2}{5}\,Babx^5+\frac{1}{5}\,Ab^2x^5 + \frac{2}{5}\,Aacx^5+\frac{1}{4}\,Ba^2x^4+\frac{1}{2}\,Aabx^4+\frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*B*c^2*x^8 + 2/7*B*b*c*x^7 + 1/7*A*c^2*x^7 + 1/6*B*b^2*x^6 + 1/3*B*a*c*
x^6 + 1/3*A*b*c*x^6 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 2/5*A*a*c*x^5 + 1/4*
B*a^2*x^4 + 1/2*A*a*b*x^4 + 1/3*A*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^2\,dx = x^4\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^7\left(\frac{A\,c^2}{7}+\frac{2\,B\,b\,c}{7}\right) + x^5\left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}+\frac{2\,A\,a\,c}{5}\right) + x^6\left(\frac{B\,b^2}{6}+\frac{A\,c\,b}{3}+\frac{B\,a\,c}{3}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,c^2\,x^8}{8}$$

input
```
int(x^2*(A + B*x)*(a + b*x + c*x^2)^2,x)
```

output
```
x^4*((B*a^2)/4 + (A*a*b)/2) + x^7*((A*c^2)/7 + (2*B*b*c)/7) + x^5*((A*b^2)
/5 + (2*A*a*c)/5 + (2*B*a*b)/5) + x^6*((B*b^2)/6 + (A*b*c)/3 + (B*a*c)/3)
+ (A*a^2*x^3)/3 + (B*c^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{x^3(105b\,c^2x^5+120a\,c^2x^4+240b^2c\,x^4+560abc\,x^3+140b^3x^3+336a^2c\,x^2+504a\,b^2x^2+630a^2bx+280a}{840}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^2,x)
```

output
```
(x**3*(280*a**3 + 630*a**2*b*x + 336*a**2*c*x**2 + 504*a*b**2*x**2 + 560*a
*b*c*x**3 + 120*a*c**2*x**4 + 140*b**3*x**3 + 240*b**2*c*x**4 + 105*b*c**2
*x**5))/840
```

3.14 $\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{4}\left(2abB+A\left(b^2+2ac\right)\right)x^4+\frac{1}{5}\left(b^2B+2Abc+2aBc\right)x^5+\frac{1}{6}c(2bB+Ac)x^6+\frac{1}{7}Bc^2x^7$$

output
```
1/2*a^2*A*x^2+1/3*a*(2*A*b+B*a)*x^3+1/4*(2*a*b*B+A*(2*a*c+b^2))*x^4+1/5*(2
*A*b*c+2*B*a*c+B*b^2)*x^5+1/6*c*(A*c+2*B*b)*x^6+1/7*B*c^2*x^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{4}\left(Ab^2+2abB+2aAc\right)x^4+\frac{1}{5}\left(b^2B+2Abc+2aBc\right)x^5+\frac{1}{6}c(2bB+Ac)x^6+\frac{1}{7}Bc^2x^7$$

input
```
Integrate[x*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^3)/3 + ((A*b^2 + 2*a*b*B + 2*a*A*c)*x^4
)/4 + ((b^2*B + 2*A*b*c + 2*a*B*c)*x^5)/5 + (c*(2*b*B + A*c)*x^6)/6 + (B*c
^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^2Ax+x^4\left(2aBc+2Abc+b^2B\right)+x^3\left(A\left(2ac+b^2\right)+2abB\right)+ax^2(aB+2Ab)+cx^5(Ac+2bB)+Bc^2x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^2Ax^2+\frac{1}{5}x^5\left(2aBc+2Abc+b^2B\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abB\right)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{6}cx^6(Ac+2bB)+\frac{1}{7}Bc^2x^7$$

input
```
Int[x*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^3)/3 + ((2*a*b*B + A*(b^2 + 2*a*c))*x^4
)/4 + ((b^2*B + 2*A*b*c + 2*a*B*c)*x^5)/5 + (c*(2*b*B + A*c)*x^6)/6 + (B*c
^2*x^7)/7
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

method	result
default	$\frac{B\,c^2x^7}{7}+\frac{\left(A\,c^2+2Bbc\right)x^6}{6}+\frac{\left(2Abc+B\left(2ac+b^2\right)\right)x^5}{5}+\frac{\left(2abB+A\left(2ac+b^2\right)\right)x^4}{4}+\frac{\left(2abA+a^2B\right)x^3}{3}+\frac{a^2A\,x^2}{2}$
norman	$\frac{B\,c^2x^7}{7}+\left(\frac{1}{6}A\,c^2+\frac{1}{3}Bbc\right)x^6+\left(\frac{2}{5}Abc+\frac{2}{5}aBc+\frac{1}{5}B\,b^2\right)x^5+\left(\frac{1}{2}Aac+\frac{1}{4}b^2A+\frac{1}{2}abB\right)x^4+($
orering	$\frac{x^2\left(60B\,c^2x^5+70x^4A\,c^2+140x^4Bbc+168x^3Abc+168Bac\,x^3+84x^3B\,b^2+210Aac\,x^2+105x^2b^2A+210Ba\,x^2b+280abAx+140a\right.}{420}$
gosper	$\frac{1}{7}B\,c^2x^7+\frac{1}{6}x^6A\,c^2+\frac{1}{3}x^6Bbc+\frac{2}{5}x^5Abc+\frac{2}{5}x^5aBc+\frac{1}{5}B\,b^2x^5+\frac{1}{2}x^4Aac+\frac{1}{4}b^2A\,x^4+\frac{1}{2}x^4a$
risch	$\frac{1}{7}B\,c^2x^7+\frac{1}{6}x^6A\,c^2+\frac{1}{3}x^6Bbc+\frac{2}{5}x^5Abc+\frac{2}{5}x^5aBc+\frac{1}{5}B\,b^2x^5+\frac{1}{2}x^4Aac+\frac{1}{4}b^2A\,x^4+\frac{1}{2}x^4a$
parallelrisch	$\frac{1}{7}B\,c^2x^7+\frac{1}{6}x^6A\,c^2+\frac{1}{3}x^6Bbc+\frac{2}{5}x^5Abc+\frac{2}{5}x^5aBc+\frac{1}{5}B\,b^2x^5+\frac{1}{2}x^4Aac+\frac{1}{4}b^2A\,x^4+\frac{1}{2}x^4a$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*B*c^2*x^7+1/6*(A*c^2+2*B*b*c)*x^6+1/5*(2*A*b*c+B*(2*a*c+b^2))*x^5+1/4*
(2*a*b*B+A*(2*a*c+b^2))*x^4+1/3*(2*A*a*b+B*a^2)*x^3+1/2*a^2*A*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{1}{7}Bc^2x^7 + \frac{1}{6}\left(2Bbc+Ac^2\right)x^6 + \frac{1}{5}\left(Bb^2+2(Ba+Ab)c\right)x^5 + \frac{1}{2}Aa^2x^2 + \frac{1}{4}\left(2Bab+Ab^2+2Aac\right)x^4 + \frac{1}{3}\left(Ba^2+2Aab\right)x^3$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
1/7*B*c^2*x^7 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/5*(B*b^2 + 2*(B*a + A*b)*c)*
x^5 + 1/2*A*a^2*x^2 + 1/4*(2*B*a*b + A*b^2 + 2*A*a*c)*x^4 + 1/3*(B*a^2 + 2
*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{Aa^2x^2}{2} + \frac{Bc^2x^7}{7} + x^6\left(\frac{Ac^2}{6}+\frac{Bbc}{3}\right) + x^5\cdot\left(\frac{2Abc}{5}+\frac{2Bac}{5}+\frac{Bb^2}{5}\right) + x^4\left(\frac{Aac}{2}+\frac{Ab^2}{4}+\frac{Bab}{2}\right) + x^3\cdot\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x+a)**2,x)
```

output
```
A*a**2*x**2/2 + B*c**2*x**7/7 + x**6*(A*c**2/6 + B*b*c/3) + x**5*(2*A*b*c/
5 + 2*B*a*c/5 + B*b**2/5) + x**4*(A*a*c/2 + A*b**2/4 + B*a*b/2) + x**3*(2*
A*a*b/3 + B*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{1}{7}\,Bc^2x^7+\frac{1}{6}\left(2\,Bbc+Ac^2\right)x^6+\frac{1}{5}\left(Bb^2+2\,(Ba+Ab)c\right)x^5+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\left(2\,Bab+Ab^2+2\,Aac\right)x^4+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/7*B*c^2*x^7 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/5*(B*b^2 + 2*(B*a + A*b)*c)*
x^5 + 1/2*A*a^2*x^2 + 1/4*(2*B*a*b + A*b^2 + 2*A*a*c)*x^4 + 1/3*(B*a^2 + 2
*A*a*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{1}{7}\,Bc^2x^7+\frac{1}{3}\,Bbcx^6+\frac{1}{6}\,Ac^2x^6+\frac{1}{5}\,Bb^2x^5+\frac{2}{5}\,Bacx^5+\frac{2}{5}\,Abcx^5+\frac{1}{2}\,Babx^4+\frac{1}{4}\,Ab^2x^4+\frac{1}{2}\,Aacx^4+\frac{1}{3}\,Ba^2x^3+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/7*B*c^2*x^7 + 1/3*B*b*c*x^6 + 1/6*A*c^2*x^6 + 1/5*B*b^2*x^5 + 2/5*B*a*c*
x^5 + 2/5*A*b*c*x^5 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*A*a*c*x^4 + 1/3*
B*a^2*x^3 + 2/3*A*a*b*x^3 + 1/2*A*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx = x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^6\left(\frac{A\,c^2}{6}+\frac{B\,b\,c}{3}\right)+x^4\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}+\frac{A\,a\,c}{2}\right)+x^5\left(\frac{B\,b^2}{5}+\frac{2\,A\,c\,b}{5}+\frac{2\,B\,a\,c}{5}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,c^2\,x^7}{7}$$

input
```
int(x*(A + B*x)*(a + b*x + c*x^2)^2,x)
```

output
```
x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^6*((A*c^2)/6 + (B*b*c)/3) + x^4*((A*b^2)
/4 + (A*a*c)/2 + (B*a*b)/2) + x^5*((B*b^2)/5 + (2*A*b*c)/5 + (2*B*a*c)/5)
+ (A*a^2*x^2)/2 + (B*c^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int x(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{x^2(60b\,c^2x^5+70a\,c^2x^4+140b^2c\,x^4+336abc\,x^3+84b^3x^3+210a^2c\,x^2+315a\,b^2x^2+420a^2bx+210a^3)}{420}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^2,x)
```

output
```
(x**2*(210*a**3 + 420*a**2*b*x + 210*a**2*c*x**2 + 315*a*b**2*x**2 + 336*a
*b*c*x**3 + 70*a*c**2*x**4 + 84*b**3*x**3 + 140*b**2*c*x**4 + 60*b*c**2*x*
*5))/420
```

3.15 $\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 96

$$\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx = a^2Ax+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{3}\left(2abB+A\left(b^2+2ac\right)\right)x^3 +\frac{1}{4}\left(b^2B+2Abc+2aBc\right)x^4 +\frac{1}{5}c(2bB+Ac)x^5+\frac{1}{6}Bc^2x^6$$

output
```
a^2*A*x+1/2*a*(2*A*b+B*a)*x^2+1/3*(2*a*b*B+A*(2*a*c+b^2))*x^3+1/4*(2*A*b*c
+2*B*a*c+B*b^2)*x^4+1/5*c*(A*c+2*B*b)*x^5+1/6*B*c^2*x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx = a^2Ax+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{3}\left(Ab^2+2abB+2aAc\right)x^3 +\frac{1}{4}\left(b^2B+2Abc+2aBc\right)x^4 +\frac{1}{5}c(2bB+Ac)x^5+\frac{1}{6}Bc^2x^6$$

input
```
Integrate[(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^2)/2 + ((A*b^2 + 2*a*b*B + 2*a*A*c)*x^3)/3 +
((b^2*B + 2*A*b*c + 2*a*B*c)*x^4)/4 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^6
)/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^2A+x^3\left(2aBc+2Abc+b^2B\right)+x^2\left(A\left(2ac+b^2\right)+2abB\right)+ax(aB+2Ab)+cx^4(Ac+2bB)+Bc^2x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{4}x^4\left(2aBc+2Abc+b^2B\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abB\right)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{5}cx^5(Ac+2bB)+\frac{1}{6}Bc^2x^6$$

input
```
Int[(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^2)/2 + ((2*a*b*B + A*(b^2 + 2*a*c))*x^3)/3 +
((b^2*B + 2*A*b*c + 2*a*B*c)*x^4)/4 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^6
)/6
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

method	result
norman	$\frac{B\,c^2x^6}{6}+\left(\frac{1}{5}A\,c^2+\frac{2}{5}Bbc\right)x^5+\left(\frac{1}{2}Abc+\frac{1}{2}aBc+\frac{1}{4}B\,b^2\right)x^4+\left(\frac{2}{3}Aac+\frac{1}{3}b^2A+\frac{2}{3}abB\right)x^3+($
default	$\frac{B\,c^2x^6}{6}+\frac{(A\,c^2+2Bbc)x^5}{5}+\frac{(2Abc+B(2ac+b^2))x^4}{4}+\frac{(2abB+A(2ac+b^2))x^3}{3}+\frac{(2abA+a^2B)x^2}{2}+a^2Ax$
gosper	$\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{2}{5}x^5Bbc+\frac{1}{2}x^4Abc+\frac{1}{2}Bac\,x^4+\frac{1}{4}x^4B\,b^2+\frac{2}{3}aAc\,x^3+\frac{1}{3}A\,x^3b^2+\frac{2}{3}ab$
risch	$\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{2}{5}x^5Bbc+\frac{1}{2}x^4Abc+\frac{1}{2}Bac\,x^4+\frac{1}{4}x^4B\,b^2+\frac{2}{3}aAc\,x^3+\frac{1}{3}A\,x^3b^2+\frac{2}{3}ab$
parallelrisch	$\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{2}{5}x^5Bbc+\frac{1}{2}x^4Abc+\frac{1}{2}Bac\,x^4+\frac{1}{4}x^4B\,b^2+\frac{2}{3}aAc\,x^3+\frac{1}{3}A\,x^3b^2+\frac{2}{3}ab$
orering	$\frac{x(10B\,c^2x^5+12x^4A\,c^2+24x^4Bbc+30x^3Abc+30Bac\,x^3+15x^3B\,b^2+40Aac\,x^2+20x^2b^2A+40Ba\,x^2b+60abAx+30a^2Bx+60a}{60}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*B*c^2*x^6+(1/5*A*c^2+2/5*B*b*c)*x^5+(1/2*A*b*c+1/2*a*B*c+1/4*B*b^2)*x^
4+(2/3*A*a*c+1/3*b^2*A+2/3*a*b*B)*x^3+(a*b*A+1/2*a^2*B)*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{1}{6}\,Bc^2x^6+\frac{1}{5}\,\left(2\,Bbc+Ac^2\right)x^5+\frac{1}{4}\,\left(Bb^2+2\,(Ba+Ab)c\right)x^4+Aa^2x+\frac{1}{3}\,\left(2\,Bab+Ab^2+2\,Aac\right)x^3+\frac{1}{2}\,\left(Ba^2+2\,Aab\right)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
1/6*B*c^2*x^6 + 1/5*(2*B*b*c + A*c^2)*x^5 + 1/4*(B*b^2 + 2*(B*a + A*b)*c)*
x^4 + A*a^2*x + 1/3*(2*B*a*b + A*b^2 + 2*A*a*c)*x^3 + 1/2*(B*a^2 + 2*A*a*b
)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx = Aa^2x + \frac{Bc^2x^6}{6} + x^5\left(\frac{Ac^2}{5} + \frac{2Bbc}{5}\right) + x^4\left(\frac{Abc}{2} + \frac{Bac}{2} + \frac{Bb^2}{4}\right) + x^3 \cdot \left(\frac{2Aac}{3} + \frac{Ab^2}{3} + \frac{2Bab}{3}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2,x)
```

output
```
A*a**2*x + B*c**2*x**6/6 + x**5*(A*c**2/5 + 2*B*b*c/5) + x**4*(A*b*c/2 + B
*a*c/2 + B*b**2/4) + x**3*(2*A*a*c/3 + A*b**2/3 + 2*B*a*b/3) + x**2*(A*a*b
 + B*a**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{1}{6}\,Bc^2x^6 + \frac{1}{5}\,\left(2\,Bbc + Ac^2\right)x^5 + \frac{1}{4}\,\left(Bb^2 + 2\,(Ba + Ab)c\right)x^4 + Aa^2x + \frac{1}{3}\,\left(2\,Bab + Ab^2 + 2\,Aac\right)x^3 + \frac{1}{2}\,\left(Ba^2 + 2\,Aab\right)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6*B*c^2*x^6 + 1/5*(2*B*b*c + A*c^2)*x^5 + 1/4*(B*b^2 + 2*(B*a + A*b)*c)*
x^4 + A*a^2*x + 1/3*(2*B*a*b + A*b^2 + 2*A*a*c)*x^3 + 1/2*(B*a^2 + 2*A*a*b
)*x^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{1}{6}\,Bc^2x^6+\frac{2}{5}\,Bbcx^5+\frac{1}{5}\,Ac^2x^5+\frac{1}{4}\,Bb^2x^4+\frac{1}{2}\,Bacx^4+\frac{1}{2}\,Abcx^4+\frac{2}{3}\,Babx^3+\frac{1}{3}\,Ab^2x^3+\frac{2}{3}\,Aacx^3+\frac{1}{2}\,Ba^2x^2+Aabx^2+Aa^2x$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*B*c^2*x^6 + 2/5*B*b*c*x^5 + 1/5*A*c^2*x^5 + 1/4*B*b^2*x^4 + 1/2*B*a*c*
x^4 + 1/2*A*b*c*x^4 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + 2/3*A*a*c*x^3 + 1/2*
B*a^2*x^2 + A*a*b*x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

$$\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx = x^2\left(\frac{B\,a^2}{2}+A\,b\,a\right)+x^5\left(\frac{A\,c^2}{5}+\frac{2\,B\,b\,c}{5}\right)+x^3\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}+\frac{2\,A\,a\,c}{3}\right)+x^4\left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}+\frac{B\,a\,c}{2}\right)+\frac{B\,c^2\,x^6}{6}+A\,a^2\,x$$

input
```
int((A + B*x)*(a + b*x + c*x^2)^2,x)
```

output
```
x^2*((B*a^2)/2 + A*a*b) + x^5*((A*c^2)/5 + (2*B*b*c)/5) + x^3*((A*b^2)/3 +
 (2*A*a*c)/3 + (2*B*a*b)/3) + x^4*((B*b^2)/4 + (A*b*c)/2 + (B*a*c)/2) + (B
*c^2*x^6)/6 + A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int (A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{x(10b\,c^2x^5+12a\,c^2x^4+24b^2c\,x^4+60abc\,x^3+15b^3x^3+40a^2c\,x^2+60a\,b^2x^2+90a^2bx+60a^3)}{60}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2,x)
```

output
```
(x*(60*a**3 + 90*a**2*b*x + 40*a**2*c*x**2 + 60*a*b**2*x**2 + 60*a*b*c*x**
3 + 12*a*c**2*x**4 + 15*b**3*x**3 + 24*b**2*c*x**4 + 10*b*c**2*x**5))/60
```

3.16 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x} \, dx = a(2Ab+aB)x+\frac{1}{2}\left(2abB+A\left(b^2+2ac\right)\right)x^2+\frac{1}{3}\left(b^2B+2Abc+2aBc\right)x^3+\frac{1}{4}c(2bB+Ac)x^4+\frac{1}{5}Bc^2x^5+a^2A\log(x)$$

output
```
a*(2*A*b+B*a)*x+1/2*(2*a*b*B+A*(2*a*c+b^2))*x^2+1/3*(2*A*b*c+2*B*a*c+B*b^2
)*x^3+1/4*c*(A*c+2*B*b)*x^4+1/5*B*c^2*x^5+a^2*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x} \, dx = a(2Ab+aB)x+\frac{1}{2}\left(Ab^2+2abB+2aAc\right)x^2+\frac{1}{3}\left(b^2B+2Abc+2aBc\right)x^3+\frac{1}{4}c(2bB+Ac)x^4+\frac{1}{5}Bc^2x^5+a^2A\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x,x]
```

output
```
a*(2*A*b + a*B)*x + ((A*b^2 + 2*a*b*B + 2*a*A*c)*x^2)/2 + ((b^2*B + 2*A*b*
c + 2*a*B*c)*x^3)/3 + (c*(2*b*B + A*c)*x^4)/4 + (B*c^2*x^5)/5 + a^2*A*Log[
x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^2A}{x}+x^2\left(2aBc+2Abc+b^2B\right)+x\left(A\left(2ac+b^2\right)+2abB\right)+a(aB+2Ab)+cx^3(Ac+2bB)+Bc^2x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2A\log(x)+\frac{1}{3}x^3\left(2aBc+2Abc+b^2B\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abB\right)+ax(aB+2Ab)+\frac{1}{4}cx^4(Ac+2bB)+\frac{1}{5}Bc^2x^5$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x,x]
```

output
```
a*(2*A*b + a*B)*x + ((2*a*b*B + A*(b^2 + 2*a*c))*x^2)/2 + ((b^2*B + 2*A*b*
c + 2*a*B*c)*x^3)/3 + (c*(2*b*B + A*c)*x^4)/4 + (B*c^2*x^5)/5 + a^2*A*Log[
x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result
norman	$\left(\frac{1}{4}A\,c^{2}+\frac{1}{2}Bbc\right)x^{4}+\left(Aac+\frac{1}{2}b^{2}A+abB\right)x^{2}+\left(\frac{2}{3}Abc+\frac{2}{3}aBc+\frac{1}{3}B\,b^{2}\right)x^{3}+\left(2abA+a^{2}\right.$
default	$\frac{B\,c^{2}x^{5}}{5}+\frac{x^{4}A\,c^{2}}{4}+\frac{x^{4}Bbc}{2}+\frac{2x^{3}Abc}{3}+\frac{2Bac\,x^{3}}{3}+\frac{x^{3}B\,b^{2}}{3}+Aac\,x^{2}+\frac{x^{2}b^{2}A}{2}+Ba\,x^{2}b+2abAx+a^{2}$
risch	$\frac{B\,c^{2}x^{5}}{5}+\frac{x^{4}A\,c^{2}}{4}+\frac{x^{4}Bbc}{2}+\frac{2x^{3}Abc}{3}+\frac{2Bac\,x^{3}}{3}+\frac{x^{3}B\,b^{2}}{3}+Aac\,x^{2}+\frac{x^{2}b^{2}A}{2}+Ba\,x^{2}b+2abAx+a^{2}$
parallelrisch	$\frac{B\,c^{2}x^{5}}{5}+\frac{x^{4}A\,c^{2}}{4}+\frac{x^{4}Bbc}{2}+\frac{2x^{3}Abc}{3}+\frac{2Bac\,x^{3}}{3}+\frac{x^{3}B\,b^{2}}{3}+Aac\,x^{2}+\frac{x^{2}b^{2}A}{2}+Ba\,x^{2}b+2abAx+a^{2}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(1/4*A*c^2+1/2*B*b*c)*x^4+(A*a*c+1/2*b^2*A+a*b*B)*x^2+(2/3*A*b*c+2/3*a*B*c
+1/3*B*b^2)*x^3+(2*A*a*b+B*a^2)*x+1/5*B*c^2*x^5+a^2*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^{2}\right)^{2}}{x}\,dx=\frac{1}{5}\,Bc^{2}x^{5}+\frac{1}{4}\,(2\,Bbc+Ac^{2})x^{4}+\frac{1}{3}\,(Bb^{2}+2\,(Ba+Ab)c)x^{3}+Aa^{2}\log\left(x\right)+\frac{1}{2}\,(2\,Bab+Ab^{2}+2\,Aac)x^{2}+(Ba^{2}+2\,Aab)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x,x, algorithm="fricas")
```

output
```
1/5*B*c^2*x^5 + 1/4*(2*B*b*c + A*c^2)*x^4 + 1/3*(B*b^2 + 2*(B*a + A*b)*c)*
x^3 + A*a^2*log(x) + 1/2*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + (B*a^2 + 2*A*a*
b)*x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x}\,dx = Aa^2\log(x) + \frac{Bc^2x^5}{5} + x^4\left(\frac{Ac^2}{4} + \frac{Bbc}{2}\right) + x^3\cdot\left(\frac{2Abc}{3} + \frac{2Bac}{3} + \frac{Bb^2}{3}\right) + x^2\left(Aac + \frac{Ab^2}{2} + Bab\right) + x\left(2Aab + Ba^2\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x,x)
```

output
```
A*a**2*log(x) + B*c**2*x**5/5 + x**4*(A*c**2/4 + B*b*c/2) + x**3*(2*A*b*c/
3 + 2*B*a*c/3 + B*b**2/3) + x**2*(A*a*c + A*b**2/2 + B*a*b) + x*(2*A*a*b +
 B*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x}\,dx = \frac{1}{5}\,Bc^2x^5 + \frac{1}{4}\left(2\,Bbc + Ac^2\right)x^4 + \frac{1}{3}\left(Bb^2 + 2\left(Ba + Ab\right)c\right)x^3 + Aa^2\log(x) + \frac{1}{2}\left(2\,Bab + Ab^2 + 2\,Aac\right)x^2 + \left(Ba^2 + 2\,Aab\right)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x,x, algorithm="maxima")
```

output
```
1/5*B*c^2*x^5 + 1/4*(2*B*b*c + A*c^2)*x^4 + 1/3*(B*b^2 + 2*(B*a + A*b)*c)*
x^3 + A*a^2*log(x) + 1/2*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + (B*a^2 + 2*A*a*
b)*x
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x}\,dx = \frac{1}{5}\,Bc^2x^5+\frac{1}{2}\,Bbcx^4+\frac{1}{4}\,Ac^2x^4+\frac{1}{3}\,Bb^2x^3+\frac{2}{3}\,Bacx^3+\frac{2}{3}\,Abcx^3+Babx^2+\frac{1}{2}\,Ab^2x^2+Aacx^2+Ba^2x+2\,Aabx+Aa^2\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x,x, algorithm="giac")
```

output
```
1/5*B*c^2*x^5 + 1/2*B*b*c*x^4 + 1/4*A*c^2*x^4 + 1/3*B*b^2*x^3 + 2/3*B*a*c*
x^3 + 2/3*A*b*c*x^3 + B*a*b*x^2 + 1/2*A*b^2*x^2 + A*a*c*x^2 + B*a^2*x + 2*
A*a*b*x + A*a^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x}\,dx = x^4\left(\frac{A\,c^2}{4}+\frac{B\,b\,c}{2}\right)+x^2\left(\frac{A\,b^2}{2}+B\,a\,b+A\,a\,c\right)+x^3\left(\frac{B\,b^2}{3}+\frac{2\,A\,c\,b}{3}+\frac{2\,B\,a\,c}{3}\right)+x\left(B\,a^2+2\,A\,b\,a\right)+\frac{B\,c^2\,x^5}{5}+A\,a^2\,\ln\left(x\right)$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x,x)
```

output
```
x^4*((A*c^2)/4 + (B*b*c)/2) + x^2*((A*b^2)/2 + A*a*c + B*a*b) + x^3*((B*b^
2)/3 + (2*A*b*c)/3 + (2*B*a*c)/3) + x*(B*a^2 + 2*A*a*b) + (B*c^2*x^5)/5 +
A*a^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x}\,dx = \log(x)\,a^3+3a^2bx+a^2c\,x^2+\frac{3a\,b^2x^2}{2}+\frac{4abc\,x^3}{3}+\frac{a\,c^2x^4}{4}+\frac{b^3x^3}{3}+\frac{b^2c\,x^4}{2}+\frac{b\,c^2x^5}{5}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x,x)
```

output
```
(60*log(x)*a**3 + 180*a**2*b*x + 60*a**2*c*x**2 + 90*a*b**2*x**2 + 80*a*b*
c*x**3 + 15*a*c**2*x**4 + 20*b**3*x**3 + 30*b**2*c*x**4 + 12*b*c**2*x**5)/
60
```

3.17 $\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^2} dx = -\frac{a^2A}{x} + \left(2abB + A\left(b^2+2ac\right)\right)x + \frac{1}{2}\left(b^2B + 2Abc + 2aBc\right)x^2 + \frac{1}{3}c(2bB + Ac)x^3 + \frac{1}{4}Bc^2x^4 + a(2Ab + aB)\log(x)$$

output
```
-a^2*A/x+(2*a*b*B+A*(2*a*c+b^2))*x+1/2*(2*A*b*c+2*B*a*c+B*b^2)*x^2+1/3*c*(
A*c+2*B*b)*x^3+1/4*B*c^2*x^4+a*(2*A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^2} dx = -\frac{a^2A}{x} + ax(2bB + 2Ac + Bcx) + \frac{1}{12}x\left(4A\left(3b^2 + 3bcx + c^2x^2\right) + Bx\left(6b^2 + 8bcx + 3c^2x^2\right)\right) + a(2Ab + aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^2,x]
```

output
```
-((a^2*A)/x) + a*x*(2*b*B + 2*A*c + B*c*x) + (x*(4*A*(3*b^2 + 3*b*c*x + c^
2*x^2) + B*x*(6*b^2 + 8*b*c*x + 3*c^2*x^2)))/12 + a*(2*A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^2}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{a^2A}{x^2}+x\left(2aBc+2Abc+b^2B\right)+Ab^2\left(\frac{2a(Ac+bB)}{Ab^2}+1\right)+\frac{a(aB+2Ab)}{x}+cx^2(Ac+2bB)+Bc^2x^3\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2A}{x}+\frac{1}{2}x^2\left(2aBc+2Abc+b^2B\right)+x\left(A\left(2ac+b^2\right)+2abB\right)+a\log(x)(aB+2Ab)+\frac{1}{3}cx^3(Ac+2bB)+\frac{1}{4}Bc^2x^4$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^2,x]
```

output
```
-((a^2*A)/x) + (2*a*b*B + A*(b^2 + 2*a*c))*x + ((b^2*B + 2*A*b*c + 2*a*B*c
)*x^2)/2 + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^4)/4 + a*(2*A*b + a*B)*Log[x
]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result
default	$\frac{B c^2 x^4}{4}+\frac{A c^2 x^3}{3}+\frac{2 B b c x^3}{3}+A b c x^2+B a c x^2+\frac{x^2 B b^2}{2}+2 A a c x+x b^2 A+2 x a b B+a(2 A b+$
norman	$\frac{\left(\frac{1}{3} A c^2+\frac{2}{3} B b c\right) x^4+\left(A b c+a B c+\frac{1}{2} B b^2\right) x^3+\left(2 A a c+b^2 A+2 a b B\right) x^2-a^2 A+\frac{B c^2 x^5}{4}}{x}+\left(2 a b A+a^2 B\right) \ln (x)$
risch	$\frac{B c^2 x^4}{4}+\frac{A c^2 x^3}{3}+\frac{2 B b c x^3}{3}+A b c x^2+B a c x^2+\frac{x^2 B b^2}{2}+2 A a c x+x b^2 A+2 x a b B-\frac{a^2 A}{x}+2 A$
parallelrisch	$\frac{3 B c^2 x^5+4 x^4 A c^2+8 x^4 B b c+12 x^3 A b c+12 B a c x^3+6 x^3 B b^2+24 A \ln (x) x a b+24 A a c x^2+12 x^2 b^2 A+12 B \ln (x) x a^2+24 B a x^2 b-}{12 x}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*c^2*x^4+1/3*A*c^2*x^3+2/3*B*b*c*x^3+A*b*c*x^2+B*a*c*x^2+1/2*x^2*B*b^
2+2*A*a*c*x+x*b^2*A+2*x*a*b*B+a*(2*A*b+B*a)*ln(x)-a^2*A/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^2}\,dx$$
$$=\frac{3\,Bc^2x^5+4\,(2\,Bbc+Ac^2)x^4+6\,(Bb^2+2\,(Ba+Ab)c)x^3-12\,Aa^2+12\,(2\,Bab+Ab^2+2\,Aac)x^2+12\,(}{12\,x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/12*(3*B*c^2*x^5 + 4*(2*B*b*c + A*c^2)*x^4 + 6*(B*b^2 + 2*(B*a + A*b)*c)*
x^3 - 12*A*a^2 + 12*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 12*(B*a^2 + 2*A*a*b)
*x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^2}\,dx = -\frac{Aa^2}{x} + \frac{Bc^2x^4}{4} + a(2Ab+Ba)\log(x) + x^3\left(\frac{Ac^2}{3} + \frac{2Bbc}{3}\right) + x^2\left(Abc + Bac + \frac{Bb^2}{2}\right) + x\left(2Aac + Ab^2 + 2Bab\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**2,x)
```

output
```
-A*a**2/x + B*c**2*x**4/4 + a*(2*A*b + B*a)*log(x) + x**3*(A*c**2/3 + 2*B*
b*c/3) + x**2*(A*b*c + B*a*c + B*b**2/2) + x*(2*A*a*c + A*b**2 + 2*B*a*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^2}\,dx = \frac{1}{4}Bc^2x^4 + \frac{1}{3}\left(2Bbc + Ac^2\right)x^3 + \frac{1}{2}\left(Bb^2 + 2(Ba+Ab)c\right)x^2 - \frac{Aa^2}{x} + \left(2Bab + Ab^2 + 2Aac\right)x + \left(Ba^2 + 2Aab\right)\log(x)$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/4*B*c^2*x^4 + 1/3*(2*B*b*c + A*c^2)*x^3 + 1/2*(B*b^2 + 2*(B*a + A*b)*c)*
x^2 - A*a^2/x + (2*B*a*b + A*b^2 + 2*A*a*c)*x + (B*a^2 + 2*A*a*b)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^2}\,dx = \frac{1}{4}\,Bc^2x^4 + \frac{2}{3}\,Bbcx^3 + \frac{1}{3}\,Ac^2x^3 + \frac{1}{2}\,Bb^2x^2 + Bacx^2 + Abcx^2 + 2\,Babx + Ab^2x + 2\,Aacx - \frac{Aa^2}{x} + \left(Ba^2 + 2\,Aab\right)\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
1/4*B*c^2*x^4 + 2/3*B*b*c*x^3 + 1/3*A*c^2*x^3 + 1/2*B*b^2*x^2 + B*a*c*x^2
+ A*b*c*x^2 + 2*B*a*b*x + A*b^2*x + 2*A*a*c*x - A*a^2/x + (B*a^2 + 2*A*a*b
)*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 10.62 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^2}\,dx = x^3\left(\frac{A\,c^2}{3} + \frac{2\,B\,b\,c}{3}\right) + x\left(A\,b^2 + 2\,B\,a\,b + 2\,A\,a\,c\right) + \ln\left(x\right)\,\left(B\,a^2 + 2\,A\,b\,a\right) + x^2\left(\frac{B\,b^2}{2} + A\,c\,b + B\,a\,c\right) - \frac{A\,a^2}{x} + \frac{B\,c^2\,x^4}{4}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^2,x)
```

output
```
x^3*((A*c^2)/3 + (2*B*b*c)/3) + x*(A*b^2 + 2*A*a*c + 2*B*a*b) + log(x)*(B*
a^2 + 2*A*a*b) + x^2*((B*b^2)/2 + A*b*c + B*a*c) - (A*a^2)/x + (B*c^2*x^4)
/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^2}\,dx$$
$$=\frac{36\log(x)\,a^2bx-12a^3+24a^2c\,x^2+36a\,b^2x^2+24abc\,x^3+4a\,c^2x^4+6b^3x^3+8b^2c\,x^4+3b\,c^2x^5}{12x}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^2,x)
```

output
```
(36*log(x)*a**2*b*x - 12*a**3 + 24*a**2*c*x**2 + 36*a*b**2*x**2 + 24*a*b*c
*x**3 + 4*a*c**2*x**4 + 6*b**3*x**3 + 8*b**2*c*x**4 + 3*b*c**2*x**5)/(12*x
)
```

3.18 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^3}\,dx = -\frac{a^2A}{2x^2}-\frac{a(2Ab+aB)}{x}+\left(b^2B+2Abc+2aBc\right)x+\frac{1}{2}c(2bB+Ac)x^2+\frac{1}{3}Bc^2x^3+\left(2abB+A\left(b^2+2ac\right)\right)\log(x)$$

output
```
-1/2*a^2*A/x^2-a*(2*A*b+B*a)/x+(2*A*b*c+2*B*a*c+B*b^2)*x+1/2*c*(A*c+2*B*b)
*x^2+1/3*B*c^2*x^3+(2*a*b*B+A*(2*a*c+b^2))*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^3}\,dx = b^2Bx+bcx(2A+Bx)-\frac{a^2(A+2Bx)}{2x^2}+\frac{1}{6}c^2x^2(3A+2Bx)+a\left(-\frac{2Ab}{x}+2Bcx\right)+2abB\log(x)+A\left(b^2+2ac\right)\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^3,x]
```

output
```
b^2*B*x + b*c*x*(2*A + B*x) - (a^2*(A + 2*B*x))/(2*x^2) + (c^2*x^2*(3*A +
2*B*x))/6 + a*((-2*A*b)/x + 2*B*c*x) + 2*a*b*B*Log[x] + A*(b^2 + 2*a*c)*Lo
g[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^3}dx$$

$$\Big\downarrow \text{ 1195}$$

$$\int \left(\frac{a^2A}{x^3}+\frac{A\left(2ac+b^2\right)+2abB}{x}+b^2B\left(\frac{2c(aB+Ab)}{b^2B}+1\right)+\frac{a(aB+2Ab)}{x^2}+cx(Ac+2bB)+Bc^2x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2A}{2x^2}+x\left(2aBc+2Abc+b^2B\right)+\log(x)\left(A\left(2ac+b^2\right)+2abB\right)-\frac{a(aB+2Ab)}{x}+\frac{1}{2}cx^2(Ac+2bB)+\frac{1}{3}Bc^2x^3$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 - (a*(2*A*b + a*B))/x + (b^2*B + 2*A*b*c + 2*a*B*c)*x + (
c*(2*b*B + A*c)*x^2)/2 + (B*c^2*x^3)/3 + (2*a*b*B + A*(b^2 + 2*a*c))*Log[x
]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result
default	$\frac{B\,c^2x^3}{3}+\frac{A\,c^2x^2}{2}+x^2Bbc+2Abcx+2Bacx+xB\,b^2-\frac{a^2A}{2x^2}+\left(2Aac+b^2A+2abB\right)\ln\left(x\right)-$
norman	$\frac{\left(\frac{1}{2}A\,c^2+Bbc\right)x^4+\left(-2abA-a^2B\right)x+\left(2Abc+2aBc+B\,b^2\right)x^3-\frac{a^2A}{2}+\frac{B\,c^2x^5}{3}}{x^2}+\left(2Aac+b^2A+2abB\right)\ln\left(x\right)$
risch	$\frac{B\,c^2x^3}{3}+\frac{A\,c^2x^2}{2}+x^2Bbc+2Abcx+2Bacx+xB\,b^2+\frac{\left(-2abA-a^2B\right)x-\frac{a^2A}{2}}{x^2}+2A\ln\left(x\right)ac+A$
parallelrisch	$\frac{2B\,c^2x^5+3x^4A\,c^2+6x^4Bbc+12A\ln(x)x^2ac+6A\ln(x)x^2b^2+12x^3Abc+12B\ln(x)x^2ab+12Bac\,x^3+6x^3B\,b^2-12abAx-6a^2B}{6x^2}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*c^2*x^3+1/2*A*c^2*x^2+x^2*B*b*c+2*A*b*c*x+2*B*a*c*x+x*B*b^2-1/2*a^2*
A/x^2+(2*A*a*c+A*b^2+2*B*a*b)*ln(x)-a*(2*A*b+B*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^3}\,dx$$
$$=\frac{2\,Bc^2x^5+3\,(2\,Bbc+Ac^2)x^4+6\,(Bb^2+2\,(Ba+Ab)c)x^3+6\,(2\,Bab+Ab^2+2\,Aac)x^2\log\left(x\right)-3\,Aa^2-}{6\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/6*(2*B*c^2*x^5 + 3*(2*B*b*c + A*c^2)*x^4 + 6*(B*b^2 + 2*(B*a + A*b)*c)*x
^3 + 6*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2*log(x) - 3*A*a^2 - 6*(B*a^2 + 2*A*a
*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^2}{x^3}\,dx = \frac{Bc^2x^3}{3} + x^2\left(\frac{Ac^2}{2} + Bbc\right) + x\left(2Abc + 2Bac + Bb^2\right) + \left(2Aac + Ab^2 + 2Bab\right)\log\left(x\right) + \frac{-Aa^2 + x(-4Aab - 2Ba^2)}{2x^2}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**3,x)
```

output
```
B*c**2*x**3/3 + x**2*(A*c**2/2 + B*b*c) + x*(2*A*b*c + 2*B*a*c + B*b**2) +
 (2*A*a*c + A*b**2 + 2*B*a*b)*log(x) + (-A*a**2 + x*(-4*A*a*b - 2*B*a**2))
/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^2}{x^3}\,dx = \frac{1}{3}\,Bc^2x^3 + \frac{1}{2}\,(2\,Bbc + Ac^2)x^2 + \left(Bb^2 + 2\,(Ba + Ab)c\right)x + \left(2\,Bab + Ab^2 + 2\,Aac\right)\log\left(x\right) - \frac{Aa^2 + 2\,(Ba^2 + 2\,Aab)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/3*B*c^2*x^3 + 1/2*(2*B*b*c + A*c^2)*x^2 + (B*b^2 + 2*(B*a + A*b)*c)*x +
(2*B*a*b + A*b^2 + 2*A*a*c)*log(x) - 1/2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)*x)/x
^2
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^3}\,dx = \frac{1}{3}\,Bc^2x^3 + Bbcx^2 + \frac{1}{2}\,Ac^2x^2 + Bb^2x + 2\,Bacx + 2\,Abcx + \left(2\,Bab + Ab^2 + 2\,Aac\right)\log\left(|x|\right) - \frac{Aa^2 + 2\,(Ba^2 + 2\,Aab)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^3,x, algorithm="giac")
```

output
```
1/3*B*c^2*x^3 + B*b*c*x^2 + 1/2*A*c^2*x^2 + B*b^2*x + 2*B*a*c*x + 2*A*b*c*
x + (2*B*a*b + A*b^2 + 2*A*a*c)*log(abs(x)) - 1/2*(A*a^2 + 2*(B*a^2 + 2*A*
a*b)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^3}\,dx = x^2\left(\frac{A\,c^2}{2} + B\,b\,c\right) + x\left(B\,b^2 + 2\,A\,c\,b + 2\,B\,a\,c\right) + \ln\left(x\right)\,\left(A\,b^2 + 2\,B\,a\,b + 2\,A\,a\,c\right) - \frac{\frac{A\,a^2}{2} + x\left(B\,a^2 + 2\,A\,b\,a\right)}{x^2} + \frac{B\,c^2\,x^3}{3}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^3,x)
```

output
```
x^2*((A*c^2)/2 + B*b*c) + x*(B*b^2 + 2*A*b*c + 2*B*a*c) + log(x)*(A*b^2 +
2*A*a*c + 2*B*a*b) - ((A*a^2)/2 + x*(B*a^2 + 2*A*a*b))/x^2 + (B*c^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^3}\,dx$$
$$=\frac{12\log(x)\,a^2c\,x^2+18\log(x)\,a\,b^2x^2-3a^3-18a^2bx+24abc\,x^3+3a\,c^2x^4+6b^3x^3+6b^2c\,x^4+2b\,c^2x^5}{6x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^3,x)
```

output
```
(12*log(x)*a**2*c*x**2 + 18*log(x)*a*b**2*x**2 - 3*a**3 - 18*a**2*b*x + 24
*a*b*c*x**3 + 3*a*c**2*x**4 + 6*b**3*x**3 + 6*b**2*c*x**4 + 2*b*c**2*x**5)
/(6*x**2)
```

3.19 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a(2Ab+aB)}{2x^2}-\frac{2abB+A(b^2+2ac)}{x} + c(2bB+Ac)x+\frac{1}{2}Bc^2x^2 + \left(b^2B+2Abc+2aBc\right)\log(x)$$

output
```
-1/3*a^2*A/x^3-1/2*a*(2*A*b+B*a)/x^2-(2*a*b*B+A*(2*a*c+b^2))/x+c*(A*c+2*B*
b)*x+1/2*B*c^2*x^2+(2*A*b*c+2*B*a*c+B*b^2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}\,dx = -\frac{Ab^2}{x}+2bBcx+Ac^2x+\frac{1}{2}Bc^2x^2-\frac{a^2(2A+3Bx)}{6x^3} -\frac{a(Ab+2bBx+2Acx)}{x^2}+\left(b^2B+2Abc+2aBc\right)\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^4,x]
```

output
```
-((A*b^2)/x) + 2*b*B*c*x + A*c^2*x + (B*c^2*x^2)/2 - (a^2*(2*A + 3*B*x))/(
6*x^3) - (a*(A*b + 2*b*B*x + 2*A*c*x))/x^2 + (b^2*B + 2*A*b*c + 2*a*B*c)*L
og[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^2A}{x^4}+\frac{A(2ac+b^2)+2abB}{x^2}+\frac{2aBc+2Abc+b^2B}{x}+\frac{a(aB+2Ab)}{x^3}+c(Ac+2bB)+Bc^2x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{3x^3}-\frac{A(2ac+b^2)+2abB}{x}+\log(x)\left(2aBc+2Abc+b^2B\right)-\frac{a(aB+2Ab)}{2x^2}+cx(Ac+2bB)+\frac{1}{2}Bc^2x^2$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a*(2*A*b + a*B))/(2*x^2) - (2*a*b*B + A*(b^2 + 2*a*c))
/x + c*(2*b*B + A*c)*x + (B*c^2*x^2)/2 + (b^2*B + 2*A*b*c + 2*a*B*c)*Log[x
]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

method	result
default	$\frac{B\,c^2x^2}{2}+A\,c^2x+2Bbcx-\frac{a^2A}{3x^3}-\frac{a(2Ab+Ba)}{2x^2}+\left(2Abc+2aBc+B\,b^2\right)\ln\left(x\right)-\frac{2Aac+b^2A+2abB}{x}$
risch	$\frac{B\,c^2x^2}{2}+A\,c^2x+2Bbcx+\frac{\left(-2Aac-b^2A-2abB\right)x^2+\left(-abA-\frac{1}{2}a^2B\right)x-\frac{a^2A}{3}}{x^3}+2A\ln\left(x\right)bc+2B\ln\left(x\right)$
norman	$\frac{\left(-abA-\frac{1}{2}a^2B\right)x+\left(A\,c^2+2Bbc\right)x^4+\left(-2Aac-b^2A-2abB\right)x^2-\frac{a^2A}{3}+\frac{B\,c^2x^5}{2}}{x^3}+\left(2Abc+2aBc+B\,b^2\right)\ln\left(x\right)$
parallelrisch	$\frac{3B\,c^2x^5+12A\ln(x)x^3bc+6x^4A\,c^2+12B\ln(x)x^3ac+6B\ln(x)x^3b^2+12x^4Bbc-12Aac\,x^2-6x^2b^2A-12Ba\,x^2b-6abAx-3a^2B}{6x^3}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*c^2*x^2+A*c^2*x+2*B*b*c*x-1/3*a^2*A/x^3-1/2*a*(2*A*b+B*a)/x^2+(2*A*b
*c+2*B*a*c+B*b^2)*ln(x)-(2*A*a*c+A*b^2+2*B*a*b)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}\,dx$$
$$=\frac{3\,Bc^2x^5+6\,(2\,Bbc+Ac^2)x^4+6\,(Bb^2+2\,(Ba+Ab)c)x^3\log\left(x\right)-2\,Aa^2-6\,(2\,Bab+Ab^2+2\,Aac)x^2-}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*B*c^2*x^5 + 6*(2*B*b*c + A*c^2)*x^4 + 6*(B*b^2 + 2*(B*a + A*b)*c)*x
^3*log(x) - 2*A*a^2 - 6*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 - 3*(B*a^2 + 2*A*a
*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}\,dx = \frac{Bc^2x^2}{2} + x\left(Ac^2+2Bbc\right) + \left(2Abc+2Bac+Bb^2\right)\log\left(x\right) + \frac{-2Aa^2+x^2(-12Aac-6Ab^2-12Bab)+x(-6Aab-3Ba^2)}{6x^3}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**4,x)
```

output
```
B*c**2*x**2/2 + x*(A*c**2 + 2*B*b*c) + (2*A*b*c + 2*B*a*c + B*b**2)*log(x)
 + (-2*A*a**2 + x**2*(-12*A*a*c - 6*A*b**2 - 12*B*a*b) + x*(-6*A*a*b - 3*B
*a**2))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}\,dx = \frac{1}{2}\,Bc^2x^2 + \left(2\,Bbc+Ac^2\right)x + \left(Bb^2+2\,(Ba+Ab)c\right)\log\left(x\right) - \frac{2\,Aa^2+6\,(2\,Bab+Ab^2+2\,Aac)x^2+3\,(Ba^2+2\,Aab)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^4,x, algorithm="maxima")
```

output
```
1/2*B*c^2*x^2 + (2*B*b*c + A*c^2)*x + (B*b^2 + 2*(B*a + A*b)*c)*log(x) - 1
/6*(2*A*a^2 + 6*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 3*(B*a^2 + 2*A*a*b)*x)/x
^3
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}\,dx = \frac{1}{2}\,Bc^2x^2+2\,Bbcx+Ac^2x+\left(Bb^2+2\,Bac+2\,Abc\right)\log\left(|x|\right) - \frac{2\,Aa^2+6\,(2\,Bab+Ab^2+2\,Aac)x^2+3\,(Ba^2+2\,Aab)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^4,x, algorithm="giac")
```

output
```
1/2*B*c^2*x^2 + 2*B*b*c*x + A*c^2*x + (B*b^2 + 2*B*a*c + 2*A*b*c)*log(abs(
x)) - 1/6*(2*A*a^2 + 6*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 3*(B*a^2 + 2*A*a*
b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 10.51 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}\,dx = x\,\left(A\,c^2+2\,B\,b\,c\right)-\frac{\frac{A\,a^2}{3}+x^2\,\left(A\,b^2+2\,B\,a\,b+2\,A\,a\,c\right)+x\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)}{x^3} + \ln\left(x\right)\,\left(B\,b^2+2\,A\,c\,b+2\,B\,a\,c\right)+\frac{B\,c^2\,x^2}{2}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^4,x)
```

output
```
x*(A*c^2 + 2*B*b*c) - ((A*a^2)/3 + x^2*(A*b^2 + 2*A*a*c + 2*B*a*b) + x*((B
*a^2)/2 + A*a*b))/x^3 + log(x)*(B*b^2 + 2*A*b*c + 2*B*a*c) + (B*c^2*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^4}\,dx$$
$$=\frac{24\log(x)\,abc\,x^3+6\log(x)\,b^3x^3-2a^3-9a^2bx-12a^2c\,x^2-18a\,b^2x^2+6a\,c^2x^4+12b^2c\,x^4+3b\,c^2x^5}{6x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^4,x)
```

output
```
(24*log(x)*a*b*c*x**3 + 6*log(x)*b**3*x**3 - 2*a**3 - 9*a**2*b*x - 12*a**2
*c*x**2 - 18*a*b**2*x**2 + 6*a*c**2*x**4 + 12*b**2*c*x**4 + 3*b*c**2*x**5)
/(6*x**3)
```

3.20 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^5} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^5} \, dx = -\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{3x^3}-\frac{2abB+A(b^2+2ac)}{2x^2} -\frac{b^2B+2Abc+2aBc}{x}+Bc^2x+c(2bB+Ac)\log(x)$$

output
```
-1/4*a^2*A/x^4-1/3*a*(2*A*b+B*a)/x^3-1/2*(2*a*b*B+A*(2*a*c+b^2))/x^2-(2*A*
b*c+2*B*a*c+B*b^2)/x+B*c^2*x+c*(A*c+2*B*b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^5} \, dx = -\frac{a^2(3A+4Bx)+4ax(3Bx(b+2cx)+A(2b+3cx))+6x^2(Ab(b+4cx)+2Bx(b^2-c^2x^2))-12c(2bB}{12x^4}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^5,x]
```

output
```
-1/12*(a^2*(3*A + 4*B*x) + 4*a*x*(3*B*x*(b + 2*c*x) + A*(2*b + 3*c*x)) + 6
*x^2*(A*b*(b + 4*c*x) + 2*B*x*(b^2 - c^2*x^2)) - 12*c*(2*b*B + A*c)*x^4*Lo
g[x])/x^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^5}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^2A}{x^5}+\frac{A\left(2ac+b^2\right)+2abB}{x^3}+\frac{2aBc+2Abc+b^2B}{x^2}+\frac{a(aB+2Ab)}{x^4}+\frac{c(Ac+2bB)}{x}+Bc^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{4x^4}-\frac{A\left(2ac+b^2\right)+2abB}{2x^2}-\frac{2aBc+2Abc+b^2B}{x}-\frac{a(aB+2Ab)}{3x^3}+c\log(x)(Ac+2bB)+Bc^2x$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^5,x]
```

output
```
-1/4*(a^2*A)/x^4 - (a*(2*A*b + a*B))/(3*x^3) - (2*a*b*B + A*(b^2 + 2*a*c))
/(2*x^2) - (b^2*B + 2*A*b*c + 2*a*B*c)/x + B*c^2*x + c*(2*b*B + A*c)*Log[x
]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

method	result
default	$c^2xB-\frac{a(2Ab+Ba)}{3x^3}-\frac{2Aac+b^2A+2abB}{2x^2}-\frac{a^2A}{4x^4}+c(Ac+2Bb)\ln(x)-\frac{2Abc+2aBc+Bb^2}{x}$
risch	$c^2xB+\frac{(-2Abc-2aBc-Bb^2)x^3+(-Aac-\frac{1}{2}b^2A-abB)x^2+(-\frac{2}{3}abA-\frac{1}{3}a^2B)x-\frac{a^2A}{4}}{x^4}+A\ln(x)c^2+2B\ln(x)$
norman	$\frac{(-\frac{2}{3}abA-\frac{1}{3}a^2B)x+(-Aac-\frac{1}{2}b^2A-abB)x^2+(-2Abc-2aBc-Bb^2)x^3+Bc^2x^5-\frac{a^2A}{4}}{x^4}+(Ac^2+2Bbc)\ln(x)$
parallelrisch	$\frac{12A\ln(x)x^4c^2+24B\ln(x)x^4bc+12Bc^2x^5-24x^3Abc-24Bacx^3-12x^3Bb^2-12Aacx^2-6x^2b^2A-12Bax^2b-8abAx-4a^2Bx-}{12x^4}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^2*x*B-1/3*a*(2*A*b+B*a)/x^3-1/2*(2*A*a*c+A*b^2+2*B*a*b)/x^2-1/4*a^2*A/x^
4+c*(A*c+2*B*b)*ln(x)-(2*A*b*c+2*B*a*c+B*b^2)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^5}\,dx$$
$$=\frac{12\,Bc^2x^5+12\,(2\,Bbc+Ac^2)x^4\log(x)-12\,(Bb^2+2\,(Ba+Ab)c)x^3-3\,Aa^2-6\,(2\,Bab+Ab^2+2\,Aac)}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(12*B*c^2*x^5 + 12*(2*B*b*c + A*c^2)*x^4*log(x) - 12*(B*b^2 + 2*(B*a
+ A*b)*c)*x^3 - 3*A*a^2 - 6*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 - 4*(B*a^2 + 2
*A*a*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^5}\,dx = Bc^2x + c(Ac+2Bb)\log(x) + \frac{-3Aa^2 + x^3(-24Abc-24Bac-12Bb^2) + x^2(-12Aac-6Ab^2-12Bab) + x(-8Aab-4Ba^2)}{12x^4}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**5,x)
```

output
```
B*c**2*x + c*(A*c + 2*B*b)*log(x) + (-3*A*a**2 + x**3*(-24*A*b*c - 24*B*a*
c - 12*B*b**2) + x**2*(-12*A*a*c - 6*A*b**2 - 12*B*a*b) + x*(-8*A*a*b - 4*
B*a**2))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^5}\,dx = Bc^2x + \left(2\,Bbc + Ac^2\right)\log(x) - \frac{12\,(Bb^2+2\,(Ba+Ab)c)x^3 + 3\,Aa^2 + 6\,(2\,Bab+Ab^2+2\,Aac)x^2 + 4\,(Ba^2+2\,Aab)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^5,x, algorithm="maxima")
```

output
```
B*c^2*x + (2*B*b*c + A*c^2)*log(x) - 1/12*(12*(B*b^2 + 2*(B*a + A*b)*c)*x^
3 + 3*A*a^2 + 6*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 4*(B*a^2 + 2*A*a*b)*x)/x
^4
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^5}\,dx = Bc^2x + \left(2\,Bbc + Ac^2\right)\log\left(|x|\right) - \frac{12\,(Bb^2 + 2\,Bac + 2\,Abc)x^3 + 3\,Aa^2 + 6\,(2\,Bab + Ab^2 + 2\,Aac)x^2 + 4\,(Ba^2 + 2\,Aab)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^5,x, algorithm="giac")
```

output
```
B*c^2*x + (2*B*b*c + A*c^2)*log(abs(x)) - 1/12*(12*(B*b^2 + 2*B*a*c + 2*A*
b*c)*x^3 + 3*A*a^2 + 6*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 4*(B*a^2 + 2*A*a*
b)*x)/x^4
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^5}\,dx = \ln(x)\,\left(A\,c^2 + 2\,B\,b\,c\right) - \frac{\frac{A\,a^2}{4} + x^2\left(\frac{A\,b^2}{2} + B\,a\,b + A\,a\,c\right) + x^3\left(B\,b^2 + 2\,A\,c\,b + 2\,B\,a\,c\right) + x\left(\frac{B\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right)}{x^4} + B\,c^2\,x$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^5,x)
```

output
```
log(x)*(A*c^2 + 2*B*b*c) - ((A*a^2)/4 + x^2*((A*b^2)/2 + A*a*c + B*a*b) +
x^3*(B*b^2 + 2*A*b*c + 2*B*a*c) + x*((B*a^2)/3 + (2*A*a*b)/3))/x^4 + B*c^2
*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^5}\,dx$$
$$=\frac{4\log(x)\,a\,c^2x^4+8\log(x)\,b^2c\,x^4-a^3-4a^2bx-4a^2c\,x^2-6a\,b^2x^2-16abc\,x^3-4b^3x^3+4b\,c^2x^5}{4x^4}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^5,x)
```

output
```
(4*log(x)*a*c**2*x**4 + 8*log(x)*b**2*c*x**4 - a**3 - 4*a**2*b*x - 4*a**2*
c*x**2 - 6*a*b**2*x**2 - 16*a*b*c*x**3 - 4*b**3*x**3 + 4*b*c**2*x**5)/(4*x
**4)
```

3.21 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6} \, dx = -\frac{a^2 A}{5x^5} - \frac{a(2Ab+aB)}{4x^4} - \frac{2abB+A(b^2+2ac)}{3x^3} - \frac{b^2B+2Abc+2aBc}{2x^2} - \frac{c(2bB+Ac)}{x} + Bc^2\log(x)$$

output
```
-1/5*a^2*A/x^5-1/4*a*(2*A*b+B*a)/x^4-1/3*(2*a*b*B+A*(2*a*c+b^2))/x^3-1/2*(
2*A*b*c+2*B*a*c+B*b^2)/x^2-c*(A*c+2*B*b)/x+B*c^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6} \, dx = -\frac{3a^2(4A+5Bx)+10ax(3Ab+4bBx+4Acx+6Bcx^2)+10x^2(3bBx(b+4cx)+2A(b^2+3bcx+3c^2x}{60x^5} + Bc^2\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^6,x]
```

output
```
-1/60*(3*a^2*(4*A + 5*B*x) + 10*a*x*(3*A*b + 4*b*B*x + 4*A*c*x + 6*B*c*x^2
) + 10*x^2*(3*b*B*x*(b + 4*c*x) + 2*A*(b^2 + 3*b*c*x + 3*c^2*x^2)))/x^5 +
B*c^2*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^2A}{x^6}+\frac{A\left(2ac+b^2\right)+2abB}{x^4}+\frac{2aBc+2Abc+b^2B}{x^3}+\frac{a(aB+2Ab)}{x^5}+\frac{c(Ac+2bB)}{x^2}+\frac{Bc^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{5x^5}-\frac{A\left(2ac+b^2\right)+2abB}{3x^3}-\frac{2aBc+2Abc+b^2B}{2x^2}-\frac{a(aB+2Ab)}{4x^4}-\frac{c(Ac+2bB)}{x}+Bc^2\log(x)$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(4*x^4) - (2*a*b*B + A*(b^2 + 2*a*c))
/(3*x^3) - (b^2*B + 2*A*b*c + 2*a*B*c)/(2*x^2) - (c*(2*b*B + A*c))/x + B*c
^2*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result
default	$-\frac{a^2 A}{5x^5}-\frac{2Aac+b^2A+2abB}{3x^3}-\frac{2Abc+2aBc+B\,b^2}{2x^2}-\frac{a(2Ab+Ba)}{4x^4}+B\,c^2\ln\left(x\right)-\frac{c(Ac+2Bb)}{x}$
norman	$\frac{\left(-\frac{1}{2}abA-\frac{1}{4}a^2B\right)x+\left(-\frac{2}{3}Aac-\frac{1}{3}b^2A-\frac{2}{3}abB\right)x^2+\left(-Abc-aBc-\frac{1}{2}B\,b^2\right)x^3+\left(-A\,c^2-2Bbc\right)x^4-\frac{a^2A}{5}}{x^5}+B\,c^2\ln\left(x\right)$
risch	$\frac{\left(-\frac{1}{2}abA-\frac{1}{4}a^2B\right)x+\left(-\frac{2}{3}Aac-\frac{1}{3}b^2A-\frac{2}{3}abB\right)x^2+\left(-Abc-aBc-\frac{1}{2}B\,b^2\right)x^3+\left(-A\,c^2-2Bbc\right)x^4-\frac{a^2A}{5}}{x^5}+B\,c^2\ln\left(x\right)$
parallelrisch	$-\frac{-60B\,c^2\ln(x)x^5+60x^4A\,c^2+120x^4Bbc+60x^3Abc+60Bac\,x^3+30x^3B\,b^2+40Aac\,x^2+20x^2b^2A+40Ba\,x^2b+30abAx+15a^2}{60x^5}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2*A/x^5-1/3*(2*A*a*c+A*b^2+2*B*a*b)/x^3-1/2*(2*A*b*c+2*B*a*c+B*b^2)
/x^2-1/4*a*(2*A*b+B*a)/x^4+B*c^2*ln(x)-c*(A*c+2*B*b)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6}\,dx$$
$$=\frac{60\,Bc^2x^5\log\left(x\right)-60\,(2\,Bbc+Ac^2)x^4-30\,(Bb^2+2\,(Ba+Ab)c)x^3-12\,Aa^2-20\,(2\,Bab+Ab^2+2\,Ac}{60\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^6,x, algorithm="fricas")
```

output
```
1/60*(60*B*c^2*x^5*log(x) - 60*(2*B*b*c + A*c^2)*x^4 - 30*(B*b^2 + 2*(B*a
+ A*b)*c)*x^3 - 12*A*a^2 - 20*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 - 15*(B*a^2
+ 2*A*a*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 3.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6}\,dx = Bc^2\log{(x)} + \frac{-12Aa^2 + x^4(-60Ac^2 - 120Bbc) + x^3(-60Abc - 60Bac - 30Bb^2) + x^2(-40Aac - 20Ab^2 - 40Bab)}{60x^5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**6,x)
```

output
```
B*c**2*log(x) + (-12*A*a**2 + x**4*(-60*A*c**2 - 120*B*b*c) + x**3*(-60*A*
b*c - 60*B*a*c - 30*B*b**2) + x**2*(-40*A*a*c - 20*A*b**2 - 40*B*a*b) + x*
(-30*A*a*b - 15*B*a**2))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6}\,dx = Bc^2\log{(x)} - \frac{60\,(2\,Bbc + Ac^2)x^4 + 30\,(Bb^2 + 2\,(Ba + Ab)c)x^3 + 12\,Aa^2 + 20\,(2\,Bab + Ab^2 + 2\,Aac)x^2 + 15\,(Ba^2 -}{60\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^6,x, algorithm="maxima")
```

output
```
B*c^2*log(x) - 1/60*(60*(2*B*b*c + A*c^2)*x^4 + 30*(B*b^2 + 2*(B*a + A*b)*
c)*x^3 + 12*A*a^2 + 20*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 15*(B*a^2 + 2*A*a
*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6}\,dx = Bc^2\log\left(|x|\right) - \frac{60\,(2\,Bbc+Ac^2)x^4+30\,(Bb^2+2\,Bac+2\,Abc)x^3+12\,Aa^2+20\,(2\,Bab+Ab^2+2\,Aac)x^2+15\,(Ba^2}{60\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^6,x, algorithm="giac")
```

output
```
B*c^2*log(abs(x)) - 1/60*(60*(2*B*b*c + A*c^2)*x^4 + 30*(B*b^2 + 2*B*a*c +
 2*A*b*c)*x^3 + 12*A*a^2 + 20*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 15*(B*a^2
+ 2*A*a*b)*x)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6}\,dx = B\,c^2\,\ln\left(x\right) - \frac{x^4\left(A\,c^2+2\,B\,b\,c\right)+\frac{A\,a^2}{5}+x^2\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}+\frac{2\,A\,a\,c}{3}\right)+x^3\left(\frac{B\,b^2}{2}+A\,c\,b+B\,a\,c\right)+x\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)}{x^5}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^6,x)
```

output
```
B*c^2*log(x) - (x^4*(A*c^2 + 2*B*b*c) + (A*a^2)/5 + x^2*((A*b^2)/3 + (2*A*
a*c)/3 + (2*B*a*b)/3) + x^3*((B*b^2)/2 + A*b*c + B*a*c) + x*((B*a^2)/4 + (
A*a*b)/2))/x^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^6}\,dx$$
$$=\frac{60\log(x)\,b\,c^2x^5-12a^3-45a^2bx-40a^2c\,x^2-60a\,b^2x^2-120abc\,x^3-60a\,c^2x^4-30b^3x^3-120b^2c\,x^4}{60x^5}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^6,x)
```

output
```
(60*log(x)*b*c**2*x**5 - 12*a**3 - 45*a**2*b*x - 40*a**2*c*x**2 - 60*a*b**
2*x**2 - 120*a*b*c*x**3 - 60*a*c**2*x**4 - 30*b**3*x**3 - 120*b**2*c*x**4)
/(60*x**5)
```

3.22 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^7}\,dx = -\frac{a^2A}{6x^6}-\frac{a(2Ab+aB)}{5x^5}-\frac{2abB+A(b^2+2ac)}{4x^4} -\frac{b^2B+2Abc+2aBc}{3x^3}-\frac{c(2bB+Ac)}{2x^2}-\frac{Bc^2}{x}$$

output
```
-1/6*a^2*A/x^6-1/5*a*(2*A*b+B*a)/x^5-1/4*(2*a*b*B+A*(2*a*c+b^2))/x^4-1/3*(
2*A*b*c+2*B*a*c+B*b^2)/x^3-1/2*c*(A*c+2*B*b)/x^2-B*c^2/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^7}\,dx = -\frac{2a^2(5A+6Bx)+2ax(5Bx(3b+4cx)+3A(4b+5cx))+5x^2(4Bx(b^2+3bcx+3c^2x^2)+A(3b^2+8bc}{60x^6}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^7,x]
```

output
```
-1/60*(2*a^2*(5*A + 6*B*x) + 2*a*x*(5*B*x*(3*b + 4*c*x) + 3*A*(4*b + 5*c*x
)) + 5*x^2*(4*B*x*(b^2 + 3*b*c*x + 3*c^2*x^2) + A*(3*b^2 + 8*b*c*x + 6*c^2
*x^2)))/x^6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^7}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^2A}{x^7}+\frac{A\left(2ac+b^2\right)+2abB}{x^5}+\frac{2aBc+2Abc+b^2B}{x^4}+\frac{a(aB+2Ab)}{x^6}+\frac{c(Ac+2bB)}{x^3}+\frac{Bc^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{6x^6}-\frac{A\left(2ac+b^2\right)+2abB}{4x^4}-\frac{2aBc+2Abc+b^2B}{3x^3}-\frac{a(aB+2Ab)}{5x^5}-\frac{c(Ac+2bB)}{2x^2}-\frac{Bc^2}{x}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^7,x]
```

output
```
-1/6*(a^2*A)/x^6 - (a*(2*A*b + a*B))/(5*x^5) - (2*a*b*B + A*(b^2 + 2*a*c))
/(4*x^4) - (b^2*B + 2*A*b*c + 2*a*B*c)/(3*x^3) - (c*(2*b*B + A*c))/(2*x^2)
 - (B*c^2)/x
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a(2Ab+Ba)}{5x^5}-\frac{a^2A}{6x^6}-\frac{2Abc+2aBc+B\,b^2}{3x^3}-\frac{c(Ac+2Bb)}{2x^2}-\frac{2Aac+b^2A+2abB}{4x^4}-\frac{B\,c^2}{x}$
norman	$\frac{-B\,c^2x^5+\left(-\frac{1}{2}A\,c^2-Bbc\right)x^4+\left(-\frac{2}{3}Abc-\frac{2}{3}aBc-\frac{1}{3}B\,b^2\right)x^3+\left(-\frac{1}{2}Aac-\frac{1}{4}b^2A-\frac{1}{2}abB\right)x^2+\left(-\frac{2}{5}abA-\frac{1}{5}a^2B\right)x-\frac{a^2A}{6}}{x^6}$
risch	$\frac{-B\,c^2x^5+\left(-\frac{1}{2}A\,c^2-Bbc\right)x^4+\left(-\frac{2}{3}Abc-\frac{2}{3}aBc-\frac{1}{3}B\,b^2\right)x^3+\left(-\frac{1}{2}Aac-\frac{1}{4}b^2A-\frac{1}{2}abB\right)x^2+\left(-\frac{2}{5}abA-\frac{1}{5}a^2B\right)x-\frac{a^2A}{6}}{x^6}$
gosper	$-\frac{60B\,c^2x^5+30x^4A\,c^2+60x^4Bbc+40x^3Abc+40Bac\,x^3+20x^3B\,b^2+30Aac\,x^2+15x^2b^2A+30Ba\,x^2b+24abAx+12a^2Bx+10a}{60x^6}$
parallelrisch	$-\frac{60B\,c^2x^5+30x^4A\,c^2+60x^4Bbc+40x^3Abc+40Bac\,x^3+20x^3B\,b^2+30Aac\,x^2+15x^2b^2A+30Ba\,x^2b+24abAx+12a^2Bx+10a}{60x^6}$
orering	$-\frac{60B\,c^2x^5+30x^4A\,c^2+60x^4Bbc+40x^3Abc+40Bac\,x^3+20x^3B\,b^2+30Aac\,x^2+15x^2b^2A+30Ba\,x^2b+24abAx+12a^2Bx+10a}{60x^6}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a*(2*A*b+B*a)/x^5-1/6*a^2*A/x^6-1/3*(2*A*b*c+2*B*a*c+B*b^2)/x^3-1/2*c
*(A*c+2*B*b)/x^2-1/4*(2*A*a*c+A*b^2+2*B*a*b)/x^4-B*c^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^7}\,dx = -\frac{60\,Bc^2x^5+30\,(2\,Bbc+Ac^2)x^4+20\,(Bb^2+2\,(Ba+Ab)c)x^3+10\,Aa^2+15\,(2\,Bab+Ab^2+2\,Aac)x^2}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^7,x, algorithm="fricas")
```

output
```
-1/60*(60*B*c^2*x^5 + 30*(2*B*b*c + A*c^2)*x^4 + 20*(B*b^2 + 2*(B*a + A*b)
*c)*x^3 + 10*A*a^2 + 15*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 12*(B*a^2 + 2*A*
a*b)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^7}\,dx = \frac{-10Aa^2-60Bc^2x^5+x^4(-30Ac^2-60Bbc)+x^3(-40Abc-40Bac-20Bb^2)+x^2(-30Aac-15Ab^2-}{60x^6}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**7,x)
```

output
```
(-10*A*a**2 - 60*B*c**2*x**5 + x**4*(-30*A*c**2 - 60*B*b*c) + x**3*(-40*A*
b*c - 40*B*a*c - 20*B*b**2) + x**2*(-30*A*a*c - 15*A*b**2 - 30*B*a*b) + x*
(-24*A*a*b - 12*B*a**2))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^7}\,dx = -\frac{60\,Bc^2x^5 + 30\,(2\,Bbc + Ac^2)x^4 + 20\,(Bb^2 + 2\,(Ba + Ab)c)x^3 + 10\,Aa^2 + 15\,(2\,Bab + Ab^2 + 2\,Aac)x^2}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^7,x, algorithm="maxima")
```

output
```
-1/60*(60*B*c^2*x^5 + 30*(2*B*b*c + A*c^2)*x^4 + 20*(B*b^2 + 2*(B*a + A*b)
*c)*x^3 + 10*A*a^2 + 15*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 12*(B*a^2 + 2*A*
a*b)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^7}\,dx = -\frac{60\,Bc^2x^5 + 60\,Bbcx^4 + 30\,Ac^2x^4 + 20\,Bb^2x^3 + 40\,Bacx^3 + 40\,Abcx^3 + 30\,Babx^2 + 15\,Ab^2x^2 + 30\,Aa}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^7,x, algorithm="giac")
```

output
```
-1/60*(60*B*c^2*x^5 + 60*B*b*c*x^4 + 30*A*c^2*x^4 + 20*B*b^2*x^3 + 40*B*a*
c*x^3 + 40*A*b*c*x^3 + 30*B*a*b*x^2 + 15*A*b^2*x^2 + 30*A*a*c*x^2 + 12*B*a
^2*x + 24*A*a*b*x + 10*A*a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 10.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^7}\,dx =$$
$$-\frac{x^4\left(\frac{A\,c^2}{2}+B\,b\,c\right)+\frac{A\,a^2}{6}+x^2\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}+\frac{A\,a\,c}{2}\right)+x^3\left(\frac{B\,b^2}{3}+\frac{2\,A\,c\,b}{3}+\frac{2\,B\,a\,c}{3}\right)+x\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+B}{x^6}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^7,x)
```

output
```
-(x^4*((A*c^2)/2 + B*b*c) + (A*a^2)/6 + x^2*((A*b^2)/4 + (A*a*c)/2 + (B*a*
b)/2) + x^3*((B*b^2)/3 + (2*A*b*c)/3 + (2*B*a*c)/3) + x*((B*a^2)/5 + (2*A*
a*b)/5) + B*c^2*x^5)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^7}\,dx$$
$$=\frac{-60b\,c^2x^5-30a\,c^2x^4-60b^2c\,x^4-80abc\,x^3-20b^3x^3-30a^2c\,x^2-45a\,b^2x^2-36a^2bx-10a^3}{60x^6}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^7,x)
```

output
```
( - 10*a**3 - 36*a**2*b*x - 30*a**2*c*x**2 - 45*a*b**2*x**2 - 80*a*b*c*x**
3 - 30*a*c**2*x**4 - 20*b**3*x**3 - 60*b**2*c*x**4 - 60*b*c**2*x**5)/(60*x
**6)
```

3.23 $\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^8} dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^8} dx = -\frac{a^2A}{7x^7} - \frac{a(2Ab+aB)}{6x^6} - \frac{2abB+A(b^2+2ac)}{5x^5} - \frac{b^2B+2Abc+2aBc}{4x^4} - \frac{c(2bB+Ac)}{3x^3} - \frac{Bc^2}{2x^2}$$

output
```
-1/7*a^2*A/x^7-1/6*a*(2*A*b+B*a)/x^6-1/5*(2*a*b*B+A*(2*a*c+b^2))/x^5-1/4*(
2*A*b*c+2*B*a*c+B*b^2)/x^4-1/3*c*(A*c+2*B*b)/x^3-1/2*B*c^2/x^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^8} dx = -\frac{10a^2(6A+7Bx)+14ax(3Bx(4b+5cx)+2A(5b+6cx))+7x^2(5Bx(3b^2+8bcx+6c^2x^2)+2A(6b^2}{420x^7}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^8,x]
```

output
```
-1/420*(10*a^2*(6*A + 7*B*x) + 14*a*x*(3*B*x*(4*b + 5*c*x) + 2*A*(5*b + 6*
c*x)) + 7*x^2*(5*B*x*(3*b^2 + 8*b*c*x + 6*c^2*x^2) + 2*A*(6*b^2 + 15*b*c*x
 + 10*c^2*x^2)))/x^7
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^8}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^2A}{x^8}+\frac{A\left(2ac+b^2\right)+2abB}{x^6}+\frac{2aBc+2Abc+b^2B}{x^5}+\frac{a(aB+2Ab)}{x^7}+\frac{c(Ac+2bB)}{x^4}+\frac{Bc^2}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{7x^7}-\frac{A\left(2ac+b^2\right)+2abB}{5x^5}-\frac{2aBc+2Abc+b^2B}{4x^4}-\frac{a(aB+2Ab)}{6x^6}-\frac{c(Ac+2bB)}{3x^3}-\frac{Bc^2}{2x^2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^8,x]
```

output
```
-1/7*(a^2*A)/x^7 - (a*(2*A*b + a*B))/(6*x^6) - (2*a*b*B + A*(b^2 + 2*a*c))
/(5*x^5) - (b^2*B + 2*A*b*c + 2*a*B*c)/(4*x^4) - (c*(2*b*B + A*c))/(3*x^3)
 - (B*c^2)/(2*x^2)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

method	result
default	$-\frac{2Aac+b^2A+2abB}{5x^5}-\frac{a(2Ab+Ba)}{6x^6}-\frac{c(Ac+2Bb)}{3x^3}-\frac{B\,c^2}{2x^2}-\frac{2Abc+2aBc+B\,b^2}{4x^4}-\frac{a^2A}{7x^7}$
norman	$\frac{-\frac{B\,c^2x^5}{2}+\left(-\frac{1}{3}A\,c^2-\frac{2}{3}Bbc\right)x^4+\left(-\frac{1}{2}Abc-\frac{1}{2}aBc-\frac{1}{4}B\,b^2\right)x^3+\left(-\frac{2}{5}Aac-\frac{1}{5}b^2A-\frac{2}{5}abB\right)x^2+\left(-\frac{1}{3}abA-\frac{1}{6}a^2B\right)x-\frac{a^2A}{7}}{x^7}$
risch	$\frac{-\frac{B\,c^2x^5}{2}+\left(-\frac{1}{3}A\,c^2-\frac{2}{3}Bbc\right)x^4+\left(-\frac{1}{2}Abc-\frac{1}{2}aBc-\frac{1}{4}B\,b^2\right)x^3+\left(-\frac{2}{5}Aac-\frac{1}{5}b^2A-\frac{2}{5}abB\right)x^2+\left(-\frac{1}{3}abA-\frac{1}{6}a^2B\right)x-\frac{a^2A}{7}}{x^7}$
gosper	$-\frac{210B\,c^2x^5+140x^4A\,c^2+280x^4Bbc+210x^3Abc+210Bac\,x^3+105x^3B\,b^2+168Aac\,x^2+84x^2b^2A+168Ba\,x^2b+140abAx+70}{420x^7}$
parallelrisch	$-\frac{210B\,c^2x^5+140x^4A\,c^2+280x^4Bbc+210x^3Abc+210Bac\,x^3+105x^3B\,b^2+168Aac\,x^2+84x^2b^2A+168Ba\,x^2b+140abAx+70}{420x^7}$
orering	$-\frac{210B\,c^2x^5+140x^4A\,c^2+280x^4Bbc+210x^3Abc+210Bac\,x^3+105x^3B\,b^2+168Aac\,x^2+84x^2b^2A+168Ba\,x^2b+140abAx+70}{420x^7}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(2*A*a*c+A*b^2+2*B*a*b)/x^5-1/6*a*(2*A*b+B*a)/x^6-1/3*c*(A*c+2*B*b)/x
^3-1/2*B*c^2/x^2-1/4*(2*A*b*c+2*B*a*c+B*b^2)/x^4-1/7*a^2*A/x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^8}\,dx = -\frac{210\,Bc^2x^5+140\,(2\,Bbc+Ac^2)x^4+105\,(Bb^2+2\,(Ba+Ab)c)x^3+60\,Aa^2+84\,(2\,Bab+Ab^2+2\,Aac}{420\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^8,x, algorithm="fricas")
```

output
```
-1/420*(210*B*c^2*x^5 + 140*(2*B*b*c + A*c^2)*x^4 + 105*(B*b^2 + 2*(B*a +
A*b)*c)*x^3 + 60*A*a^2 + 84*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 70*(B*a^2 +
2*A*a*b)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^8}\,dx = \frac{-60Aa^2-210Bc^2x^5+x^4(-140Ac^2-280Bbc)+x^3(-210Abc-210Bac-105Bb^2)+x^2(-168Aac-}{420x^7}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**8,x)
```

output
```
(-60*A*a**2 - 210*B*c**2*x**5 + x**4*(-140*A*c**2 - 280*B*b*c) + x**3*(-21
0*A*b*c - 210*B*a*c - 105*B*b**2) + x**2*(-168*A*a*c - 84*A*b**2 - 168*B*a
*b) + x*(-140*A*a*b - 70*B*a**2))/(420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^8}\,dx = -\frac{210\,Bc^2x^5 + 140\,(2\,Bbc + Ac^2)x^4 + 105\,(Bb^2 + 2\,(Ba + Ab)c)x^3 + 60\,Aa^2 + 84\,(2\,Bab + Ab^2 + 2\,Aac}{420\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^8,x, algorithm="maxima")
```

output
```
-1/420*(210*B*c^2*x^5 + 140*(2*B*b*c + A*c^2)*x^4 + 105*(B*b^2 + 2*(B*a +
A*b)*c)*x^3 + 60*A*a^2 + 84*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 70*(B*a^2 +
2*A*a*b)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^8}\,dx = -\frac{210\,Bc^2x^5 + 280\,Bbcx^4 + 140\,Ac^2x^4 + 105\,Bb^2x^3 + 210\,Bacx^3 + 210\,Abcx^3 + 168\,Babx^2 + 84\,Ab^2x^2}{420\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^8,x, algorithm="giac")
```

output
```
-1/420*(210*B*c^2*x^5 + 280*B*b*c*x^4 + 140*A*c^2*x^4 + 105*B*b^2*x^3 + 21
0*B*a*c*x^3 + 210*A*b*c*x^3 + 168*B*a*b*x^2 + 84*A*b^2*x^2 + 168*A*a*c*x^2
 + 70*B*a^2*x + 140*A*a*b*x + 60*A*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^8}\,dx =$$
$$-\frac{x^4\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+\frac{A\,a^2}{7}+x^2\left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}+\frac{2\,A\,a\,c}{5}\right)+x^3\left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}+\frac{B\,a\,c}{2}\right)+x\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+B}{x^7}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^8,x)
```

output
```
-(x^4*((A*c^2)/3 + (2*B*b*c)/3) + (A*a^2)/7 + x^2*((A*b^2)/5 + (2*A*a*c)/5
 + (2*B*a*b)/5) + x^3*((B*b^2)/4 + (A*b*c)/2 + (B*a*c)/2) + x*((B*a^2)/6 +
 (A*a*b)/3) + (B*c^2*x^5)/2)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^8}\,dx$$
$$=\frac{-210b\,c^2x^5-140a\,c^2x^4-280b^2c\,x^4-420abc\,x^3-105b^3x^3-168a^2c\,x^2-252a\,b^2x^2-210a^2bx-60a^3}{420x^7}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^8,x)
```

output
```
( - 60*a**3 - 210*a**2*b*x - 168*a**2*c*x**2 - 252*a*b**2*x**2 - 420*a*b*c
*x**3 - 140*a*c**2*x**4 - 105*b**3*x**3 - 280*b**2*c*x**4 - 210*b*c**2*x**
5)/(420*x**7)
```

3.24 $\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^9}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^9}\,dx = -\frac{a^2A}{8x^8}-\frac{a(2Ab+aB)}{7x^7}-\frac{2abB+A(b^2+2ac)}{6x^6} -\frac{b^2B+2Abc+2aBc}{5x^5}-\frac{c(2bB+Ac)}{4x^4}-\frac{Bc^2}{3x^3}$$

output
```
-1/8*a^2*A/x^8-1/7*a*(2*A*b+B*a)/x^7-1/6*(2*a*b*B+A*(2*a*c+b^2))/x^6-1/5*(
2*A*b*c+2*B*a*c+B*b^2)/x^5-1/4*c*(A*c+2*B*b)/x^4-1/3*B*c^2/x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^9}\,dx = -\frac{15a^2(7A+8Bx)+8ax(7Bx(5b+6cx)+5A(6b+7cx))+14x^2(2Bx(6b^2+15bcx+10c^2x^2)+A(10b}{840x^8}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^9,x]
```

output
```
-1/840*(15*a^2*(7*A + 8*B*x) + 8*a*x*(7*B*x*(5*b + 6*c*x) + 5*A*(6*b + 7*c
*x)) + 14*x^2*(2*B*x*(6*b^2 + 15*b*c*x + 10*c^2*x^2) + A*(10*b^2 + 24*b*c*
x + 15*c^2*x^2)))/x^8
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^9}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^2A}{x^9}+\frac{A(2ac+b^2)+2abB}{x^7}+\frac{2aBc+2Abc+b^2B}{x^6}+\frac{a(aB+2Ab)}{x^8}+\frac{c(Ac+2bB)}{x^5}+\frac{Bc^2}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{8x^8}-\frac{A(2ac+b^2)+2abB}{6x^6}-\frac{2aBc+2Abc+b^2B}{5x^5}-\frac{a(aB+2Ab)}{7x^7}-\frac{c(Ac+2bB)}{4x^4}-\frac{Bc^2}{3x^3}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^9,x]
```

output
```
-1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(7*x^7) - (2*a*b*B + A*(b^2 + 2*a*c))
/(6*x^6) - (b^2*B + 2*A*b*c + 2*a*B*c)/(5*x^5) - (c*(2*b*B + A*c))/(4*x^4)
 - (B*c^2)/(3*x^3)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

method	result
default	$-\frac{2Abc+2aBc+B\,b^2}{5x^5}-\frac{2Aac+b^2A+2abB}{6x^6}-\frac{B\,c^2}{3x^3}-\frac{a^2A}{8x^8}-\frac{c(Ac+2Bb)}{4x^4}-\frac{a(2Ab+Ba)}{7x^7}$
norman	$\frac{-\frac{B\,c^2x^5}{3}+\left(-\frac{1}{4}A\,c^2-\frac{1}{2}Bbc\right)x^4+\left(-\frac{2}{5}Abc-\frac{2}{5}aBc-\frac{1}{5}B\,b^2\right)x^3+\left(-\frac{1}{3}Aac-\frac{1}{6}b^2A-\frac{1}{3}abB\right)x^2+\left(-\frac{2}{7}abA-\frac{1}{7}a^2B\right)x-\frac{a^2A}{8}}{x^8}$
risch	$\frac{-\frac{B\,c^2x^5}{3}+\left(-\frac{1}{4}A\,c^2-\frac{1}{2}Bbc\right)x^4+\left(-\frac{2}{5}Abc-\frac{2}{5}aBc-\frac{1}{5}B\,b^2\right)x^3+\left(-\frac{1}{3}Aac-\frac{1}{6}b^2A-\frac{1}{3}abB\right)x^2+\left(-\frac{2}{7}abA-\frac{1}{7}a^2B\right)x-\frac{a^2A}{8}}{x^8}$
gosper	$-\frac{280B\,c^2x^5+210x^4A\,c^2+420x^4Bbc+336x^3Abc+336Bac\,x^3+168x^3B\,b^2+280Aac\,x^2+140x^2b^2A+280Ba\,x^2b+240abAx+1}{840x^8}$
parallelrisch	$-\frac{280B\,c^2x^5+210x^4A\,c^2+420x^4Bbc+336x^3Abc+336Bac\,x^3+168x^3B\,b^2+280Aac\,x^2+140x^2b^2A+280Ba\,x^2b+240abAx+1}{840x^8}$
orering	$-\frac{280B\,c^2x^5+210x^4A\,c^2+420x^4Bbc+336x^3Abc+336Bac\,x^3+168x^3B\,b^2+280Aac\,x^2+140x^2b^2A+280Ba\,x^2b+240abAx+1}{840x^8}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(2*A*b*c+2*B*a*c+B*b^2)/x^5-1/6*(2*A*a*c+A*b^2+2*B*a*b)/x^6-1/3*B*c^2
/x^3-1/8*a^2*A/x^8-1/4*c*(A*c+2*B*b)/x^4-1/7*a*(2*A*b+B*a)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^9}\,dx = -\frac{280\,Bc^2x^5+210\,(2\,Bbc+Ac^2)x^4+168\,(Bb^2+2\,(Ba+Ab)c)x^3+105\,Aa^2+140\,(2\,Bab+Ab^2+2\,A}{840\,x^8}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^9,x, algorithm="fricas")
```

output
```
-1/840*(280*B*c^2*x^5 + 210*(2*B*b*c + A*c^2)*x^4 + 168*(B*b^2 + 2*(B*a +
A*b)*c)*x^3 + 105*A*a^2 + 140*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 120*(B*a^2
 + 2*A*a*b)*x)/x^8
```

Sympy [A] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^9}\,dx = \frac{-105Aa^2-280Bc^2x^5+x^4(-210Ac^2-420Bbc)+x^3(-336Abc-336Bac-168Bb^2)+x^2(-280Aac-}{840x^8}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**9,x)
```

output
```
(-105*A*a**2 - 280*B*c**2*x**5 + x**4*(-210*A*c**2 - 420*B*b*c) + x**3*(-3
36*A*b*c - 336*B*a*c - 168*B*b**2) + x**2*(-280*A*a*c - 140*A*b**2 - 280*B
*a*b) + x*(-240*A*a*b - 120*B*a**2))/(840*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^9}\,dx = -\frac{280\,Bc^2x^5 + 210\,(2\,Bbc + Ac^2)x^4 + 168\,(Bb^2 + 2\,(Ba + Ab)c)x^3 + 105\,Aa^2 + 140\,(2\,Bab + Ab^2 + 2\,A \cdots}{840\,x^8}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^9,x, algorithm="maxima")
```

output
```
-1/840*(280*B*c^2*x^5 + 210*(2*B*b*c + A*c^2)*x^4 + 168*(B*b^2 + 2*(B*a +
A*b)*c)*x^3 + 105*A*a^2 + 140*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 120*(B*a^2
 + 2*A*a*b)*x)/x^8
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^9}\,dx = -\frac{280\,Bc^2x^5 + 420\,Bbcx^4 + 210\,Ac^2x^4 + 168\,Bb^2x^3 + 336\,Bacx^3 + 336\,Abcx^3 + 280\,Babx^2 + 140\,Ab^2x^2 \cdots}{840\,x^8}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^9,x, algorithm="giac")
```

output
```
-1/840*(280*B*c^2*x^5 + 420*B*b*c*x^4 + 210*A*c^2*x^4 + 168*B*b^2*x^3 + 33
6*B*a*c*x^3 + 336*A*b*c*x^3 + 280*B*a*b*x^2 + 140*A*b^2*x^2 + 280*A*a*c*x^
2 + 120*B*a^2*x + 240*A*a*b*x + 105*A*a^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^9}\,dx =$$
$$-\frac{x^4\left(\frac{A\,c^2}{4}+\frac{B\,b\,c}{2}\right)+\frac{A\,a^2}{8}+x^2\left(\frac{A\,b^2}{6}+\frac{B\,a\,b}{3}+\frac{A\,a\,c}{3}\right)+x^3\left(\frac{B\,b^2}{5}+\frac{2\,A\,c\,b}{5}+\frac{2\,B\,a\,c}{5}\right)+x\left(\frac{B\,a^2}{7}+\frac{2\,A\,b\,a}{7}\right)+\frac{B}{}}{x^8}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^9,x)
```

output
```
-(x^4*((A*c^2)/4 + (B*b*c)/2) + (A*a^2)/8 + x^2*((A*b^2)/6 + (A*a*c)/3 + (
B*a*b)/3) + x^3*((B*b^2)/5 + (2*A*b*c)/5 + (2*B*a*c)/5) + x*((B*a^2)/7 + (
2*A*a*b)/7) + (B*c^2*x^5)/3)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^9}\,dx$$
$$=\frac{-280b\,c^2x^5-210a\,c^2x^4-420b^2c\,x^4-672abc\,x^3-168b^3x^3-280a^2c\,x^2-420a\,b^2x^2-360a^2bx-105a^3}{840x^8}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^9,x)
```

output
```
( - 105*a**3 - 360*a**2*b*x - 280*a**2*c*x**2 - 420*a*b**2*x**2 - 672*a*b*
c*x**3 - 210*a*c**2*x**4 - 168*b**3*x**3 - 420*b**2*c*x**4 - 280*b*c**2*x*
*5)/(840*x**8)
```

3.25 $\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\begin{aligned}\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx=&\frac{1}{3}a^3Ax^3+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5\\&+\frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^6\\&+\frac{1}{7}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^7\\&+\frac{3}{8}c\left(b^2B+Abc+aBc\right)x^8\\&+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{10}Bc^3x^{10}\end{aligned}$$

output
```
1/3*a^3*A*x^3+1/4*a^2*(3*A*b+B*a)*x^4+3/5*a*(a*b*B+A*(a*c+b^2))*x^5+1/6*(3
*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^6+1/7*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b
^3)*x^7+3/8*c*(A*b*c+B*a*c+B*b^2)*x^8+1/9*c^2*(A*c+3*B*b)*x^9+1/10*B*c^3*x
^10
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{1}{3}a^3Ax^3+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5+\frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^6+\frac{1}{7}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^7+\frac{3}{8}c\left(b^2B+Abc+aBc\right)x^8+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{10}Bc^3x^{10}$$

input
```
Integrate[x^2*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^4)/4 + (3*a*(a*b*B + A*(b^2 + a*c))*x
^5)/5 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^7)/7 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8
+ (c^2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^3Ax^2+a^2x^3(aB+3Ab)+3cx^7\left(aBc+Abc+b^2B\right)+3ax^4\left(A\left(ac+b^2\right)+abB\right)+x^6\left(3aAc^2+6abBc+3A\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^2x^4(aB+3Ab)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{6}x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{9}c^2x^9(Ac+3bB)+\frac{1}{10}Bc^3x^{10}$$

input
```
Int[x^2*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^4)/4 + (3*a*(a*b*B + A*(b^2 + a*c))*x
^5)/5 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^7)/7 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8
+ (c^2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^10)/10
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

method	result
norman	$\frac{Bc^3x^{10}}{10}+\left(\frac{1}{9}Ac^3+\frac{1}{3}Bbc^2\right)x^9+\left(\frac{3}{8}Abc^2+\frac{3}{8}Bac^2+\frac{3}{8}Bb^2c\right)x^8+\left(\frac{3}{7}Aac^2+\frac{3}{7}Ab^2c+\frac{6}{7}Ba$
orering	$x^3\left(252Bc^3x^7+280Ac^3x^6+840Bbc^2x^6+945Abc^2x^5+945Bac^2x^5+945Bb^2cx^5+1080Aac^2x^4+1080Ab^2cx^4+2160Babcx^4$
gosper	$\frac{1}{10}Bc^3x^{10}+\frac{1}{9}x^9Ac^3+\frac{1}{3}x^9Bbc^2+\frac{3}{8}x^8Abc^2+\frac{3}{8}x^8Bac^2+\frac{3}{8}x^8Bb^2c+\frac{3}{7}x^7Aac^2+\frac{3}{7}x^7Ab^2$
risch	$\frac{1}{10}Bc^3x^{10}+\frac{1}{9}x^9Ac^3+\frac{1}{3}x^9Bbc^2+\frac{3}{8}x^8Abc^2+\frac{3}{8}x^8Bac^2+\frac{3}{8}x^8Bb^2c+\frac{3}{7}x^7Aac^2+\frac{3}{7}x^7Ab^2$
parallelrisch	$\frac{1}{10}Bc^3x^{10}+\frac{1}{9}x^9Ac^3+\frac{1}{3}x^9Bbc^2+\frac{3}{8}x^8Abc^2+\frac{3}{8}x^8Bac^2+\frac{3}{8}x^8Bb^2c+\frac{3}{7}x^7Aac^2+\frac{3}{7}x^7Ab^2$
default	$\frac{Bc^3x^{10}}{10}+\frac{(Ac^3+3Bbc^2)x^9}{9}+\frac{(3Abc^2+B(ac^2+2b^2c+c(2ac+b^2)))x^8}{8}+\frac{(A(ac^2+2b^2c+c(2ac+b^2))+B(4abc+b(2ac-}{7}$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*B*c^3*x^10+(1/9*A*c^3+1/3*B*b*c^2)*x^9+(3/8*A*b*c^2+3/8*B*a*c^2+3/8*B
*b^2*c)*x^8+(3/7*A*a*c^2+3/7*A*b^2*c+6/7*B*a*b*c+1/7*B*b^3)*x^7+(A*a*b*c+1
/6*A*b^3+1/2*B*a^2*c+1/2*B*a*b^2)*x^6+(3/5*a^2*A*c+3/5*A*a*b^2+3/5*B*a^2*b
)*x^5+(3/4*A*a^2*b+1/4*B*a^3)*x^4+1/3*a^3*A*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx =& \frac{1}{10}Bc^3x^{10}+\frac{1}{9}\left(3\,Bbc^2+Ac^3\right)x^9 \\ &+\frac{3}{8}\left(Bb^2c+(Ba+Ab)c^2\right)x^8 \\ &+\frac{1}{7}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^7 \\ &+\frac{1}{3}Aa^3x^3 \\ &+\frac{1}{6}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^6 \\ &+\frac{3}{5}\left(Ba^2b+Aab^2+Aa^2c\right)x^5 \\ &+\frac{1}{4}\left(Ba^3+3\,Aa^2b\right)x^4\end{aligned}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
1/10*B*c^3*x^10 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/8*(B*b^2*c + (B*a + A*b)
*c^2)*x^8 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^7 + 1/3*A*a^
3*x^3 + 1/6*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/5*(B*a^2*b
 + A*a*b^2 + A*a^2*c)*x^5 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.21

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{Aa^3x^3}{3}+\frac{Bc^3x^{10}}{10}+x^9\left(\frac{Ac^3}{9}+\frac{Bbc^2}{3}\right)+x^8\cdot\left(\frac{3Abc^2}{8}+\frac{3Bac^2}{8}+\frac{3Bb^2c}{8}\right)+x^7\cdot\left(\frac{3Aac^2}{7}+\frac{3Ab^2c}{7}+\frac{6Babc}{7}+\frac{Bb^3}{7}\right)+x^6\left(Aabc+\frac{Ab^3}{6}+\frac{Ba^2c}{2}+\frac{Bab^2}{2}\right)+x^5\cdot\left(\frac{3Aa^2c}{5}+\frac{3Aab^2}{5}+\frac{3Ba^2b}{5}\right)+x^4\cdot\left(\frac{3Aa^2b}{4}+\frac{Ba^3}{4}\right)$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x+a)**3,x)
```

output
```
A*a**3*x**3/3 + B*c**3*x**10/10 + x**9*(A*c**3/9 + B*b*c**2/3) + x**8*(3*A
*b*c**2/8 + 3*B*a*c**2/8 + 3*B*b**2*c/8) + x**7*(3*A*a*c**2/7 + 3*A*b**2*c
/7 + 6*B*a*b*c/7 + B*b**3/7) + x**6*(A*a*b*c + A*b**3/6 + B*a**2*c/2 + B*a
*b**2/2) + x**5*(3*A*a**2*c/5 + 3*A*a*b**2/5 + 3*B*a**2*b/5) + x**4*(3*A*a
**2*b/4 + B*a**3/4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{1}{10}Bc^3x^{10} + \frac{1}{9}\left(3Bbc^2+Ac^3\right)x^9 + \frac{3}{8}\left(Bb^2c+(Ba+Ab)c^2\right)x^8 + \frac{1}{7}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^7 + \frac{1}{3}Aa^3x^3 + \frac{1}{6}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^6 + \frac{3}{5}\left(Ba^2b+Aab^2+Aa^2c\right)x^5 + \frac{1}{4}\left(Ba^3+3Aa^2b\right)x^4$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/10*B*c^3*x^10 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/8*(B*b^2*c + (B*a + A*b)
*c^2)*x^8 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^7 + 1/3*A*a^
3*x^3 + 1/6*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/5*(B*a^2*b
 + A*a*b^2 + A*a^2*c)*x^5 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.16

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{1}{10}Bc^3x^{10} + \frac{1}{3}Bbc^2x^9 + \frac{1}{9}Ac^3x^9 + \frac{3}{8}Bb^2cx^8 + \frac{3}{8}Bac^2x^8 + \frac{3}{8}Abc^2x^8 + \frac{1}{7}Bb^3x^7 + \frac{6}{7}Babcx^7 + \frac{3}{7}Ab^2cx^7 + \frac{3}{7}Aac^2x^7 + \frac{1}{2}Bab^2x^6 + \frac{1}{6}Ab^3x^6 + \frac{1}{2}Ba^2cx^6 + Aabcx^6 + \frac{3}{5}Ba^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{3}{5}Aa^2cx^5 + \frac{1}{4}Ba^3x^4 + \frac{3}{4}Aa^2bx^4 + \frac{1}{3}Aa^3x^3$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/10*B*c^3*x^10 + 1/3*B*b*c^2*x^9 + 1/9*A*c^3*x^9 + 3/8*B*b^2*c*x^8 + 3/8*
B*a*c^2*x^8 + 3/8*A*b*c^2*x^8 + 1/7*B*b^3*x^7 + 6/7*B*a*b*c*x^7 + 3/7*A*b^
2*c*x^7 + 3/7*A*a*c^2*x^7 + 1/2*B*a*b^2*x^6 + 1/6*A*b^3*x^6 + 1/2*B*a^2*c*
x^6 + A*a*b*c*x^6 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/5*A*a^2*c*x^5 +
1/4*B*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/3*A*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx = x^6\left(\frac{B\,c\,a^2}{2}+\frac{B\,a\,b^2}{2}+A\,c\,a\,b+\frac{A\,b^3}{6}\right) + x^7\left(\frac{B\,b^3}{7}+\frac{3\,A\,b^2\,c}{7}+\frac{6\,B\,a\,b\,c}{7}+\frac{3\,A\,a\,c^2}{7}\right) + x^4\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)+x^9\left(\frac{A\,c^3}{9}+\frac{B\,b\,c^2}{3}\right) + x^5\left(\frac{3\,B\,a^2\,b}{5}+\frac{3\,A\,c\,a^2}{5}+\frac{3\,A\,a\,b^2}{5}\right) + x^8\left(\frac{3\,B\,b^2\,c}{8}+\frac{3\,A\,b\,c^2}{8}+\frac{3\,B\,a\,c^2}{8}\right) + \frac{A\,a^3\,x^3}{3}+\frac{B\,c^3\,x^{10}}{10}$$

input
```
int(x^2*(A + B*x)*(a + b*x + c*x^2)^3,x)
```

output
```
x^6*((A*b^3)/6 + (B*a*b^2)/2 + (B*a^2*c)/2 + A*a*b*c) + x^7*((B*b^3)/7 + (
3*A*a*c^2)/7 + (3*A*b^2*c)/7 + (6*B*a*b*c)/7) + x^4*((B*a^3)/4 + (3*A*a^2*
b)/4) + x^9*((A*c^3)/9 + (B*b*c^2)/3) + x^5*((3*A*a*b^2)/5 + (3*A*a^2*c)/5
 + (3*B*a^2*b)/5) + x^8*((3*A*b*c^2)/8 + (3*B*a*c^2)/8 + (3*B*b^2*c)/8) +
(A*a^3*x^3)/3 + (B*c^3*x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$
$$=\frac{x^3(252b\,c^3x^7+280a\,c^3x^6+840b^2c^2x^6+1890ab\,c^2x^5+945b^3c\,x^5+1080a^2c^2x^4+3240a\,b^2c\,x^4+360b^4x^4}{2520}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^3,x)
```

output
```
(x**3*(840*a**4 + 2520*a**3*b*x + 1512*a**3*c*x**2 + 3024*a**2*b**2*x**2 +
 3780*a**2*b*c*x**3 + 1080*a**2*c**2*x**4 + 1680*a*b**3*x**3 + 3240*a*b**2
*c*x**4 + 1890*a*b*c**2*x**5 + 280*a*c**3*x**6 + 360*b**4*x**4 + 945*b**3*
c*x**5 + 840*b**2*c**2*x**6 + 252*b*c**3*x**7))/2520
```

3.26 $\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 166

$$\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{1}{2}a^3Ax^2+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4+\frac{1}{5}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^5+\frac{1}{6}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^6+\frac{3}{7}c\left(b^2B+Abc+aBc\right)x^7+\frac{1}{8}c^2(3bB+Ac)x^8+\frac{1}{9}Bc^3x^9$$

output
```
1/2*a^3*A*x^2+1/3*a^2*(3*A*b+B*a)*x^3+3/4*a*(a*b*B+A*(a*c+b^2))*x^4+1/5*(3
*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^5+1/6*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b
^3)*x^6+3/7*c*(A*b*c+B*a*c+B*b^2)*x^7+1/8*c^2*(A*c+3*B*b)*x^8+1/9*B*c^3*x^
9
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{1}{2}a^3Ax^2+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4+\frac{1}{5}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^5+\frac{1}{6}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^6+\frac{3}{7}c\left(b^2B+Abc+aBc\right)x^7+\frac{1}{8}c^2(3bB+Ac)x^8+\frac{1}{9}Bc^3x^9$$

input
```
Integrate[x*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x
^4)/4 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^7)/7
+ (c^2*(3*b*B + A*c)*x^8)/8 + (B*c^3*x^9)/9
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^3Ax+a^2x^2(aB+3Ab)+3cx^6\left(aBc+Abc+b^2B\right)+3ax^3\left(A\left(ac+b^2\right)+abB\right)+x^5\left(3aAc^2+6abBc+3Ab\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^3Ax^2 + \frac{1}{3}a^2x^3(aB + 3Ab) + \frac{3}{7}cx^7\left(aBc + Abc + b^2B\right) + \frac{3}{4}ax^4\left(A\left(ac + b^2\right) + abB\right) + \frac{1}{6}x^6\left(3aAc^2 + 6abBc + 3Ab^2c + b^3B\right) + \frac{1}{5}x^5\left(A\left(6abc + b^3\right) + 3aB\left(ac + b^2\right)\right) + \frac{1}{8}c^2x^8(Ac + 3bB) + \frac{1}{9}Bc^3x^9$$

input
```
Int[x*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x
^4)/4 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^7)/7
+ (c^2*(3*b*B + A*c)*x^8)/8 + (B*c^3*x^9)/9
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

method	result
norman	$\frac{B\,c^3x^9}{9} + \left(\frac{1}{8}A\,c^3 + \frac{3}{8}Bb\,c^2\right)x^8 + \left(\frac{3}{7}Ab\,c^2 + \frac{3}{7}Ba\,c^2 + \frac{3}{7}B\,b^2c\right)x^7 + \left(\frac{1}{2}Aa\,c^2 + \frac{1}{2}A\,b^2c + Babc\right.$
gosper	$\frac{1}{9}B\,c^3x^9 + \frac{1}{8}x^8A\,c^3 + \frac{3}{8}x^8Bb\,c^2 + \frac{3}{7}x^7Ab\,c^2 + \frac{3}{7}x^7Ba\,c^2 + \frac{3}{7}x^7B\,b^2c + \frac{1}{2}x^6Aa\,c^2 + \frac{1}{2}x^6A\,b^2c$
risch	$\frac{1}{9}B\,c^3x^9 + \frac{1}{8}x^8A\,c^3 + \frac{3}{8}x^8Bb\,c^2 + \frac{3}{7}x^7Ab\,c^2 + \frac{3}{7}x^7Ba\,c^2 + \frac{3}{7}x^7B\,b^2c + \frac{1}{2}x^6Aa\,c^2 + \frac{1}{2}x^6A\,b^2c$
parallelrisch	$\frac{1}{9}B\,c^3x^9 + \frac{1}{8}x^8A\,c^3 + \frac{3}{8}x^8Bb\,c^2 + \frac{3}{7}x^7Ab\,c^2 + \frac{3}{7}x^7Ba\,c^2 + \frac{3}{7}x^7B\,b^2c + \frac{1}{2}x^6Aa\,c^2 + \frac{1}{2}x^6A\,b^2c$
orering	$x^2\left(280B\,c^3x^7+315A\,c^3x^6+945Bb\,c^2x^6+1080Ab\,c^2x^5+1080Ba\,c^2x^5+1080B\,b^2c\,x^5+1260Aa\,c^2x^4+1260A\,b^2c\,x^4+2520Bab\right.$
default	$\frac{B\,c^3x^9}{9} + \frac{\left(A\,c^3+3Bb\,c^2\right)x^8}{8} + \frac{\left(3Ab\,c^2+B\left(a\,c^2+2b^2c+c\left(2ac+b^2\right)\right)\right)x^7}{7} + \frac{\left(A\left(a\,c^2+2b^2c+c\left(2ac+b^2\right)\right)+B\left(4abc+b\left(2ac+\right.\right.\right.}{6}$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*B*c^3*x^9+(1/8*A*c^3+3/8*B*b*c^2)*x^8+(3/7*A*b*c^2+3/7*B*a*c^2+3/7*B*b
^2*c)*x^7+(1/2*A*a*c^2+1/2*A*b^2*c+B*a*b*c+1/6*B*b^3)*x^6+(6/5*A*a*b*c+1/5
*A*b^3+3/5*B*a^2*c+3/5*B*a*b^2)*x^5+(3/4*a^2*A*c+3/4*A*a*b^2+3/4*B*a^2*b)*
x^4+(A*a^2*b+1/3*B*a^3)*x^3+1/2*a^3*A*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx =& \frac{1}{9}Bc^3x^9+\frac{1}{8}\left(3Bbc^2+Ac^3\right)x^8\\&+\frac{3}{7}\left(Bb^2c+(Ba+Ab)c^2\right)x^7\\&+\frac{1}{6}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^6\\&+\frac{1}{2}Aa^3x^2\\&+\frac{1}{5}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^5\\&+\frac{3}{4}\left(Ba^2b+Aab^2+Aa^2c\right)x^4+\frac{1}{3}\left(Ba^3+3Aa^2b\right)x^3\end{aligned}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
1/9*B*c^3*x^9 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 3/7*(B*b^2*c + (B*a + A*b)*c
^2)*x^7 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^6 + 1/2*A*a^3*
x^2 + 1/5*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 + 3/4*(B*a^2*b +
 A*a*b^2 + A*a^2*c)*x^4 + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20

$$\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{Aa^3x^2}{2}+\frac{Bc^3x^9}{9}+x^8\left(\frac{Ac^3}{8}+\frac{3Bbc^2}{8}\right)+x^7\cdot\left(\frac{3Abc^2}{7}+\frac{3Bac^2}{7}+\frac{3Bb^2c}{7}\right)+x^6\left(\frac{Aac^2}{2}+\frac{Ab^2c}{2}+Babc+\frac{Bb^3}{6}\right)+x^5\cdot\left(\frac{6Aabc}{5}+\frac{Ab^3}{5}+\frac{3Ba^2c}{5}+\frac{3Bab^2}{5}\right)+x^4\cdot\left(\frac{3Aa^2c}{4}+\frac{3Aab^2}{4}+\frac{3Ba^2b}{4}\right)+x^3\left(Aa^2b+\frac{Ba^3}{3}\right)$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x+a)**3,x)
```

output
```
A*a**3*x**2/2 + B*c**3*x**9/9 + x**8*(A*c**3/8 + 3*B*b*c**2/8) + x**7*(3*A
*b*c**2/7 + 3*B*a*c**2/7 + 3*B*b**2*c/7) + x**6*(A*a*c**2/2 + A*b**2*c/2 +
 B*a*b*c + B*b**3/6) + x**5*(6*A*a*b*c/5 + A*b**3/5 + 3*B*a**2*c/5 + 3*B*a
*b**2/5) + x**4*(3*A*a**2*c/4 + 3*A*a*b**2/4 + 3*B*a**2*b/4) + x**3*(A*a**
2*b + B*a**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{1}{9}\,Bc^3x^9+\frac{1}{8}\,(3\,Bbc^2+Ac^3)x^8+\frac{3}{7}\,(Bb^2c+(Ba+Ab)c^2)x^7+\frac{1}{6}\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{5}\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^5+\frac{3}{4}\,(Ba^2b+Aab^2+Aa^2c)x^4+\frac{1}{3}\,(Ba^3+3\,Aa^2b)x^3$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/9*B*c^3*x^9 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 3/7*(B*b^2*c + (B*a + A*b)*c
^2)*x^7 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^6 + 1/2*A*a^3*
x^2 + 1/5*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 + 3/4*(B*a^2*b +
 A*a*b^2 + A*a^2*c)*x^4 + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.15

$$\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{1}{9}Bc^3x^9 + \frac{3}{8}Bbc^2x^8 + \frac{1}{8}Ac^3x^8 + \frac{3}{7}Bb^2cx^7 + \frac{3}{7}Bac^2x^7 + \frac{3}{7}Abc^2x^7 + \frac{1}{6}Bb^3x^6 + Babcx^6 + \frac{1}{2}Ab^2cx^6 + \frac{1}{2}Aac^2x^6 + \frac{3}{5}Bab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{5}Ba^2cx^5 + \frac{6}{5}Aabcx^5 + \frac{3}{4}Ba^2bx^4 + \frac{3}{4}Aab^2x^4 + \frac{3}{4}Aa^2cx^4 + \frac{1}{3}Ba^3x^3 + Aa^2bx^3 + \frac{1}{2}Aa^3x^2$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/9*B*c^3*x^9 + 3/8*B*b*c^2*x^8 + 1/8*A*c^3*x^8 + 3/7*B*b^2*c*x^7 + 3/7*B*
a*c^2*x^7 + 3/7*A*b*c^2*x^7 + 1/6*B*b^3*x^6 + B*a*b*c*x^6 + 1/2*A*b^2*c*x^
6 + 1/2*A*a*c^2*x^6 + 3/5*B*a*b^2*x^5 + 1/5*A*b^3*x^5 + 3/5*B*a^2*c*x^5 +
6/5*A*a*b*c*x^5 + 3/4*B*a^2*b*x^4 + 3/4*A*a*b^2*x^4 + 3/4*A*a^2*c*x^4 + 1/
3*B*a^3*x^3 + A*a^2*b*x^3 + 1/2*A*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx = x^5\left(\frac{3\,B\,c\,a^2}{5}+\frac{3\,B\,a\,b^2}{5}+\frac{6\,A\,c\,a\,b}{5}+\frac{A\,b^3}{5}\right)+x^6\left(\frac{B\,b^3}{6}+\frac{A\,b^2\,c}{2}+B\,a\,b\,c+\frac{A\,a\,c^2}{2}\right)+x^3\left(\frac{B\,a^3}{3}+A\,b\,a^2\right)+x^8\left(\frac{A\,c^3}{8}+\frac{3\,B\,b\,c^2}{8}\right)+x^4\left(\frac{3\,B\,a^2\,b}{4}+\frac{3\,A\,c\,a^2}{4}+\frac{3\,A\,a\,b^2}{4}\right)+x^7\left(\frac{3\,B\,b^2\,c}{7}+\frac{3\,A\,b\,c^2}{7}+\frac{3\,B\,a\,c^2}{7}\right)+\frac{A\,a^3\,x^2}{2}+\frac{B\,c^3\,x^9}{9}$$

input
```
int(x*(A + B*x)*(a + b*x + c*x^2)^3,x)
```

output
```
x^5*((A*b^3)/5 + (3*B*a*b^2)/5 + (3*B*a^2*c)/5 + (6*A*a*b*c)/5) + x^6*((B*
b^3)/6 + (A*a*c^2)/2 + (A*b^2*c)/2 + B*a*b*c) + x^3*((B*a^3)/3 + A*a^2*b)
+ x^8*((A*c^3)/8 + (3*B*b*c^2)/8) + x^4*((3*A*a*b^2)/4 + (3*A*a^2*c)/4 + (
3*B*a^2*b)/4) + x^7*((3*A*b*c^2)/7 + (3*B*a*c^2)/7 + (3*B*b^2*c)/7) + (A*a
^3*x^2)/2 + (B*c^3*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{x^2(280b\,c^3x^7+315a\,c^3x^6+945b^2c^2x^6+2160ab\,c^2x^5+1080b^3c\,x^5+1260a^2c^2x^4+3780a\,b^2c\,x^4+420b^4x}{2520}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^3,x)
```

output
```
(x**2*(1260*a**4 + 3360*a**3*b*x + 1890*a**3*c*x**2 + 3780*a**2*b**2*x**2
+ 4536*a**2*b*c*x**3 + 1260*a**2*c**2*x**4 + 2016*a*b**3*x**3 + 3780*a*b**
2*c*x**4 + 2160*a*b*c**2*x**5 + 315*a*c**3*x**6 + 420*b**4*x**4 + 1080*b**
3*c*x**5 + 945*b**2*c**2*x**6 + 280*b*c**3*x**7))/2520
```

3.27 $\int (A+Bx) \left(a+bx+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 158

$$\begin{aligned}\int (A+Bx) \left(a+bx+cx^2\right)^3 \, dx = {} & a^3Ax+\frac{1}{2}a^2(3Ab+aB)x^2+a\left(abB+A\left(b^2+ac\right)\right)x^3 \\ & +\frac{1}{4}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^4 \\ & +\frac{1}{5}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^5 \\ & +\frac{1}{2}c\left(b^2B+Abc+aBc\right)x^6 \\ & +\frac{1}{7}c^2(3bB+Ac)x^7+\frac{1}{8}Bc^3x^8\end{aligned}$$

output
```
a^3*A*x+1/2*a^2*(3*A*b+B*a)*x^2+a*(a*b*B+A*(a*c+b^2))*x^3+1/4*(3*a*B*(a*c+
b^2)+A*(6*a*b*c+b^3))*x^4+1/5*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^5+1/
2*c*(A*b*c+B*a*c+B*b^2)*x^6+1/7*c^2*(A*c+3*B*b)*x^7+1/8*B*c^3*x^8
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx = a^3Ax+\frac{1}{2}a^2(3Ab+aB)x^2+a\left(abB+A\left(b^2+ac\right)\right)x^3 +\frac{1}{4}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^4 +\frac{1}{5}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^5 +\frac{1}{2}c\left(b^2B+Abc+aBc\right)x^6 +\frac{1}{7}c^2(3bB+Ac)x^7+\frac{1}{8}Bc^3x^8$$

input
```
Integrate[(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^2)/2 + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*
a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*
B*c + 3*a*A*c^2)*x^5)/5 + (c*(b^2*B + A*b*c + a*B*c)*x^6)/2 + (c^2*(3*b*B
+ A*c)*x^7)/7 + (B*c^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^3A+a^2x(aB+3Ab)+3cx^5\left(aBc+Abc+b^2B\right)+3ax^2\left(A\left(ac+b^2\right)+abB\right)+x^4\left(3aAc^2+6abBc+3Ab^2c\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$a^3Ax+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{1}{2}cx^6(aBc+Abc+b^2B)+ax^3(A(ac+b^2)+abB)+\frac{1}{5}x^5(3aAc^2+6abBc+3Ab^2c+b^3B)+\frac{1}{4}x^4(A(6abc+b^3)+3aB(ac+b^2))+\frac{1}{7}c^2x^7(Ac+3bB)+\frac{1}{8}Bc^3x^8$$

input
```
Int[(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^2)/2 + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*
a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*
B*c + 3*a*A*c^2)*x^5)/5 + (c*(b^2*B + A*b*c + a*B*c)*x^6)/2 + (c^2*(3*b*B
+ A*c)*x^7)/7 + (B*c^3*x^8)/8
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

method	result
norman	$\frac{B\,c^3x^8}{8}+\left(\frac{1}{7}A\,c^3+\frac{3}{7}Bb\,c^2\right)x^7+\left(\frac{1}{2}Ab\,c^2+\frac{1}{2}Ba\,c^2+\frac{1}{2}B\,b^2c\right)x^6+\left(\frac{3}{5}Aa\,c^2+\frac{3}{5}A\,b^2c+\frac{6}{5}Bab\right.$
gosper	$\frac{1}{8}B\,c^3x^8+\frac{1}{7}A\,c^3x^7+\frac{3}{7}x^7Bb\,c^2+\frac{1}{2}x^6Ab\,c^2+\frac{1}{2}Ba\,c^2x^6+\frac{1}{2}x^6B\,b^2c+\frac{3}{5}aA\,c^2x^5+\frac{3}{5}x^5A\,b^2c$
risch	$\frac{1}{8}B\,c^3x^8+\frac{1}{7}A\,c^3x^7+\frac{3}{7}x^7Bb\,c^2+\frac{1}{2}x^6Ab\,c^2+\frac{1}{2}Ba\,c^2x^6+\frac{1}{2}x^6B\,b^2c+\frac{3}{5}aA\,c^2x^5+\frac{3}{5}x^5A\,b^2c$
parallelrisch	$\frac{1}{8}B\,c^3x^8+\frac{1}{7}A\,c^3x^7+\frac{3}{7}x^7Bb\,c^2+\frac{1}{2}x^6Ab\,c^2+\frac{1}{2}Ba\,c^2x^6+\frac{1}{2}x^6B\,b^2c+\frac{3}{5}aA\,c^2x^5+\frac{3}{5}x^5A\,b^2c$
orering	$\frac{x(35B\,c^3x^7+40A\,c^3x^6+120Bb\,c^2x^6+140Ab\,c^2x^5+140Ba\,c^2x^5+140B\,b^2c\,x^5+168Aa\,c^2x^4+168A\,b^2c\,x^4+336Babc\,x^4+56B}{}$
default	$\frac{B\,c^3x^8}{8}+\frac{(A\,c^3+3Bb\,c^2)x^7}{7}+\frac{(3Ab\,c^2+B(a\,c^2+2b^2c+c(2ac+b^2)))x^6}{6}+\frac{(A(a\,c^2+2b^2c+c(2ac+b^2))+B(4abc+b(2ac+}{5}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*B*c^3*x^8+(1/7*A*c^3+3/7*B*b*c^2)*x^7+(1/2*A*b*c^2+1/2*B*a*c^2+1/2*B*b
^2*c)*x^6+(3/5*A*a*c^2+3/5*A*b^2*c+6/5*B*a*b*c+1/5*B*b^3)*x^5+(3/2*A*a*b*c
+1/4*A*b^3+3/4*B*a^2*c+3/4*B*a*b^2)*x^4+(A*a^2*c+A*a*b^2+B*a^2*b)*x^3+(3/2
*A*a^2*b+1/2*B*a^3)*x^2+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

$$\begin{aligned}\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx =& \frac{1}{8}\,Bc^3x^8+\frac{1}{7}\left(3\,Bbc^2+Ac^3\right)x^7\\&+\frac{1}{2}\left(Bb^2c+(Ba+Ab)c^2\right)x^6\\&+\frac{1}{5}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^5+Aa^3x\\&+\frac{1}{4}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^4\\&+\left(Ba^2b+Aab^2+Aa^2c\right)x^3+\frac{1}{2}\left(Ba^3+3\,Aa^2b\right)x^2\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*B*c^3*x^8 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 1/2*(B*b^2*c + (B*a + A*b)*c
^2)*x^6 + 1/5*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^5 + A*a^3*x +
1/4*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 + (B*a^2*b + A*a*b^2 +
 A*a^2*c)*x^3 + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.20

$$\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx = Aa^3x+\frac{Bc^3x^8}{8}+x^7\left(\frac{Ac^3}{7}+\frac{3Bbc^2}{7}\right) + x^6\left(\frac{Abc^2}{2}+\frac{Bac^2}{2}+\frac{Bb^2c}{2}\right)+x^5 \cdot\left(\frac{3Aac^2}{5}+\frac{3Ab^2c}{5}+\frac{6Babc}{5}+\frac{Bb^3}{5}\right)+x^4 \cdot\left(\frac{3Aabc}{2}+\frac{Ab^3}{4}+\frac{3Ba^2c}{4}+\frac{3Bab^2}{4}\right) + x^3\left(Aa^2c+Aab^2+Ba^2b\right)+x^2\cdot\left(\frac{3Aa^2b}{2}+\frac{Ba^3}{2}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3,x)
```

output
```
A*a**3*x + B*c**3*x**8/8 + x**7*(A*c**3/7 + 3*B*b*c**2/7) + x**6*(A*b*c**2
/2 + B*a*c**2/2 + B*b**2*c/2) + x**5*(3*A*a*c**2/5 + 3*A*b**2*c/5 + 6*B*a*
b*c/5 + B*b**3/5) + x**4*(3*A*a*b*c/2 + A*b**3/4 + 3*B*a**2*c/4 + 3*B*a*b*
*2/4) + x**3*(A*a**2*c + A*a*b**2 + B*a**2*b) + x**2*(3*A*a**2*b/2 + B*a**
3/2)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

$$\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{1}{8}\,Bc^3x^8+\frac{1}{7}\left(3\,Bbc^2+Ac^3\right)x^7 + \frac{1}{2}\left(Bb^2c+(Ba+Ab)c^2\right)x^6 + \frac{1}{5}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^5+Aa^3x + \frac{1}{4}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^4 + \left(Ba^2b+Aab^2+Aa^2c\right)x^3+\frac{1}{2}\left(Ba^3+3\,Aa^2b\right)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*B*c^3*x^8 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 1/2*(B*b^2*c + (B*a + A*b)*c
^2)*x^6 + 1/5*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^5 + A*a^3*x +
1/4*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 + (B*a^2*b + A*a*b^2 +
 A*a^2*c)*x^3 + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

$$\begin{aligned}\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx = {} & \frac{1}{8}Bc^3x^8+\frac{3}{7}Bbc^2x^7+\frac{1}{7}Ac^3x^7+\frac{1}{2}Bb^2cx^6 \\ & +\frac{1}{2}Bac^2x^6+\frac{1}{2}Abc^2x^6+\frac{1}{5}Bb^3x^5+\frac{6}{5}Babcx^5 \\ & +\frac{3}{5}Ab^2cx^5+\frac{3}{5}Aac^2x^5+\frac{3}{4}Bab^2x^4+\frac{1}{4}Ab^3x^4 \\ & +\frac{3}{4}Ba^2cx^4+\frac{3}{2}Aabcx^4+Ba^2bx^3+Aab^2x^3 \\ & +Aa^2cx^3+\frac{1}{2}Ba^3x^2+\frac{3}{2}Aa^2bx^2+Aa^3x\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/8*B*c^3*x^8 + 3/7*B*b*c^2*x^7 + 1/7*A*c^3*x^7 + 1/2*B*b^2*c*x^6 + 1/2*B*
a*c^2*x^6 + 1/2*A*b*c^2*x^6 + 1/5*B*b^3*x^5 + 6/5*B*a*b*c*x^5 + 3/5*A*b^2*
c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*B*a*b^2*x^4 + 1/4*A*b^3*x^4 + 3/4*B*a^2*c*x^
4 + 3/2*A*a*b*c*x^4 + B*a^2*b*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3 + 1/2*B*a^3*
x^2 + 3/2*A*a^2*b*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx = x^4\left(\frac{3\,B\,c\,a^2}{4}+\frac{3\,B\,a\,b^2}{4}+\frac{3\,A\,c\,a\,b}{2}+\frac{A\,b^3}{4}\right) + x^5\left(\frac{B\,b^3}{5}+\frac{3\,A\,b^2\,c}{5}+\frac{6\,B\,a\,b\,c}{5}+\frac{3\,A\,a\,c^2}{5}\right) + x^2\left(\frac{B\,a^3}{2}+\frac{3\,A\,b\,a^2}{2}\right)+x^7\left(\frac{A\,c^3}{7}+\frac{3\,B\,b\,c^2}{7}\right) + x^3\left(B\,a^2\,b+A\,c\,a^2+A\,a\,b^2\right) + x^6\left(\frac{B\,b^2\,c}{2}+\frac{A\,b\,c^2}{2}+\frac{B\,a\,c^2}{2}\right)+\frac{B\,c^3\,x^8}{8}+A\,a^3\,x$$

input
```
int((A + B*x)*(a + b*x + c*x^2)^3,x)
```

output
```
x^4*((A*b^3)/4 + (3*B*a*b^2)/4 + (3*B*a^2*c)/4 + (3*A*a*b*c)/2) + x^5*((B*
b^3)/5 + (3*A*a*c^2)/5 + (3*A*b^2*c)/5 + (6*B*a*b*c)/5) + x^2*((B*a^3)/2 +
 (3*A*a^2*b)/2) + x^7*((A*c^3)/7 + (3*B*b*c^2)/7) + x^3*(A*a*b^2 + A*a^2*c
 + B*a^2*b) + x^6*((A*b*c^2)/2 + (B*a*c^2)/2 + (B*b^2*c)/2) + (B*c^3*x^8)/
8 + A*a^3*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{x(35b\,c^3x^7+40a\,c^3x^6+120b^2c^2x^6+280ab\,c^2x^5+140b^3c\,x^5+168a^2c^2x^4+504a\,b^2c\,x^4+56b^4x^4+630a}{280}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3,x)
```

output
```
(x*(280*a**4 + 560*a**3*b*x + 280*a**3*c*x**2 + 560*a**2*b**2*x**2 + 630*a
**2*b*c*x**3 + 168*a**2*c**2*x**4 + 280*a*b**3*x**3 + 504*a*b**2*c*x**4 +
280*a*b*c**2*x**5 + 40*a*c**3*x**6 + 56*b**4*x**4 + 140*b**3*c*x**5 + 120*
b**2*c**2*x**6 + 35*b*c**3*x**7))/280
```

3.28 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x} dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x} dx = {} & a^2(3Ab+aB)x+\frac{3}{2}a\left(abB+A\left(b^2+ac\right)\right)x^2 \\ & +\frac{1}{3}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^3 \\ & +\frac{1}{4}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^4 \\ & +\frac{3}{5}c\left(b^2B+Abc+aBc\right)x^5 \\ & +\frac{1}{6}c^2(3bB+Ac)x^6+\frac{1}{7}Bc^3x^7+a^3A\log(x)\end{aligned}$$

output
```
a^2*(3*A*b+B*a)*x+3/2*a*(a*b*B+A*(a*c+b^2))*x^2+1/3*(3*a*B*(a*c+b^2)+A*(6*
a*b*c+b^3))*x^3+1/4*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^4+3/5*c*(A*b*c
+B*a*c+B*b^2)*x^5+1/6*c^2*(A*c+3*B*b)*x^6+1/7*B*c^3*x^7+a^3*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx = a^2(3Ab+aB)x+\frac{3}{2}a\left(abB+A\left(b^2+ac\right)\right)x^2 +\frac{1}{3}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^3 +\frac{1}{4}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^4 +\frac{3}{5}c\left(b^2B+Abc+aBc\right)x^5 +\frac{1}{6}c^2(3bB+Ac)x^6+\frac{1}{7}Bc^3x^7+a^3A\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x,x]
```

output
```
a^2*(3*A*b + a*B)*x + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 +
 a*c) + A*(b^3 + 6*a*b*c))*x^3)/3 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*
A*c^2)*x^4)/4 + (3*c*(b^2*B + A*b*c + a*B*c)*x^5)/5 + (c^2*(3*b*B + A*c)*x
^6)/6 + (B*c^3*x^7)/7 + a^3*A*Log[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{a^3A}{x}+a^2(aB+3Ab)+3cx^4\left(aBc+Abc+b^2B\right)+3ax\left(A\left(ac+b^2\right)+abB\right)+x^3\left(3aAc^2+6abBc+3Ab^2c\right.\right.$$

$$\downarrow \text{2009}$$

$$a^3 A \log(x) + a^2 x(aB + 3Ab) + \frac{3}{5} c x^5 \left(aBc + Abc + b^2 B\right) + \frac{3}{2} a x^2 \left(A\left(ac + b^2\right) + abB\right) + \frac{1}{4} x^4 \left(3aAc^2 + 6abBc + 3Ab^2 c + b^3 B\right) + \frac{1}{3} x^3 \left(A\left(6abc + b^3\right) + 3aB\left(ac + b^2\right)\right) + \frac{1}{6} c^2 x^6 (Ac + 3bB) + \frac{1}{7} B c^3 x^7$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x,x]
```

output
```
a^2*(3*A*b + a*B)*x + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 +
 a*c) + A*(b^3 + 6*a*b*c))*x^3)/3 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*
A*c^2)*x^4)/4 + (3*c*(b^2*B + A*b*c + a*B*c)*x^5)/5 + (c^2*(3*b*B + A*c)*x
^6)/6 + (B*c^3*x^7)/7 + a^3*A*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

method	result
norman	$\left(\frac{1}{6}A c^3 + \frac{1}{2}Bb c^2\right) x^6 + \left(\frac{3}{5}Ab c^2 + \frac{3}{5}Ba c^2 + \frac{3}{5}B b^2 c\right) x^5 + \left(\frac{3}{2}a^2 Ac + \frac{3}{2}Aa b^2 + \frac{3}{2}B a^2 b\right) x^2 +$
default	$\frac{B c^3 x^7}{7} + \frac{A c^3 x^6}{6} + \frac{Bb c^2 x^6}{2} + \frac{3Ab c^2 x^5}{5} + \frac{3Ba c^2 x^5}{5} + \frac{3B b^2 c x^5}{5} + \frac{3Aa c^2 x^4}{4} + \frac{3A b^2 c x^4}{4} + \frac{3Babc x^4}{2} +$
risch	$\frac{B c^3 x^7}{7} + \frac{A c^3 x^6}{6} + \frac{Bb c^2 x^6}{2} + \frac{3Ab c^2 x^5}{5} + \frac{3Ba c^2 x^5}{5} + \frac{3B b^2 c x^5}{5} + \frac{3Aa c^2 x^4}{4} + \frac{3A b^2 c x^4}{4} + \frac{3Babc x^4}{2} +$
parallelrisch	$\frac{B c^3 x^7}{7} + \frac{A c^3 x^6}{6} + \frac{Bb c^2 x^6}{2} + \frac{3Ab c^2 x^5}{5} + \frac{3Ba c^2 x^5}{5} + \frac{3B b^2 c x^5}{5} + \frac{3Aa c^2 x^4}{4} + \frac{3A b^2 c x^4}{4} + \frac{3Babc x^4}{2} +$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(1/6*A*c^3+1/2*B*b*c^2)*x^6+(3/5*A*b*c^2+3/5*B*a*c^2+3/5*B*b^2*c)*x^5+(3/2
*a^2*A*c+3/2*A*a*b^2+3/2*B*a^2*b)*x^2+(3/4*A*a*c^2+3/4*A*b^2*c+3/2*B*a*b*c
+1/4*B*b^3)*x^4+(2*A*a*b*c+1/3*A*b^3+B*a^2*c+B*a*b^2)*x^3+(3*A*a^2*b+B*a^3
)*x+1/7*B*c^3*x^7+a^3*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx = \frac{1}{7}\,Bc^3x^7 + \frac{1}{6}\left(3\,Bbc^2 + Ac^3\right)x^6 + \frac{3}{5}\left(Bb^2c + (Ba+Ab)c^2\right)x^5 + \frac{1}{4}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^4 + Aa^3\log\left(x\right) + \frac{1}{3}\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x^3 + \frac{3}{2}\left(Ba^2b + Aab^2 + Aa^2c\right)x^2 + \left(Ba^3 + 3\,Aa^2b\right)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x,x, algorithm="fricas")
```

output
```
1/7*B*c^3*x^7 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/5*(B*b^2*c + (B*a + A*b)*c
^2)*x^5 + 1/4*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 + A*a^3*log(
x) + 1/3*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 3/2*(B*a^2*b +
A*a*b^2 + A*a^2*c)*x^2 + (B*a^3 + 3*A*a^2*b)*x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx = Aa^3\log(x) + \frac{Bc^3x^7}{7} + x^6\left(\frac{Ac^3}{6} + \frac{Bbc^2}{2}\right) + x^5\cdot\left(\frac{3Abc^2}{5} + \frac{3Bac^2}{5} + \frac{3Bb^2c}{5}\right) + x^4\cdot\left(\frac{3Aac^2}{4} + \frac{3Ab^2c}{4} + \frac{3Babc}{2} + \frac{Bb^3}{4}\right) + x^3\cdot\left(2Aabc + \frac{Ab^3}{3} + Ba^2c + Bab^2\right) + x^2\cdot\left(\frac{3Aa^2c}{2} + \frac{3Aab^2}{2} + \frac{3Ba^2b}{2}\right) + x\left(3Aa^2b + Ba^3\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x,x)
```

output
```
A*a**3*log(x) + B*c**3*x**7/7 + x**6*(A*c**3/6 + B*b*c**2/2) + x**5*(3*A*b
*c**2/5 + 3*B*a*c**2/5 + 3*B*b**2*c/5) + x**4*(3*A*a*c**2/4 + 3*A*b**2*c/4
 + 3*B*a*b*c/2 + B*b**3/4) + x**3*(2*A*a*b*c + A*b**3/3 + B*a**2*c + B*a*b
**2) + x**2*(3*A*a**2*c/2 + 3*A*a*b**2/2 + 3*B*a**2*b/2) + x*(3*A*a**2*b +
 B*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx = \frac{1}{7}Bc^3x^7 + \frac{1}{6}\left(3Bbc^2 + Ac^3\right)x^6 + \frac{3}{5}\left(Bb^2c + (Ba+Ab)c^2\right)x^5 + \frac{1}{4}\left(Bb^3 + 3Aac^2 + 3\left(2Bab + Ab^2\right)c\right)x^4 + Aa^3\log(x) + \frac{1}{3}\left(3Bab^2 + Ab^3 + 3\left(Ba^2 + 2Aab\right)c\right)x^3 + \frac{3}{2}\left(Ba^2b + Aab^2 + Aa^2c\right)x^2 + \left(Ba^3 + 3Aa^2b\right)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x,x, algorithm="maxima")
```

output
```
1/7*B*c^3*x^7 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/5*(B*b^2*c + (B*a + A*b)*c
^2)*x^5 + 1/4*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 + A*a^3*log(
x) + 1/3*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 3/2*(B*a^2*b +
A*a*b^2 + A*a^2*c)*x^2 + (B*a^3 + 3*A*a^2*b)*x
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx = \frac{1}{7}\,Bc^3x^7 + \frac{1}{2}\,Bbc^2x^6 + \frac{1}{6}\,Ac^3x^6 + \frac{3}{5}\,Bb^2cx^5 + \frac{3}{5}\,Bac^2x^5 + \frac{3}{5}\,Abc^2x^5 + \frac{1}{4}\,Bb^3x^4 + \frac{3}{2}\,Babcx^4 + \frac{3}{4}\,Ab^2cx^4 + \frac{3}{4}\,Aac^2x^4 + Bab^2x^3 + \frac{1}{3}\,Ab^3x^3 + Ba^2cx^3 + 2\,Aabcx^3 + \frac{3}{2}\,Ba^2bx^2 + \frac{3}{2}\,Aab^2x^2 + \frac{3}{2}\,Aa^2cx^2 + Ba^3x + 3\,Aa^2bx + Aa^3\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x,x, algorithm="giac")
```

output
```
1/7*B*c^3*x^7 + 1/2*B*b*c^2*x^6 + 1/6*A*c^3*x^6 + 3/5*B*b^2*c*x^5 + 3/5*B*
a*c^2*x^5 + 3/5*A*b*c^2*x^5 + 1/4*B*b^3*x^4 + 3/2*B*a*b*c*x^4 + 3/4*A*b^2*
c*x^4 + 3/4*A*a*c^2*x^4 + B*a*b^2*x^3 + 1/3*A*b^3*x^3 + B*a^2*c*x^3 + 2*A*
a*b*c*x^3 + 3/2*B*a^2*b*x^2 + 3/2*A*a*b^2*x^2 + 3/2*A*a^2*c*x^2 + B*a^3*x
+ 3*A*a^2*b*x + A*a^3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 10.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx = x^3\left(B\,c\,a^2+B\,a\,b^2+2\,A\,c\,a\,b+\frac{A\,b^3}{3}\right) + x^4\left(\frac{B\,b^3}{4}+\frac{3\,A\,b^2\,c}{4}+\frac{3\,B\,a\,b\,c}{2}+\frac{3\,A\,a\,c^2}{4}\right) + x\left(B\,a^3+3\,A\,b\,a^2\right)+x^6\left(\frac{A\,c^3}{6}+\frac{B\,b\,c^2}{2}\right) + x^2\left(\frac{3\,B\,a^2\,b}{2}+\frac{3\,A\,c\,a^2}{2}+\frac{3\,A\,a\,b^2}{2}\right) + x^5\left(\frac{3\,B\,b^2\,c}{5}+\frac{3\,A\,b\,c^2}{5}+\frac{3\,B\,a\,c^2}{5}\right) + \frac{B\,c^3\,x^7}{7}+A\,a^3\,\ln\left(x\right)$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x,x)
```

output
```
x^3*((A*b^3)/3 + B*a*b^2 + B*a^2*c + 2*A*a*b*c) + x^4*((B*b^3)/4 + (3*A*a*
c^2)/4 + (3*A*b^2*c)/4 + (3*B*a*b*c)/2) + x*(B*a^3 + 3*A*a^2*b) + x^6*((A*
c^3)/6 + (B*b*c^2)/2) + x^2*((3*A*a*b^2)/2 + (3*A*a^2*c)/2 + (3*B*a^2*b)/2
) + x^5*((3*A*b*c^2)/5 + (3*B*a*c^2)/5 + (3*B*b^2*c)/5) + (B*c^3*x^7)/7 +
A*a^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx = \log(x)\,a^4+4a^3bx+\frac{3a^3c\,x^2}{2}+3a^2b^2x^2+3a^2bc\,x^3 + \frac{3a^2c^2x^4}{4}+\frac{4a\,b^3x^3}{3}+\frac{9a\,b^2c\,x^4}{4}+\frac{6ab\,c^2x^5}{5} + \frac{a\,c^3x^6}{6}+\frac{b^4x^4}{4}+\frac{3b^3c\,x^5}{5}+\frac{b^2c^2x^6}{2}+\frac{b\,c^3x^7}{7}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x,x)
```

output
```
(420*log(x)*a**4 + 1680*a**3*b*x + 630*a**3*c*x**2 + 1260*a**2*b**2*x**2 +
 1260*a**2*b*c*x**3 + 315*a**2*c**2*x**4 + 560*a*b**3*x**3 + 945*a*b**2*c*
x**4 + 504*a*b*c**2*x**5 + 70*a*c**3*x**6 + 105*b**4*x**4 + 252*b**3*c*x**
5 + 210*b**2*c**2*x**6 + 60*b*c**3*x**7)/420
```

3.29 $\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx = &-\frac{a^3A}{x}+3a\big(abB+A\big(b^2+ac\big)\big)\,x\\&+\frac{1}{2}\big(3aB\big(b^2+ac\big)+A\big(b^3+6abc\big)\big)\,x^2\\&+\frac{1}{3}\big(b^3B+3Ab^2c+6abBc+3aAc^2\big)\,x^3\\&+\frac{3}{4}c\big(b^2B+Abc+aBc\big)\,x^4+\frac{1}{5}c^2(3bB+Ac)x^5\\&+\frac{1}{6}Bc^3x^6+a^2(3Ab+aB)\log(x)\end{aligned}$$

output
```
-a^3*A/x+3*a*(a*b*B+A*(a*c+b^2))*x+1/2*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x
^2+1/3*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^3+3/4*c*(A*b*c+B*a*c+B*b^2)
*x^4+1/5*c^2*(A*c+3*B*b)*x^5+1/6*B*c^3*x^6+a^2*(3*A*b+B*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx = -\frac{a^3A}{x}+3a\left(abB+A\left(b^2+ac\right)\right)x+\frac{1}{2}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^2+\frac{1}{3}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^3+\frac{3}{4}c\left(b^2B+Abc+aBc\right)x^4+\frac{1}{5}c^2(3bB+Ac)x^5+\frac{1}{6}Bc^3x^6+a^2(3Ab+aB)\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^2,x]
```

output
```
-((a^3*A)/x) + 3*a*(a*b*B + A*(b^2 + a*c))*x + ((3*a*B*(b^2 + a*c) + A*(b^
3 + 6*a*b*c))*x^2)/2 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^3)/3
 + (3*c*(b^2*B + A*b*c + a*B*c)*x^4)/4 + (c^2*(3*b*B + A*c)*x^5)/5 + (B*c^
3*x^6)/6 + a^2*(3*A*b + a*B)*Log[x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^2}+\frac{a^2(aB+3Ab)}{x}+3cx^3\left(aBc+Abc+b^2B\right)+3a\left(A\left(ac+b^2\right)+abB\right)+x^2\left(3aAc^2+6abBc+3Ab^2c+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{x}+a^2\log(x)(aB+3Ab)+\frac{3}{4}cx^4\left(aBc+Abc+b^2B\right)+3ax\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{3}x^3\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{2}x^2\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{5}c^2x^5(Ac+3bB)+\frac{1}{6}Bc^3x^6$$

input

```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^2,x]
```

output

```
-((a^3*A)/x) + 3*a*(a*b*B + A*(b^2 + a*c))*x + ((3*a*B*(b^2 + a*c) + A*(b^
3 + 6*a*b*c))*x^2)/2 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^3)/3
 + (3*c*(b^2*B + A*b*c + a*B*c)*x^4)/4 + (c^2*(3*b*B + A*c)*x^5)/5 + (B*c^
3*x^6)/6 + a^2*(3*A*b + a*B)*Log[x]
```

Defintions of rubi rules used

rule 1195

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

method	result
norman	$\frac{\left(\frac{1}{5}A\,c^3+\frac{3}{5}Bb\,c^2\right)x^6+\left(\frac{3}{4}Ab\,c^2+\frac{3}{4}Ba\,c^2+\frac{3}{4}B\,b^2c\right)x^5+\left(Aa\,c^2+A\,b^2c+2Babc+\frac{1}{3}B\,b^3\right)x^4+\left(3Aabc+\frac{1}{2}A\,b^3+\frac{3}{2}B\,a^2c+\frac{3}{2}Ba\,b^2\right)}{x}$
default	$\frac{B\,c^3x^6}{6}+\frac{A\,c^3x^5}{5}+\frac{3Bb\,c^2x^5}{5}+\frac{3Ab\,c^2x^4}{4}+\frac{3Ba\,c^2x^4}{4}+\frac{3x^4B\,b^2c}{4}+Aa\,c^2x^3+A\,b^2c\,x^3+2Babc\,x^3+$
risch	$\frac{B\,c^3x^6}{6}+\frac{A\,c^3x^5}{5}+\frac{3Bb\,c^2x^5}{5}+\frac{3Ab\,c^2x^4}{4}+\frac{3Ba\,c^2x^4}{4}+\frac{3x^4B\,b^2c}{4}+Aa\,c^2x^3+A\,b^2c\,x^3+2Babc\,x^3+$
parallelrisch	$10B\,c^3x^7+12A\,c^3x^6+36Bb\,c^2x^6+45Ab\,c^2x^5+45Ba\,c^2x^5+45B\,b^2c\,x^5+60Aa\,c^2x^4+60A\,b^2c\,x^4+120Babc\,x^4+20B\,b^3x^4+180$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
((1/5*A*c^3+3/5*B*b*c^2)*x^6+(3/4*A*b*c^2+3/4*B*a*c^2+3/4*B*b^2*c)*x^5+(A*
a*c^2+A*b^2*c+2*B*a*b*c+1/3*B*b^3)*x^4+(3*A*a*b*c+1/2*A*b^3+3/2*B*a^2*c+3/
2*B*a*b^2)*x^3+(3*A*a^2*c+3*A*a*b^2+3*B*a^2*b)*x^2-a^3*A+1/6*B*c^3*x^7)/x+
(3*A*a^2*b+B*a^3)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx = \frac{10\,Bc^3x^7 + 12\,(3\,Bbc^2 + Ac^3)x^6 + 45\,(Bb^2c + (Ba + Ab)c^2)x^5 + 20\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab + Ab^2)c)x}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^2,x, algorithm="fricas")
```

output
```
1/60*(10*B*c^3*x^7 + 12*(3*B*b*c^2 + A*c^3)*x^6 + 45*(B*b^2*c + (B*a + A*b
)*c^2)*x^5 + 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 - 60*A*a^3
 + 30*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 180*(B*a^2*b + A*a
*b^2 + A*a^2*c)*x^2 + 60*(B*a^3 + 3*A*a^2*b)*x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx = -\frac{Aa^3}{x}+\frac{Bc^3x^6}{6}+a^2\cdot(3Ab+Ba)\log(x) + x^5\left(\frac{Ac^3}{5}+\frac{3Bbc^2}{5}\right)+x^4 \cdot\left(\frac{3Abc^2}{4}+\frac{3Bac^2}{4}+\frac{3Bb^2c}{4}\right) + x^3\left(Aac^2+Ab^2c+2Babc+\frac{Bb^3}{3}\right) + x^2\cdot\left(3Aabc+\frac{Ab^3}{2}+\frac{3Ba^2c}{2}+\frac{3Bab^2}{2}\right) + x\left(3Aa^2c+3Aab^2+3Ba^2b\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**2,x)
```

output
```
-A*a**3/x + B*c**3*x**6/6 + a**2*(3*A*b + B*a)*log(x) + x**5*(A*c**3/5 + 3
*B*b*c**2/5) + x**4*(3*A*b*c**2/4 + 3*B*a*c**2/4 + 3*B*b**2*c/4) + x**3*(A
*a*c**2 + A*b**2*c + 2*B*a*b*c + B*b**3/3) + x**2*(3*A*a*b*c + A*b**3/2 +
3*B*a**2*c/2 + 3*B*a*b**2/2) + x*(3*A*a**2*c + 3*A*a*b**2 + 3*B*a**2*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx = \frac{1}{6}\,Bc^3x^6+\frac{1}{5}\,\left(3\,Bbc^2+Ac^3\right)x^5 + \frac{3}{4}\,\left(Bb^2c+(Ba+Ab)c^2\right)x^4 + \frac{1}{3}\,\left(Bb^3+3\,Aac^2+3\,\left(2\,Bab+Ab^2\right)c\right)x^3-\frac{Aa^3}{x} + \frac{1}{2}\,\left(3\,Bab^2+Ab^3+3\,\left(Ba^2+2\,Aab\right)c\right)x^2 + 3\,\left(Ba^2b+Aab^2+Aa^2c\right)x+\left(Ba^3+3\,Aa^2b\right)\log(x)$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/6*B*c^3*x^6 + 1/5*(3*B*b*c^2 + A*c^3)*x^5 + 3/4*(B*b^2*c + (B*a + A*b)*c
^2)*x^4 + 1/3*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^3 - A*a^3/x +
1/2*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^2 + 3*(B*a^2*b + A*a*b^2
 + A*a^2*c)*x + (B*a^3 + 3*A*a^2*b)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx = \frac{1}{6}\,Bc^3x^6+\frac{3}{5}\,Bbc^2x^5+\frac{1}{5}\,Ac^3x^5+\frac{3}{4}\,Bb^2cx^4+\frac{3}{4}\,Bac^2x^4 +\frac{3}{4}\,Abc^2x^4+\frac{1}{3}\,Bb^3x^3+2\,Babcx^3+Ab^2cx^3+Aac^2x^3 +\frac{3}{2}\,Bab^2x^2+\frac{1}{2}\,Ab^3x^2+\frac{3}{2}\,Ba^2cx^2+3\,Aabcx^2+3\,Ba^2bx +3\,Aab^2x+3\,Aa^2cx-\frac{Aa^3}{x}+\left(Ba^3+3\,Aa^2b\right)\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^2,x, algorithm="giac")
```

output
```
1/6*B*c^3*x^6 + 3/5*B*b*c^2*x^5 + 1/5*A*c^3*x^5 + 3/4*B*b^2*c*x^4 + 3/4*B*
a*c^2*x^4 + 3/4*A*b*c^2*x^4 + 1/3*B*b^3*x^3 + 2*B*a*b*c*x^3 + A*b^2*c*x^3
+ A*a*c^2*x^3 + 3/2*B*a*b^2*x^2 + 1/2*A*b^3*x^2 + 3/2*B*a^2*c*x^2 + 3*A*a*
b*c*x^2 + 3*B*a^2*b*x + 3*A*a*b^2*x + 3*A*a^2*c*x - A*a^3/x + (B*a^3 + 3*A
*a^2*b)*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx = x^2\left(\frac{3\,B\,c\,a^2}{2}+\frac{3\,B\,a\,b^2}{2}+3\,A\,c\,a\,b+\frac{A\,b^3}{2}\right) + x^3\left(\frac{B\,b^3}{3}+A\,b^2\,c+2\,B\,a\,b\,c+A\,a\,c^2\right) + x\left(3\,B\,a^2\,b+3\,A\,c\,a^2+3\,A\,a\,b^2\right) + x^5\left(\frac{A\,c^3}{5}+\frac{3\,B\,b\,c^2}{5}\right)+\ln(x)\,\left(B\,a^3+3\,A\,b\,a^2\right) + x^4\left(\frac{3\,B\,b^2\,c}{4}+\frac{3\,A\,b\,c^2}{4}+\frac{3\,B\,a\,c^2}{4}\right)-\frac{A\,a^3}{x}+\frac{B\,c^3\,x^6}{6}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^2,x)
```

output
```
x^2*((A*b^3)/2 + (3*B*a*b^2)/2 + (3*B*a^2*c)/2 + 3*A*a*b*c) + x^3*((B*b^3)
/3 + A*a*c^2 + A*b^2*c + 2*B*a*b*c) + x*(3*A*a*b^2 + 3*A*a^2*c + 3*B*a^2*b
) + x^5*((A*c^3)/5 + (3*B*b*c^2)/5) + log(x)*(B*a^3 + 3*A*a^2*b) + x^4*((3
*A*b*c^2)/4 + (3*B*a*c^2)/4 + (3*B*b^2*c)/4) - (A*a^3)/x + (B*c^3*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx = \frac{240\log(x)\,a^3bx-60a^4+180a^3c\,x^2+360a^2b^2x^2+270a^2bc\,x^3+60a^2c^2x^4+120a\,b^3x^3+180a\,b^2c\,x^4+90}{60x}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^2,x)
```

output
```
(240*log(x)*a**3*b*x - 60*a**4 + 180*a**3*c*x**2 + 360*a**2*b**2*x**2 + 27
0*a**2*b*c*x**3 + 60*a**2*c**2*x**4 + 120*a*b**3*x**3 + 180*a*b**2*c*x**4
+ 90*a*b*c**2*x**5 + 12*a*c**3*x**6 + 20*b**4*x**4 + 45*b**3*c*x**5 + 36*b
**2*c**2*x**6 + 10*b*c**3*x**7)/(60*x)
```

3.30 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx = -\frac{a^3A}{2x^2}-\frac{a^2(3Ab+aB)}{x}+\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x+\frac{1}{2}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^2+c\left(b^2B+Abc+aBc\right)x^3+\frac{1}{4}c^2(3bB+Ac)x^4+\frac{1}{5}Bc^3x^5+3a\left(abB+A\left(b^2+ac\right)\right)\log(x)$$

output
```
-1/2*a^3*A/x^2-a^2*(3*A*b+B*a)/x+(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x+1/2*(
3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^2+c*(A*b*c+B*a*c+B*b^2)*x^3+1/4*c^2
*(A*c+3*B*b)*x^4+1/5*B*c^3*x^5+3*a*(a*b*B+A*(a*c+b^2))*ln(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx = -\frac{a^3A}{2x^2}-\frac{a^2(3Ab+aB)}{x} + \left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x + \frac{1}{2}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^2 + c\left(b^2B+Abc+aBc\right)x^3+\frac{1}{4}c^2(3bB+Ac)x^4 + \frac{1}{5}Bc^3x^5+3a\left(abB+A\left(b^2+ac\right)\right)\log(x)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^3,x]
```

output
```
-1/2*(a^3*A)/x^2 - (a^2*(3*A*b + a*B))/x + (3*a*B*(b^2 + a*c) + A*(b^3 + 6
*a*b*c))*x + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^2)/2 + c*(b^2*
B + A*b*c + a*B*c)*x^3 + (c^2*(3*b*B + A*c)*x^4)/4 + (B*c^3*x^5)/5 + 3*a*(
a*b*B + A*(b^2 + a*c))*Log[x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^3}+\frac{a^2(aB+3Ab)}{x^2}+3cx^2\left(aBc+Abc+b^2B\right)+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x}+x\left(3aAc^2+6abBc+3Ab^2c+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{2x^2}-\frac{a^2(aB+3Ab)}{x}+cx^3\left(aBc+Abc+b^2B\right)+3a\log(x)\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{2}x^2\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+x\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{4}c^2x^4(Ac+3bB)+\frac{1}{5}Bc^3x^5$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^3,x]
```

output
```
-1/2*(a^3*A)/x^2 - (a^2*(3*A*b + a*B))/x + (3*a*B*(b^2 + a*c) + A*(b^3 + 6
*a*b*c))*x + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^2)/2 + c*(b^2*
B + A*b*c + a*B*c)*x^3 + (c^2*(3*b*B + A*c)*x^4)/4 + (B*c^3*x^5)/5 + 3*a*(
a*b*B + A*(b^2 + a*c))*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08

method	result
norman	$\frac{\left(\frac{1}{4}A\,c^3+\frac{3}{4}Bb\,c^2\right)x^6+\left(\frac{3}{2}Aa\,c^2+\frac{3}{2}A\,b^2c+3Babc+\frac{1}{2}B\,b^3\right)x^4+\left(-3A\,a^2b-B\,a^3\right)x+\left(Ab\,c^2+Ba\,c^2+B\,b^2c\right)x^5+\left(6Aabc+A\,b^3+3\right.}{x^2}$
default	$\frac{B\,c^3x^5}{5}+\frac{A\,c^3x^4}{4}+\frac{3Bb\,c^2x^4}{4}+Ab\,c^2x^3+Ba\,c^2x^3+B\,b^2c\,x^3+\frac{3Aa\,c^2x^2}{2}+\frac{3A\,b^2c\,x^2}{2}+3Babc\,x^2+$
risch	$\frac{B\,c^3x^5}{5}+\frac{A\,c^3x^4}{4}+\frac{3Bb\,c^2x^4}{4}+Ab\,c^2x^3+Ba\,c^2x^3+B\,b^2c\,x^3+\frac{3Aa\,c^2x^2}{2}+\frac{3A\,b^2c\,x^2}{2}+3Babc\,x^2+$
parallelrisch	$\frac{4B\,c^3x^7+5A\,c^3x^6+15Bb\,c^2x^6+20Ab\,c^2x^5+20Ba\,c^2x^5+20B\,b^2c\,x^5+30Aa\,c^2x^4+30A\,b^2c\,x^4+60Babc\,x^4+10B\,b^3x^4+60A\ln}{20}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
((1/4*A*c^3+3/4*B*b*c^2)*x^6+(3/2*A*a*c^2+3/2*A*b^2*c+3*B*a*b*c+1/2*B*b^3)
*x^4+(-3*A*a^2*b-B*a^3)*x+(A*b*c^2+B*a*c^2+B*b^2*c)*x^5+(6*A*a*b*c+A*b^3+3
*B*a^2*c+3*B*a*b^2)*x^3-1/2*a^3*A+1/5*B*c^3*x^7)/x^2+(3*A*a^2*c+3*A*a*b^2+
3*B*a^2*b)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx = \frac{4\,Bc^3x^7 + 5\,(3\,Bbc^2 + Ac^3)x^6 + 20\,(Bb^2c + (Ba + Ab)c^2)x^5 + 10\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab + Ab^2)c)x^4 +$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^3,x, algorithm="fricas")
```

output
```
1/20*(4*B*c^3*x^7 + 5*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + (B*a + A*b)*
c^2)*x^5 + 10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 - 10*A*a^3 +
 20*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 60*(B*a^2*b + A*a*b^
2 + A*a^2*c)*x^2*log(x) - 20*(B*a^3 + 3*A*a^2*b)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx = {} & \frac{Bc^3x^5}{5} + 3a\left(Aac + Ab^2 + Bab\right)\log\left(x\right) \\ & + x^4\left(\frac{Ac^3}{4} + \frac{3Bbc^2}{4}\right) + x^3\left(Abc^2 + Bac^2 + Bb^2c\right) \\ & + x^2\cdot\left(\frac{3Aac^2}{2} + \frac{3Ab^2c}{2} + 3Babc + \frac{Bb^3}{2}\right) \\ & + x\left(6Aabc + Ab^3 + 3Ba^2c + 3Bab^2\right) \\ & + \frac{-Aa^3 + x(-6Aa^2b - 2Ba^3)}{2x^2}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**3,x)
```

output
```
B*c**3*x**5/5 + 3*a*(A*a*c + A*b**2 + B*a*b)*log(x) + x**4*(A*c**3/4 + 3*B
*b*c**2/4) + x**3*(A*b*c**2 + B*a*c**2 + B*b**2*c) + x**2*(3*A*a*c**2/2 +
3*A*b**2*c/2 + 3*B*a*b*c + B*b**3/2) + x*(6*A*a*b*c + A*b**3 + 3*B*a**2*c
+ 3*B*a*b**2) + (-A*a**3 + x*(-6*A*a**2*b - 2*B*a**3))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx = \frac{1}{5}\,Bc^3x^5 + \frac{1}{4}\left(3\,Bbc^2 + Ac^3\right)x^4 + \left(Bb^2c + (Ba+Ab)c^2\right)x^3 + \frac{1}{2}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^2 + \left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x + 3\left(Ba^2b + Aab^2 + Aa^2c\right)\log\left(x\right) - \frac{Aa^3 + 2\,(Ba^3 + 3\,Aa^2b)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^3,x, algorithm="maxima")
```

output
```
1/5*B*c^3*x^5 + 1/4*(3*B*b*c^2 + A*c^3)*x^4 + (B*b^2*c + (B*a + A*b)*c^2)*
x^3 + 1/2*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^2 + (3*B*a*b^2 + A
*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*log(x) -
 1/2*(A*a^3 + 2*(B*a^3 + 3*A*a^2*b)*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx = \frac{1}{5}Bc^3x^5 + \frac{3}{4}Bbc^2x^4 + \frac{1}{4}Ac^3x^4 + Bb^2cx^3 + Bac^2x^3 + Abc^2x^3 + \frac{1}{2}Bb^3x^2 + 3\,Babcx^2 + \frac{3}{2}Ab^2cx^2 + \frac{3}{2}Aac^2x^2 + 3\,Bab^2x + Ab^3x + 3\,Ba^2cx + 6\,Aabcx + 3\left(Ba^2b + Aab^2 + Aa^2c\right)\log\left(|x|\right) - \frac{Aa^3 + 2\,(Ba^3 + 3\,Aa^2b)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^3,x, algorithm="giac")
```

output
```
1/5*B*c^3*x^5 + 3/4*B*b*c^2*x^4 + 1/4*A*c^3*x^4 + B*b^2*c*x^3 + B*a*c^2*x^
3 + A*b*c^2*x^3 + 1/2*B*b^3*x^2 + 3*B*a*b*c*x^2 + 3/2*A*b^2*c*x^2 + 3/2*A*
a*c^2*x^2 + 3*B*a*b^2*x + A*b^3*x + 3*B*a^2*c*x + 6*A*a*b*c*x + 3*(B*a^2*b
 + A*a*b^2 + A*a^2*c)*log(abs(x)) - 1/2*(A*a^3 + 2*(B*a^3 + 3*A*a^2*b)*x)/
x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx = x^2\left(\frac{B\,b^3}{2} + \frac{3\,A\,b^2\,c}{2} + 3\,B\,a\,b\,c + \frac{3\,A\,a\,c^2}{2}\right) - \frac{x\,(B\,a^3 + 3\,A\,b\,a^2) + \frac{A\,a^3}{2}}{x^2} + x^4\left(\frac{A\,c^3}{4} + \frac{3\,B\,b\,c^2}{4}\right) + x^3\left(B\,b^2\,c + A\,b\,c^2 + B\,a\,c^2\right) + x\left(3\,B\,c\,a^2 + 3\,B\,a\,b^2 + 6\,A\,c\,a\,b + A\,b^3\right) + \ln(x)\,\left(3\,B\,a^2\,b + 3\,A\,c\,a^2 + 3\,A\,a\,b^2\right) + \frac{B\,c^3\,x^5}{5}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^3,x)
```

output
```
x^2*((B*b^3)/2 + (3*A*a*c^2)/2 + (3*A*b^2*c)/2 + 3*B*a*b*c) - (x*(B*a^3 +
3*A*a^2*b) + (A*a^3)/2)/x^2 + x^4*((A*c^3)/4 + (3*B*b*c^2)/4) + x^3*(A*b*c
^2 + B*a*c^2 + B*b^2*c) + x*(A*b^3 + 3*B*a*b^2 + 3*B*a^2*c + 6*A*a*b*c) +
log(x)*(3*A*a*b^2 + 3*A*a^2*c + 3*B*a^2*b) + (B*c^3*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^3}\,dx$$
$$= \frac{60\log(x)\,a^3c\,x^2+120\log(x)\,a^2b^2x^2-10a^4-80a^3bx+180a^2bc\,x^3+30a^2c^2x^4+80a\,b^3x^3+90a\,b^2c\,x^4+}{20x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^3,x)
```

output
```
(60*log(x)*a**3*c*x**2 + 120*log(x)*a**2*b**2*x**2 - 10*a**4 - 80*a**3*b*x
 + 180*a**2*b*c*x**3 + 30*a**2*c**2*x**4 + 80*a*b**3*x**3 + 90*a*b**2*c*x*
*4 + 40*a*b*c**2*x**5 + 5*a*c**3*x**6 + 10*b**4*x**4 + 20*b**3*c*x**5 + 15
*b**2*c**2*x**6 + 4*b*c**3*x**7)/(20*x**2)
```

3.31 $\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^4}\,dx = -\frac{a^3A}{3x^3} - \frac{a^2(3Ab+aB)}{2x^2} - \frac{3a(abB+A(b^2+ac))}{x} + \left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x + \frac{3}{2}c\left(b^2B+Abc+aBc\right)x^2 + \frac{1}{3}c^2(3bB+Ac)x^3 + \frac{1}{4}Bc^3x^4 + \left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)\log(x)$$

output
```
-1/3*a^3*A/x^3-1/2*a^2*(3*A*b+B*a)/x^2-3*a*(a*b*B+A*(a*c+b^2))/x+(3*A*a*c^
2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x+3/2*c*(A*b*c+B*a*c+B*b^2)*x^2+1/3*c^2*(A*c+
3*B*b)*x^3+1/4*B*c^3*x^4+(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^4}\,dx = \frac{-2a^3(2A+3Bx)+x^4(12b^3B+18b^2c(2A+Bx)+6bc^2x(3A+2Bx)+c^3x^2(4A+3Bx))-18a^2x(2bB}{12}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^4,x]
```

output
```
(-2*a^3*(2*A + 3*B*x) + x^4*(12*b^3*B + 18*b^2*c*(2*A + B*x) + 6*b*c^2*x*(
3*A + 2*B*x) + c^3*x^2*(4*A + 3*B*x)) - 18*a^2*x*(2*b*B*x + A*(b + 2*c*x))
 + 18*a*x^2*(B*c*x^2*(4*b + c*x) - 2*A*(b^2 - c^2*x^2)) + 12*(3*a*B*(b^2 +
 a*c) + A*(b^3 + 6*a*b*c))*x^3*Log[x])/(12*x^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^4}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^4}+\frac{a^2(aB+3Ab)}{x^3}+\frac{3a(A(ac+b^2)+abB)}{x^2}+3cx(aBc+Abc+b^2B)+\frac{A(6abc+b^3)+3aB(ac+b^2)}{x}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{3x^3}-\frac{a^2(aB+3Ab)}{2x^2}+\frac{3}{2}cx^2\left(aBc+Abc+b^2B\right)-\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x}+ x\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\log(x)\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{3}c^2x^3(Ac+ 3bB)+\frac{1}{4}Bc^3x^4$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^4,x]
```

output
```
-1/3*(a^3*A)/x^3 - (a^2*(3*A*b + a*B))/(2*x^2) - (3*a*(a*b*B + A*(b^2 + a*
c)))/x + (b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x + (3*c*(b^2*B + A*b
*c + a*B*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^3)/3 + (B*c^3*x^4)/4 + (3*a*B*(b
^2 + a*c) + A*(b^3 + 6*a*b*c))*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

method	result
default	$\frac{B\,c^3x^4}{4}+\frac{A\,c^3x^3}{3}+x^3Bb\,c^2+\frac{3Ab\,c^2x^2}{2}+\frac{3Ba\,c^2x^2}{2}+\frac{3x^2B\,b^2c}{2}+3Aa\,c^2x+3A\,b^2cx+6Babcx+$
norman	$\frac{\left(\frac{1}{3}A\,c^3+Bb\,c^2\right)x^6+\left(-\frac{3}{2}A\,a^2b-\frac{1}{2}B\,a^3\right)x+\left(\frac{3}{2}Ab\,c^2+\frac{3}{2}Ba\,c^2+\frac{3}{2}B\,b^2c\right)x^5+\left(-3a^2Ac-3Aa\,b^2-3B\,a^2b\right)x^2+\left(3Aa\,c^2+3A\,b^2c+\right.}{x^3}$
risch	$\frac{B\,c^3x^4}{4}+\frac{A\,c^3x^3}{3}+x^3Bb\,c^2+\frac{3Ab\,c^2x^2}{2}+\frac{3Ba\,c^2x^2}{2}+\frac{3x^2B\,b^2c}{2}+3Aa\,c^2x+3A\,b^2cx+6Babcx+$
parallelrisch	$\frac{3B\,c^3x^7+4A\,c^3x^6+12Bb\,c^2x^6+18Ab\,c^2x^5+18Ba\,c^2x^5+18B\,b^2c\,x^5+72A\ln(x)x^3abc+12A\ln(x)x^3b^3+36Aa\,c^2x^4+36A\,b^2c\,x}{12}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*c^3*x^4+1/3*A*c^3*x^3+x^3*B*b*c^2+3/2*A*b*c^2*x^2+3/2*B*a*c^2*x^2+3/
2*x^2*B*b^2*c+3*A*a*c^2*x+3*A*b^2*c*x+6*B*a*b*c*x+x*B*b^3-1/3*a^3*A/x^3-1/
2*a^2*(3*A*b+B*a)/x^2+(6*A*a*b*c+A*b^3+3*B*a^2*c+3*B*a*b^2)*ln(x)-3*a*(A*a
*c+A*b^2+B*a*b)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^4}\,dx = \frac{3\,Bc^3x^7+4\,(3\,Bbc^2+Ac^3)x^6+18\,(Bb^2c+(Ba+Ab)c^2)x^5+12\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^4+}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^4,x, algorithm="fricas")
```

output
```
1/12*(3*B*c^3*x^7 + 4*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + (B*a + A*b)*
c^2)*x^5 + 12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 + 12*(3*B*a*
b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3*log(x) - 4*A*a^3 - 36*(B*a^2*b +
A*a*b^2 + A*a^2*c)*x^2 - 6*(B*a^3 + 3*A*a^2*b)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^4}\,dx &= \frac{Bc^3x^4}{4}+x^3\left(\frac{Ac^3}{3}+Bbc^2\right)+x^2\cdot\left(\frac{3Abc^2}{2}+\frac{3Bac^2}{2}+\frac{3Bb^2c}{2}\right)\\ &\quad+x\left(3Aac^2+3Ab^2c+6Babc+Bb^3\right)+\left(6Aabc+Ab^3+3Ba^2c+3Bab^2\right)\log(x)\\ &\quad+\frac{-2Aa^3+x^2(-18Aa^2c-18Aab^2-18Ba^2b)+x(-9Aa^2b-3Ba^3)}{6x^3}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**4,x)
```

output
```
B*c**3*x**4/4 + x**3*(A*c**3/3 + B*b*c**2) + x**2*(3*A*b*c**2/2 + 3*B*a*c*
*2/2 + 3*B*b**2*c/2) + x*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3) +
(6*A*a*b*c + A*b**3 + 3*B*a**2*c + 3*B*a*b**2)*log(x) + (-2*A*a**3 + x**2*
(-18*A*a**2*c - 18*A*a*b**2 - 18*B*a**2*b) + x*(-9*A*a**2*b - 3*B*a**3))/(
6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^4}\,dx$$
$$= \frac{1}{4}\,Bc^3x^4 + \frac{1}{3}\left(3\,Bbc^2 + Ac^3\right)x^3 + \frac{3}{2}\left(Bb^2c + (Ba + Ab)c^2\right)x^2 + \left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x + \left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)\log\left(x\right) - \frac{2\,Aa^3 + 18\,(Ba^2b + Aab^2 + Aa^2c)x^2 + 3\,(Ba^3 + 3\,Aa^2b)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^4,x, algorithm="maxima")
```

output
```
1/4*B*c^3*x^4 + 1/3*(3*B*b*c^2 + A*c^3)*x^3 + 3/2*(B*b^2*c + (B*a + A*b)*c
^2)*x^2 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x + (3*B*a*b^2 + A*b
^3 + 3*(B*a^2 + 2*A*a*b)*c)*log(x) - 1/6*(2*A*a^3 + 18*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^2 + 3*(B*a^3 + 3*A*a^2*b)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^4}\,dx$$
$$=\frac{1}{4}Bc^3x^4+Bbc^2x^3+\frac{1}{3}Ac^3x^3+\frac{3}{2}Bb^2cx^2+\frac{3}{2}Bac^2x^2+\frac{3}{2}Abc^2x^2+Bb^3x+6Babcx+3Ab^2cx+3Aac^2x+\left(3Bab^2+Ab^3+3Ba^2c+6Aabc\right)\log\left(|x|\right)-\frac{2Aa^3+18\left(Ba^2b+Aab^2+Aa^2c\right)x^2+3\left(Ba^3+3Aa^2b\right)x}{6x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^4,x, algorithm="giac")
```

output
```
1/4*B*c^3*x^4 + B*b*c^2*x^3 + 1/3*A*c^3*x^3 + 3/2*B*b^2*c*x^2 + 3/2*B*a*c^
2*x^2 + 3/2*A*b*c^2*x^2 + B*b^3*x + 6*B*a*b*c*x + 3*A*b^2*c*x + 3*A*a*c^2*
x + (3*B*a*b^2 + A*b^3 + 3*B*a^2*c + 6*A*a*b*c)*log(abs(x)) - 1/6*(2*A*a^3
 + 18*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 3*(B*a^3 + 3*A*a^2*b)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^4}\,dx$$
$$=\ln(x)\left(3Bca^2+3Bab^2+6Acab+Ab^3\right)-\frac{x\left(\frac{Ba^3}{2}+\frac{3Aba^2}{2}\right)+\frac{Aa^3}{3}+x^2\left(3Ba^2b+3Aca^2+3Aab^2\right)}{x^3}+x^3\left(\frac{Ac^3}{3}+Bbc^2\right)+x^2\left(\frac{3Bb^2c}{2}+\frac{3Abc^2}{2}+\frac{3Bac^2}{2}\right)+x\left(Bb^3+3Ab^2c+6Babc+3Aac^2\right)+\frac{Bc^3x^4}{4}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^4,x)
```

output
```
log(x)*(A*b^3 + 3*B*a*b^2 + 3*B*a^2*c + 6*A*a*b*c) - (x*((B*a^3)/2 + (3*A*
a^2*b)/2) + (A*a^3)/3 + x^2*(3*A*a*b^2 + 3*A*a^2*c + 3*B*a^2*b))/x^3 + x^3
*((A*c^3)/3 + B*b*c^2) + x^2*((3*A*b*c^2)/2 + (3*B*a*c^2)/2 + (3*B*b^2*c)/
2) + x*(B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*b*c) + (B*c^3*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^4}\,dx$$
$$= \frac{108\log(x)\,a^2bc\,x^3+48\log(x)\,a\,b^3x^3-4a^4-24a^3bx-36a^3c\,x^2-72a^2b^2x^2+36a^2c^2x^4+108a\,b^2c\,x^4+3}{12x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^4,x)
```

output
```
(108*log(x)*a**2*b*c*x**3 + 48*log(x)*a*b**3*x**3 - 4*a**4 - 24*a**3*b*x -
 36*a**3*c*x**2 - 72*a**2*b**2*x**2 + 36*a**2*c**2*x**4 + 108*a*b**2*c*x**
4 + 36*a*b*c**2*x**5 + 4*a*c**3*x**6 + 12*b**4*x**4 + 18*b**3*c*x**5 + 12*
b**2*c**2*x**6 + 3*b*c**3*x**7)/(12*x**3)
```

3.32 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^5}\,dx = -\frac{a^3A}{4x^4}-\frac{a^2(3Ab+aB)}{3x^3}-\frac{3a(abB+A(b^2+ac))}{2x^2} - \frac{3aB(b^2+ac)+A(b^3+6abc)}{x} + 3c\left(b^2B+Abc+aBc\right)x+\frac{1}{2}c^2(3bB+Ac)x^2 + \frac{1}{3}Bc^3x^3+\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)\log(x)$$

output
```
-1/4*a^3*A/x^4-1/3*a^2*(3*A*b+B*a)/x^3-3/2*a*(a*b*B+A*(a*c+b^2))/x^2-(3*a*
B*(a*c+b^2)+A*(6*a*b*c+b^3))/x+3*c*(A*b*c+B*a*c+B*b^2)*x+1/2*c^2*(A*c+3*B*
b)*x^2+1/3*B*c^3*x^3+(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^5}\,dx$$
$$= \frac{-3a^3A-4a^2(3Ab+aB)x-18a(abB+A(b^2+ac))x^2-12(3aB(b^2+ac)+A(b^3+6abc))x^3+36c(b^2}{12x^4}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^5,x]
```

output
```
(-3*a^3*A - 4*a^2*(3*A*b + a*B)*x - 18*a*(a*b*B + A*(b^2 + a*c))*x^2 - 12*
(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^3 + 36*c*(b^2*B + A*b*c + a*B*c)
*x^5 + 6*c^2*(3*b*B + A*c)*x^6 + 4*B*c^3*x^7 + 12*(b^3*B + 3*A*b^2*c + 6*a
*b*B*c + 3*a*A*c^2)*x^4*Log[x])/(12*x^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^5}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{a^3A}{x^5}+\frac{a^2(aB+3Ab)}{x^4}+\frac{3a(A(ac+b^2)+abB)}{x^3}+3c(aBc+Abc+b^2B)+\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{x}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^3 A}{4x^4} - \frac{a^2(aB+3Ab)}{3x^3} - \frac{3a\left(A\left(ac+b^2\right)+abB\right)}{2x^2} + 3cx\left(aBc+Abc+b^2B\right) + \log(x)\left(3aAc^2+6abBc+3Ab^2c+b^3B\right) - \frac{A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)}{x} + \frac{1}{2}c^2x^2(Ac+3bB) + \frac{1}{3}Bc^3x^3$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^5,x]
```

output
```
-1/4*(a^3*A)/x^4 - (a^2*(3*A*b + a*B))/(3*x^3) - (3*a*(a*b*B + A*(b^2 + a*
c)))/(2*x^2) - (3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))/x + 3*c*(b^2*B + A*
b*c + a*B*c)*x + (c^2*(3*b*B + A*c)*x^2)/2 + (B*c^3*x^3)/3 + (b^3*B + 3*A*
b^2*c + 6*a*b*B*c + 3*a*A*c^2)*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02

method	result
default	$\frac{B\,c^3x^3}{3}+\frac{A\,c^3x^2}{2}+\frac{3Bb\,c^2x^2}{2}+3Ab\,c^2x+3Ba\,c^2x+3B\,b^2cx-\frac{a^2(3Ab+Ba)}{3x^3}-\frac{3a\left(Aac+b^2A+abB\right)}{2x^2}-$
norman	$\frac{\left(\frac{1}{2}A\,c^3+\frac{3}{2}Bb\,c^2\right)x^6+\left(-A\,a^2b-\frac{1}{3}B\,a^3\right)x+\left(-\frac{3}{2}a^2Ac-\frac{3}{2}Aa\,b^2-\frac{3}{2}B\,a^2b\right)x^2+\left(3Ab\,c^2+3Ba\,c^2+3B\,b^2c\right)x^5+\left(-6Aabc-A\,b^3-\right.}{x^4}$
risch	$\frac{B\,c^3x^3}{3}+\frac{A\,c^3x^2}{2}+\frac{3Bb\,c^2x^2}{2}+3Ab\,c^2x+3Ba\,c^2x+3B\,b^2cx+\frac{\left(-6Aabc-A\,b^3-3B\,a^2c-3Ba\,b^2\right)x^3+\left(-\right.}{}$
parallelrisch	$\frac{4B\,c^3x^7+6A\,c^3x^6+18Bb\,c^2x^6+36A\ln(x)x^4a\,c^2+36A\ln(x)x^4b^2c+36Ab\,c^2x^5+72B\ln(x)x^4abc+12B\ln(x)x^4b^3+36Ba\,c^2x^5}{12}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*c^3*x^3+1/2*A*c^3*x^2+3/2*B*b*c^2*x^2+3*A*b*c^2*x+3*B*a*c^2*x+3*B*b^
2*c*x-1/3*a^2*(3*A*b+B*a)/x^3-3/2*a*(A*a*c+A*b^2+B*a*b)/x^2-1/4*a^3*A/x^4+
(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*ln(x)-(6*A*a*b*c+A*b^3+3*B*a^2*c+3*B
*a*b^2)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^5}\,dx$$
$$= \frac{4\,Bc^3x^7 + 6\,(3\,Bbc^2 + Ac^3)x^6 + 36\,(Bb^2c + (Ba + Ab)c^2)x^5 + 12\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab + Ab^2)c)x^4\,\mathrm{l}}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*B*c^3*x^7 + 6*(3*B*b*c^2 + A*c^3)*x^6 + 36*(B*b^2*c + (B*a + A*b)*
c^2)*x^5 + 12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4*log(x) - 3*A
*a^3 - 12*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 - 18*(B*a^2*b +
A*a*b^2 + A*a^2*c)*x^2 - 4*(B*a^3 + 3*A*a^2*b)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^5}\,dx = \frac{Bc^3x^3}{3} + x^2\left(\frac{Ac^3}{2} + \frac{3Bbc^2}{2}\right) + x\left(3Abc^2 + 3Bac^2 + 3Bb^2c\right) + \left(3Aac^2 + 3Ab^2c + 6Babc + Bb^3\right)\log(x) + \frac{-3Aa^3 + x^3(-72Aabc - 12Ab^3 - 36Ba^2c - 36Bab^2) + x^2(-18Aa^2c - 18Aab^2 - 18Ba^2b) + x(-12A}{12x^4}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**5,x)
```

output
```
B*c**3*x**3/3 + x**2*(A*c**3/2 + 3*B*b*c**2/2) + x*(3*A*b*c**2 + 3*B*a*c**
2 + 3*B*b**2*c) + (3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3)*log(x) +
(-3*A*a**3 + x**3*(-72*A*a*b*c - 12*A*b**3 - 36*B*a**2*c - 36*B*a*b**2) +
x**2*(-18*A*a**2*c - 18*A*a*b**2 - 18*B*a**2*b) + x*(-12*A*a**2*b - 4*B*a*
*3))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^5}\,dx = \frac{1}{3}\,Bc^3x^3 + \frac{1}{2}\,\left(3\,Bbc^2 + Ac^3\right)x^2 + 3\,\left(Bb^2c + (Ba+Ab)c^2\right)x + \left(Bb^3 + 3\,Aac^2 + 3\,\left(2\,Bab + Ab^2\right)c\right)\log\left(x\right) - \frac{3\,Aa^3 + 12\,(3\,Bab^2 + Ab^3 + 3\,(Ba^2 + 2\,Aab)c)x^3 + 18\,(Ba^2b + Aab^2 + Aa^2c)x^2 + 4\,(Ba^3 + 3\,Aa^2b)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^5,x, algorithm="maxima")
```

output
```
1/3*B*c^3*x^3 + 1/2*(3*B*b*c^2 + A*c^3)*x^2 + 3*(B*b^2*c + (B*a + A*b)*c^2
)*x + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*log(x) - 1/12*(3*A*a^3 +
 12*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 18*(B*a^2*b + A*a*b^
2 + A*a^2*c)*x^2 + 4*(B*a^3 + 3*A*a^2*b)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^5}\,dx = \frac{1}{3}\,Bc^3x^3 + \frac{3}{2}\,Bbc^2x^2 + \frac{1}{2}\,Ac^3x^2 + 3\,Bb^2cx + 3\,Bac^2x + 3\,Abc^2x + \left(Bb^3 + 6\,Babc + 3\,Ab^2c + 3\,Aac^2\right)\log\left(|x|\right) - \frac{3\,Aa^3 + 12\,(3\,Bab^2 + Ab^3 + 3\,Ba^2c + 6\,Aabc)x^3 + 18\,(Ba^2b + Aab^2 + Aa^2c)x^2 + 4\,(Ba^3 + 3\,Aa^2b)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^5,x, algorithm="giac")
```

output
```
1/3*B*c^3*x^3 + 3/2*B*b*c^2*x^2 + 1/2*A*c^3*x^2 + 3*B*b^2*c*x + 3*B*a*c^2*
x + 3*A*b*c^2*x + (B*b^3 + 6*B*a*b*c + 3*A*b^2*c + 3*A*a*c^2)*log(abs(x))
- 1/12*(3*A*a^3 + 12*(3*B*a*b^2 + A*b^3 + 3*B*a^2*c + 6*A*a*b*c)*x^3 + 18*
(B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 4*(B*a^3 + 3*A*a^2*b)*x)/x^4
```

Mupad [B] (verification not implemented)

Time = 10.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^3}{x^5}\,dx = \ln(x)\,\left(B\,b^3+3\,A\,b^2\,c+6\,B\,a\,b\,c+3\,A\,a\,c^2\right) - \frac{x^3\,(3\,B\,c\,a^2+3\,B\,a\,b^2+6\,A\,c\,a\,b+A\,b^3)+x\,\left(\frac{B\,a^3}{3}+A\,b\,a^2\right)+\frac{A\,a^3}{4}+x^2\,\left(\frac{3\,B\,a^2\,b}{2}+\frac{3\,A\,c\,a^2}{2}+\frac{3\,A\,a\,b^2}{2}\right)}{x^4} + x\,\left(3\,B\,b^2\,c+3\,A\,b\,c^2+3\,B\,a\,c^2\right)+x^2\,\left(\frac{A\,c^3}{2}+\frac{3\,B\,b\,c^2}{2}\right)+\frac{B\,c^3\,x^3}{3}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^5,x)
```

output
```
log(x)*(B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*b*c) - (x^3*(A*b^3 + 3*B*a*b
^2 + 3*B*a^2*c + 6*A*a*b*c) + x*((B*a^3)/3 + A*a^2*b) + (A*a^3)/4 + x^2*((
3*A*a*b^2)/2 + (3*A*a^2*c)/2 + (3*B*a^2*b)/2))/x^4 + x*(3*A*b*c^2 + 3*B*a*
c^2 + 3*B*b^2*c) + x^2*((A*c^3)/2 + (3*B*b*c^2)/2) + (B*c^3*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^3}{x^5}\,dx = \frac{36\log(x)\,a^2c^2x^4+108\log(x)\,a\,b^2c\,x^4+12\log(x)\,b^4x^4-3a^4-16a^3bx-18a^3c\,x^2-36a^2b^2x^2-108a^2bc\,}{12x^4}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^5,x)
```

output
```
(36*log(x)*a**2*c**2*x**4 + 108*log(x)*a*b**2*c*x**4 + 12*log(x)*b**4*x**4
 - 3*a**4 - 16*a**3*b*x - 18*a**3*c*x**2 - 36*a**2*b**2*x**2 - 108*a**2*b*
c*x**3 - 48*a*b**3*x**3 + 72*a*b*c**2*x**5 + 6*a*c**3*x**6 + 36*b**3*c*x**
5 + 18*b**2*c**2*x**6 + 4*b*c**3*x**7)/(12*x**4)
```

3.33 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6} \, dx = & -\frac{a^3A}{5x^5}-\frac{a^2(3Ab+aB)}{4x^4}-\frac{a(abB+A(b^2+ac))}{x^3} \\ & -\frac{3aB(b^2+ac)+A(b^3+6abc)}{2x^2} \\ & -\frac{b^3B+3Ab^2c+6abBc+3aAc^2}{x}+c^2(3bB+Ac)x \\ & +\frac{1}{2}Bc^3x^2+3c\left(b^2B+Abc+aBc\right)\log(x)\end{aligned}$$

output
```
-1/5*a^3*A/x^5-1/4*a^2*(3*A*b+B*a)/x^4-a*(a*b*B+A*(a*c+b^2))/x^3-1/2*(3*a*
B*(a*c+b^2)+A*(6*a*b*c+b^3))/x^2-(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)/x+c
^2*(A*c+3*B*b)*x+1/2*B*c^3*x^2+3*c*(A*b*c+B*a*c+B*b^2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6}\,dx = -\frac{a^3(4A+5Bx)+5a^2x(3Ab+4bBx+4Acx+6Bcx^2)+10ax^2(3bBx(b+4cx)+2A(b^2+3bcx+3c^2x}{20x}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^6,x]
```

output
```
-1/20*(a^3*(4*A + 5*B*x) + 5*a^2*x*(3*A*b + 4*b*B*x + 4*A*c*x + 6*B*c*x^2)
 + 10*a*x^2*(3*b*B*x*(b + 4*c*x) + 2*A*(b^2 + 3*b*c*x + 3*c^2*x^2)) + 10*x
^3*(A*(b^3 + 6*b^2*c*x - 2*c^3*x^3) - B*x*(-2*b^3 + 6*b*c^2*x^2 + c^3*x^3)
) - 60*c*(b^2*B + A*b*c + a*B*c)*x^5*Log[x])/x^5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^6}+\frac{a^2(aB+3Ab)}{x^5}+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x^4}+\frac{3c\left(aBc+Abc+b^2B\right)}{x}+\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{x^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{5x^5}-\frac{a^2(aB+3Ab)}{4x^4}-\frac{a\left(A\left(ac+b^2\right)+abB\right)}{x^3}+3c\log(x)\left(aBc+Abc+b^2B\right)-\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{x}-\frac{A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)}{2x^2}+c^2x(Ac+3bB)+\frac{1}{2}Bc^3x^2$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^6,x]
```

output
```
-1/5*(a^3*A)/x^5 - (a^2*(3*A*b + a*B))/(4*x^4) - (a*(a*b*B + A*(b^2 + a*c)
))/x^3 - (3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))/(2*x^2) - (b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)/x + c^2*(3*b*B + A*c)*x + (B*c^3*x^2)/2 + 3*c
*(b^2*B + A*b*c + a*B*c)*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98

method	result
default	$\frac{B\,c^3x^2}{2}+A\,c^3x+3Bb\,c^2x-\frac{a^3A}{5x^5}-\frac{a\left(Aac+b^2A+abB\right)}{x^3}-\frac{6Aabc+A\,b^3+3B\,a^2c+3Ba\,b^2}{2x^2}-\frac{a^2(3Ab+Ba)}{4x^4}+3$
risch	$\frac{B\,c^3x^2}{2}+A\,c^3x+3Bb\,c^2x+\frac{\left(-3Aa\,c^2-3A\,b^2c-6Babc-B\,b^3\right)x^4+\left(-3Aabc-\frac{1}{2}A\,b^3-\frac{3}{2}B\,a^2c-\frac{3}{2}Ba\,b^2\right)x^3+\left(-a^2A\right.}{x^5}$
norman	$\frac{\left(-\frac{3}{4}A\,a^2b-\frac{1}{4}B\,a^3\right)x+\left(-3Aabc-\frac{1}{2}A\,b^3-\frac{3}{2}B\,a^2c-\frac{3}{2}Ba\,b^2\right)x^3+\left(A\,c^3+3Bb\,c^2\right)x^6+\left(-a^2Ac-Aa\,b^2-B\,a^2b\right)x^2+\left(-3Aa\,c^2-3\right.}{x^5}$
parallelrisch	$\frac{10B\,c^3x^7+60A\ln(x)x^5b\,c^2+20A\,c^3x^6+60B\ln(x)x^5a\,c^2+60B\ln(x)x^5b^2c+60Bb\,c^2x^6-60Aa\,c^2x^4-60A\,b^2c\,x^4-120Babc\,x^4}{20}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*c^3*x^2+A*c^3*x+3*B*b*c^2*x-1/5*a^3*A/x^5-a*(A*a*c+A*b^2+B*a*b)/x^3-
1/2*(6*A*a*b*c+A*b^3+3*B*a^2*c+3*B*a*b^2)/x^2-1/4*a^2*(3*A*b+B*a)/x^4+3*c*
(A*b*c+B*a*c+B*b^2)*ln(x)-(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6}\,dx$$
$$= \frac{10\,Bc^3x^7+20\,(3\,Bbc^2+Ac^3)x^6+60\,(Bb^2c+(Ba+Ab)c^2)x^5\log{(x)}-20\,(Bb^3+3\,Aac^2+3\,(2\,Bab+}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^6,x, algorithm="fricas")
```

output
```
1/20*(10*B*c^3*x^7 + 20*(3*B*b*c^2 + A*c^3)*x^6 + 60*(B*b^2*c + (B*a + A*b
)*c^2)*x^5*log(x) - 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 - 4
*A*a^3 - 10*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 - 20*(B*a^2*b
+ A*a*b^2 + A*a^2*c)*x^2 - 5*(B*a^3 + 3*A*a^2*b)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6}\,dx = \frac{Bc^3x^2}{2}+3c\left(Abc+Bac+Bb^2\right)\log{(x)}+x\left(Ac^3+3Bbc^2\right)$$
$$+\frac{-4Aa^3+x^4(-60Aac^2-60Ab^2c-120Babc-20Bb^3)+x^3(-60Aabc-10Ab^3-30Ba^2c-30Bab^2)}{20x^5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**6,x)
```

output
```
B*c**3*x**2/2 + 3*c*(A*b*c + B*a*c + B*b**2)*log(x) + x*(A*c**3 + 3*B*b*c*
*2) + (-4*A*a**3 + x**4*(-60*A*a*c**2 - 60*A*b**2*c - 120*B*a*b*c - 20*B*b
**3) + x**3*(-60*A*a*b*c - 10*A*b**3 - 30*B*a**2*c - 30*B*a*b**2) + x**2*(
-20*A*a**2*c - 20*A*a*b**2 - 20*B*a**2*b) + x*(-15*A*a**2*b - 5*B*a**3))/(
20*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6}\,dx$$
$$= \frac{1}{2}Bc^3x^2 + \left(3\,Bbc^2 + Ac^3\right)x + 3\left(Bb^2c + (Ba+Ab)c^2\right)\log\left(x\right)$$
$$- \frac{20\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab + Ab^2)c)x^4 + 4\,Aa^3 + 10\,(3\,Bab^2 + Ab^3 + 3\,(Ba^2 + 2\,Aab)c)x^3 + 20\,(Ba}{20\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^6,x, algorithm="maxima")
```

output
```
1/2*B*c^3*x^2 + (3*B*b*c^2 + A*c^3)*x + 3*(B*b^2*c + (B*a + A*b)*c^2)*log(
x) - 1/20*(20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 + 4*A*a^3 +
10*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 20*(B*a^2*b + A*a*b^2
 + A*a^2*c)*x^2 + 5*(B*a^3 + 3*A*a^2*b)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6}\,dx$$
$$= \frac{1}{2}Bc^3x^2 + 3\,Bbc^2x + Ac^3x + 3\left(Bb^2c + Bac^2 + Abc^2\right)\log\left(|x|\right)$$
$$- \frac{20\,(Bb^3 + 6\,Babc + 3\,Ab^2c + 3\,Aac^2)x^4 + 4\,Aa^3 + 10\,(3\,Bab^2 + Ab^3 + 3\,Ba^2c + 6\,Aabc)x^3 + 20\,(Ba^2b}{20\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^6,x, algorithm="giac")
```

output
```
1/2*B*c^3*x^2 + 3*B*b*c^2*x + A*c^3*x + 3*(B*b^2*c + B*a*c^2 + A*b*c^2)*lo
g(abs(x)) - 1/20*(20*(B*b^3 + 6*B*a*b*c + 3*A*b^2*c + 3*A*a*c^2)*x^4 + 4*A
*a^3 + 10*(3*B*a*b^2 + A*b^3 + 3*B*a^2*c + 6*A*a*b*c)*x^3 + 20*(B*a^2*b +
A*a*b^2 + A*a^2*c)*x^2 + 5*(B*a^3 + 3*A*a^2*b)*x)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6}\,dx = x\left(A\,c^3+3\,B\,b\,c^2\right) - \frac{x^3\left(\frac{3\,B\,c\,a^2}{2}+\frac{3\,B\,a\,b^2}{2}+3\,A\,c\,a\,b+\frac{A\,b^3}{2}\right)+x^4\left(B\,b^3+3\,A\,b^2\,c+6\,B\,a\,b\,c+3\,A\,a\,c^2\right)+x\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)}{x^5} + \ln(x)\,\left(3\,B\,b^2\,c+3\,A\,b\,c^2+3\,B\,a\,c^2\right)+\frac{B\,c^3\,x^2}{2}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^6,x)
```

output
```
x*(A*c^3 + 3*B*b*c^2) - (x^3*((A*b^3)/2 + (3*B*a*b^2)/2 + (3*B*a^2*c)/2 +
3*A*a*b*c) + x^4*(B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*b*c) + x*((B*a^3)/
4 + (3*A*a^2*b)/4) + (A*a^3)/5 + x^2*(A*a*b^2 + A*a^2*c + B*a^2*b))/x^5 +
log(x)*(3*A*b*c^2 + 3*B*a*c^2 + 3*B*b^2*c) + (B*c^3*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^6}\,dx = \frac{60\log(x)\,ab\,c^2x^5+30\log(x)\,b^3c\,x^5-2a^4-10a^3bx-10a^3c\,x^2-20a^2b^2x^2-45a^2bc\,x^3-30a^2c^2x^4-20a}{10x^5}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^6,x)
```

output
```
(60*log(x)*a*b*c**2*x**5 + 30*log(x)*b**3*c*x**5 - 2*a**4 - 10*a**3*b*x -
10*a**3*c*x**2 - 20*a**2*b**2*x**2 - 45*a**2*b*c*x**3 - 30*a**2*c**2*x**4
- 20*a*b**3*x**3 - 90*a*b**2*c*x**4 + 10*a*c**3*x**6 - 10*b**4*x**4 + 30*b
**2*c**2*x**6 + 5*b*c**3*x**7)/(10*x**5)
```

3.34 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^7}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^7}\,dx = -\frac{a^3A}{6x^6}-\frac{a^2(3Ab+aB)}{5x^5}-\frac{3a(abB+A(b^2+ac))}{4x^4} -\frac{3aB(b^2+ac)+A(b^3+6abc)}{3x^3} -\frac{b^3B+3Ab^2c+6abBc+3aAc^2}{2x^2} -\frac{3c(b^2B+Abc+aBc)}{x}+Bc^3x+c^2(3bB+Ac)\log(x)$$

output
```
-1/6*a^3*A/x^6-1/5*a^2*(3*A*b+B*a)/x^5-3/4*a*(a*b*B+A*(a*c+b^2))/x^4-1/3*(
3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))/x^3-1/2*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*
b^3)/x^2-3*c*(A*b*c+B*a*c+B*b^2)/x+B*c^3*x+c^2*(A*c+3*B*b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^7}\,dx =$$
$$-\frac{2a^3(5A+6Bx)+3a^2x(5Bx(3b+4cx)+3A(4b+5cx))+15ax^2(4Bx(b^2+3bcx+3c^2x^2)+A(3b^2+$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^7,x]
```

output
```
-1/60*(2*a^3*(5*A + 6*B*x) + 3*a^2*x*(5*B*x*(3*b + 4*c*x) + 3*A*(4*b + 5*c
*x)) + 15*a*x^2*(4*B*x*(b^2 + 3*b*c*x + 3*c^2*x^2) + A*(3*b^2 + 8*b*c*x +
6*c^2*x^2)) + 10*x^3*(A*b*(2*b^2 + 9*b*c*x + 18*c^2*x^2) + 3*B*x*(b^3 + 6*
b^2*c*x - 2*c^3*x^3)) - 60*c^2*(3*b*B + A*c)*x^6*Log[x])/x^6
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^7}\,dx$$
$$\Big\downarrow \text{1195}$$
$$\int \left(\frac{a^3A}{x^7}+\frac{a^2(aB+3Ab)}{x^6}+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x^5}+\frac{3c\left(aBc+Abc+b^2B\right)}{x^2}+\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{x^3}\right.$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^3A}{6x^6}-\frac{a^2(aB+3Ab)}{5x^5}-\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{4x^4}-\frac{3c\left(aBc+Abc+b^2B\right)}{x}-\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{2x^2}-\frac{A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)}{3x^3}+c^2\log(x)(Ac+3bB)+Bc^3x$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^7,x]
```

output
```
-1/6*(a^3*A)/x^6 - (a^2*(3*A*b + a*B))/(5*x^5) - (3*a*(a*b*B + A*(b^2 + a*
c)))/(4*x^4) - (3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))/(3*x^3) - (b^3*B +
3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)/(2*x^2) - (3*c*(b^2*B + A*b*c + a*B*c))
/x + B*c^3*x + c^2*(3*b*B + A*c)*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

method	result
default	$B\,c^3x-\frac{a^2(3Ab+Ba)}{5x^5}-\frac{a^3A}{6x^6}-\frac{6Aabc+A\,b^3+3B\,a^2c+3Ba\,b^2}{3x^3}-\frac{3Aa\,c^2+3A\,b^2c+6Babc+B\,b^3}{2x^2}-\frac{3a(Aac+b^2A+a}{4x^4}$
risch	$B\,c^3x+\frac{(-3Ab\,c^2-3Ba\,c^2-3B\,b^2c)x^5+(-\frac{3}{2}Aa\,c^2-\frac{3}{2}A\,b^2c-3Babc-\frac{1}{2}B\,b^3)x^4+(-2Aabc-\frac{1}{3}A\,b^3-B\,a^2c-Ba\,b^2)x^3+(-}{x^6}$
norman	$\frac{(-\frac{3}{5}A\,a^2b-\frac{1}{5}B\,a^3)x+(-\frac{3}{4}a^2Ac-\frac{3}{4}Aa\,b^2-\frac{3}{4}B\,a^2b)x^2+(-\frac{3}{2}Aa\,c^2-\frac{3}{2}A\,b^2c-3Babc-\frac{1}{2}B\,b^3)x^4+(-2Aabc-\frac{1}{3}A\,b^3-B\,a^2c-}{x^6}$
parallelrisch	$\frac{60A\ln(x)x^6c^3+180B\ln(x)x^6b\,c^2+60B\,c^3x^7-180Ab\,c^2x^5-180Ba\,c^2x^5-180B\,b^2c\,x^5-90Aa\,c^2x^4-90A\,b^2c\,x^4-180Babc\,x^4-}{6}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
B*c^3*x-1/5*a^2*(3*A*b+B*a)/x^5-1/6*a^3*A/x^6-1/3*(6*A*a*b*c+A*b^3+3*B*a^2
*c+3*B*a*b^2)/x^3-1/2*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)/x^2-3/4*a*(A*a
*c+A*b^2+B*a*b)/x^4+c^2*(A*c+3*B*b)*ln(x)-3*c*(A*b*c+B*a*c+B*b^2)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^7}\,dx$$
$$= \frac{60\,Bc^3x^7+60\,(3\,Bbc^2+Ac^3)x^6\log{(x)}-180\,(Bb^2c+(Ba+Ab)c^2)x^5-30\,(Bb^3+3\,Aac^2+3\,(2\,Bab+}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^7,x, algorithm="fricas")
```

output
```
1/60*(60*B*c^3*x^7 + 60*(3*B*b*c^2 + A*c^3)*x^6*log(x) - 180*(B*b^2*c + (B
*a + A*b)*c^2)*x^5 - 30*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 -
10*A*a^3 - 20*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 - 45*(B*a^2*
b + A*a*b^2 + A*a^2*c)*x^2 - 12*(B*a^3 + 3*A*a^2*b)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^7}\,dx = Bc^3x+c^2(Ac+3Bb)\log{(x)}$$
$$+\frac{-10Aa^3+x^5(-180Abc^2-180Bac^2-180Bb^2c)+x^4(-90Aac^2-90Ab^2c-180Babc-30Bb^3)+x^3(}{6}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**7,x)
```

output
```
B*c**3*x + c**2*(A*c + 3*B*b)*log(x) + (-10*A*a**3 + x**5*(-180*A*b*c**2 -
 180*B*a*c**2 - 180*B*b**2*c) + x**4*(-90*A*a*c**2 - 90*A*b**2*c - 180*B*a
*b*c - 30*B*b**3) + x**3*(-120*A*a*b*c - 20*A*b**3 - 60*B*a**2*c - 60*B*a*
b**2) + x**2*(-45*A*a**2*c - 45*A*a*b**2 - 45*B*a**2*b) + x*(-36*A*a**2*b
- 12*B*a**3))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^7}\,dx = Bc^3x + \left(3\,Bbc^2 + Ac^3\right)\log\left(x\right) - \frac{180\,(Bb^2c + (Ba+Ab)c^2)x^5 + 30\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab + Ab^2)c)x^4 + 10\,Aa^3 + 20\,(3\,Bab^2 + Ab^3}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^7,x, algorithm="maxima")
```

output
```
B*c^3*x + (3*B*b*c^2 + A*c^3)*log(x) - 1/60*(180*(B*b^2*c + (B*a + A*b)*c^
2)*x^5 + 30*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 + 10*A*a^3 + 2
0*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 45*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^2 + 12*(B*a^3 + 3*A*a^2*b)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^7}\,dx = Bc^3x + \left(3\,Bbc^2 + Ac^3\right)\log\left(|x|\right) - \frac{180\,(Bb^2c + Bac^2 + Abc^2)x^5 + 30\,(Bb^3 + 6\,Babc + 3\,Ab^2c + 3\,Aac^2)x^4 + 10\,Aa^3 + 20\,(3\,Bab^2 + Ab^3}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^7,x, algorithm="giac")
```

output
```
B*c^3*x + (3*B*b*c^2 + A*c^3)*log(abs(x)) - 1/60*(180*(B*b^2*c + B*a*c^2 +
 A*b*c^2)*x^5 + 30*(B*b^3 + 6*B*a*b*c + 3*A*b^2*c + 3*A*a*c^2)*x^4 + 10*A*
a^3 + 20*(3*B*a*b^2 + A*b^3 + 3*B*a^2*c + 6*A*a*b*c)*x^3 + 45*(B*a^2*b + A
*a*b^2 + A*a^2*c)*x^2 + 12*(B*a^3 + 3*A*a^2*b)*x)/x^6
```

Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^3}{x^7}\,dx = \ln(x)\,\left(A\,c^3+3\,B\,b\,c^2\right)$$
$$-\frac{x^3\left(B\,c\,a^2+B\,a\,b^2+2\,A\,c\,a\,b+\frac{A\,b^3}{3}\right)+x^4\left(\frac{B\,b^3}{2}+\frac{3\,A\,b^2\,c}{2}+3\,B\,a\,b\,c+\frac{3\,A\,a\,c^2}{2}\right)+x\left(\frac{B\,a^3}{5}+\frac{3\,A\,b\,a^2}{5}\right)+}{x^6}$$
$$+B\,c^3\,x$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^7,x)
```

output
```
log(x)*(A*c^3 + 3*B*b*c^2) - (x^3*((A*b^3)/3 + B*a*b^2 + B*a^2*c + 2*A*a*b
*c) + x^4*((B*b^3)/2 + (3*A*a*c^2)/2 + (3*A*b^2*c)/2 + 3*B*a*b*c) + x*((B*
a^3)/5 + (3*A*a^2*b)/5) + (A*a^3)/6 + x^2*((3*A*a*b^2)/4 + (3*A*a^2*c)/4 +
 (3*B*a^2*b)/4) + x^5*(3*A*b*c^2 + 3*B*a*c^2 + 3*B*b^2*c))/x^6 + B*c^3*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^3}{x^7}\,dx$$
$$= \frac{60\log(x)\,a\,c^3x^6+180\log(x)\,b^2c^2x^6-10a^4-48a^3bx-45a^3c\,x^2-90a^2b^2x^2-180a^2bc\,x^3-90a^2c^2x^4-}{60x^6}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^7,x)
```

output
```
(60*log(x)*a*c**3*x**6 + 180*log(x)*b**2*c**2*x**6 - 10*a**4 - 48*a**3*b*x
 - 45*a**3*c*x**2 - 90*a**2*b**2*x**2 - 180*a**2*b*c*x**3 - 90*a**2*c**2*x
**4 - 80*a*b**3*x**3 - 270*a*b**2*c*x**4 - 360*a*b*c**2*x**5 - 30*b**4*x**
4 - 180*b**3*c*x**5 + 60*b*c**3*x**7)/(60*x**6)
```

3.35 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^8} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 160

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^8} \, dx = -\frac{a^3A}{7x^7} - \frac{a^2(3Ab+aB)}{6x^6} - \frac{3a(abB+A(b^2+ac))}{5x^5} - \frac{3aB(b^2+ac)+A(b^3+6abc)}{4x^4} - \frac{b^3B+3Ab^2c+6abBc+3aAc^2}{3x^3} - \frac{3c(b^2B+Abc+aBc)}{2x^2} - \frac{c^2(3bB+Ac)}{x} + Bc^3\log(x)$$

output

```
-1/7*a^3*A/x^7-1/6*a^2*(3*A*b+B*a)/x^6-3/5*a*(a*b*B+A*(a*c+b^2))/x^5-1/4*(
3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))/x^4-1/3*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*
b^3)/x^3-3/2*c*(A*b*c+B*a*c+B*b^2)/x^2-c^2*(A*c+3*B*b)/x+B*c^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^8}\,dx =$$
$$-\frac{10a^3(6A+7Bx)+21a^2x(3Bx(4b+5cx)+2A(5b+6cx))+21ax^2(5Bx(3b^2+8bcx+6c^2x^2)+2A(6\cdots}{\cdots}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^8,x]
```

output
```
-1/420*(10*a^3*(6*A + 7*B*x) + 21*a^2*x*(3*B*x*(4*b + 5*c*x) + 2*A*(5*b +
6*c*x)) + 21*a*x^2*(5*B*x*(3*b^2 + 8*b*c*x + 6*c^2*x^2) + 2*A*(6*b^2 + 15*
b*c*x + 10*c^2*x^2)) + 35*x^3*(2*b*B*x*(2*b^2 + 9*b*c*x + 18*c^2*x^2) + 3*
A*(b^3 + 4*b^2*c*x + 6*b*c^2*x^2 + 4*c^3*x^3)) - 420*B*c^3*x^7*Log[x])/x^7
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^8}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^8}+\frac{a^2(aB+3Ab)}{x^7}+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x^6}+\frac{3c\left(aBc+Abc+b^2B\right)}{x^3}+\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{x^4}\cdots\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{7x^7}-\frac{a^2(aB+3Ab)}{6x^6}-\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{5x^5}-\frac{3c\left(aBc+Abc+b^2B\right)}{2x^2}-\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{3x^3}-\frac{A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)}{4x^4}-\frac{c^2(Ac+3bB)}{x}+Bc^3\log(x)$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^8,x]
```

output
```
-1/7*(a^3*A)/x^7 - (a^2*(3*A*b + a*B))/(6*x^6) - (3*a*(a*b*B + A*(b^2 + a*
c)))/(5*x^5) - (3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))/(4*x^4) - (b^3*B +
3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)/(3*x^3) - (3*c*(b^2*B + A*b*c + a*B*c))
/(2*x^2) - (c^2*(3*b*B + A*c))/x + B*c^3*Log[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.95

method	result
default	$-\frac{3a\left(Aac+b^{2}A+abB\right)}{5x^{5}}-\frac{a^{2}\left(3Ab+Ba\right)}{6x^{6}}-\frac{3Aa\,c^{2}+3A\,b^{2}c+6Babc+B\,b^{3}}{3x^{3}}-\frac{3c\left(Abc+aBc+B\,b^{2}\right)}{2x^{2}}-\frac{6Aabc+A\,b^{3}+3B\,a}{4x^{4}}$
norman	$\frac{\left(-\frac{1}{2}A\,a^{2}b-\frac{1}{6}B\,a^{3}\right)x+\left(-\frac{3}{2}Ab\,c^{2}-\frac{3}{2}Ba\,c^{2}-\frac{3}{2}B\,b^{2}c\right)x^{5}+\left(-\frac{3}{5}a^{2}Ac-\frac{3}{5}Aa\,b^{2}-\frac{3}{5}B\,a^{2}b\right)x^{2}+\left(-Aa\,c^{2}-A\,b^{2}c-2Babc-\frac{1}{3}B\,b^{3}\right)x}{x^{7}}$
risch	$\frac{\left(-\frac{1}{2}A\,a^{2}b-\frac{1}{6}B\,a^{3}\right)x+\left(-\frac{3}{2}Ab\,c^{2}-\frac{3}{2}Ba\,c^{2}-\frac{3}{2}B\,b^{2}c\right)x^{5}+\left(-\frac{3}{5}a^{2}Ac-\frac{3}{5}Aa\,b^{2}-\frac{3}{5}B\,a^{2}b\right)x^{2}+\left(-Aa\,c^{2}-A\,b^{2}c-2Babc-\frac{1}{3}B\,b^{3}\right)x}{x^{7}}$
parallelrisch	$-\frac{-420B\,c^{3}\ln\left(x\right)x^{7}+420A\,c^{3}x^{6}+1260Bb\,c^{2}x^{6}+630Ab\,c^{2}x^{5}+630Ba\,c^{2}x^{5}+630B\,b^{2}c\,x^{5}+420Aa\,c^{2}x^{4}+420A\,b^{2}c\,x^{4}+840Bab}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-3/5*a*(A*a*c+A*b^2+B*a*b)/x^5-1/6*a^2*(3*A*b+B*a)/x^6-1/3*(3*A*a*c^2+3*A*
b^2*c+6*B*a*b*c+B*b^3)/x^3-3/2*c*(A*b*c+B*a*c+B*b^2)/x^2-1/4*(6*A*a*b*c+A*
b^3+3*B*a^2*c+3*B*a*b^2)/x^4+B*c^3*ln(x)-1/7*a^3*A/x^7-c^2*(A*c+3*B*b)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^3}{x^8}\,dx$$
$$= \frac{420\,Bc^3x^7\log{(x)} - 420\,(3\,Bbc^2 + Ac^3)x^6 - 630\,(Bb^2c + (Ba+Ab)c^2)x^5 - 140\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bc}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^8,x, algorithm="fricas")
```

output
```
1/420*(420*B*c^3*x^7*log(x) - 420*(3*B*b*c^2 + A*c^3)*x^6 - 630*(B*b^2*c +
 (B*a + A*b)*c^2)*x^5 - 140*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^
4 - 60*A*a^3 - 105*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 - 252*(
B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 - 70*(B*a^3 + 3*A*a^2*b)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 26.69 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^3}{x^8}\,dx = Bc^3\log{(x)}$$
$$+ \frac{-60Aa^3 + x^6(-420Ac^3 - 1260Bbc^2) + x^5(-630Abc^2 - 630Bac^2 - 630Bb^2c) + x^4(-420Aac^2 - 420A}{}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**8,x)
```

output
```
B*c**3*log(x) + (-60*A*a**3 + x**6*(-420*A*c**3 - 1260*B*b*c**2) + x**5*(-
630*A*b*c**2 - 630*B*a*c**2 - 630*B*b**2*c) + x**4*(-420*A*a*c**2 - 420*A*
b**2*c - 840*B*a*b*c - 140*B*b**3) + x**3*(-630*A*a*b*c - 105*A*b**3 - 315
*B*a**2*c - 315*B*a*b**2) + x**2*(-252*A*a**2*c - 252*A*a*b**2 - 252*B*a**
2*b) + x*(-210*A*a**2*b - 70*B*a**3))/(420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^8}\,dx = Bc^3\log(x) - \frac{420\,(3\,Bbc^2+Ac^3)x^6+630\,(Bb^2c+(Ba+Ab)c^2)x^5+140\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^4+60}{42}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^8,x, algorithm="maxima")
```

output
```
B*c^3*log(x) - 1/420*(420*(3*B*b*c^2 + A*c^3)*x^6 + 630*(B*b^2*c + (B*a +
A*b)*c^2)*x^5 + 140*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 + 60*A
*a^3 + 105*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 252*(B*a^2*b
+ A*a*b^2 + A*a^2*c)*x^2 + 70*(B*a^3 + 3*A*a^2*b)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^8}\,dx = Bc^3\log(|x|) - \frac{420\,(3\,Bbc^2+Ac^3)x^6+630\,(Bb^2c+Bac^2+Abc^2)x^5+140\,(Bb^3+6\,Babc+3\,Ab^2c+3\,Aac^2)x^4+60}{420}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^8,x, algorithm="giac")
```

output
```
B*c^3*log(abs(x)) - 1/420*(420*(3*B*b*c^2 + A*c^3)*x^6 + 630*(B*b^2*c + B*
a*c^2 + A*b*c^2)*x^5 + 140*(B*b^3 + 6*B*a*b*c + 3*A*b^2*c + 3*A*a*c^2)*x^4
 + 60*A*a^3 + 105*(3*B*a*b^2 + A*b^3 + 3*B*a^2*c + 6*A*a*b*c)*x^3 + 252*(B
*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 70*(B*a^3 + 3*A*a^2*b)*x)/x^7
```

Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^8}\,dx = B\,c^3\,\ln(x) - \frac{x^3\left(\frac{3\,B\,c\,a^2}{4}+\frac{3\,B\,a\,b^2}{4}+\frac{3\,A\,c\,a\,b}{2}+\frac{A\,b^3}{4}\right)+x^4\left(\frac{B\,b^3}{3}+A\,b^2\,c+2\,B\,a\,b\,c+A\,a\,c^2\right)+x\left(\frac{B\,a^3}{6}+\frac{A\,b\,a^2}{2}\right)+\frac{A\,a^3}{7}\cdots}{x^7}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^8,x)
```

output
```
B*c^3*log(x) - (x^3*((A*b^3)/4 + (3*B*a*b^2)/4 + (3*B*a^2*c)/4 + (3*A*a*b*
c)/2) + x^4*((B*b^3)/3 + A*a*c^2 + A*b^2*c + 2*B*a*b*c) + x*((B*a^3)/6 + (
A*a^2*b)/2) + (A*a^3)/7 + x^6*(A*c^3 + 3*B*b*c^2) + x^2*((3*A*a*b^2)/5 + (
3*A*a^2*c)/5 + (3*B*a^2*b)/5) + x^5*((3*A*b*c^2)/2 + (3*B*a*c^2)/2 + (3*B*
b^2*c)/2))/x^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^8}\,dx = \frac{420\log(x)\,b\,c^3x^7-60a^4-280a^3bx-252a^3c\,x^2-504a^2b^2x^2-945a^2bc\,x^3-420a^2c^2x^4-420a\,b^3x^3-12\cdots}{420x^7}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^8,x)
```

output
```
(420*log(x)*b*c**3*x**7 - 60*a**4 - 280*a**3*b*x - 252*a**3*c*x**2 - 504*a
**2*b**2*x**2 - 945*a**2*b*c*x**3 - 420*a**2*c**2*x**4 - 420*a*b**3*x**3 -
 1260*a*b**2*c*x**4 - 1260*a*b*c**2*x**5 - 420*a*c**3*x**6 - 140*b**4*x**4
 - 630*b**3*c*x**5 - 1260*b**2*c**2*x**6)/(420*x**7)
```

3.36 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9} \, dx = -\frac{a^3A}{8x^8}-\frac{a^2(3Ab+aB)}{7x^7}-\frac{a(abB+A(b^2+ac))}{2x^6} -\frac{3aB(b^2+ac)+A(b^3+6abc)}{5x^5} -\frac{b^3B+3Ab^2c+6abBc+3aAc^2}{4x^4} -\frac{c(b^2B+Abc+aBc)}{x^3}-\frac{c^2(3bB+Ac)}{2x^2}-\frac{Bc^3}{x}$$

output

```
-1/8*a^3*A/x^8-1/7*a^2*(3*A*b+B*a)/x^7-1/2*a*(a*b*B+A*(a*c+b^2))/x^6-1/5*(
3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))/x^5-1/4*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*
b^3)/x^4-c*(A*b*c+B*a*c+B*b^2)/x^3-1/2*c^2*(A*c+3*B*b)/x^2-B*c^3/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9}\,dx = -\frac{5a^3(7A+8Bx)+4a^2x(7Bx(5b+6cx)+5A(6b+7cx))+14ax^2(2Bx(6b^2+15bcx+10c^2x^2)+A(10\cdots}{2\cdots}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^9,x]
```

output
```
-1/280*(5*a^3*(7*A + 8*B*x) + 4*a^2*x*(7*B*x*(5*b + 6*c*x) + 5*A*(6*b + 7*
c*x)) + 14*a*x^2*(2*B*x*(6*b^2 + 15*b*c*x + 10*c^2*x^2) + A*(10*b^2 + 24*b
*c*x + 15*c^2*x^2)) + 14*x^3*(5*B*x*(b^3 + 4*b^2*c*x + 6*b*c^2*x^2 + 4*c^3
*x^3) + A*(4*b^3 + 15*b^2*c*x + 20*b*c^2*x^2 + 10*c^3*x^3)))/x^8
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^9}+\frac{a^2(aB+3Ab)}{x^8}+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x^7}+\frac{3c\left(aBc+Abc+b^2B\right)}{x^4}+\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{x^5}\cdots\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{8x^8}-\frac{a^2(aB+3Ab)}{7x^7}-\frac{a\left(A\left(ac+b^2\right)+abB\right)}{2x^6}-\frac{c\left(aBc+Abc+b^2B\right)}{x^3}-\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{4x^4}-\frac{A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)}{5x^5}-\frac{c^2(Ac+3bB)}{2x^2}-\frac{Bc^3}{x}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^9,x]
```

output
```
-1/8*(a^3*A)/x^8 - (a^2*(3*A*b + a*B))/(7*x^7) - (a*(a*b*B + A*(b^2 + a*c)
))/(2*x^6) - (3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))/(5*x^5) - (b^3*B + 3*
A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)/(4*x^4) - (c*(b^2*B + A*b*c + a*B*c))/x^3
 - (c^2*(3*b*B + A*c))/(2*x^2) - (B*c^3)/x
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

method	result
default	$-\frac{6Aabc+Ab^3+3Ba^2c+3Bab^2}{5x^5}-\frac{a(Aac+b^2A+abB)}{2x^6}-\frac{c(Abc+aBc+Bb^2)}{x^3}-\frac{c^2(Ac+3Bb)}{2x^2}-\frac{a^3A}{8x^8}-$ $3Aac^2+3A$
norman	$-Bc^3x^7+\left(-\frac{1}{2}Ac^3-\frac{3}{2}Bbc^2\right)x^6+\left(-Abc^2-Bac^2-Bb^2c\right)x^5+\left(-\frac{3}{4}Aac^2-\frac{3}{4}Ab^2c-\frac{3}{2}Babc-\frac{1}{4}Bb^3\right)x^4+\left(-\frac{6}{5}Aabc-\frac{1}{5}Ab^3-\right.$ / x^8
risch	$-Bc^3x^7+\left(-\frac{1}{2}Ac^3-\frac{3}{2}Bbc^2\right)x^6+\left(-Abc^2-Bac^2-Bb^2c\right)x^5+\left(-\frac{3}{4}Aac^2-\frac{3}{4}Ab^2c-\frac{3}{2}Babc-\frac{1}{4}Bb^3\right)x^4+\left(-\frac{6}{5}Aabc-\frac{1}{5}Ab^3-\right.$ / x^8
gosper	$-$ $280Bc^3x^7+140Ac^3x^6+420Bbc^2x^6+280Abc^2x^5+280Bac^2x^5+280Bb^2cx^5+210Aac^2x^4+210Ab^2cx^4+420Babcx^4+70$
parallelrisch	$-$ $280Bc^3x^7+140Ac^3x^6+420Bbc^2x^6+280Abc^2x^5+280Bac^2x^5+280Bb^2cx^5+210Aac^2x^4+210Ab^2cx^4+420Babcx^4+70$
orering	$-$ $280Bc^3x^7+140Ac^3x^6+420Bbc^2x^6+280Abc^2x^5+280Bac^2x^5+280Bb^2cx^5+210Aac^2x^4+210Ab^2cx^4+420Babcx^4+70$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(6*A*a*b*c+A*b^3+3*B*a^2*c+3*B*a*b^2)/x^5-1/2*a*(A*a*c+A*b^2+B*a*b)/x
^6-c*(A*b*c+B*a*c+B*b^2)/x^3-1/2*c^2*(A*c+3*B*b)/x^2-1/8*a^3*A/x^8-1/4*(3*
A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)/x^4-1/7*a^2*(3*A*b+B*a)/x^7-B*c^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9}\,dx = -\frac{280\,Bc^3x^7+140\,(3\,Bbc^2+Ac^3)x^6+280\,(Bb^2c+(Ba+Ab)c^2)x^5+70\,(Bb^3+3\,Aac^2+3\,(2\,Bab+A\cdots}{\cdots}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^9,x, algorithm="fricas")
```

output
```
-1/280*(280*B*c^3*x^7 + 140*(3*B*b*c^2 + A*c^3)*x^6 + 280*(B*b^2*c + (B*a
+ A*b)*c^2)*x^5 + 70*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 + 35*
A*a^3 + 56*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 140*(B*a^2*b
+ A*a*b^2 + A*a^2*c)*x^2 + 40*(B*a^3 + 3*A*a^2*b)*x)/x^8
```

Sympy [A] (verification not implemented)

Time = 59.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9}\,dx = \frac{-35Aa^3-280Bc^3x^7+x^6(-140Ac^3-420Bbc^2)+x^5(-280Abc^2-280Bac^2-280Bb^2c)+x^4(-210Aa\cdots}{\cdots}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**9,x)
```

output
```
(-35*A*a**3 - 280*B*c**3*x**7 + x**6*(-140*A*c**3 - 420*B*b*c**2) + x**5*(
-280*A*b*c**2 - 280*B*a*c**2 - 280*B*b**2*c) + x**4*(-210*A*a*c**2 - 210*A
*b**2*c - 420*B*a*b*c - 70*B*b**3) + x**3*(-336*A*a*b*c - 56*A*b**3 - 168*
B*a**2*c - 168*B*a*b**2) + x**2*(-140*A*a**2*c - 140*A*a*b**2 - 140*B*a**2
*b) + x*(-120*A*a**2*b - 40*B*a**3))/(280*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9}\,dx = -\frac{280\,Bc^3x^7+140\,(3\,Bbc^2+Ac^3)x^6+280\,(Bb^2c+(Ba+Ab)c^2)x^5+70\,(Bb^3+3\,Aac^2+3\,(2\,Bab+A$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^9,x, algorithm="maxima")
```

output
```
-1/280*(280*B*c^3*x^7 + 140*(3*B*b*c^2 + A*c^3)*x^6 + 280*(B*b^2*c + (B*a
+ A*b)*c^2)*x^5 + 70*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 + 35*
A*a^3 + 56*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 140*(B*a^2*b
+ A*a*b^2 + A*a^2*c)*x^2 + 40*(B*a^3 + 3*A*a^2*b)*x)/x^8
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9}\,dx = -\frac{280\,Bc^3x^7+420\,Bbc^2x^6+140\,Ac^3x^6+280\,Bb^2cx^5+280\,Bac^2x^5+280\,Abc^2x^5+70\,Bb^3x^4+420\,Ba$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^9,x, algorithm="giac")
```

output
```
-1/280*(280*B*c^3*x^7 + 420*B*b*c^2*x^6 + 140*A*c^3*x^6 + 280*B*b^2*c*x^5
+ 280*B*a*c^2*x^5 + 280*A*b*c^2*x^5 + 70*B*b^3*x^4 + 420*B*a*b*c*x^4 + 210
*A*b^2*c*x^4 + 210*A*a*c^2*x^4 + 168*B*a*b^2*x^3 + 56*A*b^3*x^3 + 168*B*a^
2*c*x^3 + 336*A*a*b*c*x^3 + 140*B*a^2*b*x^2 + 140*A*a*b^2*x^2 + 140*A*a^2*
c*x^2 + 40*B*a^3*x + 120*A*a^2*b*x + 35*A*a^3)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9}\,dx = -\frac{x^3\left(\frac{3\,B\,c\,a^2}{5}+\frac{3\,B\,a\,b^2}{5}+\frac{6\,A\,c\,a\,b}{5}+\frac{A\,b^3}{5}\right)+x^4\left(\frac{B\,b^3}{4}+\frac{3\,A\,b^2\,c}{4}+\frac{3\,B\,a\,b\,c}{2}+\frac{3\,A\,a\,c^2}{4}\right)+x\left(\frac{B\,a^3}{7}+\frac{3\,A\,b\,a^2}{7}\right)+\frac{A\,a^3}{8}}{x^8}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^9,x)
```

output
```
-(x^3*((A*b^3)/5 + (3*B*a*b^2)/5 + (3*B*a^2*c)/5 + (6*A*a*b*c)/5) + x^4*((
B*b^3)/4 + (3*A*a*c^2)/4 + (3*A*b^2*c)/4 + (3*B*a*b*c)/2) + x*((B*a^3)/7 +
 (3*A*a^2*b)/7) + (A*a^3)/8 + x^6*((A*c^3)/2 + (3*B*b*c^2)/2) + x^2*((A*a*
b^2)/2 + (A*a^2*c)/2 + (B*a^2*b)/2) + x^5*(A*b*c^2 + B*a*c^2 + B*b^2*c) +
B*c^3*x^7)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^9}\,dx = \frac{-280b\,c^3x^7-140a\,c^3x^6-420b^2c^2x^6-560ab\,c^2x^5-280b^3c\,x^5-210a^2c^2x^4-630a\,b^2c\,x^4-70b^4x^4-50\cdots}{280x^8}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^9,x)
```

output
```
( - 35*a**4 - 160*a**3*b*x - 140*a**3*c*x**2 - 280*a**2*b**2*x**2 - 504*a*
*2*b*c*x**3 - 210*a**2*c**2*x**4 - 224*a*b**3*x**3 - 630*a*b**2*c*x**4 - 5
60*a*b*c**2*x**5 - 140*a*c**3*x**6 - 70*b**4*x**4 - 280*b**3*c*x**5 - 420*
b**2*c**2*x**6 - 280*b*c**3*x**7)/(280*x**8)
```

3.37 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{10}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{10}}\,dx = &-\frac{a^3A}{9x^9}-\frac{a^2(3Ab+aB)}{8x^8}-\frac{3a(abB+A(b^2+ac))}{7x^7}\\&-\frac{3aB(b^2+ac)+A(b^3+6abc)}{6x^6}\\&-\frac{b^3B+3Ab^2c+6abBc+3aAc^2}{5x^5}\\&-\frac{3c(b^2B+Abc+aBc)}{4x^4}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{2x^2}\end{aligned}$$

output

```
-1/9*a^3*A/x^9-1/8*a^2*(3*A*b+B*a)/x^8-3/7*a*(a*b*B+A*(a*c+b^2))/x^7-1/6*(
3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))/x^6-1/5*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*
b^3)/x^5-3/4*c*(A*b*c+B*a*c+B*b^2)/x^4-1/3*c^2*(A*c+3*B*b)/x^3-1/2*B*c^3/x
^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{10}}\,dx =$$
$$-\frac{35a^3(8A+9Bx)+45a^2x(4Bx(6b+7cx)+3A(7b+8cx))+18ax^2(7Bx(10b^2+24bcx+15c^2x^2)+4$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^10,x]
```

output
```
-1/2520*(35*a^3*(8*A + 9*B*x) + 45*a^2*x*(4*B*x*(6*b + 7*c*x) + 3*A*(7*b +
 8*c*x)) + 18*a*x^2*(7*B*x*(10*b^2 + 24*b*c*x + 15*c^2*x^2) + 4*A*(15*b^2
+ 35*b*c*x + 21*c^2*x^2)) + 42*x^3*(3*B*x*(4*b^3 + 15*b^2*c*x + 20*b*c^2*x
^2 + 10*c^3*x^3) + A*(10*b^3 + 36*b^2*c*x + 45*b*c^2*x^2 + 20*c^3*x^3)))/x
^9
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{10}}\,dx$$
$$\Big\downarrow \; 1195$$
$$\int \left(\frac{a^3A}{x^{10}}+\frac{a^2(aB+3Ab)}{x^9}+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x^8}+\frac{3c\left(aBc+Abc+b^2B\right)}{x^5}+\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{x^6}\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{a^3A}{9x^9}-\frac{a^2(aB+3Ab)}{8x^8}-\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{7x^7}-\frac{3c\left(aBc+Abc+b^2B\right)}{4x^4}-\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{5x^5}-\frac{A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)}{6x^6}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{Bc^3}{2x^2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^10,x]
```

output
```
-1/9*(a^3*A)/x^9 - (a^2*(3*A*b + a*B))/(8*x^8) - (3*a*(a*b*B + A*(b^2 + a*
c)))/(7*x^7) - (3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))/(6*x^6) - (b^3*B +
3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)/(5*x^5) - (3*c*(b^2*B + A*b*c + a*B*c))
/(4*x^4) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/(2*x^2)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

method	result
default	$-\frac{3Aa\,c^2+3A\,b^2c+6Babc+B\,b^3}{5x^5}-\frac{6Aabc+A\,b^3+3B\,a^2c+3Ba\,b^2}{6x^6}-\frac{c^2(Ac+3Bb)}{3x^3}-\frac{B\,c^3}{2x^2}-\frac{a^2(3Ab+Ba)}{8x^8}-\frac{3c(Abc-$
norman	$\frac{-\frac{B\,c^3x^7}{2}+\left(-\frac{1}{3}A\,c^3-Bb\,c^2\right)x^6+\left(-\frac{3}{4}Ab\,c^2-\frac{3}{4}Ba\,c^2-\frac{3}{4}B\,b^2c\right)x^5+\left(-\frac{3}{5}Aa\,c^2-\frac{3}{5}A\,b^2c-\frac{6}{5}Babc-\frac{1}{5}B\,b^3\right)x^4+\left(-Aabc-\frac{1}{6}A\,b^3-\right.}{x^9}$
risch	$\frac{-\frac{B\,c^3x^7}{2}+\left(-\frac{1}{3}A\,c^3-Bb\,c^2\right)x^6+\left(-\frac{3}{4}Ab\,c^2-\frac{3}{4}Ba\,c^2-\frac{3}{4}B\,b^2c\right)x^5+\left(-\frac{3}{5}Aa\,c^2-\frac{3}{5}A\,b^2c-\frac{6}{5}Babc-\frac{1}{5}B\,b^3\right)x^4+\left(-Aabc-\frac{1}{6}A\,b^3-\right.}{x^9}$
gosper	$-\frac{1260B\,c^3x^7+840A\,c^3x^6+2520Bb\,c^2x^6+1890Ab\,c^2x^5+1890Ba\,c^2x^5+1890B\,b^2c\,x^5+1512Aa\,c^2x^4+1512A\,b^2c\,x^4+3024Ba}{}$
parallelrisch	$-\frac{1260B\,c^3x^7+840A\,c^3x^6+2520Bb\,c^2x^6+1890Ab\,c^2x^5+1890Ba\,c^2x^5+1890B\,b^2c\,x^5+1512Aa\,c^2x^4+1512A\,b^2c\,x^4+3024Ba}{}$
orering	$-\frac{1260B\,c^3x^7+840A\,c^3x^6+2520Bb\,c^2x^6+1890Ab\,c^2x^5+1890Ba\,c^2x^5+1890B\,b^2c\,x^5+1512Aa\,c^2x^4+1512A\,b^2c\,x^4+3024Ba}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)/x^5-1/6*(6*A*a*b*c+A*b^3+3*B*a^
2*c+3*B*a*b^2)/x^6-1/3*c^2*(A*c+3*B*b)/x^3-1/2*B*c^3/x^2-1/8*a^2*(3*A*b+B*
a)/x^8-3/4*c*(A*b*c+B*a*c+B*b^2)/x^4-1/9*a^3*A/x^9-3/7*a*(A*a*c+A*b^2+B*a*
b)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{10}}\,dx = -\frac{1260\,Bc^3x^7 + 840\,(3\,Bbc^2 + Ac^3)x^6 + 1890\,(Bb^2c + (Ba+Ab)c^2)x^5 + 504\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^10,x, algorithm="fricas")
```

output
```
-1/2520*(1260*B*c^3*x^7 + 840*(3*B*b*c^2 + A*c^3)*x^6 + 1890*(B*b^2*c + (B
*a + A*b)*c^2)*x^5 + 504*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 +
 280*A*a^3 + 420*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 1080*(B
*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 315*(B*a^3 + 3*A*a^2*b)*x)/x^9
```

Sympy [A] (verification not implemented)

Time = 121.61 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{10}}\,dx = \frac{-280Aa^3 - 1260Bc^3x^7 + x^6(-840Ac^3 - 2520Bbc^2) + x^5(-1890Abc^2 - 1890Bac^2 - 1890Bb^2c) + x^4(-$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**10,x)
```

output
```
(-280*A*a**3 - 1260*B*c**3*x**7 + x**6*(-840*A*c**3 - 2520*B*b*c**2) + x**
5*(-1890*A*b*c**2 - 1890*B*a*c**2 - 1890*B*b**2*c) + x**4*(-1512*A*a*c**2
- 1512*A*b**2*c - 3024*B*a*b*c - 504*B*b**3) + x**3*(-2520*A*a*b*c - 420*A
*b**3 - 1260*B*a**2*c - 1260*B*a*b**2) + x**2*(-1080*A*a**2*c - 1080*A*a*b
**2 - 1080*B*a**2*b) + x*(-945*A*a**2*b - 315*B*a**3))/(2520*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{10}}\,dx =$$
$$-\frac{1260\,Bc^3x^7 + 840\,(3\,Bbc^2 + Ac^3)x^6 + 1890\,(Bb^2c + (Ba + Ab)c^2)x^5 + 504\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^10,x, algorithm="maxima")
```

output
```
-1/2520*(1260*B*c^3*x^7 + 840*(3*B*b*c^2 + A*c^3)*x^6 + 1890*(B*b^2*c + (B
*a + A*b)*c^2)*x^5 + 504*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 +
 280*A*a^3 + 420*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 1080*(B
*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 315*(B*a^3 + 3*A*a^2*b)*x)/x^9
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{10}}\,dx =$$
$$-\frac{1260\,Bc^3x^7 + 2520\,Bbc^2x^6 + 840\,Ac^3x^6 + 1890\,Bb^2cx^5 + 1890\,Bac^2x^5 + 1890\,Abc^2x^5 + 504\,Bb^3x^4 +$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^10,x, algorithm="giac")
```

output
```
-1/2520*(1260*B*c^3*x^7 + 2520*B*b*c^2*x^6 + 840*A*c^3*x^6 + 1890*B*b^2*c*
x^5 + 1890*B*a*c^2*x^5 + 1890*A*b*c^2*x^5 + 504*B*b^3*x^4 + 3024*B*a*b*c*x
^4 + 1512*A*b^2*c*x^4 + 1512*A*a*c^2*x^4 + 1260*B*a*b^2*x^3 + 420*A*b^3*x^
3 + 1260*B*a^2*c*x^3 + 2520*A*a*b*c*x^3 + 1080*B*a^2*b*x^2 + 1080*A*a*b^2*
x^2 + 1080*A*a^2*c*x^2 + 315*B*a^3*x + 945*A*a^2*b*x + 280*A*a^3)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{10}}\,dx = -\frac{x^3\left(\frac{B\,c\,a^2}{2}+\frac{B\,a\,b^2}{2}+A\,c\,a\,b+\frac{A\,b^3}{6}\right)+x^4\left(\frac{B\,b^3}{5}+\frac{3\,A\,b^2\,c}{5}+\frac{6\,B\,a\,b\,c}{5}+\frac{3\,A\,a\,c^2}{5}\right)+x\left(\frac{B\,a^3}{8}+\frac{3\,A\,b\,a^2}{8}\right)+\frac{A\,a^3}{9}}{x^9}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^10,x)
```

output
```
-(x^3*((A*b^3)/6 + (B*a*b^2)/2 + (B*a^2*c)/2 + A*a*b*c) + x^4*((B*b^3)/5 +
 (3*A*a*c^2)/5 + (3*A*b^2*c)/5 + (6*B*a*b*c)/5) + x*((B*a^3)/8 + (3*A*a^2*
b)/8) + (A*a^3)/9 + x^6*((A*c^3)/3 + B*b*c^2) + x^2*((3*A*a*b^2)/7 + (3*A*
a^2*c)/7 + (3*B*a^2*b)/7) + x^5*((3*A*b*c^2)/4 + (3*B*a*c^2)/4 + (3*B*b^2*
c)/4) + (B*c^3*x^7)/2)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{10}}\,dx = \frac{-630b\,c^3x^7-420a\,c^3x^6-1260b^2c^2x^6-1890ab\,c^2x^5-945b^3c\,x^5-756a^2c^2x^4-2268a\,b^2c\,x^4-252b^4x^4}{1260x^9}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^10,x)
```

output
```
( - 140*a**4 - 630*a**3*b*x - 540*a**3*c*x**2 - 1080*a**2*b**2*x**2 - 1890
*a**2*b*c*x**3 - 756*a**2*c**2*x**4 - 840*a*b**3*x**3 - 2268*a*b**2*c*x**4
 - 1890*a*b*c**2*x**5 - 420*a*c**3*x**6 - 252*b**4*x**4 - 945*b**3*c*x**5
- 1260*b**2*c**2*x**6 - 630*b*c**3*x**7)/(1260*x**9)
```

3.38 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx = -\frac{a^3A}{10x^{10}}-\frac{a^2(3Ab+aB)}{9x^9}-\frac{3a(abB+A(b^2+ac))}{8x^8} -\frac{3aB(b^2+ac)+A(b^3+6abc)}{7x^7} -\frac{b^3B+3Ab^2c+6abBc+3aAc^2}{6x^6} -\frac{3c(b^2B+Abc+aBc)}{5x^5}-\frac{c^2(3bB+Ac)}{4x^4}-\frac{Bc^3}{3x^3}$$

output
```
-1/10*a^3*A/x^10-1/9*a^2*(3*A*b+B*a)/x^9-3/8*a*(a*b*B+A*(a*c+b^2))/x^8-1/7
*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))/x^7-1/6*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+
B*b^3)/x^6-3/5*c*(A*b*c+B*a*c+B*b^2)/x^5-1/4*c^2*(A*c+3*B*b)/x^4-1/3*B*c^3
/x^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx =$$
$$-\frac{28a^3(9A+10Bx)+15a^2x(9Bx(7b+8cx)+7A(8b+9cx))+9ax^2(8Bx(15b^2+35bcx+21c^2x^2)+5$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^11,x]
```

output
```
-1/2520*(28*a^3*(9*A + 10*B*x) + 15*a^2*x*(9*B*x*(7*b + 8*c*x) + 7*A*(8*b
+ 9*c*x)) + 9*a*x^2*(8*B*x*(15*b^2 + 35*b*c*x + 21*c^2*x^2) + 5*A*(21*b^2
+ 48*b*c*x + 28*c^2*x^2)) + 6*x^3*(7*B*x*(10*b^3 + 36*b^2*c*x + 45*b*c^2*x
^2 + 20*c^3*x^3) + 3*A*(20*b^3 + 70*b^2*c*x + 84*b*c^2*x^2 + 35*c^3*x^3)))
/x^10
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^{11}}+\frac{a^2(aB+3Ab)}{x^{10}}+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x^9}+\frac{3c\left(aBc+Abc+b^2B\right)}{x^6}+\frac{3aAc^2+6abBc+3Ab^2c+b^3B$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{10x^{10}}-\frac{a^2(aB+3Ab)}{9x^9}-\frac{3a\big(A\big(ac+b^2\big)+abB\big)}{8x^8}-\frac{3c\big(aBc+Abc+b^2B\big)}{5x^5}-\frac{3aAc^2+6abBc+3Ab^2c+b^3B}{6x^6}-\frac{A\big(6abc+b^3\big)+3aB\big(ac+b^2\big)}{7x^7}-\frac{c^2(Ac+3bB)}{4x^4}-\frac{Bc^3}{3x^3}$$

input

```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^11,x]
```

output

```
-1/10*(a^3*A)/x^10 - (a^2*(3*A*b + a*B))/(9*x^9) - (3*a*(a*b*B + A*(b^2 +
a*c)))/(8*x^8) - (3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))/(7*x^7) - (b^3*B
+ 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)/(6*x^6) - (3*c*(b^2*B + A*b*c + a*B*c
))/(5*x^5) - (c^2*(3*b*B + A*c))/(4*x^4) - (B*c^3)/(3*x^3)
```

Defintions of rubi rules used

rule 1195

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

method	result
default	$-\frac{3c(Abc+aBc+B\,b^2)}{5x^5}-\frac{3Aa\,c^2+3A\,b^2c+6Babc+B\,b^3}{6x^6}-\frac{B\,c^3}{3x^3}-\frac{3a(Aac+b^2A+abB)}{8x^8}-\frac{c^2(Ac+3Bb)}{4x^4}-\frac{a^2(3Ab+}{9x^9}$
norman	$\frac{-\frac{B\,c^3x^7}{3}+(-\frac{1}{4}A\,c^3-\frac{3}{4}Bb\,c^2)x^6+(-\frac{3}{5}Ab\,c^2-\frac{3}{5}Ba\,c^2-\frac{3}{5}B\,b^2c)x^5+(-\frac{1}{2}Aa\,c^2-\frac{1}{2}A\,b^2c-Babc-\frac{1}{6}B\,b^3)x^4+(-\frac{6}{7}Aabc-\frac{1}{7}A\,b}{x^{10}}$
risch	$\frac{-\frac{B\,c^3x^7}{3}+(-\frac{1}{4}A\,c^3-\frac{3}{4}Bb\,c^2)x^6+(-\frac{3}{5}Ab\,c^2-\frac{3}{5}Ba\,c^2-\frac{3}{5}B\,b^2c)x^5+(-\frac{1}{2}Aa\,c^2-\frac{1}{2}A\,b^2c-Babc-\frac{1}{6}B\,b^3)x^4+(-\frac{6}{7}Aabc-\frac{1}{7}A\,b}{x^{10}}$
gosper	$-\frac{840B\,c^3x^7+630A\,c^3x^6+1890Bb\,c^2x^6+1512Ab\,c^2x^5+1512Ba\,c^2x^5+1512B\,b^2c\,x^5+1260Aa\,c^2x^4+1260A\,b^2c\,x^4+2520Bab}{}$
parallelrisch	$-\frac{840B\,c^3x^7+630A\,c^3x^6+1890Bb\,c^2x^6+1512Ab\,c^2x^5+1512Ba\,c^2x^5+1512B\,b^2c\,x^5+1260Aa\,c^2x^4+1260A\,b^2c\,x^4+2520Bab}{}$
orering	$-\frac{840B\,c^3x^7+630A\,c^3x^6+1890Bb\,c^2x^6+1512Ab\,c^2x^5+1512Ba\,c^2x^5+1512B\,b^2c\,x^5+1260Aa\,c^2x^4+1260A\,b^2c\,x^4+2520Bab}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^11,x,method=_RETURNVERBOSE)
```

output
```
-3/5*c*(A*b*c+B*a*c+B*b^2)/x^5-1/6*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)/x
^6-1/3*B*c^3/x^3-3/8*a*(A*a*c+A*b^2+B*a*b)/x^8-1/4*c^2*(A*c+3*B*b)/x^4-1/9
*a^2*(3*A*b+B*a)/x^9-1/7*(6*A*a*b*c+A*b^3+3*B*a^2*c+3*B*a*b^2)/x^7-1/10*a^
3*A/x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx =$$

$$-\frac{840\,Bc^3x^7 + 630\,(3\,Bbc^2 + Ac^3)x^6 + 1512\,(Bb^2c + (Ba + Ab)c^2)x^5 + 420\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab +}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^11,x, algorithm="fricas")
```

output
```
-1/2520*(840*B*c^3*x^7 + 630*(3*B*b*c^2 + A*c^3)*x^6 + 1512*(B*b^2*c + (B*
a + A*b)*c^2)*x^5 + 420*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 +
252*A*a^3 + 360*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 945*(B*a
^2*b + A*a*b^2 + A*a^2*c)*x^2 + 280*(B*a^3 + 3*A*a^2*b)*x)/x^10
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**11,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx = -\frac{840\,Bc^3x^7+630\,(3\,Bbc^2+Ac^3)x^6+1512\,(Bb^2c+(Ba+Ab)c^2)x^5+420\,(Bb^3+3\,Aac^2+3\,(2\,Bab+$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^11,x, algorithm="maxima")
```

output
```
-1/2520*(840*B*c^3*x^7 + 630*(3*B*b*c^2 + A*c^3)*x^6 + 1512*(B*b^2*c + (B*
a + A*b)*c^2)*x^5 + 420*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 +
252*A*a^3 + 360*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 945*(B*a
^2*b + A*a*b^2 + A*a^2*c)*x^2 + 280*(B*a^3 + 3*A*a^2*b)*x)/x^10
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx = -\frac{840\,Bc^3x^7+1890\,Bbc^2x^6+630\,Ac^3x^6+1512\,Bb^2cx^5+1512\,Bac^2x^5+1512\,Abc^2x^5+420\,Bb^3x^4+2$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^11,x, algorithm="giac")
```

output
```
-1/2520*(840*B*c^3*x^7 + 1890*B*b*c^2*x^6 + 630*A*c^3*x^6 + 1512*B*b^2*c*x
^5 + 1512*B*a*c^2*x^5 + 1512*A*b*c^2*x^5 + 420*B*b^3*x^4 + 2520*B*a*b*c*x^
4 + 1260*A*b^2*c*x^4 + 1260*A*a*c^2*x^4 + 1080*B*a*b^2*x^3 + 360*A*b^3*x^3
 + 1080*B*a^2*c*x^3 + 2160*A*a*b*c*x^3 + 945*B*a^2*b*x^2 + 945*A*a*b^2*x^2
 + 945*A*a^2*c*x^2 + 280*B*a^3*x + 840*A*a^2*b*x + 252*A*a^3)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx = -\frac{x^3\left(\frac{3\,B\,c\,a^2}{7}+\frac{3\,B\,a\,b^2}{7}+\frac{6\,A\,c\,a\,b}{7}+\frac{A\,b^3}{7}\right)+x^4\left(\frac{B\,b^3}{6}+\frac{A\,b^2\,c}{2}+B\,a\,b\,c+\frac{A\,a\,c^2}{2}\right)+x\left(\frac{B\,a^3}{9}+\frac{A\,b\,a^2}{3}\right)+\frac{A\,a^3}{10}+}{x^{10}}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^11,x)
```

output
```
-(x^3*((A*b^3)/7 + (3*B*a*b^2)/7 + (3*B*a^2*c)/7 + (6*A*a*b*c)/7) + x^4*((
B*b^3)/6 + (A*a*c^2)/2 + (A*b^2*c)/2 + B*a*b*c) + x*((B*a^3)/9 + (A*a^2*b)
/3) + (A*a^3)/10 + x^6*((A*c^3)/4 + (3*B*b*c^2)/4) + x^2*((3*A*a*b^2)/8 +
(3*A*a^2*c)/8 + (3*B*a^2*b)/8) + x^5*((3*A*b*c^2)/5 + (3*B*a*c^2)/5 + (3*B
*b^2*c)/5) + (B*c^3*x^7)/3)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11}}\,dx = \frac{-840b\,c^3x^7-630a\,c^3x^6-1890b^2c^2x^6-3024ab\,c^2x^5-1512b^3c\,x^5-1260a^2c^2x^4-3780a\,b^2c\,x^4-420b^4x}{2520x^{10}}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^11,x)
```

output
```
( - 252*a**4 - 1120*a**3*b*x - 945*a**3*c*x**2 - 1890*a**2*b**2*x**2 - 324
0*a**2*b*c*x**3 - 1260*a**2*c**2*x**4 - 1440*a*b**3*x**3 - 3780*a*b**2*c*x
**4 - 3024*a*b*c**2*x**5 - 630*a*c**3*x**6 - 420*b**4*x**4 - 1512*b**3*c*x
**5 - 1890*b**2*c**2*x**6 - 840*b*c**3*x**7)/(2520*x**10)
```

3.39 $\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 229

$$\begin{aligned}\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx &= \frac{(b^2cd-ac^2d-b^3e+2abce)\,x}{c^4}-\frac{(bcd-b^2e+ace)\,x^2}{2c^3}+\frac{(cd-be)x^3}{3c^2}+\frac{ex^4}{4c}\\&\quad-\frac{(b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}\\&\quad-\frac{(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)\log\left(a+bx+cx^2\right)}{2c^5}\end{aligned}$$

output
```
(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*x/c^4-1/2*(a*c*e-b^2*e+b*c*d)*x^2/c^3+1/
3*(-b*e+c*d)*x^3/c^2+1/4*e*x^4/c-(-5*a^2*b*c^2*e+2*a^2*c^3*d+5*a*b^3*c*e-4
*a*b^2*c^2*d-b^5*e+b^4*c*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*
a*c+b^2)^(1/2)-1/2*(-a^2*c^2*e+3*a*b^2*c*e-2*a*b*c^2*d-b^4*e+b^3*c*d)*ln(c
*x^2+b*x+a)/c^5
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.97

$$\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx$$

$$= \frac{-12c(-b^2cd+ac^2d+b^3e-2abce)\,x-6c^2(bcd-b^2e+ace)\,x^2+4c^3(cd-be)x^3+3c^4ex^4+\frac{12(b^4cd-4ab^2c}{}}{1}$$

input
```
Integrate[(x^4*(d + e*x))/(a + b*x + c*x^2),x]
```

output
```
(-12*c*(-(b^2*c*d) + a*c^2*d + b^3*e - 2*a*b*c*e)*x - 6*c^2*(b*c*d - b^2*e
 + a*c*e)*x^2 + 4*c^3*(c*d - b*e)*x^3 + 3*c^4*e*x^4 + (12*(b^4*c*d - 4*a*b
^2*c^2*d + 2*a^2*c^3*d - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)*ArcTan[(b +
2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 6*(-(b^3*c*d) + 2*a*b*c^2
*d + b^4*e - 3*a*b^2*c*e + a^2*c^2*e)*Log[a + x*(b + c*x)])/(12*c^5)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \ 1200$$

$$\int \left(-\frac{x\left(-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)+a\left(2abce-ac^2d+b^3(-e)+b^2cd\right)}{c^4\left(a+bx+cx^2\right)}-\frac{x\left(ace+b^2(-e)+b\right.}{c^3}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd\right)}{c^5\sqrt{b^2-4ac}}-\frac{\left(-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)\log\left(a+bx+cx^2\right)}{2c^5}-\frac{x^2\left(ace+b^2(-e)+bcd\right)}{2c^3}+\frac{x\left(2abce-ac^2d+b^3(-e)+b^2cd\right)}{c^4}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^4}{4c}$$

input
```
Int[(x^4*(d + e*x))/(a + b*x + c*x^2),x]
```

output
```
((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*x)/c^4 - ((b*c*d - b^2*e + a*c*e)
*x^2)/(2*c^3) + ((c*d - b*e)*x^3)/(3*c^2) + (e*x^4)/(4*c) - ((b^4*c*d - 4*
a*b^2*c^2*d + 2*a^2*c^3*d - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)*ArcTanh[(
b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) - ((b^3*c*d - 2*a*b
*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*Log[a + b*x + c*x^2])/(2*c^5)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{1}{4}c^3x^4e-\frac{1}{3}bc^2x^3e+\frac{1}{3}c^3dx^3-\frac{1}{2}ac^2ex^2+\frac{1}{2}b^2cex^2-\frac{1}{2}bc^2dx^2+2abcex-adxc^2-b^3ex+b^2cxd}{c^4}+\frac{\frac{\left(a^2c^2e-3ab^2ce+2abc^2d+b^4e-b^3cd\right.}{2c}}{}$
risch	Expression too large to display

input
```
int(x^4*(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*c^3*x^4*e-1/3*b*c^2*x^3*e+1/3*c^3*d*x^3-1/2*a*c^2*e*x^2+1/2*b^2
*c*e*x^2-1/2*b*c^2*d*x^2+2*a*b*c*e*x-a*d*x*c^2-b^3*e*x+b^2*c*x*d)+1/c^4*(1
/2*(a^2*c^2*e-3*a*b^2*c*e+2*a*b*c^2*d+b^4*e-b^3*c*d)/c*ln(c*x^2+b*x+a)+2*(
-2*a^2*b*c*e+a^2*c^2*d+a*b^3*e-c*d*a*b^2-1/2*(a^2*c^2*e-3*a*b^2*c*e+2*a*b*
c^2*d+b^4*e-b^3*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.19

$$\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*c^4 - 4*a*c^5)*e*x^4 + 4*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 -
 4*a*b*c^4)*e)*x^3 - 6*((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 +
 4*a^2*c^4)*e)*x^2 - 6*sqrt(b^2 - 4*a*c)*((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3
)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 12*((b^4*c^2 -
 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e)*x - 6
*((b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^
2 - 4*a^3*c^3)*e)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6), 1/12*(3*(b^2*
c^4 - 4*a*c^5)*e*x^4 + 4*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)
*x^3 - 6*((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e)
*x^2 - 12*sqrt(-b^2 + 4*a*c)*((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 -
 5*a*b^3*c + 5*a^2*b*c^2)*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 -
 4*a*c)) + 12*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*e)*x - 6*((b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d - (b^6 -
7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e)*log(c*x^2 + b*x + a))/(b^2*c^5
- 4*a*c^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. 2(236) = 472.

Time = 2.00 (sec) , antiderivative size = 1100, normalized size of antiderivative = 4.80

$$\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
x**3*(-b*e/(3*c**2) + d/(3*c)) + x**2*(-a*e/(2*c**2) + b**2*e/(2*c**3) - b
*d/(2*c**2)) + x*(2*a*b*e/c**3 - a*d/c**2 - b**3*e/c**4 + b**2*d/c**3) + (
-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e - 2*a**2*c**3*d - 5*a*b**3*c*e + 4*a
*b**2*c**2*d + b**5*e - b**4*c*d)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e -
 3*a*b**2*c*e + 2*a*b*c**2*d + b**4*e - b**3*c*d)/(2*c**5))*log(x + (2*a**
3*c**2*e - 4*a**2*b**2*c*e + 3*a**2*b*c**2*d + a*b**4*e - a*b**3*c*d - 4*a
*c**5*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e - 2*a**2*c**3*d - 5*a*b**3*c*
e + 4*a*b**2*c**2*d + b**5*e - b**4*c*d)/(2*c**5*(4*a*c - b**2)) + (a**2*c
**2*e - 3*a*b**2*c*e + 2*a*b*c**2*d + b**4*e - b**3*c*d)/(2*c**5)) + b**2*
c**4*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e - 2*a**2*c**3*d - 5*a*b**3*c*e
 + 4*a*b**2*c**2*d + b**5*e - b**4*c*d)/(2*c**5*(4*a*c - b**2)) + (a**2*c*
*2*e - 3*a*b**2*c*e + 2*a*b*c**2*d + b**4*e - b**3*c*d)/(2*c**5)))/(5*a**2
*b*c**2*e - 2*a**2*c**3*d - 5*a*b**3*c*e + 4*a*b**2*c**2*d + b**5*e - b**4
*c*d)) + (sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e - 2*a**2*c**3*d - 5*a*b**3*
c*e + 4*a*b**2*c**2*d + b**5*e - b**4*c*d)/(2*c**5*(4*a*c - b**2)) + (a**2
*c**2*e - 3*a*b**2*c*e + 2*a*b*c**2*d + b**4*e - b**3*c*d)/(2*c**5))*log(x
 + (2*a**3*c**2*e - 4*a**2*b**2*c*e + 3*a**2*b*c**2*d + a*b**4*e - a*b**3*
c*d - 4*a*c**5*(sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e - 2*a**2*c**3*d - 5*a
*b**3*c*e + 4*a*b**2*c**2*d + b**5*e - b**4*c*d)/(2*c**5*(4*a*c - b**2)) +
 (a**2*c**2*e - 3*a*b**2*c*e + 2*a*b*c**2*d + b**4*e - b**3*c*d)/(2*c**...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

$$\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx = \frac{3\,c^3ex^4+4\,c^3dx^3-4\,bc^2ex^3-6\,bc^2dx^2+6\,b^2cex^2-6\,ac^2ex^2+12\,b^2cdx-12\,ac^2dx-12\,b^3ex+24\,abce}{12\,c^4} - \frac{(b^3cd-2\,abc^2d-b^4e+3\,ab^2ce-a^2c^2e)\log\left(cx^2+bx+a\right)}{2\,c^5} + \frac{(b^4cd-4\,ab^2c^2d+2\,a^2c^3d-b^5e+5\,ab^3ce-5\,a^2bc^2e)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^5}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/12*(3*c^3*e*x^4 + 4*c^3*d*x^3 - 4*b*c^2*e*x^3 - 6*b*c^2*d*x^2 + 6*b^2*c*
e*x^2 - 6*a*c^2*e*x^2 + 12*b^2*c*d*x - 12*a*c^2*d*x - 12*b^3*e*x + 24*a*b*
c*e*x)/c^4 - 1/2*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)
*log(c*x^2 + b*x + a)/c^5 + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d - b^5*e
 + 5*a*b^3*c*e - 5*a^2*b*c^2*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sq
rt(-b^2 + 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.32

$$\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx$$
$$= x^3\left(\frac{d}{3\,c}-\frac{b\,e}{3\,c^2}\right)+x\left(\frac{b\left(\frac{b\left(\frac{d}{c}-\frac{b\,e}{c^2}\right)}{c}+\frac{a\,e}{c^2}\right)}{c}-\frac{a\left(\frac{d}{c}-\frac{b\,e}{c^2}\right)}{c}\right)-x^2\left(\frac{b\left(\frac{d}{c}-\frac{b\,e}{c^2}\right)}{2\,c}+\frac{a\,e}{2\,c^2}\right)$$
$$+\frac{\ln\left(c\,x^2+b\,x+a\right)\left(4\,e\,a^3\,c^3-13\,e\,a^2\,b^2\,c^2+8\,d\,a^2\,b\,c^3+7\,e\,a\,b^4\,c-6\,d\,a\,b^3\,c^2-e\,b^6+d\,b^5\,c\right)}{2\left(4\,a\,c^6-b^2\,c^5\right)}$$
$$+\frac{e\,x^4}{4\,c}$$
$$-\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\left(5\,e\,a^2\,b\,c^2-2\,d\,a^2\,c^3-5\,e\,a\,b^3\,c+4\,d\,a\,b^2\,c^2+e\,b^5-d\,b^4\,c\right)}{c^5\sqrt{4\,a\,c-b^2}}$$

input
```
int((x^4*(d + e*x))/(a + b*x + c*x^2),x)
```

output
```
x^3*(d/(3*c) - (b*e)/(3*c^2)) + x*((b*((b*(d/c - (b*e)/c^2))/c + (a*e)/c^2
))/c - (a*(d/c - (b*e)/c^2))/c) - x^2*((b*(d/c - (b*e)/c^2))/(2*c) + (a*e)
/(2*c^2)) + (log(a + b*x + c*x^2)*(4*a^3*c^3*e - b^6*e + b^5*c*d - 13*a^2*
b^2*c^2*e + 7*a*b^4*c*e - 6*a*b^3*c^2*d + 8*a^2*b*c^3*d))/(2*(4*a*c^6 - b^
2*c^5)) + (e*x^4)/(4*c) - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b
^2)^(1/2))*(b^5*e - 2*a^2*c^3*d - b^4*c*d - 5*a*b^3*c*e + 4*a*b^2*c^2*d +
5*a^2*b*c^2*e))/(c^5*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 587, normalized size of antiderivative = 2.56

$$\int \frac{x^4(d+ex)}{a+bx+cx^2}\,dx$$
$$= \frac{-60\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b\,c^2e+60\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^3ce-48\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right.}{}$$

input
```
int(x^4*(e*x+d)/(c*x^2+b*x+a),x)
```

output
```
( - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2
*e + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**3*
d + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c*e
- 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*d
 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*e + 12*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*c*d + 24*log(
a + b*x + c*x**2)*a**3*c**3*e - 78*log(a + b*x + c*x**2)*a**2*b**2*c**2*e
+ 48*log(a + b*x + c*x**2)*a**2*b*c**3*d + 42*log(a + b*x + c*x**2)*a*b**4
*c*e - 36*log(a + b*x + c*x**2)*a*b**3*c**2*d - 6*log(a + b*x + c*x**2)*b*
*6*e + 6*log(a + b*x + c*x**2)*b**5*c*d + 96*a**2*b*c**3*e*x - 48*a**2*c**
4*d*x - 24*a**2*c**4*e*x**2 - 72*a*b**3*c**2*e*x + 60*a*b**2*c**3*d*x + 30
*a*b**2*c**3*e*x**2 - 24*a*b*c**4*d*x**2 - 16*a*b*c**4*e*x**3 + 16*a*c**5*
d*x**3 + 12*a*c**5*e*x**4 + 12*b**5*c*e*x - 12*b**4*c**2*d*x - 6*b**4*c**2
*e*x**2 + 6*b**3*c**3*d*x**2 + 4*b**3*c**3*e*x**3 - 4*b**2*c**4*d*x**3 - 3
*b**2*c**4*e*x**4)/(12*c**5*(4*a*c - b**2))
```

3.40 $\int \frac{x^3(d+ex)}{a+bx+cx^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{x^3(d+ex)}{a+bx+cx^2} \, dx = -\frac{(bcd-b^2e+ace)\,x}{c^3}+\frac{(cd-be)x^2}{2c^2}+\frac{ex^3}{3c} + \frac{(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}} + \frac{(b^2cd-ac^2d-b^3e+2abce)\log\left(a+bx+cx^2\right)}{2c^4}$$

output
```
-(a*c*e-b^2*e+b*c*d)*x/c^3+1/2*(-b*e+c*d)*x^2/c^2+1/3*e*x^3/c+(-2*a^2*c^2*
e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/c^4/(-4*a*c+b^2)^(1/2)+1/2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*ln(c*x^2
+b*x+a)/c^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

$$\int \frac{x^3(d+ex)}{a+bx+cx^2}\,dx$$
$$= \frac{-6c(bcd-b^2e+ace)\,x+3c^2(cd-be)x^2+2c^3ex^3+\frac{6(-b^3cd+3abc^2d+b^4e-4ab^2ce+2a^2c^2e)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-3(}{6c^4}$$

input
```
Integrate[(x^3*(d + e*x))/(a + b*x + c*x^2),x]
```

output
```
(-6*c*(b*c*d - b^2*e + a*c*e)*x + 3*c^2*(c*d - b*e)*x^2 + 2*c^3*e*x^3 + (6
*(-(b^3*c*d) + 3*a*b*c^2*d + b^4*e - 4*a*b^2*c*e + 2*a^2*c^2*e)*ArcTan[(b
+ 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(-(b^2*c*d) + a*c^2*d
 + b^3*e - 2*a*b*c*e)*Log[a + x*(b + c*x)])/(6*c^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \; 1200$$
$$\int \left(-\frac{ace+b^2(-e)+bcd}{c^3}+\frac{a\left(ace+b^2(-e)+bcd\right)+x\left(2abce-ac^2d+b^3(-e)+b^2cd\right)}{c^3\left(a+bx+cx^2\right)}+\frac{x(cd-be)}{c^2}+\frac{ex^2}{c}\right)d$$
$$\Big\downarrow \; 2009$$

$$\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd\right)}{c^4\sqrt{b^2-4ac}}-\frac{x\left(ace+b^2(-e)+bcd\right)}{c^3}+\frac{\left(2abce-ac^2d+b^3(-e)+b^2cd\right)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x^2(cd-be)}{2c^2}+\frac{ex^3}{3c}$$

input
```
Int[(x^3*(d + e*x))/(a + b*x + c*x^2),x]
```

output
```
-(((b*c*d - b^2*e + a*c*e)*x)/c^3) + ((c*d - b*e)*x^2)/(2*c^2) + (e*x^3)/(
3*c) + ((b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + ((b^2*c*d - a*
c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x + c*x^2])/(2*c^4)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08

method	result
default	$-\frac{-\frac{1}{3}e\,x^3c^2+\frac{1}{2}bce\,x^2-\frac{1}{2}c^2d\,x^2+acex-b^2ex+bcdx}{c^3}+\frac{\frac{\left(2abce-a\,c^2d-e\,b^3+cd\,b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(a^2ce-ea\,b^2+abcd-\frac{\left(2abce-a\,c^2d \right.}{2}\right.}{\sqrt{4a}}}{c^3}$
risch	Expression too large to display

input
```
int(x^3*(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/3*e*x^3*c^2+1/2*b*c*e*x^2-1/2*c^2*d*x^2+a*c*e*x-b^2*e*x+b*c*d*x
)+1/c^3*(1/2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)/c*ln(c*x^2+b*x+a)+2*(a^2*c*
e-e*a*b^2+a*b*c*d-1/2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*b/c)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.33

$$\int \frac{x^3(d+ex)}{a+bx+cx^2}\,dx$$
$$= \left[\frac{2\,(b^2c^3-4\,ac^4)ex^3+3\,((b^2c^3-4\,ac^4)d-(b^3c^2-4\,abc^3)e)x^2-3\,\sqrt{b^2-4\,ac}((b^3c-3\,abc^2)d-(b^4-4\,a\right.$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c^3 - 4*a*c^4)*e*x^3 + 3*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 -
4*a*b*c^3)*e)*x^2 - 3*sqrt(b^2 - 4*a*c)*((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*
a*b^2*c + 2*a^2*c^2)*e)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2
- 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 6*((b^3*c^2 - 4*a*b*c^3)*d - (b
^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e)*x + 3*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3
)*d - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e)*log(c*x^2 + b*x + a))/(b^2*c^4 -
4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*e*x^3 + 3*((b^2*c^3 - 4*a*c^4)*d - (b
^3*c^2 - 4*a*b*c^3)*e)*x^2 + 6*sqrt(-b^2 + 4*a*c)*((b^3*c - 3*a*b*c^2)*d -
 (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) - 6*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*
c^3)*e)*x + 3*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d - (b^5 - 6*a*b^3*c + 8*
a^2*b*c^2)*e)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(168) = 336$.

Time = 1.71 (sec) , antiderivative size = 840, normalized size of antiderivative = 4.97

$$\int \frac{x^3(d+ex)}{a+bx+cx^2}\,dx = x^2\left(-\frac{be}{2c^2}+\frac{d}{2c}\right)+x\left(-\frac{ae}{c^2}+\frac{b^2e}{c^3}-\frac{bd}{c^2}\right)$$
$$+\left(-\frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2e-4ab^2ce+3abc^2d+b^4e-b^3cd)}{2c^4\cdot(4ac-b^2)}\right.$$
$$\left.+\frac{2abce-ac^2d-b^3e+b^2cd}{2c^4}\right)\log\left(x+\frac{-3a^2bce+2a^2c^2d+ab^3e-ab^2cd+4ac^4\left(-\frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2e-4ab}{2c^4\cdot(4a}\right.}{}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2e-4ab^2ce+3abc^2d+b^4e-b^3cd)}{2c^4\cdot(4ac-b^2)}\right.$$
$$\left.+\frac{2abce-ac^2d-b^3e+b^2cd}{2c^4}\right)\log\left(x+\frac{-3a^2bce+2a^2c^2d+ab^3e-ab^2cd+4ac^4\left(\frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2e-4ab^2c}{2c^4\cdot(4ac-}\right.}{}\right.$$
$$+\frac{ex^3}{3c}$$

input
```
integrate(x**3*(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
x**2*(-b*e/(2*c**2) + d/(2*c)) + x*(-a*e/c**2 + b**2*e/c**3 - b*d/c**2) +
(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e - 4*a*b**2*c*e + 3*a*b*c**2*d + b**4*
e - b**3*c*d)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e - a*c**2*d - b**3*e + b
**2*c*d)/(2*c**4))*log(x + (-3*a**2*b*c*e + 2*a**2*c**2*d + a*b**3*e - a*b
**2*c*d + 4*a*c**4*(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e - 4*a*b**2*c*e + 3
*a*b*c**2*d + b**4*e - b**3*c*d)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e - a*
c**2*d - b**3*e + b**2*c*d)/(2*c**4)) - b**2*c**3*(-sqrt(-4*a*c + b**2)*(2
*a**2*c**2*e - 4*a*b**2*c*e + 3*a*b*c**2*d + b**4*e - b**3*c*d)/(2*c**4*(4
*a*c - b**2)) + (2*a*b*c*e - a*c**2*d - b**3*e + b**2*c*d)/(2*c**4)))/(2*a
**2*c**2*e - 4*a*b**2*c*e + 3*a*b*c**2*d + b**4*e - b**3*c*d)) + (sqrt(-4*
a*c + b**2)*(2*a**2*c**2*e - 4*a*b**2*c*e + 3*a*b*c**2*d + b**4*e - b**3*c
*d)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e - a*c**2*d - b**3*e + b**2*c*d)/(
2*c**4))*log(x + (-3*a**2*b*c*e + 2*a**2*c**2*d + a*b**3*e - a*b**2*c*d +
4*a*c**4*(sqrt(-4*a*c + b**2)*(2*a**2*c**2*e - 4*a*b**2*c*e + 3*a*b*c**2*d
 + b**4*e - b**3*c*d)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e - a*c**2*d - b*
*3*e + b**2*c*d)/(2*c**4)) - b**2*c**3*(sqrt(-4*a*c + b**2)*(2*a**2*c**2*e
 - 4*a*b**2*c*e + 3*a*b*c**2*d + b**4*e - b**3*c*d)/(2*c**4*(4*a*c - b**2)
) + (2*a*b*c*e - a*c**2*d - b**3*e + b**2*c*d)/(2*c**4)))/(2*a**2*c**2*e -
 4*a*b**2*c*e + 3*a*b*c**2*d + b**4*e - b**3*c*d)) + e*x**3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00

$$\int \frac{x^3(d+ex)}{a+bx+cx^2}\,dx = \frac{2\,c^2ex^3+3\,c^2dx^2-3\,bcex^2-6\,bcdx+6\,b^2ex-6\,acex}{6\,c^3} + \frac{(b^2cd-ac^2d-b^3e+2\,abce)\log\left(cx^2+bx+a\right)}{2\,c^4} - \frac{(b^3cd-3\,abc^2d-b^4e+4\,ab^2ce-2\,a^2c^2e)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^4}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/6*(2*c^2*e*x^3 + 3*c^2*d*x^2 - 3*b*c*e*x^2 - 6*b*c*d*x + 6*b^2*e*x - 6*a
*c*e*x)/c^3 + 1/2*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*log(c*x^2 + b*x
+ a)/c^4 - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*arc
tan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.31

$$\int \frac{x^3(d+ex)}{a+bx+cx^2}\,dx = x^2\left(\frac{d}{2\,c}-\frac{b\,e}{2\,c^2}\right)-x\left(\frac{b\left(\frac{d}{c}-\frac{b\,e}{c^2}\right)}{c}+\frac{a\,e}{c^2}\right) + \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(8\,e\,a^2\,b\,c^2-4\,d\,a^2\,c^3-6\,e\,a\,b^3\,c+5\,d\,a\,b^2\,c^2+e\,b^5-d\,b^4\,c\right)}{2\,\left(4\,a\,c^5-b^2\,c^4\right)} + \frac{e\,x^3}{3\,c} + \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,e\,a^2\,c^2-4\,e\,a\,b^2\,c+3\,d\,a\,b\,c^2+e\,b^4-d\,b^3\,c\right)}{c^4\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((x^3*(d + e*x))/(a + b*x + c*x^2),x)
```

output
```
x^2*(d/(2*c) - (b*e)/(2*c^2)) - x*((b*(d/c - (b*e)/c^2))/c + (a*e)/c^2) +
(log(a + b*x + c*x^2)*(b^5*e - 4*a^2*c^3*d - b^4*c*d - 6*a*b^3*c*e + 5*a*b
^2*c^2*d + 8*a^2*b*c^2*e))/(2*(4*a*c^5 - b^2*c^4)) + (e*x^3)/(3*c) + (atan
(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^4*e + 2*a^2*c^2*e
 - b^3*c*d + 3*a*b*c^2*d - 4*a*b^2*c*e))/(c^4*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.64

$$\int \frac{x^3(d+ex)}{a+bx+cx^2}\,dx = \frac{12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2c^2e - 24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2ce + 18\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ab}{}$$

input
```
int(x^3*(e*x+d)/(c*x^2+b*x+a),x)
```

output
```
(12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*e -
24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*e + 18
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*d + 6*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*e - 6*sqrt(4*a*
c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*d + 24*log(a + b*x +
 c*x**2)*a**2*b*c**2*e - 12*log(a + b*x + c*x**2)*a**2*c**3*d - 18*log(a +
 b*x + c*x**2)*a*b**3*c*e + 15*log(a + b*x + c*x**2)*a*b**2*c**2*d + 3*log
(a + b*x + c*x**2)*b**5*e - 3*log(a + b*x + c*x**2)*b**4*c*d - 24*a**2*c**
3*e*x + 30*a*b**2*c**2*e*x - 24*a*b*c**3*d*x - 12*a*b*c**3*e*x**2 + 12*a*c
**4*d*x**2 + 8*a*c**4*e*x**3 - 6*b**4*c*e*x + 6*b**3*c**2*d*x + 3*b**3*c**
2*e*x**2 - 3*b**2*c**3*d*x**2 - 2*b**2*c**3*e*x**3)/(6*c**4*(4*a*c - b**2)
)
```

3.41 $\int \frac{x^2(d+ex)}{a+bx+cx^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{x^2(d+ex)}{a+bx+cx^2} \, dx = \frac{(cd-be)x}{c^2} + \frac{ex^2}{2c} - \frac{(b^2cd-2ac^2d-b^3e+3abce) \operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} - \frac{(bcd-b^2e+ace) \log(a+bx+cx^2)}{2c^3}$$

output
```
(-b*e+c*d)*x/c^2+1/2*e*x^2/c-(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((
2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)-1/2*(a*c*e-b^2*e+b*c*d
)*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{x^2(d+ex)}{a+bx+cx^2} \, dx = \frac{2c(cd-be)x + c^2ex^2 + \frac{2(b^2cd-2ac^2d-b^3e+3abce) \arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (-bcd+b^2e-ace) \log(a+x(b+cx))}{2c^3}$$

input
```
Integrate[(x^2*(d + e*x))/(a + b*x + c*x^2),x]
```

output
```
(2*c*(c*d - b*e)*x + c^2*e*x^2 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c
*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*c*d)
 + b^2*e - a*c*e)*Log[a + x*(b + c*x)])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(-\frac{x\left(ace+b^2(-e)+bcd\right)+a(cd-be)}{c^2\left(a+bx+cx^2\right)}+\frac{cd-be}{c^2}+\frac{ex}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(3abce-2ac^2d+b^3(-e)+b^2cd\right)}{c^3\sqrt{b^2-4ac}}-\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{x(cd-be)}{c^2}+\frac{ex^2}{2c}$$

input
```
Int[(x^2*(d + e*x))/(a + b*x + c*x^2),x]
```

output
```
((c*d - b*e)*x)/c^2 + (e*x^2)/(2*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*
b*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) - (
(b*c*d - b^2*e + a*c*e)*Log[a + b*x + c*x^2])/(2*c^3)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{-\frac{1}{2} c e \,x^{2}+b e x -c d x}{c^{2}}+\frac{\frac{\left(-a c e +e \,b^{2}-d b c \right) \ln \left(c \,x^{2}+b x +a \right)}{2 c}+\frac{2 \left(a b e -a c d -\frac{\left(-a c e +e \,b^{2}-d b c \right) b}{2 c}\right) \arctan \left(\frac{2 c x +b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}}{c^{2}}$	127
risch	Expression too large to display	2051

input
```
int(x^2*(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/2*c*e*x^2+b*e*x-c*d*x)+1/c^2*(1/2*(-a*c*e+b^2*e-b*c*d)/c*ln(c*x
^2+b*x+a)+2*(a*b*e-a*c*d-1/2*(-a*c*e+b^2*e-b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.42

$$\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)ex^2+\sqrt{b^2-4\,ac}((b^2c-2\,ac^2)d-(b^3-3\,abc)e)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+}{2\,(b^2c^3-}\right.$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2 - 4*a*c^3)*e*x^2 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d -
 (b^3 - 3*a*b*c)*e)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*
a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c -
 4*a*b*c^2)*e)*x - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*
e)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*e*x
^2 - 2*sqrt(-b^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan
(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*((b^2*c^2 - 4*a*c^3)*d
 - (b^3*c - 4*a*b*c^2)*e)*x - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c +
4*a^2*c^2)*e)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(119) = 238$.

Time = 1.12 (sec) , antiderivative size = 609, normalized size of antiderivative = 5.03

$$\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx = x\left(-\frac{be}{c^2}+\frac{d}{c}\right)+\left(-\frac{\sqrt{-4ac+b^2}\cdot(3abce-2ac^2d-b^3e+b^2cd)}{2c^3\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{ace-b^2e+bcd}{2c^3}\right)\log\left(x+\frac{2a^2ce-ab^2e+abcd+4ac^3\left(-\frac{\sqrt{-4ac+b^2}\cdot(3abce-2ac^2d-b^3e+b^2cd)}{2c^3\cdot(4ac-b^2)}-\frac{ace-b^2e+bcd}{2c^3}\right)}{3abce-2ac^2d-b^3e+}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(3abce-2ac^2d-b^3e+b^2cd)}{2c^3\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{ace-b^2e+bcd}{2c^3}\right)\log\left(x+\frac{2a^2ce-ab^2e+abcd+4ac^3\left(\frac{\sqrt{-4ac+b^2}\cdot(3abce-2ac^2d-b^3e+b^2cd)}{2c^3\cdot(4ac-b^2)}-\frac{ace-b^2e+bcd}{2c^3}\right)}{3abce-2ac^2d-b^3e+b^2}\right.$$
$$+\frac{ex^2}{2c}$$

input
```
integrate(x**2*(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
x*(-b*e/c**2 + d/c) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3
*e + b**2*c*d)/(2*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(2*c**3)
)*log(x + (2*a**2*c*e - a*b**2*e + a*b*c*d + 4*a*c**3*(-sqrt(-4*a*c + b**2
)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(2*c**3*(4*a*c - b**2)) - (
a*c*e - b**2*e + b*c*d)/(2*c**3)) - b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b
*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(2*c**3*(4*a*c - b**2)) - (a*c*e -
b**2*e + b*c*d)/(2*c**3)))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)) +
 (sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(2*c**3
*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(2*c**3))*log(x + (2*a**2*c*e
- a*b**2*e + a*b*c*d + 4*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2
*d - b**3*e + b**2*c*d)/(2*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)
/(2*c**3)) - b**2*c**2*(sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3
*e + b**2*c*d)/(2*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(2*c**3)
))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)) + e*x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

$$\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx = \frac{cex^2+2\,cdx-2\,bex}{2\,c^2} - \frac{(bcd-b^2e+ace)\log(cx^2+bx+a)}{2\,c^3} + \frac{(b^2cd-2\,ac^2d-b^3e+3\,abce)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*(c*e*x^2 + 2*c*d*x - 2*b*e*x)/c^2 - 1/2*(b*c*d - b^2*e + a*c*e)*log(c*
x^2 + b*x + a)/c^3 + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arctan((2*c
*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.39

$$\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx = x\left(\frac{d}{c}-\frac{b\,e}{c^2}\right) - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(4\,e\,a^2\,c^2-5\,e\,a\,b^2\,c+4\,d\,a\,b\,c^2+e\,b^4-d\,b^3\,c\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)} + \frac{e\,x^2}{2\,c} - \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(e\,b^3-d\,b^2\,c-3\,a\,e\,b\,c+2\,a\,d\,c^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((x^2*(d + e*x))/(a + b*x + c*x^2),x)
```

output
```
x*(d/c - (b*e)/c^2) - (log(a + b*x + c*x^2)*(b^4*e + 4*a^2*c^2*e - b^3*c*d
 + 4*a*b*c^2*d - 5*a*b^2*c*e))/(2*(4*a*c^4 - b^2*c^3)) + (e*x^2)/(2*c) - (
atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^3*e + 2*a*c^2
*d - b^2*c*d - 3*a*b*c*e))/(c^3*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.65

$$\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx = \frac{6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abce-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,c^2d-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3e+2}{}$$

input
```
int(x^2*(e*x+d)/(c*x^2+b*x+a),x)
```

output
```
(6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*e - 4*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**2*d - 2*sqrt(4*a
*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*e + 2*sqrt(4*a*c - b*
*2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*d - 4*log(a + b*x + c*x**2
)*a**2*c**2*e + 5*log(a + b*x + c*x**2)*a*b**2*c*e - 4*log(a + b*x + c*x**
2)*a*b*c**2*d - log(a + b*x + c*x**2)*b**4*e + log(a + b*x + c*x**2)*b**3*
c*d - 8*a*b*c**2*e*x + 8*a*c**3*d*x + 4*a*c**3*e*x**2 + 2*b**3*c*e*x - 2*b
**2*c**2*d*x - b**2*c**2*e*x**2)/(2*c**3*(4*a*c - b**2))
```

3.42 $\int \frac{x(d+ex)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{x(d+ex)}{a+bx+cx^2}\,dx = \frac{ex}{c}+\frac{(bcd-b^2e+2ace)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log(a+bx+cx^2)}{2c^2}$$

output
```
e*x/c+(2*a*c*e-b^2*e+b*c*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*
a*c+b^2)^(1/2)+1/2*(-b*e+c*d)*ln(c*x^2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{x(d+ex)}{a+bx+cx^2}\,dx = \frac{2cex+\frac{2(-bcd+b^2e-2ace)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(cd-be)\log(a+x(b+cx))}{2c^2}$$

input
```
Integrate[(x*(d + e*x))/(a + b*x + c*x^2),x]
```

output
```
(2*c*e*x + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + x*(b + c*x)])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)}{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{e}{c}-\frac{ae-x(cd-be)}{c\left(a+bx+cx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ace+b^2(-e)+bcd\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{ex}{c}$$

input
```
Int[(x*(d + e*x))/(a + b*x + c*x^2),x]
```

output
```
(e*x)/c + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x + c*x^2])/(2*c^2)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{ex}{c}+\frac{\frac{(-be+cd)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-ae-\frac{(-be+cd)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$	90
risch	Expression too large to display	1357

input
```
int(x*(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
e/c*x+1/c*(1/2*(-b*e+c*d)/c*ln(c*x^2+b*x+a)+2*(-a*e-1/2*(-b*e+c*d)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.40

$$\int \frac{x(d+ex)}{a+bx+cx^2}\,dx$$
$$=\left[\frac{2\,(b^2c-4\,ac^2)ex+(bcd-(b^2-2\,ac)e)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+((b^2c-4\,a\cdots}{2\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2*c - 4*a*c^2)*e*x + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a*c
)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/
(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*x^2 +
 b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*e*x + 2*(b*c*d -
(b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*x^2 + b*
x + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(82) = 164$.

Time = 0.74 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.98

$$\int \frac{x(d+ex)}{a+bx+cx^2}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{2c^2\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{be-cd}{2c^2}\right)\log\left(x+\frac{-abe-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{2c^2\cdot(4ac-b^2)}-\frac{be-cd}{2c^2}\right)+2acd+b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+}{2c^2\cdot(4ac-b^2)}\right.}{2ace-b^2e+bcd}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{2c^2\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{be-cd}{2c^2}\right)\log\left(x+\frac{-abe-4ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{2c^2\cdot(4ac-b^2)}-\frac{be-cd}{2c^2}\right)+2acd+b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{2c^2\cdot(4ac-b^2)}\right.}{2ace-b^2e+bcd}\right.$$
$$+\frac{ex}{c}$$

input
```
integrate(x*(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(2*c**2*(4*a*c - b**2)) -
 (b*e - c*d)/(2*c**2))*log(x + (-a*b*e - 4*a*c**2*(-sqrt(-4*a*c + b**2)*(2
*a*c*e - b**2*e + b*c*d)/(2*c**2*(4*a*c - b**2)) - (b*e - c*d)/(2*c**2)) +
 2*a*c*d + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(2*c**2
*(4*a*c - b**2)) - (b*e - c*d)/(2*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + (s
qrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(2*c**2*(4*a*c - b**2)) - (b
*e - c*d)/(2*c**2))*log(x + (-a*b*e - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c
*e - b**2*e + b*c*d)/(2*c**2*(4*a*c - b**2)) - (b*e - c*d)/(2*c**2)) + 2*a
*c*d + b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(2*c**2*(4*a
*c - b**2)) - (b*e - c*d)/(2*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + e*x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \frac{x(d+ex)}{a+bx+cx^2}\,dx = \frac{ex}{c} + \frac{(cd-be)\log\left(cx^2+bx+a\right)}{2\,c^2} - \frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
e*x/c + 1/2*(c*d - b*e)*log(c*x^2 + b*x + a)/c^2 - (b*c*d - b^2*e + 2*a*c*
e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.49

$$\int \frac{x(d+ex)}{a+bx+cx^2}\,dx = \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(e\,b^3-d\,b^2\,c-4\,a\,e\,b\,c+4\,a\,d\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}+\frac{e\,x}{c} - \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(-e\,b^2+c\,d\,b+2\,a\,c\,e\right)}{c^2\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((x*(d + e*x))/(a + b*x + c*x^2),x)
```

output
```
(log(a + b*x + c*x^2)*(b^3*e + 4*a*c^2*d - b^2*c*d - 4*a*b*c*e))/(2*(4*a*c
^3 - b^2*c^2)) + (e*x)/c - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c -
b^2)^(1/2))*(2*a*c*e - b^2*e + b*c*d))/(c^2*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.53

$$\int \frac{x(d+ex)}{a+bx+cx^2}\,dx = \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ace+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2e-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bcd-4$$

input
```
int(x*(e*x+d)/(c*x^2+b*x+a),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*e + 2*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*e - 2*sqrt(4*a*
c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*d - 4*log(a + b*x + c*x
**2)*a*b*c*e + 4*log(a + b*x + c*x**2)*a*c**2*d + log(a + b*x + c*x**2)*b*
*3*e - log(a + b*x + c*x**2)*b**2*c*d + 8*a*c**2*e*x - 2*b**2*c*e*x)/(2*c*
*2*(4*a*c - b**2))
```

3.43 $\int \frac{d+ex}{a+bx+cx^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{d+ex}{a+bx+cx^2} \, dx = -\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx+cx^2\right)}{2c}$$

output
```
-(-b*e+2*c*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1
/2*e*ln(c*x^2+b*x+a)/c
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{a+bx+cx^2} \, dx = \frac{-\frac{2(-2cd+be)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + e\log(a+x(b+cx))}{2c}$$

input
```
Integrate[(d + e*x)/(a + b*x + c*x^2),x]
```

output
```
((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c] + e*Log[a + x*(b + c*x)])/(2*c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \ 1142$$

$$\frac{(2cd-be)\int \frac{1}{cx^2+bx+a}dx}{2c}+\frac{e\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \ 1083$$

$$\frac{e\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{(2cd-be)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c}$$

$$\Big\downarrow \ 219$$

$$\frac{e\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}$$

$$\Big\downarrow \ 1103$$

$$\frac{e\log\left(a+bx+cx^2\right)}{2c}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}$$

input
```
Int[(d + e*x)/(a + b*x + c*x^2),x]
```

output
```
-(((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a
*c])) + (e*Log[a + b*x + c*x^2])/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

method	result
default	$\frac{e\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(d-\frac{be}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}$
risch	$\frac{2\ln\left(-4abce+8a\,c^{2}d+e\,b^{3}-2cd\,b^{2}-2\sqrt{-\left(be-2cd\right)^{2}\left(4ac-b^{2}\right)}\,cx-\sqrt{-\left(be-2cd\right)^{2}\left(4ac-b^{2}\right)}\,b\right)ae}{4ac-b^{2}}-\frac{\ln\left(-4abce+8a\,c^{2}d+e\,b^{3}-2cd\,b \right.}{}$

input
```
int((e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*ln(c*x^2+b*x+a)/c+2*(d-1/2*b/c*e)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.09

$$\int \frac{d+ex}{a+bx+cx^2}\,dx$$
$$=\left[\frac{(b^2-4\,ac)e\log\left(cx^2+bx+a\right)-\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{2\,(b^2c-4\,ac^2)},\ \frac{(b^2-4\,}{}\right.$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*((b^2 - 4*a*c)*e*log(c*x^2 + b*x + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b*
e)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))
/(c*x^2 + b*x + a)))/(b^2*c - 4*a*c^2), 1/2*((b^2 - 4*a*c)*e*log(c*x^2 + b
*x + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2
*c*x + b)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(58) = 116$.

Time = 0.41 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.24

$$\int \frac{d+ex}{a+bx+cx^2}\,dx=\left(\frac{e}{2c}\right.$$
$$\left.-\frac{\sqrt{-4ac+b^2}(be-2cd)}{2c\,(4ac-b^2)}\right)\log\left(x+\frac{-4ac\left(\frac{e}{2c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{2c(4ac-b^2)}\right)+2ae+b^2\left(\frac{e}{2c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{2c(4ac-b^2)}\right)-}{be-2cd}\right.$$
$$+\left(\frac{e}{2c}\right.$$
$$\left.+\frac{\sqrt{-4ac+b^2}(be-2cd)}{2c\,(4ac-b^2)}\right)\log\left(x+\frac{-4ac\left(\frac{e}{2c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{2c(4ac-b^2)}\right)+2ae+b^2\left(\frac{e}{2c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{2c(4ac-b^2)}\right)-}{be-2cd}\right.$$

input
```
integrate((e*x+d)/(c*x**2+b*x+a),x)
```

output
```
(e/(2*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(2*c*(4*a*c - b**2)))*log(x +
 (-4*a*c*(e/(2*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(2*c*(4*a*c - b**2))
) + 2*a*e + b**2*(e/(2*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(2*c*(4*a*c
- b**2))) - b*d)/(b*e - 2*c*d)) + (e/(2*c) + sqrt(-4*a*c + b**2)*(b*e - 2*
c*d)/(2*c*(4*a*c - b**2)))*log(x + (-4*a*c*(e/(2*c) + sqrt(-4*a*c + b**2)*
(b*e - 2*c*d)/(2*c*(4*a*c - b**2))) + 2*a*e + b**2*(e/(2*c) + sqrt(-4*a*c
+ b**2)*(b*e - 2*c*d)/(2*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{d+ex}{a+bx+cx^2}\,dx = \frac{e\log\left(cx^2+bx+a\right)}{2\,c} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,acc}}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*e*log(c*x^2 + b*x + a)/c + (2*c*d - b*e)*arctan((2*c*x + b)/sqrt(-b^2
+ 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.45

$$\int \frac{d+ex}{a+bx+cx^2}\,dx = \frac{2\,d\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}} - \frac{b^2\,e\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)} + \frac{2\,a\,c\,e\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c} - \frac{b\,e\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((d + e*x)/(a + b*x + c*x^2),x)
```

output
```
(2*d*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b
^2)^(1/2) - (b^2*e*log(a + b*x + c*x^2))/(2*(4*a*c^2 - b^2*c)) + (2*a*c*e*
log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*e*atan(b/(4*a*c - b^2)^(1/2)
+ (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.86

$$\int \frac{d+ex}{a+bx+cx^2}\,dx = \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)be+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)cd+4\log(c\,x^2+bx+a)\,ace-\log(c\,x^2+}{2c\,(4ac-b^2)}$$

input
```
int((e*x+d)/(c*x^2+b*x+a),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*e + 4*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c*d + 4*log(a + b*x +
c*x**2)*a*c*e - log(a + b*x + c*x**2)*b**2*e)/(2*c*(4*a*c - b**2))
```

3.44 $\int \frac{d+ex}{x(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx=\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx+cx^2\right)}{2a}$$

output
```
(-2*a*e+b*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+d*
ln(x)/a-1/2*d*ln(c*x^2+b*x+a)/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx=-\frac{\frac{2(bd-2ae)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+d(-2\log(x)+\log(a+x(b+cx)))}{2a}$$

input
```
Integrate[(d + e*x)/(x*(a + b*x + c*x^2)),x]
```

output
```
-1/2*((2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
 4*a*c] + d*(-2*Log[x] + Log[a + x*(b + c*x)]))/a
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{ 1200}$$

$$\int \left(\frac{ae-bd-cdx}{a\left(a+bx+cx^2\right)}+\frac{d}{ax}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx+cx^2\right)}{2a}+\frac{d\log(x)}{a}$$

input
```
Int[(d + e*x)/(x*(a + b*x + c*x^2)),x]
```

output
```
((b*d - 2*a*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c
]) + (d*Log[x])/a - (d*Log[a + b*x + c*x^2])/(2*a)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{d\ln(x)}{a}+\frac{-\frac{d\ln\left(c\,x^{2}+bx+a\right)}{2}+\frac{2\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a}$	70
risch	Expression too large to display	1709

input
```
int((e*x+d)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
d*ln(x)/a+1/a*(-1/2*d*ln(c*x^2+b*x+a)+2*(a*e-1/2*b*d)/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.21

$$\int \frac{d+ex}{x\,(a+bx+cx^2)}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)d\log\left(cx^2+bx+a\right)-2\,(b^2-4\,ac)d\log\left(x\right)+\sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac}{cx^2+b}\right.}{2\,(ab^2-4\,a^2c)}\right.$$
$$-\frac{(b^2-4\,ac)d\log\left(cx^2+bx+a\right)-2\,(b^2-4\,ac)d\log\left(x\right)-2\,\sqrt{-b^2+4\,ac}(bd-2\,ae)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}}{b^2-4}\right.}{2\,(ab^2-4\,a^2c)}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 4*a*c)*d*log(c*x^2 + b*x + a) - 2*(b^2 - 4*a*c)*d*log(x) + s
qrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sq
rt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)))/(a*b^2 - 4*a^2*c), -1/2*(
(b^2 - 4*a*c)*d*log(c*x^2 + b*x + a) - 2*(b^2 - 4*a*c)*d*log(x) - 2*sqrt(-
b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4
*a*c)))/(a*b^2 - 4*a^2*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/x/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx = -\frac{d\log\left(cx^2+bx+a\right)}{2\,a} + \frac{d\log\left(|x|\right)}{a} - \frac{\left(bd-2\,ae\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*d*log(c*x^2 + b*x + a)/a + d*log(abs(x))/a - (b*d - 2*a*e)*arctan((2*
c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

Mupad [B] (verification not implemented)

Time = 11.38 (sec) , antiderivative size = 375, normalized size of antiderivative = 5.28

$$\int \frac{d+ex}{x(a+bx+cx^2)}\,dx = \ln\left(a^2\,e\,\sqrt{b^2-4\,a\,c}-2\,a\,b^2\,d+a^2\,b\,e+6\,a^2\,c\,d-2\,b^3\,d\,x-2\,a\,b\,d\,\sqrt{b^2-4\,a\,c}+a\,b^2\,e\,x-2\,a^2\,c\,e\,x-2\,b^2\,d\,x\,\sqrt{b^2-4\,a\,c}+7\,a\,b\,c\,d\,x+a\,b\,e\,x\,\sqrt{b^2-4\,a\,c}+3\,a\,c\,d\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{2\,a\,e\,\sqrt{b^2-4\,a\,c}-b\,d\,\sqrt{b^2-4\,a\,c}}{2\,a\,b^2-8\,a^2\,c}-\frac{d}{2\,a}\right)-\ln\left(a^2\,e\,\sqrt{b^2-4\,a\,c}+2\,a\,b^2\,d-a^2\,b\,e-6\,a^2\,c\,d+2\,b^3\,d\,x-2\,a\,b\,d\,\sqrt{b^2-4\,a\,c}-a\,b^2\,e\,x+2\,a^2\,c\,e\,x-2\,b^2\,d\,x\,\sqrt{b^2-4\,a\,c}-7\,a\,b\,c\,d\,x+a\,b\,e\,x\,\sqrt{b^2-4\,a\,c}+3\,a\,c\,d\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{2\,a\,e\,\sqrt{b^2-4\,a\,c}-b\,d\,\sqrt{b^2-4\,a\,c}}{2\,a\,b^2-8\,a^2\,c}+\frac{d}{2\,a}\right)+\frac{d\,\ln(x)}{a}$$

input
```
int((d + e*x)/(x*(a + b*x + c*x^2)),x)
```

output
```
log(a^2*e*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d + a^2*b*e + 6*a^2*c*d - 2*b^3*d*
x - 2*a*b*d*(b^2 - 4*a*c)^(1/2) + a*b^2*e*x - 2*a^2*c*e*x - 2*b^2*d*x*(b^2
 - 4*a*c)^(1/2) + 7*a*b*c*d*x + a*b*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*c*d*x*(b
^2 - 4*a*c)^(1/2))*((2*a*e*(b^2 - 4*a*c)^(1/2) - b*d*(b^2 - 4*a*c)^(1/2))/
(2*a*b^2 - 8*a^2*c) - d/(2*a)) - log(a^2*e*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d
 - a^2*b*e - 6*a^2*c*d + 2*b^3*d*x - 2*a*b*d*(b^2 - 4*a*c)^(1/2) - a*b^2*e
*x + 2*a^2*c*e*x - 2*b^2*d*x*(b^2 - 4*a*c)^(1/2) - 7*a*b*c*d*x + a*b*e*x*(
b^2 - 4*a*c)^(1/2) + 3*a*c*d*x*(b^2 - 4*a*c)^(1/2))*((2*a*e*(b^2 - 4*a*c)^
(1/2) - b*d*(b^2 - 4*a*c)^(1/2))/(2*a*b^2 - 8*a^2*c) + d/(2*a)) + (d*log(x
))/a
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.93

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx$$
$$= \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ae-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bd-4\log(c\,x^2+bx+a)\,acd+\log(c\,x^2+bx}{2a\left(4ac-b^2\right)}$$

input
```
int((e*x+d)/x/(c*x^2+b*x+a),x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*e - 2*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*d - 4*log(a + b*x + c*x
**2)*a*c*d + log(a + b*x + c*x**2)*b**2*d + 8*log(x)*a*c*d - 2*log(x)*b**2
*d)/(2*a*(4*a*c - b**2))
```

3.45 $\int \frac{d+ex}{x^2(a+bx+cx^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{d+ex}{x^2(a+bx+cx^2)}\,dx = -\frac{d}{ax}-\frac{(b^2d-2acd-abe)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} -\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log(a+bx+cx^2)}{2a^2}$$

output
```
-d/a/x-(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(1/2)-(-a*e+b*d)*ln(x)/a^2+1/2*(-a*e+b*d)*ln(c*x^2+b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{d+ex}{x^2(a+bx+cx^2)}\,dx = \frac{-\frac{2ad}{x}+\frac{2(b^2d-2acd-abe)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(-bd+ae)\log(x)+(bd-ae)\log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[(d + e*x)/(x^2*(a + b*x + c*x^2)),x]
```

output
```
((-2*a*d)/x + (2*(b^2*d - 2*a*c*d - a*b*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(-(b*d) + a*e)*Log[x] + (b*d - a*e)*Log[a
+ x*(b + c*x)])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^2(a+bx+cx^2)}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{cx(bd-ae)-abe-acd+b^2d}{a^2(a+bx+cx^2)}+\frac{ae-bd}{a^2x}+\frac{d}{ax^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-abe-2acd+b^2d)}{a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log(a+bx+cx^2)}{2a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{ax}$$

input
```
Int[(d + e*x)/(x^2*(a + b*x + c*x^2)),x]
```

output
```
-(d/(a*x)) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b*d - a*e)*Lo
g[a + b*x + c*x^2])/(2*a^2)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

method	result
default	$-\frac{d}{ax}+\frac{(ae-bd)\ln(x)}{a^2}+\frac{\frac{(-ace+dbc)\ln\left(cx^2+bx+a\right)}{2c}+\frac{2\left(-abe-acd+b^2d-\frac{(-ace+dbc)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^2}$
risch	$-\frac{d}{ax}+\frac{e\ln(x)}{a}-\frac{bd\ln(x)}{a^2}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4ca^3-a^2b^2\right)\_Z^2+\left(4a^2ce-eab^2-4abcd+b^3d\right)\_Z+ace^2-bcde+c^2d^2\right)}\_R\ln($

input
```
int((e*x+d)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-d/a/x+(a*e-b*d)/a^2*ln(x)+1/a^2*(1/2*(-a*c*e+b*c*d)/c*ln(c*x^2+b*x+a)+2*(
-a*b*e-a*c*d+b^2*d-1/2*(-a*c*e+b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.47

$$\int\frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx$$
$$=\left[\frac{(abe-(b^2-2ac)d)\sqrt{b^2-4ac}x\log\left(\frac{2c^2x^2+2bcx+b^2-2ac+\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+((b^3-4abc)d-(ab^2-4a^2c}{2(a^2b^2-4a^3c)x}\right.$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*((a*b*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x*log(c*x^2 + b*x + a) - 2*((b^3 -
4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x*log(x) - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*
b^2 - 4*a^3*c)*x), 1/2*(2*(a*b*e - (b^2 - 2*a*c)*d)*sqrt(-b^2 + 4*a*c)*x*a
rctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d
- (a*b^2 - 4*a^2*c)*e)*x*log(c*x^2 + b*x + a) - 2*((b^3 - 4*a*b*c)*d - (a*
b^2 - 4*a^2*c)*e)*x*log(x) - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/x**2/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{(bd-ae)\log\left(cx^2+bx+a\right)}{2\,a^2} - \frac{(bd-ae)\log\left(|x|\right)}{a^2} + \frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^2} - \frac{d}{ax}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*(b*d - a*e)*log(c*x^2 + b*x + a)/a^2 - (b*d - a*e)*log(abs(x))/a^2 + (
b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
 + 4*a*c)*a^2) - d/(a*x)
```

Mupad [B] (verification not implemented)

Time = 12.33 (sec) , antiderivative size = 791, normalized size of antiderivative = 7.61

$$\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx = \frac{\ln\left(x\right)\,\left(a\,e-b\,d\right)}{a^2} - \frac{d}{a\,x}$$
$$+ \frac{\ln\left(\frac{b\,c^2\,d^2-a\,c^2\,d\,e}{a^2} + \frac{\left(\frac{e\,a^2\,b\,c+d\,a^2\,c^2-d\,a\,b^2\,c}{a^2} + \frac{\left(\frac{x\,\left(6\,a^3\,c^2-2\,a^2\,b^2\,c\right)}{a^2}-a\,b\,c\right)\,\left(d\,{\left(b^2-4\,a\,c\right)}^{3/2}+b^2\,d\,\sqrt{b^2-4\,a\,c}-2\,a\,e\,\left(4\,a\,c-b^2\right)+2\,b\,d\,\left(4\,a\,c-\cdots\right.\right.}{16\,a^3\,c-4\,a^2\,b^2}\cdots}{\cdots}\right)}{\cdots}$$
$$- \frac{\ln\left(\frac{b\,c^2\,d^2-a\,c^2\,d\,e}{a^2} - \frac{\left(\frac{e\,a^2\,b\,c+d\,a^2\,c^2-d\,a\,b^2\,c}{a^2} - \frac{\left(\frac{x\,\left(6\,a^3\,c^2-2\,a^2\,b^2\,c\right)}{a^2}-a\,b\,c\right)\,\left(d\,{\left(b^2-4\,a\,c\right)}^{3/2}+b^2\,d\,\sqrt{b^2-4\,a\,c}+2\,a\,e\,\left(4\,a\,c-b^2\right)-2\,b\,d\,\left(4\,a\,c-\cdots\right.\right.}{16\,a^3\,c-4\,a^2\,b^2}\cdots}{\cdots}\right)}{\cdots}$$

input
```
int((d + e*x)/(x^2*(a + b*x + c*x^2)),x)
```

output
```
(log(x)*(a*e - b*d))/a^2 - d/(a*x) + (log((b*c^2*d^2 - a*c^2*d*e)/a^2 + ((
(a^2*c^2*d - a*b^2*c*d + a^2*b*c*e)/a^2 + (((x*(6*a^3*c^2 - 2*a^2*b^2*c))/
a^2 - a*b*c)*(d*(b^2 - 4*a*c)^(3/2) + b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a*e*(4
*a*c - b^2) + 2*b*d*(4*a*c - b^2) - 2*a*b*e*(b^2 - 4*a*c)^(1/2)))/(16*a^3*
c - 4*a^2*b^2) + (x*(3*a^2*c^2*e - 2*a*b*c^2*d))/a^2)*(d*(b^2 - 4*a*c)^(3/
2) + b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a*e*(4*a*c - b^2) + 2*b*d*(4*a*c - b^2)
 - 2*a*b*e*(b^2 - 4*a*c)^(1/2)))/(16*a^3*c - 4*a^2*b^2) + (c^3*d^2*x)/a^2)
*(d*(b^2 - 4*a*c)^(3/2) + b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a*e*(4*a*c - b^2)
+ 2*b*d*(4*a*c - b^2) - 2*a*b*e*(b^2 - 4*a*c)^(1/2)))/(16*a^3*c - 4*a^2*b^
2) - (log((b*c^2*d^2 - a*c^2*d*e)/a^2 - (((a^2*c^2*d - a*b^2*c*d + a^2*b*c
*e)/a^2 - (((x*(6*a^3*c^2 - 2*a^2*b^2*c))/a^2 - a*b*c)*(d*(b^2 - 4*a*c)^(3
/2) + b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a*e*(4*a*c - b^2) - 2*b*d*(4*a*c - b^2
) - 2*a*b*e*(b^2 - 4*a*c)^(1/2)))/(16*a^3*c - 4*a^2*b^2) + (x*(3*a^2*c^2*e
 - 2*a*b*c^2*d))/a^2)*(d*(b^2 - 4*a*c)^(3/2) + b^2*d*(b^2 - 4*a*c)^(1/2) +
 2*a*e*(4*a*c - b^2) - 2*b*d*(4*a*c - b^2) - 2*a*b*e*(b^2 - 4*a*c)^(1/2)))
/(16*a^3*c - 4*a^2*b^2) + (c^3*d^2*x)/a^2)*(d*(b^2 - 4*a*c)^(3/2) + b^2*d*
(b^2 - 4*a*c)^(1/2) + 2*a*e*(4*a*c - b^2) - 2*b*d*(4*a*c - b^2) - 2*a*b*e*
(b^2 - 4*a*c)^(1/2)))/(16*a^3*c - 4*a^2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.51

$$\int \frac{d+ex}{x^2(a+bx+cx^2)}\,dx = \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abex-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)acdx+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2dx}{}$$

input
```
int((e*x+d)/x^2/(c*x^2+b*x+a),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*e*x - 4*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d*x + 2*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*d*x - 4*log(a + b*x
 + c*x**2)*a**2*c*e*x + log(a + b*x + c*x**2)*a*b**2*e*x + 4*log(a + b*x +
 c*x**2)*a*b*c*d*x - log(a + b*x + c*x**2)*b**3*d*x + 8*log(x)*a**2*c*e*x
- 2*log(x)*a*b**2*e*x - 8*log(x)*a*b*c*d*x + 2*log(x)*b**3*d*x - 8*a**2*c*
d + 2*a*b**2*d)/(2*a**2*x*(4*a*c - b**2))
```

3.46 $\int \frac{d+ex}{x^3(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{d+ex}{x^3(a+bx+cx^2)}\,dx = -\frac{d}{2ax^2}+\frac{bd-ae}{a^2x}+\frac{(b^3d-3abcd-ab^2e+2a^2ce)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{(b^2d-acd-abe)\log(x)}{a^3}-\frac{(b^2d-acd-abe)\log(a+bx+cx^2)}{2a^3}$$

output
```
-1/2*d/a/x^2+(-a*e+b*d)/a^2/x+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)*arctanh(
(2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)+(-a*b*e-a*c*d+b^2*d)*
ln(x)/a^3-1/2*(-a*b*e-a*c*d+b^2*d)*ln(c*x^2+b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{x^3\left(a+bx+cx^2\right)}\,dx$$
$$= \frac{-\frac{a^2d}{x^2}+\frac{2a(bd-ae)}{x}+\frac{2(-b^3d+3abcd+ab^2e-2a^2ce)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2d-acd-abe)\log(x)+(-b^2d+acd+\ldots}{2a^3}$$

input
```
Integrate[(d + e*x)/(x^3*(a + b*x + c*x^2)),x]
```

output
```
(-((a^2*d)/x^2) + (2*a*(b*d - a*e))/x + (2*(-(b^3*d) + 3*a*b*c*d + a*b^2*e
 - 2*a^2*c*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] +
 2*(b^2*d - a*c*d - a*b*e)*Log[x] + (-(b^2*d) + a*c*d + a*b*e)*Log[a + x*(
b + c*x)])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^3\left(a+bx+cx^2\right)}\,dx$$
$$\Big\downarrow \text{1200}$$
$$\int \left(\frac{-abe-acd+b^2d}{a^3x}+\frac{ae-bd}{a^2x^2}+\frac{-a^2ce-cx\left(-abe-acd+b^2d\right)+ab^2e+2abcd+b^3(-d)}{a^3\left(a+bx+cx^2\right)}+\frac{d}{ax^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{(-abe-acd+b^2d)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(-abe-acd+b^2d)}{a^3}+\frac{bd-ae}{a^2x}+\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2a^2ce-ab^2e-3abcd+b^3d)}{a^3\sqrt{b^2-4ac}}-\frac{d}{2ax^2}$$

input
```
Int[(d + e*x)/(x^3*(a + b*x + c*x^2)),x]
```

output
```
-1/2*d/(a*x^2) + (b*d - a*e)/(a^2*x) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a
^2*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) +
((b^2*d - a*c*d - a*b*e)*Log[x])/a^3 - ((b^2*d - a*c*d - a*b*e)*Log[a + b*
x + c*x^2])/(2*a^3)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.20

method	result
default	$-\frac{d}{2a\,x^2}-\frac{ae-bd}{a^2x}+\frac{(-abe-acd+b^2d)\ln(x)}{a^3}+\frac{\frac{(abce+a\,c^2d-cd\,b^2)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(-a^2ce+ea\,b^2+2abcd-b^3d-\frac{(abce+a\,c^2d-cd}{2c}\right.}{\sqrt{4ac-b^2}}}{a^3}$
risch	$\frac{-\frac{(ae-bd)x}{a^2}-\frac{d}{2a}}{x^2}-\frac{\ln(x)be}{a^2}-\frac{cd\ln(x)}{a^2}+\frac{\ln(x)b^2d}{a^3}+\left(\sum_{\_R=\text{RootOf}((4c\,a^4-a^3b^2)\_Z^2+(-4a^2bce-4a^2c^2d+a\,b^3e+5cda\,b^2-\right.$

input
```
int((e*x+d)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a/x^2-(a*e-b*d)/a^2/x+(-a*b*e-a*c*d+b^2*d)*ln(x)/a^3+1/a^3*(1/2*(a*
b*c*e+a*c^2*d-b^2*c*d)/c*ln(c*x^2+b*x+a)+2*(-a^2*c*e+e*a*b^2+2*a*b*c*d-b^3
*d-1/2*(a*b*c*e+a*c^2*d-b^2*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.57

$$\int \frac{d+ex}{x^3\left(a+bx+cx^2\right)}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}((b^3-3\,abc)d-(ab^2-2\,a^2c)e)x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-((b^4-5\,ab^2c+4\right.$$

input
```
integrate((e*x+d)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
 + b*x + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*x
^2*log(c*x^2 + b*x + a) + 2*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*
a^2*b*c)*e)*x^2*log(x) - (a^2*b^2 - 4*a^3*c)*d + 2*((a*b^3 - 4*a^2*b*c)*d
- (a^2*b^2 - 4*a^3*c)*e)*x)/((a^3*b^2 - 4*a^4*c)*x^2), 1/2*(2*sqrt(-b^2 +
4*a*c)*((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2*arctan(-sqrt(-b^2 + 4
*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b
^3 - 4*a^2*b*c)*e)*x^2*log(c*x^2 + b*x + a) + 2*((b^4 - 5*a*b^2*c + 4*a^2*
c^2)*d - (a*b^3 - 4*a^2*b*c)*e)*x^2*log(x) - (a^2*b^2 - 4*a^3*c)*d + 2*((a
*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^3\,(a+bx+cx^2)}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)/x**3/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^3\,(a+bx+cx^2)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{d+ex}{x^3\,(a+bx+cx^2)}\,dx=&-\frac{(b^2d-acd-abe)\log\left(cx^2+bx+a\right)}{2\,a^3}\\&+\frac{(b^2d-acd-abe)\log\left(|x|\right)}{a^3}\\&-\frac{(b^3d-3\,abcd-ab^2e+2\,a^2ce)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3}\\&-\frac{a^2d-2\,(abd-a^2e)x}{2\,a^3x^2}\end{aligned}$$

input
```
integrate((e*x+d)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(b^2*d - a*c*d - a*b*e)*log(c*x^2 + b*x + a)/a^3 + (b^2*d - a*c*d - a
*b*e)*log(abs(x))/a^3 - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3) - 1/2*(a^2*d - 2*(
a*b*d - a^2*e)*x)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 11.67 (sec) , antiderivative size = 814, normalized size of antiderivative = 5.61

$$\int \frac{d+ex}{x^3\,(a+bx+cx^2)}\,dx$$
$$= \frac{\ln\left(6\,a^3\,c^2\,d-2\,a^2\,b^3\,e+2\,a\,b^4\,d+2\,b^5\,d\,x+7\,a^3\,b\,c\,e-2\,a\,b^4\,e\,x+2\,a\,b^3\,d\,\sqrt{b^2-4\,a\,c}+a^3\,c\,e\,\sqrt{b^2-4\,a}\right.}{}$$
$$-\frac{\ln\left(x\right)\,\left(a\,\left(b\,e+c\,d\right)-b^2\,d\right)}{a^3}-\frac{\frac{d}{2\,a}+\frac{x\,\left(a\,e-b\,d\right)}{a^2}}{x^2}$$
$$+\frac{\ln\left(2\,a^2\,b^3\,e-6\,a^3\,c^2\,d-2\,a\,b^4\,d-2\,b^5\,d\,x-7\,a^3\,b\,c\,e+2\,a\,b^4\,e\,x+2\,a\,b^3\,d\,\sqrt{b^2-4\,a\,c}+a^3\,c\,e\,\sqrt{b^2-4}\right.}{}$$

input
```
int((d + e*x)/(x^3*(a + b*x + c*x^2)),x)
```

output
```
(log(6*a^3*c^2*d - 2*a^2*b^3*e + 2*a*b^4*d + 2*b^5*d*x + 7*a^3*b*c*e - 2*a
*b^4*e*x + 2*a*b^3*d*(b^2 - 4*a*c)^(1/2) + a^3*c*e*(b^2 - 4*a*c)^(1/2) + 2
*b^4*d*x*(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c*d - 2*a^3*c^2*e*x - 2*a^2*b^2*e
*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*e*x*(b^2 - 4*a*c)^(1/2) + 9*a^2*b*c^2*d*x +
 8*a^2*b^2*c*e*x + 3*a^2*c^2*d*x*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*d*x - 3*
a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*d*x*(b^2 - 4*a*c)^(1/2) + 4*a^2*
b*c*e*x*(b^2 - 4*a*c)^(1/2))*(a^2*(2*c^2*d + 2*b*c*e + c*e*(b^2 - 4*a*c)^(
1/2)) + (b^4*d)/2 - a*((b^3*e)/2 + (b^2*e*(b^2 - 4*a*c)^(1/2))/2 + (5*b^2*
c*d)/2 + (3*b*c*d*(b^2 - 4*a*c)^(1/2))/2) + (b^3*d*(b^2 - 4*a*c)^(1/2))/2)
)/(4*a^4*c - a^3*b^2) - (log(x)*(a*(b*e + c*d) - b^2*d))/a^3 - (d/(2*a) +
(x*(a*e - b*d))/a^2)/x^2 + (log(2*a^2*b^3*e - 6*a^3*c^2*d - 2*a*b^4*d - 2*
b^5*d*x - 7*a^3*b*c*e + 2*a*b^4*e*x + 2*a*b^3*d*(b^2 - 4*a*c)^(1/2) + a^3*
c*e*(b^2 - 4*a*c)^(1/2) + 2*b^4*d*x*(b^2 - 4*a*c)^(1/2) + 9*a^2*b^2*c*d +
2*a^3*c^2*e*x - 2*a^2*b^2*e*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*e*x*(b^2 - 4*a*c
)^(1/2) - 9*a^2*b*c^2*d*x - 8*a^2*b^2*c*e*x + 3*a^2*c^2*d*x*(b^2 - 4*a*c)^
(1/2) + 10*a*b^3*c*d*x - 3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*d*x*(
b^2 - 4*a*c)^(1/2) + 4*a^2*b*c*e*x*(b^2 - 4*a*c)^(1/2))*(a^2*(2*c^2*d + 2*
b*c*e - c*e*(b^2 - 4*a*c)^(1/2)) + (b^4*d)/2 - a*((b^3*e)/2 - (b^2*e*(b^2
- 4*a*c)^(1/2))/2 + (5*b^2*c*d)/2 - (3*b*c*d*(b^2 - 4*a*c)^(1/2))/2) - (b^
3*d*(b^2 - 4*a*c)^(1/2))/2))/(4*a^4*c - a^3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.81

$$\int \frac{d+ex}{x^3(a+bx+cx^2)}\,dx$$
$$= \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2ce\,x^2+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2e\,x^2+6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{}$$

input
```
int((e*x+d)/x^3/(c*x^2+b*x+a),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c*e*x**
2 + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*e*x**
2 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*d*x**2
 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*d*x**2 +
 4*log(a + b*x + c*x**2)*a**2*b*c*e*x**2 + 4*log(a + b*x + c*x**2)*a**2*c*
*2*d*x**2 - log(a + b*x + c*x**2)*a*b**3*e*x**2 - 5*log(a + b*x + c*x**2)*
a*b**2*c*d*x**2 + log(a + b*x + c*x**2)*b**4*d*x**2 - 8*log(x)*a**2*b*c*e*
x**2 - 8*log(x)*a**2*c**2*d*x**2 + 2*log(x)*a*b**3*e*x**2 + 10*log(x)*a*b*
*2*c*d*x**2 - 2*log(x)*b**4*d*x**2 - 4*a**3*c*d - 8*a**3*c*e*x + a**2*b**2
*d + 2*a**2*b**2*e*x + 8*a**2*b*c*d*x - 2*a*b**3*d*x)/(2*a**3*x**2*(4*a*c
- b**2))
```

3.47 $\int \frac{d+ex}{x^4(a+bx+cx^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{d+ex}{x^4(a+bx+cx^2)}\,dx = -\frac{d}{3ax^3}+\frac{bd-ae}{2a^2x^2}-\frac{b^2d-acd-abe}{a^3x} - \frac{(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{(b^3d-2abcd-ab^2e+a^2ce)\log(x)}{a^4} + \frac{(b^3d-2abcd-ab^2e+a^2ce)\log(a+bx+cx^2)}{2a^4}$$

output
```
-1/3*d/a/x^3+1/2*(-a*e+b*d)/a^2/x^2-(-a*b*e-a*c*d+b^2*d)/a^3/x-(3*a^2*b*c*
e+2*a^2*c^2*d-a*b^3*e-4*a*b^2*c*d+b^4*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/a^4/(-4*a*c+b^2)^(1/2)-(a^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d)*ln(x)/a^4+1/
2*(a^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d)*ln(c*x^2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.96

$$\int \frac{d+ex}{x^4\left(a+bx+cx^2\right)}\,dx$$

$$= \frac{-\frac{2a^3d}{x^3}+\frac{3a^2(bd-ae)}{x^2}+\frac{6a(-b^2d+acd+abe)}{x}+\frac{6(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-6(b^3d-2abcd-a}{6a^4}$$

input
```
Integrate[(d + e*x)/(x^4*(a + b*x + c*x^2)),x]
```

output
```
((-2*a^3*d)/x^3 + (3*a^2*(b*d - a*e))/x^2 + (6*a*(-(b^2*d) + a*c*d + a*b*e
))/x + (6*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcT
an[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6*(b^3*d - 2*a*b*
c*d - a*b^2*e + a^2*c*e)*Log[x] + 3*(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e
)*Log[a + x*(b + c*x)])/(6*a^4)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^4\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{-abe-acd+b^2d}{a^3x^2}+\frac{ae-bd}{a^2x^3}+\frac{-a^2ce+ab^2e+2abcd+b^3(-d)}{a^4x}+\frac{cx\left(a^2ce-ab^2e-2abcd+b^3d\right)+2a^2bce}{a^4\left(a+bx+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-abe-acd+b^2d}{a^3x}+\frac{bd-ae}{2a^2x^2}-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d)}{a^4\sqrt{b^2-4ac}}+\frac{(a^2ce-ab^2e-2abcd+b^3d)\log(a+bx+cx^2)}{2a^4}-\frac{\log(x)(a^2ce-ab^2e-2abcd+b^3d)}{a^4}-\frac{d}{3ax^3}$$

input
```
Int[(d + e*x)/(x^4*(a + b*x + c*x^2)),x]
```

output
```
-1/3*d/(a*x^3) + (b*d - a*e)/(2*a^2*x^2) - (b^2*d - a*c*d - a*b*e)/(a^3*x)
 - ((b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b
 + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) - ((b^3*d - 2*a*b*c*
d - a*b^2*e + a^2*c*e)*Log[x])/a^4 + ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c
*e)*Log[a + b*x + c*x^2])/(2*a^4)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.19

method	result
default	$-\frac{d}{3a\,x^3}-\frac{ae-bd}{2a^2x^2}-\frac{-abe-acd+b^2d}{a^3x}+\frac{(-a^2ce+ea\,b^2+2abcd-b^3d)\ln(x)}{a^4}+\frac{\frac{(a^2c^2e-a\,b^2ce-2ab\,c^2d+b^3cd)\ln(c\,x^2+bx+a)}{2c}+\frac{2(\cdots}{\cdots}}{\cdots}$
risch	$\frac{\frac{(abe+acd-b^2d)x^2}{a^3}-\frac{(ae-bd)x}{2a^2}-\frac{d}{3a}}{x^3}-\frac{\ln(x)ce}{a^2}+\frac{\ln(x)e\,b^2}{a^3}+\frac{2bcd\ln(x)}{a^3}-\frac{\ln(x)b^3d}{a^4}+\left(\sum_{\_R=\operatorname{RootOf}((4c\,a^5-a^4b^2)\_Z^2+(-\cdots}\right.$

input
```
int((e*x+d)/x^4/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a/x^3-1/2*(a*e-b*d)/a^2/x^2-(-a*b*e-a*c*d+b^2*d)/a^3/x+1/a^4*(-a^2*
c*e+a*b^2*e+2*a*b*c*d-b^3*d)*ln(x)+1/a^4*(1/2*(a^2*c^2*e-a*b^2*c*e-2*a*b*c
^2*d+b^3*c*d)/c*ln(c*x^2+b*x+a)+2*(2*a^2*b*c*e+a^2*c^2*d-a*b^3*e-3*c*d*a*b
^2+b^4*d-1/2*(a^2*c^2*e-a*b^2*c*e-2*a*b*c^2*d+b^3*c*d)*b/c)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 687, normalized size of antiderivative = 3.37

$$\int \frac{d+ex}{x^4(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x^4/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(b^2 - 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^
2*b*c)*e)*x^3*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(
2*c*x + b))/(c*x^2 + b*x + a)) + 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a
*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e)*x^3*log(c*x^2 + b*x + a) - 6*((b^5 - 6*
a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e)*x^3*log(x)
 - 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x^2 -
 2*(a^3*b^2 - 4*a^4*c)*d + 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^4*c
)*e)*x)/((a^4*b^2 - 4*a^5*c)*x^3), -1/6*(6*sqrt(-b^2 + 4*a*c)*((b^4 - 4*a*
b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e)*x^3*arctan(-sqrt(-b^2 + 4*a*
c)*(2*c*x + b)/(b^2 - 4*a*c)) - 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*
b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e)*x^3*log(c*x^2 + b*x + a) + 6*((b^5 - 6*a
*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e)*x^3*log(x)
+ 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x^2 +
2*(a^3*b^2 - 4*a^4*c)*d - 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^4*c)
*e)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^4 (a+bx+cx^2)} \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)/x**4/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^4 (a+bx+cx^2)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^4/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.01

$$\int \frac{d+ex}{x^4\left(a+bx+cx^2\right)}\,dx = \frac{\left(b^3d-2\,abcd-ab^2e+a^2ce\right)\log\left(cx^2+bx+a\right)}{2\,a^4} - \frac{\left(b^3d-2\,abcd-ab^2e+a^2ce\right)\log\left(|x|\right)}{a^4} + \frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-ab^3e+3\,a^2bce\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^4} - \frac{2\,a^3d+6\left(ab^2d-a^2cd-a^2be\right)x^2-3\left(a^2bd-a^3e\right)x}{6\,a^4x^3}$$

input
```
integrate((e*x+d)/x^4/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*log(c*x^2 + b*x + a)/a^4 - (b^
3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*log(abs(x))/a^4 + (b^4*d - 4*a*b^2*c*
d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/(sqrt(-b^2 + 4*a*c)*a^4) - 1/6*(2*a^3*d + 6*(a*b^2*d - a^2*c*d - a^2
*b*e)*x^2 - 3*(a^2*b*d - a^3*e)*x)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 12.10 (sec) , antiderivative size = 1063, normalized size of antiderivative = 5.21

$$\int \frac{d+ex}{x^4\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)/(x^4*(a + b*x + c*x^2)),x)
```

output
```
(log(2*a^2*b^4*e + 6*a^4*c^2*e - 2*a*b^5*d - 2*b^6*d*x + 2*a*b^5*e*x + 2*a
*b^4*d*(b^2 - 4*a*c)^(1/2) + 2*b^5*d*x*(b^2 - 4*a*c)^(1/2) + 11*a^2*b^3*c*
d - 13*a^3*b*c^2*d - 9*a^3*b^2*c*e + 2*a^3*c^3*d*x - 2*a^2*b^3*e*(b^2 - 4*
a*c)^(1/2) + a^3*c^2*d*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*e*x*(b^2 - 4*a*c)^(1/
2) - 10*a^2*b^3*c*e*x + 9*a^3*b*c^2*e*x - 5*a^2*b^2*c*d*(b^2 - 4*a*c)^(1/2
) - 3*a^3*c^2*e*x*(b^2 - 4*a*c)^(1/2) - 17*a^2*b^2*c^2*d*x + 12*a*b^4*c*d*
x + 3*a^3*b*c*e*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*d*x*(b^2 - 4*a*c)^(1/2) +
7*a^2*b*c^2*d*x*(b^2 - 4*a*c)^(1/2) + 6*a^2*b^2*c*e*x*(b^2 - 4*a*c)^(1/2))
*(b^4*d*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^3*c^2*e + a*b^4*e + 6*a*b^3*c*d
- a*b^3*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a^2*b^2*c*e + 2*a^2*c^2*
d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2
 - 4*a*c)^(1/2)))/(2*(4*a^5*c - a^4*b^2)) - (d/(3*a) + (x*(a*e - b*d))/(2*
a^2) - (x^2*(a*b*e - b^2*d + a*c*d))/a^3)/x^3 - (log(2*a^2*b^4*e + 6*a^4*c
^2*e - 2*a*b^5*d - 2*b^6*d*x + 2*a*b^5*e*x - 2*a*b^4*d*(b^2 - 4*a*c)^(1/2)
 - 2*b^5*d*x*(b^2 - 4*a*c)^(1/2) + 11*a^2*b^3*c*d - 13*a^3*b*c^2*d - 9*a^3
*b^2*c*e + 2*a^3*c^3*d*x + 2*a^2*b^3*e*(b^2 - 4*a*c)^(1/2) - a^3*c^2*d*(b^
2 - 4*a*c)^(1/2) + 2*a*b^4*e*x*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^3*c*e*x + 9*
a^3*b*c^2*e*x + 5*a^2*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a^3*c^2*e*x*(b^2 - 4
*a*c)^(1/2) - 17*a^2*b^2*c^2*d*x + 12*a*b^4*c*d*x - 3*a^3*b*c*e*(b^2 - 4*a
*c)^(1/2) + 8*a*b^3*c*d*x*(b^2 - 4*a*c)^(1/2) - 7*a^2*b*c^2*d*x*(b^2 - ...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.74

$$\int \frac{d+ex}{x^4(a+bx+cx^2)}\,dx = \frac{12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2c^2d\,x^3 - 6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^3e\,x^3 - 15\log(c\,x^2+bx+a)\,a^2b^2c\cdots}{\cdots}$$

input
```
int((e*x+d)/x^4/(c*x^2+b*x+a),x)
```

output
```
(18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c*e*x**
3 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*d
*x**3 - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*e
*x**3 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*
c*d*x**3 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*
d*x**3 + 12*log(a + b*x + c*x**2)*a**3*c**2*e*x**3 - 15*log(a + b*x + c*x*
*2)*a**2*b**2*c*e*x**3 - 24*log(a + b*x + c*x**2)*a**2*b*c**2*d*x**3 + 3*l
og(a + b*x + c*x**2)*a*b**4*e*x**3 + 18*log(a + b*x + c*x**2)*a*b**3*c*d*x
**3 - 3*log(a + b*x + c*x**2)*b**5*d*x**3 - 24*log(x)*a**3*c**2*e*x**3 + 3
0*log(x)*a**2*b**2*c*e*x**3 + 48*log(x)*a**2*b*c**2*d*x**3 - 6*log(x)*a*b*
*4*e*x**3 - 36*log(x)*a*b**3*c*d*x**3 + 6*log(x)*b**5*d*x**3 - 8*a**4*c*d
- 12*a**4*c*e*x + 2*a**3*b**2*d + 3*a**3*b**2*e*x + 12*a**3*b*c*d*x + 24*a
**3*b*c*e*x**2 + 24*a**3*c**2*d*x**2 - 3*a**2*b**3*d*x - 6*a**2*b**3*e*x**
2 - 30*a**2*b**2*c*d*x**2 + 6*a*b**4*d*x**2)/(6*a**4*x**3*(4*a*c - b**2))
```

3.48 $\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 266

$$\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx = \frac{(cd-2be)x}{c^3}+\frac{ex^2}{2c^2} - \frac{ac\left(b^3d-3abcd+4ab^2e-\frac{b^4e}{c}-2a^2ce\right)+(b^4cd-4ab^2c^2d+2a^2c^3d-b^5e+5ab^3ce-5a^2bc^2e)\,x}{c^4\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{(2b^4cd-12ab^2c^2d+12a^2c^3d-3b^5e+20ab^3ce-30a^2bc^2e)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\,(b^2-4ac)^{3/2}} - \frac{(2bcd-3b^2e+2ace)\log\left(a+bx+cx^2\right)}{2c^4}$$

output
```
(-2*b*e+c*d)*x/c^3+1/2*e*x^2/c^2-(a*c*(b^3*d-3*a*b*c*d+4*a*b^2*e-b^4*e/c-2
*a^2*c*e)+(-5*a^2*b*c^2*e+2*a^2*c^3*d+5*a*b^3*c*e-4*a*b^2*c^2*d-b^5*e+b^4*
c*d)*x)/c^4/(-4*a*c+b^2)/(c*x^2+b*x+a)-(-30*a^2*b*c^2*e+12*a^2*c^3*d+20*a*
b^3*c*e-12*a*b^2*c^2*d-3*b^5*e+2*b^4*c*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/c^4/(-4*a*c+b^2)^(3/2)-1/2*(2*a*c*e-3*b^2*e+2*b*c*d)*ln(c*x^2+b*x+a)
/c^4
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.94

$$\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{2c(cd-2be)x+c^2ex^2+\frac{2(2a^3c^2e+b^4(-cd+be)x+ab^2(b^2e+4c^2dx-bc(d+5ex))+a^2c(-4b^2e-2c^2dx+bc(3d+5ex)))}{(b^2-4ac)(a+x(b+cx))}+\frac{2(-2b^4cd+1\ldots}{\ldots}}{2c^4}$$

input
```
Integrate[(x^4*(d + e*x))/(a + b*x + c*x^2)^2,x]
```

output
```
(2*c*(c*d - 2*b*e)*x + c^2*e*x^2 + (2*(2*a^3*c^2*e + b^4*(-(c*d) + b*e)*x
+ a*b^2*(b^2*e + 4*c^2*d*x - b*c*(d + 5*e*x)) + a^2*c*(-4*b^2*e - 2*c^2*d*
x + b*c*(3*d + 5*e*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(-2*b^4*c*
d + 12*a*b^2*c^2*d - 12*a^2*c^3*d + 3*b^5*e - 20*a*b^3*c*e + 30*a^2*b*c^2*
e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (-2*b*c*
d + 3*b^2*e - 2*a*c*e)*Log[a + x*(b + c*x)])/(2*c^4)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1233, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{ 1233}$$

$$\frac{\int -\frac{x^2(3a(2cd-be)+(-3eb^2+2cdb+8ace)x)}{cx^2+bx+a}dx}{c(b^2-4ac)}+\frac{x^3(x(2ace+b^2(-e)+bcd)+a(2cd-be))}{c(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x^3\left(x\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int\frac{x^2\left(3a(2cd-be)+\left(-3eb^2+2cdb+8ace\right)x\right)}{cx^2+bx+a}dx}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{x^3\left(x\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int\left(-\frac{-3eb^3+2cdb^2+11aceb-6ac^2d}{c^2}+\frac{\left(-3eb^2+2cdb+8ace\right)x}{c}+\frac{a\left(-3eb^3+2cdb^2+11aceb-6ac^2d\right)+\left(b^2-4ac\right)\left(-3eb^2+2cdb+2ace\right)x}{c^2\left(cx^2+bx+a\right)}\right)dx}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3\left(x\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-30a^2bc^2e+12a^2c^3d+20ab^3ce-12ab^2c^2d-3b^5e+2b^4cd\right)}{c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-4ac\right)\left(2ace-3b^2e+2bcd\right)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{x^2\left(8ace-\right.}{}}{c\left(b^2-4ac\right)}$$

input
```
Int[(x^4*(d + e*x))/(a + b*x + c*x^2)^2,x]
```

output
```
(x^3*(a*(2*c*d - b*e) + (b*c*d - b^2*e + 2*a*c*e)*x))/(c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)) - (-(((2*b^2*c*d - 6*a*c^2*d - 3*b^3*e + 11*a*b*c*e)*x)/c^
2) + ((2*b*c*d - 3*b^2*e + 8*a*c*e)*x^2)/(2*c) + ((2*b^4*c*d - 12*a*b^2*c^
2*d + 12*a^2*c^3*d - 3*b^5*e + 20*a*b^3*c*e - 30*a^2*b*c^2*e)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*(2*b*
c*d - 3*b^2*e + 2*a*c*e)*Log[a + b*x + c*x^2])/(2*c^3))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.31

method	result
default	$-\frac{-\frac{1}{2}ce\,x^2+2bex-cdx}{c^3}+\frac{-\frac{\left(5a^2b\,c^2e-2a^2c^3d-5a\,b^3ce+4a\,b^2c^2d+b^5e-b^4cd\right)x}{\left(4ac-b^2\right)c}-\frac{a\left(2a^2c^2e-4a\,b^2ce+3ab\,c^2d+b^4e-b^3cd\right)}{\left(4ac-b^2\right)c}}{c\,x^2+bx+a}+$ $\left(-8a^2c^2e+1\right.$
risch	Expression too large to display

input
```
int(x^4*(e*x+d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/2*c*e*x^2+2*b*e*x-c*d*x)+1/c^3*((-(5*a^2*b*c^2*e-2*a^2*c^3*d-5*
a*b^3*c*e+4*a*b^2*c^2*d+b^5*e-b^4*c*d)/(4*a*c-b^2)/c*x-a*(2*a^2*c^2*e-4*a*
b^2*c*e+3*a*b*c^2*d+b^4*e-b^3*c*d)/(4*a*c-b^2)/c)/(c*x^2+b*x+a)+1/(4*a*c-b
^2)*(1/2*(-8*a^2*c^2*e+14*a*b^2*c*e-8*a*b*c^2*d-3*b^4*e+2*b^3*c*d)/c*ln(c*
x^2+b*x+a)+2*(11*a^2*b*c*e-6*a^2*c^2*d-3*a*b^3*e+2*c*d*a*b^2-1/2*(-8*a^2*c
^2*e+14*a*b^2*c*e-8*a*b*c^2*d-3*b^4*e+2*b^3*c*d)*b/c)/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 838 vs. $2(258) = 516$.

Time = 0.17 (sec) , antiderivative size = 1696, normalized size of antiderivative = 6.38

$$\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*e*x^4 + (2*(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*d - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e)*x^3 + (2*
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - (4*b^6*c - 33*a*b^4*c^2 + 72*a^
2*b^2*c^3 - 16*a^3*c^4)*e)*x^2 + ((2*(b^4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*d
 - (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*e)*x^2 + 2*(a*b^4*c - 6*a^2*b^2
*c^2 + 6*a^3*c^3)*d - (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*e + (2*(b^5*
c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*d - (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*e
)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 -
 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(a*b^5*c - 7*a^2*b^3*c^2 + 12*
a^3*b*c^3)*d + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*e - 2*
((b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*d - (b^7 - 11*a*b^5*c
 + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*e)*x - ((2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*d - (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*e)*x^
2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d - (3*a*b^6 - 26*a^2*b^4*c
 + 64*a^3*b^2*c^2 - 32*a^4*c^3)*e + (2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c
^3)*d - (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*e)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^
2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x), 1/2*(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*e*x^4 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 1
6*a^2*c^5)*d - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e)*x^3 + (2*(b^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. $2(272) = 544$.

Time = 3.95 (sec) , antiderivative size = 1572, normalized size of antiderivative = 5.91

$$\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(e*x+d)/(c*x**2+b*x+a)**2,x)
```

output
```
x*(-2*b*e/c**3 + d/c**2) + (-sqrt(-(4*a*c - b**2)**3)*(30*a**2*b*c**2*e -
12*a**2*c**3*d - 20*a*b**3*c*e + 12*a*b**2*c**2*d + 3*b**5*e - 2*b**4*c*d)
/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c
*e - 3*b**2*e + 2*b*c*d)/(2*c**4))*log(x + (16*a**3*c**2*e - 17*a**2*b**2*
c*e + 10*a**2*b*c**2*d + 16*a**2*c**5*(-sqrt(-(4*a*c - b**2)**3)*(30*a**2*
b*c**2*e - 12*a**2*c**3*d - 20*a*b**3*c*e + 12*a*b**2*c**2*d + 3*b**5*e -
2*b**4*c*d)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
)) - (2*a*c*e - 3*b**2*e + 2*b*c*d)/(2*c**4)) + 3*a*b**4*e - 2*a*b**3*c*d
- 8*a*b**2*c**4*(-sqrt(-(4*a*c - b**2)**3)*(30*a**2*b*c**2*e - 12*a**2*c**
3*d - 20*a*b**3*c*e + 12*a*b**2*c**2*d + 3*b**5*e - 2*b**4*c*d)/(2*c**4*(6
4*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c*e - 3*b**2
*e + 2*b*c*d)/(2*c**4)) + b**4*c**3*(-sqrt(-(4*a*c - b**2)**3)*(30*a**2*b*
c**2*e - 12*a**2*c**3*d - 20*a*b**3*c*e + 12*a*b**2*c**2*d + 3*b**5*e - 2*
b**4*c*d)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
 - (2*a*c*e - 3*b**2*e + 2*b*c*d)/(2*c**4)))/(30*a**2*b*c**2*e - 12*a**2*c
**3*d - 20*a*b**3*c*e + 12*a*b**2*c**2*d + 3*b**5*e - 2*b**4*c*d)) + (sqrt
(-(4*a*c - b**2)**3)*(30*a**2*b*c**2*e - 12*a**2*c**3*d - 20*a*b**3*c*e +
12*a*b**2*c**2*d + 3*b**5*e - 2*b**4*c*d)/(2*c**4*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c*e - 3*b**2*e + 2*b*c*d)/(2*c**4)
)*log(x + (16*a**3*c**2*e - 17*a**2*b**2*c*e + 10*a**2*b*c**2*d + 16*a*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.07

$$\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{(2\,b^4cd-12\,ab^2c^2d+12\,a^2c^3d-3\,b^5e+20\,ab^3ce-30\,a^2bc^2e)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^4-4\,ac^5)\sqrt{-b^2+4\,ac}}$$
$$-\frac{(2\,bcd-3\,b^2e+2\,ace)\log\left(cx^2+bx+a\right)}{2\,c^4}+\frac{c^2ex^2+2\,c^2dx-4\,bcex}{2\,c^4}$$
$$-\frac{ab^3cd-3\,a^2bc^2d-ab^4e+4\,a^2b^2ce-2\,a^3c^2e+(b^4cd-4\,ab^2c^2d+2\,a^2c^3d-b^5e+5\,ab^3ce-5\,a^2bc^2e)x}{(cx^2+bx+a)(b^2-4\,ac)c^4}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
(2*b^4*c*d - 12*a*b^2*c^2*d + 12*a^2*c^3*d - 3*b^5*e + 20*a*b^3*c*e - 30*a
^2*b*c^2*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^4 - 4*a*c^5)*sq
rt(-b^2 + 4*a*c)) - 1/2*(2*b*c*d - 3*b^2*e + 2*a*c*e)*log(c*x^2 + b*x + a)
/c^4 + 1/2*(c^2*e*x^2 + 2*c^2*d*x - 4*b*c*e*x)/c^4 - (a*b^3*c*d - 3*a^2*b*
c^2*d - a*b^4*e + 4*a^2*b^2*c*e - 2*a^3*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d +
 2*a^2*c^3*d - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)*x)/((c*x^2 + b*x + a)*
(b^2 - 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 11.67 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.61

$$\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx = x\left(\frac{d}{c^2}-\frac{2\,b\,e}{c^3}\right) - \frac{\frac{a\,(2\,e\,a^2\,c^2-4\,e\,a\,b^2\,c+3\,d\,a\,b\,c^2+e\,b^4-d\,b^3\,c)}{c\,(4\,a\,c-b^2)}+\frac{x\,(5\,e\,a^2\,b\,c^2-2\,d\,a^2\,c^3-5\,e\,a\,b^3\,c+4\,d\,a\,b^2\,c^2+e\,b^5-d\,b^4\,c)}{c\,(4\,a\,c-b^2)}}{c^4\,x^2+b\,c^3\,x+a\,c^3} + \frac{e\,x^2}{2\,c^2} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,(128\,e\,a^4\,c^4-288\,e\,a^3\,b^2\,c^3+128\,d\,a^3\,b\,c^4+168\,e\,a^2\,b^4\,c^2-96\,d\,a^2\,b^3\,c^3-38\,e\,a\,b^6\,c}{2\,\left(64\,a^3\,c^7-48\,a^2\,b^2\,c^6+12\,a\,b^4\,c^5-b^6\,c^4\right)} + \frac{\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^3-4\,a\,b\,c^4}{c^3\,{(4\,a\,c-b^2)}^{3/2}}\right)\,(30\,e\,a^2\,b\,c^2-12\,d\,a^2\,c^3-20\,e\,a\,b^3\,c+12\,d\,a\,b^2\,c^2+3\,e\,b^5-2\,d\,b^4\,c)}{c^4\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int((x^4*(d + e*x))/(a + b*x + c*x^2)^2,x)
```

output
```
x*(d/c^2 - (2*b*e)/c^3) - ((a*(b^4*e + 2*a^2*c^2*e - b^3*c*d + 3*a*b*c^2*d
 - 4*a*b^2*c*e))/(c*(4*a*c - b^2)) + (x*(b^5*e - 2*a^2*c^3*d - b^4*c*d - 5
*a*b^3*c*e + 4*a*b^2*c^2*d + 5*a^2*b*c^2*e))/(c*(4*a*c - b^2)))/(a*c^3 + c
^4*x^2 + b*c^3*x) + (e*x^2)/(2*c^2) - (log(a + b*x + c*x^2)*(3*b^8*e + 128
*a^4*c^4*e - 2*b^7*c*d - 96*a^2*b^3*c^3*d + 168*a^2*b^4*c^2*e - 288*a^3*b^
2*c^3*e - 38*a*b^6*c*e + 24*a*b^5*c^2*d + 128*a^3*b*c^4*d))/(2*(64*a^3*c^7
 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)) + (atan((2*c*x)/(4*a*c - b^2)
^(1/2) - (b^3*c^3 - 4*a*b*c^4)/(c^3*(4*a*c - b^2)^(3/2)))*(3*b^5*e - 12*a^
2*c^3*d - 2*b^4*c*d - 20*a*b^3*c*e + 12*a*b^2*c^2*d + 30*a^2*b*c^2*e))/(c^
4*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1610, normalized size of antiderivative = 6.05

$$\int \frac{x^4(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^4*(e*x+d)/(c*x^2+b*x+a)^2,x)
```

output

```
(60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c**2
*e - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**
3*d - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4
*c*e + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**
3*c**2*d + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2
*b**3*c**2*e*x - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**2*b**2*c**3*d*x + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a**2*b**2*c**3*e*x**2 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqr
t(4*a*c - b**2))*a**2*b*c**4*d*x**2 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x
)/sqrt(4*a*c - b**2))*a*b**6*e - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqr
t(4*a*c - b**2))*a*b**5*c*d - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(
4*a*c - b**2))*a*b**5*c*e*x + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(
4*a*c - b**2))*a*b**4*c**2*d*x - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sq
rt(4*a*c - b**2))*a*b**4*c**2*e*x**2 + 24*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*a*b**3*c**3*d*x**2 + 6*sqrt(4*a*c - b**2)*atan((b
+ 2*c*x)/sqrt(4*a*c - b**2))*b**7*e*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*b**6*c*d*x + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)
/sqrt(4*a*c - b**2))*b**6*c*e*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)
/sqrt(4*a*c - b**2))*b**5*c**2*d*x**2 - 32*log(a + b*x + c*x**2)*a**4*b*c*
*3*e + 64*log(a + b*x + c*x**2)*a**3*b**3*c**2*e - 32*log(a + b*x + c*x...
```

3.49 $\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 202

$$\begin{aligned}\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx &= \frac{ex}{c^2}+\frac{a(b^2cd-2ac^2d-b^3e+3abce)+(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e)\,x}{c^3\,(b^2-4ac)\,(a+bx+cx^2)}\\&\quad+\frac{(b^3cd-6abc^2d-2b^4e+12ab^2ce-12a^2c^2e)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}}\\&\quad+\frac{(cd-2be)\log(a+bx+cx^2)}{2c^3}\end{aligned}$$

output
```
e*x/c^2+(a*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+(-2*a^2*c^2*e+4*a*b^2*c*e-3
*a*b*c^2*d-b^4*e+b^3*c*d)*x)/c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)+(-12*a^2*c^2*e
+12*a*b^2*c*e-6*a*b*c^2*d-2*b^4*e+b^3*c*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/2*(-2*b*e+c*d)*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94

$$\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{2cex + \frac{2(b^3(cd-be)x+a^2c(3be-2c(d+ex))+ab(-b^2e-3c^2dx+bc(d+4ex)))}{(b^2-4ac)(a+x(b+cx))} - \frac{2(-b^3cd+6abc^2d+2b^4e-12ab^2ce+12a^2c^2e)\arctan\left(\frac{b+2c}{\sqrt{-b^2+}}\right.}{(-b^2+4ac)^{3/2}}}{2c^3}$$

input
```
Integrate[(x^3*(d + e*x))/(a + b*x + c*x^2)^2,x]
```

output
```
(2*c*e*x + (2*(b^3*(c*d - b*e)*x + a^2*c*(3*b*e - 2*c*(d + e*x)) + a*b*(-(
b^2*e) - 3*c^2*d*x + b*c*(d + 4*e*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
- (2*(-(b^3*c*d) + 6*a*b*c^2*d + 2*b^4*e - 12*a*b^2*c*e + 12*a^2*c^2*e)*Ar
cTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c*d - 2*b*e)
*Log[a + x*(b + c*x)])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1233, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \; 1233$$

$$\frac{\int -\frac{x(2a(2cd-be)+(-2eb^2+cdb+6ace)x)}{cx^2+bx+a}dx}{c\,(b^2-4ac)} + \frac{x^2\,(x\,(2ace+b^2(-e)+bcd)+a(2cd-be))}{c\,(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{x^2\,(x\,(2ace+b^2(-e)+bcd)+a(2cd-be))}{c\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{\int \frac{x(2a(2cd-be)+(-2eb^2+cdb+6ace)x)}{cx^2+bx+a}dx}{c\,(b^2-4ac)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{x^2\left(x\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int\left(-\frac{2eb^2}{c}+db+6ae-\frac{a\left(-2eb^2+cdb+6ace\right)+\left(b^2-4ac\right)(cd-2be)x}{c(cx^2+bx+a)}\right)dx}{c\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2\left(x\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-12a^2c^2e+12ab^2ce-6abc^2d-2b^4e+b^3cd\right)}{c^2\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)(cd-2be)\log\left(a+bx+cx^2\right)}{2c^2}+x\left(6ae-\frac{2b^2e}{c}+bd\right)}{c\left(b^2-4ac\right)}$$

input
```
Int[(x^3*(d + e*x))/(a + b*x + c*x^2)^2,x]
```

output
```
(x^2*(a*(2*c*d - b*e) + (b*c*d - b^2*e + 2*a*c*e)*x))/(c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)) - ((b*d + 6*a*e - (2*b^2*e)/c)*x - ((b^3*c*d - 6*a*b*c^2*d
 - 2*b^4*e + 12*a*b^2*c*e - 12*a^2*c^2*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)*(c*d - 2*b*e)*Log[a + b*x
 + c*x^2])/(2*c^2))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.33

method	result
default	$\frac{ex}{c^2} - \frac{\frac{-\frac{\left(2a^2c^2e-4a\,b^2ce+3ab\,c^2d+b^4e-b^3cd\right)x}{\left(4ac-b^2\right)c}+\frac{a\left(3abce-2a\,c^2d-e\,b^3+cd\,b^2\right)}{\left(4ac-b^2\right)c}}{c\,x^2+bx+a}+\frac{\frac{\left(8abce-4a\,c^2d-2e\,b^3+cd\,b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(6a^2ce-\right.}{}}{}}{c^2}$
risch	Expression too large to display

input
```
int(x^3*(e*x+d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e*x/c^2-1/c^2*((-(2*a^2*c^2*e-4*a*b^2*c*e+3*a*b*c^2*d+b^4*e-b^3*c*d)/(4*a*
c-b^2)/c*x+a*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(4*a*c-b^2)/c)/(c*x^2+b*x
+a)+1/(4*a*c-b^2)*(1/2*(8*a*b*c*e-4*a*c^2*d-2*b^3*e+b^2*c*d)/c*ln(c*x^2+b*
x+a)+2*(6*a^2*c*e-2*e*a*b^2+a*b*c*d-1/2*(8*a*b*c*e-4*a*c^2*d-2*b^3*e+b^2*c
*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(196) = 392$.

Time = 0.10 (sec) , antiderivative size = 1283, normalized size of antiderivative = 6.35

$$\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e*x^3 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*e*x^2 + (((b^3*c^2 - 6*a*b*c^3)*d - 2*(b^4*c - 6*a*b^2*c
^2 + 6*a^2*c^3)*e)*x^2 + (a*b^3*c - 6*a^2*b*c^2)*d - 2*(a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2)*e + ((b^4*c - 6*a*b^2*c^2)*d - 2*(b^5 - 6*a*b^3*c + 6*a^2*b
*c^2)*e)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqr
t(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^4*c - 6*a^2*b^2*c^
2 + 8*a^3*c^3)*d - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e + 2*((b^5*c -
7*a*b^3*c^2 + 12*a^2*b*c^3)*d - (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3
*c^3)*e)*x + (((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 2*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*e)*x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d - 2
*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e)*x)*log(c*x^2 + b*x + a))
/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2
*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), 1/2*(2*(b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*e*x^3 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e*
x^2 + 2*(((b^3*c^2 - 6*a*b*c^3)*d - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e)
*x^2 + (a*b^3*c - 6*a^2*b*c^2)*d - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e +
 ((b^4*c - 6*a*b^2*c^2)*d - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e)*x)*sqrt(-
b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*
b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(206) = 412$.

Time = 2.89 (sec) , antiderivative size = 1248, normalized size of antiderivative = 6.18

$$\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(e*x+d)/(c*x**2+b*x+a)**2,x)
```

output
```
(-sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e - 12*a*b**2*c*e + 6*a*b*c**2*d
+ 2*b**4*e - b**3*c*d)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) - (2*b*e - c*d)/(2*c**3))*log(x + (-10*a**2*b*c*e - 16*a**2*
c**4*(-sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e - 12*a*b**2*c*e + 6*a*b*c*
*2*d + 2*b**4*e - b**3*c*d)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - (2*b*e - c*d)/(2*c**3)) + 8*a**2*c**2*d + 2*a*b**3*e
+ 8*a*b**2*c**3*(-sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e - 12*a*b**2*c*e
 + 6*a*b*c**2*d + 2*b**4*e - b**3*c*d)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) - (2*b*e - c*d)/(2*c**3)) - a*b**2*c*d - b**
4*c**2*(-sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e - 12*a*b**2*c*e + 6*a*b*
c**2*d + 2*b**4*e - b**3*c*d)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) - (2*b*e - c*d)/(2*c**3)))/(12*a**2*c**2*e - 12*a*b**
2*c*e + 6*a*b*c**2*d + 2*b**4*e - b**3*c*d)) + (sqrt(-(4*a*c - b**2)**3)*(
12*a**2*c**2*e - 12*a*b**2*c*e + 6*a*b*c**2*d + 2*b**4*e - b**3*c*d)/(2*c*
*3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*b*e - c*d
)/(2*c**3))*log(x + (-10*a**2*b*c*e - 16*a**2*c**4*(sqrt(-(4*a*c - b**2)**
3)*(12*a**2*c**2*e - 12*a*b**2*c*e + 6*a*b*c**2*d + 2*b**4*e - b**3*c*d)/(
2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*b*e -
 c*d)/(2*c**3)) + 8*a**2*c**2*d + 2*a*b**3*e + 8*a*b**2*c**3*(sqrt(-(4*a*c
 - b**2)**3)*(12*a**2*c**2*e - 12*a*b**2*c*e + 6*a*b*c**2*d + 2*b**4*e ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.11

$$\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx = -\frac{(b^3cd-6\,abc^2d-2\,b^4e+12\,ab^2ce-12\,a^2c^2e)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}} + \frac{ex}{c^2} + \frac{(cd-2\,be)\log(cx^2+bx+a)}{2\,c^3} + \frac{\frac{(b^3cd-3\,abc^2d-b^4e+4\,ab^2ce-2\,a^2c^2e)x}{c} + \frac{ab^2cd-2\,a^2c^2d-ab^3e+3\,a^2bce}{c}}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(b^3*c*d - 6*a*b*c^2*d - 2*b^4*e + 12*a*b^2*c*e - 12*a^2*c^2*e)*arctan((2
*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + e
*x/c^2 + 1/2*(c*d - 2*b*e)*log(c*x^2 + b*x + a)/c^3 + ((b^3*c*d - 3*a*b*c^
2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*x/c + (a*b^2*c*d - 2*a^2*c^2*d -
a*b^3*e + 3*a^2*b*c*e)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 11.50 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.78

$$\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx = \frac{\frac{a\,(e\,b^3-d\,b^2\,c-3\,a\,e\,b\,c+2\,a\,d\,c^2)}{c\,(4\,a\,c-b^2)}+\frac{x\,(2\,e\,a^2\,c^2-4\,e\,a\,b^2\,c+3\,d\,a\,b\,c^2+e\,b^4-d\,b^3\,c)}{c\,(4\,a\,c-b^2)}}{c^3\,x^2+b\,c^2\,x+a\,c^2}$$
$$+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,e\,a^3\,b\,c^3+64\,d\,a^3\,c^4+96\,e\,a^2\,b^3\,c^2-48\,d\,a^2\,b^2\,c^3-24\,e\,a\,b^5\,c+12\,d\,a\,b^4\,c^2+\right.}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)}$$
$$+\frac{e\,x}{c^2}$$
$$-\frac{\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(12\,e\,a^2\,c^2-12\,e\,a\,b^2\,c+6\,d\,a\,b\,c^2+2\,e\,b^4-d\,b^3\,c\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int((x^3*(d + e*x))/(a + b*x + c*x^2)^2,x)
```

output
```
((a*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(c*(4*a*c - b^2)) + (x*(b^4
*e + 2*a^2*c^2*e - b^3*c*d + 3*a*b*c^2*d - 4*a*b^2*c*e))/(c*(4*a*c - b^2))
)/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c*x^2)*(2*b^7*e + 64*a^3*c^
4*d - b^6*c*d - 48*a^2*b^2*c^3*d + 96*a^2*b^3*c^2*e - 24*a*b^5*c*e + 12*a*
b^4*c^2*d - 128*a^3*b*c^3*e))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48
*a^2*b^2*c^5)) + (e*x)/c^2 - (atan((2*c*x)/(4*a*c - b^2)^(1/2) - (b^3*c^2
- 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(2*b^4*e + 12*a^2*c^2*e - b^3*c*d
+ 6*a*b*c^2*d - 12*a*b^2*c*e))/(c^3*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1294, normalized size of antiderivative = 6.41

$$\int \frac{x^3(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(e*x+d)/(c*x^2+b*x+a)^2,x)
```

output
```
( - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2
*e + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*
c*e - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2
*c**2*d - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*
b**2*c**2*e*x - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a**2*b*c**3*e*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b
**2))*a*b**5*e + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a*b**4*c*d + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a
*b**4*c*e*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a
*b**3*c**2*d*x + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a*b**3*c**2*e*x**2 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a*b**2*c**3*d*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*b**6*e*x + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*b**5*c*d*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*b**5*c*e*x**2 + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*b**4*c**2*d*x**2 - 32*log(a + b*x + c*x**2)*a**3*b**2*c**2*e + 16*log
(a + b*x + c*x**2)*a**3*b*c**3*d + 16*log(a + b*x + c*x**2)*a**2*b**4*c*e
- 8*log(a + b*x + c*x**2)*a**2*b**3*c**2*d - 32*log(a + b*x + c*x**2)*a**2
*b**3*c**2*e*x + 16*log(a + b*x + c*x**2)*a**2*b**2*c**3*d*x - 32*log(a +
b*x + c*x**2)*a**2*b**2*c**3*e*x**2 + 16*log(a + b*x + c*x**2)*a**2*b*c...
```

3.50 $\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2} dx = \frac{x(a(2cd-be)+(bcd-b^2e+2ace)x)}{c(b^2-4ac)(a+bx+cx^2)} + \frac{(b^3e+2ac(2cd-3be))\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}} + \frac{e\log(a+bx+cx^2)}{2c^2}$$

output
```
x*(a*(-b*e+2*c*d)+(2*a*c*e-b^2*e+b*c*d)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)+(b
^3*e+2*a*c*(-3*b*e+2*c*d))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a
*c+b^2)^(3/2)+1/2*e*ln(c*x^2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2}\,dx = \frac{-\frac{2(2a^2ce+b^2(cd-be)x+a(-b^2e-2c^2dx+bc(d+3ex)))}{(b^2-4ac)(a+x(b+cx))}+\frac{2(b^3e+2ac(2cd-3be))\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+e\log(a+x(b+cx))}{2c^2}$$

input
```
Integrate[(x^2*(d + e*x))/(a + b*x + c*x^2)^2,x]
```

output
```
((-2*(2*a^2*c*e + b^2*(c*d - b*e)*x + a*(-(b^2*e) - 2*c^2*d*x + b*c*(d + 3
*e*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(b^3*e + 2*a*c*(2*c*d - 3*
b*e))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + e*Log
[a + x*(b + c*x)])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1233, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{\int -\frac{a(2cd-be)-(b^2-4ac)ex}{cx^2+bx+a}dx}{c(b^2-4ac)}+\frac{x(x(2ace+b^2(-e)+bcd)+a(2cd-be))}{c(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x(x(2ace+b^2(-e)+bcd)+a(2cd-be))}{c(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{a(2cd-be)-(b^2-4ac)ex}{cx^2+bx+a}dx}{c(b^2-4ac)}$$

$$\Big\downarrow \; 1142$$

$$\frac{x(x(2ace+b^2(-e)+bcd)+a(2cd-be))}{c(b^2-4ac)(a+bx+cx^2)}-\frac{\frac{(2ac(2cd-3be)+b^3e)\int\frac{1}{cx^2+bx+a}dx}{2c}-\frac{e(b^2-4ac)\int\frac{b+2cx}{cx^2+bx+a}dx}{2c}}{c(b^2-4ac)}$$

$$\Big\downarrow \; 1083$$

$$\frac{x(x(2ace+b^2(-e)+bcd)+a(2cd-be))}{c(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{e(b^2-4ac)\int\frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{(2ac(2cd-3be)+b^3e)\int\frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c}}{c(b^2-4ac)}$$

$$\Big\downarrow \; 219$$

$$\frac{x(x(2ace+b^2(-e)+bcd)+a(2cd-be))}{c(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{e(b^2-4ac)\int\frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ac(2cd-3be)+b^3e)}{c\sqrt{b^2-4ac}}}{c(b^2-4ac)}$$

$$\Big\downarrow \; 1103$$

$$\frac{x(x(2ace+b^2(-e)+bcd)+a(2cd-be))}{c(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ac(2cd-3be)+b^3e)}{c\sqrt{b^2-4ac}}-\frac{e(b^2-4ac)\log(a+bx+cx^2)}{2c}}{c(b^2-4ac)}$$

input
```
Int[(x^2*(d + e*x))/(a + b*x + c*x^2)^2,x]
```

output
```
(x*(a*(2*c*d - b*e) + (b*c*d - b^2*e + 2*a*c*e)*x))/(c*(b^2 - 4*a*c)*(a +
b*x + c*x^2)) - (-(((b^3*e + 2*a*c*(2*c*d - 3*b*e))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)*e*Log[a + b*x +
c*x^2])/(2*c))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.52

method	result
default	$\frac{\frac{\left(3abce-2a\,c^2d-e\,b^3+cd\,b^2\right)x}{c^2\left(4ac-b^2\right)}+\frac{a\left(2ace-e\,b^2+dbc\right)}{c^2\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{\frac{\left(4ace-e\,b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-abe+2acd-\frac{\left(4ace-e\,b^2\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{(4ac-b^2)c}$
risch	Expression too large to display

input
```
int(x^2*(e*x+d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/c^2*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(4*a*c-b^2)*x+a*(2*a*c*e-b^2*e+
b*c*d)/c^2/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)/c*(1/2*(4*a*c*e-b^2*e)
/c*ln(c*x^2+b*x+a)+2*(-a*b*e+2*a*c*d-1/2*(4*a*c*e-b^2*e)*b/c)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(126) = 252.

Time = 0.09 (sec) , antiderivative size = 813, normalized size of antiderivative = 6.16

$$\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*((4*a^2*c^2*d + (4*a*c^3*d + (b^3*c - 6*a*b*c^2)*e)*x^2 + (a*b^3 - 6*
a^2*b*c)*e + (4*a*b*c^2*d + (b^4 - 6*a*b^2*c)*e)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
 + b*x + a)) - 2*(a*b^3*c - 4*a^2*b*c^2)*d + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^
3*c^2)*e - 2*((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7*a*b^3*c + 12*
a^2*b*c^2)*e)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*x^2 + (b^5 - 8*a*b
^3*c + 16*a^2*b*c^2)*e*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e)*log(c*x^2
 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*
c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x), 1/2*(2*
(4*a^2*c^2*d + (4*a*c^3*d + (b^3*c - 6*a*b*c^2)*e)*x^2 + (a*b^3 - 6*a^2*b*
c)*e + (4*a*b*c^2*d + (b^4 - 6*a*b^2*c)*e)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(a*b^3*c - 4*a^2*b*c^2)*d
 + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e - 2*((b^4*c - 6*a*b^2*c^2 + 8*a^2
*c^3)*d - (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*e)*x + ((b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*e*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*x + (a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)*e)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3
*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(124) = 248$.

Time = 1.49 (sec) , antiderivative size = 901, normalized size of antiderivative = 6.83

$$\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(e*x+d)/(c*x**2+b*x+a)**2,x)
```

output
```
(e/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e - 4*a*c**2*d - b**3*e)/(
2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (
-16*a**2*c**3*(e/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e - 4*a*c**2
*d - b**3*e)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 8*a**2*c*e + 8*a*b**2*c**2*(e/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(
6*a*b*c*e - 4*a*c**2*d - b**3*e)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6))) - a*b**2*e - 2*a*b*c*d - b**4*c*(e/(2*c**2) - sqr
t(-(4*a*c - b**2)**3)*(6*a*b*c*e - 4*a*c**2*d - b**3*e)/(2*c**2*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(6*a*b*c*e - 4*a*c**2*d -
 b**3*e)) + (e/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e - 4*a*c**2*d
 - b**3*e)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
))*log(x + (-16*a**2*c**3*(e/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*
e - 4*a*c**2*d - b**3*e)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6))) + 8*a**2*c*e + 8*a*b**2*c**2*(e/(2*c**2) + sqrt(-(4*a*c -
 b**2)**3)*(6*a*b*c*e - 4*a*c**2*d - b**3*e)/(2*c**2*(64*a**3*c**3 - 48*a*
*2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b**2*e - 2*a*b*c*d - b**4*c*(e/(2
*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e - 4*a*c**2*d - b**3*e)/(2*c**
2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(6*a*b*c*e -
4*a*c**2*d - b**3*e)) + (2*a**2*c*e - a*b**2*e + a*b*c*d + x*(3*a*b*c*e -
2*a*c**2*d - b**3*e + b**2*c*d))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.23

$$\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2}\,dx = -\frac{(4\,ac^2d+b^3e-6\,abce)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{e\log\left(cx^2+bx+a\right)}{2\,c^2} - \frac{abcd-ab^2e+2\,a^2ce+(b^2cd-2\,ac^2d-b^3e+3\,abce)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input

```
integrate(x^2*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output

```
-(4*a*c^2*d + b^3*e - 6*a*b*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((
b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/2*e*log(c*x^2 + b*x + a)/c^2 -
(a*b*c*d - a*b^2*e + 2*a^2*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)
*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 895, normalized size of antiderivative = 6.78

$$\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2}\,dx = \frac{2\,a^2\,c\,e}{4\,a^2\,c^3-a\,b^2\,c^2+4\,a\,b\,c^3\,x+4\,a\,c^4\,x^2-b^3\,c^2\,x-b^2\,c^3\,x^2} - \frac{a\,b^2\,e}{4\,a^2\,c^3-a\,b^2\,c^2+4\,a\,b\,c^3\,x+4\,a\,c^4\,x^2-b^3\,c^2\,x-b^2\,c^3\,x^2} - \frac{b^3\,e\,x}{4\,a^2\,c^3-a\,b^2\,c^2+4\,a\,b\,c^3\,x+4\,a\,c^4\,x^2-b^3\,c^2\,x-b^2\,c^3\,x^2} + \frac{4\,a\,d\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3}{(4\,a\,c-b^2)^{3/2}}+\frac{4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}\right)}{(4\,a\,c-b^2)^{3/2}} - \frac{b^6\,e\,\ln(c\,x^2+b\,x+a)}{2\,(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2)} - \frac{2\,a\,c^2\,d\,x}{4\,a^2\,c^3-a\,b^2\,c^2+4\,a\,b\,c^3\,x+4\,a\,c^4\,x^2-b^3\,c^2\,x-b^2\,c^3\,x^2} + \frac{b^2\,c\,d\,x}{4\,a^2\,c^3-a\,b^2\,c^2+4\,a\,b\,c^3\,x+4\,a\,c^4\,x^2-b^3\,c^2\,x-b^2\,c^3\,x^2} + \frac{a\,b\,c\,d}{4\,a^2\,c^3-a\,b^2\,c^2+4\,a\,b\,c^3\,x+4\,a\,c^4\,x^2-b^3\,c^2\,x-b^2\,c^3\,x^2} + \frac{32\,a^3\,c^3\,e\,\ln(c\,x^2+b\,x+a)}{64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2} + \frac{b^3\,e\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3}{(4\,a\,c-b^2)^{3/2}}+\frac{4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}\right)}{c^2\,(4\,a\,c-b^2)^{3/2}} + \frac{3\,a\,b\,c\,e\,x}{4\,a^2\,c^3-a\,b^2\,c^2+4\,a\,b\,c^3\,x+4\,a\,c^4\,x^2-b^3\,c^2\,x-b^2\,c^3\,x^2} - \frac{24\,a^2\,b^2\,c^2\,e\,\ln(c\,x^2+b\,x+a)}{64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2} + \frac{6\,a\,b^4\,c\,e\,\ln(c\,x^2+b\,x+a)}{64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2} - \frac{6\,a\,b\,e\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3}{(4\,a\,c-b^2)^{3/2}}+\frac{4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}\right)}{c\,(4\,a\,c-b^2)^{3/2}}$$

input
```
int((x^2*(d + e*x))/(a + b*x + c*x^2)^2,x)
```

output
```
(2*a^2*c*e)/(4*a^2*c^3 - a*b^2*c^2 + 4*a*c^4*x^2 - b^3*c^2*x - b^2*c^3*x^2
 + 4*a*b*c^3*x) - (a*b^2*e)/(4*a^2*c^3 - a*b^2*c^2 + 4*a*c^4*x^2 - b^3*c^2
*x - b^2*c^3*x^2 + 4*a*b*c^3*x) - (b^3*e*x)/(4*a^2*c^3 - a*b^2*c^2 + 4*a*c
^4*x^2 - b^3*c^2*x - b^2*c^3*x^2 + 4*a*b*c^3*x) + (4*a*d*atan((2*c*x)/(4*a
*c - b^2)^(1/2) - b^3/(4*a*c - b^2)^(3/2) + (4*a*b*c)/(4*a*c - b^2)^(3/2))
)/(4*a*c - b^2)^(3/2) - (b^6*e*log(a + b*x + c*x^2))/(2*(64*a^3*c^5 - b^6*
c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)) - (2*a*c^2*d*x)/(4*a^2*c^3 - a*b^2*c
^2 + 4*a*c^4*x^2 - b^3*c^2*x - b^2*c^3*x^2 + 4*a*b*c^3*x) + (b^2*c*d*x)/(4
*a^2*c^3 - a*b^2*c^2 + 4*a*c^4*x^2 - b^3*c^2*x - b^2*c^3*x^2 + 4*a*b*c^3*x
) + (a*b*c*d)/(4*a^2*c^3 - a*b^2*c^2 + 4*a*c^4*x^2 - b^3*c^2*x - b^2*c^3*x
^2 + 4*a*b*c^3*x) + (32*a^3*c^3*e*log(a + b*x + c*x^2))/(64*a^3*c^5 - b^6*
c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4) + (b^3*e*atan((2*c*x)/(4*a*c - b^2)^(
1/2) - b^3/(4*a*c - b^2)^(3/2) + (4*a*b*c)/(4*a*c - b^2)^(3/2)))/(c^2*(4*a
*c - b^2)^(3/2)) + (3*a*b*c*e*x)/(4*a^2*c^3 - a*b^2*c^2 + 4*a*c^4*x^2 - b^
3*c^2*x - b^2*c^3*x^2 + 4*a*b*c^3*x) - (24*a^2*b^2*c^2*e*log(a + b*x + c*x
^2))/(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4) + (6*a*b^4*c*e
*log(a + b*x + c*x^2))/(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c
^4) - (6*a*b*e*atan((2*c*x)/(4*a*c - b^2)^(1/2) - b^3/(4*a*c - b^2)^(3/2)
+ (4*a*b*c)/(4*a*c - b^2)^(3/2)))/(c*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 766, normalized size of antiderivative = 5.80

$$\int \frac{x^2(d+ex)}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{16a^3c^3d + 2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^4e + 2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^5ex + 16\log(c\,x^2+bx+a)\,a^3}{}$$

input
```
int(x^2*(e*x+d)/(c*x^2+b*x+a)^2,x)
```

output

```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c
*e + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2
*d + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*e -
12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c*e*x +
8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*d*x
- 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*e
*x**2 + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**3
*d*x**2 + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*e
*x + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*c*e*x*
*2 + 16*log(a + b*x + c*x**2)*a**3*b*c**2*e - 8*log(a + b*x + c*x**2)*a**2
*b**3*c*e + 16*log(a + b*x + c*x**2)*a**2*b**2*c**2*e*x + 16*log(a + b*x +
 c*x**2)*a**2*b*c**3*e*x**2 + log(a + b*x + c*x**2)*a*b**5*e - 8*log(a + b
*x + c*x**2)*a*b**4*c*e*x - 8*log(a + b*x + c*x**2)*a*b**3*c**2*e*x**2 + l
og(a + b*x + c*x**2)*b**6*e*x + log(a + b*x + c*x**2)*b**5*c*e*x**2 - 8*a*
*3*b*c**2*e + 16*a**3*c**3*d + 2*a**2*b**3*c*e - 4*a**2*b**2*c**2*d - 24*a
**2*b*c**3*e*x**2 + 16*a**2*c**4*d*x**2 + 14*a*b**3*c**2*e*x**2 - 12*a*b**
2*c**3*d*x**2 - 2*b**5*c*e*x**2 + 2*b**4*c**2*d*x**2)/(2*b*c**2*(16*a**3*c
**2 - 8*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*
b**3*c*x - 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.51 $\int \frac{x(d+ex)}{(a+bx+cx^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{x(d+ex)}{(a+bx+cx^2)^2} dx = -\frac{x(bd-2ae+(2cd-be)x)}{(b^2-4ac)(a+bx+cx^2)} - \frac{2(bd-2ae)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
-x*(b*d-2*a*e+(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*(-2*a*e+b*d)*ar
ctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{x(d+ex)}{(a+bx+cx^2)^2} dx = \frac{abe+b(-cd+be)x-2ac(d+ex)}{c(-b^2+4ac)(a+x(b+cx))} - \frac{2(bd-2ae)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[(x*(d + e*x))/(a + b*x + c*x^2)^2,x]
```

output
```
(a*b*e + b*(-(c*d) + b*e)*x - 2*a*c*(d + e*x))/(c*(-b^2 + 4*a*c)*(a + x*(b
 + c*x))) - (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2
 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1224, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1224}$$

$$\frac{(bd-2ae)\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}+\frac{x\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{x\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2(bd-2ae)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(2ace+b^2(-e)+bcd\right)+a(2cd-be)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2(bd-2ae)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

input
```
Int[(x*(d + e*x))/(a + b*x + c*x^2)^2,x]
```

output
```
(a*(2*c*d - b*e) + (b*c*d - b^2*e + 2*a*c*e)*x)/(c*(b^2 - 4*a*c)*(a + b*x
+ c*x^2)) - (2*(b*d - 2*a*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2
- 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

method	result
default	$\frac{-\frac{\left(2ace-e\,b^{2}+dbc\right)x}{\left(4ac-b^{2}\right)c}+\frac{a\left(be-2cd\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{2\left(2ae-bd\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ace-e\,b^{2}+dbc\right)x}{\left(4ac-b^{2}\right)c}+\frac{a\left(be-2cd\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{2\ln\left(\left(-8a\,c^{2}+2b^{2}c\right)x+\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)ae}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{\ln\left(\left(-8a\,c^{2}+2b^{2}c\right)x+\left(-4ac+b^{2}\right)^{\frac{3}{2}}-\right.}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$

input
```
int(x*(e*x+d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(2*a*c*e-b^2*e+b*c*d)/(4*a*c-b^2)/c*x+a*(b*e-2*c*d)/c/(4*a*c-b^2))/(c*x^
2+b*x+a)+2*(2*a*e-b*d)/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(83) = 166$.

Time = 0.08 (sec) , antiderivative size = 521, normalized size of antiderivative = 5.99

$$\int \frac{x(d+ex)}{(a+bx+cx^2)^2}\,dx$$
$$= \left[\frac{(abcd - 2\,a^2ce + (bc^2d - 2\,ac^2e)x^2 + (b^2cd - 2\,abce)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right.}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c}\right.$$
$$-\frac{2\,(abcd - 2\,a^2ce + (bc^2d - 2\,ac^2e)x^2 + (b^2cd - 2\,abce)x)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right) - 2\,(}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[((a*b*c*d - 2*a^2*c*e + (b*c^2*d - 2*a*c^2*e)*x^2 + (b^2*c*d - 2*a*b*c*e)
*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 -
4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^2*c - 4*a^2*c^2)*d - (a*b^
3 - 4*a^2*b*c)*e + ((b^3*c - 4*a*b*c^2)*d - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*
e)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(2*(a*b*c*d - 2*a
^2*c*e + (b*c^2*d - 2*a*c^2*e)*x^2 + (b^2*c*d - 2*a*b*c*e)*x)*sqrt(-b^2 +
4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(a*b^2*c
- 4*a^2*c^2)*d + (a*b^3 - 4*a^2*b*c)*e - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 6
*a*b^2*c + 8*a^2*c^2)*e)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c
^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(82) = 164$.

Time = 0.68 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.36

$$\int \frac{x(d+ex)}{(a+bx+cx^2)^2}\,dx = -\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ae-bd)\log\left(x+\frac{-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ae-bd)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ae-bd)+2abe-b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdots}{4ace-2bcd}\right) + \sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ae-bd)\log\left(x+\frac{16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ae-bd)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ae-bd)+2abe+b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdots}{4ace-2bcd}\right) + \frac{abe-2acd+x(-2ace+b^2e-bcd)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

input
```
integrate(x*(e*x+d)/(c*x**2+b*x+a)**2,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**3)*(2*a*e - b*d)*log(x + (-16*a**2*c**2*sqrt(-1/(
4*a*c - b**2)**3)*(2*a*e - b*d) + 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(2
*a*e - b*d) + 2*a*b*e - b**4*sqrt(-1/(4*a*c - b**2)**3)*(2*a*e - b*d) - b*
*2*d)/(4*a*c*e - 2*b*c*d)) + sqrt(-1/(4*a*c - b**2)**3)*(2*a*e - b*d)*log(
x + (16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(2*a*e - b*d) - 8*a*b**2*c*sq
rt(-1/(4*a*c - b**2)**3)*(2*a*e - b*d) + 2*a*b*e + b**4*sqrt(-1/(4*a*c - b
**2)**3)*(2*a*e - b*d) - b**2*d)/(4*a*c*e - 2*b*c*d)) + (a*b*e - 2*a*c*d +
 x*(-2*a*c*e + b**2*e - b*c*d))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 -
 b**2*c**2) + x*(4*a*b*c**2 - b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

$$\int \frac{x(d+ex)}{(a+bx+cx^2)^2}\,dx = \frac{2\,(bd-2\,ae)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{bcdx-b^2ex+2\,acex+2\,acd-abe}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
2*(b*d - 2*a*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt
(-b^2 + 4*a*c)) + (b*c*d*x - b^2*e*x + 2*a*c*e*x + 2*a*c*d - a*b*e)/((b^2*
c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 10.89 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.03

$$\int \frac{x(d+ex)}{(a+bx+cx^2)^2}\,dx = \frac{\frac{a\,(b\,e-2\,c\,d)}{c\,(4\,a\,c-b^2)}-\frac{x\,(-e\,b^2+c\,d\,b+2\,a\,c\,e)}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{2\,\mathrm{atan}\left(\frac{(4\,a\,c-b^2)\left(\frac{(b^3-4\,a\,b\,c)\,(2\,a\,e-b\,d)}{(4\,a\,c-b^2)^{5/2}}-\frac{2\,c\,x\,(2\,a\,e-b\,d)}{(4\,a\,c-b^2)^{3/2}}\right)}{2\,a\,e-b\,d}\right)\,(2\,a\,e-b\,d)}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int((x*(d + e*x))/(a + b*x + c*x^2)^2,x)
```

output
```
((a*(b*e - 2*c*d))/(c*(4*a*c - b^2)) - (x*(2*a*c*e - b^2*e + b*c*d))/(c*(4
*a*c - b^2)))/(a + b*x + c*x^2) - (2*atan(((4*a*c - b^2)*(((b^3 - 4*a*b*c)
*(2*a*e - b*d))/(4*a*c - b^2)^(5/2) - (2*c*x*(2*a*e - b*d))/(4*a*c - b^2)^
(3/2)))/(2*a*e - b*d))*(2*a*e - b*d))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 406, normalized size of antiderivative = 4.67

$$\int \frac{x(d+ex)}{(a+bx+cx^2)^2}\,dx = \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2be - 2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2d + 4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2ex \cdots}{}$$

input
```
int(x*(e*x+d)/(c*x^2+b*x+a)^2,x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*e - 2*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*d + 4*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*e*x + 4*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*e*x**2 - 2*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*d*x - 2*sqrt(4*a*c - b**
2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*d*x**2 + 8*a**3*c*e - 2*a**
2*b**2*e - 4*a**2*b*c*d + 8*a**2*c**2*e*x**2 + a*b**3*d - 6*a*b**2*c*e*x**
2 + 4*a*b*c**2*d*x**2 + b**4*e*x**2 - b**3*c*d*x**2)/(b*(16*a**3*c**2 - 8*
a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x
 - 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.52 $\int \frac{d+ex}{\left(a+bx+cx^2\right)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{d+ex}{(a+bx+cx^2)^2}\,dx=-\frac{bd-2ae+(2cd-be)x}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{2(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
-(b*d-2*a*e+(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(c*x^2+b*x+a)+2*(-b*e+2*c*d)*arct
anh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{d+ex}{(a+bx+cx^2)^2}\,dx=\frac{\frac{-bd+2ae-2cdx+bex}{a+x(b+cx)}+\frac{2(-2cd+be)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

input
```
Integrate[(d + e*x)/(a + b*x + c*x^2)^2,x]
```

output
```
((-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(a + x*(b + c*x)) + (2*(-2*c*d + b*e)*
ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+bx+cx^2)^2} \, dx$$

$$\Big\downarrow \text{ 1159}$$

$$-\frac{(2cd-be)\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{-2ae+x(2cd-be)+bd}{(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{2(2cd-be)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{-2ae+x(2cd-be)+bd}{(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2(2cd-be)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2ae+x(2cd-be)+bd}{(b^2-4ac)\,(a+bx+cx^2)}$$

input
```
Int[(d + e*x)/(a + b*x + c*x^2)^2,x]
```

output
```
-((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (2*
(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

method	result
default	$\frac{bd-2ae+(-be+2cd)x}{(4ac-b^2)(c\,x^2+bx+a)}+\frac{2(-be+2cd)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{(be-2cd)x}{4ac-b^2}-\frac{2ae-bd}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\ln\left(\left(-8a\,c^2+2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)be}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2\ln\left(\left(-8a\,c^2+2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)c}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$

input
```
int((e*x+d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(b*d-2*a*e+(-b*e+2*c*d)*x)/(4*a*c-b^2)/(c*x^2+b*x+a)+2*(-b*e+2*c*d)/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(83) = 166$.

Time = 0.08 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.28

$$\int \frac{d+ex}{(a+bx+cx^2)^2}\,dx$$
$$= \left[\frac{(2\,acd - abe + (2\,c^2d - bce)x^2 + (2\,bcd - b^2e)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) - (b}{ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^2 + (b^5}\right.$$

input

```
integrate((e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output

```
[((2*a*c*d - a*b*e + (2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)*x)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) - (b^3 - 4*a*b*c)*d + 2*(a*b^2 - 4*a^2*c)*e - (
2*(b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2
*b*c^2)*x), (2*(2*a*c*d - a*b*e + (2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*
c)) - (b^3 - 4*a*b*c)*d + 2*(a*b^2 - 4*a^2*c)*e - (2*(b^2*c - 4*a*c^2)*d -
 (b^3 - 4*a*b*c)*e)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(78) = 156$.

Time = 0.51 (sec) , antiderivative size = 359, normalized size of antiderivative = 4.13

$$\int \frac{d+ex}{(a+bx+cx^2)^2}\,dx = \sqrt{-\frac{1}{(4ac-b^2)^3}}(be$$
$$-2cd)\log\left(x+\frac{-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)-b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2}{2bce-4c^2d}\right.$$
$$-\sqrt{-\frac{1}{(4ac-b^2)^3}}(be$$
$$-2cd)\log\left(x+\frac{16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd)+b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}(be-2cd}{2bce-4c^2d}\right.$$
$$+\frac{-2ae+bd+x(-be+2cd)}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x(4abc-b^3)}$$

input
```
integrate((e*x+d)/(c*x**2+b*x+a)**2,x)
```

output
```
sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d)*log(x + (-16*a**2*c**2*sqrt(-1/(4
*a*c - b**2)**3)*(b*e - 2*c*d) + 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(b*
e - 2*c*d) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d) + b**2*e - 2*b*
c*d)/(2*b*c*e - 4*c**2*d)) - sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d)*log(
x + (16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d) - 8*a*b**2*c*sq
rt(-1/(4*a*c - b**2)**3)*(b*e - 2*c*d) + b**4*sqrt(-1/(4*a*c - b**2)**3)*(
b*e - 2*c*d) + b**2*e - 2*b*c*d)/(2*b*c*e - 4*c**2*d)) + (-2*a*e + b*d + x
*(-b*e + 2*c*d))/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*
c - b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{2\,(2\,cd-be)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cdx-bex+bd-2\,ae}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(2*c*d - b*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqr
t(-b^2 + 4*a*c)) - (2*c*d*x - b*e*x + b*d - 2*a*e)/((c*x^2 + b*x + a)*(b^2
 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.83

$$\int \frac{d+ex}{(a+bx+cx^2)^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{(4\,a\,c-b^2)\left(\frac{(b^3-4\,a\,b\,c)\,(b\,e-2\,c\,d)}{(4\,a\,c-b^2)^{5/2}}-\frac{2\,c\,x\,(b\,e-2\,c\,d)}{(4\,a\,c-b^2)^{3/2}}\right)}{b\,e-2\,c\,d}\right)(b\,e-2\,c\,d)}{(4\,a\,c-b^2)^{3/2}} - \frac{\frac{2\,a\,e-b\,d}{4\,a\,c-b^2}+\frac{x\,(b\,e-2\,c\,d)}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}$$

input
```
int((d + e*x)/(a + b*x + c*x^2)^2,x)
```

output
```
(2*atan(((4*a*c - b^2)*(((b^3 - 4*a*b*c)*(b*e - 2*c*d))/(4*a*c - b^2)^(5/2
) - (2*c*x*(b*e - 2*c*d))/(4*a*c - b^2)^(3/2)))/(b*e - 2*c*d))*(b*e - 2*c*
d))/(4*a*c - b^2)^(3/2) - ((2*a*e - b*d)/(4*a*c - b^2) + (x*(b*e - 2*c*d))
/(4*a*c - b^2))/(a + b*x + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 404, normalized size of antiderivative = 4.64

$$\int \frac{d+ex}{(a+bx+cx^2)^2}\,dx = \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2e+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abcd-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3ex}{}$$

input
```
int((e*x+d)/(c*x^2+b*x+a)^2,x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*e + 4
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*d - 2*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*e*x + 4*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*d*x - 2*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*e*x**2 + 4*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*d*x**2 - 4*a**2*b*c*e -
8*a**2*c**2*d + a*b**3*e + 6*a*b**2*c*d + 4*a*b*c**2*e*x**2 - 8*a*c**3*d*x
**2 - b**4*d - b**3*c*e*x**2 + 2*b**2*c**2*d*x**2)/(b*(16*a**3*c**2 - 8*a*
*2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x -
 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.53 $\int \frac{d+ex}{x(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{d+ex}{x(a+bx+cx^2)^2}\,dx = \frac{b^2d-2acd-abe+c(bd-2ae)x}{a(b^2-4ac)(a+bx+cx^2)} + \frac{(b^3d-6abcd+4a^2ce)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2-4ac)^{3/2}} + \frac{d\log(x)}{a^2} - \frac{d\log(a+bx+cx^2)}{2a^2}$$

output
```
(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+(4*a^2
*c*e-6*a*b*c*d+b^3*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^
2)^(3/2)+d*ln(x)/a^2-1/2*d*ln(c*x^2+b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)^2}\,dx = \frac{-\frac{2a\left(-b^2d+b(ae-cdx)+2ac(d+ex)\right)}{(b^2-4ac)(a+x(b+cx))}+\frac{2\left(b^3d-6abcd+4a^2ce\right)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2d\log(x)-d\log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[(d + e*x)/(x*(a + b*x + c*x^2)^2),x]
```

output
```
((-2*a*(-(b^2*d) + b*(a*e - c*d*x) + 2*a*c*(d + e*x)))/((b^2 - 4*a*c)*(a +
 x*(b + c*x))) + (2*(b^3*d - 6*a*b*c*d + 4*a^2*c*e)*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*d*Log[x] - d*Log[a + x*(b + c*x
)])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1235, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)^2}\,dx$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{cx(bd-2ae)-abe-2acd+b^2d}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{\left(b^2-4ac\right)d+c(bd-2ae)x}{x\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\left(b^2-4ac\right)d+c(bd-2ae)x}{x\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}+\frac{cx(bd-2ae)-abe-2acd+b^2d}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\int \left(\frac{-db^3+5acdb-2a^2ce-c(b^2-4ac)dx}{a(cx^2+bx+a)}-\frac{(4ac-b^2)d}{ax}\right)dx}{a(b^2-4ac)}+\frac{cx(bd-2ae)-abe-2acd+b^2d}{a(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(4a^2ce-6abcd+b^3d)}{a\sqrt{b^2-4ac}}-\frac{d(b^2-4ac)\log(a+bx+cx^2)}{2a}+\frac{d\log(x)(b^2-4ac)}{a}}{a(b^2-4ac)}+\frac{cx(bd-2ae)-abe-2acd+b^2d}{a(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[(d + e*x)/(x*(a + b*x + c*x^2)^2),x]
```

output
```
(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x)/(a*(b^2 - 4*a*c)*(a + b*x +
c*x^2)) + (((b^3*d - 6*a*b*c*d + 4*a^2*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*d*Log[x])/a - ((b^2 - 4*a
*c)*d*Log[a + b*x + c*x^2])/(2*a))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.48

method	result
default	$\frac{d\ln(x)}{a^2}+\frac{\frac{\frac{ac(2ae-bd)x}{4ac-b^2}+\frac{a\left(abe+2acd-b^2d\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{\left(-4a\,c^2d+cd\,b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(2a^2ce-5abcd+b^3d-\frac{\left(-4a\,c^2d+cd\,b^2\right)b}{2c}\right)\arctan\left(\frac{2cx}{\sqrt{4ac}}\right.}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^2}$
risch	$\frac{\frac{c(2ae-bd)x}{\left(4ac-b^2\right)a}+\frac{abe+2acd-b^2d}{a\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{d\ln(x)}{a^2}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(64a^5c^3-48a^4b^2c^2+12a^3b^4c-a^2b^6\right)\_Z^2+\left(64a^3c^3d-48a^2b^2c^2d+12a\,b\right.\right.}\right.$

input
```
int((e*x+d)/x/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*ln(x)/a^2+1/a^2*((a*c*(2*a*e-b*d)/(4*a*c-b^2)*x+a*(a*b*e+2*a*c*d-b^2*d)/
(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(-4*a*c^2*d+b^2*c*d)/c*ln(c*
x^2+b*x+a)+2*(2*a^2*c*e-5*a*b*c*d+b^3*d-1/2*(-4*a*c^2*d+b^2*c*d)*b/c)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(129) = 258$.

Time = 0.32 (sec) , antiderivative size = 955, normalized size of antiderivative = 7.07

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*((4*a^3*c*e + (4*a^2*c^2*e + (b^3*c - 6*a*b*c^2)*d)*x^2 + (a*b^3 - 6
*a^2*b*c)*d + (4*a^2*b*c*e + (b^4 - 6*a*b^2*c)*d)*x)*sqrt(b^2 - 4*a*c)*log
((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) - 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d + 2*(a^2*b^3 - 4*a^3
*b*c)*e - 2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*x +
((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*d*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c*x^2 + b*x + a) - 2*((
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)
*d*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(x))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^2*b^5 -
 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*(4*a^3*c*e + (4*a^2*c^2*e + (b^3*c
 - 6*a*b*c^2)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d + (4*a^2*b*c*e + (b^4 - 6*a*b
^2*c)*d)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
 - 4*a*c)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*
c)*e + 2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*x - ((b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*
d*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c*x^2 + b*x + a) + 2*((b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*
x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(x))/(a^3*b^4 - 8*a^4*b^2*c +
 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^2*b^5 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/x/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)^2}\,dx = -\frac{\left(b^3d-6\,abcd+4\,a^2ce\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}} - \frac{d\log\left(cx^2+bx+a\right)}{2\,a^2} + \frac{d\log\left(|x|\right)}{a^2} + \frac{ab^2d-2\,a^2cd-a^2be+\left(abcd-2\,a^2ce\right)x}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)a^2}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(b^3*d - 6*a*b*c*d + 4*a^2*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((
a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/2*d*log(c*x^2 + b*x + a)/a^2 +
d*log(abs(x))/a^2 + (a*b^2*d - 2*a^2*c*d - a^2*b*e + (a*b*c*d - 2*a^2*c*e)
*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [B] (verification not implemented)

Time = 12.20 (sec) , antiderivative size = 920, normalized size of antiderivative = 6.81

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)^2}\,dx$$
$$= \frac{\frac{-d\,b^2+a\,e\,b+2\,a\,c\,d}{a\,(4\,a\,c-b^2)}+\frac{c\,x\,(2\,a\,e-b\,d)}{a\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \ln\Bigg(96\,a^4\,c^3\,d-2\,a\,b^6\,d-2\,b^7\,d\,x-84\,a^3\,b^2\,c^2\,d+2\,a\,b^3\,d\,\sqrt{-(4\,a\,c-b^2)^3} + 23\,a^2\,b^4\,c\,d-2\,a^3\,b^3\,c\,e+8\,a^4\,b\,c^2\,e+2\,a^3\,c\,e\,\sqrt{-(4\,a\,c-b^2)^3} + 2\,b^4\,d\,x\,\sqrt{-(4\,a\,c-b^2)^3}-16\,a^4\,c^3\,e\,x-9\,a^2\,b\,c\,d\,\sqrt{-(4\,a\,c-b^2)^3}+120\,a^3\,b\,c^3\,d\,x - 2\,a^2\,b^4\,c\,e\,x-94\,a^2\,b^3\,c^2\,d\,x+12\,a^2\,c^2\,d\,x\,\sqrt{-(4\,a\,c-b^2)^3}+12\,a^3\,b^2\,c^2\,e\,x + 24\,a\,b^5\,c\,d\,x-12\,a\,b^2\,c\,d\,x\,\sqrt{-(4\,a\,c-b^2)^3}+2\,a^2\,b\,c\,e\,x\,\sqrt{-(4\,a\,c-b^2)^3}\Bigg)\left(\frac{d}{2\,a^2} - \frac{\frac{b^3\,d\,\sqrt{-(4\,a\,c-b^2)^3}}{2}+2\,a^2\,c\,e\,\sqrt{-(4\,a\,c-b^2)^3}-3\,a\,b\,c\,d\,\sqrt{-(4\,a\,c-b^2)^3}}{-64\,a^5\,c^3+48\,a^4\,b^2\,c^2-12\,a^3\,b^4\,c+a^2\,b^6}\right) - \ln\Bigg(2\,a\,b^6\,d-96\,a^4\,c^3\,d+2\,b^7\,d\,x+84\,a^3\,b^2\,c^2\,d+2\,a\,b^3\,d\,\sqrt{-(4\,a\,c-b^2)^3} - 23\,a^2\,b^4\,c\,d+2\,a^3\,b^3\,c\,e-8\,a^4\,b\,c^2\,e+2\,a^3\,c\,e\,\sqrt{-(4\,a\,c-b^2)^3} + 2\,b^4\,d\,x\,\sqrt{-(4\,a\,c-b^2)^3}+16\,a^4\,c^3\,e\,x-9\,a^2\,b\,c\,d\,\sqrt{-(4\,a\,c-b^2)^3}-120\,a^3\,b\,c^3\,d\,x + 2\,a^2\,b^4\,c\,e\,x+94\,a^2\,b^3\,c^2\,d\,x+12\,a^2\,c^2\,d\,x\,\sqrt{-(4\,a\,c-b^2)^3}-12\,a^3\,b^2\,c^2\,e\,x - 24\,a\,b^5\,c\,d\,x-12\,a\,b^2\,c\,d\,x\,\sqrt{-(4\,a\,c-b^2)^3}+2\,a^2\,b\,c\,e\,x\,\sqrt{-(4\,a\,c-b^2)^3}\Bigg)\left(\frac{d}{2\,a^2} + \frac{\frac{b^3\,d\,\sqrt{-(4\,a\,c-b^2)^3}}{2}+2\,a^2\,c\,e\,\sqrt{-(4\,a\,c-b^2)^3}-3\,a\,b\,c\,d\,\sqrt{-(4\,a\,c-b^2)^3}}{-64\,a^5\,c^3+48\,a^4\,b^2\,c^2-12\,a^3\,b^4\,c+a^2\,b^6}\right) + \frac{d\,\ln(x)}{a^2}$$

input
```
int((d + e*x)/(x*(a + b*x + c*x^2)^2),x)
```

output
```
((a*b*e - b^2*d + 2*a*c*d)/(a*(4*a*c - b^2)) + (c*x*(2*a*e - b*d))/(a*(4*a
*c - b^2)))/(a + b*x + c*x^2) - log(96*a^4*c^3*d - 2*a*b^6*d - 2*b^7*d*x -
 84*a^3*b^2*c^2*d + 2*a*b^3*d*(-(4*a*c - b^2)^3)^(1/2) + 23*a^2*b^4*c*d -
2*a^3*b^3*c*e + 8*a^4*b*c^2*e + 2*a^3*c*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^4
*d*x*(-(4*a*c - b^2)^3)^(1/2) - 16*a^4*c^3*e*x - 9*a^2*b*c*d*(-(4*a*c - b^
2)^3)^(1/2) + 120*a^3*b*c^3*d*x - 2*a^2*b^4*c*e*x - 94*a^2*b^3*c^2*d*x + 1
2*a^2*c^2*d*x*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b^2*c^2*e*x + 24*a*b^5*c*d
*x - 12*a*b^2*c*d*x*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b*c*e*x*(-(4*a*c - b^
2)^3)^(1/2))*(d/(2*a^2) - ((b^3*d*(-(4*a*c - b^2)^3)^(1/2))/2 + 2*a^2*c*e*
(-(4*a*c - b^2)^3)^(1/2) - 3*a*b*c*d*(-(4*a*c - b^2)^3)^(1/2))/(a^2*b^6 -
64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - log(2*a*b^6*d - 96*a^4*c^3*
d + 2*b^7*d*x + 84*a^3*b^2*c^2*d + 2*a*b^3*d*(-(4*a*c - b^2)^3)^(1/2) - 23
*a^2*b^4*c*d + 2*a^3*b^3*c*e - 8*a^4*b*c^2*e + 2*a^3*c*e*(-(4*a*c - b^2)^3
)^(1/2) + 2*b^4*d*x*(-(4*a*c - b^2)^3)^(1/2) + 16*a^4*c^3*e*x - 9*a^2*b*c*
d*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*d*x + 2*a^2*b^4*c*e*x + 94*a^2*
b^3*c^2*d*x + 12*a^2*c^2*d*x*(-(4*a*c - b^2)^3)^(1/2) - 12*a^3*b^2*c^2*e*x
 - 24*a*b^5*c*d*x - 12*a*b^2*c*d*x*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b*c*e*
x*(-(4*a*c - b^2)^3)^(1/2))*(d/(2*a^2) + ((b^3*d*(-(4*a*c - b^2)^3)^(1/2))
/2 + 2*a^2*c*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b*c*d*(-(4*a*c - b^2)^3)^(1/
2))/(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (d*log(x)...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 873, normalized size of antiderivative = 6.47

$$\int \frac{d+ex}{x\left(a+bx+cx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)/x/(c*x^2+b*x+a)^2,x)
```

output
```
(8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c*e - 12
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c*d + 8
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c*e*x +
 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2*e*x
**2 + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*d -
 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c*d*x -
 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*d*
x**2 + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*d*x
+ 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*c*d*x**2
- 16*log(a + b*x + c*x**2)*a**3*b*c**2*d + 8*log(a + b*x + c*x**2)*a**2*b*
*3*c*d - 16*log(a + b*x + c*x**2)*a**2*b**2*c**2*d*x - 16*log(a + b*x + c*
x**2)*a**2*b*c**3*d*x**2 - log(a + b*x + c*x**2)*a*b**5*d + 8*log(a + b*x
+ c*x**2)*a*b**4*c*d*x + 8*log(a + b*x + c*x**2)*a*b**3*c**2*d*x**2 - log(
a + b*x + c*x**2)*b**6*d*x - log(a + b*x + c*x**2)*b**5*c*d*x**2 + 32*log(
x)*a**3*b*c**2*d - 16*log(x)*a**2*b**3*c*d + 32*log(x)*a**2*b**2*c**2*d*x
+ 32*log(x)*a**2*b*c**3*d*x**2 + 2*log(x)*a*b**5*d - 16*log(x)*a*b**4*c*d*
x - 16*log(x)*a*b**3*c**2*d*x**2 + 2*log(x)*b**6*d*x + 2*log(x)*b**5*c*d*x
**2 - 16*a**4*c**2*e + 12*a**3*b**2*c*e + 24*a**3*b*c**2*d - 16*a**3*c**3*
e*x**2 - 2*a**2*b**4*e - 14*a**2*b**3*c*d + 4*a**2*b**2*c**2*e*x**2 + 8*a*
*2*b*c**3*d*x**2 + 2*a*b**5*d - 2*a*b**3*c**2*d*x**2)/(2*a**2*b*(16*a**...
```

3.54 $\int \frac{d+ex}{x^2(a+bx+cx^2)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\begin{aligned}\int \frac{d+ex}{x^2(a+bx+cx^2)^2} dx &= -\frac{d}{a^2x} - \frac{b^3d-3abcd-ab^2e+2a^2ce+c(b^2d-2acd-abe)x}{a^2(b^2-4ac)(a+bx+cx^2)} \\ &\quad - \frac{(2b^4d-12ab^2cd+12a^2c^2d-ab^3e+6a^2bce)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} \\ &\quad - \frac{(2bd-ae)\log(x)}{a^3} + \frac{(2bd-ae)\log(a+bx+cx^2)}{2a^3}\end{aligned}$$

output
```
-d/a^2/x-(b^3*d-3*a*b*c*d-a*b^2*e+2*a^2*c*e+c*(-a*b*e-2*a*c*d+b^2*d)*x)/a^
2/(-4*a*c+b^2)/(c*x^2+b*x+a)-(6*a^2*b*c*e+12*a^2*c^2*d-a*b^3*e-12*a*b^2*c*
d+2*b^4*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(-
a*e+2*b*d)*ln(x)/a^3+1/2*(-a*e+2*b*d)*ln(c*x^2+b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96

$$\int \frac{d+ex}{x^2 (a+bx+cx^2)^2} dx$$

$$= \frac{-\frac{2ad}{x} - \frac{2a(b^3d+2ac(ae-cdx)+b^2(-ae+cdx)-abc(3d+ex))}{(b^2-4ac)(a+x(b+cx))} - \frac{2(2b^4d-12ab^2cd+12a^2c^2d-ab^3e+6a^2bce)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2(-2}{2a^3}$$

input
```
Integrate[(d + e*x)/(x^2*(a + b*x + c*x^2)^2),x]
```

output
```
((-2*a*d)/x - (2*a*(b^3*d + 2*a*c*(a*e - c*d*x) + b^2*(-(a*e) + c*d*x) - a
*b*c*(3*d + e*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(2*b^4*d - 12*a*
b^2*c*d + 12*a^2*c^2*d - a*b^3*e + 6*a^2*b*c*e)*ArcTan[(b + 2*c*x)/Sqrt[-b
^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*(-2*b*d + a*e)*Log[x] + (2*b*d - a*
e)*Log[a + x*(b + c*x)])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1235, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^2 (a+bx+cx^2)^2} dx$$

$$\Big\downarrow \ 1235$$

$$\frac{cx(bd-2ae)-abe-2acd+b^2d}{ax(b^2-4ac)(a+bx+cx^2)} - \frac{\int -\frac{2db^2-aeb-6acd+2c(bd-2ae)x}{x^2(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2db^2-aeb-6acd+2c(bd-2ae)x}{x^2(cx^2+bx+a)}dx}{a(b^2-4ac)} + \frac{cx(bd-2ae)-abe-2acd+b^2d}{ax(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\int \left(-\frac{(4ac-b^2)(ae-2bd)}{a^2x}+\frac{2db^4-aeb^3-10acdb^2+5a^2ceb+6a^2c^2d+c(b^2-4ac)(2bd-ae)x}{a^2(cx^2+bx+a)}+\frac{2db^2-aeb-6acd}{ax^2}\right)dx}{a\left(b^2-4ac\right)}+\frac{cx(bd-2ae)-abe-2acd+b^2d}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(6a^2bce+12a^2c^2d-ab^3e-12ab^2cd+2b^4d\right)}{a^2\sqrt{b^2-4ac}}+\frac{(b^2-4ac)(2bd-ae)\log(a+bx+cx^2)}{2a^2}-\frac{\log(x)(b^2-4ac)(2bd-ae)}{a^2}-\frac{-ab}{\cdots}}{a\left(b^2-4ac\right)}+\frac{cx(bd-2ae)-abe-2acd+b^2d}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[(d + e*x)/(x^2*(a + b*x + c*x^2)^2),x]
```

output
```
(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x)/(a*(b^2 - 4*a*c)*x*(a + b*x
+ c*x^2)) + (-((2*b^2*d - 6*a*c*d - a*b*e)/(a*x)) - ((2*b^4*d - 12*a*b^2*c
*d + 12*a^2*c^2*d - a*b^3*e + 6*a^2*b*c*e)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)*(2*b*d - a*e)*Log[x])/a^
2 + ((b^2 - 4*a*c)*(2*b*d - a*e)*Log[a + b*x + c*x^2])/(2*a^2))/(a*(b^2 -
4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.46

method	result
default	$-\frac{d}{a^{2} x}+\frac{(a e-2 b d) \ln (x)}{a^{3}}-\frac{\frac{\frac{a c\left(a b e+2 a c d-b^{2} d\right) x}{4 a c-b^{2}}-\frac{a\left(2 a^{2} c e-e a\,b^{2}-3 a b c d+b^{3} d\right)}{4 a c-b^{2}}}{c\,x^{2}+b x+a}+\frac{\frac{\left(4 a^{2} c^{2} e-a\,b^{2} c e-8 a b\,c^{2} d+2 b^{3} c d\right) \ln \left(c\,x^{2}+b x+a\right)}{2 c}+\cdots}{\cdots}}{a^{3}}$
risch	Expression too large to display

input
```
int((e*x+d)/x^2/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d/a^2/x+(a*e-2*b*d)/a^3*ln(x)-1/a^3*((a*c*(a*b*e+2*a*c*d-b^2*d)/(4*a*c-b^
2)*x-a*(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4
*a*c-b^2)*(1/2*(4*a^2*c^2*e-a*b^2*c*e-8*a*b*c^2*d+2*b^3*c*d)/c*ln(c*x^2+b*
x+a)+2*(5*a^2*b*c*e+6*a^2*c^2*d-a*b^3*e-10*c*d*a*b^2+2*b^4*d-1/2*(4*a^2*c^
2*e-a*b^2*c*e-8*a*b*c^2*d+2*b^3*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(195) = 390$.

Time = 1.06 (sec) , antiderivative size = 1615, normalized size of antiderivative = 8.03

$$\int \frac{d+ex}{x^2 (a+bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*
b*c^2)*e)*x^2 + ((2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d - (a*b^3*c - 6*a^2
*b*c^2)*e)*x^3 + (2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d - (a*b^4 - 6*a^2*b^2
*c)*e)*x^2 + (2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d - (a^2*b^3 - 6*a^3*b*c
)*e)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a^2*b^4 - 8*a^3*b^2*c + 16
*a^4*c^2)*d + 2*((2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d - (a^2*b^4 - 6*
a^3*b^2*c + 8*a^4*c^2)*e)*x - ((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d -
 (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^3 + (2*(b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^2 + (2*(a*b^5 -
 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x
)*log(c*x^2 + b*x + a) + 2*((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a
*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^3 + (2*(b^6 - 8*a*b^4*c + 16*a^2
*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^2 + (2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x)*l
og(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^
3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x), -1/2*(2
*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*b*c^2)*e
)*x^2 + 2*((2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d - (a*b^3*c - 6*a^2*b*c^2
)*e)*x^3 + (2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d - (a*b^4 - 6*a^2*b^2*c)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^2(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/x**2/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^2(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{d+ex}{x^2(a+bx+cx^2)^2}\,dx &= \frac{(2\,b^4d-12\,ab^2cd+12\,a^2c^2d-ab^3e+6\,a^2bce)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}}\\ &\quad-\frac{2\,b^2cdx^2-6\,ac^2dx^2-abcex^2+2\,b^3dx-7\,abcdx-ab^2ex+2\,a^2cex+ab^2d-4\,a^2cd}{(a^2b^2-4\,a^3c)(cx^3+bx^2+ax)}\\ &\quad+\frac{(2\,bd-ae)\log(cx^2+bx+a)}{2\,a^3}-\frac{(2\,bd-ae)\log(|x|)}{a^3}\end{aligned}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
(2*b^4*d - 12*a*b^2*c*d + 12*a^2*c^2*d - a*b^3*e + 6*a^2*b*c*e)*arctan((2*
c*x + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2
*b^2*c*d*x^2 - 6*a*c^2*d*x^2 - a*b*c*e*x^2 + 2*b^3*d*x - 7*a*b*c*d*x - a*b
^2*e*x + 2*a^2*c*e*x + a*b^2*d - 4*a^2*c*d)/((a^2*b^2 - 4*a^3*c)*(c*x^3 +
b*x^2 + a*x)) + 1/2*(2*b*d - a*e)*log(c*x^2 + b*x + a)/a^3 - (2*b*d - a*e)
*log(abs(x))/a^3
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 1366, normalized size of antiderivative = 6.80

$$\int \frac{d+ex}{x^2 (a+bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x)/(x^2*(a + b*x + c*x^2)^2),x)
```

output

```
log(96*a^5*c^3*e - 2*a^2*b^6*e + 4*a*b^7*d + 4*b^8*d*x + 174*a^3*b^3*c^2*d
 - 2*a^2*b^3*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*c^2*d*(-(4*a*c - b^2)^3)^(
1/2) - 84*a^4*b^2*c^2*e - 2*a*b^7*e*x + 4*a*b^4*d*(-(4*a*c - b^2)^3)^(1/2)
 - 46*a^2*b^5*c*d - 216*a^4*b*c^3*d + 23*a^3*b^4*c*e + 48*a^4*c^4*d*x + 4*
b^5*d*x*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*e*(-(4*a*c - b^2)^3)^(1/2) -
2*a*b^4*e*x*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^5*c*e*x + 120*a^4*b*c^3*e*
x - 18*a^2*b^2*c*d*(-(4*a*c - b^2)^3)^(1/2) + 194*a^2*b^4*c^2*d*x - 276*a^
3*b^2*c^3*d*x - 94*a^3*b^3*c^2*e*x - 12*a^3*c^2*e*x*(-(4*a*c - b^2)^3)^(1/
2) - 48*a*b^6*c*d*x - 24*a*b^3*c*d*x*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b*c
^2*d*x*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*e*x*(-(4*a*c - b^2)^3)^(1/2
))*((b^4*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
 - (a*b^3*e*(-(4*a*c - b^2)^3)^(1/2))/2 - 6*a*b^2*c*d*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b*c*e*(-(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^
4*b^4*c + 48*a^5*b^2*c^2) - e/(2*a^2) + (b*d)/a^3) - log(2*a^2*b^6*e - 96*
a^5*c^3*e - 4*a*b^7*d - 4*b^8*d*x - 174*a^3*b^3*c^2*d - 2*a^2*b^3*e*(-(4*a
*c - b^2)^3)^(1/2) + 6*a^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 84*a^4*b^2*c^2
*e + 2*a*b^7*e*x + 4*a*b^4*d*(-(4*a*c - b^2)^3)^(1/2) + 46*a^2*b^5*c*d + 2
16*a^4*b*c^3*d - 23*a^3*b^4*c*e - 48*a^4*c^4*d*x + 4*b^5*d*x*(-(4*a*c - b^
2)^3)^(1/2) + 9*a^3*b*c*e*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^4*e*x*(-(4*a*c
- b^2)^3)^(1/2) - 24*a^2*b^5*c*e*x - 120*a^4*b*c^3*e*x - 18*a^2*b^2*c*d...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1575, normalized size of antiderivative = 7.84

$$\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)/x^2/(c*x^2+b*x+a)^2,x)
```

output

```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c
*e*x - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c
**2*d*x + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**4*e*x + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*
b**3*c*d*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b**3*c*e*x**2 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a**2*b**2*c**2*d*x**2 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a**2*b**2*c**2*e*x**3 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x
)/sqrt(4*a*c - b**2))*a**2*b*c**3*d*x**3 - 4*sqrt(4*a*c - b**2)*atan((b +
2*c*x)/sqrt(4*a*c - b**2))*a*b**5*d*x + 2*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*a*b**5*e*x**2 + 24*sqrt(4*a*c - b**2)*atan((b + 2*
c*x)/sqrt(4*a*c - b**2))*a*b**4*c*d*x**2 + 2*sqrt(4*a*c - b**2)*atan((b +
2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*e*x**3 + 24*sqrt(4*a*c - b**2)*atan((b
 + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c**2*d*x**3 - 4*sqrt(4*a*c - b**2)*at
an((b + 2*c*x)/sqrt(4*a*c - b**2))*b**6*d*x**2 - 4*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*c*d*x**3 - 16*log(a + b*x + c*x**2)*
a**4*b*c**2*e*x + 8*log(a + b*x + c*x**2)*a**3*b**3*c*e*x + 32*log(a + b*x
 + c*x**2)*a**3*b**2*c**2*d*x - 16*log(a + b*x + c*x**2)*a**3*b**2*c**2*e*
x**2 - 16*log(a + b*x + c*x**2)*a**3*b*c**3*e*x**3 - log(a + b*x + c*x**2)
*a**2*b**5*e*x - 16*log(a + b*x + c*x**2)*a**2*b**4*c*d*x + 8*log(a + b...
```

3.55 $\int \frac{d+ex}{x^3(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 266

$$
\begin{aligned}
&\int \frac{d+ex}{x^3\left(a+bx+cx^2\right)^2}\,dx \\
&= -\frac{d}{2a^2x^2}+\frac{2bd-ae}{a^3x} \\
&\quad+\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce+c(b^3d-3abcd-ab^2e+2a^2ce)\,x}{a^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)} \\
&\quad+\frac{\left(3b^5d-20ab^3cd+30a^2bc^2d-2ab^4e+12a^2b^2ce-12a^3c^2e\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}} \\
&\quad+\frac{\left(3b^2d-2acd-2abe\right)\log(x)}{a^4}-\frac{\left(3b^2d-2acd-2abe\right)\log\left(a+bx+cx^2\right)}{2a^4}
\end{aligned}
$$

output
```
-1/2*d/a^2/x^2+(-a*e+2*b*d)/a^3/x+(b^4*d-4*a*b^2*c*d+2*a^2*c^2*d-a*b^3*e+3
*a^2*b*c*e+c*(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)*x)/a^3/(-4*a*c+b^2)/(c*x^
2+b*x+a)+(-12*a^3*c^2*e+12*a^2*b^2*c*e+30*a^2*b*c^2*d-2*a*b^4*e-20*a*b^3*c
*d+3*b^5*d)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(
-2*a*b*e-2*a*c*d+3*b^2*d)*ln(x)/a^4-1/2*(-2*a*b*e-2*a*c*d+3*b^2*d)*ln(c*x^
2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.95

$$\int \frac{d+ex}{x^3 (a+bx+cx^2)^2} \, dx$$

$$= \frac{-\frac{a^2 d}{x^2} - \frac{2a(-2bd+ae)}{x} + \frac{2a(b^4d+3abc(ae-cdx)+b^3(-ae+cdx)+2a^2c^2(d+ex)-ab^2c(4d+ex))}{(b^2-4ac)(a+x(b+cx))} + \frac{2(3b^5d-20ab^3cd+30a^2bc^2d-2ab^4e+12}{(-b^2+4}}{2}$$

input
```
Integrate[(d + e*x)/(x^3*(a + b*x + c*x^2)^2),x]
```

output
```
(-((a^2*d)/x^2) - (2*a*(-2*b*d + a*e))/x + (2*a*(b^4*d + 3*a*b*c*(a*e - c*
d*x) + b^3*(-(a*e) + c*d*x) + 2*a^2*c^2*(d + e*x) - a*b^2*c*(4*d + e*x)))/
((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(3*b^5*d - 20*a*b^3*c*d + 30*a^2*b*
c^2*d - 2*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e)*ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*(3*b^2*d - 2*a*c*d - 2*a*b*e)*Lo
g[x] + (-3*b^2*d + 2*a*c*d + 2*a*b*e)*Log[a + x*(b + c*x)])/(2*a^4)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1235, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^3 (a+bx+cx^2)^2} \, dx$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{cx(bd-2ae)-abe-2acd+b^2d}{ax^2(b^2-4ac)(a+bx+cx^2)} - \frac{\int -\frac{3db^2-2aeb-8acd+3c(bd-2ae)x}{x^3(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3db^2-2aeb-8acd+3c(bd-2ae)x}{x^3(cx^2+bx+a)}dx}{a\left(b^2-4ac\right)}+\frac{cx(bd-2ae)-abe-2acd+b^2d}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\int \left(\frac{3db^2-2aeb-8acd}{ax^3}+\frac{(4ac-b^2)\left(-3db^2+2aeb+2acd\right)}{a^3x}+\frac{-3db^5+2aeb^4+17acdb^3-10a^2ceb^2-19a^2c^2db+6a^3c^2e-c\left(b^2-4ac\right)\left(3db^2-2aeb-2a\right.}{a^3(cx^2+bx+a)}\right.}{a\left(b^2-4ac\right)}$$

$$\frac{cx(bd-2ae)-abe-2acd+b^2d}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(b^2-4ac)\left(-2abe-2acd+3b^2d\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)(b^2-4ac)\left(-2abe-2acd+3b^2d\right)}{a^3}+\frac{6a^2ce-2ab^2e-11abcd+3b^3d}{a^2x}+\frac{\text{arctanh}\left(\frac{b+}{\sqrt{b^2}}\right.}{}}{a\left(b^2-4ac\right)}$$

$$\frac{cx(bd-2ae)-abe-2acd+b^2d}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[(d + e*x)/(x^3*(a + b*x + c*x^2)^2),x]
```

output
```
(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*
x + c*x^2)) + (-1/2*(3*b^2*d - 8*a*c*d - 2*a*b*e)/(a*x^2) + (3*b^3*d - 11*
a*b*c*d - 2*a*b^2*e + 6*a^2*c*e)/(a^2*x) + ((3*b^5*d - 20*a*b^3*c*d + 30*a
^2*b*c^2*d - 2*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e)*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*(3*b^2*d -
2*a*c*d - 2*a*b*e)*Log[x])/a^3 - ((b^2 - 4*a*c)*(3*b^2*d - 2*a*c*d - 2*a*b
*e)*Log[a + b*x + c*x^2])/(2*a^3))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.41

method	result
default	$-\frac{d}{2a^2x^2}-\frac{ae-2bd}{a^3x}+\frac{\left(-2abe-2acd+3b^2d\right)\ln\left(x\right)}{a^4}-\frac{\frac{ac\left(2a^2ce-ea\,b^2-3abcd+b^3d\right)x}{4ac-b^2}+\frac{a\left(3a^2bce+2a^2c^2d-a\,b^3e-4cda\,b^2+b^4d\right)}{4ac-b^2}}{c\,x^2+bx+a}+$
risch	$\frac{-\frac{c\left(6a^2ce-2ea\,b^2-11abcd+3b^3d\right)x^3}{\left(4ac-b^2\right)a^3}-\frac{\left(14a^2bce+8a^2c^2d-4a\,b^3e-25cda\,b^2+6b^4d\right)x^2}{2a^3\left(4ac-b^2\right)}-\frac{(2ae-3bd)x}{2a^2}-\frac{d}{2a}}{x^2\left(c\,x^2+bx+a\right)}-\frac{2\ln\left(x\right)be}{a^3}-\frac{2cd\ln\left(x\right)}{a^3}+$

input
```
int((e*x+d)/x^3/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a^2/x^2-(a*e-2*b*d)/a^3/x+(-2*a*b*e-2*a*c*d+3*b^2*d)*ln(x)/a^4-1/a^
4*((a*c*(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x+a*(3*a^2*b*c*e+2
*a^2*c^2*d-a*b^3*e-4*a*b^2*c*d+b^4*d)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-
b^2)*(1/2*(-8*a^2*b*c^2*e-8*a^2*c^3*d+2*a*b^3*c*e+14*a*b^2*c^2*d-3*b^4*c*d
)/c*ln(c*x^2+b*x+a)+2*(6*e*c^2*a^3-10*a^2*b^2*c*e-19*a^2*b*c^2*d+2*a*b^4*e
+17*a*b^3*c*d-3*b^5*d-1/2*(-8*a^2*b*c^2*e-8*a^2*c^3*d+2*a*b^3*c*e+14*a*b^2
*c^2*d-3*b^4*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(258) = 516$.

Time = 2.15 (sec) , antiderivative size = 2003, normalized size of antiderivative = 7.53

$$\int \frac{d+ex}{x^3\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x^3/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*
a^3*b^2*c^2 + 12*a^4*c^3)*e)*x^3 + ((6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*
c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3*c + 28*a^4*b*c^2)*e)*x^2 +
 (((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2
+ 6*a^3*c^3)*e)*x^4 + ((3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5
- 6*a^2*b^3*c + 6*a^3*b*c^2)*e)*x^3 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*
c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e)*x^2)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*d + (3*(a^2*b^5 - 8
*a^3*b^3*c + 16*a^4*b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e)*x
 - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c
 - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e)*x^4 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^
3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e)*x^3
+ ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 -
 8*a^3*b^3*c + 16*a^4*b*c^2)*e)*x^2)*log(c*x^2 + b*x + a) + 2*(((3*b^6*c -
 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^
2 + 16*a^3*b*c^3)*e)*x^4 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*
b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e)*x^3 + ((3*a*b^6 - 2
6*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*e)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^3\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/x**3/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^3\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^3/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.26

$$\int \frac{d+ex}{x^3\left(a+bx+cx^2\right)^2}\,dx$$
$$= -\frac{\left(3\,b^5d-20\,ab^3cd+30\,a^2bc^2d-2\,ab^4e+12\,a^2b^2ce-12\,a^3c^2e\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4b^2-4\,a^5c\right)\sqrt{-b^2+4\,ac}}$$
$$-\frac{\left(3\,b^2d-2\,acd-2\,abe\right)\log\left(cx^2+bx+a\right)}{2\,a^4}+\frac{\left(3\,b^2d-2\,acd-2\,abe\right)\log\left(|x|\right)}{a^4}$$
$$-\frac{a^3b^2d-4\,a^4cd-2\left(3\,ab^3cd-11\,a^2bc^2d-2\,a^2b^2ce+6\,a^3c^2e\right)x^3-\left(6\,ab^4d-25\,a^2b^2cd+8\,a^3c^2d-4\,a^2\right.}{2\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)a^4x^2}$$

input
```
integrate((e*x+d)/x^3/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(3*b^5*d - 20*a*b^3*c*d + 30*a^2*b*c^2*d - 2*a*b^4*e + 12*a^2*b^2*c*e - 1
2*a^3*c^2*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^4*b^2 - 4*a^5*c)*s
qrt(-b^2 + 4*a*c)) - 1/2*(3*b^2*d - 2*a*c*d - 2*a*b*e)*log(c*x^2 + b*x + a
)/a^4 + (3*b^2*d - 2*a*c*d - 2*a*b*e)*log(abs(x))/a^4 - 1/2*(a^3*b^2*d - 4
*a^4*c*d - 2*(3*a*b^3*c*d - 11*a^2*b*c^2*d - 2*a^2*b^2*c*e + 6*a^3*c^2*e)*
x^3 - (6*a*b^4*d - 25*a^2*b^2*c*d + 8*a^3*c^2*d - 4*a^2*b^3*e + 14*a^3*b*c
*e)*x^2 - (3*a^2*b^3*d - 12*a^3*b*c*d - 2*a^3*b^2*e + 8*a^4*c*e)*x)/((c*x^
2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 12.86 (sec) , antiderivative size = 1661, normalized size of antiderivative = 6.24

$$\int \frac{d+ex}{x^3 (a+bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x)/(x^3*(a + b*x + c*x^2)^2),x)
```

output
```
log(192*a^5*c^4*d - 4*a^2*b^7*e + 6*a*b^8*d + 6*b^9*d*x + 307*a^3*b^4*c^2*
d - 492*a^4*b^2*c^3*d + 4*a^2*b^4*e*(-(4*a*c - b^2)^3)^(1/2) - 174*a^4*b^3
*c^2*e + 6*a^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^8*e*x - 6*a*b^5*d*(-
(4*a*c - b^2)^3)^(1/2) - 73*a^2*b^6*c*d + 46*a^3*b^5*c*e + 216*a^5*b*c^3*e
 - 6*b^6*d*x*(-(4*a*c - b^2)^3)^(1/2) - 48*a^5*c^4*e*x + 312*a^4*b*c^4*d*x
 + 4*a*b^5*e*x*(-(4*a*c - b^2)^3)^(1/2) + 48*a^2*b^6*c*e*x + 31*a^2*b^3*c*
d*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 18*
a^3*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*d*x - 602*a^3*b^3*c
^3*d*x + 24*a^3*c^3*d*x*(-(4*a*c - b^2)^3)^(1/2) - 194*a^3*b^4*c^2*e*x + 2
76*a^4*b^2*c^3*e*x - 76*a*b^7*c*d*x - 69*a^2*b^2*c^2*d*x*(-(4*a*c - b^2)^3
)^(1/2) + 40*a*b^4*c*d*x*(-(4*a*c - b^2)^3)^(1/2) - 24*a^2*b^3*c*e*x*(-(4*
a*c - b^2)^3)^(1/2) + 30*a^3*b*c^2*e*x*(-(4*a*c - b^2)^3)^(1/2))*(((3*b^5*
d*(-(4*a*c - b^2)^3)^(1/2))/2 - 6*a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - a*b
^4*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 15
*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(
1/2))/(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2) - (3*b^2*d)/(
2*a^4) + (b*e)/a^3 + (c*d)/a^3) - log(4*a^2*b^7*e - 192*a^5*c^4*d - 6*a*b^
8*d - 6*b^9*d*x - 307*a^3*b^4*c^2*d + 492*a^4*b^2*c^3*d + 4*a^2*b^4*e*(-(4
*a*c - b^2)^3)^(1/2) + 174*a^4*b^3*c^2*e + 6*a^4*c^2*e*(-(4*a*c - b^2)^3)^
(1/2) + 4*a*b^8*e*x - 6*a*b^5*d*(-(4*a*c - b^2)^3)^(1/2) + 73*a^2*b^6*c...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1989, normalized size of antiderivative = 7.48

$$\int \frac{d+ex}{x^3(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/x^3/(c*x^2+b*x+a)^2,x)
```

output
```
( - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**2
*e*x**2 + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*
b**3*c*e*x**2 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a**3*b**2*c**2*d*x**2 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
 - b**2))*a**3*b**2*c**2*e*x**3 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/s
qrt(4*a*c - b**2))*a**3*b*c**3*e*x**4 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*a**2*b**5*e*x**2 - 40*sqrt(4*a*c - b**2)*atan((b +
 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4*c*d*x**2 + 24*sqrt(4*a*c - b**2)*ata
n((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**4*c*e*x**3 + 60*sqrt(4*a*c - b**
2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**2*d*x**3 + 24*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**2*e*x**4 + 6
0*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**3*d
*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*d
*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*e
*x**3 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5*
c*d*x**3 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**
5*c*e*x**4 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
b**4*c**2*d*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*b**7*d*x**3 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*b**6*c*d*x**4 + 32*log(a + b*x + c*x**2)*a**4*b**2*c**2*e*x**2 + 32*lo...
```

3.56 $\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{9}aAx^{9/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{13}(bB+Ac)x^{13/2}+\frac{2}{15}Bcx^{15/2}$$

output
```
2/9*a*A*x^(9/2)+2/11*(A*b+B*a)*x^(11/2)+2/13*(A*c+B*b)*x^(13/2)+2/15*B*c*x
^(15/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2x^{9/2}(715aA+585Abx+585aBx+495bBx^2+495Acx^2+429Bcx^3)}{6435}$$

input
```
Integrate[x^(7/2)*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(2*x^(9/2)*(715*a*A + 585*A*b*x + 585*a*B*x + 495*b*B*x^2 + 495*A*c*x^2 +
429*B*c*x^3))/6435
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(x^{9/2}(aB+Ab)+aAx^{7/2}+x^{11/2}(Ac+bB)+Bcx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{13}x^{13/2}(Ac+bB)+\frac{2}{15}Bcx^{15/2}$$

input
```
Int[x^(7/2)*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(11/2))/11 + (2*(b*B + A*c)*x^(13/2))
/13 + (2*B*c*x^(15/2))/15
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2(Ac+Bb)x^{\frac{13}{2}}}{13}+\frac{2Bc\,x^{\frac{15}{2}}}{15}$	40
default	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2(Ac+Bb)x^{\frac{13}{2}}}{13}+\frac{2Bc\,x^{\frac{15}{2}}}{15}$	40
gosper	$\frac{2x^{\frac{9}{2}}\left(429Bc\,x^{3}+495Ac\,x^{2}+495Bb\,x^{2}+585Abx+585Bax+715Aa\right)}{6435}$	42
trager	$\frac{2x^{\frac{9}{2}}\left(429Bc\,x^{3}+495Ac\,x^{2}+495Bb\,x^{2}+585Abx+585Bax+715Aa\right)}{6435}$	42
risch	$\frac{2x^{\frac{9}{2}}\left(429Bc\,x^{3}+495Ac\,x^{2}+495Bb\,x^{2}+585Abx+585Bax+715Aa\right)}{6435}$	42
orering	$\frac{2x^{\frac{9}{2}}\left(429Bc\,x^{3}+495Ac\,x^{2}+495Bb\,x^{2}+585Abx+585Bax+715Aa\right)}{6435}$	42

input
```
int(x^(7/2)*(B*x+A)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/9*a*A*x^(9/2)+2/11*(A*b+B*a)*x^(11/2)+2/13*(A*c+B*b)*x^(13/2)+2/15*B*c*x
^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{6435}\left(429\,Bcx^7+495\,(Bb+Ac)x^6+715\,Aax^4+585\,(Ba+Ab)x^5\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
2/6435*(429*B*c*x^7 + 495*(B*b + A*c)*x^6 + 715*A*a*x^4 + 585*(B*a + A*b)*
x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2Aax^{\frac{9}{2}}}{9}+\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Acx^{\frac{13}{2}}}{13}+\frac{2Bax^{\frac{11}{2}}}{11}+\frac{2Bbx^{\frac{13}{2}}}{13}+\frac{2Bcx^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(7/2)*(B*x+A)*(c*x**2+b*x+a),x)
```

output
```
2*A*a*x**(9/2)/9 + 2*A*b*x**(11/2)/11 + 2*A*c*x**(13/2)/13 + 2*B*a*x**(11/
2)/11 + 2*B*b*x**(13/2)/13 + 2*B*c*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{15}\,Bcx^{\frac{15}{2}}+\frac{2}{13}\,(Bb+Ac)x^{\frac{13}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}+\frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
2/15*B*c*x^(15/2) + 2/13*(B*b + A*c)*x^(13/2) + 2/9*A*a*x^(9/2) + 2/11*(B*
a + A*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{15}\,Bcx^{\frac{15}{2}}+\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{13}\,Acx^{\frac{13}{2}}+\frac{2}{11}\,Bax^{\frac{11}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2/15*B*c*x^(15/2) + 2/13*B*b*x^(13/2) + 2/13*A*c*x^(13/2) + 2/11*B*a*x^(11
/2) + 2/11*A*b*x^(11/2) + 2/9*A*a*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=x^{11/2}\left(\frac{2\,A\,b}{11}+\frac{2\,B\,a}{11}\right)+x^{13/2}\left(\frac{2\,A\,c}{13}+\frac{2\,B\,b}{13}\right)+\frac{2\,A\,a\,x^{9/2}}{9}+\frac{2\,B\,c\,x^{15/2}}{15}$$

input
```
int(x^(7/2)*(A + B*x)*(a + b*x + c*x^2),x)
```

output
```
x^(11/2)*((2*A*b)/11 + (2*B*a)/11) + x^(13/2)*((2*A*c)/13 + (2*B*b)/13) +
(2*A*a*x^(9/2))/9 + (2*B*c*x^(15/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2\sqrt{x}\,x^4(429bc\,x^3+495ac\,x^2+495b^2x^2+1170abx+715a^2)}{6435}$$

input
```
int(x^(7/2)*(B*x+A)*(c*x^2+b*x+a),x)
```

output
```
(2*sqrt(x)*x**4*(715*a**2 + 1170*a*b*x + 495*a*c*x**2 + 495*b**2*x**2 + 42
9*b*c*x**3))/6435
```

3.57 $\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{7}aAx^{7/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{11}(bB+Ac)x^{11/2}+\frac{2}{13}Bcx^{13/2}$$

output
```
2/7*a*A*x^(7/2)+2/9*(A*b+B*a)*x^(9/2)+2/11*(A*c+B*b)*x^(11/2)+2/13*B*c*x^(
13/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2x^{7/2}(143a(9A+7Bx)+7x(13A(11b+9cx)+9Bx(13b+11cx)))}{9009}$$

input
```
Integrate[x^(5/2)*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(2*x^(7/2)*(143*a*(9*A + 7*B*x) + 7*x*(13*A*(11*b + 9*c*x) + 9*B*x*(13*b +
 11*c*x))))/9009
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(x^{7/2}(aB+Ab)+aAx^{5/2}+x^{9/2}(Ac+bB)+Bcx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{11}x^{11/2}(Ac+bB)+\frac{2}{13}Bcx^{13/2}$$

input
```
Int[x^(5/2)*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(9/2))/9 + (2*(b*B + A*c)*x^(11/2))/1
1 + (2*B*c*x^(13/2))/13
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2(Ac+Bb)x^{\frac{11}{2}}}{11}+\frac{2Bc\,x^{\frac{13}{2}}}{13}$	40
default	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2(Ac+Bb)x^{\frac{11}{2}}}{11}+\frac{2Bc\,x^{\frac{13}{2}}}{13}$	40
gosper	$\frac{2x^{\frac{7}{2}}(693Bc\,x^{3}+819Ac\,x^{2}+819Bb\,x^{2}+1001Abx+1001Bax+1287Aa)}{9009}$	42
trager	$\frac{2x^{\frac{7}{2}}(693Bc\,x^{3}+819Ac\,x^{2}+819Bb\,x^{2}+1001Abx+1001Bax+1287Aa)}{9009}$	42
risch	$\frac{2x^{\frac{7}{2}}(693Bc\,x^{3}+819Ac\,x^{2}+819Bb\,x^{2}+1001Abx+1001Bax+1287Aa)}{9009}$	42
orering	$\frac{2x^{\frac{7}{2}}(693Bc\,x^{3}+819Ac\,x^{2}+819Bb\,x^{2}+1001Abx+1001Bax+1287Aa)}{9009}$	42

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/7*a*A*x^(7/2)+2/9*(A*b+B*a)*x^(9/2)+2/11*(A*c+B*b)*x^(11/2)+2/13*B*c*x^(
13/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{9009}\left(693\,Bcx^6+819\,(Bb+Ac)x^5+1287\,Aax^3+1001\,(Ba+Ab)x^4\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
2/9009*(693*B*c*x^6 + 819*(B*b + A*c)*x^5 + 1287*A*a*x^3 + 1001*(B*a + A*b
)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2Aax^{\frac{7}{2}}}{7}+\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Acx^{\frac{11}{2}}}{11}+\frac{2Bax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{11}{2}}}{11}+\frac{2Bcx^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(5/2)*(B*x+A)*(c*x**2+b*x+a),x)
```

output
```
2*A*a*x**(7/2)/7 + 2*A*b*x**(9/2)/9 + 2*A*c*x**(11/2)/11 + 2*B*a*x**(9/2)/
9 + 2*B*b*x**(11/2)/11 + 2*B*c*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{13}\,Bcx^{\frac{13}{2}}+\frac{2}{11}\,(Bb+Ac)x^{\frac{11}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}+\frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
2/13*B*c*x^(13/2) + 2/11*(B*b + A*c)*x^(11/2) + 2/7*A*a*x^(7/2) + 2/9*(B*a
 + A*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx = \frac{2}{13}Bcx^{\frac{13}{2}} + \frac{2}{11}Bbx^{\frac{11}{2}} + \frac{2}{11}Acx^{\frac{11}{2}} + \frac{2}{9}Bax^{\frac{9}{2}} + \frac{2}{9}Abx^{\frac{9}{2}} + \frac{2}{7}Aax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2/13*B*c*x^(13/2) + 2/11*B*b*x^(11/2) + 2/11*A*c*x^(11/2) + 2/9*B*a*x^(9/2
) + 2/9*A*b*x^(9/2) + 2/7*A*a*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx = x^{9/2}\left(\frac{2\,A\,b}{9}+\frac{2\,B\,a}{9}\right) + x^{11/2}\left(\frac{2\,A\,c}{11}+\frac{2\,B\,b}{11}\right) + \frac{2\,A\,a\,x^{7/2}}{7} + \frac{2\,B\,c\,x^{13/2}}{13}$$

input
```
int(x^(5/2)*(A + B*x)*(a + b*x + c*x^2),x)
```

output
```
x^(9/2)*((2*A*b)/9 + (2*B*a)/9) + x^(11/2)*((2*A*c)/11 + (2*B*b)/11) + (2*
A*a*x^(7/2))/7 + (2*B*c*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2\sqrt{x}\,x^3(693bc\,x^3+819ac\,x^2+819b^2x^2+2002abx+1287a^2)}{9009}$$

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x+a),x)
```

output
```
(2*sqrt(x)*x**3*(1287*a**2 + 2002*a*b*x + 819*a*c*x**2 + 819*b**2*x**2 + 6
93*b*c*x**3))/9009
```

3.58 $\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{5}aAx^{5/2}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{9}(bB+Ac)x^{9/2}+\frac{2}{11}Bcx^{11/2}$$

output
```
2/5*a*A*x^(5/2)+2/7*(A*b+B*a)*x^(7/2)+2/9*(A*c+B*b)*x^(9/2)+2/11*B*c*x^(11
/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2x^{5/2}(99a(7A+5Bx)+5x(11A(9b+7cx)+7Bx(11b+9cx)))}{3465}$$

input
```
Integrate[x^(3/2)*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(2*x^(5/2)*(99*a*(7*A + 5*B*x) + 5*x*(11*A*(9*b + 7*c*x) + 7*B*x*(11*b + 9
*c*x))))/3465
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(x^{5/2}(aB+Ab)+aAx^{3/2}+x^{7/2}(Ac+bB)+Bcx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{9}x^{9/2}(Ac+bB)+\frac{2}{11}Bcx^{11/2}$$

input
```
Int[x^(3/2)*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(7/2))/7 + (2*(b*B + A*c)*x^(9/2))/9
+ (2*B*c*x^(11/2))/11
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	40
default	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	40
gosper	$\frac{2x^{\frac{5}{2}}\left(315Bc\,x^{3}+385Ac\,x^{2}+385Bb\,x^{2}+495Abx+495Bax+693Aa\right)}{3465}$	42
trager	$\frac{2x^{\frac{5}{2}}\left(315Bc\,x^{3}+385Ac\,x^{2}+385Bb\,x^{2}+495Abx+495Bax+693Aa\right)}{3465}$	42
risch	$\frac{2x^{\frac{5}{2}}\left(315Bc\,x^{3}+385Ac\,x^{2}+385Bb\,x^{2}+495Abx+495Bax+693Aa\right)}{3465}$	42
orering	$\frac{2x^{\frac{5}{2}}\left(315Bc\,x^{3}+385Ac\,x^{2}+385Bb\,x^{2}+495Abx+495Bax+693Aa\right)}{3465}$	42

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*a*A*x^(5/2)+2/7*(A*b+B*a)*x^(7/2)+2/9*(A*c+B*b)*x^(9/2)+2/11*B*c*x^(11
/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{3465}\left(315\,Bcx^5+385\,(Bb+Ac)x^4+693\,Aax^2+495\,(Ba+Ab)x^3\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c*x^5 + 385*(B*b + A*c)*x^4 + 693*A*a*x^2 + 495*(B*a + A*b)*
x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{7}{2}}}{7}+\frac{2Acx^{\frac{9}{2}}}{9}+\frac{2Bax^{\frac{7}{2}}}{7}+\frac{2Bbx^{\frac{9}{2}}}{9}+\frac{2Bcx^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(3/2)*(B*x+A)*(c*x**2+b*x+a),x)
```

output
```
2*A*a*x**(5/2)/5 + 2*A*b*x**(7/2)/7 + 2*A*c*x**(9/2)/9 + 2*B*a*x**(7/2)/7
+ 2*B*b*x**(9/2)/9 + 2*B*c*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{9}\,(Bb+Ac)x^{\frac{9}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}+\frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
2/11*B*c*x^(11/2) + 2/9*(B*b + A*c)*x^(9/2) + 2/5*A*a*x^(5/2) + 2/7*(B*a +
 A*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{9}\,Acx^{\frac{9}{2}}+\frac{2}{7}\,Bax^{\frac{7}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2/11*B*c*x^(11/2) + 2/9*B*b*x^(9/2) + 2/9*A*c*x^(9/2) + 2/7*B*a*x^(7/2) +
2/7*A*b*x^(7/2) + 2/5*A*a*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=x^{7/2}\left(\frac{2\,A\,b}{7}+\frac{2\,B\,a}{7}\right)+x^{9/2}\left(\frac{2\,A\,c}{9}+\frac{2\,B\,b}{9}\right)+\frac{2\,A\,a\,x^{5/2}}{5}+\frac{2\,B\,c\,x^{11/2}}{11}$$

input
```
int(x^(3/2)*(A + B*x)*(a + b*x + c*x^2),x)
```

output
```
x^(7/2)*((2*A*b)/7 + (2*B*a)/7) + x^(9/2)*((2*A*c)/9 + (2*B*b)/9) + (2*A*a
*x^(5/2))/5 + (2*B*c*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2\sqrt{x}\,x^2(315bc\,x^3+385ac\,x^2+385b^2x^2+990abx+693a^2)}{3465}$$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x+a),x)
```

output
```
(2*sqrt(x)*x**2*(693*a**2 + 990*a*b*x + 385*a*c*x**2 + 385*b**2*x**2 + 315
*b*c*x**3))/3465
```

3.59 $\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{3}aAx^{3/2}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{7}(bB+Ac)x^{7/2}+\frac{2}{9}Bcx^{9/2}$$

output
```
2/3*a*A*x^(3/2)+2/5*(A*b+B*a)*x^(5/2)+2/7*(A*c+B*b)*x^(7/2)+2/9*B*c*x^(9/2
)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{315}x^{3/2}(21a(5A+3Bx)+x(9A(7b+5cx)+5Bx(9b+7cx)))$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(2*x^(3/2)*(21*a*(5*A + 3*B*x) + x*(9*A*(7*b + 5*c*x) + 5*B*x*(9*b + 7*c*x
))))/315
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{ 1195}$$

$$\int \left(x^{3/2}(aB+Ab)+aA\sqrt{x}+x^{5/2}(Ac+bB)+Bcx^{7/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}x^{5/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{7}x^{7/2}(Ac+bB)+\frac{2}{9}Bcx^{9/2}$$

input
```
Int[Sqrt[x]*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(5/2))/5 + (2*(b*B + A*c)*x^(7/2))/7
+ (2*B*c*x^(9/2))/9
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{9}{2}}}{9}$	40
default	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{9}{2}}}{9}$	40
gosper	$\frac{2x^{\frac{3}{2}}\left(35Bc\,x^{3}+45Ac\,x^{2}+45Bb\,x^{2}+63Abx+63Bax+105Aa\right)}{315}$	42
trager	$\frac{2x^{\frac{3}{2}}\left(35Bc\,x^{3}+45Ac\,x^{2}+45Bb\,x^{2}+63Abx+63Bax+105Aa\right)}{315}$	42
risch	$\frac{2x^{\frac{3}{2}}\left(35Bc\,x^{3}+45Ac\,x^{2}+45Bb\,x^{2}+63Abx+63Bax+105Aa\right)}{315}$	42
orering	$\frac{2x^{\frac{3}{2}}\left(35Bc\,x^{3}+45Ac\,x^{2}+45Bb\,x^{2}+63Abx+63Bax+105Aa\right)}{315}$	42

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*a*A*x^(3/2)+2/5*(A*b+B*a)*x^(5/2)+2/7*(A*c+B*b)*x^(7/2)+2/9*B*c*x^(9/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx$$
$$=\frac{2}{315}\left(35\,Bcx^4+45\,(Bb+Ac)x^3+105\,Aax+63\,(Ba+Ab)x^2\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
2/315*(35*B*c*x^4 + 45*(B*b + A*c)*x^3 + 105*A*a*x + 63*(B*a + A*b)*x^2)*s
qrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Bcx^{\frac{9}{2}}}{9}+\frac{2x^{\frac{7}{2}}(Ac+Bb)}{7}+\frac{2x^{\frac{5}{2}}(Ab+Ba)}{5}$$

input
```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x+a),x)
```

output
```
2*A*a*x**(3/2)/3 + 2*B*c*x**(9/2)/9 + 2*x**(7/2)*(A*c + B*b)/7 + 2*x**(5/2
)*(A*b + B*a)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{9}\,Bcx^{\frac{9}{2}}+\frac{2}{7}\,(Bb+Ac)x^{\frac{7}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}+\frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
2/9*B*c*x^(9/2) + 2/7*(B*b + A*c)*x^(7/2) + 2/3*A*a*x^(3/2) + 2/5*(B*a + A
*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{2}{9}\,Bcx^{\frac{9}{2}}+\frac{2}{7}\,Bbx^{\frac{7}{2}}+\frac{2}{7}\,Acx^{\frac{7}{2}}+\frac{2}{5}\,Bax^{\frac{5}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2/9*B*c*x^(9/2) + 2/7*B*b*x^(7/2) + 2/7*A*c*x^(7/2) + 2/5*B*a*x^(5/2) + 2/
5*A*b*x^(5/2) + 2/3*A*a*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx = x^{5/2}\left(\frac{2\,A\,b}{5}+\frac{2\,B\,a}{5}\right) + x^{7/2}\left(\frac{2\,A\,c}{7}+\frac{2\,B\,b}{7}\right)+\frac{2\,A\,a\,x^{3/2}}{3}+\frac{2\,B\,c\,x^{9/2}}{9}$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x + c*x^2),x)
```

output
```
x^(5/2)*((2*A*b)/5 + (2*B*a)/5) + x^(7/2)*((2*A*c)/7 + (2*B*b)/7) + (2*A*a
*x^(3/2))/3 + (2*B*c*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)\,dx = \frac{2\sqrt{x}\,x(35bc\,x^3+45ac\,x^2+45b^2x^2+126abx+105a^2)}{315}$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x+a),x)
```

output
```
(2*sqrt(x)*x*(105*a**2 + 126*a*b*x + 45*a*c*x**2 + 45*b**2*x**2 + 35*b*c*x
**3))/315
```

3.60 $\int \frac{(A+Bx)(a+bx+cx^2)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{(A+Bx)(a+bx+cx^2)}{\sqrt{x}}\,dx = 2aA\sqrt{x}+\frac{2}{3}(Ab+aB)x^{3/2}+\frac{2}{5}(bB+Ac)x^{5/2}+\frac{2}{7}Bcx^{7/2}$$

output
```
2*a*A*x^(1/2)+2/3*(A*b+B*a)*x^(3/2)+2/5*(A*c+B*b)*x^(5/2)+2/7*B*c*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)(a+bx+cx^2)}{\sqrt{x}}\,dx = \frac{2}{105}\sqrt{x}(35a(3A+Bx) + x(7A(5b+3cx)+3Bx(7b+5cx)))$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(35*a*(3*A + B*x) + x*(7*A*(5*b + 3*c*x) + 3*B*x*(7*b + 5*c*x))
))/105
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\sqrt{x}(aB+Ab)+\frac{aA}{\sqrt{x}}+x^{3/2}(Ac+bB)+Bcx^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}x^{3/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{5}x^{5/2}(Ac+bB)+\frac{2}{7}Bcx^{7/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/Sqrt[x],x]
```

output
```
2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3 + (2*(b*B + A*c)*x^(5/2))/5 + (2
*B*c*x^(7/2))/7
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$2aA\sqrt{x}+\frac{2(Ab+Ba)x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{7}{2}}}{7}$	40
default	$2aA\sqrt{x}+\frac{2(Ab+Ba)x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{7}{2}}}{7}$	40
trager	$\left(\frac{2}{7}Bc\,x^{3}+\frac{2}{5}Ac\,x^{2}+\frac{2}{5}Bb\,x^{2}+\frac{2}{3}Abx+\frac{2}{3}Bax+2Aa\right)\sqrt{x}$	41
gosper	$\frac{2\sqrt{x}\,(15Bc\,x^{3}+21Ac\,x^{2}+21Bb\,x^{2}+35Abx+35Bax+105Aa)}{105}$	42
risch	$\frac{2\sqrt{x}\,(15Bc\,x^{3}+21Ac\,x^{2}+21Bb\,x^{2}+35Abx+35Bax+105Aa)}{105}$	42
orering	$\frac{2\sqrt{x}\,(15Bc\,x^{3}+21Ac\,x^{2}+21Bb\,x^{2}+35Abx+35Bax+105Aa)}{105}$	42

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*A*x^(1/2)+2/3*(A*b+B*a)*x^(3/2)+2/5*(A*c+B*b)*x^(5/2)+2/7*B*c*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{\sqrt{x}}\,dx$$
$$=\frac{2}{105}\left(15\,Bcx^3+21\,(Bb+Ac)x^2+105\,Aa+35\,(Ba+Ab)x\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*c*x^3 + 21*(B*b + A*c)*x^2 + 105*A*a + 35*(B*a + A*b)*x)*sqrt(
x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)(a+bx+cx^2)}{\sqrt{x}}\,dx = 2Aa\sqrt{x}+\frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Acx^{\frac{5}{2}}}{5}+\frac{2Bax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{5}{2}}}{5}+\frac{2Bcx^{\frac{7}{2}}}{7}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**(1/2),x)
```

output
```
2*A*a*sqrt(x) + 2*A*b*x**(3/2)/3 + 2*A*c*x**(5/2)/5 + 2*B*a*x**(3/2)/3 + 2
*B*b*x**(5/2)/5 + 2*B*c*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)(a+bx+cx^2)}{\sqrt{x}}\,dx = \frac{2}{7}\,Bcx^{\frac{7}{2}}+\frac{2}{5}\,(Bb+Ac)x^{\frac{5}{2}}+2\,Aa\sqrt{x}+\frac{2}{3}\,(Ba+Ab)x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(1/2),x, algorithm="maxima")
```

output
```
2/7*B*c*x^(7/2) + 2/5*(B*b + A*c)*x^(5/2) + 2*A*a*sqrt(x) + 2/3*(B*a + A*b
)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \frac{(A+Bx)(a+bx+cx^2)}{\sqrt{x}}\,dx = {} & \frac{2}{7}\,Bcx^{\frac{7}{2}}+\frac{2}{5}\,Bbx^{\frac{5}{2}}+\frac{2}{5}\,Acx^{\frac{5}{2}} \\ & +\frac{2}{3}\,Bax^{\frac{3}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}+2\,Aa\sqrt{x}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(1/2),x, algorithm="giac")
```

output
```
2/7*B*c*x^(7/2) + 2/5*B*b*x^(5/2) + 2/5*A*c*x^(5/2) + 2/3*B*a*x^(3/2) + 2/
3*A*b*x^(3/2) + 2*A*a*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{\sqrt{x}}\,dx = x^{3/2}\left(\frac{2\,A\,b}{3}+\frac{2\,B\,a}{3}\right) + x^{5/2}\left(\frac{2\,A\,c}{5}+\frac{2\,B\,b}{5}\right) + 2\,A\,a\,\sqrt{x} + \frac{2\,B\,c\,x^{7/2}}{7}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^(1/2),x)
```

output
```
x^(3/2)*((2*A*b)/3 + (2*B*a)/3) + x^(5/2)*((2*A*c)/5 + (2*B*b)/5) + 2*A*a*
x^(1/2) + (2*B*c*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\left(15bc\,x^3+21ac\,x^2+21b^2x^2+70abx+105a^2\right)}{105}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(105*a**2 + 70*a*b*x + 21*a*c*x**2 + 21*b**2*x**2 + 15*b*c*x**3
))/105
```

3.61 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx=-\frac{2aA}{\sqrt{x}}+2(Ab+aB)\sqrt{x}+\frac{2}{3}(bB+Ac)x^{3/2}+\frac{2}{5}Bcx^{5/2}$$

output
```
-2*a*A/x^(1/2)+2*(A*b+B*a)*x^(1/2)+2/3*(A*c+B*b)*x^(3/2)+2/5*B*c*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx=-\frac{2(15aA-15Abx-15aBx-5bBx^2-5Acx^2-3Bcx^3)}{15\sqrt{x}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^(3/2),x]
```

output
```
(-2*(15*a*A - 15*A*b*x - 15*a*B*x - 5*b*B*x^2 - 5*A*c*x^2 - 3*B*c*x^3))/(1
5*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1195}$$

$$\int \left(\frac{aB+Ab}{\sqrt{x}}+\frac{aA}{x^{3/2}}+\sqrt{x}(Ac+bB)+Bcx^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{3}x^{3/2}(Ac+bB)+\frac{2}{5}Bcx^{5/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^(3/2),x]
```

output
```
(-2*a*A)/Sqrt[x] + 2*(A*b + a*B)*Sqrt[x] + (2*(b*B + A*c)*x^(3/2))/3 + (2*
B*c*x^(5/2))/5
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
gosper	$-\frac{2(-3Bc\,x^{3}-5Ac\,x^{2}-5Bb\,x^{2}-15Abx-15Bax+15Aa)}{15\sqrt{x}}$	42
trager	$-\frac{2(-3Bc\,x^{3}-5Ac\,x^{2}-5Bb\,x^{2}-15Abx-15Bax+15Aa)}{15\sqrt{x}}$	42
risch	$-\frac{2(-3Bc\,x^{3}-5Ac\,x^{2}-5Bb\,x^{2}-15Abx-15Bax+15Aa)}{15\sqrt{x}}$	42
orering	$-\frac{2(-3Bc\,x^{3}-5Ac\,x^{2}-5Bb\,x^{2}-15Abx-15Bax+15Aa)}{15\sqrt{x}}$	42
derivativedivides	$\frac{2Bc\,x^{\frac{5}{2}}}{5}+\frac{2Ac\,x^{\frac{3}{2}}}{3}+\frac{2Bb\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{\sqrt{x}}$	44
default	$\frac{2Bc\,x^{\frac{5}{2}}}{5}+\frac{2Ac\,x^{\frac{3}{2}}}{3}+\frac{2Bb\,x^{\frac{3}{2}}}{3}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{\sqrt{x}}$	44

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*c*x^3-5*A*c*x^2-5*B*b*x^2-15*A*b*x-15*B*a*x+15*A*a)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx = \frac{2\left(3\,Bcx^3+5\,(Bb+Ac)x^2-15\,Aa+15\,(Ba+Ab)x\right)}{15\,\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c*x^3 + 5*(B*b + A*c)*x^2 - 15*A*a + 15*(B*a + A*b)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx = -\frac{2Aa}{\sqrt{x}}+2Ab\sqrt{x}+\frac{2Acx^{\frac{3}{2}}}{3}+2Ba\sqrt{x}+\frac{2Bbx^{\frac{3}{2}}}{3}+\frac{2Bcx^{\frac{5}{2}}}{5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**(3/2),x)
```

output
```
-2*A*a/sqrt(x) + 2*A*b*sqrt(x) + 2*A*c*x**(3/2)/3 + 2*B*a*sqrt(x) + 2*B*b*
x**(3/2)/3 + 2*B*c*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx = \frac{2}{5}\,Bcx^{\frac{5}{2}}+\frac{2}{3}\,(Bb+Ac)x^{\frac{3}{2}}-\frac{2\,Aa}{\sqrt{x}}+2\,(Ba+Ab)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(3/2),x, algorithm="maxima")
```

output
```
2/5*B*c*x^(5/2) + 2/3*(B*b + A*c)*x^(3/2) - 2*A*a/sqrt(x) + 2*(B*a + A*b)*
sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx = \frac{2}{5}\,Bcx^{\frac{5}{2}}+\frac{2}{3}\,Bbx^{\frac{3}{2}}+\frac{2}{3}\,Acx^{\frac{3}{2}}+2\,Ba\sqrt{x}+2\,Ab\sqrt{x}-\frac{2\,Aa}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(3/2),x, algorithm="giac")
```

output
```
2/5*B*c*x^(5/2) + 2/3*B*b*x^(3/2) + 2/3*A*c*x^(3/2) + 2*B*a*sqrt(x) + 2*A*
b*sqrt(x) - 2*A*a/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx = \sqrt{x}\,(2\,A\,b+2\,B\,a) + x^{3/2}\left(\frac{2\,A\,c}{3}+\frac{2\,B\,b}{3}\right) - \frac{2\,A\,a}{\sqrt{x}} + \frac{2\,B\,c\,x^{5/2}}{5}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^(3/2),x)
```

output
```
x^(1/2)*(2*A*b + 2*B*a) + x^(3/2)*((2*A*c)/3 + (2*B*b)/3) - (2*A*a)/x^(1/2
) + (2*B*c*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{3/2}}\,dx = \frac{\frac{2}{5}bc\,x^3+\frac{2}{3}ac\,x^2+\frac{2}{3}b^2x^2+4abx-2a^2}{\sqrt{x}}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(3/2),x)
```

output
```
(2*( - 15*a**2 + 30*a*b*x + 5*a*c*x**2 + 5*b**2*x**2 + 3*b*c*x**3))/(15*sq
rt(x))
```

3.62 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{5/2}}\,dx=-\frac{2aA}{3x^{3/2}}-\frac{2(Ab+aB)}{\sqrt{x}}+2(bB+Ac)\sqrt{x}+\frac{2}{3}Bcx^{3/2}$$

output
```
-2/3*a*A/x^(3/2)-2*(A*b+B*a)/x^(1/2)+2*(A*c+B*b)*x^(1/2)+2/3*B*c*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{5/2}}\,dx=-\frac{2\left(aA+3Abx+3aBx-3bBx^2-3Acx^2-Bcx^3\right)}{3x^{3/2}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^(5/2),x]
```

output
```
(-2*(a*A + 3*A*b*x + 3*a*B*x - 3*b*B*x^2 - 3*A*c*x^2 - B*c*x^3))/(3*x^(3/2
))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1195}$$

$$\int \left(\frac{aB+Ab}{x^{3/2}}+\frac{aA}{x^{5/2}}+\frac{Ac+bB}{\sqrt{x}}+Bc\sqrt{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{3x^{3/2}}+2\sqrt{x}(Ac+bB)+\frac{2}{3}Bcx^{3/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^(5/2),x]
```

output
```
(-2*a*A)/(3*x^(3/2)) - (2*(A*b + a*B))/Sqrt[x] + 2*(b*B + A*c)*Sqrt[x] + (
2*B*c*x^(3/2))/3
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

method	result	size
gosper	$-\frac{2(-Bc\,x^{3}-3Ac\,x^{2}-3Bb\,x^{2}+3Abx+3Bax+Aa)}{3x^{\frac{3}{2}}}$	41
trager	$-\frac{2(-Bc\,x^{3}-3Ac\,x^{2}-3Bb\,x^{2}+3Abx+3Bax+Aa)}{3x^{\frac{3}{2}}}$	41
risch	$-\frac{2(-Bc\,x^{3}-3Ac\,x^{2}-3Bb\,x^{2}+3Abx+3Bax+Aa)}{3x^{\frac{3}{2}}}$	41
orering	$-\frac{2(-Bc\,x^{3}-3Ac\,x^{2}-3Bb\,x^{2}+3Abx+3Bax+Aa)}{3x^{\frac{3}{2}}}$	41
derivativedivides	$\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}+2Bb\sqrt{x}-\frac{2(Ab+Ba)}{\sqrt{x}}-\frac{2aA}{3x^{\frac{3}{2}}}$	42
default	$\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}+2Bb\sqrt{x}-\frac{2(Ab+Ba)}{\sqrt{x}}-\frac{2aA}{3x^{\frac{3}{2}}}$	42

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-B*c*x^3-3*A*c*x^2-3*B*b*x^2+3*A*b*x+3*B*a*x+A*a)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{5/2}}\,dx=\frac{2\left(Bcx^3+3\left(Bb+Ac\right)x^2-Aa-3\left(Ba+Ab\right)x\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(B*c*x^3 + 3*(B*b + A*c)*x^2 - A*a - 3*(B*a + A*b)*x)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^{5/2}}\,dx = -\frac{2Aa}{3x^{\frac{3}{2}}} - \frac{2Ab}{\sqrt{x}} + 2Ac\sqrt{x} - \frac{2Ba}{\sqrt{x}} + 2Bb\sqrt{x} + \frac{2Bcx^{\frac{3}{2}}}{3}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**(5/2),x)
```

output
```
-2*A*a/(3*x**(3/2)) - 2*A*b/sqrt(x) + 2*A*c*sqrt(x) - 2*B*a/sqrt(x) + 2*B*
b*sqrt(x) + 2*B*c*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^{5/2}}\,dx = \frac{2}{3}\,Bcx^{\frac{3}{2}} + 2\,(Bb+Ac)\sqrt{x} - \frac{2\,(Aa+3\,(Ba+Ab)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(5/2),x, algorithm="maxima")
```

output
```
2/3*B*c*x^(3/2) + 2*(B*b + A*c)*sqrt(x) - 2/3*(A*a + 3*(B*a + A*b)*x)/x^(3
/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^{5/2}}\,dx = \frac{2}{3}\,Bcx^{\frac{3}{2}} + 2\,Bb\sqrt{x} + 2\,Ac\sqrt{x} - \frac{2\,(3\,Bax + 3\,Abx + Aa)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(5/2),x, algorithm="giac")
```

output
```
2/3*B*c*x^(3/2) + 2*B*b*sqrt(x) + 2*A*c*sqrt(x) - 2/3*(3*B*a*x + 3*A*b*x +
 A*a)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{5/2}}\,dx=-\frac{2\,A\,a+6\,A\,b\,x+6\,B\,a\,x-6\,A\,c\,x^2-6\,B\,b\,x^2-2\,B\,c\,x^3}{3\,x^{3/2}}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^(5/2),x)
```

output
```
-(2*A*a + 6*A*b*x + 6*B*a*x - 6*A*c*x^2 - 6*B*b*x^2 - 2*B*c*x^3)/(3*x^(3/2
))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{5/2}}\,dx=\frac{\frac{2}{3}bc\,x^3+2ac\,x^2+2b^2x^2-4abx-\frac{2}{3}a^2}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(5/2),x)
```

output
```
(2*( - a**2 - 6*a*b*x + 3*a*c*x**2 + 3*b**2*x**2 + b*c*x**3))/(3*sqrt(x)*x
)
```

3.63 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^{7/2}}\,dx = -\frac{2aA}{5x^{5/2}} - \frac{2(Ab+aB)}{3x^{3/2}} - \frac{2(bB+Ac)}{\sqrt{x}} + 2Bc\sqrt{x}$$

output
```
-2/5*a*A/x^(5/2)-2/3*(A*b+B*a)/x^(3/2)-2*(A*c+B*b)/x^(1/2)+2*B*c*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^{7/2}}\,dx = -\frac{2(a(3A+5Bx)+5x(3Bx(b-cx)+A(b+3cx)))}{15x^{5/2}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^(7/2),x]
```

output
```
(-2*(a*(3*A + 5*B*x) + 5*x*(3*B*x*(b - c*x) + A*(b + 3*c*x))))/(15*x^(5/2)
)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{aB+Ab}{x^{5/2}}+\frac{aA}{x^{7/2}}+\frac{Ac+bB}{x^{3/2}}+\frac{Bc}{\sqrt{x}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(aB+Ab)}{3x^{3/2}}-\frac{2aA}{5x^{5/2}}-\frac{2(Ac+bB)}{\sqrt{x}}+2Bc\sqrt{x}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^(7/2),x]
```

output
```
(-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/(3*x^(3/2)) - (2*(b*B + A*c))/Sqrt[
x] + 2*B*c*Sqrt[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2aA}{5x^{\frac{5}{2}}}-\frac{2(Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2(Ac+Bb)}{\sqrt{x}}+2Bc\sqrt{x}$	40
default	$-\frac{2aA}{5x^{\frac{5}{2}}}-\frac{2(Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2(Ac+Bb)}{\sqrt{x}}+2Bc\sqrt{x}$	40
gosper	$-\frac{2(-15Bc\,x^{3}+15Ac\,x^{2}+15Bb\,x^{2}+5Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}}$	42
trager	$-\frac{2(-15Bc\,x^{3}+15Ac\,x^{2}+15Bb\,x^{2}+5Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}}$	42
risch	$-\frac{2(-15Bc\,x^{3}+15Ac\,x^{2}+15Bb\,x^{2}+5Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}}$	42
orering	$-\frac{2(-15Bc\,x^{3}+15Ac\,x^{2}+15Bb\,x^{2}+5Abx+5Bax+3Aa)}{15x^{\frac{5}{2}}}$	42

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a*A/x^(5/2)-2/3*(A*b+B*a)/x^(3/2)-2*(A*c+B*b)/x^(1/2)+2*B*c*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{7/2}}\,dx=\frac{2\,(15\,Bcx^3-15\,(Bb+Ac)x^2-3\,Aa-5\,(Ba+Ab)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(15*B*c*x^3 - 15*(B*b + A*c)*x^2 - 3*A*a - 5*(B*a + A*b)*x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^{7/2}}\,dx = -\frac{2Aa}{5x^{\frac{5}{2}}} - \frac{2Ab}{3x^{\frac{3}{2}}} - \frac{2Ac}{\sqrt{x}} - \frac{2Ba}{3x^{\frac{3}{2}}} - \frac{2Bb}{\sqrt{x}} + 2Bc\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**(7/2),x)
```

output
```
-2*A*a/(5*x**(5/2)) - 2*A*b/(3*x**(3/2)) - 2*A*c/sqrt(x) - 2*B*a/(3*x**(3/
2)) - 2*B*b/sqrt(x) + 2*B*c*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^{7/2}}\,dx = 2\,Bc\sqrt{x} - \frac{2\,(15\,(Bb+Ac)x^2+3\,Aa+5\,(Ba+Ab)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(7/2),x, algorithm="maxima")
```

output
```
2*B*c*sqrt(x) - 2/15*(15*(B*b + A*c)*x^2 + 3*A*a + 5*(B*a + A*b)*x)/x^(5/2
)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^{7/2}}\,dx = 2\,Bc\sqrt{x} - \frac{2\,(15\,Bbx^2+15\,Acx^2+5\,Bax+5\,Abx+3\,Aa)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(7/2),x, algorithm="giac")
```

output
```
2*B*c*sqrt(x) - 2/15*(15*B*b*x^2 + 15*A*c*x^2 + 5*B*a*x + 5*A*b*x + 3*A*a)
/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^{7/2}}\,dx = 2\,B\,c\,\sqrt{x} - \frac{(2\,A\,c+2\,B\,b)\;x^2+\left(\frac{2\,A\,b}{3}+\frac{2\,B\,a}{3}\right)\,x+\frac{2\,A\,a}{5}}{x^{5/2}}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^(7/2),x)
```

output
```
2*B*c*x^(1/2) - ((2*A*a)/5 + x*((2*A*b)/3 + (2*B*a)/3) + x^2*(2*A*c + 2*B*
b))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{x^{7/2}}\,dx = \frac{2bc\,x^3-2ac\,x^2-2b^2x^2-\frac{4}{3}abx-\frac{2}{5}a^2}{\sqrt{x}\,x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(7/2),x)
```

output
```
(2*( - 3*a**2 - 10*a*b*x - 15*a*c*x**2 - 15*b**2*x**2 + 15*b*c*x**3))/(15*
sqrt(x)*x**2)
```

3.64 $\int \frac{(A+Bx)(a+bx+cx^2)}{x^{9/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^{9/2}} dx = -\frac{2aA}{7x^{7/2}} - \frac{2(Ab+aB)}{5x^{5/2}} - \frac{2(bB+Ac)}{3x^{3/2}} - \frac{2Bc}{\sqrt{x}}$$

output
```
-2/7*a*A/x^(7/2)-2/5*(A*b+B*a)/x^(5/2)-2/3*(A*c+B*b)/x^(3/2)-2*B*c/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^{9/2}} dx = -\frac{2(15aA+21Abx+21aBx+35bBx^2+35Acx^2+105Bcx^3)}{105x^{7/2}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/x^(9/2),x]
```

output
```
(-2*(15*a*A + 21*A*b*x + 21*a*B*x + 35*b*B*x^2 + 35*A*c*x^2 + 105*B*c*x^3)
)/(105*x^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{9/2}}\,dx$$

$$\Big\downarrow \text{ 1195}$$

$$\int \left(\frac{aB+Ab}{x^{7/2}}+\frac{aA}{x^{9/2}}+\frac{Ac+bB}{x^{5/2}}+\frac{Bc}{x^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2(aB+Ab)}{5x^{5/2}}-\frac{2aA}{7x^{7/2}}-\frac{2(Ac+bB)}{3x^{3/2}}-\frac{2Bc}{\sqrt{x}}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/x^(9/2),x]
```

output
```
(-2*a*A)/(7*x^(7/2)) - (2*(A*b + a*B))/(5*x^(5/2)) - (2*(b*B + A*c))/(3*x^
(3/2)) - (2*B*c)/Sqrt[x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{2aA}{7x^{\frac{7}{2}}}-\frac{2(Ab+Ba)}{5x^{\frac{5}{2}}}-\frac{2(Ac+Bb)}{3x^{\frac{3}{2}}}-\frac{2Bc}{\sqrt{x}}$	40
default	$-\frac{2aA}{7x^{\frac{7}{2}}}-\frac{2(Ab+Ba)}{5x^{\frac{5}{2}}}-\frac{2(Ac+Bb)}{3x^{\frac{3}{2}}}-\frac{2Bc}{\sqrt{x}}$	40
gosper	$-\frac{2(105Bc\,x^{3}+35Ac\,x^{2}+35Bb\,x^{2}+21Abx+21Bax+15Aa)}{105x^{\frac{7}{2}}}$	42
trager	$-\frac{2(105Bc\,x^{3}+35Ac\,x^{2}+35Bb\,x^{2}+21Abx+21Bax+15Aa)}{105x^{\frac{7}{2}}}$	42
risch	$-\frac{2(105Bc\,x^{3}+35Ac\,x^{2}+35Bb\,x^{2}+21Abx+21Bax+15Aa)}{105x^{\frac{7}{2}}}$	42
orering	$-\frac{2(105Bc\,x^{3}+35Ac\,x^{2}+35Bb\,x^{2}+21Abx+21Bax+15Aa)}{105x^{\frac{7}{2}}}$	42

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*a*A/x^(7/2)-2/5*(A*b+B*a)/x^(5/2)-2/3*(A*c+B*b)/x^(3/2)-2*B*c/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{9/2}}\,dx = -\frac{2\,(105\,Bcx^3+35\,(Bb+Ac)x^2+15\,Aa+21\,(Ba+Ab)x)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(105*B*c*x^3 + 35*(B*b + A*c)*x^2 + 15*A*a + 21*(B*a + A*b)*x)/x^(7
/2)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{9/2}}\,dx = -\frac{2Aa}{7x^{\frac{7}{2}}}-\frac{2Ab}{5x^{\frac{5}{2}}}-\frac{2Ac}{3x^{\frac{3}{2}}}-\frac{2Ba}{5x^{\frac{5}{2}}}-\frac{2Bb}{3x^{\frac{3}{2}}}-\frac{2Bc}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/x**(9/2),x)
```

output
```
-2*A*a/(7*x**(7/2)) - 2*A*b/(5*x**(5/2)) - 2*A*c/(3*x**(3/2)) - 2*B*a/(5*x
**(5/2)) - 2*B*b/(3*x**(3/2)) - 2*B*c/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{9/2}}\,dx = -\frac{2\left(105\,Bcx^3+35\,(Bb+Ac)x^2+15\,Aa+21\,(Ba+Ab)x\right)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/105*(105*B*c*x^3 + 35*(B*b + A*c)*x^2 + 15*A*a + 21*(B*a + A*b)*x)/x^(7
/2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{x^{9/2}}\,dx = -\frac{2\left(105\,Bcx^3+35\,Bbx^2+35\,Acx^2+21\,Bax+21\,Abx+15\,Aa\right)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/x^(9/2),x, algorithm="giac")
```

output
```
-2/105*(105*B*c*x^3 + 35*B*b*x^2 + 35*A*c*x^2 + 21*B*a*x + 21*A*b*x + 15*A
*a)/x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^{9/2}}\,dx = -\frac{2\,B\,c\,x^3+\left(\frac{2\,A\,c}{3}+\frac{2\,B\,b}{3}\right)\,x^2+\left(\frac{2\,A\,b}{5}+\frac{2\,B\,a}{5}\right)\,x+\frac{2\,A\,a}{7}}{x^{7/2}}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/x^(9/2),x)
```

output
```
-((2*A*a)/7 + x*((2*A*b)/5 + (2*B*a)/5) + x^2*((2*A*c)/3 + (2*B*b)/3) + 2*
B*c*x^3)/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)(a+bx+cx^2)}{x^{9/2}}\,dx = \frac{-2bc\,x^3-\frac{2}{3}ac\,x^2-\frac{2}{3}b^2x^2-\frac{4}{5}abx-\frac{2}{7}a^2}{\sqrt{x}\,x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)/x^(9/2),x)
```

output
```
(2*( - 15*a**2 - 42*a*b*x - 35*a*c*x**2 - 35*b**2*x**2 - 105*b*c*x**3))/(1
05*sqrt(x)*x**3)
```

3.65 $\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{9}a^2Ax^{9/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{13}\left(2abB+A\left(b^2+2ac\right)\right)x^{13/2}+\frac{2}{15}\left(b^2B+2Abc+2aBc\right)x^{15/2}+\frac{2}{17}c(2bB+Ac)x^{17/2}+\frac{2}{19}Bc^2x^{19/2}$$

output
```
2/9*a^2*A*x^(9/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/13*(2*a*b*B+A*(2*a*c+b^2))
*x^(13/2)+2/15*(2*A*b*c+2*B*a*c+B*b^2)*x^(15/2)+2/17*c*(A*c+2*B*b)*x^(17/2
)+2/19*B*c^2*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2x^{9/2}(20995a^2(11A+9Bx)+1938ax(15A(13b+11cx)+11Bx(15b+13cx))+33x^2(19A(25}{2078505}$$

input
```
Integrate[x^(7/2)*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(2*x^(9/2)*(20995*a^2*(11*A + 9*B*x) + 1938*a*x*(15*A*(13*b + 11*c*x) + 11
*B*x*(15*b + 13*c*x)) + 33*x^2*(19*A*(255*b^2 + 442*b*c*x + 195*c^2*x^2) +
 13*B*x*(323*b^2 + 570*b*c*x + 255*c^2*x^2))))/2078505
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^2Ax^{7/2}+x^{13/2}\left(2aBc+2Abc+b^2B\right)+x^{11/2}\left(A\left(2ac+b^2\right)+2abB\right)+ax^{9/2}(aB+2Ab)+cx^{15/2}(Ac+2bB$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{15}x^{15/2}\left(2aBc+2Abc+b^2B\right)+\frac{2}{13}x^{13/2}\left(A\left(2ac+b^2\right)+2abB\right)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{17}cx^{17/2}(Ac+2bB)+\frac{2}{19}Bc^2x^{19/2}$$

input
```
Int[x^(7/2)*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*(2*a*b*B + A*(b
^2 + 2*a*c))*x^(13/2))/13 + (2*(b^2*B + 2*A*b*c + 2*a*B*c)*x^(15/2))/15 +
(2*c*(2*b*B + A*c)*x^(17/2))/17 + (2*B*c^2*x^(19/2))/19
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2Bc^2x^{\frac{19}{2}}}{19}+\frac{2(Ac^2+2Bbc)x^{\frac{17}{2}}}{17}+\frac{2(2Abc+B(2ac+b^2))x^{\frac{15}{2}}}{15}+\frac{2(2abB+A(2ac+b^2))x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}$
default	$\frac{2Bc^2x^{\frac{19}{2}}}{19}+\frac{2(Ac^2+2Bbc)x^{\frac{17}{2}}}{17}+\frac{2(2Abc+B(2ac+b^2))x^{\frac{15}{2}}}{15}+\frac{2(2abB+A(2ac+b^2))x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}$
gosper	$\frac{2x^{\frac{9}{2}}(109395Bc^2x^5+122265x^4Ac^2+244530x^4Bbc+277134x^3Abc+277134Bacx^3+138567x^3Bb^2+319770Aacx^2+15}{2078505}$
trager	$\frac{2x^{\frac{9}{2}}(109395Bc^2x^5+122265x^4Ac^2+244530x^4Bbc+277134x^3Abc+277134Bacx^3+138567x^3Bb^2+319770Aacx^2+15}{2078505}$
risch	$\frac{2x^{\frac{9}{2}}(109395Bc^2x^5+122265x^4Ac^2+244530x^4Bbc+277134x^3Abc+277134Bacx^3+138567x^3Bb^2+319770Aacx^2+15}{2078505}$
orering	$\frac{2x^{\frac{9}{2}}(109395Bc^2x^5+122265x^4Ac^2+244530x^4Bbc+277134x^3Abc+277134Bacx^3+138567x^3Bb^2+319770Aacx^2+15}{2078505}$

input
```
int(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/19*B*c^2*x^(19/2)+2/17*(A*c^2+2*B*b*c)*x^(17/2)+2/15*(2*A*b*c+B*(2*a*c+b
^2))*x^(15/2)+2/13*(2*a*b*B+A*(2*a*c+b^2))*x^(13/2)+2/11*(2*A*a*b+B*a^2)*x
^(11/2)+2/9*a^2*A*x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{2}{2078505}\left(109395\,Bc^2x^9+122265\left(2\,Bbc+Ac^2\right)x^8+138567\left(Bb^2+2\,(Ba+Ab)c\right)x^7+2309\right.$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
2/2078505*(109395*B*c^2*x^9 + 122265*(2*B*b*c + A*c^2)*x^8 + 138567*(B*b^2
 + 2*(B*a + A*b)*c)*x^7 + 230945*A*a^2*x^4 + 159885*(2*B*a*b + A*b^2 + 2*A
*a*c)*x^6 + 188955*(B*a^2 + 2*A*a*b)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.43

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{4Aacx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{4Abcx^{\frac{15}{2}}}{15}+\frac{2Ac^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{13}{2}}}{13}+\frac{4Bacx^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{15}{2}}}{15}+\frac{4Bbcx^{\frac{17}{2}}}{17}+\frac{2Bc^2x^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(7/2)*(B*x+A)*(c*x**2+b*x+a)**2,x)
```

output
```
2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(11/2)/11 + 4*A*a*c*x**(13/2)/13 + 2*A*b*
*2*x**(13/2)/13 + 4*A*b*c*x**(15/2)/15 + 2*A*c**2*x**(17/2)/17 + 2*B*a**2*
x**(11/2)/11 + 4*B*a*b*x**(13/2)/13 + 4*B*a*c*x**(15/2)/15 + 2*B*b**2*x**(
15/2)/15 + 4*B*b*c*x**(17/2)/17 + 2*B*c**2*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{19}\,Bc^2x^{\frac{19}{2}}+\frac{2}{17}\left(2\,Bbc+Ac^2\right)x^{\frac{17}{2}}+\frac{2}{15}\left(Bb^2+2\left(Ba+Ab\right)c\right)x^{\frac{15}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}+\frac{2}{13}\left(2\,Bab+Ab^2+2\,Aac\right)x^{\frac{13}{2}}+\frac{2}{11}\left(Ba^2+2\,Aab\right)x^{\frac{11}{2}}$$

input

```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output

```
2/19*B*c^2*x^(19/2) + 2/17*(2*B*b*c + A*c^2)*x^(17/2) + 2/15*(B*b^2 + 2*(B
*a + A*b)*c)*x^(15/2) + 2/9*A*a^2*x^(9/2) + 2/13*(2*B*a*b + A*b^2 + 2*A*a*
c)*x^(13/2) + 2/11*(B*a^2 + 2*A*a*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{19}\,Bc^2x^{\frac{19}{2}}+\frac{4}{17}\,Bbcx^{\frac{17}{2}}+\frac{2}{17}\,Ac^2x^{\frac{17}{2}}+\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{4}{15}\,Bacx^{\frac{15}{2}}+\frac{4}{15}\,Abcx^{\frac{15}{2}}+\frac{4}{13}\,Babx^{\frac{13}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{4}{13}\,Aacx^{\frac{13}{2}}+\frac{2}{11}\,Ba^2x^{\frac{11}{2}}+\frac{4}{11}\,Aabx^{\frac{11}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

input

```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output

```
2/19*B*c^2*x^(19/2) + 4/17*B*b*c*x^(17/2) + 2/17*A*c^2*x^(17/2) + 2/15*B*b
^2*x^(15/2) + 4/15*B*a*c*x^(15/2) + 4/15*A*b*c*x^(15/2) + 4/13*B*a*b*x^(13
/2) + 2/13*A*b^2*x^(13/2) + 4/13*A*a*c*x^(13/2) + 2/11*B*a^2*x^(11/2) + 4/
11*A*a*b*x^(11/2) + 2/9*A*a^2*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=x^{11/2}\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{17/2}\left(\frac{2\,A\,c^2}{17}+\frac{4\,B\,b\,c}{17}\right)+x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}+\frac{4\,A\,a\,c}{13}\right)+x^{15/2}\left(\frac{2\,B\,b^2}{15}+\frac{4\,A\,c\,b}{15}+\frac{4\,B\,a\,c}{15}\right)+\frac{2\,A\,a^2\,x^9}{9}$$

input
```
int(x^(7/2)*(A + B*x)*(a + b*x + c*x^2)^2,x)
```

output
```
x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(17/2)*((2*A*c^2)/17 + (4*B*b*c
)/17) + x^(13/2)*((2*A*b^2)/13 + (4*A*a*c)/13 + (4*B*a*b)/13) + x^(15/2)*(
(2*B*b^2)/15 + (4*A*b*c)/15 + (4*B*a*c)/15) + (2*A*a^2*x^(9/2))/9 + (2*B*c
^2*x^(19/2))/19
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2\sqrt{x}\,x^4(109395b\,c^2x^5+122265a\,c^2x^4+244530b^2c\,x^4+554268abc\,x^3+138567b^3x^3+319770a}{2078505}$$

input
```
int(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^2,x)
```

output
```
(2*sqrt(x)*x**4*(230945*a**3 + 566865*a**2*b*x + 319770*a**2*c*x**2 + 4796
55*a*b**2*x**2 + 554268*a*b*c*x**3 + 122265*a*c**2*x**4 + 138567*b**3*x**3
 + 244530*b**2*c*x**4 + 109395*b*c**2*x**5))/2078505
```

3.66 $\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{7}a^2Ax^{7/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{11}\left(2abB+A\left(b^2+2ac\right)\right)x^{11/2}+\frac{2}{13}\left(b^2B+2Abc+2aBc\right)x^{13/2}+\frac{2}{15}c(2bB+Ac)x^{15/2}+\frac{2}{17}Bc^2x^{17/2}$$

output
```
2/7*a^2*A*x^(7/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/11*(2*a*b*B+A*(2*a*c+b^2))*x
^(11/2)+2/13*(2*A*b*c+2*B*a*c+B*b^2)*x^(13/2)+2/15*c*(A*c+2*B*b)*x^(15/2)+
2/17*B*c^2*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2x^{7/2}(12155a^2(9A+7Bx)+1190ax(13A(11b+9cx)+9Bx(13b+11cx))+21x^2(17A(195b^2}{765765}$$

input
```
Integrate[x^(5/2)*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(2*x^(7/2)*(12155*a^2*(9*A + 7*B*x) + 1190*a*x*(13*A*(11*b + 9*c*x) + 9*B*
x*(13*b + 11*c*x)) + 21*x^2*(17*A*(195*b^2 + 330*b*c*x + 143*c^2*x^2) + 11
*B*x*(255*b^2 + 442*b*c*x + 195*c^2*x^2))))/765765
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^2Ax^{5/2}+x^{11/2}\left(2aBc+2Abc+b^2B\right)+x^{9/2}\left(A\left(2ac+b^2\right)+2abB\right)+ax^{7/2}(aB+2Ab)+cx^{13/2}(Ac+2bB$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{13}x^{13/2}\left(2aBc+2Abc+b^2B\right)+\frac{2}{11}x^{11/2}\left(A\left(2ac+b^2\right)+2abB\right)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{15}cx^{15/2}(Ac+2bB)+\frac{2}{17}Bc^2x^{17/2}$$

input
```
Int[x^(5/2)*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*(2*a*b*B + A*(b^2
 + 2*a*c))*x^(11/2))/11 + (2*(b^2*B + 2*A*b*c + 2*a*B*c)*x^(13/2))/13 + (2
*c*(2*b*B + A*c)*x^(15/2))/15 + (2*B*c^2*x^(17/2))/17
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2Bc^{2}x^{\frac{17}{2}}}{17}+\frac{2(Ac^{2}+2Bbc)x^{\frac{15}{2}}}{15}+\frac{2(2Abc+B(2ac+b^{2}))x^{\frac{13}{2}}}{13}+\frac{2(2abB+A(2ac+b^{2}))x^{\frac{11}{2}}}{11}+\frac{2(2abA+a^{2}B)x^{\frac{9}{2}}}{9}$
default	$\frac{2Bc^{2}x^{\frac{17}{2}}}{17}+\frac{2(Ac^{2}+2Bbc)x^{\frac{15}{2}}}{15}+\frac{2(2Abc+B(2ac+b^{2}))x^{\frac{13}{2}}}{13}+\frac{2(2abB+A(2ac+b^{2}))x^{\frac{11}{2}}}{11}+\frac{2(2abA+a^{2}B)x^{\frac{9}{2}}}{9}$
gosper	$\frac{2x^{\frac{7}{2}}(45045Bc^{2}x^{5}+51051x^{4}Ac^{2}+102102x^{4}Bbc+117810x^{3}Abc+117810Bacx^{3}+58905x^{3}Bb^{2}+139230Aacx^{2}+69615}{765765}$
trager	$\frac{2x^{\frac{7}{2}}(45045Bc^{2}x^{5}+51051x^{4}Ac^{2}+102102x^{4}Bbc+117810x^{3}Abc+117810Bacx^{3}+58905x^{3}Bb^{2}+139230Aacx^{2}+69615}{765765}$
risch	$\frac{2x^{\frac{7}{2}}(45045Bc^{2}x^{5}+51051x^{4}Ac^{2}+102102x^{4}Bbc+117810x^{3}Abc+117810Bacx^{3}+58905x^{3}Bb^{2}+139230Aacx^{2}+69615}{765765}$
orering	$\frac{2x^{\frac{7}{2}}(45045Bc^{2}x^{5}+51051x^{4}Ac^{2}+102102x^{4}Bbc+117810x^{3}Abc+117810Bacx^{3}+58905x^{3}Bb^{2}+139230Aacx^{2}+69615}{765765}$

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/17*B*c^2*x^(17/2)+2/15*(A*c^2+2*B*b*c)*x^(15/2)+2/13*(2*A*b*c+B*(2*a*c+b
^2))*x^(13/2)+2/11*(2*a*b*B+A*(2*a*c+b^2))*x^(11/2)+2/9*(2*A*a*b+B*a^2)*x^
(9/2)+2/7*a^2*A*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{765765}\left(45045\,Bc^2x^8+51051\left(2\,Bbc+Ac^2\right)x^7+58905\left(Bb^2+2\left(Ba+Ab\right)c\right)x^6+109395\,A\right.$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
2/765765*(45045*B*c^2*x^8 + 51051*(2*B*b*c + A*c^2)*x^7 + 58905*(B*b^2 + 2
*(B*a + A*b)*c)*x^6 + 109395*A*a^2*x^3 + 69615*(2*B*a*b + A*b^2 + 2*A*a*c)
*x^5 + 85085*(B*a^2 + 2*A*a*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.43

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{9}{2}}}{9}+\frac{4Aacx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{4Abcx^{\frac{13}{2}}}{13}+\frac{2Ac^2x^{\frac{15}{2}}}{15}+\frac{2Ba^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{4Bacx^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{13}{2}}}{13}+\frac{4Bbcx^{\frac{15}{2}}}{15}+\frac{2Bc^2x^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(5/2)*(B*x+A)*(c*x**2+b*x+a)**2,x)
```

output
```
2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(9/2)/9 + 4*A*a*c*x**(11/2)/11 + 2*A*b**2
*x**(11/2)/11 + 4*A*b*c*x**(13/2)/13 + 2*A*c**2*x**(15/2)/15 + 2*B*a**2*x*
*(9/2)/9 + 4*B*a*b*x**(11/2)/11 + 4*B*a*c*x**(13/2)/13 + 2*B*b**2*x**(13/2
)/13 + 4*B*b*c*x**(15/2)/15 + 2*B*c**2*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{17}Bc^2x^{\frac{17}{2}}+\frac{2}{15}\left(2\,Bbc+Ac^2\right)x^{\frac{15}{2}}+\frac{2}{13}\left(Bb^2+2\,(Ba+Ab)c\right)x^{\frac{13}{2}}+\frac{2}{7}Aa^2x^{\frac{7}{2}}+\frac{2}{11}\left(2\,Bab+Ab^2+2\,Aac\right)x^{\frac{11}{2}}+\frac{2}{9}\left(Ba^2+2\,Aab\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
2/17*B*c^2*x^(17/2) + 2/15*(2*B*b*c + A*c^2)*x^(15/2) + 2/13*(B*b^2 + 2*(B
*a + A*b)*c)*x^(13/2) + 2/7*A*a^2*x^(7/2) + 2/11*(2*B*a*b + A*b^2 + 2*A*a*
c)*x^(11/2) + 2/9*(B*a^2 + 2*A*a*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{17}Bc^2x^{\frac{17}{2}}+\frac{4}{15}Bbcx^{\frac{15}{2}}+\frac{2}{15}Ac^2x^{\frac{15}{2}}+\frac{2}{13}Bb^2x^{\frac{13}{2}}+\frac{4}{13}Bacx^{\frac{13}{2}}+\frac{4}{13}Abcx^{\frac{13}{2}}+\frac{4}{11}Babx^{\frac{11}{2}}+\frac{2}{11}Ab^2x^{\frac{11}{2}}+\frac{4}{11}Aacx^{\frac{11}{2}}+\frac{2}{9}Ba^2x^{\frac{9}{2}}+\frac{4}{9}Aabx^{\frac{9}{2}}+\frac{2}{7}Aa^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
2/17*B*c^2*x^(17/2) + 4/15*B*b*c*x^(15/2) + 2/15*A*c^2*x^(15/2) + 2/13*B*b
^2*x^(13/2) + 4/13*B*a*c*x^(13/2) + 4/13*A*b*c*x^(13/2) + 4/11*B*a*b*x^(11
/2) + 2/11*A*b^2*x^(11/2) + 4/11*A*a*c*x^(11/2) + 2/9*B*a^2*x^(9/2) + 4/9*
A*a*b*x^(9/2) + 2/7*A*a^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=x^{9/2}\left(\frac{2\,B\,a^2}{9}+\frac{4\,A\,b\,a}{9}\right)$$
$$+x^{15/2}\left(\frac{2\,A\,c^2}{15}+\frac{4\,B\,b\,c}{15}\right)+x^{11/2}\left(\frac{2\,A\,b^2}{11}+\frac{4\,B\,a\,b}{11}+\frac{4\,A\,a\,c}{11}\right)+x^{13/2}\left(\frac{2\,B\,b^2}{13}+\frac{4\,A\,c\,b}{13}+\frac{4\,B\,a\,c}{13}\right)+\frac{2\,A\,a^2\,x^{7}}{7}$$

input
```
int(x^(5/2)*(A + B*x)*(a + b*x + c*x^2)^2,x)
```

output
```
x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(15/2)*((2*A*c^2)/15 + (4*B*b*c)/1
5) + x^(11/2)*((2*A*b^2)/11 + (4*A*a*c)/11 + (4*B*a*b)/11) + x^(13/2)*((2*
B*b^2)/13 + (4*A*b*c)/13 + (4*B*a*c)/13) + (2*A*a^2*x^(7/2))/7 + (2*B*c^2*
x^(17/2))/17
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int x^{5/2}(A+Bx)\left(a+bx\right.$$
$$\left.+cx^2\right)^2\,dx=\frac{2\sqrt{x}\,x^3(15015b\,c^2x^5+17017a\,c^2x^4+34034b^2c\,x^4+78540abc\,x^3+19635b^3x^3+46410a^2c\,x^2+}{255255}$$

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^2,x)
```

output
```
(2*sqrt(x)*x**3*(36465*a**3 + 85085*a**2*b*x + 46410*a**2*c*x**2 + 69615*a
*b**2*x**2 + 78540*a*b*c*x**3 + 17017*a*c**2*x**4 + 19635*b**3*x**3 + 3403
4*b**2*c*x**4 + 15015*b*c**2*x**5))/255255
```

3.67 $\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{9}\left(2abB+A\left(b^2+2ac\right)\right)x^{9/2}+\frac{2}{11}\left(b^2B+2Abc+2aBc\right)x^{11/2}+\frac{2}{13}c(2bB+Ac)x^{13/2}+\frac{2}{15}Bc^2x^{15/2}$$

output
```
2/5*a^2*A*x^(5/2)+2/7*a*(2*A*b+B*a)*x^(7/2)+2/9*(2*a*b*B+A*(2*a*c+b^2))*x^
(9/2)+2/11*(2*A*b*c+2*B*a*c+B*b^2)*x^(11/2)+2/13*c*(A*c+2*B*b)*x^(13/2)+2/
15*B*c^2*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2x^{5/2}(1287a^2(7A+5Bx)+130ax(11A(9b+7cx)+7Bx(11b+9cx))+7x^2(5A(143b^2+234\cdots}{45045}$$

input
```
Integrate[x^(3/2)*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(2*x^(5/2)*(1287*a^2*(7*A + 5*B*x) + 130*a*x*(11*A*(9*b + 7*c*x) + 7*B*x*(
11*b + 9*c*x)) + 7*x^2*(5*A*(143*b^2 + 234*b*c*x + 99*c^2*x^2) + 3*B*x*(19
5*b^2 + 330*b*c*x + 143*c^2*x^2))))/45045
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^2Ax^{3/2}+x^{9/2}\left(2aBc+2Abc+b^2B\right)+x^{7/2}\left(A\left(2ac+b^2\right)+2abB\right)+ax^{5/2}(aB+2Ab)+cx^{11/2}(Ac+2bB)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{11}x^{11/2}\left(2aBc+2Abc+b^2B\right)+\frac{2}{9}x^{9/2}\left(A\left(2ac+b^2\right)+2abB\right)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{13}cx^{13/2}(Ac+2bB)+\frac{2}{15}Bc^2x^{15/2}$$

input
```
Int[x^(3/2)*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*(2*a*b*B + A*(b^2
 + 2*a*c))*x^(9/2))/9 + (2*(b^2*B + 2*A*b*c + 2*a*B*c)*x^(11/2))/11 + (2*c
*(2*b*B + A*c)*x^(13/2))/13 + (2*B*c^2*x^(15/2))/15
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2B\,c^{2}x^{\frac{15}{2}}}{15}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{13}{2}}}{13}+\frac{2(2Abc+B(2ac+b^{2}))x^{\frac{11}{2}}}{11}+\frac{2(2abB+A(2ac+b^{2}))x^{\frac{9}{2}}}{9}+\frac{2(2abA+a^{2}B)x^{\frac{7}{2}}}{7}$
default	$\frac{2B\,c^{2}x^{\frac{15}{2}}}{15}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{13}{2}}}{13}+\frac{2(2Abc+B(2ac+b^{2}))x^{\frac{11}{2}}}{11}+\frac{2(2abB+A(2ac+b^{2}))x^{\frac{9}{2}}}{9}+\frac{2(2abA+a^{2}B)x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{5}{2}}(3003B\,c^{2}x^{5}+3465x^{4}A\,c^{2}+6930x^{4}Bbc+8190x^{3}Abc+8190Bac\,x^{3}+4095x^{3}B\,b^{2}+10010Aac\,x^{2}+5005x^{2}b^{2}A+1001}{45045}$
trager	$\frac{2x^{\frac{5}{2}}(3003B\,c^{2}x^{5}+3465x^{4}A\,c^{2}+6930x^{4}Bbc+8190x^{3}Abc+8190Bac\,x^{3}+4095x^{3}B\,b^{2}+10010Aac\,x^{2}+5005x^{2}b^{2}A+1001}{45045}$
risch	$\frac{2x^{\frac{5}{2}}(3003B\,c^{2}x^{5}+3465x^{4}A\,c^{2}+6930x^{4}Bbc+8190x^{3}Abc+8190Bac\,x^{3}+4095x^{3}B\,b^{2}+10010Aac\,x^{2}+5005x^{2}b^{2}A+1001}{45045}$
orering	$\frac{2x^{\frac{5}{2}}(3003B\,c^{2}x^{5}+3465x^{4}A\,c^{2}+6930x^{4}Bbc+8190x^{3}Abc+8190Bac\,x^{3}+4095x^{3}B\,b^{2}+10010Aac\,x^{2}+5005x^{2}b^{2}A+1001}{45045}$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/15*B*c^2*x^(15/2)+2/13*(A*c^2+2*B*b*c)*x^(13/2)+2/11*(2*A*b*c+B*(2*a*c+b
^2))*x^(11/2)+2/9*(2*a*b*B+A*(2*a*c+b^2))*x^(9/2)+2/7*(2*A*a*b+B*a^2)*x^(7
/2)+2/5*a^2*A*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{45045}\left(3003\,Bc^2x^7+3465\left(2\,Bbc+Ac^2\right)x^6+4095\left(Bb^2+2\,(Ba+Ab)c\right)x^5+9009\,Aa^2x^2+\right.$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
2/45045*(3003*B*c^2*x^7 + 3465*(2*B*b*c + A*c^2)*x^6 + 4095*(B*b^2 + 2*(B*
a + A*b)*c)*x^5 + 9009*A*a^2*x^2 + 5005*(2*B*a*b + A*b^2 + 2*A*a*c)*x^4 +
6435*(B*a^2 + 2*A*a*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.43

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{7}{2}}}{7}+\frac{4Aacx^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{4Abcx^{\frac{11}{2}}}{11}+\frac{2Ac^2x^{\frac{13}{2}}}{13}+\frac{2Ba^2x^{\frac{7}{2}}}{7}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{4Bacx^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{11}{2}}}{11}+\frac{4Bbcx^{\frac{13}{2}}}{13}+\frac{2Bc^2x^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(3/2)*(B*x+A)*(c*x**2+b*x+a)**2,x)
```

output
```
2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(7/2)/7 + 4*A*a*c*x**(9/2)/9 + 2*A*b**2*x
**(9/2)/9 + 4*A*b*c*x**(11/2)/11 + 2*A*c**2*x**(13/2)/13 + 2*B*a**2*x**(7/
2)/7 + 4*B*a*b*x**(9/2)/9 + 4*B*a*c*x**(11/2)/11 + 2*B*b**2*x**(11/2)/11 +
 4*B*b*c*x**(13/2)/13 + 2*B*c**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{2}{13}\left(2\,Bbc+Ac^2\right)x^{\frac{13}{2}}+\frac{2}{11}\left(Bb^2+2\,(Ba+Ab)c\right)x^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}+\frac{2}{9}\left(2\,Bab+Ab^2+2\,Aac\right)x^{\frac{9}{2}}+\frac{2}{7}\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
2/15*B*c^2*x^(15/2) + 2/13*(2*B*b*c + A*c^2)*x^(13/2) + 2/11*(B*b^2 + 2*(B
*a + A*b)*c)*x^(11/2) + 2/5*A*a^2*x^(5/2) + 2/9*(2*B*a*b + A*b^2 + 2*A*a*c
)*x^(9/2) + 2/7*(B*a^2 + 2*A*a*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{4}{13}\,Bbcx^{\frac{13}{2}}+\frac{2}{13}\,Ac^2x^{\frac{13}{2}}+\frac{2}{11}\,Bb^2x^{\frac{11}{2}}+\frac{4}{11}\,Bacx^{\frac{11}{2}}+\frac{4}{11}\,Abcx^{\frac{11}{2}}+\frac{4}{9}\,Babx^{\frac{9}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}+\frac{4}{9}\,Aacx^{\frac{9}{2}}+\frac{2}{7}\,Ba^2x^{\frac{7}{2}}+\frac{4}{7}\,Aabx^{\frac{7}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
2/15*B*c^2*x^(15/2) + 4/13*B*b*c*x^(13/2) + 2/13*A*c^2*x^(13/2) + 2/11*B*b
^2*x^(11/2) + 4/11*B*a*c*x^(11/2) + 4/11*A*b*c*x^(11/2) + 4/9*B*a*b*x^(9/2
) + 2/9*A*b^2*x^(9/2) + 4/9*A*a*c*x^(9/2) + 2/7*B*a^2*x^(7/2) + 4/7*A*a*b*
x^(7/2) + 2/5*A*a^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right)+x^{13/2}\left(\frac{2\,A\,c^2}{13}+\frac{4\,B\,b\,c}{13}\right)+x^{9/2}\left(\frac{2\,A\,b^2}{9}+\frac{4\,B\,a\,b}{9}+\frac{4\,A\,a\,c}{9}\right)+x^{11/2}\left(\frac{2\,B\,b^2}{11}+\frac{4\,A\,c\,b}{11}+\frac{4\,B\,a\,c}{11}\right)+\frac{2\,A\,a^2\,x^{5/}}{5}$$

input
```
int(x^(3/2)*(A + B*x)*(a + b*x + c*x^2)^2,x)
```

output
```
x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(13/2)*((2*A*c^2)/13 + (4*B*b*c)/1
3) + x^(9/2)*((2*A*b^2)/9 + (4*A*a*c)/9 + (4*B*a*b)/9) + x^(11/2)*((2*B*b^
2)/11 + (4*A*b*c)/11 + (4*B*a*c)/11) + (2*A*a^2*x^(5/2))/5 + (2*B*c^2*x^(1
5/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2\sqrt{x}\,x^2(3003b\,c^2x^5+3465a\,c^2x^4+6930b^2c\,x^4+16380abc\,x^3+4095b^3x^3+10010a^2c\,x^2+150}{45045}$$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^2,x)
```

output
```
(2*sqrt(x)*x**2*(9009*a**3 + 19305*a**2*b*x + 10010*a**2*c*x**2 + 15015*a*
b**2*x**2 + 16380*a*b*c*x**3 + 3465*a*c**2*x**4 + 4095*b**3*x**3 + 6930*b*
*2*c*x**4 + 3003*b*c**2*x**5))/45045
```

3.68 $\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{2}{3}a^2Ax^{3/2}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{7}\left(2abB+A\left(b^2+2ac\right)\right)x^{7/2} + \frac{2}{9}\left(b^2B+2Abc+2aBc\right)x^{9/2}+\frac{2}{11}c(2bB+Ac)x^{11/2}+\frac{2}{13}Bc^2x^{13/2}$$

output
```
2/3*a^2*A*x^(3/2)+2/5*a*(2*A*b+B*a)*x^(5/2)+2/7*(2*a*b*B+A*(2*a*c+b^2))*x^
(7/2)+2/9*(2*A*b*c+2*B*a*c+B*b^2)*x^(9/2)+2/11*c*(A*c+2*B*b)*x^(11/2)+2/13
*B*c^2*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{2x^{3/2}(3003a^2(5A+3Bx)+286ax(9A(7b+5cx)+5Bx(9b+7cx))+5x^2(13A(99b^2+154bcx+63c^2x^2)}{45045}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(2*x^(3/2)*(3003*a^2*(5*A + 3*B*x) + 286*a*x*(9*A*(7*b + 5*c*x) + 5*B*x*(9
*b + 7*c*x)) + 5*x^2*(13*A*(99*b^2 + 154*b*c*x + 63*c^2*x^2) + 7*B*x*(143*
b^2 + 234*b*c*x + 99*c^2*x^2))))/45045
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^2A\sqrt{x}+x^{7/2}\left(2aBc+2Abc+b^2B\right)+x^{5/2}\left(A\left(2ac+b^2\right)+2abB\right)+ax^{3/2}(aB+2Ab)+cx^{9/2}(Ac+2bB)+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{9}x^{9/2}\left(2aBc+2Abc+b^2B\right)+\frac{2}{7}x^{7/2}\left(A\left(2ac+b^2\right)+2abB\right)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{11}cx^{11/2}(Ac+2bB)+\frac{2}{13}Bc^2x^{13/2}$$

input
```
Int[Sqrt[x]*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*(2*a*b*B + A*(b^2
 + 2*a*c))*x^(7/2))/7 + (2*(b^2*B + 2*A*b*c + 2*a*B*c)*x^(9/2))/9 + (2*c*(
2*b*B + A*c)*x^(11/2))/11 + (2*B*c^2*x^(13/2))/13
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2Bc^2x^{\frac{13}{2}}}{13}+\frac{2(Ac^2+2Bbc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+B(2ac+b^2))x^{\frac{9}{2}}}{9}+\frac{2(2abB+A(2ac+b^2))x^{\frac{7}{2}}}{7}+\frac{2(2abA+a^2B)x^{\frac{5}{2}}}{5}$
default	$\frac{2Bc^2x^{\frac{13}{2}}}{13}+\frac{2(Ac^2+2Bbc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+B(2ac+b^2))x^{\frac{9}{2}}}{9}+\frac{2(2abB+A(2ac+b^2))x^{\frac{7}{2}}}{7}+\frac{2(2abA+a^2B)x^{\frac{5}{2}}}{5}$
gosper	$\frac{2x^{\frac{3}{2}}(3465Bc^2x^5+4095x^4Ac^2+8190x^4Bbc+10010x^3Abc+10010Bacx^3+5005x^3Bb^2+12870Aacx^2+6435x^2b^2A+12}{45045}$
trager	$\frac{2x^{\frac{3}{2}}(3465Bc^2x^5+4095x^4Ac^2+8190x^4Bbc+10010x^3Abc+10010Bacx^3+5005x^3Bb^2+12870Aacx^2+6435x^2b^2A+12}{45045}$
risch	$\frac{2x^{\frac{3}{2}}(3465Bc^2x^5+4095x^4Ac^2+8190x^4Bbc+10010x^3Abc+10010Bacx^3+5005x^3Bb^2+12870Aacx^2+6435x^2b^2A+12}{45045}$
orering	$\frac{2x^{\frac{3}{2}}(3465Bc^2x^5+4095x^4Ac^2+8190x^4Bbc+10010x^3Abc+10010Bacx^3+5005x^3Bb^2+12870Aacx^2+6435x^2b^2A+12}{45045}$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/13*B*c^2*x^(13/2)+2/11*(A*c^2+2*B*b*c)*x^(11/2)+2/9*(2*A*b*c+B*(2*a*c+b^
2))*x^(9/2)+2/7*(2*a*b*B+A*(2*a*c+b^2))*x^(7/2)+2/5*(2*A*a*b+B*a^2)*x^(5/2
)+2/3*a^2*A*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$
$$=\frac{2}{45045}\left(3465\,Bc^2x^6+4095\left(2\,Bbc+Ac^2\right)x^5+5005\left(Bb^2+2\,(Ba+Ab)c\right)x^4+15015\,Aa^2x+6435\,(2\,B\right.$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
2/45045*(3465*B*c^2*x^6 + 4095*(2*B*b*c + A*c^2)*x^5 + 5005*(B*b^2 + 2*(B*
a + A*b)*c)*x^4 + 15015*A*a^2*x + 6435*(2*B*a*b + A*b^2 + 2*A*a*c)*x^3 + 9
009*(B*a^2 + 2*A*a*b)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{2Bc^2x^{\frac{13}{2}}}{13}+\frac{2x^{\frac{11}{2}}\left(Ac^2+2Bbc\right)}{11}+\frac{2x^{\frac{9}{2}}\cdot\left(2Abc+2Bac+Bb^2\right)}{9}+\frac{2x^{\frac{7}{2}}\cdot\left(2Aac+Ab^2+2Bab\right)}{7}+\frac{2x^{\frac{5}{2}}\cdot\left(2Aab+Ba^2\right)}{5}$$

input
```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x+a)**2,x)
```

output
```
2*A*a**2*x**(3/2)/3 + 2*B*c**2*x**(13/2)/13 + 2*x**(11/2)*(A*c**2 + 2*B*b*
c)/11 + 2*x**(9/2)*(2*A*b*c + 2*B*a*c + B*b**2)/9 + 2*x**(7/2)*(2*A*a*c +
A*b**2 + 2*B*a*b)/7 + 2*x**(5/2)*(2*A*a*b + B*a**2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{2}{13}\,Bc^2x^{\frac{13}{2}} + \frac{2}{11}\left(2\,Bbc + Ac^2\right)x^{\frac{11}{2}} + \frac{2}{9}\left(Bb^2 + 2\,(Ba + Ab)c\right)x^{\frac{9}{2}} + \frac{2}{3}\,Aa^2x^{\frac{3}{2}} + \frac{2}{7}\left(2\,Bab + Ab^2 + 2\,Aac\right)x^{\frac{7}{2}} + \frac{2}{5}\left(Ba^2 + 2\,Aab\right)x^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
2/13*B*c^2*x^(13/2) + 2/11*(2*B*b*c + A*c^2)*x^(11/2) + 2/9*(B*b^2 + 2*(B*
a + A*b)*c)*x^(9/2) + 2/3*A*a^2*x^(3/2) + 2/7*(2*B*a*b + A*b^2 + 2*A*a*c)*
x^(7/2) + 2/5*(B*a^2 + 2*A*a*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{2}{13}\,Bc^2x^{\frac{13}{2}} + \frac{4}{11}\,Bbcx^{\frac{11}{2}} + \frac{2}{11}\,Ac^2x^{\frac{11}{2}} + \frac{2}{9}\,Bb^2x^{\frac{9}{2}} + \frac{4}{9}\,Bacx^{\frac{9}{2}} + \frac{4}{9}\,Abcx^{\frac{9}{2}} + \frac{4}{7}\,Babx^{\frac{7}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}} + \frac{4}{7}\,Aacx^{\frac{7}{2}} + \frac{2}{5}\,Ba^2x^{\frac{5}{2}} + \frac{4}{5}\,Aabx^{\frac{5}{2}} + \frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
2/13*B*c^2*x^(13/2) + 4/11*B*b*c*x^(11/2) + 2/11*A*c^2*x^(11/2) + 2/9*B*b^
2*x^(9/2) + 4/9*B*a*c*x^(9/2) + 4/9*A*b*c*x^(9/2) + 4/7*B*a*b*x^(7/2) + 2/
7*A*b^2*x^(7/2) + 4/7*A*a*c*x^(7/2) + 2/5*B*a^2*x^(5/2) + 4/5*A*a*b*x^(5/2
) + 2/3*A*a^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx = x^{5/2}\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right)+x^{11/2}\left(\frac{2\,A\,c^2}{11}+\frac{4\,B\,b\,c}{11}\right) +x^{7/2}\left(\frac{2\,A\,b^2}{7}+\frac{4\,B\,a\,b}{7}+\frac{4\,A\,a\,c}{7}\right)+x^{9/2}\left(\frac{2\,B\,b^2}{9}+\frac{4\,A\,c\,b}{9}+\frac{4\,B\,a\,c}{9}\right)+\frac{2\,A\,a^2\,x^{3/2}}{3}+\frac{2\,B\,c^2\,x^{13/2}}{13}$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x + c*x^2)^2,x)
```

output
```
x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(11/2)*((2*A*c^2)/11 + (4*B*b*c)/1
1) + x^(7/2)*((2*A*b^2)/7 + (4*A*a*c)/7 + (4*B*a*b)/7) + x^(9/2)*((2*B*b^2
)/9 + (4*A*b*c)/9 + (4*B*a*c)/9) + (2*A*a^2*x^(3/2))/3 + (2*B*c^2*x^(13/2)
)/13
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.70

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^2\,dx = \frac{2\sqrt{x}\,x(3465b\,c^2x^5+4095a\,c^2x^4+8190b^2c\,x^4+20020abc\,x^3+5005b^3x^3+12870a^2c\,x^2+19305a\,b^2x^2+}{45045}$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^2,x)
```

output
```
(2*sqrt(x)*x*(15015*a**3 + 27027*a**2*b*x + 12870*a**2*c*x**2 + 19305*a*b*
*2*x**2 + 20020*a*b*c*x**3 + 4095*a*c**2*x**4 + 5005*b**3*x**3 + 8190*b**2
*c*x**4 + 3465*b*c**2*x**5))/45045
```

3.69 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx &= 2a^2A\sqrt{x}+\frac{2}{3}a(2Ab+aB)x^{3/2}\\ &\quad+\frac{2}{5}\left(2abB+A\left(b^2+2ac\right)\right)x^{5/2}\\ &\quad+\frac{2}{7}\left(b^2B+2Abc+2aBc\right)x^{7/2}\\ &\quad+\frac{2}{9}c(2bB+Ac)x^{9/2}+\frac{2}{11}Bc^2x^{11/2}\end{aligned}$$

output
```
2*a^2*A*x^(1/2)+2/3*a*(2*A*b+B*a)*x^(3/2)+2/5*(2*a*b*B+A*(2*a*c+b^2))*x^(5
/2)+2/7*(2*A*b*c+2*B*a*c+B*b^2)*x^(7/2)+2/9*c*(A*c+2*B*b)*x^(9/2)+2/11*B*c
^2*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx$$
$$= \frac{2\sqrt{x}(1155a^2(3A+Bx)+66ax(7A(5b+3cx)+3Bx(7b+5cx))+x^2(11A(63b^2+90bcx+35c^2x^2)+5B\cdots}{3465}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(1155*a^2*(3*A + B*x) + 66*a*x*(7*A*(5*b + 3*c*x) + 3*B*x*(7*b
+ 5*c*x)) + x^2*(11*A*(63*b^2 + 90*b*c*x + 35*c^2*x^2) + 5*B*x*(99*b^2 + 1
54*b*c*x + 63*c^2*x^2))))/3465
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx$$
$$\Big\downarrow \text{1195}$$
$$\int \left(\frac{a^2A}{\sqrt{x}}+x^{5/2}\left(2aBc+2Abc+b^2B\right)+x^{3/2}\left(A\left(2ac+b^2\right)+2abB\right)+a\sqrt{x}(aB+2Ab)+cx^{7/2}(Ac+2bB)+Bc\cdots\right.$$
$$\Big\downarrow \text{2009}$$
$$2a^2A\sqrt{x}+\frac{2}{7}x^{7/2}\left(2aBc+2Abc+b^2B\right)+\frac{2}{5}x^{5/2}\left(A\left(2ac+b^2\right)+2abB\right)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{9}cx^{9/2}(Ac+2bB)+\frac{2}{11}Bc^2x^{11/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/Sqrt[x],x]
```

output
```
2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*(2*a*b*B + A*(b^2 + 2
*a*c))*x^(5/2))/5 + (2*(b^2*B + 2*A*b*c + 2*a*B*c)*x^(7/2))/7 + (2*c*(2*b*
B + A*c)*x^(9/2))/9 + (2*B*c^2*x^(11/2))/11
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2Bc^2x^{\frac{11}{2}}}{11}+\frac{2(Ac^2+2Bbc)x^{\frac{9}{2}}}{9}+\frac{2(2Abc+B(2ac+b^2))x^{\frac{7}{2}}}{7}+\frac{2(2abB+A(2ac+b^2))x^{\frac{5}{2}}}{5}+\frac{2(2abA+a^2B)x^{\frac{3}{2}}}{3}+$
default	$\frac{2Bc^2x^{\frac{11}{2}}}{11}+\frac{2(Ac^2+2Bbc)x^{\frac{9}{2}}}{9}+\frac{2(2Abc+B(2ac+b^2))x^{\frac{7}{2}}}{7}+\frac{2(2abB+A(2ac+b^2))x^{\frac{5}{2}}}{5}+\frac{2(2abA+a^2B)x^{\frac{3}{2}}}{3}+$
trager	$\left(\frac{2}{11}Bc^2x^5+\frac{2}{9}x^4Ac^2+\frac{4}{9}x^4Bbc+\frac{4}{7}x^3Abc+\frac{4}{7}Bacx^3+\frac{2}{7}x^3Bb^2+\frac{4}{5}Aacx^2+\frac{2}{5}x^2b^2A\right.$
gosper	$\frac{2\sqrt{x}\,(315Bc^2x^5+385x^4Ac^2+770x^4Bbc+990x^3Abc+990Bacx^3+495x^3Bb^2+1386Aacx^2+693x^2b^2A+1386Bax^2b+}{3465}$
risch	$\frac{2\sqrt{x}\,(315Bc^2x^5+385x^4Ac^2+770x^4Bbc+990x^3Abc+990Bacx^3+495x^3Bb^2+1386Aacx^2+693x^2b^2A+1386Bax^2b+}{3465}$
orering	$\frac{2\sqrt{x}\,(315Bc^2x^5+385x^4Ac^2+770x^4Bbc+990x^3Abc+990Bacx^3+495x^3Bb^2+1386Aacx^2+693x^2b^2A+1386Bax^2b+}{3465}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*B*c^2*x^(11/2)+2/9*(A*c^2+2*B*b*c)*x^(9/2)+2/7*(2*A*b*c+B*(2*a*c+b^2)
)*x^(7/2)+2/5*(2*a*b*B+A*(2*a*c+b^2))*x^(5/2)+2/3*(2*A*a*b+B*a^2)*x^(3/2)+
2*a^2*A*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx$$
$$=\frac{2}{3465}\left(315\,Bc^2x^5+385\left(2\,Bbc+Ac^2\right)x^4+495\left(Bb^2+2\left(Ba+Ab\right)c\right)x^3+3465\,Aa^2+693\left(2\,Bab+Ab^2\right.\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c^2*x^5 + 385*(2*B*b*c + A*c^2)*x^4 + 495*(B*b^2 + 2*(B*a +
A*b)*c)*x^3 + 3465*A*a^2 + 693*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 1155*(B*a
^2 + 2*A*a*b)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx=2Aa^2\sqrt{x}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{4Aacx^{\frac{5}{2}}}{5}+\frac{2Ab^2x^{\frac{5}{2}}}{5}+\frac{4Abcx^{\frac{7}{2}}}{7}+\frac{2Ac^2x^{\frac{9}{2}}}{9}+\frac{2Ba^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{5}{2}}}{5}+\frac{4Bacx^{\frac{7}{2}}}{7}+\frac{2Bb^2x^{\frac{7}{2}}}{7}+\frac{4Bbcx^{\frac{9}{2}}}{9}+\frac{2Bc^2x^{\frac{11}{2}}}{11}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**(1/2),x)
```

output
```
2*A*a**2*sqrt(x) + 4*A*a*b*x**(3/2)/3 + 4*A*a*c*x**(5/2)/5 + 2*A*b**2*x**(
5/2)/5 + 4*A*b*c*x**(7/2)/7 + 2*A*c**2*x**(9/2)/9 + 2*B*a**2*x**(3/2)/3 +
4*B*a*b*x**(5/2)/5 + 4*B*a*c*x**(7/2)/7 + 2*B*b**2*x**(7/2)/7 + 4*B*b*c*x*
*(9/2)/9 + 2*B*c**2*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx = \frac{2}{11}\,Bc^2x^{\frac{11}{2}} + \frac{2}{9}\left(2\,Bbc + Ac^2\right)x^{\frac{9}{2}} + \frac{2}{7}\left(Bb^2 + 2\,(Ba+Ab)c\right)x^{\frac{7}{2}} + 2\,Aa^2\sqrt{x} + \frac{2}{5}\left(2\,Bab + Ab^2 + 2\,Aac\right)x^{\frac{5}{2}} + \frac{2}{3}\left(Ba^2 + 2\,Aab\right)x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/11*B*c^2*x^(11/2) + 2/9*(2*B*b*c + A*c^2)*x^(9/2) + 2/7*(B*b^2 + 2*(B*a
+ A*b)*c)*x^(7/2) + 2*A*a^2*sqrt(x) + 2/5*(2*B*a*b + A*b^2 + 2*A*a*c)*x^(5
/2) + 2/3*(B*a^2 + 2*A*a*b)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx = \frac{2}{11}\,Bc^2x^{\frac{11}{2}} + \frac{4}{9}\,Bbcx^{\frac{9}{2}} + \frac{2}{9}\,Ac^2x^{\frac{9}{2}} + \frac{2}{7}\,Bb^2x^{\frac{7}{2}} + \frac{4}{7}\,Bacx^{\frac{7}{2}} + \frac{4}{7}\,Abcx^{\frac{7}{2}} + \frac{4}{5}\,Babx^{\frac{5}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}} + \frac{4}{5}\,Aacx^{\frac{5}{2}} + \frac{2}{3}\,Ba^2x^{\frac{3}{2}} + \frac{4}{3}\,Aabx^{\frac{3}{2}} + 2\,Aa^2\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/11*B*c^2*x^(11/2) + 4/9*B*b*c*x^(9/2) + 2/9*A*c^2*x^(9/2) + 2/7*B*b^2*x^
(7/2) + 4/7*B*a*c*x^(7/2) + 4/7*A*b*c*x^(7/2) + 4/5*B*a*b*x^(5/2) + 2/5*A*
b^2*x^(5/2) + 4/5*A*a*c*x^(5/2) + 2/3*B*a^2*x^(3/2) + 4/3*A*a*b*x^(3/2) +
2*A*a^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx = x^{3/2}\left(\frac{2\,B\,a^2}{3}+\frac{4\,A\,b\,a}{3}\right)+x^{9/2}\left(\frac{2\,A\,c^2}{9}+\frac{4\,B\,b\,c}{9}\right) +x^{5/2}\left(\frac{2\,A\,b^2}{5}+\frac{4\,B\,a\,b}{5}+\frac{4\,A\,a\,c}{5}\right)+x^{7/2}\left(\frac{2\,B\,b^2}{7}+\frac{4\,A\,c\,b}{7}+\frac{4\,B\,a\,c}{7}\right)+2\,A\,a^2\,\sqrt{x}+\frac{2\,B\,c^2\,x^{11/2}}{11}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^(1/2),x)
```

output
```
x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(9/2)*((2*A*c^2)/9 + (4*B*b*c)/9)
+ x^(5/2)*((2*A*b^2)/5 + (4*A*a*c)/5 + (4*B*a*b)/5) + x^(7/2)*((2*B*b^2)/7
 + (4*A*b*c)/7 + (4*B*a*c)/7) + 2*A*a^2*x^(1/2) + (2*B*c^2*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(315b\,c^2x^5+385a\,c^2x^4+770b^2c\,x^4+1980abc\,x^3+495b^3x^3+1386a^2c\,x^2+2079a\,b^2x^2+3465a^2bx}{3465}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(1/2),x)
```

output
```
(2*sqrt(x)*(3465*a**3 + 3465*a**2*b*x + 1386*a**2*c*x**2 + 2079*a*b**2*x**
2 + 1980*a*b*c*x**3 + 385*a*c**2*x**4 + 495*b**3*x**3 + 770*b**2*c*x**4 +
315*b*c**2*x**5))/3465
```

3.70 $\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{3/2}}\,dx = -\frac{2a^2A}{\sqrt{x}} + 2a(2Ab+aB)\sqrt{x} + \frac{2}{3}\left(2abB+A\left(b^2+2ac\right)\right)x^{3/2} + \frac{2}{5}\left(b^2B+2Abc+2aBc\right)x^{5/2} + \frac{2}{7}c(2bB+Ac)x^{7/2} + \frac{2}{9}Bc^2x^{9/2}$$

output
```
-2*a^2*A/x^(1/2)+2*a*(2*A*b+B*a)*x^(1/2)+2/3*(2*a*b*B+A*(2*a*c+b^2))*x^(3/
2)+2/5*(2*A*b*c+2*B*a*c+B*b^2)*x^(5/2)+2/7*c*(A*c+2*B*b)*x^(7/2)+2/9*B*c^2
*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{3/2}}\,dx = \frac{-630a^2(A-Bx)+84ax(5A(3b+cx)+Bx(5b+3cx))+2x^2(3A(35b^2}{315\sqrt{x}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^(3/2),x]
```

output
```
(-630*a^2*(A - B*x) + 84*a*x*(5*A*(3*b + c*x) + B*x*(5*b + 3*c*x)) + 2*x^2
*(3*A*(35*b^2 + 42*b*c*x + 15*c^2*x^2) + B*x*(63*b^2 + 90*b*c*x + 35*c^2*x
^2)))/(315*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{3/2}}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{a^2A}{x^{3/2}}+x^{3/2}\left(2aBc+2Abc+b^2B\right)+\sqrt{x}\left(A\left(2ac+b^2\right)+2abB\right)+\frac{a(aB+2Ab)}{\sqrt{x}}+cx^{5/2}(Ac+2bB)+Bc^2x^7$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{5}x^{5/2}\left(2aBc+2Abc+b^2B\right)+\frac{2}{3}x^{3/2}\left(A\left(2ac+b^2\right)+2abB\right)+2a\sqrt{x}(aB+2Ab)+\frac{2}{7}cx^{7/2}(Ac+2bB)+\frac{2}{9}Bc^2x^{9/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^(3/2),x]
```

output
```
(-2*a^2*A)/Sqrt[x] + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*(2*a*b*B + A*(b^2 + 2*
a*c))*x^(3/2))/3 + (2*(b^2*B + 2*A*b*c + 2*a*B*c)*x^(5/2))/5 + (2*c*(2*b*B
 + A*c)*x^(7/2))/7 + (2*B*c^2*x^(9/2))/9
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{2(-35B\,c^2x^5-45x^4A\,c^2-90x^4Bbc-126x^3Abc-126Bac\,x^3-63x^3B\,b^2-210Aac\,x^2-105x^2b^2A-210Ba\,x^2b-630ab}{315\sqrt{x}}$
trager	$-\frac{2(-35B\,c^2x^5-45x^4A\,c^2-90x^4Bbc-126x^3Abc-126Bac\,x^3-63x^3B\,b^2-210Aac\,x^2-105x^2b^2A-210Ba\,x^2b-630ab}{315\sqrt{x}}$
risch	$-\frac{2(-35B\,c^2x^5-45x^4A\,c^2-90x^4Bbc-126x^3Abc-126Bac\,x^3-63x^3B\,b^2-210Aac\,x^2-105x^2b^2A-210Ba\,x^2b-630ab}{315\sqrt{x}}$
orering	$-\frac{2(-35B\,c^2x^5-45x^4A\,c^2-90x^4Bbc-126x^3Abc-126Bac\,x^3-63x^3B\,b^2-210Aac\,x^2-105x^2b^2A-210Ba\,x^2b-630ab}{315\sqrt{x}}$
derivativedivides	$\frac{2B\,c^2x^{\frac{9}{2}}}{9}+\frac{2A\,c^2x^{\frac{7}{2}}}{7}+\frac{4Bbc\,x^{\frac{7}{2}}}{7}+\frac{4Abc\,x^{\frac{5}{2}}}{5}+\frac{4Bac\,x^{\frac{5}{2}}}{5}+\frac{2B\,b^2x^{\frac{5}{2}}}{5}+\frac{4Aac\,x^{\frac{3}{2}}}{3}+\frac{2A\,b^2x^{\frac{3}{2}}}{3}+\frac{4Bab\,x^{\frac{3}{2}}}{3}$
default	$\frac{2B\,c^2x^{\frac{9}{2}}}{9}+\frac{2A\,c^2x^{\frac{7}{2}}}{7}+\frac{4Bbc\,x^{\frac{7}{2}}}{7}+\frac{4Abc\,x^{\frac{5}{2}}}{5}+\frac{4Bac\,x^{\frac{5}{2}}}{5}+\frac{2B\,b^2x^{\frac{5}{2}}}{5}+\frac{4Aac\,x^{\frac{3}{2}}}{3}+\frac{2A\,b^2x^{\frac{3}{2}}}{3}+\frac{4Bab\,x^{\frac{3}{2}}}{3}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(-35*B*c^2*x^5-45*A*c^2*x^4-90*B*b*c*x^4-126*A*b*c*x^3-126*B*a*c*x^
3-63*B*b^2*x^3-210*A*a*c*x^2-105*A*b^2*x^2-210*B*a*b*x^2-630*A*a*b*x-315*B
*a^2*x+315*A*a^2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{3/2}}\,dx = \frac{2\left(35\,Bc^2x^5+45\,(2\,Bbc+Ac^2)x^4+63\,(Bb^2+2\,(Ba+Ab)c)x^3-315\,A\right.}{315\,\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*c^2*x^5 + 45*(2*B*b*c + A*c^2)*x^4 + 63*(B*b^2 + 2*(B*a + A*b)
*c)*x^3 - 315*A*a^2 + 105*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 315*(B*a^2 + 2
*A*a*b)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{3/2}}\,dx = -\frac{2Aa^2}{\sqrt{x}}+4Aab\sqrt{x}+\frac{4Aacx^{\frac{3}{2}}}{3}+\frac{2Ab^2x^{\frac{3}{2}}}{3} + \frac{4Abcx^{\frac{5}{2}}}{5}+\frac{2Ac^2x^{\frac{7}{2}}}{7}+2Ba^2\sqrt{x}+\frac{4Babx^{\frac{3}{2}}}{3}+\frac{4Bacx^{\frac{5}{2}}}{5}+\frac{2Bb^2x^{\frac{5}{2}}}{5}+\frac{4Bbcx^{\frac{7}{2}}}{7}+\frac{2Bc^2x^{\frac{9}{2}}}{9}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**(3/2),x)
```

output
```
-2*A*a**2/sqrt(x) + 4*A*a*b*sqrt(x) + 4*A*a*c*x**(3/2)/3 + 2*A*b**2*x**(3/
2)/3 + 4*A*b*c*x**(5/2)/5 + 2*A*c**2*x**(7/2)/7 + 2*B*a**2*sqrt(x) + 4*B*a
*b*x**(3/2)/3 + 4*B*a*c*x**(5/2)/5 + 2*B*b**2*x**(5/2)/5 + 4*B*b*c*x**(7/2
)/7 + 2*B*c**2*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{3/2}}\,dx = \frac{2}{9}\,Bc^2x^{\frac{9}{2}} + \frac{2}{7}\left(2\,Bbc+Ac^2\right)x^{\frac{7}{2}} + \frac{2}{5}\left(Bb^2+2\,(Ba+Ab)c\right)x^{\frac{5}{2}} - \frac{2\,Aa^2}{\sqrt{x}} + \frac{2}{3}\left(2\,Bab+Ab^2+2\,Aac\right)x^{\frac{3}{2}} + 2\left(Ba^2+2\,Aab\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/9*B*c^2*x^(9/2) + 2/7*(2*B*b*c + A*c^2)*x^(7/2) + 2/5*(B*b^2 + 2*(B*a +
A*b)*c)*x^(5/2) - 2*A*a^2/sqrt(x) + 2/3*(2*B*a*b + A*b^2 + 2*A*a*c)*x^(3/2
) + 2*(B*a^2 + 2*A*a*b)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{3/2}}\,dx = \frac{2}{9}\,Bc^2x^{\frac{9}{2}} + \frac{4}{7}\,Bbcx^{\frac{7}{2}} + \frac{2}{7}\,Ac^2x^{\frac{7}{2}} + \frac{2}{5}\,Bb^2x^{\frac{5}{2}} + \frac{4}{5}\,Bacx^{\frac{5}{2}} + \frac{4}{5}\,Abcx^{\frac{5}{2}} + \frac{4}{3}\,Babx^{\frac{3}{2}} + \frac{2}{3}\,Ab^2x^{\frac{3}{2}} + \frac{4}{3}\,Aacx^{\frac{3}{2}} + 2\,Ba^2\sqrt{x} + 4\,Aab\sqrt{x} - \frac{2\,Aa^2}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/9*B*c^2*x^(9/2) + 4/7*B*b*c*x^(7/2) + 2/7*A*c^2*x^(7/2) + 2/5*B*b^2*x^(5
/2) + 4/5*B*a*c*x^(5/2) + 4/5*A*b*c*x^(5/2) + 4/3*B*a*b*x^(3/2) + 2/3*A*b^
2*x^(3/2) + 4/3*A*a*c*x^(3/2) + 2*B*a^2*sqrt(x) + 4*A*a*b*sqrt(x) - 2*A*a^
2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^{3/2}}\,dx = \sqrt{x}\left(2\,B\,a^2+4\,A\,b\,a\right)+x^{7/2}\left(\frac{2\,A\,c^2}{7}+\frac{4\,B\,b\,c}{7}\right)$$
$$+x^{3/2}\left(\frac{2\,A\,b^2}{3}+\frac{4\,B\,a\,b}{3}+\frac{4\,A\,a\,c}{3}\right)+x^{5/2}\left(\frac{2\,B\,b^2}{5}+\frac{4\,A\,c\,b}{5}+\frac{4\,B\,a\,c}{5}\right)-\frac{2\,A\,a^2}{\sqrt{x}}+\frac{2\,B\,c^2\,x^{9/2}}{9}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^(3/2),x)
```

output
```
x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(7/2)*((2*A*c^2)/7 + (4*B*b*c)/7) + x^(3/2
)*((2*A*b^2)/3 + (4*A*a*c)/3 + (4*B*a*b)/3) + x^(5/2)*((2*B*b^2)/5 + (4*A*
b*c)/5 + (4*B*a*c)/5) - (2*A*a^2)/x^(1/2) + (2*B*c^2*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^{3/2}}\,dx = \frac{\frac{2}{9}b\,c^2x^5+\frac{2}{7}a\,c^2x^4+\frac{4}{7}b^2c\,x^4+\frac{8}{5}abc\,x^3+\frac{2}{5}b^3x^3+\frac{4}{3}a^2c\,x^2+2a\,b^2x^2+6a}{\sqrt{x}}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(3/2),x)
```

output
```
(2*( - 315*a**3 + 945*a**2*b*x + 210*a**2*c*x**2 + 315*a*b**2*x**2 + 252*a
*b*c*x**3 + 45*a*c**2*x**4 + 63*b**3*x**3 + 90*b**2*c*x**4 + 35*b*c**2*x**
5))/(315*sqrt(x))
```

3.71 $\int \frac{(A+Bx)(a+bx+cx^2)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{5/2}}\,dx=-\frac{2a^2A}{3x^{3/2}}-\frac{2a(2Ab+aB)}{\sqrt{x}}+2\left(2abB+A\left(b^2+2ac\right)\right)\sqrt{x}+\frac{2}{3}\left(b^2B+2Abc+2aBc\right)x^{3/2}+\frac{2}{5}c(2bB+Ac)x^{5/2}+\frac{2}{7}Bc^2x^{7/2}$$

output
```
-2/3*a^2*A/x^(3/2)-2*a*(2*A*b+B*a)/x^(1/2)+2*(2*a*b*B+A*(2*a*c+b^2))*x^(1/
2)+2/3*(2*A*b*c+2*B*a*c+B*b^2)*x^(3/2)+2/5*c*(A*c+2*B*b)*x^(5/2)+2/7*B*c^2
*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{5/2}}\,dx=\frac{2(-35a^2(A+3Bx)+70ax(-3A(b-cx)+Bx(3b+cx))+x^2(7A(15b^2}{105x^{3/2}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^(5/2),x]
```

output
```
(2*(-35*a^2*(A + 3*B*x) + 70*a*x*(-3*A*(b - c*x) + B*x*(3*b + c*x)) + x^2*
(7*A*(15*b^2 + 10*b*c*x + 3*c^2*x^2) + B*x*(35*b^2 + 42*b*c*x + 15*c^2*x^2
))))/(105*x^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{5/2}}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^2A}{x^{5/2}}+\sqrt{x}\left(2aBc+2Abc+b^2B\right)+\frac{A\left(2ac+b^2\right)+2abB}{\sqrt{x}}+\frac{a(aB+2Ab)}{x^{3/2}}+cx^{3/2}(Ac+2bB)+Bc^2x^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{3}x^{3/2}\left(2aBc+2Abc+b^2B\right)+2\sqrt{x}\left(A\left(2ac+b^2\right)+2abB\right)-\frac{2a(aB+2Ab)}{\sqrt{x}}+\frac{2}{5}cx^{5/2}(Ac+2bB)+\frac{2}{7}Bc^2x^{7/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^(5/2),x]
```

output
```
(-2*a^2*A)/(3*x^(3/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + 2*(2*a*b*B + A*(b^2
 + 2*a*c))*Sqrt[x] + (2*(b^2*B + 2*A*b*c + 2*a*B*c)*x^(3/2))/3 + (2*c*(2*b
*B + A*c)*x^(5/2))/5 + (2*B*c^2*x^(7/2))/7
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2B\,c^2x^{\frac{7}{2}}}{7}+\frac{2A\,c^2x^{\frac{5}{2}}}{5}+\frac{4Bbc\,x^{\frac{5}{2}}}{5}+\frac{4Abc\,x^{\frac{3}{2}}}{3}+\frac{4Bac\,x^{\frac{3}{2}}}{3}+\frac{2B\,b^2x^{\frac{3}{2}}}{3}+4Aac\sqrt{x}+2A\,b^2\sqrt{x}+4ab\ldots$
default	$\frac{2B\,c^2x^{\frac{7}{2}}}{7}+\frac{2A\,c^2x^{\frac{5}{2}}}{5}+\frac{4Bbc\,x^{\frac{5}{2}}}{5}+\frac{4Abc\,x^{\frac{3}{2}}}{3}+\frac{4Bac\,x^{\frac{3}{2}}}{3}+\frac{2B\,b^2x^{\frac{3}{2}}}{3}+4Aac\sqrt{x}+2A\,b^2\sqrt{x}+4ab\ldots$
gosper	$-\frac{2(-15B\,c^2x^5-21x^4A\,c^2-42x^4Bbc-70x^3Abc-70Bac\,x^3-35x^3B\,b^2-210Aac\,x^2-105x^2b^2A-210Ba\,x^2b+210abAx\ldots}{105x^{\frac{3}{2}}}$
trager	$-\frac{2(-15B\,c^2x^5-21x^4A\,c^2-42x^4Bbc-70x^3Abc-70Bac\,x^3-35x^3B\,b^2-210Aac\,x^2-105x^2b^2A-210Ba\,x^2b+210abAx\ldots}{105x^{\frac{3}{2}}}$
risch	$-\frac{2(-15B\,c^2x^5-21x^4A\,c^2-42x^4Bbc-70x^3Abc-70Bac\,x^3-35x^3B\,b^2-210Aac\,x^2-105x^2b^2A-210Ba\,x^2b+210abAx\ldots}{105x^{\frac{3}{2}}}$
orering	$-\frac{2(-15B\,c^2x^5-21x^4A\,c^2-42x^4Bbc-70x^3Abc-70Bac\,x^3-35x^3B\,b^2-210Aac\,x^2-105x^2b^2A-210Ba\,x^2b+210abAx\ldots}{105x^{\frac{3}{2}}}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*B*c^2*x^(7/2)+2/5*A*c^2*x^(5/2)+4/5*B*b*c*x^(5/2)+4/3*A*b*c*x^(3/2)+4/
3*B*a*c*x^(3/2)+2/3*B*b^2*x^(3/2)+4*A*a*c*x^(1/2)+2*A*b^2*x^(1/2)+4*a*b*B*
x^(1/2)-2/3*a^2*A/x^(3/2)-2*a*(2*A*b+B*a)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^2}{x^{5/2}}\,dx = \frac{2\,(15\,Bc^2x^5+21\,(2\,Bbc+Ac^2)x^4+35\,(Bb^2+2\,(Ba+Ab)c)x^3-35\,A}{105\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*c^2*x^5 + 21*(2*B*b*c + A*c^2)*x^4 + 35*(B*b^2 + 2*(B*a + A*b)
*c)*x^3 - 35*A*a^2 + 105*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 - 105*(B*a^2 + 2*
A*a*b)*x)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^2}{x^{5/2}}\,dx = -\frac{2Aa^2}{3x^{\frac{3}{2}}} - \frac{4Aab}{\sqrt{x}} + 4Aac\sqrt{x} + 2Ab^2\sqrt{x} + \frac{4Abcx^{\frac{3}{2}}}{3} + \frac{2Ac^2x^{\frac{5}{2}}}{5} - \frac{2Ba^2}{\sqrt{x}} + 4Bab\sqrt{x} + \frac{4Bacx^{\frac{3}{2}}}{3} + \frac{2Bb^2x^{\frac{3}{2}}}{3} + \frac{4Bbcx^{\frac{5}{2}}}{5} + \frac{2Bc^2x^{\frac{7}{2}}}{7}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**(5/2),x)
```

output
```
-2*A*a**2/(3*x**(3/2)) - 4*A*a*b/sqrt(x) + 4*A*a*c*sqrt(x) + 2*A*b**2*sqrt
(x) + 4*A*b*c*x**(3/2)/3 + 2*A*c**2*x**(5/2)/5 - 2*B*a**2/sqrt(x) + 4*B*a*
b*sqrt(x) + 4*B*a*c*x**(3/2)/3 + 2*B*b**2*x**(3/2)/3 + 4*B*b*c*x**(5/2)/5
+ 2*B*c**2*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{5/2}}\,dx = \frac{2}{7}\,Bc^2x^{\frac{7}{2}} + \frac{2}{5}\left(2\,Bbc+Ac^2\right)x^{\frac{5}{2}} + \frac{2}{3}\left(Bb^2+2\,(Ba+Ab)c\right)x^{\frac{3}{2}} + 2\left(2\,Bab+Ab^2+2\,Aac\right)\sqrt{x} - \frac{2\left(Aa^2+3\left(Ba^2+2\,Aab\right)x\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2/7*B*c^2*x^(7/2) + 2/5*(2*B*b*c + A*c^2)*x^(5/2) + 2/3*(B*b^2 + 2*(B*a +
A*b)*c)*x^(3/2) + 2*(2*B*a*b + A*b^2 + 2*A*a*c)*sqrt(x) - 2/3*(A*a^2 + 3*(
B*a^2 + 2*A*a*b)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{5/2}}\,dx = \frac{2}{7}\,Bc^2x^{\frac{7}{2}} + \frac{4}{5}\,Bbcx^{\frac{5}{2}} + \frac{2}{5}\,Ac^2x^{\frac{5}{2}} + \frac{2}{3}\,Bb^2x^{\frac{3}{2}} + \frac{4}{3}\,Bacx^{\frac{3}{2}} + \frac{4}{3}\,Abcx^{\frac{3}{2}} + 4\,Bab\sqrt{x} + 2\,Ab^2\sqrt{x} + 4\,Aac\sqrt{x} - \frac{2\left(3\,Ba^2x+6\,Aabx+Aa^2\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(5/2),x, algorithm="giac")
```

output
```
2/7*B*c^2*x^(7/2) + 4/5*B*b*c*x^(5/2) + 2/5*A*c^2*x^(5/2) + 2/3*B*b^2*x^(3
/2) + 4/3*B*a*c*x^(3/2) + 4/3*A*b*c*x^(3/2) + 4*B*a*b*sqrt(x) + 2*A*b^2*sq
rt(x) + 4*A*a*c*sqrt(x) - 2/3*(3*B*a^2*x + 6*A*a*b*x + A*a^2)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{5/2}}\,dx = x^{5/2}\left(\frac{2\,A\,c^2}{5}+\frac{4\,B\,b\,c}{5}\right) + \sqrt{x}\left(2\,A\,b^2+4\,B\,a\,b+4\,A\,a\,c\right) + x^{3/2}\left(\frac{2\,B\,b^2}{3}+\frac{4\,A\,c\,b}{3}+\frac{4\,B\,a\,c}{3}\right) - \frac{\frac{2\,A\,a^2}{3}+x\left(2\,B\,a^2+4\,A\,b\,a\right)}{x^{3/2}} + \frac{2\,B\,c^2\,x^{7/2}}{7}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^(5/2),x)
```

output
```
x^(5/2)*((2*A*c^2)/5 + (4*B*b*c)/5) + x^(1/2)*(2*A*b^2 + 4*A*a*c + 4*B*a*b
) + x^(3/2)*((2*B*b^2)/3 + (4*A*b*c)/3 + (4*B*a*c)/3) - ((2*A*a^2)/3 + x*(
2*B*a^2 + 4*A*a*b))/x^(3/2) + (2*B*c^2*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{5/2}}\,dx = \frac{\frac{2}{7}b\,c^2x^5+\frac{2}{5}a\,c^2x^4+\frac{4}{5}b^2c\,x^4+\frac{8}{3}abc\,x^3+\frac{2}{3}b^3x^3+4a^2c\,x^2+6a\,b^2x^2-6a}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(5/2),x)
```

output
```
(2*( - 35*a**3 - 315*a**2*b*x + 210*a**2*c*x**2 + 315*a*b**2*x**2 + 140*a*
b*c*x**3 + 21*a*c**2*x**4 + 35*b**3*x**3 + 42*b**2*c*x**4 + 15*b*c**2*x**5
))/(105*sqrt(x)*x)
```

3.72 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx = -\frac{2a^2A}{5x^{5/2}} - \frac{2a(2Ab+aB)}{3x^{3/2}} - \frac{2(2abB+A(b^2+2ac))}{\sqrt{x}} + 2\left(b^2B+2Abc+2aBc\right)\sqrt{x} + \frac{2}{3}c(2bB+Ac)x^{3/2} + \frac{2}{5}Bc^2x^{5/2}$$

output
```
-2/5*a^2*A/x^(5/2)-2/3*a*(2*A*b+B*a)/x^(3/2)-2*(2*a*b*B+A*(2*a*c+b^2))/x^(
1/2)+2*(2*A*b*c+2*B*a*c+B*b^2)*x^(1/2)+2/3*c*(A*c+2*B*b)*x^(3/2)+2/5*B*c^2
*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx = \frac{-2a^2(3A+5Bx) - 20ax(3Bx(b-cx)+A(b+3cx)) + 2x^2(5A(-3b^2 -}{15x^{5/2}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^(7/2),x]
```

output
```
(-2*a^2*(3*A + 5*B*x) - 20*a*x*(3*B*x*(b - c*x) + A*(b + 3*c*x)) + 2*x^2*(
5*A*(-3*b^2 + 6*b*c*x + c^2*x^2) + B*x*(15*b^2 + 10*b*c*x + 3*c^2*x^2)))/(
15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{a^2A}{x^{7/2}}+\frac{A\left(2ac+b^2\right)+2abB}{x^{3/2}}+\frac{2aBc+2Abc+b^2B}{\sqrt{x}}+\frac{a(aB+2Ab)}{x^{5/2}}+c\sqrt{x}(Ac+2bB)+Bc^2x^{3/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a^2A}{5x^{5/2}}+2\sqrt{x}\left(2aBc+2Abc+b^2B\right)-\frac{2\left(A\left(2ac+b^2\right)+2abB\right)}{\sqrt{x}}-\frac{2a(aB+2Ab)}{3x^{3/2}}+\frac{2}{3}cx^{3/2}(Ac+2bB)+\frac{2}{5}Bc^2x^{5/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^(7/2),x]
```

output
```
(-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/(3*x^(3/2)) - (2*(2*a*b*B + A
*(b^2 + 2*a*c)))/Sqrt[x] + 2*(b^2*B + 2*A*b*c + 2*a*B*c)*Sqrt[x] + (2*c*(2
*b*B + A*c)*x^(3/2))/3 + (2*B*c^2*x^(5/2))/5
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2Bc^2x^{\frac{5}{2}}}{5}+\frac{2Ac^2x^{\frac{3}{2}}}{3}+\frac{4Bbc\,x^{\frac{3}{2}}}{3}+4Abc\sqrt{x}+4Bac\sqrt{x}+2B\,b^2\sqrt{x}-\frac{2a(2Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2a^2A}{5x^{\frac{5}{2}}}-$
default	$\frac{2Bc^2x^{\frac{5}{2}}}{5}+\frac{2Ac^2x^{\frac{3}{2}}}{3}+\frac{4Bbc\,x^{\frac{3}{2}}}{3}+4Abc\sqrt{x}+4Bac\sqrt{x}+2B\,b^2\sqrt{x}-\frac{2a(2Ab+Ba)}{3x^{\frac{3}{2}}}-\frac{2a^2A}{5x^{\frac{5}{2}}}-$
gosper	$-\frac{2(-3Bc^2x^5-5x^4Ac^2-10x^4Bbc-30x^3Abc-30Bacx^3-15x^3Bb^2+30Aacx^2+15x^2b^2A+30Bax^2b+10abAx+5a^2B}{15x^{\frac{5}{2}}}$
trager	$-\frac{2(-3Bc^2x^5-5x^4Ac^2-10x^4Bbc-30x^3Abc-30Bacx^3-15x^3Bb^2+30Aacx^2+15x^2b^2A+30Bax^2b+10abAx+5a^2B}{15x^{\frac{5}{2}}}$
risch	$-\frac{2(-3Bc^2x^5-5x^4Ac^2-10x^4Bbc-30x^3Abc-30Bacx^3-15x^3Bb^2+30Aacx^2+15x^2b^2A+30Bax^2b+10abAx+5a^2B}{15x^{\frac{5}{2}}}$
orering	$-\frac{2(-3Bc^2x^5-5x^4Ac^2-10x^4Bbc-30x^3Abc-30Bacx^3-15x^3Bb^2+30Aacx^2+15x^2b^2A+30Bax^2b+10abAx+5a^2B}{15x^{\frac{5}{2}}}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*B*c^2*x^(5/2)+2/3*A*c^2*x^(3/2)+4/3*B*b*c*x^(3/2)+4*A*b*c*x^(1/2)+4*B*
a*c*x^(1/2)+2*B*b^2*x^(1/2)-2/3*a*(2*A*b+B*a)/x^(3/2)-2/5*a^2*A/x^(5/2)-2*
(2*A*a*c+A*b^2+2*B*a*b)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx = \frac{2\,(3\,Bc^2x^5+5\,(2\,Bbc+Ac^2)x^4+15\,(Bb^2+2\,(Ba+Ab)c)x^3-3\,Aa^2-}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c^2*x^5 + 5*(2*B*b*c + A*c^2)*x^4 + 15*(B*b^2 + 2*(B*a + A*b)*c)
*x^3 - 3*A*a^2 - 15*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 - 5*(B*a^2 + 2*A*a*b)*
x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx = -\frac{2Aa^2}{5x^{\frac{5}{2}}}-\frac{4Aab}{3x^{\frac{3}{2}}}-\frac{4Aac}{\sqrt{x}}-\frac{2Ab^2}{\sqrt{x}}+4Abc\sqrt{x}+\frac{2Ac^2x^{\frac{3}{2}}}{3}-\frac{2Ba^2}{3x^{\frac{3}{2}}}-\frac{4Bab}{\sqrt{x}}+4Bac\sqrt{x}+2Bb^2\sqrt{x}+\frac{4Bbcx^{\frac{3}{2}}}{3}+\frac{2Bc^2x^{\frac{5}{2}}}{5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**(7/2),x)
```

output
```
-2*A*a**2/(5*x**(5/2)) - 4*A*a*b/(3*x**(3/2)) - 4*A*a*c/sqrt(x) - 2*A*b**2
/sqrt(x) + 4*A*b*c*sqrt(x) + 2*A*c**2*x**(3/2)/3 - 2*B*a**2/(3*x**(3/2)) -
 4*B*a*b/sqrt(x) + 4*B*a*c*sqrt(x) + 2*B*b**2*sqrt(x) + 4*B*b*c*x**(3/2)/3
 + 2*B*c**2*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx = \frac{2}{5}\,Bc^2x^{\frac{5}{2}} + \frac{2}{3}\left(2\,Bbc+Ac^2\right)x^{\frac{3}{2}} + 2\left(Bb^2+2\,(Ba+Ab)c\right)\sqrt{x} - \frac{2\,(3\,Aa^2+15\,(2\,Bab+Ab^2+2\,Aac)x^2+5\,(Ba^2+2\,Aab)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(7/2),x, algorithm="maxima")
```

output
```
2/5*B*c^2*x^(5/2) + 2/3*(2*B*b*c + A*c^2)*x^(3/2) + 2*(B*b^2 + 2*(B*a + A*
b)*c)*sqrt(x) - 2/15*(3*A*a^2 + 15*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 5*(B*
a^2 + 2*A*a*b)*x)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx = \frac{2}{5}\,Bc^2x^{\frac{5}{2}} + \frac{4}{3}\,Bbcx^{\frac{3}{2}} + \frac{2}{3}\,Ac^2x^{\frac{3}{2}} + 2\,Bb^2\sqrt{x} + 4\,Bac\sqrt{x} + 4\,Abc\sqrt{x} - \frac{2\,(30\,Babx^2+15\,Ab^2x^2+30\,Aacx^2+5\,Ba^2x+10\,Aabx+3\,Aa^2)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(7/2),x, algorithm="giac")
```

output
```
2/5*B*c^2*x^(5/2) + 4/3*B*b*c*x^(3/2) + 2/3*A*c^2*x^(3/2) + 2*B*b^2*sqrt(x
) + 4*B*a*c*sqrt(x) + 4*A*b*c*sqrt(x) - 2/15*(30*B*a*b*x^2 + 15*A*b^2*x^2
+ 30*A*a*c*x^2 + 5*B*a^2*x + 10*A*a*b*x + 3*A*a^2)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx = x^{3/2}\left(\frac{2\,A\,c^2}{3}+\frac{4\,B\,b\,c}{3}\right) - \frac{\frac{2\,A\,a^2}{5}+x^2\,\left(2\,A\,b^2+4\,B\,a\,b+4\,A\,a\,c\right)+x\,\left(\frac{2\,B\,a^2}{3}+\frac{4\,A\,b\,a}{3}\right)}{x^{5/2}} + \sqrt{x}\,\left(2\,B\,b^2+4\,A\,c\,b+4\,B\,a\,c\right)+\frac{2\,B\,c^2\,x^{5/2}}{5}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^(7/2),x)
```

output
```
x^(3/2)*((2*A*c^2)/3 + (4*B*b*c)/3) - ((2*A*a^2)/5 + x^2*(2*A*b^2 + 4*A*a*
c + 4*B*a*b) + x*((2*B*a^2)/3 + (4*A*a*b)/3))/x^(5/2) + x^(1/2)*(2*B*b^2 +
 4*A*b*c + 4*B*a*c) + (2*B*c^2*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{7/2}}\,dx = \frac{\frac{2}{5}b\,c^2x^5+\frac{2}{3}a\,c^2x^4+\frac{4}{3}b^2c\,x^4+8abc\,x^3+2b^3x^3-4a^2c\,x^2-6a\,b^2x^2-2a}{\sqrt{x}\,x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(7/2),x)
```

output
```
(2*( - 3*a**3 - 15*a**2*b*x - 30*a**2*c*x**2 - 45*a*b**2*x**2 + 60*a*b*c*x
**3 + 5*a*c**2*x**4 + 15*b**3*x**3 + 10*b**2*c*x**4 + 3*b*c**2*x**5))/(15*
sqrt(x)*x**2)
```

3.73 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}} \, dx = -\frac{2a^2A}{7x^{7/2}} - \frac{2a(2Ab+aB)}{5x^{5/2}} - \frac{2(2abB+A(b^2+2ac))}{3x^{3/2}} - \frac{2(b^2B+2Abc+2aBc)}{\sqrt{x}} + 2c(2bB+Ac)\sqrt{x} + \frac{2}{3}Bc^2x^{3/2}$$

output
```
-2/7*a^2*A/x^(7/2)-2/5*a*(2*A*b+B*a)/x^(5/2)-2/3*(2*a*b*B+A*(2*a*c+b^2))/x
^(3/2)-2*(2*A*b*c+2*B*a*c+B*b^2)/x^(1/2)+2*c*(A*c+2*B*b)*x^(1/2)+2/3*B*c^2
*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}} \, dx = -\frac{2(3a^2(5A+7Bx)+14ax(5Bx(b+3cx)+A(3b+5cx))+35x^2(A(b^2+6bcx-3c^2x^2)-Bx(-3b^2+6bc}{105x^{7/2}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/x^(9/2),x]
```

output
```
(-2*(3*a^2*(5*A + 7*B*x) + 14*a*x*(5*B*x*(b + 3*c*x) + A*(3*b + 5*c*x)) +
35*x^2*(A*(b^2 + 6*b*c*x - 3*c^2*x^2) - B*x*(-3*b^2 + 6*b*c*x + c^2*x^2)))
)/(105*x^(7/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}}\,dx$$

$$\Big\downarrow \ 1195$$

$$\int \left(\frac{a^2A}{x^{9/2}}+\frac{2aBc+2Abc+b^2B}{x^{3/2}}+\frac{A\left(2ac+b^2\right)+2abB}{x^{5/2}}+\frac{a(aB+2Ab)}{x^{7/2}}+\frac{c(Ac+2bB)}{\sqrt{x}}+Bc^2\sqrt{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^2A}{7x^{7/2}}-\frac{2\left(A\left(2ac+b^2\right)+2abB\right)}{3x^{3/2}}-\frac{2\left(2aBc+2Abc+b^2B\right)}{\sqrt{x}}-\frac{2a(aB+2Ab)}{5x^{5/2}}+2c\sqrt{x}(Ac+2bB)+\frac{2}{3}Bc^2x^{3/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/x^(9/2),x]
```

output
```
(-2*a^2*A)/(7*x^(7/2)) - (2*a*(2*A*b + a*B))/(5*x^(5/2)) - (2*(2*a*b*B + A
*(b^2 + 2*a*c)))/(3*x^(3/2)) - (2*(b^2*B + 2*A*b*c + 2*a*B*c))/Sqrt[x] + 2
*c*(2*b*B + A*c)*Sqrt[x] + (2*B*c^2*x^(3/2))/3
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2B\,c^2x^{\frac{3}{2}}}{3}+2A\,c^2\sqrt{x}+4Bbc\sqrt{x}-\frac{2(2Abc+2aBc+B\,b^2)}{\sqrt{x}}-\frac{2(2Aac+b^2A+2abB)}{3x^{\frac{3}{2}}}-\frac{2a^2A}{7x^{\frac{7}{2}}}-\frac{2a(2Ab+}{5x^{\frac{5}{2}}}$
default	$\frac{2B\,c^2x^{\frac{3}{2}}}{3}+2A\,c^2\sqrt{x}+4Bbc\sqrt{x}-\frac{2(2Abc+2aBc+B\,b^2)}{\sqrt{x}}-\frac{2(2Aac+b^2A+2abB)}{3x^{\frac{3}{2}}}-\frac{2a^2A}{7x^{\frac{7}{2}}}-\frac{2a(2Ab+}{5x^{\frac{5}{2}}}$
gosper	$-\frac{2(-35B\,c^2x^5-105x^4A\,c^2-210x^4Bbc+210x^3Abc+210Bac\,x^3+105x^3B\,b^2+70Aac\,x^2+35x^2b^2A+70Ba\,x^2b+42abA}{105x^{\frac{7}{2}}}$
trager	$-\frac{2(-35B\,c^2x^5-105x^4A\,c^2-210x^4Bbc+210x^3Abc+210Bac\,x^3+105x^3B\,b^2+70Aac\,x^2+35x^2b^2A+70Ba\,x^2b+42abA}{105x^{\frac{7}{2}}}$
risch	$-\frac{2(-35B\,c^2x^5-105x^4A\,c^2-210x^4Bbc+210x^3Abc+210Bac\,x^3+105x^3B\,b^2+70Aac\,x^2+35x^2b^2A+70Ba\,x^2b+42abA}{105x^{\frac{7}{2}}}$
orering	$-\frac{2(-35B\,c^2x^5-105x^4A\,c^2-210x^4Bbc+210x^3Abc+210Bac\,x^3+105x^3B\,b^2+70Aac\,x^2+35x^2b^2A+70Ba\,x^2b+42abA}{105x^{\frac{7}{2}}}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*B*c^2*x^(3/2)+2*A*c^2*x^(1/2)+4*B*b*c*x^(1/2)-2*(2*A*b*c+2*B*a*c+B*b^2
)/x^(1/2)-2/3*(2*A*a*c+A*b^2+2*B*a*b)/x^(3/2)-2/7*a^2*A/x^(7/2)-2/5*a*(2*A
*b+B*a)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}}\,dx = \frac{2\,(35\,Bc^2x^5+105\,(2\,Bbc+Ac^2)x^4-105\,(Bb^2+2\,(Ba+Ab)c)x^3-15}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(9/2),x, algorithm="fricas")
```

output
```
2/105*(35*B*c^2*x^5 + 105*(2*B*b*c + A*c^2)*x^4 - 105*(B*b^2 + 2*(B*a + A*
b)*c)*x^3 - 15*A*a^2 - 35*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 - 21*(B*a^2 + 2*
A*a*b)*x)/x^(7/2)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}}\,dx = -\frac{2Aa^2}{7x^{\frac{7}{2}}}-\frac{4Aab}{5x^{\frac{5}{2}}}-\frac{4Aac}{3x^{\frac{3}{2}}}-\frac{2Ab^2}{3x^{\frac{3}{2}}}-\frac{4Abc}{\sqrt{x}}+2Ac^2\sqrt{x}-\frac{2Ba^2}{5x^{\frac{5}{2}}}-\frac{4Bab}{3x^{\frac{3}{2}}}-\frac{4Bac}{\sqrt{x}}-\frac{2Bb^2}{\sqrt{x}}+4Bbc\sqrt{x}+\frac{2Bc^2x^{\frac{3}{2}}}{3}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/x**(9/2),x)
```

output
```
-2*A*a**2/(7*x**(7/2)) - 4*A*a*b/(5*x**(5/2)) - 4*A*a*c/(3*x**(3/2)) - 2*A
*b**2/(3*x**(3/2)) - 4*A*b*c/sqrt(x) + 2*A*c**2*sqrt(x) - 2*B*a**2/(5*x**(
5/2)) - 4*B*a*b/(3*x**(3/2)) - 4*B*a*c/sqrt(x) - 2*B*b**2/sqrt(x) + 4*B*b*
c*sqrt(x) + 2*B*c**2*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}}\,dx = \frac{2}{3}\,Bc^2x^{\frac{3}{2}} + 2\left(2\,Bbc + Ac^2\right)\sqrt{x} - \frac{2\,(105\,(Bb^2 + 2\,(Ba + Ab)c)x^3 + 15\,Aa^2 + 35\,(2\,Bab + Ab^2 + 2\,Aac)x^2 + 21\,(Ba^2 + 2\,Aab)x)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(9/2),x, algorithm="maxima")
```

output
```
2/3*B*c^2*x^(3/2) + 2*(2*B*b*c + A*c^2)*sqrt(x) - 2/105*(105*(B*b^2 + 2*(B
*a + A*b)*c)*x^3 + 15*A*a^2 + 35*(2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + 21*(B*a
^2 + 2*A*a*b)*x)/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}}\,dx = \frac{2}{3}\,Bc^2x^{\frac{3}{2}} + 4\,Bbc\sqrt{x} + 2\,Ac^2\sqrt{x} - \frac{2\,(105\,Bb^2x^3 + 210\,Bacx^3 + 210\,Abcx^3 + 70\,Babx^2 + 35\,Ab^2x^2 + 70\,Aacx^2 + 21\,Ba^2x + 42\,Aabx + 15\,A}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/x^(9/2),x, algorithm="giac")
```

output
```
2/3*B*c^2*x^(3/2) + 4*B*b*c*sqrt(x) + 2*A*c^2*sqrt(x) - 2/105*(105*B*b^2*x
^3 + 210*B*a*c*x^3 + 210*A*b*c*x^3 + 70*B*a*b*x^2 + 35*A*b^2*x^2 + 70*A*a*
c*x^2 + 21*B*a^2*x + 42*A*a*b*x + 15*A*a^2)/x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}}\,dx = \sqrt{x}\left(2\,A\,c^2+4\,B\,b\,c\right) - \frac{\frac{2\,A\,a^2}{7}+x^2\left(\frac{2\,A\,b^2}{3}+\frac{4\,B\,a\,b}{3}+\frac{4\,A\,a\,c}{3}\right)+x^3\left(2\,B\,b^2+4\,A\,c\,b+4\,B\,a\,c\right)+x\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right)}{x^{7/2}} + \frac{2\,B\,c^2\,x^{3/2}}{3}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/x^(9/2),x)
```

output
```
x^(1/2)*(2*A*c^2 + 4*B*b*c) - ((2*A*a^2)/7 + x^2*((2*A*b^2)/3 + (4*A*a*c)/
3 + (4*B*a*b)/3) + x^3*(2*B*b^2 + 4*A*b*c + 4*B*a*c) + x*((2*B*a^2)/5 + (4
*A*a*b)/5))/x^(7/2) + (2*B*c^2*x^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{x^{9/2}}\,dx = \frac{\frac{2}{3}b\,c^2x^5+2a\,c^2x^4+4b^2c\,x^4-8abc\,x^3-2b^3x^3-\frac{4}{3}a^2c\,x^2-2a\,b^2x^2-\frac{6}{5}a^{}}{\sqrt{x}\,x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/x^(9/2),x)
```

output
```
(2*( - 15*a**3 - 63*a**2*b*x - 70*a**2*c*x**2 - 105*a*b**2*x**2 - 420*a*b*
c*x**3 + 105*a*c**2*x**4 - 105*b**3*x**3 + 210*b**2*c*x**4 + 35*b*c**2*x**
5))/(105*sqrt(x)*x**3)
```

3.74 $\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{9}a^3Ax^{9/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}$$
$$+\frac{6}{13}a\left(abB+A\left(b^2+ac\right)\right)x^{13/2}+\frac{2}{15}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{15/2}+\frac{2}{17}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x$$

output
```
2/9*a^3*A*x^(9/2)+2/11*a^2*(3*A*b+B*a)*x^(11/2)+6/13*a*(a*b*B+A*(a*c+b^2))
*x^(13/2)+2/15*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^(15/2)+2/17*(3*A*a*c^2+
3*A*b^2*c+6*B*a*b*c+B*b^3)*x^(17/2)+6/19*c*(A*b*c+B*a*c+B*b^2)*x^(19/2)+2/
21*c^2*(A*c+3*B*b)*x^(21/2)+2/23*B*c^3*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2x^{9/2}(3380195a^3(11A+9Bx)+468027a^2x(15A(13b+11cx)+11Bx(15b+13cx))+15939a}{}$$

input
```
Integrate[x^(7/2)*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(2*x^(9/2)*(3380195*a^3*(11*A + 9*B*x) + 468027*a^2*x*(15*A*(13*b + 11*c*x
) + 11*B*x*(15*b + 13*c*x)) + 15939*a*x^2*(19*A*(255*b^2 + 442*b*c*x + 195
*c^2*x^2) + 13*B*x*(323*b^2 + 570*b*c*x + 255*c^2*x^2)) + 429*x^3*(23*A*(2
261*b^3 + 5985*b^2*c*x + 5355*b*c^2*x^2 + 1615*c^3*x^3) + 15*B*x*(3059*b^3
 + 8211*b^2*c*x + 7429*b*c^2*x^2 + 2261*c^3*x^3))))/334639305
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^3Ax^{7/2}+a^2x^{9/2}(aB+3Ab)+3cx^{17/2}\left(aBc+Abc+b^2B\right)+3ax^{11/2}\left(A\left(ac+b^2\right)+abB\right)+x^{15/2}\left(3aAc^2+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{6}{19}cx^{19/2}\left(aBc+Abc+b^2B\right)+\frac{6}{13}ax^{13/2}\left(A\left(ac+b^2\right)+abB\right)+\frac{2}{17}x^{17/2}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{2}{15}x^{15/2}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{2}{21}c^2x^{21/2}(Ac+3bB)+\frac{2}{23}Bc^3x^{23/2}$$

input
```
Int[x^(7/2)*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*(a*b*B + A*
(b^2 + a*c))*x^(13/2))/13 + (2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^(
15/2))/15 + (2*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^(17/2))/17 +
(6*c*(b^2*B + A*b*c + a*B*c)*x^(19/2))/19 + (2*c^2*(3*b*B + A*c)*x^(21/2))
/21 + (2*B*c^3*x^(23/2))/23
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

method	result
gosper	$\frac{2x^{\frac{9}{2}}\left(14549535Bc^3x^7+15935205Ac^3x^6+47805615Bbc^2x^6+52837785Abc^2x^5+52837785Bac^2x^5+52837785Bb^2cx^5\right.}{}$
trager	$\frac{2x^{\frac{9}{2}}\left(14549535Bc^3x^7+15935205Ac^3x^6+47805615Bbc^2x^6+52837785Abc^2x^5+52837785Bac^2x^5+52837785Bb^2cx^5\right.}{}$
risch	$\frac{2x^{\frac{9}{2}}\left(14549535Bc^3x^7+15935205Ac^3x^6+47805615Bbc^2x^6+52837785Abc^2x^5+52837785Bac^2x^5+52837785Bb^2cx^5\right.}{}$
orering	$\frac{2x^{\frac{9}{2}}\left(14549535Bc^3x^7+15935205Ac^3x^6+47805615Bbc^2x^6+52837785Abc^2x^5+52837785Bac^2x^5+52837785Bb^2cx^5\right.}{}$
derivativedivides	$\frac{2Bc^3x^{\frac{23}{2}}}{23}+\frac{2(Ac^3+3Bbc^2)x^{\frac{21}{2}}}{21}+\frac{2(3Abc^2+B(ac^2+2b^2c+c(2ac+b^2)))x^{\frac{19}{2}}}{19}+\frac{2(A(ac^2+2b^2c+c(2ac+b^2))+}{17}$
default	$\frac{2Bc^3x^{\frac{23}{2}}}{23}+\frac{2(Ac^3+3Bbc^2)x^{\frac{21}{2}}}{21}+\frac{2(3Abc^2+B(ac^2+2b^2c+c(2ac+b^2)))x^{\frac{19}{2}}}{19}+\frac{2(A(ac^2+2b^2c+c(2ac+b^2))+}{17}$

input
```
int(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/334639305*x^(9/2)*(14549535*B*c^3*x^7+15935205*A*c^3*x^6+47805615*B*b*c^
2*x^6+52837785*A*b*c^2*x^5+52837785*B*a*c^2*x^5+52837785*B*b^2*c*x^5+59053
995*A*a*c^2*x^4+59053995*A*b^2*c*x^4+118107990*B*a*b*c*x^4+19684665*B*b^3*
x^4+133855722*A*a*b*c*x^3+22309287*A*b^3*x^3+66927861*B*a^2*c*x^3+66927861
*B*a*b^2*x^3+77224455*A*a^2*c*x^2+77224455*A*a*b^2*x^2+77224455*B*a^2*b*x^
2+91265265*A*a^2*b*x+30421755*B*a^3*x+37182145*A*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{334639305}\left(14549535\,Bc^3x^{11}+15935205\left(3\,Bbc^2+Ac^3\right)x^{10}+52837785\left(Bb^2c+(Ba+Ab)\right.\right.$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
2/334639305*(14549535*B*c^3*x^11 + 15935205*(3*B*b*c^2 + A*c^3)*x^10 + 528
37785*(B*b^2*c + (B*a + A*b)*c^2)*x^9 + 19684665*(B*b^3 + 3*A*a*c^2 + 3*(2
*B*a*b + A*b^2)*c)*x^8 + 37182145*A*a^3*x^4 + 22309287*(3*B*a*b^2 + A*b^3
+ 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 77224455*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^6
+ 30421755*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.62

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{6Aa^2cx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{13}{2}}}{13}+\frac{4Aabcx^{\frac{15}{2}}}{5}+\frac{6Aac^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{6Ab^2cx^{\frac{17}{2}}}{17}+\frac{6Abc^2x^{\frac{19}{2}}}{19}+\frac{2Ac^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{13}{2}}}{13}+\frac{2Ba^2cx^{\frac{15}{2}}}{5}+\frac{2Bab^2x^{\frac{15}{2}}}{5}+\frac{12Babcx^{\frac{17}{2}}}{17}+\frac{6Bac^2x^{\frac{19}{2}}}{19}+\frac{2Bb^3x^{\frac{17}{2}}}{17}+\frac{6Bb^2cx^{\frac{19}{2}}}{19}+\frac{2Bbc^2x^{\frac{21}{2}}}{7}+\frac{2Bc^3x^{\frac{23}{2}}}{23}$$

input
```
integrate(x**(7/2)*(B*x+A)*(c*x**2+b*x+a)**3,x)
```

output
```
2*A*a**3*x**(9/2)/9 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a**2*c*x**(13/2)/13 +
6*A*a*b**2*x**(13/2)/13 + 4*A*a*b*c*x**(15/2)/5 + 6*A*a*c**2*x**(17/2)/17
+ 2*A*b**3*x**(15/2)/15 + 6*A*b**2*c*x**(17/2)/17 + 6*A*b*c**2*x**(19/2)/1
9 + 2*A*c**3*x**(21/2)/21 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(13/2)/1
3 + 2*B*a**2*c*x**(15/2)/5 + 2*B*a*b**2*x**(15/2)/5 + 12*B*a*b*c*x**(17/2)
/17 + 6*B*a*c**2*x**(19/2)/19 + 2*B*b**3*x**(17/2)/17 + 6*B*b**2*c*x**(19/
2)/19 + 2*B*b*c**2*x**(21/2)/7 + 2*B*c**3*x**(23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{23}\,Bc^3x^{\frac{23}{2}}+\frac{2}{21}\,\left(3\,Bbc^2+Ac^3\right)x^{\frac{21}{2}}+\frac{6}{19}\,\left(Bb^2c+(Ba+Ab)c^2\right)x^{\frac{19}{2}}+\frac{2}{17}\,\left(Bb^3+3\,Aac^2+3\,\left(2\,Bab+Ab^2\right)c\right)x^{\frac{17}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}+\frac{2}{15}\,\left(3\,Bab^2+Ab^3+3\,\left(Ba^2+2\,Aab\right)c\right)x^{\frac{15}{2}}+\frac{6}{13}\,\left(Ba^2b+Aab^2+Aa^2c\right)x^{\frac{13}{2}}+\frac{2}{11}\,\left(Ba^3+3\,Aa^2b\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
2/23*B*c^3*x^(23/2) + 2/21*(3*B*b*c^2 + A*c^3)*x^(21/2) + 6/19*(B*b^2*c +
(B*a + A*b)*c^2)*x^(19/2) + 2/17*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*
c)*x^(17/2) + 2/9*A*a^3*x^(9/2) + 2/15*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A
*a*b)*c)*x^(15/2) + 6/13*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^(13/2) + 2/11*(B*
a^3 + 3*A*a^2*b)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.06

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{23}\,Bc^3x^{\frac{23}{2}}+\frac{2}{7}\,Bbc^2x^{\frac{21}{2}}+\frac{2}{21}\,Ac^3x^{\frac{21}{2}}+\frac{6}{19}\,Bb^2cx^{\frac{19}{2}}+\frac{6}{19}\,Bac^2x^{\frac{19}{2}}+\frac{6}{19}\,Abc^2x^{\frac{19}{2}}+\frac{2}{17}\,Bb^3x^{\frac{17}{2}}+\frac{12}{17}\,Babcx^{\frac{17}{2}}+\frac{6}{17}\,Ab^2cx^{\frac{17}{2}}+\frac{6}{17}\,Aac^2x^{\frac{17}{2}}+\frac{2}{5}\,Bab^2x^{\frac{15}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}+\frac{2}{5}\,Ba^2cx^{\frac{15}{2}}+\frac{4}{5}\,Aabcx^{\frac{15}{2}}+\frac{6}{13}\,Ba^2bx^{\frac{13}{2}}+\frac{6}{13}\,Aab^2x^{\frac{13}{2}}+\frac{6}{13}\,Aa^2cx^{\frac{13}{2}}+\frac{2}{11}\,Ba^3x^{\frac{11}{2}}+\frac{6}{11}\,Aa^2bx^{\frac{11}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
2/23*B*c^3*x^(23/2) + 2/7*B*b*c^2*x^(21/2) + 2/21*A*c^3*x^(21/2) + 6/19*B*
b^2*c*x^(19/2) + 6/19*B*a*c^2*x^(19/2) + 6/19*A*b*c^2*x^(19/2) + 2/17*B*b^
3*x^(17/2) + 12/17*B*a*b*c*x^(17/2) + 6/17*A*b^2*c*x^(17/2) + 6/17*A*a*c^2
*x^(17/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 2/5*B*a^2*c*x^(15
/2) + 4/5*A*a*b*c*x^(15/2) + 6/13*B*a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2)
 + 6/13*A*a^2*c*x^(13/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b*x^(11/2) + 2
/9*A*a^3*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 10.92 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=x^{15/2}\left(\frac{2\,B\,c\,a^2}{5}+\frac{2\,B\,a\,b^2}{5}+\frac{4\,A\,c\,a\,b}{5}+\frac{2\,A\,b^3}{15}\right)+x^{17/2}\left(\frac{2\,B\,b^3}{17}+\frac{6\,A\,b^2\,c}{17}+\frac{12\,B\,a\,b\,c}{17}+\frac{6\,A\,a\,c^2}{17}\right)+x^{11/2}\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right)+x^{21/2}\left(\frac{2\,A\,c^3}{21}+\frac{2\,B\,b\,c^2}{7}\right)+x$$

input
```
int(x^(7/2)*(A + B*x)*(a + b*x + c*x^2)^3,x)
```

output
```
x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^2)/5 + (2*B*a^2*c)/5 + (4*A*a*b*c)/5) +
x^(17/2)*((2*B*b^3)/17 + (6*A*a*c^2)/17 + (6*A*b^2*c)/17 + (12*B*a*b*c)/17
) + x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(21/2)*((2*A*c^3)/21 + (2
*B*b*c^2)/7) + x^(13/2)*((6*A*a*b^2)/13 + (6*A*a^2*c)/13 + (6*B*a^2*b)/13)
 + x^(19/2)*((6*A*b*c^2)/19 + (6*B*a*c^2)/19 + (6*B*b^2*c)/19) + (2*A*a^3*
x^(9/2))/9 + (2*B*c^3*x^(23/2))/23
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int x^{7/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{2\sqrt{x}\,x^4(14549535b\,c^3x^7+15935205a\,c^3x^6+47805615b^2c^2x^6+105675570ab\,c^2x^5+52837785}{}$$

input
```
int(x^(7/2)*(B*x+A)*(c*x^2+b*x+a)^3,x)
```

output
```
(2*sqrt(x)*x**4*(37182145*a**4 + 121687020*a**3*b*x + 77224455*a**3*c*x**2
 + 154448910*a**2*b**2*x**2 + 200783583*a**2*b*c*x**3 + 59053995*a**2*c**2
*x**4 + 89237148*a*b**3*x**3 + 177161985*a*b**2*c*x**4 + 105675570*a*b*c**
2*x**5 + 15935205*a*c**3*x**6 + 19684665*b**4*x**4 + 52837785*b**3*c*x**5
+ 47805615*b**2*c**2*x**6 + 14549535*b*c**3*x**7))/334639305
```

3.75 $\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{7}a^3Ax^{7/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}$$
$$+\frac{6}{11}a\left(abB+A\left(b^2+ac\right)\right)x^{11/2}+\frac{2}{13}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{13/2}+\frac{2}{15}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x$$

output
```
2/7*a^3*A*x^(7/2)+2/9*a^2*(3*A*b+B*a)*x^(9/2)+6/11*a*(a*b*B+A*(a*c+b^2))*x
^(11/2)+2/13*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^(13/2)+2/15*(3*A*a*c^2+3*
A*b^2*c+6*B*a*b*c+B*b^3)*x^(15/2)+6/17*c*(A*b*c+B*a*c+B*b^2)*x^(17/2)+2/19
*c^2*(A*c+3*B*b)*x^(19/2)+2/21*B*c^3*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2x^{7/2}(230945a^3(9A+7Bx)+33915a^2x(13A(11b+9cx)+9Bx(13b+11cx))+1197ax^2(17A$$

input
```
Integrate[x^(5/2)*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(2*x^(7/2)*(230945*a^3*(9*A + 7*B*x) + 33915*a^2*x*(13*A*(11*b + 9*c*x) +
9*B*x*(13*b + 11*c*x)) + 1197*a*x^2*(17*A*(195*b^2 + 330*b*c*x + 143*c^2*x
^2) + 11*B*x*(255*b^2 + 442*b*c*x + 195*c^2*x^2)) + 33*x^3*(21*A*(1615*b^3
 + 4199*b^2*c*x + 3705*b*c^2*x^2 + 1105*c^3*x^3) + 13*B*x*(2261*b^3 + 5985
*b^2*c*x + 5355*b*c^2*x^2 + 1615*c^3*x^3))))/14549535
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$

$$\big\downarrow \text{ 1195}$$

$$\int \left(a^3Ax^{5/2}+a^2x^{7/2}(aB+3Ab)+3cx^{15/2}\left(aBc+Abc+b^2B\right)+3ax^{9/2}\left(A\left(ac+b^2\right)+abB\right)+x^{13/2}\left(3aAc^2+6\right.\right.$$

$$\big\downarrow \text{ 2009}$$

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{6}{17}cx^{17/2}\left(aBc+Abc+b^2B\right)+\frac{6}{11}ax^{11/2}\left(A\left(ac+b^2\right)+abB\right)+\frac{2}{15}x^{15/2}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{2}{13}x^{13/2}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{2}{19}c^2x^{19/2}(Ac+3bB)+\frac{2}{21}Bc^3x^{21/2}$$

input
```
Int[x^(5/2)*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*(a*b*B + A*(b
^2 + a*c))*x^(11/2))/11 + (2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^(13
/2))/13 + (2*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^(15/2))/15 + (6
*c*(b^2*B + A*b*c + a*B*c)*x^(17/2))/17 + (2*c^2*(3*b*B + A*c)*x^(19/2))/1
9 + (2*B*c^3*x^(21/2))/21
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

method	result
gosper	$2x^{\frac{7}{2}}\left(692835B\,c^3x^7+765765A\,c^3x^6+2297295Bb\,c^2x^6+2567565Ab\,c^2x^5+2567565Ba\,c^2x^5+2567565B\,b^2c\,x^5+290990\right.$
trager	$2x^{\frac{7}{2}}\left(692835B\,c^3x^7+765765A\,c^3x^6+2297295Bb\,c^2x^6+2567565Ab\,c^2x^5+2567565Ba\,c^2x^5+2567565B\,b^2c\,x^5+290990\right.$
risch	$2x^{\frac{7}{2}}\left(692835B\,c^3x^7+765765A\,c^3x^6+2297295Bb\,c^2x^6+2567565Ab\,c^2x^5+2567565Ba\,c^2x^5+2567565B\,b^2c\,x^5+290990\right.$
orering	$2x^{\frac{7}{2}}\left(692835B\,c^3x^7+765765A\,c^3x^6+2297295Bb\,c^2x^6+2567565Ab\,c^2x^5+2567565Ba\,c^2x^5+2567565B\,b^2c\,x^5+290990\right.$
derivativedivides	$\frac{2B\,c^3x^{\frac{21}{2}}}{21}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{19}{2}}}{19}+\frac{2(3Ab\,c^2+B(a\,c^2+2b^2c+c(2ac+b^2)))x^{\frac{17}{2}}}{17}+\frac{2(A(a\,c^2+2b^2c+c(2ac+b^2))+}{15}$
default	$\frac{2B\,c^3x^{\frac{21}{2}}}{21}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{19}{2}}}{19}+\frac{2(3Ab\,c^2+B(a\,c^2+2b^2c+c(2ac+b^2)))x^{\frac{17}{2}}}{17}+\frac{2(A(a\,c^2+2b^2c+c(2ac+b^2))+}{15}$

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/14549535*x^(7/2)*(692835*B*c^3*x^7+765765*A*c^3*x^6+2297295*B*b*c^2*x^6+
2567565*A*b*c^2*x^5+2567565*B*a*c^2*x^5+2567565*B*b^2*c*x^5+2909907*A*a*c^
2*x^4+2909907*A*b^2*c*x^4+5819814*B*a*b*c*x^4+969969*B*b^3*x^4+6715170*A*a
*b*c*x^3+1119195*A*b^3*x^3+3357585*B*a^2*c*x^3+3357585*B*a*b^2*x^3+3968055
*A*a^2*c*x^2+3968055*A*a*b^2*x^2+3968055*B*a^2*b*x^2+4849845*A*a^2*b*x+161
6615*B*a^3*x+2078505*A*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{14549535}\left(692835\,Bc^3x^{10}+765765\left(3\,Bbc^2+Ac^3\right)x^9+2567565\left(Bb^2c+(Ba+Ab)c^2\right)x^8+\right.$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
2/14549535*(692835*B*c^3*x^10 + 765765*(3*B*b*c^2 + A*c^3)*x^9 + 2567565*(
B*b^2*c + (B*a + A*b)*c^2)*x^8 + 969969*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b +
A*b^2)*c)*x^7 + 2078505*A*a^3*x^3 + 1119195*(3*B*a*b^2 + A*b^3 + 3*(B*a^2
+ 2*A*a*b)*c)*x^6 + 3968055*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^5 + 1616615*(B
*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.62

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{6Aa^2cx^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{11}{2}}}{11}+\frac{12Aabcx^{\frac{13}{2}}}{13}+\frac{2Aac^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{13}{2}}}{13}+\frac{2Ab^2cx^{\frac{15}{2}}}{5}+\frac{6Abc^2x^{\frac{17}{2}}}{17}+\frac{2Ac^3x^{\frac{19}{2}}}{19}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{6Ba^2bx^{\frac{11}{2}}}{11}+\frac{6Ba^2cx^{\frac{13}{2}}}{13}+\frac{6Bab^2x^{\frac{13}{2}}}{13}+\frac{4Babcx^{\frac{15}{2}}}{5}+\frac{6Bac^2x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{15}{2}}}{15}+\frac{6Bb^2cx^{\frac{17}{2}}}{17}+\frac{6Bbc^2x^{\frac{19}{2}}}{19}+\frac{2Bc^3x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(5/2)*(B*x+A)*(c*x**2+b*x+a)**3,x)
```

output
```
2*A*a**3*x**(7/2)/7 + 2*A*a**2*b*x**(9/2)/3 + 6*A*a**2*c*x**(11/2)/11 + 6*
A*a*b**2*x**(11/2)/11 + 12*A*a*b*c*x**(13/2)/13 + 2*A*a*c**2*x**(15/2)/5 +
 2*A*b**3*x**(13/2)/13 + 2*A*b**2*c*x**(15/2)/5 + 6*A*b*c**2*x**(17/2)/17
+ 2*A*c**3*x**(19/2)/19 + 2*B*a**3*x**(9/2)/9 + 6*B*a**2*b*x**(11/2)/11 +
6*B*a**2*c*x**(13/2)/13 + 6*B*a*b**2*x**(13/2)/13 + 4*B*a*b*c*x**(15/2)/5
+ 6*B*a*c**2*x**(17/2)/17 + 2*B*b**3*x**(15/2)/15 + 6*B*b**2*c*x**(17/2)/1
7 + 6*B*b*c**2*x**(19/2)/19 + 2*B*c**3*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{21}\,Bc^3x^{\frac{21}{2}}+\frac{2}{19}\left(3\,Bbc^2+Ac^3\right)x^{\frac{19}{2}}+\frac{6}{17}\left(Bb^2c+(Ba+Ab)c^2\right)x^{\frac{17}{2}}+\frac{2}{15}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{\frac{15}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}+\frac{2}{13}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^{\frac{13}{2}}+\frac{6}{11}\left(Ba^2b+Aab^2+Aa^2c\right)x^{\frac{11}{2}}+\frac{2}{9}\left(Ba^3+3\,Aa^2b\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
2/21*B*c^3*x^(21/2) + 2/19*(3*B*b*c^2 + A*c^3)*x^(19/2) + 6/17*(B*b^2*c +
(B*a + A*b)*c^2)*x^(17/2) + 2/15*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*
c)*x^(15/2) + 2/7*A*a^3*x^(7/2) + 2/13*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A
*a*b)*c)*x^(13/2) + 6/11*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^(11/2) + 2/9*(B*a
^3 + 3*A*a^2*b)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.06

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{21}Bc^3x^{\frac{21}{2}}+\frac{6}{19}Bbc^2x^{\frac{19}{2}}+\frac{2}{19}Ac^3x^{\frac{19}{2}}+\frac{6}{17}Bb^2cx^{\frac{17}{2}}+\frac{6}{17}Bac^2x^{\frac{17}{2}}+\frac{6}{17}Abc^2x^{\frac{17}{2}}+\frac{2}{15}Bb^3x^{\frac{15}{2}}+\frac{4}{5}Babcx^{\frac{15}{2}}+\frac{2}{5}Ab^2cx^{\frac{15}{2}}+\frac{2}{5}Aac^2x^{\frac{15}{2}}+\frac{6}{13}Bab^2x^{\frac{13}{2}}+\frac{2}{13}Ab^3x^{\frac{13}{2}}+\frac{6}{13}Ba^2cx^{\frac{13}{2}}+\frac{12}{13}Aabcx^{\frac{13}{2}}+\frac{6}{11}Ba^2bx^{\frac{11}{2}}+\frac{6}{11}Aab^2x^{\frac{11}{2}}+\frac{6}{11}Aa^2cx^{\frac{11}{2}}+\frac{2}{9}Ba^3x^{\frac{9}{2}}+\frac{2}{3}Aa^2bx^{\frac{9}{2}}+\frac{2}{7}Aa^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
2/21*B*c^3*x^(21/2) + 6/19*B*b*c^2*x^(19/2) + 2/19*A*c^3*x^(19/2) + 6/17*B
*b^2*c*x^(17/2) + 6/17*B*a*c^2*x^(17/2) + 6/17*A*b*c^2*x^(17/2) + 2/15*B*b
^3*x^(15/2) + 4/5*B*a*b*c*x^(15/2) + 2/5*A*b^2*c*x^(15/2) + 2/5*A*a*c^2*x^
(15/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/13*B*a^2*c*x^(13/
2) + 12/13*A*a*b*c*x^(13/2) + 6/11*B*a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2
) + 6/11*A*a^2*c*x^(11/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^(9/2) + 2/7*
A*a^3*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=x^{13/2}\left(\frac{6\,B\,c\,a^2}{13}+\frac{6\,B\,a\,b^2}{13}+\frac{12\,A\,c\,a\,b}{13}+\frac{2\,A\,b^3}{13}\right)+x^{15/2}\left(\frac{2\,B\,b^3}{15}+\frac{2\,A\,b^2\,c}{5}+\frac{4\,B\,a\,b\,c}{5}+\frac{2\,A\,a\,c^2}{5}\right)+x^{9/2}\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{19/2}\left(\frac{2\,A\,c^3}{19}+\frac{6\,B\,b\,c^2}{19}\right)+x^{11/}$$

input
```
int(x^(5/2)*(A + B*x)*(a + b*x + c*x^2)^3,x)
```

output
```
x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^2)/13 + (6*B*a^2*c)/13 + (12*A*a*b*c)/13
) + x^(15/2)*((2*B*b^3)/15 + (2*A*a*c^2)/5 + (2*A*b^2*c)/5 + (4*B*a*b*c)/5
) + x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(19/2)*((2*A*c^3)/19 + (6*B*
b*c^2)/19) + x^(11/2)*((6*A*a*b^2)/11 + (6*A*a^2*c)/11 + (6*B*a^2*b)/11) +
 x^(17/2)*((6*A*b*c^2)/17 + (6*B*a*c^2)/17 + (6*B*b^2*c)/17) + (2*A*a^3*x^
(7/2))/7 + (2*B*c^3*x^(21/2))/21
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int x^{5/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2\sqrt{x}\,x^3(692835b\,c^3x^7+765765a\,c^3x^6+2297295b^2c^2x^6+5135130ab\,c^2x^5+2567565b^3c\,x^5+2\cdots}{}$$

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x+a)^3,x)
```

output
```
(2*sqrt(x)*x**3*(2078505*a**4 + 6466460*a**3*b*x + 3968055*a**3*c*x**2 + 7
936110*a**2*b**2*x**2 + 10072755*a**2*b*c*x**3 + 2909907*a**2*c**2*x**4 +
4476780*a*b**3*x**3 + 8729721*a*b**2*c*x**4 + 5135130*a*b*c**2*x**5 + 7657
65*a*c**3*x**6 + 969969*b**4*x**4 + 2567565*b**3*c*x**5 + 2297295*b**2*c**
2*x**6 + 692835*b*c**3*x**7))/14549535
```

3.76 $\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{5}a^3Ax^{5/2}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{2}{3}a\left(abB+A\left(b^2+ac\right)\right)x^{9/2}+\frac{2}{11}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{11/2}+\frac{2}{13}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{13}$$

output
```
2/5*a^3*A*x^(5/2)+2/7*a^2*(3*A*b+B*a)*x^(7/2)+2/3*a*(a*b*B+A*(a*c+b^2))*x^
(9/2)+2/11*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^(11/2)+2/13*(3*A*a*c^2+3*A*
b^2*c+6*B*a*b*c+B*b^3)*x^(13/2)+2/5*c*(A*b*c+B*a*c+B*b^2)*x^(15/2)+2/17*c^
2*(A*c+3*B*b)*x^(17/2)+2/19*B*c^3*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2x^{5/2}(138567a^3(7A+5Bx)+20995a^2x(11A(9b+7cx)+7Bx(11b+9cx))+2261ax^2(5A(14$$

input
```
Integrate[x^(3/2)*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(2*x^(5/2)*(138567*a^3*(7*A + 5*B*x) + 20995*a^2*x*(11*A*(9*b + 7*c*x) + 7
*B*x*(11*b + 9*c*x)) + 2261*a*x^2*(5*A*(143*b^2 + 234*b*c*x + 99*c^2*x^2)
+ 3*B*x*(195*b^2 + 330*b*c*x + 143*c^2*x^2)) + 21*x^3*(19*A*(1105*b^3 + 28
05*b^2*c*x + 2431*b*c^2*x^2 + 715*c^3*x^3) + 11*B*x*(1615*b^3 + 4199*b^2*c
*x + 3705*b*c^2*x^2 + 1105*c^3*x^3))))/4849845
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(a^3Ax^{3/2}+a^2x^{5/2}(aB+3Ab)+3cx^{13/2}\left(aBc+Abc+b^2B\right)+3ax^{7/2}\left(A\left(ac+b^2\right)+abB\right)+x^{11/2}\left(3aAc^2+6\right.\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{5}cx^{15/2}\left(aBc+Abc+b^2B\right)+\frac{2}{3}ax^{9/2}\left(A\left(ac+b^2\right)+abB\right)+\frac{2}{13}x^{13/2}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{2}{11}x^{11/2}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{2}{17}c^2x^{17/2}(Ac+3bB)+\frac{2}{19}Bc^3x^{19/2}$$

input
```
Int[x^(3/2)*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (2*a*(a*b*B + A*(b
^2 + a*c))*x^(9/2))/3 + (2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^(11/2
))/11 + (2*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^(13/2))/13 + (2*c
*(b^2*B + A*b*c + a*B*c)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(17/2))/17 +
 (2*B*c^3*x^(19/2))/19
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

method	result
gosper	$\frac{2x^{\frac{5}{2}}(255255Bc^3x^7+285285Ac^3x^6+855855Bbc^2x^6+969969Abc^2x^5+969969Bac^2x^5+969969Bb^2cx^5+1119195Aac$
trager	$\frac{2x^{\frac{5}{2}}(255255Bc^3x^7+285285Ac^3x^6+855855Bbc^2x^6+969969Abc^2x^5+969969Bac^2x^5+969969Bb^2cx^5+1119195Aac$
risch	$\frac{2x^{\frac{5}{2}}(255255Bc^3x^7+285285Ac^3x^6+855855Bbc^2x^6+969969Abc^2x^5+969969Bac^2x^5+969969Bb^2cx^5+1119195Aac$
orering	$\frac{2x^{\frac{5}{2}}(255255Bc^3x^7+285285Ac^3x^6+855855Bbc^2x^6+969969Abc^2x^5+969969Bac^2x^5+969969Bb^2cx^5+1119195Aac$
derivativedivides	$\frac{2Bc^3x^{\frac{19}{2}}}{19}+\frac{2(Ac^3+3Bbc^2)x^{\frac{17}{2}}}{17}+\frac{2(3Abc^2+B(ac^2+2b^2c+c(2ac+b^2)))x^{\frac{15}{2}}}{15}+\frac{2(A(ac^2+2b^2c+c(2ac+b^2))+}{13}$
default	$\frac{2Bc^3x^{\frac{19}{2}}}{19}+\frac{2(Ac^3+3Bbc^2)x^{\frac{17}{2}}}{17}+\frac{2(3Abc^2+B(ac^2+2b^2c+c(2ac+b^2)))x^{\frac{15}{2}}}{15}+\frac{2(A(ac^2+2b^2c+c(2ac+b^2))+}{13}$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/4849845*x^(5/2)*(255255*B*c^3*x^7+285285*A*c^3*x^6+855855*B*b*c^2*x^6+96
9969*A*b*c^2*x^5+969969*B*a*c^2*x^5+969969*B*b^2*c*x^5+1119195*A*a*c^2*x^4
+1119195*A*b^2*c*x^4+2238390*B*a*b*c*x^4+373065*B*b^3*x^4+2645370*A*a*b*c*
x^3+440895*A*b^3*x^3+1322685*B*a^2*c*x^3+1322685*B*a*b^2*x^3+1616615*A*a^2
*c*x^2+1616615*A*a*b^2*x^2+1616615*B*a^2*b*x^2+2078505*A*a^2*b*x+692835*B*
a^3*x+969969*A*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{4849845}\left(255255\,Bc^3x^9+285285\left(3\,Bbc^2+Ac^3\right)x^8+969969\left(Bb^2c+(Ba+Ab)c^2\right)x^7+373\right.$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
2/4849845*(255255*B*c^3*x^9 + 285285*(3*B*b*c^2 + A*c^3)*x^8 + 969969*(B*b
^2*c + (B*a + A*b)*c^2)*x^7 + 373065*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b
^2)*c)*x^6 + 969969*A*a^3*x^2 + 440895*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A
*a*b)*c)*x^5 + 1616615*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^4 + 692835*(B*a^3 +
 3*A*a^2*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.62

$$\begin{aligned}\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx &= \frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{6Aa^2bx^{\frac{7}{2}}}{7}+\frac{2Aa^2cx^{\frac{9}{2}}}{3}\\ &+\frac{2Aab^2x^{\frac{9}{2}}}{3}+\frac{12Aabcx^{\frac{11}{2}}}{11}+\frac{6Aac^2x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{6Ab^2cx^{\frac{13}{2}}}{13}\\ &+\frac{2Abc^2x^{\frac{15}{2}}}{5}+\frac{2Ac^3x^{\frac{17}{2}}}{17}+\frac{2Ba^3x^{\frac{7}{2}}}{7}+\frac{2Ba^2bx^{\frac{9}{2}}}{3}+\frac{6Ba^2cx^{\frac{11}{2}}}{11}+\frac{6Bab^2x^{\frac{11}{2}}}{11}\\ &+\frac{12Babcx^{\frac{13}{2}}}{13}+\frac{2Bac^2x^{\frac{15}{2}}}{5}+\frac{2Bb^3x^{\frac{13}{2}}}{13}+\frac{2Bb^2cx^{\frac{15}{2}}}{5}+\frac{6Bbc^2x^{\frac{17}{2}}}{17}+\frac{2Bc^3x^{\frac{19}{2}}}{19}\end{aligned}$$

input
```
integrate(x**(3/2)*(B*x+A)*(c*x**2+b*x+a)**3,x)
```

output
```
2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(7/2)/7 + 2*A*a**2*c*x**(9/2)/3 + 2*A*
a*b**2*x**(9/2)/3 + 12*A*a*b*c*x**(11/2)/11 + 6*A*a*c**2*x**(13/2)/13 + 2*
A*b**3*x**(11/2)/11 + 6*A*b**2*c*x**(13/2)/13 + 2*A*b*c**2*x**(15/2)/5 + 2
*A*c**3*x**(17/2)/17 + 2*B*a**3*x**(7/2)/7 + 2*B*a**2*b*x**(9/2)/3 + 6*B*a
**2*c*x**(11/2)/11 + 6*B*a*b**2*x**(11/2)/11 + 12*B*a*b*c*x**(13/2)/13 + 2
*B*a*c**2*x**(15/2)/5 + 2*B*b**3*x**(13/2)/13 + 2*B*b**2*c*x**(15/2)/5 + 6
*B*b*c**2*x**(17/2)/17 + 2*B*c**3*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{2}{19}\,Bc^3x^{\frac{19}{2}} + \frac{2}{17}\left(3\,Bbc^2+Ac^3\right)x^{\frac{17}{2}} + \frac{2}{5}\left(Bb^2c+(Ba+Ab)c^2\right)x^{\frac{15}{2}} + \frac{2}{13}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{\frac{13}{2}} + \frac{2}{5}\,Aa^3x^{\frac{5}{2}} + \frac{2}{11}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^{\frac{11}{2}} + \frac{2}{3}\left(Ba^2b+Aab^2+Aa^2c\right)x^{\frac{9}{2}} + \frac{2}{7}\left(Ba^3+3\,Aa^2b\right)x^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
2/19*B*c^3*x^(19/2) + 2/17*(3*B*b*c^2 + A*c^3)*x^(17/2) + 2/5*(B*b^2*c + (
B*a + A*b)*c^2)*x^(15/2) + 2/13*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c
)*x^(13/2) + 2/5*A*a^3*x^(5/2) + 2/11*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*
a*b)*c)*x^(11/2) + 2/3*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^(9/2) + 2/7*(B*a^3
+ 3*A*a^2*b)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.06

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{2}{19}Bc^3x^{\frac{19}{2}}+\frac{6}{17}Bbc^2x^{\frac{17}{2}}+\frac{2}{17}Ac^3x^{\frac{17}{2}}+\frac{2}{5}Bb^2cx^{\frac{15}{2}}+\frac{2}{5}Bac^2x^{\frac{15}{2}}+\frac{2}{5}Abc^2x^{\frac{15}{2}}+\frac{2}{13}Bb^3x^{\frac{13}{2}}+\frac{12}{13}Babcx^{\frac{13}{2}}+\frac{6}{13}Ab^2cx^{\frac{13}{2}}+\frac{6}{13}Aac^2x^{\frac{13}{2}}+\frac{6}{11}Bab^2x^{\frac{11}{2}}+\frac{2}{11}Ab^3x^{\frac{11}{2}}+\frac{6}{11}Ba^2cx^{\frac{11}{2}}+\frac{12}{11}Aabcx^{\frac{11}{2}}+\frac{2}{3}Ba^2bx^{\frac{9}{2}}+\frac{2}{3}Aab^2x^{\frac{9}{2}}+\frac{2}{3}Aa^2cx^{\frac{9}{2}}+\frac{2}{7}Ba^3x^{\frac{7}{2}}+\frac{6}{7}Aa^2bx^{\frac{7}{2}}+\frac{2}{5}Aa^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
2/19*B*c^3*x^(19/2) + 6/17*B*b*c^2*x^(17/2) + 2/17*A*c^3*x^(17/2) + 2/5*B*
b^2*c*x^(15/2) + 2/5*B*a*c^2*x^(15/2) + 2/5*A*b*c^2*x^(15/2) + 2/13*B*b^3*
x^(13/2) + 12/13*B*a*b*c*x^(13/2) + 6/13*A*b^2*c*x^(13/2) + 6/13*A*a*c^2*x
^(13/2) + 6/11*B*a*b^2*x^(11/2) + 2/11*A*b^3*x^(11/2) + 6/11*B*a^2*c*x^(11
/2) + 12/11*A*a*b*c*x^(11/2) + 2/3*B*a^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) +
 2/3*A*a^2*c*x^(9/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^(7/2) + 2/5*A*a^3
*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx=x^{11/2}\left(\frac{6\,B\,c\,a^2}{11}+\frac{6\,B\,a\,b^2}{11}+\frac{12\,A\,c\,a\,b}{11}+\frac{2\,A\,b^3}{11}\right)+x^{13/2}\left(\frac{2\,B\,b^3}{13}+\frac{6\,A\,b^2\,c}{13}+\frac{12\,B\,a\,b\,c}{13}+\frac{6\,A\,a\,c^2}{13}\right)+x^{7/2}\left(\frac{2\,B\,a^3}{7}+\frac{6\,A\,b\,a^2}{7}\right)+x^{17/2}\left(\frac{2\,A\,c^3}{17}+\frac{6\,B\,b\,c^2}{17}\right)+x^{9/}$$

input
```
int(x^(3/2)*(A + B*x)*(a + b*x + c*x^2)^3,x)
```

output
```
x^(11/2)*((2*A*b^3)/11 + (6*B*a*b^2)/11 + (6*B*a^2*c)/11 + (12*A*a*b*c)/11
) + x^(13/2)*((2*B*b^3)/13 + (6*A*a*c^2)/13 + (6*A*b^2*c)/13 + (12*B*a*b*c
)/13) + x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(17/2)*((2*A*c^3)/17 + (
6*B*b*c^2)/17) + x^(9/2)*((2*A*a*b^2)/3 + (2*A*a^2*c)/3 + (2*B*a^2*b)/3) +
 x^(15/2)*((2*A*b*c^2)/5 + (2*B*a*c^2)/5 + (2*B*b^2*c)/5) + (2*A*a^3*x^(5/
2))/5 + (2*B*c^3*x^(19/2))/19
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int x^{3/2}(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{2\sqrt{x}\,x^2(255255b\,c^3x^7+285285a\,c^3x^6+855855b^2c^2x^6+1939938ab\,c^2x^5+969969b^3c\,x^5+111}{}$$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x+a)^3,x)
```

output
```
(2*sqrt(x)*x**2*(969969*a**4 + 2771340*a**3*b*x + 1616615*a**3*c*x**2 + 32
33230*a**2*b**2*x**2 + 3968055*a**2*b*c*x**3 + 1119195*a**2*c**2*x**4 + 17
63580*a*b**3*x**3 + 3357585*a*b**2*c*x**4 + 1939938*a*b*c**2*x**5 + 285285
*a*c**3*x**6 + 373065*b**4*x**4 + 969969*b**3*c*x**5 + 855855*b**2*c**2*x*
*6 + 255255*b*c**3*x**7))/4849845
```

3.77 $\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{2}{3}a^3Ax^{3/2}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{6}{7}a\left(abB+A\left(b^2+ac\right)\right)x^{7/2} + \frac{2}{9}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{9/2}+\frac{2}{11}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{11/2}+\frac{6}{13}c\left(b^2B+Abc+aBc\right)$$

output
```
2/3*a^3*A*x^(3/2)+2/5*a^2*(3*A*b+B*a)*x^(5/2)+6/7*a*(a*b*B+A*(a*c+b^2))*x^
(7/2)+2/9*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^(9/2)+2/11*(3*A*a*c^2+3*A*b^
2*c+6*B*a*b*c+B*b^3)*x^(11/2)+6/13*c*(A*b*c+B*a*c+B*b^2)*x^(13/2)+2/15*c^2
*(A*c+3*B*b)*x^(15/2)+2/17*B*c^3*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{2x^{3/2}(51051a^3(5A+3Bx)+7293a^2x(9A(7b+5cx)+5Bx(9b+7cx))+255ax^2(13A(99b^2+154bcx+}{}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(2*x^(3/2)*(51051*a^3*(5*A + 3*B*x) + 7293*a^2*x*(9*A*(7*b + 5*c*x) + 5*B*
x*(9*b + 7*c*x)) + 255*a*x^2*(13*A*(99*b^2 + 154*b*c*x + 63*c^2*x^2) + 7*B
*x*(143*b^2 + 234*b*c*x + 99*c^2*x^2)) + 7*x^3*(17*A*(715*b^3 + 1755*b^2*c
*x + 1485*b*c^2*x^2 + 429*c^3*x^3) + 9*B*x*(1105*b^3 + 2805*b^2*c*x + 2431
*b*c^2*x^2 + 715*c^3*x^3))))/765765
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^3A\sqrt{x}+a^2x^{3/2}(aB+3Ab)+3cx^{11/2}\left(aBc+Abc+b^2B\right)+3ax^{5/2}\left(A\left(ac+b^2\right)+abB\right)+x^{9/2}\left(3aAc^2+6a\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{6}{13}cx^{13/2}\left(aBc+Abc+b^2B\right)+\frac{6}{7}ax^{7/2}\left(A\left(ac+b^2\right)+abB\right)+\frac{2}{11}x^{11/2}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{2}{9}x^{9/2}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{2}{15}c^2x^{15/2}(Ac+3bB)+\frac{2}{17}Bc^3x^{17/2}$$

input
```
Int[Sqrt[x]*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*(a*b*B + A*(b
^2 + a*c))*x^(7/2))/7 + (2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^(9/2)
)/9 + (2*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^(11/2))/11 + (6*c*(
b^2*B + A*b*c + a*B*c)*x^(13/2))/13 + (2*c^2*(3*b*B + A*c)*x^(15/2))/15 +
(2*B*c^3*x^(17/2))/17
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

method	result
gosper	$\frac{2x^{\frac{3}{2}}\left(45045B\,c^{3}x^{7}+51051A\,c^{3}x^{6}+153153Bb\,c^{2}x^{6}+176715Ab\,c^{2}x^{5}+176715Ba\,c^{2}x^{5}+176715B\,b^{2}c\,x^{5}+208845Aa\,c^{2}x^{4}\right.}{}$
trager	$\frac{2x^{\frac{3}{2}}\left(45045B\,c^{3}x^{7}+51051A\,c^{3}x^{6}+153153Bb\,c^{2}x^{6}+176715Ab\,c^{2}x^{5}+176715Ba\,c^{2}x^{5}+176715B\,b^{2}c\,x^{5}+208845Aa\,c^{2}x^{4}\right.}{}$
risch	$\frac{2x^{\frac{3}{2}}\left(45045B\,c^{3}x^{7}+51051A\,c^{3}x^{6}+153153Bb\,c^{2}x^{6}+176715Ab\,c^{2}x^{5}+176715Ba\,c^{2}x^{5}+176715B\,b^{2}c\,x^{5}+208845Aa\,c^{2}x^{4}\right.}{}$
orering	$\frac{2x^{\frac{3}{2}}\left(45045B\,c^{3}x^{7}+51051A\,c^{3}x^{6}+153153Bb\,c^{2}x^{6}+176715Ab\,c^{2}x^{5}+176715Ba\,c^{2}x^{5}+176715B\,b^{2}c\,x^{5}+208845Aa\,c^{2}x^{4}\right.}{}$
derivativedivides	$\frac{2B\,c^{3}x^{\frac{17}{2}}}{17}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3Ab\,c^{2}+B\left(a\,c^{2}+2b^{2}c+c\left(2ac+b^{2}\right)\right)\right)x^{\frac{13}{2}}}{13}+\frac{2\left(A\left(a\,c^{2}+2b^{2}c+c\left(2ac+b^{2}\right)\right)+\right.}{11}$
default	$\frac{2B\,c^{3}x^{\frac{17}{2}}}{17}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3Ab\,c^{2}+B\left(a\,c^{2}+2b^{2}c+c\left(2ac+b^{2}\right)\right)\right)x^{\frac{13}{2}}}{13}+\frac{2\left(A\left(a\,c^{2}+2b^{2}c+c\left(2ac+b^{2}\right)\right)+\right.}{11}$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/765765*x^(3/2)*(45045*B*c^3*x^7+51051*A*c^3*x^6+153153*B*b*c^2*x^6+17671
5*A*b*c^2*x^5+176715*B*a*c^2*x^5+176715*B*b^2*c*x^5+208845*A*a*c^2*x^4+208
845*A*b^2*c*x^4+417690*B*a*b*c*x^4+69615*B*b^3*x^4+510510*A*a*b*c*x^3+8508
5*A*b^3*x^3+255255*B*a^2*c*x^3+255255*B*a*b^2*x^3+328185*A*a^2*c*x^2+32818
5*A*a*b^2*x^2+328185*B*a^2*b*x^2+459459*A*a^2*b*x+153153*B*a^3*x+255255*A*
a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$
$$=\frac{2}{765765}\left(45045\,Bc^3x^8+51051\left(3\,Bbc^2+Ac^3\right)x^7+176715\left(Bb^2c+(Ba+Ab)c^2\right)x^6+69615\left(Bb^3+3\,A\right.\right.$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
2/765765*(45045*B*c^3*x^8 + 51051*(3*B*b*c^2 + A*c^3)*x^7 + 176715*(B*b^2*
c + (B*a + A*b)*c^2)*x^6 + 69615*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*
c)*x^5 + 255255*A*a^3*x + 85085*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c
)*x^4 + 328185*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^3 + 153153*(B*a^3 + 3*A*a^2
*b)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Bc^3x^{\frac{17}{2}}}{17}+\frac{2x^{\frac{15}{2}}(Ac^3+3Bbc^2)}{15}+\frac{2x^{\frac{13}{2}}\cdot(3Abc^2+3Bac^2+3Bb^2c)}{13}+\frac{2x^{\frac{11}{2}}\cdot(3Aac^2+3Ab^2c+6Babc+Bb^3)}{11}+\frac{2x^{\frac{9}{2}}\cdot(6Aabc+Ab^3+3Ba^2c+3Bab^2)}{9}+\frac{2x^{\frac{7}{2}}\cdot(3Aa^2c+3Aab^2+3Ba^2b)}{7}+\frac{2x^{\frac{5}{2}}\cdot(3Aa^2b+Ba^3)}{5}$$

input

```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x+a)**3,x)
```

output

```
2*A*a**3*x**(3/2)/3 + 2*B*c**3*x**(17/2)/17 + 2*x**(15/2)*(A*c**3 + 3*B*b*
c**2)/15 + 2*x**(13/2)*(3*A*b*c**2 + 3*B*a*c**2 + 3*B*b**2*c)/13 + 2*x**(1
1/2)*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3)/11 + 2*x**(9/2)*(6*A*a
*b*c + A*b**3 + 3*B*a**2*c + 3*B*a*b**2)/9 + 2*x**(7/2)*(3*A*a**2*c + 3*A*
a*b**2 + 3*B*a**2*b)/7 + 2*x**(5/2)*(3*A*a**2*b + B*a**3)/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{2}{17}Bc^3x^{\frac{17}{2}} + \frac{2}{15}\left(3\,Bbc^2+Ac^3\right)x^{\frac{15}{2}} + \frac{6}{13}\left(Bb^2c+(Ba+Ab)c^2\right)x^{\frac{13}{2}} + \frac{2}{11}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{\frac{11}{2}} + \frac{2}{3}Aa^3x^{\frac{3}{2}} + \frac{2}{9}\left(3\,Bab^2+Ab^3+3\left(Ba^2+2\,Aab\right)c\right)x^{\frac{9}{2}} + \frac{6}{7}\left(Ba^2b+Aab^2+Aa^2c\right)x^{\frac{7}{2}} + \frac{2}{5}\left(Ba^3+3\,Aa^2b\right)x^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
2/17*B*c^3*x^(17/2) + 2/15*(3*B*b*c^2 + A*c^3)*x^(15/2) + 6/13*(B*b^2*c +
(B*a + A*b)*c^2)*x^(13/2) + 2/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*
c)*x^(11/2) + 2/3*A*a^3*x^(3/2) + 2/9*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*
a*b)*c)*x^(9/2) + 6/7*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^(7/2) + 2/5*(B*a^3 +
 3*A*a^2*b)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.06

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{2}{17}\,Bc^3x^{\frac{17}{2}} + \frac{2}{5}\,Bbc^2x^{\frac{15}{2}} + \frac{2}{15}\,Ac^3x^{\frac{15}{2}} + \frac{6}{13}\,Bb^2cx^{\frac{13}{2}} + \frac{6}{13}\,Bac^2x^{\frac{13}{2}} + \frac{6}{13}\,Abc^2x^{\frac{13}{2}} + \frac{2}{11}\,Bb^3x^{\frac{11}{2}} + \frac{12}{11}\,Babcx^{\frac{11}{2}} + \frac{6}{11}\,Ab^2cx^{\frac{11}{2}} + \frac{6}{11}\,Aac^2x^{\frac{11}{2}} + \frac{2}{3}\,Bab^2x^{\frac{9}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}} + \frac{2}{3}\,Ba^2cx^{\frac{9}{2}} + \frac{4}{3}\,Aabcx^{\frac{9}{2}} + \frac{6}{7}\,Ba^2bx^{\frac{7}{2}} + \frac{6}{7}\,Aab^2x^{\frac{7}{2}} + \frac{6}{7}\,Aa^2cx^{\frac{7}{2}} + \frac{2}{5}\,Ba^3x^{\frac{5}{2}} + \frac{6}{5}\,Aa^2bx^{\frac{5}{2}} + \frac{2}{3}\,Aa^3x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
2/17*B*c^3*x^(17/2) + 2/5*B*b*c^2*x^(15/2) + 2/15*A*c^3*x^(15/2) + 6/13*B*
b^2*c*x^(13/2) + 6/13*B*a*c^2*x^(13/2) + 6/13*A*b*c^2*x^(13/2) + 2/11*B*b^
3*x^(11/2) + 12/11*B*a*b*c*x^(11/2) + 6/11*A*b^2*c*x^(11/2) + 6/11*A*a*c^2
*x^(11/2) + 2/3*B*a*b^2*x^(9/2) + 2/9*A*b^3*x^(9/2) + 2/3*B*a^2*c*x^(9/2)
+ 4/3*A*a*b*c*x^(9/2) + 6/7*B*a^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 6/7*A*
a^2*c*x^(7/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^(5/2) + 2/3*A*a^3*x^(3/2
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx = x^{9/2}\left(\frac{2\,B\,c\,a^2}{3}+\frac{2\,B\,a\,b^2}{3}+\frac{4\,A\,c\,a\,b}{3}+\frac{2\,A\,b^3}{9}\right) + x^{11/2}\left(\frac{2\,B\,b^3}{11}+\frac{6\,A\,b^2\,c}{11}+\frac{12\,B\,a\,b\,c}{11}+\frac{6\,A\,a\,c^2}{11}\right) + x^{5/2}\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right) + x^{15/2}\left(\frac{2\,A\,c^3}{15}+\frac{2\,B\,b\,c^2}{5}\right) + x^{7/2}\left(\frac{6\,B\,a^2\,b}{7}+\frac{6\,A\,c\,a^2}{7}+\frac{6\,A\,a\,b^2}{7}\right) + x^{13/2}\left(\frac{6\,B\right.$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x + c*x^2)^3,x)
```

output
```
x^(9/2)*((2*A*b^3)/9 + (2*B*a*b^2)/3 + (2*B*a^2*c)/3 + (4*A*a*b*c)/3) + x^
(11/2)*((2*B*b^3)/11 + (6*A*a*c^2)/11 + (6*A*b^2*c)/11 + (12*B*a*b*c)/11)
+ x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(15/2)*((2*A*c^3)/15 + (2*B*b*
c^2)/5) + x^(7/2)*((6*A*a*b^2)/7 + (6*A*a^2*c)/7 + (6*B*a^2*b)/7) + x^(13/
2)*((6*A*b*c^2)/13 + (6*B*a*c^2)/13 + (6*B*b^2*c)/13) + (2*A*a^3*x^(3/2))/
3 + (2*B*c^3*x^(17/2))/17
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$
$$= \frac{2\sqrt{x}\,x(45045b\,c^3x^7+51051a\,c^3x^6+153153b^2c^2x^6+353430ab\,c^2x^5+176715b^3c\,x^5+208845a^2c^2x^4+62}{}$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^3,x)
```

output
```
(2*sqrt(x)*x*(255255*a**4 + 612612*a**3*b*x + 328185*a**3*c*x**2 + 656370*
a**2*b**2*x**2 + 765765*a**2*b*c*x**3 + 208845*a**2*c**2*x**4 + 340340*a*b
**3*x**3 + 626535*a*b**2*c*x**4 + 353430*a*b*c**2*x**5 + 51051*a*c**3*x**6
 + 69615*b**4*x**4 + 176715*b**3*c*x**5 + 153153*b**2*c**2*x**6 + 45045*b*
c**3*x**7))/765765
```

3.78 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 180

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{\sqrt{x}}\,dx = 2a^3A\sqrt{x}+\frac{2}{3}a^2(3Ab+aB)x^{3/2} +\frac{6}{5}a\left(abB+A\left(b^2+ac\right)\right)x^{5/2}+\frac{2}{7}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{7/2} +\frac{2}{9}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{9/2}+\frac{6}{11}c\left(b^2B+Abc+aBc\right)x^{11/2}+\frac{2}{13}c^2(3bB+Ac)x^{13/2}+\frac{2}{15}Bc^3x^{15/2}$$

output
```
2*a^3*A*x^(1/2)+2/3*a^2*(3*A*b+B*a)*x^(3/2)+6/5*a*(a*b*B+A*(a*c+b^2))*x^(5
/2)+2/7*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^(7/2)+2/9*(3*A*a*c^2+3*A*b^2*c
+6*B*a*b*c+B*b^3)*x^(9/2)+6/11*c*(A*b*c+B*a*c+B*b^2)*x^(11/2)+2/13*c^2*(A*
c+3*B*b)*x^(13/2)+2/15*B*c^3*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{\sqrt{x}}\,dx$$
$$= \frac{2\sqrt{x}(15015a^3(3A+Bx)+1287a^2x(7A(5b+3cx)+3Bx(7b+5cx))+39ax^2(11A(63b^2+90bcx+35c^2$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(15015*a^3*(3*A + B*x) + 1287*a^2*x*(7*A*(5*b + 3*c*x) + 3*B*x*
(7*b + 5*c*x)) + 39*a*x^2*(11*A*(63*b^2 + 90*b*c*x + 35*c^2*x^2) + 5*B*x*(
99*b^2 + 154*b*c*x + 63*c^2*x^2)) + x^3*(15*A*(429*b^3 + 1001*b^2*c*x + 81
9*b*c^2*x^2 + 231*c^3*x^3) + 7*B*x*(715*b^3 + 1755*b^2*c*x + 1485*b*c^2*x^
2 + 429*c^3*x^3))))/45045
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{\sqrt{x}}\,dx$$
$$\Big\downarrow \text{ 1195}$$
$$\int \left(\frac{a^3A}{\sqrt{x}}+a^2\sqrt{x}(aB+3Ab)+3cx^{9/2}\left(aBc+Abc+b^2B\right)+3ax^{3/2}\left(A\left(ac+b^2\right)+abB\right)+x^{7/2}\left(3aAc^2+6abBc\right.\right.$$
$$\Big\downarrow \text{ 2009}$$

$$2a^3A\sqrt{x}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{6}{11}cx^{11/2}\left(aBc+Abc+b^2B\right)+\frac{6}{5}ax^{5/2}\left(A\left(ac+b^2\right)+abB\right)+\frac{2}{9}x^{9/2}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{2}{7}x^{7/2}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{2}{13}c^2x^{13/2}(Ac+3bB)+\frac{2}{15}Bc^3x^{15/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/Sqrt[x],x]
```

output
```
2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (6*a*(a*b*B + A*(b^2 +
 a*c))*x^(5/2))/5 + (2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^(7/2))/7
+ (2*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^(9/2))/9 + (6*c*(b^2*B
+ A*b*c + a*B*c)*x^(11/2))/11 + (2*c^2*(3*b*B + A*c)*x^(13/2))/13 + (2*B*c
^3*x^(15/2))/15
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.06

method	result
trager	$\left(\frac{2}{15}B\,c^{3}x^{7}+\frac{2}{13}A\,c^{3}x^{6}+\frac{6}{13}Bb\,c^{2}x^{6}+\frac{6}{11}Ab\,c^{2}x^{5}+\frac{6}{11}Ba\,c^{2}x^{5}+\frac{6}{11}B\,b^{2}c\,x^{5}+\frac{2}{3}Aa\,c^{2}x^{4}\right.$
gosper	$\frac{2\sqrt{x}\,(3003B\,c^{3}x^{7}+3465A\,c^{3}x^{6}+10395Bb\,c^{2}x^{6}+12285Ab\,c^{2}x^{5}+12285Ba\,c^{2}x^{5}+12285B\,b^{2}c\,x^{5}+15015Aa\,c^{2}x^{4}+15015}{}$
risch	$\frac{2\sqrt{x}\,(3003B\,c^{3}x^{7}+3465A\,c^{3}x^{6}+10395Bb\,c^{2}x^{6}+12285Ab\,c^{2}x^{5}+12285Ba\,c^{2}x^{5}+12285B\,b^{2}c\,x^{5}+15015Aa\,c^{2}x^{4}+15015}{}$
orering	$\frac{2\sqrt{x}\,(3003B\,c^{3}x^{7}+3465A\,c^{3}x^{6}+10395Bb\,c^{2}x^{6}+12285Ab\,c^{2}x^{5}+12285Ba\,c^{2}x^{5}+12285B\,b^{2}c\,x^{5}+15015Aa\,c^{2}x^{4}+15015}{}$
derivativedivides	$\frac{2B\,c^{3}x^{\frac{15}{2}}}{15}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{13}{2}}}{13}+\frac{2(3Ab\,c^{2}+B(a\,c^{2}+2b^{2}c+c(2ac+b^{2})))x^{\frac{11}{2}}}{11}+\frac{2(A(a\,c^{2}+2b^{2}c+c(2ac+b^{2}))+}{9}$
default	$\frac{2B\,c^{3}x^{\frac{15}{2}}}{15}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{13}{2}}}{13}+\frac{2(3Ab\,c^{2}+B(a\,c^{2}+2b^{2}c+c(2ac+b^{2})))x^{\frac{11}{2}}}{11}+\frac{2(A(a\,c^{2}+2b^{2}c+c(2ac+b^{2}))+}{9}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/15*B*c^3*x^7+2/13*A*c^3*x^6+6/13*B*b*c^2*x^6+6/11*A*b*c^2*x^5+6/11*B*a*
c^2*x^5+6/11*B*b^2*c*x^5+2/3*A*a*c^2*x^4+2/3*A*b^2*c*x^4+4/3*B*a*b*c*x^4+2
/9*B*b^3*x^4+12/7*A*a*b*c*x^3+2/7*A*b^3*x^3+6/7*B*a^2*c*x^3+6/7*B*a*b^2*x^
3+6/5*A*a^2*c*x^2+6/5*A*a*b^2*x^2+6/5*B*a^2*b*x^2+2*A*a^2*b*x+2/3*B*a^3*x+
2*a^3*A)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{\sqrt{x}}\,dx$$
$$=\frac{2}{45045}\left(3003\,Bc^3x^7+3465\left(3\,Bbc^2+Ac^3\right)x^6+12285\left(Bb^2c+(Ba+Ab)c^2\right)x^5+5005\left(Bb^3+3\,Aac^2+\right.\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*B*c^3*x^7 + 3465*(3*B*b*c^2 + A*c^3)*x^6 + 12285*(B*b^2*c +
(B*a + A*b)*c^2)*x^5 + 5005*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^
4 + 45045*A*a^3 + 6435*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 2
7027*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 15015*(B*a^3 + 3*A*a^2*b)*x)*sqrt
(x)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.62

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{\sqrt{x}}\,dx = 2Aa^3\sqrt{x} + 2Aa^2bx^{\frac{3}{2}} + \frac{6Aa^2cx^{\frac{5}{2}}}{5} + \frac{6Aab^2x^{\frac{5}{2}}}{5} + \frac{12Aabcx^{\frac{7}{2}}}{7} + \frac{2Aac^2x^{\frac{9}{2}}}{3} + \frac{2Ab^3x^{\frac{7}{2}}}{7} + \frac{2Ab^2cx^{\frac{9}{2}}}{3} + \frac{6Abc^2x^{\frac{11}{2}}}{11} + \frac{2Ac^3x^{\frac{13}{2}}}{13} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{6Ba^2bx^{\frac{5}{2}}}{5} + \frac{6Ba^2cx^{\frac{7}{2}}}{7} + \frac{6Bab^2x^{\frac{7}{2}}}{7} + \frac{4Babcx^{\frac{9}{2}}}{3} + \frac{6Bac^2x^{\frac{11}{2}}}{11} + \frac{2Bb^3x^{\frac{9}{2}}}{9} + \frac{6Bb^2cx^{\frac{11}{2}}}{11} + \frac{6Bbc^2x^{\frac{13}{2}}}{13} + \frac{2Bc^3x^{\frac{15}{2}}}{15}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**(1/2),x)
```

output
```
2*A*a**3*sqrt(x) + 2*A*a**2*b*x**(3/2) + 6*A*a**2*c*x**(5/2)/5 + 6*A*a*b**
2*x**(5/2)/5 + 12*A*a*b*c*x**(7/2)/7 + 2*A*a*c**2*x**(9/2)/3 + 2*A*b**3*x*
*(7/2)/7 + 2*A*b**2*c*x**(9/2)/3 + 6*A*b*c**2*x**(11/2)/11 + 2*A*c**3*x**(
13/2)/13 + 2*B*a**3*x**(3/2)/3 + 6*B*a**2*b*x**(5/2)/5 + 6*B*a**2*c*x**(7/
2)/7 + 6*B*a*b**2*x**(7/2)/7 + 4*B*a*b*c*x**(9/2)/3 + 6*B*a*c**2*x**(11/2)
/11 + 2*B*b**3*x**(9/2)/9 + 6*B*b**2*c*x**(11/2)/11 + 6*B*b*c**2*x**(13/2)
/13 + 2*B*c**3*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{\sqrt{x}}\,dx = \frac{2}{15}\,Bc^3x^{\frac{15}{2}} + \frac{2}{13}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{13}{2}} + \frac{6}{11}\left(Bb^2c + (Ba+Ab)c^2\right)x^{\frac{11}{2}} + \frac{2}{9}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^{\frac{9}{2}} + 2\,Aa^3\sqrt{x} + \frac{2}{7}\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x^{\frac{7}{2}} + \frac{6}{5}\left(Ba^2b + Aab^2 + Aa^2c\right)x^{\frac{5}{2}} + \frac{2}{3}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/15*B*c^3*x^(15/2) + 2/13*(3*B*b*c^2 + A*c^3)*x^(13/2) + 6/11*(B*b^2*c +
(B*a + A*b)*c^2)*x^(11/2) + 2/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c
)*x^(9/2) + 2*A*a^3*sqrt(x) + 2/7*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)
*c)*x^(7/2) + 6/5*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^(5/2) + 2/3*(B*a^3 + 3*A
*a^2*b)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{\sqrt{x}}\,dx = \frac{2}{15}\,Bc^3x^{\frac{15}{2}} + \frac{6}{13}\,Bbc^2x^{\frac{13}{2}} + \frac{2}{13}\,Ac^3x^{\frac{13}{2}} + \frac{6}{11}\,Bb^2cx^{\frac{11}{2}} + \frac{6}{11}\,Bac^2x^{\frac{11}{2}} + \frac{6}{11}\,Abc^2x^{\frac{11}{2}} + \frac{2}{9}\,Bb^3x^{\frac{9}{2}} + \frac{4}{3}\,Babcx^{\frac{9}{2}} + \frac{2}{3}\,Ab^2cx^{\frac{9}{2}} + \frac{2}{3}\,Aac^2x^{\frac{9}{2}} + \frac{6}{7}\,Bab^2x^{\frac{7}{2}} + \frac{2}{7}\,Ab^3x^{\frac{7}{2}} + \frac{6}{7}\,Ba^2cx^{\frac{7}{2}} + \frac{12}{7}\,Aabcx^{\frac{7}{2}} + \frac{6}{5}\,Ba^2bx^{\frac{5}{2}} + \frac{6}{5}\,Aab^2x^{\frac{5}{2}} + \frac{6}{5}\,Aa^2cx^{\frac{5}{2}} + \frac{2}{3}\,Ba^3x^{\frac{3}{2}} + 2\,Aa^2bx^{\frac{3}{2}} + 2\,Aa^3\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/15*B*c^3*x^(15/2) + 6/13*B*b*c^2*x^(13/2) + 2/13*A*c^3*x^(13/2) + 6/11*B
*b^2*c*x^(11/2) + 6/11*B*a*c^2*x^(11/2) + 6/11*A*b*c^2*x^(11/2) + 2/9*B*b^
3*x^(9/2) + 4/3*B*a*b*c*x^(9/2) + 2/3*A*b^2*c*x^(9/2) + 2/3*A*a*c^2*x^(9/2
) + 6/7*B*a*b^2*x^(7/2) + 2/7*A*b^3*x^(7/2) + 6/7*B*a^2*c*x^(7/2) + 12/7*A
*a*b*c*x^(7/2) + 6/5*B*a^2*b*x^(5/2) + 6/5*A*a*b^2*x^(5/2) + 6/5*A*a^2*c*x
^(5/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2) + 2*A*a^3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{\sqrt{x}}\,dx = x^{7/2}\left(\frac{6\,B\,c\,a^2}{7}+\frac{6\,B\,a\,b^2}{7}+\frac{12\,A\,c\,a\,b}{7}+\frac{2\,A\,b^3}{7}\right) + x^{9/2}\left(\frac{2\,B\,b^3}{9}+\frac{2\,A\,b^2\,c}{3}+\frac{4\,B\,a\,b\,c}{3}+\frac{2\,A\,a\,c^2}{3}\right) + x^{3/2}\left(\frac{2\,B\,a^3}{3}+2\,A\,b\,a^2\right)+x^{13/2}\left(\frac{2\,A\,c^3}{13}+\frac{6\,B\,b\,c^2}{13}\right)+x^{5/2}\left(\frac{6\,B\,a^2\,b}{5}+\frac{6\,A\,c\,a^2}{5}+\frac{6\,A\,a\,b^2}{5}\right)+x^{11/2}\left(\frac{6\,B}{1}\right.$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^(1/2),x)
```

output
```
x^(7/2)*((2*A*b^3)/7 + (6*B*a*b^2)/7 + (6*B*a^2*c)/7 + (12*A*a*b*c)/7) + x
^(9/2)*((2*B*b^3)/9 + (2*A*a*c^2)/3 + (2*A*b^2*c)/3 + (4*B*a*b*c)/3) + x^(
3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(13/2)*((2*A*c^3)/13 + (6*B*b*c^2)/13)
+ x^(5/2)*((6*A*a*b^2)/5 + (6*A*a^2*c)/5 + (6*B*a^2*b)/5) + x^(11/2)*((6*A
*b*c^2)/11 + (6*B*a*c^2)/11 + (6*B*b^2*c)/11) + 2*A*a^3*x^(1/2) + (2*B*c^3
*x^(15/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(3003b\,c^3x^7+3465a\,c^3x^6+10395b^2c^2x^6+24570ab\,c^2x^5+12285b^3c\,x^5+15015a^2c^2x^4+45045a\,b^2c}{4504}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(1/2),x)
```

output
```
(2*sqrt(x)*(45045*a**4 + 60060*a**3*b*x + 27027*a**3*c*x**2 + 54054*a**2*b
**2*x**2 + 57915*a**2*b*c*x**3 + 15015*a**2*c**2*x**4 + 25740*a*b**3*x**3
+ 45045*a*b**2*c*x**4 + 24570*a*b*c**2*x**5 + 3465*a*c**3*x**6 + 5005*b**4
*x**4 + 12285*b**3*c*x**5 + 10395*b**2*c**2*x**6 + 3003*b*c**3*x**7))/4504
5
```

3.79 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{3/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{3/2}} \, dx = -\frac{2a^3A}{\sqrt{x}} + 2a^2(3Ab+aB)\sqrt{x} + 2a\left(abB+A\left(b^2+ac\right)\right)x^{3/2} + \frac{2}{5}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{5/2} + \frac{2}{7}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{7/2} + \frac{2}{3}c\left(b^2B+Abc+aBc\right)x^{9/2} + \frac{2}{11}c^2(3bB+Ac)x^{11/2} + \frac{2}{13}Bc^3x^{13/2}$$

output
```
-2*a^3*A/x^(1/2)+2*a^2*(3*A*b+B*a)*x^(1/2)+2*a*(a*b*B+A*(a*c+b^2))*x^(3/2)
+2/5*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^(5/2)+2/7*(3*A*a*c^2+3*A*b^2*c+6*
B*a*b*c+B*b^3)*x^(7/2)+2/3*c*(A*b*c+B*a*c+B*b^2)*x^(9/2)+2/11*c^2*(A*c+3*B
*b)*x^(11/2)+2/13*B*c^3*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{-30030a^3(A-Bx)+6006a^2x(5A(3b+cx)+Bx(5b+3cx))+286ax^2}{}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^(3/2),x]
```

output
```
(-30030*a^3*(A - B*x) + 6006*a^2*x*(5*A*(3*b + c*x) + B*x*(5*b + 3*c*x)) +
 286*a*x^2*(3*A*(35*b^2 + 42*b*c*x + 15*c^2*x^2) + B*x*(63*b^2 + 90*b*c*x
+ 35*c^2*x^2)) + 2*x^3*(13*A*(231*b^3 + 495*b^2*c*x + 385*b*c^2*x^2 + 105*
c^3*x^3) + 5*B*x*(429*b^3 + 1001*b^2*c*x + 819*b*c^2*x^2 + 231*c^3*x^3)))/
(15015*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^{3/2}}+\frac{a^2(aB+3Ab)}{\sqrt{x}}+3cx^{7/2}\left(aBc+Abc+b^2B\right)+3a\sqrt{x}\left(A\left(ac+b^2\right)+abB\right)+x^{5/2}\left(3aAc^2+6abBc+3\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{\sqrt{x}}+2a^2\sqrt{x}(aB+3Ab)+\frac{2}{3}cx^{9/2}\left(aBc+Abc+b^2B\right)+2ax^{3/2}\left(A\left(ac+b^2\right)+abB\right)+\frac{2}{7}x^{7/2}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{2}{5}x^{5/2}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{2}{11}c^2x^{11/2}(Ac+3bB)+\frac{2}{13}Bc^3x^{13/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^(3/2),x]
```

output
```
(-2*a^3*A)/Sqrt[x] + 2*a^2*(3*A*b + a*B)*Sqrt[x] + 2*a*(a*b*B + A*(b^2 + a
*c))*x^(3/2) + (2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^(5/2))/5 + (2*
(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^(7/2))/7 + (2*c*(b^2*B + A*b
*c + a*B*c)*x^(9/2))/3 + (2*c^2*(3*b*B + A*c)*x^(11/2))/11 + (2*B*c^3*x^(1
3/2))/13
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.09

method	result
gosper	$-\frac{2(-1155B\,c^3x^7-1365A\,c^3x^6-4095Bb\,c^2x^6-5005Ab\,c^2x^5-5005Ba\,c^2x^5-5005B\,b^2c\,x^5-6435Aa\,c^2x^4-6435A\,b^2c}{}$
trager	$-\frac{2(-1155B\,c^3x^7-1365A\,c^3x^6-4095Bb\,c^2x^6-5005Ab\,c^2x^5-5005Ba\,c^2x^5-5005B\,b^2c\,x^5-6435Aa\,c^2x^4-6435A\,b^2c}{}$
risch	$-\frac{2(-1155B\,c^3x^7-1365A\,c^3x^6-4095Bb\,c^2x^6-5005Ab\,c^2x^5-5005Ba\,c^2x^5-5005B\,b^2c\,x^5-6435Aa\,c^2x^4-6435A\,b^2c}{}$
orering	$-\frac{2(-1155B\,c^3x^7-1365A\,c^3x^6-4095Bb\,c^2x^6-5005Ab\,c^2x^5-5005Ba\,c^2x^5-5005B\,b^2c\,x^5-6435Aa\,c^2x^4-6435A\,b^2c}{}$
derivativedivides	$\frac{2B\,c^3x^{\frac{13}{2}}}{13}+\frac{2A\,c^3x^{\frac{11}{2}}}{11}+\frac{6Bb\,c^2x^{\frac{11}{2}}}{11}+\frac{2Ab\,c^2x^{\frac{9}{2}}}{3}+\frac{2Ba\,c^2x^{\frac{9}{2}}}{3}+\frac{2B\,b^2c\,x^{\frac{9}{2}}}{3}+\frac{6Aa\,c^2x^{\frac{7}{2}}}{7}+\frac{6A\,b^2c\,x^{\frac{7}{2}}}{7}+$
default	$\frac{2B\,c^3x^{\frac{13}{2}}}{13}+\frac{2A\,c^3x^{\frac{11}{2}}}{11}+\frac{6Bb\,c^2x^{\frac{11}{2}}}{11}+\frac{2Ab\,c^2x^{\frac{9}{2}}}{3}+\frac{2Ba\,c^2x^{\frac{9}{2}}}{3}+\frac{2B\,b^2c\,x^{\frac{9}{2}}}{3}+\frac{6Aa\,c^2x^{\frac{7}{2}}}{7}+\frac{6A\,b^2c\,x^{\frac{7}{2}}}{7}+$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15015*(-1155*B*c^3*x^7-1365*A*c^3*x^6-4095*B*b*c^2*x^6-5005*A*b*c^2*x^5
-5005*B*a*c^2*x^5-5005*B*b^2*c*x^5-6435*A*a*c^2*x^4-6435*A*b^2*c*x^4-12870
*B*a*b*c*x^4-2145*B*b^3*x^4-18018*A*a*b*c*x^3-3003*A*b^3*x^3-9009*B*a^2*c*
x^3-9009*B*a*b^2*x^3-15015*A*a^2*c*x^2-15015*A*a*b^2*x^2-15015*B*a^2*b*x^2
-45045*A*a^2*b*x-15015*B*a^3*x+15015*A*a^3)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{2\left(1155\,Bc^3x^7+1365\,(3\,Bbc^2+Ac^3)x^6+5005\,(Bb^2c+(Ba+Ab)c^2)x^5\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*B*c^3*x^7 + 1365*(3*B*b*c^2 + A*c^3)*x^6 + 5005*(B*b^2*c + (
B*a + A*b)*c^2)*x^5 + 2145*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4
 - 15015*A*a^3 + 3003*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 15
015*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 15015*(B*a^3 + 3*A*a^2*b)*x)/sqrt(
x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.61

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{3/2}}\,dx &= -\frac{2Aa^3}{\sqrt{x}} + 6Aa^2b\sqrt{x} + 2Aa^2cx^{\frac{3}{2}} \\ &+ 2Aab^2x^{\frac{3}{2}} + \frac{12Aabcx^{\frac{5}{2}}}{5} + \frac{6Aac^2x^{\frac{7}{2}}}{7} + \frac{2Ab^3x^{\frac{5}{2}}}{5} + \frac{6Ab^2cx^{\frac{7}{2}}}{7} \\ &+ \frac{2Abc^2x^{\frac{9}{2}}}{3} + \frac{2Ac^3x^{\frac{11}{2}}}{11} + 2Ba^3\sqrt{x} + 2Ba^2bx^{\frac{3}{2}} + \frac{6Ba^2cx^{\frac{5}{2}}}{5} + \frac{6Bab^2x^{\frac{5}{2}}}{5} \\ &+ \frac{12Babcx^{\frac{7}{2}}}{7} + \frac{2Bac^2x^{\frac{9}{2}}}{3} + \frac{2Bb^3x^{\frac{7}{2}}}{7} + \frac{2Bb^2cx^{\frac{9}{2}}}{3} + \frac{6Bbc^2x^{\frac{11}{2}}}{11} + \frac{2Bc^3x^{\frac{13}{2}}}{13}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**(3/2),x)
```

output
```
-2*A*a**3/sqrt(x) + 6*A*a**2*b*sqrt(x) + 2*A*a**2*c*x**(3/2) + 2*A*a*b**2*
x**(3/2) + 12*A*a*b*c*x**(5/2)/5 + 6*A*a*c**2*x**(7/2)/7 + 2*A*b**3*x**(5/
2)/5 + 6*A*b**2*c*x**(7/2)/7 + 2*A*b*c**2*x**(9/2)/3 + 2*A*c**3*x**(11/2)/
11 + 2*B*a**3*sqrt(x) + 2*B*a**2*b*x**(3/2) + 6*B*a**2*c*x**(5/2)/5 + 6*B*
a*b**2*x**(5/2)/5 + 12*B*a*b*c*x**(7/2)/7 + 2*B*a*c**2*x**(9/2)/3 + 2*B*b*
*3*x**(7/2)/7 + 2*B*b**2*c*x**(9/2)/3 + 6*B*b*c**2*x**(11/2)/11 + 2*B*c**3
*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{2}{13}\,Bc^3x^{\frac{13}{2}} + \frac{2}{11}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{11}{2}} + \frac{2}{3}\left(Bb^2c + (Ba+Ab)c^2\right)x^{\frac{9}{2}} + \frac{2}{7}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^{\frac{7}{2}} - \frac{2\,Aa^3}{\sqrt{x}} + \frac{2}{5}\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x^{\frac{5}{2}} + 2\left(Ba^2b + Aab^2 + Aa^2c\right)x^{\frac{3}{2}} + 2\left(Ba^3 + 3\,Aa^2b\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/13*B*c^3*x^(13/2) + 2/11*(3*B*b*c^2 + A*c^3)*x^(11/2) + 2/3*(B*b^2*c + (
B*a + A*b)*c^2)*x^(9/2) + 2/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*
x^(7/2) - 2*A*a^3/sqrt(x) + 2/5*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c
)*x^(5/2) + 2*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^(3/2) + 2*(B*a^3 + 3*A*a^2*b
)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^3}{x^{3/2}}\,dx = \frac{2}{13}\,Bc^3x^{\frac{13}{2}} + \frac{6}{11}\,Bbc^2x^{\frac{11}{2}} + \frac{2}{11}\,Ac^3x^{\frac{11}{2}} + \frac{2}{3}\,Bb^2cx^{\frac{9}{2}} + \frac{2}{3}\,Bac^2x^{\frac{9}{2}} + \frac{2}{3}\,Abc^2x^{\frac{9}{2}} + \frac{2}{7}\,Bb^3x^{\frac{7}{2}} + \frac{12}{7}\,Babcx^{\frac{7}{2}} + \frac{6}{7}\,Ab^2cx^{\frac{7}{2}} + \frac{6}{7}\,Aac^2x^{\frac{7}{2}} + \frac{6}{5}\,Bab^2x^{\frac{5}{2}} + \frac{2}{5}\,Ab^3x^{\frac{5}{2}} + \frac{6}{5}\,Ba^2cx^{\frac{5}{2}} + \frac{12}{5}\,Aabcx^{\frac{5}{2}} + 2\,Ba^2bx^{\frac{3}{2}} + 2\,Aab^2x^{\frac{3}{2}} + 2\,Aa^2cx^{\frac{3}{2}} + 2\,Ba^3\sqrt{x} + 6\,Aa^2b\sqrt{x} - \frac{2\,Aa^3}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/13*B*c^3*x^(13/2) + 6/11*B*b*c^2*x^(11/2) + 2/11*A*c^3*x^(11/2) + 2/3*B*
b^2*c*x^(9/2) + 2/3*B*a*c^2*x^(9/2) + 2/3*A*b*c^2*x^(9/2) + 2/7*B*b^3*x^(7
/2) + 12/7*B*a*b*c*x^(7/2) + 6/7*A*b^2*c*x^(7/2) + 6/7*A*a*c^2*x^(7/2) + 6
/5*B*a*b^2*x^(5/2) + 2/5*A*b^3*x^(5/2) + 6/5*B*a^2*c*x^(5/2) + 12/5*A*a*b*
c*x^(5/2) + 2*B*a^2*b*x^(3/2) + 2*A*a*b^2*x^(3/2) + 2*A*a^2*c*x^(3/2) + 2*
B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) - 2*A*a^3/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^3}{x^{3/2}}\,dx = x^{5/2}\left(\frac{6\,B\,c\,a^2}{5}+\frac{6\,B\,a\,b^2}{5}+\frac{12\,A\,c\,a\,b}{5}+\frac{2\,A\,b^3}{5}\right) + x^{7/2}\left(\frac{2\,B\,b^3}{7}+\frac{6\,A\,b^2\,c}{7}+\frac{12\,B\,a\,b\,c}{7}+\frac{6\,A\,a\,c^2}{7}\right)+\sqrt{x}\,\left(2\,B\,a^3+6\,A\,b\,a^2\right)+x^{11/2}\left(\frac{2\,A\,c^3}{11}+\frac{6\,B\,b\,c^2}{11}\right)+x^{3/2}\,(2$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^(3/2),x)
```

output
```
x^(5/2)*((2*A*b^3)/5 + (6*B*a*b^2)/5 + (6*B*a^2*c)/5 + (12*A*a*b*c)/5) + x
^(7/2)*((2*B*b^3)/7 + (6*A*a*c^2)/7 + (6*A*b^2*c)/7 + (12*B*a*b*c)/7) + x^
(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(11/2)*((2*A*c^3)/11 + (6*B*b*c^2)/11) + x
^(3/2)*(2*A*a*b^2 + 2*A*a^2*c + 2*B*a^2*b) + x^(9/2)*((2*A*b*c^2)/3 + (2*B
*a*c^2)/3 + (2*B*b^2*c)/3) - (2*A*a^3)/x^(1/2) + (2*B*c^3*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{\frac{2}{13}b\,c^3x^7+\frac{2}{11}a\,c^3x^6+\frac{6}{11}b^2c^2x^6+\frac{4}{3}ab\,c^2x^5+\frac{2}{3}b^3c\,x^5+\frac{6}{7}a^2c^2x^4+\frac{18}{7}a\,b^2c}{\sqrt{x}}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(3/2),x)
```

output
```
(2*( - 15015*a**4 + 60060*a**3*b*x + 15015*a**3*c*x**2 + 30030*a**2*b**2*x
**2 + 27027*a**2*b*c*x**3 + 6435*a**2*c**2*x**4 + 12012*a*b**3*x**3 + 1930
5*a*b**2*c*x**4 + 10010*a*b*c**2*x**5 + 1365*a*c**3*x**6 + 2145*b**4*x**4
+ 5005*b**3*c*x**5 + 4095*b**2*c**2*x**6 + 1155*b*c**3*x**7))/(15015*sqrt(
x))
```

3.80 $\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{5/2}}\,dx = -\frac{2a^3A}{3x^{3/2}} - \frac{2a^2(3Ab+aB)}{\sqrt{x}} + 6a\left(abB+A\left(b^2+ac\right)\right)\sqrt{x} + \frac{2}{3}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{3/2} + \frac{2}{5}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{5/2} + \frac{6}{7}c\left(b^2B+Abc+aBc\right)x^{7/2} + \frac{2}{9}c^2(3bB+Ac)x^{9/2} + \frac{2}{11}Bc^3x^{11/2}$$

output

```
-2/3*a^3*A/x^(3/2)-2*a^2*(3*A*b+B*a)/x^(1/2)+6*a*(a*b*B+A*(a*c+b^2))*x^(1/
2)+2/3*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^(3/2)+2/5*(3*A*a*c^2+3*A*b^2*c+
6*B*a*b*c+B*b^3)*x^(5/2)+6/7*c*(A*b*c+B*a*c+B*b^2)*x^(7/2)+2/9*c^2*(A*c+3*
B*b)*x^(9/2)+2/11*B*c^3*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2(-1155a^3(A+3Bx)+3465a^2x(-3A(b-cx)+Bx(3b+cx))+99ax$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^(5/2),x]
```

output
```
(2*(-1155*a^3*(A + 3*B*x) + 3465*a^2*x*(-3*A*(b - c*x) + B*x*(3*b + c*x))
+ 99*a*x^2*(7*A*(15*b^2 + 10*b*c*x + 3*c^2*x^2) + B*x*(35*b^2 + 42*b*c*x +
 15*c^2*x^2)) + x^3*(11*A*(105*b^3 + 189*b^2*c*x + 135*b*c^2*x^2 + 35*c^3*
x^3) + 3*B*x*(231*b^3 + 495*b^2*c*x + 385*b*c^2*x^2 + 105*c^3*x^3))))/(346
5*x^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^{5/2}}+\frac{a^2(aB+3Ab)}{x^{3/2}}+3cx^{5/2}\left(aBc+Abc+b^2B\right)+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{\sqrt{x}}+x^{3/2}\left(3aAc^2+6abBc+3Ab\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{3x^{3/2}}-\frac{2a^2(aB+3Ab)}{\sqrt{x}}+\frac{6}{7}cx^{7/2}\left(aBc+Abc+b^2B\right)+6a\sqrt{x}\left(A\left(ac+b^2\right)+abB\right)+\frac{2}{5}x^{5/2}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{2}{3}x^{3/2}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{2}{9}c^2x^{9/2}(Ac+3bB)+\frac{2}{11}Bc^3x^{11/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^(5/2),x]
```

output
```
(-2*a^3*A)/(3*x^(3/2)) - (2*a^2*(3*A*b + a*B))/Sqrt[x] + 6*a*(a*b*B + A*(b
^2 + a*c))*Sqrt[x] + (2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^(3/2))/3
 + (2*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^(5/2))/5 + (6*c*(b^2*B
 + A*b*c + a*B*c)*x^(7/2))/7 + (2*c^2*(3*b*B + A*c)*x^(9/2))/9 + (2*B*c^3*
x^(11/2))/11
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{11}{2}}}{11}+\frac{2A\,c^3x^{\frac{9}{2}}}{9}+\frac{2Bb\,c^2x^{\frac{9}{2}}}{3}+\frac{6Ab\,c^2x^{\frac{7}{2}}}{7}+\frac{6Ba\,c^2x^{\frac{7}{2}}}{7}+\frac{6B\,b^2c\,x^{\frac{7}{2}}}{7}+\frac{6Aa\,c^2x^{\frac{5}{2}}}{5}+\frac{6A\,b^2c\,x^{\frac{5}{2}}}{5}+$
default	$\frac{2B\,c^3x^{\frac{11}{2}}}{11}+\frac{2A\,c^3x^{\frac{9}{2}}}{9}+\frac{2Bb\,c^2x^{\frac{9}{2}}}{3}+\frac{6Ab\,c^2x^{\frac{7}{2}}}{7}+\frac{6Ba\,c^2x^{\frac{7}{2}}}{7}+\frac{6B\,b^2c\,x^{\frac{7}{2}}}{7}+\frac{6Aa\,c^2x^{\frac{5}{2}}}{5}+\frac{6A\,b^2c\,x^{\frac{5}{2}}}{5}+$
gosper	$-\frac{2\left(-315B\,c^3x^7-385A\,c^3x^6-1155Bb\,c^2x^6-1485Ab\,c^2x^5-1485Ba\,c^2x^5-1485B\,b^2c\,x^5-2079Aa\,c^2x^4-2079A\,b^2c\,x^4\right.}{}$
trager	$-\frac{2\left(-315B\,c^3x^7-385A\,c^3x^6-1155Bb\,c^2x^6-1485Ab\,c^2x^5-1485Ba\,c^2x^5-1485B\,b^2c\,x^5-2079Aa\,c^2x^4-2079A\,b^2c\,x^4\right.}{}$
risch	$-\frac{2\left(-315B\,c^3x^7-385A\,c^3x^6-1155Bb\,c^2x^6-1485Ab\,c^2x^5-1485Ba\,c^2x^5-1485B\,b^2c\,x^5-2079Aa\,c^2x^4-2079A\,b^2c\,x^4\right.}{}$
orering	$-\frac{2\left(-315B\,c^3x^7-385A\,c^3x^6-1155Bb\,c^2x^6-1485Ab\,c^2x^5-1485Ba\,c^2x^5-1485B\,b^2c\,x^5-2079Aa\,c^2x^4-2079A\,b^2c\,x^4\right.}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*B*c^3*x^(11/2)+2/9*A*c^3*x^(9/2)+2/3*B*b*c^2*x^(9/2)+6/7*A*b*c^2*x^(7
/2)+6/7*B*a*c^2*x^(7/2)+6/7*B*b^2*c*x^(7/2)+6/5*A*a*c^2*x^(5/2)+6/5*A*b^2*
c*x^(5/2)+12/5*B*a*b*c*x^(5/2)+2/5*B*b^3*x^(5/2)+4*A*a*b*c*x^(3/2)+2/3*A*b
^3*x^(3/2)+2*B*a^2*c*x^(3/2)+2*B*a*b^2*x^(3/2)+6*a^2*A*c*x^(1/2)+6*A*a*b^2
*x^(1/2)+6*B*a^2*b*x^(1/2)-2/3*a^3*A/x^(3/2)-2*a^2*(3*A*b+B*a)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2\,(315\,Bc^3x^7 + 385\,(3\,Bbc^2 + Ac^3)x^6 + 1485\,(Bb^2c + (Ba + Ab)c^2)x^5 +$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c^3*x^7 + 385*(3*B*b*c^2 + A*c^3)*x^6 + 1485*(B*b^2*c + (B*a
 + A*b)*c^2)*x^5 + 693*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 - 1
155*A*a^3 + 1155*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 10395*(
B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 - 3465*(B*a^3 + 3*A*a^2*b)*x)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.57

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{5/2}}\,dx = &-\frac{2Aa^3}{3x^{\frac{3}{2}}} - \frac{6Aa^2b}{\sqrt{x}} + 6Aa^2c\sqrt{x}\\ &+ 6Aab^2\sqrt{x} + 4Aabcx^{\frac{3}{2}} + \frac{6Aac^2x^{\frac{5}{2}}}{5} + \frac{2Ab^3x^{\frac{3}{2}}}{3} + \frac{6Ab^2cx^{\frac{5}{2}}}{5} + \frac{6Abc^2x^{\frac{7}{2}}}{7}\\ &+ \frac{2Ac^3x^{\frac{9}{2}}}{9} - \frac{2Ba^3}{\sqrt{x}} + 6Ba^2b\sqrt{x} + 2Ba^2cx^{\frac{3}{2}} + 2Bab^2x^{\frac{3}{2}} + \frac{12Babcx^{\frac{5}{2}}}{5}\\ &+ \frac{6Bac^2x^{\frac{7}{2}}}{7} + \frac{2Bb^3x^{\frac{5}{2}}}{5} + \frac{6Bb^2cx^{\frac{7}{2}}}{7} + \frac{2Bbc^2x^{\frac{9}{2}}}{3} + \frac{2Bc^3x^{\frac{11}{2}}}{11}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**(5/2),x)
```

output
```
-2*A*a**3/(3*x**(3/2)) - 6*A*a**2*b/sqrt(x) + 6*A*a**2*c*sqrt(x) + 6*A*a*b
**2*sqrt(x) + 4*A*a*b*c*x**(3/2) + 6*A*a*c**2*x**(5/2)/5 + 2*A*b**3*x**(3/
2)/3 + 6*A*b**2*c*x**(5/2)/5 + 6*A*b*c**2*x**(7/2)/7 + 2*A*c**3*x**(9/2)/9
 - 2*B*a**3/sqrt(x) + 6*B*a**2*b*sqrt(x) + 2*B*a**2*c*x**(3/2) + 2*B*a*b**
2*x**(3/2) + 12*B*a*b*c*x**(5/2)/5 + 6*B*a*c**2*x**(7/2)/7 + 2*B*b**3*x**(
5/2)/5 + 6*B*b**2*c*x**(7/2)/7 + 2*B*b*c**2*x**(9/2)/3 + 2*B*c**3*x**(11/2
)/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2}{11}\,Bc^3x^{\frac{11}{2}} + \frac{2}{9}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{9}{2}} + \frac{6}{7}\left(Bb^2c + (Ba + Ab)c^2\right)x^{\frac{7}{2}} + \frac{2}{5}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^{\frac{5}{2}} + \frac{2}{3}\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x^{\frac{3}{2}} + 6\left(Ba^2b + Aab^2 + Aa^2c\right)\sqrt{x} - \frac{2\left(Aa^3 + 3\left(Ba^3 + 3\,Aa^2b\right)x\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/11*B*c^3*x^(11/2) + 2/9*(3*B*b*c^2 + A*c^3)*x^(9/2) + 6/7*(B*b^2*c + (B*
a + A*b)*c^2)*x^(7/2) + 2/5*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^
(5/2) + 2/3*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^(3/2) + 6*(B*a^2
*b + A*a*b^2 + A*a^2*c)*sqrt(x) - 2/3*(A*a^3 + 3*(B*a^3 + 3*A*a^2*b)*x)/x^
(3/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2}{11}\,Bc^3x^{\frac{11}{2}} + \frac{2}{3}\,Bbc^2x^{\frac{9}{2}} + \frac{2}{9}\,Ac^3x^{\frac{9}{2}} + \frac{6}{7}\,Bb^2cx^{\frac{7}{2}} + \frac{6}{7}\,Bac^2x^{\frac{7}{2}} + \frac{6}{7}\,Abc^2x^{\frac{7}{2}} + \frac{2}{5}\,Bb^3x^{\frac{5}{2}} + \frac{12}{5}\,Babcx^{\frac{5}{2}} + \frac{6}{5}\,Ab^2cx^{\frac{5}{2}} + \frac{6}{5}\,Aac^2x^{\frac{5}{2}} + 2\,Bab^2x^{\frac{3}{2}} + \frac{2}{3}\,Ab^3x^{\frac{3}{2}} + 2\,Ba^2cx^{\frac{3}{2}} + 4\,Aabcx^{\frac{3}{2}} + 6\,Ba^2b\sqrt{x} + 6\,Aab^2\sqrt{x} + 6\,Aa^2c\sqrt{x} - \frac{2\left(3\,Ba^3x + 9\,Aa^2bx + Aa^3\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/11*B*c^3*x^(11/2) + 2/3*B*b*c^2*x^(9/2) + 2/9*A*c^3*x^(9/2) + 6/7*B*b^2*
c*x^(7/2) + 6/7*B*a*c^2*x^(7/2) + 6/7*A*b*c^2*x^(7/2) + 2/5*B*b^3*x^(5/2)
+ 12/5*B*a*b*c*x^(5/2) + 6/5*A*b^2*c*x^(5/2) + 6/5*A*a*c^2*x^(5/2) + 2*B*a
*b^2*x^(3/2) + 2/3*A*b^3*x^(3/2) + 2*B*a^2*c*x^(3/2) + 4*A*a*b*c*x^(3/2) +
 6*B*a^2*b*sqrt(x) + 6*A*a*b^2*sqrt(x) + 6*A*a^2*c*sqrt(x) - 2/3*(3*B*a^3*
x + 9*A*a^2*b*x + A*a^3)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{5/2}}\,dx = x^{3/2}\left(2\,B\,c\,a^2 + 2\,B\,a\,b^2 + 4\,A\,c\,a\,b + \frac{2\,A\,b^3}{3}\right) + x^{5/2}\left(\frac{2\,B\,b^3}{5} + \frac{6\,A\,b^2\,c}{5} + \frac{12\,B\,a\,b\,c}{5} + \frac{6\,A\,a\,c^2}{5}\right) - \frac{x\left(2\,B\,a^3 + 6\,A\,b\,a^2\right) + \frac{2\,A\,a^3}{3}}{x^{3/2}} + x^{9/2}\left(\frac{2\,A\,c^3}{9} + \frac{2\,B\,b\,c^2}{3}\right) +$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^(5/2),x)
```

output
```
x^(3/2)*((2*A*b^3)/3 + 2*B*a*b^2 + 2*B*a^2*c + 4*A*a*b*c) + x^(5/2)*((2*B*
b^3)/5 + (6*A*a*c^2)/5 + (6*A*b^2*c)/5 + (12*B*a*b*c)/5) - (x*(2*B*a^3 + 6
*A*a^2*b) + (2*A*a^3)/3)/x^(3/2) + x^(9/2)*((2*A*c^3)/9 + (2*B*b*c^2)/3) +
 x^(1/2)*(6*A*a*b^2 + 6*A*a^2*c + 6*B*a^2*b) + x^(7/2)*((6*A*b*c^2)/7 + (6
*B*a*c^2)/7 + (6*B*b^2*c)/7) + (2*B*c^3*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{\frac{2}{11}b\,c^3x^7+\frac{2}{9}a\,c^3x^6+\frac{2}{3}b^2c^2x^6+\frac{12}{7}ab\,c^2x^5+\frac{6}{7}b^3c\,x^5+\frac{6}{5}a^2c^2x^4+\frac{18}{5}a\,b^2c}{\sqrt{x}}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(5/2),x)
```

output
```
(2*( - 1155*a**4 - 13860*a**3*b*x + 10395*a**3*c*x**2 + 20790*a**2*b**2*x*
*2 + 10395*a**2*b*c*x**3 + 2079*a**2*c**2*x**4 + 4620*a*b**3*x**3 + 6237*a
*b**2*c*x**4 + 2970*a*b*c**2*x**5 + 385*a*c**3*x**6 + 693*b**4*x**4 + 1485
*b**3*c*x**5 + 1155*b**2*c**2*x**6 + 315*b*c**3*x**7))/(3465*sqrt(x)*x)
```

3.81 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{7/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{7/2}} \, dx = -\frac{2a^3A}{5x^{5/2}} - \frac{2a^2(3Ab+aB)}{3x^{3/2}} - \frac{6a(abB+A(b^2+ac))}{\sqrt{x}} + 2\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)\sqrt{x} + \frac{2}{3}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{3/2} + \frac{6}{5}c\left(b^2B+Abc+aBc\right)x^{5/2} + \frac{2}{7}$$

output
```
-2/5*a^3*A/x^(5/2)-2/3*a^2*(3*A*b+B*a)/x^(3/2)-6*a*(a*b*B+A*(a*c+b^2))/x^(
1/2)+2*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^(1/2)+2/3*(3*A*a*c^2+3*A*b^2*c+
6*B*a*b*c+B*b^3)*x^(3/2)+6/5*c*(A*b*c+B*a*c+B*b^2)*x^(5/2)+2/7*c^2*(A*c+3*
B*b)*x^(7/2)+2/9*B*c^3*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{7/2}} \, dx = \frac{2(-21a^3(3A+5Bx)-315a^2x(3Bx(b-cx)+A(b+3cx))+63ax^2(5A}{}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^(7/2),x]
```

output
```
(2*(-21*a^3*(3*A + 5*B*x) - 315*a^2*x*(3*B*x*(b - c*x) + A*(b + 3*c*x)) +
63*a*x^2*(5*A*(-3*b^2 + 6*b*c*x + c^2*x^2) + B*x*(15*b^2 + 10*b*c*x + 3*c^
2*x^2)) + x^3*(9*A*(35*b^3 + 35*b^2*c*x + 21*b*c^2*x^2 + 5*c^3*x^3) + B*x*
(105*b^3 + 189*b^2*c*x + 135*b*c^2*x^2 + 35*c^3*x^3))))/(315*x^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{7/2}}\,dx$$

$$\Big\downarrow \; 1195$$

$$\int \left(\frac{a^3A}{x^{7/2}}+\frac{a^2(aB+3Ab)}{x^{5/2}}+3cx^{3/2}\left(aBc+Abc+b^2B\right)+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x^{3/2}}+\sqrt{x}\left(3aAc^2+6abBc+3Ab^2\right.\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(aB+3Ab)}{3x^{3/2}}+\frac{6}{5}cx^{5/2}\left(aBc+Abc+b^2B\right)-\frac{6a\left(A\left(ac+b^2\right)+abB\right)}{\sqrt{x}}+\frac{2}{3}x^{3/2}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+2\sqrt{x}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{2}{7}c^2x^{7/2}(Ac+3bB)+\frac{2}{9}Bc^3x^{9/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^(7/2),x]
```

output
```
(-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/(3*x^(3/2)) - (6*a*(a*b*B +
 A*(b^2 + a*c)))/Sqrt[x] + 2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*Sqrt[
x] + (2*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*x^(3/2))/3 + (6*c*(b^2
*B + A*b*c + a*B*c)*x^(5/2))/5 + (2*c^2*(3*b*B + A*c)*x^(7/2))/7 + (2*B*c^
3*x^(9/2))/9
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{2Bc^3x^{\frac{9}{2}}}{9}+\frac{2Ac^3x^{\frac{7}{2}}}{7}+\frac{6Bbc^2x^{\frac{7}{2}}}{7}+\frac{6Abc^2x^{\frac{5}{2}}}{5}+\frac{6Bac^2x^{\frac{5}{2}}}{5}+\frac{6Bb^2cx^{\frac{5}{2}}}{5}+2Aac^2x^{\frac{3}{2}}+2Ab^2cx^{\frac{3}{2}}+$
default	$\frac{2Bc^3x^{\frac{9}{2}}}{9}+\frac{2Ac^3x^{\frac{7}{2}}}{7}+\frac{6Bbc^2x^{\frac{7}{2}}}{7}+\frac{6Abc^2x^{\frac{5}{2}}}{5}+\frac{6Bac^2x^{\frac{5}{2}}}{5}+\frac{6Bb^2cx^{\frac{5}{2}}}{5}+2Aac^2x^{\frac{3}{2}}+2Ab^2cx^{\frac{3}{2}}+$
gosper	$-\frac{2(-35Bc^3x^7-45Ac^3x^6-135Bbc^2x^6-189Abc^2x^5-189Bac^2x^5-189Bb^2cx^5-315Aac^2x^4-315Ab^2cx^4-630Bab}{}$
trager	$-\frac{2(-35Bc^3x^7-45Ac^3x^6-135Bbc^2x^6-189Abc^2x^5-189Bac^2x^5-189Bb^2cx^5-315Aac^2x^4-315Ab^2cx^4-630Bab}{}$
risch	$-\frac{2(-35Bc^3x^7-45Ac^3x^6-135Bbc^2x^6-189Abc^2x^5-189Bac^2x^5-189Bb^2cx^5-315Aac^2x^4-315Ab^2cx^4-630Bab}{}$
orering	$-\frac{2(-35Bc^3x^7-45Ac^3x^6-135Bbc^2x^6-189Abc^2x^5-189Bac^2x^5-189Bb^2cx^5-315Aac^2x^4-315Ab^2cx^4-630Bab}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*B*c^3*x^(9/2)+2/7*A*c^3*x^(7/2)+6/7*B*b*c^2*x^(7/2)+6/5*A*b*c^2*x^(5/2
)+6/5*B*a*c^2*x^(5/2)+6/5*B*b^2*c*x^(5/2)+2*A*a*c^2*x^(3/2)+2*A*b^2*c*x^(3
/2)+4*B*a*b*c*x^(3/2)+2/3*B*b^3*x^(3/2)+12*A*a*b*c*x^(1/2)+2*A*b^3*x^(1/2)
+6*B*a^2*c*x^(1/2)+6*B*a*b^2*x^(1/2)-6*a*(A*a*c+A*b^2+B*a*b)/x^(1/2)-2/5*a
^3*A/x^(5/2)-2/3*a^2*(3*A*b+B*a)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{7/2}}\,dx = \frac{2\left(35\,Bc^3x^7+45\left(3\,Bbc^2+Ac^3\right)x^6+189\left(Bb^2c+(Ba+Ab)c^2\right)x^5+10\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*c^3*x^7 + 45*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + (B*a + A
*b)*c^2)*x^5 + 105*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 - 63*A*
a^3 + 315*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 - 945*(B*a^2*b +
 A*a*b^2 + A*a^2*c)*x^2 - 105*(B*a^3 + 3*A*a^2*b)*x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{7/2}}\,dx = -\frac{2Aa^3}{5x^{\frac{5}{2}}}-\frac{2Aa^2b}{x^{\frac{3}{2}}}-\frac{6Aa^2c}{\sqrt{x}}-\frac{6Aab^2}{\sqrt{x}}+12Aabc\sqrt{x} +2Aac^2x^{\frac{3}{2}}+2Ab^3\sqrt{x}+2Ab^2cx^{\frac{3}{2}}+\frac{6Abc^2x^{\frac{5}{2}}}{5}+\frac{2Ac^3x^{\frac{7}{2}}}{7}-\frac{2Ba^3}{3x^{\frac{3}{2}}}-\frac{6Ba^2b}{\sqrt{x}}+6Ba^2c\sqrt{x} +6Bab^2\sqrt{x}+4Babcx^{\frac{3}{2}}+\frac{6Bac^2x^{\frac{5}{2}}}{5}+\frac{2Bb^3x^{\frac{3}{2}}}{3}+\frac{6Bb^2cx^{\frac{5}{2}}}{5}+\frac{6Bbc^2x^{\frac{7}{2}}}{7}+\frac{2Bc^3x^{\frac{9}{2}}}{9}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**(7/2),x)
```

output
```
-2*A*a**3/(5*x**(5/2)) - 2*A*a**2*b/x**(3/2) - 6*A*a**2*c/sqrt(x) - 6*A*a*
b**2/sqrt(x) + 12*A*a*b*c*sqrt(x) + 2*A*a*c**2*x**(3/2) + 2*A*b**3*sqrt(x)
 + 2*A*b**2*c*x**(3/2) + 6*A*b*c**2*x**(5/2)/5 + 2*A*c**3*x**(7/2)/7 - 2*B
*a**3/(3*x**(3/2)) - 6*B*a**2*b/sqrt(x) + 6*B*a**2*c*sqrt(x) + 6*B*a*b**2*
sqrt(x) + 4*B*a*b*c*x**(3/2) + 6*B*a*c**2*x**(5/2)/5 + 2*B*b**3*x**(3/2)/3
 + 6*B*b**2*c*x**(5/2)/5 + 6*B*b*c**2*x**(7/2)/7 + 2*B*c**3*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{7/2}}\,dx = \frac{2}{9}Bc^3x^{\frac{9}{2}} + \frac{2}{7}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{7}{2}} + \frac{6}{5}\left(Bb^2c + (Ba+Ab)c^2\right)x^{\frac{5}{2}} + \frac{2}{3}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^{\frac{3}{2}} + 2\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)\sqrt{x} - \frac{2\left(3\,Aa^3 + 45\left(Ba^2b + Aab^2 + Aa^2c\right)x^2 + 5\left(Ba^3 + 3\,Aa^2b\right)x\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(7/2),x, algorithm="maxima")
```

output
```
2/9*B*c^3*x^(9/2) + 2/7*(3*B*b*c^2 + A*c^3)*x^(7/2) + 6/5*(B*b^2*c + (B*a
+ A*b)*c^2)*x^(5/2) + 2/3*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^(3
/2) + 2*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*sqrt(x) - 2/15*(3*A*a^
3 + 45*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 5*(B*a^3 + 3*A*a^2*b)*x)/x^(5/2
)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{7/2}}\,dx = \frac{2}{9}Bc^3x^{\frac{9}{2}} + \frac{6}{7}Bbc^2x^{\frac{7}{2}} + \frac{2}{7}Ac^3x^{\frac{7}{2}} + \frac{6}{5}Bb^2cx^{\frac{5}{2}} + \frac{6}{5}Bac^2x^{\frac{5}{2}} + \frac{6}{5}Abc^2x^{\frac{5}{2}} + \frac{2}{3}Bb^3x^{\frac{3}{2}} + 4\,Babcx^{\frac{3}{2}} + 2\,Ab^2cx^{\frac{3}{2}} + 2\,Aac^2x^{\frac{3}{2}} + 6\,Bab^2\sqrt{x} + 2\,Ab^3\sqrt{x} + 6\,Ba^2c\sqrt{x} + 12\,Aabc\sqrt{x} - \frac{2\left(45\,Ba^2bx^2 + 45\,Aab^2x^2 + 45\,Aa^2cx^2 + 5\,Ba^3x + 15\,Aa^2bx + 3\,Aa^3\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(7/2),x, algorithm="giac")
```

output
```
2/9*B*c^3*x^(9/2) + 6/7*B*b*c^2*x^(7/2) + 2/7*A*c^3*x^(7/2) + 6/5*B*b^2*c*
x^(5/2) + 6/5*B*a*c^2*x^(5/2) + 6/5*A*b*c^2*x^(5/2) + 2/3*B*b^3*x^(3/2) +
4*B*a*b*c*x^(3/2) + 2*A*b^2*c*x^(3/2) + 2*A*a*c^2*x^(3/2) + 6*B*a*b^2*sqrt
(x) + 2*A*b^3*sqrt(x) + 6*B*a^2*c*sqrt(x) + 12*A*a*b*c*sqrt(x) - 2/15*(45*
B*a^2*b*x^2 + 45*A*a*b^2*x^2 + 45*A*a^2*c*x^2 + 5*B*a^3*x + 15*A*a^2*b*x +
 3*A*a^3)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{7/2}}\,dx = \sqrt{x}\left(6\,B\,c\,a^2+6\,B\,a\,b^2+12\,A\,c\,a\,b+2\,A\,b^3\right)$$
$$-\frac{x\left(\frac{2\,B\,a^3}{3}+2\,A\,b\,a^2\right)+\frac{2\,A\,a^3}{5}+x^2\left(6\,B\,a^2\,b+6\,A\,c\,a^2+6\,A\,a\,b^2\right)}{x^{5/2}}$$
$$+x^{3/2}\left(\frac{2\,B\,b^3}{3}+2\,A\,b^2\,c+4\,B\,a\,b\,c+2\,A\,a\,c^2\right)+x^{7/2}\left(\frac{2\,A\,c^3}{7}+\frac{6\,B\,b\,c^2}{7}\right)+x^{5/2}\left(\frac{6\,B\,b^2\,c}{5}+\frac{6\,A\,b\,c^2}{5}+\frac{6\,B\,a\,c^2}{5}\right.$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^(7/2),x)
```

output
```
x^(1/2)*(2*A*b^3 + 6*B*a*b^2 + 6*B*a^2*c + 12*A*a*b*c) - (x*((2*B*a^3)/3 +
 2*A*a^2*b) + (2*A*a^3)/5 + x^2*(6*A*a*b^2 + 6*A*a^2*c + 6*B*a^2*b))/x^(5/
2) + x^(3/2)*((2*B*b^3)/3 + 2*A*a*c^2 + 2*A*b^2*c + 4*B*a*b*c) + x^(7/2)*(
(2*A*c^3)/7 + (6*B*b*c^2)/7) + x^(5/2)*((6*A*b*c^2)/5 + (6*B*a*c^2)/5 + (6
*B*b^2*c)/5) + (2*B*c^3*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{7/2}}\,dx = \frac{\frac{2}{9}b\,c^3x^7+\frac{2}{7}a\,c^3x^6+\frac{6}{7}b^2c^2x^6+\frac{12}{5}ab\,c^2x^5+\frac{6}{5}b^3c\,x^5+2a^2c^2x^4+6a\,b^2c\,x^4}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(7/2),x)
```

output
```
(2*( - 63*a**4 - 420*a**3*b*x - 945*a**3*c*x**2 - 1890*a**2*b**2*x**2 + 28
35*a**2*b*c*x**3 + 315*a**2*c**2*x**4 + 1260*a*b**3*x**3 + 945*a*b**2*c*x*
*4 + 378*a*b*c**2*x**5 + 45*a*c**3*x**6 + 105*b**4*x**4 + 189*b**3*c*x**5
+ 135*b**2*c**2*x**6 + 35*b*c**3*x**7))/(315*sqrt(x)*x**2)
```

3.82 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}\,dx = -\frac{2a^3A}{7x^{7/2}} - \frac{2a^2(3Ab+aB)}{5x^{5/2}} - \frac{2a(abB+A(b^2+ac))}{x^{3/2}} - \frac{2(3aB(b^2+ac)+A(b^3+6abc))}{\sqrt{x}} + 2\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)\sqrt{x} + 2c\left(b^2B+Abc+aBc\right)x^{3/2} + \frac{2}{5}c^2(3bB+Ac)x^{5/2} + \frac{2}{7}Bc^3x^{7/2}$$

output
```
-2/7*a^3*A/x^(7/2)-2/5*a^2*(3*A*b+B*a)/x^(5/2)-2*a*(a*b*B+A*(a*c+b^2))/x^(
3/2)-2*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))/x^(1/2)+2*(3*A*a*c^2+3*A*b^2*c+6*
B*a*b*c+B*b^3)*x^(1/2)+2*c*(A*b*c+B*a*c+B*b^2)*x^(3/2)+2/5*c^2*(A*c+3*B*b)
*x^(5/2)+2/7*B*c^3*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{2(-a^3(5A+7Bx)-7a^2x(5Bx(b+3cx)+A(3b+5cx))-35ax^2(A(b^2$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^(9/2),x]
```

output
```
(2*(-(a^3*(5*A + 7*B*x)) - 7*a^2*x*(5*B*x*(b + 3*c*x) + A*(3*b + 5*c*x)) -
 35*a*x^2*(A*(b^2 + 6*b*c*x - 3*c^2*x^2) - B*x*(-3*b^2 + 6*b*c*x + c^2*x^2
)) + x^3*(7*A*(-5*b^3 + 15*b^2*c*x + 5*b*c^2*x^2 + c^3*x^3) + B*x*(35*b^3
+ 35*b^2*c*x + 21*b*c^2*x^2 + 5*c^3*x^3))))/(35*x^(7/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}dx$$

$$\Big\downarrow \textbf{1195}$$

$$\int \left(\frac{a^3A}{x^{9/2}}+\frac{a^2(aB+3Ab)}{x^{7/2}}+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x^{5/2}}+3c\sqrt{x}\left(aBc+Abc+b^2B\right)+\frac{3aAc^2+6abBc+3Ab^2c+b}{\sqrt{x}}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{2a^3A}{7x^{7/2}}-\frac{2a^2(aB+3Ab)}{5x^{5/2}}+2cx^{3/2}\left(aBc+Abc+b^2B\right)-\frac{2a\left(A\left(ac+b^2\right)+abB\right)}{x^{3/2}}+ 2\sqrt{x}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)-\frac{2\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)}{\sqrt{x}}+\frac{2}{5}c^2x^{5/2}(Ac+ 3bB)+\frac{2}{7}Bc^3x^{7/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^(9/2),x]
```

output
```
(-2*a^3*A)/(7*x^(7/2)) - (2*a^2*(3*A*b + a*B))/(5*x^(5/2)) - (2*a*(a*b*B +
 A*(b^2 + a*c)))/x^(3/2) - (2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c)))/Sqr
t[x] + 2*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*Sqrt[x] + 2*c*(b^2*B
+ A*b*c + a*B*c)*x^(3/2) + (2*c^2*(3*b*B + A*c)*x^(5/2))/5 + (2*B*c^3*x^(7
/2))/7
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2B c^3 x^{\frac{7}{2}}}{7}+\frac{2A c^3 x^{\frac{5}{2}}}{5}+\frac{6Bb c^2 x^{\frac{5}{2}}}{5}+2Ab c^2 x^{\frac{3}{2}}+2Ba c^2 x^{\frac{3}{2}}+2B b^2 c x^{\frac{3}{2}}+6Aa c^2\sqrt{x}+6A b^2 c$
default	$\frac{2B c^3 x^{\frac{7}{2}}}{7}+\frac{2A c^3 x^{\frac{5}{2}}}{5}+\frac{6Bb c^2 x^{\frac{5}{2}}}{5}+2Ab c^2 x^{\frac{3}{2}}+2Ba c^2 x^{\frac{3}{2}}+2B b^2 c x^{\frac{3}{2}}+6Aa c^2\sqrt{x}+6A b^2 c$
gosper	$-\frac{2(-5B c^3x^7-7A c^3x^6-21Bb c^2x^6-35Ab c^2x^5-35Ba c^2x^5-35B b^2c x^5-105Aa c^2x^4-105A b^2c x^4-210Babc x^4-3}{}$
trager	$-\frac{2(-5B c^3x^7-7A c^3x^6-21Bb c^2x^6-35Ab c^2x^5-35Ba c^2x^5-35B b^2c x^5-105Aa c^2x^4-105A b^2c x^4-210Babc x^4-3}{}$
risch	$-\frac{2(-5B c^3x^7-7A c^3x^6-21Bb c^2x^6-35Ab c^2x^5-35Ba c^2x^5-35B b^2c x^5-105Aa c^2x^4-105A b^2c x^4-210Babc x^4-3}{}$
orering	$-\frac{2(-5B c^3x^7-7A c^3x^6-21Bb c^2x^6-35Ab c^2x^5-35Ba c^2x^5-35B b^2c x^5-105Aa c^2x^4-105A b^2c x^4-210Babc x^4-3}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*B*c^3*x^(7/2)+2/5*A*c^3*x^(5/2)+6/5*B*b*c^2*x^(5/2)+2*A*b*c^2*x^(3/2)+
2*B*a*c^2*x^(3/2)+2*B*b^2*c*x^(3/2)+6*A*a*c^2*x^(1/2)+6*A*b^2*c*x^(1/2)+12
*B*a*b*c*x^(1/2)+2*B*b^3*x^(1/2)-2*a*(A*a*c+A*b^2+B*a*b)/x^(3/2)-2/5*a^2*(
3*A*b+B*a)/x^(5/2)-2*(6*A*a*b*c+A*b^3+3*B*a^2*c+3*B*a*b^2)/x^(1/2)-2/7*a^3
*A/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{2\left(5\,Bc^3x^7+7\,(3\,Bbc^2+Ac^3)x^6+35\,(Bb^2c+(Ba+Ab)c^2)x^5+35\,(Bb\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(9/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*c^3*x^7 + 7*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + (B*a + A*b)*
c^2)*x^5 + 35*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 - 5*A*a^3 -
35*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 - 35*(B*a^2*b + A*a*b^2
 + A*a^2*c)*x^2 - 7*(B*a^3 + 3*A*a^2*b)*x)/x^(7/2)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}\,dx =& -\frac{2Aa^3}{7x^{\frac{7}{2}}}-\frac{6Aa^2b}{5x^{\frac{5}{2}}}-\frac{2Aa^2c}{x^{\frac{3}{2}}}-\frac{2Aab^2}{x^{\frac{3}{2}}}-\frac{12Aabc}{\sqrt{x}}\\ &+6Aac^2\sqrt{x}-\frac{2Ab^3}{\sqrt{x}}+6Ab^2c\sqrt{x}+2Abc^2x^{\frac{3}{2}}+\frac{2Ac^3x^{\frac{5}{2}}}{5}-\frac{2Ba^3}{5x^{\frac{5}{2}}}-\frac{2Ba^2b}{x^{\frac{3}{2}}}-\frac{6Ba^2c}{\sqrt{x}}\\ &-\frac{6Bab^2}{\sqrt{x}}+12Babc\sqrt{x}+2Bac^2x^{\frac{3}{2}}+2Bb^3\sqrt{x}+2Bb^2cx^{\frac{3}{2}}+\frac{6Bbc^2x^{\frac{5}{2}}}{5}+\frac{2Bc^3x^{\frac{7}{2}}}{7}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**(9/2),x)
```

output
```
-2*A*a**3/(7*x**(7/2)) - 6*A*a**2*b/(5*x**(5/2)) - 2*A*a**2*c/x**(3/2) - 2
*A*a*b**2/x**(3/2) - 12*A*a*b*c/sqrt(x) + 6*A*a*c**2*sqrt(x) - 2*A*b**3/sq
rt(x) + 6*A*b**2*c*sqrt(x) + 2*A*b*c**2*x**(3/2) + 2*A*c**3*x**(5/2)/5 - 2
*B*a**3/(5*x**(5/2)) - 2*B*a**2*b/x**(3/2) - 6*B*a**2*c/sqrt(x) - 6*B*a*b*
*2/sqrt(x) + 12*B*a*b*c*sqrt(x) + 2*B*a*c**2*x**(3/2) + 2*B*b**3*sqrt(x) +
 2*B*b**2*c*x**(3/2) + 6*B*b*c**2*x**(5/2)/5 + 2*B*c**3*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{2}{7}\,Bc^3x^{\frac{7}{2}} + \frac{2}{5}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{5}{2}} + 2\left(Bb^2c + (Ba+Ab)c^2\right)x^{\frac{3}{2}} + 2\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)\sqrt{x} - \frac{2\left(5\,Aa^3 + 35\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x^3 + 35\left(Ba^2b + Aab^2 + Aa^2c\right)x^2 + 7\left(Ba^3 + 3\,Aa^2b\right)x\right)}{35\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(9/2),x, algorithm="maxima")
```

output
```
2/7*B*c^3*x^(7/2) + 2/5*(3*B*b*c^2 + A*c^3)*x^(5/2) + 2*(B*b^2*c + (B*a +
A*b)*c^2)*x^(3/2) + 2*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*sqrt(x)
- 2/35*(5*A*a^3 + 35*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 35*
(B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 7*(B*a^3 + 3*A*a^2*b)*x)/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{2}{7}\,Bc^3x^{\frac{7}{2}} + \frac{6}{5}\,Bbc^2x^{\frac{5}{2}} + \frac{2}{5}\,Ac^3x^{\frac{5}{2}} + 2\,Bb^2cx^{\frac{3}{2}} + 2\,Bac^2x^{\frac{3}{2}} + 2\,Abc^2x^{\frac{3}{2}} + 2\,Bb^3\sqrt{x} + 12\,Babc\sqrt{x} + 6\,Ab^2c\sqrt{x} + 6\,Aac^2\sqrt{x} - \frac{2\left(105\,Bab^2x^3 + 35\,Ab^3x^3 + 105\,Ba^2cx^3 + 210\,Aabcx^3 + 35\,Ba^2bx^2 + 35\,Aab^2x^2 + 35\,Aa^2cx^2 + 7\,Ba^3x + \cdots\right.}{35\,x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(9/2),x, algorithm="giac")
```

output
```
2/7*B*c^3*x^(7/2) + 6/5*B*b*c^2*x^(5/2) + 2/5*A*c^3*x^(5/2) + 2*B*b^2*c*x^
(3/2) + 2*B*a*c^2*x^(3/2) + 2*A*b*c^2*x^(3/2) + 2*B*b^3*sqrt(x) + 12*B*a*b
*c*sqrt(x) + 6*A*b^2*c*sqrt(x) + 6*A*a*c^2*sqrt(x) - 2/35*(105*B*a*b^2*x^3
 + 35*A*b^3*x^3 + 105*B*a^2*c*x^3 + 210*A*a*b*c*x^3 + 35*B*a^2*b*x^2 + 35*
A*a*b^2*x^2 + 35*A*a^2*c*x^2 + 7*B*a^3*x + 21*A*a^2*b*x + 5*A*a^3)/x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}\,dx = \sqrt{x}\left(2\,B\,b^3+6\,A\,b^2\,c+12\,B\,a\,b\,c+6\,A\,a\,c^2\right) - \frac{x^3\left(6\,B\,c\,a^2+6\,B\,a\,b^2+12\,A\,c\,a\,b+2\,A\,b^3\right)+x\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right)+\frac{2\,A\,a^3}{7}+x^2\left(2\,B\,a^2\,b+2\,A\,c\,a^2+2\,A\,a\right.}{x^{7/2}} + x^{5/2}\left(\frac{2\,A\,c^3}{5}+\frac{6\,B\,b\,c^2}{5}\right)+x^{3/2}\left(2\,B\,b^2\,c+2\,A\,b\,c^2+2\,B\,a\,c^2\right)+\frac{2\,B\,c^3\,x^{7/2}}{7}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^(9/2),x)
```

output
```
x^(1/2)*(2*B*b^3 + 6*A*a*c^2 + 6*A*b^2*c + 12*B*a*b*c) - (x^3*(2*A*b^3 + 6
*B*a*b^2 + 6*B*a^2*c + 12*A*a*b*c) + x*((2*B*a^3)/5 + (6*A*a^2*b)/5) + (2*
A*a^3)/7 + x^2*(2*A*a*b^2 + 2*A*a^2*c + 2*B*a^2*b))/x^(7/2) + x^(5/2)*((2*
A*c^3)/5 + (6*B*b*c^2)/5) + x^(3/2)*(2*A*b*c^2 + 2*B*a*c^2 + 2*B*b^2*c) +
(2*B*c^3*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{\frac{2}{7}b\,c^3x^7+\frac{2}{5}a\,c^3x^6+\frac{6}{5}b^2c^2x^6+4ab\,c^2x^5+2b^3c\,x^5+6a^2c^2x^4+18a\,b^2c\,x^4}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(9/2),x)
```

output
```
(2*( - 5*a**4 - 28*a**3*b*x - 35*a**3*c*x**2 - 70*a**2*b**2*x**2 - 315*a**
2*b*c*x**3 + 105*a**2*c**2*x**4 - 140*a*b**3*x**3 + 315*a*b**2*c*x**4 + 70
*a*b*c**2*x**5 + 7*a*c**3*x**6 + 35*b**4*x**4 + 35*b**3*c*x**5 + 21*b**2*c
**2*x**6 + 5*b*c**3*x**7))/(35*sqrt(x)*x**3)
```

3.83 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11/2}}\,dx = &-\frac{2a^3A}{9x^{9/2}}\\ &-\frac{2a^2(3Ab+aB)}{7x^{7/2}}-\frac{6a(abB+A(b^2+ac))}{5x^{5/2}}\\ &-\frac{2(3aB(b^2+ac)+A(b^3+6abc))}{3x^{3/2}}-\frac{2(b^3B+3Ab^2c+6abBc+3aAc^2)}{\sqrt{x}}\\ &+6c\left(b^2B+Abc+aBc\right)\sqrt{x}+\frac{2}{3}c^2(3bB+Ac)x^{3/2}+\frac{2}{5}Bc^3x^{5/2}\end{aligned}$$

output
```
-2/9*a^3*A/x^(9/2)-2/7*a^2*(3*A*b+B*a)/x^(7/2)-6/5*a*(a*b*B+A*(a*c+b^2))/x
^(5/2)-2/3*(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))/x^(3/2)-2*(3*A*a*c^2+3*A*b^2*
c+6*B*a*b*c+B*b^3)/x^(1/2)+6*c*(A*b*c+B*a*c+B*b^2)*x^(1/2)+2/3*c^2*(A*c+3*
B*b)*x^(3/2)+2/5*B*c^3*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11/2}}\,dx = -\frac{2(5a^3(7A+9Bx)+9a^2x(7Bx(3b+5cx)+3A(5b+7cx))+63ax^2(5Bx(b^2+6bcx-3c^2x^2)+A(3b^2+}{315x^{9/}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/x^(11/2),x]
```

output
```
(-2*(5*a^3*(7*A + 9*B*x) + 9*a^2*x*(7*B*x*(3*b + 5*c*x) + 3*A*(5*b + 7*c*x
)) + 63*a*x^2*(5*B*x*(b^2 + 6*b*c*x - 3*c^2*x^2) + A*(3*b^2 + 10*b*c*x + 1
5*c^2*x^2)) + 21*x^3*(5*A*(b^3 + 9*b^2*c*x - 9*b*c^2*x^2 - c^3*x^3) - 3*B*
x*(-5*b^3 + 15*b^2*c*x + 5*b*c^2*x^2 + c^3*x^3))))/(315*x^(9/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11/2}}\,dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{a^3A}{x^{11/2}}+\frac{a^2(aB+3Ab)}{x^{9/2}}+\frac{3a\left(A\left(ac+b^2\right)+abB\right)}{x^{7/2}}+\frac{3c\left(aBc+Abc+b^2B\right)}{\sqrt{x}}+\frac{3aAc^2+6abBc+3Ab^2c+b^3}{x^{3/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{9x^{9/2}}-\frac{2a^2(aB+3Ab)}{7x^{7/2}}-\frac{6a\left(A\left(ac+b^2\right)+abB\right)}{5x^{5/2}}+6c\sqrt{x}\left(aBc+Abc+b^2B\right)-\frac{2\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)}{\sqrt{x}}-\frac{2\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)}{3x^{3/2}}+\frac{2}{3}c^2x^{3/2}(Ac+3bB)+\frac{2}{5}Bc^3x^{5/2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/x^(11/2),x]
```

output
```
(-2*a^3*A)/(9*x^(9/2)) - (2*a^2*(3*A*b + a*B))/(7*x^(7/2)) - (6*a*(a*b*B +
 A*(b^2 + a*c)))/(5*x^(5/2)) - (2*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c)))
/(3*x^(3/2)) - (2*(b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2))/Sqrt[x] + 6
*c*(b^2*B + A*b*c + a*B*c)*Sqrt[x] + (2*c^2*(3*b*B + A*c)*x^(3/2))/3 + (2*
B*c^3*x^(5/2))/5
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2Bc^3x^{\frac{5}{2}}}{5}+\frac{2Ac^3x^{\frac{3}{2}}}{3}+2Bbc^2x^{\frac{3}{2}}+6Abc^2\sqrt{x}+6Bac^2\sqrt{x}+6Bb^2c\sqrt{x}-\frac{2a^3A}{9x^{\frac{9}{2}}}-\frac{6a(Aac+b}{5x}$
default	$\frac{2Bc^3x^{\frac{5}{2}}}{5}+\frac{2Ac^3x^{\frac{3}{2}}}{3}+2Bbc^2x^{\frac{3}{2}}+6Abc^2\sqrt{x}+6Bac^2\sqrt{x}+6Bb^2c\sqrt{x}-\frac{2a^3A}{9x^{\frac{9}{2}}}-\frac{6a(Aac+b}{5x}$
gosper	$-\frac{2(-63Bc^3x^7-105Ac^3x^6-315Bbc^2x^6-945Abc^2x^5-945Bac^2x^5-945Bb^2cx^5+945Aac^2x^4+945Ab^2cx^4+1890B}{}$
trager	$-\frac{2(-63Bc^3x^7-105Ac^3x^6-315Bbc^2x^6-945Abc^2x^5-945Bac^2x^5-945Bb^2cx^5+945Aac^2x^4+945Ab^2cx^4+1890B}{}$
risch	$-\frac{2(-63Bc^3x^7-105Ac^3x^6-315Bbc^2x^6-945Abc^2x^5-945Bac^2x^5-945Bb^2cx^5+945Aac^2x^4+945Ab^2cx^4+1890B}{}$
orering	$-\frac{2(-63Bc^3x^7-105Ac^3x^6-315Bbc^2x^6-945Abc^2x^5-945Bac^2x^5-945Bb^2cx^5+945Aac^2x^4+945Ab^2cx^4+1890B}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*B*c^3*x^(5/2)+2/3*A*c^3*x^(3/2)+2*B*b*c^2*x^(3/2)+6*A*b*c^2*x^(1/2)+6*
B*a*c^2*x^(1/2)+6*B*b^2*c*x^(1/2)-2/9*a^3*A/x^(9/2)-6/5*a*(A*a*c+A*b^2+B*a
*b)/x^(5/2)-2*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)/x^(1/2)-2/7*a^2*(3*A*b
+B*a)/x^(7/2)-2/3*(6*A*a*b*c+A*b^3+3*B*a^2*c+3*B*a*b^2)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11/2}}\,dx = \frac{2\left(63\,Bc^3x^7+105\,(3\,Bbc^2+Ac^3)x^6+945\,(Bb^2c+(Ba+Ab)c^2)x^5-3\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(11/2),x, algorithm="fricas")
```

output
```
2/315*(63*B*c^3*x^7 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 945*(B*b^2*c + (B*a +
A*b)*c^2)*x^5 - 315*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^4 - 35*A
*a^3 - 105*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 - 189*(B*a^2*b
+ A*a*b^2 + A*a^2*c)*x^2 - 45*(B*a^3 + 3*A*a^2*b)*x)/x^(9/2)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{11/2}}\,dx = -\frac{2Aa^3}{9x^{\frac{9}{2}}} - \frac{6Aa^2b}{7x^{\frac{7}{2}}} - \frac{6Aa^2c}{5x^{\frac{5}{2}}} - \frac{6Aab^2}{5x^{\frac{5}{2}}} - \frac{4Aabc}{x^{\frac{3}{2}}} - \frac{6Aac^2}{\sqrt{x}} - \frac{2Ab^3}{3x^{\frac{3}{2}}} - \frac{6Ab^2c}{\sqrt{x}} + 6Abc^2\sqrt{x} + \frac{2Ac^3x^{\frac{3}{2}}}{3} - \frac{2Ba^3}{7x^{\frac{7}{2}}} - \frac{6Ba^2b}{5x^{\frac{5}{2}}} - \frac{2Ba^2c}{x^{\frac{3}{2}}} - \frac{2Bab^2}{x^{\frac{3}{2}}} - \frac{12Babc}{\sqrt{x}} + 6Bac^2\sqrt{x} - \frac{2Bb^3}{\sqrt{x}} + 6Bb^2c\sqrt{x} + 2Bbc^2x^{\frac{3}{2}} + \frac{2Bc^3x^{\frac{5}{2}}}{5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**(11/2),x)
```

output
```
-2*A*a**3/(9*x**(9/2)) - 6*A*a**2*b/(7*x**(7/2)) - 6*A*a**2*c/(5*x**(5/2))
 - 6*A*a*b**2/(5*x**(5/2)) - 4*A*a*b*c/x**(3/2) - 6*A*a*c**2/sqrt(x) - 2*A
*b**3/(3*x**(3/2)) - 6*A*b**2*c/sqrt(x) + 6*A*b*c**2*sqrt(x) + 2*A*c**3*x*
*(3/2)/3 - 2*B*a**3/(7*x**(7/2)) - 6*B*a**2*b/(5*x**(5/2)) - 2*B*a**2*c/x*
*(3/2) - 2*B*a*b**2/x**(3/2) - 12*B*a*b*c/sqrt(x) + 6*B*a*c**2*sqrt(x) - 2
*B*b**3/sqrt(x) + 6*B*b**2*c*sqrt(x) + 2*B*b*c**2*x**(3/2) + 2*B*c**3*x**(
5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{x^{11/2}}\,dx = \frac{2}{5}\,Bc^3x^{\frac{5}{2}} + \frac{2}{3}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{3}{2}} + 6\left(Bb^2c + (Ba+Ab)c^2\right)\sqrt{x} - \frac{2\left(315\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^4 + 35\,Aa^3 + 105\left(3\,Bab^2 + Ab^3 + 3\left(Ba^2 + 2\,Aab\right)c\right)x^3 + 189\cdots}{315\,x^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(11/2),x, algorithm="maxima")
```

output
```
2/5*B*c^3*x^(5/2) + 2/3*(3*B*b*c^2 + A*c^3)*x^(3/2) + 6*(B*b^2*c + (B*a +
A*b)*c^2)*sqrt(x) - 2/315*(315*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)
*x^4 + 35*A*a^3 + 105*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^3 + 18
9*(B*a^2*b + A*a*b^2 + A*a^2*c)*x^2 + 45*(B*a^3 + 3*A*a^2*b)*x)/x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11/2}}\,dx = \frac{2}{5}\,Bc^3x^{\frac{5}{2}} + 2\,Bbc^2x^{\frac{3}{2}} + \frac{2}{3}\,Ac^3x^{\frac{3}{2}} + 6\,Bb^2c\sqrt{x} + 6\,Bac^2\sqrt{x} + 6\,Abc^2\sqrt{x} - \frac{2\,(315\,Bb^3x^4 + 1890\,Babcx^4 + 945\,Ab^2cx^4 + 945\,Aac^2x^4 + 315\,Bab^2x^3 + 105\,Ab^3x^3 + 315\,Ba^2cx^3 + 630}{315\,x^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/x^(11/2),x, algorithm="giac")
```

output
```
2/5*B*c^3*x^(5/2) + 2*B*b*c^2*x^(3/2) + 2/3*A*c^3*x^(3/2) + 6*B*b^2*c*sqrt
(x) + 6*B*a*c^2*sqrt(x) + 6*A*b*c^2*sqrt(x) - 2/315*(315*B*b^3*x^4 + 1890*
B*a*b*c*x^4 + 945*A*b^2*c*x^4 + 945*A*a*c^2*x^4 + 315*B*a*b^2*x^3 + 105*A*
b^3*x^3 + 315*B*a^2*c*x^3 + 630*A*a*b*c*x^3 + 189*B*a^2*b*x^2 + 189*A*a*b^
2*x^2 + 189*A*a^2*c*x^2 + 45*B*a^3*x + 135*A*a^2*b*x + 35*A*a^3)/x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11/2}}\,dx = x^{3/2}\left(\frac{2\,A\,c^3}{3} + 2\,B\,b\,c^2\right) - \frac{x^3\left(2\,B\,c\,a^2 + 2\,B\,a\,b^2 + 4\,A\,c\,a\,b + \frac{2\,A\,b^3}{3}\right) + x^4\left(2\,B\,b^3 + 6\,A\,b^2\,c + 12\,B\,a\,b\,c + 6\,A\,a\,c^2\right) + x\left(\frac{2\,B\,a^3}{7} + \frac{6\,A}{}\right.}{x^{9/2}} + \sqrt{x}\left(6\,B\,b^2\,c + 6\,A\,b\,c^2 + 6\,B\,a\,c^2\right) + \frac{2\,B\,c^3\,x^{5/2}}{5}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/x^(11/2),x)
```

output
```
x^(3/2)*((2*A*c^3)/3 + 2*B*b*c^2) - (x^3*((2*A*b^3)/3 + 2*B*a*b^2 + 2*B*a^
2*c + 4*A*a*b*c) + x^4*(2*B*b^3 + 6*A*a*c^2 + 6*A*b^2*c + 12*B*a*b*c) + x*
((2*B*a^3)/7 + (6*A*a^2*b)/7) + (2*A*a^3)/9 + x^2*((6*A*a*b^2)/5 + (6*A*a^
2*c)/5 + (6*B*a^2*b)/5))/x^(9/2) + x^(1/2)*(6*A*b*c^2 + 6*B*a*c^2 + 6*B*b^
2*c) + (2*B*c^3*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^{11/2}}\,dx = \frac{\frac{2}{5}b\,c^3x^7+\frac{2}{3}a\,c^3x^6+2b^2c^2x^6+12ab\,c^2x^5+6b^3c\,x^5-6a^2c^2x^4-18a\,b^2c\,x \cdots}{\sqrt{x}\,x \cdots}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/x^(11/2),x)
```

output
```
(2*( - 35*a**4 - 180*a**3*b*x - 189*a**3*c*x**2 - 378*a**2*b**2*x**2 - 945
*a**2*b*c*x**3 - 945*a**2*c**2*x**4 - 420*a*b**3*x**3 - 2835*a*b**2*c*x**4
 + 1890*a*b*c**2*x**5 + 105*a*c**3*x**6 - 315*b**4*x**4 + 945*b**3*c*x**5
+ 315*b**2*c**2*x**6 + 63*b*c**3*x**7))/(315*sqrt(x)*x**4)
```

3.84 $\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 347

$$\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2}\,dx = \frac{2(b^2B-Abc-aBc)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{5/2}}{5c} - \frac{\sqrt{2}\left(b^3B-Ab^2c-2abBc+aAc^2-\frac{b^4B-Ab^3c-4ab^2Bc+3aAbc^2+2a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b^3B-Ab^2c-2abBc+aAc^2+\frac{b^4B-Ab^3c-4ab^2Bc+3aAbc^2+2a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
2*(-A*b*c-B*a*c+B*b^2)*x^(1/2)/c^3-2/3*(-A*c+B*b)*x^(3/2)/c^2+2/5*B*x^(5/2
)/c-2^(1/2)*(B*b^3-A*b^2*c-2*B*a*b*c+A*a*c^2-(3*A*a*b*c^2-A*b^3*c+2*B*a^2*
c^2-4*B*a*b^2*c+B*b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/
(b-(-4*a*c+b^2)^(1/2))^(1/2))/c^(7/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)
*(B*b^3-A*b^2*c-2*B*a*b*c+A*a*c^2+(3*A*a*b*c^2-A*b^3*c+2*B*a^2*c^2-4*B*a*b
^2*c+B*b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))/c^(7/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.18

$$\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}\sqrt{x}(15b^2B-5bc(3A+Bx)+c(-15aB+cx(5A+3Bx)))-\frac{15\sqrt{2}\left(-b^4B+b^2c\left(4aB\right.\right.}{}}{}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[c]*Sqrt[x]*(15*b^2*B - 5*b*c*(3*A + B*x) + c*(-15*a*B + c*x*(5*A +
 3*B*x))) - (15*Sqrt[2]*(-(b^4*B) + b^2*c*(4*a*B - A*Sqrt[b^2 - 4*a*c]) +
a*c^2*(-2*a*B + A*Sqrt[b^2 - 4*a*c]) + b^3*(A*c + B*Sqrt[b^2 - 4*a*c]) - a
*b*c*(3*A*c + 2*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (15*Sqrt[2]*(b^4*B + a*c^2*(2*a*B + A*Sqrt[b^2 - 4*a*c]) - b^2*c*(4*a*B
 + A*Sqrt[b^2 - 4*a*c]) + a*b*c*(3*A*c - 2*B*Sqrt[b^2 - 4*a*c]) + b^3*(-(A
*c) + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(15*c^(7
/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1196, 25, 1196, 25, 1196, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 1196$$

$$\frac{\int -\frac{x^{3/2}(aB+(bB-Ac)x)}{cx^2+bx+a}dx}{c}+\frac{2Bx^{5/2}}{5c}$$

$$\Big\downarrow \; 25$$

$$\frac{2Bx^{5/2}}{5c}-\frac{\int \frac{x^{3/2}(aB+(bB-Ac)x)}{cx^2+bx+a}dx}{c}$$

$\Big\downarrow$ 1196

$$\frac{2Bx^{5/2}}{5c}-\frac{\frac{\int -\frac{\sqrt{x}\left(a(bB-Ac)+\left(Bb^2-Acb-aBc\right)x\right)}{cx^2+bx+a}dx}{c}+\frac{2x^{3/2}(bB-Ac)}{3c}}{c}$$

$\Big\downarrow$ 25

$$\frac{2Bx^{5/2}}{5c}-\frac{\frac{2x^{3/2}(bB-Ac)}{3c}-\frac{\int \frac{\sqrt{x}\left(a(bB-Ac)+\left(Bb^2-Acb-aBc\right)x\right)}{cx^2+bx+a}dx}{c}}{c}$$

$\Big\downarrow$ 1196

$$\frac{2Bx^{5/2}}{5c}-\frac{\frac{2x^{3/2}(bB-Ac)}{3c}-\frac{\frac{\int -\frac{a\left(Bb^2-Acb-aBc\right)+\left(Bb^3-Acb^2-2aBcb+aAc^2\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{c}+\frac{2\sqrt{x}\left(-aBc-Abc+b^2B\right)}{c}}{c}}{c}$$

$\Big\downarrow$ 25

$$\frac{2Bx^{5/2}}{5c}-\frac{\frac{2x^{3/2}(bB-Ac)}{3c}-\frac{\frac{2\sqrt{x}\left(-aBc-Abc+b^2B\right)}{c}-\frac{\int \frac{a\left(Bb^2-Acb-aBc\right)+\left(Bb^3-Acb^2-2aBcb+aAc^2\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{c}}{c}}{c}$$

$\Big\downarrow$ 1197

$$\frac{2Bx^{5/2}}{5c}-\frac{\frac{2x^{3/2}(bB-Ac)}{3c}-\frac{\frac{2\sqrt{x}\left(-aBc-Abc+b^2B\right)}{c}-\frac{2\int \frac{a\left(Bb^2-Acb-aBc\right)+\left(Bb^3-Acb^2-2aBcb+aAc^2\right)x}{cx^2+bx+a}d\sqrt{x}}{c}}{c}}{c}$$

$\Big\downarrow$ 1480

$$\frac{2Bx^{5/2}}{5c}-\frac{\frac{2x^{3/2}(bB-Ac)}{3c}-\frac{\frac{2\sqrt{x}\left(-aBc-Abc+b^2B\right)}{c}-\frac{2\left(\frac{1}{2}\left(-\frac{2a^2Bc^2+3aAbc^2-4ab^2Bc-Ab^3c+b^4B}{\sqrt{b^2-4ac}}+aAc^2-2abBc-Ab^2c+b^3B\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\cdots\right.}{c}}{c}}{c}$$

$\Big\downarrow$ 218

$$\frac{\frac{2Bx^{5/2}}{5c}-\frac{\frac{2x^{3/2}(bB-Ac)}{3c}-\frac{\frac{2\sqrt{x}\left(-aBc-Abc+b^2B\right)}{c}-\frac{2\left(\frac{\left(-\frac{2a^2Bc^2+3aAbc^2-4ab^2Bc-Ab^3c+b^4B}{\sqrt{b^2-4ac}}+aAc^2-2abBc-Ab^2c+b^3B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(2a^2B\cdots\right.}{\cdots}\right)}{c}}{c}}{c}}{c}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a + b*x + c*x^2),x]
```

output
```
(2*B*x^(5/2))/(5*c) - ((2*(b*B - A*c)*x^(3/2))/(3*c) - ((2*(b^2*B - A*b*c
- a*B*c)*Sqrt[x])/c - (2*(((b^3*B - A*b^2*c - 2*a*b*B*c + a*A*c^2 - (b^4*B
 - A*b^3*c - 4*a*b^2*B*c + 3*a*A*b*c^2 + 2*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B - A*b^2*c - 2*a*b*B*c + a*A*c^
2 + (b^4*B - A*b^3*c - 4*a*b^2*B*c + 3*a*A*b*c^2 + 2*a^2*B*c^2)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/c)/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{2\left(-3B\,c^{2}x^{2}-5A\,c^{2}x+5Bbcx+15Abc+15aBc-15B\,b^{2}\right)\sqrt{x}}{15c^{3}}-\frac{8\left(-\frac{\left(Aa\,c^{2}\sqrt{-4ac+b^{2}}-A\,b^{2}c\sqrt{-4ac+b^{2}}-3Aab\,c^{2}+A\,b\right.}{}\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{B\,c^{2}x^{\frac{5}{2}}}{5}-\frac{A\,c^{2}x^{\frac{3}{2}}}{3}+\frac{Bbc\,x^{\frac{3}{2}}}{3}+Abc\sqrt{x}+Bac\sqrt{x}-B\,b^{2}\sqrt{x}\right)}{c^{3}}+\frac{-\frac{\left(-Aa\,c^{2}\sqrt{-4ac+b^{2}}+A\,b^{2}c\sqrt{-4ac+b^{2}}+3Aab\,c^{2}-A\right.}{}}{}$
default	$-\frac{2\left(-\frac{B\,c^{2}x^{\frac{5}{2}}}{5}-\frac{A\,c^{2}x^{\frac{3}{2}}}{3}+\frac{Bbc\,x^{\frac{3}{2}}}{3}+Abc\sqrt{x}+Bac\sqrt{x}-B\,b^{2}\sqrt{x}\right)}{c^{3}}+\frac{-\frac{\left(-Aa\,c^{2}\sqrt{-4ac+b^{2}}+A\,b^{2}c\sqrt{-4ac+b^{2}}+3Aab\,c^{2}-A\right.}{}}{}$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*c^2*x^2-5*A*c^2*x+5*B*b*c*x+15*A*b*c+15*B*a*c-15*B*b^2)*x^(1/2
)/c^3-8/c^2*(-1/8*(A*a*c^2*(-4*a*c+b^2)^(1/2)-A*b^2*c*(-4*a*c+b^2)^(1/2)-3
*A*a*b*c^2+A*b^3*c-2*B*a*b*c*(-4*a*c+b^2)^(1/2)+B*b^3*(-4*a*c+b^2)^(1/2)-2
*B*a^2*c^2+4*B*a*b^2*c-B*b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))+1/8*(A*a*c^2*(-4*a*c+b^2)^(1/2)-A*b^2*c*(-4*a*c+b^2)^(1/2)+3*A*a*b*c
^2-A*b^3*c-2*B*a*b*c*(-4*a*c+b^2)^(1/2)+B*b^3*(-4*a*c+b^2)^(1/2)+2*B*a^2*c
^2-4*B*a*b^2*c+B*b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7707 vs. $2(297) = 594$.

Time = 12.95 (sec) , antiderivative size = 7707, normalized size of antiderivative = 22.21

$$\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40613 vs. $2(347) = 694$.

Time = 75.32 (sec) , antiderivative size = 40613, normalized size of antiderivative = 117.04

$$\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(5/2)*(B*x+A)/(c*x**2+b*x+a),x)
```

output
```
Piecewise((-A*a**3*log(sqrt(x) - sqrt(-a/b))/(b**4*sqrt(-a/b)) + A*a**3*lo
g(sqrt(x) + sqrt(-a/b))/(b**4*sqrt(-a/b)) + 2*A*a**2*sqrt(x)/b**3 - 2*A*a*
x**(3/2)/(3*b**2) + 2*A*x**(5/2)/(5*b) + B*a**4*log(sqrt(x) - sqrt(-a/b))/
(b**5*sqrt(-a/b)) - B*a**4*log(sqrt(x) + sqrt(-a/b))/(b**5*sqrt(-a/b)) - 2
*B*a**3*sqrt(x)/b**4 + 2*B*a**2*x**(3/2)/(3*b**3) - 2*B*a*x**(5/2)/(5*b**2
) + 2*B*x**(7/2)/(7*b), Eq(c, 0)), (A*b**2*log(sqrt(x) - sqrt(-b/c))/(c**3
*sqrt(-b/c)) - A*b**2*log(sqrt(x) + sqrt(-b/c))/(c**3*sqrt(-b/c)) - 2*A*b*
sqrt(x)/c**2 + 2*A*x**(3/2)/(3*c) - B*b**3*log(sqrt(x) - sqrt(-b/c))/(c**4
*sqrt(-b/c)) + B*b**3*log(sqrt(x) + sqrt(-b/c))/(c**4*sqrt(-b/c)) + 2*B*b*
*2*sqrt(x)/c**3 - 2*B*b*x**(3/2)/(3*c**2) + 2*B*x**(5/2)/(5*c), Eq(a, 0)),
 (150*sqrt(2)*A*b**3*c*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/2)/(240*b*c**4*sqr
t(-b/c) + 480*c**5*x*sqrt(-b/c)) - 150*sqrt(2)*A*b**3*c*log(sqrt(x) + sqrt
(2)*sqrt(-b/c)/2)/(240*b*c**4*sqrt(-b/c) + 480*c**5*x*sqrt(-b/c)) - 600*A*
b**2*c**2*sqrt(x)*sqrt(-b/c)/(240*b*c**4*sqrt(-b/c) + 480*c**5*x*sqrt(-b/c
)) + 300*sqrt(2)*A*b**2*c**2*x*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/2)/(240*b*
c**4*sqrt(-b/c) + 480*c**5*x*sqrt(-b/c)) - 300*sqrt(2)*A*b**2*c**2*x*log(s
qrt(x) + sqrt(2)*sqrt(-b/c)/2)/(240*b*c**4*sqrt(-b/c) + 480*c**5*x*sqrt(-b
/c)) - 800*A*b*c**3*x**(3/2)*sqrt(-b/c)/(240*b*c**4*sqrt(-b/c) + 480*c**5*
x*sqrt(-b/c)) + 320*A*c**4*x**(5/2)*sqrt(-b/c)/(240*b*c**4*sqrt(-b/c) + 48
0*c**5*x*sqrt(-b/c)) - 105*sqrt(2)*B*b**4*log(sqrt(x) - sqrt(2)*sqrt(-b...
```

Maxima [F]

$$\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2}\,dx=\int \frac{(Bx+A)x^{\frac{5}{2}}}{cx^2+bx+a}\,dx$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
2/15*(3*B*c*x^(5/2) - 5*(B*b - A*c)*x^(3/2))/c^2 - integrate(((A*b*c - (b^
2 - a*c)*B)*x^(3/2) - (B*a*b - A*a*c)*sqrt(x))/(c^3*x^2 + b*c^2*x + a*c^2)
, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5319 vs. $2(297) = 594$.

Time = 0.86 (sec) , antiderivative size = 5319, normalized size of antiderivative = 15.33

$$\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/4*((2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)
*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*A*c^2 - (2*b^7*c^2 - 20*a*b^5*c^3 +
64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*...
```

Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 14120, normalized size of antiderivative = 40.69

$$\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2} \, dx = \text{Too large to display}$$

input
```
int((x^(5/2)*(A + B*x))/(a + b*x + c*x^2),x)
```

output
```
x^(3/2)*((2*A)/(3*c) - (2*B*b)/(3*c^2)) - x^(1/2)*((b*((2*A)/c - (2*B*b)/c
^2))/c + (2*B*a)/c^2) + atan(((((8*(4*B*a^3*c^6 - A*a*b^3*c^5 + 4*A*a^2*b*
c^6 + B*a*b^4*c^4 - 5*B*a^2*b^2*c^5))/c^5 - (8*x^(1/2)*(b^3*c^7 - 4*a*b*c^
8)*(-(B^2*b^9 + A^2*b^7*c^2 + B^2*b^6*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^8
*c + 25*A^2*a^2*b^3*c^4 + A^2*a^2*c^4*(-(4*a*c - b^2)^3)^(1/2) + 42*B^2*a^
2*b^5*c^2 - 63*B^2*a^3*b^3*c^3 + A^2*b^4*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^
2*a^3*c^3*(-(4*a*c - b^2)^3)^(1/2) - 16*A*B*a^4*c^5 - 11*B^2*a*b^7*c - 9*A
^2*a*b^5*c^3 - 20*A^2*a^3*b*c^5 + 28*B^2*a^4*b*c^4 + 6*B^2*a^2*b^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) - 66*A*B*a^2*b^4*c^3 + 76*A*B*a^3*b^2*c^4 - 5*B^2*a*
b^4*c*(-(4*a*c - b^2)^3)^(1/2) - 3*A^2*a*b^2*c^3*(-(4*a*c - b^2)^3)^(1/2)
+ 20*A*B*a*b^6*c^2 - 2*A*B*b^5*c*(-(4*a*c - b^2)^3)^(1/2) + 8*A*B*a*b^3*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 6*A*B*a^2*b*c^3*(-(4*a*c - b^2)^3)^(1/2))/(2*
(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2))/c^5)*(-(B^2*b^9 + A^2*b^7*c^
2 + B^2*b^6*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^8*c + 25*A^2*a^2*b^3*c^4 +
A^2*a^2*c^4*(-(4*a*c - b^2)^3)^(1/2) + 42*B^2*a^2*b^5*c^2 - 63*B^2*a^3*b^3
*c^3 + A^2*b^4*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^3*c^3*(-(4*a*c - b^2)^
3)^(1/2) - 16*A*B*a^4*c^5 - 11*B^2*a*b^7*c - 9*A^2*a*b^5*c^3 - 20*A^2*a^3*
b*c^5 + 28*B^2*a^4*b*c^4 + 6*B^2*a^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 66
*A*B*a^2*b^4*c^3 + 76*A*B*a^3*b^2*c^4 - 5*B^2*a*b^4*c*(-(4*a*c - b^2)^3)^(
1/2) - 3*A^2*a*b^2*c^3*(-(4*a*c - b^2)^3)^(1/2) + 20*A*B*a*b^6*c^2 - 2*...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1213, normalized size of antiderivative = 3.50

$$\int \frac{x^{5/2}(A+Bx)}{a+bx+cx^2} \, dx = \text{Too large to display}$$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x+a),x)
```

output
```
(60*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**3 - 120*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*
sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2 + 30*sqrt(a)*sqrt(2*sqrt
(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sq
rt(2*sqrt(c)*sqrt(a) + b))*b**4*c - 150*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sq
rt(a) + b))*a**2*b*c**2 + 150*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a*b**3*c - 30*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**5 - 60*sq
rt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sq
rt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**3 + 120*sqrt(a)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c
))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2 - 30*sqrt(a)*sqrt(2*sqrt(c)*sq
rt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*s
qrt(c)*sqrt(a) + b))*b**4*c + 150*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a**2*b*c**2 - 150*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a...
```

3.85 $\int \frac{x^{3/2}(A+Bx)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 275

$$\int \frac{x^{3/2}(A+Bx)}{a+bx+cx^2}\,dx = -\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{3/2}}{3c} + \frac{\sqrt{2}\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2B-Abc-aBc+\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-2*(-A*c+B*b)*x^(1/2)/c^2+2/3*B*x^(3/2)/c+2^(1/2)*(B*b^2-A*b*c-B*a*c-(2*A*
a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*
x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))/c^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
+2^(1/2)*(B*b^2-A*b*c-B*a*c+(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(-4*a*c+b^
2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))/c^(
5/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.21

$$\int \frac{x^{3/2}(A+Bx)}{a+bx+cx^2}\,dx = \frac{2\sqrt{c}\sqrt{x}(-3bB+3Ac+Bcx)+\frac{3\sqrt{2}\left(-b^3B+bc\left(3aB-A\sqrt{b^2-4ac}\right)+b^2\left(Ac+B\sqrt{b^2-4ac}\right)-ac\left(2Ac+\right.\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a + b*x + c*x^2),x]
```

output
```
(2*Sqrt[c]*Sqrt[x]*(-3*b*B + 3*A*c + B*c*x) + (3*Sqrt[2]*(-(b^3*B) + b*c*(
3*a*B - A*Sqrt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c]) - a*c*(2*A*
c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2
]*(b^3*B - b*c*(3*a*B + A*Sqrt[b^2 - 4*a*c]) + a*c*(2*A*c - B*Sqrt[b^2 - 4
*a*c]) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[
x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4
*a*c]]))/(3*c^(5/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1196, 25, 1196, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 1196}$$

$$\frac{\int -\frac{\sqrt{x}(aB+(bB-Ac)x)}{cx^2+bx+a}dx}{c}+\frac{2Bx^{3/2}}{3c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2Bx^{3/2}}{3c}-\frac{\int \frac{\sqrt{x}(aB+(bB-Ac)x)}{cx^2+bx+a}dx}{c}$$

$\Big\downarrow$ 1196

$$\frac{2Bx^{3/2}}{3c}-\frac{\frac{\int -\frac{a(bB-Ac)+\left(Bb^2-Acb-aBc\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{c}+\frac{2\sqrt{x}(bB-Ac)}{c}}{c}$$

$\Big\downarrow$ 25

$$\frac{2Bx^{3/2}}{3c}-\frac{\frac{2\sqrt{x}(bB-Ac)}{c}-\frac{\int \frac{a(bB-Ac)+\left(Bb^2-Acb-aBc\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{c}}{c}$$

$\Big\downarrow$ 1197

$$\frac{2Bx^{3/2}}{3c}-\frac{\frac{2\sqrt{x}(bB-Ac)}{c}-\frac{2\int \frac{a(bB-Ac)+\left(Bb^2-Acb-aBc\right)x}{cx^2+bx+a}d\sqrt{x}}{c}}{c}$$

$\Big\downarrow$ 1480

$$\frac{2Bx^{3/2}}{3c}-\frac{\frac{2\sqrt{x}(bB-Ac)}{c}-\frac{2\left(\frac{1}{2}\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right.}{c}}{c}$$

$\Big\downarrow$ 218

$$\frac{2Bx^{3/2}}{3c}-\frac{\frac{2\sqrt{x}(bB-Ac)}{c}-\frac{2\left(\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\arctan\left(\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right.}{c}}{c}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a + b*x + c*x^2),x]
```

output
```
(2*B*x^(3/2))/(3*c) - ((2*(b*B - A*c)*Sqrt[x])/c - (2*(((b^2*B - A*b*c - a
*B*c - (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B - A*b*c - a*B*c + (b^3*B - A*b^2*c
- 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x
])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]])))/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06

method	result
risch	$\frac{2(Bcx+3Ac-3Bb)\sqrt{x}}{3c^2} - \frac{8\left(-\frac{\left(Abc\sqrt{-4ac+b^2}+2Aa\,c^2-A\,b^2c+aBc\sqrt{-4ac+b^2}-B\,b^2\sqrt{-4ac+b^2}-3Babc+B\,b^3\right)\sqrt{2}\ \mathrm{arctanh}}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}-2Bb\sqrt{x}}{c^2} + \frac{-\frac{\left(-Abc\sqrt{-4ac+b^2}-2Aa\,c^2+A\,b^2c-aBc\sqrt{-4ac+b^2}+B\,b^2\sqrt{-4ac+b^2}+3Babc-B\,b^3\right)\sqrt{2}\ \mathrm{arctan}}{c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{}$
default	$\frac{\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}-2Bb\sqrt{x}}{c^2} + \frac{-\frac{\left(-Abc\sqrt{-4ac+b^2}-2Aa\,c^2+A\,b^2c-aBc\sqrt{-4ac+b^2}+B\,b^2\sqrt{-4ac+b^2}+3Babc-B\,b^3\right)\sqrt{2}\ \mathrm{arctan}}{c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{}$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*c*x+3*A*c-3*B*b)*x^(1/2)/c^2-8/c*(-1/8*(A*b*c*(-4*a*c+b^2)^(1/2)+2*
A*a*c^2-A*b^2*c+a*B*c*(-4*a*c+b^2)^(1/2)-B*b^2*(-4*a*c+b^2)^(1/2)-3*B*a*b*
c+B*b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(A*b*c*(-4*
a*c+b^2)^(1/2)-2*A*a*c^2+A*b^2*c+a*B*c*(-4*a*c+b^2)^(1/2)-B*b^2*(-4*a*c+b^
2)^(1/2)+3*B*a*b*c-B*b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5148 vs. $2(229) = 458$.

Time = 4.42 (sec) , antiderivative size = 5148, normalized size of antiderivative = 18.72

$$\int \frac{x^{3/2}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17734 vs. $2(264) = 528$.

Time = 18.14 (sec) , antiderivative size = 17734, normalized size of antiderivative = 64.49

$$\int \frac{x^{3/2}(A + Bx)}{a + bx + cx^2} \, dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(B*x+A)/(c*x**2+b*x+a),x)
```

output
```
Piecewise((A*a**2*log(sqrt(x) - sqrt(-a/b))/(b**3*sqrt(-a/b)) - A*a**2*log
(sqrt(x) + sqrt(-a/b))/(b**3*sqrt(-a/b)) - 2*A*a*sqrt(x)/b**2 + 2*A*x**(3/
2)/(3*b) - B*a**3*log(sqrt(x) - sqrt(-a/b))/(b**4*sqrt(-a/b)) + B*a**3*log
(sqrt(x) + sqrt(-a/b))/(b**4*sqrt(-a/b)) + 2*B*a**2*sqrt(x)/b**3 - 2*B*a*x
**(3/2)/(3*b**2) + 2*B*x**(5/2)/(5*b), Eq(c, 0)), (-A*b*log(sqrt(x) - sqrt
(-b/c))/(c**2*sqrt(-b/c)) + A*b*log(sqrt(x) + sqrt(-b/c))/(c**2*sqrt(-b/c)
) + 2*A*sqrt(x)/c + B*b**2*log(sqrt(x) - sqrt(-b/c))/(c**3*sqrt(-b/c)) - B
*b**2*log(sqrt(x) + sqrt(-b/c))/(c**3*sqrt(-b/c)) - 2*B*b*sqrt(x)/c**2 + 2
*B*x**(3/2)/(3*c), Eq(a, 0)), (-18*sqrt(2)*A*b**2*c*log(sqrt(x) - sqrt(2)*
sqrt(-b/c)/2)/(24*b*c**3*sqrt(-b/c) + 48*c**4*x*sqrt(-b/c)) + 18*sqrt(2)*A
*b**2*c*log(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(24*b*c**3*sqrt(-b/c) + 48*c**
4*x*sqrt(-b/c)) + 72*A*b*c**2*sqrt(x)*sqrt(-b/c)/(24*b*c**3*sqrt(-b/c) + 4
8*c**4*x*sqrt(-b/c)) - 36*sqrt(2)*A*b*c**2*x*log(sqrt(x) - sqrt(2)*sqrt(-b
/c)/2)/(24*b*c**3*sqrt(-b/c) + 48*c**4*x*sqrt(-b/c)) + 36*sqrt(2)*A*b*c**2
*x*log(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(24*b*c**3*sqrt(-b/c) + 48*c**4*x*s
qrt(-b/c)) + 96*A*c**3*x**(3/2)*sqrt(-b/c)/(24*b*c**3*sqrt(-b/c) + 48*c**4
*x*sqrt(-b/c)) + 15*sqrt(2)*B*b**3*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/2)/(24
*b*c**3*sqrt(-b/c) + 48*c**4*x*sqrt(-b/c)) - 15*sqrt(2)*B*b**3*log(sqrt(x)
 + sqrt(2)*sqrt(-b/c)/2)/(24*b*c**3*sqrt(-b/c) + 48*c**4*x*sqrt(-b/c)) - 6
0*B*b**2*c*sqrt(x)*sqrt(-b/c)/(24*b*c**3*sqrt(-b/c) + 48*c**4*x*sqrt(-b...
```

Maxima [F]

$$\int \frac{x^{3/2}(A+Bx)}{a+bx+cx^2}\,dx = \int \frac{(Bx+A)x^{\frac{3}{2}}}{cx^2+bx+a}\,dx$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
2/3*B*x^(3/2)/c + integrate(-(B*a*sqrt(x) + (B*b - A*c)*x^(3/2))/(c^2*x^2
+ b*c*x + a*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4399 vs. $2(229) = 458$.

Time = 0.79 (sec) , antiderivative size = 4399, normalized size of antiderivative = 16.00

$$\int \frac{x^{3/2}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A
*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b
^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^...
```

Mupad [B] (verification not implemented)

Time = 11.76 (sec) , antiderivative size = 10204, normalized size of antiderivative = 37.11

$$\int \frac{x^{3/2}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(A + B*x))/(a + b*x + c*x^2),x)
```

output

```
x^(1/2)*((2*A)/c - (2*B*b)/c^2) - atan(((((8*(4*A*a^2*c^5 - A*a*b^2*c^4 +
B*a*b^3*c^3 - 4*B*a^2*b*c^4))/c^3 - (8*x^(1/2)*(b^3*c^5 - 4*a*b*c^6)*(-(B^
2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*
B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 1
2*A^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 -
36*A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4
*c^2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2
)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b
^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2
*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A
^2*a^2*b*c^4 + A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*
A*B*a^2*b^2*c^3 + 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^
2 + 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*x^(1/2)*(B^2
*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a
^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a
^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 - B^2*b^4*(-(4*a*c - b^2)^3)^(1/2
) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 - A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 975, normalized size of antiderivative = 3.55

$$\int \frac{x^{3/2}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input

```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x+a),x)
```

output

```
(18*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2 - 6*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqr
t(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c + 12*sqrt(c)*sqrt(2*sqrt(c)*sqrt
(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqr
t(c)*sqrt(a) + b))*a**2*c**2 - 24*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a*b**2*c + 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4 - 18
*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2
*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2 + 6*sqrt(a)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c
))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c - 12*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
 + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c
)*sqrt(a) + b))*a**2*c**2 + 24*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b
))*a*b**2*c - 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4 - 9*sqr
t(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b
) + sqrt(a) + sqrt(c)*x)*a*b*c**2 + 3*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - ...
```

3.86 $\int \frac{\sqrt{x}(A+Bx)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 221

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx+cx^2}\,dx = \frac{2B\sqrt{x}}{c} - \frac{\sqrt{2}\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
2*B*x^(1/2)/c-2^(1/2)*(B*b-A*c-(-A*b*c-2*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))*
arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))/c^(3/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)*(B*b-A*c+(-A*b*c-2*B*a*c+B*b^2)/(-4*a*c+b^
2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))/c^(
3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx+cx^2}\,dx = \frac{2B\sqrt{x}}{c} - \frac{\sqrt{2}\left(-b^2B+Abc+2aBc+bB\sqrt{b^2-4ac}-Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(b^2B-Abc-2aBc+bB\sqrt{b^2-4ac}-Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a + b*x + c*x^2),x]
```

output
```
(2*B*Sqrt[x])/c - (Sqrt[2]*(-(b^2*B) + A*b*c + 2*a*B*c + b*B*Sqrt[b^2 - 4*
a*c] - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (Sqrt[2]*(b^2*B - A*b*c - 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1196, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1196}$$

$$\frac{\int -\frac{aB+(bB-Ac)x}{\sqrt{x}(cx^2+bx+a)}dx}{c}+\frac{2B\sqrt{x}}{c}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{2B\sqrt{x}}{c}-\frac{\int \frac{aB+(bB-Ac)x}{\sqrt{x}(cx^2+bx+a)}dx}{c}$$

$$\Big\downarrow \textcolor{blue}{1197}$$

$$\frac{2B\sqrt{x}}{c}-\frac{2\int \frac{aB+(bB-Ac)x}{cx^2+bx+a}d\sqrt{x}}{c}$$

$$\Big\downarrow \textcolor{blue}{1480}$$

$$\frac{2B\sqrt{x}}{c}-\frac{2\left(\frac{1}{2}\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\int \frac{1}{\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)+cx}\right.}{c}$$

$$\Big\downarrow \textcolor{blue}{218}$$

$$\frac{2B\sqrt{x}}{c}-\frac{2\left(\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a + b*x + c*x^2),x]
```

output
```
(2*B*Sqrt[x])/c - (2*(((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - A*c + (b^2*B - A*b*c -
 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2B\sqrt{x}}{c}+\frac{\left(Ac\sqrt{-4ac+b^2}+Abc-Bb\sqrt{-4ac+b^2}+2aBc-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{x}\,c\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(Ac\sqrt{-4ac+b^2}-\right.}{}$
default	$\frac{2B\sqrt{x}}{c}+\frac{\left(Ac\sqrt{-4ac+b^2}+Abc-Bb\sqrt{-4ac+b^2}+2aBc-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{x}\,c\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(Ac\sqrt{-4ac+b^2}-\right.}{}$
risch	$\frac{2B\sqrt{x}}{c}+\frac{\left(Ac\sqrt{-4ac+b^2}+Abc-Bb\sqrt{-4ac+b^2}+2aBc-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{x}\,c\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(Ac\sqrt{-4ac+b^2}-\right.}{}$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/c+(A*c*(-4*a*c+b^2)^(1/2)+A*b*c-B*b*(-4*a*c+b^2)^(1/2)+2*a*B*c
-B*b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-(A*c*(-4*a*c+b^2)^(
1/2)-A*b*c-B*b*(-4*a*c+b^2)^(1/2)-2*a*B*c+B*b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2642 vs. 2(179) = 358.

Time = 0.89 (sec) , antiderivative size = 2642, normalized size of antiderivative = 11.95

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B
*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A
^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 +
 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*(B
^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^2 - (5*B^
3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b + A*b^2)*c^
4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*
A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 -
 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^
2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*
B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*
A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(B^4*a*b^2 -
A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2
)*c)*sqrt(x)) - sqrt(2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*
a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*
B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(
B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log
(-sqrt(2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)
*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b
 + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13942 vs. $2(202) = 404$.

Time = 6.90 (sec) , antiderivative size = 13942, normalized size of antiderivative = 63.09

$$\int \frac{\sqrt{x}(A + Bx)}{a + bx + cx^2} \, dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)*(B*x+A)/(c*x**2+b*x+a),x)
```

output
```
Piecewise((-A*a*log(sqrt(x) - sqrt(-a/b))/(b**2*sqrt(-a/b)) + A*a*log(sqrt
(x) + sqrt(-a/b))/(b**2*sqrt(-a/b)) + 2*A*sqrt(x)/b + B*a**2*log(sqrt(x) -
 sqrt(-a/b))/(b**3*sqrt(-a/b)) - B*a**2*log(sqrt(x) + sqrt(-a/b))/(b**3*sq
rt(-a/b)) - 2*B*a*sqrt(x)/b**2 + 2*B*x**(3/2)/(3*b), Eq(c, 0)), (A*log(sqr
t(x) - sqrt(-b/c))/(c*sqrt(-b/c)) - A*log(sqrt(x) + sqrt(-b/c))/(c*sqrt(-b
/c)) - B*b*log(sqrt(x) - sqrt(-b/c))/(c**2*sqrt(-b/c)) + B*b*log(sqrt(x) +
 sqrt(-b/c))/(c**2*sqrt(-b/c)) + 2*B*sqrt(x)/c, Eq(a, 0)), (2*sqrt(2)*A*b*
c*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/2)/(4*b*c**2*sqrt(-b/c) + 8*c**3*x*sqrt
(-b/c)) - 2*sqrt(2)*A*b*c*log(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(4*b*c**2*sq
rt(-b/c) + 8*c**3*x*sqrt(-b/c)) - 8*A*c**2*sqrt(x)*sqrt(-b/c)/(4*b*c**2*sq
rt(-b/c) + 8*c**3*x*sqrt(-b/c)) + 4*sqrt(2)*A*c**2*x*log(sqrt(x) - sqrt(2)
*sqrt(-b/c)/2)/(4*b*c**2*sqrt(-b/c) + 8*c**3*x*sqrt(-b/c)) - 4*sqrt(2)*A*c
**2*x*log(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(4*b*c**2*sqrt(-b/c) + 8*c**3*x*
sqrt(-b/c)) - 3*sqrt(2)*B*b**2*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/2)/(4*b*c*
*2*sqrt(-b/c) + 8*c**3*x*sqrt(-b/c)) + 3*sqrt(2)*B*b**2*log(sqrt(x) + sqrt
(2)*sqrt(-b/c)/2)/(4*b*c**2*sqrt(-b/c) + 8*c**3*x*sqrt(-b/c)) + 12*B*b*c*s
qrt(x)*sqrt(-b/c)/(4*b*c**2*sqrt(-b/c) + 8*c**3*x*sqrt(-b/c)) - 6*sqrt(2)*
B*b*c*x*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/2)/(4*b*c**2*sqrt(-b/c) + 8*c**3*
x*sqrt(-b/c)) + 6*sqrt(2)*B*b*c*x*log(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(4*b
*c**2*sqrt(-b/c) + 8*c**3*x*sqrt(-b/c)) + 16*B*c**2*x**(3/2)*sqrt(-b/c)...
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx+cx^2}\,dx = \int \frac{(Bx+A)\sqrt{x}}{cx^2+bx+a}\,dx$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(x)/(c*x^2 + b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3186 vs. $2(179) = 358$.

Time = 0.71 (sec) , antiderivative size = 3186, normalized size of antiderivative = 14.42

$$\int \frac{\sqrt{x}(A + Bx)}{a + bx + cx^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/c + 1/4*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c
^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B
*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + ...
```

Mupad [B] (verification not implemented)

Time = 11.92 (sec) , antiderivative size = 6401, normalized size of antiderivative = 28.96

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((x^(1/2)*(A + B*x))/(a + b*x + c*x^2),x)
```

output
```
(2*B*x^(1/2))/c - atan(((((8*(4*B*a^2*c^3 - B*a*b^2*c^2))/c - (8*x^(1/2)*(
b^3*c^3 - 4*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 -
 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2
*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(2*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B
*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c
 - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) +
 12*A*B*a*b^2*c^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*x^
(1/2)*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c -
 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 1
6*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)
^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*
b^2*c^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((8*(4*B*a^
2*c^3 - B*a*b^2*c^2))/c + (8*x^(1/2)*(b^3*c^3 - 4*a*b*c^4)*(-(B^2*b^5 + A^
2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^
(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2
*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c -...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 741, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{x}(A+Bx)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x+a),x)
```

output

```
( - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2 + 2*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqr
t(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c + 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(
a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt
(c)*sqrt(a) + b))*a*b*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*
b**3 + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2 - 2*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*
sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c - 6*sqrt(c)*sqrt(2*sqrt(c)*sq
rt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*s
qrt(c)*sqrt(a) + b))*a*b*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b
))*b**3 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt
(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*a*c**2 - sqrt(a)*sqrt(2*sqrt(c)*sq
rt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*
x)*b**2*c - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(x)*sqrt(2*sqrt(
c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*a*c**2 + sqrt(a)*sqrt(2*sqrt(c)*sqr
t(a) - b)*log(sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x...
```

3.87 $\int \frac{A+Bx}{\sqrt{x}(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 180

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)}\,dx = \frac{\sqrt{2}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
2^(1/2)*(B-(-2*A*c+B*b)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2))/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+2^(1/2
)*(B+(-2*A*c+B*b)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2))/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)}\,dx = \frac{\sqrt{2}\left(\frac{\left(-bB+2Ac+B\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(bB-2Ac+B\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a + b*x + c*x^2)),x]
```

output
```
(Sqrt[2]*(((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((b*B
 - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[c]*Sqrt[b^2 - 4*a
*c])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{1197}$$

$$2\int \frac{A+Bx}{cx^2+bx+a}\,d\sqrt{x}$$

$$\Big\downarrow \text{1480}$$

$$2\left(\frac{1}{2}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\int \frac{1}{\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}\right)$$

$$\Big\downarrow 218$$

$$2\left(\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a + b*x + c*x^2)),x]
```

output
```
2*(((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x
])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.93

method	result
derivativedivides	$8c\left(-\frac{\left(2Ac+B\sqrt{-4ac+b^2}-Bb\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{x}\,\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(Bb-2Ac+B\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\right.}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+\right.}}}\right.$
default	$8c\left(-\frac{\left(2Ac+B\sqrt{-4ac+b^2}-Bb\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{x}\,\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(Bb-2Ac+B\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\right.}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+\right.}}}\right.$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
8*c*(-1/8*(2*A*c+B*(-4*a*c+b^2)^(1/2)-B*b)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))+1/8*(B*b-2*A*c+B*(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1577 vs. 2(140) = 280.

Time = 0.41 (sec) , antiderivative size = 1577, normalized size of antiderivative = 8.76

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*s
qrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(sqrt(2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A
^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((
B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*
a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)) - 4*(B
^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*sqrt(x)) - 1/2*sqrt(2)*sqrt(-(B^
2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(
-sqrt(2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*
a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*
a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2
*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^
2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)) - 4*(B^4*a^2 - A*B^3*a*b +
 A^3*B*b*c - A^4*c^2)*sqrt(x)) + 1/2*sqrt(2)*sqrt(-(B^2*a*b - (4*A*B*a - A
^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(
a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(sqrt(2)*(A*B^2*a*b^2
 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*a^2*b)*c^2 -
(2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4488 vs. $2(165) = 330$.

Time = 9.14 (sec) , antiderivative size = 4488, normalized size of antiderivative = 24.93

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(1/2)/(c*x**2+b*x+a),x)
```

output
```
Piecewise((A*log(sqrt(x) - sqrt(-a/b))/(b*sqrt(-a/b)) - A*log(sqrt(x) + sq
rt(-a/b))/(b*sqrt(-a/b)) - B*a*log(sqrt(x) - sqrt(-a/b))/(b**2*sqrt(-a/b))
 + B*a*log(sqrt(x) + sqrt(-a/b))/(b**2*sqrt(-a/b)) + 2*B*sqrt(x)/b, Eq(c,
0)), (-A*log(sqrt(x) - sqrt(-b/c))/(b*sqrt(-b/c)) + A*log(sqrt(x) + sqrt(-
b/c))/(b*sqrt(-b/c)) - 2*A/(b*sqrt(x)) + B*log(sqrt(x) - sqrt(-b/c))/(c*sq
rt(-b/c)) - B*log(sqrt(x) + sqrt(-b/c))/(c*sqrt(-b/c)), Eq(a, 0)), (2*sqrt
(2)*A*b*c*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/2)/(2*b**2*c*sqrt(-b/c) + 4*b*c
**2*x*sqrt(-b/c)) - 2*sqrt(2)*A*b*c*log(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(2
*b**2*c*sqrt(-b/c) + 4*b*c**2*x*sqrt(-b/c)) + 8*A*c**2*sqrt(x)*sqrt(-b/c)/
(2*b**2*c*sqrt(-b/c) + 4*b*c**2*x*sqrt(-b/c)) + 4*sqrt(2)*A*c**2*x*log(sqr
t(x) - sqrt(2)*sqrt(-b/c)/2)/(2*b**2*c*sqrt(-b/c) + 4*b*c**2*x*sqrt(-b/c))
 - 4*sqrt(2)*A*c**2*x*log(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(2*b**2*c*sqrt(-
b/c) + 4*b*c**2*x*sqrt(-b/c)) + sqrt(2)*B*b**2*log(sqrt(x) - sqrt(2)*sqrt(
-b/c)/2)/(2*b**2*c*sqrt(-b/c) + 4*b*c**2*x*sqrt(-b/c)) - sqrt(2)*B*b**2*lo
g(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(2*b**2*c*sqrt(-b/c) + 4*b*c**2*x*sqrt(-
b/c)) - 4*B*b*c*sqrt(x)*sqrt(-b/c)/(2*b**2*c*sqrt(-b/c) + 4*b*c**2*x*sqrt(
-b/c)) + 2*sqrt(2)*B*b*c*x*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/2)/(2*b**2*c*s
qrt(-b/c) + 4*b*c**2*x*sqrt(-b/c)) - 2*sqrt(2)*B*b*c*x*log(sqrt(x) + sqrt(
2)*sqrt(-b/c)/2)/(2*b**2*c*sqrt(-b/c) + 4*b*c**2*x*sqrt(-b/c)), Eq(a, b**2
/(4*c))), (-sqrt(2)*A*b*c*sqrt(-b/c - sqrt(-4*a*c + b**2)/c)*log(sqrt(x...
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)}\,dx=\int \frac{Bx+A}{(cx^2+bx+a)\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
2*A*sqrt(x)/a - integrate((A*c*x^(3/2) - (B*a - A*b)*sqrt(x))/(a*c*x^2 + a
*b*x + a^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(140) = 280$.

Time = 0.66 (sec) , antiderivative size = 1404, normalized size of antiderivative = 7.80

$$\int \frac{A + Bx}{\sqrt{x}\,(a + bx + cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^
2 - 4*a*c)*b*c^2)*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arctan(2*sqrt(1/2)*sqrt(x
)/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*
a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/2*((sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 1
6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c ...
```

Mupad [B] (verification not implemented)

Time = 11.91 (sec) , antiderivative size = 4141, normalized size of antiderivative = 23.01

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(1/2)*(a + b*x + c*x^2)),x)
```

output
```
- atan((((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c
*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c
 - 4*A*B*a*b^2*c)/(2*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x^(1/
2)*(8*b^3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2)
+ A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*
c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(2*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^
4*c)))^(1/2) - 8*A*b^2*c^2 + 32*A*a*c^3) + x^(1/2)*(16*A^2*c^3 - 16*B^2*a*
c^2 + 8*B^2*b^2*c - 16*A*B*b*c^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)
^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A
^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(2*(16*a^3*c^3 - 8*a^2*b^2*c^2
 + a*b^4*c)))^(1/2)*1i + ((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) +
A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^
2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(2*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*
c)))^(1/2)*(x^(1/2)*(8*b^3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c
 - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2
*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(2*(16*a^3*c^3 - 8*a
^2*b^2*c^2 + a*b^4*c)))^(1/2) + 8*A*b^2*c^2 - 32*A*a*c^3) + x^(1/2)*(16*A^
2*c^3 - 16*B^2*a*c^2 + 8*B^2*b^2*c - 16*A*B*b*c^2))*(-(B^2*a*b^3 + B^2*a*(
-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*
A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(2*(16*a^3...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.96

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)}\,dx = \frac{-2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{x}\,\sqrt{c}}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)bc-4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{x}\,\sqrt{c}}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)ac+2\sqrt{\cdots}}{\cdots}$$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x+a),x)
```

output
```
( - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c - 4*sqrt(c)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c
))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sq
rt(a) + b))*b**2 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c + 4
*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2
*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c - 2*sqrt(c)*sqrt(2*sqrt
(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sq
rt(2*sqrt(c)*sqrt(a) + b))*b**2 + sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(
 - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*b*c - sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sq
rt(a) + sqrt(c)*x)*b*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt
(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*a*c + sqrt(c)*sqrt(
2*sqrt(c)*sqrt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a
) + sqrt(c)*x)*b**2 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(x)*sq
rt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*a*c - sqrt(c)*sqrt(2*sqrt
(c)*sqrt(a) - b)*log(sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(
c)*x)*b**2)/(2*c*(4*a*c - b**2))
```

3.88 $\int \frac{A+Bx}{x^{3/2}(a+bx+cx^2)} dx$

Optimal result

Integrand size = 23, antiderivative size = 199

$$\int \frac{A+Bx}{x^{3/2}(a+bx+cx^2)}\,dx = -\frac{2A}{a\sqrt{x}} - \frac{\sqrt{2}\sqrt{c}\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-2*A/a/x^(1/2)-2^(1/2)*c^(1/2)*(A+(A*b-2*B*a)/(-4*a*c+b^2)^(1/2))*arctan(2
^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))/a/(b-(-4*a*c+b^2)^(1/
2))^(1/2)-2^(1/2)*c^(1/2)*(A-(A*b-2*B*a)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2
)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))^(
1/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx}{x^{3/2}(a+bx+cx^2)}\,dx = -\frac{\frac{2A}{\sqrt{x}}+\frac{\sqrt{2}\sqrt{c}\left(-2aB+A\left(b+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2aB+A\left(-b+\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{a}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a + b*x + c*x^2)),x]
```

output
```
-(((2*A)/Sqrt[x] + (Sqrt[2]*Sqrt[c]*(-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*A
rcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a*B + A*(-b + Sq
rt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1198, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{1198}$$

$$\frac{\int -\frac{Ab-aB+Acx}{\sqrt{x}(cx^2+bx+a)}dx}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{Ab-aB+Acx}{\sqrt{x}(cx^2+bx+a)}dx}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{1197}$$

$$-\frac{2\int \frac{Ab-aB+Acx}{cx^2+bx+a}d\sqrt{x}}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{2\left(\frac{1}{2}c\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}c\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2\left(\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a + b*x + c*x^2)),x]
```

output
```
(-2*A)/(a*Sqrt[x]) - (2*((Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{2A}{a\sqrt{x}}-\frac{8c\left(-\frac{\left(A\sqrt{-4ac+b^2}+Ab-2Ba\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{x}\,\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(A\sqrt{-4ac+b^2}-Ab+2Ba\right)\sqrt{2}\,\arctan\left(\frac{}{\sqrt{\left(b+\right.}}\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right.}}\right.}{a}$
derivativedivides	$-\frac{2A}{a\sqrt{x}}+\frac{8c\left(\frac{\left(-A\sqrt{-4ac+b^2}+Ab-2Ba\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{x}\,c\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2Ba\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{}{\sqrt{\left(-b\right.}}\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}\right.}{a}$
default	$-\frac{2A}{a\sqrt{x}}+\frac{8c\left(\frac{\left(-A\sqrt{-4ac+b^2}+Ab-2Ba\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{x}\,c\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2Ba\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{}{\sqrt{\left(-b\right.}}\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}\right.}{a}$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*A/a/x^(1/2)-8/a*c*(-1/8*(A*(-4*a*c+b^2)^(1/2)+A*b-2*B*a)/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(A*(-4*a*c+b^2)^(1/2)-A*b+2*B*a)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2925 vs. $2(159) = 318$.

Time = 0.88 (sec) , antiderivative size = 2925, normalized size of antiderivative = 14.70

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3
*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^
2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3
*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(sq
rt(2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*
a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*
A^3*a*b^3)*c - (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a
^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c
)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*
a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6
*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3
- 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))
 + 4*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3
*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*sqrt(x)) - sqrt(2)*a*x*sqrt(-(B^2*a^2*b - 2
*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqr
t((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 +
 A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4
*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-sqrt(2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3
 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305978 vs. $2(180) = 360$.

Time = 59.49 (sec) , antiderivative size = 305978, normalized size of antiderivative = 1537.58

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Piecewise((-A*log(sqrt(x) - sqrt(-a/b))/(a*sqrt(-a/b)) + A*log(sqrt(x) + s
qrt(-a/b))/(a*sqrt(-a/b)) - 2*A/(a*sqrt(x)) + B*log(sqrt(x) - sqrt(-a/b))/
(b*sqrt(-a/b)) - B*log(sqrt(x) + sqrt(-a/b))/(b*sqrt(-a/b)), Eq(c, 0)), (-
2*A/(3*b*x**(3/2)) + A*c*log(sqrt(x) - sqrt(-b/c))/(b**2*sqrt(-b/c)) - A*c
*log(sqrt(x) + sqrt(-b/c))/(b**2*sqrt(-b/c)) + 2*A*c/(b**2*sqrt(x)) - B*lo
g(sqrt(x) - sqrt(-b/c))/(b*sqrt(-b/c)) + B*log(sqrt(x) + sqrt(-b/c))/(b*sq
rt(-b/c)) - 2*B/(b*sqrt(x)), Eq(a, 0)), (-6*sqrt(2)*A*b*c*sqrt(x)*log(sqrt
(x) - sqrt(2)*sqrt(-b/c)/2)/(b**3*sqrt(x)*sqrt(-b/c) + 2*b**2*c*x**(3/2)*s
qrt(-b/c)) + 6*sqrt(2)*A*b*c*sqrt(x)*log(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(
b**3*sqrt(x)*sqrt(-b/c) + 2*b**2*c*x**(3/2)*sqrt(-b/c)) - 8*A*b*c*sqrt(-b/
c)/(b**3*sqrt(x)*sqrt(-b/c) + 2*b**2*c*x**(3/2)*sqrt(-b/c)) - 12*sqrt(2)*A
*c**2*x**(3/2)*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/2)/(b**3*sqrt(x)*sqrt(-b/c
) + 2*b**2*c*x**(3/2)*sqrt(-b/c)) + 12*sqrt(2)*A*c**2*x**(3/2)*log(sqrt(x)
 + sqrt(2)*sqrt(-b/c)/2)/(b**3*sqrt(x)*sqrt(-b/c) + 2*b**2*c*x**(3/2)*sqrt
(-b/c)) - 24*A*c**2*x*sqrt(-b/c)/(b**3*sqrt(x)*sqrt(-b/c) + 2*b**2*c*x**(3
/2)*sqrt(-b/c)) + sqrt(2)*B*b**2*sqrt(x)*log(sqrt(x) - sqrt(2)*sqrt(-b/c)/
2)/(b**3*sqrt(x)*sqrt(-b/c) + 2*b**2*c*x**(3/2)*sqrt(-b/c)) - sqrt(2)*B*b*
*2*sqrt(x)*log(sqrt(x) + sqrt(2)*sqrt(-b/c)/2)/(b**3*sqrt(x)*sqrt(-b/c) +
2*b**2*c*x**(3/2)*sqrt(-b/c)) + 2*sqrt(2)*B*b*c*x**(3/2)*log(sqrt(x) - sqr
t(2)*sqrt(-b/c)/2)/(b**3*sqrt(x)*sqrt(-b/c) + 2*b**2*c*x**(3/2)*sqrt(-b...
```

Maxima [F]

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)}\,dx=\int \frac{Bx+A}{(cx^2+bx+a)x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
-2*(A*a/sqrt(x) - (B*a - A*b)*sqrt(x))/a^2 + integrate(-((B*a*c - A*b*c)*x
^(3/2) + (B*a*b - (b^2 - a*c)*A)*sqrt(x))/(a^2*c*x^2 + a^2*b*x + a^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2809 vs. $2(159) = 318$.

Time = 0.71 (sec) , antiderivative size = 2809, normalized size of antiderivative = 14.12

$$\int \frac{A+Bx}{x^{3/2}(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-2*A/(a*sqrt(x)) - 1/4*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)
*A*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a
*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*A*abs(a) - 2*(sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c +
16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 + 16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)...
```

Mupad [B] (verification not implemented)

Time = 12.63 (sec) , antiderivative size = 6367, normalized size of antiderivative = 31.99

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(3/2)*(a + b*x + c*x^2)),x)
```

output
```
- atan((((-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2
*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3
*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 -
 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(2*(a^3*b^4 + 16*a
^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(x^(1/2)*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A
^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c
 - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*
(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-
(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^
4*b^2*c)))^(1/2) - 32*B*a^6*c^3 + 32*A*a^5*b*c^3 - 8*A*a^4*b^3*c^2 + 8*B*a
^5*b^2*c^2) + x^(1/2)*(16*A^2*a^4*c^4 - 16*B^2*a^5*c^3 - 8*A^2*a^3*b^2*c^3
 + 16*A*B*a^4*b*c^3))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2
- 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^
2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(2*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i + ((-(A^2*b^5 + B^2*a^2*b^3
+ A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*
A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1
/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2
) + 12*A*B*a^2*b^2*c)/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.89

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)}\,dx = \frac{4\sqrt{x}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\;atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{x}\,\sqrt{c}}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)c-2\sqrt{x}\,\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\;ata$$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x+a),x)
```

output

```
(4*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*c - 2*sqrt(x)*sqr
t(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqr
t(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b - 4*sqrt(x)*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/
sqrt(2*sqrt(c)*sqrt(a) + b))*c + 2*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)
*sqrt(a) + b))*b - 2*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sq
rt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*c + 2*sqrt(x)*sqr
t(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) +
 sqrt(a) + sqrt(c)*x)*c - sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(
 - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*b + sqrt(x)*
sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b
) + sqrt(a) + sqrt(c)*x)*b - 16*a*c + 4*b**2)/(2*sqrt(x)*(4*a*c - b**2))
```

3.89 $\int \frac{A+Bx}{x^{5/2}(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 284

$$\int \frac{A+Bx}{x^{5/2}(a+bx+cx^2)}\,dx = -\frac{2A}{3ax^{3/2}}+\frac{2(Ab-aB)}{a^2\sqrt{x}} - \frac{\sqrt{2}\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-2/3*A/a/x^(3/2)+2*(A*b-B*a)/a^2/x^(1/2)-2^(1/2)*c^(1/2)*(a*B*(b+(-4*a*c+b
^2)^(1/2))-A*(b^2-2*a*c+b*(-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)*x^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)+2^(1/2)*c^(1/2)*(a*B*(b-(-4*a*c+b^2)^(1/2))-A*(b^2-2*a*c-b*(-4
*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1
/2))/a^2/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)}\,dx=\frac{\frac{6Abx-2a(A+3Bx)}{x^{3/2}}-\frac{3\sqrt{2}\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+}{3a^2}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a + b*x + c*x^2)),x]
```

output
```
((6*A*b*x - 2*a*(A + 3*B*x))/x^(3/2) - (3*Sqrt[2]*Sqrt[c]*(a*B*(b + Sqrt[b
^2 - 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b
^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
3*a^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \ 1198$$

$$\frac{\int -\frac{Ab-aB+Acx}{x^{3/2}(cx^2+bx+a)}dx}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{Ab-aB+Acx}{x^{3/2}(cx^2+bx+a)}dx}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \ 1198$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}(cx^2+bx+a)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A(b^2-ac)-(Ab-aB)cx}{\sqrt{x}(cx^2+bx+a)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{\sqrt{x}(cx^2+bx+a)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A(b^2-ac)-(Ab-aB)cx}{\sqrt{x}(cx^2+bx+a)}dx}{a}-\frac{2(Ab-aB)}{a\sqrt{x}}}{a}-\frac{2A}{3ax^{3/2}}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a + b*x + c*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{2(-3Abx+3Bax+Aa)}{3a^2x^{\frac{3}{2}}}+\frac{8c\left(-\frac{\left(Ab\sqrt{-4ac+b^2}-2Aac+b^2A-aB\sqrt{-4ac+b^2}-abB\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{x}\,\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\right.}{a^2}$
derivativedivides	$\frac{8c\left(-\frac{\left(Ab\sqrt{-4ac+b^2}-2Aac+b^2A-aB\sqrt{-4ac+b^2}-abB\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{x}\,\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(Ab\sqrt{-4ac+b^2}+2Aac-b^2A-a\right.}{8\sqrt{-}}\right.}{a^2}$
default	$\frac{8c\left(-\frac{\left(Ab\sqrt{-4ac+b^2}-2Aac+b^2A-aB\sqrt{-4ac+b^2}-abB\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{x}\,\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(Ab\sqrt{-4ac+b^2}+2Aac-b^2A-a\right.}{8\sqrt{-}}\right.}{a^2}$

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*A*b*x+3*B*a*x+A*a)/a^2/x^(3/2)+8/a^2*c*(-1/8*(A*b*(-4*a*c+b^2)^(1
/2)-2*A*a*c+b^2*A-a*B*(-4*a*c+b^2)^(1/2)-a*b*B)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))+1/8*(A*b*(-4*a*c+b^2)^(1/2)+2*A*a*c-b^2*A-a*B*(-4*a*
c+b^2)^(1/2)+a*b*B)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5453 vs. $2(230) = 460$.

Time = 1.42 (sec) , antiderivative size = 5453, normalized size of antiderivative = 19.20

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/x**(5/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)}\,dx=\int \frac{Bx+A}{(cx^2+bx+a)x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
-2/3*(A*a^2/x^(3/2) + 3*(B*a*b - (b^2 - a*c)*A)*sqrt(x) + 3*(B*a^2 - A*a*b
)/sqrt(x))/a^3 + integrate(((B*a*b*c - (b^2*c - a*c^2)*A)*x^(3/2) - ((b^3
- 2*a*b*c)*A - (a*b^2 - a^2*c)*B)*sqrt(x))/(a^3*c*x^2 + a^3*b*x + a^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2870 vs. $2(230) = 460$.

Time = 0.83 (sec) , antiderivative size = 2870, normalized size of antiderivative = 10.11

$$\int \frac{A+Bx}{x^{5/2}(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
5*c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 1
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a
^2*b^2*c^3 + 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
c^4 + 32*a^3*c^4 - 24*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^5 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^4*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 +
 2*(b^2 - 4*a*c)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*
c^2 + 8*(b^2 - 4*a*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^
5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*...
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 10133, normalized size of antiderivative = 35.68

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(5/2)*(a + b*x + c*x^2)),x)
```

output
```
atan(((x^(1/2)*(16*A^2*a^8*c^5 - 16*B^2*a^9*c^4 + 8*A^2*a^6*b^4*c^3 - 32*A
^2*a^7*b^2*c^4 + 8*B^2*a^8*b^2*c^3 - 16*A*B*a^7*b^3*c^3 + 48*A*B*a^8*b*c^4
) + (-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*
b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*
b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3
*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)
^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*
a*c - b^2)^3)^(1/2))/(2*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(32*A
*a^10*c^4 - x^(1/2)*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 + B^2*a^2*
b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2
+ A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c
+ 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c
^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^
3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(2*(
a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 32*B*a^10*b*c^3 + 8*A*a^8*b^
4*c^2 - 40*A*a^9*b^2*c^3 - 8*B*a^9*b^3*c^2))*(-(A^2*b^7 + B^2*a^2*b^5 + A^
2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^
2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) +...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)}\,dx = \frac{-6\sqrt{x}\,\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{x}\,\sqrt{c}}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)bcx+12\sqrt{x}\,\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+}$$

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x+a),x)
```

output
```
( - 6*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqr
t(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*x + 12*sqr
t(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*x + 6*sqrt(x)*sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(
x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*x - 12*sqrt(x)*sqrt(c)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c
))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*x + 3*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)
*x)*b*c*x - 3*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(x)*sqrt
(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*b*c*x + 6*sqrt(x)*sqrt(c)*s
qrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sq
rt(a) + sqrt(c)*x)*a*c*x - 6*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*l
og(sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*a*c*x - 16*a
**2*c + 4*a*b**2)/(6*sqrt(x)*a*x*(4*a*c - b**2))
```

3.90 $\int \frac{A+Bx}{x^{7/2}(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 307

$$\int \frac{A+Bx}{x^{7/2}\left(a+bx+cx^2\right)}\,dx = -\frac{2A}{5ax^{5/2}}+\frac{2(Ab-aB)}{3a^2x^{3/2}}-\frac{2(Ab^2-abB-aAc)}{a^3\sqrt{x}} \\ -\frac{\sqrt{2}\sqrt{c}\left(Ab^2-abB-aAc-\frac{aB(b^2-2ac)-A(b^3-3abc)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}} \\ -\frac{\sqrt{2}\sqrt{c}\left(Ab^2-abB-aAc+\frac{aB(b^2-2ac)-A(b^3-3abc)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^3\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-2/5*A/a/x^(5/2)+2/3*(A*b-B*a)/a^2/x^(3/2)-2*(-A*a*c+A*b^2-B*a*b)/a^3/x^(1
/2)-2^(1/2)*c^(1/2)*(A*b^2-a*b*B-A*a*c-(a*B*(-2*a*c+b^2)-A*(-3*a*b*c+b^3))
/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2))/a^3/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)*c^(1/2)*(A*b^2-a*b*B-A*a*
c+(a*B*(-2*a*c+b^2)-A*(-3*a*b*c+b^3))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c
^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))/a^3/(b+(-4*a*c+b^2)^(1/2))^(1
/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx}{x^{7/2}(a+bx+cx^2)}\,dx = \frac{\frac{-30Ab^2x^2-2a^2(3A+5Bx)+10ax(3bBx+A(b+3cx))}{x^{5/2}}+\frac{15\sqrt{2}\sqrt{c}\left(aB\left(b^2-2ac+b\sqrt{b^2-4ac}\right)-A\left(b^3-3abc-\right.\right.}{\sqrt{b^2-4ac}}}{}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(a + b*x + c*x^2)),x]
```

output
```
((-30*A*b^2*x^2 - 2*a^2*(3*A + 5*B*x) + 10*a*x*(3*b*B*x + A*(b + 3*c*x)))/
x^(5/2) + (15*Sqrt[2]*Sqrt[c]*(a*B*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]) - A
*(b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (15*Sqrt[2]*Sqrt[c]*(a*B*(-b^2 + 2*a*c + b*
Sqrt[b^2 - 4*a*c]) + A*(b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[b
^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(15*a^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \; 1198$$

$$\frac{\int -\frac{Ab-aB+Acx}{x^{5/2}(cx^2+bx+a)}dx}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{Ab-aB+Acx}{x^{5/2}(cx^2+bx+a)}dx}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 1198$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow 25$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{3ax^{3/2}}}{a}-\frac{2A}{5ax^{5/2}}$$

input
```
Int[(A + B*x)/(x^(7/2)*(a + b*x + c*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{2(-Ab+Ba)}{3a^{2}x^{\frac{3}{2}}}-\frac{2(-Aac+b^{2}A-abB)}{a^{3}\sqrt{x}}+\frac{8c\left(\frac{\left(Aac\sqrt{-4ac+b^{2}}-b^{2}A\sqrt{-4ac+b^{2}}-3Aabc+A\,b^{3}+abB\sqrt{-4ac+\right.}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{\right.}}\right.}{}$
default	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{2(-Ab+Ba)}{3a^{2}x^{\frac{3}{2}}}-\frac{2(-Aac+b^{2}A-abB)}{a^{3}\sqrt{x}}+\frac{8c\left(\frac{\left(Aac\sqrt{-4ac+b^{2}}-b^{2}A\sqrt{-4ac+b^{2}}-3Aabc+A\,b^{3}+abB\sqrt{-4ac+\right.}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{\right.}}\right.}{}$
risch	$-\frac{2(-15Aac\,x^{2}+15x^{2}b^{2}A-15Ba\,x^{2}b-5abAx+5a^{2}Bx+3a^{2}A)}{15a^{3}x^{\frac{5}{2}}}+\frac{8c\left(\frac{\left(Aac\sqrt{-4ac+b^{2}}-b^{2}A\sqrt{-4ac+b^{2}}-3Aabc+A\,b^{3}+\right.}{8\sqrt{-4ac}}\right.}{}$

input
```
int((B*x+A)/x^(7/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a/x^(5/2)-2/3*(-A*b+B*a)/a^2/x^(3/2)-2*(-A*a*c+A*b^2-B*a*b)/a^3/x^(
1/2)+8/a^3*c*(1/8*(A*a*c*(-4*a*c+b^2)^(1/2)-b^2*A*(-4*a*c+b^2)^(1/2)-3*A*a
*b*c+A*b^3+a*b*B*(-4*a*c+b^2)^(1/2)+2*B*a^2*c-B*a*b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))-1/8*(A*a*c*(-4*a*c+b^2)^(1/2)-b^2*A*(-4*a*c+b^2)
^(1/2)+3*A*a*b*c-A*b^3+a*b*B*(-4*a*c+b^2)^(1/2)-2*B*a^2*c+B*a*b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7971 vs. $2(252) = 504$.

Time = 4.32 (sec) , antiderivative size = 7971, normalized size of antiderivative = 25.96

$$\int \frac{A+Bx}{x^{7/2}\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(7/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{x^{7/2}\left(a+bx+cx^2\right)}\,dx = \int \frac{Bx+A}{(cx^2+bx+a)x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
-2/15*(3*A*a^3/x^(5/2) + 15*((b^3 - 2*a*b*c)*A - (a*b^2 - a^2*c)*B)*sqrt(x
) - 15*(B*a^2*b - (a*b^2 - a^2*c)*A)/sqrt(x) + 5*(B*a^3 - A*a^2*b)/x^(3/2)
)/a^4 - integrate(-(((b^3*c - 2*a*b*c^2)*A - (a*b^2*c - a^2*c^2)*B)*x^(3/2
) + ((b^4 - 3*a*b^2*c + a^2*c^2)*A - (a*b^3 - 2*a^2*b*c)*B)*sqrt(x))/(a^4*
c*x^2 + a^4*b*x + a^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5013 vs. $2(252) = 504$.

Time = 1.16 (sec) , antiderivative size = 5013, normalized size of antiderivative = 16.33

$$\int \frac{A + Bx}{x^{7/2} (a + bx + cx^2)} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*((2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^
2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*A*a^2 - (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 3
2*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2
- 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*B*a^2 + 2*(sqrt(2)*sqrt...
```

Mupad [B] (verification not implemented)

Time = 14.52 (sec) , antiderivative size = 13983, normalized size of antiderivative = 45.55

$$\int \frac{A+Bx}{x^{7/2}\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(7/2)*(a + b*x + c*x^2)),x)
```

output
```
atan(((x^(1/2)*(16*A^2*a^12*c^6 - 16*B^2*a^13*c^5 - 8*A^2*a^9*b^6*c^3 + 48
*A^2*a^10*b^4*c^4 - 72*A^2*a^11*b^2*c^5 - 8*B^2*a^11*b^4*c^3 + 32*B^2*a^12
*b^2*c^4 + 16*A*B*a^10*b^5*c^3 - 80*A*B*a^11*b^3*c^4 + 80*A*B*a^12*b*c^5)
+ (-(A^2*b^9 + B^2*a^2*b^7 + A^2*b^6*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^
8 + 42*A^2*a^2*b^5*c^2 - 63*A^2*a^3*b^3*c^3 - A^2*a^3*c^3*(-(4*a*c - b^2)^
3)^(1/2) + B^2*a^2*b^4*(-(4*a*c - b^2)^3)^(1/2) + 25*B^2*a^4*b^3*c^2 + B^2
*a^4*c^2*(-(4*a*c - b^2)^3)^(1/2) - 16*A*B*a^5*c^4 - 11*A^2*a*b^7*c + 28*A
^2*a^4*b*c^4 - 9*B^2*a^3*b^5*c - 20*B^2*a^5*b*c^3 + 6*A^2*a^2*b^2*c^2*(-(4
*a*c - b^2)^3)^(1/2) - 66*A*B*a^3*b^4*c^2 + 76*A*B*a^4*b^2*c^3 - 5*A^2*a*b
^4*c*(-(4*a*c - b^2)^3)^(1/2) - 3*B^2*a^3*b^2*c*(-(4*a*c - b^2)^3)^(1/2) -
 2*A*B*a*b^5*(-(4*a*c - b^2)^3)^(1/2) + 20*A*B*a^2*b^6*c + 8*A*B*a^2*b^3*c
*(-(4*a*c - b^2)^3)^(1/2) - 6*A*B*a^3*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(
a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x^(1/2)*(32*a^16*b*c^3 - 8*a^
15*b^3*c^2)*(-(A^2*b^9 + B^2*a^2*b^7 + A^2*b^6*(-(4*a*c - b^2)^3)^(1/2) -
2*A*B*a*b^8 + 42*A^2*a^2*b^5*c^2 - 63*A^2*a^3*b^3*c^3 - A^2*a^3*c^3*(-(4*a
*c - b^2)^3)^(1/2) + B^2*a^2*b^4*(-(4*a*c - b^2)^3)^(1/2) + 25*B^2*a^4*b^3
*c^2 + B^2*a^4*c^2*(-(4*a*c - b^2)^3)^(1/2) - 16*A*B*a^5*c^4 - 11*A^2*a*b^
7*c + 28*A^2*a^4*b*c^4 - 9*B^2*a^3*b^5*c - 20*B^2*a^5*b*c^3 + 6*A^2*a^2*b^
2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 66*A*B*a^3*b^4*c^2 + 76*A*B*a^4*b^2*c^3 -
 5*A^2*a*b^4*c*(-(4*a*c - b^2)^3)^(1/2) - 3*B^2*a^3*b^2*c*(-(4*a*c - b^...
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx}{x^{7/2}\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/x^(7/2)/(c*x^2+b*x+a),x)
```

output

```
( - 20*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*x**2 +
 10*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*x**2 - 10
*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x**2 + 20*sqr
t(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*x**2 - 10*sqrt(x
)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*x**2 + 10*sqrt(x)*s
qrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*s
qrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x**2 + 10*sqrt(x)*sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b)
+ sqrt(a) + sqrt(c)*x)*a*c**2*x**2 - 5*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt
(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)
*b**2*c*x**2 - 10*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(x)*
sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt(c)*x)*a*c**2*x**2 + 5*sqrt(x)
*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) -
b) + sqrt(a) + sqrt(c)*x)*b**2*c*x**2 - 5*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*s
qrt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a) + sqrt...
```

3.91 $\int \frac{A+Bx}{x^{9/2}(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 381

$$\int \frac{A+Bx}{x^{9/2}\left(a+bx+cx^2\right)}\,dx = -\frac{2A}{7ax^{7/2}}+\frac{2(Ab-aB)}{5a^2x^{5/2}} -\frac{2(Ab^2-abB-aAc)}{3a^3x^{3/2}}-\frac{2(aB(b^2-ac)-A(b^3-2abc))}{a^4\sqrt{x}} -\frac{\sqrt{2}\sqrt{c}\left(aB(b^2-ac)-A(b^3-2abc)+\frac{abB(b^2-3ac)-A(b^4-4ab^2c+2a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^4\sqrt{b-\sqrt{b^2-4ac}}} -\frac{\sqrt{2}\sqrt{c}\left(aB(b^2-ac)-A(b^3-2abc)-\frac{abB(b^2-3ac)-A(b^4-4ab^2c+2a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^4\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-2/7*A/a/x^(7/2)+2/5*(A*b-B*a)/a^2/x^(5/2)-2/3*(-A*a*c+A*b^2-B*a*b)/a^3/x^
(3/2)-2*(a*B*(-a*c+b^2)-A*(-2*a*b*c+b^3))/a^4/x^(1/2)-2^(1/2)*c^(1/2)*(a*B
*(-a*c+b^2)-A*(-2*a*b*c+b^3)+(a*b*B*(-3*a*c+b^2)-A*(2*a^2*c^2-4*a*b^2*c+b^
4))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/
2))^(1/2))/a^4/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)*c^(1/2)*(a*B*(-a*c+b^2
)-A*(-2*a*b*c+b^3)-(a*b*B*(-3*a*c+b^2)-A*(2*a^2*c^2-4*a*b^2*c+b^4))/(-4*a*
c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))
/a^4/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{x^{9/2}\left(a+bx+cx^2\right)}\,dx = \frac{\frac{210Ab^3x^3-6a^3(5A+7Bx)+14a^2x(5Bx(b+3cx)+A(3b+5cx))-70abx^2(3bBx+A(b+6cx))}{x^{7/2}}+\frac{105\sqrt{2}\sqrt{c}\left(a\right.}{}}{}$$

input
```
Integrate[(A + B*x)/(x^(9/2)*(a + b*x + c*x^2)),x]
```

output
```
((210*A*b^3*x^3 - 6*a^3*(5*A + 7*B*x) + 14*a^2*x*(5*B*x*(b + 3*c*x) + A*(3
*b + 5*c*x)) - 70*a*b*x^2*(3*b*B*x + A*(b + 6*c*x)))/x^(7/2) + (105*Sqrt[2
]*Sqrt[c]*(a*B*(-b^3 + 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[b^2 - 4*
a*c]) + A*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 2*a*b*c*S
qrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (105*Sqrt[2]*Sqr
t[c]*(a*B*(b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[b^2 - 4*a*c])
+ A*(-b^4 + 4*a*b^2*c - 2*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 2*a*b*c*Sqrt[b
^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(105*a^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{9/2}\left(a+bx+cx^2\right)}\,dx$$

$$\Big\downarrow \; 1198$$

$$\frac{\int -\frac{Ab-aB+Acx}{x^{7/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{Ab-aB+Acx}{x^{7/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \; 1198$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int -\frac{abB-A\left(b^2-ac\right)-(Ab-aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}(cx^2+bx+a)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A(b^2-ac)-(Ab-aB)cx}{x^{5/2}(cx^2+bx+a)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{Ab^2-aBb-aAc+(Ab-aB)cx}{x^{5/2}(cx^2+bx+a)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int -\frac{abB-A(b^2-ac)-(Ab-aB)cx}{x^{5/2}(cx^2+bx+a)}dx}{a}-\frac{2(Ab-aB)}{5ax^{5/2}}}{a}-\frac{2A}{7ax^{7/2}}$$

input
```
Int[(A + B*x)/(x^(9/2)*(a + b*x + c*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{8c\left(-\frac{\left(-2Aabc\sqrt{-4ac+b^2}+A\,b^3\sqrt{-4ac+b^2}+2a^2A\,c^2-4Aa\,b^2c+A\,b^4+B\,a^2c\sqrt{-4ac+b^2}-Ba\,b^2\sqrt{-4ac+b^2}+3a^2bBc-Ba\,b^3\right)\sqrt{2}}{8\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{}$
default	$\frac{8c\left(-\frac{\left(-2Aabc\sqrt{-4ac+b^2}+A\,b^3\sqrt{-4ac+b^2}+2a^2A\,c^2-4Aa\,b^2c+A\,b^4+B\,a^2c\sqrt{-4ac+b^2}-Ba\,b^2\sqrt{-4ac+b^2}+3a^2bBc-Ba\,b^3\right)\sqrt{2}}{8\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{}$
risch	$-\frac{2(210Aabc\,x^3-105A\,b^3x^3-105B\,a^2c\,x^3+105Ba\,b^2x^3-35A\,a^2c\,x^2+35Aa\,b^2x^2-35B\,a^2b\,x^2-21A\,a^2bx+21B\,a^3x+}{105a^4x^{\frac{7}{2}}}$

input

```
int((B*x+A)/x^(9/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output

```
8/a^4*c*(-1/8*(-2*A*a*b*c*(-4*a*c+b^2)^(1/2)+A*b^3*(-4*a*c+b^2)^(1/2)+2*a^
2*A*c^2-4*A*a*b^2*c+A*b^4+B*a^2*c*(-4*a*c+b^2)^(1/2)-B*a*b^2*(-4*a*c+b^2)^
(1/2)+3*a^2*b*B*c-B*a*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
+1/8*(-2*A*a*b*c*(-4*a*c+b^2)^(1/2)+A*b^3*(-4*a*c+b^2)^(1/2)-2*a^2*A*c^2+4
*A*a*b^2*c-A*b^4+B*a^2*c*(-4*a*c+b^2)^(1/2)-B*a*b^2*(-4*a*c+b^2)^(1/2)-3*a
^2*b*B*c+B*a*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-2/7*A/a/x^
(7/2)-2/5*(-A*b+B*a)/a^2/x^(5/2)-2/3*(-A*a*c+A*b^2-B*a*b)/a^3/x^(3/2)-2*(2
*A*a*b*c-A*b^3-B*a^2*c+B*a*b^2)/a^4/x^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10514 vs. $2(326) = 652$.

Time = 6.37 (sec) , antiderivative size = 10514, normalized size of antiderivative = 27.60

$$\int \frac{A+Bx}{x^{9/2}\left(a+bx+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(9/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{9/2}\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(9/2)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{x^{9/2}\left(a+bx+cx^2\right)}\,dx = \int \frac{Bx+A}{(cx^2+bx+a)x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(9/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
-2/105*(15*A*a^4/x^(7/2) - 105*((b^4 - 3*a*b^2*c + a^2*c^2)*A - (a*b^3 - 2
*a^2*b*c)*B)*sqrt(x) - 105*((a*b^3 - 2*a^2*b*c)*A - (a^2*b^2 - a^3*c)*B)/s
qrt(x) - 35*(B*a^3*b - (a^2*b^2 - a^3*c)*A)/x^(3/2) + 21*(B*a^4 - A*a^3*b)
/x^(5/2))/a^5 - integrate((((b^4*c - 3*a*b^2*c^2 + a^2*c^3)*A - (a*b^3*c -
 2*a^2*b*c^2)*B)*x^(3/2) + ((b^5 - 4*a*b^3*c + 3*a^2*b*c^2)*A - (a*b^4 - 3
*a^2*b^2*c + a^3*c^2)*B)*sqrt(x))/(a^5*c*x^2 + a^5*b*x + a^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4090 vs. $2(326) = 652$.

Time = 1.06 (sec) , antiderivative size = 4090, normalized size of antiderivative = 10.73

$$\int \frac{A+Bx}{x^{9/2}\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(9/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 11*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^7*c - 2*b^8*c + 41*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 +
14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 22*a*b^6*c^2 + 2*b^7*c^2 - 56*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 26*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^3 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
3 - 82*a^2*b^4*c^3 - 18*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 13*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 112*a^3*b^2*c^4 + 50*a
^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 32*a^4*c^
5 - 40*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^7 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 2
5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 -
10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 5*sqrt...
```

Mupad [B] (verification not implemented)

Time = 15.33 (sec) , antiderivative size = 17910, normalized size of antiderivative = 47.01

$$\int \frac{A + Bx}{x^{9/2} (a + bx + cx^2)} dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(9/2)*(a + b*x + c*x^2)),x)
```

output
```
((2*x^3*(A*b^3 - B*a*b^2 + B*a^2*c - 2*A*a*b*c))/a^4 - (2*A)/(7*a) + (2*x^
2*(A*a*c - A*b^2 + B*a*b))/(3*a^3) + (2*x*(A*b - B*a))/(5*a^2))/x^(7/2) +
atan((((-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^8*(-(4*a*c - b^2)^3)^(1/2) - 2*A*
B*a*b^10 + 63*A^2*a^2*b^7*c^2 - 138*A^2*a^3*b^5*c^3 + 129*A^2*a^4*b^3*c^4
+ A^2*a^4*c^4*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^6*(-(4*a*c - b^2)^3)^(1
/2) + 42*B^2*a^4*b^5*c^2 - 63*B^2*a^5*b^3*c^3 - B^2*a^5*c^3*(-(4*a*c - b^2
)^3)^(1/2) + 16*A*B*a^6*c^5 - 13*A^2*a*b^9*c - 36*A^2*a^5*b*c^5 - 11*B^2*a
^3*b^7*c + 28*B^2*a^6*b*c^4 + 15*A^2*a^2*b^4*c^2*(-(4*a*c - b^2)^3)^(1/2)
- 10*A^2*a^3*b^2*c^3*(-(4*a*c - b^2)^3)^(1/2) + 6*B^2*a^4*b^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) - 104*A*B*a^3*b^6*c^2 + 192*A*B*a^4*b^4*c^3 - 132*A*B*a^5
*b^2*c^4 - 7*A^2*a*b^6*c*(-(4*a*c - b^2)^3)^(1/2) - 5*B^2*a^3*b^4*c*(-(4*a
*c - b^2)^3)^(1/2) - 2*A*B*a*b^7*(-(4*a*c - b^2)^3)^(1/2) + 24*A*B*a^2*b^8
*c + 12*A*B*a^2*b^5*c*(-(4*a*c - b^2)^3)^(1/2) + 8*A*B*a^4*b*c^3*(-(4*a*c
- b^2)^3)^(1/2) - 20*A*B*a^3*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^9*b^4
 + 16*a^11*c^2 - 8*a^10*b^2*c)))^(1/2)*(32*A*a^19*c^5 + x^(1/2)*(32*a^21*b
*c^3 - 8*a^20*b^3*c^2)*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^8*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*a*b^10 + 63*A^2*a^2*b^7*c^2 - 138*A^2*a^3*b^5*c^3 + 129*
A^2*a^4*b^3*c^4 + A^2*a^4*c^4*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^6*(-(4*
a*c - b^2)^3)^(1/2) + 42*B^2*a^4*b^5*c^2 - 63*B^2*a^5*b^3*c^3 - B^2*a^5*c^
3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^6*c^5 - 13*A^2*a*b^9*c - 36*A^2*a...
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 896, normalized size of antiderivative = 2.35

$$\int \frac{A + Bx}{x^{9/2} (a + bx + cx^2)} dx = \text{Too large to display}$$

input
```
int((B*x+A)/x^(9/2)/(c*x^2+b*x+a),x)
```

output
```
(126*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2*x**3 -
 42*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c*x**3 - 84
*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**2*x**3 + 42
*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c*x**3 - 126
*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2*x**3 + 42*
sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c*x**3 + 84*sqr
t(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**2*x**3 - 42*sqr
t(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c*x**3 - 63*sqrt
(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt
(a) - b) + sqrt(a) + sqrt(c)*x)*a*b*c**2*x**3 + 21*sqrt(x)*sqrt(a)*sqrt(2*
sqrt(c)*sqrt(a) - b)*log( - sqrt(x)*sqrt(2*sqrt(c)*sqrt(a) - b) + sqrt(a)
+ sqrt(c)*x)*b**3*c*x**3 + 63*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - ...
```

3.92 $\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 403

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \frac{2B\sqrt{x}}{c^2} + \frac{\sqrt{x}(a(b^2B-Abc-2aBc)+(b^3B-Ab^2c-3abBc+2aAc^2)\,x)}{c^2\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2-\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^3B-Ab^2c-13abBc+6aAc^2+\frac{3b^4B-Ab^3c-19ab^2Bc+8aAbc^2+20a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
2*B*x^(1/2)/c^2+x^(1/2)*(a*(-A*b*c-2*B*a*c+B*b^2)+(2*A*a*c^2-A*b^2*c-3*B*a
*b*c+B*b^3)*x)/c^2/(-4*a*c+b^2)/(c*x^2+b*x+a)-1/2*(3*B*b^3-A*b^2*c-13*B*a*
b*c+6*A*a*c^2-(8*A*a*b*c^2-A*b^3*c+20*B*a^2*c^2-19*B*a*b^2*c+3*B*b^4)/(-4*
a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*2^(1/2)/c^(5/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*(3*B*b^3-
A*b^2*c-13*B*a*b*c+6*A*a*c^2+(8*A*a*b*c^2-A*b^3*c+20*B*a^2*c^2-19*B*a*b^2*
c+3*B*b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
)
```

Mathematica [A] (verified)

Time = 4.69 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.14

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \frac{-\frac{2\sqrt{c}\sqrt{x}(10a^2Bc+b^2x(-3bB+c(A-2Bx))+a(-3b^2B+2c^2x(-A+4Bx)+bc(A+11Bx)))}{(b^2-4ac)(a+x(b+cx))} - \frac{\sqrt{2}\left(-3b^4B+b^2c\right.}{}}{}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a + b*x + c*x^2)^2,x]
```

output
```
((-2*Sqrt[c]*Sqrt[x]*(10*a^2*B*c + b^2*x*(-3*b*B + c*(A - 2*B*x)) + a*(-3*
b^2*B + 2*c^2*x*(-A + 4*B*x) + b*c*(A + 11*B*x))))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) - (Sqrt[2]*(-3*b^4*B + b^2*c*(19*a*B - A*Sqrt[b^2 - 4*a*c]) + 2
*a*c^2*(-10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(A*c + 3*B*Sqrt[b^2 - 4*a*c
]) - a*b*c*(8*A*c + 13*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[
x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[2]*(3*b^4*B - b^2*c*(19*a*B + A*Sqrt[b^2 - 4*a*c]) + 2*a
*c^2*(10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + a*b*c*(8*A*c - 13*B*Sqrt[b^2 - 4*a
*c]) + b^3*(-(A*c) + 3*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[
x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]))/(2*c^(5/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1233, 27, 1196, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \; 1233$$

$$\frac{\int \frac{\sqrt{x}(3a(bB-2Ac)+(3Bb^2-Acb-10aBc)x)}{2(cx^2+bx+a)}dx}{c(b^2-4ac)} - \frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\sqrt{x}(3a(bB-2Ac)+(3Bb^2-Acb-10aBc)x)}{cx^2+bx+a}dx}{2c\left(b^2-4ac\right)}-\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow 1196$$

$$\frac{\frac{\int -\frac{a\left(3Bb^2-Acb-10aBc\right)+\left(3Bb^3-Acb^2-13aBcb+6aAc^2\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{c}+\frac{2\sqrt{x}(-10aBc-Abc+3b^2B)}{c}}{2c\left(b^2-4ac\right)}-\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow 25$$

$$\frac{\frac{2\sqrt{x}(-10aBc-Abc+3b^2B)}{c}-\frac{\int \frac{a\left(3Bb^2-Acb-10aBc\right)+\left(3Bb^3-Acb^2-13aBcb+6aAc^2\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{c}}{2c\left(b^2-4ac\right)}-\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow 1197$$

$$\frac{\frac{2\sqrt{x}(-10aBc-Abc+3b^2B)}{c}-\frac{2\int \frac{a\left(3Bb^2-Acb-10aBc\right)+\left(3Bb^3-Acb^2-13aBcb+6aAc^2\right)x}{cx^2+bx+a}d\sqrt{x}}{c}}{2c\left(b^2-4ac\right)}-\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow 1480$$

$$\frac{\frac{2\sqrt{x}(-10aBc-Abc+3b^2B)}{c}-\frac{2\left(\frac{1}{2}\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\right.}{c}}{2c\left(b^2-4ac\right)}$$

$$\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow 218$$

$$\frac{\frac{2\sqrt{x}(-10aBc-Abc+3b^2B)}{c} - \frac{2\left(\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(20a^2Bc^2+8\ldots\right.}{\ldots}\right)}{c}}{2c\left(b^2-4ac\right)}$$

$$\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a + b*x + c*x^2)^2,x]
```

output
```
-((x^(3/2)*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(c*(b^2 - 4*a*
c)*(a + b*x + c*x^2))) + ((2*(3*b^2*B - A*b*c - 10*a*B*c)*Sqrt[x])/c - (2*
(((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 - (3*b^4*B - A*b^3*c - 19*a*
b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 + (3*b^4*
B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/c)/(2*c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{2\left(-\frac{\left(2Aa\,c^2-A\,b^2c-3Babc+B\,b^3\right)x^{\frac{3}{2}}}{2\left(4ac-b^2\right)}+\frac{a\left(Abc+2aBc-B\,b^2\right)\sqrt{x}}{8ac-2b^2}\right)}{c\,x^2+bx+a}+\frac{4c\left(-\frac{\left(6Aa\,c^2\sqrt{-4ac+b^2}-A\,b^2c\sqrt{-4ac+b^2}-8Aab\,c^2+A\,b^3 \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{2\left(-\frac{\left(2Aa\,c^2-A\,b^2c-3Babc+B\,b^3\right)x^{\frac{3}{2}}}{2\left(4ac-b^2\right)}+\frac{a\left(Abc+2aBc-B\,b^2\right)\sqrt{x}}{8ac-2b^2}\right)}{c\,x^2+bx+a}+\frac{4c\left(-\frac{\left(6Aa\,c^2\sqrt{-4ac+b^2}-A\,b^2c\sqrt{-4ac+b^2}-8Aab\,c^2+A\,b^3 \cdots\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{2B\sqrt{x}}{c^2}+\frac{-\frac{\left(2Aa\,c^2-A\,b^2c-3Babc+B\,b^3\right)x^{\frac{3}{2}}}{4ac-b^2}+\frac{2a\left(Abc+2aBc-B\,b^2\right)\sqrt{x}}{8ac-2b^2}}{c\,x^2+bx+a}+\frac{4c\left(-\frac{\left(6Aa\,c^2\sqrt{-4ac+b^2}-A\,b^2c\sqrt{-4ac+b^2}-8Aab \cdots\right.}{\cdots}\right.}{\cdots}$

input

```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2/c^2*((-1/2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^(3/2)+1/2*a
*(A*b*c+2*B*a*c-B*b^2)/(4*a*c-b^2)*x^(1/2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*c*
(-1/8*(6*A*a*c^2*(-4*a*c+b^2)^(1/2)-A*b^2*c*(-4*a*c+b^2)^(1/2)-8*A*a*b*c^2
+A*b^3*c-13*B*a*b*c*(-4*a*c+b^2)^(1/2)+3*B*b^3*(-4*a*c+b^2)^(1/2)-20*B*a^2
*c^2+19*B*a*b^2*c-3*B*b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))+1/8*(6*A*a*c^2*(-4*a*c+b^2)^(1/2)-A*b^2*c*(-4*a*c+b^2)^(1/2)+8*A*a*b*c^
2-A*b^3*c-13*B*a*b*c*(-4*a*c+b^2)^(1/2)+3*B*b^3*(-4*a*c+b^2)^(1/2)+20*B*a^
2*c^2-19*B*a*b^2*c+3*B*b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
))+2*B*x^(1/2)/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7251 vs. $2(359) = 718$.

Time = 15.67 (sec) , antiderivative size = 7251, normalized size of antiderivative = 17.99

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x+A)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \int \frac{(Bx+A)x^{\frac{5}{2}}}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
((A*b*c - (b^2 - 2*a*c)*B)*x^(5/2) - (B*a*b - 2*A*a*c)*x^(3/2))/(a*b^2*c -
 4*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4*a*b*c^2)*x) + integrate(
-1/2*((A*b*c - (3*b^2 - 10*a*c)*B)*x^(3/2) - 3*(B*a*b - 2*A*a*c)*sqrt(x))/
(a*b^2*c - 4*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4*a*b*c^2)*x), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5685 vs. $2(359) = 718$.

Time = 1.16 (sec) , antiderivative size = 5685, normalized size of antiderivative = 14.11

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/c^2 + (B*b^3*x^(3/2) - 3*B*a*b*c*x^(3/2) - A*b^2*c*x^(3/2) + 2
*A*a*c^2*x^(3/2) + B*a*b^2*sqrt(x) - 2*B*a^2*c*sqrt(x) - A*a*b*c*sqrt(x))/
((b^2*c^2 - 4*a*c^3)*(c*x^2 + b*x + a)) + 1/8*((2*b^4*c^3 - 20*a*b^2*c^4 +
 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 -
 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b
^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4*a*c^3)^2*A - (6*b^5*c^2 - 50
*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)
^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)...
```

Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 16631, normalized size of antiderivative = 41.27

$$\int \frac{x^{5/2}(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(5/2)*(A + B*x))/(a + b*x + c*x^2)^2,x)
```

output
```
(2*B*x^(1/2))/c^2 - atan(((((2560*B*a^5*c^7 - 4*A*a*b^7*c^4 + 256*A*a^4*b*
c^7 + 12*B*a*b^8*c^3 + 48*A*a^2*b^5*c^5 - 192*A*a^3*b^3*c^6 - 184*B*a^2*b^
6*c^4 + 1056*B*a^3*b^4*c^5 - 2688*B*a^4*b^2*c^6)/(64*a^3*c^6 - b^6*c^3 + 1
2*a*b^4*c^4 - 48*a^2*b^2*c^5) - (2*x^(1/2)*((9*B^2*b^4*(-(4*a*c - b^2)^9)^
(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1
504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*
B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 1
5360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^
7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*
a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5
*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 -
6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(
1/2))/(8*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128
0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(4*b^7*c^5 -
 48*a*b^5*c^6 - 256*a^3*b*c^8 + 192*a^2*b^3*c^7))/(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4))*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*
b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^
2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a
^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/...
```

Reduce [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 3675, normalized size of antiderivative = 9.12

$$\int \frac{x^{5/2}(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^2,x)
```

output

```
( - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**3 + 34*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x
)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**2 - 24*sqrt(a)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c
))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**3*x - 24*sqrt(a)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2
*sqrt(c)*sqrt(a) + b))*a**2*c**4*x**2 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*
sqrt(a) + b))*a*b**4*c + 34*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*
a*b**3*c**2*x + 34*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c*
*3*x**2 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**5*c*x - 6*sqr
t(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqr
t(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*c**2*x**2 + 56*sqrt(c)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqr
t(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**2 - 40*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sq...
```

3.93 $\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 321

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx &= -\frac{\sqrt{x}(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{c\,(b^2-4ac)\,(a+bx+cx^2)} \\ &+ \frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\ &+ \frac{\left(b^2B+Abc-6aBc+\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-x^(1/2)*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b
*x+a)+1/2*(B*b^2+A*b*c-6*B*a*c-(4*A*a*c^2+A*b^2*c-8*B*a*b*c+B*b^3)/(-4*a*c
+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*
2^(1/2)/c^(3/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*(B*b^2+A*b*c
-6*B*a*c+(4*A*a*c^2+A*b^2*c-8*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))*arctan(2^
(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(-4*a*
c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 3.33 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.14

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \frac{\frac{2\sqrt{c}\sqrt{x}(-abB+b(-bB+Ac)x+2ac(A+Bx))}{(b^2-4ac)(a+x(b+cx))} + \frac{\sqrt{2}\left(-b^3B+bc\left(8aB+A\sqrt{b^2-4ac}\right)+b^2\left(-Ac+B\sqrt{b^2-4ac}\right)-2a\cdots\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b\cdots}}}}{\cdots}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a + b*x + c*x^2)^2,x]
```

output
```
((2*Sqrt[c]*Sqrt[x]*(-(a*b*B) + b*(-(b*B) + A*c)*x + 2*a*c*(A + B*x)))/((b
^2 - 4*a*c)*(a + x*(b + c*x))) + (Sqrt[2]*(-(b^3*B) + b*c*(8*a*B + A*Sqrt[
b^2 - 4*a*c]) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*a*c]) - 2*a*c*(2*A*c + 3*B*Sq
rt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*B
 + 2*a*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c])
 + b*(-8*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]))/(2*c^(3/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1233, 27, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{\int \frac{a(bB-2Ac)+(Bb^2+Acb-6aBc)x}{2\sqrt{x}(cx^2+bx+a)}dx}{c(b^2-4ac)} - \frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(bB-2Ac)+(Bb^2+Acb-6aBc)x}{\sqrt{x}(cx^2+bx+a)}dx}{2c\left(b^2-4ac\right)}-\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$\big\downarrow$ 1197

$$\frac{\int \frac{a(bB-2Ac)+(Bb^2+Acb-6aBc)x}{cx^2+bx+a}d\sqrt{x}}{c\left(b^2-4ac\right)}-\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$\big\downarrow$ 1480

$$\frac{\frac{1}{2}\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+\right.}{c\left(b^2-4ac\right)}$$
$$\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$\big\downarrow$ 218

$$\frac{\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c\left(b^2-4ac\right)}$$
$$\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a + b*x + c*x^2)^2,x]
```

output
```
-((Sqrt[x]*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(c*(b^2 - 4*a*
c)*(a + b*x + c*x^2))) + (((b^2*B + A*b*c - 6*a*B*c - (b^3*B + A*b^2*c - 8
*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-\frac{\left(Abc+2aBc-B\,b^{2}\right)x^{\frac{3}{2}}}{\left(4ac-b^{2}\right)c}-\frac{a(2Ac-Bb)\sqrt{x}}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{-\frac{\left(-Abc\sqrt{-4ac+b^{2}}+4Aa\,c^{2}+A\,b^{2}c+6aBc\sqrt{-4ac+b^{2}}-B\,b^{2}\sqrt{-4ac+b^{2}}-8Babc+\right.}{2c\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}}{}$
default	$\frac{-\frac{\left(Abc+2aBc-B\,b^{2}\right)x^{\frac{3}{2}}}{\left(4ac-b^{2}\right)c}-\frac{a(2Ac-Bb)\sqrt{x}}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{-\frac{\left(-Abc\sqrt{-4ac+b^{2}}+4Aa\,c^{2}+A\,b^{2}c+6aBc\sqrt{-4ac+b^{2}}-B\,b^{2}\sqrt{-4ac+b^{2}}-8Babc+\right.}{2c\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}}{}$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/2*(A*b*c+2*B*a*c-B*b^2)/(4*a*c-b^2)/c*x^(3/2)-1/2*a*(2*A*c-B*b)/c/(4
*a*c-b^2)*x^(1/2))/(c*x^2+b*x+a)+4/(4*a*c-b^2)*(-1/8*(-A*b*c*(-4*a*c+b^2)^
(1/2)+4*A*a*c^2+A*b^2*c+6*a*B*c*(-4*a*c+b^2)^(1/2)-B*b^2*(-4*a*c+b^2)^(1/2
)-8*B*a*b*c+B*b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(
-A*b*c*(-4*a*c+b^2)^(1/2)-4*A*a*c^2-A*b^2*c+6*a*B*c*(-4*a*c+b^2)^(1/2)-B*b
^2*(-4*a*c+b^2)^(1/2)+8*B*a*b*c-B*b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4653 vs. 2(281) = 562.

Time = 4.33 (sec) , antiderivative size = 4653, normalized size of antiderivative = 14.50

$$\int \frac{x^{3/2}(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x+A)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \int \frac{(Bx+A)x^{\frac{3}{2}}}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
((B*b - 2*A*c)*x^(5/2) + (2*B*a - A*b)*x^(3/2))/(a*b^2 - 4*a^2*c + (b^2*c
- 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x) - integrate(1/2*((B*b - 2*A*c)*x^(3/2)
 + 3*(2*B*a - A*b)*sqrt(x))/(a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^
3 - 4*a*b*c)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4544 vs. $2(281) = 562$.

Time = 1.12 (sec) , antiderivative size = 4544, normalized size of antiderivative = 14.16

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(B*b^2*x^(3/2) - 2*B*a*c*x^(3/2) - A*b*c*x^(3/2) + B*a*b*sqrt(x) - 2*A*a*
c*sqrt(x))/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a)) + 1/8*((2*b^3*c^3 - 8*a*b
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b
*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*
c)*a*c^3)*(b^2*c - 4*a*c^2)^2*B - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5
+ 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - ...
```

Mupad [B] (verification not implemented)

Time = 16.46 (sec) , antiderivative size = 12408, normalized size of antiderivative = 38.65

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(A + B*x))/(a + b*x + c*x^2)^2,x)
```

output
```
- ((x^(1/2)*(2*A*a*c - B*a*b))/(c*(4*a*c - b^2)) + (x^(3/2)*(A*b*c - B*b^2
 + 2*B*a*c))/(c*(4*a*c - b^2)))/(a + b*x + c*x^2) - atan(((((512*A*a^4*c^6
 - 8*A*a*b^6*c^3 + 4*B*a*b^7*c^2 - 256*B*a^4*b*c^5 + 96*A*a^2*b^4*c^4 - 38
4*A*a^3*b^2*c^5 - 48*B*a^2*b^5*c^3 + 192*B*a^3*b^3*c^4)/(b^6*c - 64*a^3*c^
4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3) - (2*x^(1/2)*(-(B^2*b^11 + A^2*b^9*c^2
+ A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*
A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^
2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^
2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*
B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b
^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(8*(4096*a
^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3
840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(4*b^7*c^3 - 48*a*b^5*c^4 - 25
6*a^3*b*c^6 + 192*a^2*b^3*c^5))/(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))*(-(B^2
*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c
- b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5
+ 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072
*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A
^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*
c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*...
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 3001, normalized size of antiderivative = 9.35

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^2,x)
```

output
```
( - 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2 + 2*sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(
x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c - 16*sqrt(a)*sqrt(2*sqrt
(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sq
rt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2*x - 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(
a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt
(c)*sqrt(a) + b))*a*b*c**3*x**2 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a
) + b))*b**4*c*x + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c*
*2*x**2 - 8*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**2 + 14*s
qrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c - 8*sqrt(c)*sqrt(
2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(
c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*x - 8*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2
*sqrt(c)*sqrt(a) + b))*a**2*c**3*x**2 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(...
```

3.94 $\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx = -\frac{\sqrt{x}(Ab-2aB-(bB-2Ac)x)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(bB-2Ac+\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-x^(1/2)*(A*b-2*B*a-(-2*A*c+B*b)*x)/(-4*a*c+b^2)/(c*x^2+b*x+a)+1/2*(B*b-2*
A*c-(-4*A*b*c+4*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2)+1/2*(B*b-2*A*c+(-4*A*b*c+4*B*a*c+B*b^2)/(-4*a*c+b^2)^(
1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)
/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{\sqrt{x}(-Ab+2aB+bBx-2Acx)}{(b^2-4ac)(a+bx+cx^2)}$$
$$+ \frac{\left(-b^2B+4Abc-4aBc+bB\sqrt{b^2-4ac}-2Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(b^2B-4Abc+4aBc+bB\sqrt{b^2-4ac}-2Ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a + b*x + c*x^2)^2,x]
```

output
```
(Sqrt[x]*(-(A*b) + 2*a*B + b*B*x - 2*A*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x
^2)) + ((-(b^2*B) + 4*A*b*c - 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((
b^2*B - 4*A*b*c + 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1234, 27, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx$$
$$\Big\downarrow \text{1234}$$

$$-\frac{\int -\frac{Ab-2aB+(bB-2Ac)x}{2\sqrt{x}(cx^2+bx+a)}dx}{b^2-4ac}-\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{(b^2-4ac)(a+bx+cx^2)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{Ab-2aB+(bB-2Ac)x}{\sqrt{x}(cx^2+bx+a)}dx}{2(b^2-4ac)}-\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{(b^2-4ac)(a+bx+cx^2)}$$

$\Big\downarrow$ 1197

$$\frac{\int \frac{Ab-2aB+(bB-2Ac)x}{cx^2+bx+a}d\sqrt{x}}{b^2-4ac}-\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{(b^2-4ac)(a+bx+cx^2)}$$

$\Big\downarrow$ 1480

$$\frac{\frac{1}{2}\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\int \frac{1}{\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}}{b^2-4ac}-$$
$$\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{(b^2-4ac)(a+bx+cx^2)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{b^2-4ac}-$$
$$\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a + b*x + c*x^2)^2,x]
```

output
```
-((Sqrt[x]*(A*b - 2*a*B - (b*B - 2*A*c)*x))/((b^2 - 4*a*c)*(a + b*x + c*x^
2))) + (((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[
c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - 2*A*c + (b^2*B - 4*A*b*c + 4*a*B
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\frac{2(2Ac-Bb)x^{\frac{3}{2}}}{8ac-2b^2}+\frac{2(Ab-2Ba)\sqrt{x}}{8ac-2b^2}}{c\,x^2+bx+a}+\frac{4c\left(\frac{\left(2Ac\sqrt{-4ac+b^2}+4Abc-Bb\sqrt{-4ac+b^2}-4aBc-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{x}\,c\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{4}$
default	$\frac{\frac{2(2Ac-Bb)x^{\frac{3}{2}}}{8ac-2b^2}+\frac{2(Ab-2Ba)\sqrt{x}}{8ac-2b^2}}{c\,x^2+bx+a}+\frac{4c\left(\frac{\left(2Ac\sqrt{-4ac+b^2}+4Abc-Bb\sqrt{-4ac+b^2}-4aBc-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{x}\,c\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{4}$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(1/2*(2*A*c-B*b)/(4*a*c-b^2)*x^(3/2)+1/2*(A*b-2*B*a)/(4*a*c-b^2)*x^(1/2)
)/(c*x^2+b*x+a)+4/(4*a*c-b^2)*c*(1/8*(2*A*c*(-4*a*c+b^2)^(1/2)+4*A*b*c-B*b
*(-4*a*c+b^2)^(1/2)-4*a*B*c-B*b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))-1/8*(2*A*c*(-4*a*c+b^2)^(1/2)-4*A*b*c-B*b*(-4*a*c+b^2)^(1/2)+4*a*B
*c+B*b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3462 vs. 2(235) = 470.

Time = 2.14 (sec) , antiderivative size = 3462, normalized size of antiderivative = 12.54

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)*(B*x+A)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \int \frac{(Bx+A)\sqrt{x}}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
-((2*B*a*c - A*b*c)*x^(5/2) + (B*a*b - (b^2 - 2*a*c)*A)*x^(3/2))/(a^2*b^2
- 4*a^3*c + (a*b^2*c - 4*a^2*c^2)*x^2 + (a*b^3 - 4*a^2*b*c)*x) + integrate
(1/2*((2*B*a*c - A*b*c)*x^(3/2) + (3*B*a*b - (b^2 + 2*a*c)*A)*sqrt(x))/(a^
2*b^2 - 4*a^3*c + (a*b^2*c - 4*a^2*c^2)*x^2 + (a*b^3 - 4*a^2*b*c)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3781 vs. $2(235) = 470$.

Time = 0.98 (sec) , antiderivative size = 3781, normalized size of antiderivative = 13.70

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
(B*b*x^(3/2) - 2*A*c*x^(3/2) + 2*B*a*sqrt(x) - A*b*sqrt(x))/((c*x^2 + b*x
+ a)*(b^2 - 4*a*c)) + 1/8*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2
- 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*(b^2 - 4*a*c)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 -
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2
- 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*
c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 ...
```

Mupad [B] (verification not implemented)

Time = 15.45 (sec) , antiderivative size = 9434, normalized size of antiderivative = 34.18

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(1/2)*(A + B*x))/(a + b*x + c*x^2)^2,x)
```

output
```
((x^(1/2)*(A*b - 2*B*a))/(4*a*c - b^2) + (x^(3/2)*(2*A*c - B*b))/(4*a*c -
b^2))/(a + b*x + c*x^2) - atan(((((4*A*b^7*c^2 + 512*B*a^4*c^5 - 48*A*a*b^
5*c^3 - 256*A*a^3*b*c^5 - 8*B*a*b^6*c^2 + 192*A*a^2*b^3*c^4 + 96*B*a^2*b^4
*c^3 - 384*B*a^3*b^2*c^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
 - (2*x^(1/2)*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c +
A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4
- 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^
4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 -
12*A*B*a*b^8*c)/(8*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 128
0*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(4
*b^7*c^2 - 48*a*b^5*c^3 - 256*a^3*b*c^5 + 192*a^2*b^3*c^4))/(b^4 + 16*a^2*
c^2 - 8*a*b^2*c))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*
c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*
c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^
2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^
3 - 12*A*B*a*b^8*c)/(8*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 -
 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2
) - (2*x^(1/2)*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 -
 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(b^4 + 16*a^2*c^2 - 8*a
*b^2*c))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^...
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 1813, normalized size of antiderivative = 6.57

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^2,x)
```

output
```
( - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**2 + 6*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*
sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c - 8*sqrt(a)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2
*sqrt(c)*sqrt(a) + b))*a*b*c**2*x - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*
atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt
(a) + b))*a*c**3*x**2 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b*
*3*c*x + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c**2*x**2 -
2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3 - 2*sqrt(c)*sqrt(2*
sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c)
)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*x - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*
sqrt(a) + b))*b**3*c*x**2 + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a**2*c**2 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) + 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c ...
```

3.95 $\int \frac{A+Bx}{\sqrt{x}\left(a+bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 292

$$\int \frac{A+Bx}{\sqrt{x}\left(a+bx+cx^2\right)^2}\,dx = \frac{\sqrt{x}(Ab^2-abB-2aAc+(Ab-2aB)cx)}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\sqrt{c}\left(Ab-2aB+\frac{4abB+A\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(Ab-2aB-\frac{Ab^2+4abB-12aAc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
x^(1/2)*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)
+1/2*c^(1/2)*(A*b-2*B*a+(4*a*b*B+A*(-12*a*c+b^2))/(-4*a*c+b^2)^(1/2))*arct
an(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c
+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*c^(1/2)*(A*b-2*B*a-(-12*A*a*c+A*b^2
+4*B*a*b)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)^2}\,dx$$

$$= \frac{\frac{2\sqrt{x}(aB(b+2cx)-A(b^2-2ac+bcx))}{a+x(b+cx)} - \frac{\sqrt{2}\sqrt{c}\left(-2aB\left(-2b+\sqrt{b^2-4ac}\right)+A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(2aB\right.}{}}{2a\left(-b^2+4ac\right)}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a + b*x + c*x^2)^2),x]
```

output
```
((2*Sqrt[x]*(a*B*(b + 2*c*x) - A*(b^2 - 2*a*c + b*c*x)))/(a + x*(b + c*x))
 - (Sqrt[2]*Sqrt[c]*(-2*a*B*(-2*b + Sqrt[b^2 - 4*a*c]) + A*(b^2 - 12*a*c +
 b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqr
t[c]*(2*a*B*(2*b + Sqrt[b^2 - 4*a*c]) + A*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a
*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1235, 27, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{\sqrt{x}(cx(Ab-2aB)-2aAc-abB+Ab^2)}{a(b^2-4ac)(a+bx+cx^2)} - \frac{\int -\frac{Ab^2+aBb-6aAc+(Ab-2aB)cx}{2\sqrt{x}(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{abB+A(b^2-6ac)+(Ab-2aB)cx}{\sqrt{x}(cx^2+bx+a)}dx}{2a(b^2-4ac)}+\frac{\sqrt{x}(cx(Ab-2aB)-2aAc-abB+Ab^2)}{a(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 1197$$

$$\frac{\int \frac{abB+A(b^2-6ac)+(Ab-2aB)cx}{cx^2+bx+a}d\sqrt{x}}{a(b^2-4ac)}+\frac{\sqrt{x}(cx(Ab-2aB)-2aAc-abB+Ab^2)}{a(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{1}{2}c\left(\frac{A(b^2-12ac)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}c\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}-2aB+Ab\right)\int \frac{1}{\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{a(b^2-4ac)}$$
$$\frac{\sqrt{x}(cx(Ab-2aB)-2aAc-abB+Ab^2)}{a(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}-2aB+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a(b^2-4ac)}+$$
$$\frac{\sqrt{x}(cx(Ab-2aB)-2aAc-abB+Ab^2)}{a(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a + b*x + c*x^2)^2),x]
```

output
```
(Sqrt[x]*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(a*(b^2 - 4*a*c)*(
a + b*x + c*x^2)) + ((Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A*(b^2 - 12*a*c))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*B - (A
*b^2 + 4*a*b*B - 12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt
[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.60

method	result
derivativedivides	$32c^2\left(-\frac{-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2Ba\right)\sqrt{-4ac+b^2}\,\sqrt{x}}{16ac\left(x+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(12Aac\sqrt{-4ac+b^2}-3b^2A\sqrt{-4ac+b^2}+28Aabc-3A\,b^3-8B\,a^2c-6Ba\,b\right.}{16a\left(4ac+3b^2\right)\sqrt{\left(-b+\right.}}}{4(4ac-b^2)c\sqrt{-4ac+b^2}}\right.$
default	$32c^2\left(-\frac{-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2Ba\right)\sqrt{-4ac+b^2}\,\sqrt{x}}{16ac\left(x+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(12Aac\sqrt{-4ac+b^2}-3b^2A\sqrt{-4ac+b^2}+28Aabc-3A\,b^3-8B\,a^2c-6Ba\,b\right.}{16a\left(4ac+3b^2\right)\sqrt{\left(-b+\right.}}}{4(4ac-b^2)c\sqrt{-4ac+b^2}}\right.$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
32*c^2*(-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*(-1/16*(-A*(-4*a*c+b^2)^(1/2
)-A*b+2*B*a)*(-4*a*c+b^2)^(1/2)/a/c*x^(1/2)/(x+1/2*b/c-1/2/c*(-4*a*c+b^2)^
(1/2))-1/16*(12*A*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*A*(-4*a*c+b^2)^(1/2)+28*A*a
*b*c-3*A*b^3-8*B*a^2*c-6*B*a*b^2)*(-2*b+(-4*a*c+b^2)^(1/2))/a/(4*a*c+3*b^2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*(-1/1
6*(A*(-4*a*c+b^2)^(1/2)-A*b+2*B*a)*(-4*a*c+b^2)^(1/2)/a/c*x^(1/2)/(x+1/2*b
/c+1/2/c*(-4*a*c+b^2)^(1/2))-1/16*(-28*A*a*b*c+3*A*b^3+12*A*a*c*(-4*a*c+b^
2)^(1/2)-3*b^2*A*(-4*a*c+b^2)^(1/2)+8*B*a^2*c+6*B*a*b^2)*(2*b+(-4*a*c+b^2)
^(1/2))/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^
(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4884 vs. $2(256) = 512$.

Time = 6.12 (sec) , antiderivative size = 4884, normalized size of antiderivative = 16.73

$$\int \frac{A + Bx}{\sqrt{x}\left(a + bx + cx^2\right)^2}\, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx}{\sqrt{x}\left(a + bx + cx^2\right)^2}\, dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(1/2)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + Bx}{\sqrt{x}\left(a + bx + cx^2\right)^2}\, dx = \int \frac{Bx + A}{\left(cx^2 + bx + a\right)^2\sqrt{x}}\, dx$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
((B*a*b*c + (b^2*c - 6*a*c^2)*A)*x^(5/2) + 2*(a*b^2 - 4*a^2*c)*A*sqrt(x) +
 ((b^3 - 5*a*b*c)*A + (a*b^2 - 2*a^2*c)*B)*x^(3/2))/(a^3*b^2 - 4*a^4*c + (
a^2*b^2*c - 4*a^3*c^2)*x^2 + (a^2*b^3 - 4*a^3*b*c)*x) - integrate(1/2*((B*
a*b*c + (b^2*c - 6*a*c^2)*A)*x^(3/2) + ((b^3 - 7*a*b*c)*A + (a*b^2 + 2*a^2
*c)*B)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 + (a^2*b^
3 - 4*a^3*b*c)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4434 vs. $2(256) = 512$.

Time = 1.12 (sec) , antiderivative size = 4434, normalized size of antiderivative = 15.18

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(2*B*a*c*x^(3/2) - A*b*c*x^(3/2) + B*a*b*sqrt(x) - A*b^2*sqrt(x) + 2*A*a*
c*sqrt(x))/((a*b^2 - 4*a^2*c)*(c*x^2 + b*x + a)) + 1/8*((2*b^3*c^2 - 8*a*b
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*
(a*b^2 - 4*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B + 2*
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2
 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 12
8*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a
^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b...
```

Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 12364, normalized size of antiderivative = 42.34

$$\int \frac{A+Bx}{\sqrt{x}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(1/2)*(a + b*x + c*x^2)^2),x)
```

output
```
atan(((((1536*A*a^5*c^6 + 4*A*a*b^8*c^2 - 256*B*a^5*b*c^5 - 72*A*a^2*b^6*c
^3 + 480*A*a^3*b^4*c^4 - 1408*A*a^4*b^2*c^5 + 4*B*a^2*b^7*c^2 - 48*B*a^3*b
^5*c^3 + 192*B*a^4*b^3*c^4)/(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2) - (2*x^(1/2)*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^
9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b
^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2
+ 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4
*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^
3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*
c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(8*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4
*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8
*b^2*c^5)))^(1/2)*(256*a^5*b*c^5 - 4*a^2*b^7*c^2 + 48*a^3*b^5*c^3 - 192*a^
4*b^3*c^4))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))*(-(A^2*b^11 + B^2*a^2*b^
9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) +
2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b
^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*
A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 76
8*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5
*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(8*(a^3*
b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^...
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 2372, normalized size of antiderivative = 8.12

$$\int \frac{A+Bx}{\sqrt{x}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^2,x)
```

output
```
(8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c - 4*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt
(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3 + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(
c)*sqrt(a) + b))*a*b**2*c*x + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) +
b))*a*b*c**2*x**2 - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*x
 - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c*x**2 - 24*sqrt(c
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x
)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c + 10*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**2*b**2 - 24*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqr
t(a) + b))*a**2*b*c*x - 24*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a
**2*c**2*x**2 + 10*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3...
```

3.96 $\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 406

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx=-\frac{3Ab^2-abB-10aAc}{a^2\left(b^2-4ac\right)\sqrt{x}}$$
$$+\frac{Ab^2-abB-2aAc+(Ab-2aB)cx}{a\left(b^2-4ac\right)\sqrt{x}\left(a+bx+cx^2\right)}$$
$$+\frac{\sqrt{c}\left(aB\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{c}\left(aB\left(b^2-12ac-b\sqrt{b^2-4ac}\right)-A\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-(-10*A*a*c+3*A*b^2-B*a*b)/a^2/(-4*a*c+b^2)/x^(1/2)+(A*b^2-a*b*B-2*A*a*c+(
A*b-2*B*a)*c*x)/a/(-4*a*c+b^2)/x^(1/2)/(c*x^2+b*x+a)+1/2*c^(1/2)*(a*B*(b^2
-12*a*c+b*(-4*a*c+b^2)^(1/2))-A*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)^(1/2)-1
0*a*c*(-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-
1/2*c^(1/2)*(a*B*(b^2-12*a*c-b*(-4*a*c+b^2)^(1/2))-A*(3*b^3-16*a*b*c-3*b^2
*(-4*a*c+b^2)^(1/2)+10*a*c*(-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)*x^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 3.84 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \frac{-\frac{2\left(-3Ab^2x(b+cx)+abBx(b+cx)+a^2(8Ac-2Bcx)+aA\left(-2b^2+11bcx+10c^2x^2\right)\right)}{\sqrt{x}(a+x(b+cx))} - \frac{\sqrt{2}\sqrt{c}\left(aB\left(b^2-12a\right.\right.}{}}{}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a + b*x + c*x^2)^2),x]
```

output
```
((-2*(-3*A*b^2*x*(b + c*x) + a*b*B*x*(b + c*x) + a^2*(8*A*c - 2*B*c*x) + a
*A*(-2*b^2 + 11*b*c*x + 10*c^2*x^2)))/(Sqrt[x]*(a + x*(b + c*x))) - (Sqrt[
2]*Sqrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(-3*b^3 + 16*a*b*
c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*S
qrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(b^2 - 12*a*c - b*Sqrt[b^2 - 4
*a*c]) + A*(-3*b^3 + 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2
- 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a^2*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1235, 27, 1198, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \; 1235$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{3Ab^2-aBb-10aAc+3(Ab-2aB)cx}{2x^{3/2}\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3Ab^2-aBb-10aAc+3(Ab-2aB)cx}{x^{3/2}(cx^2+bx+a)}dx}{2a\,(b^2-4ac)}+\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{a\sqrt{x}\,(b^2-4ac)\,(a+bx+cx^2)}$$

$\Big\downarrow$ 1198

$$\frac{\frac{\int \frac{aB(b^2-6ac)-A(3b^3-13abc)-c(3Ab^2-aBb-10aAc)x}{\sqrt{x}(cx^2+bx+a)}dx}{a}-\frac{2(-10aAc-abB+3Ab^2)}{a\sqrt{x}}}{2a\,(b^2-4ac)}+\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{a\sqrt{x}\,(b^2-4ac)\,(a+bx+cx^2)}$$

$\Big\downarrow$ 1197

$$\frac{\frac{2\int \frac{aB(b^2-6ac)-A(3b^3-13abc)-c(3Ab^2-aBb-10aAc)x}{cx^2+bx+a}d\sqrt{x}}{a}-\frac{2(-10aAc-abB+3Ab^2)}{a\sqrt{x}}}{2a\,(b^2-4ac)}+\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{a\sqrt{x}\,(b^2-4ac)\,(a+bx+cx^2)}$$

$\Big\downarrow$ 1480

$$\frac{\frac{2\left(\frac{c\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}}{2\sqrt{b^2-4ac}}-\frac{1}{2}c\left(\frac{aB(b^2-12ac)-A(3b^3-16abc)}{\sqrt{b^2-4ac}}-10aAc-\right.}{a}}{2a\,(b^2-4ac)}$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{a\sqrt{x}\,(b^2-4ac)\,(a+bx+cx^2)}$$

$\Big\downarrow$ 218

$$\frac{\frac{2\left(\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\frac{aB(b^2-12ac)-A(3b^3-16abc)}{\sqrt{b^2-4ac}}-10aAc-ab\right.}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}}}\right.}{a}}{2a\,(b^2-4ac)}$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{a\sqrt{x}\,(b^2-4ac)\,(a+bx+cx^2)}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a + b*x + c*x^2)^2),x]
```

output
```
(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x)/(a*(b^2 - 4*a*c)*Sqrt[x]*(a
+ b*x + c*x^2)) + ((-2*(3*A*b^2 - a*b*B - 10*a*A*c))/(a*Sqrt[x]) + (2*((Sq
rt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - A*(3*b^3 - 16*a*b*c + 3*
b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]
*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2 -
 12*a*c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*
c]])))/a)/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{2A}{a^{2}\sqrt{x}}-\frac{2\left(\frac{\frac{c\left(2Aac-b^{2}A+abB\right)x^{\frac{3}{2}}}{8ac-2b^{2}}+\frac{\left(3Aabc-A\,b^{3}-2B\,a^{2}c+Ba\,b^{2}\right)\sqrt{x}}{8ac-2b^{2}}}{c\,x^{2}+bx+a}+\frac{2c\left(\frac{\left(10Aac\sqrt{-4ac+b^{2}}-3b^{2}A\sqrt{-4ac+b^{2}}-16Ac\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2A}{a^{2}\sqrt{x}}-\frac{2\left(\frac{\frac{c\left(2Aac-b^{2}A+abB\right)x^{\frac{3}{2}}}{8ac-2b^{2}}+\frac{\left(3Aabc-A\,b^{3}-2B\,a^{2}c+Ba\,b^{2}\right)\sqrt{x}}{8ac-2b^{2}}}{c\,x^{2}+bx+a}+\frac{2c\left(\frac{\left(10Aac\sqrt{-4ac+b^{2}}-3b^{2}A\sqrt{-4ac+b^{2}}-16Ac\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
risch	$-\frac{2A}{a^{2}\sqrt{x}}-\frac{\frac{\frac{2c\left(2Aac-b^{2}A+abB\right)x^{\frac{3}{2}}}{8ac-2b^{2}}+\frac{2\left(3Aabc-A\,b^{3}-2B\,a^{2}c+Ba\,b^{2}\right)\sqrt{x}}{8ac-2b^{2}}}{c\,x^{2}+bx+a}+\frac{4c\left(\frac{\left(10Aac\sqrt{-4ac+b^{2}}-3b^{2}A\sqrt{-4ac+b^{2}}-16Aab\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2*A/x^(1/2)-2/a^2*((1/2*c*(2*A*a*c-A*b^2+B*a*b)/(4*a*c-b^2)*x^(3/2)+1
/2*(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2)/(4*a*c-b^2)*x^(1/2))/(c*x^2+b*x+a)+
2/(4*a*c-b^2)*c*(1/8*(10*A*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*A*(-4*a*c+b^2)^(1/
2)-16*A*a*b*c+3*A*b^3+a*b*B*(-4*a*c+b^2)^(1/2)+12*B*a^2*c-B*a*b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(10*A*a*c*(-4*a*c+b^2)^(1/2)-3*b
^2*A*(-4*a*c+b^2)^(1/2)+16*A*a*b*c-3*A*b^3+a*b*B*(-4*a*c+b^2)^(1/2)-12*B*a
^2*c+B*a*b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7597 vs. $2(351) = 702$.

Time = 23.34 (sec) , antiderivative size = 7597, normalized size of antiderivative = 18.71

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(3/2)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \int \frac{Bx+A}{\left(cx^2+bx+a\right)^2x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
-(((3*b^3*c - 13*a*b*c^2)*A - (a*b^2*c - 6*a^2*c^2)*B)*x^(5/2) + ((3*b^4 -
 10*a*b^2*c - 10*a^2*c^2)*A - (a*b^3 - 5*a^2*b*c)*B)*x^(3/2) + 2*(a^2*b^2
- 4*a^3*c)*A/sqrt(x) + 2*(3*(a*b^3 - 4*a^2*b*c)*A - (a^2*b^2 - 4*a^3*c)*B)
*sqrt(x))/(a^4*b^2 - 4*a^5*c + (a^3*b^2*c - 4*a^4*c^2)*x^2 + (a^3*b^3 - 4*
a^4*b*c)*x) + integrate(1/2*(((3*b^3*c - 13*a*b*c^2)*A - (a*b^2*c - 6*a^2*
c^2)*B)*x^(3/2) + ((3*b^4 - 16*a*b^2*c + 10*a^2*c^2)*A - (a*b^3 - 7*a^2*b*
c)*B)*sqrt(x))/(a^4*b^2 - 4*a^5*c + (a^3*b^2*c - 4*a^4*c^2)*x^2 + (a^3*b^3
 - 4*a^4*b*c)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5405 vs. $2(351) = 702$.

Time = 1.15 (sec) , antiderivative size = 5405, normalized size of antiderivative = 13.31

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
(B*a*b*c*x^2 - 3*A*b^2*c*x^2 + 10*A*a*c^2*x^2 + B*a*b^2*x - 3*A*b^3*x - 2*
B*a^2*c*x + 11*A*a*b*c*x - 2*A*a*b^2 + 8*A*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*
x^(5/2) + b*x^(3/2) + a*sqrt(x))) + 1/8*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^
2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 +
22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2
+ 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*A - (2*a*b^3*c^2 - 8*a^2*b
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)
*a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*B - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)...
```

Mupad [B] (verification not implemented)

Time = 16.10 (sec) , antiderivative size = 17623, normalized size of antiderivative = 43.41

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(3/2)*(a + b*x + c*x^2)^2),x)
```

output
```
- ((2*A)/a - (x*(3*A*b^3 - B*a*b^2 + 2*B*a^2*c - 11*A*a*b*c))/(a^2*(4*a*c
- b^2)) + (c*x^2*(10*A*a*c - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)))/(a*x^(
1/2) + b*x^(3/2) + c*x^(5/2)) - atan(((x^(1/2)*(25600*A^2*a^12*c^9 - 9216*
B^2*a^13*c^8 + 18*A^2*a^6*b^12*c^3 - 408*A^2*a^7*b^10*c^4 + 3764*A^2*a^8*b
^8*c^5 - 17920*A^2*a^9*b^6*c^6 + 45696*A^2*a^10*b^4*c^7 - 57344*A^2*a^11*b
^2*c^8 + 2*B^2*a^8*b^10*c^3 - 52*B^2*a^9*b^8*c^4 + 576*B^2*a^10*b^6*c^5 -
3200*B^2*a^11*b^4*c^6 + 8704*B^2*a^12*b^2*c^7 - 12*A*B*a^7*b^11*c^3 + 292*
A*B*a^8*b^9*c^4 - 2816*A*B*a^9*b^7*c^5 + 13440*A*B*a^10*b^5*c^6 - 31744*A*
B*a^11*b^3*c^7 + 29696*A*B*a^12*b*c^8) + (-(9*A^2*b^13 + B^2*a^2*b^11 + 9*
A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 1
0656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*
A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2
) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15
360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*
c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a
^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*
b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c -
 b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1
/2))/(8*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280
*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x^(1/2)*(...
```

Reduce [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 3153, normalized size of antiderivative = 7.77

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^2,x)
```

output

```
(40*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**2 - 22*s
qrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c + 40*sqrt
(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*x + 40*sqrt(
x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**3*x**2 + 4*sqrt(x
)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4 - 22*sqrt(x)*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x
)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*x - 22*sqrt(x)*sqrt(a)*sq
rt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sq
rt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2*x**2 + 4*sqrt(x)*sqrt(a)*s
qrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*s
qrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**5*x + 4*sqrt(x)*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/s
qrt(2*sqrt(c)*sqrt(a) + b))*b**4*c*x**2 + 8*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**3*b*c - 4*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*sqr...
```

3.97 $\int \frac{A+Bx}{x^{5/2}(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 521

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = -\frac{5Ab^2-3abB-14aAc}{3a^2\left(b^2-4ac\right)x^{3/2}}$$
$$-\frac{aB(3b^2-10ac)-A(5b^3-19abc)}{a^3\left(b^2-4ac\right)\sqrt{x}}+\frac{Ab^2-abB-2aAc+(Ab-2aB)cx}{a\left(b^2-4ac\right)x^{3/2}\left(a+bx+cx^2\right)}$$
$$-\frac{\sqrt{c}\left(aB\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-A\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-1\right.\right.}{\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(aB\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)-A\left(5b^4-29ab^2c+28a^2c^2-5b^3\sqrt{b^2-4ac}+19\right.\right.}{\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/3*(-14*A*a*c+5*A*b^2-3*B*a*b)/a^2/(-4*a*c+b^2)/x^(3/2)-(a*B*(-10*a*c+3*
b^2)-A*(-19*a*b*c+5*b^3))/a^3/(-4*a*c+b^2)/x^(1/2)+(A*b^2-a*b*B-2*A*a*c+(A
*b-2*B*a)*c*x)/a/(-4*a*c+b^2)/x^(3/2)/(c*x^2+b*x+a)-1/2*c^(1/2)*(a*B*(3*b^
3-16*a*b*c+3*b^2*(-4*a*c+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29
*a*b^2*c+28*a^2*c^2+5*b^3*(-4*a*c+b^2)^(1/2)-19*a*b*c*(-4*a*c+b^2)^(1/2)))
*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/
(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*c^(1/2)*(a*B*(3*b^3-16
*a*b*c-3*b^2*(-4*a*c+b^2)^(1/2)+10*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29*a*b
^2*c+28*a^2*c^2-5*b^3*(-4*a*c+b^2)^(1/2)+19*a*b*c*(-4*a*c+b^2)^(1/2)))*arc
tan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(-4*
a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 5.90 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \frac{-\frac{2\left(8a^3c(A+3Bx)+15Ab^3x^2(b+cx)+a^2\left(-2A\left(b^2+20bcx-7c^2x^2\right)+3Bx\left(-2b^2+11bcx+10c^2x^2\right)\right)-abx(9b}{x^{3/2}(a+x(b+cx))}}{}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(a + b*x + c*x^2)^2),x]
```

output
```
((-2*(8*a^3*c*(A + 3*B*x) + 15*A*b^3*x^2*(b + c*x) + a^2*(-2*A*(b^2 + 20*b
*c*x - 7*c^2*x^2) + 3*B*x*(-2*b^2 + 11*b*c*x + 10*c^2*x^2)) - a*b*x*(9*b*B
*x*(b + c*x) + A*(-10*b^2 + 62*b*c*x + 57*c^2*x^2))))/(x^(3/2)*(a + x*(b +
 c*x))) - (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*
a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*
b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3 + 16*a*b*c + 3*b^2*Sqrt[
b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4 - 29*a*b^2*c + 28*a^2*
c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[
2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(6*a^3*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1235, 27, 1198, 1198, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \ 1235$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{5Ab^2-3aBb-14aAc+5(Ab-2aB)cx}{2x^{5/2}\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{5Ab^2-3aBb-14aAc+5(Ab-2aB)cx}{x^{5/2}\left(cx^2+bx+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \ 1198$$

$$\frac{\frac{\int \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)-c\left(5Ab^2-3aBb-14aAc\right)x}{x^{3/2}\left(cx^2+bx+a\right)}dx}{a}-\frac{2\left(-14aAc-3abB+5Ab^2\right)}{3ax^{3/2}}}{2a\left(b^2-4ac\right)}+\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \ 1198$$

$$\frac{\frac{\frac{\int -\frac{abB\left(3b^2-13ac\right)-A\left(5b^4-24acb^2+14a^2c^2\right)+c\left(aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2\left(aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)\right)}{a\sqrt{x}}}{a}-\frac{2\left(-14aAc-3abB+5Ab^2\right.}{3ax^{3/2}}}{2a\left(b^2-4ac\right)}$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{ax^{3/2}\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{\frac{\int \frac{abB(3b^2-13ac)-A(5b^4-24acb^2+14a^2c^2)+c(aB(3b^2-10ac)-A(5b^3-19abc))x}{\sqrt{x}(cx^2+bx+a)}dx}{a}-\frac{2(aB(3b^2-10ac)-A(5b^3-19abc))}{a\sqrt{x}}}{a}-\frac{2(-14aAc-3abB+5Ab^2\cdots}{3ax^{3/2}}}{2a(b^2-4ac)}$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{ax^{3/2}(b^2-4ac)(a+bx+cx^2)}$$

$\downarrow$ 1197

$$\frac{-\frac{\frac{2\int \frac{abB(3b^2-13ac)-A(5b^4-24acb^2+14a^2c^2)+c(aB(3b^2-10ac)-A(5b^3-19abc))x}{cx^2+bx+a}d\sqrt{x}}{a}-\frac{2(aB(3b^2-10ac)-A(5b^3-19abc))}{a\sqrt{x}}}{a}-\frac{2(-14aAc-3abB+5\cdots}{3ax^{3/2}}}{2a(b^2-4ac)}$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{ax^{3/2}(b^2-4ac)(a+bx+cx^2)}$$

$\downarrow$ 1480

$$-\frac{2\left(\frac{c\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}}{2\sqrt{b^2-4ac}}-\frac{c\left(aB\left(-3b^2\sqrt{b\cdots\right.\right.}{\cdots}\right.}{a}\cdots$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{ax^{3/2}(b^2-4ac)(a+bx+cx^2)}$$

$\downarrow$ 218

$$-\frac{2\left(\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(aB\left(-3b^2\sqrt{\cdots}\right.\right.}{\cdots}\right.}{a}\cdots$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{ax^{3/2}(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[(A + B*x)/(x^(5/2)*(a + b*x + c*x^2)^2),x]
```

output
```
(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x)/(a*(b^2 - 4*a*c)*x^(3/2)*(a
+ b*x + c*x^2)) + ((-2*(5*A*b^2 - 3*a*b*B - 14*a*A*c))/(3*a*x^(3/2)) + ((-
2*(a*B*(3*b^2 - 10*a*c) - A*(5*b^3 - 19*a*b*c)))/(a*Sqrt[x]) - (2*((Sqrt[c
]*(a*B*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a
*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a
*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (S
qrt[c]*(a*B*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2
- 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] +
 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
))/a)/a)/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{2(-6Abx+3Bax+Aa)}{3a^3x^{\frac{3}{2}}}+\frac{\frac{2c\left(3Aabc-A\,b^3-2B\,a^2c+Ba\,b^2\right)x^{\frac{3}{2}}}{8ac-2b^2}-\frac{\left(2a^2A\,c^2-4Aa\,b^2c+A\,b^4+3a^2bBc-Ba\,b^3\right)\sqrt{x}}{4ac-b^2}}{c\,x^2+bx+a}+\frac{4c\left(-\cdots\right)}{\cdots}$
derivativedivides	$-\frac{2\left(\frac{-\frac{c\left(3Aabc-A\,b^3-2B\,a^2c+Ba\,b^2\right)x^{\frac{3}{2}}}{2\left(4ac-b^2\right)}+\frac{\left(2a^2A\,c^2-4Aa\,b^2c+A\,b^4+3a^2bBc-Ba\,b^3\right)\sqrt{x}}{8ac-2b^2}}{c\,x^2+bx+a}+\frac{2c\left(\frac{\left(-19Aabc\sqrt{-4ac+b^2}+5A\,b^3\cdots\right.}{\cdots}\right)}{\cdots}\right)}{\cdots}$
default	$-\frac{2\left(\frac{-\frac{c\left(3Aabc-A\,b^3-2B\,a^2c+Ba\,b^2\right)x^{\frac{3}{2}}}{2\left(4ac-b^2\right)}+\frac{\left(2a^2A\,c^2-4Aa\,b^2c+A\,b^4+3a^2bBc-Ba\,b^3\right)\sqrt{x}}{8ac-2b^2}}{c\,x^2+bx+a}+\frac{2c\left(\frac{\left(-19Aabc\sqrt{-4ac+b^2}+5A\,b^3\cdots\right.}{\cdots}\right)}{\cdots}\right)}{\cdots}$

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-6*A*b*x+3*B*a*x+A*a)/a^3/x^(3/2)+1/a^3*(2*(1/2*c*(3*A*a*b*c-A*b^3-2
*B*a^2*c+B*a*b^2)/(4*a*c-b^2)*x^(3/2)-1/2*(2*A*a^2*c^2-4*A*a*b^2*c+A*b^4+3
*B*a^2*b*c-B*a*b^3)/(4*a*c-b^2)*x^(1/2))/(c*x^2+b*x+a)+4/(4*a*c-b^2)*c*(-1
/8*(19*A*a*b*c*(-4*a*c+b^2)^(1/2)-5*A*b^3*(-4*a*c+b^2)^(1/2)-28*a^2*A*c^2+
29*A*a*b^2*c-5*A*b^4-10*B*a^2*c*(-4*a*c+b^2)^(1/2)+3*B*a*b^2*(-4*a*c+b^2)^
(1/2)-16*a^2*b*B*c+3*B*a*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))+1/8*(19*A*a*b*c*(-4*a*c+b^2)^(1/2)-5*A*b^3*(-4*a*c+b^2)^(1/2)+28*a^2*A
*c^2-29*A*a*b^2*c+5*A*b^4-10*B*a^2*c*(-4*a*c+b^2)^(1/2)+3*B*a*b^2*(-4*a*c+
b^2)^(1/2)+16*a^2*b*B*c-3*B*a*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10203 vs. $2(460) = 920$.

Time = 51.36 (sec) , antiderivative size = 10203, normalized size of antiderivative = 19.58

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(5/2)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx=\int \frac{Bx+A}{\left(cx^2+bx+a\right)^2x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*(3*((5*b^4*c - 24*a*b^2*c^2 + 14*a^2*c^3)*A - (3*a*b^3*c - 13*a^2*b*c^
2)*B)*x^(5/2) + 3*((5*b^5 - 19*a*b^3*c - 5*a^2*b*c^2)*A - (3*a*b^4 - 10*a^
2*b^2*c - 10*a^3*c^2)*B)*x^(3/2) + 2*((15*a*b^4 - 67*a^2*b^2*c + 28*a^3*c^
2)*A - 9*(a^2*b^3 - 4*a^3*b*c)*B)*sqrt(x) - 2*(a^3*b^2 - 4*a^4*c)*A/x^(3/2
) + 2*(5*(a^2*b^3 - 4*a^3*b*c)*A - 3*(a^3*b^2 - 4*a^4*c)*B)/sqrt(x))/(a^5*
b^2 - 4*a^6*c + (a^4*b^2*c - 4*a^5*c^2)*x^2 + (a^4*b^3 - 4*a^5*b*c)*x) + i
ntegrate(-1/2*(((5*b^4*c - 24*a*b^2*c^2 + 14*a^2*c^3)*A - (3*a*b^3*c - 13*
a^2*b*c^2)*B)*x^(3/2) + ((5*b^5 - 29*a*b^3*c + 33*a^2*b*c^2)*A - (3*a*b^4
- 16*a^2*b^2*c + 10*a^3*c^2)*B)*sqrt(x))/(a^5*b^2 - 4*a^6*c + (a^4*b^2*c -
 4*a^5*c^2)*x^2 + (a^4*b^3 - 4*a^5*b*c)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6335 vs. $2(460)=920$.

Time = 1.32 (sec) , antiderivative size = 6335, normalized size of antiderivative = 12.16

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(B*a*b^2*c*x^(3/2) - A*b^3*c*x^(3/2) - 2*B*a^2*c^2*x^(3/2) + 3*A*a*b*c^2*
x^(3/2) + B*a*b^3*sqrt(x) - A*b^4*sqrt(x) - 3*B*a^2*b*c*sqrt(x) + 4*A*a*b^
2*c*sqrt(x) - 2*A*a^2*c^2*sqrt(x))/((a^3*b^2 - 4*a^4*c)*(c*x^2 + b*x + a))
 + 1/8*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*
b*c^3)*(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^
4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*
a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*B + 2*(5*sq...
```

Mupad [B] (verification not implemented)

Time = 15.77 (sec) , antiderivative size = 21585, normalized size of antiderivative = 41.43

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(5/2)*(a + b*x + c*x^2)^2),x)
```

output
```
- ((2*A)/(3*a) - (2*x*(5*A*b - 3*B*a))/(3*a^2) + (x^2*(15*A*b^4 + 14*A*a^2
*c^2 - 9*B*a*b^3 - 62*A*a*b^2*c + 33*B*a^2*b*c))/(3*a^3*(4*a*c - b^2)) + (
c*x^3*(5*A*b^3 - 3*B*a*b^2 + 10*B*a^2*c - 19*A*a*b*c))/(a^3*(4*a*c - b^2))
)/(a*x^(3/2) + b*x^(5/2) + c*x^(7/2)) - atan((((-(25*A^2*b^15 + 9*B^2*a^2*
b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*
b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5
*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2
) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*
B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a
^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 8
0640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^
2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4
*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^
7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-
(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a
^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2
*(-(4*a*c - b^2)^9)^(1/2))/(8*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c +
240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^
5)))^(1/2)*(x^(1/2)*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c
- b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3...
```

Reduce [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 3914, normalized size of antiderivative = 7.51

$$\int \frac{A+Bx}{x^{5/2}\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x+a)^2,x)
```

output
```
( - 192*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**2*
x + 90*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c*x
 - 192*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**
2*x**2 - 192*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*
c**3*x**3 - 12*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**
5*x + 90*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*x*
*2 + 90*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c**2*
x**3 - 12*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**6*x**2
- 12*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**5*c*x**3 + 1
68*sqrt(x)*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**2*x - ...
```

3.98 $\int \frac{x^{7/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 528

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = -\frac{x^{5/2}(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{2c\,(b^2-4ac)\,(a+bx+cx^2)^2}$$
$$-\frac{\sqrt{x}(a(3b^3B+Ab^2c-24abBc+20aAc^2)+(3b^4B+Ab^3c-25ab^2Bc+8aAbc^2+28a^2Bc^2)\,x)}{4c^2\,(b^2-4ac)^2\,(a+bx+cx^2)}$$
$$+\frac{\left(3b^4B+Ab^3c-27ab^2Bc-16aAbc^2+84a^2Bc^2-\frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)\arctan\Big(}{4\sqrt{2}c^{5/2}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(3b^4B+Ab^3c-27ab^2Bc-16aAbc^2+84a^2Bc^2+\frac{3b^5B+Ab^4c-33ab^3Bc-18aAb^2c^2+132a^2bBc^2-40a^2Ac^3}{\sqrt{b^2-4ac}}\right)\arctan\Big(}{4\sqrt{2}c^{5/2}\,(b^2-4ac)^2\,\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*x^(5/2)*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x
^2+b*x+a)^2-1/4*x^(1/2)*(a*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)+(8*A*a*
b*c^2+A*b^3*c+28*B*a^2*c^2-25*B*a*b^2*c+3*B*b^4)*x)/c^2/(-4*a*c+b^2)^2/(c*
x^2+b*x+a)+1/8*(3*B*b^4+A*b^3*c-27*B*a*b^2*c-16*A*a*b*c^2+84*B*a^2*c^2-(-4
0*A*a^2*c^3-18*A*a*b^2*c^2+A*b^4*c+132*B*a^2*b*c^2-33*B*a*b^3*c+3*B*b^5)/(
-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2))*2^(1/2)/c^(5/2)/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/8*(3*B
*b^4+A*b^3*c-27*B*a*b^2*c-16*A*a*b*c^2+84*B*a^2*c^2+(-40*A*a^2*c^3-18*A*a*
b^2*c^2+A*b^4*c+132*B*a^2*b*c^2-33*B*a*b^3*c+3*B*b^5)/(-4*a*c+b^2)^(1/2))*
arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/
2)/(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 12.90 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.41

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \frac{\frac{x^{9/2}(-aB(b+2cx)+A(b^2-2ac+bcx))}{(a+x(b+cx))^2}+\frac{x^{9/2}(3aB(-3b^3+8abc-3b^2cx+4ac^2x)+A(5b^4-15ab^2c+4a^2c^2+5b^3c}{2a(-b^2+4ac)(a+x(b+cx))}}{}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(a + b*x + c*x^2)^3,x]
```

output
```
((x^(9/2)*(-(a*B*(b + 2*c*x)) + A*(b^2 - 2*a*c + b*c*x)))/(a + x*(b + c*x)
)^2 + (x^(9/2)*(3*a*B*(-3*b^3 + 8*a*b*c - 3*b^2*c*x + 4*a*c^2*x) + A*(5*b^
4 - 15*a*b^2*c + 4*a^2*c^2 + 5*b^3*c*x - 8*a*b*c^2*x)))/(2*a*(-b^2 + 4*a*c
)*(a + x*(b + c*x))) + ((-2*a^2*(3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c
^2)*Sqrt[x])/c^2 + (2*a^2*(b^2*B + 12*A*b*c - 28*a*B*c)*x^(3/2))/c + 2*a*(
-7*A*b^2 + 12*a*b*B + 4*a*A*c)*x^(5/2) + 2*(3*a*B*(-3*b^2 + 4*a*c) + A*(5*
b^3 - 8*a*b*c))*x^(7/2) + (Sqrt[2]*a^2*(-3*b^5*B + b^3*c*(33*a*B + A*Sqrt[
b^2 - 4*a*c]) - 4*a*b*c^2*(33*a*B + 4*A*Sqrt[b^2 - 4*a*c]) + 9*a*b^2*c*(2*
A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^4*(-(A*c) + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^
2*c^2*(10*A*c + 21*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (Sqrt[2]*a^2*(3*b^5*B + 4*a*b*c^2*(33*a*B - 4*A*Sqrt[b^2 - 4
*a*c]) + b^4*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - 9*a*b^2*c*(2*A*c + 3*B*Sqrt[b
^2 - 4*a*c]) + 4*a^2*c^2*(-10*A*c + 21*B*Sqrt[b^2 - 4*a*c]) + b^3*(-33*a*B
*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/(4*a*(b^2 - 4*a*c)))/(2*a*(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1233, 27, 1233, 27, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{\int \frac{x^{3/2}(5a(bB-2Ac)+(3Bb^2+Acb-14aBc)x)}{2(cx^2+bx+a)^2}dx}{2c\,(b^2-4ac)}-\frac{x^{5/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^{3/2}(5a(bB-2Ac)+(3Bb^2+Acb-14aBc)x)}{(cx^2+bx+a)^2}dx}{4c\,(b^2-4ac)}-\frac{x^{5/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

$$\Big\downarrow \; 1233$$

$$\frac{\frac{\int \frac{a\left(3Bb^3+Acb^2-24aBcb+20aAc^2\right)+\left(3Bb^4+Acb^3-27aBcb^2-16aAc^2b+84a^2Bc^2\right)x}{2\sqrt{x}\left(cx^2+bx+a\right)}dx}{c(b^2-4ac)}-\frac{\sqrt{x}\left(x\left(28a^2Bc^2+8aAbc^2-25ab^2Bc+Ab^3c+3b^4B\right)+a(20aA}{c(b^2-4ac)(a+bx+cx^2)}}{4c\left(b^2-4ac\right)}$$

$$\frac{x^{5/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{a\left(3Bb^3+Acb^2-24aBcb+20aAc^2\right)+\left(3Bb^4+Acb^3-27aBcb^2-16aAc^2b+84a^2Bc^2\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{2c(b^2-4ac)}-\frac{\sqrt{x}\left(x\left(28a^2Bc^2+8aAbc^2-25ab^2Bc+Ab^3c+3b^4B\right)+a(20aA}{c(b^2-4ac)(a+bx+cx^2)}}{4c\left(b^2-4ac\right)}$$

$$\frac{x^{5/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \; 1197$$

$$\frac{\frac{\int \frac{a\left(3Bb^3+Acb^2-24aBcb+20aAc^2\right)+\left(3Bb^4+Acb^3-27aBcb^2-16aAc^2b+84a^2Bc^2\right)x}{cx^2+bx+a}d\sqrt{x}}{c(b^2-4ac)}-\frac{\sqrt{x}\left(x\left(28a^2Bc^2+8aAbc^2-25ab^2Bc+Ab^3c+3b^4B\right)+a(20a}{c(b^2-4ac)(a+bx+cx^2)}}{4c\left(b^2-4ac\right)}$$

$$\frac{x^{5/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{\frac{1}{2}\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\frac{-40a^2}{}\right.}{c(b^2-4ac)}}{}$$

$$\frac{x^{5/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Ac^3+132a^2b}{}\right.}{}}{c(b^2-4ac)}$$

$$\frac{x^{5/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input
```
Int[(x^(7/2)*(A + B*x))/(a + b*x + c*x^2)^3,x]
```

output
```
-1/2*(x^(5/2)*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(c*(b^2 - 4
*a*c)*(a + b*x + c*x^2)^2) + (-((Sqrt[x]*(a*(3*b^3*B + A*b^2*c - 24*a*b*B*
c + 20*a*A*c^2) + (3*b^4*B + A*b^3*c - 25*a*b^2*B*c + 8*a*A*b*c^2 + 28*a^2
*B*c^2)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (((3*b^4*B + A*b^3*c -
27*a*b^2*B*c - 16*a*A*b*c^2 + 84*a^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3
*B*c - 18*a*A*b^2*c^2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c -
 16*a*A*b*c^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*
b^2*c^2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]))/(c*(b^2 - 4*a*c)))/(4*c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{\left(16Aab\,c^{2}-A\,b^{3}c+44B\,a^{2}c^{2}-37Ba\,b^{2}c+5B\,b^{4}\right)x^{\frac{7}{2}}}{4\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)c}-\frac{\left(36a^{2}A\,c^{3}+5Aa\,b^{2}c^{2}+A\,b^{4}c-4B\,a^{2}b\,c^{2}-20Ba\,b^{3}c+3b^{5}B\right)x^{\frac{5}{2}}}{4c^{2}\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}-\frac{a\left(28Aab \cdots\right.}{\cdots}}{\left(c\,x^{2}+bx+a\right)^{2}}$
default	$\frac{-\frac{\left(16Aab\,c^{2}-A\,b^{3}c+44B\,a^{2}c^{2}-37Ba\,b^{2}c+5B\,b^{4}\right)x^{\frac{7}{2}}}{4\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)c}-\frac{\left(36a^{2}A\,c^{3}+5Aa\,b^{2}c^{2}+A\,b^{4}c-4B\,a^{2}b\,c^{2}-20Ba\,b^{3}c+3b^{5}B\right)x^{\frac{5}{2}}}{4c^{2}\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}-\frac{a\left(28Aab \cdots\right.}{\cdots}}{\left(c\,x^{2}+bx+a\right)^{2}}$

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/8*(16*A*a*b*c^2-A*b^3*c+44*B*a^2*c^2-37*B*a*b^2*c+5*B*b^4)/(16*a^2*c
^2-8*a*b^2*c+b^4)/c*x^(7/2)-1/8*(36*A*a^2*c^3+5*A*a*b^2*c^2+A*b^4*c-4*B*a^
2*b*c^2-20*B*a*b^3*c+3*B*b^5)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-1/8*a
/c^2*(28*A*a*b*c^2+2*A*b^3*c+28*B*a^2*c^2-49*B*a*b^2*c+6*B*b^4)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^(3/2)-1/8*a^2*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)/c
^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2))/(c*x^2+b*x+a)^2+1/c/(16*a^2*c^2-8*a
*b^2*c+b^4)*(-1/8*(-16*A*a*b*c^2*(-4*a*c+b^2)^(1/2)+A*b^3*c*(-4*a*c+b^2)^(
1/2)+40*a^2*A*c^3+18*A*a*b^2*c^2-A*b^4*c+84*B*a^2*c^2*(-4*a*c+b^2)^(1/2)-2
7*B*a*b^2*c*(-4*a*c+b^2)^(1/2)+3*B*b^4*(-4*a*c+b^2)^(1/2)-132*B*a^2*b*c^2+
33*B*a*b^3*c-3*b^5*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/
8*(-16*A*a*b*c^2*(-4*a*c+b^2)^(1/2)+A*b^3*c*(-4*a*c+b^2)^(1/2)-40*a^2*A*c^
3-18*A*a*b^2*c^2+A*b^4*c+84*B*a^2*c^2*(-4*a*c+b^2)^(1/2)-27*B*a*b^2*c*(-4*
a*c+b^2)^(1/2)+3*B*b^4*(-4*a*c+b^2)^(1/2)+132*B*a^2*b*c^2-33*B*a*b^3*c+3*b
^5*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9631 vs. $2(476) = 952$.

Time = 39.63 (sec) , antiderivative size = 9631, normalized size of antiderivative = 18.24

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \int \frac{(Bx+A)x^{\frac{7}{2}}}{(cx^2+bx+a)^3}\,dx$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(((b^2*c^2 + 20*a*c^3)*A + 3*(b^3*c - 8*a*b*c^2)*B)*x^(9/2) + (3*(b^3*
c + 8*a*b*c^2)*A + (b^4 - 11*a*b^2*c - 44*a^2*c^2)*B)*x^(7/2) + ((17*a*b^2
*c + 4*a^2*c^2)*A + 2*(a*b^3 - 22*a^2*b*c)*B)*x^(5/2) + (12*A*a^2*b*c + (a
^2*b^2 - 28*a^3*c)*B)*x^(3/2))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + (
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^
2*b*c^4)*x^3 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^2 + 2*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3)*x) - integrate(1/8*(((b^2*c + 20*a*c^2)*A + 3*(b^
3 - 8*a*b*c)*B)*x^(3/2) + 3*(12*A*a*b*c + (a*b^2 - 28*a^2*c)*B)*sqrt(x))/(
a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3997 vs. $2(476) = 952$.

Time = 1.24 (sec) , antiderivative size = 3997, normalized size of antiderivative = 7.57

$$\int \frac{x^{7/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/16*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^5*c^2 - 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^3 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 24*a*b^4*c^3 - 2*b^5*c^3 + 320*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^4 + 288*a^2*b^2*c^4 + 112*a*b^3*c^4 - 80*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*c^5 - 640*a^3*c^5 - 416*a^2*b*c^5 + sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 56*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*(b^2 - 4*a*c)*b^4*c^2 + 32*(b^2 - 4*a*c)
*a*b^2*c^3 + 2*(b^2 - 4*a*c)*b^3*c^3 - 160*(b^2 - 4*a*c)*a^2*c^4 - 104*(b^
2 - 4*a*c)*a*b*c^4)*A + 3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 1
6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^6*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - ...
```

Mupad [B] (verification not implemented)

Time = 15.12 (sec) , antiderivative size = 22943, normalized size of antiderivative = 43.45

$$\int \frac{x^{7/2}(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(7/2)*(A + B*x))/(a + b*x + c*x^2)^3,x)
```

output
```
- ((x^(5/2)*(3*B*b^5 + 36*A*a^2*c^3 + A*b^4*c - 20*B*a*b^3*c + 5*A*a*b^2*c
^2 - 4*B*a^2*b*c^2))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(7/2)*(5*
B*b^4 + 44*B*a^2*c^2 - A*b^3*c + 16*A*a*b*c^2 - 37*B*a*b^2*c))/(4*c*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) + (x^(3/2)*(28*B*a^3*c^2 + 6*B*a*b^4 + 2*A*a*b^3
*c + 28*A*a^2*b*c^2 - 49*B*a^2*b^2*c))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)) + (a^2*x^(1/2)*(3*B*b^3 + 20*A*a*c^2 + A*b^2*c - 24*B*a*b*c))/(4*c^2*(
b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b
*x + 2*b*c*x^3) - atan(((((64*A*a*b^12*c^4 - 1310720*A*a^7*c^10 + 192*B*a*
b^13*c^3 + 1572864*B*a^7*b*c^9 - 15360*A*a^3*b^8*c^6 + 163840*A*a^4*b^6*c^
7 - 737280*A*a^5*b^4*c^8 + 1572864*A*a^6*b^2*c^9 - 5376*B*a^2*b^11*c^4 + 6
1440*B*a^3*b^9*c^5 - 368640*B*a^4*b^7*c^6 + 1228800*B*a^5*b^5*c^7 - 216268
8*B*a^6*b^3*c^8)/(64*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x^(1/2
)*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A
*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4
*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^
7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^
4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*
B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*...
```

Reduce [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 7823, normalized size of antiderivative = 14.82

$$\int \frac{x^{7/2}(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x+a)^3,x)
```

output
```
( - 232*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c**3 + 58*sqr
t(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqr
t(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c**2 - 464*sqrt(a)*sq
rt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sq
rt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**3*x - 464*sqrt(a)*sqrt(2*
sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c)
)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**4*x**2 - 6*sqrt(a)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**2*b**5*c + 116*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*
sqrt(a) + b))*a**2*b**4*c**2*x - 116*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**2*b**3*c**3*x**2 - 464*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a**2*b**2*c**4*x**3 - 232*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a**2*b*c**5*x**4 - 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a
*b**6*c*x + 46*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)...
```

3.99 $\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 459

$$\begin{aligned}\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx &= -\frac{x^{3/2}(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{2c\,(b^2-4ac)\,(a+bx+cx^2)^2} \\ &- \frac{\sqrt{x}(a(b^2B-12Abc+20aBc)-(b^3B+3Ab^2c-16abBc+12aAc^2)\,x)}{4c\,(b^2-4ac)^2\,(a+bx+cx^2)} \\ &+ \frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2-\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}c^{3/2}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}} \\ &+ \frac{\left(b^3B+3Ab^2c-16abBc+12aAc^2+\frac{b^4B+3Ab^3c-18ab^2Bc+36aAbc^2-40a^2Bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}c^{3/2}\,(b^2-4ac)^2\,\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/2*x^(3/2)*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x
^2+b*x+a)^2-1/4*x^(1/2)*(a*(-12*A*b*c+20*B*a*c+B*b^2)-(12*A*a*c^2+3*A*b^2*
c-16*B*a*b*c+B*b^3)*x)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+1/8*(B*b^3+3*A*b^2*c
-16*B*a*b*c+12*A*a*c^2-(36*A*a*b*c^2+3*A*b^3*c-40*B*a^2*c^2-18*B*a*b^2*c+B
*b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*2^(1/2)/c^(3/2)/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1
/8*(B*b^3+3*A*b^2*c-16*B*a*b*c+12*A*a*c^2+(36*A*a*b*c^2+3*A*b^3*c-40*B*a^2
*c^2-18*B*a*b^2*c+B*b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(-4*a*c+b^2)^2/(b+(-4*a*c+
b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 10.54 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.13

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \frac{\frac{2\sqrt{c}\sqrt{x}\left(-20a^3Bc+ax\left(-2b^3B+12Ac^3x^2+b^2c(19A-5Bx)+16bc^2x(A-Bx)\right)+b^2x^2\left(-b^2B+3Ac^2x+bc(5A+Bx)\right.\right.}{(b^2-4ac)^2(a+x(b+cx))^2}}{}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(a + b*x + c*x^2)^3,x]
```

output
```
((2*Sqrt[c]*Sqrt[x]*(-20*a^3*B*c + a*x*(-2*b^3*B + 12*A*c^3*x^2 + b^2*c*(1
9*A - 5*B*x) + 16*b*c^2*x*(A - B*x)) + b^2*x^2*(-(b^2*B) + 3*A*c^2*x + b*c
*(5*A + B*x)) - a^2*(b^2*B - 4*b*c*(3*A - 7*B*x) + 4*c^2*x*(A + 9*B*x))))/
((b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) + (Sqrt[2]*(-(b^4*B) + 3*b^2*c*(6*a*
B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*
(-3*A*c + B*Sqrt[b^2 - 4*a*c]) - 4*a*b*c*(9*A*c + 4*B*Sqrt[b^2 - 4*a*c]))*
ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^4*B + 3*b^2*c*(-6*a*B
 + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(-10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + 4*a*
b*c*(9*A*c - 4*B*Sqrt[b^2 - 4*a*c]) + b^3*(3*A*c + B*Sqrt[b^2 - 4*a*c]))*A
rcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*
c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(8*c^(3/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1233, 27, 1234, 27, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{\int \frac{\sqrt{x}(3a(bB-2Ac)+(Bb^2+3Acb-10aBc)x)}{2(cx^2+bx+a)^2}dx}{2c\left(b^2-4ac\right)}-\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{x}(3a(bB-2Ac)+(Bb^2+3Acb-10aBc)x)}{(cx^2+bx+a)^2}dx}{4c\left(b^2-4ac\right)}-\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1234}$$

$$\frac{-\frac{\int -\frac{a\left(Bb^2-12Acb+20aBc\right)+\left(Bb^3+3Acb^2-16aBcb+12aAc^2\right)x}{2\sqrt{x}\left(cx^2+bx+a\right)}dx}{b^2-4ac}-\frac{\sqrt{x}\left(a\left(20aBc-12Abc+b^2B\right)-x\left(12aAc^2-16abBc+3Ab^2c+b^3B\right)\right)}{(b^2-4ac)(a+bx+cx^2)}}{4c\left(b^2-4ac\right)}-\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a\left(Bb^2-12Acb+20aBc\right)+\left(Bb^3+3Acb^2-16aBcb+12aAc^2\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{2(b^2-4ac)}-\frac{\sqrt{x}\left(a\left(20aBc-12Abc+b^2B\right)-x\left(12aAc^2-16abBc+3Ab^2c+b^3B\right)\right)}{(b^2-4ac)(a+bx+cx^2)}}{4c\left(b^2-4ac\right)}-\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1197}$$

$$\frac{\frac{\int \frac{a\left(Bb^2-12Acb+20aBc\right)+\left(Bb^3+3Acb^2-16aBcb+12aAc^2\right)x}{cx^2+bx+a}d\sqrt{x}}{b^2-4ac}-\frac{\sqrt{x}(a(20aBc-12Abc+b^2B)-x(12aAc^2-16abBc+3Ab^2c+b^3B))}{(b^2-4ac)(a+bx+cx^2)}}{4c\left(b^2-4ac\right)}-\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{\frac{\frac{1}{2}\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3A}{\sqrt{b^2-4ac}}\right.}{b^2-4ac}}{4c\left(b^2-\right.}$$

$$\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+\right.}{\sqrt{2}\sqrt{}}}{b^2-4ac}}{4c\left(b^2-4ac\right)}$$

$$\frac{x^{3/2}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input
```
Int[(x^(5/2)*(A + B*x))/(a + b*x + c*x^2)^3,x]
```

output
```
-1/2*(x^(3/2)*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(c*(b^2 - 4
*a*c)*(a + b*x + c*x^2)^2) + (-((Sqrt[x]*(a*(b^2*B - 12*A*b*c + 20*a*B*c)
- (b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^2)*x))/((b^2 - 4*a*c)*(a + b*
x + c*x^2))) + (((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 - (b^4*B + 3
*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sq
rt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12
*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(b^2 - 4*a*c))/(4
*c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\frac{2\left(12Aa\,c^{2}+3A\,b^{2}c-16Babc+B\,b^{3}\right)x^{\frac{7}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{\left(16Aab\,c^{2}+5A\,b^{3}c-36B\,a^{2}c^{2}-5Ba\,b^{2}c-B\,b^{4}\right)x^{\frac{5}{2}}}{4c\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}-\frac{a\left(4Aa\,c^{2}-19A\,b^{2}c+28Babc+2B\,b^{3}\right.}{4c\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}}$
default	$\frac{\frac{2\left(12Aa\,c^{2}+3A\,b^{2}c-16Babc+B\,b^{3}\right)x^{\frac{7}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}+\frac{\left(16Aab\,c^{2}+5A\,b^{3}c-36B\,a^{2}c^{2}-5Ba\,b^{2}c-B\,b^{4}\right)x^{\frac{5}{2}}}{4c\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}-\frac{a\left(4Aa\,c^{2}-19A\,b^{2}c+28Babc+2B\,b^{3}\right.}{4c\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}}$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*(12*A*a*c^2+3*A*b^2*c-16*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^(7/2)+1/8*(16*A*a*b*c^2+5*A*b^3*c-36*B*a^2*c^2-5*B*a*b^2*c-B*b^4)/c/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-1/8*a/c*(4*A*a*c^2-19*A*b^2*c+28*B*a*b*c+2*
B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+1/8*a^2*(12*A*b*c-20*B*a*c-B*b^2
)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2))/(c*x^2+b*x+a)^2+1/(16*a^2*c^2-8*a*
b^2*c+b^4)*(1/8*(12*A*a*c^2*(-4*a*c+b^2)^(1/2)+3*A*b^2*c*(-4*a*c+b^2)^(1/2
)+36*A*a*b*c^2+3*A*b^3*c-16*B*a*b*c*(-4*a*c+b^2)^(1/2)+B*b^3*(-4*a*c+b^2)^
(1/2)-40*B*a^2*c^2-18*B*a*b^2*c+B*b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))-1/8*(12*A*a*c^2*(-4*a*c+b^2)^(1/2)+3*A*b^2*c*(-4*a*c+b^2)^(1/2
)-36*A*a*b*c^2-3*A*b^3*c-16*B*a*b*c*(-4*a*c+b^2)^(1/2)+B*b^3*(-4*a*c+b^2)^
(1/2)+40*B*a^2*c^2+18*B*a*b^2*c-B*b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7056 vs. 2(407) = 814.

Time = 11.87 (sec) , antiderivative size = 7056, normalized size of antiderivative = 15.37

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x+A)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \int \frac{(Bx+A)x^{\frac{5}{2}}}{(cx^2+bx+a)^3}\,dx$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*((12*A*b*c^2 - (b^2*c + 20*a*c^2)*B)*x^(9/2) + 3*((7*b^2*c - 4*a*c^2)
*A - (b^3 + 8*a*b*c)*B)*x^(7/2) + ((7*b^3 + 8*a*b*c)*A - (17*a*b^2 + 4*a^2
*c)*B)*x^(5/2) - (12*B*a^2*b - (5*a*b^2 + 4*a^2*c)*A)*x^(3/2))/(a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^
2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x) + integrate(1/8*((12*A*b*c -
 (b^2 + 20*a*c)*B)*x^(3/2) - 3*(12*B*a*b - (5*b^2 + 4*a*c)*A)*sqrt(x))/(a*
b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 +
(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7586 vs. $2(407) = 814$.

Time = 1.96 (sec) , antiderivative size = 7586, normalized size of antiderivative = 16.53

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/32*(3*(2*b^4*c^3 - 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4
 - 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*(b^4*c - 8*a*b^2*c^2 +
 16*a^2*c^3)^2*A + (2*b^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c
)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*B - 24*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 2*a
*b^7*c^4 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c + s...
```

Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 19073, normalized size of antiderivative = 41.55

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(5/2)*(A + B*x))/(a + b*x + c*x^2)^3,x)
```

output
```
atan(((((1310720*B*a^7*c^8 + 768*A*a*b^11*c^3 - 786432*A*a^6*b*c^8 - 64*B*
a*b^12*c^2 - 15360*A*a^2*b^9*c^4 + 122880*A*a^3*b^7*c^5 - 491520*A*a^4*b^5
*c^6 + 983040*A*a^5*b^3*c^7 + 15360*B*a^3*b^8*c^4 - 163840*B*a^4*b^6*c^5 +
 737280*B*a^5*b^4*c^6 - 1572864*B*a^6*b^2*c^7)/(64*(b^12*c + 4096*a^6*c^7
- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 -
6144*a^5*b^2*c^6)) - (x^(1/2)*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c
- 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 -
 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 1
0160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 68
0960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B
^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 -
737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 2400
0*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178
1760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^
15)^(1/2) - 180*A*B*a*b^14*c^2)/(128*(1048576*a^10*c^13 + b^20*c^3 - 40*a*
b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258
048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8
*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(64*b^11*c^3 - 1280*a*b^9*c^4 -
65536*a^5*b*c^8 + 10240*a^2*b^7*c^5 - 40960*a^3*b^5*c^6 + 81920*a^4*b^3...
```

Reduce [B] (verification not implemented)

Time = 5.02 (sec) , antiderivative size = 6753, normalized size of antiderivative = 14.71

$$\int \frac{x^{5/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x+a)^3,x)
```

output

```
( - 48*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**3 + 68*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x
)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**2 - 96*sqrt(a)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c
))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**3*x - 96*sqrt(a)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2
*sqrt(c)*sqrt(a) + b))*a**3*c**4*x**2 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*
sqrt(a) + b))*a**2*b**4*c + 136*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) +
b))*a**2*b**3*c**2*x + 88*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*2*b**2*c**3*x**2 - 96*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
b*c**4*x**3 - 48*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**5*x
**4 - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**5*c*x + 64*sqrt
(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s...
```

3.100 $\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 414

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = -\frac{\sqrt{x}(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{2c\,(b^2-4ac)\,(a+bx+cx^2)^2}$$
$$+\frac{\sqrt{x}(2b^3B-7Ab^2c+4abBc+4aAc^2+3c(b^2B-4Abc+4aBc)\,x)}{4c\,(b^2-4ac)^2\,(a+bx+cx^2)}$$
$$+\frac{3\left(b^2B-4Abc+4aBc-\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\sqrt{c}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{3\left(b^2B-4Abc+4aBc+\frac{b^3B-6Ab^2c+12abBc-8aAc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\sqrt{c}\,(b^2-4ac)^2\,\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*x^(1/2)*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x
^2+b*x+a)^2+1/4*x^(1/2)*(2*B*b^3-7*A*b^2*c+4*B*a*b*c+4*A*a*c^2+3*c*(-4*A*b
*c+4*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+3/8*(B*b^2-4*A*b*c+4*B
*a*c-(-8*A*a*c^2-6*A*b^2*c+12*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))*arctan(2^
(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*
c+b^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/8*(B*b^2-4*A*b*c+4*B*a*c+(-8*A*a*c
^2-6*A*b^2*c+12*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*
x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^2/(b+(-
4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 11.15 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.08

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \frac{1}{8}\left(\frac{2\sqrt{x}(4a^2(3bB-c(3A+Bx))+a(A(-3b^2-16bcx+4c^2x^2)+Bx(19b^2+16b\cdots}{(b^2-4ac)^2\,(a+x(b\cdots}\right.$$

$$+\frac{3\sqrt{2}\left(-b^3B-4bc\left(3aB+A\sqrt{b^2-4ac}\right)+4ac\left(2Ac+B\sqrt{b^2-4ac}\right)+b^2\left(6Ac+B\sqrt{b^2-4ac}\right)\right)\arctan\left(\cdots\right.}{\sqrt{c}\,(b^2-4ac)^{5/2}\,\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{3\sqrt{2}\left(b^3B+4bc\left(3aB-A\sqrt{b^2-4ac}\right)+b^2\left(-6Ac+B\sqrt{b^2-4ac}\right)+4ac\left(-2Ac+B\sqrt{b^2-4ac}\right)\right)\arctan\left(\cdots\right.}{\sqrt{c}\,(b^2-4ac)^{5/2}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(a + b*x + c*x^2)^3,x]
```

output
```
((2*Sqrt[x]*(4*a^2*(3*b*B - c*(3*A + B*x)) + a*(A*(-3*b^2 - 16*b*c*x + 4*c
^2*x^2) + B*x*(19*b^2 + 16*b*c*x + 12*c^2*x^2)) + b*x*(b*B*x*(5*b + 3*c*x)
 - A*(5*b^2 + 19*b*c*x + 12*c^2*x^2))))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))
^2) + (3*Sqrt[2]*(-(b^3*B) - 4*b*c*(3*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c*(
2*A*c + B*Sqrt[b^2 - 4*a*c]) + b^2*(6*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a
*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(b^3*B + 4*b*c*(3*a*B
- A*Sqrt[b^2 - 4*a*c]) + b^2*(-6*A*c + B*Sqrt[b^2 - 4*a*c]) + 4*a*c*(-2*A*
c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
8
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1233, 27, 1235, 27, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \ 1233$$

$$\frac{\int \frac{a(bB-2Ac)-(Bb^2-5Acb+6aBc)x}{2\sqrt{x}(cx^2+bx+a)^2}dx}{2c(b^2-4ac)}-\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a(bB-2Ac)-(Bb^2-5Acb+6aBc)x}{\sqrt{x}(cx^2+bx+a)^2}dx}{4c(b^2-4ac)}-\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \ 1235$$

$$\frac{\frac{\sqrt{x}\left(3cx\left(4aBc-4Abc+b^2B\right)+4aAc^2+4abBc-7Ab^2c+2b^3B\right)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{3ac\left(4abB-A\left(b^2+4ac\right)-\left(Bb^2-4Acb+4aBc\right)x\right)}{2\sqrt{x}\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}}{4c(b^2-4ac)}-\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\sqrt{x}\left(3cx\left(4aBc-4Abc+b^2B\right)+4aAc^2+4abBc-7Ab^2c+2b^3B\right)}{(b^2-4ac)(a+bx+cx^2)}-\frac{3c\int \frac{4abB-A\left(b^2+4ac\right)-\left(Bb^2-4Acb+4aBc\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{2(b^2-4ac)}}{4c(b^2-4ac)}-\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \ 1197$$

$$\frac{\frac{\sqrt{x}\left(3cx\left(4aBc-4Abc+b^2B\right)+4aAc^2+4abBc-7Ab^2c+2b^3B\right)}{(b^2-4ac)(a+bx+cx^2)}-\frac{3c\int \frac{4abB-A\left(b^2+4ac\right)-\left(Bb^2-4Acb+4aBc\right)x}{cx^2+bx+a}d\sqrt{x}}{b^2-4ac}}{4c(b^2-4ac)}-\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c(b^2-4ac)(a+bx+cx^2)^2}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{\sqrt{x}\left(3cx\left(4aBc-4Abc+b^2B\right)+4aAc^2+4abBc-7Ab^2c+2b^3B\right)}{(b^2-4ac)(a+bx+cx^2)}-\frac{3c\left(-\frac{1}{2}\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}\right.}{}}{4c(b^2-4ac)}-\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c(b^2-4ac)(a+bx+cx^2)^2}$$

$$\downarrow 218$$

$$\frac{\frac{\sqrt{x}(3cx(4aBc-4Abc+b^2B)+4aAc^2+4abBc-7Ab^2c+2b^3B)}{(b^2-4ac)(a+bx+cx^2)}-\frac{3c\left(-\frac{\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}\right.}{b^2-}}{4c\left(b^2-4ac\right)}$$

$$\frac{\sqrt{x}\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input
```
Int[(x^(3/2)*(A + B*x))/(a + b*x + c*x^2)^3,x]
```

output
```
-1/2*(Sqrt[x]*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(c*(b^2 - 4
*a*c)*(a + b*x + c*x^2)^2) + ((Sqrt[x]*(2*b^3*B - 7*A*b^2*c + 4*a*b*B*c +
4*a*A*c^2 + 3*c*(b^2*B - 4*A*b*c + 4*a*B*c)*x))/((b^2 - 4*a*c)*(a + b*x +
c*x^2)) - (3*c*(-(((b^2*B - 4*A*b*c + 4*a*B*c - (b^3*B - 6*A*b^2*c + 12*a*
b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]))
- ((b^2*B - 4*A*b*c + 4*a*B*c + (b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(b^2 - 4*a*c))/
(4*c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{3c\left(4Abc-4aBc-B\,b^{2}\right)x^{\frac{7}{2}}}{4\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}+\frac{2\left(4Aa\,c^{2}-19A\,b^{2}c+16Babc+5B\,b^{3}\right)x^{\frac{5}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}-\frac{\left(16Aabc+5A\,b^{3}+4B\,a^{2}c-19Ba\,b^{2}\right)x^{\frac{3}{2}}}{4\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}-\frac{3a\left(4Aac+b^{2}A\right.}{4\left(16a^{2}c^{2}-8c\right.}}{\left(c\,x^{2}+bx+a\right)^{2}}$
default	$\frac{-\frac{3c\left(4Abc-4aBc-B\,b^{2}\right)x^{\frac{7}{2}}}{4\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}+\frac{2\left(4Aa\,c^{2}-19A\,b^{2}c+16Babc+5B\,b^{3}\right)x^{\frac{5}{2}}}{128a^{2}c^{2}-64ca\,b^{2}+8b^{4}}-\frac{\left(16Aabc+5A\,b^{3}+4B\,a^{2}c-19Ba\,b^{2}\right)x^{\frac{3}{2}}}{4\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}-\frac{3a\left(4Aac+b^{2}A\right.}{4\left(16a^{2}c^{2}-8c\right.}}{\left(c\,x^{2}+bx+a\right)^{2}}$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-3/8*c*(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+1/8*(
4*A*a*c^2-19*A*b^2*c+16*B*a*b*c+5*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2
)-1/8*(16*A*a*b*c+5*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^(3/2)-3/8*a*(4*A*a*c+A*b^2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2))
/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*(-1/8*(-4*A*b*c*(-4*a*c+b^
2)^(1/2)+8*A*a*c^2+6*A*b^2*c+4*a*B*c*(-4*a*c+b^2)^(1/2)+B*b^2*(-4*a*c+b^2)
^(1/2)-12*B*a*b*c-B*b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
+1/8*(-4*A*b*c*(-4*a*c+b^2)^(1/2)-8*A*a*c^2-6*A*b^2*c+4*a*B*c*(-4*a*c+b^2)
^(1/2)+B*b^2*(-4*a*c+b^2)^(1/2)+12*B*a*b*c+B*b^3)/c/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5646 vs. $2(362) = 724$.

Time = 7.15 (sec) , antiderivative size = 5646, normalized size of antiderivative = 13.64

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x+A)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \int \frac{(Bx+A)x^{\frac{3}{2}}}{(cx^2+bx+a)^3}\,dx$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(3*(4*B*a*b*c^2 - (b^2*c^2 + 4*a*c^3)*A)*x^(9/2) - 3*(2*(b^3*c + 2*a*
b*c^2)*A - (7*a*b^2*c - 4*a^2*c^2)*B)*x^(7/2) - ((3*b^4 - a*b^2*c + 28*a^2
*c^2)*A - (7*a*b^3 + 8*a^2*b*c)*B)*x^(5/2) - ((a*b^3 + 8*a^2*b*c)*A - (5*a
^2*b^2 + 4*a^3*c)*B)*x^(3/2))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a*b^4
*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a
^3*b*c^3)*x^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^2 + 2*(a^2*b^5 - 8*a^
3*b^3*c + 16*a^4*b*c^2)*x) - integrate(-3/8*((4*B*a*b*c - (b^2*c + 4*a*c^2
)*A)*x^(3/2) - ((b^3 + 8*a*b*c)*A - (5*a*b^2 + 4*a^2*c)*B)*sqrt(x))/(a^2*b
^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^2
 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3170 vs. $2(362) = 724$.

Time = 1.22 (sec) , antiderivative size = 3170, normalized size of antiderivative = 7.66

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
3/16*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^5*c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2
+ 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^
2 - 4*a*c)*b^3*c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*
A - 2*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^4*c - 4*a*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 32*a^2*b^3*c^2 + 6*a*b^4*...
```

Mupad [B] (verification not implemented)

Time = 14.03 (sec) , antiderivative size = 16720, normalized size of antiderivative = 40.39

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(A + B*x))/(a + b*x + c*x^2)^3,x)
```

output
```
atan(((((3*(262144*A*a^6*c^8 - 64*A*b^12*c^2 + 1024*A*a*b^10*c^3 + 256*B*a
*b^11*c^2 - 262144*B*a^6*b*c^7 - 5120*A*a^2*b^8*c^4 + 81920*A*a^4*b^4*c^6
- 262144*A*a^5*b^2*c^7 - 5120*B*a^2*b^9*c^3 + 40960*B*a^3*b^7*c^4 - 163840
*B*a^4*b^5*c^5 + 327680*B*a^5*b^3*c^6))/(64*(b^12 + 4096*a^6*c^6 + 240*a^2
*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b
^10*c)) - (x^(1/2)*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^
2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A
^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*
a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*
b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8
 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2
*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^
8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2
*c^7 - 20*A*B*a*b^14*c))/(128*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a
^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6
 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 262144
0*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(64*b^11*c^2 - 1280*a*b^9*c^3 - 65536*
a^5*b*c^7 + 10240*a^2*b^7*c^4 - 40960*a^3*b^5*c^5 + 81920*a^4*b^3*c^6))/(8
*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(
9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-...
```

Reduce [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 5680, normalized size of antiderivative = 13.72

$$\int \frac{x^{3/2}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x+a)^3,x)
```

output
```
(24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**2 - 30*sqrt(a)
*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)
*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c + 48*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**2*x + 48*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*
sqrt(c)*sqrt(a) + b))*a**2*b*c**3*x**2 - 60*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a)
 + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c
)*sqrt(a) + b))*a*b**4*c*x - 36*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) +
b))*a*b**3*c**2*x**2 + 48*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
b**2*c**3*x**3 + 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**4
*x**4 - 30*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**5*c*x**2 - 60*
sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*
sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*c**2*x**3 - 30*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(...
```

3.101 $\int \frac{\sqrt{x}(A+Bx)}{\left(a+bx+cx^2\right)^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 446

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a+bx+cx^2\right)^3} \, dx = -\frac{\sqrt{x}(Ab-2aB-(bB-2Ac)x)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$
$$-\frac{\sqrt{x}\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2+20ac\right)\right)x\right)}{4a\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$
$$+\frac{\sqrt{c}\left(6aB\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)+A\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{b-\sqrt{\cdots}}}\right.}{4\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{c}\left(6aB\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)+A\left(b^3-52abc-b^2\sqrt{b^2-4ac}-20ac\sqrt{b^2-4ac}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{b+\sqrt{\cdots}}}\right.}{4\sqrt{2}a\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*x^(1/2)*(A*b-2*B*a-(-2*A*c+B*b)*x)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-1/4*x
^(1/2)*(a*B*(-4*a*c+7*b^2)-A*(8*a*b*c+b^3)+c*(12*a*b*B-A*(20*a*c+b^2))*x)/
a/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+1/8*c^(1/2)*(6*a*B*(3*b^2+4*a*c-2*b*(-4*a*c
+b^2)^(1/2))+A*(b^3-52*a*b*c+b^2*(-4*a*c+b^2)^(1/2)+20*a*c*(-4*a*c+b^2)^(1
/2)))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)
/a/(-4*a*c+b^2)^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/8*c^(1/2)*(6*a*B*(3*b
^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))+A*(b^3-52*a*b*c-b^2*(-4*a*c+b^2)^(1/2)-20
*a*c*(-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(5/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 7.29 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^3}\,dx$$

$$= \frac{\frac{2\sqrt{x}(-12a^3Bc+Ab^2x(b+cx)^2+a^2(-3b^2B+16bc(A-Bx)+4c^2x(9A+Bx))-a(bBx(5b^2+19bcx+12c^2x^2)+A(b^3-5b^2cx-28bc^2x^2-20c^3x^3)))}{(a+x(b+cx))^2}}{}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(a + b*x + c*x^2)^3,x]
```

output
```
((2*Sqrt[x]*(-12*a^3*B*c + A*b^2*x*(b + c*x)^2 + a^2*(-3*b^2*B + 16*b*c*(A
 - B*x) + 4*c^2*x*(9*A + B*x)) - a*(b*B*x*(5*b^2 + 19*b*c*x + 12*c^2*x^2)
+ A*(b^3 - 5*b^2*c*x - 28*b*c^2*x^2 - 20*c^3*x^3))))/(a + x*(b + c*x))^2 +
 (Sqrt[2]*Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3
- 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sq
rt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-6*a*B*(3*b^2 + 4*a*c + 2*b
*Sqrt[b^2 - 4*a*c]) + A*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(8*a*(b^2 - 4*a*
c)^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1234, 27, 1235, 27, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \; 1234$$

$$-\frac{\int -\frac{Ab-2aB+5(bB-2Ac)x}{2\sqrt{x}\left(cx^2+bx+a\right)^2}dx}{2\left(b^2-4ac\right)}-\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{Ab-2aB+5(bB-2Ac)x}{\sqrt{x}\left(cx^2+bx+a\right)^2}dx}{4\left(b^2-4ac\right)}-\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1235

$$\frac{-\frac{\int -\frac{3aB\left(b^2+4ac\right)+2A\left(\frac{b^3}{2}-8abc\right)-c\left(12abB-A\left(b^2+20ac\right)\right)x}{2\sqrt{x}\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}-\frac{\sqrt{x}\left(-A\left(8abc+b^3\right)+cx\left(12abB-A\left(20ac+b^2\right)\right)+aB\left(7b^2-4ac\right)\right)}{a(b^2-4ac)(a+bx+cx^2)}}{4\left(b^2-4ac\right)}-\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{3aB\left(b^2+4ac\right)+A\left(b^3-16abc\right)-c\left(12abB-A\left(b^2+20ac\right)\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{2a(b^2-4ac)}-\frac{\sqrt{x}\left(-A\left(8abc+b^3\right)+cx\left(12abB-A\left(20ac+b^2\right)\right)+aB\left(7b^2-4ac\right)\right)}{a(b^2-4ac)(a+bx+cx^2)}}{4\left(b^2-4ac\right)}-\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1197

$$\frac{\frac{\int \frac{3aB\left(b^2+4ac\right)+A\left(b^3-16abc\right)-c\left(12abB-A\left(b^2+20ac\right)\right)x}{cx^2+bx+a}d\sqrt{x}}{a(b^2-4ac)}-\frac{\sqrt{x}\left(-A\left(8abc+b^3\right)+cx\left(12abB-A\left(20ac+b^2\right)\right)+aB\left(7b^2-4ac\right)\right)}{a(b^2-4ac)(a+bx+cx^2)}}{4\left(b^2-4ac\right)}-\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1480

$$\frac{\frac{-\frac{1}{2}c\left(-A\left(20ac+b^2\right)-\frac{A\left(b^3-52abc\right)+6aB\left(4ac+3b^2\right)}{\sqrt{b^2-4ac}}+12abB\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}-\frac{1}{2}c\left(-A\left(20ac+b^2\right)+\frac{A\left(b^3-52abc\right)+6aB\left(4ac+3b^2\right)}{\sqrt{b^2-4ac}}+12a\cdots\right.}{a(b^2-4ac)}}{4\left(b^2-4ac\right)}$$

$$\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{-\frac{\sqrt{c}\left(-A\left(20ac+b^2\right)-\frac{A\left(b^3-52abc\right)+6aB\left(4ac+3b^2\right)}{\sqrt{b^2-4ac}}+12abB\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-A\left(20ac+b^2\right)+\frac{A\left(b^3-52abc\right)+6aB\left(4ac+3b^2\right)}{\sqrt{b^2-4ac}}+12abB\right)\text{arct}}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a(b^2-4ac)}}{4\left(b^2-4ac\right)}$$

$$\frac{\sqrt{x}(-2aB-x(bB-2Ac)+Ab)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(a + b*x + c*x^2)^3,x]
```

output
```
-1/2*(Sqrt[x]*(A*b - 2*a*B - (b*B - 2*A*c)*x))/((b^2 - 4*a*c)*(a + b*x + c
*x^2)^2) + (-((Sqrt[x]*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(12*a*
b*B - A*(b^2 + 20*a*c))*x))/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2))) + (-((Sqr
t[c]*(12*a*b*B - A*(b^2 + 20*a*c) - (6*a*B*(3*b^2 + 4*a*c) + A*(b^3 - 52*a
*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - (Sqrt[c]*(12*a*b*B
 - A*(b^2 + 20*a*c) + (6*a*B*(3*b^2 + 4*a*c) + A*(b^3 - 52*a*b*c))/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*(b^2 - 4*a*c)))/(4*(b^2 - 4*a*c
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{c^{2}\left(20Aac+b^{2}A-12abB\right)x^{\frac{7}{2}}}{4a\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}+\frac{c\left(28Aabc+2A\,b^{3}+4B\,a^{2}c-19Ba\,b^{2}\right)x^{\frac{5}{2}}}{4a\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}+\frac{\left(36a^{2}A\,c^{2}+5Aa\,b^{2}c+A\,b^{4}-16a^{2}bBc-5Ba\,b^{3}\right)x^{\frac{3}{2}}}{4a\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}+2(1}{\left(c\,x^{2}+bx+a\right)^{2}}$
default	$\frac{\frac{c^{2}\left(20Aac+b^{2}A-12abB\right)x^{\frac{7}{2}}}{4a\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}+\frac{c\left(28Aabc+2A\,b^{3}+4B\,a^{2}c-19Ba\,b^{2}\right)x^{\frac{5}{2}}}{4a\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}+\frac{\left(36a^{2}A\,c^{2}+5Aa\,b^{2}c+A\,b^{4}-16a^{2}bBc-5Ba\,b^{3}\right)x^{\frac{3}{2}}}{4a\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}+2(1}{\left(c\,x^{2}+bx+a\right)^{2}}$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*c^2*(20*A*a*c+A*b^2-12*B*a*b)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+
1/8/a*c*(28*A*a*b*c+2*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^(5/2)+1/8*(36*A*a^2*c^2+5*A*a*b^2*c+A*b^4-16*B*a^2*b*c-5*B*a*b^3)/a/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+1/8*(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b
^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2))/(c*x^2+b*x+a)^2+1/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*c*(1/8*(20*A*a*c*(-4*a*c+b^2)^(1/2)+b^2*A*(-4*a*c+b^2)^(1/2)+
52*A*a*b*c-A*b^3-12*a*b*B*(-4*a*c+b^2)^(1/2)-24*B*a^2*c-18*B*a*b^2)/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(20*A*a*c*(-4*a*c+b^2)^(1/2)+b^
2*A*(-4*a*c+b^2)^(1/2)-52*A*a*b*c+A*b^3-12*a*b*B*(-4*a*c+b^2)^(1/2)+24*B*a
^2*c+18*B*a*b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7267 vs. $2(382) = 764$.

Time = 15.52 (sec) , antiderivative size = 7267, normalized size of antiderivative = 16.29

$$\int \frac{\sqrt{x}(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)*(B*x+A)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \int \frac{(Bx+A)\sqrt{x}}{(cx^2+bx+a)^3}\,dx$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(((b^3*c^2 - 16*a*b*c^3)*A + 3*(a*b^2*c^2 + 4*a^2*c^3)*B)*x^(9/2) + ((
2*b^4*c - 31*a*b^2*c^2 + 20*a^2*c^3)*A + 6*(a*b^3*c + 2*a^2*b*c^2)*B)*x^(7
/2) + ((b^5 - 12*a*b^3*c - 4*a^2*b*c^2)*A + (3*a*b^4 - a^2*b^2*c + 28*a^3*
c^2)*B)*x^(5/2) + (3*(a*b^4 - 9*a^2*b^2*c + 12*a^3*c^2)*A + (a^2*b^3 + 8*a
^3*b*c)*B)*x^(3/2))/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + (a^2*b^4*c^2 - 8
*a^3*b^2*c^3 + 16*a^4*c^4)*x^4 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c
^3)*x^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^2 + 2*(a^3*b^5 - 8*a^4*b^
3*c + 16*a^5*b*c^2)*x) + integrate(-1/8*(((b^3*c - 16*a*b*c^2)*A + 3*(a*b^
2*c + 4*a^2*c^2)*B)*x^(3/2) + ((b^4 - 17*a*b^2*c - 20*a^2*c^2)*A + 3*(a*b^
3 + 8*a^2*b*c)*B)*sqrt(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*
c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7277 vs. $2(382) = 764$.

Time = 1.72 (sec) , antiderivative size = 7277, normalized size of antiderivative = 16.32

$$\int \frac{\sqrt{x}(A + Bx)}{(a + bx + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
-1/32*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*
a^2*b^2*c + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)^2*B - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^9 - 28*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c - 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^6*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 5
6*a^2*b^7*c^2 - 832*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 -
288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sq...
```

Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 19024, normalized size of antiderivative = 42.65

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(1/2)*(A + B*x))/(a + b*x + c*x^2)^3,x)
```

output
```
((x^(3/2)*(A*b^4 + 36*A*a^2*c^2 - 5*B*a*b^3 + 5*A*a*b^2*c - 16*B*a^2*b*c))
/(4*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^(1/2)*(A*b^3 + 3*B*a*b^2 + 12*B
*a^2*c - 16*A*a*b*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(5/2)*(4*B*a
^2*c^2 + 2*A*b^3*c + 28*A*a*b*c^2 - 19*B*a*b^2*c))/(4*a*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)) + (c*x^(7/2)*(20*A*a*c^2 + A*b^2*c - 12*B*a*b*c))/(4*a*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x +
 2*b*c*x^3) + atan(((((64*A*a*b^13*c^2 - 786432*B*a^8*c^8 + 1048576*A*a^7*
b*c^8 - 2304*A*a^2*b^11*c^3 + 30720*A*a^3*b^9*c^4 - 204800*A*a^4*b^7*c^5 +
 737280*A*a^5*b^5*c^6 - 1376256*A*a^6*b^3*c^7 + 192*B*a^2*b^12*c^2 - 3072*
B*a^3*b^10*c^3 + 15360*B*a^4*b^8*c^4 - 245760*B*a^6*b^4*c^6 + 786432*B*a^7
*b^2*c^7)/(64*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 -
 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x^(1/2)*(-(A^
2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-
(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2
*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2
*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2
*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*
a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c -
 b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*
a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*...
```

Reduce [B] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 5692, normalized size of antiderivative = 12.76

$$\int \frac{\sqrt{x}(A+Bx)}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x+a)^3,x)
```

output
```
( - 80*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**2 + 36*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x
)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c - 160*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))
/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**2*x - 160*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*
sqrt(c)*sqrt(a) + b))*a**3*c**3*x**2 + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*s
qrt(a) + b))*a**2*b**4 + 72*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**2*b**3*c*x - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*
c**2*x**2 - 160*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**3*
x**3 - 80*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**4*x**4 + 1
6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**5*x + 52*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*s...
```

3.102 $\int \frac{A+Bx}{\sqrt{x}(a+bx+cx^2)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 468

$$\int \frac{A+Bx}{\sqrt{x}\left(a+bx+cx^2\right)^3}\,dx=\frac{\sqrt{x}(Ab^2-abB-2aAc+(Ab-2aB)cx)}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$
$$+\frac{\sqrt{x}(abB(b^2+8ac)+A(3b^4-25ab^2c+28a^2c^2)+c(aB(b^2+20ac)+3A(b^3-8abc))\,x)}{4a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$
$$+\frac{\sqrt{c}\left(aB(b^2+20ac)+3A(b^3-8abc)+\frac{abB(b^2-52ac)+3A(b^4-10ab^2c+56a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(aB(b^2+20ac)+3A(b^3-8abc)-\frac{abB(b^2-52ac)+3A(b^4-10ab^2c+56a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x^(1/2)*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x)/a/(-4*a*c+b^2)/(c*x^2+b*
x+a)^2+1/4*x^(1/2)*(a*b*B*(8*a*c+b^2)+A*(28*a^2*c^2-25*a*b^2*c+3*b^4)+c*(a
*B*(20*a*c+b^2)+3*A*(-8*a*b*c+b^3))*x)/a^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+1/
8*c^(1/2)*(a*B*(20*a*c+b^2)+3*A*(-8*a*b*c+b^3)+(a*b*B*(-52*a*c+b^2)+3*A*(5
6*a^2*c^2-10*a*b^2*c+b^4))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^2/(b-(-4*a*c+b^
2)^(1/2))^(1/2)+1/8*c^(1/2)*(a*B*(20*a*c+b^2)+3*A*(-8*a*b*c+b^3)-(a*b*B*(-
52*a*c+b^2)+3*A*(56*a^2*c^2-10*a*b^2*c+b^4))/(-4*a*c+b^2)^(1/2))*arctan(2^
(1/2)*c^(1/2)*x^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^
2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 4.90 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)^3}\,dx$$

$$= \frac{\frac{2\sqrt{x}\left(3Ab^3x(b+cx)^2+4a^3c(4bB+11Ac+9Bcx)+a^2\left(-b^3B-4bc^2x(A-7Bx)+4c^3x^2(7A+5Bx)+b^2(-37Ac+5Bcx)\right)+ab(b+cx)\left(bBx(b+cx)+A\right.\right.}{(a+x(b+cx))^2}}{}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(a + b*x + c*x^2)^3),x]
```

output
```
((2*Sqrt[x]*(3*A*b^3*x*(b + c*x)^2 + 4*a^3*c*(4*b*B + 11*A*c + 9*B*c*x) +
a^2*(-(b^3*B) - 4*b*c^2*x*(A - 7*B*x) + 4*c^3*x^2*(7*A + 5*B*x) + b^2*(-37
*A*c + 5*B*c*x)) + a*b*(b + c*x)*(b*B*x*(b + c*x) + A*(5*b^2 - 25*b*c*x -
24*c^2*x^2))))/(a + x*(b + c*x))^2 + (Sqrt[2]*Sqrt[c]*(a*B*(b^3 - 52*a*b*c
 + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(b^4 - 10*a*b^2
*c + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcT
an[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a
*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^3 + 52*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(-b^4 + 10*a*b^2
*c - 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcT
an[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a
*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(8*a^2*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1235, 27, 1235, 27, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)^3}\,dx$$

$\downarrow$ 1235

$$\frac{\sqrt{x}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int -\frac{3Ab^2+aBb-14aAc+5(Ab-2aB)cx}{2\sqrt{x}(cx^2+bx+a)^2}dx}{2a\left(b^2-4ac\right)}$$

$\downarrow$ 27

$$\frac{\int \frac{3Ab^2+aBb-14aAc+5(Ab-2aB)cx}{\sqrt{x}(cx^2+bx+a)^2}dx}{4a\left(b^2-4ac\right)}+\frac{\sqrt{x}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 1235

$$\frac{\frac{\sqrt{x}\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{a(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{abB\left(b^2-16ac\right)+3A\left(b^4-9acb^2+28a^2c^2\right)+c\left(aB\left(b^2+20ac\right)+\right.}{2\sqrt{x}\left(cx^2+bx+a\right)}}{a(b^2-4ac)}}{4a\left(b^2-4ac\right)}$$
$$\frac{\sqrt{x}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{abB\left(b^2-16ac\right)+3A\left(b^4-9acb^2+28a^2c^2\right)+c\left(aB\left(b^2+20ac\right)+3A\left(b^3-8abc\right)\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{2a(b^2-4ac)}+\frac{\sqrt{x}\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\right.}{a(b^2-4ac)(a+bx+cx^2)}}{4a\left(b^2-4ac\right)}$$
$$\frac{\sqrt{x}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 1197

$$\frac{\frac{\int \frac{abB\left(b^2-16ac\right)+3A\left(b^4-9acb^2+28a^2c^2\right)+c\left(aB\left(b^2+20ac\right)+3A\left(b^3-8abc\right)\right)x}{cx^2+bx+a}d\sqrt{x}}{a(b^2-4ac)}+\frac{\sqrt{x}\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right.\right.\right.}{a(b^2-4ac)(a+bx+cx^2)}}{4a\left(b^2-4ac\right)}$$
$$\frac{\sqrt{x}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 1480

$$\frac{\frac{\frac{1}{2}c\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\int \frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}c\left(-\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-\right.}{\sqrt{b^2-4ac}}\right.}{a(b^2-4ac)}}{4a\left(\right.}$$
$$\frac{\sqrt{x}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 218

$$\frac{\frac{\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+abB\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+\cdots\right.}{\sqrt{2}\sqrt{\cdots}}}{a(b^2-4ac)}}{4a\left(b^2-\cdots\right.}$$

$$\frac{\sqrt{x}(cx(Ab-2aB)-2aAc-abB+Ab^2)}{2a\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(a + b*x + c*x^2)^3),x]
```

output
```
(Sqrt[x]*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(2*a*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^2) + ((Sqrt[x]*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4 - 25*a*b
^2*c + 28*a^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c))*x))/(a*(
b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3
 - 8*a*b*c) + (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b^2 + 20*a
*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c
+ 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*(b^2 - 4*
a*c)))/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1555 vs. $2(416) = 832$.

Time = 1.54 (sec) , antiderivative size = 1556, normalized size of antiderivative = 3.32

method	result	size
derivativedivides	Expression too large to display	1556
default	Expression too large to display	1556

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
128*c^3*(-1/64/(-4*a*c+b^2)^(5/2)/c/(4*a*c-b^2)^2*((-1/32/c^2/a^2*(20*a*c*
(-4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)+4*a*b*c-b^3)*(2880*A*(-4*a*c+b^2
)^(1/2)*a^3*c^3-1968*A*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2+444*A*(-4*a*c+b^2)^(
1/2)*a*b^4*c-33*A*(-4*a*c+b^2)^(1/2)*b^6-4416*A*a^3*b*c^3+2736*A*a^2*b^3*c
^2-540*A*a*b^5*c+33*A*b^7+3200*B*a^4*c^3-1248*B*a^3*b^2*c^2+24*B*a^2*b^4*c
+22*B*a*b^6)/(100*a*c+11*b^2)*x^(3/2)+1/16/c^2/a*(-6*b*(-4*a*c+b^2)^(1/2)+
28*a*c-7*b^2)*(4928*A*(-4*a*c+b^2)^(1/2)*a^3*c^3-3504*A*(-4*a*c+b^2)^(1/2)
*a^2*b^2*c^2+828*A*(-4*a*c+b^2)^(1/2)*a*b^4*c-65*A*(-4*a*c+b^2)^(1/2)*b^6-
6464*A*a^3*b*c^3+4272*A*a^2*b^3*c^2-924*A*a*b^5*c+65*A*b^7+6272*B*a^4*c^3-
3552*B*a^3*b^2*c^2+600*B*a^2*b^4*c-26*B*a*b^6)/(196*a*c-13*b^2)*x^(1/2))/(
x+1/2*b/c+1/2/c*(-4*a*c+b^2)^(1/2))^2-1/32*(20*a*c*(-4*a*c+b^2)^(1/2)+b^2*
(-4*a*c+b^2)^(1/2)+52*a*b*c-b^3)*(26880*A*(-4*a*c+b^2)^(1/2)*a^4*c^4-26880
*A*(-4*a*c+b^2)^(1/2)*a^3*b^2*c^3+10080*A*(-4*a*c+b^2)^(1/2)*a^2*b^4*c^2-1
680*A*(-4*a*c+b^2)^(1/2)*a*b^6*c+105*A*(-4*a*c+b^2)^(1/2)*b^8-85248*A*a^4*
b*c^4+61440*A*a^3*b^3*c^3-16416*A*a^2*b^5*c^2+2016*A*a*b^7*c-105*A*b^9+128
00*B*a^5*c^4+13312*B*a^4*b^2*c^3-10176*B*a^3*b^4*c^2+1792*B*a^2*b^6*c-70*B
*a*b^8)/a^2/(400*a^2*c^2+616*a*b^2*c-35*b^4)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+
1/64/(-4*a*c+b^2)^(5/2)/c/(4*a*c-b^2)^2*((-1/32/c^2/a^2*(-20*a*c*(-4*a*c+b
^2)^(1/2)-b^2*(-4*a*c+b^2)^(1/2)+4*a*b*c-b^3)*(-2880*A*(-4*a*c+b^2)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9907 vs. $2(417) = 834$.

Time = 41.30 (sec) , antiderivative size = 9907, normalized size of antiderivative = 21.17

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)^3}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/x**(1/2)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)^3}\,dx=\int \frac{Bx+A}{(cx^2+bx+a)^3\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*((3*(b^4*c^2 - 9*a*b^2*c^3 + 28*a^2*c^4)*A + (a*b^3*c^2 - 16*a^2*b*c^3
)*B)*x^(9/2) + (3*(2*b^5*c - 17*a*b^3*c^2 + 48*a^2*b*c^3)*A + (2*a*b^4*c -
 31*a^2*b^2*c^2 + 20*a^3*c^3)*B)*x^(7/2) + ((3*b^6 - 15*a*b^4*c - 19*a^2*b
^2*c^2 + 196*a^3*c^3)*A + (a*b^5 - 12*a^2*b^3*c - 4*a^3*b*c^2)*B)*x^(5/2)
+ 8*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*A*sqrt(x) + ((9*a*b^5 - 74*a^2*b^
3*c + 164*a^3*b*c^2)*A + 3*(a^2*b^4 - 9*a^3*b^2*c + 12*a^4*c^2)*B)*x^(3/2)
)/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*
a^5*c^4)*x^4 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^3 + (a^3*b^6
 - 6*a^4*b^4*c + 32*a^6*c^3)*x^2 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2
)*x) - integrate(1/8*((3*(b^4*c - 9*a*b^2*c^2 + 28*a^2*c^3)*A + (a*b^3*c -
 16*a^2*b*c^2)*B)*x^(3/2) + (3*(b^5 - 10*a*b^3*c + 36*a^2*b*c^2)*A + (a*b^
4 - 17*a^2*b^2*c - 20*a^3*c^2)*B)*sqrt(x))/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6
*c^2 + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^2 + (a^3*b^5 - 8*a^4*b^3
*c + 16*a^5*b*c^2)*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4621 vs. $2(417) = 834$.

Time = 1.36 (sec) , antiderivative size = 4621, normalized size of antiderivative = 9.87

$$\int \frac{A+Bx}{\sqrt{x}\,(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/16*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
2 + 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^3 - 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2
*c^4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c
^5 - 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^5*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)...
```

Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 22946, normalized size of antiderivative = 49.03

$$\int \frac{A+Bx}{\sqrt{x}\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(1/2)*(a + b*x + c*x^2)^3),x)
```

output
```
((x^(3/2)*(3*A*b^5 + 36*B*a^3*c^2 + B*a*b^4 - 20*A*a*b^3*c - 4*A*a^2*b*c^2
 + 5*B*a^2*b^2*c))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(1/2)*(5*A*
b^4 + 44*A*a^2*c^2 - B*a*b^3 - 37*A*a*b^2*c + 16*B*a^2*b*c))/(4*a*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (x^(5/2)*(28*A*a^2*c^3 + 6*A*b^4*c + 2*B*a*b^3*c
 - 49*A*a*b^2*c^2 + 28*B*a^2*b*c^2))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (c*x^(7/2)*(20*B*a^2*c^2 + 3*A*b^3*c - 24*A*a*b*c^2 + B*a*b^2*c))/(4*a
^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2
*a*b*x + 2*b*c*x^3) + atan((((1048576*B*a^9*b*c^8 - 5505024*A*a^9*c^9 + 1
92*A*a^2*b^14*c^2 - 5568*A*a^3*b^12*c^3 + 70656*A*a^4*b^10*c^4 - 506880*A*
a^5*b^8*c^5 + 2211840*A*a^6*b^6*c^6 - 5849088*A*a^7*b^4*c^7 + 8650752*A*a^
8*b^2*c^8 + 64*B*a^3*b^13*c^2 - 2304*B*a^4*b^11*c^3 + 30720*B*a^5*b^9*c^4
- 204800*B*a^6*b^7*c^5 + 737280*B*a^7*b^5*c^6 - 1376256*B*a^8*b^3*c^7)/(64
*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^
6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (x^(1/2)*(-(9*A^2*b^19 + B
^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^
2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 285177
6*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 2
7095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*
a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*
b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a...
```

Reduce [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 6756, normalized size of antiderivative = 14.44

$$\int \frac{A+Bx}{\sqrt{x}\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x+a)^3,x)
```

output
```
(184*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c**2 - 102*sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(
x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c + 368*sqrt(a)*sqrt(2*
sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c)
)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**2*x + 368*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqr
t(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**3*x**2 + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt
(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqr
t(c)*sqrt(a) + b))*a**2*b**5 - 204*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a**2*b**4*c*x - 20*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*2*b**3*c**2*x**2 + 368*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2
*b**2*c**3*x**3 + 184*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b
*c**4*x**4 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**6*x - 8
6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)...
```

3.103 $\int \frac{A+Bx}{x^{3/2}(a+bx+cx^2)^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 664

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \frac{3(abB(b^2-8ac)-A(5b^4-37ab^2c+60a^2c^2))}{4a^3\left(b^2-4ac\right)^2\sqrt{x}}$$

$$+\frac{Ab^2-abB-2aAc+(Ab-2aB)cx}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx+cx^2\right)^2}$$

$$-\frac{abB(b^2-16ac)-A(5b^4-35ab^2c+36a^2c^2)+c(aB(b^2-28ac)-A(5b^3-32abc))x}{4a^2\left(b^2-4ac\right)^2\sqrt{x}\left(a+bx+cx^2\right)}$$

$$+\frac{3\sqrt{c}\left(aB\left(b^4-10ab^2c+56a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)-A\left(5b^5-47ab^3c+124a^2bc^2+5b^4\sqrt{b^2 \cdots}\right.\right.}{4\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4a \cdots}}}$$

$$-\frac{3\sqrt{c}\left(aB\left(b^4-10ab^2c+56a^2c^2-b^3\sqrt{b^2-4ac}+8abc\sqrt{b^2-4ac}\right)-A\left(5b^5-47ab^3c+124a^2bc^2-5b^4\sqrt{b \cdots}\right.\right.}{4\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4 \cdots}}}$$

output
```
3/4*(a*b*B*(-8*a*c+b^2)-A*(60*a^2*c^2-37*a*b^2*c+5*b^4))/a^3/(-4*a*c+b^2)^
2/x^(1/2)+1/2*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x)/a/(-4*a*c+b^2)/x^(1/2)
/(c*x^2+b*x+a)^2-1/4*(a*b*B*(-16*a*c+b^2)-A*(36*a^2*c^2-35*a*b^2*c+5*b^4)+
c*(a*B*(-28*a*c+b^2)-A*(-32*a*b*c+5*b^3))*x)/a^2/(-4*a*c+b^2)^2/x^(1/2)/(c
*x^2+b*x+a)+3/8*c^(1/2)*(a*B*(b^4-10*a*b^2*c+56*a^2*c^2+b^3*(-4*a*c+b^2)^(
1/2)-8*a*b*c*(-4*a*c+b^2)^(1/2))-A*(5*b^5-47*a*b^3*c+124*a^2*b*c^2+5*b^4*(
-4*a*c+b^2)^(1/2)-37*a*b^2*c*(-4*a*c+b^2)^(1/2)+60*a^2*c^2*(-4*a*c+b^2)^(1
/2)))*arctan(2^(1/2)*c^(1/2)*x^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)
/a^3/(-4*a*c+b^2)^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*c^(1/2)*(a*B*(b^4
-10*a*b^2*c+56*a^2*c^2-b^3*(-4*a*c+b^2)^(1/2)+8*a*b*c*(-4*a*c+b^2)^(1/2))-
A*(5*b^5-47*a*b^3*c+124*a^2*b*c^2-5*b^4*(-4*a*c+b^2)^(1/2)+37*a*b^2*c*(-4*
a*c+b^2)^(1/2)-60*a^2*c^2*(-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)*x^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(-4*a*c+b^2)^(5/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 12.51 (sec) , antiderivative size = 628, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \frac{\frac{2(-aB(b+2cx)+A(b^2-2ac+bcx))}{\sqrt{x}(a+x(b+cx))^2}+\frac{aB(b^3-16abc+b^2cx-28ac^2x)+A(-5b^4+35ab^2c-36a^2c^2-5b^3cx+}{a(-b^2+4ac)\sqrt{x}(a+x(b+cx))}}{}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(a + b*x + c*x^2)^3),x]
```

output
```
((2*(-(a*B*(b + 2*c*x)) + A*(b^2 - 2*a*c + b*c*x)))/(Sqrt[x]*(a + x*(b + c
*x))^2) + (a*B*(b^3 - 16*a*b*c + b^2*c*x - 28*a*c^2*x) + A*(-5*b^4 + 35*a*
b^2*c - 36*a^2*c^2 - 5*b^3*c*x + 32*a*b*c^2*x))/(a*(-b^2 + 4*a*c)*Sqrt[x]*
(a + x*(b + c*x))) + ((3*(a*b*B*(b^2 - 8*a*c) + A*(-5*b^4 + 37*a*b^2*c - 6
0*a^2*c^2)))/Sqrt[x] + (3*Sqrt[c]*(-(((-(a*B*(b^4 - 10*a*b^2*c + 56*a^2*c^
2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])) + A*(5*b^5 - 47*a*
b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*
a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((a*B*(b^4 - 10*
a*b^2*c + 56*a^2*c^2 - b^3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])
+ A*(-5*b^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*
b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*S
qrt[c]*Sqrt[x])/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])
)/(Sqrt[2]*Sqrt[b^2 - 4*a*c]))/(a^2*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 578, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1235, 27, 1235, 27, 1198, 25, 1197, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^3}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int -\frac{5Ab^2-aBb-18aAc+7(Ab-2aB)cx}{2x^{3/2}\left(cx^2+bx+a\right)^2}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5Ab^2-aBb-18aAc+7(Ab-2aB)cx}{x^{3/2}\left(cx^2+bx+a\right)^2}dx}{4a\left(b^2-4ac\right)}+\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1235}$$

$$-\frac{\frac{\int \frac{3\left(abB\left(b^2-8ac\right)-A\left(5b^4-37acb^2+60a^2c^2\right)+c\left(aB\left(b^2-28ac\right)-A\left(5b^3-32abc\right)\right)x\right)}{2x^{3/2}\left(cx^2+bx+a\right)}dx}{a(b^2-4ac)}-\frac{-A(36a^2c^2-35ab^2c+5b^4)+cx\left(aB\left(b^2-28ac\right)-A\left(5b^3-32\cdots\right.\right.}{a\sqrt{x}(b^2-4ac)(a+bx+cx^2)}}{4a\left(b^2-4ac\right)}$$
$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{3\int \frac{abB\left(b^2-8ac\right)-A\left(5b^4-37acb^2+60a^2c^2\right)+c\left(aB\left(b^2-28ac\right)-A\left(5b^3-32abc\right)\right)x}{x^{3/2}\left(cx^2+bx+a\right)}dx}{2a(b^2-4ac)}-\frac{-A(36a^2c^2-35ab^2c+5b^4)+cx\left(aB\left(b^2-28ac\right)-A\left(5b^3-32a\cdots\right.\right.}{a\sqrt{x}(b^2-4ac)(a+bx+cx^2)}}{4a\left(b^2-4ac\right)}$$
$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \; 1198$$

$$-\frac{\frac{3\left(\frac{\int -\frac{aB\left(b^4-9acb^2+28a^2c^2\right)-A\left(5b^5-42acb^3+92a^2c^2b\right)+c\left(abB\left(b^2-8ac\right)-A\left(5b^4-37acb^2+60a^2c^2\right)\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2\left(abB\left(b^2-8ac\right)-A\left(60a^2c^2-37ab^2c+5b^4\right)\cdots\right.}{a\sqrt{x}}\right.}{2a(b^2-4ac)}}{4a\left(b^2-4ac\right)}$$
$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{3\left(-\frac{\int \frac{aB\left(b^4-9acb^2+28a^2c^2\right)-A\left(5b^5-42acb^3+92a^2c^2b\right)+c\left(abB\left(b^2-8ac\right)-A\left(5b^4-37acb^2+60a^2c^2\right)\right)x}{\sqrt{x}\left(cx^2+bx+a\right)}dx}{a}-\frac{2\left(abB\left(b^2-8ac\right)-A\left(60a^2c^2-37ab^2c+5b^4\right)\cdots\right.}{a\sqrt{x}}\right.}{2a(b^2-4ac)}}{4a\left(b^2-4ac\right)}$$
$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \; 1197$$

$$-\frac{3\left(-\frac{2\int \frac{aB\left(b^4-9acb^2+28a^2c^2\right)-A\left(5b^5-42acb^3+92a^2c^2b\right)+c\left(abB\left(b^2-8ac\right)-A\left(5b^4-37acb^2+60a^2c^2\right)\right)x}{cx^2+bx+a}d\sqrt{x}}{a}-\frac{2\left(abB\left(b^2-8ac\right)-A\left(60a^2c^2-37ab^2c+5b\right.\right.}{a\sqrt{x}}\right.}{2a(b^2-4ac)}$$

$$4a\left(b^2-4ac\right)$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 1480

$$-\frac{3\left(-\frac{2\left(\frac{1}{2}c\left(-A\left(60a^2c^2-37ab^2c+5b^4\right)+\frac{aB\left(56a^2c^2-10ab^2c+b^4\right)-A\left(124a^2bc^2-47ab^3c+5b^5\right)}{\sqrt{b^2-4ac}}+abB\left(b^2-8ac\right)\right)\int\frac{1}{\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)+cx}d\sqrt{x}+\frac{1}{2}c\left(-A\left(60\right.\right.\right.}{a}\right.}{}$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\Big\downarrow$ 218

$$-\frac{3\left(-\frac{2\left(\frac{\sqrt{c}\left(-A\left(60a^2c^2-37ab^2c+5b^4\right)+\frac{aB\left(56a^2c^2-10ab^2c+b^4\right)-A\left(124a^2bc^2-47ab^3c+5b^5\right)}{\sqrt{b^2-4ac}}+abB\left(b^2-8ac\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-A\left(60a^2\right.\right.}{}\right.}{a}\right.}{}$$

$$\frac{cx(Ab-2aB)-2aAc-abB+Ab^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input
```
Int[(A + B*x)/(x^(3/2)*(a + b*x + c*x^2)^3),x]
```

output
```
(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x)/(2*a*(b^2 - 4*a*c)*Sqrt[x]*(
a + b*x + c*x^2)^2) + (-((a*b*B*(b^2 - 16*a*c) - A*(5*b^4 - 35*a*b^2*c + 3
6*a^2*c^2) + c*(a*B*(b^2 - 28*a*c) - A*(5*b^3 - 32*a*b*c))*x)/(a*(b^2 - 4*
a*c)*Sqrt[x]*(a + b*x + c*x^2))) - (3*((-2*(a*b*B*(b^2 - 8*a*c) - A*(5*b^4
 - 37*a*b^2*c + 60*a^2*c^2)))/(a*Sqrt[x]) - (2*((Sqrt[c]*(a*b*B*(b^2 - 8*a
*c) - A*(5*b^4 - 37*a*b^2*c + 60*a^2*c^2) + (a*B*(b^4 - 10*a*b^2*c + 56*a^
2*c^2) - A*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[x])/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*b*B*(b^2 - 8*a*c) - A*(5*b^4 - 37*a*b^2
*c + 60*a^2*c^2) - (a*B*(b^4 - 10*a*b^2*c + 56*a^2*c^2) - A*(5*b^5 - 47*a*
b^3*c + 124*a^2*b*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[x]
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/a)
)/(2*a*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{c^2\left(52a^2Ac^2-47Aab^2c+7Ab^4+24a^2bBc-3Bab^3\right)x^{\frac{7}{2}}}{128a^2c^2-64cab^2+8b^4}+\frac{c\left(136Aa^2bc^2-99Aab^3c+14Ab^5-28Ba^3c^2+49Ba^2b\right.}{128a^2c^2-64cab^2+8b^4}\right.}{}$
default	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{c^2\left(52a^2Ac^2-47Aab^2c+7Ab^4+24a^2bBc-3Bab^3\right)x^{\frac{7}{2}}}{128a^2c^2-64cab^2+8b^4}+\frac{c\left(136Aa^2bc^2-99Aab^3c+14Ab^5-28Ba^3c^2+49Ba^2b\right.}{128a^2c^2-64cab^2+8b^4}\right.}{}$
risch	$-\frac{2A}{a^3\sqrt{x}}-\frac{\frac{2c^2\left(52a^2Ac^2-47Aab^2c+7Ab^4+24a^2bBc-3Bab^3\right)x^{\frac{7}{2}}}{128a^2c^2-64cab^2+8b^4}+\frac{2c\left(136Aa^2bc^2-99Aab^3c+14Ab^5-28Ba^3c^2+49Ba^2b^2\right.}{128a^2c^2-64cab^2+8b^4}}{}$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*A/a^3/x^(1/2)-2/a^3*((1/8*c^2*(52*A*a^2*c^2-47*A*a*b^2*c+7*A*b^4+24*B*a
^2*b*c-3*B*a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+1/8*c*(136*A*a^2*b*c^
2-99*A*a*b^3*c+14*A*b^5-28*B*a^3*c^2+49*B*a^2*b^2*c-6*B*a*b^4)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^(5/2)+1/8*(68*A*a^3*c^3+25*A*a^2*b^2*c^2-43*A*a*b^4*c+7*
A*b^6+4*B*a^3*b*c^2+20*B*a^2*b^3*c-3*B*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^(3/2)+1/8*a*(108*A*a^2*b*c^2-66*A*a*b^3*c+9*A*b^5-44*B*a^3*c^2+37*B*a^2*b
^2*c-5*B*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2))/(c*x^2+b*x+a)^2+3/2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*c*(1/8*(60*a^2*A*c^2*(-4*a*c+b^2)^(1/2)-37*A*a*b^
2*c*(-4*a*c+b^2)^(1/2)+5*A*b^4*(-4*a*c+b^2)^(1/2)-124*A*a^2*b*c^2+47*A*a*b
^3*c-5*A*b^5+8*a^2*b*B*c*(-4*a*c+b^2)^(1/2)-B*a*b^3*(-4*a*c+b^2)^(1/2)+56*
B*a^3*c^2-10*B*a^2*b^2*c+B*a*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(x^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))-1/8*(60*a^2*A*c^2*(-4*a*c+b^2)^(1/2)-37*A*a*b^2*c*(-4*a*c+b^2)^(1/2)+
5*A*b^4*(-4*a*c+b^2)^(1/2)+124*A*a^2*b*c^2-47*A*a*b^3*c+5*A*b^5+8*a^2*b*B*
c*(-4*a*c+b^2)^(1/2)-B*a*b^3*(-4*a*c+b^2)^(1/2)-56*B*a^3*c^2+10*B*a^2*b^2*
c-B*a*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(c*x^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12534 vs. $2(589) = 1178$.

Time = 122.44 (sec) , antiderivative size = 12534, normalized size of antiderivative = 18.88

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/x**(3/2)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9534 vs. 2(589) = 1178.

Time = 1.70 (sec) , antiderivative size = 9534, normalized size of antiderivative = 14.36

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
-3/32*((10*b^6*c^2 - 114*a*b^4*c^3 + 416*a^2*b^2*c^4 - 480*a^3*c^5 - 5*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 57*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 208*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 74*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 240*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 120*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 37*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 60*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 74*
(b^2 - 4*a*c)*a*b^2*c^3 - 120*(b^2 - 4*a*c)*a^2*c^4)*(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)^2*A - (2*a*b^5*c^2 - 24*a^2*b^3*c^3 + 64*a^3*b*c^4 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 12*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 32*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3...
```

Mupad [B] (verification not implemented)

Time = 17.82 (sec) , antiderivative size = 29137, normalized size of antiderivative = 43.88

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/(x^(3/2)*(a + b*x + c*x^2)^3),x)
```

output
```
- atan(((x^(1/2)*(33973862400*A^2*a^20*c^14 - 7398752256*B^2*a^21*c^13 - 2
8800*A^2*a^9*b^22*c^3 + 1232640*A^2*a^10*b^20*c^4 - 23879808*A^2*a^11*b^18
*c^5 + 275975424*A^2*a^12*b^16*c^6 - 2109763584*A^2*a^13*b^14*c^7 + 111718
56384*A^2*a^14*b^12*c^8 - 41653370880*A^2*a^15*b^10*c^9 + 108726976512*A^2
*a^16*b^8*c^10 - 192980975616*A^2*a^17*b^6*c^11 + 218414186496*A^2*a^18*b^
4*c^12 - 137631891456*A^2*a^19*b^2*c^13 - 1152*B^2*a^11*b^20*c^3 + 50688*B
^2*a^12*b^18*c^4 - 1025280*B^2*a^13*b^16*c^5 + 12496896*B^2*a^14*b^14*c^6
- 101744640*B^2*a^15*b^12*c^7 + 579796992*B^2*a^16*b^10*c^8 - 2346319872*B
^2*a^17*b^8*c^9 + 6653214720*B^2*a^18*b^6*c^10 - 12608077824*B^2*a^19*b^4*
c^11 + 14344519680*B^2*a^20*b^2*c^12 + 11520*A*B*a^10*b^21*c^3 - 499968*A*
B*a^11*b^19*c^4 + 9900288*A*B*a^12*b^17*c^5 - 117559296*A*B*a^13*b^15*c^6
+ 925433856*A*B*a^14*b^13*c^7 - 5038866432*A*B*a^15*b^11*c^8 + 19191693312
*A*B*a^16*b^9*c^9 - 50422874112*A*B*a^17*b^7*c^10 + 87350575104*A*B*a^18*b
^5*c^11 - 89992986624*A*B*a^19*b^3*c^12 + 41825599488*A*B*a^20*b*c^13) + (
-(9*(25*A^2*b^21 + B^2*a^2*b^19 + 25*A^2*b^6*(-(4*a*c - b^2)^15)^(1/2) - 1
0*A*B*a*b^20 + 17794*A^2*a^2*b^17*c^2 - 188095*A^2*a^3*b^15*c^3 + 1299860*
A^2*a^4*b^13*c^4 - 6126640*A^2*a^5*b^11*c^5 + 19905600*A^2*a^6*b^9*c^6 - 4
3904256*A^2*a^7*b^7*c^7 + 62684160*A^2*a^8*b^5*c^8 - 52039680*A^2*a^9*b^3*
c^9 - 225*A^2*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^4*(-(4*a*c - b
^2)^15)^(1/2) + 769*B^2*a^4*b^15*c^2 - 8620*B^2*a^5*b^13*c^3 + 63440*B^...
```

Reduce [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 8137, normalized size of antiderivative = 12.25

$$\int \frac{A+Bx}{x^{3/2}\left(a+bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x+a)^3,x)
```

output
```
(720*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*c**3 - 732
*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**2*c**2 + 14
40*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c**3*x + 1
440*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**4*x**2 +
 246*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**4*c - 1
464*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c**2*x
 - 744*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**
3*x**2 + 1440*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b
*c**4*x**3 + 720*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*a**
3*c**5*x**4 - 24*sqrt(x)*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(x)*sqrt(c))/sqrt(2*sqrt(c)*sqrt(a) + b))*...
```

3.104 $\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 367

$$\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx = -\frac{(33b^5B-42Ab^4c-120ab^3Bc+112aAb^2c^2+80a^2bBc^2-32a^2Ac^3)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^6} + \frac{(33b^2B-42Abc-32aBc)x^2(a+bx+cx^2)^{3/2}}{280c^3} - \frac{(11bB-14Ac)x^3(a+bx+cx^2)^{3/2}}{84c^2} + \frac{Bx^4(a+bx+cx^2)^{3/2}}{7c} + \frac{(1155b^4B-1470Ab^3c-3276ab^2Bc+2744aAbc^2+1024a^2Bc^2-6c(231b^3B-294Ab^2c-444abBc+}{13440c^5} + \frac{(b^2-4ac)(33b^5B-42Ab^4c-120ab^3Bc+112aAb^2c^2+80a^2bBc^2-32a^2Ac^3)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{13/2}}$$

output

```
-1/1024*(-32*A*a^2*c^3+112*A*a*b^2*c^2-42*A*b^4*c+80*B*a^2*b*c^2-120*B*a*b
^3*c+33*B*b^5)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6+1/280*(-42*A*b*c-32*B*a*c
+33*B*b^2)*x^2*(c*x^2+b*x+a)^(3/2)/c^3-1/84*(-14*A*c+11*B*b)*x^3*(c*x^2+b*
x+a)^(3/2)/c^2+1/7*B*x^4*(c*x^2+b*x+a)^(3/2)/c+1/13440*(1155*B*b^4-1470*A*
b^3*c-3276*B*a*b^2*c+2744*A*a*b*c^2+1024*B*a^2*c^2-6*c*(280*A*a*c^2-294*A*
b^2*c-444*B*a*b*c+231*B*b^3)*x)*(c*x^2+b*x+a)^(3/2)/c^5+1/2048*(-4*a*c+b^2
)*(-32*A*a^2*c^3+112*A*a*b^2*c^2-42*A*b^4*c+80*B*a^2*b*c^2-120*B*a*b^3*c+3
3*B*b^5)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)
```

Mathematica [A] (verified)

Time = 2.07 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.95

$$\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$=\frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^6B+210b^5c(21A+11Bx)+84b^4c(260aB-cx(35A+22Bx))+48b^3c^2(-\cdots}{}$$

input
```
Integrate[x^4*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^6*B + 210*b^5*c*(21*A + 11*B*x)
+ 84*b^4*c*(260*a*B - c*x*(35*A + 22*B*x)) + 48*b^3*c^2*(-14*a*(35*A + 18*
B*x) + c*x^2*(49*A + 33*B*x)) + 64*c^3*(128*a^3*B + 40*c^3*x^5*(7*A + 6*B*
x) + 2*a*c^2*x^3*(35*A + 24*B*x) - a^2*c*x*(105*A + 64*B*x)) - 16*b^2*c^2*
(2163*a^2*B + 2*c^2*x^3*(63*A + 44*B*x) - 2*a*c*x*(392*A + 243*B*x)) + 32*
b*c^3*(8*c^2*x^4*(7*A + 5*B*x) - 2*a*c*x^2*(119*A + 79*B*x) + a^2*(791*A +
 397*B*x))) - 105*(b^2 - 4*a*c)*(33*b^5*B - 42*A*b^4*c - 120*a*b^3*B*c + 1
12*a*A*b^2*c^2 + 80*a^2*b*B*c^2 - 32*a^2*A*c^3)*Log[b + 2*c*x - 2*Sqrt[c]*
Sqrt[a + x*(b + c*x)]])/(215040*c^(13/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1236, 27, 1236, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}x^3(8aB+(11bB-14Ac)x)\sqrt{cx^2+bx+a}dx}{7c}+\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\int x^3(8aB+(11bB-14Ac)x)\sqrt{cx^2+bx+a}dx}{14c}$$

$\downarrow$ 1236

$$\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{\int -\frac{3}{2}x^2(2a(11bB-14Ac)+(33Bb^2-42Acb-32aBc)x)\sqrt{cx^2+bx+a}dx}{6c}+\frac{x^3\left(a+bx+cx^2\right)^{3/2}(11bB-14Ac)}{6c}}{14c}$$

$\downarrow$ 27

$$\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{3/2}(11bB-14Ac)}{6c}-\frac{\int x^2(2a(11bB-14Ac)+(33Bb^2-42Acb-32aBc)x)\sqrt{cx^2+bx+a}dx}{4c}}{14c}$$

$\downarrow$ 1236

$$\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{3/2}(11bB-14Ac)}{6c}-\frac{\frac{\int -\frac{1}{2}x\left(4a\left(33Bb^2-42Acb-32aBc\right)+\left(231Bb^3-294Acb^2-444aBcb+280aAc^2\right)x\right)\sqrt{cx^2+bx+a}dx}{5c}+\frac{x^2\left(a+bx+cx^2\right)^{3/2}(}{}}{4c}}{14c}$$

$\downarrow$ 27

$$\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{3/2}(11bB-14Ac)}{6c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{3/2}\left(-32aBc-42Abc+33b^2B\right)}{5c}-\frac{\int x\left(4a\left(33Bb^2-42Acb-32aBc\right)+\left(231Bb^3-294Acb^2-444aBcb+280aA\right.\right.}{10c}}{4c}}{14c}$$

$\downarrow$ 1225

$$\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{3/2}(11bB-14Ac)}{6c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{3/2}\left(-32aBc-42Abc+33b^2B\right)}{5c}-\frac{\frac{35\left(-32a^2Ac^3+80a^2bBc^2+112aAb^2c^2-120ab^3Bc-42Ab^4c+33b^5B\right)}{16c^2}}{}}{}}{14c}$$

$\downarrow$ 1087

$$\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}-$$

$$\frac{x^3\left(a+bx+cx^2\right)^{3/2}(11bB-14Ac)}{6c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{3/2}\left(-32aBc-42Abc+33b^2B\right)}{5c}-\frac{35\left(-32a^2Ac^3+80a^2bBc^2+112aAb^2c^2-120ab^3Bc-42Ab^4c+33b^5B\right)\left(\right.}{16c^2}}{}$$

$\big\downarrow$ 1092

$$\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}-$$

$$\frac{x^3\left(a+bx+cx^2\right)^{3/2}(11bB-14Ac)}{6c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{3/2}\left(-32aBc-42Abc+33b^2B\right)}{5c}-\frac{35\left(-32a^2Ac^3+80a^2bBc^2+112aAb^2c^2-120ab^3Bc-42Ab^4c+33b^5B\right)\left(\right.}{16}}{}$$

$\big\downarrow$ 219

$$\frac{Bx^4\left(a+bx+cx^2\right)^{3/2}}{7c}-$$

$$\frac{x^3\left(a+bx+cx^2\right)^{3/2}(11bB-14Ac)}{6c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{3/2}\left(-32aBc-42Abc+33b^2B\right)}{5c}-\frac{35\left(-32a^2Ac^3+80a^2bBc^2+112aAb^2c^2-120ab^3Bc-42Ab^4c+33b^5B\right)\left(\right.}{16}}{}$$

input
```
Int[x^4*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(B*x^4*(a + b*x + c*x^2)^(3/2))/(7*c) - (((11*b*B - 14*A*c)*x^3*(a + b*x +
 c*x^2)^(3/2))/(6*c) - (((33*b^2*B - 42*A*b*c - 32*a*B*c)*x^2*(a + b*x + c
*x^2)^(3/2))/(5*c) - (-1/24*((1155*b^4*B - 1470*A*b^3*c - 3276*a*b^2*B*c +
 2744*a*A*b*c^2 + 1024*a^2*B*c^2 - 6*c*(231*b^3*B - 294*A*b^2*c - 444*a*b*
B*c + 280*a*A*c^2)*x)*(a + b*x + c*x^2)^(3/2))/c^2 + (35*(33*b^5*B - 42*A*
b^4*c - 120*a*b^3*B*c + 112*a*A*b^2*c^2 + 80*a^2*b*B*c^2 - 32*a^2*A*c^3)*(
((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c^2))/(10*c))/(
4*c))/(14*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(15360B\,c^6x^6+17920A\,c^6x^5+1280Bb\,c^5x^5+1792Ab\,c^5x^4+3072Ba\,c^5x^4-1408B\,b^2c^4x^4+4480Aa\,c^5x^3-2016A\,b^2c^4x^3-5056Bab\,c}{}$
default	Expression too large to display

input
```
int(x^4*(B*x+A)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/107520*(15360*B*c^6*x^6+17920*A*c^6*x^5+1280*B*b*c^5*x^5+1792*A*b*c^5*x^
4+3072*B*a*c^5*x^4-1408*B*b^2*c^4*x^4+4480*A*a*c^5*x^3-2016*A*b^2*c^4*x^3-
5056*B*a*b*c^4*x^3+1584*B*b^3*c^3*x^3-7616*A*a*b*c^4*x^2+2352*A*b^3*c^3*x^
2-4096*B*a^2*c^4*x^2+7776*B*a*b^2*c^3*x^2-1848*B*b^4*c^2*x^2-6720*A*a^2*c^
4*x+12544*A*a*b^2*c^3*x-2940*A*b^4*c^2*x+12704*B*a^2*b*c^3*x-12096*B*a*b^3
*c^2*x+2310*B*b^5*c*x+25312*A*a^2*b*c^3-23520*A*a*b^3*c^2+4410*A*b^5*c+819
2*B*a^3*c^3-34608*B*a^2*b^2*c^2+21840*B*a*b^4*c-3465*B*b^6)/c^6*(c*x^2+b*x
+a)^(1/2)+1/2048*(128*A*a^3*c^4-480*A*a^2*b^2*c^3+280*A*a*b^4*c^2-42*A*b^6
*c-320*B*a^3*b*c^3+560*B*a^2*b^3*c^2-252*B*a*b^5*c+33*B*b^7)/c^(13/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 843, normalized size of antiderivative = 2.30

$$\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/430080*(105*(33*B*b^7 + 128*A*a^3*c^4 - 160*(2*B*a^3*b + 3*A*a^2*b^2)*c
^3 + 280*(2*B*a^2*b^3 + A*a*b^4)*c^2 - 42*(6*B*a*b^5 + A*b^6)*c)*sqrt(c)*l
og(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c
) - 4*a*c) + 4*(15360*B*c^7*x^6 - 3465*B*b^6*c + 1280*(B*b*c^6 + 14*A*c^7)
*x^5 + 32*(256*B*a^3 + 791*A*a^2*b)*c^4 - 128*(11*B*b^2*c^5 - 2*(12*B*a +
7*A*b)*c^6)*x^4 - 336*(103*B*a^2*b^2 + 70*A*a*b^3)*c^3 + 16*(99*B*b^3*c^4
+ 280*A*a*c^6 - 2*(158*B*a*b + 63*A*b^2)*c^5)*x^3 + 210*(104*B*a*b^4 + 21*
A*b^5)*c^2 - 8*(231*B*b^4*c^3 + 8*(64*B*a^2 + 119*A*a*b)*c^5 - 6*(162*B*a*
b^2 + 49*A*b^3)*c^4)*x^2 + 2*(1155*B*b^5*c^2 - 3360*A*a^2*c^5 + 16*(397*B*
a^2*b + 392*A*a*b^2)*c^4 - 42*(144*B*a*b^3 + 35*A*b^4)*c^3)*x)*sqrt(c*x^2
+ b*x + a))/c^7, -1/215040*(105*(33*B*b^7 + 128*A*a^3*c^4 - 160*(2*B*a^3*b
 + 3*A*a^2*b^2)*c^3 + 280*(2*B*a^2*b^3 + A*a*b^4)*c^2 - 42*(6*B*a*b^5 + A*
b^6)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^
2*x^2 + b*c*x + a*c)) - 2*(15360*B*c^7*x^6 - 3465*B*b^6*c + 1280*(B*b*c^6
+ 14*A*c^7)*x^5 + 32*(256*B*a^3 + 791*A*a^2*b)*c^4 - 128*(11*B*b^2*c^5 - 2
*(12*B*a + 7*A*b)*c^6)*x^4 - 336*(103*B*a^2*b^2 + 70*A*a*b^3)*c^3 + 16*(99
*B*b^3*c^4 + 280*A*a*c^6 - 2*(158*B*a*b + 63*A*b^2)*c^5)*x^3 + 210*(104*B*
a*b^4 + 21*A*b^5)*c^2 - 8*(231*B*b^4*c^3 + 8*(64*B*a^2 + 119*A*a*b)*c^5 -
6*(162*B*a*b^2 + 49*A*b^3)*c^4)*x^2 + 2*(1155*B*b^5*c^2 - 3360*A*a^2*c^5 +
 16*(397*B*a^2*b + 392*A*a*b^2)*c^4 - 42*(144*B*a*b^3 + 35*A*b^4)*c^3)*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(401) = 802$.

Time = 0.70 (sec) , antiderivative size = 1292, normalized size of antiderivative = 3.52

$$\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-a*(-3*a*(A*a - 5*a*(A*c + B*b/14)/(6*c) - 9*b*(A*b + B*a/7 -
11*b*(A*c + B*b/14)/(12*c))/(10*c))/(4*c) - 5*b*(-4*a*(A*b + B*a/7 - 11*b*
(A*c + B*b/14)/(12*c))/(5*c) - 7*b*(A*a - 5*a*(A*c + B*b/14)/(6*c) - 9*b*(
A*b + B*a/7 - 11*b*(A*c + B*b/14)/(12*c))/(10*c))/(8*c))/(6*c))/(2*c) - b*
(-2*a*(-4*a*(A*b + B*a/7 - 11*b*(A*c + B*b/14)/(12*c))/(5*c) - 7*b*(A*a -
5*a*(A*c + B*b/14)/(6*c) - 9*b*(A*b + B*a/7 - 11*b*(A*c + B*b/14)/(12*c))/
(10*c))/(8*c))/(3*c) - 3*b*(-3*a*(A*a - 5*a*(A*c + B*b/14)/(6*c) - 9*b*(A*
b + B*a/7 - 11*b*(A*c + B*b/14)/(12*c))/(10*c))/(4*c) - 5*b*(-4*a*(A*b + B
*a/7 - 11*b*(A*c + B*b/14)/(12*c))/(5*c) - 7*b*(A*a - 5*a*(A*c + B*b/14)/(
6*c) - 9*b*(A*b + B*a/7 - 11*b*(A*c + B*b/14)/(12*c))/(10*c))/(8*c))/(6*c)
)/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*
x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c
*(b/(2*c) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(B*x**6/7 + x**5*(A*c
+ B*b/14)/(6*c) + x**4*(A*b + B*a/7 - 11*b*(A*c + B*b/14)/(12*c))/(5*c) +
x**3*(A*a - 5*a*(A*c + B*b/14)/(6*c) - 9*b*(A*b + B*a/7 - 11*b*(A*c + B*b/
14)/(12*c))/(10*c))/(4*c) + x**2*(-4*a*(A*b + B*a/7 - 11*b*(A*c + B*b/14)/
(12*c))/(5*c) - 7*b*(A*a - 5*a*(A*c + B*b/14)/(6*c) - 9*b*(A*b + B*a/7 - 1
1*b*(A*c + B*b/14)/(12*c))/(10*c))/(8*c))/(3*c) + x*(-3*a*(A*a - 5*a*(A*c
+ B*b/14)/(6*c) - 9*b*(A*b + B*a/7 - 11*b*(A*c + B*b/14)/(12*c))/(10*c))/(
4*c) - 5*b*(-4*a*(A*b + B*a/7 - 11*b*(A*c + B*b/14)/(12*c))/(5*c) - 7*b...
```

Maxima [F(-2)]

Exception generated.

$$\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.12

$$\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$=\frac{1}{107520}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(12\,Bx+\frac{Bbc^5+14\,Ac^6}{c^6}\right)x-\frac{11\,Bb^2c^4-24\,Bac^5-14\,Abc^5}{c^6}\right)\right.\right.\right.\right.$$
$$-\frac{(33\,Bb^7-252\,Bab^5c-42\,Ab^6c+560\,Ba^2b^3c^2+280\,Aab^4c^2-320\,Ba^3bc^3-480\,Aa^2b^2c^3+128\,Aa^3c^4}{2048\,c^{\frac{13}{2}}}$$

input
```
integrate(x^4*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*B*x + (B*b*c^5 + 14*A*c
^6)/c^6)*x - (11*B*b^2*c^4 - 24*B*a*c^5 - 14*A*b*c^5)/c^6)*x + (99*B*b^3*c
^3 - 316*B*a*b*c^4 - 126*A*b^2*c^4 + 280*A*a*c^5)/c^6)*x - (231*B*b^4*c^2
- 972*B*a*b^2*c^3 - 294*A*b^3*c^3 + 512*B*a^2*c^4 + 952*A*a*b*c^4)/c^6)*x
+ (1155*B*b^5*c - 6048*B*a*b^3*c^2 - 1470*A*b^4*c^2 + 6352*B*a^2*b*c^3 + 6
272*A*a*b^2*c^3 - 3360*A*a^2*c^4)/c^6)*x - (3465*B*b^6 - 21840*B*a*b^4*c -
 4410*A*b^5*c + 34608*B*a^2*b^2*c^2 + 23520*A*a*b^3*c^2 - 8192*B*a^3*c^3 -
 25312*A*a^2*b*c^3)/c^6) - 1/2048*(33*B*b^7 - 252*B*a*b^5*c - 42*A*b^6*c +
 560*B*a^2*b^3*c^2 + 280*A*a*b^4*c^2 - 320*B*a^3*b*c^3 - 480*A*a^2*b^2*c^3
 + 128*A*a^3*c^4)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) +
b))/c^(13/2)
```

Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 992, normalized size of antiderivative = 2.70

$$\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx=\text{Too large to display}$$

input
```
int(x^4*(A + B*x)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
(8*B*a^3*(a + b*x + c*x^2)^(1/2))/(105*c^3) - (33*B*b^6*(a + b*x + c*x^2)^
(1/2))/(1024*c^6) + (A*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) + (B*x^4*(a + b*
x + c*x^2)^(3/2))/(7*c) + (33*B*b^7*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1
/2) + 2*c*x))/(2048*c^(13/2)) + (A*a*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(
a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2)
- 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x
 + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (lo
g((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)
)))/(4*c)))/(2*c) - (3*A*b*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*
b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c
*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b
/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/
(4*c)))/(10*c) - (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2
))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
 + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5
*c)))/(4*c) - (5*B*a^3*b*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x
))/(32*c^(7/2)) - (63*B*a*b^5*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) +
2*c*x))/(512*c^(11/2)) + (35*B*a^2*b^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)
^(1/2) + 2*c*x))/(128*c^(9/2)) + (13*B*a*b^4*(a + b*x + c*x^2)^(1/2))/(...
```

Reduce [F]

$$\int x^4(A+Bx)\sqrt{a+bx+cx^2}\,dx=\int x^4(Bx+A)\sqrt{c\,x^2+bx+a}\,dx$$

input
```
int(x^4*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^4*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

3.105 $\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 280

$$\begin{aligned}\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx &= \frac{(21b^4B-28Ab^3c-56ab^2Bc+48aAbc^2+16a^2Bc^2)(b+2cx)\sqrt{a+bx+cx^2}}{512c^5} \\ &- \frac{(3bB-4Ac)x^2(a+bx+cx^2)^{3/2}}{20c^2}+\frac{Bx^3(a+bx+cx^2)^{3/2}}{6c} \\ &- \frac{(105b^3B-140Ab^2c-196abBc+128aAc^2-6c(21b^2B-28Abc-20aBc)x)(a+bx+cx^2)^{3/2}}{960c^4} \\ &- \frac{(b^2-4ac)(21b^4B-28Ab^3c-56ab^2Bc+48aAbc^2+16a^2Bc^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{11/2}}\end{aligned}$$

output
```
1/512*(48*A*a*b*c^2-28*A*b^3*c+16*B*a^2*c^2-56*B*a*b^2*c+21*B*b^4)*(2*c*x+
b)*(c*x^2+b*x+a)^(1/2)/c^5-1/20*(-4*A*c+3*B*b)*x^2*(c*x^2+b*x+a)^(3/2)/c^2
+1/6*B*x^3*(c*x^2+b*x+a)^(3/2)/c-1/960*(105*B*b^3-140*A*b^2*c-196*B*a*b*c+
128*A*a*c^2-6*c*(-28*A*b*c-20*B*a*c+21*B*b^2)*x)*(c*x^2+b*x+a)^(3/2)/c^4-1
/1024*(-4*a*c+b^2)*(48*A*a*b*c^2-28*A*b^3*c+16*B*a^2*c^2-56*B*a*b^2*c+21*B
*b^4)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.97

$$\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(315b^5B-210b^4c(2A+Bx)+56b^3c(-30aB+cx(5A+3Bx))-32c^3(-8c^2x^4(6A}{}$$

input
```
Integrate[x^3*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^5*B - 210*b^4*c*(2*A + B*x) + 56*b
^3*c*(-30*a*B + c*x*(5*A + 3*B*x)) - 32*c^3*(-8*c^2*x^4*(6*A + 5*B*x) - 2*
a*c*x^2*(8*A + 5*B*x) + a^2*(32*A + 15*B*x)) + 16*b*c^2*(113*a^2*B + 4*c^2
*x^3*(3*A + 2*B*x) - 2*a*c*x*(29*A + 17*B*x)) + 16*b^2*c^2*(-(c*x^2*(14*A
+ 9*B*x)) + a*(115*A + 56*B*x))) + 15*(b^2 - 4*a*c)*(21*b^4*B - 28*A*b^3*c
 - 56*a*b^2*B*c + 48*a*A*b*c^2 + 16*a^2*B*c^2)*Log[b + 2*c*x - 2*Sqrt[c]*S
qrt[a + x*(b + c*x)]])/(15360*c^(11/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1236, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{3}{2}x^2(2aB+(3bB-4Ac)x)\sqrt{cx^2+bx+a}dx}{6c}+\frac{Bx^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{3/2}}{6c}-\frac{\int x^2(2aB+(3bB-4Ac)x)\sqrt{cx^2+bx+a}dx}{4c}$$

$$\Big\downarrow \ \textbf{1236}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{3/2}}{6c}-\frac{\frac{\int -\frac{1}{2}x\left(4a(3bB-4Ac)+\left(21Bb^2-28Acb-20aBc\right)x\right)\sqrt{cx^2+bx+a}dx}{5c}+\frac{x^2\left(a+bx+cx^2\right)^{3/2}(3bB-4Ac)}{5c}}{4c}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{3/2}}{6c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{3/2}(3bB-4Ac)}{5c}-\frac{\int x\left(4a(3bB-4Ac)+\left(21Bb^2-28Acb-20aBc\right)x\right)\sqrt{cx^2+bx+a}dx}{10c}}{4c}$$

$$\Big\downarrow \ \textbf{1225}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{3/2}}{6c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{3/2}(3bB-4Ac)}{5c}-\frac{\frac{5\left(16a^2Bc^2+48aAbc^2-56ab^2Bc-28Ab^3c+21b^4B\right)\int \sqrt{cx^2+bx+a}dx}{16c^2}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-6cx\left(-20aBc-28Abc+21b^2B\right)\cdots}{24c^2}}{10c}}{4c}$$

$$\Big\downarrow \ \textbf{1087}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{3/2}}{6c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{3/2}(3bB-4Ac)}{5c}-\frac{\frac{5\left(16a^2Bc^2+48aAbc^2-56ab^2Bc-28Ab^3c+21b^4B\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c^2}-\frac{\left(a+bx+cx^2\cdots\right.}{\cdots}}{10c}}{4c}$$

$$\Big\downarrow \ \textbf{1092}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{3/2}}{6c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{3/2}(3bB-4Ac)}{5c}-\frac{\frac{5\left(16a^2Bc^2+48aAbc^2-56ab^2Bc-28Ab^3c+21b^4B\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c^2}\cdots}{10c}}{4c}$$

$$\Big\downarrow \ \textbf{219}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{3/2}}{6c} - \frac{\frac{x^2(a+bx+cx^2)^{3/2}(3bB-4Ac)}{5c} - \frac{\frac{5\left(16a^2Bc^2+48aAbc^2-56ab^2Bc-28Ab^3c+21b^4B\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c^2}}{10c}}{4c}$$

input
```
Int[x^3*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(B*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (((3*b*B - 4*A*c)*x^2*(a + b*x + c
*x^2)^(3/2))/(5*c) - (-1/24*((105*b^3*B - 140*A*b^2*c - 196*a*b*B*c + 128*
a*A*c^2 - 6*c*(21*b^2*B - 28*A*b*c - 20*a*B*c)*x)*(a + b*x + c*x^2)^(3/2))
/c^2 + (5*(21*b^4*B - 28*A*b^3*c - 56*a*b^2*B*c + 48*a*A*b*c^2 + 16*a^2*B*
c^2)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c^2))/(10
*c))/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\left(-1280B\,c^5x^5-1536A\,c^5x^4-128Bb\,c^4x^4-192Ab\,c^4x^3-320Ba\,c^4x^3+144B\,b^2c^3x^3-512Aa\,c^4x^2+224A\,b^2c^3x^2+544Bab\,c^3x^2-1\right.}{}$
default	$A\left(\frac{x^2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{5c}-\frac{7b\left(\frac{x\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right.}{8c}\right.}{10c}\right.$

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(-1280*B*c^5*x^5-1536*A*c^5*x^4-128*B*b*c^4*x^4-192*A*b*c^4*x^3-32
0*B*a*c^4*x^3+144*B*b^2*c^3*x^3-512*A*a*c^4*x^2+224*A*b^2*c^3*x^2+544*B*a*
b*c^3*x^2-168*B*b^3*c^2*x^2+928*A*a*b*c^3*x-280*A*b^3*c^2*x+480*B*a^2*c^3*
x-896*B*a*b^2*c^2*x+210*B*b^4*c*x+1024*A*a^2*c^3-1840*A*a*b^2*c^2+420*A*b^
4*c-1808*B*a^2*b*c^2+1680*B*a*b^3*c-315*B*b^5)/c^5*(c*x^2+b*x+a)^(1/2)+1/1
024*(192*A*a^2*b*c^3-160*A*a*b^3*c^2+28*A*b^5*c+64*B*a^3*c^3-240*B*a^2*b^2
*c^2+140*B*a*b^4*c-21*B*b^6)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 667, normalized size of antiderivative = 2.38

$$\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(21*B*b^6 - 64*(B*a^3 + 3*A*a^2*b)*c^3 + 80*(3*B*a^2*b^2 + 2
*A*a*b^3)*c^2 - 28*(5*B*a*b^4 + A*b^5)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x
 - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*B*
c^6*x^5 + 315*B*b^5*c - 1024*A*a^2*c^4 + 128*(B*b*c^5 + 12*A*c^6)*x^4 + 16
*(113*B*a^2*b + 115*A*a*b^2)*c^3 - 16*(9*B*b^2*c^4 - 4*(5*B*a + 3*A*b)*c^5
)*x^3 - 420*(4*B*a*b^3 + A*b^4)*c^2 + 8*(21*B*b^3*c^3 + 64*A*a*c^5 - 4*(17
*B*a*b + 7*A*b^2)*c^4)*x^2 - 2*(105*B*b^4*c^2 + 16*(15*B*a^2 + 29*A*a*b)*c
^4 - 28*(16*B*a*b^2 + 5*A*b^3)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/15360
*(15*(21*B*b^6 - 64*(B*a^3 + 3*A*a^2*b)*c^3 + 80*(3*B*a^2*b^2 + 2*A*a*b^3)
*c^2 - 28*(5*B*a*b^4 + A*b^5)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(1280*B*c^6*x^5 + 315*B
*b^5*c - 1024*A*a^2*c^4 + 128*(B*b*c^5 + 12*A*c^6)*x^4 + 16*(113*B*a^2*b +
 115*A*a*b^2)*c^3 - 16*(9*B*b^2*c^4 - 4*(5*B*a + 3*A*b)*c^5)*x^3 - 420*(4*
B*a*b^3 + A*b^4)*c^2 + 8*(21*B*b^3*c^3 + 64*A*a*c^5 - 4*(17*B*a*b + 7*A*b^
2)*c^4)*x^2 - 2*(105*B*b^4*c^2 + 16*(15*B*a^2 + 29*A*a*b)*c^4 - 28*(16*B*a
*b^2 + 5*A*b^3)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(299) = 598$.

Time = 0.67 (sec) , antiderivative size = 831, normalized size of antiderivative = 2.97

$$\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Too large to display}$$

input

```
integrate(x**3*(B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

output

```
Piecewise(((-a*(-3*a*(A*b + B*a/6 - 9*b*(A*c + B*b/12)/(10*c))/(4*c) - 5*b
*(A*a - 4*a*(A*c + B*b/12)/(5*c) - 7*b*(A*b + B*a/6 - 9*b*(A*c + B*b/12)/(
10*c))/(8*c))/(6*c))/(2*c) - b*(-2*a*(A*a - 4*a*(A*c + B*b/12)/(5*c) - 7*b
*(A*b + B*a/6 - 9*b*(A*c + B*b/12)/(10*c))/(8*c))/(3*c) - 3*b*(-3*a*(A*b +
 B*a/6 - 9*b*(A*c + B*b/12)/(10*c))/(4*c) - 5*b*(A*a - 4*a*(A*c + B*b/12)/
(5*c) - 7*b*(A*b + B*a/6 - 9*b*(A*c + B*b/12)/(10*c))/(8*c))/(6*c))/(4*c))
/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(
c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c
) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(B*x**5/6 + x**4*(A*c + B*b/12
)/(5*c) + x**3*(A*b + B*a/6 - 9*b*(A*c + B*b/12)/(10*c))/(4*c) + x**2*(A*a
 - 4*a*(A*c + B*b/12)/(5*c) - 7*b*(A*b + B*a/6 - 9*b*(A*c + B*b/12)/(10*c)
)/(8*c))/(3*c) + x*(-3*a*(A*b + B*a/6 - 9*b*(A*c + B*b/12)/(10*c))/(4*c) -
 5*b*(A*a - 4*a*(A*c + B*b/12)/(5*c) - 7*b*(A*b + B*a/6 - 9*b*(A*c + B*b/1
2)/(10*c))/(8*c))/(6*c))/(2*c) + (-2*a*(A*a - 4*a*(A*c + B*b/12)/(5*c) - 7
*b*(A*b + B*a/6 - 9*b*(A*c + B*b/12)/(10*c))/(8*c))/(3*c) - 3*b*(-3*a*(A*b
 + B*a/6 - 9*b*(A*c + B*b/12)/(10*c))/(4*c) - 5*b*(A*a - 4*a*(A*c + B*b/12
)/(5*c) - 7*b*(A*b + B*a/6 - 9*b*(A*c + B*b/12)/(10*c))/(8*c))/(6*c))/(4*c
))/c), Ne(c, 0)), (2*(B*(a + b*x)**(11/2)/(11*b) + (a + b*x)**(9/2)*(A*b -
 4*B*a)/(9*b) + (a + b*x)**(7/2)*(-3*A*a*b + 6*B*a**2)/(7*b) + (a + b*x)**
(5/2)*(3*A*a**2*b - 4*B*a**3)/(5*b) + (a + b*x)**(3/2)*(-A*a**3*b + B*a...
```

Maxima [F(-2)]

Exception generated.

$$\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.15

$$\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,Bx+\frac{Bbc^4+12\,Ac^5}{c^5}\right)x-\frac{9\,Bb^2c^3-20\,Bac^4-12\,Abc^4}{c^5}\right)x+\frac{21\,\ldots}{}\right.\right.\right.$$
$$+\frac{(21\,Bb^6-140\,Bab^4c-28\,Ab^5c+240\,Ba^2b^2c^2+160\,Aab^3c^2-64\,Ba^3c^3-192\,Aa^2bc^3)\log\left(\left|2\left(\sqrt{c}x-\ldots\right.\right.\right.}{1024\,c^{\frac{11}{2}}}$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*B*x + (B*b*c^4 + 12*A*c^5)/c^
5)*x - (9*B*b^2*c^3 - 20*B*a*c^4 - 12*A*b*c^4)/c^5)*x + (21*B*b^3*c^2 - 68
*B*a*b*c^3 - 28*A*b^2*c^3 + 64*A*a*c^4)/c^5)*x - (105*B*b^4*c - 448*B*a*b^
2*c^2 - 140*A*b^3*c^2 + 240*B*a^2*c^3 + 464*A*a*b*c^3)/c^5)*x + (315*B*b^5
 - 1680*B*a*b^3*c - 420*A*b^4*c + 1808*B*a^2*b*c^2 + 1840*A*a*b^2*c^2 - 10
24*A*a^2*c^3)/c^5) + 1/1024*(21*B*b^6 - 140*B*a*b^4*c - 28*A*b^5*c + 240*B
*a^2*b^2*c^2 + 160*A*a*b^3*c^2 - 64*B*a^3*c^3 - 192*A*a^2*b*c^3)*log(abs(2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/2)
```

Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.79

$$\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(A + B*x)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
(A*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (B*x^3*(a + b*x + c*x^2)^(3/2))/(6
*c) - (2*A*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x +
c*x^2)^(1/2))/(24*c^2)))/(5*c) + (7*A*b*((5*b*((log((b + 2*c*x)/c^(1/2) +
2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^
2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a +
b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) +
(log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3
/2))))/(4*c)))/(10*c) + (B*a*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
+ c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2
)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 +
 c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c
)))/(2*c) - (3*B*b*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*
x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*
b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3
/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x
)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/
(10*c) - (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3
- 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*...
```

Reduce [F]

$$\int x^3(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int x^3(Bx+A)\sqrt{c\,x^2+bx+a}dx$$

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

3.106 $\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 205

$$\begin{aligned}\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx &= -\frac{(7b^3B-10Ab^2c-12abBc+8aAc^2)\,(b+2cx)\sqrt{a+bx+cx^2}}{128c^4} \\ &\quad+\frac{Bx^2(a+bx+cx^2)^{3/2}}{5c} \\ &\quad+\frac{(35b^2B-50Abc-32aBc-6c(7bB-10Ac)x)\,(a+bx+cx^2)^{3/2}}{240c^3} \\ &\quad+\frac{(b^2-4ac)\,(7b^3B-10Ab^2c-12abBc+8aAc^2)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}}\end{aligned}$$

output
```
-1/128*(8*A*a*c^2-10*A*b^2*c-12*B*a*b*c+7*B*b^3)*(2*c*x+b)*(c*x^2+b*x+a)^(
1/2)/c^4+1/5*B*x^2*(c*x^2+b*x+a)^(3/2)/c+1/240*(35*B*b^2-50*A*b*c-32*B*a*c
-6*c*(-10*A*c+7*B*b)*x)*(c*x^2+b*x+a)^(3/2)/c^3+1/256*(-4*a*c+b^2)*(8*A*a*
c^2-10*A*b^2*c-12*B*a*b*c+7*B*b^3)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.02

$$\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^4B+10b^3c(15A+7Bx)+16c^2(-16a^2B+6c^2x^3(5A+4Bx)+acx(15A+\ldots}{}$$

input
```
Integrate[x^2*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^4*B + 10*b^3*c*(15*A + 7*B*x) + 1
6*c^2*(-16*a^2*B + 6*c^2*x^3*(5*A + 4*B*x) + a*c*x*(15*A + 8*B*x)) + 4*b^2
*c*(115*a*B - c*x*(25*A + 14*B*x)) + 8*b*c^2*(2*c*x^2*(5*A + 3*B*x) - a*(6
5*A + 29*B*x))) - 15*(b^2 - 4*a*c)*(7*b^3*B - 10*A*b^2*c - 12*a*b*B*c + 8*
a*A*c^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(9/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}x(4aB+(7bB-10Ac)x)\sqrt{cx^2+bx+a}dx}{5c}+\frac{Bx^2\left(a+bx+cx^2\right)^{3/2}}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{3/2}}{5c}-\frac{\int x(4aB+(7bB-10Ac)x)\sqrt{cx^2+bx+a}dx}{10c}$$

$$\Big\downarrow \text{1225}$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{3/2}}{5c}-\frac{\frac{5\left(8aAc^2-12abBc-10Ab^2c+7b^3B\right)\int\sqrt{cx^2+bx+a}dx}{16c^2}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-32aBc-6cx(7bB-10Ac)-50Abc+35b^2B\right)}{24c^2}}{10c}$$

$\Big\downarrow$ 1087

$$\frac{Bx^2\left(a+bx+cx^2\right)^{3/2}}{5c}-\frac{\frac{5\left(8aAc^2-12abBc-10Ab^2c+7b^3B\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c^2}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-32aBc-6cx(7bB-10Ac)-50Abc+\right.}{24c^2}}{10c}$$

$\Big\downarrow$ 1092

$$\frac{Bx^2\left(a+bx+cx^2\right)^{3/2}}{5c}-\frac{\frac{5\left(8aAc^2-12abBc-10Ab^2c+7b^3B\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c^2}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-32aBc-6cx(7bB-10\right.}{24c^2}}{10c}$$

$\Big\downarrow$ 219

$$\frac{Bx^2\left(a+bx+cx^2\right)^{3/2}}{5c}-\frac{\frac{5\left(8aAc^2-12abBc-10Ab^2c+7b^3B\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c^2}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-32aBc-6cx(7bB-10\right.}{24c^2}}{10c}$$

input
```
Int[x^2*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(B*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) - (-1/24*((35*b^2*B - 50*A*b*c - 32*
a*B*c - 6*c*(7*b*B - 10*A*c)*x)*(a + b*x + c*x^2)^(3/2))/c^2 + (5*(7*b^3*B
 - 10*A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2
])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(8*c^(3/2))))/(16*c^2))/(10*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{(-384Bc^4x^4-480Ac^4x^3-48Bc^3x^3b-80Abc^3x^2-128Bac^3x^2+56c^2x^2Bb^2-240Aac^3x+100Ab^2c^2x+232Babc^2x-70Bb^3cx+}{1920c^4}$
default	$A\left(\frac{x(cx^2+bx+a)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{(cx^2+bx+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)}{8c}-\frac{a\left(\frac{(2cx+b)\sqrt{cx^2}}{4c}\right.}{}\right.$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(-384*B*c^4*x^4-480*A*c^4*x^3-48*B*b*c^3*x^3-80*A*b*c^3*x^2-128*B*
a*c^3*x^2+56*B*b^2*c^2*x^2-240*A*a*c^3*x+100*A*b^2*c^2*x+232*B*a*b*c^2*x-7
0*B*b^3*c*x+520*A*a*b*c^2-150*A*b^3*c+256*B*a^2*c^2-460*B*a*b^2*c+105*B*b^
4)/c^4*(c*x^2+b*x+a)^(1/2)-1/256*(32*A*a^2*c^3-48*A*a*b^2*c^2+10*A*b^4*c-4
8*B*a^2*b*c^2+40*B*a*b^3*c-7*B*b^5)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.52

$$\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$= \left[-\frac{15\,(7\,Bb^5-32\,Aa^2c^3+48\,(Ba^2b+Aab^2)c^2-10\,(4\,Bab^3+Ab^4)c)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{\right.}}{}\right.$$
$$-\frac{15\,(7\,Bb^5-32\,Aa^2c^3+48\,(Ba^2b+Aab^2)c^2-10\,(4\,Bab^3+Ab^4)c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)}{}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(7*B*b^5 - 32*A*a^2*c^3 + 48*(B*a^2*b + A*a*b^2)*c^2 - 10*(4*
B*a*b^3 + A*b^4)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*B*c^5*x^4 - 105*B*b^4*c -
 8*(32*B*a^2 + 65*A*a*b)*c^3 + 48*(B*b*c^4 + 10*A*c^5)*x^3 + 10*(46*B*a*b^
2 + 15*A*b^3)*c^2 - 8*(7*B*b^2*c^3 - 2*(8*B*a + 5*A*b)*c^4)*x^2 + 2*(35*B*
b^3*c^2 + 120*A*a*c^4 - 2*(58*B*a*b + 25*A*b^2)*c^3)*x)*sqrt(c*x^2 + b*x +
 a))/c^5, -1/3840*(15*(7*B*b^5 - 32*A*a^2*c^3 + 48*(B*a^2*b + A*a*b^2)*c^2
 - 10*(4*B*a*b^3 + A*b^4)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*B*c^5*x^4 - 105*B*b^4*
c - 8*(32*B*a^2 + 65*A*a*b)*c^3 + 48*(B*b*c^4 + 10*A*c^5)*x^3 + 10*(46*B*a
*b^2 + 15*A*b^3)*c^2 - 8*(7*B*b^2*c^3 - 2*(8*B*a + 5*A*b)*c^4)*x^2 + 2*(35
*B*b^3*c^2 + 120*A*a*c^4 - 2*(58*B*a*b + 25*A*b^2)*c^3)*x)*sqrt(c*x^2 + b*
x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(214) = 428$.

Time = 0.62 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.63

$$\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-a*(A*a - 3*a*(A*c + B*b/10)/(4*c) - 5*b*(A*b + B*a/5 - 7*b*(A
*c + B*b/10)/(8*c))/(6*c))/(2*c) - b*(-2*a*(A*b + B*a/5 - 7*b*(A*c + B*b/1
0)/(8*c))/(3*c) - 3*b*(A*a - 3*a*(A*c + B*b/10)/(4*c) - 5*b*(A*b + B*a/5 -
 7*b*(A*c + B*b/10)/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(
c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2
*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)) + sqrt(a + b*x
+ c*x**2)*(B*x**4/5 + x**3*(A*c + B*b/10)/(4*c) + x**2*(A*b + B*a/5 - 7*b*
(A*c + B*b/10)/(8*c))/(3*c) + x*(A*a - 3*a*(A*c + B*b/10)/(4*c) - 5*b*(A*b
 + B*a/5 - 7*b*(A*c + B*b/10)/(8*c))/(6*c))/(2*c) + (-2*a*(A*b + B*a/5 - 7
*b*(A*c + B*b/10)/(8*c))/(3*c) - 3*b*(A*a - 3*a*(A*c + B*b/10)/(4*c) - 5*b
*(A*b + B*a/5 - 7*b*(A*c + B*b/10)/(8*c))/(6*c))/(4*c))/c), Ne(c, 0)), (2*
(B*(a + b*x)**(9/2)/(9*b) + (a + b*x)**(7/2)*(A*b - 3*B*a)/(7*b) + (a + b*
x)**(5/2)*(-2*A*a*b + 3*B*a**2)/(5*b) + (a + b*x)**(3/2)*(A*a**2*b - B*a**
3)/(3*b))/b**3, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**4/4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.19

$$\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$=\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,Bx+\frac{Bbc^3+10\,Ac^4}{c^4}\right)x-\frac{7\,Bb^2c^2-16\,Bac^3-10\,Abc^3}{c^4}\right)x+\frac{35\,Bb^3c\right.\right.\right.$$
$$-\frac{(7\,Bb^5-40\,Bab^3c-10\,Ab^4c+48\,Ba^2bc^2+48\,Aab^2c^2-32\,Aa^2c^3)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c\right.\right.}{256\,c^{\frac{9}{2}}}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*B*x + (B*b*c^3 + 10*A*c^4)/c^4)*x
 - (7*B*b^2*c^2 - 16*B*a*c^3 - 10*A*b*c^3)/c^4)*x + (35*B*b^3*c - 116*B*a*
b*c^2 - 50*A*b^2*c^2 + 120*A*a*c^3)/c^4)*x - (105*B*b^4 - 460*B*a*b^2*c -
150*A*b^3*c + 256*B*a^2*c^2 + 520*A*a*b*c^2)/c^4) - 1/256*(7*B*b^5 - 40*B*
a*b^3*c - 10*A*b^4*c + 48*B*a^2*b*c^2 + 48*A*a*b^2*c^2 - 32*A*a^2*c^3)*log
(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [B] (verification not implemented)

Time = 11.63 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.26

$$\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx = \frac{B\,x^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{5\,c} - \frac{A\,a\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} - \frac{5\,A\,b\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c} - \frac{2\,B\,a\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{5\,c} + \frac{A\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c} + \frac{7\,B\,b\,\left(\frac{5\,b\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}-\frac{x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}+\frac{a\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right) \cdots\right.}{\cdots}\right.}{10\,c}$$

input
```
int(x^2*(A + B*x)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
(B*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) - (A*a*((x/2 + b/(4*c))*(a + b*x + c
*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b
^2/4))/(2*c^(3/2))))/(4*c) - (5*A*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (2*B*a*((log((b +
2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
+ ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))
/(5*c) + (A*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (7*B*b*((5*b*((log((b + 2*c
*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + (
(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8
*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c
*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b
^2/4))/(2*c^(3/2))))/(4*c)))/(10*c)
```

Reduce [F]

$$\int x^2(A+Bx)\sqrt{a+bx+cx^2}\,dx=\int x^2(Bx+A)\sqrt{c\,x^2+bx+a}dx$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

3.107 $\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx = \frac{(5b^2B-8Abc-4aBc)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{(5bB-8Ac-6Bcx)(a+bx+cx^2)^{3/2}}{24c^2} - \frac{(b^2-4ac)(5b^2B-8Abc-4aBc)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}$$

output
```
1/64*(-8*A*b*c-4*B*a*c+5*B*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3-1/24*(-6
*B*c*x-8*A*c+5*B*b)*(c*x^2+b*x+a)^(3/2)/c^2-1/128*(-4*a*c+b^2)*(-8*A*b*c-4
*B*a*c+5*B*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07

$$\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(15b^3B-2b^2c(12A+5Bx)+8c^2(8aA+3aBx+8Acx^2+6Bcx^3)+4bc(-13aB+}{384c^{7/2}}$$

input
```
Integrate[x*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*B - 2*b^2*c*(12*A + 5*B*x) + 8*c^
2*(8*a*A + 3*a*B*x + 8*A*c*x^2 + 6*B*c*x^3) + 4*b*c*(-13*a*B + 2*c*x*(2*A
+ B*x))) + 3*(b^2 - 4*a*c)*(5*b^2*B - 8*A*b*c - 4*a*B*c)*Log[b + 2*c*x - 2
*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(7/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{(-4aBc-8Abc+5b^2B)\int\sqrt{cx^2+bx+a}dx}{16c^2}-\frac{(a+bx+cx^2)^{3/2}(-8Ac+5bB-6Bcx)}{24c^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(-4aBc-8Abc+5b^2B)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c^2}-\frac{(a+bx+cx^2)^{3/2}(-8Ac+5bB-6Bcx)}{24c^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(-4aBc-8Abc+5b^2B\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c^2}-\frac{\left(a+bx+cx^2\right)^{3/2}(-8Ac+5bB-6Bcx)}{24c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(-4aBc-8Abc+5b^2B\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c^2}-\frac{\left(a+bx+cx^2\right)^{3/2}(-8Ac+5bB-6Bcx)}{24c^2}$$

input
```
Int[x*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
-1/24*((5*b*B - 8*A*c - 6*B*c*x)*(a + b*x + c*x^2)^(3/2))/c^2 + ((5*b^2*B
- 8*A*b*c - 4*a*B*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 -
4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))
))/(16*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.16

method	result
risch	$\frac{\left(48B\,c^3x^3+64A\,c^3x^2+8Bb\,c^2x^2+16Ab\,c^2x+24Ba\,c^2x-10B\,b^2cx+64Aa\,c^2-24A\,b^2c-52Babc+15B\,b^3\right)\sqrt{c\,x^2+bx+a}}{192c^3}-\frac{(32Aab\,c\ldots}{}$
default	$A\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)+B\left(\frac{x\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{(c\,x^2\ldots}{}\right.}{}\right.$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*B*c^3*x^3+64*A*c^3*x^2+8*B*b*c^2*x^2+16*A*b*c^2*x+24*B*a*c^2*x-1
0*B*b^2*c*x+64*A*a*c^2-24*A*b^2*c-52*B*a*b*c+15*B*b^3)/c^3*(c*x^2+b*x+a)^(
1/2)-1/128*(32*A*a*b*c^2-8*A*b^3*c+16*B*a^2*c^2-24*B*a*b^2*c+5*B*b^4)/c^(7
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.73

$$\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$= \left[\frac{3\,(5\,Bb^4+16\,(Ba^2+2\,Aab)c^2-8\,(3\,Bab^2+Ab^3)c)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\right.}{}\right.$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(5*B*b^4 + 16*(B*a^2 + 2*A*a*b)*c^2 - 8*(3*B*a*b^2 + A*b^3)*c)*s
qrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(c) - 4*a*c) + 4*(48*B*c^4*x^3 + 15*B*b^3*c + 64*A*a*c^3 - 4*(13*B*a
*b + 6*A*b^2)*c^2 + 8*(B*b*c^3 + 8*A*c^4)*x^2 - 2*(5*B*b^2*c^2 - 4*(3*B*a
+ 2*A*b)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^4, 1/384*(3*(5*B*b^4 + 16*(B*a^2
 + 2*A*a*b)*c^2 - 8*(3*B*a*b^2 + A*b^3)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*B*c^4*x^3
 + 15*B*b^3*c + 64*A*a*c^3 - 4*(13*B*a*b + 6*A*b^2)*c^2 + 8*(B*b*c^3 + 8*A
*c^4)*x^2 - 2*(5*B*b^2*c^2 - 4*(3*B*a + 2*A*b)*c^3)*x)*sqrt(c*x^2 + b*x +
a))/c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(143) = 286$.

Time = 0.76 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.43

$$\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx = \begin{cases} \left(-\frac{a\left(Ab+\frac{Ba}{4}-\frac{5b\left(Ac+\frac{Bb}{8}\right)}{6c}\right)}{2c}-\frac{b\left(Aa-\frac{2a\left(Ac+\frac{Bb}{8}\right)}{3c}-\frac{3b\left(Ab+\frac{Ba}{4}-\frac{5b\left(Ac+\frac{Bb}{8}\right)}{6c}\right)}{4c}\right)}{2c}\right)\left(\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{other}\end{cases}\right.\right. \\ \frac{2\left(\frac{B(a+bx)^{\frac{7}{2}}}{7b}+\frac{(a+bx)^{\frac{5}{2}}(Ab-2Ba)}{5b}+\frac{(a+bx)^{\frac{3}{2}}\left(-Aab+Ba^2\right)}{3b}\right)}{b^2} \\ \sqrt{a}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right) \end{cases}$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-a*(A*b + B*a/4 - 5*b*(A*c + B*b/8)/(6*c))/(2*c) - b*(A*a - 2*
a*(A*c + B*b/8)/(3*c) - 3*b*(A*b + B*a/4 - 5*b*(A*c + B*b/8)/(6*c))/(4*c))
/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(
c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c
) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(B*x**3/4 + x**2*(A*c + B*b/8)
/(3*c) + x*(A*b + B*a/4 - 5*b*(A*c + B*b/8)/(6*c))/(2*c) + (A*a - 2*a*(A*c
 + B*b/8)/(3*c) - 3*b*(A*b + B*a/4 - 5*b*(A*c + B*b/8)/(6*c))/(4*c))/c), N
e(c, 0)), (2*(B*(a + b*x)**(7/2)/(7*b) + (a + b*x)**(5/2)*(A*b - 2*B*a)/(5
*b) + (a + b*x)**(3/2)*(-A*a*b + B*a**2)/(3*b))/b**2, Ne(b, 0)), (sqrt(a)*
(A*x**2/2 + B*x**3/3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.22

$$\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,Bx+\frac{Bbc^2+8\,Ac^3}{c^3}\right)x-\frac{5\,Bb^2c-12\,Bac^2-8\,Abc^2}{c^3}\right)x+\frac{15\,Bb^3-52\,Ba\right.$$
$$+\frac{(5\,Bb^4-24\,Bab^2c-8\,Ab^3c+16\,Ba^2c^2+32\,Aabc^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{7}{2}}}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*B*x + (B*b*c^2 + 8*A*c^3)/c^3)*x - (5
*B*b^2*c - 12*B*a*c^2 - 8*A*b*c^2)/c^3)*x + (15*B*b^3 - 52*B*a*b*c - 24*A*
b^2*c + 64*A*a*c^2)/c^3) + 1/128*(5*B*b^4 - 24*B*a*b^2*c - 8*A*b^3*c + 16*
B*a^2*c^2 + 32*A*a*b*c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sq
rt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 10.90 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.78

$$\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{A\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\sqrt{c\,x^2+b\,x+a}\right)(b^3-4\,a\,b\,c)}{16\,c^{5/2}}$$
$$- \frac{B\,a\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}$$
$$- \frac{5\,B\,b\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\sqrt{c\,x^2+b\,x+a}\right)(b^3-4\,a\,b\,c)}{16\,c^{5/2}}+\frac{(-3\,b^2+2\,c\,x\,b+8\,c\,(c\,x^2+a))\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}$$
$$+ \frac{A\,(-3\,b^2+2\,c\,x\,b+8\,c\,(c\,x^2+a))\sqrt{c\,x^2+b\,x+a}}{24\,c^2}+\frac{B\,x\,(c\,x^2+b\,x+a)^{3/2}}{4\,c}$$

input
```
int(x*(A + B*x)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
(A*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(
16*c^(5/2)) - (B*a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 +
c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)
 - (5*B*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*
a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x
^2)^(1/2))/(24*c^2)))/(8*c) + (A*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a +
b*x + c*x^2)^(1/2))/(24*c^2) + (B*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

Reduce [F]

$$\int x(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int x(Bx+A)\sqrt{c\,x^2+bx+a}dx$$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

3.108 $\int (A+Bx)\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int (A+Bx)\sqrt{a+bx+cx^2}\,dx = -\frac{(bB-2Ac)(b+2cx)\sqrt{a+bx+cx^2}}{8c^2} + \frac{B(a+bx+cx^2)^{3/2}}{3c} + \frac{(b^2-4ac)(bB-2Ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}}$$

output
```
-1/8*(-2*A*c+B*b)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^2+1/3*B*(c*x^2+b*x+a)^(3
/2)/c+1/16*(-4*a*c+b^2)*(-2*A*c+B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int (A+Bx)\sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-3b^2B+2bc(3A+Bx)+4c(2aB+cx(3A+2Bx)))+3(b^2-4ac)(bB-2Ac)\text{arct}}{24c^{5/2}}$$

input
```
Integrate[(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3*b^2*B + 2*b*c*(3*A + B*x) + 4*c*(2*a*B
+ c*x*(3*A + 2*B*x))) + 3*(b^2 - 4*a*c)*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x)/
(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(24*c^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3c}-\frac{(bB-2Ac)\int \sqrt{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{2c}$$

input
```
Int[(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(B*(a + b*x + c*x^2)^(3/2))/(3*c) - ((b*B - 2*A*c)*(((b + 2*c*x)*Sqrt[a +
b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])])/(8*c^(3/2))))/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

method	result
risch	$\frac{(8Bc^2x^2+12Ac^2x+2Bbcx+6Abc+8aBc-3Bb^2)\sqrt{cx^2+bx+a}}{24c^2}+\frac{(8Aac^2-2Ab^2c-4Babc+Bb^3)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{16c^{\frac{5}{2}}}$
default	$A\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+B\left(\frac{(cx^2+bx+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac}{}\right.}{}\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*B*c^2*x^2+12*A*c^2*x+2*B*b*c*x+6*A*b*c+8*B*a*c-3*B*b^2)/c^2*(c*x^2
+b*x+a)^(1/2)+1/16*(8*A*a*c^2-2*A*b^2*c-4*B*a*b*c+B*b^3)/c^(5/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.58

$$\int (A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$=\left[\frac{3\left(Bb^3+8\,Aac^2-2\left(2\,Bab+Ab^2\right)c\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,c\right.}{96\,c^3}\right.$$

$$-\frac{3\left(Bb^3+8\,Aac^2-2\left(2\,Bab+Ab^2\right)c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\left(8\,Bc^3x^2-3\,Bb^2c+2\,(4\right.}{48\,c^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b + A*b^2)*c)*sqrt(c)*log(-8*c^2*x^
2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) +
 4*(8*B*c^3*x^2 - 3*B*b^2*c + 2*(4*B*a + 3*A*b)*c^2 + 2*(B*b*c^2 + 6*A*c^3
)*x)*sqrt(c*x^2 + b*x + a))/c^3, -1/48*(3*(B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b
+ A*b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)
/(c^2*x^2 + b*c*x + a*c)) - 2*(8*B*c^3*x^2 - 3*B*b^2*c + 2*(4*B*a + 3*A*b)
*c^2 + 2*(B*b*c^2 + 6*A*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(102) = 204$.

Time = 0.61 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.98

$$\int (A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{Bx^2}{3}+\frac{x\left(Ac+\frac{Bb}{6}\right)}{2c}+\frac{Ab+\frac{Ba}{3}-\frac{3b\left(Ac+\frac{Bb}{6}\right)}{4c}}{c}\right)+\left(Aa-\frac{a\left(Ac+\frac{Bb}{6}\right)}{2c}-\frac{b\left(Ab+\frac{Ba}{3}-\frac{3b\left(Ac+\frac{Bb}{6}\right)}{4c}\right)}{2c}\right)\left(\right. \\ \frac{2\left(\frac{B(a+bx)^{\frac{5}{2}}}{5b}+\frac{(a+bx)^{\frac{3}{2}}(Ab-Ba)}{3b}\right)}{b} \\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(B*x**2/3 + x*(A*c + B*b/6)/(2*c) + (A*b
 + B*a/3 - 3*b*(A*c + B*b/6)/(4*c))/c) + (A*a - a*(A*c + B*b/6)/(2*c) - b*
(A*b + B*a/3 - 3*b*(A*c + B*b/6)/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*
c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(
B*(a + b*x)**(5/2)/(5*b) + (a + b*x)**(3/2)*(A*b - B*a)/(3*b))/b, Ne(b, 0)
), (sqrt(a)*(A*x + B*x**2/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (A+Bx)\sqrt{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int (A+Bx)\sqrt{a+bx+cx^2}\,dx = \frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(4\,Bx+\frac{Bbc+6\,Ac^2}{c^2}\right)x-\frac{3\,Bb^2-8\,Bac-6\,Abc}{c^2}\right) - \frac{(Bb^3-4\,Babc-2\,Ab^2c+8\,Aac^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*B*x + (B*b*c + 6*A*c^2)/c^2)*x - (3*B*b^2
 - 8*B*a*c - 6*A*b*c)/c^2) - 1/16*(B*b^3 - 4*B*a*b*c - 2*A*b^2*c + 8*A*a*c
^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 10.73 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.28

$$\int (A+Bx)\sqrt{a+bx+cx^2}\,dx = A\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a} + \frac{A\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}} + \frac{B\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}} + \frac{B\left(-3\,b^2+2\,c\,x\,b+8\,c\left(c\,x^2+a\right)\right)\sqrt{c\,x^2+b\,x+a}}{24\,c^2}$$

input
```
int((A + B*x)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
A*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (A*log((b/2 + c*x)/c^(1/2) + (
a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (B*log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + (B*(8*c
*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.06

$$\int (A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$= \frac{28\sqrt{c\,x^2+bx+a}\,ab\,c^2+24\sqrt{c\,x^2+bx+a}\,a\,c^3x-6\sqrt{c\,x^2+bx+a}\,b^3c+4\sqrt{c\,x^2+bx+a}\,b^2c^2x+16\sqrt{c}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
(28*sqrt(a + b*x + c*x**2)*a*b*c**2 + 24*sqrt(a + b*x + c*x**2)*a*c**3*x -
 6*sqrt(a + b*x + c*x**2)*b**3*c + 4*sqrt(a + b*x + c*x**2)*b**2*c**2*x +
16*sqrt(a + b*x + c*x**2)*b*c**3*x**2 + 24*sqrt(c)*log((2*sqrt(c)*sqrt(a +
 b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2 - 18*sqrt(c)*log
((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2
*c + 3*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a
*c - b**2))*b**4)/(48*c**3)
```

3.109 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}\,dx = \frac{(bB+4Ac+2Bcx)\sqrt{a+bx+cx^2}}{4c} - \sqrt{a}A\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - \frac{(b^2B-4Abc-4aBc)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}$$

output
```
1/4*(2*B*c*x+4*A*c+B*b)*(c*x^2+b*x+a)^(1/2)/c-a^(1/2)*A*arctanh(1/2*(b*x+2
*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))-1/8*(-4*A*b*c-4*B*a*c+B*b^2)*arctanh(1/2*
(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}\,dx = \frac{(-b^2B+4Abc+4aBc)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left((bB+4Ac+2Bcx)\sqrt{a+x(b+cx)}+8\sqrt{a}Aca\right.}{8c^{3/2}}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x,x]
```

output
```
((-(b^2*B) + 4*A*b*c + 4*a*B*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*
(b + c*x)])] + 2*Sqrt[c]*((b*B + 4*A*c + 2*B*c*x)*Sqrt[a + x*(b + c*x)] +
8*Sqrt[a]*A*c*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(8*c^
(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}dx$$

$$\Big\downarrow \text{1231}$$

$$\frac{\sqrt{a+bx+cx^2}(4Ac+bB+2Bcx)}{4c}-\frac{\int -\frac{8aAc-(Bb^2-4Acb-4aBc)x}{2x\sqrt{cx^2+bx+a}}dx}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{8aAc-(Bb^2-4Acb-4aBc)x}{x\sqrt{cx^2+bx+a}}dx}{8c}+\frac{\sqrt{a+bx+cx^2}(4Ac+bB+2Bcx)}{4c}$$

$$\Big\downarrow \text{1269}$$

$$\frac{8aAc\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\left(-4aBc-4Abc+b^2B\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}+\frac{\sqrt{a+bx+cx^2}(4Ac+bB+2Bcx)}{4c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{8aAc\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-2\left(-4aBc-4Abc+b^2B\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{8c}+\frac{\sqrt{a+bx+cx^2}(4Ac+bB+2Bcx)}{4c}$$

$$\Big\downarrow \text{219}$$

$$\frac{8aAc\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\frac{(-4aBc-4Abc+b^2B)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx+cx^2}(4Ac+bB+2Bcx)}{4c}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-16aAc\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-\frac{(-4aBc-4Abc+b^2B)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx+cx^2}(4Ac+bB+2Bcx)}{4c}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{(-4aBc-4Abc+b^2B)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-8\sqrt{a}Ac\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8c}+\frac{\sqrt{a+bx+cx^2}(4Ac+bB+2Bcx)}{4c}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x,x]
```

output
```
((b*B + 4*A*c + 2*B*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) + (-8*Sqrt[a]*A*c*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] - ((b^2*B - 4*A*b*c -
 4*a*B*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c])
/(8*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15

method	result
default	$B\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{(4ac-b^{2})\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+A\left(\sqrt{c\,x^{2}+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+\right.}{2\sqrt{c}}\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
B*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+A*((c*x^2+b*x+a)^(1/2)+1/2*b*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*
x^2+b*x+a)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 651, normalized size of antiderivative = 5.05

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}\,dx$$
$$= \left[\frac{8\,A\sqrt{a}c^2\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)-(Bb^2-4\,(Ba+Ab)c)\sqrt{c}\log\left(-8\,c^2x^2-8\,b\right.}{16\,c^2}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/16*(8*A*sqrt(a)*c^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 +
b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (B*b^2 - 4*(B*a + A*b)*c)*sqr
t(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*
sqrt(c) - 4*a*c) + 4*(2*B*c^2*x + B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x + a))/
c^2, 1/8*(4*A*sqrt(a)*c^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2
 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + (B*b^2 - 4*(B*a + A*b)*c)*
sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 +
b*c*x + a*c)) + 2*(2*B*c^2*x + B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x + a))/c^2
, 1/16*(16*A*sqrt(-a)*c^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqr
t(-a)/(a*c*x^2 + a*b*x + a^2)) - (B*b^2 - 4*(B*a + A*b)*c)*sqrt(c)*log(-8*
c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*
a*c) + 4*(2*B*c^2*x + B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x + a))/c^2, 1/8*(8*
A*sqrt(-a)*c^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*
x^2 + a*b*x + a^2)) + (B*b^2 - 4*(B*a + A*b)*c)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*B*c^2
*x + B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x + a))/c^2]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}\,dx = \int \frac{(A+Bx)\,\sqrt{a+bx+cx^2}}{x}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x,x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}\,dx = A\sqrt{c\,x^2+b\,x+a}+B\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a} - A\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right) + \frac{A\,b\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{2\,\sqrt{c}} + \frac{B\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x,x)
```

output
```
A*(a + b*x + c*x^2)^(1/2) + B*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - A*
a^(1/2)*log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^2)^(1/2))/x) + (A*b*log((b
/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/(2*c^(1/2)) + (B*log((b/2 +
c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 955, normalized size of antiderivative = 7.40

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x,x)
```

output
```
( - 8*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*a*b*c**2 - 16*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sq
rt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c
 - b**2))*a**2*c**2 - 4*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*l
og( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x +
c*x**2) + b + 2*c*x)*a*b*c**2 + 4*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c
 + b**2)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a +
 b*x + c*x**2) + b + 2*c*x)*a*b*c**2 + 8*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b
+ 4*a*c + b**2)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**2*c**2 - 8*sqrt(c)*sqrt(4*sqrt(c)
*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) +
2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**2*c**2 + 32*sqrt(a + b*x
+ c*x**2)*a**2*c**3 + 16*sqrt(a + b*x + c*x**2)*a*b*c**3*x - 2*sqrt(a + b*
x + c*x**2)*b**4*c - 4*sqrt(a + b*x + c*x**2)*b**3*c**2*x + 16*sqrt(a)*log
( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*
x**2) + b + 2*c*x)*a**2*c**3 - 4*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b +
 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*b**2*c**2
 + 16*sqrt(a)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)*a**2*c**3 - 4*sqrt(a)*log(sqrt(4*sqrt(...
```

3.110 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx = -\frac{(A-Bx)\sqrt{a+bx+cx^2}}{x} - \frac{(Ab+2aB)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}} + \frac{(bB+2Ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}}$$

output
```
-(-B*x+A)*(c*x^2+b*x+a)^(1/2)/x-1/2*(A*b+2*B*a)*arctanh(1/2*(b*x+2*a)/a^(1
/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)+1/2*(2*A*c+B*b)*arctanh(1/2*(2*c*x+b)/c^(
1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx = \frac{(-A+Bx)\sqrt{a+x(b+cx)}}{x} - \frac{(Ab+2aB)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{(bB+2Ac)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{2\sqrt{c}}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^2,x]
```

output
```
((-A + B*x)*Sqrt[a + x*(b + c*x)])/x - ((A*b + 2*a*B)*ArcTanh[(-(Sqrt[c]*x
) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] - ((b*B + 2*A*c)*Log[b + 2*c*
x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(2*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx$$

$$\Big\downarrow \text{1230}$$

$$-\frac{1}{2}\int -\frac{Ab+2aB+(bB+2Ac)x}{x\sqrt{cx^2+bx+a}}dx - \frac{(A-Bx)\sqrt{a+bx+cx^2}}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{Ab+2aB+(bB+2Ac)x}{x\sqrt{cx^2+bx+a}}dx - \frac{(A-Bx)\sqrt{a+bx+cx^2}}{x}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left((2Ac+bB)\int \frac{1}{\sqrt{cx^2+bx+a}}dx+(2aB+Ab)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx\right)-\frac{(A-Bx)\sqrt{a+bx+cx^2}}{x}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left((2aB+Ab)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+2(2Ac+bB)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)-\frac{(A-Bx)\sqrt{a+bx+cx^2}}{x}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left((2aB+Ab)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{(2Ac+bB)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)-\frac{(A-Bx)\sqrt{a+bx+cx^2}}{x}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{(2Ac+bB)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2(2aB+Ab)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)-\frac{(A-Bx)\sqrt{a+bx+cx^2}}{x}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(2Ac+bB)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-\frac{(2aB+Ab)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\right)-\frac{(A-Bx)\sqrt{a+bx+cx^2}}{x}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^2,x]
```

output
```
-(((A - B*x)*Sqrt[a + b*x + c*x^2])/x) + (-(((A*b + 2*a*B)*ArcTanh[(2*a +
b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) + ((b*B + 2*A*c)*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{A\sqrt{c\,x^2+bx+a}}{x}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)Ab}{2\sqrt{a}}-\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)B+A\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\cdots\right.$
default	$A\left(-\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{ax}+\frac{b\left(\sqrt{c\,x^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\right)}{2a}+\frac{2c\left(\frac{(2cx+b)\sqrt{c\,x^2\cdots}}{4c}\cdots\right.}{\cdots}\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-A/x*(c*x^2+b*x+a)^(1/2)-1/2/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(
1/2))/x)*A*b-a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*B+A*c^(
1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*B*b*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+B*(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 648, normalized size of antiderivative = 5.36

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx$$
$$=\left[\frac{(2\,Ba+Ab)\sqrt{a}cx\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)+(Bab+2\,Aac)\sqrt{c}x\log\left(-8\,c^2x^2-\cdots\right.}{4\,acx}\cdots\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*((2*B*a + A*b)*sqrt(a)*c*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt
(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + (B*a*b + 2*A*a*c)*sq
rt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) + 4*(B*a*c*x - A*a*c)*sqrt(c*x^2 + b*x + a))/(a*c*x),
1/4*((2*B*a + A*b)*sqrt(a)*c*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(
c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 2*(B*a*b + 2*A*a*c)*s
qrt(-c)*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 +
 b*c*x + a*c)) + 4*(B*a*c*x - A*a*c)*sqrt(c*x^2 + b*x + a))/(a*c*x), 1/4*(
2*(2*B*a + A*b)*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*
sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (B*a*b + 2*A*a*c)*sqrt(c)*x*log(-8*c^2
*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c
) + 4*(B*a*c*x - A*a*c)*sqrt(c*x^2 + b*x + a))/(a*c*x), 1/2*((2*B*a + A*b)
*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x
^2 + a*b*x + a^2)) - (B*a*b + 2*A*a*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(B*a*c*x - A*a*
c)*sqrt(c*x^2 + b*x + a))/(a*c*x)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx = \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**2,x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx = \sqrt{cx^2+bx+a}B + \frac{(2\,Ba+Ab)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{(Bb+2\,Ac)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)Ab+2\,Aa\sqrt{c}}{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x + a)*B + (2*B*a + A*b)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))/sqrt(-a))/sqrt(-a) - 1/2*(B*b + 2*A*c)*log(abs(-2*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c) + ((sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*A*b + 2*A*a*sqrt(c))/((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 11.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx = B\sqrt{c\,x^2+b\,x+a} - \frac{A\sqrt{c\,x^2+b\,x+a}}{x} - B\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right) + A\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right) + \frac{B\,b\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{2\sqrt{c}} - \frac{A\,b\,\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{2\sqrt{a}}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^2,x)
```

output
```
B*(a + b*x + c*x^2)^(1/2) - (A*(a + b*x + c*x^2)^(1/2))/x - B*a^(1/2)*log(
b/2 + a/x + (a^(1/2)*(a + b*x + c*x^2)^(1/2))/x) + A*c^(1/2)*log((b/2 + c*
x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)) + (B*b*log((b/2 + c*x)/c^(1/2) + (a
+ b*x + c*x^2)^(1/2)))/(2*c^(1/2)) - (A*b*log(b/2 + a/x + (a^(1/2)*(a + b*
x + c*x^2)^(1/2))/x))/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^2}\,dx = \frac{-2\sqrt{c\,x^2+bx+a}\,ac+2\sqrt{c\,x^2+bx+a}\,bcx+3\sqrt{a}\log\left(2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)bcx-3\sqrt{a}\log(x}{2cx}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(a + b*x + c*x**2)*a*c + 2*sqrt(a + b*x + c*x**2)*b*c*x + 3*sqrt
(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b*c*x - 3*sqrt(a)*lo
g(x)*b*c*x + 2*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x
)*a*c*x + sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*b**
2*x)/(2*c*x)
```

3.111 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 133

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx = -\frac{(2aA+(Ab+4aB)x)\sqrt{a+bx+cx^2}}{4ax^2} + \frac{(Ab^2-4abB-4aAc)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}} + B\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)$$

output
```
-1/4*(2*a*A+(A*b+4*B*a)*x)*(c*x^2+b*x+a)^(1/2)/a/x^2+1/8*(-4*A*a*c+A*b^2-4
*B*a*b)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)+B*c^(1/
2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx = -\frac{(Abx+2a(A+2Bx))\sqrt{a+x(b+cx)}}{4ax^2} - \frac{Ab^2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{(bB+Ac)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} - B\sqrt{c}\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^3,x]
```

output
```
-1/4*((A*b*x + 2*a*(A + 2*B*x))*Sqrt[a + x*(b + c*x)])/(a*x^2) - (A*b^2*Ar
cTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(4*a^(3/2)) - ((b*B +
A*c)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] - B*
Sqrt[c]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{2x\sqrt{cx^2+bx+a}}dx}{4a} - \frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a} - \frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\int -\frac{4abB+8acxB-A(b^2-4ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int -\frac{Ab^2-4aBb-4aAc-8aBcx}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(x(4aB+Ab)+2aA)}{4ax^2}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,\left(A b x+4 B a x+2 A a\right)}{4 x^{2} a}+\frac{-\frac{\left(4 A a c-b^{2} A+4 a b B\right) \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{\sqrt{a}}+8 a B \sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{8 a}$
default	$A\left(-\frac{\left(c\,x^{2}+b x+a\right)^{\frac{3}{2}}}{2 a\,x^{2}}-\frac{b\left(-\frac{\left(c\,x^{2}+b x+a\right)^{\frac{3}{2}}}{a x}+\frac{b\left(\sqrt{c\,x^{2}+b x+a}+\frac{b \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{2 \sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)\right)}{2 a}+\cdots\right)}{4 a}\cdots\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(A*b*x+4*B*a*x+2*A*a)/x^2/a+1/8/a*(-(4*A*a*c-A*b^
2+4*B*a*b)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+8*a*B*c^(
1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 699, normalized size of antiderivative = 5.26

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/16*(8*B*a^2*sqrt(c)*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
 b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + (4*B*a*b - A*b^2 + 4*A*a*c)*sqrt(
a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(2*A*a^2 + (4*B*a^2 + A*a*b)*x)*sqrt(c*x^2
+ b*x + a))/(a^2*x^2), -1/16*(16*B*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - (4*B*a*b - A*b^
2 + 4*A*a*c)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2
+ b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(2*A*a^2 + (4*B*a^2 + A*a
*b)*x)*sqrt(c*x^2 + b*x + a))/(a^2*x^2), 1/8*(4*B*a^2*sqrt(c)*x^2*log(-8*c
^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a
*c) + (4*B*a*b - A*b^2 + 4*A*a*c)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x
 + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*A*a^2 + (4*B*a^
2 + A*a*b)*x)*sqrt(c*x^2 + b*x + a))/(a^2*x^2), -1/8*(8*B*a^2*sqrt(-c)*x^2
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
a*c)) - (4*B*a*b - A*b^2 + 4*A*a*c)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(2*A*a^2 + (4*B*
a^2 + A*a*b)*x)*sqrt(c*x^2 + b*x + a))/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx = \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**3,x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(112) = 224.

Time = 0.27 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.68

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx = -B\sqrt{c}\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right) + \frac{(4\,Bab-Ab^2+4\,Aac)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a} + \frac{4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Bab+\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Ab^2+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Aac+8}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-B*sqrt(c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b)) + 1
/4*(4*B*a*b - A*b^2 + 4*A*a*c)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
/sqrt(-a))/(sqrt(-a)*a) + 1/4*(4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a
*b + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*b^2 + 4*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^3*A*a*c + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B*a^2*sqr
t(c) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*A*a*b*sqrt(c) - 4*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*B*a^2*b + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A
*a*b^2 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*c - 8*B*a^3*sqrt(c))/
(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^2*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx = \int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{x^3}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^3,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^3}\,dx = \frac{-4\sqrt{c\,x^2+bx+a}\,a^2-10\sqrt{c\,x^2+bx+a}\,abx+4\sqrt{a}\log\left(2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)ac\,x^2+3\sqrt{a}\,\mathrm{lo}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^3,x)
```

output
```
( - 4*sqrt(a + b*x + c*x**2)*a**2 - 10*sqrt(a + b*x + c*x**2)*a*b*x + 4*sq
rt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*c*x**2 + 3*sqrt(
a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**2*x**2 - 4*sqrt(a)
*log(x)*a*c*x**2 - 3*sqrt(a)*log(x)*b**2*x**2 + 8*sqrt(c)*log( - 2*sqrt(c)
*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a*b*x**2)/(8*a*x**2)
```

3.112 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}\,dx = \frac{(Ab-2aB)(2a+bx)\sqrt{a+bx+cx^2}}{8a^2x^2} - \frac{A(a+bx+cx^2)^{3/2}}{3ax^3} - \frac{(Ab-2aB)\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{5/2}}$$

output
```
1/8*(A*b-2*B*a)*(b*x+2*a)*(c*x^2+b*x+a)^(1/2)/a^2/x^2-1/3*A*(c*x^2+b*x+a)^
(3/2)/a/x^3-1/16*(A*b-2*B*a)*(-4*a*c+b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c
*x^2+b*x+a)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}\,dx = \frac{\sqrt{a}\sqrt{a+x(b+cx)}(3Ab^2x^2-4a^2(2A+3Bx)-2ax(3bBx+A(b+4cx)))+3(Ab^3+8a^2Bc)\,x^3\operatorname{arctanh}}{24a^{5/2}x^3}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^4,x]
```

output
```
(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(3*A*b^2*x^2 - 4*a^2*(2*A + 3*B*x) - 2*a*x*
(3*b*B*x + A*(b + 4*c*x))) + 3*(A*b^3 + 8*a^2*B*c)*x^3*ArcTanh[(Sqrt[c]*x
- Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 6*a*b*(b*B + 2*A*c)*x^3*ArcTanh[(-(Sqr
t[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(24*a^(5/2)*x^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}dx$$

$$\Big\downarrow \text{1228}$$

$$-\frac{(Ab-2aB)\int \frac{\sqrt{cx^2+bx+a}}{x^3}dx}{2a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{1152}$$

$$-\frac{(Ab-2aB)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{(Ab-2aB)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(Ab-2aB)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{3ax^3}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^4,x]
```

output
```
-1/3*(A*(a + b*x + c*x^2)^(3/2))/(a*x^3) - ((A*b - 2*a*B)*(-1/4*((2*a + b*
x)*Sqrt[a + b*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2))))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(8Aac\,x^2-3x^2b^2A+6Ba\,x^2b+2abAx+12a^2Bx+8a^2A)}{24x^3a^2}+\frac{(4Aabc-A\,b^3-8B\,a^2c+2Ba\,b^2)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+}}{x}\right.}{16a^{\frac{5}{2}}}$
default	$A\left(-\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{3a\,x^3}-\frac{b\left(-\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{2a\,x^2}-\frac{b\left(-\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{ax}+\frac{b\left(\sqrt{c\,x^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{}}{}\right.\right.}{2a}\right.}{}\right.}{}\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+b*x+a)^(1/2)*(8*A*a*c*x^2-3*A*b^2*x^2+6*B*a*b*x^2+2*A*a*b*x+1
2*B*a^2*x+8*A*a^2)/x^3/a^2+1/16*(4*A*a*b*c-A*b^3-8*B*a^2*c+2*B*a*b^2)/a^(5
/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.62

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}\,dx$$
$$= \left[\frac{3\,(2\,Bab^2 - Ab^3 - 4\,(2\,Ba^2 - Aab)c)\sqrt{a}x^3\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2+4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right) - 4\,(8\,Aa^3}{96\,a^3x^3}\right.$$
$$-\frac{3\,(2\,Bab^2 - Ab^3 - 4\,(2\,Ba^2 - Aab)c)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right) + 2\,(8\,Aa^3 + (6\,Ba^2b - 3}{48\,a^3x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/96*(3*(2*B*a*b^2 - A*b^3 - 4*(2*B*a^2 - A*a*b)*c)*sqrt(a)*x^3*log(-(8*a
*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8
*a^2)/x^2) - 4*(8*A*a^3 + (6*B*a^2*b - 3*A*a*b^2 + 8*A*a^2*c)*x^2 + 2*(6*B
*a^3 + A*a^2*b)*x)*sqrt(c*x^2 + b*x + a))/(a^3*x^3), -1/48*(3*(2*B*a*b^2 -
 A*b^3 - 4*(2*B*a^2 - A*a*b)*c)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(8*A*a^3 + (6*B*a^2*
b - 3*A*a*b^2 + 8*A*a^2*c)*x^2 + 2*(6*B*a^3 + A*a^2*b)*x)*sqrt(c*x^2 + b*x
 + a))/(a^3*x^3)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}\,dx = \int \frac{(A+Bx)\,\sqrt{a+bx+cx^2}}{x^4}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**4,x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(105) = 210$.

Time = 0.29 (sec) , antiderivative size = 524, normalized size of antiderivative = 4.33

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}\,dx$$
$$= -\frac{(2\,Bab^2 - Ab^3 - 8\,Ba^2c + 4\,Aabc)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^2}$$
$$+ \frac{6\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5 Bab^2 - 3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5 Ab^3 + 24\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5 Ba^2c}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/8*(2*B*a*b^2 - A*b^3 - 8*B*a^2*c + 4*A*a*b*c)*arctan(-(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/24*(6*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^5*B*a*b^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*b^3
+ 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^2*c + 12*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*A*a*b*c + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*B*a
^2*b*sqrt(c) + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*A*a^2*c^(3/2) + 8*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a*b^3 + 48*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*A*a^2*b*c - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B*a^3*
b*sqrt(c) + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*A*a^2*b^2*sqrt(c) - 6
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^3*b^2 + 3*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*A*a^2*b^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^4*c +
36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^3*b*c + 16*A*a^4*c^(3/2))/(((sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^3*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}\,dx=\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{x^4}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^4,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^4}\,dx$$
$$=\frac{-16\sqrt{c\,x^2+bx+a}\,a^3-28\sqrt{c\,x^2+bx+a}\,a^2bx-16\sqrt{c\,x^2+bx+a}\,a^2c\,x^2-6\sqrt{c\,x^2+bx+a}\,a\,b^2x^2+1}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x + c*x**2)*a**3 - 28*sqrt(a + b*x + c*x**2)*a**2*b*x -
16*sqrt(a + b*x + c*x**2)*a**2*c*x**2 - 6*sqrt(a + b*x + c*x**2)*a*b**2*x*
*2 + 12*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b*c*x*
*3 - 3*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**3*x**3
 - 12*sqrt(a)*log(x)*a*b*c*x**3 + 3*sqrt(a)*log(x)*b**3*x**3)/(48*a**2*x**
3)
```

3.113 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 172

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx = -\frac{(5Ab^2-8abB-4aAc)\,(2a+bx)\sqrt{a+bx+cx^2}}{64a^3x^2} - \frac{A(a+bx+cx^2)^{3/2}}{4ax^4} + \frac{(5Ab-8aB)\,(a+bx+cx^2)^{3/2}}{24a^2x^3} + \frac{(b^2-4ac)\,(5Ab^2-8abB-4aAc)\,\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{7/2}}$$

output

```
-1/64*(-4*A*a*c+5*A*b^2-8*B*a*b)*(b*x+2*a)*(c*x^2+b*x+a)^(1/2)/a^3/x^2-1/4
*A*(c*x^2+b*x+a)^(3/2)/a/x^4+1/24*(5*A*b-8*B*a)*(c*x^2+b*x+a)^(3/2)/a^2/x^
3+1/128*(-4*a*c+b^2)*(-4*A*a*c+5*A*b^2-8*B*a*b)*arctanh(1/2*(b*x+2*a)/a^(1
/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+x(b+cx)}(15Ab^3x^3+16a^3(3A+4Bx)-2abx^2(5Ab+12bBx+26Acx)+8a^2x(A(b+3cx)+2Bx(b+4cx)))}{x^4} - 15Ab^4\text{arctanh}\left(\frac{\sqrt{c}x-}{}\right.}{192a^{7/2}}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^5,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(15*A*b^3*x^3 + 16*a^3*(3*A + 4*B*x) - 2
*a*b*x^2*(5*A*b + 12*b*B*x + 26*A*c*x) + 8*a^2*x*(A*(b + 3*c*x) + 2*B*x*(b
 + 4*c*x))))/x^4) - 15*A*b^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/S
qrt[a]] + 24*a*(-(b^3*B) - 3*A*b^2*c + 4*a*b*B*c + 2*a*A*c^2)*ArcTanh[(-(S
qrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(192*a^(7/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx$$

$$\Big\downarrow \ 1237$$

$$-\frac{\int \frac{(5Ab-8aB+2Acx)\sqrt{cx^2+bx+a}}{2x^4}dx}{4a} - \frac{A(a+bx+cx^2)^{3/2}}{4ax^4}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{(5Ab-8aB+2Acx)\sqrt{cx^2+bx+a}}{x^4}dx}{8a} - \frac{A(a+bx+cx^2)^{3/2}}{4ax^4}$$

$$\Big\downarrow \; 1228$$

$$-\frac{-\frac{\left(-4aAc-8abB+5Ab^2\right)\int \frac{\sqrt{cx^2+bx+a}}{x^3}dx}{2a}-\frac{(5Ab-8aB)\left(a+bx+cx^2\right)^{3/2}}{3ax^3}}{8a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \; 1152$$

$$-\frac{-\frac{\left(-4aAc-8abB+5Ab^2\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{(5Ab-8aB)\left(a+bx+cx^2\right)^{3/2}}{3ax^3}}{8a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{\left(-4aAc-8abB+5Ab^2\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{(5Ab-8aB)\left(a+bx+cx^2\right)^{3/2}}{3ax^3}}{8a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\left(-4aAc-8abB+5Ab^2\right)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{(5Ab-8aB)\left(a+bx+cx^2\right)^{3/2}}{3ax^3}}{8a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{4ax^4}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^5,x]
```

output
```
-1/4*(A*(a + b*x + c*x^2)^(3/2))/(a*x^4) - (-1/3*((5*A*b - 8*a*B)*(a + b*x
 + c*x^2)^(3/2))/(a*x^3) - ((5*A*b^2 - 8*a*b*B - 4*a*A*c)*(-1/4*((2*a + b*
x)*Sqrt[a + b*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2))))/(2*a))/(8*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-52Aabc\,x^3+15A\,b^3x^3+64B\,a^2c\,x^3-24Ba\,b^2x^3+24A\,a^2c\,x^2-10Aa\,b^2x^2+16B\,a^2b\,x^2+8A\,a^2bx+64B\,a^3x+48a^3}{192x^4a^3}$
default	$A\Bigg(-\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{5b\left(-\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{3a\,x^3}-\frac{b\left(-\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{2a\,x^2}-\frac{b\left(-\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{ax}+\frac{b\left(\sqrt{c\,x^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+\cdots}\right)}{2\sqrt{c}}\right.}{}\right.}{}\right.}{}\right.}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x^2+b*x+a)^(1/2)*(-52*A*a*b*c*x^3+15*A*b^3*x^3+64*B*a^2*c*x^3-24
*B*a*b^2*x^3+24*A*a^2*c*x^2-10*A*a*b^2*x^2+16*B*a^2*b*x^2+8*A*a^2*b*x+64*B
*a^3*x+48*A*a^3)/x^4/a^3+1/128*(16*A*a^2*c^2-24*A*a*b^2*c+5*A*b^4+32*B*a^2
*b*c-8*B*a*b^3)/a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.47

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx$$
$$= \left[-\frac{3\,(8\,Bab^3-5\,Ab^4-16\,Aa^2c^2-8\,(4\,Ba^2b-3\,Aab^2)c)\sqrt{a}x^4\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2+4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{}}{x^2}\right.}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[-1/768*(3*(8*B*a*b^3 - 5*A*b^4 - 16*A*a^2*c^2 - 8*(4*B*a^2*b - 3*A*a*b^2)
*c)*sqrt(a)*x^4*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a
)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(48*A*a^4 - (24*B*a^2*b^2 - 15*A*a
*b^3 - 4*(16*B*a^3 - 13*A*a^2*b)*c)*x^3 + 2*(8*B*a^3*b - 5*A*a^2*b^2 + 12*
A*a^3*c)*x^2 + 8*(8*B*a^4 + A*a^3*b)*x)*sqrt(c*x^2 + b*x + a))/(a^4*x^4),
1/384*(3*(8*B*a*b^3 - 5*A*b^4 - 16*A*a^2*c^2 - 8*(4*B*a^2*b - 3*A*a*b^2)*c
)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*
x^2 + a*b*x + a^2)) - 2*(48*A*a^4 - (24*B*a^2*b^2 - 15*A*a*b^3 - 4*(16*B*a
^3 - 13*A*a^2*b)*c)*x^3 + 2*(8*B*a^3*b - 5*A*a^2*b^2 + 12*A*a^3*c)*x^2 + 8
*(8*B*a^4 + A*a^3*b)*x)*sqrt(c*x^2 + b*x + a))/(a^4*x^4)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx = \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**5,x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(150) = 300$.

Time = 0.26 (sec) , antiderivative size = 991, normalized size of antiderivative = 5.76

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/64*(8*B*a*b^3 - 5*A*b^4 - 32*B*a^2*b*c + 24*A*a*b^2*c - 16*A*a^2*c^2)*ar
ctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/192
*(24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a*b^3 - 15*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^7*A*b^4 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a^2
*b*c + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a*b^2*c - 48*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^7*A*a^2*c^2 - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*B*a^3*c^(3/2) - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^2*b^3
+ 55*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a*b^4 - 288*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*B*a^3*b*c - 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*A*a^2*b^2*c - 336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a^3*c^2 - 384*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*B*a^3*b^2*sqrt(c) + 384*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^4*B*a^4*c^(3/2) - 1152*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^4*A*a^3*b*c^(3/2) + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a^3
*b^3 - 73*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a^2*b^4 + 96*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^3*B*a^4*b*c - 648*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*A*a^3*b^2*c - 336*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a^4*c^2 +
 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B*a^4*b^2*sqrt(c) - 384*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*A*a^3*b^3*sqrt(c) - 128*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*B*a^5*c^(3/2) - 256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*A*a^4*b*c^(3/2) + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^4*b^3 -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx=\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{x^5}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^5,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^5}\,dx$$
$$= \frac{-32\sqrt{c\,x^2+bx+a}\,a^4-48\sqrt{c\,x^2+bx+a}\,a^3bx-16\sqrt{c\,x^2+bx+a}\,a^3c\,x^2-4\sqrt{c\,x^2+bx+a}\,a^2b^2x^2-8}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^5,x)
```

output
```
( - 32*sqrt(a + b*x + c*x**2)*a**4 - 48*sqrt(a + b*x + c*x**2)*a**3*b*x -
16*sqrt(a + b*x + c*x**2)*a**3*c*x**2 - 4*sqrt(a + b*x + c*x**2)*a**2*b**2
*x**2 - 8*sqrt(a + b*x + c*x**2)*a**2*b*c*x**3 + 6*sqrt(a + b*x + c*x**2)*
a*b**3*x**3 + 16*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b
*x)*a**2*c**2*x**4 + 8*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2
*a - b*x)*a*b**2*c*x**4 - 3*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2
) - 2*a - b*x)*b**4*x**4 - 16*sqrt(a)*log(x)*a**2*c**2*x**4 - 8*sqrt(a)*lo
g(x)*a*b**2*c*x**4 + 3*sqrt(a)*log(x)*b**4*x**4)/(128*a**3*x**4)
```

3.114 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 235

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6} \, dx &= \frac{(7Ab^3-10ab^2B-12aAbc+8a^2Bc)\,(2a+bx)\sqrt{a+bx+cx^2}}{128a^4x^2} - \frac{A(a+bx+cx^2)^{3/2}}{5ax^5} \\ &\quad + \frac{(7Ab-10aB)\,(a+bx+cx^2)^{3/2}}{40a^2x^4} - \frac{(35Ab^2-50abB-32aAc)\,(a+bx+cx^2)^{3/2}}{240a^3x^3} \\ &\quad + \frac{(b^2-4ac)\,(2aB(5b^2-4ac)-A(7b^3-12abc))\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{256a^{9/2}}\end{aligned}$$

output

```
1/128*(-12*A*a*b*c+7*A*b^3+8*B*a^2*c-10*B*a*b^2)*(b*x+2*a)*(c*x^2+b*x+a)^(
1/2)/a^4/x^2-1/5*A*(c*x^2+b*x+a)^(3/2)/a/x^5+1/40*(7*A*b-10*B*a)*(c*x^2+b*
x+a)^(3/2)/a^2/x^4-1/240*(-32*A*a*c+35*A*b^2-50*B*a*b)*(c*x^2+b*x+a)^(3/2)
/a^3/x^3+1/256*(-4*a*c+b^2)*(2*a*B*(-4*a*c+5*b^2)-A*(-12*a*b*c+7*b^3))*arc
tanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6}\,dx$$
$$= \frac{\sqrt{a}\sqrt{a+x(b+cx)}(105Ab^4x^4-96a^4(4A+5Bx)-10ab^2x^3(7Ab+15bBx+46Acx)-16a^3x(5Bx(b+$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^6,x]
```

output
```
(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(105*A*b^4*x^4 - 96*a^4*(4*A + 5*B*x) - 10*
a*b^2*x^3*(7*A*b + 15*b*B*x + 46*A*c*x) - 16*a^3*x*(5*B*x*(b + 3*c*x) + A*
(3*b + 8*c*x)) + 4*a^2*x^2*(5*b*B*x*(5*b + 26*c*x) + 2*A*(7*b^2 + 29*b*c*x
 + 32*c^2*x^2))) + 15*(7*A*b^5 - 32*a^3*B*c^2)*x^5*ArcTanh[(Sqrt[c]*x - Sq
rt[a + x*(b + c*x)])/Sqrt[a]] - 30*a*b*(-5*b^3*B - 20*A*b^2*c + 24*a*b*B*c
 + 24*a*A*c^2)*x^5*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]]
)/(1920*a^(9/2)*x^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1237, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6}\,dx$$
$$\Big\downarrow \ 1237$$
$$-\frac{\int \frac{(7Ab-10aB+4Acx)\sqrt{cx^2+bx+a}}{2x^5}dx}{5a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{5ax^5}$$
$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{(7Ab-10aB+4Acx)\sqrt{cx^2+bx+a}}{x^5}dx}{10a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 1237

$$-\frac{-\frac{\int \frac{\left(35Ab^2-50aBb-32aAc+2(7Ab-10aB)cx\right)\sqrt{cx^2+bx+a}}{2x^4}dx}{4a}-\frac{(7Ab-10aB)\left(a+bx+cx^2\right)^{3/2}}{4ax^4}}{10a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{\left(35Ab^2-50aBb-32aAc+2(7Ab-10aB)cx\right)\sqrt{cx^2+bx+a}}{x^4}dx}{8a}-\frac{(7Ab-10aB)\left(a+bx+cx^2\right)^{3/2}}{4ax^4}}{10a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 1228

$$-\frac{-\frac{-\frac{5\left(8a^2Bc-12aAbc-10ab^2B+7Ab^3\right)\int \frac{\sqrt{cx^2+bx+a}}{x^3}dx}{2a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-32aAc-50abB+35Ab^2\right)}{3ax^3}}{8a}-\frac{(7Ab-10aB)\left(a+bx+cx^2\right)^{3/2}}{4ax^4}}{10a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 1152

$$-\frac{-\frac{-\frac{5\left(8a^2Bc-12aAbc-10ab^2B+7Ab^3\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-32aAc-50abB+35Ab^2\right)}{3ax^3}}{8a}-\frac{(7Ab-}{}}{10a}$$

$$\frac{A\left(a+bx+cx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 1154

$$-\frac{-\frac{-\frac{5\left(8a^2Bc-12aAbc-10ab^2B+7Ab^3\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-32aAc-50abB+35Ab^2\right)}{3ax^3}}{8a}}{10a}$$

$$\frac{A\left(a+bx+cx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{-\frac{5\left(8a^2Bc-12aAbc-10ab^2B+7Ab^3\right)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-32aAc-50abB+35Ab^2\right)}{3ax^3}}{8a}}{10a}$$

$$\frac{A\left(a+bx+cx^2\right)^{3/2}}{5ax^5}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^6,x]
```

output
```
-1/5*(A*(a + b*x + c*x^2)^(3/2))/(a*x^5) - (-1/4*((7*A*b - 10*a*B)*(a + b*
x + c*x^2)^(3/2))/(a*x^4) - (-1/3*((35*A*b^2 - 50*a*b*B - 32*a*A*c)*(a + b
*x + c*x^2)^(3/2))/(a*x^3) - (5*(7*A*b^3 - 10*a*b^2*B - 12*a*A*b*c + 8*a^2
*B*c)*(-1/4*((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2))))/(2*a)
)/(8*a))/(10*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,\left(-256 A\,a^{2} c^{2} x^{4}+460 A a\,b^{2} c\,x^{4}-105 A\,b^{4} x^{4}-520 B\,a^{2} b c\,x^{4}+150 B a\,b^{3} x^{4}-232 A\,a^{2} b c\,x^{3}+70 A a\,b^{3} x^{3}+240 B\,a^{3} c\,x^{3}-\right.}{1920 x^{5} a^{4}}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(c*x^2+b*x+a)^(1/2)*(-256*A*a^2*c^2*x^4+460*A*a*b^2*c*x^4-105*A*b^
4*x^4-520*B*a^2*b*c*x^4+150*B*a*b^3*x^4-232*A*a^2*b*c*x^3+70*A*a*b^3*x^3+2
40*B*a^3*c*x^3-100*B*a^2*b^2*x^3+128*A*a^3*c*x^2-56*A*a^2*b^2*x^2+80*B*a^3
*b*x^2+48*A*a^3*b*x+480*B*a^4*x+384*A*a^4)/x^5/a^4-1/256*(48*A*a^2*b*c^2-4
0*A*a*b^3*c+7*A*b^5-32*B*a^3*c^2+48*B*a^2*b^2*c-10*B*a*b^4)/a^(9/2)*ln((2*
a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.35

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6}\,dx$$

$$= \Bigg[-\frac{15\,(10\,Bab^4 - 7\,Ab^5 + 16\,(2\,Ba^3 - 3\,Aa^2b)c^2 - 8\,(6\,Ba^2b^2 - 5\,Aab^3)c)\sqrt{a}x^5 \log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{}}{}\right.}{}$$

$$-\frac{15\,(10\,Bab^4 - 7\,Ab^5 + 16\,(2\,Ba^3 - 3\,Aa^2b)c^2 - 8\,(6\,Ba^2b^2 - 5\,Aab^3)c)\sqrt{-a}x^5 \arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a}{2\,(acx^2+abx+a}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
[-1/7680*(15*(10*B*a*b^4 - 7*A*b^5 + 16*(2*B*a^3 - 3*A*a^2*b)*c^2 - 8*(6*B
*a^2*b^2 - 5*A*a*b^3)*c)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4
*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(384*A*a^5 +
(150*B*a^2*b^3 - 105*A*a*b^4 - 256*A*a^3*c^2 - 20*(26*B*a^3*b - 23*A*a^2*b
^2)*c)*x^4 - 2*(50*B*a^3*b^2 - 35*A*a^2*b^3 - 4*(30*B*a^4 - 29*A*a^3*b)*c)
*x^3 + 8*(10*B*a^4*b - 7*A*a^3*b^2 + 16*A*a^4*c)*x^2 + 48*(10*B*a^5 + A*a^
4*b)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^5), -1/3840*(15*(10*B*a*b^4 - 7*A*b^
5 + 16*(2*B*a^3 - 3*A*a^2*b)*c^2 - 8*(6*B*a^2*b^2 - 5*A*a*b^3)*c)*sqrt(-a)
*x^5*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*
x + a^2)) + 2*(384*A*a^5 + (150*B*a^2*b^3 - 105*A*a*b^4 - 256*A*a^3*c^2 -
20*(26*B*a^3*b - 23*A*a^2*b^2)*c)*x^4 - 2*(50*B*a^3*b^2 - 35*A*a^2*b^3 - 4
*(30*B*a^4 - 29*A*a^3*b)*c)*x^3 + 8*(10*B*a^4*b - 7*A*a^3*b^2 + 16*A*a^4*c
)*x^2 + 48*(10*B*a^5 + A*a^4*b)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^5)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6}\,dx = \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**6,x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1407 vs. $2(209) = 418$.

Time = 0.27 (sec) , antiderivative size = 1407, normalized size of antiderivative = 5.99

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
-1/128*(10*B*a*b^4 - 7*A*b^5 - 48*B*a^2*b^2*c + 40*A*a*b^3*c + 32*B*a^3*c^
2 - 48*A*a^2*b*c^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/
(sqrt(-a)*a^4) + 1/1920*(150*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a*b^4
 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*b^5 - 720*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*B*a^2*b^2*c + 600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^9*A*a*b^3*c + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a^3*c^2 - 720*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a^2*b*c^2 - 700*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*B*a^2*b^4 + 490*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A
*a*b^5 + 3360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a^3*b^2*c - 2800*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a^2*b^3*c + 2880*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^7*B*a^4*c^2 + 3360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A
*a^3*b*c^2 + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*B*a^4*b*c^(3/2) +
 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*A*a^4*c^(5/2) + 1280*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*B*a^3*b^4 - 896*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*A*a^2*b^5 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^4*b^2*
c + 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a^3*b^3*c + 15360*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^5*A*a^4*b*c^2 + 3840*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^4*B*a^4*b^3*sqrt(c) - 8960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*B*a^5*b*c^(3/2) + 24320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*A*a^4*b^2
*c^(3/2) + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*A*a^5*c^(5/2) - 5...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6}\,dx=\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{x^6}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^6,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.46

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^6}\,dx$$
$$= \frac{-768\sqrt{c\,x^2+bx+a}\,a^5-1056\sqrt{c\,x^2+bx+a}\,a^4bx-256\sqrt{c\,x^2+bx+a}\,a^4c\,x^2-48\sqrt{c\,x^2+bx+a}\,a^3b^2}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^6,x)
```

output
```
( - 768*sqrt(a + b*x + c*x**2)*a**5 - 1056*sqrt(a + b*x + c*x**2)*a**4*b*x
 - 256*sqrt(a + b*x + c*x**2)*a**4*c*x**2 - 48*sqrt(a + b*x + c*x**2)*a**3
*b**2*x**2 - 16*sqrt(a + b*x + c*x**2)*a**3*b*c*x**3 + 512*sqrt(a + b*x +
c*x**2)*a**3*c**2*x**4 + 60*sqrt(a + b*x + c*x**2)*a**2*b**3*x**3 + 120*sq
rt(a + b*x + c*x**2)*a**2*b**2*c*x**4 - 90*sqrt(a + b*x + c*x**2)*a*b**4*x
**4 + 240*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b
*c**2*x**5 + 120*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)
*a*b**3*c*x**5 - 45*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b
*x)*b**5*x**5 - 240*sqrt(a)*log(x)*a**2*b*c**2*x**5 - 120*sqrt(a)*log(x)*a
*b**3*c*x**5 + 45*sqrt(a)*log(x)*b**5*x**5)/(3840*a**4*x**5)
```

3.115 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx &= \frac{(4abB(7b^2-12ac)-A(21b^4-56ab^2c+16a^2c^2))\,(2a+bx)\sqrt{a+bx+cx^2}}{512a^5x^2} \\ &\quad -\frac{A(a+bx+cx^2)^{3/2}}{6ax^6}+\frac{(3Ab-4aB)\,(a+bx+cx^2)^{3/2}}{20a^2x^5} \\ &\quad -\frac{(21Ab^2-28abB-20aAc)\,(a+bx+cx^2)^{3/2}}{160a^3x^4} \\ &\quad +\frac{(105Ab^3-140ab^2B-196aAbc+128a^2Bc)\,(a+bx+cx^2)^{3/2}}{960a^4x^3} \\ &\quad -\frac{(b^2-4ac)\,(4abB(7b^2-12ac)-A(21b^4-56ab^2c+16a^2c^2))\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{1024a^{11/2}}\end{aligned}$$

output

```
1/512*(4*a*b*B*(-12*a*c+7*b^2)-A*(16*a^2*c^2-56*a*b^2*c+21*b^4))*(b*x+2*a)
*(c*x^2+b*x+a)^(1/2)/a^5/x^2-1/6*A*(c*x^2+b*x+a)^(3/2)/a/x^6+1/20*(3*A*b-4
*B*a)*(c*x^2+b*x+a)^(3/2)/a^2/x^5-1/160*(-20*A*a*c+21*A*b^2-28*B*a*b)*(c*x
^2+b*x+a)^(3/2)/a^3/x^4+1/960*(-196*A*a*b*c+105*A*b^3+128*B*a^2*c-140*B*a*
b^2)*(c*x^2+b*x+a)^(3/2)/a^4/x^3-1/1024*(-4*a*c+b^2)*(4*a*b*B*(-12*a*c+7*b
^2)-A*(16*a^2*c^2-56*a*b^2*c+21*b^4))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2
+b*x+a)^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx$$
$$= \frac{-\sqrt{a}\sqrt{a+x(b+cx)}(315Ab^5x^5+256a^5(5A+6Bx)-210ab^3x^4(2bBx+A(b+8cx))+64a^4x(A(2b+5\ldots}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^7,x]
```

output
```
(-(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(315*A*b^5*x^5 + 256*a^5*(5*A + 6*B*x) -
210*a*b^3*x^4*(2*b*B*x + A*(b + 8*c*x)) + 64*a^4*x*(A*(2*b + 5*c*x) + B*x*
(3*b + 8*c*x)) - 16*a^3*x^2*(A*(9*b^2 + 34*b*c*x + 30*c^2*x^2) + 2*B*x*(7*
b^2 + 29*b*c*x + 32*c^2*x^2)) + 8*a^2*b*x^3*(5*b*B*x*(7*b + 46*c*x) + A*(2
1*b^2 + 112*b*c*x + 226*c^2*x^2)))) - 315*A*b^6*x^6*ArcTanh[(Sqrt[c]*x - S
qrt[a + x*(b + c*x)])/Sqrt[a]] - 60*a*(7*b^5*B + 35*A*b^4*c - 40*a*b^3*B*c
 - 60*a*A*b^2*c^2 + 48*a^2*b*B*c^2 + 16*a^2*A*c^3)*x^6*ArcTanh[(-(Sqrt[c]*
x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(7680*a^(11/2)*x^6)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1237, 27, 1237, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int \frac{3(3Ab-4aB+2Acx)\sqrt{cx^2+bx+a}}{2x^6}dx}{6a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(3Ab-4aB+2Acx)\sqrt{cx^2+bx+a}}{x^6}dx}{4a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 1237

$$-\frac{-\frac{\int \frac{\left(21Ab^2-28aBb-20aAc+4(3Ab-4aB)cx\right)\sqrt{cx^2+bx+a}}{2x^5}dx}{5a}-\frac{(3Ab-4aB)\left(a+bx+cx^2\right)^{3/2}}{5ax^5}}{4a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{\left(21Ab^2-28aBb-20aAc+4(3Ab-4aB)cx\right)\sqrt{cx^2+bx+a}}{x^5}dx}{10a}-\frac{(3Ab-4aB)\left(a+bx+cx^2\right)^{3/2}}{5ax^5}}{4a}-\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 1237

$$-\frac{-\frac{-\frac{\int \frac{\left(105Ab^3-140aBb^2-196aAcb+128a^2Bc+2c\left(21Ab^2-28aBb-20aAc\right)x\right)\sqrt{cx^2+bx+a}}{2x^4}dx}{4a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-20aAc-28abB+21Ab^2\right)}{4ax^4}}{10a}-\frac{(3Ab-4aB}{}}{4a}$$

$$\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{\int \frac{\left(105Ab^3-140aBb^2-196aAcb+128a^2Bc+2c\left(21Ab^2-28aBb-20aAc\right)x\right)\sqrt{cx^2+bx+a}}{x^4}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-20aAc-28abB+21Ab^2\right)}{4ax^4}}{10a}-\frac{(3Ab-4aB}{}}{4a}$$

$$\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 1228

$$-\frac{-\frac{-\frac{\frac{5\left(4abB\left(7b^2-12ac\right)-A\left(16a^2c^2-56ab^2c+21b^4\right)\right)\int \frac{\sqrt{cx^2+bx+a}}{x^3}dx}{2a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(128a^2Bc-196aAbc-140ab^2B+105Ab^3\right)}{3ax^3}}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(-2\right.}{4a}}{10a}}{4a}$$

$$\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 1152

$$-\frac{-\frac{-\frac{\frac{5\left(4abB\left(7b^2-12ac\right)-A\left(16a^2c^2-56ab^2c+21b^4\right)\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(128a^2Bc-196aAbc-14\right.}{3ax^3}}{8a}}{10a}}{4a}$$

$$\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{-\frac{\frac{5\left(4abB\left(7b^2-12ac\right)-A\left(16a^2c^2-56ab^2c+21b^4\right)\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(128a^2Bc-1\right.}{3a}}{8a}}{10a}}{4a}$$

$$\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{-\frac{\frac{5\left(4abB\left(7b^2-12ac\right)-A\left(16a^2c^2-56ab^2c+21b^4\right)\right)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{2a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(128a^2Bc-1\right.}{3ax}}{8a}}{10a}}{4a}$$

$$\frac{A\left(a+bx+cx^2\right)^{3/2}}{6ax^6}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^7,x]
```

output
```
-1/6*(A*(a + b*x + c*x^2)^(3/2))/(a*x^6) - (-1/5*((3*A*b - 4*a*B)*(a + b*x
 + c*x^2)^(3/2))/(a*x^5) - (-1/4*((21*A*b^2 - 28*a*b*B - 20*a*A*c)*(a + b*
x + c*x^2)^(3/2))/(a*x^4) - (-1/3*((105*A*b^3 - 140*a*b^2*B - 196*a*A*b*c
+ 128*a^2*B*c)*(a + b*x + c*x^2)^(3/2))/(a*x^3) + (5*(4*a*b*B*(7*b^2 - 12*
a*c) - A*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2))*(-1/4*((2*a + b*x)*Sqrt[a + b
*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[
a + b*x + c*x^2])])/(8*a^(3/2))))/(2*a))/(8*a))/(10*a))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(1808A\,a^2b\,c^2x^5-1680Aa\,b^3c\,x^5+315A\,b^5x^5-1024B\,a^3c^2x^5+1840B\,a^2b^2c\,x^5-420Ba\,b^4x^5-480A\,a^3c^2x^4+896A\,a}{}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(c*x^2+b*x+a)^(1/2)*(1808*A*a^2*b*c^2*x^5-1680*A*a*b^3*c*x^5+315*A
*b^5*x^5-1024*B*a^3*c^2*x^5+1840*B*a^2*b^2*c*x^5-420*B*a*b^4*x^5-480*A*a^3
*c^2*x^4+896*A*a^2*b^2*c*x^4-210*A*a*b^4*x^4-928*B*a^3*b*c*x^4+280*B*a^2*b
^3*x^4-544*A*a^3*b*c*x^3+168*A*a^2*b^3*x^3+512*B*a^4*c*x^3-224*B*a^3*b^2*x
^3+320*A*a^4*c*x^2-144*A*a^3*b^2*x^2+192*B*a^4*b*x^2+128*A*a^4*b*x+1536*B*
a^5*x+1280*A*a^5)/x^6/a^5-1/1024*(64*A*a^3*c^3-240*A*a^2*b^2*c^2+140*A*a*b
^4*c-21*A*b^6+192*B*a^3*b*c^2-160*B*a^2*b^3*c+28*B*a*b^5)/a^(11/2)*ln((2*a
+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.29

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[1/30720*(15*(28*B*a*b^5 - 21*A*b^6 + 64*A*a^3*c^3 + 48*(4*B*a^3*b - 5*A*a
^2*b^2)*c^2 - 20*(8*B*a^2*b^3 - 7*A*a*b^4)*c)*sqrt(a)*x^6*log(-(8*a*b*x +
(b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x
^2) - 4*(1280*A*a^6 - (420*B*a^2*b^4 - 315*A*a*b^5 + 16*(64*B*a^4 - 113*A*
a^3*b)*c^2 - 80*(23*B*a^3*b^2 - 21*A*a^2*b^3)*c)*x^5 + 2*(140*B*a^3*b^3 -
105*A*a^2*b^4 - 240*A*a^4*c^2 - 16*(29*B*a^4*b - 28*A*a^3*b^2)*c)*x^4 - 8*
(28*B*a^4*b^2 - 21*A*a^3*b^3 - 4*(16*B*a^5 - 17*A*a^4*b)*c)*x^3 + 16*(12*B
*a^5*b - 9*A*a^4*b^2 + 20*A*a^5*c)*x^2 + 128*(12*B*a^6 + A*a^5*b)*x)*sqrt(
c*x^2 + b*x + a))/(a^6*x^6), 1/15360*(15*(28*B*a*b^5 - 21*A*b^6 + 64*A*a^3
*c^3 + 48*(4*B*a^3*b - 5*A*a^2*b^2)*c^2 - 20*(8*B*a^2*b^3 - 7*A*a*b^4)*c)*
sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^
2 + a*b*x + a^2)) - 2*(1280*A*a^6 - (420*B*a^2*b^4 - 315*A*a*b^5 + 16*(64*
B*a^4 - 113*A*a^3*b)*c^2 - 80*(23*B*a^3*b^2 - 21*A*a^2*b^3)*c)*x^5 + 2*(14
0*B*a^3*b^3 - 105*A*a^2*b^4 - 240*A*a^4*c^2 - 16*(29*B*a^4*b - 28*A*a^3*b^
2)*c)*x^4 - 8*(28*B*a^4*b^2 - 21*A*a^3*b^3 - 4*(16*B*a^5 - 17*A*a^4*b)*c)*
x^3 + 16*(12*B*a^5*b - 9*A*a^4*b^2 + 20*A*a^5*c)*x^2 + 128*(12*B*a^6 + A*a
^5*b)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^6)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx=\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**7,x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1955 vs. $2(280) = 560$.

Time = 0.25 (sec) , antiderivative size = 1955, normalized size of antiderivative = 6.31

$$\int \frac{(A + Bx)\sqrt{a + bx + cx^2}}{x^7} dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
1/512*(28*B*a*b^5 - 21*A*b^6 - 160*B*a^2*b^3*c + 140*A*a*b^4*c + 192*B*a^3
*b*c^2 - 240*A*a^2*b^2*c^2 + 64*A*a^3*c^3)*arctan(-(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^5) - 1/7680*(420*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^11*B*a*b^5 - 315*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*b^
6 - 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*B*a^2*b^3*c + 2100*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^11*A*a*b^4*c + 2880*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^11*B*a^3*b*c^2 - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*
a^2*b^2*c^2 + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*a^3*c^3 - 2380*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a^2*b^5 + 1785*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^9*A*a*b^6 + 13600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B
*a^3*b^3*c - 11900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a^2*b^4*c - 163
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a^4*b*c^2 + 20400*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^9*A*a^3*b^2*c^2 - 5440*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*A*a^4*c^3 - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*B*a^5*c^
(5/2) + 5544*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a^3*b^5 - 4158*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^7*A*a^2*b^6 - 31680*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*B*a^4*b^3*c + 27720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*
a^3*b^4*c - 48000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a^5*b*c^2 - 4752
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a^4*b^2*c^2 - 36480*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^7*A*a^5*c^3 - 97280*(sqrt(c)*x - sqrt(c*x^2 + b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx = \int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{x^7}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^7,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.51

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^7}\,dx = \frac{-2560\sqrt{c\,x^2+bx+a}\,a^6-3328\sqrt{c\,x^2+bx+a}\,a^5bx-640\sqrt{c\,x^2+bx+a}\,a^5c\,x^2-96\sqrt{c\,x^2+bx+a}\,a^4b}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^7,x)
```

output
```
( - 2560*sqrt(a + b*x + c*x**2)*a**6 - 3328*sqrt(a + b*x + c*x**2)*a**5*b*
x - 640*sqrt(a + b*x + c*x**2)*a**5*c*x**2 - 96*sqrt(a + b*x + c*x**2)*a**
4*b**2*x**2 + 64*sqrt(a + b*x + c*x**2)*a**4*b*c*x**3 + 960*sqrt(a + b*x +
 c*x**2)*a**4*c**2*x**4 + 112*sqrt(a + b*x + c*x**2)*a**3*b**3*x**3 + 64*s
qrt(a + b*x + c*x**2)*a**3*b**2*c*x**4 - 1568*sqrt(a + b*x + c*x**2)*a**3*
b*c**2*x**5 - 140*sqrt(a + b*x + c*x**2)*a**2*b**4*x**4 - 320*sqrt(a + b*x
 + c*x**2)*a**2*b**3*c*x**5 + 210*sqrt(a + b*x + c*x**2)*a*b**5*x**5 + 960
*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*c**3*x**6
- 720*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**2*
c**2*x**6 - 300*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*
a*b**4*c*x**6 + 105*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b
*x)*b**6*x**6 - 960*sqrt(a)*log(x)*a**3*c**3*x**6 + 720*sqrt(a)*log(x)*a**
2*b**2*c**2*x**6 + 300*sqrt(a)*log(x)*a*b**4*c*x**6 - 105*sqrt(a)*log(x)*b
**6*x**6)/(15360*a**5*x**6)
```

3.116 $\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 455

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{(b^2-4ac)\left(143b^5B-198Ab^4c-440ab^3Bc+432aAb^2c^2+240a^2bBc^2-96a^2Ac^3\right)(b+2cx)}{32768c^7}$$

$$-\frac{\left(143b^5B-198Ab^4c-440ab^3Bc+432aAb^2c^2+240a^2bBc^2-96a^2Ac^3\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{12288c^6}$$

$$+\frac{\left(143b^2B-198Abc-128aBc\right)x^2\left(a+bx+cx^2\right)^{5/2}}{2016c^3}$$

$$-\frac{(13bB-18Ac)x^3\left(a+bx+cx^2\right)^{5/2}}{144c^2}+\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}$$

$$+\frac{(3003b^4B-4158Ab^3c-7524ab^2Bc+6696aAbc^2+2048a^2Bc^2-10c(429b^3B-594Ab^2c-748abBc+5}{80640c^5}$$

$$-\frac{(b^2-4ac)^2\left(143b^5B-198Ab^4c-440ab^3Bc+432aAb^2c^2+240a^2bBc^2-96a^2Ac^3\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx}}\right.}{65536c^{15/2}}$$

output
```
1/32768*(-4*a*c+b^2)*(-96*A*a^2*c^3+432*A*a*b^2*c^2-198*A*b^4*c+240*B*a^2*
b*c^2-440*B*a*b^3*c+143*B*b^5)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^7-1/12288*(
-96*A*a^2*c^3+432*A*a*b^2*c^2-198*A*b^4*c+240*B*a^2*b*c^2-440*B*a*b^3*c+14
3*B*b^5)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^6+1/2016*(-198*A*b*c-128*B*a*c+14
3*B*b^2)*x^2*(c*x^2+b*x+a)^(5/2)/c^3-1/144*(-18*A*c+13*B*b)*x^3*(c*x^2+b*x
+a)^(5/2)/c^2+1/9*B*x^4*(c*x^2+b*x+a)^(5/2)/c+1/80640*(3003*B*b^4-4158*A*b
^3*c-7524*B*a*b^2*c+6696*A*a*b*c^2+2048*B*a^2*c^2-10*c*(504*A*a*c^2-594*A*
b^2*c-748*B*a*b*c+429*B*b^3)*x)*(c*x^2+b*x+a)^(5/2)/c^5-1/65536*(-4*a*c+b^
2)^2*(-96*A*a^2*c^3+432*A*a*b^2*c^2-198*A*b^4*c+240*B*a^2*b*c^2-440*B*a*b^
3*c+143*B*b^5)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(15/2)
```

Mathematica [A] (verified)

Time = 4.12 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.11

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(45045b^8B-2310b^7c(27A+13Bx)+924b^6c(-475aB+cx(45A+26B$$

input
```
Integrate[x^4*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(45045*b^8*B - 2310*b^7*c*(27*A + 13*B*x)
 + 924*b^6*c*(-475*a*B + c*x*(45*A + 26*B*x)) + 256*c^4*(1024*a^4*B + 560*
c^4*x^7*(9*A + 8*B*x) + 6*a^2*c^2*x^3*(105*A + 64*B*x) + 40*a*c^3*x^5*(189
*A + 160*B*x) - a^3*c*x*(945*A + 512*B*x)) + 72*b^5*c^2*(-22*c*x^2*(21*A +
 13*B*x) + 7*a*(1095*A + 517*B*x)) + 16*b^4*c^2*(86499*a^2*B + 22*c^2*x^3*
(81*A + 52*B*x) - 9*a*c*x*(2247*A + 1276*B*x)) - 192*b^2*c^3*(7641*a^3*B -
 40*c^3*x^5*(3*A + 2*B*x) + 4*a*c^2*x^3*(213*A + 134*B*x) - a^2*c*x*(3543*
A + 1970*B*x)) - 32*b^3*c^3*(8*c^2*x^4*(99*A + 65*B*x) - 4*a*c*x^2*(1755*A
 + 1067*B*x) + 9*a^2*(5103*A + 2353*B*x)) + 128*b*c^4*(24*a*c^2*x^4*(39*A
+ 25*B*x) + 80*c^3*x^6*(153*A + 133*B*x) - 6*a^2*c*x^2*(453*A + 269*B*x) +
 a^3*(8271*A + 3701*B*x))) + 315*(b^2 - 4*a*c)^2*(143*b^5*B - 198*A*b^4*c
- 440*a*b^3*B*c + 432*a*A*b^2*c^2 + 240*a^2*b*B*c^2 - 96*a^2*A*c^3)*Log[b
+ 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(20643840*c^(15/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.86, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1236, 27, 1236, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{\int -\frac{1}{2}x^3(8aB+(13bB-18Ac)x)\left(cx^2+bx+a\right)^{3/2}dx}{9c}+\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-\frac{\int x^3(8aB+(13bB-18Ac)x)\left(cx^2+bx+a\right)^{3/2}dx}{18c}$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-\frac{\frac{\int -\frac{1}{2}x^2\left(6a(13bB-18Ac)+\left(143Bb^2-198Acb-128aBc\right)x\right)\left(cx^2+bx+a\right)^{3/2}dx}{8c}+\frac{x^3\left(a+bx+cx^2\right)^{5/2}(13bB-18Ac)}{8c}}{18c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{5/2}(13bB-18Ac)}{8c}-\frac{\int x^2\left(6a(13bB-18Ac)+\left(143Bb^2-198Acb-128aBc\right)x\right)\left(cx^2+bx+a\right)^{3/2}dx}{16c}}{18c}$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{5/2}(13bB-18Ac)}{8c}-\frac{\frac{\int -\frac{1}{2}x\left(4a\left(143Bb^2-198Acb-128aBc\right)+3\left(429Bb^3-594Acb^2-748aBcb+504aAc^2\right)x\right)\left(cx^2+bx+a\right)^{3/2}dx}{7c}+\frac{x^2\left(a+bx+c\cdots\right.}{\cdots}}{16c}}{18c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{5/2}(13bB-18Ac)}{8c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{5/2}\left(-128aBc-198Abc+143b^2B\right)}{7c}-\frac{\int x\left(4a\left(143Bb^2-198Acb-128aBc\right)+3\left(429Bb^3-594Acb^2-748aBc\right.\right.}{14c}}{16c}}{18c}$$

$\downarrow$ 1225

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{5/2}(13bB-18Ac)}{8c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{5/2}\left(-128aBc-198Abc+143b^2B\right)}{7c}-\frac{21\left(-96a^2Ac^3+240a^2bBc^2+432aAb^2c^2-440ab^3Bc-198Ab^4c+143\right.}{8c^2}}{}}{18c}$$

$\downarrow$ 1087

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{5/2}(13bB-18Ac)}{8c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{5/2}\left(-128aBc-198Abc+143b^2B\right)}{7c}-\frac{21\left(-96a^2Ac^3+240a^2bBc^2+432aAb^2c^2-440ab^3Bc-198Ab^4c+143\right.}{8c}}{}}{}$$

$\downarrow$ 1087

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{5/2}(13bB-18Ac)}{8c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{5/2}\left(-128aBc-198Abc+143b^2B\right)}{7c}-\frac{21\left(-96a^2Ac^3+240a^2bBc^2+432aAb^2c^2-440ab^3Bc-198Ab^4c+143\right.}{}}{}}{}$$

$\downarrow$ 1092

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{5/2}(13bB-18Ac)}{8c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{5/2}\left(-128aBc-198Abc+143b^2B\right)}{7c}-\frac{21\left(-96a^2Ac^3+240a^2bBc^2+432aAb^2c^2-440ab^3Bc-198Ab^4c+143\right.}{}}{}}{}$$

$$\Big\downarrow \text{219}$$

$$\frac{Bx^4\left(a+bx+cx^2\right)^{5/2}}{9c}-$$

$$21\Big(-96a^2Ac^3+240a^2bBc^2+432aAb^2c^2-440ab^3Bc-198Ab^4c+143$$

$$\frac{\frac{x^3\left(a+bx+cx^2\right)^{5/2}(13bB-18Ac)}{8c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{5/2}\left(-128aBc-198Abc+143b^2B\right)}{7c}-}{}}{}$$

input

```
Int[x^4*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output

```
(B*x^4*(a + b*x + c*x^2)^(5/2))/(9*c) - (((13*b*B - 18*A*c)*x^3*(a + b*x +
 c*x^2)^(5/2))/(8*c) - (((143*b^2*B - 198*A*b*c - 128*a*B*c)*x^2*(a + b*x
+ c*x^2)^(5/2))/(7*c) - (-1/20*((3003*b^4*B - 4158*A*b^3*c - 7524*a*b^2*B*
c + 6696*a*A*b*c^2 + 2048*a^2*B*c^2 - 10*c*(429*b^3*B - 594*A*b^2*c - 748*
a*b*B*c + 504*a*A*c^2)*x)*(a + b*x + c*x^2)^(5/2))/c^2 + (21*(143*b^5*B -
198*A*b^4*c - 440*a*b^3*B*c + 432*a*A*b^2*c^2 + 240*a^2*b*B*c^2 - 96*a^2*A
*c^3)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b
 + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(8*c^2))/(14*
c))/(16*c))/(18*c)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.43

method	result
risch	$\frac{(1146880B\,c^8x^8+1290240A\,c^8x^7+1361920Bb\,c^7x^7+1566720Ab\,c^7x^6+1638400Ba\,c^7x^6+15360B\,b^2c^6x^6+1935360Aa\,c^7x^5+23040}{}$
default	Expression too large to display

input
```
int(x^4*(B*x+A)*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10321920/c^7*(1146880*B*c^8*x^8+1290240*A*c^8*x^7+1361920*B*b*c^7*x^7+15
66720*A*b*c^7*x^6+1638400*B*a*c^7*x^6+15360*B*b^2*c^6*x^6+1935360*A*a*c^7*
x^5+23040*A*b^2*c^6*x^5+76800*B*a*b*c^6*x^5-16640*B*b^3*c^5*x^5+119808*A*a
*b*c^6*x^4-25344*A*b^3*c^5*x^4+98304*B*a^2*c^6*x^4-102912*B*a*b^2*c^5*x^4+
18304*B*b^4*c^4*x^4+161280*A*a^2*c^6*x^3-163584*A*a*b^2*c^5*x^3+28512*A*b^
4*c^4*x^3-206592*B*a^2*b*c^5*x^3+136576*B*a*b^3*c^4*x^3-20592*B*b^5*c^3*x^
3-347904*A*a^2*b*c^5*x^2+224640*A*a*b^3*c^4*x^2-33264*A*b^5*c^3*x^2-131072
*B*a^3*c^5*x^2+378240*B*a^2*b^2*c^4*x^2-183744*B*a*b^4*c^3*x^2+24024*B*b^6
*c^2*x^2-241920*A*a^3*c^5*x+680256*A*a^2*b^2*c^4*x-323568*A*a*b^4*c^3*x+41
580*A*b^6*c^2*x+473728*B*a^3*b*c^4*x-677664*B*a^2*b^3*c^3*x+260568*B*a*b^5
*c^2*x-30030*B*b^7*c*x+1058688*A*a^3*b*c^4-1469664*A*a^2*b^3*c^3+551880*A*
a*b^5*c^2-62370*A*b^7*c+262144*B*a^4*c^4-1467072*B*a^3*b^2*c^3+1383984*B*a
^2*b^4*c^2-438900*B*a*b^6*c+45045*B*b^8)*(c*x^2+b*x+a)^(1/2)+1/65536*(1536
*A*a^4*c^5-7680*A*a^3*b^2*c^4+6720*A*a^2*b^4*c^3-2016*A*a*b^6*c^2+198*A*b^
8*c-3840*B*a^4*b*c^4+8960*B*a^3*b^3*c^3-6048*B*a^2*b^5*c^2+1584*B*a*b^7*c-
143*B*b^9)/c^(15/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1263, normalized size of antiderivative = 2.78

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/41287680*(315*(143*B*b^9 - 1536*A*a^4*c^5 + 3840*(B*a^4*b + 2*A*a^3*b^
2)*c^4 - 2240*(4*B*a^3*b^3 + 3*A*a^2*b^4)*c^3 + 2016*(3*B*a^2*b^5 + A*a*b^
6)*c^2 - 198*(8*B*a*b^7 + A*b^8)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1146880*B*c^9
*x^8 + 45045*B*b^8*c + 71680*(19*B*b*c^8 + 18*A*c^9)*x^7 + 5120*(3*B*b^2*c
^7 + 2*(160*B*a + 153*A*b)*c^8)*x^6 + 128*(2048*B*a^4 + 8271*A*a^3*b)*c^5
- 1280*(13*B*b^3*c^6 - 1512*A*a*c^8 - 6*(10*B*a*b + 3*A*b^2)*c^7)*x^5 - 25
92*(566*B*a^3*b^2 + 567*A*a^2*b^3)*c^4 + 128*(143*B*b^4*c^5 + 24*(32*B*a^2
 + 39*A*a*b)*c^7 - 6*(134*B*a*b^2 + 33*A*b^3)*c^6)*x^4 + 504*(2746*B*a^2*b
^4 + 1095*A*a*b^5)*c^3 - 16*(1287*B*b^5*c^4 - 10080*A*a^2*c^7 + 48*(269*B*
a^2*b + 213*A*a*b^2)*c^6 - 22*(388*B*a*b^3 + 81*A*b^4)*c^5)*x^3 - 2310*(19
0*B*a*b^6 + 27*A*b^7)*c^2 + 8*(3003*B*b^6*c^3 - 32*(512*B*a^3 + 1359*A*a^2
*b)*c^6 + 240*(197*B*a^2*b^2 + 117*A*a*b^3)*c^5 - 198*(116*B*a*b^4 + 21*A*
b^5)*c^4)*x^2 - 2*(15015*B*b^7*c^2 + 120960*A*a^3*c^6 - 32*(7402*B*a^3*b +
 10629*A*a^2*b^2)*c^5 + 72*(4706*B*a^2*b^3 + 2247*A*a*b^4)*c^4 - 1386*(94*
B*a*b^5 + 15*A*b^6)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^8, 1/20643840*(315*(1
43*B*b^9 - 1536*A*a^4*c^5 + 3840*(B*a^4*b + 2*A*a^3*b^2)*c^4 - 2240*(4*B*a
^3*b^3 + 3*A*a^2*b^4)*c^3 + 2016*(3*B*a^2*b^5 + A*a*b^6)*c^2 - 198*(8*B*a*
b^7 + A*b^8)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt
(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(1146880*B*c^9*x^8 + 45045*B*b^8*c + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4983 vs. $2(496) = 992$.

Time = 0.78 (sec) , antiderivative size = 4983, normalized size of antiderivative = 10.95

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(B*x+A)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Piecewise(((-a*(-3*a*(A*a**2 - 5*a*(2*A*a*c + A*b**2 + 2*B*a*b - 7*a*(A*c*
*2 + 19*B*b*c/18)/(8*c) - 13*b*(2*A*b*c + 10*B*a*c/9 + B*b**2 - 15*b*(A*c*
*2 + 19*B*b*c/18)/(16*c))/(14*c))/(6*c) - 9*b*(2*A*a*b + B*a**2 - 6*a*(2*A
*b*c + 10*B*a*c/9 + B*b**2 - 15*b*(A*c**2 + 19*B*b*c/18)/(16*c))/(7*c) - 1
1*b*(2*A*a*c + A*b**2 + 2*B*a*b - 7*a*(A*c**2 + 19*B*b*c/18)/(8*c) - 13*b*
(2*A*b*c + 10*B*a*c/9 + B*b**2 - 15*b*(A*c**2 + 19*B*b*c/18)/(16*c))/(14*c
))/(12*c))/(10*c))/(4*c) - 5*b*(-4*a*(2*A*a*b + B*a**2 - 6*a*(2*A*b*c + 10
*B*a*c/9 + B*b**2 - 15*b*(A*c**2 + 19*B*b*c/18)/(16*c))/(7*c) - 11*b*(2*A*
a*c + A*b**2 + 2*B*a*b - 7*a*(A*c**2 + 19*B*b*c/18)/(8*c) - 13*b*(2*A*b*c
+ 10*B*a*c/9 + B*b**2 - 15*b*(A*c**2 + 19*B*b*c/18)/(16*c))/(14*c))/(12*c)
)/(5*c) - 7*b*(A*a**2 - 5*a*(2*A*a*c + A*b**2 + 2*B*a*b - 7*a*(A*c**2 + 19
*B*b*c/18)/(8*c) - 13*b*(2*A*b*c + 10*B*a*c/9 + B*b**2 - 15*b*(A*c**2 + 19
*B*b*c/18)/(16*c))/(14*c))/(6*c) - 9*b*(2*A*a*b + B*a**2 - 6*a*(2*A*b*c +
10*B*a*c/9 + B*b**2 - 15*b*(A*c**2 + 19*B*b*c/18)/(16*c))/(7*c) - 11*b*(2*
A*a*c + A*b**2 + 2*B*a*b - 7*a*(A*c**2 + 19*B*b*c/18)/(8*c) - 13*b*(2*A*b*
c + 10*B*a*c/9 + B*b**2 - 15*b*(A*c**2 + 19*B*b*c/18)/(16*c))/(14*c))/(12*
c))/(10*c))/(8*c))/(6*c))/(2*c) - b*(-2*a*(-4*a*(2*A*a*b + B*a**2 - 6*a*(2
*A*b*c + 10*B*a*c/9 + B*b**2 - 15*b*(A*c**2 + 19*B*b*c/18)/(16*c))/(7*c) -
 11*b*(2*A*a*c + A*b**2 + 2*B*a*b - 7*a*(A*c**2 + 19*B*b*c/18)/(8*c) - 13*
b*(2*A*b*c + 10*B*a*c/9 + B*b**2 - 15*b*(A*c**2 + 19*B*b*c/18)/(16*c))/...
```

Maxima [F(-2)]

Exception generated.

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.40

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{1}{10321920}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\left(16\,Bcx+\frac{19\,Bbc^8+18\,Ac^9}{c^8}\right)x+\frac{3\,Bb^2\cdots}{\cdots}\right.\right.\right.\right.\right.\right.\right.$$
$$+\frac{(143\,Bb^9-1584\,Bab^7c-198\,Ab^8c+6048\,Ba^2b^5c^2+2016\,Aab^6c^2-8960\,Ba^3b^3c^3-6720\,Aa^2b^4c^3+384\cdots}{65536\,c^{\frac{15}{2}}}$$

input
```
integrate(x^4*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/10321920*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*(16*B*c*x + (19*B*
b*c^8 + 18*A*c^9)/c^8)*x + (3*B*b^2*c^7 + 320*B*a*c^8 + 306*A*b*c^8)/c^8)*
x - (13*B*b^3*c^6 - 60*B*a*b*c^7 - 18*A*b^2*c^7 - 1512*A*a*c^8)/c^8)*x + (
143*B*b^4*c^5 - 804*B*a*b^2*c^6 - 198*A*b^3*c^6 + 768*B*a^2*c^7 + 936*A*a*
b*c^7)/c^8)*x - (1287*B*b^5*c^4 - 8536*B*a*b^3*c^5 - 1782*A*b^4*c^5 + 1291
2*B*a^2*b*c^6 + 10224*A*a*b^2*c^6 - 10080*A*a^2*c^7)/c^8)*x + (3003*B*b^6*
c^3 - 22968*B*a*b^4*c^4 - 4158*A*b^5*c^4 + 47280*B*a^2*b^2*c^5 + 28080*A*a
*b^3*c^5 - 16384*B*a^3*c^6 - 43488*A*a^2*b*c^6)/c^8)*x - (15015*B*b^7*c^2
- 130284*B*a*b^5*c^3 - 20790*A*b^6*c^3 + 338832*B*a^2*b^3*c^4 + 161784*A*a
*b^4*c^4 - 236864*B*a^3*b*c^5 - 340128*A*a^2*b^2*c^5 + 120960*A*a^3*c^6)/c
^8)*x + (45045*B*b^8*c - 438900*B*a*b^6*c^2 - 62370*A*b^7*c^2 + 1383984*B*
a^2*b^4*c^3 + 551880*A*a*b^5*c^3 - 1467072*B*a^3*b^2*c^4 - 1469664*A*a^2*b
^3*c^4 + 262144*B*a^4*c^5 + 1058688*A*a^3*b*c^5)/c^8) + 1/65536*(143*B*b^9
 - 1584*B*a*b^7*c - 198*A*b^8*c + 6048*B*a^2*b^5*c^2 + 2016*A*a*b^6*c^2 -
8960*B*a^3*b^3*c^3 - 6720*A*a^2*b^4*c^3 + 3840*B*a^4*b*c^4 + 7680*A*a^3*b^
2*c^4 - 1536*A*a^4*c^5)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt
(c) + b))/c^(15/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int x^4\,(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,dx$$

input
```
int(x^4*(A + B*x)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(x^4*(A + B*x)*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int x^4(Bx+A)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}dx$$

input
```
int(x^4*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x^4*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

3.117 $\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 356

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = -\frac{3(b^2-4ac)\left(33b^4B-48Ab^3c-72ab^2Bc+64aAbc^2+16a^2Bc^2\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^6}+\frac{\left(33b^4B-48Ab^3c-72ab^2Bc+64aAbc^2+16a^2Bc^2\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{2048c^5}-\frac{(11bB-16Ac)x^2(a+bx+cx^2)^{5/2}}{112c^2}+\frac{Bx^3(a+bx+cx^2)^{5/2}}{8c}-\frac{\left(231b^3B-336Ab^2c-372abBc+256aAc^2-10c(33b^2B-48Abc-28aBc)\,x\right)\left(a+bx+cx^2\right)^{5/2}}{4480c^4}+\frac{3(b^2-4ac)^2\left(33b^4B-48Ab^3c-72ab^2Bc+64aAbc^2+16a^2Bc^2\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}}$$

output

```
-3/16384*(-4*a*c+b^2)*(64*A*a*b*c^2-48*A*b^3*c+16*B*a^2*c^2-72*B*a*b^2*c+3
3*B*b^4)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6+1/2048*(64*A*a*b*c^2-48*A*b^3*c
+16*B*a^2*c^2-72*B*a*b^2*c+33*B*b^4)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^5-1/1
12*(-16*A*c+11*B*b)*x^2*(c*x^2+b*x+a)^(5/2)/c^2+1/8*B*x^3*(c*x^2+b*x+a)^(5
/2)/c-1/4480*(231*B*b^3-336*A*b^2*c-372*B*a*b*c+256*A*a*c^2-10*c*(-48*A*b*
c-28*B*a*c+33*B*b^2)*x)*(c*x^2+b*x+a)^(5/2)/c^4+3/32768*(-4*a*c+b^2)^2*(64
*A*a*b*c^2-48*A*b^3*c+16*B*a^2*c^2-72*B*a*b^2*c+33*B*b^4)*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)
```

Mathematica [A] (verified)

Time = 2.89 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.16

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^7B+210b^6c(24A+11Bx)+84b^5c(365aB-2cx(20A+11Bx$$

input
```
Integrate[x^3*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^7*B + 210*b^6*c*(24*A + 11*B*x)
+ 84*b^5*c*(365*a*B - 2*c*x*(20*A + 11*B*x)) - 16*b^3*c^2*(5103*a^2*B + 8*
c^2*x^3*(18*A + 11*B*x) - 52*a*c*x*(28*A + 15*B*x)) + 128*c^4*(80*c^3*x^6*
(8*A + 7*B*x) + 2*a^2*c*x^2*(64*A + 35*B*x) + 8*a*c^2*x^4*(128*A + 105*B*x
) - a^3*(256*A + 105*B*x)) + 24*b^4*c^2*(2*c*x^2*(56*A + 33*B*x) - 7*a*(24
0*A + 107*B*x)) + 64*b*c^3*(919*a^3*B + 8*a*c^2*x^3*(22*A + 13*B*x) + 80*c
^3*x^5*(20*A + 17*B*x) - 2*a^2*c*x*(292*A + 151*B*x)) + 32*b^2*c^3*(8*c^2*
x^4*(8*A + 5*B*x) - 4*a*c*x^2*(124*A + 71*B*x) + a^2*(2744*A + 1181*B*x)))
 - 105*(b^2 - 4*a*c)^2*(33*b^4*B - 48*A*b^3*c - 72*a*b^2*B*c + 64*a*A*b*c^
2 + 16*a^2*B*c^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(11468
80*c^(13/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1236, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}x^2(6aB+(11bB-16Ac)x)\left(cx^2+bx+a\right)^{3/2}dx}{8c}+\frac{Bx^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$$\Big\downarrow \; 27$$

$$\frac{Bx^3(a+bx+cx^2)^{5/2}}{8c}-\frac{\int x^2(6aB+(11bB-16Ac)x)\left(cx^2+bx+a\right)^{3/2}dx}{16c}$$

$$\Big\downarrow \; 1236$$

$$\frac{Bx^3(a+bx+cx^2)^{5/2}}{8c}-\frac{\frac{\int -\frac{1}{2}x(4a(11bB-16Ac)+3(33Bb^2-48Acb-28aBc)x)\left(cx^2+bx+a\right)^{3/2}dx}{7c}+\frac{x^2(a+bx+cx^2)^{5/2}(11bB-16Ac)}{7c}}{16c}$$

$$\Big\downarrow \; 27$$

$$\frac{Bx^3(a+bx+cx^2)^{5/2}}{8c}-\frac{\frac{x^2(a+bx+cx^2)^{5/2}(11bB-16Ac)}{7c}-\frac{\int x(4a(11bB-16Ac)+3(33Bb^2-48Acb-28aBc)x)\left(cx^2+bx+a\right)^{3/2}dx}{14c}}{16c}$$

$$\Big\downarrow \; 1225$$

$$\frac{Bx^3(a+bx+cx^2)^{5/2}}{8c}-\frac{\frac{x^2(a+bx+cx^2)^{5/2}(11bB-16Ac)}{7c}-\frac{\frac{7\left(16a^2Bc^2+64aAbc^2-72ab^2Bc-48Ab^3c+33b^4B\right)\int\left(cx^2+bx+a\right)^{3/2}dx}{8c^2}-\frac{(a+bx+cx^2)^{5/2}\left(-10cx\left(-28aBc-48Abc+3\right.\right.}{}}{14c}}{16c}$$

$$\Big\downarrow \; 1087$$

$$\frac{Bx^3(a+bx+cx^2)^{5/2}}{8c}-\frac{\frac{x^2(a+bx+cx^2)^{5/2}(11bB-16Ac)}{7c}-\frac{\frac{7\left(16a^2Bc^2+64aAbc^2-72ab^2Bc-48Ab^3c+33b^4B\right)\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\int\sqrt{cx^2+bx+a}dx}{16c}\right)}{8c^2}-\frac{(a+b}{}}{14c}}{16c}$$

$$\Big\downarrow \; 1087$$

$$\frac{Bx^3(a+bx+cx^2)^{5/2}}{8c}-\frac{\frac{x^2(a+bx+cx^2)^{5/2}(11bB-16Ac)}{7c}-\frac{7\left(16a^2Bc^2+64aAbc^2-72ab^2Bc-48Ab^3c+33b^4B\right)\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b}{}\right.}{16c}\right.}{8c^2}}{16c}$$

$$\Big\downarrow \; 1092$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{5/2}}{8c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{5/2}(11bB-16Ac)}{7c}-\frac{7\left(16a^2Bc^2+64aAbc^2-72ab^2Bc-48Ab^3c+33b^4B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b}{}\right.}{}\right.}{8c^2}}{16c}$$

$\downarrow$ 219

$$\frac{Bx^3\left(a+bx+cx^2\right)^{5/2}}{8c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{5/2}(11bB-16Ac)}{7c}-\frac{7\left(16a^2Bc^2+64aAbc^2-72ab^2Bc-48Ab^3c+33b^4B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b}{}\right.}{}\right.}{8c^2}}{16c}$$

input
```
Int[x^3*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(B*x^3*(a + b*x + c*x^2)^(5/2))/(8*c) - (((11*b*B - 16*A*c)*x^2*(a + b*x +
 c*x^2)^(5/2))/(7*c) - (-1/20*((231*b^3*B - 336*A*b^2*c - 372*a*b*B*c + 25
6*a*A*c^2 - 10*c*(33*b^2*B - 48*A*b*c - 28*a*B*c)*x)*(a + b*x + c*x^2)^(5/
2))/c^2 + (7*(33*b^4*B - 48*A*b^3*c - 72*a*b^2*B*c + 64*a*A*b*c^2 + 16*a^2
*B*c^2)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*((
(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(8*c^2))/(1
4*c))/(16*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{(-71680 B\,c^7 x^7 - 81920 A\,c^7 x^6 - 87040 B b\,c^6 x^6 - 102400 A b\,c^6 x^5 - 107520 B a\,c^6 x^5 - 1280 B\,b^2 c^5 x^5 - 131072 A a\,c^6 x^4 - 2048 A\,b^2 c^5 x^4$
default	Expression too large to display

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/573440/c^6*(-71680*B*c^7*x^7-81920*A*c^7*x^6-87040*B*b*c^6*x^6-102400*A
*b*c^6*x^5-107520*B*a*c^6*x^5-1280*B*b^2*c^5*x^5-131072*A*a*c^6*x^4-2048*A
*b^2*c^5*x^4-6656*B*a*b*c^5*x^4+1408*B*b^3*c^4*x^4-11264*A*a*b*c^5*x^3+230
4*A*b^3*c^4*x^3-8960*B*a^2*c^5*x^3+9088*B*a*b^2*c^4*x^3-1584*B*b^4*c^3*x^3
-16384*A*a^2*c^5*x^2+15872*A*a*b^2*c^4*x^2-2688*A*b^4*c^3*x^2+19328*B*a^2*
b*c^4*x^2-12480*B*a*b^3*c^3*x^2+1848*B*b^5*c^2*x^2+37376*A*a^2*b*c^4*x-232
96*A*a*b^3*c^3*x+3360*A*b^5*c^2*x+13440*B*a^3*c^4*x-37792*B*a^2*b^2*c^3*x+
17976*B*a*b^4*c^2*x-2310*B*b^6*c*x+32768*A*a^3*c^4-87808*A*a^2*b^2*c^3+403
20*A*a*b^4*c^2-5040*A*b^6*c-58816*B*a^3*b*c^3+81648*B*a^2*b^3*c^2-30660*B*
a*b^5*c+3465*B*b^7)*(c*x^2+b*x+a)^(1/2)+3/32768*(1024*A*a^3*b*c^4-1280*A*a
^2*b^3*c^3+448*A*a*b^5*c^2-48*A*b^7*c+256*B*a^4*c^4-1280*B*a^3*b^2*c^3+112
0*B*a^2*b^4*c^2-336*B*a*b^6*c+33*B*b^8)/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1037, normalized size of antiderivative = 2.91

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2293760*(105*(33*B*b^8 + 256*(B*a^4 + 4*A*a^3*b)*c^4 - 1280*(B*a^3*b^2
+ A*a^2*b^3)*c^3 + 224*(5*B*a^2*b^4 + 2*A*a*b^5)*c^2 - 48*(7*B*a*b^6 + A*b
^7)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) + 4*(71680*B*c^8*x^7 - 3465*B*b^7*c - 32768*A*a
^3*c^5 + 5120*(17*B*b*c^7 + 16*A*c^8)*x^6 + 1280*(B*b^2*c^6 + 4*(21*B*a +
20*A*b)*c^7)*x^5 + 64*(919*B*a^3*b + 1372*A*a^2*b^2)*c^4 - 128*(11*B*b^3*c
^5 - 1024*A*a*c^7 - 4*(13*B*a*b + 4*A*b^2)*c^6)*x^4 - 1008*(81*B*a^2*b^3 +
 40*A*a*b^4)*c^3 + 16*(99*B*b^4*c^4 + 16*(35*B*a^2 + 44*A*a*b)*c^6 - 8*(71
*B*a*b^2 + 18*A*b^3)*c^5)*x^3 + 420*(73*B*a*b^5 + 12*A*b^6)*c^2 - 8*(231*B
*b^5*c^3 - 2048*A*a^2*c^6 + 16*(151*B*a^2*b + 124*A*a*b^2)*c^5 - 24*(65*B*
a*b^3 + 14*A*b^4)*c^4)*x^2 + 2*(1155*B*b^6*c^2 - 64*(105*B*a^3 + 292*A*a^2
*b)*c^5 + 16*(1181*B*a^2*b^2 + 728*A*a*b^3)*c^4 - 84*(107*B*a*b^4 + 20*A*b
^5)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^7, -1/1146880*(105*(33*B*b^8 + 256*(B
*a^4 + 4*A*a^3*b)*c^4 - 1280*(B*a^3*b^2 + A*a^2*b^3)*c^3 + 224*(5*B*a^2*b^
4 + 2*A*a*b^5)*c^2 - 48*(7*B*a*b^6 + A*b^7)*c)*sqrt(-c)*arctan(1/2*sqrt(c*
x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(71680*B*
c^8*x^7 - 3465*B*b^7*c - 32768*A*a^3*c^5 + 5120*(17*B*b*c^7 + 16*A*c^8)*x^
6 + 1280*(B*b^2*c^6 + 4*(21*B*a + 20*A*b)*c^7)*x^5 + 64*(919*B*a^3*b + 137
2*A*a^2*b^2)*c^4 - 128*(11*B*b^3*c^5 - 1024*A*a*c^7 - 4*(13*B*a*b + 4*A*b^
2)*c^6)*x^4 - 1008*(81*B*a^2*b^3 + 40*A*a*b^4)*c^3 + 16*(99*B*b^4*c^4 +...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3089 vs. $2(384) = 768$.

Time = 0.76 (sec) , antiderivative size = 3089, normalized size of antiderivative = 8.68

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(B*x+A)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Piecewise(((-a*(-3*a*(2*A*a*b + B*a**2 - 5*a*(2*A*b*c + 9*B*a*c/8 + B*b**2
 - 13*b*(A*c**2 + 17*B*b*c/16)/(14*c))/(6*c) - 9*b*(2*A*a*c + A*b**2 + 2*B
*a*b - 6*a*(A*c**2 + 17*B*b*c/16)/(7*c) - 11*b*(2*A*b*c + 9*B*a*c/8 + B*b*
*2 - 13*b*(A*c**2 + 17*B*b*c/16)/(14*c))/(12*c))/(10*c))/(4*c) - 5*b*(A*a*
*2 - 4*a*(2*A*a*c + A*b**2 + 2*B*a*b - 6*a*(A*c**2 + 17*B*b*c/16)/(7*c) -
11*b*(2*A*b*c + 9*B*a*c/8 + B*b**2 - 13*b*(A*c**2 + 17*B*b*c/16)/(14*c))/(
12*c))/(5*c) - 7*b*(2*A*a*b + B*a**2 - 5*a*(2*A*b*c + 9*B*a*c/8 + B*b**2 -
 13*b*(A*c**2 + 17*B*b*c/16)/(14*c))/(6*c) - 9*b*(2*A*a*c + A*b**2 + 2*B*a
*b - 6*a*(A*c**2 + 17*B*b*c/16)/(7*c) - 11*b*(2*A*b*c + 9*B*a*c/8 + B*b**2
 - 13*b*(A*c**2 + 17*B*b*c/16)/(14*c))/(12*c))/(10*c))/(8*c))/(6*c))/(2*c)
 - b*(-2*a*(A*a**2 - 4*a*(2*A*a*c + A*b**2 + 2*B*a*b - 6*a*(A*c**2 + 17*B*
b*c/16)/(7*c) - 11*b*(2*A*b*c + 9*B*a*c/8 + B*b**2 - 13*b*(A*c**2 + 17*B*b
*c/16)/(14*c))/(12*c))/(5*c) - 7*b*(2*A*a*b + B*a**2 - 5*a*(2*A*b*c + 9*B*
a*c/8 + B*b**2 - 13*b*(A*c**2 + 17*B*b*c/16)/(14*c))/(6*c) - 9*b*(2*A*a*c
+ A*b**2 + 2*B*a*b - 6*a*(A*c**2 + 17*B*b*c/16)/(7*c) - 11*b*(2*A*b*c + 9*
B*a*c/8 + B*b**2 - 13*b*(A*c**2 + 17*B*b*c/16)/(14*c))/(12*c))/(10*c))/(8*
c))/(3*c) - 3*b*(-3*a*(2*A*a*b + B*a**2 - 5*a*(2*A*b*c + 9*B*a*c/8 + B*b**
2 - 13*b*(A*c**2 + 17*B*b*c/16)/(14*c))/(6*c) - 9*b*(2*A*a*c + A*b**2 + 2*
B*a*b - 6*a*(A*c**2 + 17*B*b*c/16)/(7*c) - 11*b*(2*A*b*c + 9*B*a*c/8 + B*b
**2 - 13*b*(A*c**2 + 17*B*b*c/16)/(14*c))/(12*c))/(10*c))/(4*c) - 5*b*(...
```

Maxima [F(-2)]

Exception generated.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.47

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{1}{573440}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\,Bcx+\frac{17\,Bbc^7+16\,Ac^8}{c^7}\right)x+\frac{Bb^2c^6+84\,\ldots}{\ldots}\right.\right.\right.\right.\right.\right.$$
$$-\frac{3\left(33\,Bb^8-336\,Bab^6c-48\,Ab^7c+1120\,Ba^2b^4c^2+448\,Aab^5c^2-1280\,Ba^3b^2c^3-1280\,Aa^2b^3c^3+256\,Bc\ldots\right.}{32768\,c^{\frac{13}{2}}}$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/573440*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*B*c*x + (17*B*b*c^7
+ 16*A*c^8)/c^7)*x + (B*b^2*c^6 + 84*B*a*c^7 + 80*A*b*c^7)/c^7)*x - (11*B*
b^3*c^5 - 52*B*a*b*c^6 - 16*A*b^2*c^6 - 1024*A*a*c^7)/c^7)*x + (99*B*b^4*c
^4 - 568*B*a*b^2*c^5 - 144*A*b^3*c^5 + 560*B*a^2*c^6 + 704*A*a*b*c^6)/c^7)
*x - (231*B*b^5*c^3 - 1560*B*a*b^3*c^4 - 336*A*b^4*c^4 + 2416*B*a^2*b*c^5
+ 1984*A*a*b^2*c^5 - 2048*A*a^2*c^6)/c^7)*x + (1155*B*b^6*c^2 - 8988*B*a*b
^4*c^3 - 1680*A*b^5*c^3 + 18896*B*a^2*b^2*c^4 + 11648*A*a*b^3*c^4 - 6720*B
*a^3*c^5 - 18688*A*a^2*b*c^5)/c^7)*x - (3465*B*b^7*c - 30660*B*a*b^5*c^2 -
 5040*A*b^6*c^2 + 81648*B*a^2*b^3*c^3 + 40320*A*a*b^4*c^3 - 58816*B*a^3*b*
c^4 - 87808*A*a^2*b^2*c^4 + 32768*A*a^3*c^5)/c^7) - 3/32768*(33*B*b^8 - 33
6*B*a*b^6*c - 48*A*b^7*c + 1120*B*a^2*b^4*c^2 + 448*A*a*b^5*c^2 - 1280*B*a
^3*b^2*c^3 - 1280*A*a^2*b^3*c^3 + 256*B*a^4*c^4 + 1024*A*a^3*b*c^4)*log(ab
s(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int x^3\,\left(A+B\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}\,dx$$

input
```
int(x^3*(A + B*x)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(x^3*(A + B*x)*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int x^3(Bx+A)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}dx$$

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

3.118 $\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 269

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{(b^2-4ac)\left(9b^3B-14Ab^2c-12abBc+8aAc^2\right)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^5}-\frac{\left(9b^3B-14Ab^2c-12abBc+8aAc^2\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{384c^4}+\frac{Bx^2\left(a+bx+cx^2\right)^{5/2}}{7c}+\frac{(63b^2B-98Abc-48aBc-10c(9bB-14Ac)x)\left(a+bx+cx^2\right)^{5/2}}{840c^3}-\frac{(b^2-4ac)^2\left(9b^3B-14Ab^2c-12abBc+8aAc^2\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}}$$

output
```
1/1024*(-4*a*c+b^2)*(8*A*a*c^2-14*A*b^2*c-12*B*a*b*c+9*B*b^3)*(2*c*x+b)*(c
*x^2+b*x+a)^(1/2)/c^5-1/384*(8*A*a*c^2-14*A*b^2*c-12*B*a*b*c+9*B*b^3)*(2*c
*x+b)*(c*x^2+b*x+a)^(3/2)/c^4+1/7*B*x^2*(c*x^2+b*x+a)^(5/2)/c+1/840*(63*B*
b^2-98*A*b*c-48*B*a*c-10*c*(-14*A*c+9*B*b)*x)*(c*x^2+b*x+a)^(5/2)/c^3-1/20
48*(-4*a*c+b^2)^2*(8*A*a*c^2-14*A*b^2*c-12*B*a*b*c+9*B*b^3)*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.22

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(945b^6B-210b^5c(7A+3Bx)+28b^4c(-270aB+cx(35A+18Bx))+}{}$$

input
```
Integrate[x^2*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^6*B - 210*b^5*c*(7*A + 3*B*x) + 28
*b^4*c*(-270*a*B + c*x*(35*A + 18*B*x)) + 48*b^2*c^2*(343*a^2*B + 2*c^2*x^
3*(7*A + 4*B*x) - 2*a*c*x*(63*A + 31*B*x)) + 16*b^3*c^2*(-(c*x^2*(49*A + 2
7*B*x)) + 7*a*(95*A + 39*B*x)) + 32*b*c^3*(6*a*c*x^2*(21*A + 11*B*x) - 3*a
^2*(189*A + 73*B*x) + 8*c^2*x^4*(91*A + 75*B*x)) + 64*c^3*(-96*a^3*B + 40*
c^3*x^5*(7*A + 6*B*x) + 3*a^2*c*x*(35*A + 16*B*x) + 2*a*c^2*x^3*(245*A + 1
92*B*x))) + 105*(b^2 - 4*a*c)^2*(9*b^3*B - 14*A*b^2*c - 12*a*b*B*c + 8*a*A
*c^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(215040*c^(11/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}x(4aB+(9bB-14Ac)x)\left(cx^2+bx+a\right)^{3/2}dx}{7c}+\frac{Bx^2\left(a+bx+cx^2\right)^{5/2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{5/2}}{7c}-\frac{\int x(4aB+(9bB-14Ac)x)\left(cx^2+bx+a\right)^{3/2}dx}{14c}$$

$\big\downarrow$ 1225

$$\frac{Bx^2\left(a+bx+cx^2\right)^{5/2}}{7c}-\frac{\frac{7\left(8aAc^2-12abBc-14Ab^2c+9b^3B\right)\int\left(cx^2+bx+a\right)^{3/2}dx}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(-48aBc-10cx(9bB-14Ac)-98Abc+63b^2B\right)}{60c^2}}{14c}$$

$\big\downarrow$ 1087

$$\frac{Bx^2\left(a+bx+cx^2\right)^{5/2}}{7c}-\frac{\frac{7\left(8aAc^2-12abBc-14Ab^2c+9b^3B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^2+bx+a}\,dx}{16c}\right)}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(-48aBc-10cx(9bB-14Ac)-98\right.}{60c^2}}{14c}$$

$\big\downarrow$ 1087

$$\frac{Bx^2\left(a+bx+cx^2\right)^{5/2}}{7c}-\frac{\frac{7\left(8aAc^2-12abBc-14Ab^2c+9b^3B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{24c^2}-\left(a+bx+cx^2\right)^{5/}}{14c}$$

$\big\downarrow$ 1092

$$\frac{Bx^2\left(a+bx+cx^2\right)^{5/2}}{7c}-\frac{\frac{7\left(8aAc^2-12abBc-14Ab^2c+9b^3B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{24c^2}-(}{14c}$$

$\big\downarrow$ 219

$$\frac{Bx^2\left(a+bx+cx^2\right)^{5/2}}{7c} - \frac{\frac{7\left(8aAc^2-12abBc-14Ab^2c+9b^3B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)}{24c^2} - \frac{(a \cdots}{\cdots}}{14c}$$

input
```
Int[x^2*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(B*x^2*(a + b*x + c*x^2)^(5/2))/(7*c) - (-1/60*((63*b^2*B - 98*A*b*c - 48*
a*B*c - 10*c*(9*b*B - 14*A*c)*x)*(a + b*x + c*x^2)^(5/2))/c^2 + (7*(9*b^3*
B - 14*A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*(((b + 2*c*x)*(a + b*x + c*x^2)^(
3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c)
- ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(3/2))))/(16*c)))/(24*c^2))/(14*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{\left(-15360 B\,c^{6} x^{6}-17920 A\,c^{6} x^{5}-19200 B b\,c^{5} x^{5}-23296 A b\,c^{5} x^{4}-24576 B a\,c^{5} x^{4}-384 B\,b^{2} c^{4} x^{4}-31360 A a\,c^{5} x^{3}-672 A\,b^{2} c^{4} x^{3}-211\right.}{}$
default	$A\left(\frac{x\left(c\,x^{2}+b x+a\right)^{\frac{5}{2}}}{6 c}-\frac{7 b\left(\frac{\left(c\,x^{2}+b x+a\right)^{\frac{5}{2}}}{5 c}-\frac{b\left(\frac{(2 c x+b)\left(c\,x^{2}+b x+a\right)^{\frac{3}{2}}}{8 c}+\frac{3\left(4 a c-b^{2}\right)\left(\frac{(2 c x+b)\sqrt{c\,x^{2}+b x+a}}{4 c}+\frac{\left(4 a c-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c}\right.}{8 c^{\frac{3}{2}}}\right.}{16 c}\right.}{2 c}\right.}{12 c}\right.$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/107520/c^5*(-15360*B*c^6*x^6-17920*A*c^6*x^5-19200*B*b*c^5*x^5-23296*A*
b*c^5*x^4-24576*B*a*c^5*x^4-384*B*b^2*c^4*x^4-31360*A*a*c^5*x^3-672*A*b^2*
c^4*x^3-2112*B*a*b*c^4*x^3+432*B*b^3*c^3*x^3-4032*A*a*b*c^4*x^2+784*A*b^3*
c^3*x^2-3072*B*a^2*c^4*x^2+2976*B*a*b^2*c^3*x^2-504*B*b^4*c^2*x^2-6720*A*a
^2*c^4*x+6048*A*a*b^2*c^3*x-980*A*b^4*c^2*x+7008*B*a^2*b*c^3*x-4368*B*a*b^
3*c^2*x+630*B*b^5*c*x+18144*A*a^2*b*c^3-10640*A*a*b^3*c^2+1470*A*b^5*c+614
4*B*a^3*c^3-16464*B*a^2*b^2*c^2+7560*B*a*b^4*c-945*B*b^6)*(c*x^2+b*x+a)^(1
/2)-1/2048*(128*A*a^3*c^4-288*A*a^2*b^2*c^3+120*A*a*b^4*c^2-14*A*b^6*c-192
*B*a^3*b*c^3+240*B*a^2*b^3*c^2-84*B*a*b^5*c+9*B*b^7)/c^(11/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 845, normalized size of antiderivative = 3.14

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/430080*(105*(9*B*b^7 + 128*A*a^3*c^4 - 96*(2*B*a^3*b + 3*A*a^2*b^2)*c^3
 + 120*(2*B*a^2*b^3 + A*a*b^4)*c^2 - 14*(6*B*a*b^5 + A*b^6)*c)*sqrt(c)*log
(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c) + 4*(15360*B*c^7*x^6 + 945*B*b^6*c + 1280*(15*B*b*c^6 + 14*A*c^7)
*x^5 - 96*(64*B*a^3 + 189*A*a^2*b)*c^4 + 128*(3*B*b^2*c^5 + 2*(96*B*a + 91
*A*b)*c^6)*x^4 + 112*(147*B*a^2*b^2 + 95*A*a*b^3)*c^3 - 16*(27*B*b^3*c^4 -
 1960*A*a*c^6 - 6*(22*B*a*b + 7*A*b^2)*c^5)*x^3 - 210*(36*B*a*b^4 + 7*A*b^
5)*c^2 + 8*(63*B*b^4*c^3 + 24*(16*B*a^2 + 21*A*a*b)*c^5 - 2*(186*B*a*b^2 +
 49*A*b^3)*c^4)*x^2 - 2*(315*B*b^5*c^2 - 3360*A*a^2*c^5 + 48*(73*B*a^2*b +
 63*A*a*b^2)*c^4 - 14*(156*B*a*b^3 + 35*A*b^4)*c^3)*x)*sqrt(c*x^2 + b*x +
a))/c^6, 1/215040*(105*(9*B*b^7 + 128*A*a^3*c^4 - 96*(2*B*a^3*b + 3*A*a^2*
b^2)*c^3 + 120*(2*B*a^2*b^3 + A*a*b^4)*c^2 - 14*(6*B*a*b^5 + A*b^6)*c)*sqr
t(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c
*x + a*c)) + 2*(15360*B*c^7*x^6 + 945*B*b^6*c + 1280*(15*B*b*c^6 + 14*A*c^
7)*x^5 - 96*(64*B*a^3 + 189*A*a^2*b)*c^4 + 128*(3*B*b^2*c^5 + 2*(96*B*a +
91*A*b)*c^6)*x^4 + 112*(147*B*a^2*b^2 + 95*A*a*b^3)*c^3 - 16*(27*B*b^3*c^4
 - 1960*A*a*c^6 - 6*(22*B*a*b + 7*A*b^2)*c^5)*x^3 - 210*(36*B*a*b^4 + 7*A*
b^5)*c^2 + 8*(63*B*b^4*c^3 + 24*(16*B*a^2 + 21*A*a*b)*c^5 - 2*(186*B*a*b^2
 + 49*A*b^3)*c^4)*x^2 - 2*(315*B*b^5*c^2 - 3360*A*a^2*c^5 + 48*(73*B*a^2*b
 + 63*A*a*b^2)*c^4 - 14*(156*B*a*b^3 + 35*A*b^4)*c^3)*x)*sqrt(c*x^2 + b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1911 vs. $2(280) = 560$.

Time = 0.70 (sec) , antiderivative size = 1911, normalized size of antiderivative = 7.10

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Piecewise(((-a*(A*a**2 - 3*a*(2*A*a*c + A*b**2 + 2*B*a*b - 5*a*(A*c**2 + 1
5*B*b*c/14)/(6*c) - 9*b*(2*A*b*c + 8*B*a*c/7 + B*b**2 - 11*b*(A*c**2 + 15*
B*b*c/14)/(12*c))/(10*c))/(4*c) - 5*b*(2*A*a*b + B*a**2 - 4*a*(2*A*b*c + 8
*B*a*c/7 + B*b**2 - 11*b*(A*c**2 + 15*B*b*c/14)/(12*c))/(5*c) - 7*b*(2*A*a
*c + A*b**2 + 2*B*a*b - 5*a*(A*c**2 + 15*B*b*c/14)/(6*c) - 9*b*(2*A*b*c +
8*B*a*c/7 + B*b**2 - 11*b*(A*c**2 + 15*B*b*c/14)/(12*c))/(10*c))/(8*c))/(6
*c))/(2*c) - b*(-2*a*(2*A*a*b + B*a**2 - 4*a*(2*A*b*c + 8*B*a*c/7 + B*b**2
 - 11*b*(A*c**2 + 15*B*b*c/14)/(12*c))/(5*c) - 7*b*(2*A*a*c + A*b**2 + 2*B
*a*b - 5*a*(A*c**2 + 15*B*b*c/14)/(6*c) - 9*b*(2*A*b*c + 8*B*a*c/7 + B*b**
2 - 11*b*(A*c**2 + 15*B*b*c/14)/(12*c))/(10*c))/(8*c))/(3*c) - 3*b*(A*a**2
 - 3*a*(2*A*a*c + A*b**2 + 2*B*a*b - 5*a*(A*c**2 + 15*B*b*c/14)/(6*c) - 9*
b*(2*A*b*c + 8*B*a*c/7 + B*b**2 - 11*b*(A*c**2 + 15*B*b*c/14)/(12*c))/(10*
c))/(4*c) - 5*b*(2*A*a*b + B*a**2 - 4*a*(2*A*b*c + 8*B*a*c/7 + B*b**2 - 11
*b*(A*c**2 + 15*B*b*c/14)/(12*c))/(5*c) - 7*b*(2*A*a*c + A*b**2 + 2*B*a*b
- 5*a*(A*c**2 + 15*B*b*c/14)/(6*c) - 9*b*(2*A*b*c + 8*B*a*c/7 + B*b**2 - 1
1*b*(A*c**2 + 15*B*b*c/14)/(12*c))/(10*c))/(8*c))/(6*c))/(4*c))/(2*c))*Pie
cewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a -
b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2),
 True)) + sqrt(a + b*x + c*x**2)*(B*c*x**6/7 + x**5*(A*c**2 + 15*B*b*c/14)
/(6*c) + x**4*(2*A*b*c + 8*B*a*c/7 + B*b**2 - 11*b*(A*c**2 + 15*B*b*c/1...
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.56

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{1}{107520}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(12\,Bcx+\frac{15\,Bbc^6+14\,Ac^7}{c^6}\right)x+\frac{3\,Bb^2c^5+192\,B\ldots}{c\ldots}\right.\right.\right.\right.\right.$$
$$+\frac{(9\,Bb^7-84\,Bab^5c-14\,Ab^6c+240\,Ba^2b^3c^2+120\,Aab^4c^2-192\,Ba^3bc^3-288\,Aa^2b^2c^3+128\,Aa^3c^4)\log(\ldots}{2048\,c^{\frac{11}{2}}}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*B*c*x + (15*B*b*c^6 + 1
4*A*c^7)/c^6)*x + (3*B*b^2*c^5 + 192*B*a*c^6 + 182*A*b*c^6)/c^6)*x - (27*B
*b^3*c^4 - 132*B*a*b*c^5 - 42*A*b^2*c^5 - 1960*A*a*c^6)/c^6)*x + (63*B*b^4
*c^3 - 372*B*a*b^2*c^4 - 98*A*b^3*c^4 + 384*B*a^2*c^5 + 504*A*a*b*c^5)/c^6
)*x - (315*B*b^5*c^2 - 2184*B*a*b^3*c^3 - 490*A*b^4*c^3 + 3504*B*a^2*b*c^4
 + 3024*A*a*b^2*c^4 - 3360*A*a^2*c^5)/c^6)*x + (945*B*b^6*c - 7560*B*a*b^4
*c^2 - 1470*A*b^5*c^2 + 16464*B*a^2*b^2*c^3 + 10640*A*a*b^3*c^3 - 6144*B*a
^3*c^4 - 18144*A*a^2*b*c^4)/c^6) + 1/2048*(9*B*b^7 - 84*B*a*b^5*c - 14*A*b
^6*c + 240*B*a^2*b^3*c^2 + 120*A*a*b^4*c^2 - 192*B*a^3*b*c^3 - 288*A*a^2*b
^2*c^3 + 128*A*a^3*c^4)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt
(c) + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int x^2\,(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,dx$$

input
```
int(x^2*(A + B*x)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(x^2*(A + B*x)*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int x^2(Bx+A)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}dx$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

3.119 $\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 198

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = -\frac{(b^2-4ac)\,(7b^2B-12Abc-4aBc)\,(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} + \frac{(7b^2B-12Abc-4aBc)\,(b+2cx)\,(a+bx+cx^2)^{3/2}}{192c^3} - \frac{(7bB-12Ac-10Bcx)\,(a+bx+cx^2)^{5/2}}{60c^2} + \frac{(b^2-4ac)^2\,(7b^2B-12Abc-4aBc)\,\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}$$

output

```
-1/512*(-4*a*c+b^2)*(-12*A*b*c-4*B*a*c+7*B*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(1
/2)/c^4+1/192*(-12*A*b*c-4*B*a*c+7*B*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^
3-1/60*(-10*B*c*x-12*A*c+7*B*b)*(c*x^2+b*x+a)^(5/2)/c^2+1/1024*(-4*a*c+b^2
)^2*(-12*A*b*c-4*B*a*c+7*B*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.28

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5B+10b^4c(18A+7Bx)+8b^3c(95aB-cx(15A+7Bx))+48b}{}$$

input
```
Integrate[x*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5*B + 10*b^4*c*(18*A + 7*B*x) + 8
*b^3*c*(95*a*B - c*x*(15*A + 7*B*x)) + 48*b^2*c^2*(c*x^2*(2*A + B*x) - a*(
25*A + 9*B*x)) + 16*b*c^2*(-81*a^2*B + 6*a*c*x*(7*A + 3*B*x) + 4*c^2*x^3*(
33*A + 26*B*x)) + 32*c^3*(8*c^2*x^4*(6*A + 5*B*x) + 3*a^2*(16*A + 5*B*x) +
 2*a*c*x^2*(48*A + 35*B*x))) - 15*(b^2 - 4*a*c)^2*(7*b^2*B - 12*A*b*c - 4*
a*B*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(15360*c^(9/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \; 1225$$

$$\frac{\left(-4aBc-12Abc+7b^2B\right)\int\left(cx^2+bx+a\right)^{3/2}dx}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

$$\Big\downarrow \; 1087$$

$$\frac{\left(-4aBc-12Abc+7b^2B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(-4aBc-12Abc+7b^2B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(-4aBc-12Abc+7b^2B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(-4aBc-12Abc+7b^2B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

input
```
Int[x*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
-1/60*((7*b*B - 12*A*c - 10*B*c*x)*(a + b*x + c*x^2)^(5/2))/c^2 + ((7*b^2*
B - 12*A*b*c - 4*a*B*c)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*
(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*
c)))/(24*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.60

method	result
risch	$\frac{(1280B\,c^5x^5+1536A\,c^5x^4+1664Bb\,c^4x^4+2112Ab\,c^4x^3+2240Ba\,c^4x^3+48B\,b^2c^3x^3+3072Aa\,c^4x^2+96A\,b^2c^3x^2+288Bab\,c^3x^2-56}{}$
default	$A\left(\frac{(c\,x^2+bx+a)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{2c}\right)+$

input

```
int(x*(B*x+A)*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/7680/c^4*(1280*B*c^5*x^5+1536*A*c^5*x^4+1664*B*b*c^4*x^4+2112*A*b*c^4*x^
3+2240*B*a*c^4*x^3+48*B*b^2*c^3*x^3+3072*A*a*c^4*x^2+96*A*b^2*c^3*x^2+288*
B*a*b*c^3*x^2-56*B*b^3*c^2*x^2+672*A*a*b*c^3*x-120*A*b^3*c^2*x+480*B*a^2*c
^3*x-432*B*a*b^2*c^2*x+70*B*b^4*c*x+1536*A*a^2*c^3-1200*A*a*b^2*c^2+180*A*
b^4*c-1296*B*a^2*b*c^2+760*B*a*b^3*c-105*B*b^5)*(c*x^2+b*x+a)^(1/2)-1/1024
*(192*A*a^2*b*c^3-96*A*a*b^3*c^2+12*A*b^5*c+64*B*a^3*c^3-144*B*a^2*b^2*c^2
+60*B*a*b^4*c-7*B*b^6)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 669, normalized size of antiderivative = 3.38

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(7*B*b^6 - 64*(B*a^3 + 3*A*a^2*b)*c^3 + 48*(3*B*a^2*b^2 + 2*
A*a*b^3)*c^2 - 12*(5*B*a*b^4 + A*b^5)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x
- b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*B*c
^6*x^5 - 105*B*b^5*c + 1536*A*a^2*c^4 + 128*(13*B*b*c^5 + 12*A*c^6)*x^4 -
48*(27*B*a^2*b + 25*A*a*b^2)*c^3 + 16*(3*B*b^2*c^4 + 4*(35*B*a + 33*A*b)*c
^5)*x^3 + 20*(38*B*a*b^3 + 9*A*b^4)*c^2 - 8*(7*B*b^3*c^3 - 384*A*a*c^5 - 1
2*(3*B*a*b + A*b^2)*c^4)*x^2 + 2*(35*B*b^4*c^2 + 48*(5*B*a^2 + 7*A*a*b)*c^
4 - 12*(18*B*a*b^2 + 5*A*b^3)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360
*(15*(7*B*b^6 - 64*(B*a^3 + 3*A*a^2*b)*c^3 + 48*(3*B*a^2*b^2 + 2*A*a*b^3)*
c^2 - 12*(5*B*a*b^4 + A*b^5)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*B*c^6*x^5 - 105*B*
b^5*c + 1536*A*a^2*c^4 + 128*(13*B*b*c^5 + 12*A*c^6)*x^4 - 48*(27*B*a^2*b
+ 25*A*a*b^2)*c^3 + 16*(3*B*b^2*c^4 + 4*(35*B*a + 33*A*b)*c^5)*x^3 + 20*(3
8*B*a*b^3 + 9*A*b^4)*c^2 - 8*(7*B*b^3*c^3 - 384*A*a*c^5 - 12*(3*B*a*b + A*
b^2)*c^4)*x^2 + 2*(35*B*b^4*c^2 + 48*(5*B*a^2 + 7*A*a*b)*c^4 - 12*(18*B*a*
b^2 + 5*A*b^3)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. 2(199) = 398.

Time = 0.90 (sec) , antiderivative size = 1175, normalized size of antiderivative = 5.93

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Piecewise(((-a*(2*A*a*b + B*a**2 - 3*a*(2*A*b*c + 7*B*a*c/6 + B*b**2 - 9*b
*(A*c**2 + 13*B*b*c/12)/(10*c))/(4*c) - 5*b*(2*A*a*c + A*b**2 + 2*B*a*b -
4*a*(A*c**2 + 13*B*b*c/12)/(5*c) - 7*b*(2*A*b*c + 7*B*a*c/6 + B*b**2 - 9*b
*(A*c**2 + 13*B*b*c/12)/(10*c))/(8*c))/(6*c))/(2*c) - b*(A*a**2 - 2*a*(2*A
*a*c + A*b**2 + 2*B*a*b - 4*a*(A*c**2 + 13*B*b*c/12)/(5*c) - 7*b*(2*A*b*c
+ 7*B*a*c/6 + B*b**2 - 9*b*(A*c**2 + 13*B*b*c/12)/(10*c))/(8*c))/(3*c) - 3
*b*(2*A*a*b + B*a**2 - 3*a*(2*A*b*c + 7*B*a*c/6 + B*b**2 - 9*b*(A*c**2 + 1
3*B*b*c/12)/(10*c))/(4*c) - 5*b*(2*A*a*c + A*b**2 + 2*B*a*b - 4*a*(A*c**2
+ 13*B*b*c/12)/(5*c) - 7*b*(2*A*b*c + 7*B*a*c/6 + B*b**2 - 9*b*(A*c**2 + 1
3*B*b*c/12)/(10*c))/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(
c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2
*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)) + sqrt(a + b*x
+ c*x**2)*(B*c*x**5/6 + x**4*(A*c**2 + 13*B*b*c/12)/(5*c) + x**3*(2*A*b*c
+ 7*B*a*c/6 + B*b**2 - 9*b*(A*c**2 + 13*B*b*c/12)/(10*c))/(4*c) + x**2*(2*
A*a*c + A*b**2 + 2*B*a*b - 4*a*(A*c**2 + 13*B*b*c/12)/(5*c) - 7*b*(2*A*b*c
 + 7*B*a*c/6 + B*b**2 - 9*b*(A*c**2 + 13*B*b*c/12)/(10*c))/(8*c))/(3*c) +
x*(2*A*a*b + B*a**2 - 3*a*(2*A*b*c + 7*B*a*c/6 + B*b**2 - 9*b*(A*c**2 + 13
*B*b*c/12)/(10*c))/(4*c) - 5*b*(2*A*a*c + A*b**2 + 2*B*a*b - 4*a*(A*c**2 +
 13*B*b*c/12)/(5*c) - 7*b*(2*A*b*c + 7*B*a*c/6 + B*b**2 - 9*b*(A*c**2 + 13
*B*b*c/12)/(10*c))/(8*c))/(6*c))/(2*c) + (A*a**2 - 2*a*(2*A*a*c + A*b**...
```

Maxima [F(-2)]

Exception generated.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.67

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,Bcx+\frac{13\,Bbc^5+12\,Ac^6}{c^5}\right)x+\frac{3\,Bb^2c^4+140\,Bac^5+\cdots}{c^5}\right.\right.\right.\right.$$
$$-\frac{\left(7\,Bb^6-60\,Bab^4c-12\,Ab^5c+144\,Ba^2b^2c^2+96\,Aab^3c^2-64\,Ba^3c^3-192\,Aa^2bc^3\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2\cdots}\right.\right.\right.}{1024\,c^{\frac{9}{2}}}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*B*c*x + (13*B*b*c^5 + 12*A*c^
6)/c^5)*x + (3*B*b^2*c^4 + 140*B*a*c^5 + 132*A*b*c^5)/c^5)*x - (7*B*b^3*c^
3 - 36*B*a*b*c^4 - 12*A*b^2*c^4 - 384*A*a*c^5)/c^5)*x + (35*B*b^4*c^2 - 21
6*B*a*b^2*c^3 - 60*A*b^3*c^3 + 240*B*a^2*c^4 + 336*A*a*b*c^4)/c^5)*x - (10
5*B*b^5*c - 760*B*a*b^3*c^2 - 180*A*b^4*c^2 + 1296*B*a^2*b*c^3 + 1200*A*a*
b^2*c^3 - 1536*A*a^2*c^4)/c^5) - 1/1024*(7*B*b^6 - 60*B*a*b^4*c - 12*A*b^5
*c + 144*B*a^2*b^2*c^2 + 96*A*a*b^3*c^2 - 64*B*a^3*c^3 - 192*A*a^2*b*c^3)*
log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \int x\,\left(A+B\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}\,dx$$

input
```
int(x*(A + B*x)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(x*(A + B*x)*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int x(Bx+A)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}dx$$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

3.120 $\int (A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 158

$$\int (A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{3(b^2-4ac)\,(bB-2Ac)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3}-\frac{(bB-2Ac)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{16c^2}+\frac{B\left(a+bx+cx^2\right)^{5/2}}{5c}-\frac{3\left(b^2-4ac\right)^2(bB-2Ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}}$$

output
```
3/128*(-4*a*c+b^2)*(-2*A*c+B*b)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3-1/16*(-2
*A*c+B*b)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^2+1/5*B*(c*x^2+b*x+a)^(5/2)/c-3/
256*(-4*a*c+b^2)^2*(-2*A*c+B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.22

$$\int (A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(15b^4B-10b^3c(3A+Bx)+8bc^2(25aA+7aBx+30Acx^2+22Bcx^3)+$$

input
```
Integrate[(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4*B - 10*b^3*c*(3*A + B*x) + 8*b*c^2*
(25*a*A + 7*a*B*x + 30*A*c*x^2 + 22*B*c*x^3) + 4*b^2*c*(-25*a*B + c*x*(5*A
 + 2*B*x)) + 16*c^2*(8*a^2*B + 2*c^2*x^3*(5*A + 4*B*x) + a*c*x*(25*A + 16*
B*x))) - 15*(b^2 - 4*a*c)^2*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] +
Sqrt[a + x*(b + c*x)])])/(640*c^(7/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{B\left(a+bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\int \left(cx^2+bx+a\right)^{3/2}dx}{2c}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{B\left(a+bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{2c}$$

$$\Big\downarrow \ 1087$$

$$\frac{B\left(a+bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \ 1092$$

$$\frac{B\left(a+bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \ 219$$

$$\frac{B\left(a+bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)}{2c}$$

input
```
Int[(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(B*(a + b*x + c*x^2)^(5/2))/(5*c) - ((b*B - 2*A*c)*(((b + 2*c*x)*(a + b*x
+ c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^
2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(8*c^(3/2))))/(16*c)))/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(128 B c^4 x^4+160 A c^4 x^3+176 B c^3 x^3 b+240 A b c^3 x^2+256 B a c^3 x^2+8 c^2 x^2 B b^2+400 A a c^3 x+20 A b^2 c^2 x+56 B a b c^2 x-10 B b^3 c x+200 A}{640 c^3}$
default	$A\left(\frac{(2cx+b)(c x^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+B\left(\frac{(c x^2+bx+a)}{5c}\right.$

input

```
int((B*x+A)*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/640/c^3*(128*B*c^4*x^4+160*A*c^4*x^3+176*B*b*c^3*x^3+240*A*b*c^3*x^2+256
*B*a*c^3*x^2+8*B*b^2*c^2*x^2+400*A*a*c^3*x+20*A*b^2*c^2*x+56*B*a*b*c^2*x-1
0*B*b^3*c*x+200*A*a*b*c^2-30*A*b^3*c+128*B*a^2*c^2-100*B*a*b^2*c+15*B*b^4)
*(c*x^2+b*x+a)^(1/2)+3/256*(32*A*a^2*c^3-16*A*a*b^2*c^2+2*A*b^4*c-16*B*a^2
*b*c^2+8*B*a*b^3*c-B*b^5)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 515, normalized size of antiderivative = 3.26

$$\int (A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \left[-\frac{15\left(Bb^5-32\,Aa^2c^3+16\,(Ba^2b+Aab^2)c^2-2\,(4\,Bab^3+Ab^4)c\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx\right.}{}\right.$$

input

```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2560*(15*(B*b^5 - 32*A*a^2*c^3 + 16*(B*a^2*b + A*a*b^2)*c^2 - 2*(4*B*a
*b^3 + A*b^4)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(128*B*c^5*x^4 + 15*B*b^4*c + 8*(
16*B*a^2 + 25*A*a*b)*c^3 + 16*(11*B*b*c^4 + 10*A*c^5)*x^3 - 10*(10*B*a*b^2
 + 3*A*b^3)*c^2 + 8*(B*b^2*c^3 + 2*(16*B*a + 15*A*b)*c^4)*x^2 - 2*(5*B*b^3
*c^2 - 200*A*a*c^4 - 2*(14*B*a*b + 5*A*b^2)*c^3)*x)*sqrt(c*x^2 + b*x + a))
/c^4, 1/1280*(15*(B*b^5 - 32*A*a^2*c^3 + 16*(B*a^2*b + A*a*b^2)*c^2 - 2*(4
*B*a*b^3 + A*b^4)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(128*B*c^5*x^4 + 15*B*b^4*c + 8*(16
*B*a^2 + 25*A*a*b)*c^3 + 16*(11*B*b*c^4 + 10*A*c^5)*x^3 - 10*(10*B*a*b^2 +
 3*A*b^3)*c^2 + 8*(B*b^2*c^3 + 2*(16*B*a + 15*A*b)*c^4)*x^2 - 2*(5*B*b^3*c
^2 - 200*A*a*c^4 - 2*(14*B*a*b + 5*A*b^2)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c
^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(151) = 302$.

Time = 0.60 (sec) , antiderivative size = 711, normalized size of antiderivative = 4.50

$$\int (A + Bx) \left(a + bx + cx^2\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(B*c*x**4/5 + x**3*(A*c**2 + 11*B*b*c/10
)/(4*c) + x**2*(2*A*b*c + 6*B*a*c/5 + B*b**2 - 7*b*(A*c**2 + 11*B*b*c/10)/
(8*c))/(3*c) + x*(2*A*a*c + A*b**2 + 2*B*a*b - 3*a*(A*c**2 + 11*B*b*c/10)/
(4*c) - 5*b*(2*A*b*c + 6*B*a*c/5 + B*b**2 - 7*b*(A*c**2 + 11*B*b*c/10)/(8*
c))/(6*c))/(2*c) + (2*A*a*b + B*a**2 - 2*a*(2*A*b*c + 6*B*a*c/5 + B*b**2 -
 7*b*(A*c**2 + 11*B*b*c/10)/(8*c))/(3*c) - 3*b*(2*A*a*c + A*b**2 + 2*B*a*b
 - 3*a*(A*c**2 + 11*B*b*c/10)/(4*c) - 5*b*(2*A*b*c + 6*B*a*c/5 + B*b**2 -
7*b*(A*c**2 + 11*B*b*c/10)/(8*c))/(6*c))/(4*c))/c) + (A*a**2 - a*(2*A*a*c
+ A*b**2 + 2*B*a*b - 3*a*(A*c**2 + 11*B*b*c/10)/(4*c) - 5*b*(2*A*b*c + 6*B
*a*c/5 + B*b**2 - 7*b*(A*c**2 + 11*B*b*c/10)/(8*c))/(6*c))/(2*c) - b*(2*A*
a*b + B*a**2 - 2*a*(2*A*b*c + 6*B*a*c/5 + B*b**2 - 7*b*(A*c**2 + 11*B*b*c/
10)/(8*c))/(3*c) - 3*b*(2*A*a*c + A*b**2 + 2*B*a*b - 3*a*(A*c**2 + 11*B*b*
c/10)/(4*c) - 5*b*(2*A*b*c + 6*B*a*c/5 + B*b**2 - 7*b*(A*c**2 + 11*B*b*c/1
0)/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x +
 c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2
*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(B*(a + b*x)**(7/
2)/(7*b) + (a + b*x)**(5/2)*(A*b - B*a)/(5*b))/b, Ne(b, 0)), (a**(3/2)*(A*
x + B*x**2/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (A + Bx) \left(a + bx + cx^2\right)^{3/2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.58

$$\int (A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{1}{640}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,Bcx+\frac{11\,Bbc^4+10\,Ac^5}{c^4}\right)x+\frac{Bb^2c^3+32\,Bac^4+30\,Abc^4}{c^4}\right.\right.\right. + \frac{3\left(Bb^5-8\,Bab^3c-2\,Ab^4c+16\,Ba^2bc^2+16\,Aab^2c^2-32\,Aa^2c^3\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*B*c*x + (11*B*b*c^4 + 10*A*c^5)/c^
4)*x + (B*b^2*c^3 + 32*B*a*c^4 + 30*A*b*c^4)/c^4)*x - (5*B*b^3*c^2 - 28*B*
a*b*c^3 - 10*A*b^2*c^3 - 200*A*a*c^4)/c^4)*x + (15*B*b^4*c - 100*B*a*b^2*c
^2 - 30*A*b^3*c^2 + 128*B*a^2*c^3 + 200*A*a*b*c^3)/c^4) + 3/256*(B*b^5 - 8
*B*a*b^3*c - 2*A*b^4*c + 16*B*a^2*b*c^2 + 16*A*a*b^2*c^2 - 32*A*a^2*c^3)*l
og(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 10.98 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.93

$$\int (A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{B\left(c\,x^2+b\,x+a\right)^{5/2}}{5\,c} + \frac{A\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)\left(3\,a\,c-\frac{3\,b^2}{4}\right)}{4\,c} - \frac{B\,b\left(\frac{3\,a\left(\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{(b+2\,c\,x)\,\sqrt{c\,x^2+b\,x+a}}{4\,c}\right)}{4}+\frac{x\left(c\,x^2+b\,x+a\right)^{3/2}}{4}+\frac{b\left(c\,x^2+b\,x+a\right)^{3/2}}{8\,c}-\frac{3\,b^2\left(\ln\left(\frac{b}{2}\right.\right.}{}\right.}{2\,c} + \frac{A\left(\frac{b}{2}+c\,x\right)\left(c\,x^2+b\,x+a\right)^{3/2}}{4\,c}$$

input
```
int((A + B*x)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
(B*(a + b*x + c*x^2)^(5/2))/(5*c) + (A*((x/2 + b/(4*c))*(a + b*x + c*x^2)^
(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))
/(2*c^(3/2)))*(3*a*c - (3*b^2)/4))/(4*c) - (B*b*((3*a*(log((b/2 + c*x)/c^(
1/2) + (a + b*x + c*x^2)^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b +
2*c*x)*(a + b*x + c*x^2)^(1/2))/(4*c)))/4 + (x*(a + b*x + c*x^2)^(3/2))/4
+ (b*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*(log((b/2 + c*x)/c^(1/2) + (a
 + b*x + c*x^2)^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x)*(a
 + b*x + c*x^2)^(1/2))/(4*c)))/(16*c)))/(2*c) + (A*(b/2 + c*x)*(a + b*x +
c*x^2)^(3/2))/(4*c)
```

Reduce [F]

$$\int (A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int (Bx+A)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}dx$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

3.121 $\int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 218

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x} \, dx = -\frac{(3b^3B-8Ab^2c-12abBc-64aAc^2+2c(3b^2B-8Abc-12aBc)\,x)\sqrt{a+bx+cx^2}}{64c^2} + \frac{(3bB+8Ac+6Bcx)\left(a+bx+cx^2\right)^{3/2}}{24c} - a^{3/2}A\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) + \frac{(64aAbc^2+(b^2-4ac)\,(3b^2B-8Abc-12aBc))\operatorname{arctanh}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{128c^{5/2}}$$

output
```
-1/64*(3*B*b^3-8*A*b^2*c-12*B*a*b*c-64*A*a*c^2+2*c*(-8*A*b*c-12*B*a*c+3*B*
b^2)*x)*(c*x^2+b*x+a)^(1/2)/c^2+1/24*(6*B*c*x+8*A*c+3*B*b)*(c*x^2+b*x+a)^(
3/2)/c-a^(3/2)*A*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))+1/128*
(64*A*a*b*c^2+(-4*a*c+b^2)*(-8*A*b*c-12*B*a*c+3*B*b^2))*arctanh(1/2*(2*c*x
+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x}\,dx = \frac{\sqrt{a+x(b+cx)}(-9b^3B+6b^2c(4A+Bx)+8c^2(32aA+15aBx+8A}{192c^2}$$
$$+2a^{3/2}A\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)$$
$$-\frac{(3b^4B-8Ab^3c-24ab^2Bc+96aAbc^2+48a^2Bc^2)\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{128c^{5/2}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x,x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(-9*b^3*B + 6*b^2*c*(4*A + B*x) + 8*c^2*(32*a*A + 1
5*a*B*x + 8*A*c*x^2 + 6*B*c*x^3) + 4*b*c*(15*a*B + 2*c*x*(14*A + 9*B*x)))
/(192*c^2) + 2*a^(3/2)*A*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[
a]] - ((3*b^4*B - 8*A*b^3*c - 24*a*b^2*B*c + 96*a*A*b*c^2 + 48*a^2*B*c^2)*
Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(8Ac+3bB+6Bcx)}{24c}-\frac{\int -\frac{(16aAc+(8Abc-3B(b^2-4ac))x)\sqrt{cx^2+bx+a}}{2x}dx}{8c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(16aAc-(3Bb^2-8Acb-12aBc)x)\sqrt{cx^2+bx+a}}{x}dx}{16c}+\frac{(a+bx+cx^2)^{3/2}(8Ac+3bB+6Bcx)}{24c}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{-\frac{\int -\frac{128a^2Ac^2+\left(64aAbc^2+\left(b^2-4ac\right)\left(3Bb^2-8Acb-12aBc\right)\right)x}{2x\sqrt{cx^2+bx+a}}dx}{4c}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-12aBc-8Abc+3b^2B\right)-64aAc^2-12abBc-8Ab^2c+3b^3B\right)}{4c}}{16c}+$$
$$\frac{(a+bx+cx^2)^{3/2}(8Ac+3bB+6Bcx)}{24c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{128a^2Ac^2+\left(64aAbc^2+\left(b^2-4ac\right)\left(3Bb^2-8Acb-12aBc\right)\right)x}{x\sqrt{cx^2+bx+a}}dx}{8c}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-12aBc-8Abc+3b^2B\right)-64aAc^2-12abBc-8Ab^2c+3b^3B\right)}{4c}}{16c}+$$
$$\frac{(a+bx+cx^2)^{3/2}(8Ac+3bB+6Bcx)}{24c}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{\frac{128a^2Ac^2\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+\left(\left(b^2-4ac\right)\left(-12aBc-8Abc+3b^2B\right)+64aAbc^2\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-12aBc-8Abc+3b^2B\right)-64\cdots\right.}{4c}}{16c}$$
$$\frac{(a+bx+cx^2)^{3/2}(8Ac+3bB+6Bcx)}{24c}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\frac{128a^2Ac^2\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+2\left(\left(b^2-4ac\right)\left(-12aBc-8Abc+3b^2B\right)+64aAbc^2\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{8c}-\sqrt{a+bx+cx^2}\left(2cx\left(-12aBc-8A\cdots\right.\right.}{16c}$$
$$\frac{(a+bx+cx^2)^{3/2}(8Ac+3bB+6Bcx)}{24c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{128a^2Ac^2\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{\left(\left(b^2-4ac\right)\left(-12aBc-8Abc+3b^2B\right)+64aAbc^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{8c}-\sqrt{a+bx+cx^2}\left(2cx\left(-12aBc-8Abc+3b\cdots\right.\right.}{16c}$$
$$\frac{(a+bx+cx^2)^{3/2}(8Ac+3bB+6Bcx)}{24c}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\frac{\frac{\left((b^2-4ac)\left(-12aBc-8Abc+3b^2B\right)+64aAbc^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-256a^2Ac^2\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{8c}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-12aB\right.\right.}{}}{16c}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(8Ac+3bB+6Bcx)}{24c}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\left((b^2-4ac)\left(-12aBc-8Abc+3b^2B\right)+64aAbc^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-128a^{3/2}Ac^2\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8c}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-12a\right.\right.}{}}{16c}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}(8Ac+3bB+6Bcx)}{24c}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x,x]
```

output
```
((3*b*B + 8*A*c + 6*B*c*x)*(a + b*x + c*x^2)^(3/2))/(24*c) + (-1/4*((3*b^3
*B - 8*A*b^2*c - 12*a*b*B*c - 64*a*A*c^2 + 2*c*(3*b^2*B - 8*A*b*c - 12*a*B
*c)*x)*Sqrt[a + b*x + c*x^2])/c + (-128*a^(3/2)*A*c^2*ArcTanh[(2*a + b*x)/
(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] + ((64*a*A*b*c^2 + (b^2 - 4*a*c)*(3*b^2
*B - 8*A*b*c - 12*a*B*c))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/Sqrt[c])/(8*c))/(16*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.24

method	result
default	$B\left(\frac{(2cx+b)(c\,x^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+A\left(\frac{(c\,x^2+bx+a)}{3}\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
B*(1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)*(
c*x^2+b*x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))))+A*(1/3*(c*x^2+b*x+a)^(3/2)+1/2*b*(1/4*(2*c*x+b)*(c*x^2+b*
x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2)))+a*((c*x^2+b*x+a)^(1/2)+1/2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))/c^(1/2)-a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))
```

Fricas [A] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 1023, normalized size of antiderivative = 4.69

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/768*(384*A*a^(3/2)*c^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2
 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 3*(3*B*b^4 + 48*(B*a^2 + 2
*A*a*b)*c^2 - 8*(3*B*a*b^2 + A*b^3)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x -
b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*B*c^4*x
^3 - 9*B*b^3*c + 256*A*a*c^3 + 12*(5*B*a*b + 2*A*b^2)*c^2 + 8*(9*B*b*c^3 +
 8*A*c^4)*x^2 + 2*(3*B*b^2*c^2 + 4*(15*B*a + 14*A*b)*c^3)*x)*sqrt(c*x^2 +
b*x + a))/c^3, 1/384*(192*A*a^(3/2)*c^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2
- 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 3*(3*B*b^4 +
 48*(B*a^2 + 2*A*a*b)*c^2 - 8*(3*B*a*b^2 + A*b^3)*c)*sqrt(-c)*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48
*B*c^4*x^3 - 9*B*b^3*c + 256*A*a*c^3 + 12*(5*B*a*b + 2*A*b^2)*c^2 + 8*(9*B
*b*c^3 + 8*A*c^4)*x^2 + 2*(3*B*b^2*c^2 + 4*(15*B*a + 14*A*b)*c^3)*x)*sqrt(
c*x^2 + b*x + a))/c^3, 1/768*(768*A*sqrt(-a)*a*c^3*arctan(1/2*sqrt(c*x^2 +
 b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 3*(3*B*b^4 + 48*
(B*a^2 + 2*A*a*b)*c^2 - 8*(3*B*a*b^2 + A*b^3)*c)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(
48*B*c^4*x^3 - 9*B*b^3*c + 256*A*a*c^3 + 12*(5*B*a*b + 2*A*b^2)*c^2 + 8*(9
*B*b*c^3 + 8*A*c^4)*x^2 + 2*(3*B*b^2*c^2 + 4*(15*B*a + 14*A*b)*c^3)*x)*sqr
t(c*x^2 + b*x + a))/c^3, 1/384*(384*A*sqrt(-a)*a*c^3*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 3*(3*B*b^4 ...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x}\,dx=\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/x,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x}\,dx = \int \frac{\left(A+B\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{x}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x, x)
```

Reduce [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x}\,dx = \int \frac{(Bx+A)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x,x)
```

output
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x,x)
```

3.122 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 193

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx = \frac{\left(b^2B+18Abc+8aBc+2c(bB+6Ac)x\right)\sqrt{a+bx+cx^2}}{8c} - \frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x} - \frac{1}{2}\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) - \frac{\left(b^3B-6Ab^2c-12abBc-24aAc^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}}$$

output
```
1/8*(B*b^2+18*A*b*c+8*B*a*c+2*c*(6*A*c+B*b)*x)*(c*x^2+b*x+a)^(1/2)/c-1/3*(
-B*x+3*A)*(c*x^2+b*x+a)^(3/2)/x-1/2*a^(1/2)*(3*A*b+2*B*a)*arctanh(1/2*(b*x
+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))-1/16*(-24*A*a*c^2-6*A*b^2*c-12*B*a*b*c+
B*b^3)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{a+x(b+cx)}(-8ac(3A-4Bx)+x(3b^2B+4c^2x(3A+2Bx)+2b}{24cx}$$
$$+\frac{(-b^3B+6Ab^2c+12abBc+24aAc^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{16c^{3/2}}$$
$$-\sqrt{a}(3Ab+2aB)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(-8*a*c*(3*A - 4*B*x) + x*(3*b^2*B + 4*c^2*x*(3*A +
 2*B*x) + 2*b*c*(15*A + 7*B*x))))/(24*c*x) + ((-(b^3*B) + 6*A*b^2*c + 12*a
*b*B*c + 24*a*A*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])
])/(16*c^(3/2)) - Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a +
 x*(b + c*x)])/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1230, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{ 1230}$$

$$-\frac{1}{2}\int -\frac{(3Ab+2aB+(bB+6Ac)x)\sqrt{cx^2+bx+a}}{x}dx-\frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\int \frac{(3Ab+2aB+(bB+6Ac)x)\sqrt{cx^2+bx+a}}{x}dx-\frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx+cx^2}\left(8aBc+2cx(6Ac+bB)+18Abc+b^2B\right)}{4c}-\frac{\int -\frac{8a(3Ab+2aB)c-\left(Bb^3-6Acb^2-12aBcb-24aAc^2\right)x}{2x\sqrt{cx^2+bx+a}}dx}{4c}\right)$$

$$\frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{8a(3Ab+2aB)c-\left(Bb^3-6Acb^2-12aBcb-24aAc^2\right)x}{x\sqrt{cx^2+bx+a}}dx}{8c}+\frac{\sqrt{a+bx+cx^2}\left(8aBc+2cx(6Ac+bB)+18Abc+b^2B\right)}{4c}\right)-$$

$$\frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{8ac(2aB+3Ab)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\left(-24aAc^2-12abBc-6Ab^2c+b^3B\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}+\frac{\sqrt{a+bx+cx^2}(8}{}\right.$$

$$\frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{8ac(2aB+3Ab)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-2\left(-24aAc^2-12abBc-6Ab^2c+b^3B\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{8c}+\frac{\sqrt{a+}}{}\right.$$

$$\frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{8ac(2aB+3Ab)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\frac{\left(-24aAc^2-12abBc-6Ab^2c+b^3B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx+cx^2}(8aI}{}\right.$$

$$\frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{-16ac(2aB+3Ab)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-\frac{(-24aAc^2-12abBc-6Ab^2c+b^3B)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+b}}{}\right.$$

$$\frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{-\frac{(-24aAc^2-12abBc-6Ab^2c+b^3B)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-8\sqrt{a}c(2aB+3Ab)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8c}+\frac{\sqrt{a+b}}{}\right.$$

$$\frac{(3A-Bx)\left(a+bx+cx^2\right)^{3/2}}{3x}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^2,x]
```

output
```
-1/3*((3*A - B*x)*(a + b*x + c*x^2)^(3/2))/x + ((((b^2*B + 18*A*b*c + 8*a*B
*c + 2*c*(b*B + 6*A*c)*x)*Sqrt[a + b*x + c*x^2])/(4*c) + (-8*Sqrt[a]*(3*A*
b + 2*a*B)*c*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] - ((b^
3*B - 6*A*b^2*c - 12*a*b*B*c - 24*a*A*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/Sqrt[c])/(8*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{aA\sqrt{c\,x^{2}+bx+a}}{x}+\frac{3Aa\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2}+\frac{B\,b^{2}\sqrt{c\,x^{2}+bx+a}}{8c}-\frac{B\,b^{3}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{16c^{\frac{3}{2}}}+\frac{Bc\,x^{2}\sqrt{c\,}}{3}$
default	$A\left(-\frac{\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}}{ax}+\frac{3b\left(\frac{\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{3}+\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2}\right)+a\left(\sqrt{c\,x^{2}+bx+a}+\frac{b}{}\right.}{2a}\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a*A*(c*x^2+b*x+a)^(1/2)/x+3/2*A*a*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))+1/8*B*b^2/c*(c*x^2+b*x+a)^(1/2)-1/16*B*b^3/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*B*c*x^2*(c*x^2+b*x+a)^(1/2)+7/12*B*b
*x*(c*x^2+b*x+a)^(1/2)+3/4*a*b*B*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))/c^(1/2)+5/4*A*b*(c*x^2+b*x+a)^(1/2)-3/2*A*a^(1/2)*ln((2*a+b*x+2*a^(1/2)
*(c*x^2+b*x+a)^(1/2))/x)*b-B*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(
1/2))/x)+3/8*b^2*A*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/2
*A*c*x*(c*x^2+b*x+a)^(1/2)+4/3*B*a*(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 917, normalized size of antiderivative = 4.75

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/96*(24*(2*B*a + 3*A*b)*sqrt(a)*c^2*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2
- 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 3*(B*b^3 - 2
4*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^
2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*B*c^3*x^3
- 24*A*a*c^2 + 2*(7*B*b*c^2 + 6*A*c^3)*x^2 + (3*B*b^2*c + 2*(16*B*a + 15*A
*b)*c^2)*x)*sqrt(c*x^2 + b*x + a))/(c^2*x), 1/48*(12*(2*B*a + 3*A*b)*sqrt(
a)*c^2*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x
+ 2*a)*sqrt(a) + 8*a^2)/x^2) + 3*(B*b^3 - 24*A*a*c^2 - 6*(2*B*a*b + A*b^2)
*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*
x^2 + b*c*x + a*c)) + 2*(8*B*c^3*x^3 - 24*A*a*c^2 + 2*(7*B*b*c^2 + 6*A*c^3
)*x^2 + (3*B*b^2*c + 2*(16*B*a + 15*A*b)*c^2)*x)*sqrt(c*x^2 + b*x + a))/(c
^2*x), 1/96*(48*(2*B*a + 3*A*b)*sqrt(-a)*c^2*x*arctan(1/2*sqrt(c*x^2 + b*x
 + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 3*(B*b^3 - 24*A*a*c^
2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sq
rt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*B*c^3*x^3 - 24*A*a
*c^2 + 2*(7*B*b*c^2 + 6*A*c^3)*x^2 + (3*B*b^2*c + 2*(16*B*a + 15*A*b)*c^2)
*x)*sqrt(c*x^2 + b*x + a))/(c^2*x), 1/48*(24*(2*B*a + 3*A*b)*sqrt(-a)*c^2*
x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x +
 a^2)) + 3*(B*b^3 - 24*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(-c)*x*arctan(
1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c))...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx = \int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/x**2,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(3/2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx = \frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(4\,Bcx+\frac{7\,Bbc^2+6\,Ac^3}{c^2}\right)x+\frac{3\,Bb^2c+32\,Bac}{c^2}\right.$$
$$+\frac{\left(2\,Ba^2+3\,Aab\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$+\frac{\left(Bb^3-12\,Babc-6\,Ab^2c-24\,Aac^2\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{3}{2}}}$$
$$+\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)Aab+2\,Aa^2\sqrt{c}}{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*B*c*x + (7*B*b*c^2 + 6*A*c^3)/c^2)*x + (3
*B*b^2*c + 32*B*a*c^2 + 30*A*b*c^2)/c^2) + (2*B*a^2 + 3*A*a*b)*arctan(-(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) + 1/16*(B*b^3 - 12*B*a
*b*c - 6*A*b^2*c - 24*A*a*c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*sqrt(c) + b))/c^(3/2) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a*b + 2*
A*a^2*sqrt(c))/((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{x^2}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^2,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx=\frac{-48\sqrt{c\,x^2+bx+a}\,a^2c^2+124\sqrt{c\,x^2+bx+a}\,ab\,c^2x+24\sqrt{c\,x^2+bx-}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^2,x)
```

output
```
( - 48*sqrt(a + b*x + c*x**2)*a**2*c**2 + 124*sqrt(a + b*x + c*x**2)*a*b*c
**2*x + 24*sqrt(a + b*x + c*x**2)*a*c**3*x**2 + 6*sqrt(a + b*x + c*x**2)*b
**3*c*x + 28*sqrt(a + b*x + c*x**2)*b**2*c**2*x**2 + 16*sqrt(a + b*x + c*x
**2)*b*c**3*x**3 + 120*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
- b*x)*a*b*c**2*x - 120*sqrt(a)*log(x)*a*b*c**2*x + 72*sqrt(c)*log( - 2*sq
rt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*c**2*x + 54*sqrt(c)*log( -
2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a*b**2*c*x - 3*sqrt(c)*log(
- 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*b**4*x)/(48*c**2*x)
```

3.123 $\int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 179

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx = -\frac{3(Ab+2aB-(bB+2Ac)x)\sqrt{a+bx+cx^2}}{4x}-\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2} -\frac{3(4abB+A(b^2+4ac))\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}} +\frac{3(b^2B+4Abc+4aBc)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}$$

output
```
-3/4*(A*b+2*B*a-(2*A*c+B*b)*x)*(c*x^2+b*x+a)^(1/2)/x-1/2*(-B*x+A)*(c*x^2+b
*x+a)^(3/2)/x^2-3/8*(4*a*b*B+A*(4*a*c+b^2))*arctanh(1/2*(b*x+2*a)/a^(1/2)/
(c*x^2+b*x+a)^(1/2))/a^(1/2)+3/8*(4*A*b*c+4*B*a*c+B*b^2)*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx = \frac{1}{8}\left(\frac{2\sqrt{a+x(b+cx)}(-2a(A+2Bx)+x(Bx(5b+2cx)+A(-5b+4}{x^2}\right.$$

$$-\frac{6(4abB+A(b^2+4ac))\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

$$\left.-\frac{3(b^2B+4Abc+4aBc)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{\sqrt{c}}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^3,x]
```

output
```
((2*Sqrt[a + x*(b + c*x)]*(-2*a*(A + 2*B*x) + x*(B*x*(5*b + 2*c*x) + A*(-5
*b + 4*c*x))))/x^2 - (6*(4*a*b*B + A*(b^2 + 4*a*c))*ArcTanh[(-(Sqrt[c]*x)
+ Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] - (3*(b^2*B + 4*A*b*c + 4*a*B*c
)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c])/8
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1230, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \ 1230$$

$$-\frac{3}{8}\int -\frac{2(Ab+2aB+(bB+2Ac)x)\sqrt{cx^2+bx+a}}{x^2}dx-\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{4}\int \frac{(Ab+2aB+(bB+2Ac)x)\sqrt{cx^2+bx+a}}{x^2}dx-\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{1230}$$

$$\frac{3}{4}\left(-\frac{1}{2}\int -\frac{4abB+A\left(b^2+4ac\right)+\left(Bb^2+4Acb+4aBc\right)x}{x\sqrt{cx^2+bx+a}}dx-\frac{\sqrt{a+bx+cx^2}(2aB-x(2Ac+bB)+Ab)}{x}\right)-\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{4abB+A\left(b^2+4ac\right)+\left(Bb^2+4Acb+4aBc\right)x}{x\sqrt{cx^2+bx+a}}dx-\frac{\sqrt{a+bx+cx^2}(2aB-x(2Ac+bB)+Ab)}{x}\right)-\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\left(4aBc+4Abc+b^2B\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx+\left(A\left(4ac+b^2\right)+4abB\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx\right)-\frac{\sqrt{a+bx}\cdots}{\cdots}\right.\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\left(A\left(4ac+b^2\right)+4abB\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+2\left(4aBc+4Abc+b^2B\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\cdots\right.\right.\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\left(A\left(4ac+b^2\right)+4abB\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{\left(4aBc+4Abc+b^2B\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)-\sqrt{a\cdots}\right.\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\left(4aBc+4Abc+b^2B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2\left(A\left(4ac+b^2\right)+4abB\right)\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx}}\right.\right.$$
$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\left(4aBc+4Abc+b^2B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-\frac{\left(A\left(4ac+b^2\right)+4abB\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\right)-\right.$$
$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{3/2}}{2x^2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^3,x]
```

output
```
-1/2*((A - B*x)*(a + b*x + c*x^2)^(3/2))/x^2 + (3*(-(((A*b + 2*a*B - (b*B
+ 2*A*c)*x)*Sqrt[a + b*x + c*x^2])/x) + (-(((4*a*b*B + A*(b^2 + 4*a*c))*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) + ((b^2*B +
 4*A*b*c + 4*a*B*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/Sqrt[c])/2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(5Abx+4Bax+2Aa)}{4x^2}-\frac{3\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)Ac}{2}-\frac{3\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)b^2A}{8\sqrt{a}}-\frac{3\sqrt{a}\,\ln$
default	$A\left(-\frac{(c\,x^2+bx+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{b\left(-\frac{(c\,x^2+bx+a)^{\frac{5}{2}}}{ax}+\frac{3b\left(\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{3}+\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2}\right.}{2}\right.}{}\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(5*A*b*x+4*B*a*x+2*A*a)/x^2-3/2*a^(1/2)*ln((2*a+b
*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*A*c-3/8/a^(1/2)*ln((2*a+b*x+2*a^(1/2)
*(c*x^2+b*x+a)^(1/2))/x)*b^2*A-3/2*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)*b*B+3/8*B*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c
^(1/2)+A*c*(c*x^2+b*x+a)^(1/2)+3/2*A*b*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+1/2*B*c*x*(c*x^2+b*x+a)^(1/2)+5/4*B*b*(c*x^2+b*x+a)^(1/2)
+3/2*a*B*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 921, normalized size of antiderivative = 5.15

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/16*(3*(B*a*b^2 + 4*(B*a^2 + A*a*b)*c)*sqrt(c)*x^2*log(-8*c^2*x^2 - 8*b*
c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 3*(4*A*
a*c^2 + (4*B*a*b + A*b^2)*c)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2
 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(2*B*a*c^
2*x^3 - 2*A*a^2*c - (4*B*a^2 + 5*A*a*b)*c*x + (5*B*a*b*c + 4*A*a*c^2)*x^2)
*sqrt(c*x^2 + b*x + a))/(a*c*x^2), -1/16*(6*(B*a*b^2 + 4*(B*a^2 + A*a*b)*c
)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*
x^2 + b*c*x + a*c)) - 3*(4*A*a*c^2 + (4*B*a*b + A*b^2)*c)*sqrt(a)*x^2*log(
-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a
) + 8*a^2)/x^2) - 4*(2*B*a*c^2*x^3 - 2*A*a^2*c - (4*B*a^2 + 5*A*a*b)*c*x +
 (5*B*a*b*c + 4*A*a*c^2)*x^2)*sqrt(c*x^2 + b*x + a))/(a*c*x^2), 1/16*(6*(4
*A*a*c^2 + (4*B*a*b + A*b^2)*c)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 3*(B*a*b^2 + 4*(B*a^2
+ A*a*b)*c)*sqrt(c)*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*B*a*c^2*x^3 - 2*A*a^2*c - (4*B*
a^2 + 5*A*a*b)*c*x + (5*B*a*b*c + 4*A*a*c^2)*x^2)*sqrt(c*x^2 + b*x + a))/(
a*c*x^2), 1/8*(3*(4*A*a*c^2 + (4*B*a*b + A*b^2)*c)*sqrt(-a)*x^2*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 3*(
B*a*b^2 + 4*(B*a^2 + A*a*b)*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*B*a*c^2*x^3 - 2...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx = \int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/x**3,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(3/2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(152) = 304.

Time = 0.29 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.30

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx = \frac{1}{4}\left(2\,Bcx + \frac{5\,Bbc+4\,Ac^2}{c}\right)\sqrt{cx^2+bx+a}$$
$$+\frac{3\left(4\,Bab+Ab^2+4\,Aac\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}}$$
$$-\frac{3\left(Bb^2+4\,Bac+4\,Abc\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\sqrt{c}}$$
$$+\frac{4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3 Bab+5\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3 Ab^2+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3 Aac+8\,(\sqrt{}}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/4*(2*B*c*x + (5*B*b*c + 4*A*c^2)/c)*sqrt(c*x^2 + b*x + a) + 3/4*(4*B*a*b
 + A*b^2 + 4*A*a*c)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/
sqrt(-a) - 3/8*(B*b^2 + 4*B*a*c + 4*A*b*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*sqrt(c) + b))/sqrt(c) + 1/4*(4*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*B*a*b + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*b^2 + 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*A*a*c + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*B*a^2*sqrt(c) + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*A*a*b*sqrt(
c) - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^2*b - 3*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*A*a*b^2 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*c -
 8*B*a^3*sqrt(c) - 8*A*a^2*b*sqrt(c))/((sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2 - a)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx=\int \frac{\left(A+B\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{x^3}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^3,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.49

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx=\frac{-4\sqrt{c\,x^2+bx+a}\,a^2c-18\sqrt{c\,x^2+bx+a}\,abcx+8\sqrt{c\,x^2+bx+a}\,a\,c$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^3,x)
```

output
```
( - 4*sqrt(a + b*x + c*x**2)*a**2*c - 18*sqrt(a + b*x + c*x**2)*a*b*c*x +
8*sqrt(a + b*x + c*x**2)*a*c**2*x**2 + 10*sqrt(a + b*x + c*x**2)*b**2*c*x*
*2 + 4*sqrt(a + b*x + c*x**2)*b*c**2*x**3 + 12*sqrt(a)*log(2*sqrt(a)*sqrt(
a + b*x + c*x**2) - 2*a - b*x)*a*c**2*x**2 + 15*sqrt(a)*log(2*sqrt(a)*sqrt
(a + b*x + c*x**2) - 2*a - b*x)*b**2*c*x**2 - 12*sqrt(a)*log(x)*a*c**2*x**
2 - 15*sqrt(a)*log(x)*b**2*c*x**2 + 24*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b
*x + c*x**2) - b - 2*c*x)*a*b*c*x**2 + 3*sqrt(c)*log( - 2*sqrt(c)*sqrt(a +
 b*x + c*x**2) - b - 2*c*x)*b**3*x**2)/(8*c*x**2)
```

3.124 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 206

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx=\frac{\left(Ab^2-6abB-8aAc+2(Ab+6aB)cx\right)\sqrt{a+bx+cx^2}}{8ax}$$
$$-\frac{(4aA+3(Ab+2aB)x)\left(a+bx+cx^2\right)^{3/2}}{12ax^3}$$
$$-\frac{\left(6aB(b^2+4ac)-A(b^3-12abc)\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}}$$
$$+\frac{1}{2}\sqrt{c}(3bB+2Ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)$$

output
```
1/8*(A*b^2-6*a*b*B-8*A*a*c+2*(A*b+6*B*a)*c*x)*(c*x^2+b*x+a)^(1/2)/a/x-1/12
*(4*a*A+3*(A*b+2*B*a)*x)*(c*x^2+b*x+a)^(3/2)/a/x^3-1/16*(6*a*B*(4*a*c+b^2)
-A*(-12*a*b*c+b^3))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(
3/2)+1/2*c^(1/2)*(2*A*c+3*B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)
^(1/2))
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx = -\frac{\sqrt{a+x(b+cx)}(3Ab^2x^2+4a^2(2A+3Bx)+2ax(3Bx(5b-4cx)+A(7b+16cx)))}{24ax^3} + \frac{(-6aB(b^2+4ac)+A(b^3-12abc))\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{8a^{3/2}} - \frac{1}{2}\sqrt{c}(3bB+2Ac)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^4,x]
```

output
```
-1/24*(Sqrt[a + x*(b + c*x)]*(3*A*b^2*x^2 + 4*a^2*(2*A + 3*B*x) + 2*a*x*(3
*B*x*(5*b - 4*c*x) + A*(7*b + 16*c*x))))/(a*x^3) + ((-6*a*B*(b^2 + 4*a*c)
+ A*(b^3 - 12*a*b*c))*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[
a]])/(8*a^(3/2)) - (Sqrt[c]*(3*b*B + 2*A*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt
[a + x*(b + c*x)]])/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{2x^2}dx}{4a} - \frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a} - \frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -\frac{(6abB-A(b^2-8ac)+2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{(Ab^2-6aBb-8aAc-2(Ab+6aB)cx)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}(3x(2aB+Ab)+4aA)}{12ax^3}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,\left(32 A a c\,x^{2}+3 x^{2} b^{2} A+30 B a\,x^{2} b+14 a b A x+12 a^{2} B x+8 a^{2} A\right)}{24 x^{3} a}+\frac{-\frac{\left(12 A a b c-A\,b^{3}+24 B\,a^{2} c+6 B a\,b^{2}\right) \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{\cdots}}{x}\right.}{\sqrt{a}}}{\cdots}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+b*x+a)^(1/2)*(32*A*a*c*x^2+3*A*b^2*x^2+30*B*a*b*x^2+14*A*a*b*
x+12*B*a^2*x+8*A*a^2)/x^3/a+1/16/a*(-(12*A*a*b*c-A*b^3+24*B*a^2*c+6*B*a*b^
2)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+16*A*a*c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+32*B*a*b*c^(1/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))+16*B*a*c^2*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(
3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 953, normalized size of antiderivative = 4.63

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/96*(24*(3*B*a^2*b + 2*A*a^2*c)*sqrt(c)*x^3*log(-8*c^2*x^2 - 8*b*c*x - b
^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 3*(6*B*a*b^2 -
 A*b^3 + 12*(2*B*a^2 + A*a*b)*c)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)
*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(24*B
*a^2*c*x^3 - 8*A*a^3 - (30*B*a^2*b + 3*A*a*b^2 + 32*A*a^2*c)*x^2 - 2*(6*B*
a^3 + 7*A*a^2*b)*x)*sqrt(c*x^2 + b*x + a))/(a^2*x^3), -1/96*(48*(3*B*a^2*b
 + 2*A*a^2*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sq
rt(-c)/(c^2*x^2 + b*c*x + a*c)) - 3*(6*B*a*b^2 - A*b^3 + 12*(2*B*a^2 + A*a
*b)*c)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
+ a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(24*B*a^2*c*x^3 - 8*A*a^3 - (30
*B*a^2*b + 3*A*a*b^2 + 32*A*a^2*c)*x^2 - 2*(6*B*a^3 + 7*A*a^2*b)*x)*sqrt(c
*x^2 + b*x + a))/(a^2*x^3), 1/48*(3*(6*B*a*b^2 - A*b^3 + 12*(2*B*a^2 + A*a
*b)*c)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/
(a*c*x^2 + a*b*x + a^2)) + 12*(3*B*a^2*b + 2*A*a^2*c)*sqrt(c)*x^3*log(-8*c
^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a
*c) + 2*(24*B*a^2*c*x^3 - 8*A*a^3 - (30*B*a^2*b + 3*A*a*b^2 + 32*A*a^2*c)*
x^2 - 2*(6*B*a^3 + 7*A*a^2*b)*x)*sqrt(c*x^2 + b*x + a))/(a^2*x^3), 1/48*(3
*(6*B*a*b^2 - A*b^3 + 12*(2*B*a^2 + A*a*b)*c)*sqrt(-a)*x^3*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 24*(3*B*
a^2*b + 2*A*a^2*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x...
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{x^4}\,dx = \int \frac{(A+Bx)\,(a+bx+cx^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/x**4,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(3/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(179) = 358$.

Time = 0.29 (sec) , antiderivative size = 627, normalized size of antiderivative = 3.04

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx = \sqrt{cx^2+bx+a}Bc$$
$$-\frac{(3\,Bbc+2\,Ac^2)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{2\,\sqrt{c}}$$
$$+\frac{(6\,Bab^2-Ab^3+24\,Ba^2c+12\,Aabc)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a}$$
$$+\frac{30\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5Bab^2+3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5Ab^3+24\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5Ba^2c+}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x + a)*B*c - 1/2*(3*B*b*c + 2*A*c^2)*log(abs(-2*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c) + 1/8*(6*B*a*b^2 - A*b^3 +
24*B*a^2*c + 12*A*a*b*c)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(
-a))/(sqrt(-a)*a) + 1/24*(30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a*b^2
 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*b^3 + 24*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^5*B*a^2*c + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a*b
*c + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*B*a^2*b*sqrt(c) + 48*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^4*A*a*b^2*sqrt(c) + 96*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*A*a^2*c^(3/2) - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B
*a^2*b^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a*b^3 - 144*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*B*a^3*b*sqrt(c) - 96*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*A*a^3*c^(3/2) + 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^3
*b^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*b^3 - 24*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*B*a^4*c + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a
^3*b*c + 48*B*a^4*b*sqrt(c) + 64*A*a^4*c^(3/2))/(((sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2 - a)^3*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{x^4}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^4,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^4}\,dx=\frac{-16\sqrt{c\,x^2+bx+a}\,a^3-52\sqrt{c\,x^2+bx+a}\,a^2bx-64\sqrt{c\,x^2+bx+a}\,a}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x + c*x**2)*a**3 - 52*sqrt(a + b*x + c*x**2)*a**2*b*x -
64*sqrt(a + b*x + c*x**2)*a**2*c*x**2 - 66*sqrt(a + b*x + c*x**2)*a*b**2*x
**2 + 48*sqrt(a + b*x + c*x**2)*a*b*c*x**3 + 108*sqrt(a)*log(2*sqrt(a)*sqr
t(a + b*x + c*x**2) - 2*a - b*x)*a*b*c*x**3 + 15*sqrt(a)*log(2*sqrt(a)*sqr
t(a + b*x + c*x**2) - 2*a - b*x)*b**3*x**3 - 108*sqrt(a)*log(x)*a*b*c*x**3
 - 15*sqrt(a)*log(x)*b**3*x**3 + 48*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x
+ c*x**2) - b - 2*c*x)*a**2*c*x**3 + 72*sqrt(c)*log( - 2*sqrt(c)*sqrt(a +
b*x + c*x**2) - b - 2*c*x)*a*b**2*x**3)/(48*a*x**3)
```

3.125 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 217

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx = -\frac{\left(2a(8abB-3A(b^2-4ac))+(8aB(b^2+8ac)-3A(b^3-4abc))\,x\right)\sqrt{a+bx+cx^2}}{64a^2x^2} - \frac{(6aA+(3Ab+8aB)x)\left(a+bx+cx^2\right)^{3/2}}{24ax^4} + \frac{\left(8abB(b^2-12ac)-3A(b^2-4ac)^2\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{5/2}} + Bc^{3/2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)$$

output
```
-1/64*(2*a*(8*a*b*B-3*A*(-4*a*c+b^2))+(8*a*B*(8*a*c+b^2)-3*A*(-4*a*b*c+b^3
))*x)*(c*x^2+b*x+a)^(1/2)/a^2/x^2-1/24*(6*a*A+(3*A*b+8*B*a)*x)*(c*x^2+b*x+
a)^(3/2)/a/x^4+1/128*(8*a*b*B*(-12*a*c+b^2)-3*A*(-4*a*c+b^2)^2)*arctanh(1/
2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)+B*c^(3/2)*arctanh(1/2*(2*
c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx =$$
$$-\frac{\sqrt{a+x(b+cx)}\left(-9Ab^3x^3+16a^3(3A+4Bx)+6abx^2(4bBx+A(b+10cx))+8a^2x(3A(3b+5cx)+2B\right.}{192a^2x^4}$$
$$+\frac{3Ab^4\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{64a^{5/2}}$$
$$+\frac{\left(b^3B+3Ab^2c-12abBc-6aAc^2\right)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{8a^{3/2}}$$
$$-Bc^{3/2}\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^5,x]
```

output
```
-1/192*(Sqrt[a + x*(b + c*x)]*(-9*A*b^3*x^3 + 16*a^3*(3*A + 4*B*x) + 6*a*b
*x^2*(4*b*B*x + A*(b + 10*c*x)) + 8*a^2*x*(3*A*(3*b + 5*c*x) + 2*B*x*(7*b
+ 16*c*x))))/(a^2*x^4) + (3*A*b^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x
)])/Sqrt[a]])/(64*a^(5/2)) + ((b^3*B + 3*A*b^2*c - 12*a*b*B*c - 6*a*A*c^2)
*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(8*a^(3/2)) - B*
c^(3/2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1229, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx$$
$$\Big\downarrow \ 1229$$

$$-\frac{\int -\frac{(8abB+16acxB-3A(b^2-4ac))\sqrt{cx^2+bx+a}}{2x^3}dx}{8a}-\frac{(a+bx+cx^2)^{3/2}(x(8aB+3Ab)+6aA)}{24ax^4}$$

$\big\downarrow$ 27

$$\frac{\int \frac{(8abB+16acxB-3A(b^2-4ac))\sqrt{cx^2+bx+a}}{x^3}dx}{16a}-\frac{(a+bx+cx^2)^{3/2}(x(8aB+3Ab)+6aA)}{24ax^4}$$

$\big\downarrow$ 1229

$$\frac{-\frac{\int \frac{-128a^2Bxc^2-3A(b^2-4ac)^2+8abB(b^2-12ac)}{2x\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}(2a(8abB-3A(b^2-4ac))+x(8aB(8ac+b^2)-3A(b^3-4abc)))}{4ax^2}}{16a}-\frac{(a+bx+cx^2)^{3/2}(x(8aB+3Ab)+6aA)}{24ax^4}$$

$\big\downarrow$ 27

$$\frac{-\frac{\int \frac{-128a^2Bxc^2-3A(b^2-4ac)^2+8abB(b^2-12ac)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(2a(8abB-3A(b^2-4ac))+x(8aB(8ac+b^2)-3A(b^3-4abc)))}{4ax^2}}{16a}-\frac{(a+bx+cx^2)^{3/2}(x(8aB+3Ab)+6aA)}{24ax^4}$$

$\big\downarrow$ 1269

$$\frac{-\frac{(8abB(b^2-12ac)-3A(b^2-4ac)^2)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-128a^2Bc^2\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(2a(8abB-3A(b^2-4ac))+x(8aB(8ac+b^2)}{4ax^2}}{16a}-\frac{(a+bx+cx^2)^{3/2}(x(8aB+3Ab)+6aA)}{24ax^4}$$

$\big\downarrow$ 1092

$$\frac{-\frac{(8abB(b^2-12ac)-3A(b^2-4ac)^2)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-256a^2Bc^2\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{8a}-\frac{\sqrt{a+bx+cx^2}(2a(8abB-3A(b^2-4ac))+x(8}{4ax^2}}{16a}-\frac{(a+bx+cx^2)^{3/2}(x(8aB+3Ab)+6aA)}{24ax^4}$$

$\big\downarrow$ 219

$$-\frac{-\frac{\left(8abB\left(b^2-12ac\right)-3A\left(b^2-4ac\right)^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-128a^2Bc^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(8abB-3A\left(b^2-4ac\right)\right)+x\right.}{4ax^2}}{16a}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(x(8aB+3Ab)+6aA\right)}{24ax^4}$$

$\Big\downarrow$ 1154

$$-\frac{-\frac{-2\left(8abB\left(b^2-12ac\right)-3A\left(b^2-4ac\right)^2\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-128a^2Bc^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(8abB-3\right.\right.}{}}{16a}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(x(8aB+3Ab)+6aA\right)}{24ax^4}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{-128a^2Bc^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-\frac{\left(8abB\left(b^2-12ac\right)-3A\left(b^2-4ac\right)^2\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}}{8a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(8abB-3A\left(b^2\right.\right.\right.}{}}{16a}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(x(8aB+3Ab)+6aA\right)}{24ax^4}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^5,x]
```

output
```
-1/24*((6*a*A + (3*A*b + 8*a*B)*x)*(a + b*x + c*x^2)^(3/2))/(a*x^4) + (-1/
4*((2*a*(8*a*b*B - 3*A*(b^2 - 4*a*c)) + (8*a*B*(b^2 + 8*a*c) - 3*A*(b^3 -
4*a*b*c))*x)*Sqrt[a + b*x + c*x^2])/(a*x^2) - (-(((8*a*b*B*(b^2 - 12*a*c)
- 3*A*(b^2 - 4*a*c)^2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2
])])/Sqrt[a]) - 128*a^2*B*c^(3/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(8*a))/(16*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,\left(60 A a b c\,x^{3}-9 A\,b^{3} x^{3}+256 B\,a^{2} c\,x^{3}+24 B a\,b^{2} x^{3}+120 A\,a^{2} c\,x^{2}+6 A a\,b^{2} x^{2}+112 B\,a^{2} b\,x^{2}+72 A\,a^{2} b x+64 B\,a^{3} x+48 a^{3}\right.}{192 x^{4} a^{2}}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x^2+b*x+a)^(1/2)*(60*A*a*b*c*x^3-9*A*b^3*x^3+256*B*a^2*c*x^3+24*
B*a*b^2*x^3+120*A*a^2*c*x^2+6*A*a*b^2*x^2+112*B*a^2*b*x^2+72*A*a^2*b*x+64*
B*a^3*x+48*A*a^3)/x^4/a^2+1/128/a^2*(-(48*A*a^2*c^2-24*A*a*b^2*c+3*A*b^4+9
6*B*a^2*b*c-8*B*a*b^3)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)+128*B*a^2*c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 1083, normalized size of antiderivative = 4.99

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/768*(384*B*a^3*c^(3/2)*x^4*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^
2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 3*(8*B*a*b^3 - 3*A*b^4 - 48*A*
a^2*c^2 - 24*(4*B*a^2*b - A*a*b^2)*c)*sqrt(a)*x^4*log(-(8*a*b*x + (b^2 + 4
*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*
(48*A*a^4 + (24*B*a^2*b^2 - 9*A*a*b^3 + 4*(64*B*a^3 + 15*A*a^2*b)*c)*x^3 +
 2*(56*B*a^3*b + 3*A*a^2*b^2 + 60*A*a^3*c)*x^2 + 8*(8*B*a^4 + 9*A*a^3*b)*x
)*sqrt(c*x^2 + b*x + a))/(a^3*x^4), -1/768*(768*B*a^3*sqrt(-c)*c*x^4*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c))
+ 3*(8*B*a*b^3 - 3*A*b^4 - 48*A*a^2*c^2 - 24*(4*B*a^2*b - A*a*b^2)*c)*sqrt
(a)*x^4*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(48*A*a^4 + (24*B*a^2*b^2 - 9*A*a*b^3 + 4*
(64*B*a^3 + 15*A*a^2*b)*c)*x^3 + 2*(56*B*a^3*b + 3*A*a^2*b^2 + 60*A*a^3*c)
*x^2 + 8*(8*B*a^4 + 9*A*a^3*b)*x)*sqrt(c*x^2 + b*x + a))/(a^3*x^4), 1/384*
(192*B*a^3*c^(3/2)*x^4*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 3*(8*B*a*b^3 - 3*A*b^4 - 48*A*a^2*c^2
 - 24*(4*B*a^2*b - A*a*b^2)*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(48*A*a^4 + (24*B*a^2
*b^2 - 9*A*a*b^3 + 4*(64*B*a^3 + 15*A*a^2*b)*c)*x^3 + 2*(56*B*a^3*b + 3*A*
a^2*b^2 + 60*A*a^3*c)*x^2 + 8*(8*B*a^4 + 9*A*a^3*b)*x)*sqrt(c*x^2 + b*x +
a))/(a^3*x^4), -1/384*(384*B*a^3*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^2 +...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx=\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/x**5,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(3/2)/x**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(191) = 382$.

Time = 0.35 (sec) , antiderivative size = 1016, normalized size of antiderivative = 4.68

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-B*c^(3/2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b)) - 1
/64*(8*B*a*b^3 - 3*A*b^4 - 96*B*a^2*b*c + 24*A*a*b^2*c - 48*A*a^2*c^2)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/192*
(24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a*b^3 - 9*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^7*A*b^4 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a^2*
b*c + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a*b^2*c + 240*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^7*A*a^2*c^2 + 384*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^6*B*a^2*b^2*sqrt(c) + 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*B*a^3
*c^(3/2) + 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*A*a^2*b*c^(3/2) + 40*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^2*b^3 + 33*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*A*a*b^4 - 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^3
*b*c + 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a^2*b^2*c + 144*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^5*A*a^3*c^2 - 384*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^4*B*a^3*b^2*sqrt(c) + 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*A
*a^2*b^3*sqrt(c) - 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*B*a^4*c^(3/2
) - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a^3*b^3 + 33*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*A*a^2*b^4 + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^3*B*a^4*b*c + 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a^3*b^2*c + 14
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a^4*c^2 + 1280*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*B*a^5*c^(3/2) + 768*(sqrt(c)*x - sqrt(c*x^2 + b*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{x^5}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^5,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx = \frac{-96\sqrt{c\,x^2+bx+a}\,a^4-272\sqrt{c\,x^2+bx+a}\,a^3bx-240\sqrt{c\,x^2+bx+a}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^5,x)
```

output
```
( - 96*sqrt(a + b*x + c*x**2)*a**4 - 272*sqrt(a + b*x + c*x**2)*a**3*b*x -
 240*sqrt(a + b*x + c*x**2)*a**3*c*x**2 - 236*sqrt(a + b*x + c*x**2)*a**2*
b**2*x**2 - 632*sqrt(a + b*x + c*x**2)*a**2*b*c*x**3 - 30*sqrt(a + b*x + c
*x**2)*a*b**3*x**3 + 144*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*
a - b*x)*a**2*c**2*x**4 + 216*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2)
 - 2*a - b*x)*a*b**2*c*x**4 - 15*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x*
*2) - 2*a - b*x)*b**4*x**4 - 144*sqrt(a)*log(x)*a**2*c**2*x**4 - 216*sqrt(
a)*log(x)*a*b**2*c*x**4 + 15*sqrt(a)*log(x)*b**4*x**4 + 384*sqrt(c)*log( -
 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*b*c*x**4)/(384*a**2*x*
*4)
```

3.126 $\int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{x^6} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 170

$$\int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{x^6} \, dx = -\frac{3(Ab-2aB)(b^2-4ac)(2a+bx)\sqrt{a+bx+cx^2}}{128a^3x^2} + \frac{(Ab-2aB)(2a+bx)(a+bx+cx^2)^{3/2}}{16a^2x^4} - \frac{A(a+bx+cx^2)^{5/2}}{5ax^5} + \frac{3(Ab-2aB)(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{256a^{7/2}}$$

output
```
-3/128*(A*b-2*B*a)*(-4*a*c+b^2)*(b*x+2*a)*(c*x^2+b*x+a)^(1/2)/a^3/x^2+1/16
*(A*b-2*B*a)*(b*x+2*a)*(c*x^2+b*x+a)^(3/2)/a^2/x^4-1/5*A*(c*x^2+b*x+a)^(5/
2)/a/x^5+3/256*(A*b-2*B*a)*(-4*a*c+b^2)^2*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c
*x^2+b*x+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx = \frac{-\sqrt{a}\sqrt{a+x(b+cx)}(15Ab^4x^4+32a^4(4A+5Bx)-10ab^2x^3(3bBx+$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^6,x]
```

output
```
(-(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(15*A*b^4*x^4 + 32*a^4*(4*A + 5*B*x) - 10
*a*b^2*x^3*(3*b*B*x + A*(b + 10*c*x)) + 16*a^3*x*(5*B*x*(3*b + 5*c*x) + A*
(11*b + 16*c*x)) + 4*a^2*x^2*(5*b*B*x*(b + 10*c*x) + 2*A*(b^2 + 7*b*c*x +
16*c^2*x^2)))) - 15*(A*b^5 - 32*a^3*B*c^2)*x^5*ArcTanh[(Sqrt[c]*x - Sqrt[a
 + x*(b + c*x)])/Sqrt[a]] + 30*a*b*(-(b^3*B) - 4*A*b^2*c + 8*a*b*B*c + 8*a
*A*c^2)*x^5*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(640*
a^(7/2)*x^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{ 1228}$$

$$-\frac{(Ab-2aB)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{x^5}dx}{2a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 1152}$$

$$-\frac{(Ab-2aB)\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^2+bx+a}}{x^3}dx}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{2a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{1152}$$

$$-\frac{(Ab-2aB)\left(-\frac{3(b^2-4ac)\left(-\frac{(b^2-4ac)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)(a+bx+cx^2)^{3/2}}{8ax^4}\right)}{2a}-\frac{A(a+bx+cx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{(Ab-2aB)\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)(a+bx+cx^2)^{3/2}}{8ax^4}\right)}{2a}-\frac{A(a+bx+cx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(Ab-2aB)\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)(a+bx+cx^2)^{3/2}}{8ax^4}\right)}{2a}-\frac{A(a+bx+cx^2)^{5/2}}{5ax^5}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^6,x]
```

output
```
-1/5*(A*(a + b*x + c*x^2)^(5/2))/(a*x^5) - ((A*b - 2*a*B)*(-1/8*((2*a + b*
x)*(a + b*x + c*x^2)^(3/2))/(a*x^4) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x)*
Sqrt[a + b*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqr
t[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2))))/(16*a)))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(128A\,a^2c^2x^4-100Aa\,b^2c\,x^4+15A\,b^4x^4+200B\,a^2bc\,x^4-30Ba\,b^3x^4+56A\,a^2bc\,x^3-10Aa\,b^3x^3+400B\,a^3c\,x^3+20B\,a}{640x^5a^3}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/640*(c*x^2+b*x+a)^(1/2)*(128*A*a^2*c^2*x^4-100*A*a*b^2*c*x^4+15*A*b^4*x
^4+200*B*a^2*b*c*x^4-30*B*a*b^3*x^4+56*A*a^2*b*c*x^3-10*A*a*b^3*x^3+400*B*
a^3*c*x^3+20*B*a^2*b^2*x^3+256*A*a^3*c*x^2+8*A*a^2*b^2*x^2+240*B*a^3*b*x^2
+176*A*a^3*b*x+160*B*a^4*x+128*A*a^4)/x^5/a^3+3/256*(16*A*a^2*b*c^2-8*A*a*
b^3*c+A*b^5-32*B*a^3*c^2+16*B*a^2*b^2*c-2*B*a*b^4)/a^(7/2)*ln((2*a+b*x+2*a
^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.26

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx = \left[-\frac{15\left(2\,Bab^4-Ab^5+16\left(2\,Ba^3-Aa^2b\right)c^2-8\left(2\,Ba^2b^2-Aab^3\right)c\right)\sqrt{}}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[-1/2560*(15*(2*B*a*b^4 - A*b^5 + 16*(2*B*a^3 - A*a^2*b)*c^2 - 8*(2*B*a^2*
b^2 - A*a*b^3)*c)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c
*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(128*A*a^5 - (30*B*a
^2*b^3 - 15*A*a*b^4 - 128*A*a^3*c^2 - 100*(2*B*a^3*b - A*a^2*b^2)*c)*x^4 +
 2*(10*B*a^3*b^2 - 5*A*a^2*b^3 + 4*(50*B*a^4 + 7*A*a^3*b)*c)*x^3 + 8*(30*B
*a^4*b + A*a^3*b^2 + 32*A*a^4*c)*x^2 + 16*(10*B*a^5 + 11*A*a^4*b)*x)*sqrt(
c*x^2 + b*x + a))/(a^4*x^5), 1/1280*(15*(2*B*a*b^4 - A*b^5 + 16*(2*B*a^3 -
 A*a^2*b)*c^2 - 8*(2*B*a^2*b^2 - A*a*b^3)*c)*sqrt(-a)*x^5*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(128*A*
a^5 - (30*B*a^2*b^3 - 15*A*a*b^4 - 128*A*a^3*c^2 - 100*(2*B*a^3*b - A*a^2*
b^2)*c)*x^4 + 2*(10*B*a^3*b^2 - 5*A*a^2*b^3 + 4*(50*B*a^4 + 7*A*a^3*b)*c)*
x^3 + 8*(30*B*a^4*b + A*a^3*b^2 + 32*A*a^4*c)*x^2 + 16*(10*B*a^5 + 11*A*a^
4*b)*x)*sqrt(c*x^2 + b*x + a))/(a^4*x^5)]
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{x^6}\,dx = \int \frac{(A+Bx)\,(a+bx+cx^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/x**6,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(3/2)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1357 vs. $2(151) = 302$.

Time = 0.25 (sec) , antiderivative size = 1357, normalized size of antiderivative = 7.98

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^6,x, algorithm="giac")
```

output
```
3/128*(2*B*a*b^4 - A*b^5 - 16*B*a^2*b^2*c + 8*A*a*b^3*c + 32*B*a^3*c^2 - 1
6*A*a^2*b*c^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt
(-a)*a^3) - 1/640*(30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a*b^4 - 15*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*b^5 - 240*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^9*B*a^2*b^2*c + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a*b
^3*c - 800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a^3*c^2 - 240*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^9*A*a^2*b*c^2 - 2560*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^8*B*a^3*b*c^(3/2) - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*A
*a^3*c^(5/2) - 140*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a^2*b^4 + 70*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a*b^5 - 1440*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^7*B*a^3*b^2*c - 560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a
^2*b^3*c + 320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a^4*c^2 - 2720*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a^3*b*c^2 - 1280*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^6*B*a^3*b^3*sqrt(c) + 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^6*B*a^4*b*c^(3/2) - 5120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*A*a^3*b
^2*c^(3/2) - 128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a^2*b^5 - 2560*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a^3*b^3*c - 3840*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*A*a^4*b*c^2 + 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*B*a^4*b^3*sqrt(c) - 1280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*A*a^3*b^4
*sqrt(c) - 2560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*B*a^5*b*c^(3/2) -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{x^6}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^6,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.02

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx = \frac{-256\sqrt{c\,x^2+bx+a}\,a^5-672\sqrt{c\,x^2+bx+a}\,a^4bx-512\sqrt{c\,x^2+bx+}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^6,x)
```

output
```
( - 256*sqrt(a + b*x + c*x**2)*a**5 - 672*sqrt(a + b*x + c*x**2)*a**4*b*x
- 512*sqrt(a + b*x + c*x**2)*a**4*c*x**2 - 496*sqrt(a + b*x + c*x**2)*a**3
*b**2*x**2 - 912*sqrt(a + b*x + c*x**2)*a**3*b*c*x**3 - 256*sqrt(a + b*x +
 c*x**2)*a**3*c**2*x**4 - 20*sqrt(a + b*x + c*x**2)*a**2*b**3*x**3 - 200*s
qrt(a + b*x + c*x**2)*a**2*b**2*c*x**4 + 30*sqrt(a + b*x + c*x**2)*a*b**4*
x**4 + 240*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*
b*c**2*x**5 - 120*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x
)*a*b**3*c*x**5 + 15*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a -
b*x)*b**5*x**5 - 240*sqrt(a)*log(x)*a**2*b*c**2*x**5 + 120*sqrt(a)*log(x)*
a*b**3*c*x**5 - 15*sqrt(a)*log(x)*b**5*x**5)/(1280*a**3*x**5)
```

3.127 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 230

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx = \frac{(b^2-4ac)\left(7Ab^2-12abB-4aAc\right)(2a+bx)\sqrt{a+bx+cx^2}}{512a^4x^2} - \frac{\left(7Ab^2-12abB-4aAc\right)(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{192a^3x^4} - \frac{A\left(a+bx+cx^2\right)^{5/2}}{6ax^6} + \frac{(7Ab-12aB)\left(a+bx+cx^2\right)^{5/2}}{60a^2x^5} - \frac{\left(b^2-4ac\right)^2\left(7Ab^2-12abB-4aAc\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{1024a^{9/2}}$$

output
```
1/512*(-4*a*c+b^2)*(-4*A*a*c+7*A*b^2-12*B*a*b)*(b*x+2*a)*(c*x^2+b*x+a)^(1/
2)/a^4/x^2-1/192*(-4*A*a*c+7*A*b^2-12*B*a*b)*(b*x+2*a)*(c*x^2+b*x+a)^(3/2)
/a^3/x^4-1/6*A*(c*x^2+b*x+a)^(5/2)/a/x^6+1/60*(7*A*b-12*B*a)*(c*x^2+b*x+a)
^(5/2)/a^2/x^5-1/1024*(-4*a*c+b^2)^2*(-4*A*a*c+7*A*b^2-12*B*a*b)*arctanh(1
/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 4.10 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx = \frac{-\sqrt{a}\sqrt{a+x(b+cx)}(-105Ab^5x^5+256a^5(5A+6Bx)+10ab^3x^4(7Ab$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^7,x]
```

output
```
(-(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-105*A*b^5*x^5 + 256*a^5*(5*A + 6*B*x) +
 10*a*b^3*x^4*(7*A*b + 18*b*B*x + 76*A*c*x) + 64*a^4*x*(26*A*b + 33*b*B*x
+ 35*A*c*x + 48*B*c*x^2) + 48*a^3*x^2*(A*(b^2 + 6*b*c*x + 10*c^2*x^2) + 2*
B*x*(b^2 + 7*b*c*x + 16*c^2*x^2)) - 8*a^2*b*x^3*(15*b*B*x*(b + 10*c*x) + A
*(7*b^2 + 54*b*c*x + 162*c^2*x^2)))) + 105*A*b^6*x^6*ArcTanh[(Sqrt[c]*x -
Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 60*a*(3*b^5*B + 15*A*b^4*c - 24*a*b^3*B*
c - 36*a*A*b^2*c^2 + 48*a^2*b*B*c^2 + 16*a^2*A*c^3)*x^6*ArcTanh[(-(Sqrt[c]
*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(7680*a^(9/2)*x^6)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx$$

$$\Big\downarrow \; 1237$$

$$-\frac{\int \frac{(7Ab-12aB+2Acx)\left(cx^2+bx+a\right)^{3/2}}{2x^6}dx}{6a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{(7Ab-12aB+2Acx)\left(cx^2+bx+a\right)^{3/2}}{x^6}dx}{12a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 1228$$

$$-\frac{-\frac{\left(-4aAc-12abB+7Ab^2\right)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{x^5}dx}{2a}-\frac{(7Ab-12aB)\left(a+bx+cx^2\right)^{5/2}}{5ax^5}}{12a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 1152$$

$$-\frac{-\frac{\left(-4aAc-12abB+7Ab^2\right)\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^2+bx+a}}{x^3}dx}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{2a}-\frac{(7Ab-12aB)\left(a+bx+cx^2\right)^{5/2}}{5ax^5}}{12a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 1152$$

$$-\frac{-\frac{\left(-4aAc-12abB+7Ab^2\right)\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{2a}-\frac{(7Ab-12aB)\left(a+bx+cx^2\right)^{5/2}}{5ax^5}}{12a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 1154$$

$$-\frac{-\frac{\left(-4aAc-12abB+7Ab^2\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{2a}-\frac{(7Ab-12aB)\left(a+bx+cx^2\right)^{5/2}}{5ax^5}}{12a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 219$$

$$-\frac{-\frac{\left(-4aAc-12abB+7Ab^2\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{2a}-\frac{(7Ab-}{}}{12a}$$

$$\frac{A\left(a+bx+cx^2\right)^{5/2}}{6ax^6}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^7,x]
```

output
```
-1/6*(A*(a + b*x + c*x^2)^(5/2))/(a*x^6) - (-1/5*((7*A*b - 12*a*B)*(a + b*
x + c*x^2)^(5/2))/(a*x^5) - ((7*A*b^2 - 12*a*b*B - 4*a*A*c)*(-1/8*((2*a +
b*x)*(a + b*x + c*x^2)^(3/2))/(a*x^4) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x
)*Sqrt[a + b*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*S
qrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2))))/(16*a)))/(2*a))/(12*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x +a}\,\left(-1296 A\,a^{2} b\,c^{2} x^{5}+760 A a\,b^{3} c\,x^{5}-105 A\,b^{5} x^{5}+1536 B\,a^{3} c^{2} x^{5}-1200 B\,a^{2} b^{2} c\,x^{5}+180 B a\,b^{4} x^{5}+480 A\,a^{3} c^{2} x^{4}-432 A\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(c*x^2+b*x+a)^(1/2)*(-1296*A*a^2*b*c^2*x^5+760*A*a*b^3*c*x^5-105*A
*b^5*x^5+1536*B*a^3*c^2*x^5-1200*B*a^2*b^2*c*x^5+180*B*a*b^4*x^5+480*A*a^3
*c^2*x^4-432*A*a^2*b^2*c*x^4+70*A*a*b^4*x^4+672*B*a^3*b*c*x^4-120*B*a^2*b^
3*x^4+288*A*a^3*b*c*x^3-56*A*a^2*b^3*x^3+3072*B*a^4*c*x^3+96*B*a^3*b^2*x^3
+2240*A*a^4*c*x^2+48*A*a^3*b^2*x^2+2112*B*a^4*b*x^2+1664*A*a^4*b*x+1536*B*
a^5*x+1280*A*a^5)/x^6/a^4+1/1024*(64*A*a^3*c^3-144*A*a^2*b^2*c^2+60*A*a*b^
4*c-7*A*b^6+192*B*a^3*b*c^2-96*B*a^2*b^3*c+12*B*a*b^5)/a^(9/2)*ln((2*a+b*x
+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 709, normalized size of antiderivative = 3.08

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/30720*(15*(12*B*a*b^5 - 7*A*b^6 + 64*A*a^3*c^3 + 48*(4*B*a^3*b - 3*A*a^
2*b^2)*c^2 - 12*(8*B*a^2*b^3 - 5*A*a*b^4)*c)*sqrt(a)*x^6*log(-(8*a*b*x + (
b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^
2) - 4*(1280*A*a^6 + (180*B*a^2*b^4 - 105*A*a*b^5 + 48*(32*B*a^4 - 27*A*a^
3*b)*c^2 - 40*(30*B*a^3*b^2 - 19*A*a^2*b^3)*c)*x^5 - 2*(60*B*a^3*b^3 - 35*
A*a^2*b^4 - 240*A*a^4*c^2 - 24*(14*B*a^4*b - 9*A*a^3*b^2)*c)*x^4 + 8*(12*B
*a^4*b^2 - 7*A*a^3*b^3 + 12*(32*B*a^5 + 3*A*a^4*b)*c)*x^3 + 16*(132*B*a^5*
b + 3*A*a^4*b^2 + 140*A*a^5*c)*x^2 + 128*(12*B*a^6 + 13*A*a^5*b)*x)*sqrt(c
*x^2 + b*x + a))/(a^5*x^6), -1/15360*(15*(12*B*a*b^5 - 7*A*b^6 + 64*A*a^3*
c^3 + 48*(4*B*a^3*b - 3*A*a^2*b^2)*c^2 - 12*(8*B*a^2*b^3 - 5*A*a*b^4)*c)*s
qrt(-a)*x^6*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2
 + a*b*x + a^2)) + 2*(1280*A*a^6 + (180*B*a^2*b^4 - 105*A*a*b^5 + 48*(32*B
*a^4 - 27*A*a^3*b)*c^2 - 40*(30*B*a^3*b^2 - 19*A*a^2*b^3)*c)*x^5 - 2*(60*B
*a^3*b^3 - 35*A*a^2*b^4 - 240*A*a^4*c^2 - 24*(14*B*a^4*b - 9*A*a^3*b^2)*c)
*x^4 + 8*(12*B*a^4*b^2 - 7*A*a^3*b^3 + 12*(32*B*a^5 + 3*A*a^4*b)*c)*x^3 +
16*(132*B*a^5*b + 3*A*a^4*b^2 + 140*A*a^5*c)*x^2 + 128*(12*B*a^6 + 13*A*a^
5*b)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^6)]
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{x^7}\,dx = \int \frac{(A+Bx)\,(a+bx+cx^2)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/x**7,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(3/2)/x**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2059 vs. 2(204) = 408.

Time = 0.28 (sec) , antiderivative size = 2059, normalized size of antiderivative = 8.95

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/512*(12*B*a*b^5 - 7*A*b^6 - 96*B*a^2*b^3*c + 60*A*a*b^4*c + 192*B*a^3*b
*c^2 - 144*A*a^2*b^2*c^2 + 64*A*a^3*c^3)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) + 1/7680*(180*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^11*B*a*b^5 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*b^6
- 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*B*a^2*b^3*c + 900*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^11*A*a*b^4*c + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^11*B*a^3*b*c^2 - 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*a^2
*b^2*c^2 + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*a^3*c^3 + 15360*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^10*B*a^4*c^(5/2) - 1020*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^9*B*a^2*b^5 + 595*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
9*A*a*b^6 + 8160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a^3*b^3*c - 5100*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a^2*b^4*c + 29760*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^9*B*a^4*b*c^2 + 12240*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^9*A*a^3*b^2*c^2 + 15040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a^4*c^
3 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*B*a^4*b^2*c^(3/2) - 15360*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*B*a^5*c^(5/2) + 76800*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^8*A*a^4*b*c^(5/2) + 2376*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*B*a^3*b^5 - 1386*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a^2*b^
6 + 24000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a^4*b^3*c + 11880*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^7*A*a^3*b^4*c + 13440*(sqrt(c)*x - sqrt(c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^7}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{x^7}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^7,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.03

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{3/2}}{x^7}\,dx = \frac{-512\sqrt{c\,x^2+bx+a}\,a^6-1280\sqrt{c\,x^2+bx+a}\,a^5bx-896\sqrt{c\,x^2+bx+}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^7,x)
```

output
```
( - 512*sqrt(a + b*x + c*x**2)*a**6 - 1280*sqrt(a + b*x + c*x**2)*a**5*b*x
 - 896*sqrt(a + b*x + c*x**2)*a**5*c*x**2 - 864*sqrt(a + b*x + c*x**2)*a**
4*b**2*x**2 - 1344*sqrt(a + b*x + c*x**2)*a**4*b*c*x**3 - 192*sqrt(a + b*x
 + c*x**2)*a**4*c**2*x**4 - 16*sqrt(a + b*x + c*x**2)*a**3*b**3*x**3 - 96*
sqrt(a + b*x + c*x**2)*a**3*b**2*c*x**4 - 96*sqrt(a + b*x + c*x**2)*a**3*b
*c**2*x**5 + 20*sqrt(a + b*x + c*x**2)*a**2*b**4*x**4 + 176*sqrt(a + b*x +
 c*x**2)*a**2*b**3*c*x**5 - 30*sqrt(a + b*x + c*x**2)*a*b**5*x**5 + 192*sq
rt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*c**3*x**6
+ 144*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b*
*2*c**2*x**6 - 108*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a -
 b*x)*a*b**4*c*x**6 + 15*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) -
 2*a - b*x)*b**6*x**6 - 192*sqrt(a)*log(x)*a**3*c**3*x**6 - 144*sqrt(a)*lo
g(x)*a**2*b**2*c**2*x**6 + 108*sqrt(a)*log(x)*a*b**4*c*x**6 - 15*sqrt(a)*l
og(x)*b**6*x**6)/(3072*a**4*x**6)
```

3.128 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 303

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx = &-\frac{(b^2-4ac)\left(9Ab^3-14ab^2B-12aAbc+8a^2Bc\right)(2a+bx)\sqrt{a+bx+cx^2}}{1024a^5x^2}\\&+\frac{\left(9Ab^3-14ab^2B-12aAbc+8a^2Bc\right)(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{384a^4x^4}\\&-\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}+\frac{(9Ab-14aB)\left(a+bx+cx^2\right)^{5/2}}{84a^2x^6}\\&-\frac{\left(63Ab^2-98abB-48aAc\right)\left(a+bx+cx^2\right)^{5/2}}{840a^3x^5}\\&-\frac{(b^2-4ac)^2\left(2aB(7b^2-4ac)-A(9b^3-12abc)\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2048a^{11/2}}\end{aligned}$$

output

```
-1/1024*(-4*a*c+b^2)*(-12*A*a*b*c+9*A*b^3+8*B*a^2*c-14*B*a*b^2)*(b*x+2*a)*
(c*x^2+b*x+a)^(1/2)/a^5/x^2+1/384*(-12*A*a*b*c+9*A*b^3+8*B*a^2*c-14*B*a*b^
2)*(b*x+2*a)*(c*x^2+b*x+a)^(3/2)/a^4/x^4-1/7*A*(c*x^2+b*x+a)^(5/2)/a/x^7+1
/84*(9*A*b-14*B*a)*(c*x^2+b*x+a)^(5/2)/a^2/x^6-1/840*(-48*A*a*c+63*A*b^2-9
8*B*a*b)*(c*x^2+b*x+a)^(5/2)/a^3/x^5-1/2048*(-4*a*c+b^2)^2*(2*a*B*(-4*a*c+
7*b^2)-A*(-12*a*b*c+9*b^3))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1
/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx = \frac{-\sqrt{a}\sqrt{a+x(b+cx)}\left(945Ab^6x^6+2560a^6(6A+7Bx)-210ab^4x^5(7bB\right.}{}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^8,x]
```

output
```
(-(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(945*A*b^6*x^6 + 2560*a^6*(6*A + 7*B*x) -
 210*a*b^4*x^5*(7*b*B*x + 3*A*(b + 12*c*x)) + 128*a^5*x*(6*A*(25*b + 32*c*
x) + 7*B*x*(26*b + 35*c*x)) + 96*a^4*x^2*(7*B*x*(b^2 + 6*b*c*x + 10*c^2*x^
2) + A*(4*b^2 + 22*b*c*x + 32*c^2*x^2)) + 28*a^2*b^2*x^4*(5*b*B*x*(7*b + 7
6*c*x) + 6*A*(3*b^2 + 26*b*c*x + 98*c^2*x^2)) - 16*a^3*x^3*(7*b*B*x*(7*b^2
 + 54*b*c*x + 162*c^2*x^2) + 3*A*(9*b^3 + 62*b^2*c*x + 146*b*c^2*x^2 + 128
*c^3*x^3)))) - 105*(9*A*b^7 + 128*a^4*B*c^3)*x^7*ArcTanh[(Sqrt[c]*x - Sqrt
[a + x*(b + c*x)])/Sqrt[a]] - 210*a*b*(7*b^5*B + 42*A*b^4*c - 60*a*b^3*B*c
 - 120*a*A*b^2*c^2 + 144*a^2*b*B*c^2 + 96*a^2*A*c^3)*x^7*ArcTanh[(-(Sqrt[c
]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(107520*a^(11/2)*x^7)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1237, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int \frac{(9Ab-14aB+4Acx)\left(cx^2+bx+a\right)^{3/2}}{2x^7}dx}{7a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(9Ab-14aB+4Acx)\left(cx^2+bx+a\right)^{3/2}}{x^7}dx}{14a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}$$

$\Big\downarrow$ 1237

$$-\frac{-\frac{\int \frac{\left(63Ab^2-98aBb-48aAc+2(9Ab-14aB)cx\right)\left(cx^2+bx+a\right)^{3/2}}{2x^6}dx}{6a}-\frac{(9Ab-14aB)\left(a+bx+cx^2\right)^{5/2}}{6ax^6}}{14a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{\left(63Ab^2-98aBb-48aAc+2(9Ab-14aB)cx\right)\left(cx^2+bx+a\right)^{3/2}}{x^6}dx}{12a}-\frac{(9Ab-14aB)\left(a+bx+cx^2\right)^{5/2}}{6ax^6}}{14a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}$$

$\Big\downarrow$ 1228

$$-\frac{-\frac{-\frac{7\left(8a^2Bc-12aAbc-14ab^2B+9Ab^3\right)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{x^5}dx}{2a}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(-48aAc-98abB+63Ab^2\right)}{5ax^5}}{12a}-\frac{(9Ab-14aB)\left(a+bx+cx^2\right)^{5/2}}{6ax^6}}{14a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}$$

$\Big\downarrow$ 1152

$$-\frac{-\frac{-\frac{7\left(8a^2Bc-12aAbc-14ab^2B+9Ab^3\right)\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^2+bx+a}}{x^3}dx}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{2a}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(-48aAc-98abB+63Ab^2\right)}{5ax^5}}{12a}-(9\cdots}{14a}-\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}$$

$\Big\downarrow$ 1152

$$-\frac{-\frac{-\frac{7\left(8a^2Bc-12aAbc-14ab^2B+9Ab^3\right)\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{2a}-\frac{\left(a+bx+cx\cdots\right.}{\cdots}}{12a}\cdots}{14a}$$

$$\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{-\frac{7\left(8a^2Bc-12aAbc-14ab^2B+9Ab^3\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{2a}\cdots}{12a}\cdots}{14a}$$

$$\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{-\frac{7\left(8a^2Bc-12aAbc-14ab^2B+9Ab^3\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{2a}\cdots}{12a}\cdots}{14a}$$

$$\frac{A\left(a+bx+cx^2\right)^{5/2}}{7ax^7}$$

input

```
Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^8,x]
```

output
```
-1/7*(A*(a + b*x + c*x^2)^(5/2))/(a*x^7) - (-1/6*((9*A*b - 14*a*B)*(a + b*
x + c*x^2)^(5/2))/(a*x^6) - (-1/5*((63*A*b^2 - 98*a*b*B - 48*a*A*c)*(a + b
*x + c*x^2)^(5/2))/(a*x^5) - (7*(9*A*b^3 - 14*a*b^2*B - 12*a*A*b*c + 8*a^2
*B*c)*(-1/8*((2*a + b*x)*(a + b*x + c*x^2)^(3/2))/(a*x^4) - (3*(b^2 - 4*a*
c)*(-1/4*((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*ArcT
anh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2))))/(16*a)))
/(2*a))/(12*a))/(14*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{\sqrt{c x^2+bx+a}\,(-6144 A a^3 c^3 x^6+16464 A a^2 b^2 c^2 x^6-7560 A a b^4 c x^6+945 A b^6 x^6-18144 B a^3 b c^2 x^6+10640 B a^2 b^3 c x^6-1470 B a b^5 x}{}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/107520*(c*x^2+b*x+a)^(1/2)*(-6144*A*a^3*c^3*x^6+16464*A*a^2*b^2*c^2*x^6
-7560*A*a*b^4*c*x^6+945*A*b^6*x^6-18144*B*a^3*b*c^2*x^6+10640*B*a^2*b^3*c*
x^6-1470*B*a*b^5*x^6-7008*A*a^3*b*c^2*x^5+4368*A*a^2*b^3*c*x^5-630*A*a*b^5
*x^5+6720*B*a^4*c^2*x^5-6048*B*a^3*b^2*c*x^5+980*B*a^2*b^4*x^5+3072*A*a^4*
c^2*x^4-2976*A*a^3*b^2*c*x^4+504*A*a^2*b^4*x^4+4032*B*a^4*b*c*x^4-784*B*a^
3*b^3*x^4+2112*A*a^4*b*c*x^3-432*A*a^3*b^3*x^3+31360*B*a^5*c*x^3+672*B*a^4
*b^2*x^3+24576*A*a^5*c*x^2+384*A*a^4*b^2*x^2+23296*B*a^5*b*x^2+19200*A*a^5
*b*x+17920*B*a^6*x+15360*A*a^6)/x^7/a^5-1/2048*(192*A*a^3*b*c^3-240*A*a^2*
b^3*c^2+84*A*a*b^5*c-9*A*b^7-128*B*a^4*c^3+288*B*a^3*b^2*c^2-120*B*a^2*b^4
*c+14*B*a*b^6)/a^(11/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
[1/430080*(105*(14*B*a*b^6 - 9*A*b^7 - 64*(2*B*a^4 - 3*A*a^3*b)*c^3 + 48*(
6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2 - 12*(10*B*a^2*b^4 - 7*A*a*b^5)*c)*sqrt(a)*
x^7*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a
)*sqrt(a) + 8*a^2)/x^2) - 4*(15360*A*a^7 - (1470*B*a^2*b^5 - 945*A*a*b^6 +
 6144*A*a^4*c^3 + 336*(54*B*a^4*b - 49*A*a^3*b^2)*c^2 - 280*(38*B*a^3*b^3
- 27*A*a^2*b^4)*c)*x^6 + 2*(490*B*a^3*b^4 - 315*A*a^2*b^5 + 48*(70*B*a^5 -
 73*A*a^4*b)*c^2 - 168*(18*B*a^4*b^2 - 13*A*a^3*b^3)*c)*x^5 - 8*(98*B*a^4*
b^3 - 63*A*a^3*b^4 - 384*A*a^5*c^2 - 12*(42*B*a^5*b - 31*A*a^4*b^2)*c)*x^4
 + 16*(42*B*a^5*b^2 - 27*A*a^4*b^3 + 4*(490*B*a^6 + 33*A*a^5*b)*c)*x^3 + 1
28*(182*B*a^6*b + 3*A*a^5*b^2 + 192*A*a^6*c)*x^2 + 1280*(14*B*a^7 + 15*A*a
^6*b)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^7), 1/215040*(105*(14*B*a*b^6 - 9*A
*b^7 - 64*(2*B*a^4 - 3*A*a^3*b)*c^3 + 48*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2 -
 12*(10*B*a^2*b^4 - 7*A*a*b^5)*c)*sqrt(-a)*x^7*arctan(1/2*sqrt(c*x^2 + b*x
 + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(15360*A*a^7 - (14
70*B*a^2*b^5 - 945*A*a*b^6 + 6144*A*a^4*c^3 + 336*(54*B*a^4*b - 49*A*a^3*b
^2)*c^2 - 280*(38*B*a^3*b^3 - 27*A*a^2*b^4)*c)*x^6 + 2*(490*B*a^3*b^4 - 31
5*A*a^2*b^5 + 48*(70*B*a^5 - 73*A*a^4*b)*c^2 - 168*(18*B*a^4*b^2 - 13*A*a^
3*b^3)*c)*x^5 - 8*(98*B*a^4*b^3 - 63*A*a^3*b^4 - 384*A*a^5*c^2 - 12*(42*B*
a^5*b - 31*A*a^4*b^2)*c)*x^4 + 16*(42*B*a^5*b^2 - 27*A*a^4*b^3 + 4*(490*B*
a^6 + 33*A*a^5*b)*c)*x^3 + 128*(182*B*a^6*b + 3*A*a^5*b^2 + 192*A*a^6*c...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx = \int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/x**8,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(3/2)/x**8, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2713 vs. $2(273) = 546$.

Time = 0.28 (sec) , antiderivative size = 2713, normalized size of antiderivative = 8.95

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/1024*(14*B*a*b^6 - 9*A*b^7 - 120*B*a^2*b^4*c + 84*A*a*b^5*c + 288*B*a^3*
b^2*c^2 - 240*A*a^2*b^3*c^2 - 128*B*a^4*c^3 + 192*A*a^3*b*c^3)*arctan(-(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^5) - 1/107520*(1470
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*B*a*b^6 - 945*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^13*A*b^7 - 12600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*B*
a^2*b^4*c + 8820*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*A*a*b^5*c + 30240*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*B*a^3*b^2*c^2 - 25200*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^13*A*a^2*b^3*c^2 - 13440*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^13*B*a^4*c^3 + 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*A*a^
3*b*c^3 - 9800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*B*a^2*b^6 + 6300*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*a*b^7 + 84000*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^11*B*a^3*b^4*c - 58800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^1
1*A*a^2*b^5*c - 201600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*B*a^4*b^2*c^
2 + 168000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*a^3*b^3*c^2 - 197120*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*B*a^5*c^3 - 134400*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^11*A*a^4*b*c^3 - 1075200*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^10*B*a^5*b*c^(5/2) - 430080*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*A
*a^5*c^(7/2) + 27734*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a^3*b^6 - 178
29*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a^2*b^7 - 237720*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^9*B*a^4*b^4*c + 166404*(sqrt(c)*x - sqrt(c*x^2 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{x^8}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^8,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.85

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{x^8}\,dx = \frac{-30720\sqrt{c\,x^2+bx+a}\,a^7-74240\sqrt{c\,x^2+bx+a}\,a^6bx-49152\sqrt{c\,x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/x^8,x)
```

output
```
( - 30720*sqrt(a + b*x + c*x**2)*a**7 - 74240*sqrt(a + b*x + c*x**2)*a**6*
b*x - 49152*sqrt(a + b*x + c*x**2)*a**6*c*x**2 - 47360*sqrt(a + b*x + c*x*
*2)*a**5*b**2*x**2 - 66944*sqrt(a + b*x + c*x**2)*a**5*b*c*x**3 - 6144*sqr
t(a + b*x + c*x**2)*a**5*c**2*x**4 - 480*sqrt(a + b*x + c*x**2)*a**4*b**3*
x**3 - 2112*sqrt(a + b*x + c*x**2)*a**4*b**2*c*x**4 + 576*sqrt(a + b*x + c
*x**2)*a**4*b*c**2*x**5 + 12288*sqrt(a + b*x + c*x**2)*a**4*c**3*x**6 + 56
0*sqrt(a + b*x + c*x**2)*a**3*b**4*x**4 + 3360*sqrt(a + b*x + c*x**2)*a**3
*b**3*c*x**5 + 3360*sqrt(a + b*x + c*x**2)*a**3*b**2*c**2*x**6 - 700*sqrt(
a + b*x + c*x**2)*a**2*b**5*x**5 - 6160*sqrt(a + b*x + c*x**2)*a**2*b**4*c
*x**6 + 1050*sqrt(a + b*x + c*x**2)*a*b**6*x**6 + 6720*sqrt(a)*log(2*sqrt(
a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*b*c**3*x**7 + 5040*sqrt(a)*log
(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**3*c**2*x**7 - 3780*
sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**5*c*x**7 +
525*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**7*x**7 -
6720*sqrt(a)*log(x)*a**3*b*c**3*x**7 - 5040*sqrt(a)*log(x)*a**2*b**3*c**2*
x**7 + 3780*sqrt(a)*log(x)*a*b**5*c*x**7 - 525*sqrt(a)*log(x)*b**7*x**7)/(
215040*a**5*x**7)
```

3.129 $\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 543

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx =$$

$$-\frac{(b^2-4ac)^2\,(195b^5B-286Ab^4c-520ab^3Bc+528aAb^2c^2+240a^2bBc^2-96a^2Ac^3)\,(b+2cx)\sqrt{a+bx+}}{262144c^8}$$

$$+\frac{(b^2-4ac)\,(195b^5B-286Ab^4c-520ab^3Bc+528aAb^2c^2+240a^2bBc^2-96a^2Ac^3)\,(b+2cx)\,(a+bx+cx}{98304c^7}$$

$$-\frac{(195b^5B-286Ab^4c-520ab^3Bc+528aAb^2c^2+240a^2bBc^2-96a^2Ac^3)\,(b+2cx)\,(a+bx+cx^2)^{5/2}}{30720c^6}$$

$$+\frac{(195b^2B-286Abc-160aBc)\,x^2(a+bx+cx^2)^{7/2}}{3960c^3}$$

$$-\frac{(15bB-22Ac)x^3(a+bx+cx^2)^{7/2}}{220c^2}+\frac{Bx^4(a+bx+cx^2)^{7/2}}{11c}$$

$$+\frac{(19305b^4B-28314Ab^3c-42900ab^2Bc+39688aAbc^2+10240a^2Bc^2-14c(2145b^3B-3146Ab^2c-338}{887040c^5}$$

$$+\frac{(b^2-4ac)^3\,(195b^5B-286Ab^4c-520ab^3Bc+528aAb^2c^2+240a^2bBc^2-96a^2Ac^3)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx}}\right.}{524288c^{17/2}}$$

output
```
-1/262144*(-4*a*c+b^2)^2*(-96*A*a^2*c^3+528*A*a*b^2*c^2-286*A*b^4*c+240*B*
a^2*b*c^2-520*B*a*b^3*c+195*B*b^5)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^8+1/983
04*(-4*a*c+b^2)*(-96*A*a^2*c^3+528*A*a*b^2*c^2-286*A*b^4*c+240*B*a^2*b*c^2
-520*B*a*b^3*c+195*B*b^5)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^7-1/30720*(-96*A
*a^2*c^3+528*A*a*b^2*c^2-286*A*b^4*c+240*B*a^2*b*c^2-520*B*a*b^3*c+195*B*b
^5)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c^6+1/3960*(-286*A*b*c-160*B*a*c+195*B*b
^2)*x^2*(c*x^2+b*x+a)^(7/2)/c^3-1/220*(-22*A*c+15*B*b)*x^3*(c*x^2+b*x+a)^(
7/2)/c^2+1/11*B*x^4*(c*x^2+b*x+a)^(7/2)/c+1/887040*(19305*B*b^4-28314*A*b^
3*c-42900*B*a*b^2*c+39688*A*a*b*c^2+10240*B*a^2*c^2-14*c*(2376*A*a*c^2-314
6*A*b^2*c-3380*B*a*b*c+2145*B*b^3)*x)*(c*x^2+b*x+a)^(7/2)/c^5+1/524288*(-4
*a*c+b^2)^3*(-96*A*a^2*c^3+528*A*a*b^2*c^2-286*A*b^4*c+240*B*a^2*b*c^2-520
*B*a*b^3*c+195*B*b^5)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c
^(17/2)
```

Mathematica [A] (verified)

Time = 9.79 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.29

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-675675b^{10}B+90090b^9c(11A+5Bx)+60060b^8c(150aB-cx(11A+$$

input
```
Integrate[x^4*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-675675*b^10*B + 90090*b^9*c*(11*A + 5*B
*x) + 60060*b^8*c*(150*a*B - c*x*(11*A + 6*B*x)) - 528*b^7*c^2*(-13*c*x^2*
(77*A + 45*B*x) + 35*a*(671*A + 299*B*x)) + 64*b^5*c^3*(52*c^2*x^4*(121*A
+ 75*B*x) - 33*a*c*x^2*(2607*A + 1495*B*x) + 198*a^2*(4319*A + 1880*B*x))
- 1056*b^6*c^2*(41510*a^2*B + 13*c^2*x^3*(33*A + 20*B*x) - a*c*x*(7161*A +
 3835*B*x)) + 640*b^4*c^3*(143847*a^3*B - 4*c^3*x^5*(143*A + 90*B*x) - 99*
a^2*c*x*(460*A + 241*B*x) + a*c^2*x^3*(6655*A + 3952*B*x)) - 1280*b^3*c^4*
(-24*c^3*x^6*(11*A + 7*B*x) + 80*a*c^2*x^4*(33*A + 20*B*x) + 33*a^3*(2295*
A + 971*B*x) - a^2*c*x^2*(14223*A + 7967*B*x)) + 256*b^2*c^4*(-285945*a^4*
B + 240*a*c^3*x^5*(44*A + 27*B*x) - 30*a^2*c^2*x^3*(1529*A + 884*B*x) + 11
2*c^4*x^7*(4213*A + 3720*B*x) + 10*a^3*c*x*(15763*A + 8037*B*x)) + 1024*c^
5*(10240*a^5*B + 8064*c^5*x^9*(11*A + 10*B*x) + 30*a^3*c^2*x^3*(231*A + 12
8*B*x) - 5*a^4*c*x*(2079*A + 1024*B*x) + 112*a*c^4*x^7*(2079*A + 1840*B*x)
 + 8*a^2*c^3*x^5*(21483*A + 18080*B*x)) + 512*b*c^5*(120*a^2*c^2*x^4*(121*
A + 71*B*x) + 896*c^4*x^8*(451*A + 405*B*x) - 10*a^3*c*x^2*(3553*A + 1929*
B*x) + 5*a^4*(20449*A + 8347*B*x) + 16*a*c^3*x^6*(34991*A + 30275*B*x))) -
 3465*(b^2 - 4*a*c)^3*(195*b^5*B - 286*A*b^4*c - 520*a*b^3*B*c + 528*a*A*b
^2*c^2 + 240*a^2*b*B*c^2 - 96*a^2*A*c^3)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)]])/(1816657920*c^(17/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.80, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {1236, 27, 1236, 27, 1236, 27, 1225, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}x^3(8aB+(15bB-22Ac)x)\left(cx^2+bx+a\right)^{5/2}dx}{11c}+\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-\frac{\int x^3(8aB+(15bB-22Ac)x)\left(cx^2+bx+a\right)^{5/2}dx}{22c}$$

$\Big\downarrow$ 1236

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-\frac{\frac{\int -\frac{1}{2}x^2(6a(15bB-22Ac)+(195Bb^2-286Acb-160aBc)x)\left(cx^2+bx+a\right)^{5/2}dx}{10c}+\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}}{22c}$$

$\Big\downarrow$ 27

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}-\frac{\int x^2(6a(15bB-22Ac)+(195Bb^2-286Acb-160aBc)x)\left(cx^2+bx+a\right)^{5/2}dx}{20c}}{22c}$$

$\Big\downarrow$ 1236

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}-\frac{\frac{\int -\frac{1}{2}x\left(4a\left(195Bb^2-286Acb-160aBc\right)+\left(2145Bb^3-3146Acb^2-3380aBcb+2376aAc^2\right)x\right)\left(cx^2+bx+a\right)^{5/2}dx}{9c}+\frac{x^2\left(a+b\right.}{}}{20c}}{22c}$$

$\Big\downarrow$ 27

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}\left(-160aBc-286Abc+195b^2B\right)}{9c}-\frac{\int x\left(4a\left(195Bb^2-286Acb-160aBc\right)+\left(2145Bb^3-3146Acb^2-3380a\right.\right.}{18c}}{20c}}{22c}$$

$\Big\downarrow$ 1225

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}\left(-160aBc-286Abc+195b^2B\right)}{9c}-\frac{99\left(-96a^2Ac^3+240a^2bBc^2+528aAb^2c^2-520ab^3Bc-286Ab^4c+195\right.}{32c^2}}{}}{}$$

$\Big\downarrow$ 1087

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-\frac{\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}\left(-160aBc-286Abc+195b^2B\right)}{9c}-\frac{99\left(-96a^2Ac^3+240a^2bBc^2+528aAb^2c^2-520ab^3Bc-286Ab^4c+195\right.}{}}{}}{}$$

$\Big\downarrow$ 1087

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-$$

$$99\left(-96a^2Ac^3+240a^2bBc^2+528aAb^2c^2-520ab^3Bc-286Ab^4c+195\right.$$

$$\frac{\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}\left(-160aBc-286Abc+195b^2B\right)}{9c}-}{}}{}$$

$\downarrow$ 1087

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-$$

$$99\left(-96a^2Ac^3+240a^2bBc^2+528aAb^2c^2-520ab^3Bc-286Ab^4c+195\right.$$

$$\frac{\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}\left(-160aBc-286Abc+195b^2B\right)}{9c}-}{}}{}$$

$\downarrow$ 1092

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-$$

$$99\left(-96a^2Ac^3+240a^2bBc^2+528aAb^2c^2-520ab^3Bc-286Ab^4c+195\right.$$

$$\frac{\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}\left(-160aBc-286Abc+195b^2B\right)}{9c}-}{}}{}$$

$\downarrow$ 219

$$\frac{Bx^4\left(a+bx+cx^2\right)^{7/2}}{11c}-$$

$$99\left(-96a^2Ac^3+240a^2bBc^2+528aAb^2c^2-520ab^3Bc-286Ab^4c+195\right.$$

$$\frac{x^3\left(a+bx+cx^2\right)^{7/2}(15bB-22Ac)}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}\left(-160aBc-286Abc+195b^2B\right)}{9c}-}{}$$

input
```
Int[x^4*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(B*x^4*(a + b*x + c*x^2)^(7/2))/(11*c) - (((15*b*B - 22*A*c)*x^3*(a + b*x
+ c*x^2)^(7/2))/(10*c) - (((195*b^2*B - 286*A*b*c - 160*a*B*c)*x^2*(a + b*
x + c*x^2)^(7/2))/(9*c) - (-1/112*((19305*b^4*B - 28314*A*b^3*c - 42900*a*
b^2*B*c + 39688*a*A*b*c^2 + 10240*a^2*B*c^2 - 14*c*(2145*b^3*B - 3146*A*b^
2*c - 3380*a*b*B*c + 2376*a*A*c^2)*x)*(a + b*x + c*x^2)^(7/2))/c^2 + (99*(
195*b^5*B - 286*A*b^4*c - 520*a*b^3*B*c + 528*a*A*b^2*c^2 + 240*a^2*b*B*c^
2 - 96*a^2*A*c^3)*(((b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(12*c) - (5*(b^2
- 4*a*c)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(
((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(24*c)))/(
32*c^2))/(18*c))/(20*c))/(22*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(82575360B\,c^{10}x^{10}+90832896A\,c^{10}x^{9}+185794560Bb\,c^{9}x^{9}+206897152Ab\,c^{9}x^{8}+211025920Ba\,c^{9}x^{8}+106659840B\,b^{2}c^{8}x^{8}+238436}{}$
default	Expression too large to display

input
```
int(x^4*(B*x+A)*(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/908328960/c^8*(82575360*B*c^10*x^10+90832896*A*c^10*x^9+185794560*B*b*c^
9*x^9+206897152*A*b*c^9*x^8+211025920*B*a*c^9*x^8+106659840*B*b^2*c^8*x^8+
238436352*A*a*c^9*x^7+120795136*A*b^2*c^8*x^7+248012800*B*a*b*c^8*x^7+2150
40*B*b^3*c^7*x^7+286646272*A*a*b*c^8*x^6+337920*A*b^3*c^7*x^6+148111360*B*
a^2*c^8*x^6+1658880*B*a*b^2*c^7*x^6-230400*B*b^4*c^6*x^6+175988736*A*a^2*c
^8*x^5+2703360*A*a*b^2*c^7*x^5-366080*A*b^4*c^6*x^5+4362240*B*a^2*b*c^7*x^
5-2048000*B*a*b^3*c^6*x^5+249600*B*b^5*c^5*x^5+7434240*A*a^2*b*c^7*x^4-337
9200*A*a*b^3*c^6*x^4+402688*A*b^5*c^5*x^4+3932160*B*a^3*c^7*x^4-6789120*B*
a^2*b^2*c^6*x^4+2529280*B*a*b^4*c^5*x^4-274560*B*b^6*c^4*x^4+7096320*A*a^3
*c^7*x^3-11742720*A*a^2*b^2*c^6*x^3+4259200*A*a*b^4*c^5*x^3-453024*A*b^6*c
^4*x^3-9876480*B*a^3*b*c^6*x^3+10197760*B*a^2*b^3*c^5*x^3-3157440*B*a*b^5*
c^4*x^3+308880*B*b^7*c^3*x^3-18191360*A*a^3*b*c^6*x^2+18205440*A*a^2*b^3*c
^5*x^2-5505984*A*a*b^5*c^4*x^2+528528*A*b^7*c^3*x^2-5242880*B*a^4*c^6*x^2+
20574720*B*a^3*b^2*c^5*x^2-15269760*B*a^2*b^4*c^4*x^2+4049760*B*a*b^6*c^3*
x^2-360360*B*b^8*c^2*x^2-10644480*A*a^4*c^6*x+40353280*A*a^3*b^2*c^5*x-291
45600*A*a^2*b^4*c^4*x+7562016*A*a*b^6*c^3*x-660660*A*b^8*c^2*x+21368320*B*
a^4*b*c^5*x-41015040*B*a^3*b^3*c^4*x+23823360*B*a^2*b^5*c^3*x-5525520*B*a*
b^7*c^2*x+450450*B*b^9*c*x+52349440*A*a^4*b*c^5-96940800*A*a^3*b^3*c^4+547
30368*A*a^2*b^5*c^3-12400080*A*a*b^7*c^2+990990*A*b^9*c+10485760*B*a^5*c^5
-73201920*B*a^4*b^2*c^4+92062080*B*a^3*b^4*c^3-43834560*B*a^2*b^6*c^2+9...
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1775, normalized size of antiderivative = 3.27

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^4*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/3633315840*(3465*(195*B*b^11 + 6144*A*a^5*c^6 - 7680*(2*B*a^5*b + 5*A*a
^4*b^2)*c^5 + 44800*(B*a^4*b^3 + A*a^3*b^4)*c^4 - 20160*(2*B*a^3*b^5 + A*a
^2*b^6)*c^3 + 3960*(4*B*a^2*b^7 + A*a*b^8)*c^2 - 286*(10*B*a*b^9 + A*b^10)
*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(c) - 4*a*c) + 4*(82575360*B*c^11*x^10 - 675675*B*b^10*c + 4128
768*(45*B*b*c^10 + 22*A*c^11)*x^9 + 229376*(465*B*b^2*c^9 + 2*(460*B*a + 4
51*A*b)*c^10)*x^8 + 14336*(15*B*b^3*c^8 + 16632*A*a*c^10 + 2*(8650*B*a*b +
 4213*A*b^2)*c^9)*x^7 + 2560*(4096*B*a^5 + 20449*A*a^4*b)*c^6 - 1024*(225*
B*b^4*c^7 - 8*(18080*B*a^2 + 34991*A*a*b)*c^9 - 30*(54*B*a*b^2 + 11*A*b^3)
*c^8)*x^6 - 42240*(1733*B*a^4*b^2 + 2295*A*a^3*b^3)*c^5 + 256*(975*B*b^5*c
^6 + 687456*A*a^2*c^9 + 240*(71*B*a^2*b + 44*A*a*b^2)*c^8 - 10*(800*B*a*b^
3 + 143*A*b^4)*c^7)*x^5 + 12672*(7265*B*a^3*b^4 + 4319*A*a^2*b^5)*c^4 - 12
8*(2145*B*b^6*c^5 - 480*(64*B*a^3 + 121*A*a^2*b)*c^8 + 240*(221*B*a^2*b^2
+ 110*A*a*b^3)*c^7 - 26*(760*B*a*b^4 + 121*A*b^5)*c^6)*x^4 - 18480*(2372*B
*a^2*b^6 + 671*A*a*b^7)*c^3 + 16*(19305*B*b^7*c^4 + 443520*A*a^3*c^8 - 480
*(1286*B*a^3*b + 1529*A*a^2*b^2)*c^7 + 40*(15934*B*a^2*b^3 + 6655*A*a*b^4)
*c^6 - 858*(230*B*a*b^5 + 33*A*b^6)*c^5)*x^3 + 90090*(100*B*a*b^8 + 11*A*b
^9)*c^2 - 8*(45045*B*b^8*c^3 + 640*(1024*B*a^4 + 3553*A*a^3*b)*c^7 - 480*(
5358*B*a^3*b^2 + 4741*A*a^2*b^3)*c^6 + 792*(2410*B*a^2*b^4 + 869*A*a*b^5)*
c^5 - 858*(590*B*a*b^6 + 77*A*b^7)*c^4)*x^2 + 2*(225225*B*b^9*c^2 - 532...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17279 vs. 2(590) = 1180.

Time = 0.98 (sec) , antiderivative size = 17279, normalized size of antiderivative = 31.82

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate(x**4*(B*x+A)*(c*x**2+b*x+a)**(5/2),x)
```

output
```
Piecewise(((-a*(-3*a*(A*a**3 - 5*a*(3*A*a**2*c + 3*A*a*b**2 + 3*B*a**2*b -
 7*a*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 9*a*(A*c**3 + 45*B*b*
c**2/22)/(10*c) - 17*b*(3*A*b*c**2 + 23*B*a*c**2/11 + 3*B*b**2*c - 19*b*(A
*c**3 + 45*B*b*c**2/22)/(20*c))/(18*c))/(8*c) - 13*b*(6*A*a*b*c + A*b**3 +
 3*B*a**2*c + 3*B*a*b**2 - 8*a*(3*A*b*c**2 + 23*B*a*c**2/11 + 3*B*b**2*c -
 19*b*(A*c**3 + 45*B*b*c**2/22)/(20*c))/(9*c) - 15*b*(3*A*a*c**2 + 3*A*b**
2*c + 6*B*a*b*c + B*b**3 - 9*a*(A*c**3 + 45*B*b*c**2/22)/(10*c) - 17*b*(3*
A*b*c**2 + 23*B*a*c**2/11 + 3*B*b**2*c - 19*b*(A*c**3 + 45*B*b*c**2/22)/(2
0*c))/(18*c))/(16*c))/(14*c))/(6*c) - 9*b*(3*A*a**2*b + B*a**3 - 6*a*(6*A*
a*b*c + A*b**3 + 3*B*a**2*c + 3*B*a*b**2 - 8*a*(3*A*b*c**2 + 23*B*a*c**2/1
1 + 3*B*b**2*c - 19*b*(A*c**3 + 45*B*b*c**2/22)/(20*c))/(9*c) - 15*b*(3*A*
a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 9*a*(A*c**3 + 45*B*b*c**2/22)/(
10*c) - 17*b*(3*A*b*c**2 + 23*B*a*c**2/11 + 3*B*b**2*c - 19*b*(A*c**3 + 45
*B*b*c**2/22)/(20*c))/(18*c))/(16*c))/(7*c) - 11*b*(3*A*a**2*c + 3*A*a*b**
2 + 3*B*a**2*b - 7*a*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 9*a*(
A*c**3 + 45*B*b*c**2/22)/(10*c) - 17*b*(3*A*b*c**2 + 23*B*a*c**2/11 + 3*B*
b**2*c - 19*b*(A*c**3 + 45*B*b*c**2/22)/(20*c))/(18*c))/(8*c) - 13*b*(6*A*
a*b*c + A*b**3 + 3*B*a**2*c + 3*B*a*b**2 - 8*a*(3*A*b*c**2 + 23*B*a*c**2/1
1 + 3*B*b**2*c - 19*b*(A*c**3 + 45*B*b*c**2/22)/(20*c))/(9*c) - 15*b*(3*A*
a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 9*a*(A*c**3 + 45*B*b*c**2/22...
```

Maxima [F(-2)]

Exception generated.

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 906, normalized size of antiderivative = 1.67

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/908328960*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(4*(14*(16*(18*(20*B*c^2*
x + (45*B*b*c^11 + 22*A*c^12)/c^10)*x + (465*B*b^2*c^10 + 920*B*a*c^11 + 9
02*A*b*c^11)/c^10)*x + (15*B*b^3*c^9 + 17300*B*a*b*c^10 + 8426*A*b^2*c^10
+ 16632*A*a*c^11)/c^10)*x - (225*B*b^4*c^8 - 1620*B*a*b^2*c^9 - 330*A*b^3*
c^9 - 144640*B*a^2*c^10 - 279928*A*a*b*c^10)/c^10)*x + (975*B*b^5*c^7 - 80
00*B*a*b^3*c^8 - 1430*A*b^4*c^8 + 17040*B*a^2*b*c^9 + 10560*A*a*b^2*c^9 +
687456*A*a^2*c^10)/c^10)*x - (2145*B*b^6*c^6 - 19760*B*a*b^4*c^7 - 3146*A*
b^5*c^7 + 53040*B*a^2*b^2*c^8 + 26400*A*a*b^3*c^8 - 30720*B*a^3*c^9 - 5808
0*A*a^2*b*c^9)/c^10)*x + (19305*B*b^7*c^5 - 197340*B*a*b^5*c^6 - 28314*A*b
^6*c^6 + 637360*B*a^2*b^3*c^7 + 266200*A*a*b^4*c^7 - 617280*B*a^3*b*c^8 -
733920*A*a^2*b^2*c^8 + 443520*A*a^3*c^9)/c^10)*x - (45045*B*b^8*c^4 - 5062
20*B*a*b^6*c^5 - 66066*A*b^7*c^5 + 1908720*B*a^2*b^4*c^6 + 688248*A*a*b^5*
c^6 - 2571840*B*a^3*b^2*c^7 - 2275680*A*a^2*b^3*c^7 + 655360*B*a^4*c^8 + 2
273920*A*a^3*b*c^8)/c^10)*x + (225225*B*b^9*c^3 - 2762760*B*a*b^7*c^4 - 33
0330*A*b^8*c^4 + 11911680*B*a^2*b^5*c^5 + 3781008*A*a*b^6*c^5 - 20507520*B
*a^3*b^3*c^6 - 14572800*A*a^2*b^4*c^6 + 10684160*B*a^4*b*c^7 + 20176640*A*
a^3*b^2*c^7 - 5322240*A*a^4*c^8)/c^10)*x - (675675*B*b^10*c^2 - 9009000*B*
a*b^8*c^3 - 990990*A*b^9*c^3 + 43834560*B*a^2*b^6*c^4 + 12400080*A*a*b^7*c
^4 - 92062080*B*a^3*b^4*c^5 - 54730368*A*a^2*b^5*c^5 + 73201920*B*a^4*b^2*
c^6 + 96940800*A*a^3*b^3*c^6 - 10485760*B*a^5*c^7 - 52349440*A*a^4*b*c^...
```

Mupad [F(-1)]

Timed out.

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \int x^4\,\left(A+B\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2}\,dx$$

input
```
int(x^4*(A + B*x)*(a + b*x + c*x^2)^(5/2),x)
```

output
```
int(x^4*(A + B*x)*(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^4(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int x^4(Bx+A)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}dx$$

input
```
int(x^4*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x^4*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

3.130 $\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 432

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{\left(b^2-4ac\right)^2\left(143b^4B-220Ab^3c-264ab^2Bc+240aAbc^2+48a^2Bc^2\right)(b+2cx)\sqrt{a+bx+cx^2}}{131072c^7}$$
$$-\frac{\left(b^2-4ac\right)\left(143b^4B-220Ab^3c-264ab^2Bc+240aAbc^2+48a^2Bc^2\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{49152c^6}$$
$$+\frac{\left(143b^4B-220Ab^3c-264ab^2Bc+240aAbc^2+48a^2Bc^2\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{15360c^5}$$
$$-\frac{(13bB-20Ac)x^2\left(a+bx+cx^2\right)^{7/2}}{180c^2}+\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}$$
$$-\frac{\left(1287b^3B-1980Ab^2c-1804abBc+1280aAc^2-14c\left(143b^2B-220Abc-108aBc\right)x\right)\left(a+bx+cx^2\right)^{7/2}}{40320c^4}$$
$$-\frac{\left(b^2-4ac\right)^3\left(143b^4B-220Ab^3c-264ab^2Bc+240aAbc^2+48a^2Bc^2\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{262144c^{15/2}}$$

output
```
1/131072*(-4*a*c+b^2)^2*(240*A*a*b*c^2-220*A*b^3*c+48*B*a^2*c^2-264*B*a*b^
2*c+143*B*b^4)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^7-1/49152*(-4*a*c+b^2)*(240
*A*a*b*c^2-220*A*b^3*c+48*B*a^2*c^2-264*B*a*b^2*c+143*B*b^4)*(2*c*x+b)*(c*
x^2+b*x+a)^(3/2)/c^6+1/15360*(240*A*a*b*c^2-220*A*b^3*c+48*B*a^2*c^2-264*B
*a*b^2*c+143*B*b^4)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c^5-1/180*(-20*A*c+13*B*
b)*x^2*(c*x^2+b*x+a)^(7/2)/c^2+1/10*B*x^3*(c*x^2+b*x+a)^(7/2)/c-1/40320*(1
287*B*b^3-1980*A*b^2*c-1804*B*a*b*c+1280*A*a*c^2-14*c*(-220*A*b*c-108*B*a*
c+143*B*b^2)*x)*(c*x^2+b*x+a)^(7/2)/c^4-1/262144*(-4*a*c+b^2)^3*(240*A*a*b
*c^2-220*A*b^3*c+48*B*a^2*c^2-264*B*a*b^2*c+143*B*b^4)*arctanh(1/2*(2*c*x+
b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(15/2)
```

Mathematica [A] (verified)

Time = 6.28 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.35

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{2\sqrt{c}\sqrt{a+x(b+cx)}(45045b^9B-2310b^8c(30A+13Bx)+1848b^7c(-305aB+cx(25A+13\cdots}{}$$

input
```
Integrate[x^3*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(45045*b^9*B - 2310*b^8*c*(30*A + 13*B*x)
 + 1848*b^7*c*(-305*a*B + c*x*(25*A + 13*B*x)) - 640*b^3*c^3*(6885*a^3*B -
 8*c^3*x^5*(5*A + 3*B*x) + 4*a*c^2*x^3*(107*A + 60*B*x) - 3*a^2*c*x*(879*A
 + 431*B*x)) - 320*b^4*c^3*(4*c^2*x^4*(22*A + 13*B*x) + 207*a^2*(49*A + 20
*B*x) - a*c*x^2*(1116*A + 605*B*x)) + 32*b^5*c^2*(77742*a^2*B + 22*c^2*x^3
*(45*A + 26*B*x) - 9*a*c*x*(1715*A + 869*B*x)) + 48*b^6*c^2*(-11*c*x^2*(70
*A + 39*B*x) + 7*a*(2425*A + 1023*B*x)) + 512*c^5*(896*c^4*x^8*(10*A + 9*B
*x) + 10*a^3*c*x^2*(128*A + 63*B*x) - 5*a^4*(512*A + 189*B*x) + 24*a^2*c^2
*x^4*(800*A + 651*B*x) + 16*a*c^3*x^6*(1520*A + 1323*B*x)) + 256*b^2*c^4*(
120*a*c^2*x^4*(7*A + 4*B*x) - 15*a^2*c*x^2*(266*A + 139*B*x) + 5*a^3*(3663
*A + 1433*B*x) + 8*c^3*x^6*(3090*A + 2681*B*x)) + 256*b*c^4*(9295*a^4*B +
60*a^2*c^2*x^3*(41*A + 22*B*x) + 224*c^4*x^7*(185*A + 164*B*x) - 10*a^3*c*
x*(689*A + 323*B*x) + 16*a*c^3*x^5*(3765*A + 3181*B*x))) + 315*(b^2 - 4*a*
c)^3*(143*b^4*B - 220*A*b^3*c - 264*a*b^2*B*c + 240*a*A*b*c^2 + 48*a^2*B*c
^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(82575360*c^(15/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1236, 27, 1236, 27, 1225, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}x^2(6aB+(13bB-20Ac)x)\left(cx^2+bx+a\right)^{5/2}dx}{10c}+\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}-\frac{\int x^2(6aB+(13bB-20Ac)x)\left(cx^2+bx+a\right)^{5/2}dx}{20c}$$

$$\Big\downarrow \text{1236}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}-\frac{\frac{\int -\frac{1}{2}x\left(4a(13bB-20Ac)+\left(143Bb^2-220Acb-108aBc\right)x\right)\left(cx^2+bx+a\right)^{5/2}dx}{9c}+\frac{x^2\left(a+bx+cx^2\right)^{7/2}(13bB-20Ac)}{9c}}{20c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}(13bB-20Ac)}{9c}-\frac{\int x\left(4a(13bB-20Ac)+\left(143Bb^2-220Acb-108aBc\right)x\right)\left(cx^2+bx+a\right)^{5/2}dx}{18c}}{20c}$$

$$\Big\downarrow \text{1225}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}(13bB-20Ac)}{9c}-\frac{\frac{9\left(48a^2Bc^2+240aAbc^2-264ab^2Bc-220Ab^3c+143b^4B\right)\int\left(cx^2+bx+a\right)^{5/2}dx}{32c^2}-\frac{\left(a+bx+cx^2\right)^{7/2}\left(-14cx\left(-108aBc-220\right.\right.}{}}{18c}}{20c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}(13bB-20Ac)}{9c}-\frac{\frac{9\left(48a^2Bc^2+240aAbc^2-264ab^2Bc-220Ab^3c+143b^4B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\int\left(cx^2+bx+a\right)^{3/2}dx}{24c}\right)}{32c^2}\cdots}{18c}}{20c}$$

$\downarrow$ 1087

$$\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}(13bB-20Ac)}{9c}-\frac{\frac{9\left(48a^2Bc^2+240aAbc^2-264ab^2Bc-220Ab^3c+143b^4B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)\cdots}{8c}\cdots\right)}{\cdots}\right)}{32c^2}\cdots}{\cdots}}{20c}$$

$\downarrow$ 1087

$$\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{7/2}(13bB-20Ac)}{9c}-\frac{\frac{9\left(48a^2Bc^2+240aAbc^2-264ab^2Bc-220Ab^3c+143b^4B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)\cdots}{8c}\cdots\right)}{\cdots}\right)}{32c^2}\cdots}{\cdots}}{\cdots}$$

$\downarrow$ 1092

$$\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}-$$

$$\frac{x^2\left(a+bx+cx^2\right)^{7/2}(13bB-20Ac)}{9c}-\frac{9\left(48a^2Bc^2+240aAbc^2-264ab^2Bc-220Ab^3c+143b^4B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)}{8c}\cdots\right.}{\cdots}\right.}{32c^2\cdots}$$

$\downarrow$ 219

$$\frac{Bx^3\left(a+bx+cx^2\right)^{7/2}}{10c}-$$

$$\frac{x^2\left(a+bx+cx^2\right)^{7/2}(13bB-20Ac)}{9c}-\frac{9\left(48a^2Bc^2+240aAbc^2-264ab^2Bc-220Ab^3c+143b^4B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)}{8c}\cdots\right.}{\cdots}\right.}{32c^2\cdots}$$

input
```
Int[x^3*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(B*x^3*(a + b*x + c*x^2)^(7/2))/(10*c) - (((13*b*B - 20*A*c)*x^2*(a + b*x
+ c*x^2)^(7/2))/(9*c) - (-1/112*((1287*b^3*B - 1980*A*b^2*c - 1804*a*b*B*c
 + 1280*a*A*c^2 - 14*c*(143*b^2*B - 220*A*b*c - 108*a*B*c)*x)*(a + b*x + c
*x^2)^(7/2))/c^2 + (9*(143*b^4*B - 220*A*b^3*c - 264*a*b^2*B*c + 240*a*A*b
*c^2 + 48*a^2*B*c^2)*(((b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(12*c) - (5*(b
^2 - 4*a*c)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c
)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(24*c))
)/(32*c^2))/(18*c))/(20*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{(-4128768B\,c^9x^9-4587520A\,c^9x^8-9404416Bb\,c^8x^8-10608640Ab\,c^8x^7-10838016Ba\,c^8x^7-5490688B\,b^2c^7x^7-12451840Aa\,c^8}{}$
default	Expression too large to display

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/41287680/c^7*(-4128768*B*c^9*x^9-4587520*A*c^9*x^8-9404416*B*b*c^8*x^8-
10608640*A*b*c^8*x^7-10838016*B*a*c^8*x^7-5490688*B*b^2*c^7*x^7-12451840*A
*a*c^8*x^6-6328320*A*b^2*c^7*x^6-13029376*B*a*b*c^7*x^6-15360*B*b^3*c^6*x^
6-15421440*A*a*b*c^7*x^5-25600*A*b^3*c^6*x^5-7999488*B*a^2*c^7*x^5-122880*
B*a*b^2*c^6*x^5+16640*B*b^4*c^5*x^5-9830400*A*a^2*c^7*x^4-215040*A*a*b^2*c
^6*x^4+28160*A*b^4*c^5*x^4-337920*B*a^2*b*c^6*x^4+153600*B*a*b^3*c^5*x^4-1
8304*B*b^5*c^4*x^4-629760*A*a^2*b*c^6*x^3+273920*A*a*b^3*c^5*x^3-31680*A*b
^5*c^4*x^3-322560*B*a^3*c^6*x^3+533760*B*a^2*b^2*c^5*x^3-193600*B*a*b^4*c^
4*x^3+20592*B*b^6*c^3*x^3-655360*A*a^3*c^6*x^2+1021440*A*a^2*b^2*c^5*x^2-3
57120*A*a*b^4*c^4*x^2+36960*A*b^6*c^3*x^2+826880*B*a^3*b*c^5*x^2-827520*B*
a^2*b^3*c^4*x^2+250272*B*a*b^5*c^3*x^2-24024*B*b^7*c^2*x^2+1763840*A*a^3*b
*c^5*x-1687680*A*a^2*b^3*c^4*x+493920*A*a*b^5*c^3*x-46200*A*b^7*c^2*x+4838
40*B*a^4*c^5*x-1834240*B*a^3*b^2*c^4*x+1324800*B*a^2*b^4*c^3*x-343728*B*a*
b^6*c^2*x+30030*B*b^8*c*x+1310720*A*a^4*c^5-4688640*A*a^3*b^2*c^4+3245760*
A*a^2*b^4*c^3-814800*A*a*b^6*c^2+69300*A*b^8*c-2379520*B*a^4*b*c^4+4406400
*B*a^3*b^3*c^3-2487744*B*a^2*b^5*c^2+563640*B*a*b^7*c-45045*B*b^9)*(c*x^2+
b*x+a)^(1/2)+1/262144*(15360*A*a^4*b*c^5-25600*A*a^3*b^3*c^4+13440*A*a^2*b
^5*c^3-2880*A*a*b^7*c^2+220*A*b^9*c+3072*B*a^5*c^5-19200*B*a^4*b^2*c^4+224
00*B*a^3*b^4*c^3-10080*B*a^2*b^6*c^2+1980*B*a*b^8*c-143*B*b^10)/c^(15/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1511, normalized size of antiderivative = 3.50

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/165150720*(315*(143*B*b^10 - 3072*(B*a^5 + 5*A*a^4*b)*c^5 + 6400*(3*B*
a^4*b^2 + 4*A*a^3*b^3)*c^4 - 4480*(5*B*a^3*b^4 + 3*A*a^2*b^5)*c^3 + 1440*(
7*B*a^2*b^6 + 2*A*a*b^7)*c^2 - 220*(9*B*a*b^8 + A*b^9)*c)*sqrt(c)*log(-8*c
^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a
*c) - 4*(4128768*B*c^10*x^9 + 45045*B*b^9*c - 1310720*A*a^4*c^6 + 229376*(
41*B*b*c^9 + 20*A*c^10)*x^8 + 14336*(383*B*b^2*c^8 + 4*(189*B*a + 185*A*b)
*c^9)*x^7 + 1024*(15*B*b^3*c^7 + 12160*A*a*c^9 + 4*(3181*B*a*b + 1545*A*b^
2)*c^8)*x^6 + 14080*(169*B*a^4*b + 333*A*a^3*b^2)*c^5 - 256*(65*B*b^4*c^6
- 48*(651*B*a^2 + 1255*A*a*b)*c^8 - 20*(24*B*a*b^2 + 5*A*b^3)*c^7)*x^5 - 2
880*(1530*B*a^3*b^3 + 1127*A*a^2*b^4)*c^4 + 128*(143*B*b^5*c^5 + 76800*A*a
^2*c^8 + 240*(11*B*a^2*b + 7*A*a*b^2)*c^7 - 20*(60*B*a*b^3 + 11*A*b^4)*c^6
)*x^4 + 336*(7404*B*a^2*b^5 + 2425*A*a*b^6)*c^3 - 16*(1287*B*b^6*c^4 - 960
*(21*B*a^3 + 41*A*a^2*b)*c^7 + 80*(417*B*a^2*b^2 + 214*A*a*b^3)*c^6 - 220*
(55*B*a*b^4 + 9*A*b^5)*c^5)*x^3 - 4620*(122*B*a*b^7 + 15*A*b^8)*c^2 + 8*(3
003*B*b^7*c^3 + 81920*A*a^3*c^7 - 6080*(17*B*a^3*b + 21*A*a^2*b^2)*c^6 + 2
40*(431*B*a^2*b^3 + 186*A*a*b^4)*c^5 - 132*(237*B*a*b^5 + 35*A*b^6)*c^4)*x
^2 - 2*(15015*B*b^8*c^2 + 1280*(189*B*a^4 + 689*A*a^3*b)*c^6 - 320*(2866*B
*a^3*b^2 + 2637*A*a^2*b^3)*c^5 + 720*(920*B*a^2*b^4 + 343*A*a*b^5)*c^4 - 9
24*(186*B*a*b^6 + 25*A*b^7)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^8, 1/82575360
*(315*(143*B*b^10 - 3072*(B*a^5 + 5*A*a^4*b)*c^5 + 6400*(3*B*a^4*b^2 + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10654 vs. $2(461) = 922$.

Time = 0.91 (sec) , antiderivative size = 10654, normalized size of antiderivative = 24.66

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(B*x+A)*(c*x**2+b*x+a)**(5/2),x)
```

output
```
Piecewise(((-a*(-3*a*(3*A*a**2*b + B*a**3 - 5*a*(6*A*a*b*c + A*b**3 + 3*B*
a**2*c + 3*B*a*b**2 - 7*a*(3*A*b*c**2 + 21*B*a*c**2/10 + 3*B*b**2*c - 17*b
*(A*c**3 + 41*B*b*c**2/20)/(18*c))/(8*c) - 13*b*(3*A*a*c**2 + 3*A*b**2*c +
 6*B*a*b*c + B*b**3 - 8*a*(A*c**3 + 41*B*b*c**2/20)/(9*c) - 15*b*(3*A*b*c*
*2 + 21*B*a*c**2/10 + 3*B*b**2*c - 17*b*(A*c**3 + 41*B*b*c**2/20)/(18*c))/
(16*c))/(14*c))/(6*c) - 9*b*(3*A*a**2*c + 3*A*a*b**2 + 3*B*a**2*b - 6*a*(3
*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 8*a*(A*c**3 + 41*B*b*c**2/20
)/(9*c) - 15*b*(3*A*b*c**2 + 21*B*a*c**2/10 + 3*B*b**2*c - 17*b*(A*c**3 +
41*B*b*c**2/20)/(18*c))/(16*c))/(7*c) - 11*b*(6*A*a*b*c + A*b**3 + 3*B*a**
2*c + 3*B*a*b**2 - 7*a*(3*A*b*c**2 + 21*B*a*c**2/10 + 3*B*b**2*c - 17*b*(A
*c**3 + 41*B*b*c**2/20)/(18*c))/(8*c) - 13*b*(3*A*a*c**2 + 3*A*b**2*c + 6*
B*a*b*c + B*b**3 - 8*a*(A*c**3 + 41*B*b*c**2/20)/(9*c) - 15*b*(3*A*b*c**2
+ 21*B*a*c**2/10 + 3*B*b**2*c - 17*b*(A*c**3 + 41*B*b*c**2/20)/(18*c))/(16
*c))/(14*c))/(12*c))/(10*c))/(4*c) - 5*b*(A*a**3 - 4*a*(3*A*a**2*c + 3*A*a
*b**2 + 3*B*a**2*b - 6*a*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 8
*a*(A*c**3 + 41*B*b*c**2/20)/(9*c) - 15*b*(3*A*b*c**2 + 21*B*a*c**2/10 + 3
*B*b**2*c - 17*b*(A*c**3 + 41*B*b*c**2/20)/(18*c))/(16*c))/(7*c) - 11*b*(6
*A*a*b*c + A*b**3 + 3*B*a**2*c + 3*B*a*b**2 - 7*a*(3*A*b*c**2 + 21*B*a*c**
2/10 + 3*B*b**2*c - 17*b*(A*c**3 + 41*B*b*c**2/20)/(18*c))/(8*c) - 13*b*(3
*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 8*a*(A*c**3 + 41*B*b*c**2...
```

Maxima [F(-2)]

Exception generated.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.78

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/41287680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(4*(14*(16*(18*B*c^2*x + (
41*B*b*c^10 + 20*A*c^11)/c^9)*x + (383*B*b^2*c^9 + 756*B*a*c^10 + 740*A*b*
c^10)/c^9)*x + (15*B*b^3*c^8 + 12724*B*a*b*c^9 + 6180*A*b^2*c^9 + 12160*A*
a*c^10)/c^9)*x - (65*B*b^4*c^7 - 480*B*a*b^2*c^8 - 100*A*b^3*c^8 - 31248*B
*a^2*c^9 - 60240*A*a*b*c^9)/c^9)*x + (143*B*b^5*c^6 - 1200*B*a*b^3*c^7 - 2
20*A*b^4*c^7 + 2640*B*a^2*b*c^8 + 1680*A*a*b^2*c^8 + 76800*A*a^2*c^9)/c^9)
*x - (1287*B*b^6*c^5 - 12100*B*a*b^4*c^6 - 1980*A*b^5*c^6 + 33360*B*a^2*b^
2*c^7 + 17120*A*a*b^3*c^7 - 20160*B*a^3*c^8 - 39360*A*a^2*b*c^8)/c^9)*x +
(3003*B*b^7*c^4 - 31284*B*a*b^5*c^5 - 4620*A*b^6*c^5 + 103440*B*a^2*b^3*c^
6 + 44640*A*a*b^4*c^6 - 103360*B*a^3*b*c^7 - 127680*A*a^2*b^2*c^7 + 81920*
A*a^3*c^8)/c^9)*x - (15015*B*b^8*c^3 - 171864*B*a*b^6*c^4 - 23100*A*b^7*c^
4 + 662400*B*a^2*b^4*c^5 + 246960*A*a*b^5*c^5 - 917120*B*a^3*b^2*c^6 - 843
840*A*a^2*b^3*c^6 + 241920*B*a^4*c^7 + 881920*A*a^3*b*c^7)/c^9)*x + (45045
*B*b^9*c^2 - 563640*B*a*b^7*c^3 - 69300*A*b^8*c^3 + 2487744*B*a^2*b^5*c^4
+ 814800*A*a*b^6*c^4 - 4406400*B*a^3*b^3*c^5 - 3245760*A*a^2*b^4*c^5 + 237
9520*B*a^4*b*c^6 + 4688640*A*a^3*b^2*c^6 - 1310720*A*a^4*c^7)/c^9) + 1/262
144*(143*B*b^10 - 1980*B*a*b^8*c - 220*A*b^9*c + 10080*B*a^2*b^6*c^2 + 288
0*A*a*b^7*c^2 - 22400*B*a^3*b^4*c^3 - 13440*A*a^2*b^5*c^3 + 19200*B*a^4*b^
2*c^4 + 25600*A*a^3*b^3*c^4 - 3072*B*a^5*c^5 - 15360*A*a^4*b*c^5)*log(abs(
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(15/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int x^3\,\left(A+B\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2}\,dx$$

input
```
int(x^3*(A + B*x)*(a + b*x + c*x^2)^(5/2),x)
```

output
```
int(x^3*(A + B*x)*(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int x^3(Bx+A)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}dx$$

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

3.131 $\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 333

$$\begin{aligned}
\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx =
&-\frac{5(b^2-4ac)^2\left(11b^3B-18Ab^2c-12abBc+8aAc^2\right)(b+2cx)\sqrt{a+bx+cx^2}}{32768c^6}\\
&+\frac{5(b^2-4ac)\left(11b^3B-18Ab^2c-12abBc+8aAc^2\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{12288c^5}\\
&-\frac{\left(11b^3B-18Ab^2c-12abBc+8aAc^2\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{768c^4}\\
&+\frac{Bx^2\left(a+bx+cx^2\right)^{7/2}}{9c}\\
&+\frac{\left(99b^2B-162Abc-64aBc-14c(11bB-18Ac)x\right)\left(a+bx+cx^2\right)^{7/2}}{2016c^3}\\
&+\frac{5(b^2-4ac)^3\left(11b^3B-18Ab^2c-12abBc+8aAc^2\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{65536c^{13/2}}
\end{aligned}$$

output
```
-5/32768*(-4*a*c+b^2)^2*(8*A*a*c^2-18*A*b^2*c-12*B*a*b*c+11*B*b^3)*(2*c*x+
b)*(c*x^2+b*x+a)^(1/2)/c^6+5/12288*(-4*a*c+b^2)*(8*A*a*c^2-18*A*b^2*c-12*B
*a*b*c+11*B*b^3)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^5-1/768*(8*A*a*c^2-18*A*b
^2*c-12*B*a*b*c+11*B*b^3)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c^4+1/9*B*x^2*(c*x
^2+b*x+a)^(7/2)/c+1/2016*(99*B*b^2-162*A*b*c-64*B*a*c-14*c*(-18*A*c+11*B*b
)*x)*(c*x^2+b*x+a)^(7/2)/c^3+5/65536*(-4*a*c+b^2)^3*(8*A*a*c^2-18*A*b^2*c-
12*B*a*b*c+11*B*b^3)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^
(13/2)
```

Mathematica [A] (verified)

Time = 4.06 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.45

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^8B+210b^7c(27A+11Bx)+84b^6c(485aB-cx(45A+22Bx)}{}$$

input
```
Integrate[x^2*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^8*B + 210*b^7*c*(27*A + 11*B*x)
+ 84*b^6*c*(485*a*B - c*x*(45*A + 22*B*x)) + 72*b^5*c^2*(2*c*x^2*(21*A + 1
1*B*x) - 7*a*(125*A + 49*B*x)) + 128*b*c^4*(6*a^2*c*x^2*(87*A + 41*B*x) -
13*a^3*(153*A + 53*B*x) + 24*a*c^2*x^4*(307*A + 251*B*x) + 16*c^3*x^6*(297
*A + 259*B*x)) + 192*b^2*c^3*(1221*a^3*B + 4*a*c^2*x^3*(27*A + 14*B*x) + 8
*c^3*x^5*(243*A + 206*B*x) - a^2*c*x*(597*A + 266*B*x)) + 256*c^4*(-256*a^
4*B + 112*c^4*x^7*(9*A + 8*B*x) + a^3*c*x*(315*A + 128*B*x) + 8*a*c^3*x^5*
(357*A + 304*B*x) + 6*a^2*c^2*x^3*(413*A + 320*B*x)) - 16*b^4*c^2*(10143*a
^2*B + 2*c^2*x^3*(81*A + 44*B*x) - 3*a*c*x*(791*A + 372*B*x)) + 32*b^3*c^3
*(8*c^2*x^4*(9*A + 5*B*x) - 4*a*c*x^2*(213*A + 107*B*x) + 3*a^2*(2359*A +
879*B*x))) - 315*(b^2 - 4*a*c)^3*(11*b^3*B - 18*A*b^2*c - 12*a*b*B*c + 8*a
*A*c^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(4128768*c^(13/2
))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1236, 27, 1225, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{\int -\frac{1}{2}x(4aB+(11bB-18Ac)x)\left(cx^2+bx+a\right)^{5/2}dx}{9c}+\frac{Bx^2\left(a+bx+cx^2\right)^{7/2}}{9c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{7/2}}{9c}-\frac{\int x(4aB+(11bB-18Ac)x)\left(cx^2+bx+a\right)^{5/2}dx}{18c}$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{7/2}}{9c}-\frac{\frac{9\left(8aAc^2-12abBc-18Ab^2c+11b^3B\right)\int \left(cx^2+bx+a\right)^{5/2}dx}{32c^2}-\frac{\left(a+bx+cx^2\right)^{7/2}\left(-64aBc-14cx(11bB-18Ac)-162Abc+99b^2B\right)}{112c^2}}{18c}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{7/2}}{9c}-\frac{\frac{9\left(8aAc^2-12abBc-18Ab^2c+11b^3B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{24c}\right)}{32c^2}-\frac{\left(a+bx+cx^2\right)^{7/2}\left(-64aBc-14cx(11bB-18A\ldots\right.}{112c^2}}{18c}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{7/2}}{9c}-\frac{\frac{9\left(8aAc^2-12abBc-18Ab^2c+11b^3B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{24c}\right)}{32c^2}-\frac{(a+bx+c\ldots}{\ldots}}{18c}$$

$\downarrow$ 1087

$$\frac{Bx^2\left(a+bx+cx^2\right)^{7/2}}{9c}-\frac{9\left(8aAc^2-12abBc-18Ab^2c+11b^3B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int}{}\right.}{16c}\right.}{24c}\right.}{32c^2}}{18c}$$

$\downarrow$ 1092

$$\frac{Bx^2\left(a+bx+cx^2\right)^{7/2}}{9c}-\frac{9\left(8aAc^2-12abBc-18Ab^2c+11b^3B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int}{}\right.}{16c}\right.}{24c}\right.}{32c^2}}{18c}$$

$\downarrow$ 219

$$\frac{Bx^2\left(a+bx+cx^2\right)^{7/2}}{9c}-\frac{\frac{9\left(8aAc^2-12abBc-18Ab^2c+11b^3B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{a}}{}\right)}{16c}\right)}{24c}\right)}{32c^2}}{18c}$$

input
```
Int[x^2*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(B*x^2*(a + b*x + c*x^2)^(7/2))/(9*c) - (-1/112*((99*b^2*B - 162*A*b*c - 6
4*a*B*c - 14*c*(11*b*B - 18*A*c)*x)*(a + b*x + c*x^2)^(7/2))/c^2 + (9*(11*
b^3*B - 18*A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*(((b + 2*c*x)*(a + b*x + c*x^
2)^(5/2))/(12*c) - (5*(b^2 - 4*a*c)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))
/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(
3/2))))/(16*c)))/(24*c)))/(32*c^2))/(18*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(303) = 606$.

Time = 1.29 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.96

method	result
risch	$-\frac{\left(-229376B\,c^{8}x^{8}-258048A\,c^{8}x^{7}-530432Bb\,c^{7}x^{7}-608256Ab\,c^{7}x^{6}-622592Ba\,c^{7}x^{6}-316416B\,b^{2}c^{6}x^{6}-731136Aa\,c^{7}x^{5}-373248\ldots\right.}{\ldots}$
default	$A\left(\frac{x\left(c\,x^{2}+bx+a\right)^{\frac{7}{2}}}{8c}-\frac{9b\left(\frac{\left(c\,x^{2}+bx+a\right)^{\frac{7}{2}}}{7c}-\frac{b\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}}{12c}+\frac{5\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\sqrt{\ldots}}{\ldots}\ldots\right)}{\ldots}\right)}{24c}\right)}{2c}\right)}{16c}\ldots\right.$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2064384/c^6*(-229376*B*c^8*x^8-258048*A*c^8*x^7-530432*B*b*c^7*x^7-6082
56*A*b*c^7*x^6-622592*B*a*c^7*x^6-316416*B*b^2*c^6*x^6-731136*A*a*c^7*x^5-
373248*A*b^2*c^6*x^5-771072*B*a*b*c^6*x^5-1280*B*b^3*c^5*x^5-943104*A*a*b*
c^6*x^4-2304*A*b^3*c^5*x^4-491520*B*a^2*c^6*x^4-10752*B*a*b^2*c^5*x^4+1408
*B*b^4*c^4*x^4-634368*A*a^2*c^6*x^3-20736*A*a*b^2*c^5*x^3+2592*A*b^4*c^4*x
^3-31488*B*a^2*b*c^5*x^3+13696*B*a*b^3*c^4*x^3-1584*B*b^5*c^3*x^3-66816*A*
a^2*b*c^5*x^2+27264*A*a*b^3*c^4*x^2-3024*A*b^5*c^3*x^2-32768*B*a^3*c^5*x^2
+51072*B*a^2*b^2*c^4*x^2-17856*B*a*b^4*c^3*x^2+1848*B*b^6*c^2*x^2-80640*A*
a^3*c^5*x+114624*A*a^2*b^2*c^4*x-37968*A*a*b^4*c^3*x+3780*A*b^6*c^2*x+8819
2*B*a^3*b*c^4*x-84384*B*a^2*b^3*c^3*x+24696*B*a*b^5*c^2*x-2310*B*b^7*c*x+2
54592*A*a^3*b*c^4-226464*A*a^2*b^3*c^3+63000*A*a*b^5*c^2-5670*A*b^7*c+6553
6*B*a^4*c^4-234432*B*a^3*b^2*c^3+162288*B*a^2*b^4*c^2-40740*B*a*b^6*c+3465
*B*b^8)*(c*x^2+b*x+a)^(1/2)-5/65536*(512*A*a^4*c^5-1536*A*a^3*b^2*c^4+960*
A*a^2*b^4*c^3-224*A*a*b^6*c^2+18*A*b^8*c-768*B*a^4*b*c^4+1280*B*a^3*b^3*c^
3-672*B*a^2*b^5*c^2+144*B*a*b^7*c-11*B*b^9)/c^(13/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(303) = 606$.

Time = 0.22 (sec) , antiderivative size = 1263, normalized size of antiderivative = 3.79

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/8257536*(315*(11*B*b^9 - 512*A*a^4*c^5 + 768*(B*a^4*b + 2*A*a^3*b^2)*c
^4 - 320*(4*B*a^3*b^3 + 3*A*a^2*b^4)*c^3 + 224*(3*B*a^2*b^5 + A*a*b^6)*c^2
 - 18*(8*B*a*b^7 + A*b^8)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sq
rt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(229376*B*c^9*x^8 - 3
465*B*b^8*c + 14336*(37*B*b*c^8 + 18*A*c^9)*x^7 + 1024*(309*B*b^2*c^7 + 2*
(304*B*a + 297*A*b)*c^8)*x^6 - 128*(512*B*a^4 + 1989*A*a^3*b)*c^5 + 256*(5
*B*b^3*c^6 + 2856*A*a*c^8 + 6*(502*B*a*b + 243*A*b^2)*c^7)*x^5 + 96*(2442*
B*a^3*b^2 + 2359*A*a^2*b^3)*c^4 - 128*(11*B*b^4*c^5 - 24*(160*B*a^2 + 307*
A*a*b)*c^7 - 6*(14*B*a*b^2 + 3*A*b^3)*c^6)*x^4 - 504*(322*B*a^2*b^4 + 125*
A*a*b^5)*c^3 + 16*(99*B*b^5*c^4 + 39648*A*a^2*c^7 + 48*(41*B*a^2*b + 27*A*
a*b^2)*c^6 - 2*(428*B*a*b^3 + 81*A*b^4)*c^5)*x^3 + 210*(194*B*a*b^6 + 27*A
*b^7)*c^2 - 8*(231*B*b^6*c^3 - 32*(128*B*a^3 + 261*A*a^2*b)*c^6 + 48*(133*
B*a^2*b^2 + 71*A*a*b^3)*c^5 - 18*(124*B*a*b^4 + 21*A*b^5)*c^4)*x^2 + 2*(11
55*B*b^7*c^2 + 40320*A*a^3*c^6 - 32*(1378*B*a^3*b + 1791*A*a^2*b^2)*c^5 +
24*(1758*B*a^2*b^3 + 791*A*a*b^4)*c^4 - 126*(98*B*a*b^5 + 15*A*b^6)*c^3)*x
)*sqrt(c*x^2 + b*x + a))/c^7, -1/4128768*(315*(11*B*b^9 - 512*A*a^4*c^5 +
768*(B*a^4*b + 2*A*a^3*b^2)*c^4 - 320*(4*B*a^3*b^3 + 3*A*a^2*b^4)*c^3 + 22
4*(3*B*a^2*b^5 + A*a*b^6)*c^2 - 18*(8*B*a*b^7 + A*b^8)*c)*sqrt(-c)*arctan(
1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) -
2*(229376*B*c^9*x^8 - 3465*B*b^8*c + 14336*(37*B*b*c^8 + 18*A*c^9)*x^7 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6552 vs. $2(355) = 710$.

Time = 0.94 (sec) , antiderivative size = 6552, normalized size of antiderivative = 19.68

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x+a)**(5/2),x)
```

output
```
Piecewise(((-a*(A*a**3 - 3*a*(3*A*a**2*c + 3*A*a*b**2 + 3*B*a**2*b - 5*a*(
3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 7*a*(A*c**3 + 37*B*b*c**2/1
8)/(8*c) - 13*b*(3*A*b*c**2 + 19*B*a*c**2/9 + 3*B*b**2*c - 15*b*(A*c**3 +
37*B*b*c**2/18)/(16*c))/(14*c))/(6*c) - 9*b*(6*A*a*b*c + A*b**3 + 3*B*a**2
*c + 3*B*a*b**2 - 6*a*(3*A*b*c**2 + 19*B*a*c**2/9 + 3*B*b**2*c - 15*b*(A*c
**3 + 37*B*b*c**2/18)/(16*c))/(7*c) - 11*b*(3*A*a*c**2 + 3*A*b**2*c + 6*B*
a*b*c + B*b**3 - 7*a*(A*c**3 + 37*B*b*c**2/18)/(8*c) - 13*b*(3*A*b*c**2 +
19*B*a*c**2/9 + 3*B*b**2*c - 15*b*(A*c**3 + 37*B*b*c**2/18)/(16*c))/(14*c)
)/(12*c))/(10*c))/(4*c) - 5*b*(3*A*a**2*b + B*a**3 - 4*a*(6*A*a*b*c + A*b*
*3 + 3*B*a**2*c + 3*B*a*b**2 - 6*a*(3*A*b*c**2 + 19*B*a*c**2/9 + 3*B*b**2*
c - 15*b*(A*c**3 + 37*B*b*c**2/18)/(16*c))/(7*c) - 11*b*(3*A*a*c**2 + 3*A*
b**2*c + 6*B*a*b*c + B*b**3 - 7*a*(A*c**3 + 37*B*b*c**2/18)/(8*c) - 13*b*(
3*A*b*c**2 + 19*B*a*c**2/9 + 3*B*b**2*c - 15*b*(A*c**3 + 37*B*b*c**2/18)/(
16*c))/(14*c))/(12*c))/(5*c) - 7*b*(3*A*a**2*c + 3*A*a*b**2 + 3*B*a**2*b -
 5*a*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 7*a*(A*c**3 + 37*B*b*
c**2/18)/(8*c) - 13*b*(3*A*b*c**2 + 19*B*a*c**2/9 + 3*B*b**2*c - 15*b*(A*c
**3 + 37*B*b*c**2/18)/(16*c))/(14*c))/(6*c) - 9*b*(6*A*a*b*c + A*b**3 + 3*
B*a**2*c + 3*B*a*b**2 - 6*a*(3*A*b*c**2 + 19*B*a*c**2/9 + 3*B*b**2*c - 15*
b*(A*c**3 + 37*B*b*c**2/18)/(16*c))/(7*c) - 11*b*(3*A*a*c**2 + 3*A*b**2*c
+ 6*B*a*b*c + B*b**3 - 7*a*(A*c**3 + 37*B*b*c**2/18)/(8*c) - 13*b*(3*A*...
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(303) = 606$.

Time = 0.27 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.92

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \frac{1}{2064384}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(4\left(14\left(16\,Bc^2x+\frac{37\,Bbc^9+18\,Ac^{10}}{c^8}\right)x+\frac{309\,B\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{5\left(11\,Bb^9-144\,Bab^7c-18\,Ab^8c+672\,Ba^2b^5c^2+224\,Aab^6c^2-1280\,Ba^3b^3c^3-960\,Aa^2b^4c^3+768\,Ba^4b\right.}{65536\,c^{\frac{13}{2}}}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/2064384*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(4*(14*(16*B*c^2*x + (37*B*
b*c^9 + 18*A*c^10)/c^8)*x + (309*B*b^2*c^8 + 608*B*a*c^9 + 594*A*b*c^9)/c^
8)*x + (5*B*b^3*c^7 + 3012*B*a*b*c^8 + 1458*A*b^2*c^8 + 2856*A*a*c^9)/c^8)
*x - (11*B*b^4*c^6 - 84*B*a*b^2*c^7 - 18*A*b^3*c^7 - 3840*B*a^2*c^8 - 7368
*A*a*b*c^8)/c^8)*x + (99*B*b^5*c^5 - 856*B*a*b^3*c^6 - 162*A*b^4*c^6 + 196
8*B*a^2*b*c^7 + 1296*A*a*b^2*c^7 + 39648*A*a^2*c^8)/c^8)*x - (231*B*b^6*c^
4 - 2232*B*a*b^4*c^5 - 378*A*b^5*c^5 + 6384*B*a^2*b^2*c^6 + 3408*A*a*b^3*c
^6 - 4096*B*a^3*c^7 - 8352*A*a^2*b*c^7)/c^8)*x + (1155*B*b^7*c^3 - 12348*B
*a*b^5*c^4 - 1890*A*b^6*c^4 + 42192*B*a^2*b^3*c^5 + 18984*A*a*b^4*c^5 - 44
096*B*a^3*b*c^6 - 57312*A*a^2*b^2*c^6 + 40320*A*a^3*c^7)/c^8)*x - (3465*B*
b^8*c^2 - 40740*B*a*b^6*c^3 - 5670*A*b^7*c^3 + 162288*B*a^2*b^4*c^4 + 6300
0*A*a*b^5*c^4 - 234432*B*a^3*b^2*c^5 - 226464*A*a^2*b^3*c^5 + 65536*B*a^4*
c^6 + 254592*A*a^3*b*c^6)/c^8) - 5/65536*(11*B*b^9 - 144*B*a*b^7*c - 18*A*
b^8*c + 672*B*a^2*b^5*c^2 + 224*A*a*b^6*c^2 - 1280*B*a^3*b^3*c^3 - 960*A*a
^2*b^4*c^3 + 768*B*a^4*b*c^4 + 1536*A*a^3*b^2*c^4 - 512*A*a^4*c^5)*log(abs
(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int x^2\,(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}\,dx$$

input
```
int(x^2*(A + B*x)*(a + b*x + c*x^2)^(5/2),x)
```

output
```
int(x^2*(A + B*x)*(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int x^2(Bx+A)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}dx$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

3.132 $\int x(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 252

$$\begin{aligned}\int x(A+Bx)\left(a+bx\right.&\left.+cx^2\right)^{5/2}\,dx=\frac{5(b^2-4ac)^2\left(9b^2B-16Abc-4aBc\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5}\\&-\frac{5(b^2-4ac)\left(9b^2B-16Abc-4aBc\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{6144c^4}\\&+\frac{\left(9b^2B-16Abc-4aBc\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{384c^3}\\&-\frac{(9bB-16Ac-14Bcx)\left(a+bx+cx^2\right)^{7/2}}{112c^2}\\&-\frac{5(b^2-4ac)^3\left(9b^2B-16Abc-4aBc\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}}\end{aligned}$$

output
```
5/16384*(-4*a*c+b^2)^2*(-16*A*b*c-4*B*a*c+9*B*b^2)*(2*c*x+b)*(c*x^2+b*x+a)
^(1/2)/c^5-5/6144*(-4*a*c+b^2)*(-16*A*b*c-4*B*a*c+9*B*b^2)*(2*c*x+b)*(c*x^
2+b*x+a)^(3/2)/c^4+1/384*(-16*A*b*c-4*B*a*c+9*B*b^2)*(2*c*x+b)*(c*x^2+b*x+
a)^(5/2)/c^3-1/112*(-14*B*c*x-16*A*c+9*B*b)*(c*x^2+b*x+a)^(7/2)/c^2-5/3276
8*(-4*a*c+b^2)^3*(-16*A*b*c-4*B*a*c+9*B*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)
/(c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.55

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(945b^7B-210b^6c(8A+3Bx)+28b^5c(-375aB+2cx(20A+9Bx))+}{}$$

input
```
Integrate[x*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^7*B - 210*b^6*c*(8*A + 3*B*x) + 28
*b^5*c*(-375*a*B + 2*c*x*(20*A + 9*B*x)) + 16*b^3*c^2*(2359*a^2*B + 24*c^2
*x^3*(2*A + B*x) - 4*a*c*x*(168*A + 71*B*x)) + 8*b^4*c^2*(-2*c*x^2*(56*A +
 27*B*x) + 7*a*(320*A + 113*B*x)) + 32*b^2*c^3*(12*a*c*x^2*(20*A + 9*B*x)
- 3*a^2*(616*A + 199*B*x) + 8*c^2*x^4*(296*A + 243*B*x)) + 64*b*c^3*(-663*
a^3*B + 6*a^2*c*x*(76*A + 29*B*x) + 16*c^3*x^5*(116*A + 99*B*x) + 8*a*c^2*
x^3*(394*A + 307*B*x)) + 128*c^4*(48*c^3*x^6*(8*A + 7*B*x) + 3*a^3*(128*A
+ 35*B*x) + 8*a*c^2*x^4*(144*A + 119*B*x) + 2*a^2*c*x^2*(576*A + 413*B*x))
) + 105*(b^2 - 4*a*c)^3*(9*b^2*B - 16*A*b*c - 4*a*B*c)*Log[b + 2*c*x - 2*S
qrt[c]*Sqrt[a + x*(b + c*x)]])/(688128*c^(11/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1225, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx$$

$$\Big\downarrow \; 1225$$

$$\frac{\left(-4aBc-16Abc+9b^2B\right)\int \left(cx^2+bx+a\right)^{5/2}dx}{32c^2}-\frac{\left(a+bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$\Big\downarrow$ 1087

$$\frac{\left(-4aBc-16Abc+9b^2B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{24c}\right)}{32c^2}-\frac{\left(a+bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$\Big\downarrow$ 1087

$$\frac{\left(-4aBc-16Abc+9b^2B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{24c}\right)}{32c^2}-\frac{\left(a+bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$\Big\downarrow$ 1087

$$\frac{\left(-4aBc-16Abc+9b^2B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4a\cdots\right.}{\cdots}\right.}{16c}\right.}{24c}\cdots\right)}{32c^2}-\frac{\left(a+bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$\Big\downarrow$ 1092

$$\frac{\left(-4aBc-16Abc+9b^2B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\cdots}{\cdots}\right)}{16c}\right)}{24c}\right)}{32c^2}$$

$$\frac{\left(a+bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$\Big\downarrow$ 219

$$\frac{\left(-4aBc-16Abc+9b^2B\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\cdots}{\cdots}\right)}{16c}\right)}{24c}\right)}{32c^2}$$

$$\frac{\left(a+bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

input
```
Int[x*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
-1/112*((9*b*B - 16*A*c - 14*B*c*x)*(a + b*x + c*x^2)^(7/2))/c^2 + ((9*b^2
*B - 16*A*b*c - 4*a*B*c)*(((b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(12*c) - (
5*(b^2 - 4*a*c)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4
*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(24
*c)))/(32*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(226) = 452$.

Time = 1.23 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.03

method	result
default	$A\left(\frac{(c x^2+bx+a)^{\frac{7}{2}}}{7c}-\frac{b\left(\frac{(2cx+b)\left(c x^2+bx+a\right)^{\frac{5}{2}}}{12c}+\frac{5\left(4ac-b^2\right)\left(\frac{(2cx+b)\left(c x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)}{}\right.}{16c}\right.}{24c}\right.}{2c}\right.$
risch	$\frac{(43008B\,c^7x^7+49152A\,c^7x^6+101376Bb\,c^6x^6+118784Ab\,c^6x^5+121856Ba\,c^6x^5+62208B\,b^2c^5x^5+147456Aa\,c^6x^4+75776A\,b^2c^5x^4}{}$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/7*(c*x^2+b*x+a)^(7/2)/c-1/2*b/c*(1/12*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c
+5/24*(4*a*c-b^2)/c*(1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c+3/16*(4*a*c-b^2)/
c*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+B*(1/8*x*(c*x^2+b*x+a)^(7/2)/c-9/16*b
/c*(1/7*(c*x^2+b*x+a)^(7/2)/c-1/2*b/c*(1/12*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/
c+5/24*(4*a*c-b^2)/c*(1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c+3/16*(4*a*c-b^2)
/c*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))-1/8*a/c*(1/12*(2*c*x+b)*(c*x^2+b*x+a
)^(5/2)/c+5/24*(4*a*c-b^2)/c*(1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c+3/16*(4*
a*c-b^2)/c*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(226) = 452$.

Time = 0.17 (sec) , antiderivative size = 1039, normalized size of antiderivative = 4.12

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/1376256*(105*(9*B*b^8 + 256*(B*a^4 + 4*A*a^3*b)*c^4 - 768*(B*a^3*b^2 +
A*a^2*b^3)*c^3 + 96*(5*B*a^2*b^4 + 2*A*a*b^5)*c^2 - 16*(7*B*a*b^6 + A*b^7)
*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(c) - 4*a*c) + 4*(43008*B*c^8*x^7 + 945*B*b^7*c + 49152*A*a^3*c
^5 + 3072*(33*B*b*c^7 + 16*A*c^8)*x^6 + 256*(243*B*b^2*c^6 + 4*(119*B*a +
116*A*b)*c^7)*x^5 - 192*(221*B*a^3*b + 308*A*a^2*b^2)*c^4 + 128*(3*B*b^3*c
^5 + 1152*A*a*c^7 + 4*(307*B*a*b + 148*A*b^2)*c^6)*x^4 + 112*(337*B*a^2*b^
3 + 160*A*a*b^4)*c^3 - 16*(27*B*b^4*c^4 - 16*(413*B*a^2 + 788*A*a*b)*c^6 -
 24*(9*B*a*b^2 + 2*A*b^3)*c^5)*x^3 - 420*(25*B*a*b^5 + 4*A*b^6)*c^2 + 8*(6
3*B*b^5*c^3 + 18432*A*a^2*c^6 + 48*(29*B*a^2*b + 20*A*a*b^2)*c^5 - 8*(71*B
*a*b^3 + 14*A*b^4)*c^4)*x^2 - 2*(315*B*b^6*c^2 - 192*(35*B*a^3 + 76*A*a^2*
b)*c^5 + 48*(199*B*a^2*b^2 + 112*A*a*b^3)*c^4 - 28*(113*B*a*b^4 + 20*A*b^5
)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/688128*(105*(9*B*b^8 + 256*(B*a^4
+ 4*A*a^3*b)*c^4 - 768*(B*a^3*b^2 + A*a^2*b^3)*c^3 + 96*(5*B*a^2*b^4 + 2*A
*a*b^5)*c^2 - 16*(7*B*a*b^6 + A*b^7)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(43008*B*c^8*x^7
 + 945*B*b^7*c + 49152*A*a^3*c^5 + 3072*(33*B*b*c^7 + 16*A*c^8)*x^6 + 256*
(243*B*b^2*c^6 + 4*(119*B*a + 116*A*b)*c^7)*x^5 - 192*(221*B*a^3*b + 308*A
*a^2*b^2)*c^4 + 128*(3*B*b^3*c^5 + 1152*A*a*c^7 + 4*(307*B*a*b + 148*A*b^2
)*c^6)*x^4 + 112*(337*B*a^2*b^3 + 160*A*a*b^4)*c^3 - 16*(27*B*b^4*c^4 -...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4009 vs. $2(262) = 524$.

Time = 0.96 (sec) , antiderivative size = 4009, normalized size of antiderivative = 15.91

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x+a)**(5/2),x)
```

output
```
Piecewise(((-a*(3*A*a**2*b + B*a**3 - 3*a*(6*A*a*b*c + A*b**3 + 3*B*a**2*c
 + 3*B*a*b**2 - 5*a*(3*A*b*c**2 + 17*B*a*c**2/8 + 3*B*b**2*c - 13*b*(A*c**
3 + 33*B*b*c**2/16)/(14*c))/(6*c) - 9*b*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b
*c + B*b**3 - 6*a*(A*c**3 + 33*B*b*c**2/16)/(7*c) - 11*b*(3*A*b*c**2 + 17*
B*a*c**2/8 + 3*B*b**2*c - 13*b*(A*c**3 + 33*B*b*c**2/16)/(14*c))/(12*c))/(
10*c))/(4*c) - 5*b*(3*A*a**2*c + 3*A*a*b**2 + 3*B*a**2*b - 4*a*(3*A*a*c**2
 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 6*a*(A*c**3 + 33*B*b*c**2/16)/(7*c) -
 11*b*(3*A*b*c**2 + 17*B*a*c**2/8 + 3*B*b**2*c - 13*b*(A*c**3 + 33*B*b*c**
2/16)/(14*c))/(12*c))/(5*c) - 7*b*(6*A*a*b*c + A*b**3 + 3*B*a**2*c + 3*B*a
*b**2 - 5*a*(3*A*b*c**2 + 17*B*a*c**2/8 + 3*B*b**2*c - 13*b*(A*c**3 + 33*B
*b*c**2/16)/(14*c))/(6*c) - 9*b*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b
**3 - 6*a*(A*c**3 + 33*B*b*c**2/16)/(7*c) - 11*b*(3*A*b*c**2 + 17*B*a*c**2
/8 + 3*B*b**2*c - 13*b*(A*c**3 + 33*B*b*c**2/16)/(14*c))/(12*c))/(10*c))/(
8*c))/(6*c))/(2*c) - b*(A*a**3 - 2*a*(3*A*a**2*c + 3*A*a*b**2 + 3*B*a**2*b
 - 4*a*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 6*a*(A*c**3 + 33*B*
b*c**2/16)/(7*c) - 11*b*(3*A*b*c**2 + 17*B*a*c**2/8 + 3*B*b**2*c - 13*b*(A
*c**3 + 33*B*b*c**2/16)/(14*c))/(12*c))/(5*c) - 7*b*(6*A*a*b*c + A*b**3 +
3*B*a**2*c + 3*B*a*b**2 - 5*a*(3*A*b*c**2 + 17*B*a*c**2/8 + 3*B*b**2*c - 1
3*b*(A*c**3 + 33*B*b*c**2/16)/(14*c))/(6*c) - 9*b*(3*A*a*c**2 + 3*A*b**2*c
 + 6*B*a*b*c + B*b**3 - 6*a*(A*c**3 + 33*B*b*c**2/16)/(7*c) - 11*b*(3*A...
```

Maxima [F(-2)]

Exception generated.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(226) = 452.

Time = 0.28 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.09

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{1}{344064}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(12\left(14\,Bc^2x+\frac{33\,Bbc^8+16\,Ac^9}{c^7}\right)x+\frac{243\,Bb^2c^7\cdots}{\cdots}\right.\right.\right.\right.\right.\right.$$
$$+\frac{5\left(9\,Bb^8-112\,Bab^6c-16\,Ab^7c+480\,Ba^2b^4c^2+192\,Aab^5c^2-768\,Ba^3b^2c^3-768\,Aa^2b^3c^3+256\,Ba^4c^4+\cdots\right.}{32768\,c^{\frac{11}{2}}}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/344064*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(14*B*c^2*x + (33*B*b*c^
8 + 16*A*c^9)/c^7)*x + (243*B*b^2*c^7 + 476*B*a*c^8 + 464*A*b*c^8)/c^7)*x
+ (3*B*b^3*c^6 + 1228*B*a*b*c^7 + 592*A*b^2*c^7 + 1152*A*a*c^8)/c^7)*x - (
27*B*b^4*c^5 - 216*B*a*b^2*c^6 - 48*A*b^3*c^6 - 6608*B*a^2*c^7 - 12608*A*a
*b*c^7)/c^7)*x + (63*B*b^5*c^4 - 568*B*a*b^3*c^5 - 112*A*b^4*c^5 + 1392*B*
a^2*b*c^6 + 960*A*a*b^2*c^6 + 18432*A*a^2*c^7)/c^7)*x - (315*B*b^6*c^3 - 3
164*B*a*b^4*c^4 - 560*A*b^5*c^4 + 9552*B*a^2*b^2*c^5 + 5376*A*a*b^3*c^5 -
6720*B*a^3*c^6 - 14592*A*a^2*b*c^6)/c^7)*x + (945*B*b^7*c^2 - 10500*B*a*b^
5*c^3 - 1680*A*b^6*c^3 + 37744*B*a^2*b^3*c^4 + 17920*A*a*b^4*c^4 - 42432*B
*a^3*b*c^5 - 59136*A*a^2*b^2*c^5 + 49152*A*a^3*c^6)/c^7) + 5/32768*(9*B*b^
8 - 112*B*a*b^6*c - 16*A*b^7*c + 480*B*a^2*b^4*c^2 + 192*A*a*b^5*c^2 - 768
*B*a^3*b^2*c^3 - 768*A*a^2*b^3*c^3 + 256*B*a^4*c^4 + 1024*A*a^3*b*c^4)*log
(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int x\,\left(A+B\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2}\,dx$$

input
```
int(x*(A + B*x)*(a + b*x + c*x^2)^(5/2),x)
```

output
```
int(x*(A + B*x)*(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int x(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int x(Bx+A)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}dx$$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

3.133 $\int (A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 203

$$\begin{aligned}\int (A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = &-\frac{5(b^2-4ac)^2(bB-2Ac)(b+2cx)\sqrt{a+bx+cx^2}}{1024c^4}\\ &+\frac{5(b^2-4ac)(bB-2Ac)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{384c^3}\\ &-\frac{(bB-2Ac)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{24c^2}+\frac{B\left(a+bx+cx^2\right)^{7/2}}{7c}\\ &+\frac{5(b^2-4ac)^3(bB-2Ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{9/2}}\end{aligned}$$

output
```
-5/1024*(-4*a*c+b^2)^2*(-2*A*c+B*b)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4+5/38
4*(-4*a*c+b^2)*(-2*A*c+B*b)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^3-1/24*(-2*A*c
+B*b)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c^2+1/7*B*(c*x^2+b*x+a)^(7/2)/c+5/2048
*(-4*a*c+b^2)^3*(-2*A*c+B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 4.24 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.53

$$\int (A+Bx) \left(a+bx+cx^2\right)^{5/2} dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-105b^6B+70b^5c(3A+Bx)+28b^4c(40aB-cx(5A+2Bx))+16b^3c^2(\ldots$$

input
```
Integrate[(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^6*B + 70*b^5*c*(3*A + B*x) + 28*b^4
*c*(40*a*B - c*x*(5*A + 2*B*x)) + 16*b^3*c^2*(c*x^2*(7*A + 3*B*x) - 14*a*(
10*A + 3*B*x)) + 64*c^3*(48*a^3*B + 8*c^3*x^5*(7*A + 6*B*x) + 3*a^2*c*x*(7
7*A + 48*B*x) + 2*a*c^2*x^3*(91*A + 72*B*x)) + 16*b^2*c^2*(-231*a^2*B + 6*
a*c*x*(14*A + 5*B*x) + 2*c^2*x^3*(189*A + 148*B*x)) + 32*b*c^3*(3*a^2*(77*
A + 19*B*x) + 8*c^2*x^4*(35*A + 29*B*x) + 2*a*c*x^2*(273*A + 197*B*x))) +
105*(b^2 - 4*a*c)^3*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a +
 x*(b + c*x)])])/(21504*c^(9/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1160, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx) \left(a+bx+cx^2\right)^{5/2} dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{B\left(a+bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\int \left(cx^2+bx+a\right)^{5/2}dx}{2c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{B\left(a+bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\int\left(cx^2+bx+a\right)^{3/2}dx}{24c}\right)}{2c}$$

$\downarrow$ 1087

$$\frac{B\left(a+bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^2+bx+a}dx}{16c}\right)}{24c}\right)}{2c}$$

$\downarrow$ 1087

$$\frac{B\left(a+bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{24c}\right)}{2c}$$

$\downarrow$ 1092

$$\frac{B\left(a+bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d}{4c}\right)}{16c}\right)}{24c}\right)}{2c}$$

$\downarrow$ 219

$$\frac{B\left(a+bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)}{24c}\right)}{2c}$$

input
```
Int[(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(B*(a + b*x + c*x^2)^(7/2))/(7*c) - ((b*B - 2*A*c)*(((b + 2*c*x)*(a + b*x
+ c*x^2)^(5/2))/(12*c) - (5*(b^2 - 4*a*c)*(((b + 2*c*x)*(a + b*x + c*x^2)^
(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c)
 - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(8*c^(3/2))))/(16*c)))/(24*c)))/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.55

method	result
default	$A\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}{12c}+\frac{5\left(4ac-b^2\right)\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{24c}\right.$
risch	$\frac{(3072B\,c^6x^6+3584A\,c^6x^5+7424Bb\,c^5x^5+8960Ab\,c^5x^4+9216Ba\,c^5x^4+4736B\,b^2c^4x^4+11648Aa\,c^5x^3+6048A\,b^2c^4x^3+12608Bab\,c}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/12*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c+5/24*(4*a*c-b^2)/c*(1/8*(2*c*x+b)*
(c*x^2+b*x+a)^(3/2)/c+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2
)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
+B*(1/7*(c*x^2+b*x+a)^(7/2)/c-1/2*b/c*(1/12*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/
c+5/24*(4*a*c-b^2)/c*(1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c+3/16*(4*a*c-b^2)
/c*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(177) = 354$.

Time = 0.15 (sec) , antiderivative size = 843, normalized size of antiderivative = 4.15

$$\int (A + Bx) \left(a + bx + cx^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/86016*(105*(B*b^7 + 128*A*a^3*c^4 - 32*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 +
24*(2*B*a^2*b^3 + A*a*b^4)*c^2 - 2*(6*B*a*b^5 + A*b^6)*c)*sqrt(c)*log(-8*c
^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a
*c) + 4*(3072*B*c^7*x^6 - 105*B*b^6*c + 256*(29*B*b*c^6 + 14*A*c^7)*x^5 +
96*(32*B*a^3 + 77*A*a^2*b)*c^4 + 128*(37*B*b^2*c^5 + 2*(36*B*a + 35*A*b)*c
^6)*x^4 - 112*(33*B*a^2*b^2 + 20*A*a*b^3)*c^3 + 16*(3*B*b^3*c^4 + 728*A*a*
c^6 + 2*(394*B*a*b + 189*A*b^2)*c^5)*x^3 + 70*(16*B*a*b^4 + 3*A*b^5)*c^2 -
 8*(7*B*b^4*c^3 - 24*(48*B*a^2 + 91*A*a*b)*c^5 - 2*(30*B*a*b^2 + 7*A*b^3)*
c^4)*x^2 + 2*(35*B*b^5*c^2 + 7392*A*a^2*c^5 + 48*(19*B*a^2*b + 14*A*a*b^2)
*c^4 - 14*(24*B*a*b^3 + 5*A*b^4)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/43
008*(105*(B*b^7 + 128*A*a^3*c^4 - 32*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 24*(2
*B*a^2*b^3 + A*a*b^4)*c^2 - 2*(6*B*a*b^5 + A*b^6)*c)*sqrt(-c)*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(30
72*B*c^7*x^6 - 105*B*b^6*c + 256*(29*B*b*c^6 + 14*A*c^7)*x^5 + 96*(32*B*a^
3 + 77*A*a^2*b)*c^4 + 128*(37*B*b^2*c^5 + 2*(36*B*a + 35*A*b)*c^6)*x^4 - 1
12*(33*B*a^2*b^2 + 20*A*a*b^3)*c^3 + 16*(3*B*b^3*c^4 + 728*A*a*c^6 + 2*(39
4*B*a*b + 189*A*b^2)*c^5)*x^3 + 70*(16*B*a*b^4 + 3*A*b^5)*c^2 - 8*(7*B*b^4
*c^3 - 24*(48*B*a^2 + 91*A*a*b)*c^5 - 2*(30*B*a*b^2 + 7*A*b^3)*c^4)*x^2 +
2*(35*B*b^5*c^2 + 7392*A*a^2*c^5 + 48*(19*B*a^2*b + 14*A*a*b^2)*c^4 - 14*(
24*B*a*b^3 + 5*A*b^4)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2428 vs. $2(197) = 394$.

Time = 0.70 (sec) , antiderivative size = 2428, normalized size of antiderivative = 11.96

$$\int (A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(B*c**2*x**6/7 + x**5*(A*c**3 + 29*B*b*c
**2/14)/(6*c) + x**4*(3*A*b*c**2 + 15*B*a*c**2/7 + 3*B*b**2*c - 11*b*(A*c*
*3 + 29*B*b*c**2/14)/(12*c))/(5*c) + x**3*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a
*b*c + B*b**3 - 5*a*(A*c**3 + 29*B*b*c**2/14)/(6*c) - 9*b*(3*A*b*c**2 + 15
*B*a*c**2/7 + 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(10*c))/
(4*c) + x**2*(6*A*a*b*c + A*b**3 + 3*B*a**2*c + 3*B*a*b**2 - 4*a*(3*A*b*c*
*2 + 15*B*a*c**2/7 + 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(
5*c) - 7*b*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 5*a*(A*c**3 + 2
9*B*b*c**2/14)/(6*c) - 9*b*(3*A*b*c**2 + 15*B*a*c**2/7 + 3*B*b**2*c - 11*b
*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(10*c))/(8*c))/(3*c) + x*(3*A*a**2*c +
3*A*a*b**2 + 3*B*a**2*b - 3*a*(3*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**
3 - 5*a*(A*c**3 + 29*B*b*c**2/14)/(6*c) - 9*b*(3*A*b*c**2 + 15*B*a*c**2/7
+ 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(10*c))/(4*c) - 5*b*
(6*A*a*b*c + A*b**3 + 3*B*a**2*c + 3*B*a*b**2 - 4*a*(3*A*b*c**2 + 15*B*a*c
**2/7 + 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(5*c) - 7*b*(3
*A*a*c**2 + 3*A*b**2*c + 6*B*a*b*c + B*b**3 - 5*a*(A*c**3 + 29*B*b*c**2/14
)/(6*c) - 9*b*(3*A*b*c**2 + 15*B*a*c**2/7 + 3*B*b**2*c - 11*b*(A*c**3 + 29
*B*b*c**2/14)/(12*c))/(10*c))/(8*c))/(6*c))/(2*c) + (3*A*a**2*b + B*a**3 -
 2*a*(6*A*a*b*c + A*b**3 + 3*B*a**2*c + 3*B*a*b**2 - 4*a*(3*A*b*c**2 + 15*
B*a*c**2/7 + 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(5*c) ...
```

Maxima [F(-2)]

Exception generated.

$$\int (A + Bx) \left(a + bx + cx^2\right)^{5/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(177) = 354$.

Time = 0.27 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.08

$$\int (A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \frac{1}{21504}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(12\,Bc^2x+\frac{29\,Bbc^7+14\,Ac^8}{c^6}\right)x+\frac{37\,Bb^2c^6+72\,Bc\right.\right.\right.\right.\right.\right. - \frac{5\left(Bb^7-12\,Bab^5c-2\,Ab^6c+48\,Ba^2b^3c^2+24\,Aab^4c^2-64\,Ba^3bc^3-96\,Aa^2b^2c^3+128\,Aa^3c^4\right)\log\left(\left|2\left(\sqrt{\right.\right.\right.}{2048\,c^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/21504*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*B*c^2*x + (29*B*b*c^7 + 1
4*A*c^8)/c^6)*x + (37*B*b^2*c^6 + 72*B*a*c^7 + 70*A*b*c^7)/c^6)*x + (3*B*b
^3*c^5 + 788*B*a*b*c^6 + 378*A*b^2*c^6 + 728*A*a*c^7)/c^6)*x - (7*B*b^4*c^
4 - 60*B*a*b^2*c^5 - 14*A*b^3*c^5 - 1152*B*a^2*c^6 - 2184*A*a*b*c^6)/c^6)*
x + (35*B*b^5*c^3 - 336*B*a*b^3*c^4 - 70*A*b^4*c^4 + 912*B*a^2*b*c^5 + 672
*A*a*b^2*c^5 + 7392*A*a^2*c^6)/c^6)*x - (105*B*b^6*c^2 - 1120*B*a*b^4*c^3
- 210*A*b^5*c^3 + 3696*B*a^2*b^2*c^4 + 2240*A*a*b^3*c^4 - 3072*B*a^3*c^5 -
 7392*A*a^2*b*c^5)/c^6) - 5/2048*(B*b^7 - 12*B*a*b^5*c - 2*A*b^6*c + 48*B*
a^2*b^3*c^2 + 24*A*a*b^4*c^2 - 64*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 128*A*a
^3*c^4)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2
)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \int \left(A+B\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2}\,dx$$

input
```
int((A + B*x)*(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((A + B*x)*(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int (A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int (Bx+A)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}dx$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

output
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

3.134 $\int \frac{(A+Bx)(a+bx+cx^2)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 346

$$\int \frac{(A+Bx)(a+bx+cx^2)^{5/2}}{x}\,dx = \frac{(64aAb^2c^2+512a^2Ac^3+b(b^2-4ac)(5b^2B-12Abc-20aBc)+2c(6}{512c^3}$$

$$-\frac{(5b^3B-12Ab^2c-20abBc-64aAc^2+2c(5b^2B-12Abc-20aBc)x)(a+bx+cx^2)^{3/2}}{192c^2}$$

$$+\frac{(5bB+12Ac+10Bcx)(a+bx+cx^2)^{5/2}}{60c}$$

$$-a^{5/2}A\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)+\frac{(512a^2Abc^3-(b^2-4ac)(64aAbc^2+(b^2-4ac)(5b^2B-12Abc-}{1024c^{7/2}}$$

output
```
1/512*(64*A*a*b^2*c^2+512*A*a^2*c^3+b*(-4*a*c+b^2)*(-12*A*b*c-20*B*a*c+5*B
*b^2)+2*c*(64*A*a*b*c^2+(-4*a*c+b^2)*(-12*A*b*c-20*B*a*c+5*B*b^2))*x)*(c*x
^2+b*x+a)^(1/2)/c^3-1/192*(5*B*b^3-12*A*b^2*c-20*B*a*b*c-64*A*a*c^2+2*c*(-
12*A*b*c-20*B*a*c+5*B*b^2)*x)*(c*x^2+b*x+a)^(3/2)/c^2+1/60*(10*B*c*x+12*A*
c+5*B*b)*(c*x^2+b*x+a)^(5/2)/c-a^(5/2)*A*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*
x^2+b*x+a)^(1/2))+1/1024*(512*a^2*A*b*c^3-(-4*a*c+b^2)*(64*A*a*b*c^2+(-4*a
*c+b^2)*(-12*A*b*c-20*B*a*c+5*B*b^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^
2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x}\,dx = \frac{\sqrt{a+x(b+cx)}(75b^5B-10b^4c(18A+5Bx)+40b^3c(-20aB+cx(3A \\ +2a^{5/2}A\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) \\ -\frac{(-5b^6B+12Ab^5c+60ab^4Bc-160aAb^3c^2-240a^2b^2Bc^2+960a^2Abc^3+320a^3Bc^3)\log\left(b+2cx-2\sqrt{c}\right.}{1024c^{7/2}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x,x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(75*b^5*B - 10*b^4*c*(18*A + 5*B*x) + 40*b^3*c*(-20
*a*B + c*x*(3*A + B*x)) + 48*b^2*c^2*(5*a*(9*A + 2*B*x) + c*x^2*(62*A + 45
*B*x)) + 32*c^3*(8*c^2*x^4*(6*A + 5*B*x) + 2*a*c*x^2*(88*A + 65*B*x) + a^2
*(368*A + 165*B*x)) + 16*b*c^2*(165*a^2*B + 4*c^2*x^3*(63*A + 50*B*x) + 2*
a*c*x*(311*A + 195*B*x))))/(7680*c^3) + 2*a^(5/2)*A*ArcTanh[(Sqrt[c]*x - S
qrt[a + x*(b + c*x)])/Sqrt[a]] - ((-5*b^6*B + 12*A*b^5*c + 60*a*b^4*B*c -
160*a*A*b^3*c^2 - 240*a^2*b^2*B*c^2 + 960*a^2*A*b*c^3 + 320*a^3*B*c^3)*Log
[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(1024*c^(7/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1231, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x}dx$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(12Ac+5bB+10Bcx)}{60c}-\frac{\int -\frac{(24aAc+(12Abc-5B(b^2-4ac))x)\left(cx^2+bx+a\right)^{3/2}}{2x}dx}{12c}$$

$\downarrow$ 27

$$\frac{\int \frac{(24aAc-(5Bb^2-12Acb-20aBc)x)\left(cx^2+bx+a\right)^{3/2}}{x}dx}{24c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12Ac+5bB+10Bcx)}{60c}$$

$\downarrow$ 1231

$$\frac{-\frac{\int -\frac{3\left(128a^2Ac^2+\left(64aAbc^2+\left(b^2-4ac\right)\left(5Bb^2-12Acb-20aBc\right)\right)x\right)\sqrt{cx^2+bx+a}}{2x}dx}{8c}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(2cx\left(-20aBc-12Abc+5b^2B\right)-64aAc^2-20ab\cdots\right.}{8c}}{24c}$$
$$\frac{\left(a+bx+cx^2\right)^{5/2}(12Ac+5bB+10Bcx)}{60c}$$

$\downarrow$ 27

$$\frac{\frac{3\int \frac{\left(128a^2Ac^2+\left(64aAbc^2+\left(b^2-4ac\right)\left(5Bb^2-12Acb-20aBc\right)\right)x\right)\sqrt{cx^2+bx+a}}{x}dx}{16c}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(2cx\left(-20aBc-12Abc+5b^2B\right)-64aAc^2-20abBc\cdots\right.}{8c}}{24c}$$
$$\frac{\left(a+bx+cx^2\right)^{5/2}(12Ac+5bB+10Bcx)}{60c}$$

$\downarrow$ 1231

$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(512a^2Ac^3+2cx\left(\left(b^2-4ac\right)\left(-20aBc-12Abc+5b^2B\right)+64aAbc^2\right)+b\left(\left(b^2-4ac\right)\left(-20aBc-12Abc+5b^2B\right)+64aAbc^2\right)\right)}{4c}-\frac{\int -\frac{1024a^3Ac^3+\left(512a^2\cdots\right.}{\cdots}}{\cdots}\right.}{16c}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(12Ac+5bB+10Bcx)}{60c}$$

$\downarrow$ 27

$$\frac{3\left(\frac{\int \frac{1024a^3Ac^3+\left(512a^2Abc^3-\left(b^2-4ac\right)\left(5Bb^4-12Acb^3-40aBcb^2+112aAc^2b+80a^2Bc^2\right)\right)x}{x\sqrt{cx^2+bx+a}}dx}{8c}+\sqrt{a+bx+cx^2}\left(512a^2Ac^3+2cx\left(\left(b^2-4ac\right)\left(-20aBc-12Abc+\cdots\right.\right.\right.\right.}{16c}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(12Ac+5bB+10Bcx)}{60c}$$

$\downarrow$ 1269

$$3\left(\frac{1024a^3Ac^3\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\left(512a^2Abc^3-\left(b^2-4ac\right)\left(80a^2Bc^2+112aAbc^2-40ab^2Bc-12Ab^3c+5b^4B\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}+\frac{\sqrt{a+bx+cx^2}\left(512a^2Ac^3+2c\right.}{16c}\right.$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(12Ac+5bB+10Bcx\right)}{60c}$$

$$\Big\downarrow \text{ 1092}$$

$$3\left(\frac{1024a^3Ac^3\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+2\left(512a^2Abc^3-\left(b^2-4ac\right)\left(80a^2Bc^2+112aAbc^2-40ab^2Bc-12Ab^3c+5b^4B\right)\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{8c}+\frac{\sqrt{a+bx+cx^2}}{16c}\right.$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(12Ac+5bB+10Bcx\right)}{60c}$$

$$\Big\downarrow \text{ 219}$$

$$3\left(\frac{1024a^3Ac^3\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{\left(512a^2Abc^3-\left(b^2-4ac\right)\left(80a^2Bc^2+112aAbc^2-40ab^2Bc-12Ab^3c+5b^4B\right)\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx+cx^2}\left(\right.}{16c}\right.$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(12Ac+5bB+10Bcx\right)}{60c}$$

$$\Big\downarrow \text{ 1154}$$

$$3\left(\frac{\frac{\left(512a^2Abc^3-\left(b^2-4ac\right)\left(80a^2Bc^2+112aAbc^2-40ab^2Bc-12Ab^3c+5b^4B\right)\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2048a^3Ac^3\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{8c}+\right.$$

$$16c$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(12Ac+5bB+10Bcx\right)}{60c}$$

$$\Big\downarrow \text{ 219}$$

$$3\left(\frac{\sqrt{a+bx+cx^2}\left(512a^2Ac^3+2cx\left(\left(b^2-4ac\right)\left(-20aBc-12Abc+5b^2B\right)+64aAbc^2\right)+b\left(\left(b^2-4ac\right)\left(-20aBc-12Abc+5b^2B\right)+64aAbc^2\right)\right)}{4c}+\frac{\left(512a^2Abc^3-\left(b^2-4ac\right)\right.}{}\right.$$

$$\overline{\qquad 16c \qquad}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(12Ac+5bB+10Bcx)}{60c}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x,x]
```

output
```
((5*b*B + 12*A*c + 10*B*c*x)*(a + b*x + c*x^2)^(5/2))/(60*c) + (-1/8*((5*b
^3*B - 12*A*b^2*c - 20*a*b*B*c - 64*a*A*c^2 + 2*c*(5*b^2*B - 12*A*b*c - 20
*a*B*c)*x)*(a + b*x + c*x^2)^(3/2))/c + (3*(((512*a^2*A*c^3 + b*(64*a*A*b*
c^2 + (b^2 - 4*a*c)*(5*b^2*B - 12*A*b*c - 20*a*B*c)) + 2*c*(64*a*A*b*c^2 +
 (b^2 - 4*a*c)*(5*b^2*B - 12*A*b*c - 20*a*B*c))*x)*Sqrt[a + b*x + c*x^2])/
(4*c) + (-1024*a^(5/2)*A*c^3*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x +
 c*x^2])] + ((512*a^2*A*b*c^3 - (b^2 - 4*a*c)*(5*b^4*B - 12*A*b^3*c - 40*a
*b^2*B*c + 112*a*A*b*c^2 + 80*a^2*B*c^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/Sqrt[c])/(8*c)))/(16*c))/(24*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.25

method	result
default	$B\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}}{12c}+\frac{5\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}-\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{24c}\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
B*(1/12*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c+5/24*(4*a*c-b^2)/c*(1/8*(2*c*x+b)*
(c*x^2+b*x+a)^(3/2)/c+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2
)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
+A*(1/5*(c*x^2+b*x+a)^(5/2)+1/2*b*(1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c+3/1
6*(4*a*c-b^2)/c*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+a*(1/3*(c*x^2+b*x+a)^(3/2
)+1/2*b*(1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c+1/8*(4*a*c-b^2)/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+a*((c*x^2+b*x+a)^(1/2)+1/2*b*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-a^(1/2)*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x))))
```

Fricas [A] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 1575, normalized size of antiderivative = 4.55

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/30720*(15360*A*a^(5/2)*c^4*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c
*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 15*(5*B*b^6 - 320*(B*a
^3 + 3*A*a^2*b)*c^3 + 80*(3*B*a^2*b^2 + 2*A*a*b^3)*c^2 - 12*(5*B*a*b^4 + A
*b^5)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*B*c^6*x^5 + 75*B*b^5*c + 11776*A*a^
2*c^4 + 128*(25*B*b*c^5 + 12*A*c^6)*x^4 + 240*(11*B*a^2*b + 9*A*a*b^2)*c^3
 + 16*(135*B*b^2*c^4 + 4*(65*B*a + 63*A*b)*c^5)*x^3 - 20*(40*B*a*b^3 + 9*A
*b^4)*c^2 + 8*(5*B*b^3*c^3 + 704*A*a*c^5 + 12*(65*B*a*b + 31*A*b^2)*c^4)*x
^2 - 2*(25*B*b^4*c^2 - 16*(165*B*a^2 + 311*A*a*b)*c^4 - 60*(4*B*a*b^2 + A*
b^3)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^4, 1/15360*(7680*A*a^(5/2)*c^4*log(-
(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a)
 + 8*a^2)/x^2) + 15*(5*B*b^6 - 320*(B*a^3 + 3*A*a^2*b)*c^3 + 80*(3*B*a^2*b
^2 + 2*A*a*b^3)*c^2 - 12*(5*B*a*b^4 + A*b^5)*c)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(1280*B*
c^6*x^5 + 75*B*b^5*c + 11776*A*a^2*c^4 + 128*(25*B*b*c^5 + 12*A*c^6)*x^4 +
 240*(11*B*a^2*b + 9*A*a*b^2)*c^3 + 16*(135*B*b^2*c^4 + 4*(65*B*a + 63*A*b
)*c^5)*x^3 - 20*(40*B*a*b^3 + 9*A*b^4)*c^2 + 8*(5*B*b^3*c^3 + 704*A*a*c^5
+ 12*(65*B*a*b + 31*A*b^2)*c^4)*x^2 - 2*(25*B*b^4*c^2 - 16*(165*B*a^2 + 31
1*A*a*b)*c^4 - 60*(4*B*a*b^2 + A*b^3)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^4,
1/30720*(30720*A*sqrt(-a)*a^2*c^4*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x...
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x}\,dx = \int \frac{(A+Bx)\,(a+bx+cx^2)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x, x)
```

Reduce [F]

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x}\,dx=\int \frac{(Bx+A)\,(c\,x^2+bx+a)^{\frac{5}{2}}}{x}dx$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x,x)
```

output
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x,x)
```

3.135 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^2}\,dx =$$

$$-\frac{\left(3b^4B-10Ab^3c-28ab^2Bc-440aAbc^2-128a^2Bc^2+2c(3b^3B-10Ab^2c-28abBc-120aAc^2)\,x\right)\sqrt{a}\ \cdots}{128c^2}$$

$$+\frac{\left(3b^2B+70Abc+16aBc+6c(bB+10Ac)x\right)\left(a+bx+cx^2\right)^{3/2}}{48c}$$

$$-\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$$-\frac{1}{2}a^{3/2}(5Ab+2aB)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)+\frac{(3b^5B-10Ab^4c-40ab^3Bc+240aAb^2c^2+240a^2bBc\ \cdots}{256c^{5/2}}$$

output

```
-1/128*(3*B*b^4-10*A*b^3*c-28*B*a*b^2*c-440*A*a*b*c^2-128*B*a^2*c^2+2*c*(-
120*A*a*c^2-10*A*b^2*c-28*B*a*b*c+3*B*b^3)*x)*(c*x^2+b*x+a)^(1/2)/c^2+1/48
*(3*B*b^2+70*A*b*c+16*B*a*c+6*c*(10*A*c+B*b)*x)*(c*x^2+b*x+a)^(3/2)/c-1/5*
(-B*x+5*A)*(c*x^2+b*x+a)^(5/2)/x-1/2*a^(3/2)*(5*A*b+2*B*a)*arctanh(1/2*(b*
x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))+1/256*(480*A*a^2*c^3+240*A*a*b^2*c^2-1
0*A*b^4*c+240*B*a^2*b*c^2-40*B*a*b^3*c+3*B*b^5)*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^2}\,dx = \frac{\sqrt{a+x(b+cx)}(-128a^2c^2(15A-23Bx)+x(-45b^4B+30b^3c(5A+$$

$$+a^{3/2}(5Ab+2aB)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)$$

$$-\frac{(3b^5B-10Ab^4c-40ab^3Bc+240aAb^2c^2+240a^2bBc^2+480a^2Ac^3)\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+\right.\right.}{256c^{5/2}}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^2,x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(-128*a^2*c^2*(15*A - 23*B*x) + x*(-45*b^4*B + 30*b
^3*c*(5*A + B*x) + 96*c^4*x^3*(5*A + 4*B*x) + 16*b*c^3*x^2*(85*A + 63*B*x)
 + 4*b^2*c^2*x*(295*A + 186*B*x)) + 4*a*c*x*(135*b^2*B + 4*c^2*x*(135*A +
88*B*x) + 2*b*c*(695*A + 311*B*x))))/(1920*c^2*x) + a^(3/2)*(5*A*b + 2*a*B
)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - ((3*b^5*B - 10*A*
b^4*c - 40*a*b^3*B*c + 240*a*A*b^2*c^2 + 240*a^2*b*B*c^2 + 480*a^2*A*c^3)*
Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(256*c^(5/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {1230, 25, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \; 1230$$

$$-\frac{1}{2}\int -\frac{(5Ab+2aB+(bB+10Ac)x)\left(cx^2+bx+a\right)^{3/2}}{x}dx-\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{(5Ab+2aB+(bB+10Ac)x)\left(cx^2+bx+a\right)^{3/2}}{x}dx-\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{\left(a+bx+cx^2\right)^{3/2}\left(16aBc+6cx(10Ac+bB)+70Abc+3b^2B\right)}{24c}-\frac{\int -\frac{(16a(5Ab+2aB)c-(3Bb^3-10Acb^2-28aBcb-12}{2x}}{8c}\right.$$

$$\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{(16a(5Ab+2aB)c-(3Bb^3-10Acb^2-28aBcb-120aAc^2)x)\sqrt{cx^2+bx+a}}{x}dx}{16c}+\frac{\left(a+bx+cx^2\right)^{3/2}\left(16aBc+6cx(10Ac+bB)\right.}{24c}\right.$$

$$\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{-\frac{\int -\frac{128a^2(5Ab+2aB)c^2+\left(3Bb^5-10Acb^4-40aBcb^3+240aAc^2b^2+240a^2Bc^2b+480a^2Ac^3\right)x}{2x\sqrt{cx^2+bx+a}}dx}{4c}-\frac{\sqrt{a+bx+cx^2}\left(-128a^2Bc^2+2cx\left(-120aAc^2-28\right.\right.}{}}{16c}\right.$$

$$\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{128a^2(5Ab+2aB)c^2+\left(3Bb^5-10Acb^4-40aBcb^3+240aAc^2b^2+240a^2Bc^2b+480a^2Ac^3\right)x}{x\sqrt{cx^2+bx+a}}dx}{8c}-\frac{\sqrt{a+bx+cx^2}\left(-128a^2Bc^2+2cx\left(-120aAc^2-28abB\right.\right.}{}}{16c}\right.$$

$$\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{\frac{\left(480a^2Ac^3+240a^2bBc^2+240aAb^2c^2-40ab^3Bc-10Ab^4c+3b^5B\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx+128a^2c^2(2aB+5Ab)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx}{8c}-\frac{\sqrt{a+bx+cx^2}(\ldots}{}}{16c}\right.$$

$$\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$\downarrow$ 1092

$$\frac{1}{2}\left(\frac{\frac{2\left(480a^2Ac^3+240a^2bBc^2+240aAb^2c^2-40ab^3Bc-10Ab^4c+3b^5B\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}+128a^2c^2(2aB+5Ab)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx}{8c}-}{16c}\right.$$

$$\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\frac{128a^2c^2(2aB+5Ab)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{\left(480a^2Ac^3+240a^2bBc^2+240aAb^2c^2-40ab^3Bc-10Ab^4c+3b^5B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{8c}-\frac{\sqrt{a+bx}}{}}{16c}\right.$$

$$\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$\downarrow$ 1154

$$\frac{1}{2}\left(\frac{\frac{\frac{\left(480a^2Ac^3+240a^2bBc^2+240aAb^2c^2-40ab^3Bc-10Ab^4c+3b^5B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-256a^2c^2(2aB+5Ab)\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{8c}}{16c}\right.$$

$$\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\frac{\frac{\left(480a^2Ac^3+240a^2bBc^2+240aAb^2c^2-40ab^3Bc-10Ab^4c+3b^5B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-128a^{3/2}c^2(2aB+5Ab)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right.}{8c}}{16c}\right.$$

$$\frac{(5A-Bx)\left(a+bx+cx^2\right)^{5/2}}{5x}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^2,x]
```

output
```
-1/5*((5*A - B*x)*(a + b*x + c*x^2)^(5/2))/x + (((3*b^2*B + 70*A*b*c + 16*
a*B*c + 6*c*(b*B + 10*A*c)*x)*(a + b*x + c*x^2)^(3/2))/(24*c) + (-1/4*((3*
b^4*B - 10*A*b^3*c - 28*a*b^2*B*c - 440*a*A*b*c^2 - 128*a^2*B*c^2 + 2*c*(3
*b^3*B - 10*A*b^2*c - 28*a*b*B*c - 120*a*A*c^2)*x)*Sqrt[a + b*x + c*x^2])/
c + (-128*a^(3/2)*(5*A*b + 2*a*B)*c^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[
a + b*x + c*x^2])] + ((3*b^5*B - 10*A*b^4*c - 40*a*b^3*B*c + 240*a*A*b^2*c
^2 + 240*a^2*b*B*c^2 + 480*a^2*A*c^3)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])])/Sqrt[c])/(8*c))/(16*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(278) = 556.

Time = 1.28 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.94

method	result
risch	$-\frac{a^2 A\sqrt{c\,x^2+bx+a}}{x}+\frac{B\,c^2x^4\sqrt{c\,x^2+bx+a}}{5}-\frac{5b^3a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)B}{32c^{\frac{3}{2}}}+\frac{15a^2A\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8}+\frac{9\cdots}{}$
default	$A\left(-\frac{(c\,x^2+bx+a)^{\frac{7}{2}}}{ax}+\frac{5b\left(\frac{(c\,x^2+bx+a)^{\frac{5}{2}}}{5}+\frac{b\left(\frac{(2cx+b)(c\,x^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{\cdots}\right)}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{2}\right.}{}\right.$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*A*(c*x^2+b*x+a)^(1/2)/x+1/5*B*c^2*x^4*(c*x^2+b*x+a)^(1/2)-5/32/c^(3/2
)*b^3*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*B+15/8*a^2*A*c^(1/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+9/8*c*a*x*(c*x^2+b*x+a)^(1/2)*A
+311/240*a*x*(c*x^2+b*x+a)^(1/2)*B*b-5/2*A*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(
c*x^2+b*x+a)^(1/2))/x)*b-B*a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)+15/16*A*a*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1
5/16*B*a^2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-5/128/c^(
3/2)*b^4*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*A+3/256/c^(5/2)*b^5*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*B+9/32/c*b^2*a*(c*x^2+b*x+a)^(1
/2)*B+1/4*c^2*x^3*(c*x^2+b*x+a)^(1/2)*A+31/80*b^2*x^2*(c*x^2+b*x+a)^(1/2)*
B+59/96*b^2*x*(c*x^2+b*x+a)^(1/2)*A+5/64/c*b^3*(c*x^2+b*x+a)^(1/2)*A-3/128
/c^2*b^4*(c*x^2+b*x+a)^(1/2)*B+11/15*c*x^2*(c*x^2+b*x+a)^(1/2)*a*B+139/48*
b*a*(c*x^2+b*x+a)^(1/2)*A+21/40*c*x^3*(c*x^2+b*x+a)^(1/2)*B*b+17/24*c*b*x^
2*(c*x^2+b*x+a)^(1/2)*A+1/64/c*b^3*x*(c*x^2+b*x+a)^(1/2)*B+23/15*a^2*(c*x^
2+b*x+a)^(1/2)*B
```

Fricas [A] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 1393, normalized size of antiderivative = 4.49

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
[1/7680*(1920*(2*B*a^2 + 5*A*a*b)*sqrt(a)*c^3*x*log(-(8*a*b*x + (b^2 + 4*a
*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 15*(
3*B*b^5 + 480*A*a^2*c^3 + 240*(B*a^2*b + A*a*b^2)*c^2 - 10*(4*B*a*b^3 + A*
b^4)*c)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(384*B*c^5*x^5 - 1920*A*a^2*c^3 + 48*(21
*B*b*c^4 + 10*A*c^5)*x^4 + 8*(93*B*b^2*c^3 + 2*(88*B*a + 85*A*b)*c^4)*x^3
+ 2*(15*B*b^3*c^2 + 1080*A*a*c^4 + 2*(622*B*a*b + 295*A*b^2)*c^3)*x^2 - (4
5*B*b^4*c - 8*(368*B*a^2 + 695*A*a*b)*c^3 - 30*(18*B*a*b^2 + 5*A*b^3)*c^2)
*x)*sqrt(c*x^2 + b*x + a))/(c^3*x), 1/3840*(960*(2*B*a^2 + 5*A*a*b)*sqrt(a
)*c^3*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
 2*a)*sqrt(a) + 8*a^2)/x^2) - 15*(3*B*b^5 + 480*A*a^2*c^3 + 240*(B*a^2*b +
 A*a*b^2)*c^2 - 10*(4*B*a*b^3 + A*b^4)*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(384*B*c^5*x
^5 - 1920*A*a^2*c^3 + 48*(21*B*b*c^4 + 10*A*c^5)*x^4 + 8*(93*B*b^2*c^3 + 2
*(88*B*a + 85*A*b)*c^4)*x^3 + 2*(15*B*b^3*c^2 + 1080*A*a*c^4 + 2*(622*B*a*
b + 295*A*b^2)*c^3)*x^2 - (45*B*b^4*c - 8*(368*B*a^2 + 695*A*a*b)*c^3 - 30
*(18*B*a*b^2 + 5*A*b^3)*c^2)*x)*sqrt(c*x^2 + b*x + a))/(c^3*x), 1/7680*(38
40*(2*B*a^2 + 5*A*a*b)*sqrt(-a)*c^3*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*
x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 15*(3*B*b^5 + 480*A*a^2*c^3 +
 240*(B*a^2*b + A*a*b^2)*c^2 - 10*(4*B*a*b^3 + A*b^4)*c)*sqrt(c)*x*log(...
```

Sympy [F]

$$\int \frac{(A + Bx)\,(a + bx + cx^2)^{5/2}}{x^2}\,dx = \int \frac{(A + Bx)\,(a + bx + cx^2)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x**2,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^2}\,dx = \frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,Bc^2x+\frac{21\,Bbc^5+10\,Ac^6}{c^4}\right)x+\frac{93\,B\right.\right.\right.$$
$$+\frac{\left(2\,Ba^3+5\,Aa^2b\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)Aa^2b+2\,Aa^3\sqrt{c}}{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a}$$
$$-\frac{\left(3\,Bb^5-40\,Bab^3c-10\,Ab^4c+240\,Ba^2bc^2+240\,Aab^2c^2+480\,Aa^2c^3\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{\right.\right.}{256\,c^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*B*c^2*x + (21*B*b*c^5 + 10*A*c^6)
/c^4)*x + (93*B*b^2*c^4 + 176*B*a*c^5 + 170*A*b*c^5)/c^4)*x + (15*B*b^3*c^
3 + 1244*B*a*b*c^4 + 590*A*b^2*c^4 + 1080*A*a*c^5)/c^4)*x - (45*B*b^4*c^2
- 540*B*a*b^2*c^3 - 150*A*b^3*c^3 - 2944*B*a^2*c^4 - 5560*A*a*b*c^4)/c^4)
+ (2*B*a^3 + 5*A*a^2*b)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-
a))/sqrt(-a) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*b + 2*A*a^3*sqrt
(c))/((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a) - 1/256*(3*B*b*b^5 - 40*B*a
*b^3*c - 10*A*b^4*c + 240*B*a^2*b*c^2 + 240*A*a*b^2*c^2 + 480*A*a^2*c^3)*l
og(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^2}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x^2}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^2,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^2}\,dx=\frac{-3840\sqrt{c\,x^2+bx+a}\,a^3c^3+17008\sqrt{c\,x^2+bx+a}\,a^2b\,c^3x+4320\sqrt{c\,x}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^2,x)
```

output
```
( - 3840*sqrt(a + b*x + c*x**2)*a**3*c**3 + 17008*sqrt(a + b*x + c*x**2)*a
**2*b*c**3*x + 4320*sqrt(a + b*x + c*x**2)*a**2*c**4*x**2 + 1380*sqrt(a +
b*x + c*x**2)*a*b**3*c**2*x + 7336*sqrt(a + b*x + c*x**2)*a*b**2*c**3*x**2
 + 5536*sqrt(a + b*x + c*x**2)*a*b*c**4*x**3 + 960*sqrt(a + b*x + c*x**2)*
a*c**5*x**4 - 90*sqrt(a + b*x + c*x**2)*b**5*c*x + 60*sqrt(a + b*x + c*x**
2)*b**4*c**2*x**2 + 1488*sqrt(a + b*x + c*x**2)*b**3*c**3*x**3 + 2016*sqrt
(a + b*x + c*x**2)*b**2*c**4*x**4 + 768*sqrt(a + b*x + c*x**2)*b*c**5*x**5
 + 13440*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b*
c**3*x - 13440*sqrt(a)*log(x)*a**2*b*c**3*x + 7200*sqrt(c)*log( - 2*sqrt(c
)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**3*c**3*x + 7200*sqrt(c)*log( - 2*
sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*b**2*c**2*x - 750*sqrt(c)
*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a*b**4*c*x + 45*sqrt
(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*b**6*x)/(3840*c**
3*x)
```

3.136 $\int \frac{(A+Bx)(a+bx+cx^2)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 273

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^3}\,dx = \frac{5(b^3B+40Ab^2c+44abBc+32aAc^2+2c(b^2B+16Abc+12aBc)\,x)}{64c}$$
$$-\frac{5(6(Ab+aB)-(bB+4Ac)x)\left(a+bx+cx^2\right)^{3/2}}{24x}-\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$
$$-\frac{5}{8}\sqrt{a}\left(3Ab^2+4abB+4aAc\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)-\frac{5(b^4B-8Ab^3c-24ab^2Bc-96aAbc^2-48a}{128c^{3/2}}$$

output
```
5/64*(B*b^3+40*A*b^2*c+44*B*a*b*c+32*A*a*c^2+2+2*c*(16*A*b*c+12*B*a*c+B*b^2)
*x)*(c*x^2+b*x+a)^(1/2)/c-5/24*(6*A*b+6*B*a-(4*A*c+B*b)*x)*(c*x^2+b*x+a)^(
3/2)/x-1/4*(-B*x+2*A)*(c*x^2+b*x+a)^(5/2)/x^2-5/8*a^(1/2)*(4*A*a*c+3*A*b^2
+4*B*a*b)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))-5/128*(-96*A*
a*b*c^2-8*A*b^3*c-48*B*a^2*c^2-24*B*a*b^2*c+B*b^4)*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^3}\,dx = \frac{\sqrt{a+x(b+cx)}(-96a^2c(A+2Bx)+x^2(15b^3B+16c^3x^2(4A+3Bx)}{}$$
$$+\frac{5(-b^4B+8Ab^3c+24ab^2Bc+96aAbc^2+48a^2Bc^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{128c^{3/2}}$$
$$-\frac{5}{4}\sqrt{a}(3Ab^2+4abB+4aAc)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^3,x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(-96*a^2*c*(A + 2*B*x) + x^2*(15*b^3*B + 16*c^3*x^2
*(4*A + 3*B*x) + 8*b*c^2*x*(26*A + 17*B*x) + 2*b^2*c*(132*A + 59*B*x)) + 4
*a*c*x*(-4*A*(27*b - 28*c*x) + B*x*(139*b + 54*c*x))))/(192*c*x^2) + (5*(-
(b^4*B) + 8*A*b^3*c + 24*a*b^2*B*c + 96*a*A*b*c^2 + 48*a^2*B*c^2)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(3/2)) - (5*Sqrt[a]
*(3*A*b^2 + 4*a*b*B + 4*a*A*c)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x
)])/Sqrt[a]])/4
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {1230, 27, 1230, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^3}\,dx$$
$$\Big\downarrow \text{1230}$$
$$-\frac{5}{16}\int -\frac{2(2(Ab+aB)+(bB+4Ac)x)\left(cx^2+bx+a\right)^{3/2}}{x^2}dx-\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}\int \frac{(2(Ab+aB)+(bB+4Ac)x)\left(cx^2+bx+a\right)^{3/2}}{x^2}dx-\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{1230}$$

$$\frac{5}{8}\left(-\frac{1}{2}\int -\frac{\left(2\left(3Ab^2+4aBb+4aAc\right)+\left(Bb^2+16Acb+12aBc\right)x\right)\sqrt{cx^2+bx+a}}{x}dx-\frac{\left(a+bx+cx^2\right)^{3/2}(6(a\right.$$
$$\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{\left(2\left(3Ab^2+4aBb+4aAc\right)+\left(Bb^2+16Acb+12aBc\right)x\right)\sqrt{cx^2+bx+a}}{x}dx-\frac{\left(a+bx+cx^2\right)^{3/2}(6(aB+}{3x}\right.$$
$$\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{\sqrt{a+bx+cx^2}\left(2cx\left(12aBc+16Abc+b^2B\right)+32aAc^2+44abBc+40Ab^2c+b^3B\right)}{4c}-\frac{\int -\frac{16ac(3Ab^2+4aBb-}{}}{}\right.\right.$$
$$\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{\int \frac{16ac(3Ab^2+4aBb+4aAc)-(Bb^4-8Acb^3-24aBcb^2-96aAc^2b-48a^2Bc^2)x}{x\sqrt{cx^2+bx+a}}dx}{8c}+\frac{\sqrt{a+bx+cx^2}\left(2cx\left(12aBc+16Abc+\right.\right.}{}\right.\right.$$
$$\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{16ac\left(4aAc+4abB+3Ab^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\left(-48a^2Bc^2-96aAbc^2-24ab^2Bc-8Ab^3c+b^4B\right)\int \frac{}{\sqrt{cx^2}}}{8c}\right.\right.$$
$$\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{16ac\left(4aAc+4abB+3Ab^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-2\left(-48a^2Bc^2-96aAbc^2-24ab^2Bc-8Ab^3c+b^4B\right)\int \frac{}{4c-}}{8c}\right.\right.$$

$$\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{16ac\left(4aAc+4abB+3Ab^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\frac{\left(-48a^2Bc^2-96aAbc^2-24ab^2Bc-8Ab^3c+b^4B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx}}\right.}{\sqrt{c}}}{8c}\right.\right.$$

$$\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{-32ac\left(4aAc+4abB+3Ab^2\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-\frac{\left(-48a^2Bc^2-96aAbc^2-24ab^2Bc-8Ab^3c+b^4B\right)\text{arctanh}\left(\right.}{\sqrt{c}}}{8c}\right.\right.$$

$$\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{-\frac{\left(-48a^2Bc^2-96aAbc^2-24ab^2Bc-8Ab^3c+b^4B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-16\sqrt{a}c\left(4aAc+4abB+3Ab^2\right)\text{arctanh}\left(\right.}{8c}\right.\right.$$

$$\frac{(2A-Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^3,x]
```

output
```
-1/4*((2*A - B*x)*(a + b*x + c*x^2)^(5/2))/x^2 + (5*(-1/3*((6*(A*b + a*B)
- (b*B + 4*A*c)*x)*(a + b*x + c*x^2)^(3/2))/x + (((b^3*B + 40*A*b^2*c + 44
*a*b*B*c + 32*a*A*c^2 + 2*c*(b^2*B + 16*A*b*c + 12*a*B*c)*x)*Sqrt[a + b*x
+ c*x^2])/(4*c) + (-16*Sqrt[a]*c*(3*A*b^2 + 4*a*b*B + 4*a*A*c)*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] - ((b^4*B - 8*A*b^3*c - 24*a*b
^2*B*c - 96*a*A*b*c^2 - 48*a^2*B*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])])/Sqrt[c])/(8*c))/2))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(240) = 480$.

Time = 1.35 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.85

method	result
risch	$-\frac{a\sqrt{c\,x^2+bx+a}\,(9Abx+4Bax+2Aa)}{4x^2}+\frac{c^2x^2\sqrt{c\,x^2+bx+a}\,A}{3}+\frac{59b^2x\sqrt{c\,x^2+bx+a}\,B}{96}+\frac{5b^3\sqrt{c\,x^2+bx+a}\,B}{64c}+\frac{7ca\sqrt{c\,x^2+bx}}{3}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*(c*x^2+b*x+a)^(1/2)*(9*A*b*x+4*B*a*x+2*A*a)/x^2+1/3*c^2*x^2*(c*x^2+
b*x+a)^(1/2)*A+59/96*b^2*x*(c*x^2+b*x+a)^(1/2)*B+5/64/c*b^3*(c*x^2+b*x+a)^
(1/2)*B+7/3*c*a*(c*x^2+b*x+a)^(1/2)*A+139/48*a*(c*x^2+b*x+a)^(1/2)*B*b-5/2
*A*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*c-5/2*B*a^(3/2)*l
n((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b+17/24*c*x^2*(c*x^2+b*x+a)^(
1/2)*B*b+13/12*c*b*x*(c*x^2+b*x+a)^(1/2)*A-5/128/c^(3/2)*b^4*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*B+15/16*B*a*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))/c^(1/2)+15/8*B*a^2*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))+15/4*A*a*b*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))+1/4*B*c^2*x^3*(c*x^2+b*x+a)^(1/2)-15/8*A*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*
(c*x^2+b*x+a)^(1/2))/x)*b^2+9/8*c*x*(c*x^2+b*x+a)^(1/2)*a*B+5/16*A*b^3*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+11/8*b^2*(c*x^2+b*x+a)^(1
/2)*A
```

Fricas [A] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 1269, normalized size of antiderivative = 4.65

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^3}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/768*(15*(B*b^4 - 48*(B*a^2 + 2*A*a*b)*c^2 - 8*(3*B*a*b^2 + A*b^3)*c)*s
qrt(c)*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(c) - 4*a*c) - 240*(4*A*a*c^3 + (4*B*a*b + 3*A*b^2)*c^2)*sqrt(a)
*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2) - 4*(48*B*c^4*x^5 - 96*A*a^2*c^2 + 8*(17*B*b*c^3
 + 8*A*c^4)*x^4 - 48*(4*B*a^2 + 9*A*a*b)*c^2*x + 2*(59*B*b^2*c^2 + 4*(27*B*
a + 26*A*b)*c^3)*x^3 + (15*B*b^3*c + 448*A*a*c^3 + 4*(139*B*a*b + 66*A*b^2
)*c^2)*x^2)*sqrt(c*x^2 + b*x + a))/(c^2*x^2), 1/384*(15*(B*b^4 - 48*(B*a^2
 + 2*A*a*b)*c^2 - 8*(3*B*a*b^2 + A*b^3)*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*
x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 120*(4*A*a*
c^3 + (4*B*a*b + 3*A*b^2)*c^2)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x
^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 2*(48*B*c
^4*x^5 - 96*A*a^2*c^2 + 8*(17*B*b*c^3 + 8*A*c^4)*x^4 - 48*(4*B*a^2 + 9*A*a
*b)*c^2*x + 2*(59*B*b^2*c^2 + 4*(27*B*a + 26*A*b)*c^3)*x^3 + (15*B*b^3*c +
 448*A*a*c^3 + 4*(139*B*a*b + 66*A*b^2)*c^2)*x^2)*sqrt(c*x^2 + b*x + a))/(
c^2*x^2), 1/768*(480*(4*A*a*c^3 + (4*B*a*b + 3*A*b^2)*c^2)*sqrt(-a)*x^2*ar
ctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2
)) - 15*(B*b^4 - 48*(B*a^2 + 2*A*a*b)*c^2 - 8*(3*B*a*b^2 + A*b^3)*c)*sqrt(
c)*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(c) - 4*a*c) + 4*(48*B*c^4*x^5 - 96*A*a^2*c^2 + 8*(17*B*b*c^3 + 8...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^3}\,dx=\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x**3,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(239) = 478$.

Time = 0.29 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.93

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x^3}\,dx = \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,Bc^2x+\frac{17\,Bbc^4+8\,Ac^5}{c^3}\right)x+\frac{59\,Bb^2c^3\ldots}{\ldots}\right.\right.$$
$$+\frac{5\,(4\,Ba^2b+3\,Aab^2+4\,Aa^2c)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}}$$
$$+\frac{5\,(Bb^4-24\,Bab^2c-8\,Ab^3c-48\,Ba^2c^2-96\,Aabc^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{3}{2}}}$$
$$+\frac{4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Ba^2b+9\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Aab^2+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Aa^2c+\ldots}{\ldots}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*B*c^2*x + (17*B*b*c^4 + 8*A*c^5)/c^3)
*x + (59*B*b^2*c^3 + 108*B*a*c^4 + 104*A*b*c^4)/c^3)*x + (15*B*b^3*c^2 + 5
56*B*a*b*c^3 + 264*A*b^2*c^3 + 448*A*a*c^4)/c^3) + 5/4*(4*B*a^2*b + 3*A*a*
b^2 + 4*A*a^2*c)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqr
t(-a) + 5/128*(B*b^4 - 24*B*a*b^2*c - 8*A*b^3*c - 48*B*a^2*c^2 - 96*A*a*b*
c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2) +
 1/4*(4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a^2*b + 9*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*A*a*b^2 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*
a^2*c + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B*a^3*sqrt(c) + 24*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*A*a^2*b*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*B*a^3*b - 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*b^2 + 4
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^3*c - 8*B*a^4*sqrt(c) - 16*A*a^3*
b*sqrt(c))/((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^3}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x^3}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^3,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^3}\,dx=\frac{-192\sqrt{c\,x^2+bx+a}\,a^3c^2-1248\sqrt{c\,x^2+bx+a}\,a^2b\,c^2x+896\sqrt{c\,x^2+}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^3,x)
```

output

```
( - 192*sqrt(a + b*x + c*x**2)*a**3*c**2 - 1248*sqrt(a + b*x + c*x**2)*a**
2*b*c**2*x + 896*sqrt(a + b*x + c*x**2)*a**2*c**3*x**2 + 1640*sqrt(a + b*x
 + c*x**2)*a*b**2*c**2*x**2 + 848*sqrt(a + b*x + c*x**2)*a*b*c**3*x**3 + 1
28*sqrt(a + b*x + c*x**2)*a*c**4*x**4 + 30*sqrt(a + b*x + c*x**2)*b**4*c*x
**2 + 236*sqrt(a + b*x + c*x**2)*b**3*c**2*x**3 + 272*sqrt(a + b*x + c*x**
2)*b**2*c**3*x**4 + 96*sqrt(a + b*x + c*x**2)*b*c**4*x**5 + 960*sqrt(a)*lo
g(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*c**3*x**2 + 1680*sqrt
(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**2*c**2*x**2 - 9
60*sqrt(a)*log(x)*a**2*c**3*x**2 - 1680*sqrt(a)*log(x)*a*b**2*c**2*x**2 +
2160*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*b*c
**2*x**2 + 480*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x
)*a*b**3*c*x**2 - 15*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b -
 2*c*x)*b**5*x**2)/(384*c**2*x**2)
```

3.137 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 255

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^4}\,dx = -\frac{5(4abB+A(b^2+4ac)-(b^2B+4Abc+4aBc)\,x)\sqrt{a+bx+cx^2}}{8x} - \frac{5(Ab+2aB-(bB+2Ac)x)\left(a+bx+cx^2\right)^{3/2}}{12x^2} - \frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3} - \frac{5(2aB(3b^2+4ac)+A(b^3+12abc))\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16\sqrt{a}} + \frac{5(b^3B+6Ab^2c+12abBc+8aAc^2)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16\sqrt{c}}$$

output

```
-5/8*(4*a*b*B+A*(4*a*c+b^2)-(4*A*b*c+4*B*a*c+B*b^2)*x)*(c*x^2+b*x+a)^(1/2)
/x-5/12*(A*b+2*B*a-(2*A*c+B*b)*x)*(c*x^2+b*x+a)^(3/2)/x^2-1/3*(-B*x+A)*(c*
x^2+b*x+a)^(5/2)/x^3-5/16*(2*a*B*(4*a*c+3*b^2)+A*(12*a*b*c+b^3))*arctanh(1
/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)+5/16*(8*A*a*c^2+6*A*b^2*
c+12*B*a*b*c+B*b^3)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(
1/2)
```

Mathematica [A] (verified)

Time = 2.70 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^4}\,dx=\frac{1}{48}\left(-\frac{2\sqrt{a+x(b+cx)}(4a^2(2A+3Bx)+2ax(Bx(27b-28cx)+A(\right.$$
$$-\frac{30(2aB(3b^2+4ac)+A(b^3+12abc))\,\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}$$
$$\left.-\frac{15(b^3B+6Ab^2c+12abBc+8aAc^2)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{\sqrt{c}}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^4,x]
```

output
```
((-2*Sqrt[a + x*(b + c*x)]*(4*a^2*(2*A + 3*B*x) + 2*a*x*(B*x*(27*b - 28*c*
x) + A*(13*b + 28*c*x)) - x^2*(B*x*(33*b^2 + 26*b*c*x + 8*c^2*x^2) + A*(-3
3*b^2 + 54*b*c*x + 12*c^2*x^2))))/x^3 - (30*(2*a*B*(3*b^2 + 4*a*c) + A*(b^
3 + 12*a*b*c))*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sq
rt[a] - (15*(b^3*B + 6*A*b^2*c + 12*a*b*B*c + 8*a*A*c^2)*Log[b + 2*c*x - 2
*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c])/48
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1230, 27, 1230, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^4}\,dx$$
$$\Big\downarrow \text{1230}$$
$$-\frac{5}{18}\int -\frac{3(Ab+2aB+(bB+2Ac)x)\left(cx^2+bx+a\right)^{3/2}}{x^3}dx-\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{6}\int \frac{(Ab+2aB+(bB+2Ac)x)\left(cx^2+bx+a\right)^{3/2}}{x^3}dx-\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$$\Big\downarrow \; 1230$$

$$\frac{5}{6}\left(-\frac{3}{8}\int -\frac{2\left(4abB+A\left(b^2+4ac\right)+\left(Bb^2+4Acb+4aBc\right)x\right)\sqrt{cx^2+bx+a}}{x^2}dx-\frac{\left(a+bx+cx^2\right)^{3/2}\left(2aB-x(\right.}{2x^2}\right.$$

$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\left(4abB+A\left(b^2+4ac\right)+\left(Bb^2+4Acb+4aBc\right)x\right)\sqrt{cx^2+bx+a}}{x^2}dx-\frac{\left(a+bx+cx^2\right)^{3/2}\left(2aB-x(2Ac\right.}{2x^2}\right.$$

$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$$\Big\downarrow \; 1230$$

$$\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\int -\frac{2aB\left(3b^2+4ac\right)+A\left(b^3+12acb\right)+\left(Bb^3+6Acb^2+12aBcb+8aAc^2\right)x}{x\sqrt{cx^2+bx+a}}dx-\frac{\sqrt{a+bx+cx^2}\left(-\right.}{}\right.\right.$$

$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$$\Big\downarrow \; 25$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{2aB\left(3b^2+4ac\right)+A\left(b^3+12acb\right)+\left(Bb^3+6Acb^2+12aBcb+8aAc^2\right)x}{x\sqrt{cx^2+bx+a}}dx-\frac{\sqrt{a+bx+cx^2}\left(-x(4\right.}{}\right.\right.$$

$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$$\Big\downarrow \; 1269$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(8aAc^2+12abBc+6Ab^2c+b^3B\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx+\left(A\left(12abc+b^3\right)+2aB\left(4ac+3b^2\right)\right)\int \frac{}{x\sqrt{c}}\right.\right.\right.$$

$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$$\Big\downarrow \; 1092$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(2\left(8aAc^2+12abBc+6Ab^2c+b^3B\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}+\left(A\left(12abc+b^3\right)+2aB\left(4ac+\right.\right.\right.\right.\right.$$
$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$\downarrow$ 219

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(A\left(12abc+b^3\right)+2aB\left(4ac+3b^2\right)\right)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{\left(8aAc^2+12abBc+6Ab^2c+b^3B\right)\text{arcta}}{\sqrt{c}}\right.\right.\right.$$
$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$\downarrow$ 1154

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\left(8aAc^2+12abBc+6Ab^2c+b^3B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2\left(A\left(12abc+b^3\right)+2aB\left(4ac+3b^2\right)\right)\int\right.\right.\right.$$
$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

$\downarrow$ 219

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\left(\frac{\left(8aAc^2+12abBc+6Ab^2c+b^3B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-\frac{\left(A\left(12abc+b^3\right)+2aB\left(4ac+3b^2\right)\right)\text{ar}}{\sqrt{a}}\right.\right.\right.$$
$$\frac{(A-Bx)\left(a+bx+cx^2\right)^{5/2}}{3x^3}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^4,x]
```

output
```
-1/3*((A - B*x)*(a + b*x + c*x^2)^(5/2))/x^3 + (5*(-1/2*((A*b + 2*a*B - (b
*B + 2*A*c)*x)*(a + b*x + c*x^2)^(3/2))/x^2 + (3*(-(((4*a*b*B + A*(b^2 + 4
*a*c) - (b^2*B + 4*A*b*c + 4*a*B*c)*x)*Sqrt[a + b*x + c*x^2])/x) + (-(((2*
a*B*(3*b^2 + 4*a*c) + A*(b^3 + 12*a*b*c))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + b*x + c*x^2])])/Sqrt[a]) + ((b^3*B + 6*A*b^2*c + 12*a*b*B*c + 8*a*
A*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c])/2)
)/4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(223) = 446.

Time = 1.32 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.79

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(56Aac\,x^2+33x^2b^2A+54Ba\,x^2b+26abAx+12a^2Bx+8a^2A)}{24x^3}-\frac{15\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)Abc}{4}+\frac{15A\,b}{}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+b*x+a)^(1/2)*(56*A*a*c*x^2+33*A*b^2*x^2+54*B*a*b*x^2+26*A*a*b
*x+12*B*a^2*x+8*A*a^2)/x^3-15/4*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a
)^(1/2))/x)*A*b*c+15/8*A*b^2*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))+5/2*A*a*c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*c^2
*x*(c*x^2+b*x+a)^(1/2)*A+9/4*c*b*(c*x^2+b*x+a)^(1/2)*A-5/16/a^(1/2)*ln((2*
a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*A*b^3+5/16*B*b^3*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+11/8*b^2*(c*x^2+b*x+a)^(1/2)*B-5/2*a^(3
/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*B*c-15/8*a^(1/2)*ln((2*a
+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*B*b^2+13/12*c*x*(c*x^2+b*x+a)^(1/2)
*B*b+15/4*B*a*b*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*B*
c^2*x^2*(c*x^2+b*x+a)^(1/2)+7/3*c*(c*x^2+b*x+a)^(1/2)*a*B
```

Fricas [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 1293, normalized size of antiderivative = 5.07

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
[1/96*(15*(B*a*b^3 + 8*A*a^2*c^2 + 6*(2*B*a^2*b + A*a*b^2)*c)*sqrt(c)*x^3*
log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(
c) - 4*a*c) + 15*(4*(2*B*a^2 + 3*A*a*b)*c^2 + (6*B*a*b^2 + A*b^3)*c)*sqrt(
a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(8*B*a*c^3*x^5 - 8*A*a^3*c + 2*(13*B*a*b*c^
2 + 6*A*a*c^3)*x^4 + (33*B*a*b^2*c + 2*(28*B*a^2 + 27*A*a*b)*c^2)*x^3 - 2*
(6*B*a^3 + 13*A*a^2*b)*c*x - (56*A*a^2*c^2 + 3*(18*B*a^2*b + 11*A*a*b^2)*c
)*x^2)*sqrt(c*x^2 + b*x + a))/(a*c*x^3), -1/96*(30*(B*a*b^3 + 8*A*a^2*c^2
+ 6*(2*B*a^2*b + A*a*b^2)*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 15*(4*(2*B*a^2 + 3*A*a*b)
*c^2 + (6*B*a*b^2 + A*b^3)*c)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^
2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(8*B*a*c
^3*x^5 - 8*A*a^3*c + 2*(13*B*a*b*c^2 + 6*A*a*c^3)*x^4 + (33*B*a*b^2*c + 2*
(28*B*a^2 + 27*A*a*b)*c^2)*x^3 - 2*(6*B*a^3 + 13*A*a^2*b)*c*x - (56*A*a^2*
c^2 + 3*(18*B*a^2*b + 11*A*a*b^2)*c)*x^2)*sqrt(c*x^2 + b*x + a))/(a*c*x^3)
, 1/96*(30*(4*(2*B*a^2 + 3*A*a*b)*c^2 + (6*B*a*b^2 + A*b^3)*c)*sqrt(-a)*x^
3*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x +
 a^2)) + 15*(B*a*b^3 + 8*A*a^2*c^2 + 6*(2*B*a^2*b + A*a*b^2)*c)*sqrt(c)*x^
3*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqr
t(c) - 4*a*c) + 4*(8*B*a*c^3*x^5 - 8*A*a^3*c + 2*(13*B*a*b*c^2 + 6*A*a*...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^4}\,dx=\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x**4,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(224) = 448$.

Time = 0.33 (sec) , antiderivative size = 765, normalized size of antiderivative = 3.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*B*c^2*x + (13*B*b*c^3 + 6*A*c^4)/c^2)*x +
 (33*B*b^2*c^2 + 56*B*a*c^3 + 54*A*b*c^3)/c^2) + 5/8*(6*B*a*b^2 + A*b^3 +
8*B*a^2*c + 12*A*a*b*c)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-
a))/sqrt(-a) - 5/16*(B*b^3 + 12*B*a*b*c + 6*A*b^2*c + 8*A*a*c^2)*log(abs(2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/sqrt(c) + 1/24*(54*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a*b^2 + 33*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*A*b^3 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^2*c + 108*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a*b*c + 144*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*B*a^2*b*sqrt(c) + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*A*a*b^2*sqrt(c) + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*A*a^2*c^(3/2
) - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a^2*b^2 - 40*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*A*a*b^3 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*A*a^2*b*c - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B*a^3*b*sqrt(c) - 1
44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*A*a^2*b^2*sqrt(c) - 192*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*A*a^3*c^(3/2) + 42*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*B*a^3*b^2 + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*b^3 - 2
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^4*c + 36*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*A*a^3*b*c + 96*B*a^4*b*sqrt(c) + 48*A*a^3*b^2*sqrt(c) + 112*A
*a^4*c^(3/2))/((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^4}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x^4}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^4,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^4}\,dx = \frac{-16\sqrt{c\,x^2+bx+a}\,a^3c-76\sqrt{c\,x^2+bx+a}\,a^2bcx-112\sqrt{c\,x^2+bx+}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^4,x)
```

output
```
( - 16*sqrt(a + b*x + c*x**2)*a**3*c - 76*sqrt(a + b*x + c*x**2)*a**2*b*c*
x - 112*sqrt(a + b*x + c*x**2)*a**2*c**2*x**2 - 174*sqrt(a + b*x + c*x**2)
*a*b**2*c*x**2 + 220*sqrt(a + b*x + c*x**2)*a*b*c**2*x**3 + 24*sqrt(a + b*
x + c*x**2)*a*c**3*x**4 + 66*sqrt(a + b*x + c*x**2)*b**3*c*x**3 + 52*sqrt(
a + b*x + c*x**2)*b**2*c**2*x**4 + 16*sqrt(a + b*x + c*x**2)*b*c**3*x**5 +
 300*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b*c**2*x*
*3 + 105*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**3*c*
x**3 - 300*sqrt(a)*log(x)*a*b*c**2*x**3 - 105*sqrt(a)*log(x)*b**3*c*x**3 +
 120*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*c**
2*x**3 + 270*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*
a*b**2*c*x**3 + 15*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2
*c*x)*b**4*x**3)/(48*c*x**3)
```

3.138 $\int \frac{(A+Bx)(a+bx+cx^2)^{5/2}}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 284

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^5}\,dx &= -\frac{5(8aB(b^2+4ac)-A(b^3-20abc)-2c(16abB+A(b^2+12ac))\,x)\sqrt{a+bx+cx^2}}{64ax} \\ &- \frac{5(4a(Ab+4aB)+3(8abB+A(b^2+4ac))\,x)\left(a+bx+cx^2\right)^{3/2}}{96ax^3} \\ &- \frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4} \\ &- \frac{5(8abB(b^2+12ac)-A(b^4-24ab^2c-48a^2c^2))\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{3/2}} \\ &+ \frac{5}{8}\sqrt{c}(3b^2B+4Abc+4aBc)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\end{aligned}$$

output
```
-5/64*(8*a*B*(4*a*c+b^2)-A*(-20*a*b*c+b^3)-2*c*(16*a*b*B+A*(12*a*c+b^2))*x
)*(c*x^2+b*x+a)^(1/2)/a/x-5/96*(4*a*(A*b+4*B*a)+3*(8*a*b*B+A*(4*a*c+b^2))*
x)*(c*x^2+b*x+a)^(3/2)/a/x^3-1/4*(-2*B*x+A)*(c*x^2+b*x+a)^(5/2)/x^4-5/128*
(8*a*b*B*(12*a*c+b^2)-A*(-48*a^2*c^2-24*a*b^2*c+b^4))*arctanh(1/2*(b*x+2*a
)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)+5/8*c^(1/2)*(4*A*b*c+4*B*a*c+3*B*b^
2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^5}\,dx =$$

$$-\frac{\sqrt{a+x(b+cx)}(15Ab^3x^3+16a^3(3A+4Bx)+8a^2x(17Ab+26bBx+27Acx+56Bcx^2)+2ax^2(A(59b}{192ax^4}$$

$$-\frac{5Ab^4\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{64a^{3/2}}$$

$$-\frac{5(b^3B+3Ab^2c+12abBc+6aAc^2)\,\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

$$-\frac{5}{8}\sqrt{c}(3b^2B+4Abc+4aBc)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^5,x]
```

output
```
-1/192*(Sqrt[a + x*(b + c*x)]*(15*A*b^3*x^3 + 16*a^3*(3*A + 4*B*x) + 8*a^2
*x*(17*A*b + 26*b*B*x + 27*A*c*x + 56*B*c*x^2) + 2*a*x^2*(A*(59*b^2 + 278*
b*c*x - 96*c^2*x^2) - 12*B*x*(-11*b^2 + 18*b*c*x + 4*c^2*x^2))))/(a*x^4) -
 (5*A*b^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(64*a^(3/2
)) - (5*(b^3*B + 3*A*b^2*c + 12*a*b*B*c + 6*a*A*c^2)*ArcTanh[(-(Sqrt[c]*x)
 + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(8*Sqrt[a]) - (5*Sqrt[c]*(3*b^2*B + 4*
A*b*c + 4*a*B*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/8
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1230, 27, 1229, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^5}\,dx$$

$$\Big\downarrow \text{1230}$$

$$-\frac{5}{16}\int -\frac{2(Ab+4aB+2(bB+Ac)x)\left(cx^2+bx+a\right)^{3/2}}{x^4}dx-\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}\int \frac{(Ab+4aB+2(bB+Ac)x)\left(cx^2+bx+a\right)^{3/2}}{x^4}dx-\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{1229}$$

$$\frac{5}{8}\left(-\frac{\int -\frac{\left(8aB\left(b^2+4ac\right)-A\left(b^3-20abc\right)+2c\left(16abB+A\left(b^2+12ac\right)\right)x\right)\sqrt{cx^2+bx+a}}{2x^2}dx}{4a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(3x\left(A\left(4ac+b^2\right)+8a\right.\right.}{12ax^3}\right.$$

$$\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}\left(\frac{\int \frac{\left(8aB\left(b^2+4ac\right)-A\left(b^3-20abc\right)+2c\left(16abB+A\left(b^2+12ac\right)\right)x\right)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(3x\left(A\left(4ac+b^2\right)+8abB\right.\right.}{12ax^3}\right.$$

$$\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{1230}$$

$$\frac{5}{8}\left(\frac{-\frac{1}{2}\int -\frac{8abB\left(b^2+12ac\right)-A\left(b^4-24acb^2-48a^2c^2\right)+16ac\left(3Bb^2+4Acb+4aBc\right)x}{x\sqrt{cx^2+bx+a}}dx-\frac{\sqrt{a+bx+cx^2}\left(-A\left(b^3-20abc\right)-2cx\left(A\left(12ac+b^2\right)+1\right.\right.}{x}}{8a}\right.$$

$$\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{8}\left(\frac{\frac{1}{2}\int \frac{8abB\left(b^2+12ac\right)-A\left(b^4-24acb^2-48a^2c^2\right)+16ac\left(3Bb^2+4Acb+4aBc\right)x}{x\sqrt{cx^2+bx+a}}dx-\frac{\sqrt{a+bx+cx^2}\left(-A\left(b^3-20abc\right)-2cx\left(A\left(12ac+b^2\right)+16abB\right.\right.}{x}}{8a}\right.$$

$$\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$$\Big\downarrow \text{1269}$$

$$\frac{5}{8}\left(\frac{\frac{1}{2}\left(\left(8abB\left(12ac+b^2\right)-A\left(-48a^2c^2-24ab^2c+b^4\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+16ac\left(4aBc+4Abc+3b^2B\right)\int \frac{1}{\sqrt{cx^2+b\ldots}}\right.}{8a}\right.$$

$$\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$\Big\downarrow$ 1092

$$\frac{5}{8}\left(\frac{\frac{1}{2}\left(\left(8abB\left(12ac+b^2\right)-A\left(-48a^2c^2-24ab^2c+b^4\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+32ac\left(4aBc+4Abc+3b^2B\right)\int \frac{1}{4c-\frac{(b+\ldots}{cx^2\ldots}}\right.}{8a}\right.$$

$$\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$\Big\downarrow$ 219

$$\frac{5}{8}\left(\frac{\frac{1}{2}\left(\left(8abB\left(12ac+b^2\right)-A\left(-48a^2c^2-24ab^2c+b^4\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+16a\sqrt{c}\left(4aBc+4Abc+3b^2B\right)\text{arctanh}\ldots\right.}{8a}\right.$$

$$\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$\Big\downarrow$ 1154

$$\frac{5}{8}\left(\frac{\frac{1}{2}\left(16a\sqrt{c}\left(4aBc+4Abc+3b^2B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-2\left(8abB\left(12ac+b^2\right)-A\left(-48a^2c^2-24ab^2c+b^4\ldots\right.\right.\right.}{8a}\right.$$

$$\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

$\Big\downarrow$ 219

$$\frac{5}{8}\left(\frac{\frac{1}{2}\left(16a\sqrt{c}\left(4aBc+4Abc+3b^2B\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-\frac{\left(8abB\left(12ac+b^2\right)-A\left(-48a^2c^2-24ab^2c+b^4\right)\right)\operatorname{arctanh}\left(\frac{}{2\sqrt{}}\right.}{\sqrt{a}}\right.}{8a}\right.$$

$$\frac{(A-2Bx)\left(a+bx+cx^2\right)^{5/2}}{4x^4}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^5,x]
```

output
```
-1/4*((A - 2*B*x)*(a + b*x + c*x^2)^(5/2))/x^4 + (5*(-1/12*((4*a*(A*b + 4*
a*B) + 3*(8*a*b*B + A*(b^2 + 4*a*c))*x)*(a + b*x + c*x^2)^(3/2))/(a*x^3) +
 (-(((8*a*B*(b^2 + 4*a*c) - A*(b^3 - 20*a*b*c) - 2*c*(16*a*b*B + A*(b^2 +
12*a*c))*x)*Sqrt[a + b*x + c*x^2])/x) + (-(((8*a*b*B*(b^2 + 12*a*c) - A*(b
^4 - 24*a*b^2*c - 48*a^2*c^2))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x
 + c*x^2])])/Sqrt[a]) + 16*a*Sqrt[c]*(3*b^2*B + 4*A*b*c + 4*a*B*c)*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/2)/(8*a)))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(556Aabc\,x^3+15A\,b^3x^3+448B\,a^2c\,x^3+264Ba\,b^2x^3+216A\,a^2c\,x^2+118Aa\,b^2x^2+208B\,a^2b\,x^2+136A\,a^2bx+64B\,a^3}{192x^4a}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x^2+b*x+a)^(1/2)*(556*A*a*b*c*x^3+15*A*b^3*x^3+448*B*a^2*c*x^3+2
64*B*a*b^2*x^3+216*A*a^2*c*x^2+118*A*a*b^2*x^2+208*B*a^2*b*x^2+136*A*a^2*b
*x+64*B*a^3*x+48*A*a^3)/x^4/a-15/8*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)*A*c^2-15/16/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)*A*b^2*c+5/128/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
*A*b^4-15/4*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b*B*c-5/
16/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*B*b^3+5/2*a*B*c^(
3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+c^2*(c*x^2+b*x+a)^(1/2)*A
+5/2*A*b*c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+15/8*B*b^2*c^
(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*B*c^2*x*(c*x^2+b*x+a
)^(1/2)+9/4*c*(c*x^2+b*x+a)^(1/2)*B*b
```

Fricas [A] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 1305, normalized size of antiderivative = 4.60

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
[1/768*(240*(3*B*a^2*b^2 + 4*(B*a^3 + A*a^2*b)*c)*sqrt(c)*x^4*log(-8*c^2*x
^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c)
+ 15*(8*B*a*b^3 - A*b^4 + 48*A*a^2*c^2 + 24*(4*B*a^2*b + A*a*b^2)*c)*sqrt(
a)*x^4*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(96*B*a^2*c^2*x^5 - 48*A*a^4 + 48*(9*B*a^2*
b*c + 4*A*a^2*c^2)*x^4 - (264*B*a^2*b^2 + 15*A*a*b^3 + 4*(112*B*a^3 + 139*
A*a^2*b)*c)*x^3 - 2*(104*B*a^3*b + 59*A*a^2*b^2 + 108*A*a^3*c)*x^2 - 8*(8*
B*a^4 + 17*A*a^3*b)*x)*sqrt(c*x^2 + b*x + a))/(a^2*x^4), -1/768*(480*(3*B*
a^2*b^2 + 4*(B*a^3 + A*a^2*b)*c)*sqrt(-c)*x^4*arctan(1/2*sqrt(c*x^2 + b*x
+ a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 15*(8*B*a*b^3 - A*b^4
 + 48*A*a^2*c^2 + 24*(4*B*a^2*b + A*a*b^2)*c)*sqrt(a)*x^4*log(-(8*a*b*x +
(b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x
^2) - 4*(96*B*a^2*c^2*x^5 - 48*A*a^4 + 48*(9*B*a^2*b*c + 4*A*a^2*c^2)*x^4
- (264*B*a^2*b^2 + 15*A*a*b^3 + 4*(112*B*a^3 + 139*A*a^2*b)*c)*x^3 - 2*(10
4*B*a^3*b + 59*A*a^2*b^2 + 108*A*a^3*c)*x^2 - 8*(8*B*a^4 + 17*A*a^3*b)*x)*
sqrt(c*x^2 + b*x + a))/(a^2*x^4), 1/384*(15*(8*B*a*b^3 - A*b^4 + 48*A*a^2*
c^2 + 24*(4*B*a^2*b + A*a*b^2)*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^2 + b*x
 + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 120*(3*B*a^2*b^2 + 4
*(B*a^3 + A*a^2*b)*c)*sqrt(c)*x^4*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*(96*B*a^2*c^2*x^5 - 4...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^5}\,dx = \int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x**5,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(254) = 508$.

Time = 0.35 (sec) , antiderivative size = 1163, normalized size of antiderivative = 4.10

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^5,x, algorithm="giac")
```

output
```
1/4*(2*B*c^2*x + (9*B*b*c^2 + 4*A*c^3)/c)*sqrt(c*x^2 + b*x + a) - 5/8*(3*B
*b^2*c + 4*B*a*c^2 + 4*A*b*c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*sqrt(c) + b))/sqrt(c) + 5/64*(8*B*a*b^3 - A*b^4 + 96*B*a^2*b*c + 24*A*
a*b^2*c + 48*A*a^2*c^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-
a))/(sqrt(-a)*a) + 1/192*(264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a*b^
3 + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*b^4 + 864*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^7*B*a^2*b*c + 792*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7
*A*a*b^2*c + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a^2*c^2 + 1152*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^6*B*a^2*b^2*sqrt(c) + 384*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^6*A*a*b^3*sqrt(c) + 1152*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^6*B*a^3*c^(3/2) + 2304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*A*a^
2*b*c^(3/2) - 584*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^2*b^3 + 73*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a*b^4 - 1248*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^5*B*a^3*b*c - 600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a^2*
b^2*c - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a^3*c^2 - 2304*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^4*B*a^3*b^2*sqrt(c) - 2688*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*B*a^4*c^(3/2) - 3456*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*A*a^3*b*c^(3/2) + 440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a^3*b^3
- 55*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a^2*b^4 + 672*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*B*a^4*b*c + 1320*(sqrt(c)*x - sqrt(c*x^2 + b*x +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^5}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x^5}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^5,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.44

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^5}\,dx = \frac{-96\sqrt{c\,x^2+bx+a}\,a^4-400\sqrt{c\,x^2+bx+a}\,a^3bx-432\sqrt{c\,x^2+bx+a}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^5,x)
```

output
```
( - 96*sqrt(a + b*x + c*x**2)*a**4 - 400*sqrt(a + b*x + c*x**2)*a**3*b*x -
 432*sqrt(a + b*x + c*x**2)*a**3*c*x**2 - 652*sqrt(a + b*x + c*x**2)*a**2*
b**2*x**2 - 2008*sqrt(a + b*x + c*x**2)*a**2*b*c*x**3 + 384*sqrt(a + b*x +
 c*x**2)*a**2*c**2*x**4 - 558*sqrt(a + b*x + c*x**2)*a*b**3*x**3 + 864*sqr
t(a + b*x + c*x**2)*a*b**2*c*x**4 + 192*sqrt(a + b*x + c*x**2)*a*b*c**2*x*
*5 + 720*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*c*
*2*x**4 + 1800*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a
*b**2*c*x**4 + 105*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*
x)*b**4*x**4 - 720*sqrt(a)*log(x)*a**2*c**2*x**4 - 1800*sqrt(a)*log(x)*a*b
**2*c*x**4 - 105*sqrt(a)*log(x)*b**4*x**4 + 1920*sqrt(c)*log( - 2*sqrt(c)*
sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*b*c*x**4 + 720*sqrt(c)*log( - 2*s
qrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a*b**3*x**4)/(384*a*x**4)
```

3.139 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 346

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx = \frac{\left(10abB(b^2-20ac)-A(3b^4-28ab^2c+128a^2c^2)+2c(10aB(b^2+12a\right.}{128a^2x}$$
$$+\frac{\left(4a(3Ab^2-10abB-16aAc)-3(10aB(b^2+4ac)-A(3b^3-20abc))\,x\right)\left(a+bx+cx^2\right)^{3/2}}{192a^2x^3}$$
$$-\frac{(8aA+5(Ab+2aB)x)\left(a+bx+cx^2\right)^{5/2}}{40ax^5}$$
$$+\frac{\left(10aB(b^4-24ab^2c-48a^2c^2)-A(3b^5-40ab^3c+240a^2bc^2)\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{256a^{5/2}}$$
$$+\frac{1}{2}c^{3/2}(5bB+2Ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)$$

output
```
1/128*(10*a*b*B*(-20*a*c+b^2)-A*(128*a^2*c^2-28*a*b^2*c+3*b^4)+2*c*(10*a*B
*(12*a*c+b^2)-A*(-28*a*b*c+3*b^3))*x)*(c*x^2+b*x+a)^(1/2)/a^2/x+1/192*(4*a
*(-16*A*a*c+3*A*b^2-10*B*a*b)-3*(10*a*B*(4*a*c+b^2)-A*(-20*a*b*c+3*b^3))*x
)*(c*x^2+b*x+a)^(3/2)/a^2/x^3-1/40*(8*a*A+5*(A*b+2*B*a)*x)*(c*x^2+b*x+a)^(
5/2)/a/x^5+1/256*(10*a*B*(-48*a^2*c^2-24*a*b^2*c+b^4)-A*(240*a^2*b*c^2-40*
a*b^3*c+3*b^5))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)
+1/2*c^(3/2)*(2*A*c+5*B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/
2))
```

Mathematica [A] (verified)

Time = 4.98 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx = \frac{-\frac{\sqrt{a+x(b+cx)}\left(-45Ab^4x^4+96a^4(4A+5Bx)+30ab^2x^3(5bBx+A(b+18cx))+16a^3x(5Bx(17b+\cdots\right.}{a^2x\cdots}}{\cdots}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^6,x]
```

output
```
(-((Sqrt[a + x*(b + c*x)]*(-45*A*b^4*x^4 + 96*a^4*(4*A + 5*B*x) + 30*a*b^2
*x^3*(5*b*B*x + A*(b + 18*c*x)) + 16*a^3*x*(5*B*x*(17*b + 27*c*x) + A*(63*
b + 88*c*x)) + 4*a^2*x^2*(5*B*x*(59*b^2 + 278*b*c*x - 96*c^2*x^2) + 2*A*(9
3*b^2 + 311*b*c*x + 368*c^2*x^2))))/(a^2*x^5)) + (45*(A*b^5 + 160*a^3*B*c^
2)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(5/2) + (150*b*
(b^3*B + 4*A*b^2*c - 24*a*b*B*c - 24*a*A*c^2)*ArcTanh[(-(Sqrt[c]*x) + Sqrt
[a + x*(b + c*x)])/Sqrt[a]])/a^(3/2) - 960*c^(3/2)*(5*b*B + 2*A*c)*Log[b +
 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/1920
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {1229, 27, 1229, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{\int \frac{\left(3Ab^2-10aBb-16aAc-2(Ab+10aB)cx\right)\left(cx^2+bx+a\right)^{3/2}}{2x^4}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(3Ab^2-10aBb-16aAc-2(Ab+10aB)cx\right)\left(cx^2+bx+a\right)^{3/2}}{x^4}dx}{16a}-\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$\Big\downarrow$ 1229

$$-\frac{-\frac{\int -\frac{\left(10abB\left(b^2-20ac\right)-2A\left(\frac{3b^4}{2}-14acb^2+64a^2c^2\right)-2c\left(10aB\left(b^2+12ac\right)-A\left(3b^3-28abc\right)\right)x\right)\sqrt{cx^2+bx+a}}{2x^2}dx}{4a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(4a\left(-16aAc-10a\right.\right.}{}}{16a}-\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{\left(10abB\left(b^2-20ac\right)-A\left(3b^4-28acb^2+128a^2c^2\right)-2c\left(10aB\left(b^2+12ac\right)-A\left(3b^3-28abc\right)\right)x\right)\sqrt{cx^2+bx+a}}{x^2}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(4a\left(-16aAc-10abB+3\right.\right.}{1}}{16a}-\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$\Big\downarrow$ 1230

$$-\frac{\frac{-\frac{1}{2}\int -\frac{-128a^2(5bB+2Ac)xc^2+10aB\left(b^4-24acb^2-48a^2c^2\right)-A\left(3b^5-40acb^3+240a^2c^2b\right)}{x\sqrt{cx^2+bx+a}}dx-\frac{\sqrt{a+bx+cx^2}\left(-A\left(128a^2c^2-28ab^2c+3b^4\right)+2cx\left(10aB\left(12ac+\right.\right.\right.}{x}}{8a}}{16a}-\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{1}{2}\int \frac{-128a^2(5bB+2Ac)xc^2+10aB\left(b^4-24acb^2-48a^2c^2\right)-A\left(3b^5-40acb^3+240a^2c^2b\right)}{x\sqrt{cx^2+bx+a}}dx-\frac{\sqrt{a+bx+cx^2}\left(-A\left(128a^2c^2-28ab^2c+3b^4\right)+2cx\left(10aB\left(12ac+b^2\right)-\right.\right.}{x}}{8a}}{16a}-\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$\Big\downarrow$ 1269

$$-\frac{\frac{1}{2}\left(\left(10aB\left(-48a^2c^2-24ab^2c+b^4\right)-A\left(240a^2bc^2-40ab^3c+3b^5\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-128a^2c^2(2Ac+5bB)\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)-\frac{\sqrt{a+bx+cx^2}\left(-A\left(1\right.\right.}{}}{8a}-\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$$\Big\downarrow \ 1092$$

$$-\frac{\frac{\frac{1}{2}\left(\left(10aB\left(-48a^2c^2-24ab^2c+b^4\right)-A\left(240a^2bc^2-40ab^3c+3b^5\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-256a^2c^2(2Ac+5bB)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)-\frac{\sqrt{a+}}{}}{8a}}{} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{\frac{1}{2}\left(\left(10aB\left(-48a^2c^2-24ab^2c+b^4\right)-A\left(240a^2bc^2-40ab^3c+3b^5\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-128a^2c^{3/2}(2Ac+5bB)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)-\frac{\sqrt{}}{}}{8a}}{} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$$\Big\downarrow \ 1154$$

$$-\frac{\frac{\frac{1}{2}\left(-2\left(10aB\left(-48a^2c^2-24ab^2c+b^4\right)-A\left(240a^2bc^2-40ab^3c+3b^5\right)\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-128a^2c^{3/2}(2Ac+5bB)\text{arctanh}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.\right.}{8a}}{} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{\frac{1}{2}\left(-\frac{\left(10aB\left(-48a^2c^2-24ab^2c+b^4\right)-A\left(240a^2bc^2-40ab^3c+3b^5\right)\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}-128a^2c^{3/2}(2Ac+5bB)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right.\right.}{8a}}{} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(5x(2aB+Ab)+8aA)}{40ax^5}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^6,x]
```

output
```
-1/40*((8*a*A + 5*(A*b + 2*a*B)*x)*(a + b*x + c*x^2)^(5/2))/(a*x^5) - (-1/
12*((4*a*(3*A*b^2 - 10*a*b*B - 16*a*A*c) - 3*(10*a*B*(b^2 + 4*a*c) - A*(3*
b^3 - 20*a*b*c))*x)*(a + b*x + c*x^2)^(3/2))/(a*x^3) + (-(((10*a*b*B*(b^2
- 20*a*c) - A*(3*b^4 - 28*a*b^2*c + 128*a^2*c^2) + 2*c*(10*a*B*(b^2 + 12*a
*c) - A*(3*b^3 - 28*a*b*c))*x)*Sqrt[a + b*x + c*x^2])/x) + (-(((10*a*B*(b^
4 - 24*a*b^2*c - 48*a^2*c^2) - A*(3*b^5 - 40*a*b^3*c + 240*a^2*b*c^2))*Arc
Tanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) - 128*a^2*c^
(3/2)*(5*b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/2)/(8*a))/(16*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,\left(2944 A\,a^{2} c^{2} x^{4}+540 A a\,b^{2} c\,x^{4}-45 A\,b^{4} x^{4}+5560 B\,a^{2} b c\,x^{4}+150 B a\,b^{3} x^{4}+2488 A\,a^{2} b c\,x^{3}+30 A a\,b^{3} x^{3}+2160 B\,a^{3} c\,x^{3}\right.}{1920 x^{5} a^{2}}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(c*x^2+b*x+a)^(1/2)*(2944*A*a^2*c^2*x^4+540*A*a*b^2*c*x^4-45*A*b^4
*x^4+5560*B*a^2*b*c*x^4+150*B*a*b^3*x^4+2488*A*a^2*b*c*x^3+30*A*a*b^3*x^3+
2160*B*a^3*c*x^3+1180*B*a^2*b^2*x^3+1408*A*a^3*c*x^2+744*A*a^2*b^2*x^2+136
0*B*a^3*b*x^2+1008*A*a^3*b*x+480*B*a^4*x+384*A*a^4)/x^5/a^2+1/256/a^2*(-(2
40*A*a^2*b*c^2-40*A*a*b^3*c+3*A*b^5+480*B*a^3*c^2+240*B*a^2*b^2*c-10*B*a*b
^4)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+256*a^2*A*c^(5/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+768*B*a^2*b*c^(3/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+256*B*a^2*c^3*(1/c*(c*x^2+b*x+a)^(1/2)
-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 1445, normalized size of antiderivative = 4.18

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
[1/7680*(1920*(5*B*a^3*b*c + 2*A*a^3*c^2)*sqrt(c)*x^5*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 15*(10
*B*a*b^4 - 3*A*b^5 - 240*(2*B*a^3 + A*a^2*b)*c^2 - 40*(6*B*a^2*b^2 - A*a*b
^3)*c)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
+ a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(1920*B*a^3*c^2*x^5 - 384*A*a^5
 - (150*B*a^2*b^3 - 45*A*a*b^4 + 2944*A*a^3*c^2 + 20*(278*B*a^3*b + 27*A*a
^2*b^2)*c)*x^4 - 2*(590*B*a^3*b^2 + 15*A*a^2*b^3 + 4*(270*B*a^4 + 311*A*a^
3*b)*c)*x^3 - 8*(170*B*a^4*b + 93*A*a^3*b^2 + 176*A*a^4*c)*x^2 - 48*(10*B*
a^5 + 21*A*a^4*b)*x)*sqrt(c*x^2 + b*x + a))/(a^3*x^5), -1/7680*(3840*(5*B*
a^3*b*c + 2*A*a^3*c^2)*sqrt(-c)*x^5*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 15*(10*B*a*b^4 - 3*A*b^5 - 240*
(2*B*a^3 + A*a^2*b)*c^2 - 40*(6*B*a^2*b^2 - A*a*b^3)*c)*sqrt(a)*x^5*log(-(
8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a)
+ 8*a^2)/x^2) - 4*(1920*B*a^3*c^2*x^5 - 384*A*a^5 - (150*B*a^2*b^3 - 45*A*
a*b^4 + 2944*A*a^3*c^2 + 20*(278*B*a^3*b + 27*A*a^2*b^2)*c)*x^4 - 2*(590*B
*a^3*b^2 + 15*A*a^2*b^3 + 4*(270*B*a^4 + 311*A*a^3*b)*c)*x^3 - 8*(170*B*a^
4*b + 93*A*a^3*b^2 + 176*A*a^4*c)*x^2 - 48*(10*B*a^5 + 21*A*a^4*b)*x)*sqrt
(c*x^2 + b*x + a))/(a^3*x^5), -1/3840*(15*(10*B*a*b^4 - 3*A*b^5 - 240*(2*B
*a^3 + A*a^2*b)*c^2 - 40*(6*B*a^2*b^2 - A*a*b^3)*c)*sqrt(-a)*x^5*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) -...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx=\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x**6,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1525 vs. $2(314) = 628$.

Time = 0.41 (sec) , antiderivative size = 1525, normalized size of antiderivative = 4.41

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^6,x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x + a)*B*c^2 - 1/2*(5*B*b*c^2 + 2*A*c^3)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c) - 1/128*(10*B*a*b^4 - 3
*A*b^5 - 240*B*a^2*b^2*c + 40*A*a*b^3*c - 480*B*a^3*c^2 - 240*A*a^2*b*c^2)
*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/
1920*(150*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a*b^4 - 45*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^9*A*b^5 + 7920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
9*B*a^2*b^2*c + 600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a*b^3*c + 4320
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a^3*c^2 + 7920*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*A*a^2*b*c^2 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^8*B*a^2*b^3*sqrt(c) + 23040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*B*a^3*b
*c^(3/2) + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*A*a^2*b^2*c^(3/2) +
 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*A*a^3*c^(5/2) + 580*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^7*B*a^2*b^4 + 210*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^7*A*a*b^5 - 13920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a^3*b^2*c
 + 6160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a^2*b^3*c - 4800*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^7*B*a^4*c^2 - 2400*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^7*A*a^3*b*c^2 - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*B*a^3*b
^3*sqrt(c) + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*A*a^2*b^4*sqrt(c)
- 57600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*B*a^4*b*c^(3/2) - 23040*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^6*A*a^4*c^(5/2) - 1280*(sqrt(c)*x - sqr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x^6}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^6,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^6}\,dx = \frac{-768\sqrt{c\,x^2+bx+a}\,a^5-2976\sqrt{c\,x^2+bx+a}\,a^4bx-2816\sqrt{c\,x^2+bx}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^6,x)
```

output
```
( - 768*sqrt(a + b*x + c*x**2)*a**5 - 2976*sqrt(a + b*x + c*x**2)*a**4*b*x
 - 2816*sqrt(a + b*x + c*x**2)*a**4*c*x**2 - 4208*sqrt(a + b*x + c*x**2)*a
**3*b**2*x**2 - 9296*sqrt(a + b*x + c*x**2)*a**3*b*c*x**3 - 5888*sqrt(a +
b*x + c*x**2)*a**3*c**2*x**4 - 2420*sqrt(a + b*x + c*x**2)*a**2*b**3*x**3
- 12200*sqrt(a + b*x + c*x**2)*a**2*b**2*c*x**4 + 3840*sqrt(a + b*x + c*x*
*2)*a**2*b*c**2*x**5 - 210*sqrt(a + b*x + c*x**2)*a*b**4*x**4 + 10800*sqrt
(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b*c**2*x**5 + 3
000*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**3*c*x**
5 - 105*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**5*x**
5 - 10800*sqrt(a)*log(x)*a**2*b*c**2*x**5 - 3000*sqrt(a)*log(x)*a*b**3*c*x
**5 + 105*sqrt(a)*log(x)*b**5*x**5 + 3840*sqrt(c)*log( - 2*sqrt(c)*sqrt(a
+ b*x + c*x**2) - b - 2*c*x)*a**3*c**2*x**5 + 9600*sqrt(c)*log( - 2*sqrt(c
)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*b**2*c*x**5)/(3840*a**2*x**5)
```

3.140 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^7}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 332

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^7}\,dx = \frac{\left(2a\left(4abB(3b^2-28ac)-5A(b^2-4ac)^2\right)-\left(5Ab(b^2-4ac)^2-4aB(\ldots\right.\right.}{512a^3x^2}$$
$$-\frac{\left(2a(12abB-5A(b^2-4ac))+\left(4aB(3b^2+16ac)-5A(b^3-4abc)\right)x\right)\left(a+bx+cx^2\right)^{3/2}}{192a^2x^4}$$
$$-\frac{(10aA+(5Ab+12aB)x)\left(a+bx+cx^2\right)^{5/2}}{60ax^6}$$
$$+\frac{\left(5A(b^2-4ac)^3-4abB(3b^4-40ab^2c+240a^2c^2)\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{1024a^{7/2}}$$
$$+Bc^{5/2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)$$

output
```
1/512*(2*a*(4*a*b*B*(-28*a*c+3*b^2)-5*A*(-4*a*c+b^2)^2)-(5*A*b*(-4*a*c+b^2
)^2-4*a*B*(-128*a^2*c^2-28*a*b^2*c+3*b^4))*x)*(c*x^2+b*x+a)^(1/2)/a^3/x^2-
1/192*(2*a*(12*a*b*B-5*A*(-4*a*c+b^2))+(4*a*B*(16*a*c+3*b^2)-5*A*(-4*a*b*c
+b^3))*x)*(c*x^2+b*x+a)^(3/2)/a^2/x^4-1/60*(10*a*A+(5*A*b+12*B*a)*x)*(c*x^
2+b*x+a)^(5/2)/a/x^6+1/1024*(5*A*(-4*a*c+b^2)^3-4*a*b*B*(240*a^2*c^2-40*a*
b^2*c+3*b^4))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)+B
*c^(5/2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 4.68 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^7}\,dx =$$

$$-\frac{\sqrt{a+x(b+cx)}(75Ab^5x^5+256a^5(5A+6Bx)-10ab^3x^4(18bBx+5A(b+16cx))+64a^4x(A(50b+65c\ldots}{}$$

$$-\frac{5Ab^6\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{512a^{7/2}}$$

$$-\frac{(3b^5B+15Ab^4c-40ab^3Bc-60aAb^2c^2+240a^2bBc^2+80a^2Ac^3)\,\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{128a^{5/2}}$$

$$-Bc^{5/2}\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^7,x]
```

output
```
-1/7680*(Sqrt[a + x*(b + c*x)]*(75*A*b^5*x^5 + 256*a^5*(5*A + 6*B*x) - 10*
a*b^3*x^4*(18*b*B*x + 5*A*(b + 16*c*x)) + 64*a^4*x*(A*(50*b + 65*c*x) + B*
x*(63*b + 88*c*x)) + 40*a^2*b*x^3*(3*b*B*x*(b + 18*c*x) + A*(b^2 + 12*b*c*
x + 66*c^2*x^2)) + 16*a^3*x^2*(15*A*(9*b^2 + 26*b*c*x + 22*c^2*x^2) + 2*B*
x*(93*b^2 + 311*b*c*x + 368*c^2*x^2))))/(a^3*x^6) - (5*A*b^6*ArcTanh[(Sqrt
[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(512*a^(7/2)) - ((3*b^5*B + 15*A*
b^4*c - 40*a*b^3*B*c - 60*a*A*b^2*c^2 + 240*a^2*b*B*c^2 + 80*a^2*A*c^3)*Ar
cTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(128*a^(5/2)) - B*c
^(5/2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {1229, 27, 1229, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^7}\,dx$$

$$\Big\downarrow \; 1229$$

$$-\frac{\int -\frac{(12abB+24acxB-5A(b^2-4ac))\left(cx^2+bx+a\right)^{3/2}}{2x^5}dx}{12a}-\frac{\left(a+bx+cx^2\right)^{5/2}(x(12aB+5Ab)+10aA)}{60ax^6}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(12abB+24acxB-5A(b^2-4ac))\left(cx^2+bx+a\right)^{3/2}}{x^5}dx}{24a}-\frac{\left(a+bx+cx^2\right)^{5/2}(x(12aB+5Ab)+10aA)}{60ax^6}$$

$$\Big\downarrow \; 1229$$

$$\frac{-\frac{\int \frac{3\left(-128a^2Bxc^2-5A\left(b^2-4ac\right)^2+4abB\left(3b^2-28ac\right)\right)\sqrt{cx^2+bx+a}}{2x^3}dx}{8a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(2a(12abB-5A(b^2-4ac))+x(4aB(16ac+3b^2)-5A(b^3-\right.}{8ax^4}}{24a}$$
$$\frac{\left(a+bx+cx^2\right)^{5/2}(x(12aB+5Ab)+10aA)}{60ax^6}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{3\int \frac{\left(-128a^2Bxc^2-5A\left(b^2-4ac\right)^2+4abB\left(3b^2-28ac\right)\right)\sqrt{cx^2+bx+a}}{x^3}dx}{16a}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(2a(12abB-5A(b^2-4ac))+x(4aB(16ac+3b^2)-5A(b^3-\right.}{8ax^4}}{24a}$$
$$\frac{\left(a+bx+cx^2\right)^{5/2}(x(12aB+5Ab)+10aA)}{60ax^6}$$

$$\Big\downarrow \; 1229$$

$$\frac{-\frac{3\left(-\frac{\int -\frac{-1024a^3Bxc^3+5A\left(b^2-4ac\right)^3-4abB\left(3b^4-40acb^2+240a^2c^2\right)}{2x\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(4abB\left(3b^2-28ac\right)-5A\left(b^2-4ac\right)^2\right)-x\left(5Ab\left(b^2-4ac\right)^2-4aB\right.\right.}{4ax^2}\right.}{16a}}{24a}$$
$$\frac{\left(a+bx+cx^2\right)^{5/2}(x(12aB+5Ab)+10aA)}{60ax^6}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\left(\frac{\int \frac{-1024a^3Bxc^3+5A\left(b^2-4ac\right)^3-4abB\left(3b^4-40acb^2+240a^2c^2\right)}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(4abB\left(3b^2-28ac\right)-5A\left(b^2-4ac\right)^2\right)-x\left(5Ab\left(b^2-4ac\right)^2-4aB\left(-12\cdots\right.\right.\right.}{4ax^2}\cdots\right.}{16a}\cdots}{24a}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(x(12aB+5Ab)+10aA\right)}{60ax^6}$$

$$\Big\downarrow \; 1269$$

$$-\frac{3\left(\frac{\left(5A\left(b^2-4ac\right)^3-4abB\left(240a^2c^2-40ab^2c+3b^4\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-1024a^3Bc^3\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(4abB\left(3b^2-28ac\right)-5A\left(b^2-4ac\right)^2\cdots\right.\right.}{\cdots}\right.}{16a}\cdots}{24a}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(x(12aB+5Ab)+10aA\right)}{60ax^6}$$

$$\Big\downarrow \; 1092$$

$$-\frac{3\left(\frac{\left(5A\left(b^2-4ac\right)^3-4abB\left(240a^2c^2-40ab^2c+3b^4\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-2048a^3Bc^3\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{8a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(4abB\left(3b^2-28ac\right)\cdots\right.\right.}{\cdots}\right.}{16a}\cdots}{\cdots}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(x(12aB+5Ab)+10aA\right)}{60ax^6}$$

$$\Big\downarrow \; 219$$

$$-\frac{3\left(\frac{\left(5A\left(b^2-4ac\right)^3-4abB\left(240a^2c^2-40ab^2c+3b^4\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-1024a^3Bc^{5/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(4abB\left(3b^2-28ac\right)\cdots\right.\right.}{\cdots}\right.}{16a}\cdots}{\cdots}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(x(12aB+5Ab)+10aA\right)}{60ax^6}$$

$$\Big\downarrow \; 1154$$

$$-\frac{3\left(\frac{-2\left(5A\left(b^2-4ac\right)^3-4abB\left(240a^2c^2-40ab^2c+3b^4\right)\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-1024a^3Bc^{5/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(\cdots\right.\right.}{\cdots}\right.}{16a}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(x(12aB+5Ab)+10aA)}{60ax^6}$$

$\downarrow$ 219

$$-\frac{3\left(\frac{-1024a^3Bc^{5/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-\frac{\left(5A\left(b^2-4ac\right)^3-4abB\left(240a^2c^2-40ab^2c+3b^4\right)\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}}{8a}-\frac{\sqrt{a+bx+cx^2}\left(2a\left(4\cdots\right.\right.}{\cdots}\right.}{16a}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}(x(12aB+5Ab)+10aA)}{60ax^6}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^7,x]
```

output
```
-1/60*((10*a*A + (5*A*b + 12*a*B)*x)*(a + b*x + c*x^2)^(5/2))/(a*x^6) + (-
1/8*((2*a*(12*a*b*B - 5*A*(b^2 - 4*a*c)) + (4*a*B*(3*b^2 + 16*a*c) - 5*A*(
b^3 - 4*a*b*c))*x)*(a + b*x + c*x^2)^(3/2))/(a*x^4) - (3*(-1/4*((2*a*(4*a*
b*B*(3*b^2 - 28*a*c) - 5*A*(b^2 - 4*a*c)^2) - (5*A*b*(b^2 - 4*a*c)^2 - 4*a
*B*(3*b^4 - 28*a*b^2*c - 128*a^2*c^2))*x)*Sqrt[a + b*x + c*x^2])/(a*x^2) +
 (-(((5*A*(b^2 - 4*a*c)^3 - 4*a*b*B*(3*b^4 - 40*a*b^2*c + 240*a^2*c^2))*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) - 1024*a^3*
B*c^(5/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*a)))/
(16*a))/(24*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(2640A\,a^2b\,c^2x^5-800Aa\,b^3c\,x^5+75A\,b^5x^5+11776B\,a^3c^2x^5+2160B\,a^2b^2c\,x^5-180Ba\,b^4x^5+5280A\,a^3c^2x^4+480A\,a}{}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(c*x^2+b*x+a)^(1/2)*(2640*A*a^2*b*c^2*x^5-800*A*a*b^3*c*x^5+75*A*b
^5*x^5+11776*B*a^3*c^2*x^5+2160*B*a^2*b^2*c*x^5-180*B*a*b^4*x^5+5280*A*a^3
*c^2*x^4+480*A*a^2*b^2*c*x^4-50*A*a*b^4*x^4+9952*B*a^3*b*c*x^4+120*B*a^2*b
^3*x^4+6240*A*a^3*b*c*x^3+40*A*a^2*b^3*x^3+5632*B*a^4*c*x^3+2976*B*a^3*b^2
*x^3+4160*A*a^4*c*x^2+2160*A*a^3*b^2*x^2+4032*B*a^4*b*x^2+3200*A*a^4*b*x+1
536*B*a^5*x+1280*A*a^5)/x^6/a^3+1/1024/a^3*(-(320*A*a^3*c^3-240*A*a^2*b^2*
c^2+60*A*a*b^4*c-5*A*b^6+960*B*a^3*b*c^2-160*B*a^2*b^3*c+12*B*a*b^5)/a^(1/
2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1024*B*a^3*c^(5/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 1659, normalized size of antiderivative = 5.00

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
[1/30720*(15360*B*a^4*c^(5/2)*x^6*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 15*(12*B*a*b^5 - 5*A*b^6 +
 320*A*a^3*c^3 + 240*(4*B*a^3*b - A*a^2*b^2)*c^2 - 20*(8*B*a^2*b^3 - 3*A*a
*b^4)*c)*sqrt(a)*x^6*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*
x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(1280*A*a^6 - (180*B*a^2*b^4
- 75*A*a*b^5 - 16*(736*B*a^4 + 165*A*a^3*b)*c^2 - 80*(27*B*a^3*b^2 - 10*A*
a^2*b^3)*c)*x^5 + 2*(60*B*a^3*b^3 - 25*A*a^2*b^4 + 2640*A*a^4*c^2 + 16*(31
1*B*a^4*b + 15*A*a^3*b^2)*c)*x^4 + 8*(372*B*a^4*b^2 + 5*A*a^3*b^3 + 4*(176
*B*a^5 + 195*A*a^4*b)*c)*x^3 + 16*(252*B*a^5*b + 135*A*a^4*b^2 + 260*A*a^5
*c)*x^2 + 128*(12*B*a^6 + 25*A*a^5*b)*x)*sqrt(c*x^2 + b*x + a))/(a^4*x^6),
 -1/30720*(30720*B*a^4*sqrt(-c)*c^2*x^6*arctan(1/2*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 15*(12*B*a*b^5 - 5*A*b^6 +
320*A*a^3*c^3 + 240*(4*B*a^3*b - A*a^2*b^2)*c^2 - 20*(8*B*a^2*b^3 - 3*A*a*
b^4)*c)*sqrt(a)*x^6*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
 + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(1280*A*a^6 - (180*B*a^2*b^4 -
 75*A*a*b^5 - 16*(736*B*a^4 + 165*A*a^3*b)*c^2 - 80*(27*B*a^3*b^2 - 10*A*a
^2*b^3)*c)*x^5 + 2*(60*B*a^3*b^3 - 25*A*a^2*b^4 + 2640*A*a^4*c^2 + 16*(311
*B*a^4*b + 15*A*a^3*b^2)*c)*x^4 + 8*(372*B*a^4*b^2 + 5*A*a^3*b^3 + 4*(176*
B*a^5 + 195*A*a^4*b)*c)*x^3 + 16*(252*B*a^5*b + 135*A*a^4*b^2 + 260*A*a^5*
c)*x^2 + 128*(12*B*a^6 + 25*A*a^5*b)*x)*sqrt(c*x^2 + b*x + a))/(a^4*x^6...
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x^7}\,dx=\int \frac{(A+Bx)\,(a+bx+cx^2)^{\frac{5}{2}}}{x^7}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x**7,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A + Bx)(a + bx + cx^2)^{5/2}}{x^7} dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2086 vs. $2(302) = 604$.

Time $= 0.54$ (sec) , antiderivative size $= 2086$, normalized size of antiderivative $= 6.28$

$$\int \frac{(A + Bx)(a + bx + cx^2)^{5/2}}{x^7} dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^7,x, algorithm="giac")
```

output
```
-B*c^(5/2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b)) + 1
/512*(12*B*a*b^5 - 5*A*b^6 - 160*B*a^2*b^3*c + 60*A*a*b^4*c + 960*B*a^3*b*
c^2 - 240*A*a^2*b^2*c^2 + 320*A*a^3*c^3)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/7680*(180*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^11*B*a*b^5 - 75*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*b^6 -
 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*B*a^2*b^3*c + 900*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^11*A*a*b^4*c - 31680*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^11*B*a^3*b*c^2 - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*a^2
*b^2*c^2 - 10560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*a^3*c^3 - 46080*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*B*a^3*b^2*c^(3/2) - 46080*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^10*B*a^4*c^(5/2) - 46080*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^10*A*a^3*b*c^(5/2) - 1020*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^9*B*a^2*b^5 + 425*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a*b^6 - 22240*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*B*a^3*b^3*c - 5100*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*A*a^2*b^4*c + 41280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^9*B*a^4*b*c^2 - 56400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a^3*b^2*c
^2 - 1600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*A*a^4*c^3 - 15360*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^8*B*a^3*b^4*sqrt(c) + 46080*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^8*B*a^4*b^2*c^(3/2) - 76800*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^8*A*a^3*b^3*c^(3/2) + 138240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^7}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x^7}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^7,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.52

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^7}\,dx = \frac{-2560\sqrt{c\,x^2+bx+a}\,a^6-9472\sqrt{c\,x^2+bx+a}\,a^5bx-8320\sqrt{c\,x^2+b}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^7,x)
```

output
```
( - 2560*sqrt(a + b*x + c*x**2)*a**6 - 9472*sqrt(a + b*x + c*x**2)*a**5*b*
x - 8320*sqrt(a + b*x + c*x**2)*a**5*c*x**2 - 12384*sqrt(a + b*x + c*x**2)
*a**4*b**2*x**2 - 23744*sqrt(a + b*x + c*x**2)*a**4*b*c*x**3 - 10560*sqrt(
a + b*x + c*x**2)*a**4*c**2*x**4 - 6032*sqrt(a + b*x + c*x**2)*a**3*b**3*x
**3 - 20864*sqrt(a + b*x + c*x**2)*a**3*b**2*c*x**4 - 28832*sqrt(a + b*x +
 c*x**2)*a**3*b*c**2*x**5 - 140*sqrt(a + b*x + c*x**2)*a**2*b**4*x**4 - 27
20*sqrt(a + b*x + c*x**2)*a**2*b**3*c*x**5 + 210*sqrt(a + b*x + c*x**2)*a*
b**5*x**5 + 4800*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)
*a**3*c**3*x**6 + 10800*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
 - b*x)*a**2*b**2*c**2*x**6 - 1500*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*
x**2) - 2*a - b*x)*a*b**4*c*x**6 + 105*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x
+ c*x**2) - 2*a - b*x)*b**6*x**6 - 4800*sqrt(a)*log(x)*a**3*c**3*x**6 - 10
800*sqrt(a)*log(x)*a**2*b**2*c**2*x**6 + 1500*sqrt(a)*log(x)*a*b**4*c*x**6
 - 105*sqrt(a)*log(x)*b**6*x**6 + 15360*sqrt(c)*log( - 2*sqrt(c)*sqrt(a +
b*x + c*x**2) - b - 2*c*x)*a**3*b*c**2*x**6)/(15360*a**3*x**6)
```

3.141 $\int \frac{(A+Bx)(a+bx+cx^2)^{5/2}}{x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^8}\,dx = \frac{5(Ab-2aB)\left(b^2-4ac\right)^2(2a+bx)\sqrt{a+bx+cx^2}}{1024a^4x^2} - \frac{5(Ab-2aB)\left(b^2-4ac\right)(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{384a^3x^4} + \frac{(Ab-2aB)(2a+bx)\left(a+bx+cx^2\right)^{5/2}}{24a^2x^6} - \frac{A\left(a+bx+cx^2\right)^{7/2}}{7ax^7} - \frac{5(Ab-2aB)\left(b^2-4ac\right)^3\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2048a^{9/2}}$$

output
```
5/1024*(A*b-2*B*a)*(-4*a*c+b^2)^2*(b*x+2*a)*(c*x^2+b*x+a)^(1/2)/a^4/x^2-5/
384*(A*b-2*B*a)*(-4*a*c+b^2)*(b*x+2*a)*(c*x^2+b*x+a)^(3/2)/a^3/x^4+1/24*(A
*b-2*B*a)*(b*x+2*a)*(c*x^2+b*x+a)^(5/2)/a^2/x^6-1/7*A*(c*x^2+b*x+a)^(7/2)/
a/x^7-5/2048*(A*b-2*B*a)*(-4*a*c+b^2)^3*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x
^2+b*x+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.84

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x^8}\,dx = \frac{-\sqrt{a}\sqrt{a+x(b+cx)}(-105Ab^6x^6+512a^6(6A+7Bx)+128a^5x(58A\ldots}{}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^8,x]
```

output
```
(-(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-105*A*b^6*x^6 + 512*a^6*(6*A + 7*B*x) +
 128*a^5*x*(58*A*b + 70*b*B*x + 72*A*c*x + 91*B*c*x^2) + 70*a*b^4*x^5*(3*b
*B*x + A*(b + 16*c*x)) - 28*a^2*b^2*x^4*(5*b*B*x*(b + 16*c*x) + 2*A*(b^2 +
 12*b*c*x + 66*c^2*x^2)) + 32*a^4*x^2*(21*B*x*(9*b^2 + 26*b*c*x + 22*c^2*x
^2) + 2*A*(74*b^2 + 197*b*c*x + 144*c^2*x^2)) + 16*a^3*x^3*(7*b*B*x*(b^2 +
 12*b*c*x + 66*c^2*x^2) + 3*A*(b^3 + 10*b^2*c*x + 38*b*c^2*x^2 + 64*c^3*x^
3)))) + 105*(A*b^7 + 128*a^4*B*c^3)*x^7*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b
 + c*x)])/Sqrt[a]] + 210*a*b*(b^5*B + 6*A*b^4*c - 12*a*b^3*B*c - 24*a*A*b^
2*c^2 + 48*a^2*b*B*c^2 + 32*a^2*A*c^3)*x^7*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a
+ x*(b + c*x)])/Sqrt[a]])/(21504*a^(9/2)*x^7)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1228, 1152, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^{5/2}}{x^8}\,dx$$

$$\Big\downarrow \; 1228$$

$$-\frac{(Ab-2aB)\int \frac{(cx^2+bx+a)^{5/2}}{x^7}dx}{2a}-\frac{A(a+bx+cx^2)^{7/2}}{7ax^7}$$

$$\Big\downarrow \; 1152$$

$$-\frac{(Ab-2aB)\left(-\frac{5\left(b^2-4ac\right)\int\frac{\left(cx^2+bx+a\right)^{3/2}}{x^5}dx}{24a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{5/2}}{12ax^6}\right)}{2a}-\frac{A\left(a+bx+cx^2\right)^{7/2}}{7ax^7}$$

$\downarrow$ 1152

$$-\frac{(Ab-2aB)\left(-\frac{5\left(b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\int\frac{\sqrt{cx^2+bx+a}}{x^3}dx}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{24a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{5/2}}{12ax^6}\right)}{2a}-\frac{A\left(a+bx+cx^2\right)^{7/2}}{7ax^7}$$

$\downarrow$ 1152

$$-\frac{(Ab-2aB)\left(-\frac{5\left(b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{24a}-\frac{(2a+bx)(a}{12}\right.}{2a}$$

$$\frac{A\left(a+bx+cx^2\right)^{7/2}}{7ax^7}$$

$\downarrow$ 1154

$$-\frac{(Ab-2aB)\left(-\frac{5(b^2-4ac)\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)(a+bx+cx^2)^{3/2}}{8ax^4}\right)}{24a}-\cdots\right)}{2a}$$

$$\frac{A(a+bx+cx^2)^{7/2}}{7ax^7}$$

$\downarrow$ 219

$$-\frac{(Ab-2aB)\left(-\frac{5(b^2-4ac)\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)(a+bx+cx^2)^{3/2}}{8ax^4}\right)}{24a}-\cdots\right)}{2a}$$

$$\frac{A(a+bx+cx^2)^{7/2}}{7ax^7}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^8,x]
```

output
```
-1/7*(A*(a + b*x + c*x^2)^(7/2))/(a*x^7) - ((A*b - 2*a*B)*(-1/12*((2*a + b
*x)*(a + b*x + c*x^2)^(5/2))/(a*x^6) - (5*(b^2 - 4*a*c)*(-1/8*((2*a + b*x)
*(a + b*x + c*x^2)^(3/2))/(a*x^4) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x)*Sq
rt[a + b*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[
a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2))))/(16*a)))/(24*a)))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(193) = 386$.

Time = 1.82 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.04

method	result
risch	$-\frac{\sqrt{c\,x^2+b x+a}\,(3072 A\,a^3 c^3 x^6-3696 A\,a^2 b^2 c^2 x^6+1120 A a\,b^4 c\,x^6-105 A\,b^6 x^6+7392 B\,a^3 b\,c^2 x^6-2240 B\,a^2 b^3 c\,x^6+210 B a\,b^5 x^6+18\cdots}{\cdots}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21504*(c*x^2+b*x+a)^(1/2)*(3072*A*a^3*c^3*x^6-3696*A*a^2*b^2*c^2*x^6+11
20*A*a*b^4*c*x^6-105*A*b^6*x^6+7392*B*a^3*b*c^2*x^6-2240*B*a^2*b^3*c*x^6+2
10*B*a*b^5*x^6+1824*A*a^3*b*c^2*x^5-672*A*a^2*b^3*c*x^5+70*A*a*b^5*x^5+147
84*B*a^4*c^2*x^5+1344*B*a^3*b^2*c*x^5-140*B*a^2*b^4*x^5+9216*A*a^4*c^2*x^4
+480*A*a^3*b^2*c*x^4-56*A*a^2*b^4*x^4+17472*B*a^4*b*c*x^4+112*B*a^3*b^3*x^
4+12608*A*a^4*b*c*x^3+48*A*a^3*b^3*x^3+11648*B*a^5*c*x^3+6048*B*a^4*b^2*x^
3+9216*A*a^5*c*x^2+4736*A*a^4*b^2*x^2+8960*B*a^5*b*x^2+7424*A*a^5*b*x+3584
*B*a^6*x+3072*A*a^6)/x^7/a^4+5/2048*(64*A*a^3*b*c^3-48*A*a^2*b^3*c^2+12*A*
a*b^5*c-A*b^7-128*B*a^4*c^3+96*B*a^3*b^2*c^2-24*B*a^2*b^4*c+2*B*a*b^6)/a^(
9/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(197) = 394$.

Time = 2.20 (sec) , antiderivative size = 887, normalized size of antiderivative = 4.05

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
[1/86016*(105*(2*B*a*b^6 - A*b^7 - 64*(2*B*a^4 - A*a^3*b)*c^3 + 48*(2*B*a^
3*b^2 - A*a^2*b^3)*c^2 - 12*(2*B*a^2*b^4 - A*a*b^5)*c)*sqrt(a)*x^7*log(-(8
*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) +
 8*a^2)/x^2) - 4*(3072*A*a^7 + (210*B*a^2*b^5 - 105*A*a*b^6 + 3072*A*a^4*c
^3 + 3696*(2*B*a^4*b - A*a^3*b^2)*c^2 - 1120*(2*B*a^3*b^3 - A*a^2*b^4)*c)*
x^6 - 2*(70*B*a^3*b^4 - 35*A*a^2*b^5 - 48*(154*B*a^5 + 19*A*a^4*b)*c^2 - 3
36*(2*B*a^4*b^2 - A*a^3*b^3)*c)*x^5 + 8*(14*B*a^4*b^3 - 7*A*a^3*b^4 + 1152
*A*a^5*c^2 + 12*(182*B*a^5*b + 5*A*a^4*b^2)*c)*x^4 + 16*(378*B*a^5*b^2 + 3
*A*a^4*b^3 + 4*(182*B*a^6 + 197*A*a^5*b)*c)*x^3 + 128*(70*B*a^6*b + 37*A*a
^5*b^2 + 72*A*a^6*c)*x^2 + 256*(14*B*a^7 + 29*A*a^6*b)*x)*sqrt(c*x^2 + b*x
 + a))/(a^5*x^7), -1/43008*(105*(2*B*a*b^6 - A*b^7 - 64*(2*B*a^4 - A*a^3*b
)*c^3 + 48*(2*B*a^3*b^2 - A*a^2*b^3)*c^2 - 12*(2*B*a^2*b^4 - A*a*b^5)*c)*s
qrt(-a)*x^7*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2
 + a*b*x + a^2)) + 2*(3072*A*a^7 + (210*B*a^2*b^5 - 105*A*a*b^6 + 3072*A*a
^4*c^3 + 3696*(2*B*a^4*b - A*a^3*b^2)*c^2 - 1120*(2*B*a^3*b^3 - A*a^2*b^4)
*c)*x^6 - 2*(70*B*a^3*b^4 - 35*A*a^2*b^5 - 48*(154*B*a^5 + 19*A*a^4*b)*c^2
 - 336*(2*B*a^4*b^2 - A*a^3*b^3)*c)*x^5 + 8*(14*B*a^4*b^3 - 7*A*a^3*b^4 +
1152*A*a^5*c^2 + 12*(182*B*a^5*b + 5*A*a^4*b^2)*c)*x^4 + 16*(378*B*a^5*b^2
 + 3*A*a^4*b^3 + 4*(182*B*a^6 + 197*A*a^5*b)*c)*x^3 + 128*(70*B*a^6*b + 37
*A*a^5*b^2 + 72*A*a^6*c)*x^2 + 256*(14*B*a^7 + 29*A*a^6*b)*x)*sqrt(c*x^...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^8}\,dx=\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x**8,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x**8, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2598 vs. $2(197) = 394$.

Time = 0.29 (sec) , antiderivative size = 2598, normalized size of antiderivative = 11.86

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^8,x, algorithm="giac")
```

output
```
-5/1024*(2*B*a*b^6 - A*b^7 - 24*B*a^2*b^4*c + 12*A*a*b^5*c + 96*B*a^3*b^2*
c^2 - 48*A*a^2*b^3*c^2 - 128*B*a^4*c^3 + 64*A*a^3*b*c^3)*arctan(-(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) + 1/21504*(210*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^13*B*a*b^6 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^13*A*b^7 - 2520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*B*a^2*b^4*c
 + 1260*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*A*a*b^5*c + 10080*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^13*B*a^3*b^2*c^2 - 5040*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^13*A*a^2*b^3*c^2 + 29568*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
13*B*a^4*c^3 + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*A*a^3*b*c^3 + 1
29024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*B*a^4*b*c^(5/2) + 43008*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^12*A*a^4*c^(7/2) - 1400*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^11*B*a^2*b^6 + 700*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11
*A*a*b^7 + 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*B*a^3*b^4*c - 8400
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*a^2*b^5*c + 147840*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^11*B*a^4*b^2*c^2 + 33600*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^11*A*a^3*b^3*c^2 - 25088*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*
B*a^5*c^3 + 141568*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*A*a^4*b*c^3 + 21
5040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*B*a^4*b^3*c^(3/2) - 129024*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^10*B*a^5*b*c^(5/2) + 387072*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^10*A*a^4*b^2*c^(5/2) + 3962*(sqrt(c)*x - sqrt(c*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^8}\,dx = \int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x^8}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^8,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.56

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^8}\,dx = \frac{-6144\sqrt{c\,x^2+bx+a}\,a^7-22016\sqrt{c\,x^2+bx+a}\,a^6bx-18432\sqrt{c\,x^2+}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^8,x)
```

output
```
( - 6144*sqrt(a + b*x + c*x**2)*a**7 - 22016*sqrt(a + b*x + c*x**2)*a**6*b
*x - 18432*sqrt(a + b*x + c*x**2)*a**6*c*x**2 - 27392*sqrt(a + b*x + c*x**
2)*a**5*b**2*x**2 - 48512*sqrt(a + b*x + c*x**2)*a**5*b*c*x**3 - 18432*sqr
t(a + b*x + c*x**2)*a**5*c**2*x**4 - 12192*sqrt(a + b*x + c*x**2)*a**4*b**
3*x**3 - 35904*sqrt(a + b*x + c*x**2)*a**4*b**2*c*x**4 - 33216*sqrt(a + b*
x + c*x**2)*a**4*b*c**2*x**5 - 6144*sqrt(a + b*x + c*x**2)*a**4*c**3*x**6
- 112*sqrt(a + b*x + c*x**2)*a**3*b**4*x**4 - 1344*sqrt(a + b*x + c*x**2)*
a**3*b**3*c*x**5 - 7392*sqrt(a + b*x + c*x**2)*a**3*b**2*c**2*x**6 + 140*s
qrt(a + b*x + c*x**2)*a**2*b**5*x**5 + 2240*sqrt(a + b*x + c*x**2)*a**2*b*
*4*c*x**6 - 210*sqrt(a + b*x + c*x**2)*a*b**6*x**6 + 6720*sqrt(a)*log(2*sq
rt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*b*c**3*x**7 - 5040*sqrt(a)*
log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**3*c**2*x**7 + 12
60*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**5*c*x**7
 - 105*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**7*x**7
 - 6720*sqrt(a)*log(x)*a**3*b*c**3*x**7 + 5040*sqrt(a)*log(x)*a**2*b**3*c*
*2*x**7 - 1260*sqrt(a)*log(x)*a*b**5*c*x**7 + 105*sqrt(a)*log(x)*b**7*x**7
)/(43008*a**4*x**7)
```

3.142 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^9}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 288

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^9}\,dx = & -\frac{5(b^2-4ac)^2\,(9Ab^2-16abB-4aAc)\,(2a+bx)\sqrt{a+bx+cx^2}}{16384a^5x^2} \\
& + \frac{5(b^2-4ac)\,(9Ab^2-16abB-4aAc)\,(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{6144a^4x^4} \\
& - \frac{(9Ab^2-16abB-4aAc)\,(2a+bx)\left(a+bx+cx^2\right)^{5/2}}{384a^3x^6} \\
& - \frac{A\left(a+bx+cx^2\right)^{7/2}}{8ax^8} + \frac{(9Ab-16aB)\left(a+bx+cx^2\right)^{7/2}}{112a^2x^7} \\
& + \frac{5(b^2-4ac)^3\,(9Ab^2-16abB-4aAc)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{32768a^{11/2}}
\end{aligned}
$$

output

```
-5/16384*(-4*a*c+b^2)^2*(-4*A*a*c+9*A*b^2-16*B*a*b)*(b*x+2*a)*(c*x^2+b*x+a
)^(1/2)/a^5/x^2+5/6144*(-4*a*c+b^2)*(-4*A*a*c+9*A*b^2-16*B*a*b)*(b*x+2*a)*
(c*x^2+b*x+a)^(3/2)/a^4/x^4-1/384*(-4*A*a*c+9*A*b^2-16*B*a*b)*(b*x+2*a)*(c
*x^2+b*x+a)^(5/2)/a^3/x^6-1/8*A*(c*x^2+b*x+a)^(7/2)/a/x^8+1/112*(9*A*b-16*
B*a)*(c*x^2+b*x+a)^(7/2)/a^2/x^7+5/32768*(-4*a*c+b^2)^3*(-4*A*a*c+9*A*b^2-
16*B*a*b)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 7.67 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.70

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^9}\,dx = \frac{-\sqrt{a}\sqrt{a+x(b+cx)}(945Ab^7x^7+6144a^7(7A+8Bx)-210ab^5x^6(3A$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^9,x]
```

output
```
(-(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(945*A*b^7*x^7 + 6144*a^7*(7*A + 8*B*x) -
 210*a*b^5*x^6*(3*A*b + 8*b*B*x + 50*A*c*x) + 1024*a^6*x*(4*B*x*(29*b + 36
*c*x) + A*(99*b + 119*c*x)) + 256*a^5*x^2*(4*B*x*(74*b^2 + 197*b*c*x + 144
*c^2*x^2) + A*(243*b^2 + 614*b*c*x + 413*c^2*x^2)) + 56*a^2*b^3*x^5*(20*b*
B*x*(b + 16*c*x) + A*(9*b^2 + 113*b*c*x + 674*c^2*x^2)) + 384*a^4*x^3*(A*(
b^3 + 9*b^2*c*x + 29*b*c^2*x^2 + 35*c^3*x^3) + 2*B*x*(b^3 + 10*b^2*c*x + 3
8*b*c^2*x^2 + 64*c^3*x^3)) - 16*a^3*b*x^4*(56*b*B*x*(b^2 + 12*b*c*x + 66*c
^2*x^2) + A*(27*b^3 + 284*b^2*c*x + 1194*b*c^2*x^2 + 2652*c^3*x^3)))) - 94
5*A*b^8*x^8*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 1680*a*
(-(b^7*B) - 7*A*b^6*c + 12*a*b^5*B*c + 30*a*A*b^4*c^2 - 48*a^2*b^3*B*c^2 -
 48*a^2*A*b^2*c^3 + 64*a^3*b*B*c^3 + 16*a^3*A*c^4)*x^8*ArcTanh[(-(Sqrt[c]*
x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(344064*a^(11/2)*x^8)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1237, 27, 1228, 1152, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^9}\,dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int \frac{(9Ab-16aB+2Acx)\left(cx^2+bx+a\right)^{5/2}}{2x^8}dx}{8a}-\frac{A\left(a+bx+cx^2\right)^{7/2}}{8ax^8}$$

$\downarrow$ 27

$$-\frac{\int \frac{(9Ab-16aB+2Acx)(cx^2+bx+a)^{5/2}}{x^8}dx}{16a}-\frac{A(a+bx+cx^2)^{7/2}}{8ax^8}$$

$\downarrow$ 1228

$$-\frac{-\frac{(-4aAc-16abB+9Ab^2)\int \frac{(cx^2+bx+a)^{5/2}}{x^7}dx}{2a}-\frac{(9Ab-16aB)(a+bx+cx^2)^{7/2}}{7ax^7}}{16a}-\frac{A(a+bx+cx^2)^{7/2}}{8ax^8}$$

$\downarrow$ 1152

$$-\frac{-\frac{(-4aAc-16abB+9Ab^2)\left(-\frac{5(b^2-4ac)\int \frac{(cx^2+bx+a)^{3/2}}{x^5}dx}{24a}-\frac{(2a+bx)(a+bx+cx^2)^{5/2}}{12ax^6}\right)}{2a}-\frac{(9Ab-16aB)(a+bx+cx^2)^{7/2}}{7ax^7}}{16a}-\frac{A(a+bx+cx^2)^{7/2}}{8ax^8}$$

$\downarrow$ 1152

$$-\frac{-\frac{(-4aAc-16abB+9Ab^2)\left(-\frac{5(b^2-4ac)\left(-\frac{3(b^2-4ac)\int \frac{\sqrt{cx^2+bx+a}}{x^3}dx}{16a}-\frac{(2a+bx)(a+bx+cx^2)^{3/2}}{8ax^4}\right)}{24a}-\frac{(2a+bx)(a+bx+cx^2)^{5/2}}{12ax^6}\right)}{2a}-\frac{(9Ab-16aB\cdots}{\cdots}}{16a}-\frac{A(a+bx+cx^2)^{7/2}}{8ax^8}$$

$\downarrow$ 1152

$$-\frac{-\frac{(-4aAc-16abB+9Ab^2)\left(-\frac{5(b^2-4ac)\left(-\frac{3(b^2-4ac)\left(-\frac{(b^2-4ac)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)(a+bx+cx^2)^{3/2}}{8ax^4}\right)}{24a}-\frac{(2a+\cdots}{\cdots}\cdots\right)}{2a}\cdots}{16a}-\frac{A(a+bx+cx^2)^{7/2}}{8ax^8}$$

$\Big\downarrow$ 1154

$$-\frac{-\frac{(-4aAc-16abB+9Ab^2)\left(-\frac{5(b^2-4ac)\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/\cdots}}{8ax^4}\cdots\right)}{24a}\cdots\right)}{2a}\cdots}{16a}\cdots$$

$$\frac{A\left(a+bx+cx^2\right)^{7/2}}{8ax^8}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{(-4aAc-16abB+9Ab^2)\left(-\frac{5(b^2-4ac)\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/\cdots}}{8ax^4}\cdots\right)}{24a}\cdots\right)}{2a}\cdots}{16a}\cdots$$

$$\frac{A\left(a+bx+cx^2\right)^{7/2}}{8ax^8}$$

input

```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^9,x]
```

output
```
-1/8*(A*(a + b*x + c*x^2)^(7/2))/(a*x^8) - (-1/7*((9*A*b - 16*a*B)*(a + b*
x + c*x^2)^(7/2))/(a*x^7) - ((9*A*b^2 - 16*a*b*B - 4*a*A*c)*(-1/12*((2*a +
 b*x)*(a + b*x + c*x^2)^(5/2))/(a*x^6) - (5*(b^2 - 4*a*c)*(-1/8*((2*a + b*
x)*(a + b*x + c*x^2)^(3/2))/(a*x^4) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x)*
Sqrt[a + b*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqr
t[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2))))/(16*a)))/(24*a)))/(2*a))/(16*a
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(258) = 516$.

Time = 1.85 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.95

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-42432A\,a^3b\,c^3x^7+37744A\,a^2b^3c^2x^7-10500Aa\,b^5c\,x^7+945A\,b^7x^7+49152B\,a^4c^3x^7-59136B\,a^3b^2c^2x^7+17920B}{}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/344064*(c*x^2+b*x+a)^(1/2)*(-42432*A*a^3*b*c^3*x^7+37744*A*a^2*b^3*c^2*
x^7-10500*A*a*b^5*c*x^7+945*A*b^7*x^7+49152*B*a^4*c^3*x^7-59136*B*a^3*b^2*
c^2*x^7+17920*B*a^2*b^4*c*x^7-1680*B*a*b^6*x^7+13440*A*a^4*c^3*x^6-19104*A
*a^3*b^2*c^2*x^6+6328*A*a^2*b^4*c*x^6-630*A*a*b^6*x^6+29184*B*a^4*b*c^2*x^
6-10752*B*a^3*b^3*c*x^6+1120*B*a^2*b^5*x^6+11136*A*a^4*b*c^2*x^5-4544*A*a^
3*b^3*c*x^5+504*A*a^2*b^5*x^5+147456*B*a^5*c^2*x^5+7680*B*a^4*b^2*c*x^5-89
6*B*a^3*b^4*x^5+105728*A*a^5*c^2*x^4+3456*A*a^4*b^2*c*x^4-432*A*a^3*b^4*x^
4+201728*B*a^5*b*c*x^4+768*B*a^4*b^3*x^4+157184*A*a^5*b*c*x^3+384*A*a^4*b^
3*x^3+147456*B*a^6*c*x^3+75776*B*a^5*b^2*x^3+121856*A*a^6*c*x^2+62208*A*a^
5*b^2*x^2+118784*B*a^6*b*x^2+101376*A*a^6*b*x+49152*B*a^7*x+43008*A*a^7)/x
^8/a^5+5/32768*(256*A*a^4*c^4-768*A*a^3*b^2*c^3+480*A*a^2*b^4*c^2-112*A*a*
b^6*c+9*A*b^8+1024*B*a^4*b*c^3-768*B*a^3*b^3*c^2+192*B*a^2*b^5*c-16*B*a*b^
7)/a^(11/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(258) = 516$.

Time = 4.08 (sec) , antiderivative size = 1091, normalized size of antiderivative = 3.79

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
[-1/1376256*(105*(16*B*a*b^7 - 9*A*b^8 - 256*A*a^4*c^4 - 256*(4*B*a^4*b -
3*A*a^3*b^2)*c^3 + 96*(8*B*a^3*b^3 - 5*A*a^2*b^4)*c^2 - 16*(12*B*a^2*b^5 -
 7*A*a*b^6)*c)*sqrt(a)*x^8*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^
2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(43008*A*a^8 - (1680*B*
a^2*b^6 - 945*A*a*b^7 - 192*(256*B*a^5 - 221*A*a^4*b)*c^3 + 112*(528*B*a^4
*b^2 - 337*A*a^3*b^3)*c^2 - 140*(128*B*a^3*b^4 - 75*A*a^2*b^5)*c)*x^7 + 2*
(560*B*a^3*b^5 - 315*A*a^2*b^6 + 6720*A*a^5*c^3 + 48*(304*B*a^5*b - 199*A*
a^4*b^2)*c^2 - 28*(192*B*a^4*b^3 - 113*A*a^3*b^4)*c)*x^6 - 8*(112*B*a^4*b^
4 - 63*A*a^3*b^5 - 48*(384*B*a^6 + 29*A*a^5*b)*c^2 - 8*(120*B*a^5*b^2 - 71
*A*a^4*b^3)*c)*x^5 + 16*(48*B*a^5*b^3 - 27*A*a^4*b^4 + 6608*A*a^6*c^2 + 8*
(1576*B*a^6*b + 27*A*a^5*b^2)*c)*x^4 + 128*(592*B*a^6*b^2 + 3*A*a^5*b^3 +
4*(288*B*a^7 + 307*A*a^6*b)*c)*x^3 + 256*(464*B*a^7*b + 243*A*a^6*b^2 + 47
6*A*a^7*c)*x^2 + 3072*(16*B*a^8 + 33*A*a^7*b)*x)*sqrt(c*x^2 + b*x + a))/(a
^6*x^8), 1/688128*(105*(16*B*a*b^7 - 9*A*b^8 - 256*A*a^4*c^4 - 256*(4*B*a^
4*b - 3*A*a^3*b^2)*c^3 + 96*(8*B*a^3*b^3 - 5*A*a^2*b^4)*c^2 - 16*(12*B*a^2
*b^5 - 7*A*a*b^6)*c)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x +
2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(43008*A*a^8 - (1680*B*a^2*b^6
- 945*A*a*b^7 - 192*(256*B*a^5 - 221*A*a^4*b)*c^3 + 112*(528*B*a^4*b^2 - 3
37*A*a^3*b^3)*c^2 - 140*(128*B*a^3*b^4 - 75*A*a^2*b^5)*c)*x^7 + 2*(560*B*a
^3*b^5 - 315*A*a^2*b^6 + 6720*A*a^5*c^3 + 48*(304*B*a^5*b - 199*A*a^4*b...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^9}\,dx=\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x**9,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x**9, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)(a+bx+cx^2)^{5/2}}{x^9}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3603 vs. $2(258) = 516$.

Time = 0.31 (sec) , antiderivative size = 3603, normalized size of antiderivative = 12.51

$$\int \frac{(A+Bx)(a+bx+cx^2)^{5/2}}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^9,x, algorithm="giac")
```

output
```
5/16384*(16*B*a*b^7 - 9*A*b^8 - 192*B*a^2*b^5*c + 112*A*a*b^6*c + 768*B*a^
3*b^3*c^2 - 480*A*a^2*b^4*c^2 - 1024*B*a^4*b*c^3 + 768*A*a^3*b^2*c^3 - 256
*A*a^4*c^4)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a
)*a^5) - 1/344064*(1680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*B*a*b^7 - 9
45*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*A*b^8 - 20160*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^15*B*a^2*b^5*c + 11760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^15*A*a*b^6*c + 80640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*B*a^3*b^3*c
^2 - 50400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*A*a^2*b^4*c^2 - 107520*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*B*a^4*b*c^3 + 80640*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^15*A*a^3*b^2*c^3 - 26880*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^15*A*a^4*c^4 - 688128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^14*B*a^5*
c^(7/2) - 12880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*B*a^2*b^7 + 7245*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^13*A*a*b^8 + 154560*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^13*B*a^3*b^5*c - 90160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^13*A*a^2*b^6*c - 618240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*B*a^4*b^3*
c^2 + 386400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*A*a^3*b^4*c^2 - 215756
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*B*a^5*b*c^3 - 618240*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^13*A*a^4*b^2*c^3 - 711424*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^13*A*a^5*c^4 - 6193152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*
B*a^5*b^2*c^(5/2) + 688128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*B*a^6...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^9}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x^9}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^9,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 19.11 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.47

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^9}\,dx = \frac{-12288\sqrt{c\,x^2+bx+a}\,a^8-87040\sqrt{c\,x^2+bx+a}\,a^6bc\,x^3-58624\sqrt{c\,x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^9,x)
```

output
```
( - 12288*sqrt(a + b*x + c*x**2)*a**8 - 43008*sqrt(a + b*x + c*x**2)*a**7*
b*x - 34816*sqrt(a + b*x + c*x**2)*a**7*c*x**2 - 51712*sqrt(a + b*x + c*x*
*2)*a**6*b**2*x**2 - 87040*sqrt(a + b*x + c*x**2)*a**6*b*c*x**3 - 30208*sq
rt(a + b*x + c*x**2)*a**6*c**2*x**4 - 21760*sqrt(a + b*x + c*x**2)*a**5*b*
*3*x**3 - 58624*sqrt(a + b*x + c*x**2)*a**5*b**2*c*x**4 - 45312*sqrt(a + b
*x + c*x**2)*a**5*b*c**2*x**5 - 3840*sqrt(a + b*x + c*x**2)*a**5*c**3*x**6
 - 96*sqrt(a + b*x + c*x**2)*a**4*b**4*x**4 - 896*sqrt(a + b*x + c*x**2)*a
**4*b**3*c*x**5 - 2880*sqrt(a + b*x + c*x**2)*a**4*b**2*c**2*x**6 - 1920*s
qrt(a + b*x + c*x**2)*a**4*b*c**3*x**7 + 112*sqrt(a + b*x + c*x**2)*a**3*b
**5*x**5 + 1264*sqrt(a + b*x + c*x**2)*a**3*b**4*c*x**6 + 6112*sqrt(a + b*
x + c*x**2)*a**3*b**3*c**2*x**7 - 140*sqrt(a + b*x + c*x**2)*a**2*b**6*x**
6 - 2120*sqrt(a + b*x + c*x**2)*a**2*b**5*c*x**7 + 210*sqrt(a + b*x + c*x*
*2)*a*b**7*x**7 + 3840*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2
*a - b*x)*a**4*c**4*x**8 + 3840*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*
x**2) - 2*a - b*x)*a**3*b**2*c**3*x**8 - 4320*sqrt(a)*log( - 2*sqrt(a)*sqr
t(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**4*c**2*x**8 + 1200*sqrt(a)*log( -
 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**6*c*x**8 - 105*sqrt(a)
*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**8*x**8 - 3840*sqr
t(a)*log(x)*a**4*c**4*x**8 - 3840*sqrt(a)*log(x)*a**3*b**2*c**3*x**8 + 432
0*sqrt(a)*log(x)*a**2*b**4*c**2*x**8 - 1200*sqrt(a)*log(x)*a*b**6*c*x**...
```

3.143 $\int \frac{(A+Bx)(a+bx+cx^2)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 375

$$\int \frac{(A+Bx)(a+bx+cx^2)^{5/2}}{x^{10}}\,dx = -\frac{5(b^2-4ac)^2\,(2aB(9b^2-4ac)-A(11b^3-12abc))\,(2a+bx)\sqrt{a+bx+cx^2}}{32768a^6x^2} + \frac{5(b^2-4ac)\,(2aB(9b^2-4ac)-A(11b^3-12abc))\,(2a+bx)\,(a+bx+cx^2)^{3/2}}{12288a^5x^4} + \frac{(11Ab^3-18ab^2B-12aAbc+8a^2Bc)\,(2a+bx)\,(a+bx+cx^2)^{5/2}}{768a^4x^6} - \frac{A(a+bx+cx^2)^{7/2}}{9ax^9} + \frac{(11Ab-18aB)\,(a+bx+cx^2)^{7/2}}{144a^2x^8} - \frac{(99Ab^2-162abB-64aAc)\,(a+bx+cx^2)^{7/2}}{2016a^3x^7} + \frac{5(b^2-4ac)^3\,(2aB(9b^2-4ac)-A(11b^3-12abc))\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{65536a^{13/2}}$$

output
```
-5/32768*(-4*a*c+b^2)^2*(2*a*B*(-4*a*c+9*b^2)-A*(-12*a*b*c+11*b^3))*(b*x+2
*a)*(c*x^2+b*x+a)^(1/2)/a^6/x^2+5/12288*(-4*a*c+b^2)*(2*a*B*(-4*a*c+9*b^2)
-A*(-12*a*b*c+11*b^3))*(b*x+2*a)*(c*x^2+b*x+a)^(3/2)/a^5/x^4+1/768*(-12*A*
a*b*c+11*A*b^3+8*B*a^2*c-18*B*a*b^2)*(b*x+2*a)*(c*x^2+b*x+a)^(5/2)/a^4/x^6
-1/9*A*(c*x^2+b*x+a)^(7/2)/a/x^9+1/144*(11*A*b-18*B*a)*(c*x^2+b*x+a)^(7/2)
/a^2/x^8-1/2016*(-64*A*a*c+99*A*b^2-162*B*a*b)*(c*x^2+b*x+a)^(7/2)/a^3/x^7
+5/65536*(-4*a*c+b^2)^3*(2*a*B*(-4*a*c+9*b^2)-A*(-12*a*b*c+11*b^3))*arctan
h(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(13/2)
```

Mathematica [A] (verified)

Time = 10.97 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^{10}}\,dx = \frac{-\frac{A(a+x(b+cx))^{7/2}}{x^9}+\frac{(11Ab-18aB)(a+x(b+cx))^{7/2}}{16ax^8}+\frac{(-99Ab^2+162abB+64aAc)(a+x(\cdots}{224a^2x^7}\cdots}{\cdots}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^10,x]
```

output
```
(-((A*(a + x*(b + c*x))^(7/2))/x^9) + ((11*A*b - 18*a*B)*(a + x*(b + c*x))
^(7/2))/(16*a*x^8) + ((-99*A*b^2 + 162*a*b*B + 64*a*A*c)*(a + x*(b + c*x))
^(7/2))/(224*a^2*x^7) + (3*(2*a*B*(-9*b^2 + 4*a*c) + A*(11*b^3 - 12*a*b*c)
)*(256*a^(5/2)*(2*a + b*x)*(a + x*(b + c*x))^(5/2) - 5*(b^2 - 4*a*c)*x^2*(
16*a^(3/2)*(2*a + b*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*x^2*(2*Sq
rt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*x^2*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))))/(65536*a^(11/2)*x^6))/(9*a)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1237, 27, 1237, 27, 1228, 1152, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^{10}}\,dx$$

$$\Big\downarrow \; 1237$$

$$-\frac{\int \frac{(11Ab-18aB+4Acx)\left(cx^2+bx+a\right)^{5/2}}{2x^9}dx}{9a}-\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{(11Ab-18aB+4Acx)\left(cx^2+bx+a\right)^{5/2}}{x^9}dx}{18a}-\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

$$\Big\downarrow \; 1237$$

$$-\frac{-\frac{\int \frac{\left(99Ab^2-162aBb-64aAc+2(11Ab-18aB)cx\right)\left(cx^2+bx+a\right)^{5/2}}{2x^8}dx}{8a}-\frac{(11Ab-18aB)\left(a+bx+cx^2\right)^{7/2}}{8ax^8}}{18a}-\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{\left(99Ab^2-162aBb-64aAc+2(11Ab-18aB)cx\right)\left(cx^2+bx+a\right)^{5/2}}{x^8}dx}{16a}-\frac{(11Ab-18aB)\left(a+bx+cx^2\right)^{7/2}}{8ax^8}}{18a}-\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

$$\Big\downarrow \; 1228$$

$$-\frac{-\frac{-\frac{9\left(8a^2Bc-12aAbc-18ab^2B+11Ab^3\right)\int \frac{\left(cx^2+bx+a\right)^{5/2}}{x^7}dx}{2a}-\frac{\left(a+bx+cx^2\right)^{7/2}\left(-64aAc-162abB+99Ab^2\right)}{7ax^7}}{16a}-\frac{(11Ab-18aB)\left(a+bx+cx^2\right)^{7/2}}{8ax^8}}{18a}-\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

$$\Big\downarrow \; 1152$$

$$-\frac{-\frac{-\frac{9\left(8a^2Bc-12aAbc-18ab^2B+11Ab^3\right)\left(-\frac{5\left(b^2-4ac\right)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{x^5}dx}{24a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{5/2}}{12ax^6}\right)}{2a}-\frac{\left(a+bx+cx^2\right)^{7/2}\left(-64aAc-162abB+99Ab^2\right)}{7ax^7}}{16a}}{18a}$$

$$\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

$\big\downarrow$ 1152

$$-\frac{-\frac{-\frac{9\left(8a^2Bc-12aAbc-18ab^2B+11Ab^3\right)\left(-\frac{5\left(b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^2+bx+a}}{x^3}dx}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/2}}{8ax^4}\right)}{24a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{5/2}}{12ax^6}\right)}{2a}-\frac{\left(a+b\cdots\right.}{\cdots}}{16a}}{18a}$$

$$\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

$\big\downarrow$ 1152

$$-\frac{-\frac{-\frac{9\left(8a^2Bc-12aAbc-18ab^2B+11Ab^3\right)\left(-\frac{5\left(b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{8a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+bx+cx^2\right)^{3/\cdots}}{8ax^4}\cdots\right)}{24a}\cdots\right.}{2a}\cdots}{16a}}{18a}$$

$$\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

$\big\downarrow$ 1154

$$-\frac{-\frac{-\frac{9\left(8a^2Bc-12aAbc-18ab^2B+11Ab^3\right)\left(-\frac{5\left(b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{4a}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+\cdots\right.}{8a\cdots}\cdots\right)}{24a}\cdots\right.}{2a}\cdots}{16a}}{18a}$$

$$\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

$$\downarrow 219$$

$$-\frac{-\frac{-\frac{9\left(8a^2Bc-12aAbc-18ab^2B+11Ab^3\right)\left(-\frac{5\left(b^2-4ac\right)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4ax^2}\right)}{16a}-\frac{(2a+bx)\left(a+\cdots\right.}{8a\cdots}\right)}{24a}\cdots\right)}{2a}\cdots}{16a}\cdots}{18a}\cdots-\frac{A\left(a+bx+cx^2\right)^{7/2}}{9ax^9}$$

input

```
Int[((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^10,x]
```

output

```
-1/9*(A*(a + b*x + c*x^2)^(7/2))/(a*x^9) - (-1/8*((11*A*b - 18*a*B)*(a + b
*x + c*x^2)^(7/2))/(a*x^8) - (-1/7*((99*A*b^2 - 162*a*b*B - 64*a*A*c)*(a +
 b*x + c*x^2)^(7/2))/(a*x^7) - (9*(11*A*b^3 - 18*a*b^2*B - 12*a*A*b*c + 8*
a^2*B*c)*(-1/12*((2*a + b*x)*(a + b*x + c*x^2)^(5/2))/(a*x^6) - (5*(b^2 -
4*a*c)*(-1/8*((2*a + b*x)*(a + b*x + c*x^2)^(3/2))/(a*x^4) - (3*(b^2 - 4*a
*c)*(-1/4*((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(a*x^2) + ((b^2 - 4*a*c)*Arc
Tanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2))))/(16*a))
)/(24*a)))/(2*a))/(16*a))/(18*a)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(341) = 682$.

Time = 2.04 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.85

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-65536A\,a^4c^4x^8+234432A\,a^3b^2c^3x^8-162288A\,a^2b^4c^2x^8+40740Aa\,b^6c\,x^8-3465A\,b^8x^8-254592B\,a^4b\,c^3x^8+226}{}$
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/2064384*(c*x^2+b*x+a)^(1/2)*(-65536*A*a^4*c^4*x^8+234432*A*a^3*b^2*c^3*
x^8-162288*A*a^2*b^4*c^2*x^8+40740*A*a*b^6*c*x^8-3465*A*b^8*x^8-254592*B*a
^4*b*c^3*x^8+226464*B*a^3*b^3*c^2*x^8-63000*B*a^2*b^5*c*x^8+5670*B*a*b^7*x
^8-88192*A*a^4*b*c^3*x^7+84384*A*a^3*b^3*c^2*x^7-24696*A*a^2*b^5*c*x^7+231
0*A*a*b^7*x^7+80640*B*a^5*c^3*x^7-114624*B*a^4*b^2*c^2*x^7+37968*B*a^3*b^4
*c*x^7-3780*B*a^2*b^6*x^7+32768*A*a^5*c^3*x^6-51072*A*a^4*b^2*c^2*x^6+1785
6*A*a^3*b^4*c*x^6-1848*A*a^2*b^6*x^6+66816*B*a^5*b*c^2*x^6-27264*B*a^4*b^3
*c*x^6+3024*B*a^3*b^5*x^6+31488*A*a^5*b*c^2*x^5-13696*A*a^4*b^3*c*x^5+1584
*A*a^3*b^5*x^5+634368*B*a^6*c^2*x^5+20736*B*a^5*b^2*c*x^5-2592*B*a^4*b^4*x
^5+491520*A*a^6*c^2*x^4+10752*A*a^5*b^2*c*x^4-1408*A*a^4*b^4*x^4+943104*B*
a^6*b*c*x^4+2304*B*a^5*b^3*x^4+771072*A*a^6*b*c*x^3+1280*A*a^5*b^3*x^3+731
136*B*a^7*c*x^3+373248*B*a^6*b^2*x^3+622592*A*a^7*c*x^2+316416*A*a^6*b^2*x
^2+608256*B*a^7*b*x^2+530432*A*a^7*b*x+258048*B*a^8*x+229376*A*a^8)/x^9/a^
6-5/65536*(768*A*a^4*b*c^4-1280*A*a^3*b^3*c^3+672*A*a^2*b^5*c^2-144*A*a*b^
7*c+11*A*b^9-512*B*a^5*c^4+1536*B*a^4*b^2*c^3-960*B*a^3*b^4*c^2+224*B*a^2*
b^6*c-18*B*a*b^8)/a^(13/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 1315, normalized size of antiderivative = 3.51

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
[-1/8257536*(315*(18*B*a*b^8 - 11*A*b^9 + 256*(2*B*a^5 - 3*A*a^4*b)*c^4 -
256*(6*B*a^4*b^2 - 5*A*a^3*b^3)*c^3 + 96*(10*B*a^3*b^4 - 7*A*a^2*b^5)*c^2
- 16*(14*B*a^2*b^6 - 9*A*a*b^7)*c)*sqrt(a)*x^9*log(-(8*a*b*x + (b^2 + 4*a*
c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(22
9376*A*a^9 + (5670*B*a^2*b^7 - 3465*A*a*b^8 - 65536*A*a^5*c^4 - 576*(442*B
*a^5*b - 407*A*a^4*b^2)*c^3 + 336*(674*B*a^4*b^3 - 483*A*a^3*b^4)*c^2 - 42
0*(150*B*a^3*b^5 - 97*A*a^2*b^6)*c)*x^8 - 2*(1890*B*a^3*b^6 - 1155*A*a^2*b
^7 - 64*(630*B*a^6 - 689*A*a^5*b)*c^3 + 144*(398*B*a^5*b^2 - 293*A*a^4*b^3
)*c^2 - 84*(226*B*a^4*b^4 - 147*A*a^3*b^5)*c)*x^7 + 8*(378*B*a^4*b^5 - 231
*A*a^3*b^6 + 4096*A*a^6*c^3 + 48*(174*B*a^6*b - 133*A*a^5*b^2)*c^2 - 24*(1
42*B*a^5*b^3 - 93*A*a^4*b^4)*c)*x^6 - 16*(162*B*a^5*b^4 - 99*A*a^4*b^5 - 4
8*(826*B*a^7 + 41*A*a^6*b)*c^2 - 8*(162*B*a^6*b^2 - 107*A*a^5*b^3)*c)*x^5
+ 128*(18*B*a^6*b^3 - 11*A*a^5*b^4 + 3840*A*a^7*c^2 + 12*(614*B*a^7*b + 7*
A*a^6*b^2)*c)*x^4 + 256*(1458*B*a^7*b^2 + 5*A*a^6*b^3 + 12*(238*B*a^8 + 25
1*A*a^7*b)*c)*x^3 + 1024*(594*B*a^8*b + 309*A*a^7*b^2 + 608*A*a^8*c)*x^2 +
 14336*(18*B*a^9 + 37*A*a^8*b)*x)*sqrt(c*x^2 + b*x + a))/(a^7*x^9), -1/412
8768*(315*(18*B*a*b^8 - 11*A*b^9 + 256*(2*B*a^5 - 3*A*a^4*b)*c^4 - 256*(6*
B*a^4*b^2 - 5*A*a^3*b^3)*c^3 + 96*(10*B*a^3*b^4 - 7*A*a^2*b^5)*c^2 - 16*(1
4*B*a^2*b^6 - 9*A*a*b^7)*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(229376*A*a^9 + (5670...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^{10}}\,dx=\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(5/2)/x**10,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**(5/2)/x**10, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^{10}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4427 vs. $2(341) = 682$.

Time = 0.31 (sec) , antiderivative size = 4427, normalized size of antiderivative = 11.81

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^10,x, algorithm="giac")
```

output
```
-5/32768*(18*B*a*b^8 - 11*A*b^9 - 224*B*a^2*b^6*c + 144*A*a*b^7*c + 960*B*
a^3*b^4*c^2 - 672*A*a^2*b^5*c^2 - 1536*B*a^4*b^2*c^3 + 1280*A*a^3*b^3*c^3
+ 512*B*a^5*c^4 - 768*A*a^4*b*c^4)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))/sqrt(-a))/(sqrt(-a)*a^6) + 1/2064384*(5670*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^17*B*a*b^8 - 3465*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^17*A*b^9 -
 70560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^17*B*a^2*b^6*c + 45360*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^17*A*a*b^7*c + 302400*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^17*B*a^3*b^4*c^2 - 211680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
17*A*a^2*b^5*c^2 - 483840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^17*B*a^4*b^2
*c^3 + 403200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^17*A*a^3*b^3*c^3 + 16128
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^17*B*a^5*c^4 - 241920*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^17*A*a^4*b*c^4 - 49140*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^15*B*a^2*b^8 + 30030*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*A*a*b^9
 + 611520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*B*a^3*b^6*c - 393120*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^15*A*a^2*b^7*c - 2620800*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^15*B*a^4*b^4*c^2 + 1834560*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^15*A*a^3*b^5*c^2 + 4193280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*
B*a^5*b^2*c^3 - 3494400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*A*a^4*b^3*c
^3 + 4107264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^15*B*a^6*c^4 + 2096640*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^15*A*a^5*b*c^4 + 28901376*(sqrt(c)*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^{10}}\,dx=\int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{5/2}}{x^{10}}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^10,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(5/2))/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 96.89 (sec) , antiderivative size = 829, normalized size of antiderivative = 2.21

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{5/2}}{x^{10}}\,dx = \text{Too large to display}$$

input

```
int((B*x+A)*(c*x^2+b*x+a)^(5/2)/x^10,x)
```

output

```
( - 458752*sqrt(a + b*x + c*x**2)*a**9 - 1576960*sqrt(a + b*x + c*x**2)*a*
*8*b*x - 1245184*sqrt(a + b*x + c*x**2)*a**8*c*x**2 - 1849344*sqrt(a + b*x
 + c*x**2)*a**7*b**2*x**2 - 3004416*sqrt(a + b*x + c*x**2)*a**7*b*c*x**3 -
 983040*sqrt(a + b*x + c*x**2)*a**7*c**2*x**4 - 749056*sqrt(a + b*x + c*x*
*2)*a**6*b**3*x**3 - 1907712*sqrt(a + b*x + c*x**2)*a**6*b**2*c*x**4 - 133
1712*sqrt(a + b*x + c*x**2)*a**6*b*c**2*x**5 - 65536*sqrt(a + b*x + c*x**2
)*a**6*c**3*x**6 - 1792*sqrt(a + b*x + c*x**2)*a**5*b**4*x**4 - 14080*sqrt
(a + b*x + c*x**2)*a**5*b**3*c*x**5 - 31488*sqrt(a + b*x + c*x**2)*a**5*b*
*2*c**2*x**6 + 15104*sqrt(a + b*x + c*x**2)*a**5*b*c**3*x**7 + 131072*sqrt
(a + b*x + c*x**2)*a**5*c**4*x**8 + 2016*sqrt(a + b*x + c*x**2)*a**4*b**5*
x**5 + 18816*sqrt(a + b*x + c*x**2)*a**4*b**4*c*x**6 + 60480*sqrt(a + b*x
+ c*x**2)*a**4*b**3*c**2*x**7 + 40320*sqrt(a + b*x + c*x**2)*a**4*b**2*c**
3*x**8 - 2352*sqrt(a + b*x + c*x**2)*a**3*b**6*x**6 - 26544*sqrt(a + b*x +
 c*x**2)*a**3*b**5*c*x**7 - 128352*sqrt(a + b*x + c*x**2)*a**3*b**4*c**2*x
**8 + 2940*sqrt(a + b*x + c*x**2)*a**2*b**7*x**7 + 44520*sqrt(a + b*x + c*
x**2)*a**2*b**6*c*x**8 - 4410*sqrt(a + b*x + c*x**2)*a*b**8*x**8 + 80640*s
qrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**4*b*c**4*x**9
+ 80640*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*b**
3*c**3*x**9 - 90720*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b
*x)*a**2*b**5*c**2*x**9 + 25200*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*...
```

3.144 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 206

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = -\frac{(7bB-8Ac)x^2\sqrt{a+bx+cx^2}}{24c^2}+\frac{Bx^3\sqrt{a+bx+cx^2}}{4c} -\frac{(105b^3B-120Ab^2c-220abBc+128aAc^2-2c(35b^2B-40Abc-36aBc)\,x)\sqrt{a+bx+cx^2}}{192c^4} +\frac{(35b^4B-40Ab^3c-120ab^2Bc+96aAbc^2+48a^2Bc^2)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{9/2}}$$

output
```
-1/24*(-8*A*c+7*B*b)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*B*x^3*(c*x^2+b*x+a)^(
1/2)/c-1/192*(105*B*b^3-120*A*b^2*c-220*B*a*b*c+128*A*a*c^2-2*c*(-40*A*b*c
-36*B*a*c+35*B*b^2)*x)*(c*x^2+b*x+a)^(1/2)/c^4+1/128*(96*A*a*b*c^2-40*A*b^
3*c+48*B*a^2*c^2-120*B*a*b^2*c+35*B*b^4)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^3B+10b^2c(12A+7Bx)+8c^2(-16aA-9aBx+8Acx^2+6Bcx^3)+4bc(55}{}$$

input
```
Integrate[(x^3*(A + B*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*B + 10*b^2*c*(12*A + 7*B*x) + 8
*c^2*(-16*a*A - 9*a*B*x + 8*A*c*x^2 + 6*B*c*x^3) + 4*b*c*(55*a*B - 2*c*x*(
10*A + 7*B*x))) - 3*(35*b^4*B - 40*A*b^3*c - 120*a*b^2*B*c + 96*a*A*b*c^2
+ 48*a^2*B*c^2)*Log[c^4*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(3
84*c^(9/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1236, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{x^2(6aB+(7bB-8Ac)x)}{2\sqrt{cx^2+bx+a}}dx}{4c}+\frac{Bx^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^3\sqrt{a+bx+cx^2}}{4c}-\frac{\int \frac{x^2(6aB+(7bB-8Ac)x)}{\sqrt{cx^2+bx+a}}dx}{8c}$$

$$\Big\downarrow \ 1236$$

$$\frac{Bx^3\sqrt{a+bx+cx^2}}{4c}-\frac{\frac{\int -\frac{x\left(4a(7bB-8Ac)+\left(35Bb^2-40Acb-36aBc\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{x^2\sqrt{a+bx+cx^2}(7bB-8Ac)}{3c}}{8c}$$

$$\Big\downarrow \ 27$$

$$\frac{Bx^3\sqrt{a+bx+cx^2}}{4c}-\frac{\frac{x^2\sqrt{a+bx+cx^2}(7bB-8Ac)}{3c}-\frac{\int \frac{x\left(4a(7bB-8Ac)+\left(35Bb^2-40Acb-36aBc\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{6c}}{8c}$$

$$\Big\downarrow \ 1225$$

$$\frac{Bx^3\sqrt{a+bx+cx^2}}{4c}-$$

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}(7bB-8Ac)}{3c}-\frac{\frac{3\left(48a^2Bc^2+96aAbc^2-120ab^2Bc-40Ab^3c+35b^4B\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c^2}-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-36aBc-40Abc+35b^2B\right)+128a\right.}{4c^2}}{6c}}{8c}$$

$$\Big\downarrow \ 1092$$

$$\frac{Bx^3\sqrt{a+bx+cx^2}}{4c}-$$

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}(7bB-8Ac)}{3c}-\frac{\frac{3\left(48a^2Bc^2+96aAbc^2-120ab^2Bc-40Ab^3c+35b^4B\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c^2}-\sqrt{a+bx+cx^2}\left(-2cx\left(-36aBc-40Abc+\right.\right.}{6c}}{8c}$$

$$\Big\downarrow \ 219$$

$$\frac{Bx^3\sqrt{a+bx+cx^2}}{4c}-$$

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}(7bB-8Ac)}{3c}-\frac{\frac{3\left(48a^2Bc^2+96aAbc^2-120ab^2Bc-40Ab^3c+35b^4B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\sqrt{a+bx+cx^2}\left(-2cx\left(-36aBc-40Abc+\right.\right.}{6c}}{8c}$$

input
```
Int[(x^3*(A + B*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(B*x^3*Sqrt[a + b*x + c*x^2])/(4*c) - (((7*b*B - 8*A*c)*x^2*Sqrt[a + b*x +
 c*x^2])/(3*c) - (-1/4*((105*b^3*B - 120*A*b^2*c - 220*a*b*B*c + 128*a*A*c
^2 - 2*c*(35*b^2*B - 40*A*b*c - 36*a*B*c)*x)*Sqrt[a + b*x + c*x^2])/c^2 +
(3*(35*b^4*B - 40*A*b^3*c - 120*a*b^2*B*c + 96*a*A*b*c^2 + 48*a^2*B*c^2)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(6*c))
/(8*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(-48B\,c^{3}x^{3}-64A\,c^{3}x^{2}+56Bb\,c^{2}x^{2}+80Ab\,c^{2}x+72Ba\,c^{2}x-70B\,b^{2}cx+128Aa\,c^{2}-120A\,b^{2}c-220Babc+105B\,b^{3}\right)\sqrt{c\,x^{2}+bx+a}}{192c^{4}}+$
default	$A\left(\frac{x^{2}\sqrt{c\,x^{2}+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^{2}+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{6c}-\frac{2a(\ldots}{}\right.$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*B*c^3*x^3-64*A*c^3*x^2+56*B*b*c^2*x^2+80*A*b*c^2*x+72*B*a*c^2*
x-70*B*b^2*c*x+128*A*a*c^2-120*A*b^2*c-220*B*a*b*c+105*B*b^3)/c^4*(c*x^2+b
*x+a)^(1/2)+1/128*(96*A*a*b*c^2-40*A*b^3*c+48*B*a^2*c^2-120*B*a*b^2*c+35*B
*b^4)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.92

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{3\,(35\,Bb^4+48\,(Ba^2+2\,Aab)c^2-40\,(3\,Bab^2+Ab^3)c)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\right.}{}\right.$$
$$-\frac{3\,(35\,Bb^4+48\,(Ba^2+2\,Aab)c^2-40\,(3\,Bab^2+Ab^3)c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(48\,Bc^4}{}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(35*B*b^4 + 48*(B*a^2 + 2*A*a*b)*c^2 - 40*(3*B*a*b^2 + A*b^3)*c)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
 b)*sqrt(c) - 4*a*c) + 4*(48*B*c^4*x^3 - 105*B*b^3*c - 128*A*a*c^3 + 20*(1
1*B*a*b + 6*A*b^2)*c^2 - 8*(7*B*b*c^3 - 8*A*c^4)*x^2 + 2*(35*B*b^2*c^2 - 4
*(9*B*a + 10*A*b)*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(35*B*b^4
+ 48*(B*a^2 + 2*A*a*b)*c^2 - 40*(3*B*a*b^2 + A*b^3)*c)*sqrt(-c)*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(
48*B*c^4*x^3 - 105*B*b^3*c - 128*A*a*c^3 + 20*(11*B*a*b + 6*A*b^2)*c^2 - 8
*(7*B*b*c^3 - 8*A*c^4)*x^2 + 2*(35*B*b^2*c^2 - 4*(9*B*a + 10*A*b)*c^3)*x)*
sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.05

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \left(-\frac{a\left(-\frac{3Ba}{4c}-\frac{5b\left(A-\frac{7Bb}{8c}\right)}{6c}\right)}{2c}-\frac{b\left(-\frac{2a\left(A-\frac{7Bb}{8c}\right)}{3c}-\frac{3b\left(-\frac{3Ba}{4c}-\frac{5b\left(A-\frac{7Bb}{8c}\right)}{6c}\right)}{4c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c} \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right. \\ \frac{2\left(\frac{B(a+bx)^{\frac{9}{2}}}{9b}+\frac{(a+bx)^{\frac{7}{2}}(Ab-4Ba)}{7b}+\frac{(a+bx)^{\frac{5}{2}}\left(-3Aab+6Ba^2\right)}{5b}+\frac{(a+bx)^{\frac{3}{2}}\cdot\left(3Aa^2b-4Ba^3\right)}{3b}+\frac{\sqrt{a+bx}\left(-Aa^3b+Ba^4\right)}{b}\right)}{b^4} \\ \frac{\frac{Ax^4}{4}+\frac{Bx^5}{5}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**3*(B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-a*(-3*B*a/(4*c) - 5*b*(A - 7*B*b/(8*c))/(6*c))/(2*c) - b*(-2*
a*(A - 7*B*b/(8*c))/(3*c) - 3*b*(-3*B*a/(4*c) - 5*b*(A - 7*B*b/(8*c))/(6*c
))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c
*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(
c*(b/(2*c) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(B*x**3/(4*c) + x**2*
(A - 7*B*b/(8*c))/(3*c) + x*(-3*B*a/(4*c) - 5*b*(A - 7*B*b/(8*c))/(6*c))/(
2*c) + (-2*a*(A - 7*B*b/(8*c))/(3*c) - 3*b*(-3*B*a/(4*c) - 5*b*(A - 7*B*b/
(8*c))/(6*c))/(4*c))/c), Ne(c, 0)), (2*(B*(a + b*x)**(9/2)/(9*b) + (a + b*
x)**(7/2)*(A*b - 4*B*a)/(7*b) + (a + b*x)**(5/2)*(-3*A*a*b + 6*B*a**2)/(5*
b) + (a + b*x)**(3/2)*(3*A*a**2*b - 4*B*a**3)/(3*b) + sqrt(a + b*x)*(-A*a*
*3*b + B*a**4)/b)/b**4, Ne(b, 0)), ((A*x**4/4 + B*x**5/5)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.88

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,Bx}{c}-\frac{7\,Bbc^2-8\,Ac^3}{c^4}\right)x+\frac{35\,Bb^2c-36\,Bac^2-40\,Abc^2}{c^4}\right)x-\frac{105\,Bb^3-\cdots}{\cdots}\right. - \frac{(35\,Bb^4-120\,Bab^2c-40\,Ab^3c+48\,Ba^2c^2+96\,Aabc^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{9}{2}}}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*B*x/c - (7*B*b*c^2 - 8*A*c^3)/c^4)*x
+ (35*B*b^2*c - 36*B*a*c^2 - 40*A*b*c^2)/c^4)*x - (105*B*b^3 - 220*B*a*b*c
 - 120*A*b^2*c + 128*A*a*c^2)/c^4) - 1/128*(35*B*b^4 - 120*B*a*b^2*c - 40*
A*b^3*c + 48*B*a^2*c^2 + 96*A*a*b*c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{x^3\,(A+B\,x)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x^3*(A + B*x))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x^3*(A + B*x))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{x^3(Bx+A)}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^3*(B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

3.145 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{Bx^2\sqrt{a+bx+cx^2}}{3c} + \frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x)\sqrt{a+bx+cx^2}}{24c^3} - \frac{(5b^3B-6Ab^2c-12abBc+8aAc^2)\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}}$$

output
```
1/3*B*x^2*(c*x^2+b*x+a)^(1/2)/c+1/24*(15*B*b^2-18*A*b*c-16*B*a*c-2*c*(-6*A
*c+5*B*b)*x)*(c*x^2+b*x+a)^(1/2)/c^3-1/16*(8*A*a*c^2-6*A*b^2*c-12*B*a*b*c+
5*B*b^3)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(15b^2B-2bc(9A+5Bx)+4c(-4aB+cx(3A+2Bx)))+3(5b^3B-6Ab^2c-12ab}{48c^{7/2}}$$

input
```
Integrate[(x^2*(A + B*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*B - 2*b*c*(9*A + 5*B*x) + 4*c*(-4
*a*B + c*x*(3*A + 2*B*x))) + 3*(5*b^3*B - 6*A*b^2*c - 12*a*b*B*c + 8*a*A*c
^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(48*c^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{x(4aB+(5bB-6Ac)x)}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{Bx^2\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^2\sqrt{a+bx+cx^2}}{3c}-\frac{\int \frac{x(4aB+(5bB-6Ac)x)}{\sqrt{cx^2+bx+a}}dx}{6c}$$

$$\Big\downarrow \text{1225}$$

$$\frac{Bx^2\sqrt{a+bx+cx^2}}{3c}-\frac{\frac{3(8aAc^2-12abBc-6Ab^2c+5b^3B)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c^2}-\frac{\sqrt{a+bx+cx^2}(-16aBc-2cx(5bB-6Ac)-18Abc+15b^2B)}{4c^2}}{6c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{Bx^2\sqrt{a+bx+cx^2}}{3c}-\frac{\frac{3(8aAc^2-12abBc-6Ab^2c+5b^3B)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c^2}-\frac{\sqrt{a+bx+cx^2}(-16aBc-2cx(5bB-6Ac)-18Abc+15b^2B)}{4c^2}}{6c}$$

$$\Big\downarrow 219$$

$$\frac{Bx^2\sqrt{a+bx+cx^2}}{3c}-\frac{\frac{3(8aAc^2-12abBc-6Ab^2c+5b^3B)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{\sqrt{a+bx+cx^2}(-16aBc-2cx(5bB-6Ac)-18Abc+15b^2B)}{4c^2}}{6c}$$

input

```
Int[(x^2*(A + B*x))/Sqrt[a + b*x + c*x^2],x]
```

output

```
(B*x^2*Sqrt[a + b*x + c*x^2])/(3*c) - (-1/4*((15*b^2*B - 18*A*b*c - 16*a*B
*c - 2*c*(5*b*B - 6*A*c)*x)*Sqrt[a + b*x + c*x^2])/c^2 + (3*(5*b^3*B - 6*A
*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(8*c^(5/2)))/(6*c)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\left(-8B\,c^2x^2-12A\,c^2x+10Bbcx+18Abc+16aBc-15B\,b^2\right)\sqrt{c\,x^2+bx+a}}{24c^3}-\frac{\left(8Aa\,c^2-6A\,b^2c-12Babc+5B\,b^3\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+\right.}{16c^{\frac{7}{2}}}$
default	$A\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+B\left(\frac{x^2\sqrt{c\,x^2+bx-}}{3c}\right.$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*B*c^2*x^2-12*A*c^2*x+10*B*b*c*x+18*A*b*c+16*B*a*c-15*B*b^2)/c^3*
(c*x^2+b*x+a)^(1/2)-1/16*(8*A*a*c^2-6*A*b^2*c-12*B*a*b*c+5*B*b^3)/c^(7/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.06

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{3\left(5\,Bb^3+8\,Aac^2-6\,(2\,Bab+Ab^2)c\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\right.}{96\,c^4}\right.$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*log(-8*c^2*
x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c)
 + 4*(8*B*c^3*x^2 + 15*B*b^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*
A*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^4, 1/48*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*
B*a*b + A*b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sq
rt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(8*B*c^3*x^2 + 15*B*b^2*c - 2*(8*B*a +
 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.04

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \left(-\frac{a\left(A-\frac{5Bb}{6c}\right)}{2c}-\frac{b\left(-\frac{2Ba}{3c}-\frac{3b\left(A-\frac{5Bb}{6c}\right)}{4c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)+\sqrt{a+bx+cx^2}\,(\\ \frac{2\left(\frac{B(a+bx)^{\frac{7}{2}}}{7b}+\frac{(a+bx)^{\frac{5}{2}}(Ab-3Ba)}{5b}+\frac{(a+bx)^{\frac{3}{2}}\left(-2Aab+3Ba^2\right)}{3b}+\frac{\sqrt{a+bx}\left(Aa^2b-Ba^3\right)}{b}\right)}{b^3} \\ \frac{\frac{Ax^3}{3}+\frac{Bx^4}{4}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**2*(B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-a*(A - 5*B*b/(6*c))/(2*c) - b*(-2*B*a/(3*c) - 3*b*(A - 5*B*b/
(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) +
 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/s
qrt(c*(b/(2*c) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(B*x**2/(3*c) + x
*(A - 5*B*b/(6*c))/(2*c) + (-2*B*a/(3*c) - 3*b*(A - 5*B*b/(6*c))/(4*c))/c)
, Ne(c, 0)), (2*(B*(a + b*x)**(7/2)/(7*b) + (a + b*x)**(5/2)*(A*b - 3*B*a)
/(5*b) + (a + b*x)**(3/2)*(-2*A*a*b + 3*B*a**2)/(3*b) + sqrt(a + b*x)*(A*a
**2*b - B*a**3)/b)/b**3, Ne(b, 0)), ((A*x**3/3 + B*x**4/4)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx &= \frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,Bx}{c}-\frac{5\,Bbc-6\,Ac^2}{c^3}\right)x+\frac{15\,Bb^2-16\,Bac-18\,Abc}{c^3}\right)\\&\quad+\frac{(5\,Bb^3-12\,Babc-6\,Ab^2c+8\,Aac^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{7}{2}}}\end{aligned}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*B*x/c - (5*B*b*c - 6*A*c^2)/c^3)*x + (15*
B*b^2 - 16*B*a*c - 18*A*b*c)/c^3) + 1/16*(5*B*b^3 - 12*B*a*b*c - 6*A*b^2*c
 + 8*A*a*c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/
c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{x^2\,(A+B\,x)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x^2*(A + B*x))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x^2*(A + B*x))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{-68\sqrt{c\,x^2+bx+a}\,ab\,c^2+24\sqrt{c\,x^2+bx+a}\,a\,c^3x+30\sqrt{c\,x^2+bx+a}\,b^3c-20\sqrt{c\,x^2+bx+a}\,b^2c^2x+1}{}$$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 68*sqrt(a + b*x + c*x**2)*a*b*c**2 + 24*sqrt(a + b*x + c*x**2)*a*c**3*
x + 30*sqrt(a + b*x + c*x**2)*b**3*c - 20*sqrt(a + b*x + c*x**2)*b**2*c**2
*x + 16*sqrt(a + b*x + c*x**2)*b*c**3*x**2 - 24*sqrt(c)*log((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2 + 54*sqrt(c
)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a
*b**2*c - 15*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sq
rt(4*a*c - b**2))*b**4)/(48*c**4)
```

3.146 $\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = -\frac{(3bB-4Ac-2Bcx)\sqrt{a+bx+cx^2}}{4c^2} + \frac{(3b^2B-4(Ab+aB)c)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
-1/4*(-2*B*c*x-4*A*c+3*B*b)*(c*x^2+b*x+a)^(1/2)/c^2+1/8*(3*B*b^2-4*(A*b+B*
a)*c)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{(-3bB+4Ac+2Bcx)\sqrt{a+x(b+cx)}}{4c^2} + \frac{(-3b^2B+4Abc+4aBc)\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{8c^{5/2}}$$

input
```
Integrate[(x*(A + B*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
((-3*b*B + 4*A*c + 2*B*c*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((-3*b^2*B +
4*A*b*c + 4*a*B*c)*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
/(8*c^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{(-4aBc-4Abc+3b^2B)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c^2}-\frac{\sqrt{a+bx+cx^2}(-4Ac+3bB-2Bcx)}{4c^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(-4aBc-4Abc+3b^2B)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c^2}-\frac{\sqrt{a+bx+cx^2}(-4Ac+3bB-2Bcx)}{4c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(-4aBc-4Abc+3b^2B)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{\sqrt{a+bx+cx^2}(-4Ac+3bB-2Bcx)}{4c^2}$$

input
```
Int[(x*(A + B*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
-1/4*((3*b*B - 4*A*c - 2*B*c*x)*Sqrt[a + b*x + c*x^2])/c^2 + ((3*b^2*B - 4
*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(8*c^(5/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

method	result
risch	$\frac{\left(2 B c x +4 A c -3 B b \right) \sqrt{c \,x^{2}+b x +a}}{4 c^{2}}-\frac{\left(4 A b c +4 a B c -3 B \,b^{2}\right) \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c \,x^{2}+b x +a}\right)}{8 c^{\frac{5}{2}}}$
default	$A \left(\frac{\sqrt{c \,x^{2}+b x +a}}{c}-\frac{b \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c \,x^{2}+b x +a}\right)}{2 c^{\frac{3}{2}}}\right)+B \left(\frac{x \sqrt{c \,x^{2}+b x +a}}{2 c}-\frac{3 b \left(\frac{\sqrt{c \,x^{2}+b x +a}}{c}-\frac{b \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c \,x^{2}+b x +a}\right)}{2 c^{\frac{3}{2}}}\right)}{4 c}\right.$

input
```
int(x*(B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*B*c*x+4*A*c-3*B*b)/c^2*(c*x^2+b*x+a)^(1/2)-1/8*(4*A*b*c+4*B*a*c-3*B
*b^2)/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.32

$$\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[-\frac{(3\,Bb^2-4\,(Ba+Ab)c)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4\,(2\,B}{16\,c^3}\right.$$
$$-\frac{(3\,Bb^2-4\,(Ba+Ab)c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(2\,Bc^2x-3\,Bbc+4\,Ac^2)\sqrt{cx^2+bx+}}{8\,c^3}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((3*B*b^2 - 4*(B*a + A*b)*c)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*B*c^2*x - 3
*B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((3*B*b^2 - 4*(B*a + A*
b)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*
x^2 + b*c*x + a*c)) - 2*(2*B*c^2*x - 3*B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x +
 a))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(87) = 174.

Time = 0.83 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.22

$$\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \left(-\frac{Ba}{2c}-\frac{b\left(A-\frac{3Bb}{4c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)+\left(\frac{Bx}{2c}+\frac{A-\frac{3Bb}{4c}}{c}\right)\sqrt{a+bx+cx^2} \\ \frac{2\left(\frac{B(a+bx)^{\frac{5}{2}}}{5b}+\frac{(a+bx)^{\frac{3}{2}}(Ab-2Ba)}{3b}+\frac{\sqrt{a+bx}\left(-Aab+Ba^2\right)}{b}\right)}{b^2} \\ \frac{\frac{Ax^2}{2}+\frac{Bx^3}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x*(B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-B*a/(2*c) - b*(A - 3*B*b/(4*c))/(2*c))*Piecewise((log(b + 2*s
qrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((
b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)) + (B*x/(2*c
) + (A - 3*B*b/(4*c))/c)*sqrt(a + b*x + c*x**2), Ne(c, 0)), (2*(B*(a + b*x
)**(5/2)/(5*b) + (a + b*x)**(3/2)*(A*b - 2*B*a)/(3*b) + sqrt(a + b*x)*(-A*
a*b + B*a**2)/b)/b**2, Ne(b, 0)), ((A*x**2/2 + B*x**3/3)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,Bx}{c}-\frac{3\,Bb-4\,Ac}{c^2}\right) - \frac{(3\,Bb^2-4\,Bac-4\,Abc)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*B*x/c - (3*B*b - 4*A*c)/c^2) - 1/8*(3*B*b^2 -
 4*B*a*c - 4*A*b*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)
+ b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{x\,(A+B\,x)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x*(A + B*x))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x*(A + B*x))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.57

$$\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{8\sqrt{c\,x^2+bx+a}\,a\,c^2-6\sqrt{c\,x^2+bx+a}\,b^2c+4\sqrt{c\,x^2+bx+a}\,b\,c^2x-8\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)abc}{8c^3}$$

input
```
int(x*(B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(8*sqrt(a + b*x + c*x**2)*a*c**2 - 6*sqrt(a + b*x + c*x**2)*b**2*c + 4*sqr
t(a + b*x + c*x**2)*b*c**2*x - 8*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x
**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c + 3*sqrt(c)*log((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**3)/(8*c**3)
```

3.147 $\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx=\frac{B\sqrt{a+bx+cx^2}}{c}-\frac{(bB-2Ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}$$

output
```
B*(c*x^2+b*x+a)^(1/2)/c-1/2*(-2*A*c+B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx=\frac{B\sqrt{a+x(b+cx)}}{c}+\frac{(-bB+2Ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2c^{3/2}}$$

input
```
Integrate[(A + B*x)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(B*Sqrt[a + x*(b + c*x)])/c + ((-(b*B) + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + x*(b + c*x)])])/(2*c^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{B\sqrt{a+bx+cx^2}}{c}-\frac{(bB-2Ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{B\sqrt{a+bx+cx^2}}{c}-\frac{(bB-2Ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\sqrt{a+bx+cx^2}}{c}-\frac{(bB-2Ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}$$

input
```
Int[(A + B*x)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(B*Sqrt[a + b*x + c*x^2])/c - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{B\sqrt{c\,x^{2}+b x +a}}{c}+\frac{\left(2 A c -B b \right) \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x +a}\right)}{2 c^{\frac{3}{2}}}$	58
default	$\frac{A \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x +a}\right)}{\sqrt{c}}+B \left(\frac{\sqrt{c\,x^{2}+b x +a}}{c}-\frac{b \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x +a}\right)}{2 c^{\frac{3}{2}}}\right)$	82

input
```
int((B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B*(c*x^2+b*x+a)^(1/2)/c+1/2*(2*A*c-B*b)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.42

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{4\sqrt{cx^2+bx+a}Bc-(Bb-2\,Ac)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)}{4\,c^2}\right.$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(c*x^2 + b*x + a)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-8*c^2*x^2 -
 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/c^2
, 1/2*(2*sqrt(c*x^2 + b*x + a)*B*c + (B*b - 2*A*c)*sqrt(-c)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(56) = 112$.

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx = \begin{cases} \frac{B\sqrt{a+bx+cx^2}}{c} + \left(A - \frac{Bb}{2c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ a - \frac{b^2}{4c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right) & \text{for}\ c \neq 0 \\ \frac{2A\sqrt{a+bx}+\frac{2B\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}}{b} & \text{for}\ b \neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((B*sqrt(a + b*x + c*x**2)/c + (A - B*b/(2*c))*Piecewise((log(b +
 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0))
, ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c,
0)), ((2*A*sqrt(a + b*x) + 2*B*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3)/b)/
b, Ne(b, 0)), ((A*x + B*x**2/2)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx+a}B}{c} + \frac{(Bb-2\,Ac)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x + a)*B/c + 1/2*(B*b - 2*A*c)*log(abs(2*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 11.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx = \frac{B\sqrt{c\,x^2+b\,x+a}}{c} + \frac{A\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{\sqrt{c}} - \frac{B\,b\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{2\,c^{3/2}}$$

input
```
int((A + B*x)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
(B*(a + b*x + c*x^2)^(1/2))/c + (A*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*
x^2)^(1/2)))/c^(1/2) - (B*b*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1
/2)))/(2*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{c\,x^2+bx+a}\,bc+2\sqrt{c}\log\left(\frac{2\sqrt{c}\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)ac-\sqrt{c}\log\left(\frac{2\sqrt{c}\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)b^2}{2c^2}$$

input
```
int((B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x + c*x**2)*b*c + 2*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*
x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*c - sqrt(c)*log((2*sqrt(c)*sqrt(a
 + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**2)/(2*c**2)
```

3.148 $\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx = -\frac{A\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+\frac{B\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

output
```
-A*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)+B*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx = \frac{2A\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{B\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{\sqrt{c}}$$

input
```
Integrate[(A + B*x)/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*A*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] - (B*Lo
g[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1269}$$

$$A\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+B\int \frac{1}{\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 1092}$$

$$A\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+2B\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}$$

$$\Big\downarrow \text{ 219}$$

$$A\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{B\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{B\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2A\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{B\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-\frac{A\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

input
```
Int[(A + B*x)/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((A*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) + (B
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{B \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{\sqrt{c}}-\frac{A \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{\sqrt{a}}$	67

input
```
int((B*x+A)/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-A/a^(1/2)*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 468, normalized size of antiderivative = 6.08

$$\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{Ba\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)+A\sqrt{a}c\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2}{}\right.}{2\,ac}\right.$$
$$-\frac{2\,Ba\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-A\sqrt{a}c\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)}{2\,ac},\ \frac{2\,A}{}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(B*a*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(c) - 4*a*c) + A*sqrt(a)*c*log(-(8*a*b*x + (b^2 + 4*a*c)*
x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2))/(a*c), -1
/2*(2*B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) - A*sqrt(a)*c*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 -
4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2))/(a*c), 1/2*(2*A
*sqrt(-a)*c*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2
 + a*b*x + a^2)) + B*a*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/(a*c), (A*sqrt(-a)*c*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - B*
a*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
+ b*c*x + a*c)))/(a*c)]
```

Sympy [F]

$$\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/x/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(x*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx = \frac{B\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{\sqrt{c}} - \frac{A\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}}$$

input
```
int((A + B*x)/(x*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
(B*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(1/2) - (A*log(b/
2 + a/x + (a^(1/2)*(a + b*x + c*x^2)^(1/2))/x))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 794, normalized size of antiderivative = 10.31

$$\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/x/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*b*c - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*
sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**
2))*a*c - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*s
qrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b +
2*c*x)*b*c + sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*s
qrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b +
2*c*x)*b*c + 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqr
t(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) +
 b + 2*c*x)*a*c - 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(s
qrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2)
 + b + 2*c*x)*a*c + 4*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b*
*2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c**2 - sqrt(a)*log(
- sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x*
*2) + b + 2*c*x)*b**2*c + 4*sqrt(a)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c +
 b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c**2 - sqrt(a)*lo
g(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x*
*2) + b + 2*c*x)*b**2*c - 4*sqrt(a)*log(4*sqrt(c)*sqrt(a + b*x + c*x**2)*b
 + 8*sqrt(c)*sqrt(a + b*x + c*x**2)*c*x + 4*sqrt(c)*sqrt(a)*b + 8*b*c*x...
```

3.149 $\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx=-\frac{A\sqrt{a+bx+cx^2}}{ax}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}$$

output
```
-A*(c*x^2+b*x+a)^(1/2)/a/x+1/2*(A*b-2*B*a)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(
c*x^2+b*x+a)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx=-\frac{A\sqrt{a+x(b+cx)}}{ax}+\frac{(-Ab+2aB)\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((A*Sqrt[a + x*(b + c*x)])/(a*x)) + ((-(A*b) + 2*a*B)*ArcTanh[(Sqrt[c]*x
- Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1228}$$

$$-\frac{(Ab-2aB)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{A\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{1154}$$

$$\frac{(Ab-2aB)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{A\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$\frac{(Ab-2aB)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx+cx^2}}{ax}$$

input
```
Int[(A + B*x)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x + c*x^2])/(a*x)) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x)/(2
*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{A\sqrt{c\,x^{2}+b x +a}}{a x}+\frac{\left(A b -2 B a \right) \ln \left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x}\right)}{2 a^{\frac{3}{2}}}$	65
default	$A \left(-\frac{\sqrt{c\,x^{2}+b x +a}}{a x}+\frac{b \ln \left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)-\frac{B \ln \left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x}\right)}{\sqrt{a}}$	95

input
```
int((B*x+A)/x^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(c*x^2+b*x+a)^(1/2)/a/x+1/2*(A*b-2*B*a)/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(
c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.46

$$\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx = \left[-\frac{(2\,Ba-Ab)\sqrt{a}x\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2+4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)+4\,\sqrt{cx^2+bx+a}Aa}{4\,a^2x}, \frac{(2\,Ba-Ab}{}\right.$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*B*a - A*b)*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(
c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*sqrt(c*x^2 + b*x +
a)*A*a)/(a^2*x), 1/2*((2*B*a - A*b)*sqrt(-a)*x*arctan(1/2*sqrt(c*x^2 + b*x
 + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*sqrt(c*x^2 + b*x +
 a)*A*a)/(a^2*x)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/x**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**2*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{(2\,Ba-Ab)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)Ab+2\,Aa\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)a}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
(2*B*a - A*b)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(
-a)*a) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*b + 2*A*a*sqrt(c))/(((sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{A\,b\,\mathrm{atanh}\left(\frac{a+\frac{b\,x}{2}}{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}\right)}{2\,a^{3/2}} - \frac{A\,\sqrt{c\,x^2+b\,x+a}}{a\,x} - \frac{B\,\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}}$$

input
```
int((A + B*x)/(x^2*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
(A*b*atanh((a + (b*x)/2)/(a^(1/2)*(a + b*x + c*x^2)^(1/2))))/(2*a^(3/2)) -
 (A*(a + b*x + c*x^2)^(1/2))/(a*x) - (B*log(b/2 + a/x + (a^(1/2)*(a + b*x
+ c*x^2)^(1/2))/x))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{-2\sqrt{c\,x^2+bx+a}\,a+\sqrt{a}\log\left(2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)bx-\sqrt{a}\log(x)\,bx}{2ax}$$

input
```
int((B*x+A)/x^2/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x + c*x**2)*a + sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x*
*2) - 2*a - b*x)*b*x - sqrt(a)*log(x)*b*x)/(2*a*x)
```

3.150 $\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx=-\frac{A\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3Ab-4aB)\sqrt{a+bx+cx^2}}{4a^2x}-\frac{(3Ab^2-4abB-4aAc)\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}}$$

output
```
-1/2*A*(c*x^2+b*x+a)^(1/2)/a/x^2+1/4*(3*A*b-4*B*a)*(c*x^2+b*x+a)^(1/2)/a^2
/x-1/8*(-4*A*a*c+3*A*b^2-4*B*a*b)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x
+a)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx = \frac{\frac{\sqrt{a}(3Abx-2a(A+2Bx))\sqrt{a+x(b+cx)}}{x^2}+3Ab^2\mathrm{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+4a(bB+Ac)\mathrm{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right.}{4a^{5/2}}$$

input
```
Integrate[(A + B*x)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*(3*A*b*x - 2*a*(A + 2*B*x))*Sqrt[a + x*(b + c*x)])/x^2 + 3*A*b^2
*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 4*a*(b*B + A*c)*Ar
cTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int \frac{3Ab-4aB+2Acx}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{A\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{3Ab-4aB+2Acx}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{A\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{1228}$$

$$-\frac{-\frac{(-4aAc-4abB+3Ab^2)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{(3Ab-4aB)\sqrt{a+bx+cx^2}}{ax}}{4a}-\frac{A\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{\frac{(-4aAc-4abB+3Ab^2)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{(3Ab-4aB)\sqrt{a+bx+cx^2}}{ax}}{4a}-\frac{A\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{(-4aAc-4abB+3Ab^2)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}-\frac{(3Ab-4aB)\sqrt{a+bx+cx^2}}{ax}}{4a}-\frac{A\sqrt{a+bx+cx^2}}{2ax^2}$$

input
```
Int[(A + B*x)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x + c*x^2])/(a*x^2) - (-(((3*A*b - 4*a*B)*Sqrt[a + b*x
+ c*x^2])/(a*x)) + ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*x)/(2*S
qrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,\left(-3 A b x+4 B a x+2 A a\right)}{4 a^{2} x^{2}}+\frac{\left(4 A a c-3 b^{2} A+4 a b B\right) \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{8 a^{\frac{5}{2}}}$
default	$A\left(-\frac{\sqrt{c\,x^{2}+b x+a}}{2 a\,x^{2}}-\frac{3 b\left(-\frac{\sqrt{c\,x^{2}+b x+a}}{a x}+\frac{b \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)}{4 a}+\frac{c \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)+B\left(-\right.$

input
```
int((B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(-3*A*b*x+4*B*a*x+2*A*a)/a^2/x^2+1/8*(4*A*a*c-3*A
*b^2+4*B*a*b)/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.03

$$\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{(4\,Bab-3\,Ab^2+4\,Aac)\sqrt{a}x^2\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2+4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)-4\,(2\,Aa^2+(4\,Ba^2-3}{16\,a^3x^2} \right.$$
$$-\frac{(4\,Bab-3\,Ab^2+4\,Aac)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+2\,(2\,Aa^2+(4\,Ba^2-3\,Aab)x)\sqrt{cx^2+}{8\,a^3x^2}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*
a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(
2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x)*sqrt(c*x^2 + b*x + a))/(a^3*x^2), -1/8*((
4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)
*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(2*A*a^2 + (4*B*a^2 - 3
*A*a*b)*x)*sqrt(c*x^2 + b*x + a))/(a^3*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/x**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(98) = 196$.

Time = 0.24 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.61

$$\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{(4\,Bab-3\,Ab^2+4\,Aac)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^2} + \frac{4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Bab-3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Ab^2+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Aac+ 8}{}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(4*B*a*b - 3*A*b^2 + 4*A*a*c)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/4*(4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*B*a*b - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*b^2 + 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*A*a*c + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B
*a^2*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^2*b + 5*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*A*a*b^2 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*A*a^2*c - 8*B*a^3*sqrt(c) + 8*A*a^2*b*sqrt(c))/(((sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2 - a)^2*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+B\,x}{x^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/(x^3*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^3*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx = \frac{-4\sqrt{c\,x^2+bx+a}\,a^2-2\sqrt{c\,x^2+bx+a}\,abx+4\sqrt{a}\log\left(-2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)ac\,x^2+\sqrt{a}\log}{8a^2x^2}$$

input
```
int((B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x + c*x**2)*a**2 - 2*sqrt(a + b*x + c*x**2)*a*b*x + 4*sqr
t(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*c*x**2 + sqrt(
a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**2*x**2 - 4*sqrt
(a)*log(x)*a*c*x**2 - sqrt(a)*log(x)*b**2*x**2)/(8*a**2*x**2)
```

3.151 $\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 167

$$\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx = -\frac{A\sqrt{a+bx+cx^2}}{3ax^3}+\frac{(5Ab-6aB)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{(15Ab^2-18abB-16aAc)\sqrt{a+bx+cx^2}}{24a^3x} + \frac{(5Ab^3-6ab^2B-12aAbc+8a^2Bc)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{7/2}}$$

output
```
-1/3*A*(c*x^2+b*x+a)^(1/2)/a/x^3+1/12*(5*A*b-6*B*a)*(c*x^2+b*x+a)^(1/2)/a^
2/x^2-1/24*(-16*A*a*c+15*A*b^2-18*B*a*b)*(c*x^2+b*x+a)^(1/2)/a^3/x+1/16*(-
12*A*a*b*c+5*A*b^3+8*B*a^2*c-6*B*a*b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x
^2+b*x+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(-15Ab^2x^2-4a^2(2A+3Bx)+2ax(5Ab+9bBx+8Acx))}{x^3} - 3(5Ab^3+8a^2Bc)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) - 18}{24a^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-15*A*b^2*x^2 - 4*a^2*(2*A + 3*B*x) + 2*a
*x*(5*A*b + 9*b*B*x + 8*A*c*x)))/x^3 - 3*(5*A*b^3 + 8*a^2*B*c)*ArcTanh[(Sq
rt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - 18*a*b*(b*B + 2*A*c)*ArcTanh[(
-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(24*a^(7/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 1237}$$

$$-\frac{\int \frac{5Ab-6aB+4Acx}{2x^3\sqrt{cx^2+bx+a}}dx}{3a} - \frac{A\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{5Ab-6aB+4Acx}{x^3\sqrt{cx^2+bx+a}}dx}{6a} - \frac{A\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \text{ 1237}$$

$$-\frac{-\frac{\int \frac{15Ab^2-18aBb-16aAc+2(5Ab-6aB)cx}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{(5Ab-6aB)\sqrt{a+bx+cx^2}}{2ax^2}}{6a}-\frac{A\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{\int \frac{15Ab^2-18aBb-16aAc+2(5Ab-6aB)cx}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{(5Ab-6aB)\sqrt{a+bx+cx^2}}{2ax^2}}{6a}-\frac{A\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \text{ 1228}$$

$$-\frac{-\frac{-\frac{3\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(-16aAc-18abB+15Ab^2\right)}{ax}}{4a}-\frac{(5Ab-6aB)\sqrt{a+bx+cx^2}}{2ax^2}}{6a}-\frac{A\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \text{ 1154}$$

$$-\frac{-\frac{\frac{3\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(-16aAc-18abB+15Ab^2\right)}{ax}}{4a}-\frac{(5Ab-6aB)\sqrt{a+bx+cx^2}}{2ax^2}}{6a}-\frac{A\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{-\frac{\frac{3\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx+cx^2}\left(-16aAc-18abB+15Ab^2\right)}{ax}}{4a}-\frac{(5Ab-6aB)\sqrt{a+bx+cx^2}}{2ax^2}}{6a}-\frac{A\sqrt{a+bx+cx^2}}{3ax^3}$$

input
```
Int[(A + B*x)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/3*(A*Sqrt[a + b*x + c*x^2])/(a*x^3) - (-1/2*((5*A*b - 6*a*B)*Sqrt[a + b
*x + c*x^2])/(a*x^2) - (-(((15*A*b^2 - 18*a*b*B - 16*a*A*c)*Sqrt[a + b*x +
 c*x^2])/(a*x)) + (3*(5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)))/(4*a))/(6*a
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-16Aac\,x^2+15x^2b^2A-18Ba\,x^2b-10abAx+12a^2Bx+8a^2A)}{24a^3x^3}-\frac{(12Aabc-5A\,b^3-8B\,a^2c+6Ba\,b^2)\ln\left(\frac{2a+bx+2\cdots}{\cdots}\right.}{16a^{\frac{7}{2}}}$
default	$A\left(-\frac{\sqrt{c\,x^2+bx+a}}{3a\,x^3}-\frac{5b\left(-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\cdots\right.}{6a}\right.$

input
```
int((B*x+A)/x^4/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+b*x+a)^(1/2)*(-16*A*a*c*x^2+15*A*b^2*x^2-18*B*a*b*x^2-10*A*a*
b*x+12*B*a^2*x+8*A*a^2)/a^3/x^3-1/16*(12*A*a*b*c-5*A*b^3-8*B*a^2*c+6*B*a*b
^2)/a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$=\left[\frac{3\,(6\,Bab^2-5\,Ab^3-4\,(2\,Ba^2-3\,Aab)c)\sqrt{a}x^3\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)-4\,(8\,A\cdots}{96\,a^4x^3}\right.$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(a)*x^3*log(-
(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a)
 + 8*a^2)/x^2) - 4*(8*A*a^3 - (18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^2 +
 2*(6*B*a^3 - 5*A*a^2*b)*x)*sqrt(c*x^2 + b*x + a))/(a^4*x^3), 1/48*(3*(6*B
*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(-a)*x^3*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(8*A*a^3
 - (18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^2 + 2*(6*B*a^3 - 5*A*a^2*b)*x)
*sqrt(c*x^2 + b*x + a))/(a^4*x^3)]
```

Sympy [F]

$$\int \frac{A + Bx}{x^4\sqrt{a + bx + cx^2}}\,dx = \int \frac{A + Bx}{x^4\sqrt{a + bx + cx^2}}\,dx$$

input
```
integrate((B*x+A)/x**4/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**4*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx}{x^4\sqrt{a + bx + cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(145) = 290$.

Time = 0.25 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.06

$$\int \frac{A + Bx}{x^4\sqrt{a + bx + cx^2}}\,dx$$
$$= \frac{(6\,Bab^2 - 5\,Ab^3 - 8\,Ba^2c + 12\,Aabc)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^3}$$
$$- \frac{18\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^5 Bab^2 - 15\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^5 Ab^3 - 24\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^5 Ba}{}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*(6*B*a*b^2 - 5*A*b^3 - 8*B*a^2*c + 12*A*a*b*c)*arctan(-(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/24*(18*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*B*a*b^2 - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*
b^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^2*c + 36*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^5*A*a*b*c - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*B*a^2*b^2 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a*b^3 - 96*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*A*a^2*b*c - 48*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*B*a^3*b*sqrt(c) - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*A*a^3
*c^(3/2) + 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^3*b^2 - 33*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*A*a^2*b^3 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*B*a^4*c - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^3*b*c + 48*B*a^4*
b*sqrt(c) - 48*A*a^3*b^2*sqrt(c) + 32*A*a^4*c^(3/2))/(((sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2 - a)^3*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx}{x^4\sqrt{a + bx + cx^2}}\,dx = \int \frac{A + B\,x}{x^4\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

input
```
int((A + B*x)/(x^4*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^4*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{-16\sqrt{c\,x^2+bx+a}\,a^3-4\sqrt{c\,x^2+bx+a}\,a^2bx+32\sqrt{c\,x^2+bx+a}\,a^2c\,x^2+6\sqrt{c\,x^2+bx+a}\,a\,b^2x^2+12}{}$$

input
```
int((B*x+A)/x^4/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 16*sqrt(a + b*x + c*x**2)*a**3 - 4*sqrt(a + b*x + c*x**2)*a**2*b*x + 3
2*sqrt(a + b*x + c*x**2)*a**2*c*x**2 + 6*sqrt(a + b*x + c*x**2)*a*b**2*x**
2 + 12*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b*c*x**
3 + 3*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**3*x**3
- 12*sqrt(a)*log(x)*a*b*c*x**3 - 3*sqrt(a)*log(x)*b**3*x**3)/(48*a**3*x**3
)
```

3.152 $\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 231

$$\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx = -\frac{A\sqrt{a+bx+cx^2}}{4ax^4}+\frac{(7Ab-8aB)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{(35Ab^2-40abB-36aAc)\sqrt{a+bx+cx^2}}{96a^3x^2} + \frac{(105Ab^3-120ab^2B-220aAbc+128a^2Bc)\sqrt{a+bx+cx^2}}{192a^4x} + \frac{(8abB(5b^2-12ac)-A(35b^4-120ab^2c+48a^2c^2))\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{9/2}}$$

output

```
-1/4*A*(c*x^2+b*x+a)^(1/2)/a/x^4+1/24*(7*A*b-8*B*a)*(c*x^2+b*x+a)^(1/2)/a^
2/x^3-1/96*(-36*A*a*c+35*A*b^2-40*B*a*b)*(c*x^2+b*x+a)^(1/2)/a^3/x^2+1/192
*(-220*A*a*b*c+105*A*b^3+128*B*a^2*c-120*B*a*b^2)*(c*x^2+b*x+a)^(1/2)/a^4/
x+1/128*(8*a*b*B*(-12*a*c+5*b^2)-A*(48*a^2*c^2-120*a*b^2*c+35*b^4))*arctan
h(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(105Ab^3x^3-16a^3(3A+4Bx)-10abx^2(7Ab+12bBx+22Acx)+8a^2x(2Bx(5b+8cx)+A(7b+9cx)))}{x^4}+105Ab^4\text{arctanh}\Big(\sqrt{\ }}{192a^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(105*A*b^3*x^3 - 16*a^3*(3*A + 4*B*x) - 10
*a*b*x^2*(7*A*b + 12*b*B*x + 22*A*c*x) + 8*a^2*x*(2*B*x*(5*b + 8*c*x) + A*
(7*b + 9*c*x))))/x^4 + 105*A*b^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)
])/Sqrt[a]] - 24*a*(-5*b^3*B - 15*A*b^2*c + 12*a*b*B*c + 6*a*A*c^2)*ArcTan
h[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(192*a^(9/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1237, 27, 1237, 27, 1237, 27, 25, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \ 1237$$
$$-\frac{\int \frac{7Ab-8aB+6Acx}{2x^4\sqrt{cx^2+bx+a}}dx}{4a}-\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$
$$\Big\downarrow \ 27$$
$$-\frac{\int \frac{7Ab-8aB+6Acx}{x^4\sqrt{cx^2+bx+a}}dx}{8a}-\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$
$$\Big\downarrow \ 1237$$

$$-\frac{-\frac{\int \frac{35Ab^2-40aBb-36aAc+4(7Ab-8aB)cx}{2x^3\sqrt{cx^2+bx+a}}dx}{3a}-\frac{(7Ab-8aB)\sqrt{a+bx+cx^2}}{3ax^3}}{8a}-\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 27

$$-\frac{-\frac{\int \frac{35Ab^2-40aBb-36aAc+4(7Ab-8aB)cx}{x^3\sqrt{cx^2+bx+a}}dx}{6a}-\frac{(7Ab-8aB)\sqrt{a+bx+cx^2}}{3ax^3}}{8a}-\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 1237

$$-\frac{-\frac{-\frac{\int -\frac{8aB\left(15b^2-16ac\right)-5A\left(21b^3-44abc\right)-2c\left(35Ab^2-40aBb-36aAc\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(-36aAc-40abB+35Ab^2\right)}{2ax^2}}{6a}-\frac{(7Ab-8aB)\sqrt{a+bx+cx^2}}{3ax^3}}{8a}-\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{\int -\frac{105Ab^3-120aBb^2-220aAcb+128a^2Bc+2c\left(35Ab^2-40aBb-36aAc\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}\left(-36aAc-40abB+35Ab^2\right)}{2ax^2}}{6a}-\frac{(7Ab-8aB)\sqrt{a+bx+cx^2}}{3ax^3}}{8a}-\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 25

$$-\frac{-\frac{-\frac{\int \frac{105Ab^3-120aBb^2-220aAcb+128a^2Bc+2c\left(35Ab^2-40aBb-36aAc\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}\left(-36aAc-40abB+35Ab^2\right)}{2ax^2}}{6a}-\frac{(7Ab-8aB)\sqrt{a+bx+cx^2}}{3ax^3}}{8a}-\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 1228

$$-\frac{-\frac{-\frac{\frac{3\left(8abB\left(5b^2-12ac\right)-A\left(48a^2c^2-120ab^2c+35b^4\right)\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(128a^2Bc-220aAbc-120ab^2B+105Ab^3\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}\left(-36aAc-\right.}{2ax^2}}{6a}}{8a}$$

$$\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 1154

$$-\frac{-\frac{-\frac{-\frac{3\left(8abB\left(5b^2-12ac\right)-A\left(48a^2c^2-120ab^2c+35b^4\right)\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(128a^2Bc-220aAbc-120ab^2B+105Ab^3\right)}{ax}}{4a}-\frac{\sqrt{a+bx}\cdots}{\cdots}}{6a}\cdots}{8a}\cdots$$

$$\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{-\frac{-\frac{\sqrt{a+bx+cx^2}\left(128a^2Bc-220aAbc-120ab^2B+105Ab^3\right)}{ax}-\frac{3\left(8abB\left(5b^2-12ac\right)-A\left(48a^2c^2-120ab^2c+35b^4\right)\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}}{4a}-\frac{\sqrt{a+bx}\cdots}{\cdots}}{6a}\cdots}{8a}\cdots$$

$$\frac{A\sqrt{a+bx+cx^2}}{4ax^4}$$

input
```
Int[(A + B*x)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/4*(A*Sqrt[a + b*x + c*x^2])/(a*x^4) - (-1/3*((7*A*b - 8*a*B)*Sqrt[a + b
*x + c*x^2])/(a*x^3) - (-1/2*((35*A*b^2 - 40*a*b*B - 36*a*A*c)*Sqrt[a + b*
x + c*x^2])/(a*x^2) - (-(((105*A*b^3 - 120*a*b^2*B - 220*a*A*b*c + 128*a^2
*B*c)*Sqrt[a + b*x + c*x^2])/(a*x)) - (3*(8*a*b*B*(5*b^2 - 12*a*c) - A*(35
*b^4 - 120*a*b^2*c + 48*a^2*c^2))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
b*x + c*x^2])])/(2*a^(3/2)))/(4*a))/(6*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\sqrt{c\,x^{2}+bx+a}\,\left(220Aabc\,x^{3}-105A\,b^{3}x^{3}-128B\,a^{2}c\,x^{3}+120Ba\,b^{2}x^{3}-72A\,a^{2}c\,x^{2}+70Aa\,b^{2}x^{2}-80B\,a^{2}b\,x^{2}-56A\,a^{2}bx+64B\,a^{3}x+4\right.}{192a^{4}x^{4}}$
default	$A\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{4a\,x^{4}}-\frac{7b\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{3a\,x^{3}}-\frac{5b\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{2a\,x^{2}}-\frac{3b\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx}{\cdots}\right.}{\cdots}\right)}{6a}\right)}{8a}\right.$

input
```
int((B*x+A)/x^5/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x^2+b*x+a)^(1/2)*(220*A*a*b*c*x^3-105*A*b^3*x^3-128*B*a^2*c*x^3+
120*B*a*b^2*x^3-72*A*a^2*c*x^2+70*A*a*b^2*x^2-80*B*a^2*b*x^2-56*A*a^2*b*x+
64*B*a^3*x+48*A*a^3)/a^4/x^4-1/128*(48*A*a^2*c^2-120*A*a*b^2*c+35*A*b^4+96
*B*a^2*b*c-40*B*a*b^3)/a^(9/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)
```

Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[-\frac{3\,(40\,Bab^3-35\,Ab^4-48\,Aa^2c^2-24\,(4\,Ba^2b-5\,Aab^2)c)\sqrt{a}x^4\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2}{x^2}\right.}{}\right.$$
$$-\frac{3\,(40\,Bab^3-35\,Ab^4-48\,Aa^2c^2-24\,(4\,Ba^2b-5\,Aab^2)c)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+2\,(4}{}$$

input
```
integrate((B*x+A)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(40*B*a*b^3 - 35*A*b^4 - 48*A*a^2*c^2 - 24*(4*B*a^2*b - 5*A*a*b
^2)*c)*sqrt(a)*x^4*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
+ a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(48*A*a^4 + (120*B*a^2*b^2 - 10
5*A*a*b^3 - 4*(32*B*a^3 - 55*A*a^2*b)*c)*x^3 - 2*(40*B*a^3*b - 35*A*a^2*b^
2 + 36*A*a^3*c)*x^2 + 8*(8*B*a^4 - 7*A*a^3*b)*x)*sqrt(c*x^2 + b*x + a))/(a
^5*x^4), -1/384*(3*(40*B*a*b^3 - 35*A*b^4 - 48*A*a^2*c^2 - 24*(4*B*a^2*b -
 5*A*a*b^2)*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*s
qrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(48*A*a^4 + (120*B*a^2*b^2 - 105*A*a*
b^3 - 4*(32*B*a^3 - 55*A*a^2*b)*c)*x^3 - 2*(40*B*a^3*b - 35*A*a^2*b^2 + 36
*A*a^3*c)*x^2 + 8*(8*B*a^4 - 7*A*a^3*b)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^4
)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/x**5/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**5*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(205) = 410.

Time = 0.26 (sec) , antiderivative size = 884, normalized size of antiderivative = 3.83

$$\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/64*(40*B*a*b^3 - 35*A*b^4 - 96*B*a^2*b*c + 120*A*a*b^2*c - 48*A*a^2*c^2
)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) + 1
/192*(120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*B*a*b^3 - 105*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^7*A*b^4 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
7*B*a^2*b*c + 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a*b^2*c - 144*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^7*A*a^2*c^2 - 440*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*B*a^2*b^3 + 385*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a*
b^4 + 1056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^3*b*c - 1320*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^5*A*a^2*b^2*c + 528*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^5*A*a^3*c^2 + 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*B*a^4*c^
(3/2) + 584*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a^3*b^3 - 511*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^3*A*a^2*b^4 - 480*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*B*a^4*b*c + 1752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a^3*b^2
*c + 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a^4*c^2 + 384*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*B*a^4*b^2*sqrt(c) - 1024*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*B*a^5*c^(3/2) + 2048*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*A*a^4*b*c^(3/2) - 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^4*b^3 + 279
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^3*b^4 - 288*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*B*a^5*b*c + 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^4*b
^2*c - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^5*c^2 - 384*B*a^5*b^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+B\,x}{x^5\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/(x^5*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^5*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{-96\sqrt{c\,x^2+bx+a}\,a^4-16\sqrt{c\,x^2+bx+a}\,a^3bx+144\sqrt{c\,x^2+bx+a}\,a^3c\,x^2+20\sqrt{c\,x^2+bx+a}\,a^2b^2x^2}{}$$

input
```
int((B*x+A)/x^5/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 96*sqrt(a + b*x + c*x**2)*a**4 - 16*sqrt(a + b*x + c*x**2)*a**3*b*x +
144*sqrt(a + b*x + c*x**2)*a**3*c*x**2 + 20*sqrt(a + b*x + c*x**2)*a**2*b*
*2*x**2 - 184*sqrt(a + b*x + c*x**2)*a**2*b*c*x**3 - 30*sqrt(a + b*x + c*x
**2)*a*b**3*x**3 + 144*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
- b*x)*a**2*c**2*x**4 - 72*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) -
2*a - b*x)*a*b**2*c*x**4 - 15*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2)
 - 2*a - b*x)*b**4*x**4 - 144*sqrt(a)*log(x)*a**2*c**2*x**4 + 72*sqrt(a)*l
og(x)*a*b**2*c*x**4 + 15*sqrt(a)*log(x)*b**4*x**4)/(384*a**4*x**4)
```

3.153 $\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 280

$$\begin{aligned}\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx &= -\frac{2x^3(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}} \\ &+\frac{(7b^2B-6Abc-16aBc)\,x^2\sqrt{a+bx+cx^2}}{3c^2\,(b^2-4ac)} \\ &+\frac{(105b^4B-90Ab^3c-460ab^2Bc+312aAbc^2+256a^2Bc^2-2c(35b^3B-30Ab^2c-116abBc+72aAc^2)\,x)}{24c^4\,(b^2-4ac)} \\ &-\frac{(35b^3B-30Ab^2c-60abBc+24aAc^2)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{9/2}}\end{aligned}$$

output

```
-2*x^3*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x
+a)^(1/2)+1/3*(-6*A*b*c-16*B*a*c+7*B*b^2)*x^2*(c*x^2+b*x+a)^(1/2)/c^2/(-4*
a*c+b^2)+1/24*(105*B*b^4-90*A*b^3*c-460*B*a*b^2*c+312*A*a*b*c^2+256*B*a^2*
c^2-2*c*(72*A*a*c^2-30*A*b^2*c-116*B*a*b*c+35*B*b^3)*x)*(c*x^2+b*x+a)^(1/2
)/c^4/(-4*a*c+b^2)-1/16*(24*A*a*c^2-30*A*b^2*c-60*B*a*b*c+35*B*b^3)*arctan
h(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.06

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{c}(-256a^3Bc^2+b^2x(-105b^3B+5b^2c(18A-7Bx)-4c^3x^2(3A+2Bx)+2bc}{}$$

input
```
Integrate[(x^4*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[c]*(-256*a^3*B*c^2 + b^2*x*(-105*b^3*B + 5*b^2*c*(18*A - 7*B*x) -
4*c^3*x^2*(3*A + 2*B*x) + 2*b*c^2*x*(15*A + 7*B*x)) + a*(-105*b^4*B + 16*c
^4*x^3*(3*A + 2*B*x) - 8*b*c^3*x^2*(15*A + 7*B*x) + 4*b^2*c^2*x*(-93*A + 4
3*B*x) + 10*b^3*c*(9*A + 53*B*x)) + 4*a^2*c*(115*b^2*B + 4*c^2*x*(9*A - 8*
B*x) - 2*b*c*(39*A + 61*B*x))) - 3*(b^2 - 4*a*c)*(35*b^3*B - 30*A*b^2*c -
60*a*b*B*c + 24*a*A*c^2)*Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x - 2*Sqrt[c]*S
qrt[a + x*(b + c*x)]])/(48*c^(9/2)*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1233, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2\int \frac{x^2(6a(bB-2Ac)+(7Bb^2-6Acb-16aBc)x)}{2\sqrt{cx^2+bx+a}}dx}{c(b^2-4ac)}-\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^2(6a(bB-2Ac)+(7Bb^2-6Acb-16aBc)x)}{\sqrt{cx^2+bx+a}}dx}{c(b^2-4ac)}-\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ \textbf{1236}$$

$$\frac{\frac{\int -\frac{x\left(4a\left(7Bb^2-6Acb-16aBc\right)+\left(35Bb^3-30Acb^2-116aBcb+72aAc^2\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{x^2\sqrt{a+bx+cx^2}\left(-16aBc-6Abc+7b^2B\right)}{3c}}{c\left(b^2-4ac\right)}-\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-16aBc-6Abc+7b^2B\right)}{3c}-\frac{\int \frac{x\left(4a\left(7Bb^2-6Acb-16aBc\right)+\left(35Bb^3-30Acb^2-116aBcb+72aAc^2\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{6c}}{c\left(b^2-4ac\right)}-\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ \textbf{1225}$$

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-16aBc-6Abc+7b^2B\right)}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(24aAc^2-60abBc-30Ab^2c+35b^3B\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c^2}-\frac{\sqrt{a+bx+cx^2}\left(256a^2Bc^2-2cx\left(72aAc^2-11\right.\right.}{}}{6c}}{c\left(b^2-4ac\right)}-\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ \textbf{1092}$$

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-16aBc-6Abc+7b^2B\right)}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(24aAc^2-60abBc-30Ab^2c+35b^3B\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c^2}-\frac{\sqrt{a+bx+cx^2}\left(256a^2Bc^2-2c\right.}{}}{6c}}{c\left(b^2-4ac\right)}-\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ \textbf{219}$$

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-16aBc-6Abc+7b^2B\right)}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(24aAc^2-60abBc-30Ab^2c+35b^3B\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{\sqrt{a+bx+cx^2}\left(256a^2Bc^2-2c\right.}{}}{6c}}{c\left(b^2-4ac\right)}-\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[(x^4*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-2*x^3*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(c*(b^2 - 4*a*c)*
Sqrt[a + b*x + c*x^2]) + (((7*b^2*B - 6*A*b*c - 16*a*B*c)*x^2*Sqrt[a + b*x
 + c*x^2])/(3*c) - (-1/4*((105*b^4*B - 90*A*b^3*c - 460*a*b^2*B*c + 312*a*
A*b*c^2 + 256*a^2*B*c^2 - 2*c*(35*b^3*B - 30*A*b^2*c - 116*a*b*B*c + 72*a*
A*c^2)*x)*Sqrt[a + b*x + c*x^2])/c^2 + (3*(b^2 - 4*a*c)*(35*b^3*B - 30*A*b
^2*c - 60*a*b*B*c + 24*a*A*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*c^(5/2)))/(6*c))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{\left(-8B\,c^{2}x^{2}-12A\,c^{2}x+22Bbcx+42Abc+40aBc-57B\,b^{2}\right)\sqrt{c\,x^{2}+bx+a}}{24c^{4}}-\frac{c\left(24Aa\,c^{2}-30A\,b^{2}c-60Babc+35B\,b^{3}\right)\left(-\frac{x}{c\sqrt{c\,x^{2}+bx}}\right.}{}$
default	$A\left(\frac{x^{3}}{2c\sqrt{c\,x^{2}+bx+a}}-\frac{5b\left(\frac{x^{2}}{c\sqrt{c\,x^{2}+bx+a}}-\frac{3b\left(-\frac{x}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+b}\right.}{c^{\frac{3}{2}}}\right.}{2c}\right.}{4c}\right.$

input
```
int(x^4*(B*x+A)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*B*c^2*x^2-12*A*c^2*x+22*B*b*c*x+42*A*b*c+40*B*a*c-57*B*b^2)/c^4*
(c*x^2+b*x+a)^(1/2)-1/16/c^4*(c*(24*A*a*c^2-30*A*b^2*c-60*B*a*b*c+35*B*b^3
)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b
)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2)))+(-8*A*a*b*c^2-14*A*b^3*c-16*B*a^2*c^2-12*B*a*b^2*c+19*B*b^
4)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
)+38*B*a*b^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+16*a^2*A*c^2*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-28*A*a*b^2*c*(2*c*x+b)/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)-56*a^2*b*B*c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1035, normalized size of antiderivative = 3.70

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(35*B*a*b^5 - 96*A*a^3*c^3 + 48*(5*B*a^3*b + 3*A*a^2*b^2)*c^2 + (
35*B*b^5*c - 96*A*a^2*c^4 + 48*(5*B*a^2*b + 3*A*a*b^2)*c^3 - 10*(20*B*a*b^
3 + 3*A*b^4)*c^2)*x^2 - 10*(20*B*a^2*b^3 + 3*A*a*b^4)*c + (35*B*b^6 - 96*A
*a^2*b*c^3 + 48*(5*B*a^2*b^2 + 3*A*a*b^3)*c^2 - 10*(20*B*a*b^4 + 3*A*b^5)*
c)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) + 4*(105*B*a*b^4*c + 8*(B*b^2*c^4 - 4*B*a*c^5)*x
^4 + 8*(32*B*a^3 + 39*A*a^2*b)*c^3 - 2*(7*B*b^3*c^3 + 24*A*a*c^5 - 2*(14*B
*a*b + 3*A*b^2)*c^4)*x^3 - 10*(46*B*a^2*b^2 + 9*A*a*b^3)*c^2 + (35*B*b^4*c
^2 + 8*(16*B*a^2 + 15*A*a*b)*c^4 - 2*(86*B*a*b^2 + 15*A*b^3)*c^3)*x^2 + (1
05*B*b^5*c - 144*A*a^2*c^4 + 4*(122*B*a^2*b + 93*A*a*b^2)*c^3 - 10*(53*B*a
*b^3 + 9*A*b^4)*c^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*a^2*c^6 + (b
^2*c^6 - 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x), 1/48*(3*(35*B*a*b^5 - 96
*A*a^3*c^3 + 48*(5*B*a^3*b + 3*A*a^2*b^2)*c^2 + (35*B*b^5*c - 96*A*a^2*c^4
 + 48*(5*B*a^2*b + 3*A*a*b^2)*c^3 - 10*(20*B*a*b^3 + 3*A*b^4)*c^2)*x^2 - 1
0*(20*B*a^2*b^3 + 3*A*a*b^4)*c + (35*B*b^6 - 96*A*a^2*b*c^3 + 48*(5*B*a^2*
b^2 + 3*A*a*b^3)*c^2 - 10*(20*B*a*b^4 + 3*A*b^5)*c)*x)*sqrt(-c)*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(
105*B*a*b^4*c + 8*(B*b^2*c^4 - 4*B*a*c^5)*x^4 + 8*(32*B*a^3 + 39*A*a^2*b)*
c^3 - 2*(7*B*b^3*c^3 + 24*A*a*c^5 - 2*(14*B*a*b + 3*A*b^2)*c^4)*x^3 - 10*(
46*B*a^2*b^2 + 9*A*a*b^3)*c^2 + (35*B*b^4*c^2 + 8*(16*B*a^2 + 15*A*a*b)...
```

Sympy [F]

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(B*x+A)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(x**4*(A + B*x)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.30

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4\,(Bb^2c^3-4\,Bac^4)x}{b^2c^4-4\,ac^5}-\frac{7\,Bb^3c^2-28\,Babc^3-6\,Ab^2c^3+24\,Aac^4}{b^2c^4-4\,ac^5}\right)x+\frac{35\,Bb^4c-172\,Bab^2c^2-30\,Ab^3c}{b^2c^4-4}\right.\right.}{}$$
$$+\frac{(35\,Bb^3-60\,Babc-30\,Ab^2c+24\,Aac^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{9}{2}}}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/24*(((2*(4*(B*b^2*c^3 - 4*B*a*c^4)*x/(b^2*c^4 - 4*a*c^5) - (7*B*b^3*c^2
- 28*B*a*b*c^3 - 6*A*b^2*c^3 + 24*A*a*c^4)/(b^2*c^4 - 4*a*c^5))*x + (35*B*
b^4*c - 172*B*a*b^2*c^2 - 30*A*b^3*c^2 + 128*B*a^2*c^3 + 120*A*a*b*c^3)/(b
^2*c^4 - 4*a*c^5))*x + (105*B*b^5 - 530*B*a*b^3*c - 90*A*b^4*c + 488*B*a^2
*b*c^2 + 372*A*a*b^2*c^2 - 144*A*a^2*c^3)/(b^2*c^4 - 4*a*c^5))*x + (105*B*
a*b^4 - 460*B*a^2*b^2*c - 90*A*a*b^3*c + 256*B*a^3*c^2 + 312*A*a^2*b*c^2)/
(b^2*c^4 - 4*a*c^5))/sqrt(c*x^2 + b*x + a) + 1/16*(35*B*b^3 - 60*B*a*b*c -
 30*A*b^2*c + 24*A*a*c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sq
rt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^4\,(A+B\,x)}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((x^4*(A + B*x))/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1083, normalized size of antiderivative = 3.87

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(B*x+A)/(c*x^2+b*x+a)^(3/2),x)
```

output

```
( - 1136*sqrt(a + b*x + c*x**2)*a**3*b*c**3 + 288*sqrt(a + b*x + c*x**2)*a
**3*c**4*x + 1100*sqrt(a + b*x + c*x**2)*a**2*b**3*c**2 - 1720*sqrt(a + b*
x + c*x**2)*a**2*b**2*c**3*x - 496*sqrt(a + b*x + c*x**2)*a**2*b*c**4*x**2
 + 96*sqrt(a + b*x + c*x**2)*a**2*c**5*x**3 - 210*sqrt(a + b*x + c*x**2)*a
*b**5*c + 1240*sqrt(a + b*x + c*x**2)*a*b**4*c**2*x + 404*sqrt(a + b*x + c
*x**2)*a*b**3*c**3*x**2 - 136*sqrt(a + b*x + c*x**2)*a*b**2*c**4*x**3 + 64
*sqrt(a + b*x + c*x**2)*a*b*c**5*x**4 - 210*sqrt(a + b*x + c*x**2)*b**6*c*
x - 70*sqrt(a + b*x + c*x**2)*b**5*c**2*x**2 + 28*sqrt(a + b*x + c*x**2)*b
**4*c**3*x**3 - 16*sqrt(a + b*x + c*x**2)*b**3*c**4*x**4 - 288*sqrt(c)*log
((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*c
**3 + 1152*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt
(4*a*c - b**2))*a**3*b**2*c**2 - 288*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x +
 c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**3*x - 288*sqrt(c)*log(
(2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*c*
*4*x**2 - 690*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/s
qrt(4*a*c - b**2))*a**2*b**4*c + 1152*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x
+ c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**2*x + 1152*sqrt(c)
*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b**2*c**3*x**2 + 105*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b
+ 2*c*x)/sqrt(4*a*c - b**2))*a*b**6 - 690*sqrt(c)*log((2*sqrt(c)*sqrt(a...
```

3.154 $\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 197

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}} - \frac{(15b^3B-12Ab^2c-52abBc+32aAc^2-2c(5b^2B-4Abc-12aBc)\,x)\,\sqrt{a+bx+cx^2}}{4c^3\,(b^2-4ac)} + \frac{3(5b^2B-4Abc-4aBc)\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}}$$

output

```
-2*x^2*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x
+a)^(1/2)-1/4*(15*B*b^3-12*A*b^2*c-52*B*a*b*c+32*A*a*c^2-2*c*(-4*A*b*c-12*
B*a*c+5*B*b^2)*x)*(c*x^2+b*x+a)^(1/2)/c^3/(-4*a*c+b^2)+3/8*(-4*A*b*c-4*B*a
*c+5*B*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{c}(4a^2c(-13bB+8Ac+6Bcx)+a(15b^3B-20bc^2x(-2A+Bx)+8c^3x^2(2A}{}$$

input
```
Integrate[(x^3*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[c]*(4*a^2*c*(-13*b*B + 8*A*c + 6*B*c*x) + a*(15*b^3*B - 20*b*c^2*x
*(-2*A + B*x) + 8*c^3*x^2*(2*A + B*x) - 2*b^2*c*(6*A + 31*B*x)) + b^2*x*(1
5*b^2*B - 2*c^2*x*(2*A + B*x) + b*(-12*A*c + 5*B*c*x))) + 3*(b^2 - 4*a*c)*
(5*b^2*B - 4*A*b*c - 4*a*B*c)*Sqrt[a + x*(b + c*x)]*Log[c^3*(b + 2*c*x - 2
*Sqrt[c]*Sqrt[a + x*(b + c*x)]])])/(8*c^(7/2)*(-b^2 + 4*a*c)*Sqrt[a + x*(b
+ c*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1233, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1233$$

$$\frac{2\int \frac{x\left(4a(bB-2Ac)+\left(5Bb^2-4Acb-12aBc\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{c\left(b^2-4ac\right)}-\frac{2x^2\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x\left(4a(bB-2Ac)+\left(5Bb^2-4Acb-12aBc\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{c\left(b^2-4ac\right)}-\frac{2x^2\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1225}$$

$$\frac{\frac{3(b^2-4ac)(-4aBc-4Abc+5b^2B)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c^2}-\frac{\sqrt{a+bx+cx^2}(-2cx(-12aBc-4Abc+5b^2B)+32aAc^2-52abBc-12Ab^2c+15b^3B)}{4c^2}}{c(b^2-4ac)}-\frac{2x^2(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{3(b^2-4ac)(-4aBc-4Abc+5b^2B)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c^2}-\frac{\sqrt{a+bx+cx^2}(-2cx(-12aBc-4Abc+5b^2B)+32aAc^2-52abBc-12Ab^2c+15}{4c^2}}{c(b^2-4ac)}$$

$$\frac{2x^2(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3(b^2-4ac)(-4aBc-4Abc+5b^2B)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{\sqrt{a+bx+cx^2}(-2cx(-12aBc-4Abc+5b^2B)+32aAc^2-52abBc-12Ab^2c+15b}{4c^2}}{c(b^2-4ac)}$$

$$\frac{2x^2(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[(x^3*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-2*x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(c*(b^2 - 4*a*c)*
Sqrt[a + b*x + c*x^2]) + (-1/4*((15*b^3*B - 12*A*b^2*c - 52*a*b*B*c + 32*a
*A*c^2 - 2*c*(5*b^2*B - 4*A*b*c - 12*a*B*c)*x)*Sqrt[a + b*x + c*x^2])/c^2
+ (3*(b^2 - 4*a*c)*(5*b^2*B - 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.82

method	result
risch	$\frac{(2Bcx+4Ac-7Bb)\sqrt{c\,x^2+bx+a}}{4c^3} - \frac{3c(4Abc+4aBc-5B\,b^2)\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}} - \frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}} - \frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c} + \frac{\ln\left(\right.}{}\right.}{}$
default	$A\left(\frac{x^2}{c\sqrt{c\,x^2+bx+a}} - \frac{3b\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}} - \frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}} - \frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c} + \frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}\right)}{2c} - \frac{2a\left(-\right.}{}\right.$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*B*c*x+4*A*c-7*B*b)/c^3*(c*x^2+b*x+a)^(1/2)-1/8/c^3*(3*c*(4*A*b*c+4*
B*a*c-5*B*b^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)
-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))+(8*A*a*c^2+4*A*b^2*c-4*B*a*b*c-7*B*b^3)*(-1/c
/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))-14*B*a
*b^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+8*B*a^2*c*(2*c*x+b)/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)+8*A*a*b*c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(181) = 362$.

Time = 0.25 (sec) , antiderivative size = 793, normalized size of antiderivative = 4.03

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*(5*B*a*b^4 + 16*(B*a^3 + A*a^2*b)*c^2 + (5*B*b^4*c + 16*(B*a^2 +
 A*a*b)*c^3 - 4*(6*B*a*b^2 + A*b^3)*c^2)*x^2 - 4*(6*B*a^2*b^2 + A*a*b^3)*c
 + (5*B*b^5 + 16*(B*a^2*b + A*a*b^2)*c^2 - 4*(6*B*a*b^3 + A*b^4)*c)*x)*sqr
t(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*
sqrt(c) - 4*a*c) + 4*(15*B*a*b^3*c + 32*A*a^2*c^3 - 2*(B*b^2*c^3 - 4*B*a*c
^4)*x^3 - 4*(13*B*a^2*b + 3*A*a*b^2)*c^2 + (5*B*b^3*c^2 + 16*A*a*c^4 - 4*(
5*B*a*b + A*b^2)*c^3)*x^2 + (15*B*b^4*c + 8*(3*B*a^2 + 5*A*a*b)*c^3 - 2*(3
1*B*a*b^2 + 6*A*b^3)*c^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5
 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*(3*(5*B*a*b^4
+ 16*(B*a^3 + A*a^2*b)*c^2 + (5*B*b^4*c + 16*(B*a^2 + A*a*b)*c^3 - 4*(6*B*
a*b^2 + A*b^3)*c^2)*x^2 - 4*(6*B*a^2*b^2 + A*a*b^3)*c + (5*B*b^5 + 16*(B*a
^2*b + A*a*b^2)*c^2 - 4*(6*B*a*b^3 + A*b^4)*c)*x)*sqrt(-c)*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(15*B*
a*b^3*c + 32*A*a^2*c^3 - 2*(B*b^2*c^3 - 4*B*a*c^4)*x^3 - 4*(13*B*a^2*b + 3
*A*a*b^2)*c^2 + (5*B*b^3*c^2 + 16*A*a*c^4 - 4*(5*B*a*b + A*b^2)*c^3)*x^2 +
 (15*B*b^4*c + 8*(3*B*a^2 + 5*A*a*b)*c^3 - 2*(31*B*a*b^2 + 6*A*b^3)*c^2)*x
)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2
+ (b^3*c^4 - 4*a*b*c^5)*x)]
```

Sympy [F]

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(B*x+A)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(x**3*(A + B*x)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.35

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,(Bb^2c^2-4\,Bac^3)x}{b^2c^3-4\,ac^4} - \frac{5\,Bb^3c-20\,Babc^2-4\,Ab^2c^2+16\,Aac^3}{b^2c^3-4\,ac^4}\right)x - \frac{15\,Bb^4-62\,Bab^2c-12\,Ab^3c+24\,Ba^2\cdots}{b^2c^3-4\,ac^4}\cdots}{4\,\sqrt{cx^2+bx+a}} - \frac{3\,(5\,Bb^2-4\,Bac-4\,Abc)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{7}{2}}}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*(((2*(B*b^2*c^2 - 4*B*a*c^3)*x/(b^2*c^3 - 4*a*c^4) - (5*B*b^3*c - 20*B
*a*b*c^2 - 4*A*b^2*c^2 + 16*A*a*c^3)/(b^2*c^3 - 4*a*c^4))*x - (15*B*b^4 -
62*B*a*b^2*c - 12*A*b^3*c + 24*B*a^2*c^2 + 40*A*a*b*c^2)/(b^2*c^3 - 4*a*c^
4))*x - (15*B*a*b^3 - 52*B*a^2*b*c - 12*A*a*b^2*c + 32*A*a^2*c^2)/(b^2*c^3
 - 4*a*c^4))/sqrt(c*x^2 + b*x + a) - 3/8*(5*B*b^2 - 4*B*a*c - 4*A*b*c)*log
(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((x^3*(A + B*x))/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{x^3(Bx+A)}{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int(x^3*(B*x+A)/(c*x^2+b*x+a)^(3/2),x)
```

3.155 $\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2x(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}} + \frac{(3b^2B-2Abc-8aBc)\,\sqrt{a+bx+cx^2}}{c^2\,(b^2-4ac)} - \frac{(3bB-2Ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}}$$

output
```
-2*x*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(1/2)+(-2*A*b*c-8*B*a*c+3*B*b^2)*(c*x^2+b*x+a)^(1/2)/c^2/(-4*a*c+b^2)-1/
2*(-2*A*c+3*B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2
)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\frac{2\sqrt{c}(8a^2Bc-b^2x(3bB-2Ac+Bcx)+a(-3b^2B+4c^2x(-A+Bx)+2bc(A+5Bx)))}{\sqrt{a+x(b+cx)}} - (b^2-4ac)\,(3bB-}{2c^{5/2}\,(-b^2+4ac)}$$

input
```
Integrate[(x^2*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
((2*Sqrt[c]*(8*a^2*B*c - b^2*x*(3*b*B - 2*A*c + B*c*x) + a*(-3*b^2*B + 4*c
^2*x*(-A + B*x) + 2*b*c*(A + 5*B*x))))/Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*
c)*(3*b*B - 2*A*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(2*c^
(5/2)*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1233, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2\int \frac{2a(bB-2Ac)+(3Bb^2-2Acb-8aBc)x}{2\sqrt{cx^2+bx+a}}dx}{c(b^2-4ac)}-\frac{2x(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2a(bB-2Ac)+(3Bb^2-2Acb-8aBc)x}{\sqrt{cx^2+bx+a}}dx}{c(b^2-4ac)}-\frac{2x(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{\sqrt{a+bx+cx^2}(-8aBc-2Abc+3b^2B)}{c}-\frac{(b^2-4ac)(3bB-2Ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}}{c(b^2-4ac)}-\frac{2x(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{\sqrt{a+bx+cx^2}(-8aBc-2Abc+3b^2B)}{c}-\frac{(b^2-4ac)(3bB-2Ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}}{c(b^2-4ac)}-\frac{2x(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sqrt{a+bx+cx^2}(-8aBc-2Abc+3b^2B)}{c}-\frac{(b^2-4ac)(3bB-2Ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}}{c\left(b^2-4ac\right)}-\frac{2x\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[(x^2*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-2*x*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(c*(b^2 - 4*a*c)*Sq
rt[a + b*x + c*x^2]) + (((3*b^2*B - 2*A*b*c - 8*a*B*c)*Sqrt[a + b*x + c*x^
2])/c - ((b^2 - 4*a*c)*(3*b*B - 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(2*c^(3/2)))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.59

method	result
risch	$\frac{B\sqrt{c\,x^2+bx+a}}{c^2}+\frac{c(2Ac-3Bb)\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}\right)-B(2}{2c^2}$
default	$A\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}\right)+B\left(\frac{x^2}{c\sqrt{c\,x^2+bx+a}}\right.$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B/c^2*(c*x^2+b*x+a)^(1/2)+1/2/c^2*(c*(2*A*c-3*B*b)*(-x/c/(c*x^2+b*x+a)^(1/
2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+
a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-B*(2*a*c+
b^2)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2))-2*a*b*B*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(139) = 278$.

Time = 0.19 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.94

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \left[-\frac{(3\,Bab^3+8\,Aa^2c^2+(3\,Bb^3c+8\,Aac^3-2\,(6\,Bab+Ab^2)c^2)x^2-2\,(6\,Ba^2b+\right.$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((3*B*a*b^3 + 8*A*a^2*c^2 + (3*B*b^3*c + 8*A*a*c^3 - 2*(6*B*a*b + A*
b^2)*c^2)*x^2 - 2*(6*B*a^2*b + A*a*b^2)*c + (3*B*b^4 + 8*A*a*b*c^2 - 2*(6*
B*a*b^2 + A*b^3)*c)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(3*B*a*b^2*c - 2*(4*B*a^2 +
 A*a*b)*c^2 + (B*b^2*c^2 - 4*B*a*c^3)*x^2 + (3*B*b^3*c + 4*A*a*c^3 - 2*(5*
B*a*b + A*b^2)*c^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^
2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), 1/2*((3*B*a*b^3 + 8*A*a^2
*c^2 + (3*B*b^3*c + 8*A*a*c^3 - 2*(6*B*a*b + A*b^2)*c^2)*x^2 - 2*(6*B*a^2*
b + A*a*b^2)*c + (3*B*b^4 + 8*A*a*b*c^2 - 2*(6*B*a*b^2 + A*b^3)*c)*x)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*
x + a*c)) + 2*(3*B*a*b^2*c - 2*(4*B*a^2 + A*a*b)*c^2 + (B*b^2*c^2 - 4*B*a*
c^3)*x^2 + (3*B*b^3*c + 4*A*a*c^3 - 2*(5*B*a*b + A*b^2)*c^2)*x)*sqrt(c*x^2
 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 -
 4*a*b*c^4)*x)]
```

Sympy [F]

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(B*x+A)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(x**2*(A + B*x)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\frac{(Bb^2c-4\,Bac^2)x}{b^2c^2-4\,ac^3}+\frac{3\,Bb^3-10\,Babc-2\,Ab^2c+4\,Aac^2}{b^2c^2-4\,ac^3}\right)x+\frac{3\,Bab^2-8\,Ba^2c-2\,Aabc}{b^2c^2-4\,ac^3}}{\sqrt{cx^2+bx+a}} + \frac{(3\,Bb-2\,Ac)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{5}{2}}}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
(((B*b^2*c - 4*B*a*c^2)*x/(b^2*c^2 - 4*a*c^3) + (3*B*b^3 - 10*B*a*b*c - 2*
A*b^2*c + 4*A*a*c^2)/(b^2*c^2 - 4*a*c^3))*x + (3*B*a*b^2 - 8*B*a^2*c - 2*A
*a*b*c)/(b^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a) + 1/2*(3*B*b - 2*A*c)*l
og(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x^2\,(A+B\,x)}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((x^2*(A + B*x))/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.63

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{-8\sqrt{c}\,a^3c^2 - 4\sqrt{c}\,a\,b^4 - 4\sqrt{c}\,b^5x + 24\sqrt{c\,x^2+bx+a}\,a\,b^2c^2x + 8\sqrt{c\,x^2+bx+a}}{}$$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(20*sqrt(a + b*x + c*x**2)*a**2*b*c**2 - 8*sqrt(a + b*x + c*x**2)*a**2*c**
3*x - 6*sqrt(a + b*x + c*x**2)*a*b**3*c + 24*sqrt(a + b*x + c*x**2)*a*b**2
*c**2*x + 8*sqrt(a + b*x + c*x**2)*a*b*c**3*x**2 - 6*sqrt(a + b*x + c*x**2
)*b**4*c*x - 2*sqrt(a + b*x + c*x**2)*b**3*c**2*x**2 + 8*sqrt(c)*log((2*sq
rt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*c**2 -
14*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c -
 b**2))*a**2*b**2*c + 8*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b
+ 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2*x + 8*sqrt(c)*log((2*sqrt(c)*sqrt
(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**3*x**2 + 3*sqr
t(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2)
)*a*b**4 - 14*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/s
qrt(4*a*c - b**2))*a*b**3*c*x - 14*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c
*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*x**2 + 3*sqrt(c)*log((
2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*x +
 3*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c -
 b**2))*b**4*c*x**2 - 8*sqrt(c)*a**3*c**2 + 16*sqrt(c)*a**2*b**2*c - 8*sqr
t(c)*a**2*b*c**2*x - 8*sqrt(c)*a**2*c**3*x**2 - 4*sqrt(c)*a*b**4 + 16*sqrt
(c)*a*b**3*c*x + 16*sqrt(c)*a*b**2*c**2*x**2 - 4*sqrt(c)*b**5*x - 4*sqrt(c
)*b**4*c*x**2)/(2*c**3*(4*a**2*c - a*b**2 + 4*a*b*c*x + 4*a*c**2*x**2 - b*
*3*x - b**2*c*x**2))
```

3.156 $\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}} + \frac{B\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

output
```
(-2*a*(-2*A*c+B*b)-2*(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(1/2)+B*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\frac{2\sqrt{c}(abB+b(bB-Ac)x-2ac(A+Bx))}{\sqrt{a+x(b+cx)}} - B(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}\,(-b^2+4ac)}$$

input
```
Integrate[(x*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
((2*Sqrt[c]*(a*b*B + b*(b*B - A*c)*x - 2*a*c*(A + B*x)))/Sqrt[a + x*(b + c
*x)] - B*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])/(c^(3/2)*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1224, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1224}$$

$$\frac{B\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{c}-\frac{2\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2B\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}-\frac{2\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}-\frac{2\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[(x*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(c*(b^2 - 4*a*c)*Sqrt
[a + b*x + c*x^2]) + (B*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/c^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.75

method	result
default	$A\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^{2})\sqrt{c\,x^{2}+bx+a}}\right)+B\left(-\frac{x}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}\right)}{2c}+\right.$

input
```
int(x*(B*x+A)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
+B*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+
b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(86) = 172$.

Time = 0.18 (sec) , antiderivative size = 405, normalized size of antiderivative = 4.22

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \left[\frac{(Bab^2-4\,Ba^2c+(Bb^2c-4\,Bac^2)x^2+(Bb^3-4\,Babc)x)\sqrt{c}\log\left(-8\,c^2x^2-8\right.}{2\,(ab^2c^2-}\right.$$

$$-\frac{(Bab^2-4\,Ba^2c+(Bb^2c-4\,Bac^2)x^2+(Bb^3-4\,Babc)x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(Babc}{ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^2+(b^3c^2-4\,abc^3)x}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((B*a*b^2 - 4*B*a^2*c + (B*b^2*c - 4*B*a*c^2)*x^2 + (B*b^3 - 4*B*a*b*
c)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) - 4*(B*a*b*c - 2*A*a*c^2 + (B*b^2*c - (2*B*a + A
*b)*c^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a
*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -((B*a*b^2 - 4*B*a^2*c + (B*b^2*c -
4*B*a*c^2)*x^2 + (B*b^3 - 4*B*a*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(B*a*b*c - 2*A*a
*c^2 + (B*b^2*c - (2*B*a + A*b)*c^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2
- 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)]
```

Sympy [F]

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{x(A+Bx)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(B*x+A)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral(x*(A + B*x)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{(Bb^2-2\,Bac-Abc)x}{b^2c-4\,ac^2}+\frac{Bab-2\,Aac}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}} - \frac{B\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((B*b^2 - 2*B*a*c - A*b*c)*x/(b^2*c - 4*a*c^2) + (B*a*b - 2*A*a*c)/(b^2
*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - B*log(abs(2*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{B\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}} - \frac{A\,(4\,a+2\,b\,x)}{(4\,a\,c-b^2)\,\sqrt{c\,x^2+b\,x+a}} + \frac{B\left(\frac{a\,b}{2}-x\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

input
```
int((x*(A + B*x))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
(B*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (A*(4*a +
 2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (B*((a*b)/2 - x*(a*c -
b^2/2)))/(c*(a*c - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.75

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{-4\sqrt{c\,x^2+bx+a}\,a^2c^2+2\sqrt{c\,x^2+bx+a}\,a\,b^2c-6\sqrt{c\,x^2+bx+a}\,ab\,c^2x+2\sqrt{c}}{}$$

input
```
int(x*(B*x+A)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
( - 4*sqrt(a + b*x + c*x**2)*a**2*c**2 + 2*sqrt(a + b*x + c*x**2)*a*b**2*c
 - 6*sqrt(a + b*x + c*x**2)*a*b*c**2*x + 2*sqrt(a + b*x + c*x**2)*b**3*c*x
 + 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c
 - b**2))*a**2*b*c - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2
*c*x)/sqrt(4*a*c - b**2))*a*b**3 + 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x +
 c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x + 4*sqrt(c)*log((2*sq
rt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*x**
2 - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c
- b**2))*b**4*x - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*
x)/sqrt(4*a*c - b**2))*b**3*c*x**2 - 6*sqrt(c)*a**2*b*c + 2*sqrt(c)*a*b**3
 - 6*sqrt(c)*a*b**2*c*x - 6*sqrt(c)*a*b*c**2*x**2 + 2*sqrt(c)*b**4*x + 2*s
qrt(c)*b**3*c*x**2)/(c**2*(4*a**2*c - a*b**2 + 4*a*b*c*x + 4*a*c**2*x**2 -
 b**3*x - b**2*c*x**2))
```

3.157 $\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2(Ab-2aB-(bB-2Ac)x)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

output
```
(-2*A*b+4*B*a+2*(-2*A*c+B*b)*x)/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2B(2a+bx)-2A(b+2cx)}{(b^2-4ac)\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*B*(2*a + b*x) - 2*A*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1158}$$

$$-\frac{2(-2aB-x(bB-2Ac)+Ab)}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[(A + B*x)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(A*b - 2*a*B - (b*B - 2*A*c)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{4Acx-2Bbx+2Ab-4Ba}{\sqrt{c\,x^{2}+bx+a}\,(4ac-b^{2})}$	45
trager	$\frac{4Acx-2Bbx+2Ab-4Ba}{\sqrt{c\,x^{2}+bx+a}\,(4ac-b^{2})}$	45
orering	$\frac{4Acx-2Bbx+2Ab-4Ba}{\sqrt{c\,x^{2}+bx+a}\,(4ac-b^{2})}$	45
default	$\frac{2A(2cx+b)}{(4ac-b^{2})\sqrt{c\,x^{2}+bx+a}}+B\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^{2})\sqrt{c\,x^{2}+bx+a}}\right)$	91

input
```
int((B*x+A)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/(c*x^2+b*x+a)^(1/2)*(2*A*c*x-B*b*x+A*b-2*B*a)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{cx^2+bx+a}(2\,Ba-Ab+(Bb-2\,Ac)x)}{ab^2-4\,a^2c+(b^2c-4\,ac^2)x^2+(b^3-4\,abc)x}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*x^2 + b*x + a)*(2*B*a - A*b + (B*b - 2*A*c)*x)/(a*b^2 - 4*a^2*c +
 (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)
```

Sympy [F]

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{A+Bx}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((A + B*x)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left(\frac{(Bb-2\,Ac)x}{b^2-4\,ac}+\frac{2\,Ba-Ab}{b^2-4\,ac}\right)}{\sqrt{cx^2+bx+a}}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*((B*b - 2*A*c)*x/(b^2 - 4*a*c) + (2*B*a - A*b)/(b^2 - 4*a*c))/sqrt(c*x^2
 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\,A\,b-4\,B\,a+4\,A\,c\,x-2\,B\,b\,x}{(4\,a\,c-b^2)\,\sqrt{c\,x^2+b\,x+a}}$$

input
```
int((A + B*x)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
(2*A*b - 4*B*a + 4*A*c*x - 2*B*b*x)/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.49

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}}\,dx = \frac{-2\sqrt{c\,x^2+bx+a}\,abc+4\sqrt{c\,x^2+bx+a}\,a\,c^2x-2\sqrt{c\,x^2+bx+a}\,b^2cx+4\sqrt{c}\,a^2}{c\,(4a\,c^2x^2-b^2c\,x^2+4abcx-b^3x}$$

input
```
int((B*x+A)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(2*( - sqrt(a + b*x + c*x**2)*a*b*c + 2*sqrt(a + b*x + c*x**2)*a*c**2*x -
sqrt(a + b*x + c*x**2)*b**2*c*x + 2*sqrt(c)*a**2*c - sqrt(c)*a*b**2 + 2*sq
rt(c)*a*b*c*x + 2*sqrt(c)*a*c**2*x**2 - sqrt(c)*b**3*x - sqrt(c)*b**2*c*x*
*2))/(c*(4*a**2*c - a*b**2 + 4*a*b*c*x + 4*a*c**2*x**2 - b**3*x - b**2*c*x
**2))
```

3.158 $\int \frac{A+Bx}{x(a+bx+cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{3/2}} \, dx = \frac{2(Ab^2-abB-2aAc+(Ab-2aB)cx)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{A\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}}$$

output
```
2*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)
-A*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{3/2}} \, dx = \frac{2\left(\frac{\sqrt{a}(aB(b+2cx)-A(b^2-2ac+bcx))}{\sqrt{a+x(b+cx)}} - A(b^2-4ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{a^{3/2}\left(-b^2+4ac\right)}$$

input
```
Integrate[(A + B*x)/(x*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*((Sqrt[a]*(a*B*(b + 2*c*x) - A*(b^2 - 2*a*c + b*c*x)))/Sqrt[a + x*(b +
c*x)] - A*(b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a
]]))/(a^(3/2)*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1235$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{A\left(b^2-4ac\right)}{2x\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{A\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{a}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1154$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2A\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}$$

$$\Big\downarrow \; 219$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{A\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}}$$

input
```
Int[(A + B*x)/(x*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a
+ b*x + c*x^2]) - (A*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])
])/a^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{2B(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+A\left(\frac{1}{a\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{a(4ac-b^2)\sqrt{c\,x^2+bx+a}}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{a^{\frac{3}{2}}}\right)$	124

input
```
int((B*x+A)/x/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*B*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+A*(1/a/(c*x^2+b*x+a)^(1/2)-b
/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(87) = 174$.

Time = 0.20 (sec) , antiderivative size = 412, normalized size of antiderivative = 4.29

$$\int \frac{A+Bx}{x\,(a+bx+cx^2)^{3/2}}\,dx = \left[\frac{(Aab^2-4\,Aa^2c+(Ab^2c-4\,Aac^2)x^2+(Ab^3-4\,Aabc)x)\sqrt{a}\log\left(-\frac{8\,abx+(b^2+\cdots}{\cdots}\right.}{2\,(a^3b^2-4\,a^4c+(\cdots}\right.$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((A*a*b^2 - 4*A*a^2*c + (A*b^2*c - 4*A*a*c^2)*x^2 + (A*b^3 - 4*A*a*b*
c)*x)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(B*a^2*b - A*a*b^2 + 2*A*a^2*c + (2*
B*a^2 - A*a*b)*c*x)*sqrt(c*x^2 + b*x + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c
 - 4*a^3*c^2)*x^2 + (a^2*b^3 - 4*a^3*b*c)*x), ((A*a*b^2 - 4*A*a^2*c + (A*b
^2*c - 4*A*a*c^2)*x^2 + (A*b^3 - 4*A*a*b*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*
x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(B*a^2*b
- A*a*b^2 + 2*A*a^2*c + (2*B*a^2 - A*a*b)*c*x)*sqrt(c*x^2 + b*x + a))/(a^3
*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 + (a^2*b^3 - 4*a^3*b*c)*x)]
```

Sympy [F]

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((A + B*x)/(x*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{3/2}}\,dx=-\frac{2\left(\frac{(2\,Ba^2c-Aabc)x}{a^2b^2-4\,a^3c}+\frac{Ba^2b-Aab^2+2\,Aa^2c}{a^2b^2-4\,a^3c}\right)}{\sqrt{cx^2+bx+a}}+\frac{2\,A\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*B*a^2*c - A*a*b*c)*x/(a^2*b^2 - 4*a^3*c) + (B*a^2*b - A*a*b^2 + 2*A
*a^2*c)/(a^2*b^2 - 4*a^3*c))/sqrt(c*x^2 + b*x + a) + 2*A*arctan(-(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{x\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((A + B*x)/(x*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 2230, normalized size of antiderivative = 23.23

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)/x/(c*x^2+b*x+a)^(3/2),x)
```

output

```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*a*b - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*
sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**
2))*b**2*x - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sq
rt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c
 - b**2))*b*c*x**2 - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*at
an((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b
 - 4*a*c - b**2))*a**2 - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2
)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(
a)*b - 4*a*c - b**2))*a*b*x - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c -
 b**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*
sqrt(a)*b - 4*a*c - b**2))*a*c*x**2 - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4
*a*c + b**2)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*s
qrt(a + b*x + c*x**2) + b + 2*c*x)*a*b - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b
+ 4*a*c + b**2)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b**2*x - sqrt(a)*sqrt(4*sqrt(c)*sqrt
(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*
sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b*c*x**2 + sqrt(a)*sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b*...
```

3.159 $\int \frac{A+Bx}{x^2(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{A+Bx}{x^2(a+bx+cx^2)^{3/2}}\,dx = -\frac{A}{ax\sqrt{a+bx+cx^2}} + \frac{2aB(b^2-2ac)-A(3b^3-10abc)-c(3Ab^2-2abB-8aAc)\,x}{a^2(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{(3Ab-2aB)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}}$$

output
```
-A/a/x/(c*x^2+b*x+a)^(1/2)+(2*a*B*(-2*a*c+b^2)-A*(-10*a*b*c+3*b^3)-c*(-8*A
*a*c+3*A*b^2-2*B*a*b)*x)/a^2/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+1/2*(3*A*b-2
*B*a)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{\frac{\sqrt{a}\left(-4a^2c(A-Bx)+3Ab^2x(b+cx)-2abBx(b+cx)+aA\left(b^2-10bcx-8c^2x^2\right)\right)}{x\sqrt{a+x(b+cx)}}+(3Ab-2aB)\left(b^2-\right.}{a^{5/2}\left(-b^2+4ac\right)}$$

input
```
Integrate[(A + B*x)/(x^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
((Sqrt[a]*(-4*a^2*c*(A - B*x) + 3*A*b^2*x*(b + c*x) - 2*a*b*B*x*(b + c*x)
+ a*A*(b^2 - 10*b*c*x - 8*c^2*x^2)))/(x*Sqrt[a + x*(b + c*x)]) + (3*A*b -
2*a*B)*(b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])
/(a^(5/2)*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{3Ab^2-2aBb-8aAc+2(Ab-2aB)cx}{2x^2\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3Ab^2-2aBb-8aAc+2(Ab-2aB)cx}{x^2\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{-\frac{(b^2-4ac)(3Ab-2aB)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}(-8aAc-2abB+3Ab^2)}{ax}}{a(b^2-4ac)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{(b^2-4ac)(3Ab-2aB)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}(-8aAc-2abB+3Ab^2)}{ax}}{a(b^2-4ac)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(b^2-4ac)(3Ab-2aB)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx+cx^2}(-8aAc-2abB+3Ab^2)}{ax}}{a(b^2-4ac)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[(A + B*x)/(x^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[
a + b*x + c*x^2]) + (-(((3*A*b^2 - 2*a*b*B - 8*a*A*c)*Sqrt[a + b*x + c*x^2
])/(a*x)) + ((3*A*b - 2*a*B)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.63

method	result
default	$A\left(-\frac{1}{ax\sqrt{c\,x^2+bx+a}}-\frac{3b\left(\frac{1}{a\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{a\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}-\frac{4c(2cx+b)}{a(4ac-b^2)\sqrt{c\,x^2+bx+}}\right.$
risch	$-\frac{A\sqrt{c\,x^2+bx+a}}{a^2x}-\frac{\frac{2b^2A(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+a(3Ab-2Ba)\left(\frac{1}{a\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{a\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{a^{\frac{3}{2}}}\right.}{2a^2}$

input
```
int((B*x+A)/x^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/a/x/(c*x^2+b*x+a)^(1/2)-3/2*b/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+
b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+
b*x+a)^(1/2))/x))-4*c/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+B*(1/a/
(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2
)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(134) = 268.

Time = 0.30 (sec) , antiderivative size = 657, normalized size of antiderivative = 4.41

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \left[\frac{((4\,(2\,Ba^2-3\,Aab)c^2-(2\,Bab^2-3\,Ab^3)c)x^3-(2\,Bab^3-3\,Ab^4-4\,(2\,Ba^2}{}\right.$$

$$-\frac{((4\,(2\,Ba^2-3\,Aab)c^2-(2\,Bab^2-3\,Ab^3)c)x^3-(2\,Bab^3-3\,Ab^4-4\,(2\,Ba^2b-3\,Aab^2)c)x^2-(2\,Ba^2b^2}{}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((4*(2*B*a^2 - 3*A*a*b)*c^2 - (2*B*a*b^2 - 3*A*b^3)*c)*x^3 - (2*B*a*
b^3 - 3*A*b^4 - 4*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 - (2*B*a^2*b^2 - 3*A*a*b^
3 - 4*(2*B*a^3 - 3*A*a^2*b)*c)*x)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^
2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(A*a^2*b
^2 - 4*A*a^3*c - (8*A*a^2*c^2 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 - (2*B*a^2*
b^2 - 3*A*a*b^3 - 2*(2*B*a^3 - 5*A*a^2*b)*c)*x)*sqrt(c*x^2 + b*x + a))/((a
^3*b^2*c - 4*a^4*c^2)*x^3 + (a^3*b^3 - 4*a^4*b*c)*x^2 + (a^4*b^2 - 4*a^5*c
)*x), -1/2*(((4*(2*B*a^2 - 3*A*a*b)*c^2 - (2*B*a*b^2 - 3*A*b^3)*c)*x^3 - (
2*B*a*b^3 - 3*A*b^4 - 4*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 - (2*B*a^2*b^2 - 3*
A*a*b^3 - 4*(2*B*a^3 - 3*A*a^2*b)*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(A*a^2*b^2 - 4*A
*a^3*c - (8*A*a^2*c^2 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 - (2*B*a^2*b^2 - 3*
A*a*b^3 - 2*(2*B*a^3 - 5*A*a^2*b)*c)*x)*sqrt(c*x^2 + b*x + a))/((a^3*b^2*c
 - 4*a^4*c^2)*x^3 + (a^3*b^3 - 4*a^4*b*c)*x^2 + (a^4*b^2 - 4*a^5*c)*x)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**2/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**2*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(\frac{\left(Ba^3bc-Aa^2b^2c+2\,Aa^3c^2\right)x}{a^4b^2-4\,a^5c}+\frac{Ba^3b^2-Aa^2b^3-2\,Ba^4c+3\,Aa^3bc}{a^4b^2-4\,a^5c}\right)}{\sqrt{cx^2+bx+a}} + \frac{\left(2\,Ba-3\,Ab\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)Ab+2\,Aa\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)a^2}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*((B*a^3*b*c - A*a^2*b^2*c + 2*A*a^3*c^2)*x/(a^4*b^2 - 4*a^5*c) + (B*a^3*
b^2 - A*a^2*b^3 - 2*B*a^4*c + 3*A*a^3*b*c)/(a^4*b^2 - 4*a^5*c))/sqrt(c*x^2
 + b*x + a) + (2*B*a - 3*A*b)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/
sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*b + 2*A*
a*sqrt(c))/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{A+B\,x}{x^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^2*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((A + B*x)/(x^2*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.03

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{-8\sqrt{c\,x^2+bx+a}\,a^3c+2\sqrt{c\,x^2+bx+a}\,a^2b^2-12\sqrt{c\,x^2+bx+a}\,a^2bcx-16}{}$$

input
```
int((B*x+A)/x^2/(c*x^2+b*x+a)^(3/2),x)
```

output
```
( - 8*sqrt(a + b*x + c*x**2)*a**3*c + 2*sqrt(a + b*x + c*x**2)*a**2*b**2 -
 12*sqrt(a + b*x + c*x**2)*a**2*b*c*x - 16*sqrt(a + b*x + c*x**2)*a**2*c**
2*x**2 + 2*sqrt(a + b*x + c*x**2)*a*b**3*x + 2*sqrt(a + b*x + c*x**2)*a*b*
*2*c*x**2 + 4*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)
*a**2*b*c*x - sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)
*a*b**3*x + 4*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)
*a*b**2*c*x**2 + 4*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a -
 b*x)*a*b*c**2*x**3 - sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*
a - b*x)*b**4*x**2 - sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
 - b*x)*b**3*c*x**3 - 4*sqrt(a)*log(x)*a**2*b*c*x + sqrt(a)*log(x)*a*b**3*
x - 4*sqrt(a)*log(x)*a*b**2*c*x**2 - 4*sqrt(a)*log(x)*a*b*c**2*x**3 + sqrt
(a)*log(x)*b**4*x**2 + sqrt(a)*log(x)*b**3*c*x**3)/(2*a**2*x*(4*a**2*c - a
*b**2 + 4*a*b*c*x + 4*a*c**2*x**2 - b**3*x - b**2*c*x**2))
```

3.160 $\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{A}{2ax^2\sqrt{a+bx+cx^2}}+\frac{5Ab-4aB}{4a^2x\sqrt{a+bx+cx^2}} -\frac{4abB(3b^2-10ac)-A(15b^4-62ab^2c+24a^2c^2)+c(4aB(3b^2-8ac)-A(15b^3-52abc))\,x}{4a^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} -\frac{3(5Ab^2-4abB-4aAc)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{7/2}}$$

output
```
-1/2*A/a/x^2/(c*x^2+b*x+a)^(1/2)+1/4*(5*A*b-4*B*a)/a^2/x/(c*x^2+b*x+a)^(1/
2)-1/4*(4*a*b*B*(-10*a*c+3*b^2)-A*(24*a^2*c^2-62*a*b^2*c+15*b^4)+c*(4*a*B*
(-8*a*c+3*b^2)-A*(-52*a*b*c+15*b^3))*x)/a^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/
2)-3/8*(-4*A*a*c+5*A*b^2-4*B*a*b)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x
+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{\frac{\sqrt{a}\left(-8a^3c(A+2Bx)-15Ab^3x^2(b+cx)+2a^2\left(A\left(b^2+10bcx-12c^2x^2\right)+2Bx\left(b^2-10bcx-8c^2x^2\right)\right)+abx(12bB\right.}{x^2\sqrt{a+x(b+cx)}}}{4a}$$

input
```
Integrate[(A + B*x)/(x^3*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
((Sqrt[a]*(-8*a^3*c*(A + 2*B*x) - 15*A*b^3*x^2*(b + c*x) + 2*a^2*(A*(b^2 +
 10*b*c*x - 12*c^2*x^2) + 2*B*x*(b^2 - 10*b*c*x - 8*c^2*x^2)) + a*b*x*(12*
b*B*x*(b + c*x) + A*(-5*b^2 + 62*b*c*x + 52*c^2*x^2))))/(x^2*Sqrt[a + x*(b
 + c*x)]) - 3*(b^2 - 4*a*c)*(5*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(Sqrt[c]
*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(4*a^(7/2)*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1235, 27, 1237, 27, 25, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1235$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{5Ab^2-4aBb-12aAc+4(Ab-2aB)cx}{2x^3\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5Ab^2-4aBb-12aAc+4(Ab-2aB)cx}{x^3\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1237$$

$$\frac{-\frac{\int -\frac{4aB\left(3b^2-8ac\right)-2A\left(\frac{15b^3}{2}-26abc\right)-2c\left(5Ab^2-4aBb-12aAc\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}(-12aAc-4abB+5Ab^2)}{2ax^2}}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int -\frac{15Ab^3-12aBb^2-52aAcb+32a^2Bc+2c\left(5Ab^2-4aBb-12aAc\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}(-12aAc-4abB+5Ab^2)}{2ax^2}}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{15Ab^3-12aBb^2-52aAcb+32a^2Bc+2c\left(5Ab^2-4aBb-12aAc\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}(-12aAc-4abB+5Ab^2)}{2ax^2}}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1228$$

$$\frac{-\frac{\frac{\sqrt{a+bx+cx^2}\left(4aB\left(3b^2-8ac\right)-A\left(15b^3-52abc\right)\right)}{ax}-\frac{3\left(b^2-4ac\right)\left(-4aAc-4abB+5Ab^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}}{4a}-\frac{\sqrt{a+bx+cx^2}(-12aAc-4abB+5Ab^2)}{2ax^2}}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1154$$

$$\frac{-\frac{\frac{3\left(b^2-4ac\right)\left(-4aAc-4abB+5Ab^2\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}+\frac{\sqrt{a+bx+cx^2}\left(4aB\left(3b^2-8ac\right)-A\left(15b^3-52abc\right)\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}(-12aAc-4abB}{2ax^2}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{\frac{3(b^2-4ac)(-4aAc-4abB+5Ab^2)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx+cx^2}\left(4aB(3b^2-8ac)-A(15b^3-52abc)\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}(-12aAc-4abB+}{2ax^2}}{a(b^2-4ac)}$$

$$\frac{2(cx(Ab-2aB)-2aAc-abB+Ab^2)}{ax^2(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[(A + B*x)/(x^3*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqr
t[a + b*x + c*x^2]) + (-1/2*((5*A*b^2 - 4*a*b*B - 12*a*A*c)*Sqrt[a + b*x +
 c*x^2])/(a*x^2) - (((4*a*B*(3*b^2 - 8*a*c) - A*(15*b^3 - 52*a*b*c))*Sqrt[
a + b*x + c*x^2])/(a*x) + (3*(b^2 - 4*a*c)*(5*A*b^2 - 4*a*b*B - 4*a*A*c)*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)))/(4*a))
/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.68

method	result
risch	$-\frac{\sqrt{c\,x^{2}+bx+a}\,(-7Abx+4Bax+2Aa)}{4a^{3}x^{2}}-\frac{c(4Aac-7b^{2}A+4abB)\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}\right)-\frac{14A\,b^{3}(2cx+b)}{\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+}}}{}$
default	$A\left(-\frac{1}{2a\,x^{2}\sqrt{c\,x^{2}+bx+a}}-\frac{5b\left(-\frac{1}{ax\sqrt{c\,x^{2}+bx+a}}-\frac{3b\left(\frac{1}{a\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{a\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}-\right.}{4a}\right.$

input
```
int((B*x+A)/x^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(-7*A*b*x+4*B*a*x+2*A*a)/a^3/x^2-1/8/a^3*(c*(4*A*
a*c-7*A*b^2+4*B*a*b)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2))-14*A*b^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+8*B
*a*b^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+16*B*a^2*c*(2*c*x+b)/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2)+3*a*(4*A*a*c-5*A*b^2+4*B*a*b)*(1/a/(c*x^2+b*x+
a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b
*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-8*A*a*b*c*(2*c*x+b)/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(197) = 394$.

Time = 0.46 (sec) , antiderivative size = 869, normalized size of antiderivative = 3.97

$$\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*((16*A*a^2*c^3 + 8*(2*B*a^2*b - 3*A*a*b^2)*c^2 - (4*B*a*b^3 - 5*
A*b^4)*c)*x^4 - (4*B*a*b^4 - 5*A*b^5 - 16*A*a^2*b*c^2 - 8*(2*B*a^2*b^2 - 3
*A*a*b^3)*c)*x^3 - (4*B*a^2*b^3 - 5*A*a*b^4 - 16*A*a^3*c^2 - 8*(2*B*a^3*b
- 3*A*a^2*b^2)*c)*x^2)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(
c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(2*A*a^3*b^2 - 8*A*
a^4*c - (4*(8*B*a^3 - 13*A*a^2*b)*c^2 - 3*(4*B*a^2*b^2 - 5*A*a*b^3)*c)*x^3
 + (12*B*a^2*b^3 - 15*A*a*b^4 - 24*A*a^3*c^2 - 2*(20*B*a^3*b - 31*A*a^2*b^
2)*c)*x^2 + (4*B*a^3*b^2 - 5*A*a^2*b^3 - 4*(4*B*a^4 - 5*A*a^3*b)*c)*x)*sqr
t(c*x^2 + b*x + a))/((a^4*b^2*c - 4*a^5*c^2)*x^4 + (a^4*b^3 - 4*a^5*b*c)*x
^3 + (a^5*b^2 - 4*a^6*c)*x^2), 1/8*(3*((16*A*a^2*c^3 + 8*(2*B*a^2*b - 3*A*
a*b^2)*c^2 - (4*B*a*b^3 - 5*A*b^4)*c)*x^4 - (4*B*a*b^4 - 5*A*b^5 - 16*A*a^
2*b*c^2 - 8*(2*B*a^2*b^2 - 3*A*a*b^3)*c)*x^3 - (4*B*a^2*b^3 - 5*A*a*b^4 -
16*A*a^3*c^2 - 8*(2*B*a^3*b - 3*A*a^2*b^2)*c)*x^2)*sqrt(-a)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*A*
a^3*b^2 - 8*A*a^4*c - (4*(8*B*a^3 - 13*A*a^2*b)*c^2 - 3*(4*B*a^2*b^2 - 5*A
*a*b^3)*c)*x^3 + (12*B*a^2*b^3 - 15*A*a*b^4 - 24*A*a^3*c^2 - 2*(20*B*a^3*b
 - 31*A*a^2*b^2)*c)*x^2 + (4*B*a^3*b^2 - 5*A*a^2*b^3 - 4*(4*B*a^4 - 5*A*a^
3*b)*c)*x)*sqrt(c*x^2 + b*x + a))/((a^4*b^2*c - 4*a^5*c^2)*x^4 + (a^4*b^3
- 4*a^5*b*c)*x^3 + (a^5*b^2 - 4*a^6*c)*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^3 (a+bx+cx^2)^{3/2}}\,dx=\int \frac{A+Bx}{x^3 (a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**3/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**3*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x^3(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(197) = 394$.

Time = 0.25 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx}{x^3(a+bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{(Ba^4b^2c-Aa^3b^3c-2\,Ba^5c^2+3\,Aa^4bc^2)x}{a^6b^2-4\,a^7c}+\frac{Ba^4b^3-Aa^3b^4-3\,Ba^5bc+4\,Aa^4b^2c-2\,Aa^5c^2}{a^6b^2-4\,a^7c}\right)}{\sqrt{cx^2+bx+a}} - \frac{3\,(4\,Bab-5\,Ab^2+4\,Aac)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3} + \frac{4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Bab-7\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Ab^2+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Aac+8\,(\cdots}{\cdots}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((B*a^4*b^2*c - A*a^3*b^3*c - 2*B*a^5*c^2 + 3*A*a^4*b*c^2)*x/(a^6*b^2 -
 4*a^7*c) + (B*a^4*b^3 - A*a^3*b^4 - 3*B*a^5*b*c + 4*A*a^4*b^2*c - 2*A*a^5
*c^2)/(a^6*b^2 - 4*a^7*c))/sqrt(c*x^2 + b*x + a) - 3/4*(4*B*a*b - 5*A*b^2
+ 4*A*a*c)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)
*a^3) + 1/4*(4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a*b - 7*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^3*A*b^2 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*A*a*c + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B*a^2*sqrt(c) - 8*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*A*a*b*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*B*a^2*b + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a*b^2 + 4*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*c - 8*B*a^3*sqrt(c) + 16*A*a^2*b*sq
rt(c))/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^2*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^3 (a+bx+cx^2)^{3/2}}\,dx=\int \frac{A+B\,x}{x^3\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^3*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((A + B*x)/(x^3*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 696, normalized size of antiderivative = 3.18

$$\int \frac{A+Bx}{x^3 (a+bx+cx^2)^{3/2}}\,dx=\frac{-16\sqrt{c\,x^2+bx+a}\,a^4c+4\sqrt{c\,x^2+bx+a}\,a^3b^2-48\sqrt{c\,x^2+bx+a}\,a^3c^2x^2-$$

input
```
int((B*x+A)/x^3/(c*x^2+b*x+a)^(3/2),x)
```

output

```
( - 16*sqrt(a + b*x + c*x**2)*a**4*c + 4*sqrt(a + b*x + c*x**2)*a**3*b**2
+ 8*sqrt(a + b*x + c*x**2)*a**3*b*c*x - 48*sqrt(a + b*x + c*x**2)*a**3*c**
2*x**2 - 2*sqrt(a + b*x + c*x**2)*a**2*b**3*x + 44*sqrt(a + b*x + c*x**2)*
a**2*b**2*c*x**2 + 40*sqrt(a + b*x + c*x**2)*a**2*b*c**2*x**3 - 6*sqrt(a +
 b*x + c*x**2)*a*b**4*x**2 - 6*sqrt(a + b*x + c*x**2)*a*b**3*c*x**3 + 48*s
qrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*c**2*x**2
 - 24*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b*
*2*c*x**2 + 48*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x
)*a**2*b*c**2*x**3 + 48*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) -
2*a - b*x)*a**2*c**3*x**4 + 3*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x*
*2) - 2*a - b*x)*a*b**4*x**2 - 24*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x +
c*x**2) - 2*a - b*x)*a*b**3*c*x**3 - 24*sqrt(a)*log( - 2*sqrt(a)*sqrt(a +
b*x + c*x**2) - 2*a - b*x)*a*b**2*c**2*x**4 + 3*sqrt(a)*log( - 2*sqrt(a)*s
qrt(a + b*x + c*x**2) - 2*a - b*x)*b**5*x**3 + 3*sqrt(a)*log( - 2*sqrt(a)*
sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**4*c*x**4 - 48*sqrt(a)*log(x)*a**3*c
**2*x**2 + 24*sqrt(a)*log(x)*a**2*b**2*c*x**2 - 48*sqrt(a)*log(x)*a**2*b*c
**2*x**3 - 48*sqrt(a)*log(x)*a**2*c**3*x**4 - 3*sqrt(a)*log(x)*a*b**4*x**2
 + 24*sqrt(a)*log(x)*a*b**3*c*x**3 + 24*sqrt(a)*log(x)*a*b**2*c**2*x**4 -
3*sqrt(a)*log(x)*b**5*x**3 - 3*sqrt(a)*log(x)*b**4*c*x**4)/(8*a**3*x**2*(4
*a**2*c - a*b**2 + 4*a*b*c*x + 4*a*c**2*x**2 - b**3*x - b**2*c*x**2))
```

3.161 $\int \frac{A+Bx}{x^4(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 292

$$\int \frac{A+Bx}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{A}{3ax^3\sqrt{a+bx+cx^2}}$$
$$+\frac{7Ab-6aB}{12a^2x^2\sqrt{a+bx+cx^2}}-\frac{35Ab^2-30abB-32aAc}{24a^3x\sqrt{a+bx+cx^2}}$$
$$+\frac{6aB(15b^4-62ab^2c+24a^2c^2)-A(105b^5-530ab^3c+488a^2bc^2)+c(6abB(15b^2-52ac)-A(105b^4-46}{24a^4\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$
$$+\frac{(35Ab^3-30ab^2B-60aAbc+24a^2Bc)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{9/2}}$$

output

```
-1/3*A/a/x^3/(c*x^2+b*x+a)^(1/2)+1/12*(7*A*b-6*B*a)/a^2/x^2/(c*x^2+b*x+a)^
(1/2)-1/24*(-32*A*a*c+35*A*b^2-30*B*a*b)/a^3/x/(c*x^2+b*x+a)^(1/2)+1/24*(6
*a*B*(24*a^2*c^2-62*a*b^2*c+15*b^4)-A*(488*a^2*b*c^2-530*a*b^3*c+105*b^5)+
c*(6*a*b*B*(-52*a*c+15*b^2)-A*(256*a^2*c^2-460*a*b^2*c+105*b^4))*x)/a^4/(-
4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+1/16*(-60*A*a*b*c+35*A*b^3+24*B*a^2*c-30*B*
a*b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx=\frac{\sqrt{a}\left(-16a^4c(2A+3Bx)+105Ab^4x^3(b+cx)+4a^3\left(3Bx\left(b^2+10bcx-12c^2x^2\right)+2A\left(b^2+7bcx+16c^2x^2\right)\right)-5ab^2x\right.}{}$$

input
```
Integrate[(A + B*x)/(x^4*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
((Sqrt[a]*(-16*a^4*c*(2*A + 3*B*x) + 105*A*b^4*x^3*(b + c*x) + 4*a^3*(3*B*
x*(b^2 + 10*b*c*x - 12*c^2*x^2) + 2*A*(b^2 + 7*b*c*x + 16*c^2*x^2)) - 5*a*
b^2*x^2*(18*b*B*x*(b + c*x) + A*(-7*b^2 + 106*b*c*x + 92*c^2*x^2)) + 2*a^2
*x*(3*b*B*x*(-5*b^2 + 62*b*c*x + 52*c^2*x^2) + A*(-7*b^3 - 86*b^2*c*x + 24
4*b*c^2*x^2 + 128*c^3*x^3))))/(x^3*Sqrt[a + x*(b + c*x)]) + 3*(b^2 - 4*a*c
)*(6*a*B*(-5*b^2 + 4*a*c) + 5*A*(7*b^3 - 12*a*b*c))*ArcTanh[(Sqrt[c]*x - S
qrt[a + x*(b + c*x)])/Sqrt[a]])/(24*a^(9/2)*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {1235, 27, 1237, 27, 25, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{7Ab^2-6aBb-16aAc+6(Ab-2aB)cx}{2x^4\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7Ab^2-6aBb-16aAc+6(Ab-2aB)cx}{x^4\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1237$$

$$\frac{-\frac{\int -\frac{6aB\left(5b^2-12ac\right)-2A\left(\frac{35b^3}{2}-58abc\right)-4c\left(7Ab^2-6aBb-16aAc\right)x}{2x^3\sqrt{cx^2+bx+a}}dx}{3a}-\frac{\sqrt{a+bx+cx^2}\left(-16aAc-6abB+7Ab^2\right)}{3ax^3}}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int -\frac{35Ab^3-30aBb^2-116aAcb+72a^2Bc+4c\left(7Ab^2-6aBb-16aAc\right)x}{x^3\sqrt{cx^2+bx+a}}dx}{6a}-\frac{\sqrt{a+bx+cx^2}\left(-16aAc-6abB+7Ab^2\right)}{3ax^3}}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{35Ab^3-30aBb^2-116aAcb+72a^2Bc+4c\left(7Ab^2-6aBb-16aAc\right)x}{x^3\sqrt{cx^2+bx+a}}dx}{6a}-\frac{\sqrt{a+bx+cx^2}\left(-16aAc-6abB+7Ab^2\right)}{3ax^3}}{a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1237$$

$$\frac{-\frac{-\frac{\int -\frac{6abB\left(15b^2-52ac\right)-2A\left(\frac{105b^4}{2}-230acb^2+128a^2c^2\right)-2c\left(35Ab^3-30aBb^2-116aAcb+72a^2Bc\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(72a^2Bc-116aAbc-30ab^2B+35Ab^3\right)}{2ax^2}\cdots}{6a}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\frac{\int \frac{6abB\left(15b^2-52ac\right)-A\left(105b^4-460acb^2+256a^2c^2\right)-2c\left(35Ab^3-30aBb^2-116aAcb+72a^2Bc\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}\left(72a^2Bc-116aAbc-30ab^2B+35Ab^3\right)}{2ax^2}}{6a}-\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ax^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1228$$

$$-\frac{-\frac{-\frac{3(b^2-4ac)(6aB(5b^2-4ac)-5A(7b^3-12abc))\int\frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}(6abB(15b^2-52ac)-A(256a^2c^2-460ab^2c+105b^4))}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}(72a}{}}{6a}}{a(b^2-4ac)}$$

$$\frac{2(cx(Ab-2aB)-2aAc-abB+Ab^2)}{ax^3(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{\frac{3(b^2-4ac)(6aB(5b^2-4ac)-5A(7b^3-12abc))\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}(6abB(15b^2-52ac)-A(256a^2c^2-460ab^2c+105b^4))}{ax}}{4a}-\frac{\sqrt{a+b}}{}}{6a}}{a(b^2-4ac)}$$

$$\frac{2(cx(Ab-2aB)-2aAc-abB+Ab^2)}{ax^3(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\frac{3(b^2-4ac)(6aB(5b^2-4ac)-5A(7b^3-12abc))\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx+cx^2}(6abB(15b^2-52ac)-A(256a^2c^2-460ab^2c+105b^4))}{ax}}{4a}-\frac{\sqrt{a+b}}{}}{6a}}{a(b^2-4ac)}$$

$$\frac{2(cx(Ab-2aB)-2aAc-abB+Ab^2)}{ax^3(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input

```
Int[(A + B*x)/(x^4*(a + b*x + c*x^2)^(3/2)),x]
```

output

```
(2*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqr
t[a + b*x + c*x^2]) + (-1/3*((7*A*b^2 - 6*a*b*B - 16*a*A*c)*Sqrt[a + b*x +
 c*x^2])/(a*x^3) - (-1/2*((35*A*b^3 - 30*a*b^2*B - 116*a*A*b*c + 72*a^2*B*
c)*Sqrt[a + b*x + c*x^2])/(a*x^2) + (-(((6*a*b*B*(15*b^2 - 52*a*c) - A*(10
5*b^4 - 460*a*b^2*c + 256*a^2*c^2))*Sqrt[a + b*x + c*x^2])/(a*x)) + (3*(b^
2 - 4*a*c)*(6*a*B*(5*b^2 - 4*a*c) - 5*A*(7*b^3 - 12*a*b*c))*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)))/(4*a))/(6*a))/(a*(b
^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{\sqrt{c x^2+bx+a}\,(-40Aac\,x^2+57x^2b^2A-42Ba\,x^2b-22abAx+12a^2Bx+8a^2A)}{24a^4x^3}+\frac{c(28Aabc-19A\,b^3-8B\,a^2c+14Ba\,b^2)\left(-\frac{\ldots}{c\sqrt{c\,x^2\ldots}}\right.}{\ldots}$
default	$A\left(-\frac{1}{3a\,x^3\sqrt{c\,x^2+bx+a}}-\frac{7b\left(-\frac{1}{2a\,x^2\sqrt{c\,x^2+bx+a}}-\frac{5b\left(-\frac{1}{ax\sqrt{c\,x^2+bx+a}}-\frac{3b\left(\frac{1}{a\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{a\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{\ln\left(\frac{2a+b\ldots}{\ldots}\right)}{\ldots}\right)}{2a}\ldots}{4a}\ldots\right)}{\ldots}\ldots\right.$

input
```
int((B*x+A)/x^4/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+b*x+a)^(1/2)*(-40*A*a*c*x^2+57*A*b^2*x^2-42*B*a*b*x^2-22*A*a*
b*x+12*B*a^2*x+8*A*a^2)/a^4/x^3+1/16/a^4*(c*(28*A*a*b*c-19*A*b^3-8*B*a^2*c
+14*B*a*b^2)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*
x+a)^(1/2))+a*(60*A*a*b*c-35*A*b^3-24*B*a^2*c+30*B*a*b^2)*(1/a/(c*x^2+b*x+
a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b
*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-38*A*b^4*(2*c*x+b)/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)+28*B*a*b^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+32*a^2
*A*c^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+24*A*a*b^2*c*(2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+16*a^2*b*B*c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*
x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(266) = 532$.

Time = 0.97 (sec) , antiderivative size = 1093, normalized size of antiderivative = 3.74

$$\int \frac{A+Bx}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(3*((48*(2*B*a^3 - 5*A*a^2*b)*c^3 - 8*(18*B*a^2*b^2 - 25*A*a*b^3)*c^
2 + 5*(6*B*a*b^4 - 7*A*b^5)*c)*x^5 + (30*B*a*b^5 - 35*A*b^6 + 48*(2*B*a^3*
b - 5*A*a^2*b^2)*c^2 - 8*(18*B*a^2*b^3 - 25*A*a*b^4)*c)*x^4 + (30*B*a^2*b^
4 - 35*A*a*b^5 + 48*(2*B*a^4 - 5*A*a^3*b)*c^2 - 8*(18*B*a^3*b^2 - 25*A*a^2
*b^3)*c)*x^3)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b
*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(8*A*a^4*b^2 - 32*A*a^5*c +
(256*A*a^3*c^3 + 4*(78*B*a^3*b - 115*A*a^2*b^2)*c^2 - 15*(6*B*a^2*b^3 - 7*
A*a*b^4)*c)*x^4 - (90*B*a^2*b^4 - 105*A*a*b^5 + 8*(18*B*a^4 - 61*A*a^3*b)*
c^2 - 2*(186*B*a^3*b^2 - 265*A*a^2*b^3)*c)*x^3 - (30*B*a^3*b^3 - 35*A*a^2*
b^4 - 128*A*a^4*c^2 - 4*(30*B*a^4*b - 43*A*a^3*b^2)*c)*x^2 + 2*(6*B*a^4*b^
2 - 7*A*a^3*b^3 - 4*(6*B*a^5 - 7*A*a^4*b)*c)*x)*sqrt(c*x^2 + b*x + a))/((a
^5*b^2*c - 4*a^6*c^2)*x^5 + (a^5*b^3 - 4*a^6*b*c)*x^4 + (a^6*b^2 - 4*a^7*c
)*x^3), 1/48*(3*((48*(2*B*a^3 - 5*A*a^2*b)*c^3 - 8*(18*B*a^2*b^2 - 25*A*a*
b^3)*c^2 + 5*(6*B*a*b^4 - 7*A*b^5)*c)*x^5 + (30*B*a*b^5 - 35*A*b^6 + 48*(2
*B*a^3*b - 5*A*a^2*b^2)*c^2 - 8*(18*B*a^2*b^3 - 25*A*a*b^4)*c)*x^4 + (30*B
*a^2*b^4 - 35*A*a*b^5 + 48*(2*B*a^4 - 5*A*a^3*b)*c^2 - 8*(18*B*a^3*b^2 - 2
5*A*a^2*b^3)*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)
*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(8*A*a^4*b^2 - 32*A*a^5*c + (256*A*
a^3*c^3 + 4*(78*B*a^3*b - 115*A*a^2*b^2)*c^2 - 15*(6*B*a^2*b^3 - 7*A*a*b^4
)*c)*x^4 - (90*B*a^2*b^4 - 105*A*a*b^5 + 8*(18*B*a^4 - 61*A*a^3*b)*c^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^4 (a+bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**4/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(266) = 532$.

Time = 0.25 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.73

$$\int \frac{A+Bx}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
2*((B*a^5*b^3*c - A*a^4*b^4*c - 3*B*a^6*b*c^2 + 4*A*a^5*b^2*c^2 - 2*A*a^6*
c^3)*x/(a^8*b^2 - 4*a^9*c) + (B*a^5*b^4 - A*a^4*b^5 - 4*B*a^6*b^2*c + 5*A*
a^5*b^3*c + 2*B*a^7*c^2 - 5*A*a^6*b*c^2)/(a^8*b^2 - 4*a^9*c))/sqrt(c*x^2 +
 b*x + a) + 1/8*(30*B*a*b^2 - 35*A*b^3 - 24*B*a^2*c + 60*A*a*b*c)*arctan(-
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/24*(42*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a*b^2 - 57*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^5*A*b^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*B*a^2*c + 84
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*A*a*b*c + 48*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^4*B*a^2*b*sqrt(c) - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*A*a*b^2*sqrt(c) + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*A*a^2*c^(3/2)
- 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a^2*b^2 + 136*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*A*a*b^3 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*A*a^2*b*c - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B*a^3*b*sqrt(c) + 1
44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*A*a^2*b^2*sqrt(c) - 192*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*A*a^3*c^(3/2) + 54*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*B*a^3*b^2 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*b^3 + 2
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^4*c - 36*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*A*a^3*b*c + 96*B*a^4*b*sqrt(c) - 144*A*a^3*b^2*sqrt(c) + 80*A
*a^4*c^(3/2))/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^3*a^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{x^4\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^4*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((A + B*x)/(x^4*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 796, normalized size of antiderivative = 2.73

$$\int \frac{A+Bx}{x^4 \left(a+bx+cx^2\right)^{3/2}} \, dx = \text{Too large to display}$$

input

```
int((B*x+A)/x^4/(c*x^2+b*x+a)^(3/2),x)
```

output

```
( - 64*sqrt(a + b*x + c*x**2)*a**5*c + 16*sqrt(a + b*x + c*x**2)*a**4*b**2
 + 16*sqrt(a + b*x + c*x**2)*a**4*b*c*x + 256*sqrt(a + b*x + c*x**2)*a**4*
c**2*x**2 - 4*sqrt(a + b*x + c*x**2)*a**3*b**3*x - 104*sqrt(a + b*x + c*x*
*2)*a**3*b**2*c*x**2 + 688*sqrt(a + b*x + c*x**2)*a**3*b*c**2*x**3 + 512*s
qrt(a + b*x + c*x**2)*a**3*c**3*x**4 + 10*sqrt(a + b*x + c*x**2)*a**2*b**4
*x**2 - 316*sqrt(a + b*x + c*x**2)*a**2*b**3*c*x**3 - 296*sqrt(a + b*x + c
*x**2)*a**2*b**2*c**2*x**4 + 30*sqrt(a + b*x + c*x**2)*a*b**5*x**3 + 30*sq
rt(a + b*x + c*x**2)*a*b**4*c*x**4 + 432*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*
x + c*x**2) - 2*a - b*x)*a**3*b*c**2*x**3 - 168*sqrt(a)*log(2*sqrt(a)*sqrt
(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**3*c*x**3 + 432*sqrt(a)*log(2*sqrt(
a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**2*c**2*x**4 + 432*sqrt(a)*l
og(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b*c**3*x**5 + 15*sqr
t(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**5*x**3 - 168*s
qrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**4*c*x**4 - 1
68*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**3*c**2*x
**5 + 15*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**6*x*
*4 + 15*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**5*c*x
**5 - 432*sqrt(a)*log(x)*a**3*b*c**2*x**3 + 168*sqrt(a)*log(x)*a**2*b**3*c
*x**3 - 432*sqrt(a)*log(x)*a**2*b**2*c**2*x**4 - 432*sqrt(a)*log(x)*a**2*b
*c**3*x**5 - 15*sqrt(a)*log(x)*a*b**5*x**3 + 168*sqrt(a)*log(x)*a*b**4*...
```

3.162 $\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 285

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2x^3(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{3c\,(b^2-4ac)\,(a+bx+cx^2)^{3/2}}$$
$$-\frac{2x(a(5b^3B-2Ab^2c-28abBc+24aAc^2)+(5b^4B-2Ab^3c-32ab^2Bc+16aAbc^2+32a^2Bc^2)\,x)}{3c^2\,(b^2-4ac)^2\sqrt{a+bx+cx^2}}$$
$$+\frac{(15b^4B-6Ab^3c-100ab^2Bc+40aAbc^2+128a^2Bc^2)\sqrt{a+bx+cx^2}}{3c^3\,(b^2-4ac)^2}$$
$$-\frac{(5bB-2Ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{7/2}}$$

output
```
-2/3*x^3*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b
*x+a)^(3/2)-2/3*x*(a*(24*A*a*c^2-2*A*b^2*c-28*B*a*b*c+5*B*b^3)+(16*A*a*b*c
^2-2*A*b^3*c+32*B*a^2*c^2-32*B*a*b^2*c+5*B*b^4)*x)/c^2/(-4*a*c+b^2)^2/(c*x
^2+b*x+a)^(1/2)+1/3*(40*A*a*b*c^2-6*A*b^3*c+128*B*a^2*c^2-100*B*a*b^2*c+15
*B*b^4)*(c*x^2+b*x+a)^(1/2)/c^3/(-4*a*c+b^2)^2-1/2*(-2*A*c+5*B*b)*arctanh(
1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.01

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{128a^4Bc^2+2ab^2x(15b^3B+2bc^2x(9A-37Bx)+4c^3x^2(7A-3Bx)-3b^2c(2A+\dots}{\dots}$$
$$+\frac{(5bB-2Ac)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{2c^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(128*a^4*B*c^2 + 2*a*b^2*x*(15*b^3*B + 2*b*c^2*x*(9*A - 37*B*x) + 4*c^3*x^
2*(7*A - 3*B*x) - 3*b^2*c*(2*A + 15*B*x)) + a^2*(15*b^4*B + 256*b*B*c^3*x^
3 + 16*c^4*x^3*(-4*A + 3*B*x) + 12*b^2*c^2*x*(7*A + 4*B*x) - 6*b^3*c*(A +
35*B*x)) - 4*a^3*c*(25*b^2*B + 12*c^2*x*(A - 4*B*x) - 2*b*c*(5*A + 39*B*x)
) + b^4*x^2*(15*b^2*B + c^2*x*(-8*A + 3*B*x) + b*(-6*A*c + 20*B*c*x)))/(3*
c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + ((5*b*B - 2*A*c)*Log[b + 2*
c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(2*c^(7/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1233, 27, 1233, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2\int \frac{x^2(6a(bB-2Ac)+(5Bb^2-2Acb-16aBc)x)}{2(cx^2+bx+a)^{3/2}}dx}{3c(b^2-4ac)}-\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^2\left(6a(bB-2Ac)+\left(5Bb^2-2Acb-16aBc\right)x\right)}{\left(cx^2+bx+a\right)^{3/2}}dx}{3c\left(b^2-4ac\right)}-\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 1233

$$\frac{\frac{2\int \frac{2a\left(5Bb^3-2Acb^2-28aBcb+24aAc^2\right)+\left(15Bb^4-6Acb^3-100aBcb^2+40aAc^2b+128a^2Bc^2\right)x}{2\sqrt{cx^2+bx+a}}dx}{c(b^2-4ac)}-\frac{2x\left(x\left(32a^2Bc^2+16aAbc^2-32ab^2Bc-2Ab^3c+5b^4B\right)\cdots\right.}{c(b^2-4ac)\sqrt{a+bx\cdots}}}{3c\left(b^2-4ac\right)}$$

$$\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{2a\left(5Bb^3-2Acb^2-28aBcb+24aAc^2\right)+\left(15Bb^4-6Acb^3-100aBcb^2+40aAc^2b+128a^2Bc^2\right)x}{\sqrt{cx^2+bx+a}}dx}{c(b^2-4ac)}-\frac{2x\left(x\left(32a^2Bc^2+16aAbc^2-32ab^2Bc-2Ab^3c+5b^4B\right)+\cdots\right.}{c(b^2-4ac)\sqrt{a+bx+\cdots}}}{3c\left(b^2-4ac\right)}$$

$$\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 1160

$$\frac{\frac{\frac{\sqrt{a+bx+cx^2}\left(128a^2Bc^2+40aAbc^2-100ab^2Bc-6Ab^3c+15b^4B\right)}{c}-\frac{3\left(b^2-4ac\right)^2(5bB-2Ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}}{c(b^2-4ac)}-\frac{2x\left(x\left(32a^2Bc^2+16aAbc^2-32ab^2Bc-2Ab\cdots\right.\right.}{c(b^2-4a\cdots}}{3c\left(b^2-4ac\right)}$$

$$\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\frac{\sqrt{a+bx+cx^2}\left(128a^2Bc^2+40aAbc^2-100ab^2Bc-6Ab^3c+15b^4B\right)}{c}-\frac{3\left(b^2-4ac\right)^2(5bB-2Ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}}{c(b^2-4ac)}-\frac{2x\left(x\left(32a^2Bc^2+16aAbc^2-3\cdots\right.\right.}{\cdots}}{3c\left(b^2-4ac\right)}$$

$$\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\sqrt{a+bx+cx^2}\left(128a^2Bc^2+40aAbc^2-100ab^2Bc-6Ab^3c+15b^4B\right)}{c}-\frac{3\left(b^2-4ac\right)^2(5bB-2Ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}}{c(b^2-4ac)}-\frac{2x(x(32a^2Bc^2+16aAbc^2-32}{}}{3c\left(b^2-4ac\right)}$$

$$\frac{2x^3\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

input
```
Int[(x^4*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*x^3*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(3*c*(b^2 - 4*a*c
)*(a + b*x + c*x^2)^(3/2)) + ((-2*x*(a*(5*b^3*B - 2*A*b^2*c - 28*a*b*B*c +
 24*a*A*c^2) + (5*b^4*B - 2*A*b^3*c - 32*a*b^2*B*c + 16*a*A*b*c^2 + 32*a^2
*B*c^2)*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (((15*b^4*B - 6*A*b^
3*c - 100*a*b^2*B*c + 40*a*A*b*c^2 + 128*a^2*B*c^2)*Sqrt[a + b*x + c*x^2])
/c - (3*(b^2 - 4*a*c)^2*(5*b*B - 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*c^(3/2)))/(c*(b^2 - 4*a*c)))/(3*c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. $2(263) = 526$.

Time = 1.34 (sec) , antiderivative size = 1246, normalized size of antiderivative = 4.37

method	result	size
default	Expression too large to display	1246
risch	Expression too large to display	5483

input
```
int(x^4*(B*x+A)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*
b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*
b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2
*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*
x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/
3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(
3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))+1/c*(-x/c/(c*x^
2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)))+B*(x^4/c/(c*x^2+b*x+a)^(3/2)-5/2*b/c*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1
/2*b/c*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1
/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x
^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a
/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*
c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/
3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)
/(c*x^2+b*x+a)^(1/2))))+1/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2
+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-4*a/c*(-x^2/c/(c*x^2+b*x+a)^(3/
2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(263) = 526$.

Time = 0.65 (sec) , antiderivative size = 1601, normalized size of antiderivative = 5.62

$$\int \frac{x^4(A+Bx)}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(5*B*a^2*b^5 - 32*A*a^4*c^3 + (5*B*b^5*c^2 - 32*A*a^2*c^5 + 16*(
5*B*a^2*b + A*a*b^2)*c^4 - 2*(20*B*a*b^3 + A*b^4)*c^3)*x^4 + 2*(5*B*b^6*c
- 32*A*a^2*b*c^4 + 16*(5*B*a^2*b^2 + A*a*b^3)*c^3 - 2*(20*B*a*b^4 + A*b^5)
*c^2)*x^3 + 16*(5*B*a^4*b + A*a^3*b^2)*c^2 + (5*B*b^7 + 12*A*a*b^4*c^2 + 1
60*B*a^3*b*c^3 - 64*A*a^3*c^4 - 2*(15*B*a*b^5 + A*b^6)*c)*x^2 - 2*(20*B*a^
3*b^3 + A*a^2*b^4)*c + 2*(5*B*a*b^6 - 32*A*a^3*b*c^3 + 16*(5*B*a^3*b^2 + A
*a^2*b^3)*c^2 - 2*(20*B*a^2*b^4 + A*a*b^5)*c)*x)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(
15*B*a^2*b^4*c + 3*(B*b^4*c^3 - 8*B*a*b^2*c^4 + 16*B*a^2*c^5)*x^4 + 8*(16*
B*a^4 + 5*A*a^3*b)*c^3 + 4*(5*B*b^5*c^2 - 16*A*a^2*c^5 + 2*(32*B*a^2*b + 7
*A*a*b^2)*c^4 - (37*B*a*b^3 + 2*A*b^4)*c^3)*x^3 - 2*(50*B*a^3*b^2 + 3*A*a^
2*b^3)*c^2 + 3*(5*B*b^6*c + 64*B*a^3*c^4 + 4*(4*B*a^2*b^2 + 3*A*a*b^3)*c^3
 - 2*(15*B*a*b^4 + A*b^5)*c^2)*x^2 + 6*(5*B*a*b^5*c - 8*A*a^3*c^4 + 2*(26*
B*a^3*b + 7*A*a^2*b^2)*c^3 - (35*B*a^2*b^3 + 2*A*a*b^4)*c^2)*x)*sqrt(c*x^2
 + b*x + a))/(a^2*b^4*c^4 - 8*a^3*b^2*c^5 + 16*a^4*c^6 + (b^4*c^6 - 8*a*b^
2*c^7 + 16*a^2*c^8)*x^4 + 2*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*x^3 + (
b^6*c^4 - 6*a*b^4*c^5 + 32*a^3*c^7)*x^2 + 2*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 1
6*a^3*b*c^6)*x), 1/6*(3*(5*B*a^2*b^5 - 32*A*a^4*c^3 + (5*B*b^5*c^2 - 32*A*
a^2*c^5 + 16*(5*B*a^2*b + A*a*b^2)*c^4 - 2*(20*B*a*b^3 + A*b^4)*c^3)*x^4 +
 2*(5*B*b^6*c - 32*A*a^2*b*c^4 + 16*(5*B*a^2*b^2 + A*a*b^3)*c^3 - 2*(20...
```

Sympy [F]

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(B*x+A)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(x**4*(A + B*x)/(a + b*x + c*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(A + Bx)}{(a + bx + cx^2)^{5/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.60

$$\int \frac{x^4(A + Bx)}{(a + bx + cx^2)^{5/2}}\, dx = \frac{\left(\left(\left(\frac{3\,(B b^4 c^2 - 8\,B a b^2 c^3 + 16\,B a^2 c^4)x}{b^4 c^3 - 8\,a b^2 c^4 + 16\,a^2 c^5} + \frac{4\,(5\,B b^5 c - 37\,B a b^3 c^2 - 2\,A b^4 c^2 + 64\,B a^2 b c^3 + 14\,A a b^2 c^3 - 16\,A a^2 c^4}{b^4 c^3 - 8\,a b^2 c^4 + 16\,a^2 c^5}\right.\right.\right.}{} + \frac{(5\,B b - 2\,A c)\log\left(\left|2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} + b\right|\right)}{2\,c^{\frac{7}{2}}}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((3*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c^4)*x/(b^4*c^3 - 8*a*b^2*
c^4 + 16*a^2*c^5) + 4*(5*B*b^5*c - 37*B*a*b^3*c^2 - 2*A*b^4*c^2 + 64*B*a^2
*b*c^3 + 14*A*a*b^2*c^3 - 16*A*a^2*c^4)/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5))*x + 3*(5*B*b^6 - 30*B*a*b^4*c - 2*A*b^5*c + 16*B*a^2*b^2*c^2 + 12*A*a*
b^3*c^2 + 64*B*a^3*c^3)/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))*x + 6*(5*B*a
*b^5 - 35*B*a^2*b^3*c - 2*A*a*b^4*c + 52*B*a^3*b*c^2 + 14*A*a^2*b^2*c^2 -
8*A*a^3*c^3)/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))*x + (15*B*a^2*b^4 - 100
*B*a^3*b^2*c - 6*A*a^2*b^3*c + 128*B*a^4*c^2 + 40*A*a^3*b*c^2)/(b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5))/(c*x^2 + b*x + a)^(3/2) + 1/2*(5*B*b - 2*A*c)*l
og(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x^4\,(A+B\,x)}{(c\,x^2+b\,x+a)^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x))/(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((x^4*(A + B*x))/(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x^4(Bx+A)}{(c\,x^2+bx+a)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x^4*(B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

3.163 $\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 189

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}} dx = -\frac{2x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)x)}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}} - \frac{2(a(3b^3B-20abBc+16aAc^2)+(3b^4B-22ab^2Bc+8aAbc^2+24a^2Bc^2)x)}{3c^2(b^2-4ac)^2\sqrt{a+bx+cx^2}} + \frac{B\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

output
```
-2/3*x^2*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b
*x+a)^(3/2)-2/3*(a*(16*A*a*c^2-20*B*a*b*c+3*B*b^3)+(8*A*a*b*c^2+24*B*a^2*c
^2-22*B*a*b^2*c+3*B*b^4)*x)/c^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)+B*arcta
nh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.08

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx =$$
$$-\frac{2(4a^3c(-5bB+4Ac+6Bcx)+b^3x^2(3b^2B+4bBcx-Ac^2x)+2abx(3b^3B-9b^2Bcx+6Ac^3x^2+bc^2x(3}{3c^2\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$
$$-\frac{B\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{5/2}}$$

input
```
Integrate[(x^3*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(4*a^3*c*(-5*b*B + 4*A*c + 6*B*c*x) + b^3*x^2*(3*b^2*B + 4*b*B*c*x - A
*c^2*x) + 2*a*b*x*(3*b^3*B - 9*b^2*B*c*x + 6*A*c^3*x^2 + b*c^2*x*(3*A - 14
*B*x)) + a^2*(3*b^3*B - 42*b^2*B*c*x + 24*A*b*c^2*x + 8*c^3*x^2*(3*A + 4*B
*x))))/(3*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) - (B*Log[c^2*(b + 2
*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])])/c^(5/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1233, 27, 1224, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{1233}$$
$$\frac{2\int \frac{x\left(4a(bB-2Ac)+3B\left(b^2-4ac\right)x\right)}{2\left(cx^2+bx+a\right)^{3/2}}dx}{3c\left(b^2-4ac\right)}-\frac{2x^2\left(x\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x(4a(bB-2Ac)+3B(b^2-4ac)x)}{(cx^2+bx+a)^{3/2}}dx}{3c(b^2-4ac)}-\frac{2x^2(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\big\downarrow$ 1224

$$\frac{\frac{3B(b^2-4ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{c}-\frac{2(x(24a^2Bc^2+8aAbc^2-22ab^2Bc+3b^4B)+a(16aAc^2-20abBc+3b^3B))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}}{3c(b^2-4ac)}-\frac{2x^2(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\big\downarrow$ 1092

$$\frac{\frac{6B(b^2-4ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}-\frac{2(x(24a^2Bc^2+8aAbc^2-22ab^2Bc+3b^4B)+a(16aAc^2-20abBc+3b^3B))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}}{3c(b^2-4ac)}-\frac{2x^2(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\big\downarrow$ 219

$$\frac{\frac{3B(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}-\frac{2(x(24a^2Bc^2+8aAbc^2-22ab^2Bc+3b^4B)+a(16aAc^2-20abBc+3b^3B))}{c(b^2-4ac)\sqrt{a+bx+cx^2}}}{3c(b^2-4ac)}-\frac{2x^2(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

input
```
Int[(x^3*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(3*c*(b^2 - 4*a*c
)*(a + b*x + c*x^2)^(3/2)) + ((-2*(a*(3*b^3*B - 20*a*b*B*c + 16*a*A*c^2) +
 (3*b^4*B - 22*a*b^2*B*c + 8*a*A*b*c^2 + 24*a^2*B*c^2)*x))/(c*(b^2 - 4*a*c
)*Sqrt[a + b*x + c*x^2]) + (3*B*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/c^(3/2))/(3*c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(173) = 346.

Time = 1.22 (sec) , antiderivative size = 765, normalized size of antiderivative = 4.05

method	result
default	$A\left(-\frac{x^2}{c(c\,x^2+bx+a)^{\frac{3}{2}}}+\frac{b\left(-\frac{x}{2c\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^2\right)^2\sqrt{c\,x^2+bx+a}}\right)}{2c}\right)}{4c}\right.}{2c}\right.$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/
c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*
x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2
/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b
)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*
c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x
^2+b*x+a)^(1/2))))+B*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(-x^2/c/(c*x^
2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^
2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3
*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)
^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b
^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)
)))+1/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2
*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(173) = 346$.

Time = 0.61 (sec) , antiderivative size = 1061, normalized size of antiderivative = 5.61

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output

```
[1/6*(3*(B*a^2*b^4 - 8*B*a^3*b^2*c + 16*B*a^4*c^2 + (B*b^4*c^2 - 8*B*a*b^2
*c^3 + 16*B*a^2*c^4)*x^4 + 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^
3 + (B*b^6 - 6*B*a*b^4*c + 32*B*a^3*c^3)*x^2 + 2*(B*a*b^5 - 8*B*a^2*b^3*c
+ 16*B*a^3*b*c^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(3*B*a^2*b^3*c - 20*B*a^3*b*c
^2 + 16*A*a^3*c^3 + (4*B*b^4*c^2 + 4*(8*B*a^2 + 3*A*a*b)*c^4 - (28*B*a*b^2
 + A*b^3)*c^3)*x^3 + 3*(B*b^5*c - 6*B*a*b^3*c^2 + 2*A*a*b^2*c^3 + 8*A*a^2*
c^4)*x^2 + 6*(B*a*b^4*c - 7*B*a^2*b^2*c^2 + 4*(B*a^3 + A*a^2*b)*c^3)*x)*sq
rt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5
- 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)
*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3
*c^4 + 16*a^3*b*c^5)*x), -1/3*(3*(B*a^2*b^4 - 8*B*a^3*b^2*c + 16*B*a^4*c^2
 + (B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c^4)*x^4 + 2*(B*b^5*c - 8*B*a*b^3
*c^2 + 16*B*a^2*b*c^3)*x^3 + (B*b^6 - 6*B*a*b^4*c + 32*B*a^3*c^3)*x^2 + 2*
(B*a*b^5 - 8*B*a^2*b^3*c + 16*B*a^3*b*c^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(3*B*a^2*b
^3*c - 20*B*a^3*b*c^2 + 16*A*a^3*c^3 + (4*B*b^4*c^2 + 4*(8*B*a^2 + 3*A*a*b
)*c^4 - (28*B*a*b^2 + A*b^3)*c^3)*x^3 + 3*(B*b^5*c - 6*B*a*b^3*c^2 + 2*A*a
*b^2*c^3 + 8*A*a^2*c^4)*x^2 + 6*(B*a*b^4*c - 7*B*a^2*b^2*c^2 + 4*(B*a^3 +
A*a^2*b)*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + ...
```

Sympy [F]

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{\frac{5}{2}}}\,dx$$

input

```
integrate(x**3*(B*x+A)/(c*x**2+b*x+a)**(5/2),x)
```

output

```
Integral(x**3*(A + B*x)/(a + b*x + c*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.65

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2\left(\left(\left(\frac{(4\,Bb^4c-28\,Bab^2c^2-Ab^3c^2+32\,Ba^2c^3+12\,Aabc^3)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{3\,(Bb^5-6\,Bab^3c+2\,Aab^2c^2+8\,Aa^2c^3)}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x+\frac{6\,(Bab^4-7\,Ba^2b^2c+4\,Ba^3c^2+4}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right.\right.}{3\,(cx^2+bx+a)^{\frac{3}{2}}} - \frac{B\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{5}{2}}}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*((((4*B*b^4*c - 28*B*a*b^2*c^2 - A*b^3*c^2 + 32*B*a^2*c^3 + 12*A*a*b*
c^3)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(B*b^5 - 6*B*a*b^3*c + 2*A
*a*b^2*c^2 + 8*A*a^2*c^3)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 6*(B*a
*b^4 - 7*B*a^2*b^2*c + 4*B*a^3*c^2 + 4*A*a^2*b*c^2)/(b^4*c^2 - 8*a*b^2*c^3
 + 16*a^2*c^4))*x + (3*B*a^2*b^3 - 20*B*a^3*b*c + 16*A*a^3*c^2)/(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2) - B*log(abs(2*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{(c\,x^2+b\,x+a)^{5/2}}\,dx$$

input
```
int((x^3*(A + B*x))/(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((x^3*(A + B*x))/(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x^3(Bx+A)}{(c\,x^2+bx+a)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
int(x^3*(B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

3.164 $\int \frac{x^2(A+Bx)}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{x^2(A+Bx)}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2x^2(Ab-2aB-(bB-2Ac)x)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{8(Ab-2aB)(2a+bx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

output
```
-2/3*x^2*(A*b-2*B*a-(-2*A*c+B*b)*x)/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+8/3*(
A*b-2*B*a)*(b*x+2*a)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17

$$\int \frac{x^2(A+Bx)}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2(-16a^3B+b^2x^2(3Ab+bBx+2Acx)+8a^2(Ab-3Bx(b+cx))+2ax(-3bBx}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(-16*a^3*B + b^2*x^2*(3*A*b + b*B*x + 2*A*c*x) + 8*a^2*(A*b - 3*B*x*(b
+ c*x)) + 2*a*x*(-3*b*B*x*(b + 2*c*x) + A*(6*b^2 + 6*b*c*x + 4*c^2*x^2))))
/(3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1227, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1227}$$

$$\frac{4(Ab-2aB)\int \frac{x}{(cx^2+bx+a)^{3/2}}dx}{3\,(b^2-4ac)}-\frac{2x^2(-2aB-x(bB-2Ac)+Ab)}{3\,(b^2-4ac)\,(a+bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1158}$$

$$\frac{8(2a+bx)(Ab-2aB)}{3\,(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2x^2(-2aB-x(bB-2Ac)+Ab)}{3\,(b^2-4ac)\,(a+bx+cx^2)^{3/2}}$$

input
```
Int[(x^2*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*x^2*(A*b - 2*a*B - (b*B - 2*A*c)*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2
)^(3/2)) + (8*(A*b - 2*a*B)*(2*a + b*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x +
 c*x^2])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1227
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(b*f - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[m*((b*(
e*f + d*g) - 2*(c*d*f + a*e*g))/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m
 - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.40

method	result
trager	$\frac{\frac{16}{3}Aa\,c^2x^3+\frac{4}{3}A\,b^2c\,x^3-8Babc\,x^3+\frac{2}{3}x^3B\,b^3+8Aabc\,x^2+2A\,b^3x^2-16B\,a^2c\,x^2-4Ba\,b^2x^2+8Aa\,b^2x-16B\,a^2bx+\frac{16}{3}A\,a^2b-\frac{32}{3}B\,a^3}{(4ac-b^2)^2(c\,x^2+bx+a)^{\frac{3}{2}}}$
gosper	$\frac{\frac{16}{3}Aa\,c^2x^3+\frac{4}{3}A\,b^2c\,x^3-8Babc\,x^3+\frac{2}{3}x^3B\,b^3+8Aabc\,x^2+2A\,b^3x^2-16B\,a^2c\,x^2-4Ba\,b^2x^2+8Aa\,b^2x-16B\,a^2bx+\frac{16}{3}A\,a^2b-\frac{32}{3}B\,a^3}{(c\,x^2+bx+a)^{\frac{3}{2}}(16a^2c^2-8ca\,b^2+b^4)}$
orering	$\frac{\frac{16}{3}Aa\,c^2x^3+\frac{4}{3}A\,b^2c\,x^3-8Babc\,x^3+\frac{2}{3}x^3B\,b^3+8Aabc\,x^2+2A\,b^3x^2-16B\,a^2c\,x^2-4Ba\,b^2x^2+8Aa\,b^2x-16B\,a^2bx+\frac{16}{3}A\,a^2b-\frac{32}{3}B\,a^3}{(c\,x^2+bx+a)^{\frac{3}{2}}(16a^2c^2-8ca\,b^2+b^4)}$
default	$A\left(-\frac{x}{2c(c\,x^2+bx+a)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^2\right)^2\sqrt{c\,x^2+bx+a}}\right)}{2c}\right)}{4c}+\frac{a\left(\frac{\frac{4cx}{3}+\cdots}{\left(4ac-b^2\right)\left(c\,x\cdots\right.}\right.}{\cdots}\right.$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(8*A*a*c^2*x^3+2*A*b^2*c*x^3-12*B*a*b*c*x^3+B*b^3*x^3+12*A*a*b*c*x^2+3
*A*b^3*x^2-24*B*a^2*c*x^2-6*B*a*b^2*x^2+12*A*a*b^2*x-24*B*a^2*b*x+8*A*a^2*
b-16*B*a^3)/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(87) = 174$.

Time = 0.46 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.64

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx =$$
$$-\frac{2\,(16\,Ba^3-8\,Aa^2b-(Bb^3+8\,Aac^2-2\,(6\,Bab-Ab^2)c)x^3+3\,(2\,Bab^2-Ab^3+4\,(2\,Ba^2-Aab)c)x}{3\,(a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^4+2\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^3+(b^6-6\,ab^4c}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(16*B*a^3 - 8*A*a^2*b - (B*b^3 + 8*A*a*c^2 - 2*(6*B*a*b - A*b^2)*c)*x
^3 + 3*(2*B*a*b^2 - A*b^3 + 4*(2*B*a^2 - A*a*b)*c)*x^2 + 12*(2*B*a^2*b - A
*a*b^2)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*x)
```

Sympy [F]

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(B*x+A)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(x**2*(A + B*x)/(a + b*x + c*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(87) = 174$.

Time = 0.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.07

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(\left(\left(\left(\frac{(Bb^3-12\,Babc+2\,Ab^2c+8\,Aac^2)x}{b^4-8\,ab^2c+16\,a^2c^2} - \frac{3\,(2\,Bab^2-Ab^3+8\,Ba^2c-4\,Aabc)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x - \frac{12\,(2\,Ba^2b-Aab^2)}{b^4-8\,ab^2c+16\,a^2c^2}\right.\right.\right.}{3\,(cx^2+bx+a)^{\frac{3}{2}}}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*((((B*b^3 - 12*B*a*b*c + 2*A*b^2*c + 8*A*a*c^2)*x/(b^4 - 8*a*b^2*c + 1
6*a^2*c^2) - 3*(2*B*a*b^2 - A*b^3 + 8*B*a^2*c - 4*A*a*b*c)/(b^4 - 8*a*b^2*
c + 16*a^2*c^2))*x - 12*(2*B*a^2*b - A*a*b^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^
2))*x - 8*(2*B*a^3 - A*a^2*b)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x
 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 10.90 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(-16\,B\,a^3-24\,B\,a^2\,b\,x+8\,A\,a^2\,b-24\,B\,a^2\,c\,x^2-6\,B\,a\,b^2\,x^2+12\,A\,a\,b^2\,x-1\right.}{3\,\left(4\,a\,c-b^2\right)^2\,\left(c\,x^2\right.}$$

input
```
int((x^2*(A + B*x))/(a + b*x + c*x^2)^(5/2),x)
```

output
```
(2*(3*A*b^3*x^2 - 16*B*a^3 + B*b^3*x^3 + 8*A*a^2*b + 12*A*a*b^2*x - 24*B*a
^2*b*x - 6*B*a*b^2*x^2 + 8*A*a*c^2*x^3 - 24*B*a^2*c*x^2 + 2*A*b^2*c*x^3 +
12*A*a*b*c*x^2 - 12*B*a*b*c*x^3))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/
2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 503, normalized size of antiderivative = 5.35

$$\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{-\frac{16\sqrt{c\,x^2+bx+a}\,a^3b\,c^2}{3}-8\sqrt{c\,x^2+bx+a}\,a^2b^2c^2x-8\sqrt{c\,x^2+bx+a}\,a^2b\,c^3x^2+\frac{16\sqrt{c}}{}}{}$$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
(2*( - 8*sqrt(a + b*x + c*x**2)*a**3*b*c**2 - 12*sqrt(a + b*x + c*x**2)*a*
*2*b**2*c**2*x - 12*sqrt(a + b*x + c*x**2)*a**2*b*c**3*x**2 + 8*sqrt(a + b
*x + c*x**2)*a**2*c**4*x**3 - 3*sqrt(a + b*x + c*x**2)*a*b**3*c**2*x**2 -
10*sqrt(a + b*x + c*x**2)*a*b**2*c**3*x**3 + sqrt(a + b*x + c*x**2)*b**4*c
**2*x**3 + 8*sqrt(c)*a**4*c**2 - 18*sqrt(c)*a**3*b**2*c + 16*sqrt(c)*a**3*
b*c**2*x + 16*sqrt(c)*a**3*c**3*x**2 + 5*sqrt(c)*a**2*b**4 - 36*sqrt(c)*a*
*2*b**3*c*x - 28*sqrt(c)*a**2*b**2*c**2*x**2 + 16*sqrt(c)*a**2*b*c**3*x**3
 + 8*sqrt(c)*a**2*c**4*x**4 + 10*sqrt(c)*a*b**5*x - 8*sqrt(c)*a*b**4*c*x**
2 - 36*sqrt(c)*a*b**3*c**2*x**3 - 18*sqrt(c)*a*b**2*c**3*x**4 + 5*sqrt(c)*
b**6*x**2 + 10*sqrt(c)*b**5*c*x**3 + 5*sqrt(c)*b**4*c**2*x**4))/(3*c**2*(1
6*a**4*c**2 - 8*a**3*b**2*c + 32*a**3*b*c**2*x + 32*a**3*c**3*x**2 + a**2*
b**4 - 16*a**2*b**3*c*x + 32*a**2*b*c**3*x**3 + 16*a**2*c**4*x**4 + 2*a*b*
*5*x - 6*a*b**4*c*x**2 - 16*a*b**3*c**2*x**3 - 8*a*b**2*c**3*x**4 + b**6*x
**2 + 2*b**5*c*x**3 + b**4*c**2*x**4))
```

3.165 $\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x)}{3c\,(b^2-4ac)\,(a+bx+cx^2)^{3/2}} - \frac{2(4Abc-B(b^2+4ac))\,(b+2cx)}{3c\,(b^2-4ac)^2\sqrt{a+bx+cx^2}}$$

output
```
1/3*(-2*a*(-2*A*c+B*b)-2*(-A*b*c-2*B*a*c+B*b^2)*x)/c/(-4*a*c+b^2)/(c*x^2+b
*x+a)^(3/2)-2/3*(4*A*b*c-B*(4*a*c+b^2))*(2*c*x+b)/c/(-4*a*c+b^2)^2/(c*x^2+
b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(8a^2(bB-Ac)-2aAb(b+6cx)+4aBx(3b^2+3bcx+2c^2x^2)+bx(bBx(3b+2}{3\,(b^2-4ac)^2\,(a+x(b+cx))^{3/2}}$$

input
```
Integrate[(x*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(8*a^2*(b*B - A*c) - 2*a*A*b*(b + 6*c*x) + 4*a*B*x*(3*b^2 + 3*b*c*x + 2
*c^2*x^2) + b*x*(b*B*x*(3*b + 2*c*x) - A*(3*b^2 + 12*b*c*x + 8*c^2*x^2))))
/(3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1224, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \ 1224$$

$$-\frac{(4aBc-4Abc+b^2B)\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{3c(b^2-4ac)}-\frac{2(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\Big\downarrow \ 1088$$

$$\frac{2(b+2cx)(4aBc-4Abc+b^2B)}{3c(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(x(-2aBc-Abc+b^2B)+a(bB-2Ac))}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

input
```
Int[(x*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x))/(3*c*(b^2 - 4*a*c)*(a
 + b*x + c*x^2)^(3/2)) + (2*(b^2*B - 4*A*b*c + 4*a*B*c)*(b + 2*c*x))/(3*c*
(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

method	result
trager	$-\frac{2\left(8Ab\,c^{2}x^{3}-8Ba\,c^{2}x^{3}-2B\,b^{2}c\,x^{3}+12A\,b^{2}c\,x^{2}-12Babc\,x^{2}-3x^{2}B\,b^{3}+12Aabcx+3A\,b^{3}x-12Ba\,b^{2}x+8a^{2}Ac+2Aa\,b^{2}-8B\,a^{2}b\right)}{3\left(4ac-b^{2}\right)^{2}\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}$
gosper	$-\frac{2\left(8Ab\,c^{2}x^{3}-8Ba\,c^{2}x^{3}-2B\,b^{2}c\,x^{3}+12A\,b^{2}c\,x^{2}-12Babc\,x^{2}-3x^{2}B\,b^{3}+12Aabcx+3A\,b^{3}x-12Ba\,b^{2}x+8a^{2}Ac+2Aa\,b^{2}-8B\,a^{2}b\right)}{3\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}$
orering	$-\frac{2\left(8Ab\,c^{2}x^{3}-8Ba\,c^{2}x^{3}-2B\,b^{2}c\,x^{3}+12A\,b^{2}c\,x^{2}-12Babc\,x^{2}-3x^{2}B\,b^{3}+12Aabcx+3A\,b^{3}x-12Ba\,b^{2}x+8a^{2}Ac+2Aa\,b^{2}-8B\,a^{2}b\right)}{3\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}\left(16a^{2}c^{2}-8ca\,b^{2}+b^{4}\right)}$
default	$A\left(-\frac{1}{3c\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}+\frac{16c\left(2cx+b\right)}{3\left(4ac-b^{2}\right)^{2}\sqrt{c\,x^{2}+bx+a}}\right)}{2c}\right)+B\left(-\frac{x}{2c\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{}{3c\left(\right.}\right.}{}\right.$

input
```
int(x*(B*x+A)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(8*A*b*c^2*x^3-8*B*a*c^2*x^3-2*B*b^2*c*x^3+12*A*b^2*c*x^2-12*B*a*b*c*
x^2-3*B*b^3*x^2+12*A*a*b*c*x+3*A*b^3*x-12*B*a*b^2*x+8*A*a^2*c+2*A*a*b^2-8*
B*a^2*b)/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(107) = 214.

Time = 0.51 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.12

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\,(8\,Ba^2b-2\,Aab^2-8\,Aa^2c+2\,(Bb^2c+4\,(Ba-Ab)c^2)x^3+3\,(Bb^3+4\,(}{3\,(a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^4+2\,(b^5c-8\,ab^3c^2+16}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(8*B*a^2*b - 2*A*a*b^2 - 8*A*a^2*c + 2*(B*b^2*c + 4*(B*a - A*b)*c^2)*x
^3 + 3*(B*b^3 + 4*(B*a*b - A*b^2)*c)*x^2 + 3*(4*B*a*b^2 - A*b^3 - 4*A*a*b*
c)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3
*b*c^2)*x)
```

Sympy [F]

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{x(A+Bx)}{(a+bx+cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(B*x+A)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral(x*(A + B*x)/(a + b*x + c*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.70

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{2\,(Bb^2c+4\,Bac^2-4\,Abc^2)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\,(Bb^3+4\,Babc-4\,Ab^2c)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{3\,(4\,Bab^2-Ab^3-4\,Aabc)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{2\,(\cdots}{}\right.}{3\,(cx^2+bx+a)^{\frac{3}{2}}}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*(((2*(B*b^2*c + 4*B*a*c^2 - 4*A*b*c^2)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2
) + 3*(B*b^3 + 4*B*a*b*c - 4*A*b^2*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x +
3*(4*B*a*b^2 - A*b^3 - 4*A*a*b*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 2*(4
*B*a^2*b - A*a*b^2 - 4*A*a^2*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b
*x + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\,(8\,B\,a^2\,b-8\,A\,a^2\,c+12\,B\,a\,b^2\,x-2\,A\,a\,b^2+12\,B\,a\,b\,c\,x^2-12\,A\,a\,b\,c\,x+8\,B\,a}{3\,(4\,a\,c-b^2)^2\,(c\,x^2+a}$$

input
```
int((x*(A + B*x))/(a + b*x + c*x^2)^(5/2),x)
```

output
```
(2*(3*B*b^3*x^2 - 2*A*a*b^2 - 8*A*a^2*c + 8*B*a^2*b - 3*A*b^3*x + 12*B*a*b
^2*x - 12*A*b^2*c*x^2 - 8*A*b*c^2*x^3 + 8*B*a*c^2*x^3 + 2*B*b^2*c*x^3 - 12
*A*a*b*c*x + 12*B*a*b*c*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.49

$$\int \frac{x(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{-\frac{16\sqrt{c\,x^2+bx+a}\,a^3c^2}{3}+4\sqrt{c\,x^2+bx+a}\,a^2b^2c-8\sqrt{c\,x^2+bx+a}\,a^2b\,c^2x+6\sqrt{c\,x^2-}}{c\,(16a^2c^4x^4-8a\,b^2c^3}$$

input
```
int(x*(B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
(2*( - 8*sqrt(a + b*x + c*x**2)*a**3*c**2 + 6*sqrt(a + b*x + c*x**2)*a**2*
b**2*c - 12*sqrt(a + b*x + c*x**2)*a**2*b*c**2*x + 9*sqrt(a + b*x + c*x**2
)*a*b**3*c*x + 3*sqrt(a + b*x + c*x**2)*b**4*c*x**2 + 2*sqrt(a + b*x + c*x
**2)*b**3*c**2*x**3 + 16*sqrt(c)*a**3*b*c - 6*sqrt(c)*a**2*b**3 + 32*sqrt(
c)*a**2*b**2*c*x + 32*sqrt(c)*a**2*b*c**2*x**2 - 12*sqrt(c)*a*b**4*x + 4*s
qrt(c)*a*b**3*c*x**2 + 32*sqrt(c)*a*b**2*c**2*x**3 + 16*sqrt(c)*a*b*c**3*x
**4 - 6*sqrt(c)*b**5*x**2 - 12*sqrt(c)*b**4*c*x**3 - 6*sqrt(c)*b**3*c**2*x
**4))/(3*c*(16*a**4*c**2 - 8*a**3*b**2*c + 32*a**3*b*c**2*x + 32*a**3*c**3
*x**2 + a**2*b**4 - 16*a**2*b**3*c*x + 32*a**2*b*c**3*x**3 + 16*a**2*c**4*
x**4 + 2*a*b**5*x - 6*a*b**4*c*x**2 - 16*a*b**3*c**2*x**3 - 8*a*b**2*c**3*
x**4 + b**6*x**2 + 2*b**5*c*x**3 + b**4*c**2*x**4))
```

3.166 $\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx=-\frac{2(Ab-2aB-(bB-2Ac)x)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}-\frac{8(bB-2Ac)(b+2cx)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}}$$

output
```
1/3*(-2*A*b+4*B*a+2*(-2*A*c+B*b)*x)/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)-8/3*(
-2*A*c+B*b)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2(A(b+2cx)(b^2-8bcx-4c(3a+2cx^2))+B(8a^2c+2ab(b+6cx)+bx(3b^2+12bcx+8c^2x^2)))}{3(b^2-4ac)^2(a+x(b+cx))^{3/2}}$$

input
```
Integrate[(A + B*x)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(A*(b + 2*c*x)*(b^2 - 8*b*c*x - 4*c*(3*a + 2*c*x^2)) + B*(8*a^2*c + 2*
a*b*(b + 6*c*x) + b*x*(3*b^2 + 12*b*c*x + 8*c^2*x^2))))/(3*(b^2 - 4*a*c)^2
*(a + x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{4(bB-2Ac)\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{3\,(b^2-4ac)}-\frac{2(-2aB-x(bB-2Ac)+Ab)}{3\,(b^2-4ac)\,(a+bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{8(b+2cx)(bB-2Ac)}{3\,(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(-2aB-x(bB-2Ac)+Ab)}{3\,(b^2-4ac)\,(a+bx+cx^2)^{3/2}}$$

input
```
Int[(A + B*x)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(A*b - 2*a*B - (b*B - 2*A*c)*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3
/2)) - (8*(b*B - 2*A*c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x
^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37

method	result
trager	$\frac{\frac{32}{3} A c^3 x^3-\frac{16}{3} x^3 B b c^2+16 A b c^2 x^2-8 x^2 B b^2 c+16 A a c^2 x+4 A b^2 c x-8 B a b c x-2 x B b^3+8 A a b c-\frac{2}{3} A b^3-\frac{16}{3} B a^2 c-\frac{4}{3} B a b^2}{\left(4 a c-b^2\right)^2 \left(c x^2+b x+a\right)^{\frac{3}{2}}}$
gosper	$\frac{\frac{32}{3} A c^3 x^3-\frac{16}{3} x^3 B b c^2+16 A b c^2 x^2-8 x^2 B b^2 c+16 A a c^2 x+4 A b^2 c x-8 B a b c x-2 x B b^3+8 A a b c-\frac{2}{3} A b^3-\frac{16}{3} B a^2 c-\frac{4}{3} B a b^2}{\left(c x^2+b x+a\right)^{\frac{3}{2}} \left(16 a^2 c^2-8 c a b^2+b^4\right)}$
orering	$\frac{\frac{32}{3} A c^3 x^3-\frac{16}{3} x^3 B b c^2+16 A b c^2 x^2-8 x^2 B b^2 c+16 A a c^2 x+4 A b^2 c x-8 B a b c x-2 x B b^3+8 A a b c-\frac{2}{3} A b^3-\frac{16}{3} B a^2 c-\frac{4}{3} B a b^2}{\left(c x^2+b x+a\right)^{\frac{3}{2}} \left(16 a^2 c^2-8 c a b^2+b^4\right)}$
default	$A \left(\frac{\frac{4 c x}{3}+\frac{2 b}{3}}{\left(4 a c-b^2\right) \left(c x^2+b x+a\right)^{\frac{3}{2}}}+\frac{16 c (2 c x+b)}{3 \left(4 a c-b^2\right)^2 \sqrt{c x^2+b x+a}}\right)+B \left(-\frac{1}{3 c \left(c x^2+b x+a\right)^{\frac{3}{2}}}-\frac{b \left(\frac{\frac{4 c x}{3}+\frac{2 b}{3}}{\left(4 a c-b^2\right) \left(c x^2+b x+a\right)^{\frac{3}{2}}}+\frac{\cdots}{3 \left(4 a c \cdots\right.}\right.}{2 c}\right.$

input
```
int((B*x+A)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(16*A*c^3*x^3-8*B*b*c^2*x^3+24*A*b*c^2*x^2-12*B*b^2*c*x^2+24*A*a*c^2*x
+6*A*b^2*c*x-12*B*a*b*c*x-3*B*b^3*x+12*A*a*b*c-A*b^3-8*B*a^2*c-2*B*a*b^2)/
(4*a*c-b^2)^2/(c*x^2+b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(82) = 164$.

Time = 0.48 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.72

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2\,(2\,Bab^2+Ab^3+8\,(Bbc^2-2\,Ac^3)x^3+12\,(Bb^2c-2\,Abc^2)x^2+4\,(2\,Ba^2-3\,Aab)c+3\,(Bb^3-8\,Ac}{3\,(a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^4+2\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^3+(b^6-6\,ab^4c}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*B*a*b^2 + A*b^3 + 8*(B*b*c^2 - 2*A*c^3)*x^3 + 12*(B*b^2*c - 2*A*b*
c^2)*x^2 + 4*(2*B*a^2 - 3*A*a*b)*c + 3*(B*b^3 - 8*A*a*c^2 + 2*(2*B*a*b - A
*b^2)*c)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c +
16*a^3*b*c^2)*x)
```

Sympy [F]

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{A+Bx}{(a+bx+cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral((A + B*x)/(a + b*x + c*x**2)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(82) = 164$.

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2\left(\left(4\left(\frac{2\,(Bbc^2-2\,Ac^3)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\,(Bb^2c-2\,Abc^2)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{3\,(Bb^3+4\,Babc-2\,Ab^2c-8\,Aac^2)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{2\,Bab^2+Ab^3+8\,Ba^2c-12\,Aabc}{b^4-8\,ab^2c+16\,a^2c^2}\right)}{3\,(cx^2+bx+a)^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*((4*(2*(B*b*c^2 - 2*A*c^3)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) + 3*(B*b^
2*c - 2*A*b*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 3*(B*b^3 + 4*B*a*b*c
- 2*A*b^2*c - 8*A*a*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + (2*B*a*b^2 +
A*b^3 + 8*B*a^2*c - 12*A*a*b*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b
*x + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2\,(8\,B\,a^2\,c+2\,B\,a\,b^2+12\,B\,a\,b\,c\,x-12\,A\,a\,b\,c-24\,A\,a\,c^2\,x+3\,B\,b^3\,x+A\,b^3+12\,B\,b^2\,c\,x^2-6\,A\,b^2\,c\,x}{3\,(4\,a\,c-b^2)^2\,(c\,x^2+b\,x+a)^{3/2}}$$

input
```
int((A + B*x)/(a + b*x + c*x^2)^(5/2),x)
```

output
```
-(2*(A*b^3 - 16*A*c^3*x^3 + 2*B*a*b^2 + 8*B*a^2*c + 3*B*b^3*x - 24*A*a*c^2
*x - 6*A*b^2*c*x - 24*A*b*c^2*x^2 + 12*B*b^2*c*x^2 + 8*B*b*c^2*x^3 - 12*A*
a*b*c + 12*B*a*b*c*x))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.83

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}}\,dx = \frac{8\sqrt{c\,x^2+bx+a}\,a^2bc+48\sqrt{c\,x^2+bx+a}\,a^2c^2x-6\sqrt{c\,x^2+bx+a}\,a\,b^3-12\sqrt{c}}{}$$

input
```
int((B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
(2*(4*sqrt(a + b*x + c*x**2)*a**2*b*c + 24*sqrt(a + b*x + c*x**2)*a**2*c**
2*x - 3*sqrt(a + b*x + c*x**2)*a*b**3 - 6*sqrt(a + b*x + c*x**2)*a*b**2*c*
x + 24*sqrt(a + b*x + c*x**2)*a*b*c**2*x**2 + 16*sqrt(a + b*x + c*x**2)*a*
c**3*x**3 - 3*sqrt(a + b*x + c*x**2)*b**4*x - 12*sqrt(a + b*x + c*x**2)*b*
*3*c*x**2 - 8*sqrt(a + b*x + c*x**2)*b**2*c**2*x**3 - 16*sqrt(c)*a**3*c +
8*sqrt(c)*a**2*b**2 - 32*sqrt(c)*a**2*b*c*x - 32*sqrt(c)*a**2*c**2*x**2 +
16*sqrt(c)*a*b**3*x - 32*sqrt(c)*a*b*c**2*x**3 - 16*sqrt(c)*a*c**3*x**4 +
8*sqrt(c)*b**4*x**2 + 16*sqrt(c)*b**3*c*x**3 + 8*sqrt(c)*b**2*c**2*x**4))/
(3*(16*a**4*c**2 - 8*a**3*b**2*c + 32*a**3*b*c**2*x + 32*a**3*c**3*x**2 +
a**2*b**4 - 16*a**2*b**3*c*x + 32*a**2*b*c**3*x**3 + 16*a**2*c**4*x**4 + 2
*a*b**5*x - 6*a*b**4*c*x**2 - 16*a*b**3*c**2*x**3 - 8*a*b**2*c**3*x**4 + b
**6*x**2 + 2*b**5*c*x**3 + b**4*c**2*x**4))
```

3.167 $\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2(Ab^2-abB-2aAc+(Ab-2aB)cx)}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2(8a^2bBc+A(3b^4-22ab^2c+24a^2c^2)+c(3Ab^3-20aAbc+16a^2Bc)\,x)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}} - \frac{A\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{5/2}}$$

output
```
2/3*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/
2)+2/3*(8*a^2*b*B*c+A*(24*a^2*c^2-22*a*b^2*c+3*b^4)+c*(-20*A*a*b*c+3*A*b^3
+16*B*a^2*c)*x)/a^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)-A*arctanh(1/2*(b*x+
2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{6Ab^3x(b+cx)^2+8a^3c(3bB+8Ac+6Bcx)+4aAb(2b^3-9b^2cx-21bc^2x^2-}{3a^2\left(b^2-4ac\right)^2(a}$$
$$+\frac{2A\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x)/(x*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(6*A*b^3*x*(b + c*x)^2 + 8*a^3*c*(3*b*B + 8*A*c + 6*B*c*x) + 4*a*A*b*(2*b^
3 - 9*b^2*c*x - 21*b*c^2*x^2 - 10*c^3*x^3) + 2*a^2*(-(b^3*B) + 24*b*B*c^2*
x^2 + 8*c^3*x^2*(3*A + 2*B*x) + b^2*(-28*A*c + 6*B*c*x)))/(3*a^2*(b^2 - 4*
a*c)^2*(a + x*(b + c*x))^(3/2)) + (2*A*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b
+ c*x)])/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1235, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 1235}$$
$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2\int -\frac{3A\left(b^2-4ac\right)+4(Ab-2aB)cx}{2x\left(cx^2+bx+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{3A\left(b^2-4ac\right)+4(Ab-2aB)cx}{x\left(cx^2+bx+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1235$$

$$\frac{\frac{2\left(cx\left(16a^2Bc-20aAbc+3Ab^3\right)+A\left(24a^2c^2-22ab^2c+3b^4\right)+8a^2bBc\right)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{3A\left(b^2-4ac\right)^2}{2x\sqrt{cx^2+bx+a}}dx}{a\left(b^2-4ac\right)}}{3a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3A\left(b^2-4ac\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{a}+\frac{2\left(cx\left(16a^2Bc-20aAbc+3Ab^3\right)+A\left(24a^2c^2-22ab^2c+3b^4\right)+8a^2bBc\right)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}}{3a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{2\left(cx\left(16a^2Bc-20aAbc+3Ab^3\right)+A\left(24a^2c^2-22ab^2c+3b^4\right)+8a^2bBc\right)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{6A\left(b^2-4ac\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}}{3a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2\left(cx\left(16a^2Bc-20aAbc+3Ab^3\right)+A\left(24a^2c^2-22ab^2c+3b^4\right)+8a^2bBc\right)}{a\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{3A\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}}}{3a\left(b^2-4ac\right)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(x*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(3*a*(b^2 - 4*a*c)*(a +
b*x + c*x^2)^(3/2)) + ((2*(8*a^2*b*B*c + A*(3*b^4 - 22*a*b^2*c + 24*a^2*c^
2) + c*(3*A*b^3 - 20*a*A*b*c + 16*a^2*B*c)*x))/(a*(b^2 - 4*a*c)*Sqrt[a + b
*x + c*x^2]) - (3*A*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
b*x + c*x^2])])/a^(3/2))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.38

method	result
default	$B\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{(4ac-b^2)(cx^2+bx+a)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3(4ac-b^2)^2\sqrt{cx^2+bx+a}}\right)+A\left(\frac{1}{3a(cx^2+bx+a)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)^{\frac{3}{2}}}+\frac{16}{3\left(4ac-b\right.}\right.}{2a}\right.$

input
```
int((B*x+A)/x/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c
*x+b)/(c*x^2+b*x+a)^(1/2))+A*(1/3/a/(c*x^2+b*x+a)^(3/2)-1/2*b/a*(2/3*(2*c*
x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2
+b*x+a)^(1/2))+1/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(169) = 338$.

Time = 1.15 (sec) , antiderivative size = 1077, normalized size of antiderivative = 5.85

$$\int \frac{A + Bx}{x\left(a + bx + cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(A*a^2*b^4 - 8*A*a^3*b^2*c + 16*A*a^4*c^2 + (A*b^4*c^2 - 8*A*a*b^2
*c^3 + 16*A*a^2*c^4)*x^4 + 2*(A*b^5*c - 8*A*a*b^3*c^2 + 16*A*a^2*b*c^3)*x^
3 + (A*b^6 - 6*A*a*b^4*c + 32*A*a^3*c^3)*x^2 + 2*(A*a*b^5 - 8*A*a^2*b^3*c
+ 16*A*a^3*b*c^2)*x)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*
x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(B*a^3*b^3 - 4*A*a^2*
b^4 - 32*A*a^4*c^2 - (3*A*a*b^3*c^2 + 4*(4*B*a^3 - 5*A*a^2*b)*c^3)*x^3 - 6
*(A*a*b^4*c + 4*A*a^3*c^3 + (4*B*a^3*b - 7*A*a^2*b^2)*c^2)*x^2 - 4*(3*B*a^
4*b - 7*A*a^3*b^2)*c - 3*(A*a*b^5 + 8*B*a^4*c^2 + 2*(B*a^3*b^2 - 3*A*a^2*b
^3)*c)*x)*sqrt(c*x^2 + b*x + a))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + (a^
3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^4 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2
 + 16*a^5*b*c^3)*x^3 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^2 + 2*(a^4*b
^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x), 1/3*(3*(A*a^2*b^4 - 8*A*a^3*b^2*c + 1
6*A*a^4*c^2 + (A*b^4*c^2 - 8*A*a*b^2*c^3 + 16*A*a^2*c^4)*x^4 + 2*(A*b^5*c
- 8*A*a*b^3*c^2 + 16*A*a^2*b*c^3)*x^3 + (A*b^6 - 6*A*a*b^4*c + 32*A*a^3*c^
3)*x^2 + 2*(A*a*b^5 - 8*A*a^2*b^3*c + 16*A*a^3*b*c^2)*x)*sqrt(-a)*arctan(1
/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2
*(B*a^3*b^3 - 4*A*a^2*b^4 - 32*A*a^4*c^2 - (3*A*a*b^3*c^2 + 4*(4*B*a^3 - 5
*A*a^2*b)*c^3)*x^3 - 6*(A*a*b^4*c + 4*A*a^3*c^3 + (4*B*a^3*b - 7*A*a^2*b^2
)*c^2)*x^2 - 4*(3*B*a^4*b - 7*A*a^3*b^2)*c - 3*(A*a*b^5 + 8*B*a^4*c^2 + 2*
(B*a^3*b^2 - 3*A*a^2*b^3)*c)*x)*sqrt(c*x^2 + b*x + a))/(a^5*b^4 - 8*a^6...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.81

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(\left(\left(\left(\frac{(3\,Aa^5b^3c^2+16\,Ba^7c^3-20\,Aa^6bc^3)x}{a^7b^4-8\,a^8b^2c+16\,a^9c^2}+\frac{6\,(Aa^5b^4c+4\,Ba^7bc^2-7\,Aa^6b^2c^2+4\,Aa^7c^3)}{a^7b^4-8\,a^8b^2c+16\,a^9c^2}\right)x+\frac{3\,(Aa\cdots}{\cdots}\right.\right.\right.}{3\,(cx^2+bx+\cdots} + \frac{2\,A\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*((((3*A*a^5*b^3*c^2 + 16*B*a^7*c^3 - 20*A*a^6*b*c^3)*x/(a^7*b^4 - 8*a^
8*b^2*c + 16*a^9*c^2) + 6*(A*a^5*b^4*c + 4*B*a^7*b*c^2 - 7*A*a^6*b^2*c^2 +
 4*A*a^7*c^3)/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))*x + 3*(A*a^5*b^5 + 2*B
*a^7*b^2*c - 6*A*a^6*b^3*c + 8*B*a^8*c^2)/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2))*x - (B*a^7*b^3 - 4*A*a^6*b^4 - 12*B*a^8*b*c + 28*A*a^7*b^2*c - 32*A*
a^8*c^2)/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))/(c*x^2 + b*x + a)^(3/2) + 2
*A*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{5/2}}\,dx=\int \frac{A+B\,x}{x\,\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x)/(x*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int((A + B*x)/(x*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 7582, normalized size of antiderivative = 41.21

$$\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)/x/(c*x^2+b*x+a)^(5/2),x)
```

output
```
( - 24*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sq
rt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)
)*a**3*b*c + 6*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sq
rt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c
 - b**2))*a**2*b**3 - 48*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*
atan((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)
*b - 4*a*c - b**2))*a**2*b**2*c*x - 48*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b -
4*a*c - b**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*s
qrt(c)*sqrt(a)*b - 4*a*c - b**2))*a**2*b*c**2*x**2 + 12*sqrt(a)*sqrt(4*sqr
t(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b
+ 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))*a*b**4*x - 12*sqrt(a)*s
qrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x
**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))*a*b**3*c*x**2
- 48*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqrt
(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))*
a*b**2*c**2*x**3 - 24*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*ata
n((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b
- 4*a*c - b**2))*a*b*c**3*x**4 + 6*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*
c - b**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(
c)*sqrt(a)*b - 4*a*c - b**2))*b**5*x**2 + 12*sqrt(a)*sqrt(4*sqrt(c)*sqr...
```

3.168 $\int \frac{A+Bx}{x^2(a+bx+cx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 277

$$\int \frac{A+Bx}{x^2(a+bx+cx^2)^{5/2}} \, dx = -\frac{A}{ax(a+bx+cx^2)^{3/2}}$$
$$+\frac{2aB(b^2-2ac)-A(5b^3-18abc)-c(5Ab^2-2abB-16aAc)x}{3a^2(b^2-4ac)(a+bx+cx^2)^{3/2}}$$
$$-\frac{3(5Ab-2aB)(b^2-4ac)(b^2-2ac)-4abc(5Ab^2-2abB-16aAc)-c(2abB(3b^2-20ac)-A(15b^4-1}{3a^3(b^2-4ac)^2\sqrt{a+bx+cx^2}}$$
$$+\frac{(5Ab-2aB)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{7/2}}$$

output
```
-A/a/x/(c*x^2+b*x+a)^(3/2)+1/3*(2*a*B*(-2*a*c+b^2)-A*(-18*a*b*c+5*b^3)-c*(
-16*A*a*c+5*A*b^2-2*B*a*b)*x)/a^2/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)-1/3*(3*
(5*A*b-2*B*a)*(-4*a*c+b^2)*(-2*a*c+b^2)-4*a*b*c*(-16*A*a*c+5*A*b^2-2*B*a*b
)-c*(2*a*b*B*(-20*a*c+3*b^2)-A*(128*a^2*c^2-100*a*b^2*c+15*b^4))*x)/a^3/(-
4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)+1/2*(5*A*b-2*B*a)*arctanh(1/2*(b*x+2*a)/a
^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{x^2(a+bx+cx^2)^{5/2}}\,dx = \frac{16a^4c^2(-3A+4Bx)-15Ab^4x^2(b+cx)^2+8a^3c(-7b^2Bx+6Bc^2x^3+A(3b^2}{}$$
$$+\frac{(-5Ab+2aB)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^2*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(16*a^4*c^2*(-3*A + 4*B*x) - 15*A*b^4*x^2*(b + c*x)^2 + 8*a^3*c*(-7*b^2*B*
x + 6*B*c^2*x^3 + A*(3*b^2 - 32*b*c*x - 24*c^2*x^2)) + 2*a*b^2*x*(b + c*x)
*(3*b*B*x*(b + c*x) + A*(-10*b^2 + 55*b*c*x + 50*c^2*x^2)) - a^2*(4*b*B*x*
(-2*b^3 + 9*b^2*c*x + 21*b*c^2*x^2 + 10*c^3*x^3) + A*(3*b^4 - 148*b^3*c*x
+ 48*b^2*c^2*x^2 + 312*b*c^3*x^3 + 128*c^4*x^4)))/(3*a^3*(b^2 - 4*a*c)^2*x
*(a + x*(b + c*x))^(3/2)) + ((-5*A*b + 2*a*B)*ArcTanh[(Sqrt[c]*x - Sqrt[a
+ x*(b + c*x)])/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1235, 27, 1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2(a+bx+cx^2)^{5/2}}\,dx$$
$$\Big\downarrow \ 1235$$
$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax(b^2-4ac)(a+bx+cx^2)^{3/2}}-\frac{2\int -\frac{5Ab^2-2aBb-16aAc+6(Ab-2aB)cx}{2x^2(cx^2+bx+a)^{3/2}}dx}{3a(b^2-4ac)}$$
$$\Big\downarrow \ 27$$

$$\frac{\int \frac{5Ab^2-2aBb-16aAc+6(Ab-2aB)cx}{x^2(cx^2+bx+a)^{3/2}}dx}{3a(b^2-4ac)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 1235

$$\frac{-\frac{2\int \frac{2abB\left(3b^2-20ac\right)-2A\left(\frac{15b^4}{2}-50acb^2+64a^2c^2\right)-2c\left(5Ab^3-2aBb^2-28aAcb+24a^2Bc\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{a(b^2-4ac)}-\frac{2(-A(32a^2c^2-32ab^2c+5b^4)+cx(2aB(b^2-12ac)-\ldots}{ax(b^2-4ac)\sqrt{a+bx+c\ldots}}}{3a(b^2-4ac)}$$
$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{2abB\left(3b^2-20ac\right)-A\left(15b^4-100acb^2+128a^2c^2\right)-2c\left(5Ab^3-2aBb^2-28aAcb+24a^2Bc\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{a(b^2-4ac)}-\frac{2(-A(32a^2c^2-32ab^2c+5b^4)+cx(2aB(b^2-12ac)-A(\ldots}{ax(b^2-4ac)\sqrt{a+bx+cx\ldots}}}{3a(b^2-4ac)}$$
$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 1228

$$\frac{-\frac{\frac{3\left(b^2-4ac\right)^2(5Ab-2aB)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(2abB\left(3b^2-20ac\right)-A\left(128a^2c^2-100ab^2c+15b^4\right)\right)}{ax}}{a(b^2-4ac)}-\frac{2(-A(32a^2c^2-32ab^2c+5b^4)+cx(2aB\ldots}{ax(b^2-\ldots}}{3a(b^2-4ac)}$$
$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 1154

$$\frac{-\frac{-\frac{3\left(b^2-4ac\right)^2(5Ab-2aB)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(2abB\left(3b^2-20ac\right)-A\left(128a^2c^2-100ab^2c+15b^4\right)\right)}{ax}}{a(b^2-4ac)}-2(-A(32a^2c^2-32ab^2c+\ldots}{3a(b^2-4ac)}$$
$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{-\frac{2\left(-A\left(32a^2c^2-32ab^2c+5b^4\right)+cx\left(2aB\left(b^2-12ac\right)-A\left(5b^3-28abc\right)\right)+2abB\left(b^2-8ac\right)\right)}{ax(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{-\frac{3\left(b^2-4ac\right)^2(5Ab-2aB)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}}{a(}}{3a\left(b^2-4ac\right)}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(x^2*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(3*a*(b^2 - 4*a*c)*x*(a
+ b*x + c*x^2)^(3/2)) + ((-2*(2*a*b*B*(b^2 - 8*a*c) - A*(5*b^4 - 32*a*b^2*
c + 32*a^2*c^2) + c*(2*a*B*(b^2 - 12*a*c) - A*(5*b^3 - 28*a*b*c))*x))/(a*(
b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]) - (-(((2*a*b*B*(3*b^2 - 20*a*c) - A*
(15*b^4 - 100*a*b^2*c + 128*a^2*c^2))*Sqrt[a + b*x + c*x^2])/(a*x)) - (3*(
5*A*b - 2*a*B)*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x
 + c*x^2])])/(2*a^(3/2)))/(a*(b^2 - 4*a*c)))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.69

method	result
default	$A\left(-\frac{1}{ax(cx^2+bx+a)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(cx^2+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^2\right)^2\sqrt{cx^2+bx+a}}\right)}{2a}+\frac{1}{a\sqrt{cx^2+bx+a}}-\frac{\cdots}{a\left(4ac\cdots\right.}\cdots\right.}{2a}\cdots\right.$
risch	Expression too large to display

input
```
int((B*x+A)/x^2/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/a/x/(c*x^2+b*x+a)^(3/2)-5/2*b/a*(1/3/a/(c*x^2+b*x+a)^(3/2)-1/2*b/a*(
2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+
b)/(c*x^2+b*x+a)^(1/2))+1/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)))-4*c/a*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*
c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+B*(1/3/a/(c*x^2+b*x+a)^(3/2)-1/2*
b/a*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2
*c*x+b)/(c*x^2+b*x+a)^(1/2))+1/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a
)^(1/2))/x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(256) = 512$.

Time = 1.61 (sec) , antiderivative size = 1655, normalized size of antiderivative = 5.97

$$\int \frac{A + Bx}{x^2 (a + bx + cx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((16*(2*B*a^3 - 5*A*a^2*b)*c^4 - 8*(2*B*a^2*b^2 - 5*A*a*b^3)*c^3
 + (2*B*a*b^4 - 5*A*b^5)*c^2)*x^5 + 2*(16*(2*B*a^3*b - 5*A*a^2*b^2)*c^3 -
8*(2*B*a^2*b^3 - 5*A*a*b^4)*c^2 + (2*B*a*b^5 - 5*A*b^6)*c)*x^4 + (2*B*a*b^
6 - 5*A*b^7 + 32*(2*B*a^4 - 5*A*a^3*b)*c^3 - 6*(2*B*a^2*b^4 - 5*A*a*b^5)*c
)*x^3 + 2*(2*B*a^2*b^5 - 5*A*a*b^6 + 16*(2*B*a^4*b - 5*A*a^3*b^2)*c^2 - 8*
(2*B*a^3*b^3 - 5*A*a^2*b^4)*c)*x^2 + (2*B*a^3*b^4 - 5*A*a^2*b^5 + 16*(2*B*
a^5 - 5*A*a^4*b)*c^2 - 8*(2*B*a^4*b^2 - 5*A*a^3*b^3)*c)*x)*sqrt(a)*log(-(8
*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) +
 8*a^2)/x^2) + 4*(3*A*a^3*b^4 - 24*A*a^4*b^2*c + 48*A*a^5*c^2 + (128*A*a^3
*c^4 + 20*(2*B*a^3*b - 5*A*a^2*b^2)*c^3 - 3*(2*B*a^2*b^3 - 5*A*a*b^4)*c^2)
*x^4 - 6*(4*(2*B*a^4 - 13*A*a^3*b)*c^3 - 7*(2*B*a^3*b^2 - 5*A*a^2*b^3)*c^2
 + (2*B*a^2*b^4 - 5*A*a*b^5)*c)*x^3 - 3*(2*B*a^2*b^5 - 5*A*a*b^6 - 16*A*a^
3*b^2*c^2 - 64*A*a^4*c^3 - 6*(2*B*a^3*b^3 - 5*A*a^2*b^4)*c)*x^2 - 4*(2*B*a
^3*b^4 - 5*A*a^2*b^5 + 16*(B*a^5 - 4*A*a^4*b)*c^2 - (14*B*a^4*b^2 - 37*A*a
^3*b^3)*c)*x)*sqrt(c*x^2 + b*x + a))/((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^
6*c^4)*x^5 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x^4 + (a^4*b^6 -
 6*a^5*b^4*c + 32*a^7*c^3)*x^3 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*
x^2 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x), 1/6*(3*((16*(2*B*a^3 - 5*A*
a^2*b)*c^4 - 8*(2*B*a^2*b^2 - 5*A*a*b^3)*c^3 + (2*B*a*b^4 - 5*A*b^5)*c^2)*
x^5 + 2*(16*(2*B*a^3*b - 5*A*a^2*b^2)*c^3 - 8*(2*B*a^2*b^3 - 5*A*a*b^4)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/x**2/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.74

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{\left(3\,Ba^9b^3c^2-6\,Aa^8b^4c^2-20\,Ba^{10}bc^3+38\,Aa^9b^2c^3-40\,Aa^{10}c^4\right)x}{a^{11}b^4-8\,a^{12}b^2c+16\,a^{13}c^2}+\frac{3\left(2\,Ba^9b^4c-4\,Aa^8b^5c-14\,B\cdots\right.}{a^{11}b^{4}\cdots}\right.\right.\right.}{\cdots}$$
$$+\frac{\left(2\,Ba-5\,Ab\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)Ab+2\,Aa\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)a^3}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*((((3*B*a^9*b^3*c^2 - 6*A*a^8*b^4*c^2 - 20*B*a^10*b*c^3 + 38*A*a^9*b^2
*c^3 - 40*A*a^10*c^4)*x/(a^11*b^4 - 8*a^12*b^2*c + 16*a^13*c^2) + 3*(2*B*a
^9*b^4*c - 4*A*a^8*b^5*c - 14*B*a^10*b^2*c^2 + 27*A*a^9*b^3*c^2 + 8*B*a^11
*c^3 - 36*A*a^10*b*c^3)/(a^11*b^4 - 8*a^12*b^2*c + 16*a^13*c^2))*x + 3*(B*
a^9*b^5 - 2*A*a^8*b^6 - 6*B*a^10*b^3*c + 12*A*a^9*b^4*c - 8*A*a^10*b^2*c^2
 - 16*A*a^11*c^3)/(a^11*b^4 - 8*a^12*b^2*c + 16*a^13*c^2))*x + (4*B*a^10*b
^4 - 7*A*a^9*b^5 - 28*B*a^11*b^2*c + 50*A*a^10*b^3*c + 32*B*a^12*c^2 - 80*
A*a^11*b*c^2)/(a^11*b^4 - 8*a^12*b^2*c + 16*a^13*c^2))/(c*x^2 + b*x + a)^(
3/2) + (2*B*a - 5*A*b)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a
))/(sqrt(-a)*a^3) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*b + 2*A*a*sqrt(
c))/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{5/2}}\,dx=\int \frac{A+B\,x}{x^2\,\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x)/(x^2*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int((A + B*x)/(x^2*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1284, normalized size of antiderivative = 4.64

$$\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)/x^2/(c*x^2+b*x+a)^(5/2),x)
```

output

```
( - 96*sqrt(a + b*x + c*x**2)*a**5*c**2 + 48*sqrt(a + b*x + c*x**2)*a**4*b
**2*c - 384*sqrt(a + b*x + c*x**2)*a**4*b*c**2*x - 384*sqrt(a + b*x + c*x*
*2)*a**4*c**3*x**2 - 6*sqrt(a + b*x + c*x**2)*a**3*b**4 + 184*sqrt(a + b*x
 + c*x**2)*a**3*b**3*c*x - 96*sqrt(a + b*x + c*x**2)*a**3*b**2*c**2*x**2 -
 528*sqrt(a + b*x + c*x**2)*a**3*b*c**3*x**3 - 256*sqrt(a + b*x + c*x**2)*
a**3*c**4*x**4 - 24*sqrt(a + b*x + c*x**2)*a**2*b**5*x + 108*sqrt(a + b*x
+ c*x**2)*a**2*b**4*c*x**2 + 252*sqrt(a + b*x + c*x**2)*a**2*b**3*c**2*x**
3 + 120*sqrt(a + b*x + c*x**2)*a**2*b**2*c**3*x**4 - 18*sqrt(a + b*x + c*x
**2)*a*b**6*x**2 - 36*sqrt(a + b*x + c*x**2)*a*b**5*c*x**3 - 18*sqrt(a + b
*x + c*x**2)*a*b**4*c**2*x**4 + 144*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x
+ c*x**2) - 2*a - b*x)*a**4*b*c**2*x - 72*sqrt(a)*log( - 2*sqrt(a)*sqrt(a
+ b*x + c*x**2) - 2*a - b*x)*a**3*b**3*c*x + 288*sqrt(a)*log( - 2*sqrt(a)*
sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*b**2*c**2*x**2 + 288*sqrt(a)*log(
 - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*b*c**3*x**3 + 9*sqrt
(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**5*x - 144
*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**4*c*
x**2 + 288*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*
*2*b**2*c**3*x**4 + 144*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) -
2*a - b*x)*a**2*b*c**4*x**5 + 18*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c
*x**2) - 2*a - b*x)*a*b**6*x**2 - 54*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + ...
```

3.169 $\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 366

$$\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{A}{2ax^2\left(a+bx+cx^2\right)^{3/2}}+\frac{7Ab-4aB}{4a^2x\left(a+bx+cx^2\right)^{3/2}}$$
$$-\frac{4abB(5b^2-18ac)-A(35b^4-146ab^2c+40a^2c^2)+c(4aB(5b^2-16ac)-A(35b^3-132abc))x}{12a^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$-\frac{4abB(15b^4-110ab^2c+184a^2c^2)-A(105b^6-830ab^4c+1728a^2b^2c^2-480a^3c^3)+c(4aB(15b^4-100ab^2}{12a^4\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$
$$-\frac{5(7Ab^2-4abB-4aAc)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{9/2}}$$

output
```
-1/2*A/a/x^2/(c*x^2+b*x+a)^(3/2)+1/4*(7*A*b-4*B*a)/a^2/x/(c*x^2+b*x+a)^(3/
2)-1/12*(4*a*b*B*(-18*a*c+5*b^2)-A*(40*a^2*c^2-146*a*b^2*c+35*b^4)+c*(4*a*
B*(-16*a*c+5*b^2)-A*(-132*a*b*c+35*b^3))*x)/a^3/(-4*a*c+b^2)/(c*x^2+b*x+a)
^(3/2)-1/12*(4*a*b*B*(184*a^2*c^2-110*a*b^2*c+15*b^4)-A*(-480*a^3*c^3+1728
*a^2*b^2*c^2-830*a*b^4*c+105*b^6)+c*(4*a*B*(128*a^2*c^2-100*a*b^2*c+15*b^4
)-A*(1296*a^2*b*c^2-760*a*b^3*c+105*b^5))*x)/a^4/(-4*a*c+b^2)^2/(c*x^2+b*x
+a)^(1/2)-5/8*(-4*A*a*c+7*A*b^2-4*B*a*b)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*
x^2+b*x+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 4.08 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{\sqrt{a}\left(-96a^5c^2(A+2Bx)+105Ab^5x^3(b+cx)^2+16a^4c\left(A\left(3b^2+21bcx-40c^2x^2\right)-2Bx\left(-3b^2+32bcx+24c^2x^2\right)\right)\right.}{}$$

input
```
Integrate[(A + B*x)/(x^3*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
((Sqrt[a]*(-96*a^5*c^2*(A + 2*B*x) + 105*A*b^5*x^3*(b + c*x)^2 + 16*a^4*c*
(A*(3*b^2 + 21*b*c*x - 40*c^2*x^2) - 2*B*x*(-3*b^2 + 32*b*c*x + 24*c^2*x^2
)) - 10*a*b^3*x^2*(b + c*x)*(6*b*B*x*(b + c*x) + A*(-14*b^2 + 83*b*c*x + 7
6*c^2*x^2)) - 2*a^3*(3*A*(b^4 + 28*b^3*c*x - 392*b^2*c^2*x^2 - 224*b*c^3*x
^3 + 80*c^4*x^4) + 2*B*x*(3*b^4 - 148*b^3*c*x + 48*b^2*c^2*x^2 + 312*b*c^3
*x^3 + 128*c^4*x^4)) + a^2*b*x*(40*b*B*x*(-2*b^3 + 9*b^2*c*x + 21*b*c^2*x^
2 + 10*c^3*x^3) + 3*A*(7*b^4 - 372*b^3*c*x + 232*b^2*c^2*x^2 + 1008*b*c^3*
x^3 + 432*c^4*x^4))))/((b^2 - 4*a*c)^2*x^2*(a + x*(b + c*x))^(3/2)) + 105*
A*b^2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 60*a*(b*B + A
*c)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(12*a^(9/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1235, 27, 1235, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 1235$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2\int -\frac{7Ab^2-4aBb-20aAc+8(Ab-2aB)cx}{2x^3\left(cx^2+bx+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{7Ab^2-4aBb-20aAc+8(Ab-2aB)cx}{x^3(cx^2+bx+a)^{3/2}}dx}{3a(b^2-4ac)}+\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax^2(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 1235

$$\frac{-\frac{2\int \frac{4abB\left(5b^2-28ac\right)-2A\left(\frac{35b^4}{2}-108acb^2+120a^2c^2\right)-4c\left(7Ab^3-4aBb^2-36aAcb+32a^2Bc\right)x}{2x^3\sqrt{cx^2+bx+a}}dx}{a(b^2-4ac)}-\frac{2\left(-A\left(40a^2c^2-42ab^2c+7b^4\right)+cx\left(4aB\left(b^2-8ac\right)-\cdots\right.\right.}{ax^2(b^2-4ac)\sqrt{a+bx\cdots}}}{3a(b^2-4ac)}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax^2(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{4abB\left(5b^2-28ac\right)-A\left(35b^4-216acb^2+240a^2c^2\right)-4c\left(7Ab^3-4aBb^2-36aAcb+32a^2Bc\right)x}{x^3\sqrt{cx^2+bx+a}}dx}{a(b^2-4ac)}-\frac{2\left(-A\left(40a^2c^2-42ab^2c+7b^4\right)+cx\left(4aB\left(b^2-8ac\right)-A\left(7\cdots\right.\right.\right.}{ax^2(b^2-4ac)\sqrt{a+bx+cx\cdots}}}{3a(b^2-4ac)}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax^2(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 1237

$$\frac{-\frac{-\frac{\int \frac{4aB\left(15b^4-100acb^2+128a^2c^2\right)-2A\left(\frac{105b^5}{2}-380acb^3+648a^2c^2b\right)+2c\left(4abB\left(5b^2-28ac\right)-A\left(35b^4-216acb^2+240a^2c^2\right)\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(4abB\left(5b^2-\cdots\right.\right.}{\cdots}}{a(b^2-4ac)}}{3a(b^2-4ac)}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax^2(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{-\frac{\int \frac{4aB\left(15b^4-100acb^2+128a^2c^2\right)-A\left(105b^5-760acb^3+1296a^2c^2b\right)+2c\left(4abB\left(5b^2-28ac\right)-A\left(35b^4-216acb^2+240a^2c^2\right)\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}\left(4abB\left(5b^2-2\cdots\right.\right.}{\cdots}}{a(b^2-4ac)}}{3a(b^2-4ac)}$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax^2(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 1228

$$-\frac{-\frac{\frac{15\left(b^2-4ac\right)^2\left(-4aAc-4abB+7Ab^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(4aB\left(128a^2c^2-100ab^2c+15b^4\right)-A\left(1296a^2bc^2-760ab^3c+105b^5\right)\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}}{}}{a(b^2-4ac)}$$

$$3a\left(b^2-4a\right.$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{-\frac{-\frac{15\left(b^2-4ac\right)^2\left(-4aAc-4abB+7Ab^2\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(4aB\left(128a^2c^2-100ab^2c+15b^4\right)-A\left(1296a^2bc^2-760ab^3c+105b^5\right)\right)}{ax}}{4a}}{a(b^2-4ac)}$$

$$3a\left(b^2\right.$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(-A\left(40a^2c^2-42ab^2c+7b^4\right)+cx\left(4aB\left(b^2-8ac\right)-A\left(7b^3-36abc\right)\right)+4abB\left(b^2-6ac\right)\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{-\frac{\sqrt{a+bx+cx^2}\left(4abB\left(5b^2-28ac\right)-A\left(240a^2c^2-216ab^2c+3\right.\right.}{2ax^2}}{}$$

$$3a\left(b^2\right.$$

$$\frac{2\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{3ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(x^3*(a + b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(3*a*(b^2 - 4*a*c)*x^2*(
a + b*x + c*x^2)^(3/2)) + ((-2*(4*a*b*B*(b^2 - 6*a*c) - A*(7*b^4 - 42*a*b^
2*c + 40*a^2*c^2) + c*(4*a*B*(b^2 - 8*a*c) - A*(7*b^3 - 36*a*b*c))*x))/(a*
(b^2 - 4*a*c)*x^2*Sqrt[a + b*x + c*x^2]) - (-1/2*((4*a*b*B*(5*b^2 - 28*a*c
) - A*(35*b^4 - 216*a*b^2*c + 240*a^2*c^2))*Sqrt[a + b*x + c*x^2])/(a*x^2)
 - (-(((4*a*B*(15*b^4 - 100*a*b^2*c + 128*a^2*c^2) - A*(105*b^5 - 760*a*b^
3*c + 1296*a^2*b*c^2))*Sqrt[a + b*x + c*x^2])/(a*x)) - (15*(b^2 - 4*a*c)^2
*(7*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x
 + c*x^2])])/(2*a^(3/2)))/(4*a))/(a*(b^2 - 4*a*c)))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(340) = 680$.

Time = 1.26 (sec) , antiderivative size = 780, normalized size of antiderivative = 2.13

method	result
default	$A\left(-\frac{1}{2a\,x^2(c\,x^2+bx+a)^{\frac{3}{2}}}-\frac{7b\left(-\frac{1}{ax\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^2\right)^2\sqrt{c\,x^2+bx+}}\right.}{2a}\right.}{}\right.}{}\right.$
risch	Expression too large to display

input
```
int((B*x+A)/x^3/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/2/a/x^2/(c*x^2+b*x+a)^(3/2)-7/4*b/a*(-1/a/x/(c*x^2+b*x+a)^(3/2)-5/2*
b/a*(1/3/a/(c*x^2+b*x+a)^(3/2)-1/2*b/a*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b
*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))+1/a*(1/a/(
c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)
*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))-4*c/a*(2/3*(2*c*x+b)/(4*a
*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(
1/2)))-5/2*c/a*(1/3/a/(c*x^2+b*x+a)^(3/2)-1/2*b/a*(2/3*(2*c*x+b)/(4*a*c-b^
2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))
+1/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))))+B*(-1/a/x/(c*
x^2+b*x+a)^(3/2)-5/2*b/a*(1/3/a/(c*x^2+b*x+a)^(3/2)-1/2*b/a*(2/3*(2*c*x+b)
/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x
+a)^(1/2))+1/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b
*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))-4*c/
a*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c
*x+b)/(c*x^2+b*x+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. $2(340) = 680$.

Time = 4.43 (sec) , antiderivative size = 2057, normalized size of antiderivative = 5.62

$$\int \frac{A + Bx}{x^3 (a + bx + cx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(15*((64*A*a^3*c^5 + 16*(4*B*a^3*b - 9*A*a^2*b^2)*c^4 - 4*(8*B*a^2*b
^3 - 15*A*a*b^4)*c^3 + (4*B*a*b^5 - 7*A*b^6)*c^2)*x^6 + 2*(64*A*a^3*b*c^4
+ 16*(4*B*a^3*b^2 - 9*A*a^2*b^3)*c^3 - 4*(8*B*a^2*b^4 - 15*A*a*b^5)*c^2 +
(4*B*a*b^6 - 7*A*b^7)*c)*x^5 + (4*B*a*b^7 - 7*A*b^8 - 24*A*a^2*b^4*c^2 + 1
28*A*a^4*c^4 + 32*(4*B*a^4*b - 7*A*a^3*b^2)*c^3 - 2*(12*B*a^2*b^5 - 23*A*a
*b^6)*c)*x^4 + 2*(4*B*a^2*b^6 - 7*A*a*b^7 + 64*A*a^4*b*c^3 + 16*(4*B*a^4*b
^2 - 9*A*a^3*b^3)*c^2 - 4*(8*B*a^3*b^4 - 15*A*a^2*b^5)*c)*x^3 + (4*B*a^3*b
^5 - 7*A*a^2*b^6 + 64*A*a^5*c^3 + 16*(4*B*a^5*b - 9*A*a^4*b^2)*c^2 - 4*(8*
B*a^4*b^3 - 15*A*a^3*b^4)*c)*x^2)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^
2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(6*A*a^4
*b^4 - 48*A*a^5*b^2*c + 96*A*a^6*c^2 + (16*(32*B*a^4 - 81*A*a^3*b)*c^4 - 4
0*(10*B*a^3*b^2 - 19*A*a^2*b^3)*c^3 + 15*(4*B*a^2*b^4 - 7*A*a*b^5)*c^2)*x^
5 + 6*(80*A*a^4*c^4 + 8*(26*B*a^4*b - 63*A*a^3*b^2)*c^3 - 5*(28*B*a^3*b^3
- 53*A*a^2*b^4)*c^2 + 5*(4*B*a^2*b^5 - 7*A*a*b^6)*c)*x^4 + 3*(20*B*a^2*b^6
 - 35*A*a*b^7 + 64*(4*B*a^5 - 7*A*a^4*b)*c^3 + 8*(8*B*a^4*b^2 - 29*A*a^3*b
^3)*c^2 - 10*(12*B*a^3*b^4 - 23*A*a^2*b^5)*c)*x^3 + 4*(20*B*a^3*b^5 - 35*A
*a^2*b^6 + 160*A*a^5*c^3 + 4*(64*B*a^5*b - 147*A*a^4*b^2)*c^2 - (148*B*a^4
*b^3 - 279*A*a^3*b^4)*c)*x^2 + 3*(4*B*a^4*b^4 - 7*A*a^3*b^5 + 16*(4*B*a^6
- 7*A*a^5*b)*c^2 - 8*(4*B*a^5*b^2 - 7*A*a^4*b^3)*c)*x)*sqrt(c*x^2 + b*x +
a))/((a^5*b^4*c^2 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*x^6 + 2*(a^5*b^5*c - 8*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**3/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x^3 (a+bx+cx^2)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(340) = 680.

Time = 0.28 (sec) , antiderivative size = 765, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx}{x^3 (a+bx+cx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-2/3*((((6*B*a^12*b^4*c^2 - 9*A*a^11*b^5*c^2 - 38*B*a^13*b^2*c^3 + 62*A*a^
12*b^3*c^3 + 40*B*a^14*c^4 - 96*A*a^13*b*c^4)*x/(a^15*b^4 - 8*a^16*b^2*c +
 16*a^17*c^2) + 3*(4*B*a^12*b^5*c - 6*A*a^11*b^6*c - 27*B*a^13*b^3*c^2 + 4
4*A*a^12*b^4*c^2 + 36*B*a^14*b*c^3 - 80*A*a^13*b^2*c^3 + 16*A*a^14*c^4)/(a
^15*b^4 - 8*a^16*b^2*c + 16*a^17*c^2))*x + 3*(2*B*a^12*b^6 - 3*A*a^11*b^7
- 12*B*a^13*b^4*c + 20*A*a^12*b^5*c + 8*B*a^14*b^2*c^2 - 25*A*a^13*b^3*c^2
 + 16*B*a^15*c^3 - 20*A*a^14*b*c^3)/(a^15*b^4 - 8*a^16*b^2*c + 16*a^17*c^2
))*x + (7*B*a^13*b^5 - 10*A*a^12*b^6 - 50*B*a^14*b^3*c + 78*A*a^13*b^4*c +
 80*B*a^15*b*c^2 - 162*A*a^14*b^2*c^2 + 56*A*a^15*c^3)/(a^15*b^4 - 8*a^16*
b^2*c + 16*a^17*c^2))/(c*x^2 + b*x + a)^(3/2) - 5/4*(4*B*a*b - 7*A*b^2 + 4
*A*a*c)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^
4) + 1/4*(4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a*b - 11*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*A*b^2 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A
*a*c + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B*a^2*sqrt(c) - 16*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*A*a*b*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*B*a^2*b + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a*b^2 + 4*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*c - 8*B*a^3*sqrt(c) + 24*A*a^2*b*sq
rt(c))/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^2*a^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^3\left(a+bx+cx^2\right)^{5/2}}\,dx=\int \frac{A+B\,x}{x^3\,{\left(c\,x^2+b\,x+a\right)}^{5/2}}\,dx$$

input
```
int((A + B*x)/(x^3*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int((A + B*x)/(x^3*(a + b*x + c*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1748, normalized size of antiderivative = 4.78

$$\int \frac{A+Bx}{x^3 (a+bx+cx^2)^{5/2}}\, dx = \text{Too large to display}$$

input

```
int((B*x+A)/x^3/(c*x^2+b*x+a)^(5/2),x)
```

output

```
( - 192*sqrt(a + b*x + c*x**2)*a**6*c**2 + 96*sqrt(a + b*x + c*x**2)*a**5*
b**2*c + 288*sqrt(a + b*x + c*x**2)*a**5*b*c**2*x - 1280*sqrt(a + b*x + c*
x**2)*a**5*c**3*x**2 - 12*sqrt(a + b*x + c*x**2)*a**4*b**4 - 144*sqrt(a +
b*x + c*x**2)*a**4*b**3*c*x + 2656*sqrt(a + b*x + c*x**2)*a**4*b**2*c**2*x
**2 + 1152*sqrt(a + b*x + c*x**2)*a**4*b*c**3*x**3 - 960*sqrt(a + b*x + c*
x**2)*a**4*c**4*x**4 + 18*sqrt(a + b*x + c*x**2)*a**3*b**5*x - 1048*sqrt(a
 + b*x + c*x**2)*a**3*b**4*c*x**2 + 1008*sqrt(a + b*x + c*x**2)*a**3*b**3*
c**2*x**3 + 3552*sqrt(a + b*x + c*x**2)*a**3*b**2*c**3*x**4 + 1568*sqrt(a
+ b*x + c*x**2)*a**3*b*c**4*x**5 + 120*sqrt(a + b*x + c*x**2)*a**2*b**6*x*
*2 - 660*sqrt(a + b*x + c*x**2)*a**2*b**5*c*x**3 - 1500*sqrt(a + b*x + c*x
**2)*a**2*b**4*c**2*x**4 - 720*sqrt(a + b*x + c*x**2)*a**2*b**3*c**3*x**5
+ 90*sqrt(a + b*x + c*x**2)*a*b**7*x**3 + 180*sqrt(a + b*x + c*x**2)*a*b**
6*c*x**4 + 90*sqrt(a + b*x + c*x**2)*a*b**5*c**2*x**5 + 960*sqrt(a)*log( -
 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**5*c**3*x**2 - 1200*sqrt(
a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**4*b**2*c**2*x**
2 + 1920*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**4
*b*c**3*x**3 + 1920*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
- b*x)*a**4*c**4*x**4 + 420*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2
) - 2*a - b*x)*a**3*b**4*c*x**2 - 2400*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b
*x + c*x**2) - 2*a - b*x)*a**3*b**3*c**2*x**3 - 1440*sqrt(a)*log( - 2*s...
```

3.170 $\int \frac{d+ex}{(a+bx+cx^2)^{7/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\int \frac{d+ex}{(a+bx+cx^2)^{7/2}} dx = -\frac{2(bd-2ae+(2cd-be)x)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}} + \frac{16(2cd-be)(b+2cx)}{15(b^2-4ac)^2(a+bx+cx^2)^{3/2}} - \frac{128c(2cd-be)(b+2cx)}{15(b^2-4ac)^3\sqrt{a+bx+cx^2}}$$

output
```
1/5*(-2*b*d+4*a*e-2*(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(c*x^2+b*x+a)^(5/2)+16/15
*(-b*e+2*c*d)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(3/2)-128/15*c*(-b*e+
2*c*d)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.59

$$\int \frac{d+ex}{(a+bx+cx^2)^{7/2}} dx = \frac{-2b^5(3d+5ex) - 64c^2(-3a^3e+15a^2cdx+20ac^2dx^3+8c^3dx^5) - 32bc^2(15a^2(d}{}$$

input
```
Integrate[(d + e*x)/(a + b*x + c*x^2)^(7/2),x]
```

output
```
(-2*b^5*(3*d + 5*e*x) - 64*c^2*(-3*a^3*e + 15*a^2*c*d*x + 20*a*c^2*d*x^3 +
 8*c^3*d*x^5) - 32*b*c^2*(15*a^2*(d - e*x) - 8*c^2*x^4*(-5*d + e*x) - 20*a
*c*x^2*(-3*d + e*x)) + 32*b^2*c*(3*a^2*e - 15*a*c*x*(d - 2*e*x) + 10*c^2*x
^3*(-3*d + 2*e*x)) + 80*b^3*c*(2*c*x^2*(-d + 3*e*x) + a*(d + 3*e*x)) + b^4
*(-4*a*e + 20*c*x*(d + 4*e*x)))/(15*(b^2 - 4*a*c)^3*(a + x*(b + c*x))^(5/2
))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1159, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+bx+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{1159}$$

$$-\frac{8(2cd-be)\int \frac{1}{(cx^2+bx+a)^{5/2}}dx}{5(b^2-4ac)}-\frac{2(-2ae+x(2cd-be)+bd)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{8(2cd-be)\left(-\frac{8c\int \frac{1}{\left(cx^2+bx+a\right)^{3/2}}dx}{3(b^2-4ac)}-\frac{2(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}\right)}{5(b^2-4ac)}-\frac{2(-2ae+x(2cd-be)+bd)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(-2ae+x(2cd-be)+bd)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}-\frac{8\left(\frac{16c(b+2cx)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}\right)(2cd-be)}{5(b^2-4ac)}$$

input
```
Int[(d + e*x)/(a + b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/(5*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(5
/2)) - (8*(2*c*d - b*e)*((-2*(b + 2*c*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^
2)^(3/2)) + (16*c*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])))
/(5*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(125) = 250$.

Time = 1.27 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.91

method	result
default	$d\left(\frac{\frac{4cx}{5}+\frac{2b}{5}}{(4ac-b^2)(c\,x^2+bx+a)^{\frac{5}{2}}}+\frac{16c\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^2\right)^2\sqrt{c\,x^2+bx+a}}\right)}{5(4ac-b^2)}\right)+e\left(-\frac{1}{5c(c\,x^2+bx+a)^{\frac{5}{2}}}-\right.$
trager	$-\frac{2(128b\,c^4e\,x^5-256c^5d\,x^5+320b^2c^3e\,x^4-640b\,c^4d\,x^4+320ab\,c^3e\,x^3-640a\,c^4d\,x^3+240b^3c^2e\,x^3-480c^3b^2d\,x^3+480a\,b^2c^2e\,x^2-960}{}$
gosper	$-\frac{2(128b\,c^4e\,x^5-256c^5d\,x^5+320b^2c^3e\,x^4-640b\,c^4d\,x^4+320ab\,c^3e\,x^3-640a\,c^4d\,x^3+240b^3c^2e\,x^3-480c^3b^2d\,x^3+480a\,b^2c^2e\,x^2-960}{15(c\,x^2}$
orering	$-\frac{2(128b\,c^4e\,x^5-256c^5d\,x^5+320b^2c^3e\,x^4-640b\,c^4d\,x^4+320ab\,c^3e\,x^3-640a\,c^4d\,x^3+240b^3c^2e\,x^3-480c^3b^2d\,x^3+480a\,b^2c^2e\,x^2-960}{15(c\,x^2}$

input
```
int((e*x+d)/(c*x^2+b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
d*(2/5*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(
2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c
*x^2+b*x+a)^(1/2)))+e*(-1/5/c/(c*x^2+b*x+a)^(5/2)-1/2*b/c*(2/5*(2*c*x+b)/(
4*a*c-b^2)/(c*x^2+b*x+a)^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(2*c*x+b)/(4*a*c-b^
2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(123) = 246$.

Time = 2.62 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.07

$$\int \frac{d+ex}{(a+bx+cx^2)^{7/2}}\,dx =$$

$$-\frac{2\,(128\,(2\,c^5d-bc^4e)x^5+320\,(2\,bc^4d-b^2c^3e)x^4+80\,(2\,(3\,b^2c^3+4\,ac^4)d-(}{15\,(a^3b^6-12\,a^4b^4c+48\,a^5b^2c^2-64\,a^6c^3+(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6)x^6+3\,(b^7c^2-12\,ab^5c^3}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(128*(2*c^5*d - b*c^4*e)*x^5 + 320*(2*b*c^4*d - b^2*c^3*e)*x^4 + 80*
(2*(3*b^2*c^3 + 4*a*c^4)*d - (3*b^3*c^2 + 4*a*b*c^3)*e)*x^3 + 40*(2*(b^3*c
^2 + 12*a*b*c^3)*d - (b^4*c + 12*a*b^2*c^2)*e)*x^2 + (3*b^5 - 40*a*b^3*c +
 240*a^2*b*c^2)*d + 2*(a*b^4 - 24*a^2*b^2*c - 48*a^3*c^2)*e - 5*(2*(b^4*c
- 24*a*b^2*c^2 - 48*a^2*c^3)*d - (b^5 - 24*a*b^3*c - 48*a^2*b*c^2)*e)*x)*s
qrt(c*x^2 + b*x + a)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3
 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^6 + 3*(b^7*c^2
 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 3*(b^8*c - 11*a*b^6
*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64*a^4*c^5)*x^4 + (b^9 - 6*a*b^7*
c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a^4*b*c^4)*x^3 + 3*(a*b^8 - 11*
a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 - 64*a^5*c^4)*x^2 + 3*(a^2*b^7
 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{(a+bx+cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/(c*x**2+b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{(a+bx+cx^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(123) = 246$.

Time = 0.27 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.23

$$\int \frac{d+ex}{\left(a+bx+cx^2\right)^{7/2}}\,dx =$$

$$-\frac{2\left(\left(8\left(2\left(4\left(\frac{2\left(2c^5d-bc^4e\right)x}{b^6-12ab^4c+48a^2b^2c^2-64a^3c^3}+\frac{5\left(2bc^4d-b^2c^3e\right)}{b^6-12ab^4c+48a^2b^2c^2-64a^3c^3}\right)x+\frac{5\left(6b^2c^3d+8ac^4d-3b^3c^2e-4abc^3e\right)}{b^6-12ab^4c+48a^2b^2c^2-64a^3c^3}\right)x+\frac{5\left(\right.}{}\right.\right.\right.}{}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a)^(7/2),x, algorithm="giac")
```

output
```
-2/15*((8*(2*(4*(2*(2*c^5*d - b*c^4*e)*x/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^
2 - 64*a^3*c^3) + 5*(2*b*c^4*d - b^2*c^3*e)/(b^6 - 12*a*b^4*c + 48*a^2*b^2
*c^2 - 64*a^3*c^3))*x + 5*(6*b^2*c^3*d + 8*a*c^4*d - 3*b^3*c^2*e - 4*a*b*c
^3*e)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(2*b^3*c^2*d
 + 24*a*b*c^3*d - b^4*c*e - 12*a*b^2*c^2*e)/(b^6 - 12*a*b^4*c + 48*a^2*b^2
*c^2 - 64*a^3*c^3))*x - 5*(2*b^4*c*d - 48*a*b^2*c^2*d - 96*a^2*c^3*d - b^5
*e + 24*a*b^3*c*e + 48*a^2*b*c^2*e)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 6
4*a^3*c^3))*x + (3*b^5*d - 40*a*b^3*c*d + 240*a^2*b*c^2*d + 2*a*b^4*e - 48
*a^2*b^2*c*e - 96*a^3*c^2*e)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c
^3))/(c*x^2 + b*x + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 11.32 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.93

$$\int \frac{d+ex}{(a+bx+cx^2)^{7/2}}\,dx = \frac{x\left(\frac{4c^2d}{5(4ac^2-b^2c)}-\frac{2bce}{5(4ac^2-b^2c)}\right)-\frac{4ace}{5(4ac^2-b^2c)}+\frac{2bcd}{5(4ac^2-b^2c)}}{(cx^2+bx+a)^{5/2}} - \frac{x\left(\frac{2c^2(20be-32cd)}{15(4ac^2-b^2c)(4ac-b^2)}-\frac{8bc^2e}{15(4ac^2-b^2c)(4ac-b^2)}\right)+\frac{bc(20be-32cd)}{15(4ac^2-b^2c)(4ac-b^2)}-\frac{16ac^2e}{15(4ac^2-b^2c)(4ac-b^2)}}{(cx^2+bx+a)^{3/2}} + \frac{\frac{bc(256c^2d-128bce)}{15(4ac^2-b^2c)(4ac-b^2)^2}+\frac{2c^2x(256c^2d-128bce)}{15(4ac^2-b^2c)(4ac-b^2)^2}}{\sqrt{cx^2+bx+a}} - \frac{4e}{(60ac-15b^2)(cx^2+bx+a)^{3/2}}$$

input
```
int((d + e*x)/(a + b*x + c*x^2)^(7/2),x)
```

output
```
(x*((4*c^2*d)/(5*(4*a*c^2 - b^2*c)) - (2*b*c*e)/(5*(4*a*c^2 - b^2*c))) - (
4*a*c*e)/(5*(4*a*c^2 - b^2*c)) + (2*b*c*d)/(5*(4*a*c^2 - b^2*c)))/(a + b*x
 + c*x^2)^(5/2) - (x*((2*c^2*(20*b*e - 32*c*d))/(15*(4*a*c^2 - b^2*c)*(4*a
*c - b^2)) - (8*b*c^2*e)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*c*(20*
b*e - 32*c*d))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (16*a*c^2*e)/(15*(4*
a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(3/2) + ((b*c*(256*c^2*d
- 128*b*c*e))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (2*c^2*x*(256*c^2*d
 - 128*b*c*e))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(
1/2) - (4*e)/((60*a*c - 15*b^2)*(a + b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 1120, normalized size of antiderivative = 8.30

$$\int \frac{d+ex}{(a+bx+cx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/(c*x^2+b*x+a)^(7/2),x)
```

output
```
(2*( - 96*sqrt(a + b*x + c*x**2)*a**3*c**2*e - 48*sqrt(a + b*x + c*x**2)*a
**2*b**2*c*e + 240*sqrt(a + b*x + c*x**2)*a**2*b*c**2*d - 240*sqrt(a + b*x
 + c*x**2)*a**2*b*c**2*e*x + 480*sqrt(a + b*x + c*x**2)*a**2*c**3*d*x + 2*
sqrt(a + b*x + c*x**2)*a*b**4*e - 40*sqrt(a + b*x + c*x**2)*a*b**3*c*d - 1
20*sqrt(a + b*x + c*x**2)*a*b**3*c*e*x + 240*sqrt(a + b*x + c*x**2)*a*b**2
*c**2*d*x - 480*sqrt(a + b*x + c*x**2)*a*b**2*c**2*e*x**2 + 960*sqrt(a + b
*x + c*x**2)*a*b*c**3*d*x**2 - 320*sqrt(a + b*x + c*x**2)*a*b*c**3*e*x**3
+ 640*sqrt(a + b*x + c*x**2)*a*c**4*d*x**3 + 3*sqrt(a + b*x + c*x**2)*b**5
*d + 5*sqrt(a + b*x + c*x**2)*b**5*e*x - 10*sqrt(a + b*x + c*x**2)*b**4*c*
d*x - 40*sqrt(a + b*x + c*x**2)*b**4*c*e*x**2 + 80*sqrt(a + b*x + c*x**2)*
b**3*c**2*d*x**2 - 240*sqrt(a + b*x + c*x**2)*b**3*c**2*e*x**3 + 480*sqrt(
a + b*x + c*x**2)*b**2*c**3*d*x**3 - 320*sqrt(a + b*x + c*x**2)*b**2*c**3*
e*x**4 + 640*sqrt(a + b*x + c*x**2)*b*c**4*d*x**4 - 128*sqrt(a + b*x + c*x
**2)*b*c**4*e*x**5 + 256*sqrt(a + b*x + c*x**2)*c**5*d*x**5 + 128*sqrt(c)*
a**3*b*c*e - 256*sqrt(c)*a**3*c**2*d + 384*sqrt(c)*a**2*b**2*c*e*x - 768*s
qrt(c)*a**2*b*c**2*d*x + 384*sqrt(c)*a**2*b*c**2*e*x**2 - 768*sqrt(c)*a**2
*c**3*d*x**2 + 384*sqrt(c)*a*b**3*c*e*x**2 - 768*sqrt(c)*a*b**2*c**2*d*x**
2 + 768*sqrt(c)*a*b**2*c**2*e*x**3 - 1536*sqrt(c)*a*b*c**3*d*x**3 + 384*sq
rt(c)*a*b*c**3*e*x**4 - 768*sqrt(c)*a*c**4*d*x**4 + 128*sqrt(c)*b**4*c*e*x
**3 - 256*sqrt(c)*b**3*c**2*d*x**3 + 384*sqrt(c)*b**3*c**2*e*x**4 - 768...
```

3.171 $\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 181

$$\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx = -\frac{2(bd-2ae+(2cd-be)x)}{7(b^2-4ac)(a+bx+cx^2)^{7/2}} + \frac{24(2cd-be)(b+2cx)}{35(b^2-4ac)^2(a+bx+cx^2)^{5/2}} - \frac{128c(2cd-be)(b+2cx)}{35(b^2-4ac)^3(a+bx+cx^2)^{3/2}} + \frac{1024c^2(2cd-be)(b+2cx)}{35(b^2-4ac)^4\sqrt{a+bx+cx^2}}$$

output
```
1/7*(-2*b*d+4*a*e-2*(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(c*x^2+b*x+a)^(7/2)+24/35
*(-b*e+2*c*d)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(5/2)-128/35*c*(-b*e+
2*c*d)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)^(3/2)+1024/35*c^2*(-b*e+2*c*
d)*(2*c*x+b)/(-4*a*c+b^2)^4/(c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 8.13 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.93

$$\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx =$$
$$-\frac{2(b^7(5d+7ex)-128c^3(-5a^4e+35a^3cdx+70a^2c^2dx^3+56ac^3dx^5+16c^4dx^7)-64bc^3(35a^3(d-ex)-1}{}$$

input
```
Integrate[(d + e*x)/(a + b*x + c*x^2)^(9/2),x]
```

output
```
(-2*(b^7*(5*d + 7*e*x) - 128*c^3*(-5*a^4*e + 35*a^3*c*d*x + 70*a^2*c^2*d*x
^3 + 56*a*c^3*d*x^5 + 16*c^4*d*x^7) - 64*b*c^3*(35*a^3*(d - e*x) - 16*c^3*
x^6*(-7*d + e*x) - 56*a*c^2*x^4*(-5*d + e*x) - 70*a^2*c*x^2*(-3*d + e*x))
+ 32*b^2*c^2*(15*a^3*e - 105*a^2*c*x*(d - 2*e*x) + 56*c^3*x^5*(-5*d + 2*e*
x) + 140*a*c^2*x^3*(-3*d + 2*e*x)) + 2*b^6*(a*e - 7*c*x*(d + 2*e*x)) + 560
*b^3*c^2*(-4*a*c*x^2*(d - 3*e*x) + 8*c^2*x^4*(-d + e*x) + a^2*(d + 3*e*x))
 - 40*b^4*c*(a^2*e + 14*c^2*x^3*(d - 4*e*x) - 7*a*c*x*(d + 4*e*x)) - 28*b^
5*c*(-2*c*x^2*(d + 5*e*x) + a*(3*d + 5*e*x))))/(35*(b^2 - 4*a*c)^4*(a + x*
(b + c*x))^(7/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1159, 1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx$$
$$\Big\downarrow \; 1159$$
$$-\frac{12(2cd-be)\int \frac{1}{(cx^2+bx+a)^{7/2}}dx}{7(b^2-4ac)}-\frac{2(-2ae+x(2cd-be)+bd)}{7(b^2-4ac)(a+bx+cx^2)^{7/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{12(2cd-be)\left(-\frac{16c\int \frac{1}{(cx^2+bx+a)^{5/2}}dx}{5(b^2-4ac)}-\frac{2(b+2cx)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}\right)}{7(b^2-4ac)}-\frac{2(-2ae+x(2cd-be)+bd)}{7(b^2-4ac)(a+bx+cx^2)^{7/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{12(2cd-be)\left(-\frac{16c\left(-\frac{8c\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}-\frac{2(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}\right)}{5(b^2-4ac)}-\frac{2(b+2cx)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}\right)}{7(b^2-4ac)}-\frac{2(-2ae+x(2cd-be)+bd)}{7(b^2-4ac)(a+bx+cx^2)^{7/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(-2ae+x(2cd-be)+bd)}{7(b^2-4ac)(a+bx+cx^2)^{7/2}}-\frac{12\left(-\frac{2(b+2cx)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}-\frac{16c\left(\frac{16c(b+2cx)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}\right)}{5(b^2-4ac)}\right)(2cd-be)}{7(b^2-4ac)}$$

input

```
Int[(d + e*x)/(a + b*x + c*x^2)^(9/2),x]
```

output

```
(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/(7*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(7
/2)) - (12*(2*c*d - b*e)*((-2*(b + 2*c*x))/(5*(b^2 - 4*a*c)*(a + b*x + c*x
^2)^(5/2)) - (16*c*((-2*(b + 2*c*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3
/2)) + (16*c*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])))/(5*(
b^2 - 4*a*c))))/(7*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && 
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p + 
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(167) = 334$.

Time = 1.21 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.96

method	result
default	$d\left(\frac{\frac{4cx}{7}+\frac{2b}{7}}{(4ac-b^2)(c\,x^2+bx+a)^{\frac{7}{2}}}+\frac{24c\left(\frac{\frac{4cx}{5}+\frac{2b}{5}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}+\frac{16c\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^2\right)^2\sqrt{c\,x^2+bx+a}}\right)}{5\left(4ac-b^2\right)}\right)}{7(4ac-b^2)}\right)+e$
trager	$-\frac{2(1024b\,c^6e\,x^7-2048c^7d\,x^7+3584b^2c^5e\,x^6-7168b\,c^6d\,x^6+3584ab\,c^5e\,x^5-7168a\,c^6d\,x^5+4480b^3c^4e\,x^5-8960b^2c^5d\,x^5+8960a\,b^2c}{}$
gosper	$-\frac{2(1024b\,c^6e\,x^7-2048c^7d\,x^7+3584b^2c^5e\,x^6-7168b\,c^6d\,x^6+3584ab\,c^5e\,x^5-7168a\,c^6d\,x^5+4480b^3c^4e\,x^5-8960b^2c^5d\,x^5+8960a\,b^2c}{}$
orering	$-\frac{2(1024b\,c^6e\,x^7-2048c^7d\,x^7+3584b^2c^5e\,x^6-7168b\,c^6d\,x^6+3584ab\,c^5e\,x^5-7168a\,c^6d\,x^5+4480b^3c^4e\,x^5-8960b^2c^5d\,x^5+8960a\,b^2c}{}$

input
```
int((e*x+d)/(c*x^2+b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
d*(2/7*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(7/2)+24/7*c/(4*a*c-b^2)*(2/5*(
2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(2*c*x+b)
/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x
+a)^(1/2))))+e*(-1/7/c/(c*x^2+b*x+a)^(7/2)-1/2*b/c*(2/7*(2*c*x+b)/(4*a*c-b
^2)/(c*x^2+b*x+a)^(7/2)+24/7*c/(4*a*c-b^2)*(2/5*(2*c*x+b)/(4*a*c-b^2)/(c*x
^2+b*x+a)^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(165) = 330$.

Time = 13.60 (sec) , antiderivative size = 941, normalized size of antiderivative = 5.20

$$\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
2/35*(1024*(2*c^7*d - b*c^6*e)*x^7 + 3584*(2*b*c^6*d - b^2*c^5*e)*x^6 + 89
6*(2*(5*b^2*c^5 + 4*a*c^6)*d - (5*b^3*c^4 + 4*a*b*c^5)*e)*x^5 + 2240*(2*(b
^3*c^4 + 4*a*b*c^5)*d - (b^4*c^3 + 4*a*b^2*c^4)*e)*x^4 + 280*(2*(b^4*c^3 +
 24*a*b^2*c^4 + 16*a^2*c^5)*d - (b^5*c^2 + 24*a*b^3*c^3 + 16*a^2*b*c^4)*e)
*x^3 - 28*(2*(b^5*c^2 - 40*a*b^3*c^3 - 240*a^2*b*c^4)*d - (b^6*c - 40*a*b^
4*c^2 - 240*a^2*b^2*c^3)*e)*x^2 - (5*b^7 - 84*a*b^5*c + 560*a^2*b^3*c^2 -
2240*a^3*b*c^3)*d - 2*(a*b^6 - 20*a^2*b^4*c + 240*a^3*b^2*c^2 + 320*a^4*c^
3)*e + 7*(2*(b^6*c - 20*a*b^4*c^2 + 240*a^2*b^2*c^3 + 320*a^3*c^4)*d - (b^
7 - 20*a*b^5*c + 240*a^2*b^3*c^2 + 320*a^3*b*c^3)*e)*x)*sqrt(c*x^2 + b*x +
 a)/(a^4*b^8 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3 + 256*a^8*c
^4 + (b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 256*a^4*
c^8)*x^8 + 4*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 +
256*a^4*b*c^7)*x^7 + 2*(3*b^10*c^2 - 46*a*b^8*c^3 + 256*a^2*b^6*c^4 - 576*
a^3*b^4*c^5 + 256*a^4*b^2*c^6 + 512*a^5*c^7)*x^6 + 4*(b^11*c - 13*a*b^9*c^
2 + 48*a^2*b^7*c^3 + 32*a^3*b^5*c^4 - 512*a^4*b^3*c^5 + 768*a^5*b*c^6)*x^5
 + (b^12 - 4*a*b^10*c - 90*a^2*b^8*c^2 + 800*a^3*b^6*c^3 - 2240*a^4*b^4*c^
4 + 1536*a^5*b^2*c^5 + 1536*a^6*c^6)*x^4 + 4*(a*b^11 - 13*a^2*b^9*c + 48*a
^3*b^7*c^2 + 32*a^4*b^5*c^3 - 512*a^5*b^3*c^4 + 768*a^6*b*c^5)*x^3 + 2*(3*
a^2*b^10 - 46*a^3*b^8*c + 256*a^4*b^6*c^2 - 576*a^5*b^4*c^3 + 256*a^6*b^2*
c^4 + 512*a^7*c^5)*x^2 + 4*(a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^5*c^2 - 2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/(c*x**2+b*x+a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(165) = 330$.

Time = 0.25 (sec) , antiderivative size = 768, normalized size of antiderivative = 4.24

$$\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^2+b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/35*((4*(2*(8*(2*(4*(2*(2*c^7*d - b*c^6*e)*x/(b^8 - 16*a*b^6*c + 96*a^2*b
^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4) + 7*(2*b*c^6*d - b^2*c^5*e)/(b^8 -
 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x + 7*(10*b
^2*c^5*d + 8*a*c^6*d - 5*b^3*c^4*e - 4*a*b*c^5*e)/(b^8 - 16*a*b^6*c + 96*a
^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x + 35*(2*b^3*c^4*d + 8*a*b*c
^5*d - b^4*c^3*e - 4*a*b^2*c^4*e)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 + 256*a^4*c^4))*x + 35*(2*b^4*c^3*d + 48*a*b^2*c^4*d + 32*a^2
*c^5*d - b^5*c^2*e - 24*a*b^3*c^3*e - 16*a^2*b*c^4*e)/(b^8 - 16*a*b^6*c +
96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x - 7*(2*b^5*c^2*d - 80*a
*b^3*c^3*d - 480*a^2*b*c^4*d - b^6*c*e + 40*a*b^4*c^2*e + 240*a^2*b^2*c^3*
e)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x
+ 7*(2*b^6*c*d - 40*a*b^4*c^2*d + 480*a^2*b^2*c^3*d + 640*a^3*c^4*d - b^7*
e + 20*a*b^5*c*e - 240*a^2*b^3*c^2*e - 320*a^3*b*c^3*e)/(b^8 - 16*a*b^6*c
+ 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x - (5*b^7*d - 84*a*b^5
*c*d + 560*a^2*b^3*c^2*d - 2240*a^3*b*c^3*d + 2*a*b^6*e - 40*a^2*b^4*c*e +
 480*a^3*b^2*c^2*e + 640*a^4*c^3*e)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 2
56*a^3*b^2*c^3 + 256*a^4*c^4))/(c*x^2 + b*x + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 11.79 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.31

$$\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx = \frac{x\left(\frac{2\,c^2\,(768\,c^2\,d-368\,b\,c\,e)}{105\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^2}-\frac{32\,b\,c^3\,e}{105\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^2}\right)+\frac{b\,c\,(768\,c^2\,d-368\,b\,c\,e)}{105\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^2}-\frac{\cdots}{105\,(\cdots}}{(c\,x^2+b\,x+a)^{3/2}}$$

$$+\frac{x\left(\frac{4\,c^2\,d}{7\,(4\,a\,c^2-b^2\,c)}-\frac{2\,b\,c\,e}{7\,(4\,a\,c^2-b^2\,c)}\right)-\frac{4\,a\,c\,e}{7\,(4\,a\,c^2-b^2\,c)}+\frac{2\,b\,c\,d}{7\,(4\,a\,c^2-b^2\,c)}}{(c\,x^2+b\,x+a)^{7/2}}$$

$$-\frac{x\left(\frac{2\,c^2\,(28\,b\,e-48\,c\,d)}{35\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}-\frac{8\,b\,c^2\,e}{35\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}\right)+\frac{b\,c\,(28\,b\,e-48\,c\,d)}{35\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}-\frac{16\,a\,c^2\,e}{35\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}}{(c\,x^2+b\,x+a)^{5/2}}$$

$$+\frac{\frac{2\,c^2\,x\,(2048\,c^3\,d-1024\,b\,c^2\,e)}{35\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^3}+\frac{b\,c\,(2048\,c^3\,d-1024\,b\,c^2\,e)}{35\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^3}}{\sqrt{c\,x^2+b\,x+a}}$$

$$-\frac{4\,e}{(140\,a\,c-35\,b^2)\,(c\,x^2+b\,x+a)^{5/2}}+\frac{16\,c\,e}{105\,(4\,a\,c-b^2)^2\,(c\,x^2+b\,x+a)^{3/2}}$$

input
```
int((d + e*x)/(a + b*x + c*x^2)^(9/2),x)
```

output
```
(x*((2*c^2*(768*c^2*d - 368*b*c*e))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2
) - (32*b*c^3*e)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2)) + (b*c*(768*c^2*
d - 368*b*c*e))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) - (64*a*c^3*e)/(10
5*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(3/2) + (x*((4*c^2
*d)/(7*(4*a*c^2 - b^2*c)) - (2*b*c*e)/(7*(4*a*c^2 - b^2*c))) - (4*a*c*e)/(
7*(4*a*c^2 - b^2*c)) + (2*b*c*d)/(7*(4*a*c^2 - b^2*c)))/(a + b*x + c*x^2)^
(7/2) - (x*((2*c^2*(28*b*e - 48*c*d))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2))
 - (8*b*c^2*e)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*c*(28*b*e - 48*c
*d))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (16*a*c^2*e)/(35*(4*a*c^2 - b^
2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(5/2) + ((2*c^2*x*(2048*c^3*d - 102
4*b*c^2*e))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3) + (b*c*(2048*c^3*d - 10
24*b*c^2*e))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x + c*x^2)^(1/
2) - (4*e)/((140*a*c - 35*b^2)*(a + b*x + c*x^2)^(5/2)) + (16*c*e)/(105*(4
*a*c - b^2)^2*(a + b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 41.34 (sec) , antiderivative size = 1920, normalized size of antiderivative = 10.61

$$\int \frac{d+ex}{(a+bx+cx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/(c*x^2+b*x+a)^(9/2),x)
```

output

```
(2*( - 640*sqrt(a + b*x + c*x**2)*a**4*c**3*e - 480*sqrt(a + b*x + c*x**2)
*a**3*b**2*c**2*e + 2240*sqrt(a + b*x + c*x**2)*a**3*b*c**3*d - 2240*sqrt(
a + b*x + c*x**2)*a**3*b*c**3*e*x + 4480*sqrt(a + b*x + c*x**2)*a**3*c**4*
d*x + 40*sqrt(a + b*x + c*x**2)*a**2*b**4*c*e - 560*sqrt(a + b*x + c*x**2)
*a**2*b**3*c**2*d - 1680*sqrt(a + b*x + c*x**2)*a**2*b**3*c**2*e*x + 3360*
sqrt(a + b*x + c*x**2)*a**2*b**2*c**3*d*x - 6720*sqrt(a + b*x + c*x**2)*a*
*2*b**2*c**3*e*x**2 + 13440*sqrt(a + b*x + c*x**2)*a**2*b*c**4*d*x**2 - 44
80*sqrt(a + b*x + c*x**2)*a**2*b*c**4*e*x**3 + 8960*sqrt(a + b*x + c*x**2)
*a**2*c**5*d*x**3 - 2*sqrt(a + b*x + c*x**2)*a*b**6*e + 84*sqrt(a + b*x +
c*x**2)*a*b**5*c*d + 140*sqrt(a + b*x + c*x**2)*a*b**5*c*e*x - 280*sqrt(a
+ b*x + c*x**2)*a*b**4*c**2*d*x - 1120*sqrt(a + b*x + c*x**2)*a*b**4*c**2*
e*x**2 + 2240*sqrt(a + b*x + c*x**2)*a*b**3*c**3*d*x**2 - 6720*sqrt(a + b*
x + c*x**2)*a*b**3*c**3*e*x**3 + 13440*sqrt(a + b*x + c*x**2)*a*b**2*c**4*
d*x**3 - 8960*sqrt(a + b*x + c*x**2)*a*b**2*c**4*e*x**4 + 17920*sqrt(a + b
*x + c*x**2)*a*b*c**5*d*x**4 - 3584*sqrt(a + b*x + c*x**2)*a*b*c**5*e*x**5
 + 7168*sqrt(a + b*x + c*x**2)*a*c**6*d*x**5 - 5*sqrt(a + b*x + c*x**2)*b*
*7*d - 7*sqrt(a + b*x + c*x**2)*b**7*e*x + 14*sqrt(a + b*x + c*x**2)*b**6*
c*d*x + 28*sqrt(a + b*x + c*x**2)*b**6*c*e*x**2 - 56*sqrt(a + b*x + c*x**2
)*b**5*c**2*d*x**2 - 280*sqrt(a + b*x + c*x**2)*b**5*c**2*e*x**3 + 560*sqr
t(a + b*x + c*x**2)*b**4*c**3*d*x**3 - 2240*sqrt(a + b*x + c*x**2)*b**4...
```

3.172 $\int \frac{1-x}{x\sqrt{1+3x+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \frac{1-x}{x\sqrt{1+3x+x^2}}\,dx = -2\operatorname{arctanh}\left(\frac{1+x}{\sqrt{1+3x+x^2}}\right)$$

output
```
-2*arctanh((1+x)/(x^2+3*x+1)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \frac{1-x}{x\sqrt{1+3x+x^2}}\,dx = 2\operatorname{arctanh}\left(x-\sqrt{1+3x+x^2}\right)+\log\left(-3-2x+2\sqrt{1+3x+x^2}\right)$$

input
```
Integrate[(1 - x)/(x*Sqrt[1 + 3*x + x^2]),x]
```

output
```
2*ArcTanh[x - Sqrt[1 + 3*x + x^2]] + Log[-3 - 2*x + 2*Sqrt[1 + 3*x + x^2]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1239, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{x\sqrt{x^2+3x+1}}dx$$
$$\Big\downarrow \text{1239}$$
$$-4\int \frac{1}{4-\frac{4(x+1)^2}{x^2+3x+1}}d\frac{2(x+1)}{\sqrt{x^2+3x+1}}$$
$$\Big\downarrow \text{219}$$
$$-2\text{arctanh}\left(\frac{x+1}{\sqrt{x^2+3x+1}}\right)$$

input
```
Int[(1 - x)/(x*Sqrt[1 + 3*x + x^2]),x]
```

output
```
-2*ArcTanh[(1 + x)/Sqrt[1 + 3*x + x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1239
```
Int[((f_) + (g_.)*(x_))/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_
.)*(x_)^2]), x_Symbol] :> Simp[4*f*((a - d)/(b*d - a*e))   Subst[Int[1/(4*(
a - d) - x^2), x], x, (2*(a - d) + (b - e)*x)/Sqrt[a + b*x + c*x^2]], x] /;
 FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[4*c*(a - d) - (b - e)^2, 0] && EqQ[
e*f*(b - e) - 2*g*(b*d - a*e), 0] && NeQ[b*d - a*e, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.98 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

method	result	size
default	$-\ln\left(x+\frac{3}{2}+\sqrt{x^2+3x+1}\right)-\operatorname{arctanh}\left(\frac{3x+2}{2\sqrt{x^2+3x+1}}\right)$	38
trager	$-\ln\left(\frac{2\sqrt{x^2+3x+1}\,x+2x^2+2\sqrt{x^2+3x+1}+5x+2}{x}\right)$	43

input
```
int((1-x)/x/(x^2+3*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(x+3/2+(x^2+3*x+1)^(1/2))-arctanh(1/2*(3*x+2)/(x^2+3*x+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\int \frac{1-x}{x\sqrt{1+3x+x^2}}\,dx = \log\left(4\,x^2-\sqrt{x^2+3\,x+1}(4\,x+5)+11\,x+5\right) - \log\left(-x+\sqrt{x^2+3\,x+1}+1\right)$$

input
```
integrate((1-x)/x/(x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

output
```
log(4*x^2 - sqrt(x^2 + 3*x + 1)*(4*x + 5) + 11*x + 5) - log(-x + sqrt(x^2
+ 3*x + 1) + 1)
```

Sympy [F]

$$\int \frac{1-x}{x\sqrt{1+3x+x^2}}\,dx = -\int \left(-\frac{1}{x\sqrt{x^2+3x+1}}\right)dx - \int \frac{1}{\sqrt{x^2+3x+1}}\,dx$$

input
```
integrate((1-x)/x/(x**2+3*x+1)**(1/2),x)
```

output
```
-Integral(-1/(x*sqrt(x**2 + 3*x + 1)), x) - Integral(1/sqrt(x**2 + 3*x + 1
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{1-x}{x\sqrt{1+3x+x^2}}\,dx = -\log\left(2\,x + 2\,\sqrt{x^2+3\,x+1} + 3\right) - \log\left(\frac{2\,\sqrt{x^2+3\,x+1}}{|x|} + \frac{2}{|x|} + 3\right)$$

input
```
integrate((1-x)/x/(x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-log(2*x + 2*sqrt(x^2 + 3*x + 1) + 3) - log(2*sqrt(x^2 + 3*x + 1)/abs(x) +
 2/abs(x) + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int \frac{1-x}{x\sqrt{1+3x+x^2}}\,dx = -\log\left(\left|-x + \sqrt{x^2+3\,x+1} + 1\right|\right) + \log\left(\left|-x + \sqrt{x^2+3\,x+1} - 1\right|\right) + \log\left(\left|-2\,x + 2\,\sqrt{x^2+3\,x+1} - 3\right|\right)$$

input
```
integrate((1-x)/x/(x^2+3*x+1)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 + 3*x + 1) + 1)) + log(abs(-x + sqrt(x^2 + 3*x + 1)
 - 1)) + log(abs(-2*x + 2*sqrt(x^2 + 3*x + 1) - 3))
```

Mupad [B] (verification not implemented)

Time = 11.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{1-x}{x\sqrt{1+3x+x^2}}\,dx = -\ln\left(\frac{3\,x+2\,\sqrt{x^2+3\,x+1}+2}{x}\right) - \ln\left(x+\sqrt{x^2+3\,x+1}+\frac{3}{2}\right)$$

input
```
int(-(x - 1)/(x*(3*x + x^2 + 1)^(1/2)),x)
```

output
```
- log((3*x + 2*(3*x + x^2 + 1)^(1/2) + 2)/x) - log(x + (3*x + x^2 + 1)^(1/
2) + 3/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.74

$$\int \frac{1-x}{x\sqrt{1+3x+x^2}}\,dx = -\log\left(\frac{10\sqrt{x^2+3x+1}+10x+10}{\sqrt{5}}\right) + \log\left(\frac{2\sqrt{x^2+3x+1}+2x-2}{\sqrt{5}}\right) - \log\left(\frac{2\sqrt{x^2+3x+1}+2x+3}{\sqrt{5}}\right)$$

input
```
int((1-x)/x/(x^2+3*x+1)^(1/2),x)
```

output
```
 - log((10*sqrt(x**2 + 3*x + 1) + 10*x + 10)/sqrt(5)) + log((2*sqrt(x**2 +
 3*x + 1) + 2*x - 2)/sqrt(5)) - log((2*sqrt(x**2 + 3*x + 1) + 2*x + 3)/sqr
t(5))
```

3.173 $\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 548

$$\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx = -\frac{2(4b^2B-7Abc-10aBc)\sqrt{x}\sqrt{a+bx+cx^2}}{105c^2}$$
$$+\frac{2x^{3/2}(bB+7Ac+5Bcx)\sqrt{a+bx+cx^2}}{35c}$$
$$+\frac{(8b^3B-14Ab^2c-29abBc+42aAc^2)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c}{b+\sqrt{b^2}}}\cdots}{105\sqrt{2}c^{7/2}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\left(8b^3B-14Ab^2c-29abBc+42aAc^2-\frac{2ac(4b^2B-7Abc-10aBc)}{b+\sqrt{b^2-4ac}}\right)\sqrt{1\cdots}}{105\sqrt{2}c^{7/2}\sqrt{a+bx+cx^2}}$$

output
```
-2/105*(-7*A*b*c-10*B*a*c+4*B*b^2)*x^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2+2/35*x^
(3/2)*(5*B*c*x+7*A*c+B*b)*(c*x^2+b*x+a)^(1/2)/c+1/210*(42*A*a*c^2-14*A*b^2
*c-29*B*a*b*c+8*B*b^3)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2)
)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^
(1/2)*EllipticE(2^(1/2)*c^(1/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-
(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(7/2)/(c*x^2+
b*x+a)^(1/2)-1/210*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(8
*B*b^3-14*A*b^2*c-29*B*a*b*c+42*A*a*c^2-2*a*c*(-7*A*b*c-10*B*a*c+4*B*b^2)/
(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*x^(1/2)/(-b+(-4*a*c+
b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2
^(1/2)/c^(7/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.30 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.16

$$\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{2\sqrt{x}\sqrt{a+x(b+cx)}(-4b^2B+bc(7A+3Bx)+c(10aB+3cx(7A+5Bx)))}{105c^2}$$
$$- \frac{-4(8b^3B-14Ab^2c-29abBc+42aAc^2)\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(b+cx))+i(8b^3B-14Ab^2c-29abBc+}{}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[x]*Sqrt[a + x*(b + c*x)]*(-4*b^2*B + b*c*(7*A + 3*B*x) + c*(10*a*B
 + 3*c*x*(7*A + 5*B*x))))/(105*c^2) - (-4*(8*b^3*B - 14*A*b^2*c - 29*a*b*B
*c + 42*a*A*c^2)*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(a + x*(b + c*x)) + I*(8*
b^3*B - 14*A*b^2*c - 29*a*b*B*c + 42*a*A*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqr
t[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqr
t[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2
]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])] + I*(8*b^4*B + 2*a*c^2*(10*a*B - 21*A*Sqrt[b^2 - 4*a*c
]) - 2*b^3*(7*A*c + 4*B*Sqrt[b^2 - 4*a*c]) + a*b*c*(56*A*c + 29*B*Sqrt[b^2
 - 4*a*c]) + b^2*(-37*a*B*c + 14*A*c*Sqrt[b^2 - 4*a*c]))*Sqrt[1 + (2*a)/((
b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]
*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b +
Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a
*c])])/(210*c^3*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x]*Sqrt[a + x*(b + c*
x)])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1236, 27, 1231, 27, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2\int -\frac{(aB+(4bB-7Ac)x)\sqrt{cx^2+bx+a}}{2\sqrt{x}}dx}{7c}+\frac{2B\sqrt{x}\left(a+bx+cx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\sqrt{x}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\int \frac{(aB+(4bB-7Ac)x)\sqrt{cx^2+bx+a}}{\sqrt{x}}dx}{7c}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2B\sqrt{x}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{2\sqrt{x}\sqrt{a+bx+cx^2}\left(5aBc+3cx(4bB-7Ac)-7Abc+4b^2B\right)}{15c}-\frac{2\int \frac{a\left(4Bb^2-7Acb-10aBc\right)-\left(5abBc-2\left(b^2-3ac\right)(4bB-7Ac)\right)x}{2\sqrt{x}\sqrt{cx^2+bx+a}}dx}{15c}}{7c}$$

$\Big\downarrow$ 27

$$\frac{2B\sqrt{x}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{2\sqrt{x}\sqrt{a+bx+cx^2}\left(5aBc+3cx(4bB-7Ac)-7Abc+4b^2B\right)}{15c}-\frac{\int \frac{a\left(4Bb^2-7Acb-10aBc\right)-\left(5abBc-2\left(b^2-3ac\right)(4bB-7Ac)\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{15c}}{7c}$$

$\Big\downarrow$ 1240

$$\frac{2B\sqrt{x}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{2\sqrt{x}\sqrt{a+bx+cx^2}\left(5aBc+3cx(4bB-7Ac)-7Abc+4b^2B\right)}{15c}-\frac{2\int \frac{a\left(4Bb^2-7Acb-10aBc\right)-\left(5abBc-2\left(b^2-3ac\right)(4bB-7Ac)\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{15c}}{7c}$$

$\Big\downarrow$ 1511

$$\frac{2B\sqrt{x}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{2\sqrt{x}\sqrt{a+bx+cx^2}\left(5aBc+3cx(4bB-7Ac)-7Abc+4b^2B\right)}{15c}-\frac{2\left(\frac{\sqrt{a}\left(5abBc-2\left(b^2-3ac\right)(4bB-7Ac)\right)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}\left(-\sqrt{a}\sqrt{c}\left(-10aBc-7Ab\cdots\right)\right)}{\cdots}\right)}{15c}}{7c}$$

$\Big\downarrow$ 27

$$\frac{2B\sqrt{x}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{2\sqrt{x}\sqrt{a+bx+cx^2}\left(5aBc+3cx(4bB-7Ac)-7Abc+4b^2B\right)}{15c}-\frac{2\left(\frac{\left(5abBc-2\left(b^2-3ac\right)(4bB-7Ac)\right)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}\left(-\sqrt{a}\sqrt{c}\left(-10aBc-7Abc+4b^2\cdots\right)\right)}{\cdots}\right)}{15c}}{7c}$$

$\Big\downarrow$ 1416

$$\frac{2B\sqrt{x}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{2\sqrt{x}\sqrt{a+bx+cx^2}\left(5aBc+3cx(4bB-7Ac)-7Abc+4b^2B\right)}{15c}-\frac{2\left(\frac{\left(5abBc-2\left(b^2-3ac\right)(4bB-7Ac)\right)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\left(-\cdots\right)}{\cdots}\right)}{\cdots}}{7c}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2B\sqrt{x}\left(a+bx+cx^2\right)^{3/2}}{7c}-\frac{\frac{2\sqrt{x}\sqrt{a+bx+cx^2}\left(5aBc+3cx(4bB-7Ac)-7Abc+4b^2B\right)}{15c}-\frac{2\left(\frac{\left(5abBc-2\left(b^2-3ac\right)(4bB-7Ac)\right)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right)\right.}{\sqrt[4]{c}\sqrt{a+bx+cx^2}}\right.}{\sqrt{c}}\right.}{}}{}$$

input
```
Int[Sqrt[x]*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*B*Sqrt[x]*(a + b*x + c*x^2)^(3/2))/(7*c) - ((2*Sqrt[x]*(4*b^2*B - 7*A*b
*c + 5*a*B*c + 3*c*(4*b*B - 7*A*c)*x)*Sqrt[a + b*x + c*x^2])/(15*c) - (2*(
((5*a*b*B*c - 2*(b^2 - 3*a*c)*(4*b*B - 7*A*c))*(-((Sqrt[x]*Sqrt[a + b*x +
c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a +
b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))
/Sqrt[c] - (a^(1/4)*(5*a*b*B*c - 2*(b^2 - 3*a*c)*(4*b*B - 7*A*c) - Sqrt[a]
*Sqrt[c]*(4*b^2*B - 7*A*b*c - 10*a*B*c))*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b
*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x + c*x^2]))
)/(15*c))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.59

method	result
elliptic	$\frac{\sqrt{x(c\,x^2+bx+a)}\left(\frac{2B\,x^2\sqrt{c\,x^3+b\,x^2+ax}}{7}+\frac{2\left(Ac+\frac{Bb}{7}\right)x\sqrt{c\,x^3+b\,x^2+ax}}{5c}+\frac{2\left(Ab+\frac{2Ba}{7}-\frac{4b\left(Ac+\frac{Bb}{7}\right)}{5c}\right)\sqrt{c\,x^3+b\,x^2+ax}}{3c}-\frac{a\left(Ab+\frac{2Ba}{7}-\frac{4b(}{}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(c*x^2+b*x+a))^(1/2)/x^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*B*x^2*(c*x^3+b*x^
2+a*x)^(1/2)+2/5*(A*c+1/7*B*b)/c*x*(c*x^3+b*x^2+a*x)^(1/2)+2/3*(A*b+2/7*B*
a-4/5*b/c*(A*c+1/7*B*b))/c*(c*x^3+b*x^2+a*x)^(1/2)-1/3*a/c^2*(A*b+2/7*B*a-
4/5*b/c*(A*c+1/7*B*b))*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*((x+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)*EllipticF(
2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),
1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2))+(A*a-3/5*a/c*(A*c+1/7*B*b)-2/3*b/c*(A*b+2/7*B
*a-4/5*b/c*(A*c+1/7*B*b)))*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)*((-1
/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(
-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((
x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b
+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.48

$$\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx =$$

$$-\frac{2\left(\left(8\,Bb^4+3\,(10\,Ba^2+21\,Aab)c^2-(41\,Bab^2+14\,Ab^3)c\right)\sqrt{c}\,\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9}{27\,c^3}\right.\right.}{}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*((8*B*b^4 + 3*(10*B*a^2 + 21*A*a*b)*c^2 - (41*B*a*b^2 + 14*A*b^3)*c
)*sqrt(c)*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*
c)/c^3, 1/3*(3*c*x + b)/c) + 3*(8*B*b^3*c + 42*A*a*c^3 - (29*B*a*b + 14*A*
b^2)*c^2)*sqrt(c)*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*
a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*
b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(15*B*c^4*x^2 - 4*B*b^2*c^2 + (10*B*a +
7*A*b)*c^3 + 3*(B*b*c^3 + 7*A*c^4)*x)*sqrt(c*x^2 + b*x + a)*sqrt(x))/c^4
```

Sympy [F]

$$\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{x}(A+Bx)\,\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(Bx+A)\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(Bx+A)\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{x}\,(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int(x^(1/2)*(A + B*x)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(x^(1/2)*(A + B*x)*(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{x}(A+Bx)\sqrt{a+bx+cx^2}\,dx$$
$$= \frac{28\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a^2c-6\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a\,b^2+28\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,abcx+4\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{}$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
(28*sqrt(x)*sqrt(a + b*x + c*x**2)*a**2*c - 6*sqrt(x)*sqrt(a + b*x + c*x**
2)*a*b**2 + 28*sqrt(x)*sqrt(a + b*x + c*x**2)*a*b*c*x + 4*sqrt(x)*sqrt(a +
 b*x + c*x**2)*b**3*x + 20*sqrt(x)*sqrt(a + b*x + c*x**2)*b**2*c*x**2 - 42
*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a**2*c**2 +
43*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a*b**2*c -
 8*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*b**4 - 14*
int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**3*c + 3
*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**2*b**2
)/(70*b*c)
```

3.174 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 476

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(bB+5Ac+3Bcx)\sqrt{a+bx+cx^2}}{15c}$$
$$-\frac{(2b^2B-5Abc-6aBc)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\right.\right.}{15\sqrt{2}c^{5/2}\sqrt{a+bx+cx^2}}$$
$$+\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\left(2b^2B-5Abc-6aBc-\frac{2ac(bB-10Ac)}{b+\sqrt{b^2-4ac}}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c}{b+\sqrt{b}}}}{15\sqrt{2}c^{5/2}\sqrt{a+bx+cx^2}}$$

output
```
2/15*x^(1/2)*(3*B*c*x+5*A*c+B*b)*(c*x^2+b*x+a)^(1/2)/c-1/30*(-5*A*b*c-6*B*
a*c+2*B*b^2)*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x
/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*Elli
pticE(2^(1/2)*c^(1/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^
2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(5/2)/(c*x^2+b*x+a)^(1/
2)+1/30*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(2*B*b^2-5*A*
b*c-6*B*a*c-2*a*c*(-10*A*c+B*b)/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1
/2)*c^(1/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2))/
(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(5/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.27 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx$$

$$= \frac{4\sqrt{x}(bB+5Ac+3Bcx)(a+x(b+cx))}{c} + \frac{x\left(-\frac{4(2b^2B-5Abc-6aBc)(a+x(b+cx))}{x^{3/2}} + \frac{i(2b^2B-5Abc-6aBc)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{2+\frac{4a}{\left(b+\sqrt{b^2-4ac}\right)x}}\sqrt{\frac{2a+bx}{bx-}}}{\sqrt{\frac{a}{b+\sqrt{b^2}}}}\right.}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/Sqrt[x],x]
```

output
```
((4*Sqrt[x]*(b*B + 5*A*c + 3*B*c*x)*(a + x*(b + c*x)))/c + (x*((-4*(2*b^2*
B - 5*A*b*c - 6*a*B*c)*(a + x*(b + c*x)))/x^(3/2) + (I*(2*b^2*B - 5*A*b*c
- 6*a*B*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c]
)*x)]*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*
EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a/(b + Sqrt[b^2 - 4*a
*c])] + (I*(2*b^3*B - b^2*(5*A*c + 2*B*Sqrt[b^2 - 4*a*c]) + 2*a*c*(10*A*c
+ 3*B*Sqrt[b^2 - 4*a*c]) + b*(-8*a*B*c + 5*A*c*Sqrt[b^2 - 4*a*c]))*Sqrt[2
+ (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x
)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sq
rt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c
])])/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]))/c^2)/(30*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1231, 27, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{1231}$$

$$\frac{2\sqrt{x}\sqrt{a+bx+cx^2}(5Ac+bB+3Bcx)}{15c}-\frac{2\int \frac{a(bB-10Ac)+(2Bb^2-5Acb-6aBc)x}{2\sqrt{x}\sqrt{cx^2+bx+a}}dx}{15c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{x}\sqrt{a+bx+cx^2}(5Ac+bB+3Bcx)}{15c}-\frac{\int \frac{a(bB-10Ac)+(2Bb^2-5Acb-6aBc)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{15c}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2\sqrt{x}\sqrt{a+bx+cx^2}(5Ac+bB+3Bcx)}{15c}-\frac{2\int \frac{a(bB-10Ac)+(2Bb^2-5Acb-6aBc)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{15c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2\sqrt{x}\sqrt{a+bx+cx^2}(5Ac+bB+3Bcx)}{15c}-\frac{2\left(\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(-3\sqrt{a}B\sqrt{c}-5Ac+2bB)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}(-6aBc-5Abc+2b^2B)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{15c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{x}\sqrt{a+bx+cx^2}(5Ac+bB+3Bcx)}{15c}-\frac{2\left(\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(-3\sqrt{a}B\sqrt{c}-5Ac+2bB)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{(-6aBc-5Abc+2b^2B)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{15c}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2\sqrt{x}\sqrt{a+bx+cx^2}(5Ac+bB+3Bcx)}{15c}-\frac{2\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}(-3\sqrt{a}B\sqrt{c}-5Ac+2bB)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}}-\frac{(-6aBc-5Abc+2b^2 \cdots}{\sqrt{\cdots}}\right)}{15c}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2\sqrt{x}\sqrt{a+bx+cx^2}(5Ac+bB+3Bcx)}{15c} -$$

$$\frac{2\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}(-3\sqrt{a}B\sqrt{c}-5Ac+2bB)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx+cx^2}} - \frac{(-6aBc-5Abc+2b^2 \ldots}{\ldots}\right.}{15c}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(b*B + 5*A*c + 3*B*c*x)*Sqrt[a + b*x + c*x^2])/(15*c) - (2*(-((
(2*b^2*B - 5*A*b*c - 6*a*B*c)*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a]
+ Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqr
t[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + (a^(1
/4)*(b + 2*Sqrt[a]*Sqrt[c])*(2*b*B - 3*Sqrt[a]*B*Sqrt[c] - 5*A*c)*(Sqrt[a]
 + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4
)*Sqrt[a + b*x + c*x^2])))/(15*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.67

method	result
elliptic	$\frac{\sqrt{x(c\,x^2+bx+a)}\left(\frac{2Bx\sqrt{c\,x^3+b\,x^2+ax}}{5}+\frac{2\left(Ac+\frac{Bb}{5}\right)\sqrt{c\,x^3+b\,x^2+ax}}{3c}+\frac{\left(Aa-\frac{a\left(Ac+\frac{Bb}{5}\right)}{3c}\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\ldots}}{\ldots}\right.}{\ldots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(c*x^2+b*x+a))^(1/2)/x^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*B*x*(c*x^3+b*x^2+
a*x)^(1/2)+2/3*(A*c+1/5*B*b)/c*(c*x^3+b*x^2+a*x)^(1/2)+(A*a-1/3*a/c*(A*c+1
/5*B*b))*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2
*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)*EllipticF(2^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(
-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2))+(A*b+2/5*B*a-2/3*b/c*(A*c+1/5*B*b))*(b+(-4*a*c+b^2)^(1/2)
)/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/
c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c
*x^3+b*x^2+a*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^
2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+
b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2
)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(
1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^
(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/
2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.45

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx$$
$$= \frac{2\left(\left(2\,Bb^3+30\,Aac^2-(9\,Bab+5\,Ab^2)c\right)\sqrt{c}\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+3\,(2\,B\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/45*((2*B*b^3 + 30*A*a*c^2 - (9*B*a*b + 5*A*b^2)*c)*sqrt(c)*weierstrassPI
nverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)
/c) + 3*(2*B*b^2*c - (6*B*a + 5*A*b)*c^2)*sqrt(c)*weierstrassZeta(4/3*(b^2
 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 -
 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + 3*(3*B*c^3
*x + B*b*c^2 + 5*A*c^3)*sqrt(c*x^2 + b*x + a)*sqrt(x))/c^3
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx = \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**(1/2),x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/sqrt(x), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/sqrt(x), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx=\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(1/2),x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{\sqrt{x}}\,dx=\frac{7\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,a}{5}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,bx}{5}-\frac{11\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,x}{c\,x^2+bx+a}dx\right)ac}{10}+\frac{\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,x}{c\,x^2+bx+a}dx\right)b^2}{5}+\frac{3\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^3+b\,x^2+ax}dx\right)a^2}{10}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(1/2),x)
```

output
```
(14*sqrt(x)*sqrt(a + b*x + c*x**2)*a + 4*sqrt(x)*sqrt(a + b*x + c*x**2)*b*
x - 11*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*a*c +
2*int((sqrt(x)*sqrt(a + b*x + c*x**2)*x)/(a + b*x + c*x**2),x)*b**2 + 3*in
t((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a**2)/10
```

3.175 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 432

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx = -\frac{2(3A-Bx)\sqrt{a+bx+cx^2}}{3\sqrt{x}}$$
$$+\frac{(bB+6Ac)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)\right.}{3\sqrt{2}c^{3/2}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b^2B-4aBc+\sqrt{b^2-4ac}(bB+6Ac)\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\text{arc}\right.}{3\sqrt{2}c^{3/2}\sqrt{a+bx+cx^2}}$$

output
```
-2/3*(-B*x+3*A)*(c*x^2+b*x+a)^(1/2)/x^(1/2)+1/6*(6*A*c+B*b)*(-b+(-4*a*c+b^
2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1
/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*x^(1/
2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(
1/2)))^(1/2))*2^(1/2)/c^(3/2)/(c*x^2+b*x+a)^(1/2)-1/6*(-b+(-4*a*c+b^2)^(1/
2))^(1/2)*(B*b^2-4*B*a*c+(-4*a*c+b^2)^(1/2)*(6*A*c+B*b))*(1+2*c*x/(b-(-4*a
*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(
1/2)*c^(1/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2))
/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(3/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.50 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx = \frac{\frac{4(bB+6Ac)(a+x(b+cx))}{c\sqrt{x}}+\frac{4(-3A+Bx)(a+x(b+cx))}{\sqrt{x}}-\frac{i(bB+6Ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{2+\frac{}{(b+\sqrt{}}}}{}}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^(3/2),x]
```

output
```
((4*(b*B + 6*A*c)*(a + x*(b + c*x)))/(c*Sqrt[x]) + (4*(-3*A + B*x)*(a + x*
(b + c*x)))/Sqrt[x] - (I*(b*B + 6*A*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (
4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)
/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqr
t[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)])/(c*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]) + (I*(-(b^2*B) + 4*a*B*c + b*B*Sqr
t[b^2 - 4*a*c] + 6*A*c*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 -
4*a*c])*x)]*x*Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a
*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt
[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(c*Sqrt[a/(b + Sqr
t[b^2 - 4*a*c])]))/(6*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1230, 27, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx$$

$$\Big\downarrow \; 1230$$

$$-\frac{2}{3}\int -\frac{3Ab+2aB+(bB+6Ac)x}{2\sqrt{x}\sqrt{cx^2+bx+a}}dx-\frac{2(3A-Bx)\sqrt{a+bx+cx^2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3Ab+2aB+(bB+6Ac)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx-\frac{2(3A-Bx)\sqrt{a+bx+cx^2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2}{3}\int \frac{3Ab+2aB+(bB+6Ac)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{2(3A-Bx)\sqrt{a+bx+cx^2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2}{3}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}B+3A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}(6Ac+bB)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)-\frac{2(3A-Bx)\sqrt{a+bx+cx^2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}B+3A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{(6Ac+bB)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)-\frac{2(3A-Bx)\sqrt{a+bx+cx^2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2}{3}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x\right)\left(\sqrt{a}B+3A\sqrt{c}\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx+cx^2}}-\frac{(6\cdots}{\cdots}\right)-\frac{2(3A-Bx)\sqrt{a+bx+cx^2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{2}{3}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x\right)\left(\sqrt{a}B+3A\sqrt{c}\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx+cx^2}}-\right. \quad (6$$

$$\frac{2(3A-Bx)\sqrt{a+bx+cx^2}}{3\sqrt{x}}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^(3/2),x]
```

output
```
(-2*(3*A - B*x)*Sqrt[a + b*x + c*x^2])/(3*Sqrt[x]) + (2*(-(((b*B + 6*A*c)*
(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt
[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE
[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/
4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + ((b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a]*B
 + 3*A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sq
rt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(2*a^(1/4)*c^(3/4)*Sqrt[a + b*x + c*x^2])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(356) = 712$.

Time = 1.70 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.78

method	result
elliptic	$\sqrt{x(cx^2+bx+a)}\left(-\frac{2(cx^2+bx+a)A}{\sqrt{x(cx^2+bx+a)}}+\frac{2B\sqrt{cx^3+bx^2+ax}}{3}+\frac{\left(Ab+\frac{2Ba}{3}\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{}{}}}}{}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(c*x^2+b*x+a))^(1/2)/x^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*x^2+b*x+a)*A/(x
*(c*x^2+b*x+a))^(1/2)+2/3*B*(c*x^3+b*x^2+a*x)^(1/2)+(A*b+2/3*B*a)*(b+(-4*a
*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^
(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/
2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2)
)/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+
(2*A*c+1/3*B*b)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2
*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)*((-1/2*(b+(-4*a
*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*
(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a
*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-
4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^
2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx =$$
$$-\frac{2\left(\left(Bb^2-3\left(2\,Ba+Ab\right)c\right)\sqrt{c}x\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+3\left(Bbc+6\,Ac^2\right)\sqrt{c}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
-2/9*((B*b^2 - 3*(2*B*a + A*b)*c)*sqrt(c)*x*weierstrassPInverse(4/3*(b^2 -
 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 3*(B*b*c +
6*A*c^2)*sqrt(c)*x*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9
*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a
*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(B*c^2*x - 3*A*c^2)*sqrt(c*x^2 + b*x +
a)*sqrt(x))/(c^2*x)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx = \int \frac{(A+Bx)\,\sqrt{a+bx+cx^2}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**(3/2),x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/x^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx=\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{x^{3/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(3/2),x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{3/2}}\,dx=\frac{-6\sqrt{c\,x^2+bx+a}\,a+2\sqrt{c\,x^2+bx+a}\,bx+5\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^3+b\,x^2+ax}dx\right)ab}{3\sqrt{x}}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(3/2),x)
```

output
```
( - 6*sqrt(a + b*x + c*x**2)*a + 2*sqrt(a + b*x + c*x**2)*b*x + 5*sqrt(x)*
int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*a*b + 6*sq
rt(x)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a + b*x + c*x**2),x)*a*c + sqr
t(x)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a + b*x + c*x**2),x)*b**2)/(3*s
qrt(x))
```

3.176 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 456

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx = -\frac{2(aA+(Ab+3aB)x)\sqrt{a+bx+cx^2}}{3ax^{3/2}}$$

$$+\frac{(Ab+6aB)\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{x}}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)\right.}{3\sqrt{2}a\sqrt{c}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(6aB\sqrt{b^2-4ac}+A\left(b^2-4ac+b\sqrt{b^2-4ac}\right)\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\text{Ellipti}}{3\sqrt{2}a\sqrt{c}\sqrt{a+bx+cx^2}}$$

output
```
-2/3*(a*A+(A*b+3*B*a)*x)*(c*x^2+b*x+a)^(1/2)/a/x^(3/2)+1/6*(A*b+6*B*a)*(-b
+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^
(1/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4
*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a/c^(1/2)/(c*x^2+b*x+a)^(1/2)-1/6*(-b+(-4
*a*c+b^2)^(1/2))^(1/2)*(6*a*B*(-4*a*c+b^2)^(1/2)+A*(b^2-4*a*c+b*(-4*a*c+b^
2)^(1/2)))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)
^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(
1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a/c^(1
/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.86 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx = \frac{-4(Abx+a(A+3Bx))(a+x(b+cx)) + \frac{x\left(4(Ab+6aB)\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(b+cx\right.}{}}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^(5/2),x]
```

output
```
(-4*(A*b*x + a*(A + 3*B*x))*(a + x*(b + c*x)) + (x*(4*(A*b + 6*a*B)*Sqrt[a
/(b + Sqrt[b^2 - 4*a*c])]*(a + x*(b + c*x)) + I*(A*b + 6*a*B)*(b - Sqrt[b^
2 - 4*a*c])*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a
+ 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticE[I*
ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(6*a*B*Sqrt[b^2 - 4*a*c] + A*(-b^2
+ 4*a*c + b*Sqrt[b^2 - 4*a*c]))*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)
]*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a
*c]*x)]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt
[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[a/(b + Sqrt[
b^2 - 4*a*c])])/(6*a*x^(3/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1229, 27, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 1229$$

$$-\frac{2\int -\frac{a(3bB+2Ac)+(Ab+6aB)cx}{2\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3a}-\frac{2\sqrt{a+bx+cx^2}(x(3aB+Ab)+aA)}{3ax^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a(3bB+2Ac)+(Ab+6aB)cx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{3a}-\frac{2\sqrt{a+bx+cx^2}(x(3aB+Ab)+aA)}{3ax^{3/2}}$$

$$\Big\downarrow \ 1240$$

$$\frac{2\int \frac{a(3bB+2Ac)+(Ab+6aB)cx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{3a}-\frac{2\sqrt{a+bx+cx^2}(x(3aB+Ab)+aA)}{3ax^{3/2}}$$

$$\Big\downarrow \ 1511$$

$$\frac{2\left(\sqrt{a}\left(\sqrt{c}(6aB+Ab)+\sqrt{a}(2Ac+3bB)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\sqrt{a}\sqrt{c}(6aB+Ab)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{3a}-\frac{2\sqrt{a+bx+cx^2}(x(3aB+Ab)+aA)}{3ax^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{2\left(\sqrt{a}\left(\sqrt{c}(6aB+Ab)+\sqrt{a}(2Ac+3bB)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\sqrt{c}(6aB+Ab)\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}\right)}{3a}-\frac{2\sqrt{a+bx+cx^2}(x(3aB+Ab)+aA)}{3ax^{3/2}}$$

$$\Big\downarrow \ 1416$$

$$\frac{2\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\left(\sqrt{c}(6aB+Ab)+\sqrt{a}(2Ac+3bB)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\sqrt{c}(6aB+Ab)\int \frac{\sqrt{}}{\sqrt{cx}}\right.}{3a}-\frac{2\sqrt{a+bx+cx^2}(x(3aB+Ab)+aA)}{3ax^{3/2}}$$

$$\Big\downarrow \ 1509$$

$$\frac{2\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\left(\sqrt{c}(6aB+Ab)+\sqrt{a}(2Ac+3bB)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\sqrt{c}(6aB+Ab)\left(\frac{\sqrt[4]{c}}{}\right.\right.}{3a}$$

$$\frac{2\sqrt{a+bx+cx^2}(x(3aB+Ab)+aA)}{3ax^{3/2}}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^(5/2),x]
```

output
```
(-2*(a*A + (A*b + 3*a*B)*x)*Sqrt[a + b*x + c*x^2])/(3*a*x^(3/2)) + (2*(-((
A*b + 6*a*B)*Sqrt[c]*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]
*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sq
rt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2]))) + (a^(1/4)*((A*b + 6*a*B)
*Sqrt[c] + Sqrt[a]*(3*b*B + 2*A*c))*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x +
c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x + c*x^2])))/(3*
a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(378) = 756$.

Time = 1.86 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.73

method	result
elliptic	$\sqrt{x(cx^2+bx+a)}\left(-\frac{2A\sqrt{cx^3+bx^2+ax}}{3x^2}-\frac{2\left(cx^2+bx+a\right)(Ab+3Ba)}{3a\sqrt{x\left(cx^2+bx+a\right)}}+\frac{\left(\frac{2Ac}{3}+Bb\right)\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\cdots}{-\frac{b+\sqrt{-4ac\cdots}}{2c}}}\cdots}{\cdots}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(c*x^2+b*x+a))^(1/2)/x^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3*A/x^2*(c*x^3+b*x
^2+a*x)^(1/2)-2/3*(c*x^2+b*x+a)*(A*b+3*B*a)/a/(x*(c*x^2+b*x+a))^(1/2)+(2/3
*A*c+B*b)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/
2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)*EllipticF(2^(1/2)*((x
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+
(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2))+(B*c+1/3*(A*b+3*B*a)*c/a)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1
/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4
*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^
2+a*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2
))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/
c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*Ell
ipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)
^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx =$$

$$-\frac{2\left(3\,(6\,Ba+Ab)c^{\frac{3}{2}}x^2\text{weierstrassZeta}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,a}{27\,c^3}\right.\right.\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(3*(6*B*a + A*b)*c^(3/2)*x^2*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -
4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/
27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - (3*B*a*b - A*b^2 + 6*A*a*c
)*sqrt(c)*x^2*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*
a*b*c)/c^3, 1/3*(3*c*x + b)/c) + 3*(A*a*c + (3*B*a + A*b)*c*x)*sqrt(c*x^2
+ b*x + a)*sqrt(x))/(a*c*x^2)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx = \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**(5/2),x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/x^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx = \int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{x^{5/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(5/2),x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{5/2}}\,dx = \frac{-6\sqrt{c\,x^2+bx+a}\,a+4\sqrt{c\,x^2+bx+a}\,bx-7\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^5+b\,x^4+a\,x^3}dx\right)a^2}{2\sqrt{x}\,x}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(5/2),x)
```

output
```
( - 6*sqrt(a + b*x + c*x**2)*a + 4*sqrt(a + b*x + c*x**2)*b*x - 7*sqrt(x)*
int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x**3 + b*x**4 + c*x**5),x)*a**2*x
- sqrt(x)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*
a*c*x + 2*sqrt(x)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x
**3),x)*b**2*x)/(2*sqrt(x)*x)
```

3.177 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 527

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx = \frac{2(2Ab^2-5abB-6aAc)\sqrt{a+bx+cx^2}}{15a^2\sqrt{x}}$$
$$-\frac{2(3aA+(Ab+5aB)x)\sqrt{a+bx+cx^2}}{15ax^{5/2}}$$
$$+\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\left(5abB-2A(b^2-3ac)\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{}{\sqrt{}}\right.\right.}{15\sqrt{2}a^2\sqrt{c}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\left(5abB-2A(b^2-3ac)+\frac{2a(Ab-10aB)c}{b+\sqrt{b^2-4ac}}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-}}}}{15\sqrt{2}a^2\sqrt{c}\sqrt{a+bx+cx^2}}$$

output
```
2/15*(-6*A*a*c+2*A*b^2-5*B*a*b)*(c*x^2+b*x+a)^(1/2)/a^2/x^(1/2)-2/15*(3*a*
A+(A*b+5*B*a)*x)*(c*x^2+b*x+a)^(1/2)/a/x^(5/2)+1/30*(-b+(-4*a*c+b^2)^(1/2)
)^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(5*a*b*B-2*A*(-3*a*c+b^2))*(1+2*c*x/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^
(1/2)*c^(1/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2)
)/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)-1
/30*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(5*a*b*B-2*A*(-3*
a*c+b^2)+2*a*(A*b-10*B*a)*c/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*
c^(1/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(
-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.21 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx = \frac{-4(a+x(b+cx))\left(-2Ab^2x^2+a^2(3A+5Bx)+ax(5bBx+A(b+6cx)\right)}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^(7/2),x]
```

output
```
(-4*(a + x*(b + c*x))*(-2*A*b^2*x^2 + a^2*(3*A + 5*B*x) + a*x*(5*b*B*x + A
*(b + 6*c*x))) + (x^2*(4*(-2*A*b^2 + 5*a*b*B + 6*a*A*c)*Sqrt[a/(b + Sqrt[b
^2 - 4*a*c])]*(a + x*(b + c*x)) + I*(-b + Sqrt[b^2 - 4*a*c])*(-5*a*b*B + 2
*A*(b^2 - 3*a*c))*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt
[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Ellipt
icE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqr
t[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(5*a*B*(b^2 - 4*a*c - b*Sqrt[
b^2 - 4*a*c]) + 2*A*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b
^2 - 4*a*c]))*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*
a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[
I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/(3
0*a^2*x^(5/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{ 1229}$$

$$-\frac{2\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{2x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$\Big\downarrow$ 25

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$\Big\downarrow$ 25

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$\Big\downarrow$ 25

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$\Big\downarrow$ 25

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -\frac{5abB-2A(b^2-3ac)-(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{2Ab^2-5aBb-6aAc+(Ab-10aB)cx}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{15a}-\frac{2\sqrt{a+bx+cx^2}(x(5aB+Ab)+3aA)}{15ax^{5/2}}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 842, normalized size of antiderivative = 1.60

method	result
elliptic	$\frac{\sqrt{x(c\,x^2+bx+a)}\left(-\frac{2A\sqrt{c\,x^3+b\,x^2+ax}}{5x^3}-\frac{2(Ab+5Ba)\sqrt{c\,x^3+b\,x^2+ax}}{15a\,x^2}-\frac{2\left(c\,x^2+bx+a\right)\left(6Aac-2b^2A+5abB\right)}{15a^2\sqrt{x\left(c\,x^2+bx+a\right)}}+\frac{\left(Bc-\frac{c(Ab+5Ba)}{15a}\right)\left(b+\sqrt{-4a\right.}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(c*x^2+b*x+a))^(1/2)/x^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/5*A/x^3*(c*x^3+b*x
^2+a*x)^(1/2)-2/15*(A*b+5*B*a)/a*(c*x^3+b*x^2+a*x)^(1/2)/x^2-2/15*(c*x^2+b
*x+a)/a^2*(6*A*a*c-2*A*b^2+5*B*a*b)/(x*(c*x^2+b*x+a))^(1/2)+(B*c-1/15*c*(A
*b+5*B*a)/a)*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(
-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*
x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)*EllipticF(2^(1/2)*
((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*
(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2))+1/15*(6*A*a*c-2*A*b^2+5*B*a*b)/a^2*(b+(-4*a*c+b^2)^(1
/2))*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(
c*x^3+b*x^2+a*x)^(1/2)*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b
^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c
+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^
2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^
(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)
^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1
/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx =$$

$$-\frac{2\left(\left(5\,Bab^2-2\,Ab^3-3\,(10\,Ba^2-3\,Aab)c\right)\sqrt{c}x^3\mathrm{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx+b}{3\,c}\right)+3\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/45*((5*B*a*b^2 - 2*A*b^3 - 3*(10*B*a^2 - 3*A*a*b)*c)*sqrt(c)*x^3*weiers
trassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c
*x + b)/c) + 3*(6*A*a*c^2 + (5*B*a*b - 2*A*b^2)*c)*sqrt(c)*x^3*weierstrass
Zeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, weierstrassPInver
se(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c))
 + 3*(3*A*a^2*c + (5*B*a^2 + A*a*b)*c*x + (6*A*a*c^2 + (5*B*a*b - 2*A*b^2)
*c)*x^2)*sqrt(c*x^2 + b*x + a)*sqrt(x))/(a^2*c*x^3)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx = \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**(7/2),x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**(7/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/x^(7/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx = \int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{x^{7/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(7/2),x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(7/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{7/2}}\,dx = \frac{-8\sqrt{c\,x^2+bx+a}\,a^2-40\sqrt{c\,x^2+bx+a}\,abx-16\sqrt{c\,x^2+bx+a}\,ac\,x^2 +}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(7/2),x)
```

output
```
( - 8*sqrt(a + b*x + c*x**2)*a**2 - 40*sqrt(a + b*x + c*x**2)*a*b*x - 16*s
qrt(a + b*x + c*x**2)*a*c*x**2 + 40*sqrt(a + b*x + c*x**2)*b**2*x**2 - 36*
sqrt(x)*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a*x**2 + sqrt(x)*b*x**3 + sqrt
(x)*c*x**4),x)*a**2*b*x**2 - 27*sqrt(x)*int((sqrt(a + b*x + c*x**2)*x)/(sq
rt(x)*a + sqrt(x)*b*x + sqrt(x)*c*x**2),x)*a*c**2*x**2 + 35*sqrt(x)*int((s
qrt(x)*sqrt(a + b*x + c*x**2))/(a + b*x + c*x**2),x)*a*c**2*x**2 - 20*sqrt
(x)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a + b*x + c*x**2),x)*b**2*c*x**2
)/(20*sqrt(x)*a*x**2)
```

3.178 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 634

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}} \, dx = -\frac{2(3Ab-14aB)\sqrt{a+bx+cx^2}}{105ax^{5/2}}$$
$$+\frac{2(4Ab^2-7abB-10aAc)\sqrt{a+bx+cx^2}}{105a^2x^{3/2}}$$
$$+\frac{2(14aB(b^2-3ac)-A(8b^3-29abc))\sqrt{a+bx+cx^2}}{105a^3\sqrt{x}}-\frac{2(3A+7Bx)\sqrt{a+bx+cx^2}}{21x^{7/2}}$$
$$-\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)(14aB(b^2-3ac)-A(8b^3-29abc))\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}}{105\sqrt{2}a^3\sqrt{c}\sqrt{a+bx+cx^2}}$$
$$+\frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\left(14aB(b^2-3ac)-A(8b^3-29abc)+\frac{2ac(4Ab^2-7abB-10aAc)}{b+\sqrt{b^2-4ac}}\right)\sqrt{1+\frac{}{b-}}}{105\sqrt{2}a^3\sqrt{c}\sqrt{a+bx+cx^2}}$$

output
```
-2/105*(3*A*b-14*B*a)*(c*x^2+b*x+a)^(1/2)/a/x^(5/2)+2/105*(-10*A*a*c+4*A*b
^2-7*B*a*b)*(c*x^2+b*x+a)^(1/2)/a^2/x^(3/2)+2/105*(14*a*B*(-3*a*c+b^2)-A*(
-29*a*b*c+8*b^3))*(c*x^2+b*x+a)^(1/2)/a^3/x^(1/2)-2/21*(7*B*x+3*A)*(c*x^2+
b*x+a)^(1/2)/x^(7/2)-1/210*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(
1/2))*(14*a*B*(-3*a*c+b^2)-A*(-29*a*b*c+8*b^3))*(1+2*c*x/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1
/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a
*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a^3/c^(1/2)/(c*x^2+b*x+a)^(1/2)+1/210*(-b+(
-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(14*a*B*(-3*a*c+b^2)-A*(-2
9*a*b*c+8*b^3)+2*a*c*(-10*A*a*c+4*A*b^2-7*B*a*b)/(b+(-4*a*c+b^2)^(1/2)))*(
1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/
2)*EllipticF(2^(1/2)*c^(1/2)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4
*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a^3/c^(1/2)/(c*x^2
+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.22 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}}\,dx = \frac{-4(a+x(b+cx))\left(8Ab^3x^3+3a^3(5A+7Bx)-abx^2(4Ab+14bBx+29\right.}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^(9/2),x]
```

output
```
(-4*(a + x*(b + c*x))*(8*A*b^3*x^3 + 3*a^3*(5*A + 7*B*x) - a*b*x^2*(4*A*b
+ 14*b*B*x + 29*A*c*x) + a^2*x*(7*B*x*(b + 6*c*x) + A*(3*b + 10*c*x))) + (
x^3*(4*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(14*a*B*(-b^2 + 3*a*c) + A*(8*b^3 -
 29*a*b*c))*(a + x*(b + c*x)) + I*(b - Sqrt[b^2 - 4*a*c])*(14*a*B*(-b^2 +
3*a*c) + A*(8*b^3 - 29*a*b*c))*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]
*x^(3/2)*Sqrt[(4*a + 2*b*x - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*
c]*x)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[
x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(14*a*B*(b^3 - 4
*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 3*a*c*Sqrt[b^2 - 4*a*c]) + A*(-8*b^4 + 37
*a*b^2*c - 20*a^2*c^2 + 8*b^3*Sqrt[b^2 - 4*a*c] - 29*a*b*c*Sqrt[b^2 - 4*a*
c]))*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x^(3/2)*Sqrt[(4*a + 2*b*x
 - 2*Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcSinh
[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/(210*a^3*x^
(7/2)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1229, 27, 1237, 27, 1237, 27, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}} \, dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{2\int \frac{4Ab^2-7aBb-10aAc+(3Ab-14aB)cx}{2x^{5/2}\sqrt{cx^2+bx+a}}dx}{35a}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{4Ab^2-7aBb-10aAc+(3Ab-14aB)cx}{x^{5/2}\sqrt{cx^2+bx+a}}dx}{35a}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{-\frac{2\int -\frac{14aB\left(b^2-3ac\right)-2A\left(4b^3-\frac{29abc}{2}\right)-c\left(4Ab^2-7aBb-10aAc\right)x}{2x^{3/2}\sqrt{cx^2+bx+a}}dx}{3a}-\frac{2\sqrt{a+bx+cx^2}(-10aAc-7abB+4Ab^2)}{3ax^{3/2}}}{35a}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)-c\left(4Ab^2-7aBb-10aAc\right)x}{x^{3/2}\sqrt{cx^2+bx+a}}dx}{3a}-\frac{2\sqrt{a+bx+cx^2}(-10aAc-7abB+4Ab^2)}{3ax^{3/2}}}{35a}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$\Big\downarrow$ 1237

$$-\frac{\frac{-\frac{2\int \frac{c\left(a\left(4Ab^2-7aBb-10aAc\right)-\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)x\right)}{2\sqrt{x}\sqrt{cx^2+bx+a}}dx}{a}-\frac{2\sqrt{a+bx+cx^2}\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)}{a\sqrt{x}}}{3a}-\frac{2\sqrt{a+bx+cx^2}(-10aAc-}{3ax^{3/2}}}{35a}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{-\frac{c\int \frac{a\left(4Ab^2-7aBb-10aAc\right)-\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)x}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{a}-\frac{2\sqrt{a+bx+cx^2}\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)}{a\sqrt{x}}}{3a}-\frac{2\sqrt{a+bx+cx^2}(-10aAc-7a}{3ax^{3/2}}}{35a}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$\Big\downarrow$ 1240

$$-\frac{\frac{-\frac{2c\int \frac{a\left(4Ab^2-7aBb-10aAc\right)-\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{a}-\frac{2\sqrt{a+bx+cx^2}\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)}{a\sqrt{x}}}{3a}-\frac{2\sqrt{a+bx+cx^2}(-10aAc-}{3ax^{3/2}}}{35a}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$\Big\downarrow$ 1511

$$-\frac{-\frac{-\frac{2c\left(\frac{\sqrt{a}\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}\left(-A\left(8b^3-29abc\right)-\sqrt{a}\sqrt{c}\left(-10aAc-7abB+4Ab^2\right)+14aB\left(b^2-3ac\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}}{\sqrt{c}}\right.}{a}}{3a}}{35a}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$\downarrow$ 27

$$-\frac{-\frac{-\frac{2c\left(\frac{\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{a}\left(-A\left(8b^3-29abc\right)-\sqrt{a}\sqrt{c}\left(-10aAc-7abB+4Ab^2\right)+14aB\left(b^2-3ac\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{a}}{3a}}{35a}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$\downarrow$ 1416

$$-\frac{-\frac{-\frac{2c\left(\frac{\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)\int\frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\left(-A\left(8b^3-29abc\right)-\sqrt{a}\sqrt{c}\left(-10aAc-7abB+4Ab^2\right)+14aB\left(b^2-\right.\right.}{2c^{3/4}\sqrt{a+bx+cx^2}}\right.}{a}}{3a}}{35}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

$\downarrow$ 1509

$$-\frac{-\frac{2c\left(\frac{\left(14aB\left(b^2-3ac\right)-A\left(8b^3-29abc\right)\right)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{\sqrt{x}\sqrt{a+bx+cx^2}}{\sqrt{a}+\sqrt{c}x}\right)}{\sqrt{c}}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\right.}{a}}{}-\frac{2\sqrt{a+bx+cx^2}(x(7aB+Ab)+5aA)}{35ax^{7/2}}$$

input

```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/x^(9/2),x]
```

output
```
(-2*(5*a*A + (A*b + 7*a*B)*x)*Sqrt[a + b*x + c*x^2])/(35*a*x^(7/2)) - ((-2
*(4*A*b^2 - 7*a*b*B - 10*a*A*c)*Sqrt[a + b*x + c*x^2])/(3*a*x^(3/2)) + ((-
2*(14*a*B*(b^2 - 3*a*c) - A*(8*b^3 - 29*a*b*c))*Sqrt[a + b*x + c*x^2])/(a*
Sqrt[x]) - (2*c*(((14*a*B*(b^2 - 3*a*c) - A*(8*b^3 - 29*a*b*c))*(-((Sqrt[x
]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt
[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[
(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a
+ b*x + c*x^2])))/Sqrt[c] - (a^(1/4)*(14*a*B*(b^2 - 3*a*c) - Sqrt[a]*Sqrt[
c]*(4*A*b^2 - 7*a*b*B - 10*a*A*c) - A*(8*b^3 - 29*a*b*c))*(Sqrt[a] + Sqrt[
c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a
 + b*x + c*x^2])))/a)/(3*a))/(35*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.42

method	result
elliptic	$\frac{\sqrt{x(c\,x^2+bx+a)}\left(-\frac{2A\sqrt{c\,x^3+b\,x^2+ax}}{7x^4}-\frac{2(Ab+7Ba)\sqrt{c\,x^3+b\,x^2+ax}}{35a\,x^3}-\frac{2\left(10Aac-4b^2A+7abB\right)\sqrt{c\,x^3+b\,x^2+ax}}{105a^2x^2}+\frac{2\left(c\,x^2+bx+a\right)\left(29Aabc-8\right.}{105a^3\sqrt{x\left(c\right.}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(c*x^2+b*x+a))^(1/2)/x^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/7*A/x^4*(c*x^3+b*x
^2+a*x)^(1/2)-2/35*(A*b+7*B*a)/a*(c*x^3+b*x^2+a*x)^(1/2)/x^3-2/105/a^2*(10
*A*a*c-4*A*b^2+7*B*a*b)*(c*x^3+b*x^2+a*x)^(1/2)/x^2+2/105*(c*x^2+b*x+a)/a^
3*(29*A*a*b*c-8*A*b^3-42*B*a^2*c+14*B*a*b^2)/(x*(c*x^2+b*x+a))^(1/2)-1/105
*(10*A*a*c-4*A*b^2+7*B*a*b)/a^2*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*((x+1/2*(b+
(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a
*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2
))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)*E
llipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*
c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-1/105*(29*A*a*b*c-8*A*b^3-42*B*a^2*c
+14*B*a*b^2)/a^3*(b+(-4*a*c+b^2)^(1/2))*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*
c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)*((-1/2*(b+(-4*a*
c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(2^(1/2)*((x+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*
c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2
))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(2^(1/2)*((x+1/2*(b+(-4
*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.51

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}}\,dx = \frac{2\left(\left(14\,Bab^3-8\,Ab^4-30\,Aa^2c^2-\left(63\,Ba^2b-41\,Aab^2\right)c\right)\sqrt{c}x^4\text{weierstras}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
2/315*((14*B*a*b^3 - 8*A*b^4 - 30*A*a^2*c^2 - (63*B*a^2*b - 41*A*a*b^2)*c)
*sqrt(c)*x^4*weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a
*b*c)/c^3, 1/3*(3*c*x + b)/c) - 3*((42*B*a^2 - 29*A*a*b)*c^2 - 2*(7*B*a*b^
2 - 4*A*b^3)*c)*sqrt(c)*x^4*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(
2*b^3 - 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*
b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) - 3*(15*A*a^3*c + ((42*B*a^2 - 29*
A*a*b)*c^2 - 2*(7*B*a*b^2 - 4*A*b^3)*c)*x^3 + 3*(7*B*a^3 + A*a^2*b)*c*x +
(10*A*a^2*c^2 + (7*B*a^2*b - 4*A*a*b^2)*c)*x^2)*sqrt(c*x^2 + b*x + a)*sqrt
(x))/(a^3*c*x^4)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}}\,dx=\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/x**(9/2),x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/x**(9/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}}\,dx=\int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/x^(9/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}}\,dx=\int \frac{\sqrt{cx^2+bx+a}(Bx+A)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}}\,dx=\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{x^{9/2}}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(9/2),x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/x^(9/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{x^{9/2}}\,dx=\frac{-36\sqrt{c\,x^2+bx+a}\,a-84\sqrt{c\,x^2+bx+a}\,bx-66\sqrt{x}\left(\int \frac{\sqrt{c\,x^2+bx+a}}{\sqrt{x}\,a\,x^3+\sqrt{x}\,b\,x^4+\sqrt{}}\right.}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/x^(9/2),x)
```

output
```
( - 36*sqrt(a + b*x + c*x**2)*a - 84*sqrt(a + b*x + c*x**2)*b*x - 66*sqrt(
x)*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a*x**3 + sqrt(x)*b*x**4 + sqrt(x)*c
*x**5),x)*a*b*x**3 - 55*sqrt(x)*int(sqrt(a + b*x + c*x**2)/(sqrt(x)*a*x**2
 + sqrt(x)*b*x**3 + sqrt(x)*c*x**4),x)*a*c*x**3 + 91*sqrt(x)*int((sqrt(x)*
sqrt(a + b*x + c*x**2))/(a*x**3 + b*x**4 + c*x**5),x)*a*c*x**3 - 42*sqrt(x
)*int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x**3 + b*x**4 + c*x**5),x)*b**2*
x**3)/(126*sqrt(x)*x**3)
```

3.179 $\int (2-5x)x^{7/2}\sqrt{2+5x+3x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 247

$$\int (2-5x)x^{7/2}\sqrt{2+5x+3x^2}\,dx = \frac{1543648\sqrt{x}(2+3x)}{6567561\sqrt{2+5x+3x^2}} - \frac{349240\sqrt{x}\sqrt{2+5x+3x^2}}{2189187} + \frac{2776x^{3/2}\sqrt{2+5x+3x^2}}{18711} - \frac{37252x^{5/2}\sqrt{2+5x+3x^2}}{243243} + \frac{1960x^{7/2}\sqrt{2+5x+3x^2}}{11583} + \frac{2}{429}(53-165x)x^{9/2}\sqrt{2+5x+3x^2} - \frac{1543648\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{6567561\sqrt{1+x}\sqrt{2+3x}} + \frac{349240\sqrt{2}\sqrt{1+x}\cdots}{21\cdots}$$

output
```
1543648/6567561*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-349240/2189187*x^(1/2)
*(3*x^2+5*x+2)^(1/2)+2776/18711*x^(3/2)*(3*x^2+5*x+2)^(1/2)-37252/243243*x
^(5/2)*(3*x^2+5*x+2)^(1/2)+1960/11583*x^(7/2)*(3*x^2+5*x+2)^(1/2)+2/429*(5
3-165*x)*x^(9/2)*(3*x^2+5*x+2)^(1/2)-1543648/6567561*2^(1/2)*(3*x^2+5*x+2)
^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1
/2)+349240/2189187*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arcta
n(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.72

$$\int (2 -5x)x^{7/2}\sqrt{2+5x+3x^2}\,dx = \frac{2(1543648+2811400x+670548x^2-141444x^3+58374x^4+2892348x^5+6\cdots}{\cdots}$$

input
```
Integrate[(2 - 5*x)*x^(7/2)*Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(2*(1543648 + 2811400*x + 670548*x^2 - 141444*x^3 + 58374*x^4 + 2892348*x^
5 + 671895*x^6 - 10195794*x^7 - 7577955*x^8) + (1543648*I)*Sqrt[2]*Sqrt[1
+ x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/
2] - (495928*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I
*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(6567561*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {1236, 1236, 27, 1236, 27, 1236, 27, 1231, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-5x)x^{7/2}\sqrt{3x^2+5x+2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{39}\int x^{5/2}(164x+35)\sqrt{3x^2+5x+2}dx-\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{39}\left(\frac{2}{33}\int -\frac{5}{2}x^{3/2}(1081x+328)\sqrt{3x^2+5x+2}dx+\frac{328}{33}\left(3x^2+5x+2\right)^{3/2}x^{5/2}\right)-\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\int x^{3/2}(1081x+328)\sqrt{3x^2+5x+2}dx\right)-\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 1236

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\left(\frac{2}{27}\int -3\sqrt{x}(3929x+1081)\sqrt{3x^2+5x+2}dx+\frac{2162}{27}x^{3/2}\left(3x^2+5x+2\right)^{3}\right.\right.$$
$$\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\left(\frac{2162}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}-\frac{2}{9}\int \sqrt{x}(3929x+1081)\sqrt{3x^2+5x+2}dx\right)\right.$$
$$\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 1236

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\left(\frac{2162}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}-\frac{2}{9}\left(\frac{2}{21}\int -\frac{(55879x+7858)\sqrt{3x^2+5x+2}}{2\sqrt{x}}\right.\right.\right.$$
$$\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\left(\frac{2162}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}-\frac{2}{9}\left(\frac{7858}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}-\frac{1}{21}\int \frac{(558}{}\right.\right.\right.$$
$$\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 1231

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\left(\frac{2162}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}-\frac{2}{9}\left(\frac{1}{21}\left(\frac{2}{45}\int \frac{96478x+43655}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2}{45}\sqrt{}\right.\right.\right.\right.$$
$$\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\downarrow$ 1240

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\left(\frac{2162}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}-\frac{2}{9}\left(\frac{1}{21}\left(\frac{4}{45}\int\frac{96478x+43655}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{2}{45}\sqrt{x}\right.\right.\right.\right.$$
$$\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\downarrow$ 1503

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\left(\frac{2162}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}-\frac{2}{9}\left(\frac{1}{21}\left(\frac{4}{45}\left(43655\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\right.\right.\right.\right.\right.$$
$$\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\downarrow$ 1413

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\left(\frac{2162}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}-\frac{2}{9}\left(\frac{1}{21}\left(\frac{4}{45}\left(96478\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right.\right.\right.\right.\right.$$
$$\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

$\downarrow$ 1456

$$\frac{2}{39}\left(\frac{328}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}-\frac{5}{33}\left(\frac{2162}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}-\frac{2}{9}\left(\frac{1}{21}\left(\frac{4}{45}\left(\frac{43655(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}}{\sqrt{2}\sqrt{3x^2+5x}}\right.\right.\right.\right.\right.$$
$$\frac{10}{39}x^{7/2}\left(3x^2+5x+2\right)^{3/2}$$

input
```
Int[(2 - 5*x)*x^(7/2)*Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-10*x^(7/2)*(2 + 5*x + 3*x^2)^(3/2))/39 + (2*((328*x^(5/2)*(2 + 5*x + 3*x
^2)^(3/2))/33 - (5*((2162*x^(3/2)*(2 + 5*x + 3*x^2)^(3/2))/27 - (2*((7858*
Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2))/21 + ((-2*Sqrt[x]*(397265 + 502911*x)*Sqr
t[2 + 5*x + 3*x^2])/45 + (4*(96478*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x +
3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x
]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (43655*(1 + x)*Sqrt[(2 + 3*x)/(1 +
 x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])))/4
5)/21))/9))/33))/39
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.55

method	result
default	$-\frac{2\left(22733865x^{8}+30587382x^{7}-2015685x^{6}-8677044x^{5}+633876\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-385912\sqrt{6x+4}\right.}{19702683\sqrt{x}\,\sqrt{3x^{2}+5x+2}}$
risch	$-\frac{2\left(841995x^{5}-270459x^{4}-185220x^{3}+167634x^{2}-162396x+174620\right)\sqrt{x}\,\sqrt{3x^{2}+5x+2}}{2189187}-\frac{\left(-\frac{349240\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \operatorname{EllipticF}\left(\right.}{6567561\sqrt{3x^{3}+5x^{2}+2x}}\right.}{}$
elliptic	$\frac{\sqrt{\left(3x^{2}+5x+2\right)x}\,\left(-\frac{10x^{5}\sqrt{3x^{3}+5x^{2}+2x}}{13}+\frac{106x^{4}\sqrt{3x^{3}+5x^{2}+2x}}{429}+\frac{1960x^{3}\sqrt{3x^{3}+5x^{2}+2x}}{11583}-\frac{37252x^{2}\sqrt{3x^{3}+5x^{2}+2x}}{243243}+\frac{2776x\sqrt{3x^{3}+5x^{2}+2x}}{18711}\right.}{}$

input
```
int((2-5*x)*x^(7/2)*(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/19702683/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(22733865*x^8+30587382*x^7-2015685
*x^6-8677044*x^5+633876*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ell
ipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-385912*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(
1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))-175122*x^4+424332*x
^3+4934772*x^2+3143160*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.28

$$\int (2-5x)x^{7/2}\sqrt{2+5x+3x^2}\,dx = -\frac{2}{2189187}\left(841995\,x^5-270459\,x^4-185220\,x^3+167634\,x^2-162396\,x+174620\right)\sqrt{3\,x^2+5\,x+2}\sqrt{x} - \frac{204560}{8444007}\,\sqrt{3}\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) - \frac{1543648}{6567561}\,\sqrt{3}\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*x^(7/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2/2189187*(841995*x^5 - 270459*x^4 - 185220*x^3 + 167634*x^2 - 162396*x +
 174620)*sqrt(3*x^2 + 5*x + 2)*sqrt(x) - 204560/8444007*sqrt(3)*weierstras
sPInverse(28/27, 80/729, x + 5/9) - 1543648/6567561*sqrt(3)*weierstrassZet
a(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x + 5/9))
```

Sympy [F]

$$\int (2-5x)x^{7/2}\sqrt{2+5x+3x^2}\,dx = -\int \left(-2x^{\frac{7}{2}}\sqrt{3x^2+5x+2}\right)dx - \int 5x^{\frac{9}{2}}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*x**(7/2)*(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2*x**(7/2)*sqrt(3*x**2 + 5*x + 2), x) - Integral(5*x**(9/2)*sqr
t(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int (2-5x)x^{7/2}\sqrt{2+5x+3x^2}\,dx = \int -\sqrt{3\,x^2+5\,x+2}(5\,x-2)x^{\frac{7}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(7/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)*x^(7/2), x)
```

Giac [F]

$$\int (2-5x)x^{7/2}\sqrt{2+5x+3x^2}\,dx = \int -\sqrt{3\,x^2+5\,x+2}(5\,x-2)x^{\frac{7}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(7/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)*x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-5x)x^{7/2}\sqrt{2+5x+3x^2}\,dx = -\int x^{7/2}\,(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}\,dx$$

input
```
int(-x^(7/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
-int(x^(7/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int (2-5x)x^{7/2}\sqrt{2+5x+3x^2}\,dx = -\frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^5}{13}+\frac{106\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^4}{429}+\frac{1960\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3}{11583}-\frac{37252\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{243243}+\frac{2776\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{18711}-\frac{2776\sqrt{x}\,\sqrt{3x^2+5x+2}}{31185}-\frac{385912\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{1216215}+\frac{2776\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{31185}$$

input
```
int((2-5*x)*x^(7/2)*(3*x^2+5*x+2)^(1/2),x)
```

output
```
(2*( - 467775*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**5 + 150255*sqrt(x)*sqrt(3*
x**2 + 5*x + 2)*x**4 + 102900*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 - 93130*
sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 + 90220*sqrt(x)*sqrt(3*x**2 + 5*x + 2)
*x - 54132*sqrt(x)*sqrt(3*x**2 + 5*x + 2) - 192956*int((sqrt(x)*sqrt(3*x**
2 + 5*x + 2)*x)/(3*x**2 + 5*x + 2),x) + 54132*int((sqrt(x)*sqrt(3*x**2 + 5
*x + 2))/(3*x**3 + 5*x**2 + 2*x),x)))/1216215
```

3.180 $\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 224

$$\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2}\,dx = -\frac{261784\sqrt{x}(2+3x)}{841995\sqrt{2+5x+3x^2}} + \frac{13016\sqrt{x}\sqrt{2+5x+3x^2}}{56133} - \frac{7204x^{3/2}\sqrt{2+5x+3x^2}}{31185} + \frac{1580x^{5/2}\sqrt{2+5x+3x^2}}{6237} + \frac{2}{297}(41-135x)x^{7/2}\sqrt{2+5x+3x^2} + \frac{261784\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{841995\sqrt{1+x}\sqrt{2+3x}} - \frac{13016\sqrt{2}\sqrt{1+x}\sqrt{2}}{56}$$

output
```
-261784/841995*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+13016/56133*x^(1/2)*(3*
x^2+5*x+2)^(1/2)-7204/31185*x^(3/2)*(3*x^2+5*x+2)^(1/2)+1580/6237*x^(5/2)*
(3*x^2+5*x+2)^(1/2)+2/297*(41-135*x)*x^(7/2)*(3*x^2+5*x+2)^(1/2)+261784/84
1995*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/
2))/(1+x)^(1/2)/(2+3*x)^(1/2)-13016/56133*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2
)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.76

$$\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2}\,dx = \frac{-523568-918440x-198168x^2+39780x^3+947916x^4+271350x^5-3129\cdots}{\cdots}$$

input
```
Integrate[(2 - 5*x)*x^(5/2)*Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-523568 - 918440*x - 198168*x^2 + 39780*x^3 + 947916*x^4 + 271350*x^5 - 3
129840*x^6 - 2296350*x^7 - (261784*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/
x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (66544*I)*Sqrt[2
]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqr
t[x]], 3/2])/(841995*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1236, 1236, 27, 1236, 25, 1231, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-5x)x^{5/2}\sqrt{3x^2+5x+2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{33}\int x^{3/2}(133x+25)\sqrt{3x^2+5x+2}dx-\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{33}\left(\frac{2}{27}\int -\frac{3}{2}\sqrt{x}(1105x+266)\sqrt{3x^2+5x+2}dx+\frac{266}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{33}\left(\frac{266}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}-\frac{1}{9}\int \sqrt{x}(1105x+266)\sqrt{3x^2+5x+2}dx\right)-\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{33}\left(\frac{1}{9}\left(-\frac{2}{21}\int -\frac{(8257x+1105)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx-\frac{2210}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}\right)+\frac{266}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{33}\left(\frac{1}{9}\left(\frac{2}{21}\int \frac{(8257x+1105)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx-\frac{2210}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}\right)+\frac{266}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{33}\left(\frac{1}{9}\left(\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(74313x+57860)\sqrt{3x^2+5x+2}-\frac{2}{45}\int \frac{32723x+16270}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{2210}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}\right)+\frac{266}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{33}\left(\frac{1}{9}\left(\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(74313x+57860)\sqrt{3x^2+5x+2}-\frac{1}{45}\int \frac{32723x+16270}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{2210}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}\right)+\frac{266}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2}{33}\left(\frac{1}{9}\left(\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(74313x+57860)\sqrt{3x^2+5x+2}-\frac{2}{45}\int \frac{32723x+16270}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2210}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}\right)+\frac{266}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1503}$$

$$\frac{2}{33}\left(\frac{1}{9}\left(\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(74313x+57860)\sqrt{3x^2+5x+2}-\frac{2}{45}\left(16270\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+32723\int\frac{x}{\sqrt{3x^2+5x}}\right.\right.\right.\right.$$
$$\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1413}$$

$$\frac{2}{33}\left(\frac{1}{9}\left(\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(74313x+57860)\sqrt{3x^2+5x+2}-\frac{2}{45}\left(32723\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{8135\sqrt{2}(x+1)\sqrt{\frac{3x}{x}}}{\sqrt{}}\right.\right.\right.\right.$$
$$\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1456}$$

$$\frac{2}{33}\left(\frac{1}{9}\left(\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(74313x+57860)\sqrt{3x^2+5x+2}-\frac{2}{45}\left(\frac{8135\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right.\right.\right.\right.$$
$$\frac{10}{33}x^{5/2}\left(3x^2+5x+2\right)^{3/2}$$

input
```
Int[(2 - 5*x)*x^(5/2)*Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-10*x^(5/2)*(2 + 5*x + 3*x^2)^(3/2))/33 + (2*((266*x^(3/2)*(2 + 5*x + 3*x
^2)^(3/2))/27 + ((-2210*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2))/21 + (2*((2*Sqrt[
x]*(57860 + 74313*x)*Sqrt[2 + 5*x + 3*x^2])/45 - (2*(32723*((Sqrt[x]*(2 +
3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]
*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (8135*Sqrt
[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt
[2 + 5*x + 3*x^2]))/45))/21)/9))/33
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

method	result
default	$\frac{-\frac{30x^7}{11}-\frac{368x^6}{99}+\frac{670x^5}{2079}+\frac{65812\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{841995}-\frac{130892\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{2525985}+\frac{35}{3}\cdots}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
risch	$-\frac{2(127575x^4-38745x^3-35550x^2+32418x-32540)\sqrt{x}\,\sqrt{3x^2+5x+2}}{280665}-\frac{\left(\frac{13016\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{168399\sqrt{3x^3+5x^2+2x}}+\frac{13089\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{10x^4\sqrt{3x^3+5x^2+2x}}{11}+\frac{82x^3\sqrt{3x^3+5x^2+2x}}{297}+\frac{1580x^2\sqrt{3x^3+5x^2+2x}}{6237}-\frac{7204x\sqrt{3x^3+5x^2+2x}}{31185}+\frac{13016\sqrt{3x^3+5x^2+2x}}{56133}-\frac{13\cdots}{\cdots}\right.}{\sqrt{x}\,\sqrt{3x^2+\cdots}}$

input
```
int((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/2525985/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(-3444525*x^7-4694760*x^6+407025*x^5
+98718*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4
)^(1/2),I*2^(1/2))-65446*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*El
lipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+1421874*x^4+59670*x^3+880776*x^2+5857
20*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.28

$$\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2}\,dx = -\frac{2}{280665}\left(127575\,x^4-38745\,x^3-35550\,x^2+32418\,x-32540\right)\sqrt{3\,x^2+5\,x+2}\sqrt{x} + \frac{3928}{216513}\,\sqrt{3}\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) + \frac{261784}{841995}\,\sqrt{3}\mathrm{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2/280665*(127575*x^4 - 38745*x^3 - 35550*x^2 + 32418*x - 32540)*sqrt(3*x^
2 + 5*x + 2)*sqrt(x) + 3928/216513*sqrt(3)*weierstrassPInverse(28/27, 80/7
29, x + 5/9) + 261784/841995*sqrt(3)*weierstrassZeta(28/27, 80/729, weiers
trassPInverse(28/27, 80/729, x + 5/9))
```

Sympy [F]

$$\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2}\,dx = -\int \left(-2x^{\frac{5}{2}}\sqrt{3x^2+5x+2}\right)\,dx - \int 5x^{\frac{7}{2}}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*x**(5/2)*(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2*x**(5/2)*sqrt(3*x**2 + 5*x + 2), x) - Integral(5*x**(7/2)*sqr
t(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2}\,dx = \int -\sqrt{3\,x^2+5\,x+2}(5\,x-2)x^{\frac{5}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)*x^(5/2), x)
```

Giac [F]

$$\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2}\,dx = \int -\sqrt{3\,x^2+5\,x+2}(5\,x-2)x^{\frac{5}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)*x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2}\,dx = -\int x^{5/2}\,(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}\,dx$$

input
```
int(-x^(5/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
-int(x^(5/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int (2-5x)x^{5/2}\sqrt{2+5x+3x^2}\,dx = -\frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^4}{11} + \frac{82\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3}{297} + \frac{1580\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{6237} - \frac{7204\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{31185} + \frac{7204\sqrt{x}\,\sqrt{3x^2+5x+2}}{51975} + \frac{65446\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{155925} - \frac{7204\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{51975}$$

input
```
int((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(1/2),x)
```

output
```
(2*( - 70875*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**4 + 21525*sqrt(x)*sqrt(3*x*
*2 + 5*x + 2)*x**3 + 19750*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 - 18010*sqr
t(x)*sqrt(3*x**2 + 5*x + 2)*x + 10806*sqrt(x)*sqrt(3*x**2 + 5*x + 2) + 327
23*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(3*x**2 + 5*x + 2),x) - 10806*in
t((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x)))/155925
```

3.181 $\int (2-5x)x^{3/2}\sqrt{2+5x+3x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\int (2-5x)x^{3/2}\sqrt{2+5x+3x^2}\,dx = \frac{2360\sqrt{x}(2+3x)}{5103\sqrt{2+5x+3x^2}} - \frac{668\sqrt{x}\sqrt{2+5x+3x^2}}{1701} + \frac{80}{189}x^{3/2}\sqrt{2+5x+3x^2} + \frac{2}{189}(29-105x)x^{5/2}\sqrt{2+5x+3x^2} - \frac{2360\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\left|-\frac{1}{2}\right.\right)}{5103\sqrt{1+x}\sqrt{2+3x}} + \frac{668\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{1701\sqrt{2+5x+3x^2}}$$

output
```
2360/5103*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-668/1701*x^(1/2)*(3*x^2+5*x+
2)^(1/2)+80/189*x^(3/2)*(3*x^2+5*x+2)^(1/2)+2/189*(29-105*x)*x^(5/2)*(3*x^
2+5*x+2)^(1/2)-2360/5103*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+
x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+668/1701*2^(1/2)*(1+x)^(
1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5
*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.82

$$\int (2 -5x)x^{3/2}\sqrt{2+5x+3x^2}\,dx = \frac{4720 + 7792x + 1380x^2 + 7920x^3 + 2970x^4 - 23652x^5 - 17010x^6 + 2360i\sqrt{}}{}$$

input
```
Integrate[(2 - 5*x)*x^(3/2)*Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(4720 + 7792*x + 1380*x^2 + 7920*x^3 + 2970*x^4 - 23652*x^5 - 17010*x^6 +
(2360*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSin
h[Sqrt[2/3]/Sqrt[x]], 3/2] - (356*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x
]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(5103*Sqrt[x]*Sqrt
[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1236, 27, 1236, 27, 1231, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-5x)x^{3/2}\sqrt{3x^2+5x+2}\,dx$$

$$\Big\downarrow \; 1236$$

$$\frac{2}{27}\int 3\sqrt{x}(34x+5)\sqrt{3x^2+5x+2}dx - \frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{9}\int \sqrt{x}(34x+5)\sqrt{3x^2+5x+2}dx - \frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{9}\left(\frac{2}{21}\int -\frac{(575x+68)\sqrt{3x^2+5x+2}}{2\sqrt{x}}dx+\frac{68}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\left(\frac{68}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}-\frac{1}{21}\int \frac{(575x+68)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx\right)-\frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{9}\left(\frac{1}{21}\left(\frac{2}{45}\int \frac{5(295x+167)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2}{9}\sqrt{x}(1035x+779)\sqrt{3x^2+5x+2}\right)+\frac{68}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\left(\frac{1}{21}\left(\frac{2}{9}\int \frac{295x+167}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2}{9}\sqrt{x}(1035x+779)\sqrt{3x^2+5x+2}\right)+\frac{68}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2}{9}\left(\frac{1}{21}\left(\frac{4}{9}\int \frac{295x+167}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{2}{9}\sqrt{x}(1035x+779)\sqrt{3x^2+5x+2}\right)+\frac{68}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}\right)-\frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1503}$$

$$\frac{2}{9}\left(\frac{1}{21}\left(\frac{4}{9}\left(167\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+295\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2}{9}\sqrt{x}(1035x+779)\sqrt{3x^2+5x+2}\right)+\right.$$
$$\frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1413}$$

$$\frac{2}{9}\left(\frac{1}{21}\left(\frac{4}{9}\left(295\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{167(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{2}{9}\sqrt{x}(1035x+779\right.\right.$$
$$\frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 1456

$$\frac{2}{9}\left(\frac{1}{21}\left(\frac{4}{9}\left(\frac{167(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+295\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E(\text{arct}}{3\sqrt{3x^2+5x}}\right.\right.\right.\right.$$
$$\frac{10}{27}x^{3/2}\left(3x^2+5x+2\right)^{3/2}$$

input
```
Int[(2 - 5*x)*x^(3/2)*Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-10*x^(3/2)*(2 + 5*x + 3*x^2)^(3/2))/27 + (2*((68*Sqrt[x]*(2 + 5*x + 3*x^
2)^(3/2))/21 + ((-2*Sqrt[x]*(779 + 1035*x)*Sqrt[2 + 5*x + 3*x^2])/9 + (4*(
295*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt
[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*
x^2])) + (167*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -
1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])))/9)/21))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.63

method	result
default	$-\frac{2\left(25515x^6+35478x^5+768\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-590\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right.\right.}{15309\sqrt{x}\,\sqrt{3x^2+5x+2}}$
risch	$-\frac{2(945x^3-261x^2-360x+334)\sqrt{x}\,\sqrt{3x^2+5x+2}}{1701}-\frac{\left(-\frac{668\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{5103\sqrt{3x^3+5x^2+2x}}-\frac{1180\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \left(\right.}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{10x^3\sqrt{3x^3+5x^2+2x}}{9}+\frac{58x^2\sqrt{3x^3+5x^2+2x}}{189}+\frac{80x\sqrt{3x^3+5x^2+2x}}{189}-\frac{668\sqrt{3x^3+5x^2+2x}}{1701}+\frac{668\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \text{Ellipti}}{5103\sqrt{3x^3+5x^2+}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15309/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(25515*x^6+35478*x^5+768*(6*x+4)^(1/2
)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-
590*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(
1/2),I*2^(1/2))-4455*x^4-11880*x^3+8550*x^2+6012*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

$$\int (2-5x)x^{3/2}\sqrt{2+5x+3x^2}\,dx = -\frac{2}{1701}\left(945\,x^3-261\,x^2-360\,x+334\right)\sqrt{3\,x^2+5\,x+2}\sqrt{x} + \frac{32}{6561}\,\sqrt{3}\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) - \frac{2360}{5103}\,\sqrt{3}\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2/1701*(945*x^3 - 261*x^2 - 360*x + 334)*sqrt(3*x^2 + 5*x + 2)*sqrt(x) +
32/6561*sqrt(3)*weierstrassPInverse(28/27, 80/729, x + 5/9) - 2360/5103*sq
rt(3)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x
+ 5/9))
```

Sympy [F]

$$\int (2-5x)x^{3/2}\sqrt{2+5x+3x^2}\,dx = -\int \left(-2x^{\frac{3}{2}}\sqrt{3x^2+5x+2}\right)dx - \int 5x^{\frac{5}{2}}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*x**(3/2)*(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2*x**(3/2)*sqrt(3*x**2 + 5*x + 2), x) - Integral(5*x**(5/2)*sqr
t(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int (2-5x)x^{3/2}\sqrt{2+5x+3x^2}\,dx = \int -\sqrt{3\,x^2+5\,x+2}(5\,x-2)x^{\frac{3}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)*x^(3/2), x)
```

Giac [F]

$$\int (2-5x)x^{3/2}\sqrt{2+5x+3x^2}\,dx = \int -\sqrt{3\,x^2+5\,x+2}(5\,x-2)x^{\frac{3}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-5x)x^{3/2}\sqrt{2+5x+3x^2}\,dx = -\int x^{3/2}\,(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}\,dx$$

input
```
int(-x^(3/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
-int(x^(3/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int (2-5x)x^{3/2}\sqrt{2+5x+3x^2}\,dx = -\frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3}{9} + \frac{58\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{189} + \frac{80\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{189} - \frac{16\sqrt{x}\,\sqrt{3x^2+5x+2}}{63} - \frac{118\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{189} + \frac{16\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{63}$$

input
```
int((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(1/2),x)
```

output
```
(2*( - 105*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 + 29*sqrt(x)*sqrt(3*x**2 +
5*x + 2)*x**2 + 40*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x - 24*sqrt(x)*sqrt(3*x*
*2 + 5*x + 2) - 59*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(3*x**2 + 5*x +
2),x) + 24*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x)
))/189
```

3.182 $\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\begin{aligned}\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx &= -\frac{2476\sqrt{x}(2+3x)}{2835\sqrt{2+5x+3x^2}}+\frac{164}{189}\sqrt{x}\sqrt{2+5x+3x^2}\\ &\quad+\frac{2}{105}(17-75x)x^{3/2}\sqrt{2+5x+3x^2}+\frac{2476\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{2835\sqrt{1+x}\sqrt{2+3x}}\\ &\quad-\frac{164\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{189\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
-2476/2835*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+164/189*x^(1/2)*(3*x^2+5*x+
2)^(1/2)+2/105*(17-75*x)*x^(3/2)*(3*x^2+5*x+2)^(1/2)+2476/2835*2^(1/2)*(3*
x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/
(2+3*x)^(1/2)-164/189*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(ar
ctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.92

$$\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx$$
$$= \frac{-2(2476+3730x-3354x^2-1935x^3+8748x^4+6075x^5)-2476i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\Big(i\text{arcsinh}\Big(}{2835\sqrt{x}\sqrt{2+5x+3x^2}}$$

input
```
Integrate[(2 - 5*x)*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-2*(2476 + 3730*x - 3354*x^2 - 1935*x^3 + 8748*x^4 + 6075*x^5) - (2476*I)
*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2
/3]/Sqrt[x]], 3/2] + (16*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)
*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(2835*Sqrt[x]*Sqrt[2 + 5*x
+ 3*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1236, 1231, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-5x)\sqrt{x}\sqrt{3x^2+5x+2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{21}\int \frac{(71x+5)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx-\frac{10}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(639x+430)\sqrt{3x^2+5x+2}-\frac{2}{45}\int\frac{619x+410}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{10}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(639x+430)\sqrt{3x^2+5x+2}-\frac{1}{45}\int\frac{619x+410}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{10}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 1240

$$\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(639x+430)\sqrt{3x^2+5x+2}-\frac{2}{45}\int\frac{619x+410}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{10}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 1503

$$\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(639x+430)\sqrt{3x^2+5x+2}-\frac{2}{45}\left(410\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+619\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{10}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 1413

$$\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(639x+430)\sqrt{3x^2+5x+2}-\frac{2}{45}\left(619\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{205\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\text{arc}\right.}{\sqrt{3x^2+5x+2}}\right.\right.$$
$$\frac{10}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}$$

$\Big\downarrow$ 1456

$$\frac{2}{21}\left(\frac{2}{45}\sqrt{x}(639x+430)\sqrt{3x^2+5x+2}-\frac{2}{45}\left(\frac{205\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+619\left(\frac{\sqrt{x}}{3\sqrt{3x}}\right.\right.\right.$$
$$\frac{10}{21}\sqrt{x}\left(3x^2+5x+2\right)^{3/2}$$

input
```
Int[(2 - 5*x)*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-10*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2))/21 + (2*((2*Sqrt[x]*(430 + 639*x)*Sq
rt[2 + 5*x + 3*x^2])/45 - (2*(619*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3
*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]
], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (205*Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)
/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/45))/2
1
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

method	result
default	$\frac{-\frac{30x^5}{7}+\frac{418\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{2835}-\frac{1238\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{8505}-\frac{216x^4}{35}+\frac{86x^3}{63}+\frac{4712x^2}{945}}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
risch	$-\frac{2(675x^2-153x-410)\sqrt{x}\,\sqrt{3x^2+5x+2}}{945}-\frac{\left(\frac{164\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{567\sqrt{3x^3+5x^2+2x}}+\frac{1238\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\text{EllipticE}\left(\right.\right.}{2835\sqrt{3x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{10x^2\sqrt{3x^3+5x^2+2x}}{7}+\frac{34x\sqrt{3x^3+5x^2+2x}}{105}+\frac{164\sqrt{3x^3+5x^2+2x}}{189}-\frac{164\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{567\sqrt{3x^3+5x^2+2x}}-\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/8505/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(-18225*x^5+627*(6*x+4)^(1/2)*(3+3*x)^(
1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-619*(6*x+4)
^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1
/2))-26244*x^4+5805*x^3+21204*x^2+7380*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.30

$$\begin{aligned}
&\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx \\
&= -\frac{2}{945}\left(675\,x^2-153\,x-410\right)\sqrt{3\,x^2+5\,x+2}\sqrt{x} \\
&\quad -\frac{68}{729}\sqrt{3}\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) \\
&\quad +\frac{2476}{2835}\sqrt{3}\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)
\end{aligned}$$

input
```
integrate((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2/945*(675*x^2 - 153*x - 410)*sqrt(3*x^2 + 5*x + 2)*sqrt(x) - 68/729*sqrt
(3)*weierstrassPInverse(28/27, 80/729, x + 5/9) + 2476/2835*sqrt(3)*weiers
trassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x + 5/9))
```

Sympy [F]

$$\begin{aligned}
\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx = &-\int \left(-2\sqrt{x}\sqrt{3x^2+5x+2}\right)\,dx \\
&-\int 5x^{\frac{3}{2}}\sqrt{3x^2+5x+2}\,dx
\end{aligned}$$

input
```
integrate((2-5*x)*x**(1/2)*(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2*sqrt(x)*sqrt(3*x**2 + 5*x + 2), x) - Integral(5*x**(3/2)*sqrt
(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx = \int -\sqrt{3\,x^2+5\,x+2}(5\,x-2)\sqrt{x}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)*sqrt(x), x)
```

Giac [F]

$$\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx = \int -\sqrt{3\,x^2+5\,x+2}(5\,x-2)\sqrt{x}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx = -\int \sqrt{x}\,(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}\,dx$$

input
```
int(-x^(1/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
-int(x^(1/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int (2-5x)\sqrt{x}\sqrt{2+5x+3x^2}\,dx = -\frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{7}+\frac{34\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{105}+\frac{106\sqrt{x}\,\sqrt{3x^2+5x+2}}{175}+\frac{619\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{525}-\frac{106\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{175}$$

input
```
int((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(1/2),x)
```

output
```
( - 750*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 + 170*sqrt(x)*sqrt(3*x**2 + 5*
x + 2)*x + 318*sqrt(x)*sqrt(3*x**2 + 5*x + 2) + 619*int((sqrt(x)*sqrt(3*x*
*2 + 5*x + 2)*x)/(3*x**2 + 5*x + 2),x) - 318*int((sqrt(x)*sqrt(3*x**2 + 5*
x + 2))/(3*x**3 + 5*x**2 + 2*x),x))/525
```

3.183 $\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\begin{aligned}\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx &= \frac{88\sqrt{x}(2+3x)}{27\sqrt{2+5x+3x^2}}+\frac{2}{9}(1-9x)\sqrt{x}\sqrt{2+5x+3x^2}\\ &\quad-\frac{88\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{27\sqrt{1+x}\sqrt{2+3x}}\\ &\quad+\frac{34\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\mathrm{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{9\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
88/27*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+2/9*(1-9*x)*x^(1/2)*(3*x^2+5*x+2
)^(1/2)-88/27*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/
2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+34/9*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1
/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.02

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx = \frac{2(88+226x+93x^2-126x^3-81x^4)+88i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)+14i\sqrt{2}\sqrt{1+}}{27\sqrt{x}\sqrt{2+5x+3x^2}}$$

input
```
Integrate[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x],x]
```

output
```
(2*(88 + 226*x + 93*x^2 - 126*x^3 - 81*x^4) + (88*I)*Sqrt[2]*Sqrt[1 + x^(-
1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (
14*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[S
qrt[2/3]/Sqrt[x]], 3/2])/(27*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1231, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\sqrt{3x^2+5x+2}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{9}(1-9x)\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{45}\int -\frac{5(22x+17)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\int \frac{22x+17}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(1-9x)$$

$$\Big\downarrow \ 1240$$

$$\frac{4}{9}\int \frac{22x+17}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(1-9x)$$

$$\Big\downarrow \ 1503$$

$$\frac{4}{9}\left(17\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+22\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(1-9x)$$

$$\Big\downarrow \ 1413$$

$$\frac{4}{9}\left(22\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{17(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(1-9x)$$

$$\Big\downarrow \ 1456$$

$$\frac{4}{9}\left(\frac{17(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+22\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right)|-\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$
$$\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(1-9x)$$

input
```
Int[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x],x]
```

output
```
(2*(1 - 9*x)*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])/9 + (4*(22*((Sqrt[x]*(2 + 3*x)
)/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*Ell
ipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (17*(1 + x)*Sq
rt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 +
5*x + 3*x^2])))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2\left(15\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-22\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)+243x^4+378x^3+117\right.}{81\sqrt{3x^2+5x+2}\,\sqrt{x}}$
risch	$-\frac{2(-1+9x)\sqrt{x}\,\sqrt{3x^2+5x+2}}{9}-\frac{\left(-\frac{34\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{27\sqrt{3x^3+5x^2+2x}}-\frac{44\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\frac{\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3}\right.}{27\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-2x\sqrt{3x^3+5x^2+2x}+\frac{2\sqrt{3x^3+5x^2+2x}}{9}+\frac{34\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{27\sqrt{3x^3+5x^2+2x}}+\frac{44\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\right.}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/81/(3*x^2+5*x+2)^(1/2)/x^(1/2)*(15*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*
(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-22*(6*x+4)^(1/2)*(3+3*x)
^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+243*x^4+3
78*x^3+117*x^2-18*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx = -\frac{2}{9}\sqrt{3x^2+5x+2}(9x-1)\sqrt{x}+\frac{172}{243}\sqrt{3}\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) - \frac{88}{27}\sqrt{3}\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-2/9*sqrt(3*x^2 + 5*x + 2)*(9*x - 1)*sqrt(x) + 172/243*sqrt(3)*weierstrass
PInverse(28/27, 80/729, x + 5/9) - 88/27*sqrt(3)*weierstrassZeta(28/27, 80
/729, weierstrassPInverse(28/27, 80/729, x + 5/9))
```

Sympy [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx = -\int \left(-\frac{2\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)dx - \int 5\sqrt{x}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(1/2)/x**(1/2),x)
```

output
```
-Integral(-2*sqrt(3*x**2 + 5*x + 2)/sqrt(x), x) - Integral(5*sqrt(x)*sqrt(
3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/sqrt(x), x)
```

Giac [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx = -\int \frac{(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}}{\sqrt{x}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(1/2),x)
```

output
```
-int(((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(1/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{\sqrt{x}}\,dx = -2\sqrt{x}\,\sqrt{3x^2+5x+2}\,x+\frac{6\sqrt{x}\,\sqrt{3x^2+5x+2}}{5} - \frac{22\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{5}+\frac{14\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{5}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(1/2),x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x + 3*sqrt(x)*sqrt(3*x**2 + 5*x +
2) - 11*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(3*x**2 + 5*x + 2),x) + 7*i
nt((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x)))/5
```

3.184 $\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx = \frac{22\sqrt{x}(2+3x)}{9\sqrt{2+5x+3x^2}} - \frac{2(6+5x)\sqrt{2+5x+3x^2}}{3\sqrt{x}} - \frac{22\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{9\sqrt{1+x}\sqrt{2+3x}} + \frac{10\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{3\sqrt{2+5x+3x^2}}$$

output
```
22/9*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-2/3*(6+5*x)*(3*x^2+5*x+2)^(1/2)/x
^(1/2)-22/9*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*
I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+10/3*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2
)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx = \frac{-2(14+65x+96x^2+45x^3)+22i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{9\sqrt{x}\sqrt{2+5x}}$$

input
```
Integrate[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(3/2),x]
```

output
```
(-2*(14 + 65*x + 96*x^2 + 45*x^3) + (22*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3
 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (8*I)*Sqrt[
2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sq
rt[x]], 3/2])/(9*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1230, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\sqrt{3x^2+5x+2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{1230}$$

$$-\frac{2}{3}\int -\frac{11x+10}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2\sqrt{3x^2+5x+2}(5x+6)}{3\sqrt{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{11x+10}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2(5x+6)\sqrt{3x^2+5x+2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2}{3}\int \frac{11x+10}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{2(5x+6)\sqrt{3x^2+5x+2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{2}{3}\left(10\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+11\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2(5x+6)\sqrt{3x^2+5x+2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{1413}$$

$$\frac{2}{3}\left(11\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{5\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)-\frac{2(5x+6)\sqrt{3x^2+5x+2}}{3\sqrt{x}}$$

$$\Big\downarrow \text{1456}$$

$$\frac{2}{3}\left(\frac{5\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+11\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right)\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$
$$\frac{2(5x+6)\sqrt{3x^2+5x+2}}{3\sqrt{x}}$$

input
```
Int[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(3/2),x]
```

output
```
(-2*(6 + 5*x)*Sqrt[2 + 5*x + 3*x^2])/(3*Sqrt[x]) + (2*(11*((Sqrt[x]*(2 + 3
*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*
EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (5*Sqrt[2]*
(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 +
 5*x + 3*x^2]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

method	result
default	$-\frac{3\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-11\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)+270x^3+774x^2+720x+}{27\sqrt{3x^2+5x+2}\,\sqrt{x}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{4(3x^2+5x+2)}{\sqrt{(3x^2+5x+2)x}}-\frac{10\sqrt{3x^3+5x^2+2x}}{3}+\frac{10\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{9\sqrt{3x^3+5x^2+2x}}+\frac{11\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\text{Ellip}\right.}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*(3*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x
+4)^(1/2),I*2^(1/2))-11*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ell
ipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+270*x^3+774*x^2+720*x+216)/(3*x^2+5*x+
2)^(1/2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.35

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx = \frac{2\left(35\sqrt{3}x\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-99\sqrt{3}x\text{weierstrassZeta}\left(\frac{2}{2}\right.\right.}{}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/81*(35*sqrt(3)*x*weierstrassPInverse(28/27, 80/729, x + 5/9) - 99*sqrt(3
)*x*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x +
5/9)) - 27*sqrt(3*x^2 + 5*x + 2)*(5*x + 6)*sqrt(x))/x
```

Sympy [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx = -\int \left(-\frac{2\sqrt{3x^2+5x+2}}{x^{\frac{3}{2}}}\right)dx - \int \frac{5\sqrt{3x^2+5x+2}}{\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(1/2)/x**(3/2),x)
```

output
```
-Integral(-2*sqrt(3*x**2 + 5*x + 2)/x**(3/2), x) - Integral(5*sqrt(3*x**2
+ 5*x + 2)/sqrt(x), x)
```

Maxima [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(3/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx=\int -\frac{(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}}{x^{3/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(3/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(3/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{3/2}}\,dx=\frac{-10\sqrt{3x^2+5x+2}\,x-12\sqrt{3x^2+5x+2}+10\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)+}{3\sqrt{x}}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(3/2),x)
```

output
```
( - 10*sqrt(3*x**2 + 5*x + 2)*x - 12*sqrt(3*x**2 + 5*x + 2) + 10*sqrt(x)*i
nt((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x) + 11*sqrt(x
)*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**2 + 5*x + 2),x))/(3*sqrt(x))
```

3.185 $\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\begin{aligned}\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx =& -\frac{50\sqrt{x}(2+3x)}{3\sqrt{2+5x+3x^2}} \\ &- \frac{4(1-5x)\sqrt{2+5x+3x^2}}{3x^{3/2}} + \frac{50\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{3\sqrt{1+x}\sqrt{2+3x}} \\ &- \frac{21\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
-50/3*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-4/3*(1-5*x)*(3*x^2+5*x+2)^(1/2)/
x^(3/2)+50/3*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2
*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-21*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)
*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx = \frac{-2(4+40x+81x^2+45x^3)-50i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{5/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right.\right.}{3x^{3/2}\sqrt{2+5x}}$$

input
```
Integrate[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(5/2),x]
```

output
```
(-2*(4 + 40*x + 81*x^2 + 45*x^3) - (50*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3
+ 2/x]*x^(5/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (13*I)*Sqrt[
2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(5/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sq
rt[x]], 3/2])/(3*x^(3/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1229, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\sqrt{3x^2+5x+2}}{x^{5/2}}dx$$

$$\Big\downarrow \; 1229$$

$$-\frac{1}{3}\int \frac{3(25x+21)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{4\sqrt{3x^2+5x+2}(1-5x)}{3x^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\int \frac{25x+21}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{4\sqrt{3x^2+5x+2}(1-5x)}{3x^{3/2}}$$

$$\Big\downarrow \; 1240$$

$$-2\int \frac{25x+21}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{4\sqrt{3x^2+5x+2}(1-5x)}{3x^{3/2}}$$

$$\Big\downarrow \text{1503}$$

$$-2\left(21\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+25\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{4\sqrt{3x^2+5x+2}(1-5x)}{3x^{3/2}}$$

$$\Big\downarrow \text{1413}$$

$$-2\left(25\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{21(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{4\sqrt{3x^2+5x+2}(1-5x)}{3x^{3/2}}$$

$$\Big\downarrow \text{1456}$$

$$-2\left(\frac{21(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+25\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right)\right|}{3\sqrt{3x^2+5x+2}}\right.\right.$$
$$\frac{4\sqrt{3x^2+5x+2}(1-5x)}{3x^{3/2}}$$

input
```
Int[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(5/2),x]
```

output
```
(-4*(1 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/(3*x^(3/2)) - 2*(25*((Sqrt[x]*(2 + 3*
x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*E
llipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (21*(1 + x)*
Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2
+ 5*x + 3*x^2]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.75

method	result
default	$\frac{12\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x-25\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x+180x^3+264x^2+60x-}{9\sqrt{3x^2+5x+2}\,x^{\frac{3}{2}}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{4\sqrt{3x^3+5x^2+2x}}{3x^2}+\frac{20x^2+\frac{100}{3}x+\frac{40}{3}}{\sqrt{\left(3x^2+5x+2\right)x}}-\frac{7\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{\sqrt{3x^3+5x^2+2x}}-\frac{25\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\text{Ellipti}\right.}{3}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(12*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+
4)^(1/2),I*2^(1/2))*x-25*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*El
lipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*x+180*x^3+264*x^2+60*x-24)/(3*x^2+5*x
+2)^(1/2)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.39

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx =$$
$$-\frac{2\left(64\sqrt{3}x^2\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-225\sqrt{3}x^2\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\right.\right.\right.}{27\,x^2}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(64*sqrt(3)*x^2*weierstrassPInverse(28/27, 80/729, x + 5/9) - 225*sq
rt(3)*x^2*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729
, x + 5/9)) - 18*sqrt(3*x^2 + 5*x + 2)*(5*x - 1)*sqrt(x))/x^2
```

Sympy [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx = -\int \left(-\frac{2\sqrt{3x^2+5x+2}}{x^{\frac{5}{2}}}\right)dx - \int \frac{5\sqrt{3x^2+5x+2}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(1/2)/x**(5/2),x)
```

output
```
-Integral(-2*sqrt(3*x**2 + 5*x + 2)/x**(5/2), x) - Integral(5*sqrt(3*x**2
+ 5*x + 2)/x**(3/2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(5/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx = \int -\frac{(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}}{x^{5/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(5/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(5/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{5/2}}\,dx = \frac{-10\sqrt{3x^2+5x+2}\,x+2\sqrt{3x^2+5x+2}+10\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^5+5x^4+2x^3}dx\right)x-}{\sqrt{x}\,x}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(5/2),x)
```

output
```
(2*( - 5*sqrt(3*x**2 + 5*x + 2)*x + sqrt(3*x**2 + 5*x + 2) + 5*sqrt(x)*int
((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**5 + 5*x**4 + 2*x**3),x)*x - 8*sqrt
(x)*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x)*x))/(s
qrt(x)*x)
```

3.186 $\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx = -\frac{139\sqrt{x}(2+3x)}{15\sqrt{2+5x+3x^2}}-\frac{4(3-10x)\sqrt{2+5x+3x^2}}{15x^{5/2}} +\frac{139\sqrt{2+5x+3x^2}}{15\sqrt{x}}+\frac{139\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{15\sqrt{1+x}\sqrt{2+3x}} -\frac{11\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}$$

output
```
-139/15*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-4/15*(3-10*x)*(3*x^2+5*x+2)^(1
/2)/x^(5/2)+139/15*(3*x^2+5*x+2)^(1/2)/x^(1/2)+139/15*2^(1/2)*(3*x^2+5*x+2
)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(
1/2)-11*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),
1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx = \frac{4(-6+5x+41x^2+30x^3) - 139i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{7/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right.\right.}{15x^{5/2}\sqrt{2+5\ldots}}$$

input
```
Integrate[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(7/2),x]
```

output
```
(4*(-6 + 5*x + 41*x^2 + 30*x^3) - (139*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3
+ 2/x]*x^(7/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (26*I)*Sqrt[
2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(7/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sq
rt[x]], 3/2])/(15*x^(5/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1229, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\sqrt{3x^2+5x+2}}{x^{7/2}}dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{1}{15}\int \frac{165x+139}{x^{3/2}\sqrt{3x^2+5x+2}}dx - \frac{4\sqrt{3x^2+5x+2}(3-10x)}{15x^{5/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{15}\left(\int -\frac{3(139x+110)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx + \frac{139\sqrt{3x^2+5x+2}}{\sqrt{x}}\right) - \frac{4(3-10x)\sqrt{3x^2+5x+2}}{15x^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(\frac{139\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{3}{2}\int\frac{139x+110}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{4(3-10x)\sqrt{3x^2+5x+2}}{15x^{5/2}}$$

$$\Big\downarrow \text{ 1240}$$

$$\frac{1}{15}\left(\frac{139\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\int\frac{139x+110}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{4(3-10x)\sqrt{3x^2+5x+2}}{15x^{5/2}}$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{15}\left(\frac{139\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(110\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+139\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{4(3-10x)\sqrt{3x^2+5x+2}}{15x^{5/2}}$$

$$\Big\downarrow \text{ 1413}$$

$$\frac{1}{15}\left(\frac{139\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(139\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{55\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)\right)-\frac{4(3-10x)\sqrt{3x^2+5x+2}}{15x^{5/2}}$$

$$\Big\downarrow \text{ 1456}$$

$$\frac{1}{15}\left(\frac{139\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(\frac{55\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+139\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(\right.\right.\right.$$
$$\frac{4(3-10x)\sqrt{3x^2+5x+2}}{15x^{5/2}}$$

input
```
Int[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(7/2),x]
```

output
```
(-4*(3 - 10*x)*Sqrt[2 + 5*x + 3*x^2])/(15*x^(5/2)) + ((139*Sqrt[2 + 5*x +
3*x^2])/Sqrt[x] - 3*(139*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) -
(Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])
/(3*Sqrt[2 + 5*x + 3*x^2])) + (55*Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*
EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

method	result
default	$\frac{87\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-139\,\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+2502x^4+4890x^3+}{90\sqrt{3x^2+5x+2}\,x^{\frac{5}{2}}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{4\sqrt{3x^3+5x^2+2x}}{5x^3}+\frac{8\sqrt{3x^3+5x^2+2x}}{3x^2}+\frac{\frac{139}{5}x^2+\frac{139}{3}x+\frac{278}{15}}{\sqrt{\left(3x^2+5x+2\right)x}}-\frac{11\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3\sqrt{3x^3+5x^2+2x}}-\frac{139\sqrt{6x+}}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/90*(87*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))*(6*x+4)^(1/2)*(3+3*x)^(1/2
)*6^(1/2)*(-x)^(1/2)*x^2-139*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*(6*x+4
)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*x^2+2502*x^4+4890*x^3+2652*x^2+12
0*x-144)/(3*x^2+5*x+2)^(1/2)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.36

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx =$$
$$-\frac{295\,\sqrt{3}x^3\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-1251\,\sqrt{3}x^3\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{2}{2}\right.\right.}{135\,x^3}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
-1/135*(295*sqrt(3)*x^3*weierstrassPInverse(28/27, 80/729, x + 5/9) - 1251
*sqrt(3)*x^3*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/
729, x + 5/9)) - 9*(139*x^2 + 40*x - 12)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/x^
3
```

Sympy [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx = -\int \left(-\frac{2\sqrt{3x^2+5x+2}}{x^{\frac{7}{2}}}\right)\,dx - \int \frac{5\sqrt{3x^2+5x+2}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(1/2)/x**(7/2),x)
```

output
```
-Integral(-2*sqrt(3*x**2 + 5*x + 2)/x**(7/2), x) - Integral(5*sqrt(3*x**2
+ 5*x + 2)/x**(5/2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(7/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx = \int -\frac{(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}}{x^{7/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(7/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(7/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{7/2}}\,dx = \frac{-548\sqrt{3x^2+5x+2}\,x^2+200\sqrt{3x^2+5x+2}\,x-16\sqrt{3x^2+5x+2}+440\cdots}{}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(7/2),x)
```

output
```
( - 548*sqrt(3*x**2 + 5*x + 2)*x**2 + 200*sqrt(3*x**2 + 5*x + 2)*x - 16*sq
rt(3*x**2 + 5*x + 2) + 440*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x
**4 + 5*sqrt(x)*x**3 + 2*sqrt(x)*x**2),x)*x**2 + 297*sqrt(x)*int((sqrt(3*x
**2 + 5*x + 2)*x)/(3*sqrt(x)*x**2 + 5*sqrt(x)*x + 2*sqrt(x)),x)*x**2 + 525
*sqrt(x)*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**2 + 5*x + 2),x)*x**2)/
(20*sqrt(x)*x**2)
```

3.187 $\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx &= \frac{62\sqrt{x}(2+3x)}{21\sqrt{2+5x+3x^2}} \\ &- \frac{4(1-3x)\sqrt{2+5x+3x^2}}{7x^{7/2}} + \frac{43\sqrt{2+5x+3x^2}}{21x^{3/2}} \\ &- \frac{62\sqrt{2+5x+3x^2}}{21\sqrt{x}} - \frac{62\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{21\sqrt{1+x}\sqrt{2+3x}} \\ &+ \frac{43\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{7\sqrt{2}\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
62/21*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-4/7*(1-3*x)*(3*x^2+5*x+2)^(1/2)/
x^(7/2)+43/21*(3*x^2+5*x+2)^(1/2)/x^(3/2)-62/21*(3*x^2+5*x+2)^(1/2)/x^(1/2
)-62/21*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^
(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+43/14*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*I
nverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.77

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx = \frac{-48+24x+460x^2+646x^3+258x^4+124i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,x^{9/2}E\Big(i\mathrm{ar}\cdots}{42x^{7/2}\sqrt{2}\cdots}$$

input
```
Integrate[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(9/2),x]
```

output
```
(-48 + 24*x + 460*x^2 + 646*x^3 + 258*x^4 + (124*I)*Sqrt[2]*Sqrt[1 + x^(-1
)]*Sqrt[3 + 2/x]*x^(9/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (5
*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(9/2)*EllipticF[I*ArcSinh[Sqr
t[2/3]/Sqrt[x]], 3/2])/(42*x^(7/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1229, 27, 1237, 27, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\sqrt{3x^2+5x+2}}{x^{9/2}}dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{1}{35}\int \frac{5(51x+43)}{x^{5/2}\sqrt{3x^2+5x+2}}dx-\frac{4\sqrt{3x^2+5x+2}(1-3x)}{7x^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{7}\int \frac{51x+43}{x^{5/2}\sqrt{3x^2+5x+2}}dx-\frac{4\sqrt{3x^2+5x+2}(1-3x)}{7x^{7/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{7}\left(\frac{1}{3}\int \frac{129x+124}{2x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{43\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{4(1-3x)\sqrt{3x^2+5x+2}}{7x^{7/2}}$$

$\big\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{6}\int \frac{129x+124}{x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{43\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{4(1-3x)\sqrt{3x^2+5x+2}}{7x^{7/2}}$$

$\big\downarrow$ 1237

$$\frac{1}{7}\left(\frac{1}{6}\left(-\int -\frac{3(62x+43)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{124\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{43\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{4(1-3x)\sqrt{3x^2+5x+2}}{7x^{7/2}}$$

$\big\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{6}\left(3\int \frac{62x+43}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{124\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{43\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{4(1-3x)\sqrt{3x^2+5x+2}}{7x^{7/2}}$$

$\big\downarrow$ 1240

$$\frac{1}{7}\left(\frac{1}{6}\left(6\int \frac{62x+43}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{124\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{43\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{4(1-3x)\sqrt{3x^2+5x+2}}{7x^{7/2}}$$

$\big\downarrow$ 1503

$$\frac{1}{7}\left(\frac{1}{6}\left(6\left(43\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+62\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{124\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{43\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)$$
$$\frac{4(1-3x)\sqrt{3x^2+5x+2}}{7x^{7/2}}$$

$\big\downarrow$ 1413

$$\frac{1}{7}\left(\frac{1}{6}\left(6\left(62\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{43(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{124\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)\right.$$
$$\frac{4(1-3x)\sqrt{3x^2+5x+2}}{7x^{7/2}}$$

$\big\downarrow$ 1456

$$\frac{1}{7}\left(\frac{1}{6}\left(6\left(\frac{43(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+62\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\right.\right.}{3\sqrt{3x^2+5x+2}}\right.\right.\right.\right.$$

$$\frac{4(1-3x)\sqrt{3x^2+5x+2}}{7x^{7/2}}$$

input
```
Int[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(9/2),x]
```

output
```
(-4*(1 - 3*x)*Sqrt[2 + 5*x + 3*x^2])/(7*x^(7/2)) + ((43*Sqrt[2 + 5*x + 3*x
^2])/(3*x^(3/2)) + ((-124*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x] + 6*(62*((Sqrt[x]
*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1
 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (43*
(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x + 3*x^2])))/6)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

method	result
default	$\frac{62\,\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2}, i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^{3}-57\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\;\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2}, i\sqrt{2}\right)x^{3}-1116x^{5}-1086x^{4}+1}{126\sqrt{3x^{2}+5x+2}\,x^{\frac{7}{2}}}$
elliptic	$\frac{\sqrt{(3x^{2}+5x+2)x}\left(-\frac{4\sqrt{3x^{3}+5x^{2}+2x}}{7x^{4}}+\frac{12\sqrt{3x^{3}+5x^{2}+2x}}{7x^{3}}+\frac{43\sqrt{3x^{3}+5x^{2}+2x}}{21x^{2}}-\frac{62(3x^{2}+5x+2)}{21\sqrt{(3x^{2}+5x+2)x}}+\frac{43\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\;\mathrm{EllipticF}\left(\frac{\sqrt{6x}}{2}\right.}{42\sqrt{3x^{3}+5x^{2}+2x}}\right.}{\sqrt{x}\,\sqrt{3x^{2}+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/126*(62*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*(6*x+4)^(1/2)*(3+3*x)^(1/
2)*6^(1/2)*(-x)^(1/2)*x^3-57*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2
)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))*x^3-1116*x^5-1086*x^4+1194*x^3+13
80*x^2+72*x-144)/(3*x^2+5*x+2)^(1/2)/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.34

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx = \frac{77\sqrt{3}x^4\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-558\sqrt{3}x^4\mathrm{weierstrassZeta}\left(\frac{2}{2}\right.}{}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
1/189*(77*sqrt(3)*x^4*weierstrassPInverse(28/27, 80/729, x + 5/9) - 558*sq
rt(3)*x^4*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729
, x + 5/9)) - 9*(62*x^3 - 43*x^2 - 36*x + 12)*sqrt(3*x^2 + 5*x + 2)*sqrt(x
))/x^4
```

Sympy [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx = -\int \left(-\frac{2\sqrt{3x^2+5x+2}}{x^{\frac{9}{2}}}\right) dx - \int \frac{5\sqrt{3x^2+5x+2}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(1/2)/x**(9/2),x)
```

output
```
-Integral(-2*sqrt(3*x**2 + 5*x + 2)/x**(9/2), x) - Integral(5*sqrt(3*x**2
+ 5*x + 2)/x**(7/2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(9/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx = \int -\frac{(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}}{x^{9/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(9/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(9/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{9/2}}\,dx = \frac{70\sqrt{3x^2+5x+2}\,x-12\sqrt{3x^2+5x+2}+170\sqrt{x}\left(\int \frac{\sqrt{3x^2+5x+2}}{3\sqrt{x}\,x^5+5\sqrt{x}\,x^4+2\sqrt{x}\,x^3}d\right.}{21}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(9/2),x)
```

output
```
(70*sqrt(3*x**2 + 5*x + 2)*x - 12*sqrt(3*x**2 + 5*x + 2) + 170*sqrt(x)*int
(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**5 + 5*sqrt(x)*x**4 + 2*sqrt(x)*x**3)
,x)*x**3 + 85*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**4 + 5*sqrt(
x)*x**3 + 2*sqrt(x)*x**2),x)*x**3 + 126*sqrt(x)*int((sqrt(x)*sqrt(3*x**2 +
 5*x + 2))/(3*x**5 + 5*x**4 + 2*x**3),x)*x**3)/(21*sqrt(x)*x**3)
```

3.188 $\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 224

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}} \, dx = -\frac{1331\sqrt{x}(2+3x)}{630\sqrt{2+5x+3x^2}} - \frac{4(7-20x)\sqrt{2+5x+3x^2}}{63x^{9/2}} + \frac{97\sqrt{2+5x+3x^2}}{105x^{5/2}} - \frac{79\sqrt{2+5x+3x^2}}{63x^{3/2}} + \frac{1331\sqrt{2+5x+3x^2}}{630\sqrt{x}} + \frac{1331\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{315\sqrt{2}\sqrt{1+x}\sqrt{2+3x}} - \frac{79\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{21\sqrt{2}\sqrt{2+5x+3x^2}}$$

output
```
-1331/630*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-4/63*(7-20*x)*(3*x^2+5*x+2)^
(1/2)/x^(9/2)+97/105*(3*x^2+5*x+2)^(1/2)/x^(5/2)-79/63*(3*x^2+5*x+2)^(1/2)
/x^(3/2)+1331/630*(3*x^2+5*x+2)^(1/2)/x^(1/2)+1331/630*2^(1/2)*(3*x^2+5*x+
2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^
(1/2)-79/42*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/
2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.71

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}}\,dx = \frac{-560+200x+4324x^2+3730x^3-2204x^4-2370x^5-1331i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{\ }}{}$$

input
```
Integrate[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(11/2),x]
```

output
```
(-560 + 200*x + 4324*x^2 + 3730*x^3 - 2204*x^4 - 2370*x^5 - (1331*I)*Sqrt[
2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(11/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/S
qrt[x]], 3/2] + (146*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(11/2)*El
lipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(630*x^(9/2)*Sqrt[2 + 5*x + 3*
x^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1229, 27, 1237, 27, 1237, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\sqrt{3x^2+5x+2}}{x^{11/2}}\,dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{1}{63}\int \frac{3(115x+97)}{x^{7/2}\sqrt{3x^2+5x+2}}dx - \frac{4\sqrt{3x^2+5x+2}(7-20x)}{63x^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{21}\int \frac{115x+97}{x^{7/2}\sqrt{3x^2+5x+2}}dx - \frac{4\sqrt{3x^2+5x+2}(7-20x)}{63x^{9/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{21}\left(\frac{1}{5}\int \frac{873x+790}{2x^{5/2}\sqrt{3x^2+5x+2}}dx+\frac{97\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\left(\frac{1}{10}\int \frac{873x+790}{x^{5/2}\sqrt{3x^2+5x+2}}dx+\frac{97\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{9/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{21}\left(\frac{1}{10}\left(-\frac{1}{3}\int \frac{1185x+1331}{x^{3/2}\sqrt{3x^2+5x+2}}dx-\frac{790\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{97\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{9/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{21}\left(\frac{1}{10}\left(\frac{1}{3}\left(\int -\frac{3(1331x+790)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{1331\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)-\frac{790\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{97\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\left(\frac{1}{10}\left(\frac{1}{3}\left(\frac{1331\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{3}{2}\int \frac{1331x+790}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{790\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{97\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{9/2}}$$

$$\Big\downarrow \text{1240}$$

$$\frac{1}{21}\left(\frac{1}{10}\left(\frac{1}{3}\left(\frac{1331\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\int \frac{1331x+790}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{790\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{97\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{9/2}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{21}\left(\frac{1}{10}\left(\frac{1}{3}\left(\frac{1331\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(790\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+1331\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{790\sqrt{3x^2}}{3x}\right.\right.$$
$$\frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{9/2}}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{21}\left(\frac{1}{10}\left(\frac{1}{3}\left(\frac{1331\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(1331\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{395\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{\right.\right.}{\sqrt{3x^2+5x+2}}\right.\right.\right.\right.$$
$$\frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{9/2}}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{21}\left(\frac{1}{10}\left(\frac{1}{3}\left(\frac{1331\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(\frac{395\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+1331\left(\frac{\sqrt{x}(3x+}{3\sqrt{3x^2+5x}}\right.\right.\right.\right.\right.$$
$$\frac{4(7-20x)\sqrt{3x^2+5x+2}}{63x^{9/2}}$$

input
```
Int[((2 - 5*x)*Sqrt[2 + 5*x + 3*x^2])/x^(11/2),x]
```

output
```
(-4*(7 - 20*x)*Sqrt[2 + 5*x + 3*x^2])/(63*x^(9/2)) + ((97*Sqrt[2 + 5*x + 3
*x^2])/(5*x^(5/2)) + ((-790*Sqrt[2 + 5*x + 3*x^2])/(3*x^(3/2)) + ((1331*Sq
rt[2 + 5*x + 3*x^2])/Sqrt[x] - 3*(1331*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*
x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sq
rt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (395*Sqrt[2]*(1 + x)*Sqrt[(2 +
 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/3
)/10)/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.60

method	result
default	$\frac{1623\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^4-1331\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^4+23958x^6+2571}{3780\sqrt{3x^2+5x+2}\,x^{\frac{9}{2}}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{4\sqrt{3x^3+5x^2+2x}}{9x^5}+\frac{80\sqrt{3x^3+5x^2+2x}}{63x^4}+\frac{97\sqrt{3x^3+5x^2+2x}}{105x^3}-\frac{79\sqrt{3x^3+5x^2+2x}}{63x^2}+\frac{\frac{1331}{210}x^2+\frac{1331}{126}x+\frac{1331}{315}}{\sqrt{\left(3x^2+5x+2\right)x}}-\frac{79\sqrt{6x+4}\,\sqrt{3+}}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/3780*(1623*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*
(6*x+4)^(1/2),I*2^(1/2))*x^4-1331*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)
^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*x^4+23958*x^6+25710*x^5+2748
*x^4+22380*x^3+25944*x^2+1200*x-3360)/(3*x^2+5*x+2)^(1/2)/x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.33

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}}\,dx = -\frac{455\,\sqrt{3}x^5\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-11979\,\sqrt{3}x^5\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\right.\right.}{5670\,x^5}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
-1/5670*(455*sqrt(3)*x^5*weierstrassPInverse(28/27, 80/729, x + 5/9) - 119
79*sqrt(3)*x^5*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 8
0/729, x + 5/9)) - 9*(1331*x^4 - 790*x^3 + 582*x^2 + 800*x - 280)*sqrt(3*x
^2 + 5*x + 2)*sqrt(x))/x^5
```

Sympy [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}}\,dx = -\int \left(-\frac{2\sqrt{3x^2+5x+2}}{x^{\frac{11}{2}}}\right)dx - \int \frac{5\sqrt{3x^2+5x+2}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(1/2)/x**(11/2),x)
```

output
```
-Integral(-2*sqrt(3*x**2 + 5*x + 2)/x**(11/2), x) - Integral(5*sqrt(3*x**2
 + 5*x + 2)/x**(9/2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(11/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}}\,dx = \int -\frac{\sqrt{3\,x^2+5\,x+2}(5\,x-2)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(3*x^2 + 5*x + 2)*(5*x - 2)/x^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}}\,dx = \int -\frac{(5\,x-2)\,\sqrt{3\,x^2+5\,x+2}}{x^{11/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(11/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(1/2))/x^(11/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\sqrt{2+5x+3x^2}}{x^{11/2}}\,dx = \frac{11826\sqrt{3x^2+5x+2}\,x^4 - 2628\sqrt{3x^2+5x+2}\,x^2 + 7200\sqrt{3x^2+5x+2}\,x}{}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(1/2)/x^(11/2),x)
```

output
```
(11826*sqrt(3*x**2 + 5*x + 2)*x**4 - 2628*sqrt(3*x**2 + 5*x + 2)*x**2 + 72
00*sqrt(3*x**2 + 5*x + 2)*x - 1600*sqrt(3*x**2 + 5*x + 2) + 18400*sqrt(x)*
int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**6 + 5*sqrt(x)*x**5 + 2*sqrt(x)*x*
*4),x)*x**4 + 9660*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**5 + 5*
sqrt(x)*x**4 + 2*sqrt(x)*x**3),x)*x**4 - 26280*sqrt(x)*int(sqrt(3*x**2 + 5
*x + 2)/(3*sqrt(x)*x**4 + 5*sqrt(x)*x**3 + 2*sqrt(x)*x**2),x)*x**4 - 17739
*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x)/(3*sqrt(x)*x**2 + 5*sqrt(x)*x + 2*
sqrt(x)),x)*x**4)/(3600*sqrt(x)*x**4)
```

3.189 $\int (2-5x)x^{5/2}(2+5x+3x^2)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int (2-5x)x^{5/2}(2+5x+3x^2)^{3/2}\,dx = -\frac{497824\sqrt{x}(2+3x)}{32837805\sqrt{2+5x+3x^2}} + \frac{61736\sqrt{x}\sqrt{2+5x+3x^2}}{2189187} - \frac{3688x^{3/2}\sqrt{2+5x+3x^2}}{93555} + \frac{13004x^{5/2}\sqrt{2+5x+3x^2}}{243243} + \frac{4x^{7/2}(2803+2484x)\sqrt{2+5x+3x^2}}{11583} + \frac{2}{39}(1-13x)x^{7/2}(2+5x+3x^2)^{3/2} + \frac{497824\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{32837805\sqrt{1+x}\sqrt{2+3x}} - \frac{61736\sqrt{2}\sqrt{1+x}\sqrt{2+}}{21891}$$

output

```
-497824/32837805*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+61736/2189187*x^(1/2)
*(3*x^2+5*x+2)^(1/2)-3688/93555*x^(3/2)*(3*x^2+5*x+2)^(1/2)+13004/243243*x
^(5/2)*(3*x^2+5*x+2)^(1/2)+4/11583*x^(7/2)*(2803+2484*x)*(3*x^2+5*x+2)^(1/
2)+2/39*(1-13*x)*x^(7/2)*(3*x^2+5*x+2)^(3/2)+497824/32837805*2^(1/2)*(3*x^
2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2
+3*x)^(1/2)-61736/2189187*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiA
M(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.24 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.73

$$\int (2-5x)x^{5/2}\left(2+5x+3x^2\right)^{3/2}\,dx = \frac{-497824i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)-2\left(497824+318520x-273876x\right.}{}$$

input
```
Integrate[(2 - 5*x)*x^(5/2)*(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
((-497824*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*Ar
cSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - 2*(497824 + 318520*x - 273876*x^2 + 91620
*x^3 - 37601118*x^4 - 83323080*x^5 + 69664455*x^6 + 337486905*x^7 + 320800
095*x^8 + 98513415*x^9 + (214108*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]
*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2]))/(32837805*Sqrt[x]*
Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {1236, 27, 1236, 27, 1236, 25, 1231, 27, 1231, 25, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-5x)x^{5/2}\left(3x^2+5x+2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{45}\int 5x^{3/2}(34x+5)\left(3x^2+5x+2\right)^{3/2}dx-\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\int x^{3/2}(34x+5)\left(3x^2+5x+2\right)^{3/2}dx-\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{9}\left(\frac{2}{39}\int -\frac{1}{2}\sqrt{x}(1165x+204)\left(3x^2+5x+2\right)^{3/2}dx+\frac{68}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}\right)-\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\left(\frac{68}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}-\frac{1}{39}\int \sqrt{x}(1165x+204)\left(3x^2+5x+2\right)^{3/2}dx\right)-\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{9}\left(\frac{1}{39}\left(-\frac{2}{33}\int -\frac{(14109x+1165)\left(3x^2+5x+2\right)^{3/2}}{\sqrt{x}}dx-\frac{2330}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}\right)+\frac{68}{39}x^{3/2}\left(3x^2+5x+2\right)\right. \frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{9}\left(\frac{1}{39}\left(\frac{2}{33}\int \frac{(14109x+1165)\left(3x^2+5x+2\right)^{3/2}}{\sqrt{x}}dx-\frac{2330}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}\right)+\frac{68}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}\right. \frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{9}\left(\frac{1}{39}\left(\frac{2}{33}\left(\frac{2}{21}\sqrt{x}(32921x+27010)\left(3x^2+5x+2\right)^{3/2}-\frac{2}{63}\int \frac{3(22823x+5090)\sqrt{3x^2+5x+2}}{2\sqrt{x}}dx\right)-\frac{2330}{33}\sqrt{}\right.\right. \frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\left(\frac{1}{39}\left(\frac{2}{33}\left(\frac{2}{21}\sqrt{x}(32921x+27010)\left(3x^2+5x+2\right)^{3/2}-\frac{1}{21}\int \frac{(22823x+5090)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx\right)-\frac{2330}{33}\sqrt{x}\right.\right. \frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{9}\left(\frac{1}{39}\left(\frac{2}{33}\left(\frac{1}{21}\left(\frac{2}{45}\int -\frac{31114x+38585}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2}{45}\sqrt{x}(205407x+190465)\sqrt{3x^2+5x+2}\right)+\frac{2}{21}\sqrt{x}(32921x+\right.\right.\right.$$
$$\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$\downarrow$ 25

$$\frac{2}{9}\left(\frac{1}{39}\left(\frac{2}{33}\left(\frac{1}{21}\left(-\frac{2}{45}\int \frac{31114x+38585}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2}{45}\sqrt{x}\sqrt{3x^2+5x+2}(205407x+190465)\right)+\frac{2}{21}\sqrt{x}(32921x+\right.\right.\right.$$
$$\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$\downarrow$ 1240

$$\frac{2}{9}\left(\frac{1}{39}\left(\frac{2}{33}\left(\frac{1}{21}\left(-\frac{4}{45}\int \frac{31114x+38585}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{2}{45}\sqrt{x}\sqrt{3x^2+5x+2}(205407x+190465)\right)+\frac{2}{21}\sqrt{x}(32921x+\right.\right.\right.$$
$$\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$\downarrow$ 1503

$$\frac{2}{9}\left(\frac{1}{39}\left(\frac{2}{33}\left(\frac{1}{21}\left(-\frac{4}{45}\left(38585\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+31114\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2}{45}\sqrt{x}\sqrt{3x^2+5x+2}(2\right.\right.\right.\right.$$
$$\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$\downarrow$ 1413

$$\frac{2}{9}\left(\frac{1}{39}\left(\frac{2}{33}\left(\frac{1}{21}\left(-\frac{4}{45}\left(31114\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{38585(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\right.\right.\right.\right.$$
$$\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

$\downarrow$ 1456

$$\frac{2}{9}\left(\frac{1}{39}\left(\frac{2}{33}\left(\frac{1}{21}\left(-\frac{4}{45}\left(\frac{38585(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+31114\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x\right.\right.\right.\right.\right.\right.$$
$$\frac{2}{9}x^{5/2}\left(3x^2+5x+2\right)^{5/2}$$

input
```
Int[(2 - 5*x)*x^(5/2)*(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(-2*x^(5/2)*(2 + 5*x + 3*x^2)^(5/2))/9 + (2*((68*x^(3/2)*(2 + 5*x + 3*x^2)
^(5/2))/39 + ((-2330*Sqrt[x]*(2 + 5*x + 3*x^2)^(5/2))/33 + (2*((2*Sqrt[x]*
(27010 + 32921*x)*(2 + 5*x + 3*x^2)^(3/2))/21 + ((-2*Sqrt[x]*(190465 + 205
407*x)*Sqrt[2 + 5*x + 3*x^2])/45 - (4*(31114*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[
2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[Arc
Tan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (38585*(1 + x)*Sqrt[(2 +
 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3
*x^2])))/45)/21))/33)/39))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.56

method	result
default	$-\frac{2\left(295540245x^9+962400285x^8+1012460715x^7+208993365x^6-249969240x^5+89652\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},\right.\right.}{98513415\sqrt{x}\,\sqrt{3x^2}}$
risch	$-\frac{2(10945935x^6+17401230x^5+1199205x^4-5859000x^3-292590x^2+215748x-154340)\sqrt{x}\,\sqrt{3x^2+5x+2}}{10945935}-\frac{\left(\frac{61736\sqrt{6x+4}\,\sqrt{3+3x}}{656756}\right.}{}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-2x^6\sqrt{3x^3+5x^2+2x}-\frac{124x^5\sqrt{3x^3+5x^2+2x}}{39}-\frac{94x^4\sqrt{3x^3+5x^2+2x}}{429}+\frac{12400x^3\sqrt{3x^3+5x^2+2x}}{11583}+\frac{13004x^2\sqrt{3x^3+5x^2+2x}}{243243}\right.}{}$

input
```
int((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/98513415/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(295540245*x^9+962400285*x^8+10124
60715*x^7+208993365*x^6-249969240*x^5+89652*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^
(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))+124456*(6*x+4)^(1/
2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))
-112803354*x^4+274860*x^3-3061836*x^2-2778120*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

$$\int (2-5x)x^{5/2}\left(2+5x+3x^2\right)^{3/2}\,dx =$$
$$-\frac{2}{10945935}\left(10945935\,x^6+17401230\,x^5+1199205\,x^4-5859000\,x^3-292590\,x^2+215748\,x-154340\right)\sqrt{}$$
$$-\frac{87632}{8444007}\sqrt{3}\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)$$
$$+\frac{497824}{32837805}\sqrt{3}\mathrm{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-2/10945935*(10945935*x^6 + 17401230*x^5 + 1199205*x^4 - 5859000*x^3 - 292
590*x^2 + 215748*x - 154340)*sqrt(3*x^2 + 5*x + 2)*sqrt(x) - 87632/8444007
*sqrt(3)*weierstrassPInverse(28/27, 80/729, x + 5/9) + 497824/32837805*sqr
t(3)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x +
 5/9))
```

Sympy [F]

$$\int (2-5x)x^{5/2}\left(2+5x+3x^2\right)^{3/2}\,dx = -\int \left(-4x^{\frac{5}{2}}\sqrt{3x^2+5x+2}\right)dx - \int 19x^{\frac{9}{2}}\sqrt{3x^2+5x+2}\,dx - \int 15x^{\frac{11}{2}}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*x**(5/2)*(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(-4*x**(5/2)*sqrt(3*x**2 + 5*x + 2), x) - Integral(19*x**(9/2)*sq
rt(3*x**2 + 5*x + 2), x) - Integral(15*x**(11/2)*sqrt(3*x**2 + 5*x + 2), x
)
```

Maxima [F]

$$\int (2-5x)x^{5/2}\left(2+5x+3x^2\right)^{3/2}\,dx = \int -\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)x^{\frac{5}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)*x^(5/2), x)
```

Giac [F]

$$\int (2-5x)x^{5/2}\left(2+5x+3x^2\right)^{3/2}\,dx=\int -\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)x^{\frac{5}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)*x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-5x)x^{5/2}\left(2+5x+3x^2\right)^{3/2}\,dx=-\int x^{5/2}\,(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}\,dx$$

input
```
int(-x^(5/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(3/2),x)
```

output
```
-int(x^(5/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int (2-5x)x^{5/2}\left(2+5x+3x^2\right)^{3/2}\,dx&=-2\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^6\\&-\frac{124\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^5}{39}-\frac{94\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^4}{429}\\&+\frac{12400\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3}{11583}+\frac{13004\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{243243}\\&-\frac{3688\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{93555}+\frac{3688\sqrt{x}\,\sqrt{3x^2+5x+2}}{155925}\\&+\frac{124456\left(\int\frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{6081075}-\frac{3688\left(\int\frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{155925}\end{aligned}$$

input
```
int((2-5*x)*x^(5/2)*(3*x^2+5*x+2)^(3/2),x)
```

output
```
(2*( - 6081075*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**6 - 9667350*sqrt(x)*sqrt(
3*x**2 + 5*x + 2)*x**5 - 666225*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**4 + 3255
000*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 + 162550*sqrt(x)*sqrt(3*x**2 + 5*x
 + 2)*x**2 - 119860*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x + 71916*sqrt(x)*sqrt(
3*x**2 + 5*x + 2) + 62228*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(3*x**2 +
 5*x + 2),x) - 71916*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2
 + 2*x),x)))/6081075
```

3.190 $\int (2-5x)x^{3/2}(2+5x+3x^2)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 229

$$\int (2-5x)x^{3/2}\left(2+5x+3x^2\right)^{3/2}\,dx=\frac{55112\sqrt{x}(2+3x)}{729729\sqrt{2+5x+3x^2}}-\frac{25448\sqrt{x}\sqrt{2+5x+3x^2}}{243243}+\frac{284x^{3/2}\sqrt{2+5x+3x^2}}{2079}+\frac{4x^{5/2}(9412+7665x)\sqrt{2+5x+3x^2}}{27027}+\frac{2}{143}(1-55x)x^{5/2}\left(2+5x+3x^2\right)^{3/2}-\frac{55112\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{729729\sqrt{1+x}\sqrt{2+3x}}+\frac{25448\sqrt{2}\sqrt{1+x}\sqrt{2+}}{24324}$$

output
```
55112/729729*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-25448/243243*x^(1/2)*(3*x
^2+5*x+2)^(1/2)+284/2079*x^(3/2)*(3*x^2+5*x+2)^(1/2)+4/27027*x^(5/2)*(9412
+7665*x)*(3*x^2+5*x+2)^(1/2)+2/143*(1-55*x)*x^(5/2)*(3*x^2+5*x+2)^(3/2)-55
112/729729*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I
*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+25448/243243*2^(1/2)*(1+x)^(1/2)*(2+3*
x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.23 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int (2-5x)x^{3/2}(2+5x+3x^2)^{3/2}\,dx = \frac{-2(-55112-61436x+8508x^2-1171602x^3-2497986x^4+1830195x^5+8989785x^6+83}{}$$

input
```
Integrate[(2 - 5*x)*x^(3/2)*(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(-2*(-55112 - 61436*x + 8508*x^2 - 1171602*x^3 - 2497986*x^4 + 1830195*x^5
 + 8989785*x^6 + 8374023*x^7 + 2525985*x^8) + (55112*I)*Sqrt[2]*Sqrt[1 + x
^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2]
+ (21232*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*Arc
Sinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(729729*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1236, 1236, 27, 1231, 27, 1231, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-5x)x^{3/2}\left(3x^2+5x+2\right)^{3/2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{39}\int \sqrt{x}(139x+15)\left(3x^2+5x+2\right)^{3/2}dx-\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{39}\left(\frac{2}{33}\int -\frac{(3675x+278)\left(3x^2+5x+2\right)^{3/2}}{2\sqrt{x}}dx+\frac{278}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}\right)-\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{39}\left(\frac{278}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}-\frac{1}{33}\int \frac{(3675x+278)\left(3x^2+5x+2\right)^{3/2}}{\sqrt{x}}dx\right)-\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{39}\left(\frac{1}{33}\left(\frac{2}{63}\int \frac{3(3545x+1121)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx-\frac{2}{21}\sqrt{x}(8575x+6959)\left(3x^2+5x+2\right)^{3/2}\right)+\frac{278}{33}\sqrt{x}\left(3x^2+\right.\right.$$
$$\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{39}\left(\frac{1}{33}\left(\frac{2}{21}\int \frac{(3545x+1121)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx-\frac{2}{21}\sqrt{x}(8575x+6959)\left(3x^2+5x+2\right)^{3/2}\right)+\frac{278}{33}\sqrt{x}\left(3x^2+5\right.\right.$$
$$\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{2}{9}\sqrt{x}(6381x+6908)\sqrt{3x^2+5x+2}-\frac{2}{45}\int -\frac{5(6889x+6362)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{2}{21}\sqrt{x}(8575x+6959)\left(3\right.\right.\right.$$
$$\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{9}\int \frac{6889x+6362}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(6381x+6908)\right)-\frac{2}{21}\sqrt{x}(8575x+6959)\left(3x^2+\right.\right.\right.$$
$$\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{2}{9}\int\frac{6889x+6362}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(6381x+6908)\right)-\frac{2}{21}\sqrt{x}(8575x+6959)\left(3x^2+5\right.\right.\right.$$
$$\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$\downarrow$ 1503

$$\frac{2}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{2}{9}\left(6362\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+6889\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(6381x+69\right.\right.\right.$$
$$\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$\downarrow$ 1413

$$\frac{2}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{2}{9}\left(6889\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{3181\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)+\frac{2}{9}\sqrt{x}\right.\right.\right.$$
$$\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

$\downarrow$ 1456

$$\frac{2}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{2}{9}\left(\frac{3181\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+6889\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{}}{3}\right.\right.\right.\right.\right.$$
$$\frac{10}{39}x^{3/2}\left(3x^2+5x+2\right)^{5/2}$$

input
```
Int[(2 - 5*x)*x^(3/2)*(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(-10*x^(3/2)*(2 + 5*x + 3*x^2)^(5/2))/39 + (2*((278*Sqrt[x]*(2 + 5*x + 3*x
^2)^(5/2))/33 + ((-2*Sqrt[x]*(6959 + 8575*x)*(2 + 5*x + 3*x^2)^(3/2))/21 +
 (2*((2*Sqrt[x]*(6908 + 6381*x)*Sqrt[2 + 5*x + 3*x^2])/9 + (2*(6889*((Sqrt
[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)
/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (
3181*Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1
/2])/Sqrt[2 + 5*x + 3*x^2]))/9))/21)/33))/39
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.60

method	result
default	$-\frac{2\left(7577955x^8+25122069x^7+26969355x^6+5490585x^5+3162\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-13778\sqrt{6x+4}\,\sqrt{3}\right.}{2189187\sqrt{x}\,\sqrt{3x^2+5x+2}}$
risch	$-\frac{2\left(280665x^5+462672x^4+40635x^3-172818x^2-16614x+12724\right)\sqrt{x}\,\sqrt{3x^2+5x+2}}{243243}-\frac{\left(-\frac{25448\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \text{EllipticF}\left(\frac{\sqrt{6x+}}{2}\right.}{729729\sqrt{3x^3+5x^2+2x}}\right.}{}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{30x^5\sqrt{3x^3+5x^2+2x}}{13}-\frac{544x^4\sqrt{3x^3+5x^2+2x}}{143}-\frac{430x^3\sqrt{3x^3+5x^2+2x}}{1287}+\frac{38404x^2\sqrt{3x^3+5x^2+2x}}{27027}+\frac{284x\sqrt{3x^3+5x^2+2x}}{2079}-\right.}{}$

input
```
int((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/2189187/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(7577955*x^8+25122069*x^7+26969355*
x^6+5490585*x^5+3162*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ellipt
icF(1/2*(6*x+4)^(1/2),I*2^(1/2))-13778*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)
*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))-7493958*x^4-3514806*x^3
+273528*x^2+229032*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.30

$$\int (2-5x)x^{3/2}\left(2+5x+3x^2\right)^{3/2}\,dx = -\frac{2}{243243}\left(280665\,x^5+462672\,x^4+40635\,x^3-172818\,x^2-16614\,x+12724\right)\sqrt{3\,x^2+5\,x+2}\sqrt{x} + \frac{26072}{938223}\,\sqrt{3}\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) - \frac{55112}{729729}\,\sqrt{3}\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-2/243243*(280665*x^5 + 462672*x^4 + 40635*x^3 - 172818*x^2 - 16614*x + 12
724)*sqrt(3*x^2 + 5*x + 2)*sqrt(x) + 26072/938223*sqrt(3)*weierstrassPInve
rse(28/27, 80/729, x + 5/9) - 55112/729729*sqrt(3)*weierstrassZeta(28/27,
80/729, weierstrassPInverse(28/27, 80/729, x + 5/9))
```

Sympy [F]

$$\int (2-5x)x^{3/2}\left(2+5x+3x^2\right)^{3/2}\,dx = -\int\left(-4x^{\frac{3}{2}}\sqrt{3x^2+5x+2}\right)dx - \int 19x^{\frac{7}{2}}\sqrt{3x^2+5x+2}\,dx - \int 15x^{\frac{9}{2}}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*x**(3/2)*(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(-4*x**(3/2)*sqrt(3*x**2 + 5*x + 2), x) - Integral(19*x**(7/2)*sq
rt(3*x**2 + 5*x + 2), x) - Integral(15*x**(9/2)*sqrt(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int (2-5x)x^{3/2}\left(2+5x+3x^2\right)^{3/2}\,dx = \int -\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)x^{\frac{3}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)*x^(3/2), x)
```

Giac [F]

$$\int (2-5x)x^{3/2}\left(2+5x+3x^2\right)^{3/2}\,dx = \int -\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)x^{\frac{3}{2}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-5x)x^{3/2}\left(2+5x+3x^2\right)^{3/2}\,dx = -\int x^{3/2}\,(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}\,dx$$

input
```
int(-x^(3/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(3/2),x)
```

output
```
-int(x^(3/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int (2-5x)x^{3/2}\left(2+5x+3x^2\right)^{3/2}\,dx = -\frac{30\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^5}{13} - \frac{544\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^4}{143} - \frac{430\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3}{1287} + \frac{38404\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{27027} + \frac{284\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{2079} - \frac{284\sqrt{x}\,\sqrt{3x^2+5x+2}}{3465} - \frac{13778\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{135135} + \frac{284\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{3465}$$

input

```
int((2-5*x)*x^(3/2)*(3*x^2+5*x+2)^(3/2),x)
```

output

```
(2*( - 155925*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**5 - 257040*sqrt(x)*sqrt(3*
x**2 + 5*x + 2)*x**4 - 22575*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 + 96010*s
qrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 + 9230*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x
 - 5538*sqrt(x)*sqrt(3*x**2 + 5*x + 2) - 6889*int((sqrt(x)*sqrt(3*x**2 + 5
*x + 2)*x)/(3*x**2 + 5*x + 2),x) + 5538*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2
))/(3*x**3 + 5*x**2 + 2*x),x)))/135135
```

3.191 $\int (2-5x)\sqrt{x}(2+5x+3x^2)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int (2-5x)\sqrt{x}(2+5x+3x^2)^{3/2}\,dx = -\frac{424\sqrt{x}(2+3x)}{1155\sqrt{2+5x+3x^2}} + \frac{36}{77}\sqrt{x}\sqrt{2+5x+3x^2} + \frac{4}{385}x^{3/2}(211+150x)\sqrt{2+5x+3x^2} - \frac{2}{33}x^{3/2}(1+15x)\left(2+5x+3x^2\right)^{3/2} + \frac{424\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{1155\sqrt{1+x}\sqrt{2+3x}} - \frac{36\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\mathrm{El}}{77\sqrt{2+}}$$

output

```
-424/1155*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+36/77*x^(1/2)*(3*x^2+5*x+2)^
(1/2)+4/385*x^(3/2)*(211+150*x)*(3*x^2+5*x+2)^(1/2)-2/33*x^(3/2)*(1+15*x)*
(3*x^2+5*x+2)^(3/2)+424/1155*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)
/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-36/77*2^(1/2)*(1+x)^
(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+
5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.23 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int (2-5x)\sqrt{x}(2+5x+3x^2)^{3/2}\,dx = \frac{-424i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)-2\left(424+520x-3106x^2-6140x^3+\cdots\right.}{1155\sqrt{\cdots}}$$

input
```
Integrate[(2 - 5*x)*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
((-424*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSi
nh[Sqrt[2/3]/Sqrt[x]], 3/2] - 2*(424 + 520*x - 3106*x^2 - 6140*x^3 + 3497*
x^4 + 17775*x^5 + 16065*x^6 + 4725*x^7 + (58*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*S
qrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2]))/(1155*
Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1236, 1231, 27, 1231, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-5x)\sqrt{x}(3x^2+5x+2)^{3/2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{33}\int \frac{(108x+5)(3x^2+5x+2)^{3/2}}{\sqrt{x}}dx-\frac{10}{33}\sqrt{x}(3x^2+5x+2)^{5/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{33}\left(\frac{2}{7}\sqrt{x}(84x+65)\left(3x^2+5x+2\right)^{3/2}-\frac{2}{63}\int \frac{27(39x+20)\sqrt{3x^2+5x+2}}{2\sqrt{x}}dx\right)-\frac{10}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{33}\left(\frac{2}{7}\sqrt{x}(84x+65)\left(3x^2+5x+2\right)^{3/2}-\frac{3}{7}\int \frac{(39x+20)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx\right)-\frac{10}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \; 1231$$

$$\frac{2}{33}\left(\frac{2}{7}\sqrt{x}(84x+65)\left(3x^2+5x+2\right)^{3/2}-\frac{3}{7}\left(\frac{2}{5}\sqrt{x}(39x+55)\sqrt{3x^2+5x+2}-\frac{2}{45}\int -\frac{9(53x+45)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)\right)-\frac{10}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{33}\left(\frac{2}{7}\sqrt{x}(84x+65)\left(3x^2+5x+2\right)^{3/2}-\frac{3}{7}\left(\frac{2}{5}\int \frac{53x+45}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{2}{5}\sqrt{x}\sqrt{3x^2+5x+2}(39x+55)\right)\right)-\frac{10}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \; 1240$$

$$\frac{2}{33}\left(\frac{2}{7}\sqrt{x}(84x+65)\left(3x^2+5x+2\right)^{3/2}-\frac{3}{7}\left(\frac{4}{5}\int \frac{53x+45}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{2}{5}\sqrt{x}\sqrt{3x^2+5x+2}(39x+55)\right)\right)-\frac{10}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \; 1503$$

$$\frac{2}{33}\left(\frac{2}{7}\sqrt{x}(84x+65)\left(3x^2+5x+2\right)^{3/2}-\frac{3}{7}\left(\frac{4}{5}\left(45\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+53\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)+\frac{2}{5}\sqrt{x}\right.\right.$$
$$\frac{10}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \; 1413$$

$$\frac{2}{33}\left(\frac{2}{7}\sqrt{x}(84x+65)\left(3x^2+5x+2\right)^{3/2}-\frac{3}{7}\left(\frac{4}{5}\left(53\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{45(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\right.}{\sqrt{2}\sqrt{3x^2+5x+2}}\right.\right.\right.$$
$$\frac{10}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}$$

$$\Big\downarrow \text{1456}$$

$$\frac{2}{33}\left(\frac{2}{7}\sqrt{x}(84x+65)\left(3x^2+5x+2\right)^{3/2}-\frac{3}{7}\left(\frac{4}{5}\left(\frac{45(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+53\left(\frac{\sqrt{x}(3x}{3\sqrt{3x^2+}}\right.\right.\right.\right.$$
$$\frac{10}{33}\sqrt{x}\left(3x^2+5x+2\right)^{5/2}$$

input
```
Int[(2 - 5*x)*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(-10*Sqrt[x]*(2 + 5*x + 3*x^2)^(5/2))/33 + (2*((2*Sqrt[x]*(65 + 84*x)*(2 +
 5*x + 3*x^2)^(3/2))/7 - (3*((2*Sqrt[x]*(55 + 39*x)*Sqrt[2 + 5*x + 3*x^2])
/5 + (4*(53*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 +
 x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 +
5*x + 3*x^2])) + (45*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt
[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])))/5))/7))/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.64

method	result
default	$\frac{-\frac{90x^7}{11}-\frac{306x^6}{11}-\frac{2370x^5}{77}+\frac{32\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{1155}-\frac{212\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3465}-\frac{6994x^4}{1155}}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
risch	$-\frac{2(1575x^4+2730x^3+325x^2-1196x-270)\sqrt{x}\,\sqrt{3x^2+5x+2}}{1155}-\frac{\left(\frac{12\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{77\sqrt{3x^3+5x^2+2x}}+\frac{212\sqrt{6x+4}\,\sqrt{3+3x}}{}\right.}{\sqrt{x}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{30x^4\sqrt{3x^3+5x^2+2x}}{11}-\frac{52x^3\sqrt{3x^3+5x^2+2x}}{11}-\frac{130x^2\sqrt{3x^3+5x^2+2x}}{231}+\frac{2392x\sqrt{3x^3+5x^2+2x}}{1155}+\frac{36\sqrt{3x^3+5x^2+2x}}{77}-\frac{12\sqrt{6x}}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(-14175*x^7-48195*x^6-53325*x^5+48*(6*x
+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2
^(1/2))-106*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(
6*x+4)^(1/2),I*2^(1/2))-10491*x^4+18420*x^3+11226*x^2+1620*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.31

$$\int (2-5x)\sqrt{x}\left(2+5x+3x^2\right)^{3/2}\,dx = -\frac{2}{1155}\,(1575\,x^4+2730\,x^3+325\,x^2-1196\,x-270)\sqrt{3\,x^2+5\,x+2}\sqrt{x} - \frac{32}{297}\,\sqrt{3}\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) + \frac{424}{1155}\,\sqrt{3}\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-2/1155*(1575*x^4 + 2730*x^3 + 325*x^2 - 1196*x - 270)*sqrt(3*x^2 + 5*x +
2)*sqrt(x) - 32/297*sqrt(3)*weierstrassPInverse(28/27, 80/729, x + 5/9) +
424/1155*sqrt(3)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27,
 80/729, x + 5/9))
```

Sympy [F]

$$\int (2-5x)\sqrt{x}\left(2+5x+3x^2\right)^{3/2}\,dx = -\int \left(-4\sqrt{x}\sqrt{3x^2+5x+2}\right)dx - \int 19x^{\frac{5}{2}}\sqrt{3x^2+5x+2}\,dx - \int 15x^{\frac{7}{2}}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*x**(1/2)*(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(-4*sqrt(x)*sqrt(3*x**2 + 5*x + 2), x) - Integral(19*x**(5/2)*sqr
t(3*x**2 + 5*x + 2), x) - Integral(15*x**(7/2)*sqrt(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int (2-5x)\sqrt{x}\left(2+5x+3x^2\right)^{3/2}\,dx=\int -\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)\sqrt{x}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)*sqrt(x), x)
```

Giac [F]

$$\int (2-5x)\sqrt{x}\left(2+5x+3x^2\right)^{3/2}\,dx=\int -\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)\sqrt{x}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-5x)\sqrt{x}\left(2+5x+3x^2\right)^{3/2}\,dx=-\int \sqrt{x}\,(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}\,dx$$

input
```
int(-x^(1/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(3/2),x)
```

output
```
-int(x^(1/2)*(5*x - 2)*(5*x + 3*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int (2-5x)\sqrt{x}\left(2+5x+3x^2\right)^{3/2}\,dx = -\frac{30\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^4}{11} - \frac{52\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3}{11} - \frac{130\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{231} + \frac{2392\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{1155} + \frac{688\sqrt{x}\,\sqrt{3x^2+5x+2}}{1925} + \frac{954\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{1925} - \frac{688\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{1925}$$

input
```
int((2-5*x)*x^(1/2)*(3*x^2+5*x+2)^(3/2),x)
```

output
```
(2*( - 7875*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**4 - 13650*sqrt(x)*sqrt(3*x**
2 + 5*x + 2)*x**3 - 1625*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 + 5980*sqrt(x
)*sqrt(3*x**2 + 5*x + 2)*x + 1032*sqrt(x)*sqrt(3*x**2 + 5*x + 2) + 1431*in
t((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(3*x**2 + 5*x + 2),x) - 1032*int((sqr
t(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x)))/5775
```

3.192 $\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{\sqrt{x}}\,dx = \frac{860\sqrt{x}(2+3x)}{243\sqrt{2+5x+3x^2}} + \frac{4}{81}\sqrt{x}(82+45x)\sqrt{2+5x+3x^2} - \frac{2}{9}\sqrt{x}(1+5x)\left(2+5x+3x^2\right)^{3/2} - \frac{860\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{243\sqrt{1+x}\sqrt{2+3x}} + \frac{356\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{81\sqrt{2+5x+3x^2}}$$

output

```
860/243*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+4/81*x^(1/2)*(82+45*x)*(3*x^2+
5*x+2)^(1/2)-2/9*x^(1/2)*(1+5*x)*(3*x^2+5*x+2)^(3/2)-860/243*2^(1/2)*(3*x^
2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2
+3*x)^(1/2)+356/81*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arcta
n(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{\sqrt{x}}\,dx = \frac{1720+6052x+6420x^2-1746x^3-9990x^4-8586x^5-2430x^6+860i}{}$$

input
```
Integrate[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/Sqrt[x],x]
```

output
```
(1720 + 6052*x + 6420*x^2 - 1746*x^3 - 9990*x^4 - 8586*x^5 - 2430*x^6 + (8
60*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[S
qrt[2/3]/Sqrt[x]], 3/2] + (208*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x
^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(243*Sqrt[x]*Sqrt[2 +
 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1231, 27, 1231, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\left(3x^2+5x+2\right)^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \ 1231$$

$$-\frac{2}{63}\int -\frac{7(25x+19)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx-\frac{2}{9}\sqrt{x}(5x+1)\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{2}{9}\int \frac{(25x+19)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx-\frac{2}{9}\sqrt{x}(5x+1)\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2}{9}\left(\frac{2}{9}\sqrt{x}(45x+82)\sqrt{3x^2+5x+2}-\frac{2}{45}\int -\frac{5(215x+178)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{2}{9}\sqrt{x}(5x+1)\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\left(\frac{1}{9}\int \frac{215x+178}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(45x+82)\right)-\frac{2}{9}\sqrt{x}(5x+1)\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2}{9}\left(\frac{2}{9}\int \frac{215x+178}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(45x+82)\right)-\frac{2}{9}\sqrt{x}(5x+1)\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1503}$$

$$\frac{2}{9}\left(\frac{2}{9}\left(178\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+215\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2}(45x+82)\right)-\frac{2}{9}\sqrt{x}(5x+1)\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1413}$$

$$\frac{2}{9}\left(\frac{2}{9}\left(215\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{89\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)+\frac{2}{9}\sqrt{x}\sqrt{3x^2+5x+2(}\right.$$
$$\frac{2}{9}\sqrt{x}(5x+1)\left(3x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1456}$$

$$\frac{2}{9}\left(\frac{2}{9}\left(\frac{89\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+215\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E(\arctan}{3\sqrt{3x^2+5x+2}}\right.\right.\right.$$
$$\frac{2}{9}\sqrt{x}(5x+1)\left(3x^2+5x+2\right)^{3/2}$$

input
```
Int[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/Sqrt[x],x]
```

output
```
(-2*Sqrt[x]*(1 + 5*x)*(2 + 5*x + 3*x^2)^(3/2))/9 + (2*((2*Sqrt[x]*(82 + 45
*x)*Sqrt[2 + 5*x + 3*x^2])/9 + (2*(215*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*
x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sq
rt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (89*Sqrt[2]*(1 + x)*Sqrt[(2 +
3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/9)
)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

method	result
default	$-\frac{2\left(3645x^6+12879x^5+111\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-215\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\right.}{729\sqrt{3x^2+5x+2}\,\sqrt{x}}$
risch	$-\frac{2(135x^3+252x^2+45x-146)\sqrt{x}\,\sqrt{3x^2+5x+2}}{81}-\frac{\left(-\frac{356\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{243\sqrt{3x^3+5x^2+2x}}-\frac{430\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\left(\mathrm{Ell}\right.}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{10x^3\sqrt{3x^3+5x^2+2x}}{3}-\frac{56x^2\sqrt{3x^3+5x^2+2x}}{9}-\frac{10x\sqrt{3x^3+5x^2+2x}}{9}+\frac{292\sqrt{3x^3+5x^2+2x}}{81}+\frac{356\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\mathrm{Ellipti}}{243\sqrt{3x^3+5x^2+}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/729/(3*x^2+5*x+2)^(1/2)/x^(1/2)*(3645*x^6+12879*x^5+111*(6*x+4)^(1/2)*(
3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-215
*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2
),I*2^(1/2))+14985*x^4+2619*x^3-5760*x^2-2628*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.32

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{\sqrt{x}}\,dx = -\frac{2}{81}\left(135\,x^3+252\,x^2+45\,x-146\right)\sqrt{3\,x^2+5\,x+2}\sqrt{x} + \frac{2108}{2187}\,\sqrt{3}\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) - \frac{860}{243}\,\sqrt{3}\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-2/81*(135*x^3 + 252*x^2 + 45*x - 146)*sqrt(3*x^2 + 5*x + 2)*sqrt(x) + 210
8/2187*sqrt(3)*weierstrassPInverse(28/27, 80/729, x + 5/9) - 860/243*sqrt(
3)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x + 5
/9))
```

Sympy [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{\sqrt{x}}\,dx = -\int \left(-\frac{4\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)dx - \int 19x^{\frac{3}{2}}\sqrt{3x^2+5x+2}\,dx - \int 15x^{\frac{5}{2}}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(3/2)/x**(1/2),x)
```

output
```
-Integral(-4*sqrt(3*x**2 + 5*x + 2)/sqrt(x), x) - Integral(19*x**(3/2)*sqr
t(3*x**2 + 5*x + 2), x) - Integral(15*x**(5/2)*sqrt(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{\sqrt{x}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/sqrt(x), x)
```

Giac [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{\sqrt{x}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{\sqrt{x}}\,dx = -\int \frac{(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}}{\sqrt{x}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(1/2),x)
```

output
```
-int(((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(1/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{\sqrt{x}}\,dx = -\frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3}{3} - \frac{56\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{9} - \frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{9} + \frac{14\sqrt{x}\,\sqrt{3x^2+5x+2}}{3} - \frac{43\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{9} + \frac{10\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{3}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(1/2),x)
```

output
```
( - 30*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 - 56*sqrt(x)*sqrt(3*x**2 + 5*x
+ 2)*x**2 - 10*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x + 42*sqrt(x)*sqrt(3*x**2 +
 5*x + 2) - 43*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(3*x**2 + 5*x + 2),x
) + 30*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x))/9
```

3.193 $\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{3/2}}\,dx=\frac{5848\sqrt{x}(2+3x)}{315\sqrt{2+5x+3x^2}}+\frac{2}{105}\sqrt{x}(1045+531x)\sqrt{2+5x+3x^2}-\frac{2(14+5x)\left(2+5x+3x^2\right)^{3/2}}{7\sqrt{x}}-\frac{5848\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{315\sqrt{1+x}\sqrt{2+3x}}+\frac{482\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{21\sqrt{2+5x+3x^2}}$$

output
```
5848/315*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+2/105*x^(1/2)*(1045+531*x)*(3
*x^2+5*x+2)^(1/2)-2/7*(14+5*x)*(3*x^2+5*x+2)^(3/2)/x^(1/2)-5848/315*2^(1/2
)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(
1/2)/(2+3*x)^(1/2)+482/21*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiA
M(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{3/2}}\,dx = \frac{-2(-3328-7390x+177x^2+9855x^3+7641x^4+2025x^5)+5848i\sqrt{2}}{}$$

input
```
Integrate[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(3/2),x]
```

output
```
(-2*(-3328 - 7390*x + 177*x^2 + 9855*x^3 + 7641*x^4 + 2025*x^5) + (5848*I)
*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2
/3]/Sqrt[x]], 3/2] + (1382*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/
2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(315*Sqrt[x]*Sqrt[2 + 5*x
 + 3*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1230, 27, 1231, 25, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)(3x^2+5x+2)^{3/2}}{x^{3/2}}\,dx$$

$$\Big\downarrow \ 1230$$

$$-\frac{6}{7}\int -\frac{(59x+50)\sqrt{3x^2+5x+2}}{2\sqrt{x}}dx - \frac{2(5x+14)(3x^2+5x+2)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{7}\int \frac{(59x+50)\sqrt{3x^2+5x+2}}{\sqrt{x}}dx - \frac{2(5x+14)(3x^2+5x+2)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow \text{1231}$$

$$\frac{3}{7}\left(\frac{2}{45}\sqrt{x}(531x+1045)\sqrt{3x^2+5x+2}-\frac{2}{45}\int -\frac{1462x+1205}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{2(5x+14)\left(3x^2+5x+2\right)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{7}\left(\frac{2}{45}\int \frac{1462x+1205}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{2}{45}\sqrt{x}\sqrt{3x^2+5x+2}(531x+1045)\right)-\frac{2(5x+14)\left(3x^2+5x+2\right)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow \text{1240}$$

$$\frac{3}{7}\left(\frac{4}{45}\int \frac{1462x+1205}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{2}{45}\sqrt{x}\sqrt{3x^2+5x+2}(531x+1045)\right)-\frac{2(5x+14)\left(3x^2+5x+2\right)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{3}{7}\left(\frac{4}{45}\left(1205\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+1462\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)+\frac{2}{45}\sqrt{x}\sqrt{3x^2+5x+2}(531x+1045)\right)-\frac{2(5x+14)\left(3x^2+5x+2\right)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow \text{1413}$$

$$\frac{3}{7}\left(\frac{4}{45}\left(1462\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{1205(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)+\frac{2}{45}\sqrt{x}\sqrt{3x^2+5x+\right.$$

$$\frac{2(5x+14)\left(3x^2+5x+2\right)^{3/2}}{7\sqrt{x}}$$

$$\Big\downarrow \text{1456}$$

$$\frac{3}{7}\left(\frac{4}{45}\left(\frac{1205(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+1462\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E(\text{arcta}}{3\sqrt{3x^2+5x+}}\right.\right.\right.$$

$$\frac{2(5x+14)\left(3x^2+5x+2\right)^{3/2}}{7\sqrt{x}}$$

input
```
Int[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(3/2),x]
```

output
```
(-2*(14 + 5*x)*(2 + 5*x + 3*x^2)^(3/2))/(7*Sqrt[x]) + (3*((2*Sqrt[x]*(1045
 + 531*x)*Sqrt[2 + 5*x + 3*x^2])/45 + (4*(1462*((Sqrt[x]*(2 + 3*x))/(3*Sqr
t[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[A
rcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (1205*(1 + x)*Sqrt[(2
+ 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x +
3*x^2])))/45))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\left(6075x^5+771\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-1462\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)+22923x\right.}{945\sqrt{3x^2+5x+2}\,\sqrt{x}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{8(3x^2+5x+2)}{\sqrt{(3x^2+5x+2)x}}-\frac{30x^2\sqrt{3x^3+5x^2+2x}}{7}-\frac{316x\sqrt{3x^3+5x^2+2x}}{35}-\frac{62\sqrt{3x^3+5x^2+2x}}{21}+\frac{482\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \text{EllipticF}\left(\right.}{63\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/945*(6075*x^5+771*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ellipt
icF(1/2*(6*x+4)^(1/2),I*2^(1/2))-1462*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*
(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+22923*x^4+29565*x^3+2684
7*x^2+21690*x+7560)/(3*x^2+5*x+2)^(1/2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.36

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{3/2}}\,dx = \frac{2\left(7070\sqrt{3}x\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-26316\sqrt{3}x\text{weierstra}\right.}{}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/2835*(7070*sqrt(3)*x*weierstrassPInverse(28/27, 80/729, x + 5/9) - 26316
*sqrt(3)*x*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/72
9, x + 5/9)) - 27*(225*x^3 + 474*x^2 + 155*x + 420)*sqrt(3*x^2 + 5*x + 2)*
sqrt(x))/x
```

Sympy [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{3/2}}\,dx = -\int \left(-\frac{4\sqrt{3x^2+5x+2}}{x^{\frac{3}{2}}}\right)dx - \int 19\sqrt{x}\sqrt{3x^2+5x+2}\,dx - \int 15x^{\frac{3}{2}}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(3/2)/x**(3/2),x)
```

output
```
-Integral(-4*sqrt(3*x**2 + 5*x + 2)/x**(3/2), x) - Integral(19*sqrt(x)*sqr
t(3*x**2 + 5*x + 2), x) - Integral(15*x**(3/2)*sqrt(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{3/2}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(3/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{3/2}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{3/2}}\,dx = \int -\frac{(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}}{x^{3/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(3/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(3/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{3/2}}\,dx = \frac{-\frac{30\sqrt{3x^2+5x+2}\,x^3}{7}-\frac{316\sqrt{3x^2+5x+2}\,x^2}{35}-\frac{62\sqrt{3x^2+5x+2}\,x}{21}-8\sqrt{3x^2+5x+2}+}{\sqrt{x}}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(3/2),x)
```

output
```
(2*( - 225*sqrt(3*x**2 + 5*x + 2)*x**3 - 474*sqrt(3*x**2 + 5*x + 2)*x**2 -
 155*sqrt(3*x**2 + 5*x + 2)*x - 420*sqrt(3*x**2 + 5*x + 2) + 1205*sqrt(x)*
int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x) + 1462*sqr
t(x)*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**2 + 5*x + 2),x)))/(105*sqr
t(x))
```

3.194 $\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\begin{aligned}\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{5/2}}\,dx &= -\frac{34\sqrt{x}(2+3x)}{3\sqrt{2+5x+3x^2}}-6\sqrt{x}(2+x)\sqrt{2+5x+3x^2} \\ &\quad -\frac{4(2+5x+3x^2)^{3/2}}{3x^{3/2}}+\frac{34\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x}\sqrt{2+3x}} \\ &\quad -\frac{14\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
-34/3*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-6*x^(1/2)*(2+x)*(3*x^2+5*x+2)^(1
/2)-4/3*(3*x^2+5*x+2)^(3/2)/x^(3/2)+34/3*2^(1/2)*(3*x^2+5*x+2)^(1/2)*Ellip
ticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-14*2^(1/
2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2)
)/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{5/2}}\,dx = \frac{-34i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{5/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)-2\left(8+74x+195\right.}{3}$$

input
```
Integrate[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(5/2),x]
```

output
```
((-34*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(5/2)*EllipticE[I*ArcSin
h[Sqrt[2/3]/Sqrt[x]], 3/2] - 2*(8 + 74*x + 195*x^2 + 219*x^3 + 117*x^4 + 2
7*x^5 + (4*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(5/2)*EllipticF[I*A
rcSinh[Sqrt[2/3]/Sqrt[x]], 3/2]))/(3*x^(3/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1230, 27, 1230, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\left(3x^2+5x+2\right)^{3/2}}{x^{5/2}}dx$$

$$\Big\downarrow \text{1230}$$

$$-\frac{2}{5}\int \frac{5(3x+2)\sqrt{3x^2+5x+2}}{2x^{3/2}}dx-\frac{2(3x+2)\left(3x^2+5x+2\right)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{(3x+2)\sqrt{3x^2+5x+2}}{x^{3/2}}dx-\frac{2(3x+2)\left(3x^2+5x+2\right)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \text{1230}$$

$$\frac{2}{3}\int -\frac{3(17x+14)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{2(2-x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{2(3x+2)\left(3x^2+5x+2\right)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\int \frac{17x+14}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{2(2-x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{2(3x+2)\left(3x^2+5x+2\right)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \ 1240$$

$$-2\int \frac{17x+14}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{2(2-x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{2(3x+2)\left(3x^2+5x+2\right)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \ 1503$$

$$-2\left(14\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+17\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)+\frac{2(2-x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{2(3x+2)\left(3x^2+5x+2\right)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \ 1413$$

$$-2\left(17\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{7\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)+\frac{2(2-x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{2(3x+2)\left(3x^2+5x+2\right)^{3/2}}{3x^{3/2}}$$

$$\Big\downarrow \ 1456$$

$$-2\left(\frac{7\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+17\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right.\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$
$$\frac{2(2-x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{2(3x+2)\left(3x^2+5x+2\right)^{3/2}}{3x^{3/2}}$$

input
```
Int[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(5/2),x]
```

output
```
(2*(2 - x)*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x] - (2*(2 + 3*x)*(2 + 5*x + 3*x^2)
^(3/2))/(3*x^(3/2)) - 2*(17*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2])
 - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/
2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (7*Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)
]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.73

method	result
default	$\frac{9\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x-17\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x-162x^5-702x^4-1008x^3}{9\sqrt{3x^2+5x+2}\,x^{\frac{3}{2}}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{8\sqrt{3x^3+5x^2+2x}}{3x^2}-\frac{20\left(3x^2+5x+2\right)}{3\sqrt{\left(3x^2+5x+2\right)x}}-6x\sqrt{3x^3+5x^2+2x}-16\sqrt{3x^3+5x^2+2x}-\frac{14\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\sqrt{6}\right.}{3\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(9*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4
)^(1/2),I*2^(1/2))*x-17*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ell
ipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*x-162*x^5-702*x^4-1008*x^3-660*x^2-240
*x-48)/(3*x^2+5*x+2)^(1/2)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.40

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{5/2}}\,dx =$$
$$-\frac{2\left(41\sqrt{3}x^2\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-153\sqrt{3}x^2\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\right.\right.\right.}{27\,x^2}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/27*(41*sqrt(3)*x^2*weierstrassPInverse(28/27, 80/729, x + 5/9) - 153*sq
rt(3)*x^2*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729
, x + 5/9)) + 9*(9*x^3 + 24*x^2 + 10*x + 4)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))
/x^2
```

Sympy [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{5/2}}\,dx = -\int \left(-\frac{4\sqrt{3x^2+5x+2}}{x^{\frac{5}{2}}}\right)dx - \int \frac{19\sqrt{3x^2+5x+2}}{\sqrt{x}}\,dx - \int 15\sqrt{x}\sqrt{3x^2+5x+2}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(3/2)/x**(5/2),x)
```

output
```
-Integral(-4*sqrt(3*x**2 + 5*x + 2)/x**(5/2), x) - Integral(19*sqrt(3*x**2
 + 5*x + 2)/sqrt(x), x) - Integral(15*sqrt(x)*sqrt(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{5/2}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(5/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{5/2}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{5/2}}\,dx = \int -\frac{(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}}{x^{5/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(5/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(5/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{5/2}}\,dx = \frac{-30\sqrt{3x^2+5x+2}\,x^3-80\sqrt{3x^2+5x+2}\,x^2-90\sqrt{3x^2+5x+2}\,x-2}{}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(5/2),x)
```

output
```
( - 30*sqrt(3*x**2 + 5*x + 2)*x**3 - 80*sqrt(3*x**2 + 5*x + 2)*x**2 - 90*s
qrt(3*x**2 + 5*x + 2)*x - 2*sqrt(3*x**2 + 5*x + 2) + 34*sqrt(x)*int((sqrt(
x)*sqrt(3*x**2 + 5*x + 2))/(3*x**5 + 5*x**4 + 2*x**3),x)*x - 53*sqrt(x)*in
t((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x)*x)/(5*sqrt(x
)*x)
```

3.195 $\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 181

$$\begin{aligned}\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{7/2}}\,dx &= -\frac{1418\sqrt{x}(2+3x)}{15\sqrt{2+5x+3x^2}} \\ &+ \frac{2(89-35x)\sqrt{2+5x+3x^2}}{5\sqrt{x}} - \frac{4(3-5x)\left(2+5x+3x^2\right)^{3/2}}{15x^{5/2}} \\ &+ \frac{1418\sqrt{2}\sqrt{2+5x+3x^2}\,E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{15\sqrt{1+x}\sqrt{2+3x}} \\ &- \frac{117\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
-1418/15*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+2/5*(89-35*x)*(3*x^2+5*x+2)^(
1/2)/x^(1/2)-4/15*(3-5*x)*(3*x^2+5*x+2)^(3/2)/x^(5/2)+1418/15*2^(1/2)*(3*x
^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(
2+3*x)^(1/2)-117*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(
x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.90

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{7/2}}\,dx = \frac{-2(24+80x+906x^2+2230x^3+1605x^4+225x^5)-1418i\sqrt{2}\sqrt{1+\frac{1}{x}}}{}$$

input
```
Integrate[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(7/2),x]
```

output
```
(-2*(24 + 80*x + 906*x^2 + 2230*x^3 + 1605*x^4 + 225*x^5) - (1418*I)*Sqrt[
2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(7/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sq
rt[x]], 3/2] - (337*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(7/2)*Elli
pticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(15*x^(5/2)*Sqrt[2 + 5*x + 3*x^2
])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1229, 1230, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\left(3x^2+5x+2\right)^{3/2}}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{1}{5}\int \frac{(105x+89)\sqrt{3x^2+5x+2}}{x^{3/2}}dx-\frac{4(3-5x)\left(3x^2+5x+2\right)^{3/2}}{15x^{5/2}}$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{5}\left(\frac{2}{3}\int -\frac{3(709x+585)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{2\sqrt{3x^2+5x+2}(89-35x)}{\sqrt{x}}\right)-\frac{4(3-5x)\left(3x^2+5x+2\right)^{3/2}}{15x^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{2(89-35x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-\int \frac{709x+585}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{4(3-5x)\left(3x^2+5x+2\right)^{3/2}}{15x^{5/2}}$$

$$\Big\downarrow \; 1240$$

$$\frac{1}{5}\left(\frac{2(89-35x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-2\int \frac{709x+585}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{4(3-5x)\left(3x^2+5x+2\right)^{3/2}}{15x^{5/2}}$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{5}\left(\frac{2(89-35x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-2\left(585\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+709\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{4(3-5x)\left(3x^2+5x+2\right)^{3/2}}{15x^{5/2}}$$

$$\Big\downarrow \; 1413$$

$$\frac{1}{5}\left(\frac{2(89-35x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-2\left(709\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{585(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\right.}{\sqrt{2}\sqrt{3x^2+5x+2}}\right.\right.$$
$$-\frac{4(3-5x)\left(3x^2+5x+2\right)^{3/2}}{15x^{5/2}}$$

$$\Big\downarrow \; 1456$$

$$\frac{1}{5}\left(\frac{2(89-35x)\sqrt{3x^2+5x+2}}{\sqrt{x}}-2\left(\frac{585(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+709\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\right.\right.\right.$$
$$-\frac{4(3-5x)\left(3x^2+5x+2\right)^{3/2}}{15x^{5/2}}$$

input
```
Int[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(7/2),x]
```

output
```
(-4*(3 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/(15*x^(5/2)) + ((2*(89 - 35*x)*Sqrt
[2 + 5*x + 3*x^2])/Sqrt[x] - 2*(709*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x +
 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[
x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (585*(1 + x)*Sqrt[(2 + 3*x)/(1 +
x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.71

method	result
default	$\frac{372\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-709\,\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-1350x^5+3132x^4}{45\sqrt{3x^2+5x+2}\,x^{\frac{5}{2}}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{8\sqrt{3x^3+5x^2+2x}}{5x^3}-\frac{4\sqrt{3x^3+5x^2+2x}}{3x^2}+\frac{\frac{598}{5}x^2+\frac{598}{3}x+\frac{1196}{15}}{\sqrt{\left(3x^2+5x+2\right)x}}-10\sqrt{3x^3+5x^2+2x}-\frac{39\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\text{EllipticF}\left(\right.}{\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/45*(372*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))*(6*x+4)^(1/2)*(3+3*x)^(1/
2)*6^(1/2)*(-x)^(1/2)*x^2-709*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*(6*x+
4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*x^2-1350*x^5+3132*x^4+7890*x^3+3
072*x^2-480*x-144)/(3*x^2+5*x+2)^(1/2)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.38

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{7/2}}\,dx = -\frac{2\left(1720\sqrt{3}x^3\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-6381\sqrt{3}x^3\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInver}\right.\right.}{135\,x^3}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/135*(1720*sqrt(3)*x^3*weierstrassPInverse(28/27, 80/729, x + 5/9) - 638
1*sqrt(3)*x^3*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80
/729, x + 5/9)) + 9*(75*x^3 - 299*x^2 + 10*x + 12)*sqrt(3*x^2 + 5*x + 2)*s
qrt(x))/x^3
```

Sympy [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{7/2}}\,dx = -\int\left(-\frac{4\sqrt{3x^2+5x+2}}{x^{\frac{7}{2}}}\right)dx-\int\frac{19\sqrt{3x^2+5x+2}}{x^{\frac{3}{2}}}\,dx-\int\frac{15\sqrt{3x^2+5x+2}}{\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(3/2)/x**(7/2),x)
```

output
```
-Integral(-4*sqrt(3*x**2 + 5*x + 2)/x**(7/2), x) - Integral(19*sqrt(3*x**2
 + 5*x + 2)/x**(3/2), x) - Integral(15*sqrt(3*x**2 + 5*x + 2)/sqrt(x), x)
```

Maxima [F]

$$\int \frac{(2-5x)\,(2+5x+3x^2)^{3/2}}{x^{7/2}}\,dx = \int -\frac{(3\,x^2+5\,x+2)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(7/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(7/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\,(2+5x+3x^2)^{3/2}}{x^{7/2}}\,dx = \int -\frac{(3\,x^2+5\,x+2)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(7/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\,(2+5x+3x^2)^{3/2}}{x^{7/2}}\,dx = \int -\frac{(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}}{x^{7/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(7/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(7/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{7/2}}\,dx = \frac{-600\sqrt{3x^2+5x+2}\,x^3 - 21008\sqrt{3x^2+5x+2}\,x^2 + 4600\sqrt{3x^2+5x+}}{}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(7/2),x)
```

output
```
( - 600*sqrt(3*x**2 + 5*x + 2)*x**3 - 21008*sqrt(3*x**2 + 5*x + 2)*x**2 +
4600*sqrt(3*x**2 + 5*x + 2)*x - 96*sqrt(3*x**2 + 5*x + 2) + 14040*sqrt(x)*
int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**4 + 5*sqrt(x)*x**3 + 2*sqrt(x)*x*
*2),x)*x**2 + 9477*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x)/(3*sqrt(x)*x**2
+ 5*sqrt(x)*x + 2*sqrt(x)),x)*x**2 + 17115*sqrt(x)*int((sqrt(x)*sqrt(3*x**
2 + 5*x + 2))/(3*x**2 + 5*x + 2),x)*x**2)/(60*sqrt(x)*x**2)
```

3.196 $\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx = -\frac{633\sqrt{x}(2+3x)}{7\sqrt{2+5x+3x^2}} + \frac{3(22+133x)\sqrt{2+5x+3x^2}}{7x^{3/2}} - \frac{4(1-2x)\left(2+5x+3x^2\right)^{3/2}}{7x^{7/2}} + \frac{633\sqrt{2}\sqrt{2+5x+3x^2}\,E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{7\sqrt{1+x}\sqrt{2+3x}} - \frac{783\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{7\sqrt{2+5x+3x^2}}$$

output
```
-633/7*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+3/7*(22+133*x)*(3*x^2+5*x+2)^(1
/2)/x^(3/2)-4/7*(1-2*x)*(3*x^2+5*x+2)^(3/2)/x^(7/2)+633/7*2^(1/2)*(3*x^2+5
*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*
x)^(1/2)-783/7*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^
(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx = \frac{-2(8+24x-72x^2-19x^3+384x^4+315x^5)-633i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+}}{}$$

input
```
Integrate[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(9/2),x]
```

output
```
(-2*(8 + 24*x - 72*x^2 - 19*x^3 + 384*x^4 + 315*x^5) - (633*I)*Sqrt[2]*Sqr
t[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(9/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]]
, 3/2] - (150*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(9/2)*EllipticF[
I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(7*x^(7/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1229, 27, 1229, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\left(3x^2+5x+2\right)^{3/2}}{x^{9/2}}dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{3}{35}\int \frac{15(13x+11)\sqrt{3x^2+5x+2}}{x^{5/2}}dx-\frac{4(1-2x)\left(3x^2+5x+2\right)^{3/2}}{7x^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{9}{7}\int \frac{(13x+11)\sqrt{3x^2+5x+2}}{x^{5/2}}dx-\frac{4(1-2x)\left(3x^2+5x+2\right)^{3/2}}{7x^{7/2}}$$

$$\Big\downarrow \text{1229}$$

$$-\frac{9}{7}\left(-\frac{1}{3}\int -\frac{3(211x+174)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{\sqrt{3x^2+5x+2}(133x+22)}{3x^{3/2}}\right)-\frac{4(1-2x)\left(3x^2+5x+2\right)^{3/2}}{7x^{7/2}}$$

$\big\downarrow$ 27

$$-\frac{9}{7}\left(\frac{1}{2}\int \frac{211x+174}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{(133x+22)\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{4(1-2x)\left(3x^2+5x+2\right)^{3/2}}{7x^{7/2}}$$

$\big\downarrow$ 1240

$$-\frac{9}{7}\left(\int \frac{211x+174}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{(133x+22)\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{4(1-2x)\left(3x^2+5x+2\right)^{3/2}}{7x^{7/2}}$$

$\big\downarrow$ 1503

$$-\frac{9}{7}\left(174\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+211\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{\sqrt{3x^2+5x+2}(133x+22)}{3x^{3/2}}\right)-\frac{4(1-2x)\left(3x^2+5x+2\right)^{3/2}}{7x^{7/2}}$$

$\big\downarrow$ 1413

$$-\frac{9}{7}\left(211\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{87\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}-\frac{\sqrt{3x^2+5x+2}(133x+2}{3x^{3/2}}\right.$$
$$\frac{4(1-2x)\left(3x^2+5x+2\right)^{3/2}}{7x^{7/2}}$$

$\big\downarrow$ 1456

$$-\frac{9}{7}\left(\frac{87\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+211\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\right.\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$
$$\frac{4(1-2x)\left(3x^2+5x+2\right)^{3/2}}{7x^{7/2}}$$

input
```
Int[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(9/2),x]
```

output
```
(-4*(1 - 2*x)*(2 + 5*x + 3*x^2)^(3/2))/(7*x^(7/2)) - (9*(-1/3*((22 + 133*x
)*Sqrt[2 + 5*x + 3*x^2])/x^(3/2) + 211*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*
x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sq
rt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (87*Sqrt[2]*(1 + x)*Sqrt[(2 +
3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

method	result
default	$\frac{111\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^{3}-211\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^{3}+2538x^{5}+4794x^{4}\cdots}{14\sqrt{3x^{2}+5x+2}\,x^{\frac{7}{2}}}$
elliptic	$\frac{\sqrt{(3x^{2}+5x+2)x}\,\left(-\frac{8\sqrt{3x^{3}+5x^{2}+2x}}{7x^{4}}-\frac{4\sqrt{3x^{3}+5x^{2}+2x}}{7x^{3}}+\frac{94\sqrt{3x^{3}+5x^{2}+2x}}{7x^{2}}+\frac{\frac{1269}{7}x^{2}+\frac{2115}{7}x+\frac{846}{7}}{\sqrt{\left(3x^{2}+5x+2\right)x}}-\frac{261\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\cdots\right.}{7\sqrt{3x^{3}+5x^{2}+2x}}\cdots\right.}{\sqrt{x}\,\sqrt{3x^{2}+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/14*(111*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*
x+4)^(1/2),I*2^(1/2))*x^3-211*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*(6*x+
4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*x^3+2538*x^5+4794*x^4+2608*x^3+2
88*x^2-96*x-32)/(3*x^2+5*x+2)^(1/2)/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.38

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx = -\frac{511\,\sqrt{3}x^4\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-1899\,\sqrt{3}x^4\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)-3\left(423x^3+94x^2-4x-8\right)\sqrt{3x^2+5x+2}\sqrt{x}}{21\,x^4}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-1/21*(511*sqrt(3)*x^4*weierstrassPInverse(28/27, 80/729, x + 5/9) - 1899*
sqrt(3)*x^4*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/7
29, x + 5/9)) - 3*(423*x^3 + 94*x^2 - 4*x - 8)*sqrt(3*x^2 + 5*x + 2)*sqrt(
x))/x^4
```

Sympy [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx = -\int \left(-\frac{4\sqrt{3x^2+5x+2}}{x^{\frac{9}{2}}}\right)dx - \int \frac{19\sqrt{3x^2+5x+2}}{x^{\frac{5}{2}}}\,dx - \int \frac{15\sqrt{3x^2+5x+2}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(3/2)/x**(9/2),x)
```

output
```
-Integral(-4*sqrt(3*x**2 + 5*x + 2)/x**(9/2), x) - Integral(19*sqrt(3*x**2
 + 5*x + 2)/x**(5/2), x) - Integral(15*sqrt(3*x**2 + 5*x + 2)/x**(3/2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(9/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx = \int -\frac{(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}}{x^{9/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(9/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(9/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{9/2}}\,dx = \frac{-630\sqrt{3x^2+5x+2}\,x^3+1848\sqrt{3x^2+5x+2}\,x^2-1330\sqrt{3x^2+5x+2}}{}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(9/2),x)
```

output
```
( - 630*sqrt(3*x**2 + 5*x + 2)*x**3 + 1848*sqrt(3*x**2 + 5*x + 2)*x**2 - 1
330*sqrt(3*x**2 + 5*x + 2)*x - 24*sqrt(3*x**2 + 5*x + 2) - 6590*sqrt(x)*in
t(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**5 + 5*sqrt(x)*x**4 + 2*sqrt(x)*x**3
),x)*x**3 - 3295*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**4 + 5*sq
rt(x)*x**3 + 2*sqrt(x)*x**2),x)*x**3 - 5187*sqrt(x)*int((sqrt(x)*sqrt(3*x*
*2 + 5*x + 2))/(3*x**5 + 5*x**4 + 2*x**3),x)*x**3)/(21*sqrt(x)*x**3)
```

3.197 $\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{11/2}}\,dx = -\frac{5438\sqrt{x}(2+3x)}{315\sqrt{2+5x+3x^2}} + \frac{5438\sqrt{2+5x+3x^2}}{315\sqrt{x}} + \frac{(1446+4055x)\sqrt{2+5x+3x^2}}{315x^{5/2}} - \frac{4(7-15x)(2+5x+3x^2)^{3/2}}{63x^{9/2}} + \frac{5438\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{315\sqrt{1+x}\sqrt{2+3x}} - \frac{899\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{21\sqrt{2}\sqrt{2+5x+3x^2}}$$

output
```
-5438/315*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+5438/315*(3*x^2+5*x+2)^(1/2)
/x^(1/2)+1/315*(1446+4055*x)*(3*x^2+5*x+2)^(1/2)/x^(5/2)-4/63*(7-15*x)*(3*
x^2+5*x+2)^(3/2)/x^(9/2)+5438/315*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^
(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-899/42*2^(1/2)*
(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(
3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.78

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{11/2}} dx = \frac{-1120-3200x+7424x^2+44480x^3+64706x^4+29730x^5-10876i\sqrt{}}{}$$

input
```
Integrate[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(11/2),x]
```

output
```
(-1120 - 3200*x + 7424*x^2 + 44480*x^3 + 64706*x^4 + 29730*x^5 - (10876*I)
*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(11/2)*EllipticE[I*ArcSinh[Sqrt[
2/3]/Sqrt[x]], 3/2] - (2609*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(1
1/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(630*x^(9/2)*Sqrt[2 + 5
*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1229, 1229, 27, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)(3x^2+5x+2)^{3/2}}{x^{11/2}} dx$$

$$\Big\downarrow \text{ 1229}$$

$$-\frac{1}{21}\int \frac{(285x+241)\sqrt{3x^2+5x+2}}{x^{7/2}}dx-\frac{4(7-15x)(3x^2+5x+2)^{3/2}}{63x^{9/2}}$$

$$\Big\downarrow \text{ 1229}$$

$$\frac{1}{21}\left(\frac{1}{15}\int -\frac{13485x+10876}{2x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{\sqrt{3x^2+5x+2}(4055x+1446)}{15x^{5/2}}\right)-\frac{4(7-15x)\left(3x^2+5x+2\right)^{3/2}}{63x^{9/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{21}\left(\frac{(4055x+1446)\sqrt{3x^2+5x+2}}{15x^{5/2}}-\frac{1}{30}\int \frac{13485x+10876}{x^{3/2}\sqrt{3x^2+5x+2}}dx\right)-\frac{4(7-15x)\left(3x^2+5x+2\right)^{3/2}}{63x^{9/2}}$$

$$\Big\downarrow \; 1237$$

$$\frac{1}{21}\left(\frac{1}{30}\left(\int -\frac{3(5438x+4495)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{10876\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{\sqrt{3x^2+5x+2}(4055x+1446)}{15x^{5/2}}\right)-\frac{4(7-15x)\left(3x^2+5x+2\right)^{3/2}}{63x^{9/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{21}\left(\frac{1}{30}\left(\frac{10876\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\int \frac{5438x+4495}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)+\frac{\sqrt{3x^2+5x+2}(4055x+1446)}{15x^{5/2}}\right)-\frac{4(7-15x)\left(3x^2+5x+2\right)^{3/2}}{63x^{9/2}}$$

$$\Big\downarrow \; 1240$$

$$\frac{1}{21}\left(\frac{1}{30}\left(\frac{10876\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\int \frac{5438x+4495}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)+\frac{\sqrt{3x^2+5x+2}(4055x+1446)}{15x^{5/2}}\right)-\frac{4(7-15x)\left(3x^2+5x+2\right)^{3/2}}{63x^{9/2}}$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{21}\left(\frac{1}{30}\left(\frac{10876\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(4495\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+5438\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)+\frac{\sqrt{3x^2+5x}}{}\right.-\frac{4(7-15x)\left(3x^2+5x+2\right)^{3/2}}{63x^{9/2}}$$

$$\Big\downarrow \; 1413$$

$$\frac{1}{21}\left(\frac{1}{30}\left(\frac{10876\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(5438\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{4495(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),\right.}{\sqrt{2}\sqrt{3x^2+5x+2}}\right.\right.\right.$$

$$\frac{4(7-15x)\left(3x^2+5x+2\right)^{3/2}}{63x^{9/2}}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{21}\left(\frac{1}{30}\left(\frac{10876\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(\frac{4495(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+5438\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}\right.\right.\right.\right.$$

$$\frac{4(7-15x)\left(3x^2+5x+2\right)^{3/2}}{63x^{9/2}}$$

input
```
Int[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(11/2),x]
```

output
```
(-4*(7 - 15*x)*(2 + 5*x + 3*x^2)^(3/2))/(63*x^(9/2)) + (((1446 + 4055*x)*S
qrt[2 + 5*x + 3*x^2])/(15*x^(5/2)) + ((10876*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x
] - 6*(5438*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 +
 x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 +
5*x + 3*x^2])) + (4495*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sq
rt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])))/30)/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.65

method	result
default	$\frac{2829\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^{4}-5438\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^{4}+97884x^{6}+2523}{1890\sqrt{3x^{2}+5x+2}\,x^{\frac{9}{2}}}$
elliptic	$\frac{\sqrt{(3x^{2}+5x+2)x}\left(-\frac{8\sqrt{3x^{3}+5x^{2}+2x}}{9x^{5}}-\frac{20\sqrt{3x^{3}+5x^{2}+2x}}{63x^{4}}+\frac{842\sqrt{3x^{3}+5x^{2}+2x}}{105x^{3}}+\frac{991\sqrt{3x^{3}+5x^{2}+2x}}{63x^{2}}+\frac{\frac{5438}{105}x^{2}+\frac{5438}{63}x+\frac{10876}{315}}{\sqrt{\left(3x^{2}+5x+2\right)x}}-\frac{899\sqrt{6x+4}}{}\right.}{\sqrt{x}\,\sqrt{3x^{2}+5x+2}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/1890*(2829*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*
(6*x+4)^(1/2),I*2^(1/2))*x^4-5438*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)
^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*x^4+97884*x^6+252330*x^5+259
374*x^4+133440*x^3+22272*x^2-9600*x-3360)/(3*x^2+5*x+2)^(1/2)/x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.36

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{11/2}}\,dx =$$
$$-\frac{13265\,\sqrt{3}x^5\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-48942\,\sqrt{3}x^5\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInver}\right.}{2835\,x^5}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

output
```
-1/2835*(13265*sqrt(3)*x^5*weierstrassPInverse(28/27, 80/729, x + 5/9) - 4
8942*sqrt(3)*x^5*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27,
 80/729, x + 5/9)) - 9*(5438*x^4 + 4955*x^3 + 2526*x^2 - 100*x - 280)*sqrt
(3*x^2 + 5*x + 2)*sqrt(x))/x^5
```

Sympy [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{11/2}}\,dx =$$
$$-\int\left(-\frac{4\sqrt{3x^2+5x+2}}{x^{\frac{11}{2}}}\right)dx-\int\frac{19\sqrt{3x^2+5x+2}}{x^{\frac{7}{2}}}\,dx-\int\frac{15\sqrt{3x^2+5x+2}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(3/2)/x**(11/2),x)
```

output
```
-Integral(-4*sqrt(3*x**2 + 5*x + 2)/x**(11/2), x) - Integral(19*sqrt(3*x**
2 + 5*x + 2)/x**(7/2), x) - Integral(15*sqrt(3*x**2 + 5*x + 2)/x**(5/2), x
)
```

Maxima [F]

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{11/2}}\,dx = \int -\frac{(3\,x^2+5\,x+2)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(11/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(11/2), x)
```

Giac [F]

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{11/2}}\,dx = \int -\frac{(3\,x^2+5\,x+2)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(11/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{11/2}}\,dx = \int -\frac{(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}}{x^{11/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(11/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(11/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{11/2}}\,dx = \frac{36774\sqrt{3x^2+5x+2}\,x^4+21600\sqrt{3x^2+5x+2}\,x^3-11052\sqrt{3x^2+5x}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(11/2),x)
```

output
```
(36774*sqrt(3*x**2 + 5*x + 2)*x**4 + 21600*sqrt(3*x**2 + 5*x + 2)*x**3 - 1
1052*sqrt(3*x**2 + 5*x + 2)*x**2 + 7200*sqrt(3*x**2 + 5*x + 2)*x - 640*sqr
t(3*x**2 + 5*x + 2) + 52000*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*
x**6 + 5*sqrt(x)*x**5 + 2*sqrt(x)*x**4),x)*x**4 + 27300*sqrt(x)*int(sqrt(3
*x**2 + 5*x + 2)/(3*sqrt(x)*x**5 + 5*sqrt(x)*x**4 + 2*sqrt(x)*x**3),x)*x**
4 - 81720*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**4 + 5*sqrt(x)*x
**3 + 2*sqrt(x)*x**2),x)*x**4 - 55161*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*
x)/(3*sqrt(x)*x**2 + 5*sqrt(x)*x + 2*sqrt(x)),x)*x**4)/(720*sqrt(x)*x**4)
```

3.198 $\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{13/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 229

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{13/2}}\,dx = \frac{3229\sqrt{x}(2+3x)}{1386\sqrt{2+5x+3x^2}} + \frac{1357\sqrt{2+5x+3x^2}}{693x^{3/2}} - \frac{3229\sqrt{2+5x+3x^2}}{1386\sqrt{x}} + \frac{(634+1367x)\sqrt{2+5x+3x^2}}{231x^{7/2}} - \frac{4(9-20x)(2+5x+3x^2)^{3/2}}{99x^{11/2}} - \frac{3229\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{693\sqrt{2}\sqrt{1+x}\sqrt{2+3x}} + \frac{1357\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{231\sqrt{2}\sqrt{2+5x+3x^2}}$$

output
```
3229/1386*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+1357/693*(3*x^2+5*x+2)^(1/2)
/x^(3/2)-3229/1386*(3*x^2+5*x+2)^(1/2)/x^(1/2)+1/231*(634+1367*x)*(3*x^2+5
*x+2)^(1/2)/x^(7/2)-4/99*(9-20*x)*(3*x^2+5*x+2)^(3/2)/x^(11/2)-3229/1386*2
^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1
+x)^(1/2)/(2+3*x)^(1/2)+1357/462*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*Inverse
JacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.72

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{13/2}}\,dx = \frac{-2016-5600x+11360x^2+61744x^3+86914x^4+48256x^5+8142x^6}{}$$

input

```
Integrate[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(13/2),x]
```

output

```
(-2016 - 5600*x + 11360*x^2 + 61744*x^3 + 86914*x^4 + 48256*x^5 + 8142*x^6
 + (3229*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(13/2)*EllipticE[I*Ar
cSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (842*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 +
 2/x]*x^(13/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(1386*x^(11/2
)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1229, 1229, 27, 1237, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\left(3x^2+5x+2\right)^{3/2}}{x^{13/2}}\,dx$$

$$\Big\downarrow \; 1229$$

$$-\frac{1}{33}\int \frac{(375x+317)\sqrt{3x^2+5x+2}}{x^{9/2}}dx-\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

$$\Big\downarrow \; 1229$$

$$\frac{1}{33}\left(\frac{1}{35}\int -\frac{5(3447x+2714)}{2x^{5/2}\sqrt{3x^2+5x+2}}dx+\frac{\sqrt{3x^2+5x+2}(1367x+634)}{7x^{7/2}}\right)-\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{33}\left(\frac{(1367x+634)\sqrt{3x^2+5x+2}}{7x^{7/2}}-\frac{1}{14}\int \frac{3447x+2714}{x^{5/2}\sqrt{3x^2+5x+2}}dx\right)-\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

$$\Big\downarrow \; 1237$$

$$\frac{1}{33}\left(\frac{1}{14}\left(\frac{1}{3}\int \frac{4071x+3229}{x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{2714\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{\sqrt{3x^2+5x+2}(1367x+634)}{7x^{7/2}}\right)-\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

$$\Big\downarrow \; 1237$$

$$\frac{1}{33}\left(\frac{1}{14}\left(\frac{1}{3}\left(-\int -\frac{3(3229x+2714)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{3229\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{2714\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{\sqrt{3x^2+5x+2}(1}{7x^{7/2}}\right.$$
$$-\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{33}\left(\frac{1}{14}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{3229x+2714}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{3229\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{2714\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{\sqrt{3x^2+5x+2}(1367}{7x^{7/2}}\right.$$
$$-\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

$$\Big\downarrow \; 1240$$

$$\frac{1}{33}\left(\frac{1}{14}\left(\frac{1}{3}\left(3\int \frac{3229x+2714}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{3229\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{2714\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{\sqrt{3x^2+5x+2}(1367x}{7x^{7/2}}\right.$$
$$-\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{33}\left(\frac{1}{14}\left(\frac{1}{3}\left(3\left(2714\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+3229\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{3229\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{2714\sqrt{3}}{3}\right.\right.$$

$$\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

$\downarrow$ 1413

$$\frac{1}{33}\left(\frac{1}{14}\left(\frac{1}{3}\left(3\left(3229\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{1357\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)-\frac{3229\sqrt{3}}{}\right.\right.\right.$$

$$\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

$\downarrow$ 1456

$$\frac{1}{33}\left(\frac{1}{14}\left(\frac{1}{3}\left(3\left(\frac{1357\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+3229\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3}{x}}}{3\sqrt{}}\right.\right.\right.\right.\right.$$

$$\frac{4(9-20x)\left(3x^2+5x+2\right)^{3/2}}{99x^{11/2}}$$

input
```
Int[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(13/2),x]
```

output
```
(-4*(9 - 20*x)*(2 + 5*x + 3*x^2)^(3/2))/(99*x^(11/2)) + (((634 + 1367*x)*S
qrt[2 + 5*x + 3*x^2])/(7*x^(7/2)) + ((2714*Sqrt[2 + 5*x + 3*x^2])/(3*x^(3/
2)) + ((-3229*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x] + 3*(3229*((Sqrt[x]*(2 + 3*x)
)/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*Ell
ipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (1357*Sqrt[2]*
(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 +
 5*x + 3*x^2]))/3)/14)/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.61

method	result
default	$-\frac{1545\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^{5}-3229\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^{5}+58122x^{7}+48}{8316\sqrt{3x^{2}+5x+2}\,x^{\frac{11}{2}}}$
elliptic	$\frac{\sqrt{(3x^{2}+5x+2)x}\left(-\frac{8\sqrt{3x^{3}+5x^{2}+2x}}{11x^{6}}-\frac{20\sqrt{3x^{3}+5x^{2}+2x}}{99x^{5}}+\frac{3946\sqrt{3x^{3}+5x^{2}+2x}}{693x^{4}}+\frac{1927\sqrt{3x^{3}+5x^{2}+2x}}{231x^{3}}+\frac{1357\sqrt{3x^{3}+5x^{2}+2x}}{693x^{2}}-\frac{3229(3x^{2}+5}{1386\sqrt{(3x^{2}+}}\right.}{\sqrt{x}\,\sqrt{3}}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8316*(1545*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2
*(6*x+4)^(1/2),I*2^(1/2))*x^5-3229*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x
)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*x^5+58122*x^7+48018*x^6-250
788*x^5-521484*x^4-370464*x^3-68160*x^2+33600*x+12096)/(3*x^2+5*x+2)^(1/2)
/x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.34

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{13/2}}\,dx = \frac{8281\sqrt{3}x^6\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-29061\sqrt{3}x^6\text{weierstras}}{}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

output
```
1/12474*(8281*sqrt(3)*x^6*weierstrassPInverse(28/27, 80/729, x + 5/9) - 29
061*sqrt(3)*x^6*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27,
80/729, x + 5/9)) - 9*(3229*x^5 - 2714*x^4 - 11562*x^3 - 7892*x^2 + 280*x
+ 1008)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/x^6
```

Sympy [F]

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{13/2}}\,dx = -\int\left(-\frac{4\sqrt{3x^2+5x+2}}{x^{\frac{13}{2}}}\right)dx-\int\frac{19\sqrt{3x^2+5x+2}}{x^{\frac{9}{2}}}\,dx-\int\frac{15\sqrt{3x^2+5x+2}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(3/2)/x**(13/2),x)
```

output
```
-Integral(-4*sqrt(3*x**2 + 5*x + 2)/x**(13/2), x) - Integral(19*sqrt(3*x**
2 + 5*x + 2)/x**(9/2), x) - Integral(15*sqrt(3*x**2 + 5*x + 2)/x**(7/2), x
)
```

Maxima [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{13/2}}\,dx=\int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(13/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(13/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{13/2}}\,dx=\int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(13/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{13/2}}\,dx=\int -\frac{(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}}{x^{13/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(13/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(13/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\,(2+5x+3x^2)^{3/2}}{x^{13/2}}\,dx = \frac{808500\sqrt{3x^2+5x+2}\,x^3+241846\sqrt{3x^2+5x+2}\,x^2+84700\sqrt{3x^2+5}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(13/2),x)
```

output
```
(808500*sqrt(3*x**2 + 5*x + 2)*x**3 + 241846*sqrt(3*x**2 + 5*x + 2)*x**2 +
 84700*sqrt(3*x**2 + 5*x + 2)*x - 58800*sqrt(3*x**2 + 5*x + 2) + 909300*sq
rt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**7 + 5*sqrt(x)*x**6 + 2*sqrt
(x)*x**5),x)*x**5 + 491022*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x
**6 + 5*sqrt(x)*x**5 + 2*sqrt(x)*x**4),x)*x**5 - 1546710*sqrt(x)*int(sqrt(
3*x**2 + 5*x + 2)/(3*sqrt(x)*x**5 + 5*sqrt(x)*x**4 + 2*sqrt(x)*x**3),x)*x*
*5 - 773355*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**4 + 5*sqrt(x)
*x**3 + 2*sqrt(x)*x**2),x)*x**5)/(80850*sqrt(x)*x**5)
```

3.199 $\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{15/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{15/2}}\,dx = -\frac{6907\sqrt{x}(2+3x)}{10010\sqrt{2+5x+3x^2}}+\frac{204\sqrt{2+5x+3x^2}}{385x^{5/2}} -\frac{1231\sqrt{2+5x+3x^2}}{2002x^{3/2}}+\frac{6907\sqrt{2+5x+3x^2}}{10010\sqrt{x}}+\frac{(1834+3445x)\sqrt{2+5x+3x^2}}{1001x^{9/2}} -\frac{4(11-25x)\left(2+5x+3x^2\right)^{3/2}}{143x^{13/2}}+\frac{6907\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{5005\sqrt{2}\sqrt{1+x}\sqrt{2+3x}} -\frac{3693\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{2002\sqrt{2}\sqrt{2+5x+3x^2}}$$

output
```
-6907/10010*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+204/385*(3*x^2+5*x+2)^(1/2
)/x^(5/2)-1231/2002*(3*x^2+5*x+2)^(1/2)/x^(3/2)+6907/10010*(3*x^2+5*x+2)^(
1/2)/x^(1/2)+1/1001*(1834+3445*x)*(3*x^2+5*x+2)^(1/2)/x^(9/2)-4/143*(11-25
*x)*(3*x^2+5*x+2)^(3/2)/x^(13/2)+6907/10010*2^(1/2)*(3*x^2+5*x+2)^(1/2)*El
lipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-3693/
4004*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2
*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.67

$$\int \frac{(2-5x)(2+5x+3x^2)^{3/2}}{x^{15/2}}\,dx = \frac{-24640-67200x+125440x^2+654400x^3+840316x^4+361120x^5-2}{}$$

input
```
Integrate[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(15/2),x]
```

output
```
(-24640 - 67200*x + 125440*x^2 + 654400*x^3 + 840316*x^4 + 361120*x^5 - 29
726*x^6 - 36930*x^7 - (13814*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(
15/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (4651*I)*Sqrt[2]*Sqrt
[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]]
, 3/2])/(20020*x^(13/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {1229, 27, 1229, 27, 1237, 1237, 27, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\left(3x^2+5x+2\right)^{3/2}}{x^{15/2}}\,dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{3}{143}\int \frac{3(155x+131)\sqrt{3x^2+5x+2}}{x^{11/2}}dx-\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{9}{143}\int \frac{(155x+131)\sqrt{3x^2+5x+2}}{x^{11/2}}dx-\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$$\Big\downarrow \; 1229$$

$$-\frac{9}{143}\left(-\frac{1}{63}\int -\frac{3(2305x+1768)}{2x^{7/2}\sqrt{3x^2+5x+2}}dx-\frac{\sqrt{3x^2+5x+2}(3445x+1834)}{63x^{9/2}}\right)-\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{9}{143}\left(\frac{1}{42}\int \frac{2305x+1768}{x^{7/2}\sqrt{3x^2+5x+2}}dx-\frac{(3445x+1834)\sqrt{3x^2+5x+2}}{63x^{9/2}}\right)-\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$$\Big\downarrow \; 1237$$

$$-\frac{9}{143}\left(\frac{1}{42}\left(-\frac{1}{5}\int \frac{7956x+6155}{x^{5/2}\sqrt{3x^2+5x+2}}dx-\frac{1768\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{(3445x+1834)\sqrt{3x^2+5x+2}}{63x^{9/2}}\right)-\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$$\Big\downarrow \; 1237$$

$$-\frac{9}{143}\left(\frac{1}{42}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{18465x+13814}{2x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{6155\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{1768\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{(3445x+1834}{63x}\right.$$
$$\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{9}{143}\left(\frac{1}{42}\left(\frac{1}{5}\left(\frac{1}{6}\int \frac{18465x+13814}{x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{6155\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{1768\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{(3445x+1834)}{63x}\right.$$
$$\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$$\Big\downarrow \; 1237$$

$$-\frac{9}{143}\left(\frac{1}{42}\left(\frac{1}{5}\left(\frac{1}{6}\left(-\int -\frac{3(6907x+6155)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{13814\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{6155\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{1768\sqrt{3x^2}}{5x}\right.\right.$$
$$\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$\big\downarrow$ 27

$$-\frac{9}{143}\left(\frac{1}{42}\left(\frac{1}{5}\left(\frac{1}{6}\left(3\int \frac{6907x+6155}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{13814\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{6155\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{1768\sqrt{3x^2+}}{5x^{5/2}}\right.\right.\right.$$
$$\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$\big\downarrow$ 1240

$$-\frac{9}{143}\left(\frac{1}{42}\left(\frac{1}{5}\left(\frac{1}{6}\left(6\int \frac{6907x+6155}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{13814\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{6155\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{1768\sqrt{3x^2+5}}{5x^{5/2}}\right.\right.\right.$$
$$\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$\big\downarrow$ 1503

$$-\frac{9}{143}\left(\frac{1}{42}\left(\frac{1}{5}\left(\frac{1}{6}\left(6\left(6155\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+6907\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{13814\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\right.\right.\right.\right.$$
$$\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$\big\downarrow$ 1413

$$-\frac{9}{143}\left(\frac{1}{42}\left(\frac{1}{5}\left(\frac{1}{6}\left(6\left(6907\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{6155(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{1381}{}\right.\right.\right.\right.$$
$$\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

$\big\downarrow$ 1456

$$-\frac{9}{143}\left(\frac{1}{42}\left(\frac{1}{5}\left(\frac{1}{6}\left(6\left(\frac{6155(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+6907\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)}{}\right.\right.\right.\right.\right.\right.$$
$$\frac{4(11-25x)\left(3x^2+5x+2\right)^{3/2}}{143x^{13/2}}$$

input
```
Int[((2 - 5*x)*(2 + 5*x + 3*x^2)^(3/2))/x^(15/2),x]
```

output
```
(-4*(11 - 25*x)*(2 + 5*x + 3*x^2)^(3/2))/(143*x^(13/2)) - (9*(-1/63*((1834
 + 3445*x)*Sqrt[2 + 5*x + 3*x^2])/x^(9/2) + ((-1768*Sqrt[2 + 5*x + 3*x^2])
/(5*x^(5/2)) + ((6155*Sqrt[2 + 5*x + 3*x^2])/(3*x^(3/2)) + ((-13814*Sqrt[2
 + 5*x + 3*x^2])/Sqrt[x] + 6*(6907*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x +
3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x
]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (6155*(1 + x)*Sqrt[(2 + 3*x)/(1 +
x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])))/6)
/5)/42))/143
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.57

method	result
default	$\frac{2256\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^{6}-6907\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^{6}+124326x^{8}+964}{60060\sqrt{3x^{2}+5x+2}\,x^{\frac{13}{2}}}$
elliptic	$\frac{\sqrt{(3x^{2}+5x+2)x}\left(-\frac{8\sqrt{3x^{3}+5x^{2}+2x}}{13x^{7}}-\frac{20\sqrt{3x^{3}+5x^{2}+2x}}{143x^{6}}+\frac{630\sqrt{3x^{3}+5x^{2}+2x}}{143x^{5}}+\frac{5545\sqrt{3x^{3}+5x^{2}+2x}}{1001x^{4}}+\frac{204\sqrt{3x^{3}+5x^{2}+2x}}{385x^{3}}-\frac{1231\sqrt{3x^{3}+5x^{2}}}{2002x^{2}}\right.}{}$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
1/60060*(2256*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2
*(6*x+4)^(1/2),I*2^(1/2))*x^6-6907*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x
)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*x^6+124326*x^8+96420*x^7-62
94*x^6+1083360*x^5+2520948*x^4+1963200*x^3+376320*x^2-201600*x-73920)/(3*x
^2+5*x+2)^(1/2)/x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.33

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{15/2}}\,dx =$$

$$-\frac{20860\sqrt{3}x^7\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-62163\sqrt{3}x^7\mathrm{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\mathrm{weierstrassPInver}\right.}{9}$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

output
```
-1/90090*(20860*sqrt(3)*x^7*weierstrassPInverse(28/27, 80/729, x + 5/9) -
62163*sqrt(3)*x^7*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27
, 80/729, x + 5/9)) - 9*(6907*x^6 - 6155*x^5 + 5304*x^4 + 55450*x^3 + 4410
0*x^2 - 1400*x - 6160)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/x^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-5*x)*(3*x**2+5*x+2)**(3/2)/x**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{15/2}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

output
```
-integrate((3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(15/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{15/2}}\,dx = \int -\frac{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}(5\,x-2)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(15/2),x, algorithm="giac")
```

output
```
integrate(-(3*x^2 + 5*x + 2)^(3/2)*(5*x - 2)/x^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{15/2}}\,dx = \int -\frac{(5\,x-2)\,\left(3\,x^2+5\,x+2\right)^{3/2}}{x^{15/2}}\,dx$$

input
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(15/2),x)
```

output
```
int(-((5*x - 2)*(5*x + 3*x^2 + 2)^(3/2))/x^(15/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\left(2+5x+3x^2\right)^{3/2}}{x^{15/2}}\,dx = \frac{126360\sqrt{3x^2+5x+2}\,x^3+76726\sqrt{3x^2+5x+2}\,x^2+4680\sqrt{3x^2+5x}}{}$$

input
```
int((2-5*x)*(3*x^2+5*x+2)^(3/2)/x^(15/2),x)
```

output
```
(126360*sqrt(3*x**2 + 5*x + 2)*x**3 + 76726*sqrt(3*x**2 + 5*x + 2)*x**2 +
4680*sqrt(3*x**2 + 5*x + 2)*x - 12960*sqrt(3*x**2 + 5*x + 2) + 83880*sqrt(
x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**8 + 5*sqrt(x)*x**7 + 2*sqrt(x)
*x**6),x)*x**6 + 46134*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**7
+ 5*sqrt(x)*x**6 + 2*sqrt(x)*x**5),x)*x**6 - 150280*sqrt(x)*int(sqrt(3*x**
2 + 5*x + 2)/(3*sqrt(x)*x**6 + 5*sqrt(x)*x**5 + 2*sqrt(x)*x**4),x)*x**6 -
78897*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**5 + 5*sqrt(x)*x**4
+ 2*sqrt(x)*x**3),x)*x**6)/(21060*sqrt(x)*x**6)
```

3.200 $\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 401

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx = \frac{B\sqrt{-b+\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex}}{\sqrt{-b+\sqrt{b^2-4ac}}\sqrt{e}}\right)|\frac{b-\sqrt{b^2-}}{b+\sqrt{b^2-}}\right.}{\sqrt{2}c^{3/2}\sqrt{e}\sqrt{a+x(b+cx)}} - \frac{\sqrt{-b+\sqrt{b^2-4ac}}\left(bB-2Ac+B\sqrt{b^2-4ac}\right)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{-b}}\right.\right.}{\sqrt{2}c^{3/2}\sqrt{e}\sqrt{a+x(b+cx)}}$$

output
```
1/2*B*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(b+(-4*a*c+b^2)^(1/2))*(1+2*c*x/(b-(-4
*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2
^(1/2)*c^(1/2)*(e*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/2)/e^(1/2),((b-(-4*a
*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(3/2)/e^(1/2)/(a+x
*(c*x+b))^(1/2)-1/2*(-b+(-4*a*c+b^2)^(1/2))^(1/2)*(B*b-2*A*c+B*(-4*a*c+b^2
)^(1/2))*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(
1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*(e*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))
^(1/2)/e^(1/2),((b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1
/2)/c^(3/2)/e^(1/2)/(a+x*(c*x+b))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.96 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{x^2\left(-\frac{4B\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}(a+x(b+cx))}{x^2}+\frac{iB\left(-b+\sqrt{b^2-4ac}\right)\sqrt{2+\frac{4a}{\left(b+\sqrt{b^2-4ac}\right)x}}\sqrt{\frac{2a+bx-\sqrt{b^2-4ac}x}{bx-\sqrt{b^2-4ac}x}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}{\sqrt{x}}\right)\right.}{\sqrt{x}}\right.}{2c\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}$$

input
```
Integrate[(A + B*x)/(Sqrt[e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*(x^2*((-4*B*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*(a + x*(b + c*x)))/x^2 +
(I*B*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*
Sqrt[(2*a + b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*Ellipt
icE[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqr
t[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[x] - (I*(-(b*B) + 2*A*c + B
*Sqrt[b^2 - 4*a*c])*Sqrt[2 + (4*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*Sqrt[(2*a
+ b*x - Sqrt[b^2 - 4*a*c]*x)/(b*x - Sqrt[b^2 - 4*a*c]*x)]*EllipticF[I*ArcS
inh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[x]))/(c*Sqrt[a/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[e*x]*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1241, 1240, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}dx$$

$\big\downarrow$ 1241

$$\frac{\sqrt{x}\int \frac{A+Bx}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{\sqrt{ex}}$$

$\big\downarrow$ 1240

$$\frac{2\sqrt{x}\int \frac{A+Bx}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{ex}}$$

$\big\downarrow$ 1511

$$\frac{2\sqrt{x}\left(\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{\sqrt{a}B\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{a}\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{ex}}$$

$\big\downarrow$ 27

$$\frac{2\sqrt{x}\left(\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}-\frac{B\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{ex}}$$

$\big\downarrow$ 1416

$$\frac{2\sqrt{x}\left(\frac{(\sqrt{a}+\sqrt{c}x)\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{B\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{ex}}$$

$\big\downarrow$ 1509

$$\frac{2\sqrt{x}\left(\frac{(\sqrt{a}+\sqrt{c}x)\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx+cx^2}}-\frac{B\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x)\sqrt{\frac{a+bx+cx^2}{(\sqrt{a}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{\sqrt[4]{c}\sqrt{a+bx+cx^2}}\cdots\right)}{\sqrt{c}}\cdots\right)}{\sqrt{ex}}$$

input
```
Int[(A + B*x)/(Sqrt[e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[x]*(-((B*(-((Sqrt[x]*Sqrt[a + b*x + c*x^2])/(Sqrt[a] + Sqrt[c]*x))
 + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c
]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4])/(c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[c]) + ((A + (Sqrt[a]*B)/
Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x + c*x^2])))/Sqrt[e*x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1241
```
Int[((f_) + (g_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_
)^2]), x_Symbol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(f + g*x)/(Sqrt[x]*Sqrt[a
+ b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{-2cx+\sqrt{-4ac+b^2}-b}{\sqrt{-4ac+b^2}}}\sqrt{-\frac{cx}{b+\sqrt{-4ac+b^2}}}\left(A\,\text{EllipticF}\left(\sqrt{\frac{2cx+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)c\sqrt{-4ac+b^2}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(cx^2+bx+a)ex}\left(\frac{A\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\,\text{EllipticF}\left(\sqrt{2}\sqrt{\cdots}\right.}{c\sqrt{ce\,x^3+be\,x^2+aex}}\cdots\right.}{\cdots}$

input
```
int((B*x+A)/(e*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^2+b*x+a)^(1/2)*((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2))
)^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(1/2)*(-c*x/(b+
(-4*a*c+b^2)^(1/2)))^(1/2)*(A*EllipticF(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(
-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2))*c*(-4*a*c+b^2)^(1/2)+A*EllipticF(((2*c*x+(-4*a*c+b^2)^(1/2)
+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2))*c*b-B*(-4*a*c+b^2)^(1/2)*EllipticE(((2*c*x+(-4*a*c+
b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(1/2))^(1/2))*b-2*B*EllipticF(((2*c*x+(-4*a*c+b^2)^(1/2
)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2))*a*c+4*B*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b
+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2))*a*c-B*EllipticE(((2*c*x+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+
b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))
^(1/2))*b^2)/(e*x)^(1/2)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(3\sqrt{ce}Bc\text{weierstrassZeta}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\text{weierstrassPInverse}\left(\frac{4\,(b^2-3\,ac)}{3\,c^2},-\frac{4\,(2\,b^3-9\,abc)}{27\,c^3},\frac{3\,cx-}{3\,c}\right.\right.\right.}{3\,c^2e}$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*e)*B*c*weierstrassZeta(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3
- 9*a*b*c)/c^3, weierstrassPInverse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 -
9*a*b*c)/c^3, 1/3*(3*c*x + b)/c)) + (B*b - 3*A*c)*sqrt(c*e)*weierstrassPIn
verse(4/3*(b^2 - 3*a*c)/c^2, -4/27*(2*b^3 - 9*a*b*c)/c^3, 1/3*(3*c*x + b)/
c))/(c^2*e)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+B\,x}{\sqrt{e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/((e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/((e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx+cx^2}}\,dx=\frac{\sqrt{e}\left(\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^3+b\,x^2+ax}dx\right)a+\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}}{c\,x^2+bx+a}dx\right)b\right)}{e}$$

input
```
int((B*x+A)/(e*x)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(e)*(int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a*x + b*x**2 + c*x**3),x)*
a + int((sqrt(x)*sqrt(a + b*x + c*x**2))/(a + b*x + c*x**2),x)*b))/e
```

3.201 $\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 219

$$\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx = -\frac{68920\sqrt{x}(2+3x)}{15309\sqrt{2+5x+3x^2}}+\frac{11320\sqrt{x}\sqrt{2+5x+3x^2}}{5103} - \frac{820}{567}x^{3/2}\sqrt{2+5x+3x^2}+\frac{508}{567}x^{5/2}\sqrt{2+5x+3x^2} - \frac{10}{27}x^{7/2}\sqrt{2+5x+3x^2}+\frac{68920\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{15309\sqrt{1+x}\sqrt{2+3x}} - \frac{11320\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{5103\sqrt{2+5x+3x^2}}$$

output
```
-68920/15309*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+11320/5103*x^(1/2)*(3*x^2
+5*x+2)^(1/2)-820/567*x^(3/2)*(3*x^2+5*x+2)^(1/2)+508/567*x^(5/2)*(3*x^2+5
*x+2)^(1/2)-10/27*x^(7/2)*(3*x^2+5*x+2)^(1/2)+68920/15309*2^(1/2)*(3*x^2+5
*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*
x)^(1/2)-11320/5103*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arct
an(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

$$\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx = \frac{-2(68920+138340x+40620x^2-9306x^3+4590x^4-6399x^5+8505x^6)-68920}{}$$

input
```
Integrate[((2 - 5*x)*x^(7/2))/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-2*(68920 + 138340*x + 40620*x^2 - 9306*x^3 + 4590*x^4 - 6399*x^5 + 8505*
x^6) - (68920*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[
I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (34960*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sq
rt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(15309*S
qrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1236, 1236, 27, 1236, 27, 1236, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{7/2}}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{27}\int \frac{x^{5/2}(127x+35)}{\sqrt{3x^2+5x+2}}dx-\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{27}\left(\frac{2}{21}\int -\frac{5x^{3/2}(615x+254)}{2\sqrt{3x^2+5x+2}}dx+\frac{254}{21}\sqrt{3x^2+5x+2}x^{5/2}\right)-\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 27$$

$$\frac{2}{27}\left(\frac{254}{21}x^{5/2}\sqrt{3x^2+5x+2}-\frac{5}{21}\int \frac{x^{3/2}(615x+254)}{\sqrt{3x^2+5x+2}}dx\right)-\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 1236$$

$$\frac{2}{27}\left(\frac{254}{21}x^{5/2}\sqrt{3x^2+5x+2}-\frac{5}{21}\left(\frac{2}{15}\int -\frac{15\sqrt{x}(283x+123)}{\sqrt{3x^2+5x+2}}dx+82\sqrt{3x^2+5x+2}x^{3/2}\right)\right)-\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 27$$

$$\frac{2}{27}\left(\frac{254}{21}x^{5/2}\sqrt{3x^2+5x+2}-\frac{5}{21}\left(82x^{3/2}\sqrt{3x^2+5x+2}-2\int \frac{\sqrt{x}(283x+123)}{\sqrt{3x^2+5x+2}}dx\right)\right)-\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 1236$$

$$\frac{2}{27}\left(\frac{254}{21}x^{5/2}\sqrt{3x^2+5x+2}-\frac{5}{21}\left(82x^{3/2}\sqrt{3x^2+5x+2}-2\left(\frac{2}{9}\int -\frac{1723x+566}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{566}{9}\sqrt{x}\sqrt{3x^2+5\right.\right.\right.$$
$$\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 27$$

$$\frac{2}{27}\left(\frac{254}{21}x^{5/2}\sqrt{3x^2+5x+2}-\frac{5}{21}\left(82x^{3/2}\sqrt{3x^2+5x+2}-2\left(\frac{566}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{1}{9}\int \frac{1723x+566}{\sqrt{x}\sqrt{3x^2+5x+2}\right.\right.\right.$$
$$\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 1240$$

$$\frac{2}{27}\left(\frac{254}{21}x^{5/2}\sqrt{3x^2+5x+2}-\frac{5}{21}\left(82x^{3/2}\sqrt{3x^2+5x+2}-2\left(\frac{566}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\int \frac{1723x+566}{\sqrt{3x^2+5x+2}}d\sqrt{\right.\right.\right.$$
$$\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 1503$$

$$\frac{2}{27}\left(\frac{254}{21}x^{5/2}\sqrt{3x^2+5x+2}-\frac{5}{21}\left(82x^{3/2}\sqrt{3x^2+5x+2}-2\left(\frac{566}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(566\int \frac{1}{\sqrt{3x^2+5x+\right.\right.\right.\right.$$
$$\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \; 1413$$

$$\frac{2}{27}\left(\frac{254}{21}x^{5/2}\sqrt{3x^2+5x+2}-\frac{5}{21}\left(82x^{3/2}\sqrt{3x^2+5x+2}-2\left(\frac{566}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(1723\int\frac{x}{\sqrt{3x^2+5x}}\right.\right.\right.\right.$$
$$\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \; 1456$$

$$\frac{2}{27}\left(\frac{254}{21}x^{5/2}\sqrt{3x^2+5x+2}-\frac{5}{21}\left(82x^{3/2}\sqrt{3x^2+5x+2}-2\left(\frac{566}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(\frac{283\sqrt{2}(x+1)\sqrt{\frac{3x}{x}}}{}\right.\right.\right.\right.$$
$$\frac{10}{27}x^{7/2}\sqrt{3x^2+5x+2}$$

input
```
Int[((2 - 5*x)*x^(7/2))/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-10*x^(7/2)*Sqrt[2 + 5*x + 3*x^2])/27 + (2*((254*x^(5/2)*Sqrt[2 + 5*x + 3
*x^2])/21 - (5*(82*x^(3/2)*Sqrt[2 + 5*x + 3*x^2] - 2*((566*Sqrt[x]*Sqrt[2
+ 5*x + 3*x^2])/9 - (2*(1723*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]
) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1
/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (283*Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 +
 x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/9)))/21))/2
7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.58

method	result
default	$\frac{-\frac{10x^6}{9}+\frac{158x^5}{189}+\frac{23140\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{15309}-\frac{34460\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{45927}-\frac{340x^4}{567}+\frac{206}{17}}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
risch	$-\frac{2\left(945x^3-2286x^2+3690x-5660\right)\sqrt{x}\,\sqrt{3x^2+5x+2}}{5103}-\frac{\left(\frac{11320\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{15309\sqrt{3x^3+5x^2+2x}}+\frac{34460\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-}}{}\right.}{\sqrt{x}\,\sqrt{3x^2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{10x^3\sqrt{3x^3+5x^2+2x}}{27}+\frac{508x^2\sqrt{3x^3+5x^2+2x}}{567}-\frac{820x\sqrt{3x^3+5x^2+2x}}{567}+\frac{11320\sqrt{3x^3+5x^2+2x}}{5103}-\frac{11320\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}}{15309\sqrt{3x^3}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45927/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(-25515*x^6+19197*x^5+34710*(6*x+4)^(1
/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2)
)-17230*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+
4)^(1/2),I*2^(1/2))-13770*x^4+27918*x^3+188280*x^2+101880*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.26

$$
\begin{aligned}
\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx =& \\
&-\frac{2}{5103}\left(945\,x^3-2286\,x^2+3690\,x-5660\right)\sqrt{3\,x^2+5\,x+2}\sqrt{x} \\
&+\frac{20120}{19683}\sqrt{3}\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) \\
&+\frac{68920}{15309}\sqrt{3}\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)
\end{aligned}
$$

input
```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2/5103*(945*x^3 - 2286*x^2 + 3690*x - 5660)*sqrt(3*x^2 + 5*x + 2)*sqrt(x)
 + 20120/19683*sqrt(3)*weierstrassPInverse(28/27, 80/729, x + 5/9) + 68920
/15309*sqrt(3)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 8
0/729, x + 5/9))
```

Sympy [F]

$$\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx = -\int \left(-\frac{2x^{\frac{7}{2}}}{\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5x^{\frac{9}{2}}}{\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)*x**(7/2)/(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2*x**(7/2)/sqrt(3*x**2 + 5*x + 2), x) - Integral(5*x**(9/2)/sqr
t(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{7}{2}}}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(7/2)/sqrt(3*x^2 + 5*x + 2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{7}{2}}}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(7/2)/sqrt(3*x^2 + 5*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx = -\int \frac{x^{7/2}\,(5\,x-2)}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
int(-(x^(7/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
-int((x^(7/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(2-5x)x^{7/2}}{\sqrt{2+5x+3x^2}}\,dx =& -\frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3}{27}+\frac{508\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{567}\\&-\frac{820\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{567}+\frac{164\sqrt{x}\,\sqrt{3x^2+5x+2}}{189}\\&+\frac{3446\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{567}-\frac{164\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{189}\end{aligned}$$

input
```
int((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(1/2),x)
```

output
```
(2*( - 105*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 + 254*sqrt(x)*sqrt(3*x**2 +
 5*x + 2)*x**2 - 410*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x + 246*sqrt(x)*sqrt(3
*x**2 + 5*x + 2) + 1723*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(3*x**2 + 5
*x + 2),x) - 246*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2
*x),x)))/567
```

3.202 $\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 196

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx = \frac{13688\sqrt{x}(2+3x)}{2835\sqrt{2+5x+3x^2}} - \frac{412}{189}\sqrt{x}\sqrt{2+5x+3x^2} + \frac{128}{105}x^{3/2}\sqrt{2+5x+3x^2} - \frac{10}{21}x^{5/2}\sqrt{2+5x+3x^2} - \frac{13688\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{2835\sqrt{1+x}\sqrt{2+3x}} + \frac{412\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{189\sqrt{2+5x+3x^2}}$$

output
```
13688/2835*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-412/189*x^(1/2)*(3*x^2+5*x+
2)^(1/2)+128/105*x^(3/2)*(3*x^2+5*x+2)^(1/2)-10/21*x^(5/2)*(3*x^2+5*x+2)^(
1/2)-13688/2835*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),
1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+412/189*2^(1/2)*(1+x)^(1/2)*(2+3*
x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx = \frac{27376+56080x+17076x^2-3960x^3+3618x^4-4050x^5+13688i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3}}{2835\sqrt{}}$$

input
```
Integrate[((2 - 5*x)*x^(5/2))/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(27376 + 56080*x + 17076*x^2 - 3960*x^3 + 3618*x^4 - 4050*x^5 + (13688*I)*
Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/
3]/Sqrt[x]], 3/2] - (7508*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2
)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(2835*Sqrt[x]*Sqrt[2 + 5*x
 + 3*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1236, 1236, 27, 1236, 25, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{3x^2+5x+2}}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{21}\int \frac{x^{3/2}(96x+25)}{\sqrt{3x^2+5x+2}}dx - \frac{10}{21}x^{5/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{21}\left(\frac{2}{15}\int -\frac{3\sqrt{x}(515x+192)}{2\sqrt{3x^2+5x+2}}dx+\frac{64}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)-\frac{10}{21}x^{5/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 27$$

$$\frac{2}{21}\left(\frac{64}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{1}{5}\int \frac{\sqrt{x}(515x+192)}{\sqrt{3x^2+5x+2}}dx\right)-\frac{10}{21}x^{5/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 1236$$

$$\frac{2}{21}\left(\frac{1}{5}\left(-\frac{2}{9}\int -\frac{1711x+515}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{1030}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)+\frac{64}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)-\frac{10}{21}x^{5/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 25$$

$$\frac{2}{21}\left(\frac{1}{5}\left(\frac{2}{9}\int \frac{1711x+515}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{1030}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)+\frac{64}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)-\frac{10}{21}x^{5/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 1240$$

$$\frac{2}{21}\left(\frac{1}{5}\left(\frac{4}{9}\int \frac{1711x+515}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{1030}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)+\frac{64}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)-\frac{10}{21}x^{5/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 1503$$

$$\frac{2}{21}\left(\frac{1}{5}\left(\frac{4}{9}\left(515\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+1711\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{1030}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)+\frac{64}{5}\sqrt{3x^2 +\right. \quad \frac{10}{21}x^{5/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 1413$$

$$\frac{2}{21}\left(\frac{1}{5}\left(\frac{4}{9}\left(1711\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{515(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{1030}{9}\sqrt{x}\sqrt{3x^2 +\right.\right. \quad \frac{10}{21}x^{5/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \ 1456$$

$$\frac{2}{21}\left(\frac{1}{5}\left(\frac{4}{9}\left(\frac{515(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+1711\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E(\text{ar}}{3\sqrt{3x^2+5x}}\right.\right.\right.\right. \quad \frac{10}{21}x^{5/2}\sqrt{3x^2+5x+2}$$

input
```
Int[((2 - 5*x)*x^(5/2))/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-10*x^(5/2)*Sqrt[2 + 5*x + 3*x^2])/21 + (2*((64*x^(3/2)*Sqrt[2 + 5*x + 3*
x^2])/5 + ((-1030*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])/9 + (4*(1711*((Sqrt[x]*(2
 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 +
x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (515*(1
 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*S
qrt[2 + 5*x + 3*x^2])))/9)/5))/21
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.62

method	result
default	$-\frac{2\left(6075x^5+7176\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-3422\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-5427x \cdots\right.}{8505\sqrt{x}\,\sqrt{3x^2+5x+2}}$
risch	$-\frac{2(225x^2-576x+1030)\sqrt{x}\,\sqrt{3x^2+5x+2}}{945}-\frac{\left(-\frac{412\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{567\sqrt{3x^3+5x^2+2x}}-\frac{6844\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\operatorname{Elliptic}\cdots\right.}{2835\cdots}\cdots\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}\cdots}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{10x^2\sqrt{3x^3+5x^2+2x}}{21}+\frac{128x\sqrt{3x^3+5x^2+2x}}{105}-\frac{412\sqrt{3x^3+5x^2+2x}}{189}+\frac{412\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{567\sqrt{3x^3+5x^2+2x}}+\cdots\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/8505/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(6075*x^5+7176*(6*x+4)^(1/2)*(3+3*x)^(
1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-3422*(6*x+4
)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(
1/2))-5427*x^4+5940*x^3+35982*x^2+18540*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.27

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx = -\frac{2}{945}\left(225\,x^2-576\,x+1030\right)\sqrt{3\,x^2+5\,x+2}\sqrt{x} - \frac{896}{729}\sqrt{3}\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) - \frac{13688}{2835}\sqrt{3}\mathrm{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2/945*(225*x^2 - 576*x + 1030)*sqrt(3*x^2 + 5*x + 2)*sqrt(x) - 896/729*sq
rt(3)*weierstrassPInverse(28/27, 80/729, x + 5/9) - 13688/2835*sqrt(3)*wei
erstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x + 5/9))
```

Sympy [F]

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx = -\int \left(-\frac{2x^{\frac{5}{2}}}{\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5x^{\frac{7}{2}}}{\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)*x**(5/2)/(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2*x**(5/2)/sqrt(3*x**2 + 5*x + 2), x) - Integral(5*x**(7/2)/sqr
t(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{5}{2}}}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(5/2)/sqrt(3*x^2 + 5*x + 2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{5}{2}}}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(5/2)/sqrt(3*x^2 + 5*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx = -\int \frac{x^{5/2}\,(5\,x-2)}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
int(-(x^(5/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
-int((x^(5/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{5/2}}{\sqrt{2+5x+3x^2}}\,dx = -\frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2}{21} + \frac{128\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{105} - \frac{128\sqrt{x}\,\sqrt{3x^2+5x+2}}{175} - \frac{3422\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{525} + \frac{128\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{175}$$

input

```
int((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(1/2),x)
```

output

```
(2*( - 125*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 + 320*sqrt(x)*sqrt(3*x**2 +
 5*x + 2)*x - 192*sqrt(x)*sqrt(3*x**2 + 5*x + 2) - 1711*int((sqrt(x)*sqrt(
3*x**2 + 5*x + 2)*x)/(3*x**2 + 5*x + 2),x) + 192*int((sqrt(x)*sqrt(3*x**2
+ 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x)))/525
```

3.203 $\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx = -\frac{412\sqrt{x}(2+3x)}{81\sqrt{2+5x+3x^2}}+\frac{52}{27}\sqrt{x}\sqrt{2+5x+3x^2} - \frac{2}{3}x^{3/2}\sqrt{2+5x+3x^2}+\frac{412\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{81\sqrt{1+x}\sqrt{2+3x}} - \frac{52\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{27\sqrt{2+5x+3x^2}}$$

output
```
-412/81*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+52/27*x^(1/2)*(3*x^2+5*x+2)^(1
/2)-2/3*x^(3/2)*(3*x^2+5*x+2)^(1/2)+412/81*2^(1/2)*(3*x^2+5*x+2)^(1/2)*Ell
ipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-52/27*
2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^
(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.91

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx = \frac{-2(412+874x+282x^2-99x^3+81x^4)-412i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\Big(i\text{arcsinh}}{81\sqrt{x}\sqrt{2+5x}}$$

input
```
Integrate[((2 - 5*x)*x^(3/2))/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-2*(412 + 874*x + 282*x^2 - 99*x^3 + 81*x^4) - (412*I)*Sqrt[2]*Sqrt[1 + x
^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2]
+ (256*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSi
nh[Sqrt[2/3]/Sqrt[x]], 3/2])/(81*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1236, 27, 1236, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{15}\int \frac{5\sqrt{x}(13x+3)}{\sqrt{3x^2+5x+2}}dx-\frac{2}{3}x^{3/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{\sqrt{x}(13x+3)}{\sqrt{3x^2+5x+2}}dx-\frac{2}{3}x^{3/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{3}\left(\frac{2}{9}\int -\frac{103x+26}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{26}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)-\frac{2}{3}x^{3/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\left(\frac{26}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{1}{9}\int \frac{103x+26}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{2}{3}x^{3/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1240}$$

$$\frac{2}{3}\left(\frac{26}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\int \frac{103x+26}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2}{3}x^{3/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1503}$$

$$\frac{2}{3}\left(\frac{26}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(26\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+103\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{2}{3}x^{3/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1413}$$

$$\frac{2}{3}\left(\frac{26}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(103\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{13\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)\right) \\ \frac{2}{3}x^{3/2}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1456}$$

$$\frac{2}{3}\left(\frac{26}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(\frac{13\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+103\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}}{}\right.\right.\right. \\ \frac{2}{3}x^{3/2}\sqrt{3x^2+5x+2}$$

input
```
Int[((2 - 5*x)*x^(3/2))/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-2*x^(3/2)*Sqrt[2 + 5*x + 3*x^2])/3 + (2*((26*Sqrt[x]*Sqrt[2 + 5*x + 3*x^
2])/9 - (2*(103*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*
(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[
2 + 5*x + 3*x^2])) + (13*Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF
[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/9))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

method	result
default	$\frac{\frac{154\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{81}-\frac{206\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{243}-2x^4+\frac{22x^3}{9}+\frac{224x^2}{27}+\frac{104x}{27}}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
risch	$-\frac{2(-26+9x)\sqrt{x}\,\sqrt{3x^2+5x+2}}{27}-\frac{\Big(\frac{52\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{81\sqrt{3x^3+5x^2+2x}}+\frac{206\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\Big(\frac{\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3}}{81\sqrt{3x^3+5x^2+2x}}}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\,\Big(-\frac{2x\sqrt{3x^3+5x^2+2x}}{3}+\frac{52\sqrt{3x^3+5x^2+2x}}{27}-\frac{52\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{81\sqrt{3x^3+5x^2+2x}}-\frac{206\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\Big(\mathrm{El}}{}}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/243/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(231*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)
*(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-103*(6*x+4)^(1/2)*(3+3*
x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))-243*x^4
+297*x^3+1008*x^2+468*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.28

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx = -\frac{2}{27}\sqrt{3x^2+5x+2}(9x-26)\sqrt{x} + \frac{1124}{729}\sqrt{3}\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) + \frac{412}{81}\sqrt{3}\mathrm{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2/27*sqrt(3*x^2 + 5*x + 2)*(9*x - 26)*sqrt(x) + 1124/729*sqrt(3)*weierstr
assPInverse(28/27, 80/729, x + 5/9) + 412/81*sqrt(3)*weierstrassZeta(28/27
, 80/729, weierstrassPInverse(28/27, 80/729, x + 5/9))
```

Sympy [F]

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx = -\int \left(-\frac{2x^{\frac{3}{2}}}{\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5x^{\frac{5}{2}}}{\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)*x**(3/2)/(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2*x**(3/2)/sqrt(3*x**2 + 5*x + 2), x) - Integral(5*x**(5/2)/sqr
t(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{3}{2}}}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(3/2)/sqrt(3*x^2 + 5*x + 2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{3}{2}}}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(3/2)/sqrt(3*x^2 + 5*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx = -\int \frac{x^{3/2}\,(5\,x-2)}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
int(-(x^(3/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
-int((x^(3/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{3/2}}{\sqrt{2+5x+3x^2}}\,dx = -\frac{2\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3}+\frac{2\sqrt{x}\,\sqrt{3x^2+5x+2}}{5} + \frac{103\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{15} - \frac{2\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{5}$$

input
```
int((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(1/2),x)
```

output
```
( - 10*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x + 6*sqrt(x)*sqrt(3*x**2 + 5*x + 2)
 + 103*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(3*x**2 + 5*x + 2),x) - 6*in
t((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x))/15
```

3.204 $\int \frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx = \frac{136\sqrt{x}(2+3x)}{27\sqrt{2+5x+3x^2}}-\frac{10}{9}\sqrt{x}\sqrt{2+5x+3x^2} - \frac{136\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{27\sqrt{1+x}\sqrt{2+3x}} + \frac{10\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{9\sqrt{2+5x+3x^2}}$$

output
```
136/27*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-10/9*x^(1/2)*(3*x^2+5*x+2)^(1/2
)-136/27*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2
^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+10/9*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*I
nverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00

$$\int \frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx = \frac{272+620x+258x^2-90x^3+136i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)-106i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3}}{27\sqrt{x}\sqrt{2+5x+3x^2}}$$

input
```
Integrate[((2 - 5*x)*Sqrt[x])/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(272 + 620*x + 258*x^2 - 90*x^3 + (136*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3
+ 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (106*I)*Sqrt
[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/S
qrt[x]], 3/2])/(27*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1236, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\sqrt{x}}{\sqrt{3x^2+5x+2}}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2}{9}\int \frac{34x+5}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{10}{9}\sqrt{x}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1240}$$

$$\frac{4}{9}\int \frac{34x+5}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{10}{9}\sqrt{x}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1503}$$

$$\frac{4}{9}\left(5\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+34\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{10}{9}\sqrt{x}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1413}$$

$$\frac{4}{9}\left(34\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{5(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{10}{9}\sqrt{x}\sqrt{3x^2+5x+2}$$

$$\Big\downarrow \text{1456}$$

$$\frac{4}{9}\left(\frac{5(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+34\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right)|-\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$
$$\frac{10}{9}\sqrt{x}\sqrt{3x^2+5x+2}$$

input
```
Int[((2 - 5*x)*Sqrt[x])/Sqrt[2 + 5*x + 3*x^2],x]
```

output
```
(-10*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])/9 + (4*(34*((Sqrt[x]*(2 + 3*x))/(3*Sqr
t[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[A
rcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (5*(1 + x)*Sqrt[(2 + 3
*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x
^2])))/9
```

Defintions of rubi rules used

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2\left(87\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-34\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)+135x^3+225x^2+90x\right)}{81\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{10\sqrt{3x^3+5x^2+2x}}{9}+\frac{10\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{27\sqrt{3x^3+5x^2+2x}}+\frac{68\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\frac{\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3}\right.}{27\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
risch	$-\frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}}{9}-\frac{\left(-\frac{10\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{27\sqrt{3x^3+5x^2+2x}}-\frac{68\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\frac{\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3}-\mathrm{Elliptic}\right.}{27\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/81/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(87*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*
(-x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-34*(6*x+4)^(1/2)*(3+3*x)
^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+135*x^3+2
25*x^2+90*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.29

$$\int \frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx$$
$$= -\frac{500}{243}\,\sqrt{3}\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)$$
$$-\frac{136}{27}\,\sqrt{3}\mathrm{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$
$$-\frac{10}{9}\,\sqrt{3\,x^2+5\,x+2}\sqrt{x}$$

input
```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-500/243*sqrt(3)*weierstrassPInverse(28/27, 80/729, x + 5/9) - 136/27*sqrt
(3)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x +
5/9)) - 10/9*sqrt(3*x^2 + 5*x + 2)*sqrt(x)
```

Sympy [F]

$$\int \frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx = -\int \left(-\frac{2\sqrt{x}}{\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5x^{\frac{3}{2}}}{\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)*x**(1/2)/(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2*sqrt(x)/sqrt(3*x**2 + 5*x + 2), x) - Integral(5*x**(3/2)/sqrt
(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int \frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx = \int -\frac{(5\,x-2)\sqrt{x}}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*sqrt(x)/sqrt(3*x^2 + 5*x + 2), x)
```

Giac [F]

$$\int \frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx = \int -\frac{(5\,x-2)\sqrt{x}}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*sqrt(x)/sqrt(3*x^2 + 5*x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx = -\int \frac{\sqrt{x}\,(5\,x-2)}{\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
int(-(x^(1/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(1/2),x)
```

output
```
-int((x^(1/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\sqrt{x}}{\sqrt{2+5x+3x^2}}\,dx = \frac{2\sqrt{x}\,\sqrt{3x^2+5x+2}}{5} - \frac{34\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}\,x}{3x^2+5x+2}dx\right)}{5} - \frac{2\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)}{5}$$

input
```
int((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(1/2),x)
```

output
```
(2*(sqrt(x)*sqrt(3*x**2 + 5*x + 2) - 17*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2
)*x)/(3*x**2 + 5*x + 2),x) - int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3
+ 5*x**2 + 2*x),x)))/5
```

3.205 $\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 125

$$\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}} \, dx = -\frac{10\sqrt{x}(2+3x)}{3\sqrt{2+5x+3x^2}} + \frac{10\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x}\sqrt{2+3x}} + \frac{2\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}$$

output

```
-10/3*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+10/3*2^(1/2)*(3*x^2+5*x+2)^(1/2)
*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+2*
2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^
(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.20

$$\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}}\,dx = -\frac{2x^{3/2}\left(5\left(3+\frac{2}{x^2}+\frac{5}{x}\right)+\frac{5i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)}{\sqrt{x}}-\frac{8i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right),\frac{3}{2}\right)}{\sqrt{x}}\right)}{3\sqrt{2+5x+3x^2}}$$

input
```
Integrate[(2 - 5*x)/(Sqrt[x]*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
(-2*x^(3/2)*(5*(3 + 2/x^2 + 5/x) + ((5*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3
+ 2/x]*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/Sqrt[x] - ((8*I)*Sqrt
[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]],
 3/2])/Sqrt[x]))/(3*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{\sqrt{x}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{ 1240}$$

$$2\int \frac{2-5x}{\sqrt{3x^2+5x+2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1503}$$

$$2\left(2\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}-5\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1413}$$

$$2\left(\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}-5\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1456}$$

$$2\left(\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}-5\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right)|-\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$

input
```
Int[(2 - 5*x)/(Sqrt[x]*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
2*(-5*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sq
rt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x +
3*x^2])) + (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[
x]], -1/2])/Sqrt[2 + 5*x + 3*x^2])
```

Defintions of rubi rules used

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

method	result
default	$\frac{\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\left(21\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-5\,\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\right)}{9\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(\frac{2\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3\sqrt{3x^3+5x^2+2x}}-\frac{5\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\frac{\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3}-\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{\right.\right.}}{3\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(
1/2)*(21*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-5*EllipticE(1/2*(6*x+4)^(1
/2),I*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.21

$$\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}}\,dx = \frac{86}{27}\sqrt{3}\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right) + \frac{10}{3}\sqrt{3}\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right)$$

input
```
integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
86/27*sqrt(3)*weierstrassPInverse(28/27, 80/729, x + 5/9) + 10/3*sqrt(3)*w
eierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x + 5/9))
```

Sympy [F]

$$\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}}\,dx = -\int \left(-\frac{2}{\sqrt{x}\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5\sqrt{x}}{\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)/x**(1/2)/(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2/(sqrt(x)*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5*sqrt(x)/sqr
t(3*x**2 + 5*x + 2), x)
```

Maxima [F]

$$\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{\sqrt{3\,x^2+5\,x+2}\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/(sqrt(3*x^2 + 5*x + 2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{\sqrt{3\,x^2+5\,x+2}\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/(sqrt(3*x^2 + 5*x + 2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}}\,dx = -\int \frac{5\,x-2}{\sqrt{x}\,\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(1/2)*(5*x + 3*x^2 + 2)^(1/2)),x)
```

output
```
-int((5*x - 2)/(x^(1/2)*(5*x + 3*x^2 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{\sqrt{x}\sqrt{2+5x+3x^2}}\,dx = 2\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right) - 5\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^2+5x+2}dx\right)$$

input
```
int((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(1/2),x)
```

output
```
2*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**3 + 5*x**2 + 2*x),x) - 5*int(
(sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(3*x**2 + 5*x + 2),x)
```

3.206 $\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx=\frac{2\sqrt{x}(2+3x)}{\sqrt{2+5x+3x^2}}-\frac{2\sqrt{2+5x+3x^2}}{\sqrt{x}}-\frac{2\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{\sqrt{1+x}\sqrt{2+3x}}-\frac{5\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}$$

output

```
2*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-2*(3*x^2+5*x+2)^(1/2)/x^(1/2)-2*2^(1
/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)
^(1/2)/(2+3*x)^(1/2)-5*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(a
rctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.63

$$\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx = \frac{i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}x\left(2E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)-7\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right),\frac{3}{2}\right.\right.}{\sqrt{2+5x+3x^2}}$$

input
```
Integrate[(2 - 5*x)/(x^(3/2)*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
(I*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x*(2*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]]
, 3/2] - 7*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2]))/Sqrt[2 + 5*x + 3
*x^2]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1237, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{x^{3/2}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1237}$$

$$-\int \frac{5-3x}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2\sqrt{3x^2+5x+2}}{\sqrt{x}}$$

$$\Big\downarrow \text{1240}$$

$$-2\int \frac{5-3x}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{2\sqrt{3x^2+5x+2}}{\sqrt{x}}$$

$$\Big\downarrow \text{1503}$$

$$-2\left(5\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}-3\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2\sqrt{3x^2+5x+2}}{\sqrt{x}}$$

$$\Big\downarrow \text{1413}$$

$$-2\left(\frac{5(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}-3\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2\sqrt{3x^2+5x+2}}{\sqrt{x}}$$

$$\Big\downarrow \text{1456}$$

$$-2\left(\frac{5(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}-3\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right)|-\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$

$$\frac{2\sqrt{3x^2+5x+2}}{\sqrt{x}}$$

input
```
Int[(2 - 5*x)/(x^(3/2)*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
(-2*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x] - 2*(-3*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2
+ 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTa
n[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (5*(1 + x)*Sqrt[(2 + 3*x)/
(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])
)
```

Defintions of rubi rules used

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

method	result
default	$-\frac{8\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)+18x^2+30x+12}{3\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{2\left(3x^2+5x+2\right)}{\sqrt{\left(3x^2+5x+2\right)x}}-\frac{5\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3\sqrt{3x^3+5x^2+2x}}+\frac{\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\frac{\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3}-\mathrm{E}\right.}{\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(8*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+
4)^(1/2),I*2^(1/2))-(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ellipti
cE(1/2*(6*x+4)^(1/2),I*2^(1/2))+18*x^2+30*x+12)/x^(1/2)/(3*x^2+5*x+2)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.35

$$\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx = -\frac{2\left(20\sqrt{3}x\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)+9\sqrt{3}x\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{8}{72}\right.\right.}{9\,x}$$

input
```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*(20*sqrt(3)*x*weierstrassPInverse(28/27, 80/729, x + 5/9) + 9*sqrt(3)
*x*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x + 5
/9)) + 9*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/x
```

Sympy [F]

$$\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx = -\int\left(-\frac{2}{x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}\right)dx-\int\frac{5}{\sqrt{x}\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)/x**(3/2)/(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2/(x**(3/2)*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5/(sqrt(x)*s
qrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{\sqrt{3\,x^2+5\,x+2}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/(sqrt(3*x^2 + 5*x + 2)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{\sqrt{3\,x^2+5\,x+2}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/(sqrt(3*x^2 + 5*x + 2)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{x^{3/2}\,\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(3/2)*(5*x + 3*x^2 + 2)^(1/2)),x)
```

output
```
int(-(5*x - 2)/(x^(3/2)*(5*x + 3*x^2 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{x^{3/2}\sqrt{2+5x+3x^2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{3x^2+5x+2}-5\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^3+5x^2+2x}dx\right)x+3\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^2+5x+2}dx\right)x}{x}$$

input
```
int((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(1/2),x)
```

output
```
( - 2*sqrt(x)*sqrt(3*x**2 + 5*x + 2) - 5*int((sqrt(x)*sqrt(3*x**2 + 5*x +
2))/(3*x**3 + 5*x**2 + 2*x),x)*x + 3*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/
(3*x**2 + 5*x + 2),x)*x)/x
```

3.207 $\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 171

$$\begin{aligned}\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx &= -\frac{25\sqrt{x}(2+3x)}{3\sqrt{2+5x+3x^2}}-\frac{2\sqrt{2+5x+3x^2}}{3x^{3/2}}\\ &+\frac{25\sqrt{2+5x+3x^2}}{3\sqrt{x}}+\frac{25\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x}\sqrt{2+3x}}\\ &-\frac{\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
-25/3*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-2/3*(3*x^2+5*x+2)^(1/2)/x^(3/2)+
25/3*(3*x^2+5*x+2)^(1/2)/x^(1/2)+25/3*2^(1/2)*(3*x^2+5*x+2)^(1/2)*Elliptic
E(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-2^(1/2)*(1+
x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x
^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx = \frac{-2(2+5x+3x^2)-25i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{5/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)+22i\sqrt{}}{3x^{3/2}\sqrt{2+5x+3x^2}}$$

input
```
Integrate[(2 - 5*x)/(x^(5/2)*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
(-2*(2 + 5*x + 3*x^2) - (25*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(5
/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (22*I)*Sqrt[2]*Sqrt[1 +
 x^(-1)]*Sqrt[3 + 2/x]*x^(5/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2
])/(3*x^(3/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1237, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{x^{5/2}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{ 1237}$$

$$-\frac{1}{3}\int \frac{3x+25}{x^{3/2}\sqrt{3x^2+5x+2}}dx-\frac{2\sqrt{3x^2+5x+2}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{1}{3}\left(\int -\frac{3(25x+2)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{25\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)-\frac{2\sqrt{3x^2+5x+2}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{25\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{3}{2}\int\frac{25x+2}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{2\sqrt{3x^2+5x+2}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 1240}$$

$$\frac{1}{3}\left(\frac{25\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\int\frac{25x+2}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2\sqrt{3x^2+5x+2}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{3}\left(\frac{25\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(2\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+25\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{2\sqrt{3x^2+5x+2}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 1413}$$

$$\frac{1}{3}\left(\frac{25\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(25\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)\right)-\frac{2\sqrt{3x^2+5x+2}}{3x^{3/2}}$$

$$\Big\downarrow \text{ 1456}$$

$$\frac{1}{3}\left(\frac{25\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+25\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\ldots}{\ldots}\right.\right.\right.$$
$$\frac{2\sqrt{3x^2+5x+2}}{3x^{3/2}}$$

input
```
Int[(2 - 5*x)/(x^(5/2)*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
(-2*Sqrt[2 + 5*x + 3*x^2])/(3*x^(3/2)) + ((25*Sqrt[2 + 5*x + 3*x^2])/Sqrt[
x] - 3*(25*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 +
x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5
*x + 3*x^2])) + (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[
Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

method	result
default	$\frac{69\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x-25\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x+450x^3+714x^2+240x-}{18\sqrt{3x^2+5x+2}\,x^{\frac{3}{2}}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{2\sqrt{3x^3+5x^2+2x}}{3x^2}+\frac{25x^2+\frac{125}{3}x+\frac{50}{3}}{\sqrt{(3x^2+5x+2)x}}-\frac{\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3\sqrt{3x^3+5x^2+2x}}-\frac{25\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\frac{\text{Elliptic}}{}\right.}{6\sqrt{}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(69*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x
+4)^(1/2),I*2^(1/2))*x-25*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*E
llipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*x+450*x^3+714*x^2+240*x-24)/(3*x^2+5
*x+2)^(1/2)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.35

$$\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx = \frac{107\sqrt{3}x^2\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)+225\sqrt{3}x^2\text{weierstrassZeta}\left(\frac{28}{27},\frac{8}{7}\right.}{27\,x^2}$$

input
```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(107*sqrt(3)*x^2*weierstrassPInverse(28/27, 80/729, x + 5/9) + 225*sq
rt(3)*x^2*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729
, x + 5/9)) + 9*sqrt(3*x^2 + 5*x + 2)*(25*x - 2)*sqrt(x))/x^2
```

Sympy [F]

$$\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx = -\int \left(-\frac{2}{x^{\frac{5}{2}}\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5}{x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)/x**(5/2)/(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2/(x**(5/2)*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5/(x**(3/2)*
sqrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{\sqrt{3\,x^2+5\,x+2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/(sqrt(3*x^2 + 5*x + 2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{\sqrt{3\,x^2+5\,x+2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/(sqrt(3*x^2 + 5*x + 2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx=\int -\frac{5\,x-2}{x^{5/2}\,\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(5/2)*(5*x + 3*x^2 + 2)^(1/2)),x)
```

output
```
int(-(5*x - 2)/(x^(5/2)*(5*x + 3*x^2 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{x^{5/2}\sqrt{2+5x+3x^2}}\,dx=\frac{10\sqrt{3x^2+5x+2}+4\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^5+5x^4+2x^3}dx\right)-15\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{3x^2+5x+2}dx\right)}{2\sqrt{x}}$$

input
```
int((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(1/2),x)
```

output
```
(10*sqrt(3*x**2 + 5*x + 2) + 4*sqrt(x)*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)
)/(3*x**5 + 5*x**4 + 2*x**3),x) - 15*sqrt(x)*int((sqrt(x)*sqrt(3*x**2 + 5*
x + 2))/(3*x**2 + 5*x + 2),x))/(2*sqrt(x))
```

3.208 $\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 192

$$\begin{aligned}\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx =& \frac{66\sqrt{x}(2+3x)}{5\sqrt{2+5x+3x^2}} \\ &- \frac{2\sqrt{2+5x+3x^2}}{5x^{5/2}} + \frac{3\sqrt{2+5x+3x^2}}{x^{3/2}} - \frac{66\sqrt{2+5x+3x^2}}{5\sqrt{x}} \\ &- \frac{66\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\left|-\frac{1}{2}\right.\right)}{5\sqrt{1+x}\sqrt{2+3x}} \\ &+ \frac{9\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x+3x^2}}\end{aligned}$$

output

```
66/5*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-2/5*(3*x^2+5*x+2)^(1/2)/x^(5/2)+3
*(3*x^2+5*x+2)^(1/2)/x^(3/2)-66/5*(3*x^2+5*x+2)^(1/2)/x^(1/2)-66/5*2^(1/2)
*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1
/2)/(2+3*x)^(1/2)+9/2*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(ar
ctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.78

$$\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx = \frac{-8+40x+138x^2+90x^3+132i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{7/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right.}{10x^{5/2}\sqrt{2+5x+3}}$$

input
```
Integrate[(2 - 5*x)/(x^(7/2)*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
(-8 + 40*x + 138*x^2 + 90*x^3 + (132*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 +
2/x]*x^(7/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (87*I)*Sqrt[2]
*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(7/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt
[x]], 3/2])/(10*x^(5/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1237, 27, 1237, 27, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{x^{7/2}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{1}{5}\int \frac{9(x+5)}{x^{5/2}\sqrt{3x^2+5x+2}}dx-\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{9}{5}\int \frac{x+5}{x^{5/2}\sqrt{3x^2+5x+2}}dx-\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{9}{5}\left(-\frac{1}{3}\int \frac{15x+44}{2x^{3/2}\sqrt{3x^2+5x+2}}dx-\frac{5\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{9}{5}\left(-\frac{1}{6}\int \frac{15x+44}{x^{3/2}\sqrt{3x^2+5x+2}}dx-\frac{5\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{9}{5}\left(\frac{1}{6}\left(\int -\frac{3(22x+5)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{44\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)-\frac{5\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{9}{5}\left(\frac{1}{6}\left(\frac{44\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\int \frac{22x+5}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{5\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{1240}$$

$$-\frac{9}{5}\left(\frac{1}{6}\left(\frac{44\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\int \frac{22x+5}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{5\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{1503}$$

$$-\frac{9}{5}\left(\frac{1}{6}\left(\frac{44\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(5\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+22\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{5\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{1413}$$

$$-\frac{9}{5}\left(\frac{1}{6}\left(\frac{44\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(22\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{5(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)\right)\right.-\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$$\Big\downarrow \text{1456}$$

$$-\frac{9}{5}\left(\frac{1}{6}\left(\frac{44\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(\frac{5(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+22\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x\right.\right.\right.\right.$$

$$\frac{2\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

input
```
Int[(2 - 5*x)/(x^(7/2)*Sqrt[2 + 5*x + 3*x^2]),x]
```

output
```
(-2*Sqrt[2 + 5*x + 3*x^2])/(5*x^(5/2)) - (9*((-5*Sqrt[2 + 5*x + 3*x^2])/(3
*x^(3/2)) + ((44*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x] - 6*(22*((Sqrt[x]*(2 + 3*x
))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*El
lipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (5*(1 + x)*Sq
rt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 +
5*x + 3*x^2])))/6))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

method	result
default	$-\frac{51\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^{2}-22\,\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^{2}+396x^{4}+570x^{3}+1}{10\sqrt{3x^{2}+5x+2}\,x^{\frac{5}{2}}}$
elliptic	$\frac{\sqrt{\left(3x^{2}+5x+2\right)x}\left(-\frac{2\sqrt{3x^{3}+5x^{2}+2x}}{5x^{3}}+\frac{3\sqrt{3x^{3}+5x^{2}+2x}}{x^{2}}-\frac{66\left(3x^{2}+5x+2\right)}{5\sqrt{\left(3x^{2}+5x+2\right)x}}+\frac{3\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{2\sqrt{3x^{3}+5x^{2}+2x}}+\frac{33\sqrt{6x+4}\,\sqrt{}}{}\right.}{\sqrt{x}\,\sqrt{3x^{2}+5x+2}}$

input
```
int((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(51*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))*(6*x+4)^(1/2)*(3+3*x)^(1/
2)*6^(1/2)*(-x)^(1/2)*x^2-22*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*(6*x+4
)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*x^2+396*x^4+570*x^3+126*x^2-40*x+
8)/(3*x^2+5*x+2)^(1/2)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.33

$$\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx = -\frac{65\sqrt{3}x^3\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)+198\sqrt{3}x^3\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\right.\right.}{15\,x^3}$$

input
```
integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(65*sqrt(3)*x^3*weierstrassPInverse(28/27, 80/729, x + 5/9) + 198*sq
rt(3)*x^3*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729
, x + 5/9)) + 3*(66*x^2 - 15*x + 2)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/x^3
```

Sympy [F]

$$\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx = -\int \left(-\frac{2}{x^{\frac{7}{2}}\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5}{x^{\frac{5}{2}}\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)/x**(7/2)/(3*x**2+5*x+2)**(1/2),x)
```

output
```
-Integral(-2/(x**(7/2)*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5/(x**(5/2)*
sqrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{\sqrt{3\,x^2+5\,x+2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/(sqrt(3*x^2 + 5*x + 2)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{\sqrt{3\,x^2+5\,x+2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/(sqrt(3*x^2 + 5*x + 2)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx = \int -\frac{5\,x-2}{x^{7/2}\,\sqrt{3\,x^2+5\,x+2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(7/2)*(5*x + 3*x^2 + 2)^(1/2)),x)
```

output
```
int(-(5*x - 2)/(x^(7/2)*(5*x + 3*x^2 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{x^{7/2}\sqrt{2+5x+3x^2}}\,dx = \frac{18\sqrt{3x^2+5x+2}\,x^2 - 4\sqrt{3x^2+5x+2} - 40\sqrt{x}\left(\int \frac{\sqrt{3x^2+5x+2}}{3\sqrt{x}\,x^4+5\sqrt{x}\,x^3+2\sqrt{x}\,x^2}dx\right)x^2}{10\sqrt{x}\,x^2}$$

input
```
int((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(1/2),x)
```

output
```
(18*sqrt(3*x**2 + 5*x + 2)*x**2 - 4*sqrt(3*x**2 + 5*x + 2) - 40*sqrt(x)*in
t(sqrt(3*x**2 + 5*x + 2)/(3*sqrt(x)*x**4 + 5*sqrt(x)*x**3 + 2*sqrt(x)*x**2
),x)*x**2 - 27*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x)/(3*sqrt(x)*x**2 + 5*
sqrt(x)*x + 2*sqrt(x)),x)*x**2 - 50*sqrt(x)*int((sqrt(x)*sqrt(3*x**2 + 5*x
 + 2))/(3*x**5 + 5*x**4 + 2*x**3),x)*x**2)/(10*sqrt(x)*x**2)
```

3.209 $\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 195

$$\begin{aligned}\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}}\,dx &= -\frac{24\sqrt{x}(2+3x)}{\sqrt{2+5x+3x^2}}+\frac{2\sqrt{x}(506+695x)}{27\sqrt{2+5x+3x^2}} \\ &+\frac{34}{27}\sqrt{x}\sqrt{2+5x+3x^2}-\frac{2}{9}x^{3/2}\sqrt{2+5x+3x^2} \\ &+\frac{24\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{\sqrt{1+x}\sqrt{2+3x}} \\ &-\frac{20\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
-24*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+2/27*x^(1/2)*(506+695*x)/(3*x^2+5*
x+2)^(1/2)+34/27*x^(1/2)*(3*x^2+5*x+2)^(1/2)-2/9*x^(3/2)*(3*x^2+5*x+2)^(1/
2)+24*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1
/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-20*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*Invers
eJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{-2(72+120x+22x^2-4x^3+x^4)-72i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\right.}{3\sqrt{x}\sqrt{2+5x+\ldots}}$$

input
```
Integrate[((2 - 5*x)*x^(7/2))/(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(-2*(72 + 120*x + 22*x^2 - 4*x^3 + x^4) - (72*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*
Sqrt[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (12*I
)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[
2/3]/Sqrt[x]], 3/2])/(3*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1233, 27, 1236, 27, 1236, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{7/2}}{(3x^2+5x+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2}{3}\int -\frac{5x^{3/2}(48x+37)}{\sqrt{3x^2+5x+2}}dx+\frac{2(95x+74)x^{5/2}}{3\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^{5/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{10}{3}\int \frac{x^{3/2}(48x+37)}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2x^{5/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{10}{3}\left(\frac{2}{15}\int -\frac{9\sqrt{x}(45x+32)}{2\sqrt{3x^2+5x+2}}dx+\frac{32}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)$$

$\downarrow$ 27

$$\frac{2x^{5/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{10}{3}\left(\frac{32}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{3}{5}\int \frac{\sqrt{x}(45x+32)}{\sqrt{3x^2+5x+2}}dx\right)$$

$\downarrow$ 1236

$$\frac{2x^{5/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{10}{3}\left(\frac{32}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{3}{5}\left(\frac{2}{9}\int -\frac{9(9x+5)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+10\sqrt{x}\sqrt{3x^2+5x+2}\right)\right)$$

$\downarrow$ 27

$$\frac{2x^{5/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{10}{3}\left(\frac{32}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{3}{5}\left(10\sqrt{x}\sqrt{3x^2+5x+2}-2\int \frac{9x+5}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)\right)$$

$\downarrow$ 1240

$$\frac{2x^{5/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{10}{3}\left(\frac{32}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{3}{5}\left(10\sqrt{x}\sqrt{3x^2+5x+2}-4\int \frac{9x+5}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)$$

$\downarrow$ 1503

$$\frac{2x^{5/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{10}{3}\left(\frac{32}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{3}{5}\left(10\sqrt{x}\sqrt{3x^2+5x+2}-4\left(5\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+9\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right.\right.\right.$$

$\downarrow$ 1413

$$\frac{2x^{5/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{10}{3}\left(\frac{32}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{3}{5}\left(10\sqrt{x}\sqrt{3x^2+5x+2}-4\left(9\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{5(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{Elliptic}}{\sqrt{2}\sqrt{3x^2+}}\right.\right.\right.$$

$\downarrow$ 1456

$$\frac{2x^{5/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-$$

$$\frac{10}{3}\left(\frac{32}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{3}{5}\left(10\sqrt{x}\sqrt{3x^2+5x+2}-4\left(\frac{5(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+9\left(\right.\right.\right.\right.$$

input

```
Int[((2 - 5*x)*x^(7/2))/(2 + 5*x + 3*x^2)^(3/2),x]
```

output

```
(2*x^(5/2)*(74 + 95*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (10*((32*x^(3/2)*Sqrt[
2 + 5*x + 3*x^2])/5 - (3*(10*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2] - 4*(9*((Sqrt[x
]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(
1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (5*
(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x + 3*x^2]))))/5))/3
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1233

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

method	result
default	$\frac{\frac{16\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3}-4\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-\frac{2x^4}{3}+\frac{8x^3}{3}+\frac{172x^2}{3}+40x}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{2x\left(-\frac{506}{81}-\frac{695x}{81}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{2x\sqrt{3x^3+5x^2+2x}}{9}+\frac{34\sqrt{3x^3+5x^2+2x}}{27}-\frac{20\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3\sqrt{3x^3+5x^2+2x}}-\frac{12\sqrt{6}}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(8*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4
)^(1/2),I*2^(1/2))-6*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ellipt
icE(1/2*(6*x+4)^(1/2),I*2^(1/2))-x^4+4*x^3+86*x^2+60*x)/x^(1/2)/(3*x^2+5*x
+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.35

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{2\left(36\sqrt{3}(3x^2+5x+2)\text{weierstrassZeta}\left(\frac{28}{27},\frac{80}{729},\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\right.\right.\right.}{3\left(3x^2+5x+2\right)}$$

input
```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(36*sqrt(3)*(3*x^2 + 5*x + 2)*weierstrassZeta(28/27, 80/729, weierstra
ssPInverse(28/27, 80/729, x + 5/9)) - (x^3 - 4*x^2 - 86*x - 60)*sqrt(3*x^2
 + 5*x + 2)*sqrt(x))/(3*x^2 + 5*x + 2)
```

Sympy [F]

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}}\,dx = -\int \left(-\frac{2x^{\frac{7}{2}}}{3x^2\sqrt{3x^2+5x+2}+5x\sqrt{3x^2+5x+2}+2\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5x^{\frac{9}{2}}}{3x^2\sqrt{3x^2+5x+2}+5x\sqrt{3x^2+5x+2}+2\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)*x**(7/2)/(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(-2*x**(7/2)/(3*x**2*sqrt(3*x**2 + 5*x + 2) + 5*x*sqrt(3*x**2 + 5
*x + 2) + 2*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5*x**(9/2)/(3*x**2*sqrt
(3*x**2 + 5*x + 2) + 5*x*sqrt(3*x**2 + 5*x + 2) + 2*sqrt(3*x**2 + 5*x + 2)
), x)
```

Maxima [F]

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{7}{2}}}{(3\,x^2+5\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(7/2)/(3*x^2 + 5*x + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{7}{2}}}{(3\,x^2+5\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(7/2)/(3*x^2 + 5*x + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}}\,dx = -\int \frac{x^{7/2}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{3/2}}\,dx$$

input
```
int(-(x^(7/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(3/2),x)
```

output
```
-int((x^(7/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{-60\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3+42\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2+140\sqrt{x}\,\sqrt{3x^2+5x+2}\,-}{}$$

input
```
int((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(3/2),x)
```

output
```
( - 60*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 + 42*sqrt(x)*sqrt(3*x**2 + 5*x
+ 2)*x**2 + 140*sqrt(x)*sqrt(3*x**2 + 5*x + 2) - 420*int(sqrt(3*x**2 + 5*x
 + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x +
 4*sqrt(x)),x)*x**2 - 700*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*
sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x - 280*int(
sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2
 + 20*sqrt(x)*x + 4*sqrt(x)),x) + 2673*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)
*x**3)/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x)*x**2 + 4455*int((sqrt(x)
*sqrt(3*x**2 + 5*x + 2)*x**3)/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x)*x
 + 1782*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3)/(9*x**4 + 30*x**3 + 37*x
**2 + 20*x + 4),x))/(90*(3*x**2 + 5*x + 2))
```

3.210 $\int \frac{(2-5x)x^{5/2}}{\left(2+5x+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \frac{(2-5x)x^{5/2}}{\left(2+5x+3x^2\right)^{3/2}}\,dx = \frac{1804\sqrt{x}(2+3x)}{81\sqrt{2+5x+3x^2}} - \frac{2\sqrt{x}(190+253x)}{9\sqrt{2+5x+3x^2}} - \frac{10}{27}\sqrt{x}\sqrt{2+5x+3x^2} - \frac{1804\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{81\sqrt{1+x}\sqrt{2+3x}} + \frac{580\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{27\sqrt{2+5x+3x^2}}$$

output

```
1804/81*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-2/9*x^(1/2)*(190+253*x)/(3*x^2
+5*x+2)^(1/2)-10/27*x^(1/2)*(3*x^2+5*x+2)^(1/2)-1804/81*2^(1/2)*(3*x^2+5*x
+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)
^(1/2)+580/27*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(
1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.84

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{3608+5540x+708x^2-90x^3+1804i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)\right.}{81\sqrt{x}\sqrt{2+5x+}}$$

input
```
Integrate[((2 - 5*x)*x^(5/2))/(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(3608 + 5540*x + 708*x^2 - 90*x^3 + (1804*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt
[3 + 2/x]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (64*I)*Sq
rt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]
/Sqrt[x]], 3/2])/(81*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1233, 25, 1236, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{5/2}}{(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2}{3}\int -\frac{\sqrt{x}(145x+111)}{\sqrt{3x^2+5x+2}}dx + \frac{2(95x+74)x^{3/2}}{3\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2x^{3/2}(95x+74)}{3\sqrt{3x^2+5x+2}} - \frac{2}{3}\int \frac{\sqrt{x}(145x+111)}{\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{2x^{3/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\left(\frac{2}{9}\int -\frac{451x+290}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{290}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)$$

$\big\downarrow$ 27

$$\frac{2x^{3/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\left(\frac{290}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{1}{9}\int \frac{451x+290}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)$$

$\big\downarrow$ 1240

$$\frac{2x^{3/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\left(\frac{290}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\int \frac{451x+290}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)$$

$\big\downarrow$ 1503

$$\frac{2x^{3/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\left(\frac{290}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(290\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+451\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)$$

$\big\downarrow$ 1413

$$\frac{2x^{3/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\left(\frac{290}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(451\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{145\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)\right)$$

$\big\downarrow$ 1456

$$\frac{2x^{3/2}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\left(\frac{290}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(\frac{145\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+451\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\right.\right.\right.$$

input
```
Int[((2 - 5*x)*x^(5/2))/(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(2*x^(3/2)*(74 + 95*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (2*((290*Sqrt[x]*Sqrt[
2 + 5*x + 3*x^2])/9 - (2*(451*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2
]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -
1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (145*Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1
+ x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/9))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

method	result
default	$-\frac{2\left(483\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-451\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)+135x^3+7056x^2+\right.}{243\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{2x\left(\frac{190}{27}+\frac{253x}{27}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{10\sqrt{3x^3+5x^2+2x}}{27}+\frac{580\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{81\sqrt{3x^3+5x^2+2x}}+\frac{902\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}}{\right.}}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/243*(483*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(
6*x+4)^(1/2),I*2^(1/2))-451*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)
*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+135*x^3+7056*x^2+5220*x)/x^(1/2)/(
3*x^2+5*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.49

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{2\left(710\sqrt{3}(3x^2+5x+2)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-8118\sqrt{3}(3x^2+\right.}{}$$

input
```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
2/729*(710*sqrt(3)*(3*x^2 + 5*x + 2)*weierstrassPInverse(28/27, 80/729, x
+ 5/9) - 8118*sqrt(3)*(3*x^2 + 5*x + 2)*weierstrassZeta(28/27, 80/729, wei
erstrassPInverse(28/27, 80/729, x + 5/9)) - 27*(15*x^2 + 784*x + 580)*sqrt
(3*x^2 + 5*x + 2)*sqrt(x))/(3*x^2 + 5*x + 2)
```

Sympy [F]

$$\begin{aligned}\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{3/2}}\,dx &= \\ &-\int\left(-\frac{2x^{\frac{5}{2}}}{3x^2\sqrt{3x^2+5x+2}+5x\sqrt{3x^2+5x+2}+2\sqrt{3x^2+5x+2}}\right)dx \\ &-\int\frac{5x^{\frac{7}{2}}}{3x^2\sqrt{3x^2+5x+2}+5x\sqrt{3x^2+5x+2}+2\sqrt{3x^2+5x+2}}\,dx\end{aligned}$$

input
```
integrate((2-5*x)*x**(5/2)/(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(-2*x**(5/2)/(3*x**2*sqrt(3*x**2 + 5*x + 2) + 5*x*sqrt(3*x**2 + 5
*x + 2) + 2*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5*x**(7/2)/(3*x**2*sqrt
(3*x**2 + 5*x + 2) + 5*x*sqrt(3*x**2 + 5*x + 2) + 2*sqrt(3*x**2 + 5*x + 2)
), x)
```

Maxima [F]

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{5}{2}}}{(3\,x^2+5\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(5/2)/(3*x^2 + 5*x + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{5}{2}}}{(3\,x^2+5\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(5/2)/(3*x^2 + 5*x + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{3/2}}\,dx = -\int \frac{x^{5/2}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{3/2}}\,dx$$

input
```
int(-(x^(5/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(3/2),x)
```

output
```
-int((x^(5/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{6\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2+20\sqrt{x}\,\sqrt{3x^2+5x+2}-60\left(\int \frac{\sqrt{3x^2+5x+2}}{9\sqrt{x}\,x^4+30\sqrt{x}\,x^3+37\sqrt{x}\,x^2+20}\right.}{}$$

input
```
int((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(3/2),x)
```

output
```
(6*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 + 20*sqrt(x)*sqrt(3*x**2 + 5*x + 2)
 - 60*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sq
rt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x**2 - 100*int(sqrt(3*x**2 + 5*x
 + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x +
 4*sqrt(x)),x)*x - 40*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt
(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x) - 531*int((sqrt(
x)*sqrt(3*x**2 + 5*x + 2)*x**3)/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x)
*x**2 - 885*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3)/(9*x**4 + 30*x**3 +
37*x**2 + 20*x + 4),x)*x - 354*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3)/(
9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x))/(30*(3*x**2 + 5*x + 2))
```

3.211 $\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}}\,dx = -\frac{200\sqrt{x}(2+3x)}{9\sqrt{2+5x+3x^2}}+\frac{2\sqrt{x}(74+95x)}{3\sqrt{2+5x+3x^2}} + \frac{200\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{9\sqrt{1+x}\sqrt{2+3x}} - \frac{74\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{3\sqrt{2+5x+3x^2}}$$

output
```
-200/9*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+2/3*x^(1/2)*(74+95*x)/(3*x^2+5*
x+2)^(1/2)+200/9*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2)
,1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-74/3*2^(1/2)*(1+x)^(1/2)*(2+3*x)
^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{-400-556x-30x^2-200i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)-22}{9\sqrt{x}\sqrt{2+5x+3x^2}}$$

input
```
Integrate[((2 - 5*x)*x^(3/2))/(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(-400 - 556*x - 30*x^2 - (200*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^
(3/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (22*I)*Sqrt[2]*Sqrt[1
 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3
/2])/(9*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1233, 25, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{3/2}}{(3x^2+5x+2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1233$$

$$\frac{2}{3}\int -\frac{50x+37}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{2\sqrt{x}(95x+74)}{3\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2\sqrt{x}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\int \frac{50x+37}{\sqrt{x}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \ 1240$$

$$\frac{2\sqrt{x}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{4}{3}\int \frac{50x+37}{\sqrt{3x^2+5x+2}}d\sqrt{x}$$

$$\Big\downarrow \text{1503}$$

$$\frac{2\sqrt{x}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{4}{3}\left(37\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+50\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1413}$$

$$\frac{2\sqrt{x}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{4}{3}\left(50\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{37(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{1456}$$

$$\frac{2\sqrt{x}(95x+74)}{3\sqrt{3x^2+5x+2}}-\frac{4}{3}\left(\frac{37(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+50\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right)|-\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$

input
```
Int[((2 - 5*x)*x^(3/2))/(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(2*Sqrt[x]*(74 + 95*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (4*(50*((Sqrt[x]*(2 +
3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]
*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (37*(1 + x
)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[
2 + 5*x + 3*x^2])))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

method	result
default	$\frac{\frac{26\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{9}-\frac{100\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{27}+\frac{190x^2}{3}+\frac{148x}{3}}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{2x\left(-\frac{74}{9}-\frac{95x}{9}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{74\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{9\sqrt{3x^3+5x^2+2x}}-\frac{100\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\frac{\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{\cdots}\right)}{3}\cdots\right.}{9\sqrt{3x^3+5x^2+2x}\cdots}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*(39*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x
+4)^(1/2),I*2^(1/2))-50*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ell
ipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+855*x^2+666*x)/x^(1/2)/(3*x^2+5*x+2)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}}\,dx =$$
$$-\frac{2\left(166\sqrt{3}(3x^2+5x+2)\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-900\sqrt{3}(3x^2+5x+2)\mathrm{weierstrassZeta}\left(\frac{2}{2}\cdots\right.\right.}{81\left(3x^2+5x+2\right)}$$

input
```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-2/81*(166*sqrt(3)*(3*x^2 + 5*x + 2)*weierstrassPInverse(28/27, 80/729, x
+ 5/9) - 900*sqrt(3)*(3*x^2 + 5*x + 2)*weierstrassZeta(28/27, 80/729, weie
rstrassPInverse(28/27, 80/729, x + 5/9)) - 27*sqrt(3*x^2 + 5*x + 2)*(95*x
+ 74)*sqrt(x))/(3*x^2 + 5*x + 2)
```

Sympy [F]

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}}\,dx = -\int \left(-\frac{2x^{\frac{3}{2}}}{3x^2\sqrt{3x^2+5x+2}+5x\sqrt{3x^2+5x+2}+2\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5x^{\frac{5}{2}}}{3x^2\sqrt{3x^2+5x+2}+5x\sqrt{3x^2+5x+2}+2\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)*x**(3/2)/(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(-2*x**(3/2)/(3*x**2*sqrt(3*x**2 + 5*x + 2) + 5*x*sqrt(3*x**2 + 5
*x + 2) + 2*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5*x**(5/2)/(3*x**2*sqrt
(3*x**2 + 5*x + 2) + 5*x*sqrt(3*x**2 + 5*x + 2) + 2*sqrt(3*x**2 + 5*x + 2)
), x)
```

Maxima [F]

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{3}{2}}}{(3\,x^2+5\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(3/2)/(3*x^2 + 5*x + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{3}{2}}}{(3\,x^2+5\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(3/2)/(3*x^2 + 5*x + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}}\,dx = -\int \frac{x^{3/2}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{3/2}}\,dx$$

input
```
int(-(x^(3/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(3/2),x)
```

output
```
-int((x^(3/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2-12\sqrt{x}\,\sqrt{3x^2+5x+2}+36\left(\int \frac{\sqrt{3x^2+5x+2}}{9\sqrt{x}\,x^4+30\sqrt{x}\,x^3+37\sqrt{x}\,x^2+\cdots}\right.\cdots}{\cdots}$$

input
```
int((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(3/2),x)
```

output

```
( - 2*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 - 12*sqrt(x)*sqrt(3*x**2 + 5*x +
 2) + 36*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37
*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x**2 + 60*int(sqrt(3*x**2 + 5
*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x
 + 4*sqrt(x)),x)*x + 24*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sq
rt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x) + 27*int((sqrt
(x)*sqrt(3*x**2 + 5*x + 2)*x**3)/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x
)*x**2 + 45*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3)/(9*x**4 + 30*x**3 +
37*x**2 + 20*x + 4),x)*x + 18*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3)/(9
*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x))/(4*(3*x**2 + 5*x + 2))
```

3.212 $\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\begin{aligned}\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}}\,dx &= \frac{74\sqrt{x}(2+3x)}{3\sqrt{2+5x+3x^2}}-\frac{2\sqrt{x}(30+37x)}{\sqrt{2+5x+3x^2}}\\&\quad-\frac{74\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{3\sqrt{1+x}\sqrt{2+3x}}\\&\quad+\frac{30\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
74/3*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-2*x^(1/2)*(30+37*x)/(3*x^2+5*x+2)
^(1/2)-74/3*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*
I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+30*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*
InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{148+190x+74i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)+16i\sqrt{2}\sqrt{1+}}{3\sqrt{x}\sqrt{2+5x+3x^2}}$$

input
```
Integrate[((2 - 5*x)*Sqrt[x])/(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(148 + 190*x + (74*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*Ellip
ticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (16*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*
Sqrt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(3*Sqr
t[x]*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1234, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\sqrt{x}}{(3x^2+5x+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1234}$$

$$-2\int -\frac{37x+30}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx - \frac{2\sqrt{x}(37x+30)}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{37x+30}{\sqrt{x}\sqrt{3x^2+5x+2}}dx - \frac{2\sqrt{x}(37x+30)}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1240}$$

$$2\int \frac{37x+30}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{2\sqrt{x}(37x+30)}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1503}$$

$$2\left(30\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+37\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2\sqrt{x}(37x+30)}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1413}$$

$$2\left(37\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{15\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)-$$
$$\frac{2\sqrt{x}(37x+30)}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1456}$$

$$2\left(\frac{15\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+37\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right)\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$
$$\frac{2\sqrt{x}(37x+30)}{\sqrt{3x^2+5x+2}}$$

input
```
Int[((2 - 5*x)*Sqrt[x])/(2 + 5*x + 3*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[x]*(30 + 37*x))/Sqrt[2 + 5*x + 3*x^2] + 2*(37*((Sqrt[x]*(2 + 3*x)
)/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*Ell
ipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (15*Sqrt[2]*(1
 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5
*x + 3*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.71

method	result
default	$-\frac{21\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-37\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)+666x^2+540x}{9\sqrt{x}\,\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{2x\left(10+\frac{37x}{3}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}+\frac{10\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{\sqrt{3x^3+5x^2+2x}}+\frac{37\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\frac{\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3}\right.}{3\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(21*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x
+4)^(1/2),I*2^(1/2))-37*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ell
ipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+666*x^2+540*x)/x^(1/2)/(3*x^2+5*x+2)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{2\left(85\sqrt{3}(3x^2+5x+2)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-333\sqrt{3}(3x^2+5x\right.}{}$$

input
```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
2/27*(85*sqrt(3)*(3*x^2 + 5*x + 2)*weierstrassPInverse(28/27, 80/729, x +
5/9) - 333*sqrt(3)*(3*x^2 + 5*x + 2)*weierstrassZeta(28/27, 80/729, weiers
trassPInverse(28/27, 80/729, x + 5/9)) - 27*sqrt(3*x^2 + 5*x + 2)*(37*x +
30)*sqrt(x))/(3*x^2 + 5*x + 2)
```

Sympy [F]

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}}\,dx = -\int \left(-\frac{2\sqrt{x}}{3x^2\sqrt{3x^2+5x+2}+5x\sqrt{3x^2+5x+2}+2\sqrt{3x^2+5x+2}}\right)dx - \int \frac{5x^{\frac{3}{2}}}{3x^2\sqrt{3x^2+5x+2}+5x\sqrt{3x^2+5x+2}+2\sqrt{3x^2+5x+2}}\,dx$$

input
```
integrate((2-5*x)*x**(1/2)/(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(-2*sqrt(x)/(3*x**2*sqrt(3*x**2 + 5*x + 2) + 5*x*sqrt(3*x**2 + 5*
x + 2) + 2*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5*x**(3/2)/(3*x**2*sqrt(
3*x**2 + 5*x + 2) + 5*x*sqrt(3*x**2 + 5*x + 2) + 2*sqrt(3*x**2 + 5*x + 2))
, x)
```

Maxima [F]

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{(5\,x-2)\sqrt{x}}{(3\,x^2+5\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*sqrt(x)/(3*x^2 + 5*x + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{(5\,x-2)\sqrt{x}}{(3\,x^2+5\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*sqrt(x)/(3*x^2 + 5*x + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}}\,dx = -\int \frac{\sqrt{x}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{3/2}}\,dx$$

input
```
int(-(x^(1/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(3/2),x)
```

output
```
-int((x^(1/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{3/2}}\,dx = \frac{-18\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2+120\sqrt{x}\,\sqrt{3x^2+5x+2}\,x+940\sqrt{x}\,\sqrt{3x^2+5x+2}\,-}{}$$

input
```
int((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(3/2),x)
```

output
```
( - 18*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 + 120*sqrt(x)*sqrt(3*x**2 + 5*x
 + 2)*x + 940*sqrt(x)*sqrt(3*x**2 + 5*x + 2) - 2820*int(sqrt(3*x**2 + 5*x
+ 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x +
4*sqrt(x)),x)*x**2 - 4700*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*
sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x - 1880*int
(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**
2 + 20*sqrt(x)*x + 4*sqrt(x)),x) + 243*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)
*x**3)/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x)*x**2 + 405*int((sqrt(x)*
sqrt(3*x**2 + 5*x + 2)*x**3)/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x)*x
+ 162*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3)/(9*x**4 + 30*x**3 + 37*x**
2 + 20*x + 4),x))/(180*(3*x**2 + 5*x + 2))
```

3.213 $\int \frac{2-5x}{\sqrt{x}(2+5x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{2-5x}{\sqrt{x}\,(2+5x+3x^2)^{3/2}}\,dx = -\frac{30\sqrt{x}(2+3x)}{\sqrt{2+5x+3x^2}} + \frac{2\sqrt{x}(38+45x)}{\sqrt{2+5x+3x^2}} + \frac{30\sqrt{2}\sqrt{2+5x+3x^2}\,E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{\sqrt{1+x}\sqrt{2+3x}} - \frac{37\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}$$

output
```
-30*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+2*x^(1/2)*(38+45*x)/(3*x^2+5*x+2)^
(1/2)+30*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2
^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-37*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*Inv
erseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int \frac{2-5x}{\sqrt{x}\left(2+5x+3x^2\right)^{3/2}}\,dx = \frac{-60-74x-30i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)-7i\sqrt{2}\sqrt{1+\cdots}}{\sqrt{x}\sqrt{2+5x+3x^2}}$$

input
```
Integrate[(2 - 5*x)/(Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2)),x]
```

output
```
(-60 - 74*x - (30*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*Ellipt
icE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (7*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sq
rt[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(Sqrt[x]
*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1235, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{2\sqrt{x}(45x+38)}{\sqrt{3x^2+5x+2}}-\int \frac{45x+37}{\sqrt{x}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{ 1240}$$

$$\frac{2\sqrt{x}(45x+38)}{\sqrt{3x^2+5x+2}}-2\int \frac{45x+37}{\sqrt{3x^2+5x+2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{2\sqrt{x}(45x+38)}{\sqrt{3x^2+5x+2}}-2\left(37\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+45\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 1413}$$

$$\frac{2\sqrt{x}(45x+38)}{\sqrt{3x^2+5x+2}}-2\left(45\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{37(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{ 1456}$$

$$\frac{2\sqrt{x}(45x+38)}{\sqrt{3x^2+5x+2}}-2\left(\frac{37(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+45\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right)|-\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$

input
```
Int[(2 - 5*x)/(Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2)),x]
```

output
```
(2*Sqrt[x]*(38 + 45*x))/Sqrt[2 + 5*x + 3*x^2] - 2*(45*((Sqrt[x]*(2 + 3*x))
/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*Elli
pticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (37*(1 + x)*Sqr
t[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5
*x + 3*x^2]))
```

Defintions of rubi rules used

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

method	result
default	$\frac{8\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-15\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\ \mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)+270x^{2}+228x}{3\sqrt{x}\,\sqrt{3x^{2}+5x+2}}$
elliptic	$\frac{\sqrt{(3x^{2}+5x+2)x}\left(-\frac{2x\left(-\frac{38}{3}-15x\right)\sqrt{3}}{\sqrt{x\left(x^{2}+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{37\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3\sqrt{3x^{3}+5x^{2}+2x}}-\frac{15\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\frac{\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right.}{3}\right.}{\sqrt{3x^{3}+5x^{2}+2x}}}{\sqrt{x}\,\sqrt{3x^{2}+5x+2}}$

input
```
int((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(8*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+4
)^(1/2),I*2^(1/2))-15*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Ellip
ticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+270*x^2+228*x)/x^(1/2)/(3*x^2+5*x+2)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int \frac{2-5x}{\sqrt{x}\left(2+5x+3x^2\right)^{3/2}}\,dx = -\frac{2\left(4\sqrt{3}(3\,x^2+5\,x+2)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-15\sqrt{3}(3\,x^2+5\,x+2)\text{weierstrassZeta}\left(\frac{28}{27},\right.\right.}{3\,x^2+5\,x+2}$$

input
```
integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-2*(4*sqrt(3)*(3*x^2 + 5*x + 2)*weierstrassPInverse(28/27, 80/729, x + 5/9
) - 15*sqrt(3)*(3*x^2 + 5*x + 2)*weierstrassZeta(28/27, 80/729, weierstras
sPInverse(28/27, 80/729, x + 5/9)) - sqrt(3*x^2 + 5*x + 2)*(45*x + 38)*sqr
t(x))/(3*x^2 + 5*x + 2)
```

Sympy [F]

$$\int \frac{2-5x}{\sqrt{x}\left(2+5x+3x^2\right)^{3/2}}\,dx = -\int \frac{5\sqrt{x}}{3x^2\sqrt{3x^2+5x+2}+5x\sqrt{3x^2+5x+2}+2\sqrt{3x^2+5x+2}}\,dx - \int \left(-\frac{2}{3x^{\frac{5}{2}}\sqrt{3x^2+5x+2}+5x^{\frac{3}{2}}\sqrt{3x^2+5x+2}+2\sqrt{x}\sqrt{3x^2+5x+2}}\right)dx$$

input
```
integrate((2-5*x)/x**(1/2)/(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(5*sqrt(x)/(3*x**2*sqrt(3*x**2 + 5*x + 2) + 5*x*sqrt(3*x**2 + 5*x
 + 2) + 2*sqrt(3*x**2 + 5*x + 2)), x) - Integral(-2/(3*x**(5/2)*sqrt(3*x**
2 + 5*x + 2) + 5*x**(3/2)*sqrt(3*x**2 + 5*x + 2) + 2*sqrt(x)*sqrt(3*x**2 +
 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{\sqrt{x}\,(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{5\,x-2}{(3\,x^2+5\,x+2)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/((3*x^2 + 5*x + 2)^(3/2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{2-5x}{\sqrt{x}\,(2+5x+3x^2)^{3/2}}\,dx = \int -\frac{5\,x-2}{(3\,x^2+5\,x+2)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/((3*x^2 + 5*x + 2)^(3/2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{\sqrt{x}\,(2+5x+3x^2)^{3/2}}\,dx = -\int \frac{5\,x-2}{\sqrt{x}\,(3\,x^2+5\,x+2)^{3/2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(1/2)*(5*x + 3*x^2 + 2)^(3/2)),x)
```

output
```
-int((5*x - 2)/(x^(1/2)*(5*x + 3*x^2 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{\sqrt{x}\,(2+5x+3x^2)^{3/2}}\,dx = 2\left(\int \frac{\sqrt{3x^2+5x+2}}{9\sqrt{x}\,x^4+30\sqrt{x}\,x^3+37\sqrt{x}\,x^2+20\sqrt{x}\,x+4\sqrt{x}}dx\right) - 5\left(\int \frac{\sqrt{x}\,\sqrt{3x^2+5x+2}}{9x^4+30x^3+37x^2+20x+4}dx\right)$$

input
```
int((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(3/2),x)
```

output
```
2*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x
)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x) - 5*int((sqrt(x)*sqrt(3*x**2 + 5*x +
 2))/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x)
```

3.214 $\int \frac{2-5x}{x^{3/2}(2+5x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{3/2}}\,dx=-\frac{2}{\sqrt{x}\sqrt{2+5x+3x^2}}+\frac{39\sqrt{x}(2+3x)}{\sqrt{2+5x+3x^2}}-\frac{3\sqrt{x}(35+39x)}{\sqrt{2+5x+3x^2}}-\frac{39\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{\sqrt{1+x}\sqrt{2+3x}}+\frac{45\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}$$

output

```
-2/x^(1/2)/(3*x^2+5*x+2)^(1/2)+39*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-3*x^
(1/2)*(35+39*x)/(3*x^2+5*x+2)^(1/2)-39*2^(1/2)*(3*x^2+5*x+2)^(1/2)*Ellipti
cE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+45*2^(1/2)
*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/
(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.82

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \frac{76+90x+39i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)+6i\sqrt{2}\sqrt{1+}}{\sqrt{x}\sqrt{2+5x+3x^2}}$$

input
```
Integrate[(2 - 5*x)/(x^(3/2)*(2 + 5*x + 3*x^2)^(3/2)),x]
```

output
```
(76 + 90*x + (39*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(3/2)*Ellipti
cE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (6*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqr
t[3 + 2/x]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(Sqrt[x]*
Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1235, 27, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{x^{3/2}\left(3x^2+5x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2(45x+38)}{\sqrt{x}\sqrt{3x^2+5x+2}}-\int -\frac{3(15x+13)}{x^{3/2}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{27}$$

$$3\int \frac{15x+13}{x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{2(45x+38)}{\sqrt{x}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1237}$$

$$3\left(-\int -\frac{3(13x+10)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{13\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{2(45x+38)}{\sqrt{x}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{3}{2}\int \frac{13x+10}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{13\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{2(45x+38)}{\sqrt{x}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1240}$$

$$3\left(3\int \frac{13x+10}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{13\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{2(45x+38)}{\sqrt{x}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1503}$$

$$3\left(3\left(10\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+13\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{13\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{2(45x+38)}{\sqrt{x}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1413}$$

$$3\left(3\left(13\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{5\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)-\frac{13\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{2(45x+38)}{\sqrt{x}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1456}$$

$$3\left(3\left(\frac{5\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+13\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E(\arctan(\sqrt{}}{3\sqrt{3x^2+5x+2}}\right.\right.\right.$$
$$\frac{2(45x+38)}{\sqrt{x}\sqrt{3x^2+5x+2}}$$

input
```
Int[(2 - 5*x)/(x^(3/2)*(2 + 5*x + 3*x^2)^(3/2)),x]
```

output
```
(2*(38 + 45*x))/(Sqrt[x]*Sqrt[2 + 5*x + 3*x^2]) + 3*((-13*Sqrt[2 + 5*x + 3
*x^2])/Sqrt[x] + 3*(13*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (S
qrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(
3*Sqrt[2 + 5*x + 3*x^2])) + (5*Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*Ell
ipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

method	result
default	$-\frac{9\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)-13\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)+234x^2+210x+4}{2\sqrt{3x^2+5x+2}\,\sqrt{x}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{3x^2+5x+2}{\sqrt{\left(3x^2+5x+2\right)x}}-\frac{2x\left(\frac{50}{3}+19x\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}+\frac{15\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{\sqrt{3x^3+5x^2+2x}}+\frac{39\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\left(\text{Ellip}\ldots\right.}{\ldots}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(9*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*x+
4)^(1/2),I*2^(1/2))-13*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*Elli
pticE(1/2*(6*x+4)^(1/2),I*2^(1/2))+234*x^2+210*x+4)/(3*x^2+5*x+2)^(1/2)/x^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \frac{25\,\sqrt{3}(3\,x^3+5\,x^2+2\,x)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-117\,\sqrt{3}(3\,x^3$$

input
```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(25*sqrt(3)*(3*x^3 + 5*x^2 + 2*x)*weierstrassPInverse(28/27, 80/729, x
 + 5/9) - 117*sqrt(3)*(3*x^3 + 5*x^2 + 2*x)*weierstrassZeta(28/27, 80/729,
 weierstrassPInverse(28/27, 80/729, x + 5/9)) - 3*(117*x^2 + 105*x + 2)*sq
rt(3*x^2 + 5*x + 2)*sqrt(x))/(3*x^3 + 5*x^2 + 2*x)
```

Sympy [F]

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = -\int \frac{5}{3x^{\frac{5}{2}}\sqrt{3x^2+5x+2}+5x^{\frac{3}{2}}\sqrt{3x^2+5x+2}+2\sqrt{x}\sqrt{3x^2+5x+2}}\,dx - \int \left(-\frac{2}{3x^{\frac{7}{2}}\sqrt{3x^2+5x+2}+5x^{\frac{5}{2}}\sqrt{3x^2+5x+2}+2x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}\right)dx$$

input
```
integrate((2-5*x)/x**(3/2)/(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(5/(3*x**(5/2)*sqrt(3*x**2 + 5*x + 2) + 5*x**(3/2)*sqrt(3*x**2 +
5*x + 2) + 2*sqrt(x)*sqrt(3*x**2 + 5*x + 2)), x) - Integral(-2/(3*x**(7/2)
*sqrt(3*x**2 + 5*x + 2) + 5*x**(5/2)*sqrt(3*x**2 + 5*x + 2) + 2*x**(3/2)*s
qrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/((3*x^2 + 5*x + 2)^(3/2)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/((3*x^2 + 5*x + 2)^(3/2)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \int -\frac{5\,x-2}{x^{3/2}\,\left(3\,x^2+5\,x+2\right)^{3/2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(3/2)*(5*x + 3*x^2 + 2)^(3/2)),x)
```

output
```
int(-(5*x - 2)/(x^(3/2)*(5*x + 3*x^2 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \frac{-6\sqrt{3x^2+5x+2}\,x^2-20\sqrt{3x^2+5x+2}\,x-8\sqrt{3x^2+5x+2}-120\sqrt{x}\left(\int\right.}{}$$

input
```
int((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(3/2),x)
```

output
```
( - 6*sqrt(3*x**2 + 5*x + 2)*x**2 - 20*sqrt(3*x**2 + 5*x + 2)*x - 8*sqrt(3
*x**2 + 5*x + 2) - 120*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4
+ 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x**2 -
200*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 +
 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x - 80*sqrt(x)*int(sqrt(3*
x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*s
qrt(x)*x + 4*sqrt(x)),x) - 108*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x)/(9*s
qrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)
),x)*x**2 - 180*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x)/(9*sqrt(x)*x**4 + 3
0*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x - 72*sqr
t(x)*int((sqrt(3*x**2 + 5*x + 2)*x)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37
*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x) + 27*sqrt(x)*int((sqrt(x)*sqr
t(3*x**2 + 5*x + 2)*x**2)/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x)*x**2
+ 45*sqrt(x)*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2)/(9*x**4 + 30*x**3 +
 37*x**2 + 20*x + 4),x)*x + 18*sqrt(x)*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)
*x**2)/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x) + 54*sqrt(x)*int((sqrt(x
)*sqrt(3*x**2 + 5*x + 2))/(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x)*x**2
+ 90*sqrt(x)*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(9*x**4 + 30*x**3 + 37*x
**2 + 20*x + 4),x)*x + 36*sqrt(x)*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2))/(9*
x**4 + 30*x**3 + 37*x**2 + 20*x + 4),x))/(4*sqrt(x)*(3*x**2 + 5*x + 2))
```

3.215 $\int \frac{2-5x}{x^{5/2}(2+5x+3x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{3/2}} \, dx = -\frac{2}{3x^{3/2}\sqrt{2+5x+3x^2}} + \frac{35}{3\sqrt{x}\sqrt{2+5x+3x^2}} - \frac{170\sqrt{x}(2+3x)}{3\sqrt{2+5x+3x^2}} + \frac{5\sqrt{x}(101+102x)}{3\sqrt{2+5x+3x^2}} + \frac{170\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x}\sqrt{2+3x}} - \frac{115\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x+3x^2}}$$

output
```
-2/3/x^(3/2)/(3*x^2+5*x+2)^(1/2)+35/3/x^(1/2)/(3*x^2+5*x+2)^(1/2)-170/3*x^
(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+5/3*x^(1/2)*(101+102*x)/(3*x^2+5*x+2)^(1
/2)+170/3*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*
2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-115/2*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)
*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \frac{-4-610x-690x^2-340i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{5/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)-}{6x^{3/2}\sqrt{2+5x+3x}}$$

input
```
Integrate[(2 - 5*x)/(x^(5/2)*(2 + 5*x + 3*x^2)^(3/2)),x]
```

output
```
(-4 - 610*x - 690*x^2 - (340*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(
5/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (5*I)*Sqrt[2]*Sqrt[1 +
 x^(-1)]*Sqrt[3 + 2/x]*x^(5/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2
])/(6*x^(3/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1235, 27, 1237, 27, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{x^{5/2}\left(3x^2+5x+2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}-\int -\frac{5(27x+23)}{x^{5/2}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$5\int \frac{27x+23}{x^{5/2}\sqrt{3x^2+5x+2}}dx+\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{ 1237}$$

$$5\left(-\frac{1}{3}\int \frac{69x+68}{2x^{3/2}\sqrt{3x^2+5x+2}}dx-\frac{23\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{27}$$

$$5\left(-\frac{1}{6}\int \frac{69x+68}{x^{3/2}\sqrt{3x^2+5x+2}}dx-\frac{23\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1237}$$

$$5\left(\frac{1}{6}\left(\int -\frac{3(34x+23)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{68\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)-\frac{23\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{27}$$

$$5\left(\frac{1}{6}\left(\frac{68\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\int \frac{34x+23}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{23\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1240}$$

$$5\left(\frac{1}{6}\left(\frac{68\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\int \frac{34x+23}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{23\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1503}$$

$$5\left(\frac{1}{6}\left(\frac{68\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(23\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+34\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{23\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1413}$$

$$5\left(\frac{1}{6}\left(\frac{68\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(34\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{23(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)\right)-\right.$$
$$\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1456}$$

$$5\left(\frac{1}{6}\left(\frac{68\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(\frac{23(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+34\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+\right.\right.\right.\right.$$

$$\frac{2(45x+38)}{x^{3/2}\sqrt{3x^2+5x+2}}$$

input
```
Int[(2 - 5*x)/(x^(5/2)*(2 + 5*x + 3*x^2)^(3/2)),x]
```

output
```
(2*(38 + 45*x))/(x^(3/2)*Sqrt[2 + 5*x + 3*x^2]) + 5*((-23*Sqrt[2 + 5*x + 3
*x^2])/(3*x^(3/2)) + ((68*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x] - 6*(34*((Sqrt[x]
*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1
 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (23*
(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x + 3*x^2])))/6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240

```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

method	result
default	$\frac{165\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x-170\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x+3060x^3+3030x^2+2}{18x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{\sqrt{3x^3+5x^2+2x}}{3x^2}+\frac{20x^2+\frac{100}{3}x+\frac{40}{3}}{\sqrt{(3x^2+5x+2)x}}-\frac{2x\left(-\frac{68}{3}-25x\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{115\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{6\sqrt{3x^3+5x^2+2x}}-\frac{85\sqrt{6x+4}}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(165*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*EllipticF(1/2*(6*
x+4)^(1/2),I*2^(1/2))*x-170*(6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)
*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*x+3060*x^3+3030*x^2+210*x-12)/x^(3
/2)/(3*x^2+5*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.55

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{3/2}}\,dx =$$
$$-\frac{185\sqrt{3}(3x^4+5x^3+2x^2)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-1530\sqrt{3}(3x^4+5x^3+2x^2)\text{weierstrassZ}}{27\,(3x^4+5x}$$

input
```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/27*(185*sqrt(3)*(3*x^4 + 5*x^3 + 2*x^2)*weierstrassPInverse(28/27, 80/7
29, x + 5/9) - 1530*sqrt(3)*(3*x^4 + 5*x^3 + 2*x^2)*weierstrassZeta(28/27,
 80/729, weierstrassPInverse(28/27, 80/729, x + 5/9)) - 9*(510*x^3 + 505*x
^2 + 35*x - 2)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/(3*x^4 + 5*x^3 + 2*x^2)
```

Sympy [F]

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = -\int \frac{5}{3x^{\frac{7}{2}}\sqrt{3x^2+5x+2}+5x^{\frac{5}{2}}\sqrt{3x^2+5x+2}+2x^{\frac{3}{2}}\sqrt{3x^2+5x+2}}\,dx - \int \left(-\frac{2}{3x^{\frac{9}{2}}\sqrt{3x^2+5x+2}+5x^{\frac{7}{2}}\sqrt{3x^2+5x+2}+2x^{\frac{5}{2}}\sqrt{3x^2+5x+2}}\right)dx$$

input
```
integrate((2-5*x)/x**(5/2)/(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(5/(3*x**(7/2)*sqrt(3*x**2 + 5*x + 2) + 5*x**(5/2)*sqrt(3*x**2 +
5*x + 2) + 2*x**(3/2)*sqrt(3*x**2 + 5*x + 2)), x) - Integral(-2/(3*x**(9/2
)*sqrt(3*x**2 + 5*x + 2) + 5*x**(7/2)*sqrt(3*x**2 + 5*x + 2) + 2*x**(5/2)*
sqrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/((3*x^2 + 5*x + 2)^(3/2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/((3*x^2 + 5*x + 2)^(3/2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \int -\frac{5\,x-2}{x^{5/2}\left(3\,x^2+5\,x+2\right)^{3/2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(5/2)*(5*x + 3*x^2 + 2)^(3/2)),x)
```

output
```
int(-(5*x - 2)/(x^(5/2)*(5*x + 3*x^2 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(3/2),x)
```

output
```
( - 30*sqrt(3*x**2 + 5*x + 2)*x**2 + 30*sqrt(3*x**2 + 5*x + 2)*x - 4*sqrt(
3*x**2 + 5*x + 2) - 120*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**5
 + 30*sqrt(x)*x**4 + 37*sqrt(x)*x**3 + 20*sqrt(x)*x**2 + 4*sqrt(x)*x),x)*x
**3 - 200*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**5 + 30*sqrt(x)*
x**4 + 37*sqrt(x)*x**3 + 20*sqrt(x)*x**2 + 4*sqrt(x)*x),x)*x**2 - 80*sqrt(
x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**5 + 30*sqrt(x)*x**4 + 37*sqrt(
x)*x**3 + 20*sqrt(x)*x**2 + 4*sqrt(x)*x),x)*x + 450*sqrt(x)*int(sqrt(3*x**
2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt
(x)*x + 4*sqrt(x)),x)*x**3 + 750*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqr
t(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),
x)*x**2 + 300*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**4 + 30*sqrt
(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x - 135*sqrt(x)*
int((sqrt(3*x**2 + 5*x + 2)*x**2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*s
qrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x**3 - 225*sqrt(x)*int((sqrt(3*
x**2 + 5*x + 2)*x**2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2
+ 20*sqrt(x)*x + 4*sqrt(x)),x)*x**2 - 90*sqrt(x)*int((sqrt(3*x**2 + 5*x +
2)*x**2)/(9*sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*
x + 4*sqrt(x)),x)*x + 405*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x)/(9*sqrt(x
)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*
x**3 + 675*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x)/(9*sqrt(x)*x**4 + 30*...
```

3.216 $\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = -\frac{2}{5x^{5/2}\sqrt{2+5x+3x^2}} + \frac{11}{3x^{3/2}\sqrt{2+5x+3x^2}} - \frac{487}{15\sqrt{x}\sqrt{2+5x+3x^2}} + \frac{2693\sqrt{x}(2+3x)}{30\sqrt{2+5x+3x^2}} - \frac{\sqrt{x}(8755+8079x)}{30\sqrt{2+5x+3x^2}} - \frac{2693\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{15\sqrt{2}\sqrt{1+x}\sqrt{2+3x}} + \frac{157\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x+3x^2}}$$

output

```
-2/5/x^(5/2)/(3*x^2+5*x+2)^(1/2)+11/3/x^(3/2)/(3*x^2+5*x+2)^(1/2)-487/15/x
^(1/2)/(3*x^2+5*x+2)^(1/2)+2693/30*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-1/3
0*x^(1/2)*(8755+8079*x)/(3*x^2+5*x+2)^(1/2)-2693/30*2^(1/2)*(3*x^2+5*x+2)^
(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/
2)+157/2*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2))
,1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.68

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \frac{-12+110x+4412x^2+4710x^3+2693i\sqrt{2}\sqrt{1+\frac{1}{x}}\sqrt{3+\frac{2}{x}}x^{7/2}E\left(i\text{arcsinh}\right.}{30x^{5/2}\sqrt{2+}}$$

input
```
Integrate[(2 - 5*x)/(x^(7/2)*(2 + 5*x + 3*x^2)^(3/2)),x]
```

output
```
(-12 + 110*x + 4412*x^2 + 4710*x^3 + (2693*I)*Sqrt[2]*Sqrt[1 + x^(-1)]*Sqr
t[3 + 2/x]*x^(7/2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (338*I)*
Sqrt[2]*Sqrt[1 + x^(-1)]*Sqrt[3 + 2/x]*x^(7/2)*EllipticF[I*ArcSinh[Sqrt[2/
3]/Sqrt[x]], 3/2])/(30*x^(5/2)*Sqrt[2 + 5*x + 3*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1235, 25, 1237, 27, 1237, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{x^{7/2}\left(3x^2+5x+2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\int -\frac{225x+191}{x^{7/2}\sqrt{3x^2+5x+2}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{225x+191}{x^{7/2}\sqrt{3x^2+5x+2}}dx+\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{1}{5}\int \frac{1719x+1570}{2x^{5/2}\sqrt{3x^2+5x+2}}dx+\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\frac{191\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$\downarrow$ 27

$$-\frac{1}{10}\int \frac{1719x+1570}{x^{5/2}\sqrt{3x^2+5x+2}}dx+\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\frac{191\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$\downarrow$ 1237

$$\frac{1}{10}\left(\frac{1}{3}\int \frac{2355x+2693}{x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{1570\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\frac{191\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$\downarrow$ 1237

$$\frac{1}{10}\left(\frac{1}{3}\left(-\int -\frac{3(2693x+1570)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2693\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{1570\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\frac{191\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$\downarrow$ 27

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{2693x+1570}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2693\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{1570\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\frac{191\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$\downarrow$ 1240

$$\frac{1}{10}\left(\frac{1}{3}\left(3\int \frac{2693x+1570}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{2693\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{1570\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)+\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\frac{191\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$\downarrow$ 1503

$$\frac{1}{10}\left(\frac{1}{3}\left(3\left(1570\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+2693\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2693\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{1570\sqrt{3x^2+}}{3x^{3/2}}\right.$$
$$\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\frac{191\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$\downarrow$ 1413

$$\frac{1}{10}\left(\frac{1}{3}\left(3\left(2693\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{785\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)-\frac{2693\sqrt{3x^2+5}}{\sqrt{x}}\right.\right.$$

$$\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\frac{191\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

$\Big\downarrow$ 1456

$$\frac{1}{10}\left(\frac{1}{3}\left(3\left(\frac{785\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+2693\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E(}{3\sqrt{3x^2+}}\right.\right.\right.\right.$$

$$\frac{2(45x+38)}{x^{5/2}\sqrt{3x^2+5x+2}}-\frac{191\sqrt{3x^2+5x+2}}{5x^{5/2}}$$

input
```
Int[(2 - 5*x)/(x^(7/2)*(2 + 5*x + 3*x^2)^(3/2)),x]
```

output
```
(2*(38 + 45*x))/(x^(5/2)*Sqrt[2 + 5*x + 3*x^2]) - (191*Sqrt[2 + 5*x + 3*x^
2])/(5*x^(5/2)) + ((1570*Sqrt[2 + 5*x + 3*x^2])/(3*x^(3/2)) + ((-2693*Sqrt
[2 + 5*x + 3*x^2])/Sqrt[x] + 3*(2693*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x
+ 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt
[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (785*Sqrt[2]*(1 + x)*Sqrt[(2 + 3
*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))/3)/
10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.56

method	result
default	$-\frac{3369\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-2693\,\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+48474x^4+52}{180x^{\frac{5}{2}}\sqrt{3x^2+5x+2}}$
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{\sqrt{3x^3+5x^2+2x}}{5x^3}+\frac{7\sqrt{3x^3+5x^2+2x}}{3x^2}-\frac{653(3x^2+5x+2)}{30\sqrt{(3x^2+5x+2)x}}-\frac{2x\left(\frac{95}{3}+34x\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}+\frac{157\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\text{EllipticF}\left(\frac{\sqrt{6x+}}{2}\right.}{6\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$

input
```
int((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/180*(3369*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))*(6*x+4)^(1/2)*(3+3*x)^
(1/2)*6^(1/2)*(-x)^(1/2)*x^2-2693*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))*(
6*x+4)^(1/2)*(3+3*x)^(1/2)*6^(1/2)*(-x)^(1/2)*x^2+48474*x^4+52530*x^3+5844
*x^2-660*x+72)/x^(5/2)/(3*x^2+5*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \frac{665\sqrt{3}(3x^5+5x^4+2x^3)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-24237\sqrt{3}(}{}$$

input
```
integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/270*(665*sqrt(3)*(3*x^5 + 5*x^4 + 2*x^3)*weierstrassPInverse(28/27, 80/7
29, x + 5/9) - 24237*sqrt(3)*(3*x^5 + 5*x^4 + 2*x^3)*weierstrassZeta(28/27
, 80/729, weierstrassPInverse(28/27, 80/729, x + 5/9)) - 9*(8079*x^4 + 875
5*x^3 + 974*x^2 - 110*x + 12)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/(3*x^5 + 5*x^
4 + 2*x^3)
```

Sympy [F]

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = -\int \frac{5}{3x^{\frac{9}{2}}\sqrt{3x^2+5x+2}+5x^{\frac{7}{2}}\sqrt{3x^2+5x+2}+2x^{\frac{5}{2}}\sqrt{3x^2+5x+2}}\,dx - \int \left(-\frac{2}{3x^{\frac{11}{2}}\sqrt{3x^2+5x+2}+5x^{\frac{9}{2}}\sqrt{3x^2+5x+2}+2x^{\frac{7}{2}}\sqrt{3x^2+5x+2}}\right)dx$$

input
```
integrate((2-5*x)/x**(7/2)/(3*x**2+5*x+2)**(3/2),x)
```

output
```
-Integral(5/(3*x**(9/2)*sqrt(3*x**2 + 5*x + 2) + 5*x**(7/2)*sqrt(3*x**2 +
5*x + 2) + 2*x**(5/2)*sqrt(3*x**2 + 5*x + 2)), x) - Integral(-2/(3*x**(11/
2)*sqrt(3*x**2 + 5*x + 2) + 5*x**(9/2)*sqrt(3*x**2 + 5*x + 2) + 2*x**(7/2)
*sqrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/((3*x^2 + 5*x + 2)^(3/2)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx=\int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/((3*x^2 + 5*x + 2)^(3/2)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx=\int -\frac{5\,x-2}{x^{7/2}\,\left(3\,x^2+5\,x+2\right)^{3/2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(7/2)*(5*x + 3*x^2 + 2)^(3/2)),x)
```

output
```
int(-(5*x - 2)/(x^(7/2)*(5*x + 3*x^2 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{3/2}}\,dx=\frac{750\sqrt{3x^2+5x+2}\,x^3+100\sqrt{3x^2+5x+2}\,x-24\sqrt{3x^2+5x+2}-1080\sqrt{3}$$

input
```
int((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(3/2),x)
```

output
```
(750*sqrt(3*x**2 + 5*x + 2)*x**3 + 100*sqrt(3*x**2 + 5*x + 2)*x - 24*sqrt(
3*x**2 + 5*x + 2) - 1080*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**
6 + 30*sqrt(x)*x**5 + 37*sqrt(x)*x**4 + 20*sqrt(x)*x**3 + 4*sqrt(x)*x**2),
x)*x**4 - 1800*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**6 + 30*sqr
t(x)*x**5 + 37*sqrt(x)*x**4 + 20*sqrt(x)*x**3 + 4*sqrt(x)*x**2),x)*x**3 -
720*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**6 + 30*sqrt(x)*x**5 +
 37*sqrt(x)*x**4 + 20*sqrt(x)*x**3 + 4*sqrt(x)*x**2),x)*x**2 + 2244*sqrt(x
)*int(sqrt(3*x**2 + 5*x + 2)/(9*sqrt(x)*x**5 + 30*sqrt(x)*x**4 + 37*sqrt(x
)*x**3 + 20*sqrt(x)*x**2 + 4*sqrt(x)*x),x)*x**4 + 3740*sqrt(x)*int(sqrt(3*
x**2 + 5*x + 2)/(9*sqrt(x)*x**5 + 30*sqrt(x)*x**4 + 37*sqrt(x)*x**3 + 20*s
qrt(x)*x**2 + 4*sqrt(x)*x),x)*x**3 + 1496*sqrt(x)*int(sqrt(3*x**2 + 5*x +
2)/(9*sqrt(x)*x**5 + 30*sqrt(x)*x**4 + 37*sqrt(x)*x**3 + 20*sqrt(x)*x**2 +
 4*sqrt(x)*x),x)*x**2 + 3375*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x**2)/(9*
sqrt(x)*x**4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x
)),x)*x**4 + 5625*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x**2)/(9*sqrt(x)*x**
4 + 30*sqrt(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x**3
+ 2250*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x**2)/(9*sqrt(x)*x**4 + 30*sqrt
(x)*x**3 + 37*sqrt(x)*x**2 + 20*sqrt(x)*x + 4*sqrt(x)),x)*x**2)/(60*sqrt(x
)*x**2*(3*x**2 + 5*x + 2))
```

3.217 $\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\begin{aligned}\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}}\,dx &= -\frac{2\sqrt{x}(11230+16333x)}{2187\,(2+5x+3x^2)^{3/2}}-\frac{1521056\sqrt{x}(2+3x)}{76545\sqrt{2+5x+3x^2}} \\ &+\frac{10\sqrt{x}(20017+20076x)}{2187\sqrt{2+5x+3x^2}}-\frac{9286\sqrt{x}\sqrt{2+5x+3x^2}}{5103}+\frac{1084x^{3/2}\sqrt{2+5x+3x^2}}{2835} \\ &-\frac{10}{189}x^{5/2}\sqrt{2+5x+3x^2}+\frac{1521056\sqrt{2}\sqrt{2+5x+3x^2}\,E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{76545\sqrt{1+x}\sqrt{2+3x}} \\ &-\frac{211144\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{5103\sqrt{2+5x+3x^2}}\end{aligned}$$

output

```
-2/2187*x^(1/2)*(11230+16333*x)/(3*x^2+5*x+2)^(3/2)-1521056/76545*x^(1/2)*
(2+3*x)/(3*x^2+5*x+2)^(1/2)+10/2187*x^(1/2)*(20017+20076*x)/(3*x^2+5*x+2)^
(1/2)-9286/5103*x^(1/2)*(3*x^2+5*x+2)^(1/2)+1084/2835*x^(3/2)*(3*x^2+5*x+2
)^(1/2)-10/189*x^(5/2)*(3*x^2+5*x+2)^(1/2)+1521056/76545*2^(1/2)*(3*x^2+5*
x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x
)^(1/2)-211144/5103*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arct
an(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74

$$\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{-2(3042112+8876240x+5504080x^2-2967300x^3-2106756x^4+262710x^5-}{}$$

input
```
Integrate[((2 - 5*x)*x^(13/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(-2*(3042112 + 8876240*x + 5504080*x^2 - 2967300*x^3 - 2106756*x^4 + 26271
0*x^5 - 70956*x^6 + 18225*x^7) - (1521056*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x
^(3/2)*(2 + 5*x + 3*x^2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (1
646104*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*EllipticF[
I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(76545*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {1233, 25, 1233, 27, 1236, 25, 1236, 27, 1236, 25, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{13/2}}{(3x^2+5x+2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1233$$

$$\frac{2}{9}\int -\frac{x^{9/2}(340x+407)}{(3x^2+5x+2)^{3/2}}dx+\frac{2(95x+74)x^{11/2}}{9\,(3x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2x^{11/2}(95x+74)}{9\,(3x^2+5x+2)^{3/2}}-\frac{2}{9}\int \frac{x^{9/2}(340x+407)}{(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \; 1233$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{2}{3}\int -\frac{x^{5/2}(11455x+10388)}{2\sqrt{3x^2+5x+2}}dx+\frac{2(1685x+1484)x^{7/2}}{3\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{2x^{7/2}(1685x+1484)}{3\sqrt{3x^2+5x+2}}-\frac{1}{3}\int \frac{x^{5/2}(11455x+10388)}{\sqrt{3x^2+5x+2}}dx\right)$$

$$\Big\downarrow \; 1236$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(-\frac{2}{21}\int -\frac{x^{3/2}(62751x+57275)}{\sqrt{3x^2+5x+2}}dx-\frac{22910}{21}\sqrt{3x^2+5x+2}x^{5/2}\right)+\frac{2(1685x+1484)x^{7/2}}{3\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{2}{21}\int \frac{x^{3/2}(62751x+57275)}{\sqrt{3x^2+5x+2}}dx-\frac{22910}{21}x^{5/2}\sqrt{3x^2+5x+2}\right)+\frac{2(1685x+1484)x^{7/2}}{3\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \; 1236$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{2}{21}\left(\frac{2}{15}\int -\frac{3\sqrt{x}(131965x+125502)}{2\sqrt{3x^2+5x+2}}dx+\frac{41834}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)-\frac{22910}{21}x^{5/2}\sqrt{3x^2+5x+2}\right)+\ldots\right.$$

$$\Big\downarrow \; 27$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{2}{21}\left(\frac{41834}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{1}{5}\int \frac{\sqrt{x}(131965x+125502)}{\sqrt{3x^2+5x+2}}dx\right)-\frac{22910}{21}x^{5/2}\sqrt{3x^2+5x+2}\right)+\frac{2(168\ldots}{3\sqrt{\ldots}}\right.$$

$$\Big\downarrow \; 1236$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{2}{21}\left(\frac{1}{5}\left(-\frac{2}{9}\int -\frac{95066x+131965}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{263930}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)+\frac{41834}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)-\frac{229\ldots}{2\ldots}\right.\right.$$

$$\Big\downarrow \textbf{25}$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{2}{21}\left(\frac{1}{5}\left(\frac{2}{9}\int\frac{95066x+131965}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{263930}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)+\frac{41834}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)-\frac{22910}{21}\right.\right.\right.$$

$$\Big\downarrow \textbf{1240}$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{2}{21}\left(\frac{1}{5}\left(\frac{4}{9}\int\frac{95066x+131965}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{263930}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)+\frac{41834}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)-\frac{22910}{21}\right.\right.\right.$$

$$\Big\downarrow \textbf{1503}$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{2}{21}\left(\frac{1}{5}\left(\frac{4}{9}\left(131965\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+95066\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{263930}{9}\sqrt{x}\sqrt{3x^2+5x+2}\right)\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{1413}$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{2}{21}\left(\frac{1}{5}\left(\frac{4}{9}\left(95066\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{131965(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{2639}{9}\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{1456}$$

$$\frac{2x^{11/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{1}{3}\left(\frac{2}{21}\left(\frac{1}{5}\left(\frac{4}{9}\left(\frac{131965(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+95066\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)}{}\right.\right.\right.\right.\right.\right.$$

input
```
Int[((2 - 5*x)*x^(13/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(2*x^(11/2)*(74 + 95*x))/(9*(2 + 5*x + 3*x^2)^(3/2)) - (2*((2*x^(7/2)*(148
4 + 1685*x))/(3*Sqrt[2 + 5*x + 3*x^2]) + ((-22910*x^(5/2)*Sqrt[2 + 5*x + 3
*x^2])/21 + (2*((41834*x^(3/2)*Sqrt[2 + 5*x + 3*x^2])/5 + ((-263930*Sqrt[x
]*Sqrt[2 + 5*x + 3*x^2])/9 + (4*(95066*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*
x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sq
rt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (131965*(1 + x)*Sqrt[(2 + 3*x)
/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2]
)))/9)/5))/21)/3))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.08

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(\frac{\left(-\frac{22460}{19683}-\frac{32666x}{19683}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(-\frac{100085}{6561}-\frac{33460x}{2187}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{10x^2\sqrt{3x^3+5x^2+2x}}{189}+\frac{1084x\sqrt{3x^3+5x^2+2x}}{2835}-\frac{9286\sqrt{3x}}{5}\right.}{\sqrt{x}}$
default	$-\frac{2\left(1328364\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+1140792\,\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+54\right.}{}$

input
```
int((2-5*x)*x^(13/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*((-22460/19683-32666/1
9683*x)*(3*x^3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/3)^2-2*x*(-100085/6561-33460/
2187*x)*3^(1/2)/(x*(x^2+5/3*x+2/3))^(1/2)-10/189*x^2*(3*x^3+5*x^2+2*x)^(1/
2)+1084/2835*x*(3*x^3+5*x^2+2*x)^(1/2)-9286/5103*(3*x^3+5*x^2+2*x)^(1/2)-2
11144/15309*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/
2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-760528/76545*(6*x+4)^(1/2)*(3+3*
x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*EllipticE(1/2*(6*x+4)^(
1/2),I*2^(1/2))-EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.54

$$\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}}\,dx =$$
$$-\frac{2\left(5698840\sqrt{3}(9x^4+30x^3+37x^2+20x+4)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-6844752\sqrt{3}(9x^4+\right.}{}$$

input
```
integrate((2-5*x)*x^(13/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
-2/688905*(5698840*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*weierstras
sPInverse(28/27, 80/729, x + 5/9) - 6844752*sqrt(3)*(9*x^4 + 30*x^3 + 37*x
^2 + 20*x + 4)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 8
0/729, x + 5/9)) + 27*(6075*x^6 - 23652*x^5 + 87570*x^4 - 2983836*x^3 - 85
94380*x^2 - 7545152*x - 2111440)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/(9*x^4 + 3
0*x^3 + 37*x^2 + 20*x + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-5*x)*x**(13/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{13}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(13/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(13/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{13}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(13/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(13/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}}\,dx = -\int \frac{x^{13/2}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{5/2}}\,dx$$

input
```
int(-(x^(13/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2),x)
```

output
```
-int((x^(13/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{13/2}}{(2+5x+3x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((2-5*x)*x^(13/2)/(3*x^2+5*x+2)^(5/2),x)
```

output
```
(2*( - 91125*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**6 + 354780*sqrt(x)*sqrt(3*x
**2 + 5*x + 2)*x**5 - 1313550*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**4 + 105337
80*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 + 62343900*sqrt(x)*sqrt(3*x**2 + 5*
x + 2)*x**2 + 85656880*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x + 51394128*sqrt(x)
*sqrt(3*x**2 + 5*x + 2) - 462547152*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)
*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(
x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**4 - 1541823840*int(sqrt(3*x**2 +
 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqr
t(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**3 - 1901582
736*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*s
qrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(
x)),x)*x**2 - 1027882560*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135
*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 6
0*sqrt(x)*x + 8*sqrt(x)),x)*x - 205576512*int(sqrt(3*x**2 + 5*x + 2)/(27*s
qrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186
*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x) + 663628140*int((sqrt(x)*sqrt
(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2
 + 60*x + 8),x)*x**4 + 2212093800*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(
27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**3 +
2728249020*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 +...
```

3.218 $\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 229

$$\begin{aligned}\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}}\,dx &= \frac{2\sqrt{x}(3914+5615x)}{729\,(2+5x+3x^2)^{3/2}}+\frac{33608\sqrt{x}(2+3x)}{729\sqrt{2+5x+3x^2}}\\&\quad-\frac{2\sqrt{x}(50533+55905x)}{729\sqrt{2+5x+3x^2}}+\frac{152}{243}\sqrt{x}\sqrt{2+5x+3x^2}\\&\quad-\frac{2}{27}x^{3/2}\sqrt{2+5x+3x^2}-\frac{33608\sqrt{2}\sqrt{2+5x+3x^2}\,E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{729\sqrt{1+x}\sqrt{2+3x}}\\&\quad+\frac{16040\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{243\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
2/729*x^(1/2)*(3914+5615*x)/(3*x^2+5*x+2)^(3/2)+33608/729*x^(1/2)*(2+3*x)/
(3*x^2+5*x+2)^(1/2)-2/729*x^(1/2)*(50533+55905*x)/(3*x^2+5*x+2)^(1/2)+152/
243*x^(1/2)*(3*x^2+5*x+2)^(1/2)-2/27*x^(3/2)*(3*x^2+5*x+2)^(1/2)-33608/729
*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/
(1+x)^(1/2)/(2+3*x)^(1/2)+16040/243*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*Inve
rseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.78

$$\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{134432+479680x+534680x^2+161784x^3-21276x^4+2484x^5-486x^6+336}{}$$

input
```
Integrate[((2 - 5*x)*x^(11/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(134432 + 479680*x + 534680*x^2 + 161784*x^3 - 21276*x^4 + 2484*x^5 - 486*
x^6 + (33608*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*Elli
pticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] + (14512*I)*Sqrt[2 + 2/x]*Sqrt[3
+ 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3
/2])/(729*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {1233, 25, 1233, 27, 1236, 27, 1236, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{11/2}}{(3x^2+5x+2)^{5/2}}\,dx$$

$$\Big\downarrow \ 1233$$

$$\frac{2}{9}\int -\frac{x^{7/2}(245x+333)}{(3x^2+5x+2)^{3/2}}dx+\frac{2(95x+74)x^{9/2}}{9(3x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2x^{9/2}(95x+74)}{9(3x^2+5x+2)^{3/2}}-\frac{2}{9}\int \frac{x^{7/2}(245x+333)}{(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow 1233$$

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{2}{3}\int -\frac{5x^{3/2}(1761x+1546)}{2\sqrt{3x^2+5x+2}}dx+\frac{4(905x+773)x^{5/2}}{3\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow 27$$

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{4x^{5/2}(905x+773)}{3\sqrt{3x^2+5x+2}}-\frac{5}{3}\int \frac{x^{3/2}(1761x+1546)}{\sqrt{3x^2+5x+2}}dx\right)$$

$$\Big\downarrow 1236$$

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{4x^{5/2}(905x+773)}{3\sqrt{3x^2+5x+2}}-\frac{5}{3}\left(\frac{2}{15}\int -\frac{3\sqrt{x}(2005x+1761)}{\sqrt{3x^2+5x+2}}dx+\frac{1174}{5}\sqrt{3x^2+5x+2}x^{3/2}\right)\right)$$

$$\Big\downarrow 27$$

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{4x^{5/2}(905x+773)}{3\sqrt{3x^2+5x+2}}-\frac{5}{3}\left(\frac{1174}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{2}{5}\int \frac{\sqrt{x}(2005x+1761)}{\sqrt{3x^2+5x+2}}dx\right)\right)$$

$$\Big\downarrow 1236$$

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{4x^{5/2}(905x+773)}{3\sqrt{3x^2+5x+2}}-\frac{5}{3}\left(\frac{1174}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{2}{5}\left(\frac{2}{9}\int -\frac{4201x+4010}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{4010}{9}\sqrt{x}\sqrt{3x^2+5x}\right.\right.\right.$$

$$\Big\downarrow 27$$

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{4x^{5/2}(905x+773)}{3\sqrt{3x^2+5x+2}}-\frac{5}{3}\left(\frac{1174}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{2}{5}\left(\frac{4010}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{1}{9}\int \frac{4201x+4010}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right.\right.\right.$$

$$\Big\downarrow 1240$$

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{4x^{5/2}(905x+773)}{3\sqrt{3x^2+5x+2}}-\frac{5}{3}\left(\frac{1174}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{2}{5}\left(\frac{4010}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\int \frac{4201x+4010}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right.\right.\right.$$

$\downarrow$ 1503

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{4x^{5/2}(905x+773)}{3\sqrt{3x^2+5x+2}}-\frac{5}{3}\left(\frac{1174}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{2}{5}\left(\frac{4010}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(4010\int\frac{1}{\sqrt{3x^2+5x+}}\right.\right.\right.\right.$$

$\downarrow$ 1413

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{4x^{5/2}(905x+773)}{3\sqrt{3x^2+5x+2}}-\frac{5}{3}\left(\frac{1174}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{2}{5}\left(\frac{4010}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(4201\int\frac{x}{\sqrt{3x^2+5x+}}\right.\right.\right.\right.$$

$\downarrow$ 1456

$$\frac{2x^{9/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{4x^{5/2}(905x+773)}{3\sqrt{3x^2+5x+2}}-\frac{5}{3}\left(\frac{1174}{5}x^{3/2}\sqrt{3x^2+5x+2}-\frac{2}{5}\left(\frac{4010}{9}\sqrt{x}\sqrt{3x^2+5x+2}-\frac{2}{9}\left(\frac{2005\sqrt{2}(x+1)\sqrt{\frac{3x+}{x+}}}{\sqrt{3}}\right.\right.\right.\right.$$

input
```
Int[((2 - 5*x)*x^(11/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(2*x^(9/2)*(74 + 95*x))/(9*(2 + 5*x + 3*x^2)^(3/2)) - (2*((4*x^(5/2)*(773
+ 905*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (5*((1174*x^(3/2)*Sqrt[2 + 5*x + 3*x
^2])/5 - (2*((4010*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])/9 - (2*(4201*((Sqrt[x]*(
2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 +
 x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (2005*
Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/
Sqrt[2 + 5*x + 3*x^2]))/9))/5))/3))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(\frac{\left(\frac{7828}{6561}+\frac{11230x}{6561}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(\frac{50533}{2187}+\frac{18635x}{729}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{2x\sqrt{3x^3+5x^2+2x}}{27}+\frac{152\sqrt{3x^3+5x^2+2x}}{243}+\frac{16040\sqrt{6x+4}\,\sqrt{3+3}\cdots}{72\cdots}\cdots\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}\cdots}$
default	$-\frac{2\left(3438\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-25206\,\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+5730\sqrt{6}\cdots\right.}{\cdots}$

input
```
int((2-5*x)*x^(11/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*((7828/6561+11230/6561
*x)*(3*x^3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/3)^2-2*x*(50533/2187+18635/729*x)
*3^(1/2)/(x*(x^2+5/3*x+2/3))^(1/2)-2/27*x*(3*x^3+5*x^2+2*x)^(1/2)+152/243*
(3*x^3+5*x^2+2*x)^(1/2)+16040/729*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)
/(3*x^3+5*x^2+2*x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))+16804/729*
(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*Elli
pticE(1/2*(6*x+4)^(1/2),I*2^(1/2))-EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2)))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.58

$$\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{2\left(60340\sqrt{3}(9\,x^4+30\,x^3+37\,x^2+20\,x+4)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right.}{}$$

input
```
integrate((2-5*x)*x^(11/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/6561*(60340*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*weierstrassPInv
erse(28/27, 80/729, x + 5/9) - 151236*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 2
0*x + 4)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729,
 x + 5/9)) - 27*(81*x^5 - 414*x^4 + 53958*x^3 + 141076*x^2 + 118136*x + 32
080)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-5*x)*x**(11/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{11}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(11/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(11/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{11}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(11/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(11/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}}\,dx = -\int \frac{x^{11/2}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{5/2}}\,dx$$

input
```
int(-(x^(11/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2),x)
```

output
```
-int((x^(11/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{11/2}}{(2+5x+3x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((2-5*x)*x^(11/2)/(3*x^2+5*x+2)^(5/2),x)
```

output
```
(2*( - 1215*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**5 + 6210*sqrt(x)*sqrt(3*x**2
 + 5*x + 2)*x**4 - 53190*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 - 317340*sqrt
(x)*sqrt(3*x**2 + 5*x + 2)*x**2 - 435340*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x
- 261204*sqrt(x)*sqrt(3*x**2 + 5*x + 2) + 2350836*int(sqrt(3*x**2 + 5*x +
2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x*
*3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**4 + 7836120*int(sq
rt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**
4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**
3 + 9664548*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5
 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x +
 8*sqrt(x)),x)*x**2 + 5224080*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6
+ 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**
2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x + 1044816*int(sqrt(3*x**2 + 5*x + 2)/(2
7*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 +
186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x) - 3350970*int((sqrt(x)*sqr
t(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**
2 + 60*x + 8),x)*x**4 - 11169900*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(2
7*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**3 - 1
3776210*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x
**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**2 - 7446600*int((sqrt(x)*sq...
```

3.219 $\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\begin{aligned}\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}}\,dx = &-\frac{2\sqrt{x}(1390+1957x)}{243\,(2+5x+3x^2)^{3/2}} \\ &-\frac{17512\sqrt{x}(2+3x)}{243\sqrt{2+5x+3x^2}}+\frac{14\sqrt{x}(3335+3846x)}{243\sqrt{2+5x+3x^2}}-\frac{10}{81}\sqrt{x}\sqrt{2+5x+3x^2} \\ &+\frac{17512\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{243\sqrt{1+x}\sqrt{2+3x}} \\ &-\frac{7540\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{81\sqrt{2+5x+3x^2}}\end{aligned}$$

output

```
-2/243*x^(1/2)*(1390+1957*x)/(3*x^2+5*x+2)^(3/2)-17512/243*x^(1/2)*(2+3*x)
/(3*x^2+5*x+2)^(1/2)+14/243*x^(1/2)*(3335+3846*x)/(3*x^2+5*x+2)^(1/2)-10/8
1*x^(1/2)*(3*x^2+5*x+2)^(1/2)+17512/243*2^(1/2)*(3*x^2+5*x+2)^(1/2)*Ellipt
icE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-7540/81*2
^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(
1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

$$\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{-2(35024+129880x+155660x^2+58590x^3-1512x^4+135x^5)-17512i\sqrt{2+}}{}$$

input
```
Integrate[((2 - 5*x)*x^(9/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(-2*(35024 + 129880*x + 155660*x^2 + 58590*x^3 - 1512*x^4 + 135*x^5) - (17
512*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*EllipticE[I*A
rcSinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (5108*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(
3/2)*(2 + 5*x + 3*x^2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(243*
Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1233, 25, 1233, 27, 1236, 25, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{9/2}}{(3x^2+5x+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2}{9}\int -\frac{x^{5/2}(150x+259)}{(3x^2+5x+2)^{3/2}}dx+\frac{2(95x+74)x^{7/2}}{9\,(3x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2x^{7/2}(95x+74)}{9\,(3x^2+5x+2)^{3/2}}-\frac{2}{9}\int \frac{x^{5/2}(150x+259)}{(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2x^{7/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{2}{3}\int -\frac{3\sqrt{x}(1885x+1608)}{2\sqrt{3x^2+5x+2}}dx+\frac{2(645x+536)x^{3/2}}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^{7/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{2x^{3/2}(645x+536)}{\sqrt{3x^2+5x+2}}-\int \frac{\sqrt{x}(1885x+1608)}{\sqrt{3x^2+5x+2}}dx\right)$$

$$\Big\downarrow \text{1236}$$

$$\frac{2x^{7/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(-\frac{2}{9}\int -\frac{2189x+1885}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{3770}{9}\sqrt{3x^2+5x+2}\sqrt{x}+\frac{2(645x+536)x^{3/2}}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2x^{7/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{2}{9}\int \frac{2189x+1885}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{3770}{9}\sqrt{3x^2+5x+2}\sqrt{x}+\frac{2(645x+536)x^{3/2}}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{1240}$$

$$\frac{2x^{7/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{4}{9}\int \frac{2189x+1885}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{3770}{9}\sqrt{3x^2+5x+2}\sqrt{x}+\frac{2(645x+536)x^{3/2}}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{1503}$$

$$\frac{2x^{7/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{4}{9}\left(1885\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+2189\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{3770}{9}\sqrt{3x^2+5x+2}\sqrt{x}+\frac{2(645x+536)x^{3/2}}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{1413}$$

$$\frac{2x^{7/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{4}{9}\left(2189\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{1885(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{3770}{9}\sqrt{3x^2+5x+2}\right.$$

$$\Big\downarrow \text{1456}$$

$$\frac{2x^{7/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$

$$\frac{2}{9}\left(\frac{4}{9}\left(\frac{1885(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+2189\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E(\text{arctan}}{3\sqrt{3x^2+5x+2}}\right.\right.\right.$$

input
```
Int[((2 - 5*x)*x^(9/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(2*x^(7/2)*(74 + 95*x))/(9*(2 + 5*x + 3*x^2)^(3/2)) - (2*((2*x^(3/2)*(536
+ 645*x))/Sqrt[2 + 5*x + 3*x^2] - (3770*Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])/9 +
 (4*(2189*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x
)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*
x + 3*x^2])) + (1885*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt
[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])))/9))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(\frac{\left(-\frac{2780}{2187}-\frac{3914x}{2187}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(-\frac{23345}{729}-\frac{8974x}{243}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{10\sqrt{3x^3+5x^2+2x}}{81}-\frac{7540\sqrt{6x+4}\sqrt{3+3x}\sqrt{-6x}\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{243\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\sqrt{3x^2+5x+2}}$
default	$\frac{2\sqrt{3x^2+5x+2}\left(5472\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\sqrt{3+3x}\sqrt{6}\sqrt{-x}\,x^2-13134\,\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\sqrt{3+3x}\sqrt{6}\sqrt{-x}\,x^2\right.}{}$

input
```
int((2-5*x)*x^(9/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*((-2780/2187-3914/2187
*x)*(3*x^3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/3)^2-2*x*(-23345/729-8974/243*x)*
3^(1/2)/(x*(x^2+5/3*x+2/3))^(1/2)-10/81*(3*x^3+5*x^2+2*x)^(1/2)-7540/243*(
6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*EllipticF(
1/2*(6*x+4)^(1/2),I*2^(1/2))-8756/243*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(
1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))-E
llipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.62

$$\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}}\,dx =$$
$$-\frac{2\left(24080\sqrt{3}(9x^4+30x^3+37x^2+20x+4)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-78804\sqrt{3}(9x^4+30x\right.}{}$$

input
```
integrate((2-5*x)*x^(9/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
-2/2187*(24080*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*weierstrassPIn
verse(28/27, 80/729, x + 5/9) - 78804*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 2
0*x + 4)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729,
 x + 5/9)) + 27*(45*x^4 - 26772*x^3 - 68030*x^2 - 56104*x - 15080)*sqrt(3*
x^2 + 5*x + 2)*sqrt(x))/(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-5*x)*x**(9/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{9}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(9/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(9/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{9}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(9/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(9/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}}\,dx = -\int \frac{x^{9/2}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{5/2}}\,dx$$

input
```
int(-(x^(9/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2),x)
```

output
```
-int((x^(9/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{9/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{-1350\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^4+15120\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3+92700\sqrt{x}\,\sqrt{3x^2+}}{}$$

input
```
int((2-5*x)*x^(9/2)/(3*x^2+5*x+2)^(5/2),x)
```

output

```
(2*( - 675*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**4 + 7560*sqrt(x)*sqrt(3*x**2
+ 5*x + 2)*x**3 + 46350*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 + 63260*sqrt(x
)*sqrt(3*x**2 + 5*x + 2)*x + 37956*sqrt(x)*sqrt(3*x**2 + 5*x + 2) - 341604
*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt
(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x))
,x)*x**4 - 1138680*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(
x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt
(x)*x + 8*sqrt(x)),x)*x**3 - 1404372*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x
)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt
(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**2 - 759120*int(sqrt(3*x**2 + 5*
x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x
)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x - 151824*int(sq
rt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**
4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x) + 4
76280*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**
4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**4 + 1587600*int((sqrt(x)*sqrt(3*
x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 +
60*x + 8),x)*x**3 + 1958040*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**
6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**2 + 105840
0*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4...
```

3.220 $\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\begin{aligned}\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}}\,dx = {} & \frac{2\sqrt{x}(506+695x)}{81(2+5x+3x^2)^{3/2}}+\frac{8020\sqrt{x}(2+3x)}{81\sqrt{2+5x+3x^2}} \\ & -\frac{2\sqrt{x}(10273+12075x)}{81\sqrt{2+5x+3x^2}}-\frac{8020\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{81\sqrt{1+x}\sqrt{2+3x}} \\ & +\frac{3340\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{27\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
2/81*x^(1/2)*(506+695*x)/(3*x^2+5*x+2)^(3/2)+8020/81*x^(1/2)*(2+3*x)/(3*x^
2+5*x+2)^(1/2)-2/81*x^(1/2)*(10273+12075*x)/(3*x^2+5*x+2)^(1/2)-8020/81*2^
(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+
x)^(1/2)/(2+3*x)^(1/2)+3340/27*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJa
cobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.32 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{32080+120320x+147100x^2+58212x^3-270x^4+8020i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}(2}{}$$

input
```
Integrate[((2 - 5*x)*x^(7/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(32080 + 120320*x + 147100*x^2 + 58212*x^3 - 270*x^4 + (8020*I)*Sqrt[2 + 2
/x]*Sqrt[3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*EllipticE[I*ArcSinh[Sqrt[2/3]/
Sqrt[x]], 3/2] + (2000*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(3/2)*(2 + 5*x + 3
*x^2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(81*Sqrt[x]*(2 + 5*x +
 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1233, 27, 1233, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{7/2}}{(3x^2+5x+2)^{5/2}}\,dx$$

$$\Big\downarrow \ 1233$$

$$\frac{2}{9}\int -\frac{5x^{3/2}(11x+37)}{(3x^2+5x+2)^{3/2}}dx+\frac{2(95x+74)x^{5/2}}{9\,(3x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{2x^{5/2}(95x+74)}{9\,(3x^2+5x+2)^{3/2}}-\frac{10}{9}\int \frac{x^{3/2}(11x+37)}{(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2x^{5/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{10}{9}\left(\frac{2}{3}\int -\frac{401x+334}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{4\sqrt{x}(206x+167)}{3\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^{5/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{10}{9}\left(\frac{4\sqrt{x}(206x+167)}{3\sqrt{3x^2+5x+2}}-\frac{1}{3}\int \frac{401x+334}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)$$

$$\Big\downarrow \text{1240}$$

$$\frac{2x^{5/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{10}{9}\left(\frac{4\sqrt{x}(206x+167)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\int \frac{401x+334}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1503}$$

$$\frac{2x^{5/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{10}{9}\left(\frac{4\sqrt{x}(206x+167)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\left(334\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+401\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{1413}$$

$$\frac{2x^{5/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{10}{9}\left(\frac{4\sqrt{x}(206x+167)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\left(401\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{167\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)\right.$$

$$\Big\downarrow \text{1456}$$

$$\frac{2x^{5/2}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{10}{9}\left(\frac{4\sqrt{x}(206x+167)}{3\sqrt{3x^2+5x+2}}-\frac{2}{3}\left(\frac{167\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+401\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(\right.\right.\right.$$

input
```
Int[((2 - 5*x)*x^(7/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(2*x^(5/2)*(74 + 95*x))/(9*(2 + 5*x + 3*x^2)^(3/2)) - (10*((4*Sqrt[x]*(167
 + 206*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (2*(401*((Sqrt[x]*(2 + 3*x))/(3*Sqr
t[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[A
rcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (167*Sqrt[2]*(1 + x)*S
qrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*
x^2]))/3))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.17

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(\frac{\left(\frac{1012}{729}+\frac{1390x}{729}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(\frac{10273}{243}+\frac{4025x}{81}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}+\frac{3340\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{81\sqrt{3x^3+5x^2+2x}}+\frac{4010\sqrt{6x+\cdots}}{\cdots}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
default	$-\frac{2\left(3015\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-6015\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+5025\sqrt{6x\cdots}\right.}{\cdots}$

input
```
int((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*((1012/729+1390/729*x)
*(3*x^3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/3)^2-2*x*(10273/243+4025/81*x)*3^(1/
2)/(x*(x^2+5/3*x+2/3))^(1/2)+3340/81*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1
/2)/(3*x^3+5*x^2+2*x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))+4010/81
*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*Ell
ipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))-EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{2\left(10010\sqrt{3}(9x^4+30x^3+37x^2+20x+4)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right.}{}$$

input
```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/729*(10010*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*weierstrassPInve
rse(28/27, 80/729, x + 5/9) - 36090*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*
x + 4)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x
 + 5/9)) - 27*(12075*x^3 + 30398*x^2 + 24940*x + 6680)*sqrt(3*x^2 + 5*x +
2)*sqrt(x))/(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-5*x)*x**(7/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5x-2)x^{\frac{7}{2}}}{(3x^2+5x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(7/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{7}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(7/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}}\,dx = -\int \frac{x^{7/2}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{5/2}}\,dx$$

input
```
int(-(x^(7/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2),x)
```

output
```
-int((x^(7/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{7/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{-270\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^3-1908\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2-2540\sqrt{x}\,\sqrt{3x^2+5x}}{}$$

input
```
int((2-5*x)*x^(7/2)/(3*x^2+5*x+2)^(5/2),x)
```

output
```
(2*( - 135*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**3 - 954*sqrt(x)*sqrt(3*x**2 +
 5*x + 2)*x**2 - 1270*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x - 762*sqrt(x)*sqrt(
3*x**2 + 5*x + 2) + 6858*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135
*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 6
0*sqrt(x)*x + 8*sqrt(x)),x)*x**4 + 22860*int(sqrt(3*x**2 + 5*x + 2)/(27*sq
rt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*
sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**3 + 28194*int(sqrt(3*x**2 +
 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqr
t(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**2 + 15240*i
nt(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x
)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x
)*x + 3048*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5
+ 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x +
8*sqrt(x)),x) - 8505*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135
*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**4 - 28350*int((sq
rt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3
+ 186*x**2 + 60*x + 8),x)*x**3 - 34965*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)
*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x*
*2 - 18900*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 27
9*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x - 3780*int((sqrt(x)*sqrt(...
```

3.221 $\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\begin{aligned}\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx &= -\frac{2\sqrt{x}(190+253x)}{27\left(2+5x+3x^2\right)^{3/2}}-\frac{3464\sqrt{x}(2+3x)}{27\sqrt{2+5x+3x^2}}\\ &\quad+\frac{2\sqrt{x}(4385+5196x)}{27\sqrt{2+5x+3x^2}}+\frac{3464\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{27\sqrt{1+x}\sqrt{2+3x}}\\ &\quad-\frac{1430\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{9\sqrt{2+5x+3x^2}}\end{aligned}$$

output

```
-2/27*x^(1/2)*(190+253*x)/(3*x^2+5*x+2)^(3/2)-3464/27*x^(1/2)*(2+3*x)/(3*x
^2+5*x+2)^(1/2)+2/27*x^(1/2)*(4385+5196*x)/(3*x^2+5*x+2)^(1/2)+3464/27*2^(
1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x
)^(1/2)/(2+3*x)^(1/2)-1430/9*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJaco
biAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{-2(6928+26060x+32020x^2+12825x^3)-3464i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}(2+5x+$$

input
```
Integrate[((2 - 5*x)*x^(5/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(-2*(6928 + 26060*x + 32020*x^2 + 12825*x^3) - (3464*I)*Sqrt[2 + 2/x]*Sqrt
[3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]]
, 3/2] - (826*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*Ell
ipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(27*Sqrt[x]*(2 + 5*x + 3*x^2)^(
3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1233, 25, 1234, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{5/2}}{(3x^2+5x+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1233}$$

$$\frac{2}{9}\int -\frac{(111-40x)\sqrt{x}}{(3x^2+5x+2)^{3/2}}dx+\frac{2(95x+74)x^{3/2}}{9(3x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2x^{3/2}(95x+74)}{9(3x^2+5x+2)^{3/2}}-\frac{2}{9}\int \frac{(111-40x)\sqrt{x}}{(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \text{1234}$$

$$\frac{2x^{3/2}(95x+74)}{9(3x^2+5x+2)^{3/2}}-\frac{2}{9}\left(-2\int -\frac{866x+715}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2\sqrt{x}(866x+715)}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^{3/2}(95x+74)}{9(3x^2+5x+2)^{3/2}}-\frac{2}{9}\left(\int \frac{866x+715}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2\sqrt{x}(866x+715)}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{1240}$$

$$\frac{2x^{3/2}(95x+74)}{9(3x^2+5x+2)^{3/2}}-\frac{2}{9}\left(2\int \frac{866x+715}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{2\sqrt{x}(866x+715)}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{1503}$$

$$\frac{2x^{3/2}(95x+74)}{9(3x^2+5x+2)^{3/2}}-$$
$$\frac{2}{9}\left(2\left(715\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+866\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2\sqrt{x}(866x+715)}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{1413}$$

$$\frac{2x^{3/2}(95x+74)}{9(3x^2+5x+2)^{3/2}}-$$
$$\frac{2}{9}\left(2\left(866\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{715(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{2\sqrt{x}(866x+715)}{\sqrt{3x^2+5x+2}}\right)$$

$$\Big\downarrow \text{1456}$$

$$\frac{2x^{3/2}(95x+74)}{9(3x^2+5x+2)^{3/2}}-$$
$$\frac{2}{9}\left(2\left(\frac{715(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+866\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{\right.\right.}{3\sqrt{3x^2+5x+2}}\right.\right.\right.$$

input
```
Int[((2 - 5*x)*x^(5/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(2*x^(3/2)*(74 + 95*x))/(9*(2 + 5*x + 3*x^2)^(3/2)) - (2*((-2*Sqrt[x]*(715
 + 866*x))/Sqrt[2 + 5*x + 3*x^2] + 2*(866*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 +
 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan
[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (715*(1 + x)*Sqrt[(2 + 3*x)
/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2]
))))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.17

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(\frac{\left(-\frac{380}{243}-\frac{506x}{243}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(-\frac{4385}{81}-\frac{1732x}{27}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{1430\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{27\sqrt{3x^3+5x^2+2x}}-\frac{1732\sqrt{6x+}}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
default	$\frac{2\left(1359\,\text{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-2598\,\text{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+2265\sqrt{6x+4}\right.}{}$

input
```
int((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*((-380/243-506/243*x)*
(3*x^3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/3)^2-2*x*(-4385/81-1732/27*x)*3^(1/2)
/(x*(x^2+5/3*x+2/3))^(1/2)-1430/27*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2
)/(3*x^3+5*x^2+2*x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-1732/27*(
6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*Ellip
ticE(1/2*(6*x+4)^(1/2),I*2^(1/2))-EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx =$$
$$-\frac{2\left(4210\sqrt{3}(9x^4+30x^3+37x^2+20x+4)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-15588\sqrt{3}(9x^4+30x^3\right.}{}$$

input
```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
-2/243*(4210*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*weierstrassPInve
rse(28/27, 80/729, x + 5/9) - 15588*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*
x + 4)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x
 + 5/9)) - 27*(5196*x^3 + 13045*x^2 + 10688*x + 2860)*sqrt(3*x^2 + 5*x + 2
)*sqrt(x))/(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-5*x)*x**(5/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{5}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(5/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{5}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(5/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx = -\int \frac{x^{5/2}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{5/2}}\,dx$$

input
```
int(-(x^(5/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2),x)
```

output
```
-int((x^(5/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{5/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{450\sqrt{x}\,\sqrt{3x^2+5x+2}\,x^2+440\sqrt{x}\,\sqrt{3x^2+5x+2}\,x+264\sqrt{x}\,\sqrt{3x^2+5x+2}\,-}{}$$

input
```
int((2-5*x)*x^(5/2)/(3*x^2+5*x+2)^(5/2),x)
```

output
```
(2*(225*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2 + 220*sqrt(x)*sqrt(3*x**2 + 5*
x + 2)*x + 132*sqrt(x)*sqrt(3*x**2 + 5*x + 2) - 1188*int(sqrt(3*x**2 + 5*x
 + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)
*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**4 - 3960*int(sq
rt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**
4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**
3 - 4884*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 +
279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*
sqrt(x)),x)*x**2 - 2640*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*
sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60
*sqrt(x)*x + 8*sqrt(x)),x)*x - 528*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*
x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x
)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x) - 1215*int((sqrt(x)*sqrt(3*x**2 + 5*
x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8)
,x)*x**4 - 4050*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5
 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**3 - 4995*int((sqrt(x)*
sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*
x**2 + 60*x + 8),x)*x**2 - 2700*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27
*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x - 540*i
nt((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + ...
```

3.222 $\int \frac{(2-5x)x^{3/2}}{\left(2+5x+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{2\sqrt{x}(74+95x)}{9\,(2+5x+3x^2)^{3/2}} + \frac{1450\sqrt{x}(2+3x)}{9\sqrt{2+5x+3x^2}} - \frac{2\sqrt{x}(1831+2175x)}{9\sqrt{2+5x+3x^2}} - \frac{1450\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{9\sqrt{1+x}\sqrt{2+3x}} + \frac{598\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{3\sqrt{2+5x+3x^2}}$$

output
```
2/9*x^(1/2)*(74+95*x)/(3*x^2+5*x+2)^(3/2)+1450/9*x^(1/2)*(2+3*x)/(3*x^2+5*
x+2)^(1/2)-2/9*x^(1/2)*(1831+2175*x)/(3*x^2+5*x+2)^(1/2)-1450/9*2^(1/2)*(3
*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)
/(2+3*x)^(1/2)+598/3*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arc
tan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.90

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{5800+21824x+26830x^2+10764x^3+1450i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}(2+5x+3x^2}{9\sqrt{\ }}$$

input
```
Integrate[((2 - 5*x)*x^(3/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(5800 + 21824*x + 26830*x^2 + 10764*x^3 + (1450*I)*Sqrt[2 + 2/x]*Sqrt[3 +
2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2
] + (344*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*Elliptic
F[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(9*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1233, 25, 1235, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)x^{3/2}}{(3x^2+5x+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2}{9}\int -\frac{37-135x}{\sqrt{x}\,(3x^2+5x+2)^{3/2}}dx + \frac{2\sqrt{x}(95x+74)}{9\,(3x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{x}(95x+74)}{9\,(3x^2+5x+2)^{3/2}} - \frac{2}{9}\int \frac{37-135x}{\sqrt{x}\,(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2\sqrt{x}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{\sqrt{x}(2175x+1831)}{\sqrt{3x^2+5x+2}}-\int\frac{3(725x+598)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\sqrt{x}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{\sqrt{x}(2175x+1831)}{\sqrt{3x^2+5x+2}}-\frac{3}{2}\int\frac{725x+598}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)$$

$$\Big\downarrow \text{ 1240}$$

$$\frac{2\sqrt{x}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-\frac{2}{9}\left(\frac{\sqrt{x}(2175x+1831)}{\sqrt{3x^2+5x+2}}-3\int\frac{725x+598}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{2\sqrt{x}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{\sqrt{x}(2175x+1831)}{\sqrt{3x^2+5x+2}}-3\left(598\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+725\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{ 1413}$$

$$\frac{2\sqrt{x}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{\sqrt{x}(2175x+1831)}{\sqrt{3x^2+5x+2}}-3\left(725\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{299\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)\right)$$

$$\Big\downarrow \text{ 1456}$$

$$\frac{2\sqrt{x}(95x+74)}{9\left(3x^2+5x+2\right)^{3/2}}-$$
$$\frac{2}{9}\left(\frac{\sqrt{x}(2175x+1831)}{\sqrt{3x^2+5x+2}}-3\left(\frac{299\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+725\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(\right.\right.\right.$$

input
```
Int[((2 - 5*x)*x^(3/2))/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(2*Sqrt[x]*(74 + 95*x))/(9*(2 + 5*x + 3*x^2)^(3/2)) - (2*((Sqrt[x]*(1831 +
 2175*x))/Sqrt[2 + 5*x + 3*x^2] - 3*(725*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 +
5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[
Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (299*Sqrt[2]*(1 + x)*Sqrt[(2
 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2]))
)/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.17

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(\frac{\left(\frac{148}{81}+\frac{190x}{81}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(\frac{1831}{27}+\frac{725x}{9}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}+\frac{598\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{9\sqrt{3x^3+5x^2+2x}}+\frac{725\sqrt{6x+4}\,\sqrt{3+}}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
default	$-\frac{\left(1143\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-2175\,\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+1905\sqrt{6x+}\right.}{}$

input
```
int((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*((148/81+190/81*x)*(3*
x^3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/3)^2-2*x*(1831/27+725/9*x)*3^(1/2)/(x*(x
^2+5/3*x+2/3))^(1/2)+598/9*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3
+5*x^2+2*x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))+725/9*(6*x+4)^(1/
2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*EllipticE(1/2*(
6*x+4)^(1/2),I*2^(1/2))-EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{2\left(1757\sqrt{3}(9x^4+30x^3+37x^2+20x+4)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)\right.}{}$$

input
```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/81*(1757*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*weierstrassPInvers
e(28/27, 80/729, x + 5/9) - 6525*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x +
 4)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x +
5/9)) - 27*(2175*x^3 + 5456*x^2 + 4470*x + 1196)*sqrt(3*x^2 + 5*x + 2)*sqr
t(x))/(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
```

Sympy [F]

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{5/2}}\,dx =$$

$$-\int\left(-\frac{2x^{\frac{3}{2}}}{9x^4\sqrt{3x^2+5x+2}+30x^3\sqrt{3x^2+5x+2}+37x^2\sqrt{3x^2+5x+2}+20x\sqrt{3x^2+5x+2}+4\sqrt{3x^2+}}\right.$$

$$-\int\frac{5x^{\frac{5}{2}}}{9x^4\sqrt{3x^2+5x+2}+30x^3\sqrt{3x^2+5x+2}+37x^2\sqrt{3x^2+5x+2}+20x\sqrt{3x^2+5x+2}+4\sqrt{3x^2+5x}}$$

input
```
integrate((2-5*x)*x**(3/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
-Integral(-2*x**(3/2)/(9*x**4*sqrt(3*x**2 + 5*x + 2) + 30*x**3*sqrt(3*x**2
 + 5*x + 2) + 37*x**2*sqrt(3*x**2 + 5*x + 2) + 20*x*sqrt(3*x**2 + 5*x + 2)
 + 4*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5*x**(5/2)/(9*x**4*sqrt(3*x**2
 + 5*x + 2) + 30*x**3*sqrt(3*x**2 + 5*x + 2) + 37*x**2*sqrt(3*x**2 + 5*x +
 2) + 20*x*sqrt(3*x**2 + 5*x + 2) + 4*sqrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{3}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*x^(3/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)x^{\frac{3}{2}}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*x^(3/2)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{5/2}}\,dx = -\int \frac{x^{3/2}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{5/2}}\,dx$$

input
```
int(-(x^(3/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2),x)
```

output
```
-int((x^(3/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)x^{3/2}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x+6\sqrt{x}\,\sqrt{3x^2+5x+2}-54\Big(\int \frac{\sqrt{3x^2}}{27\sqrt{x}\,x^6+135\sqrt{x}\,x^5+279\sqrt{x}\,x^4+3}}{}$$

input
```
int((2-5*x)*x^(3/2)/(3*x^2+5*x+2)^(5/2),x)
```

output

```
(10*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x + 6*sqrt(x)*sqrt(3*x**2 + 5*x + 2) -
54*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sq
rt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x
)),x)*x**4 - 180*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)
*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x
)*x + 8*sqrt(x)),x)*x**3 - 222*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6
 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x*
*2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**2 - 120*int(sqrt(3*x**2 + 5*x + 2)/(2
7*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 +
186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x - 24*int(sqrt(3*x**2 + 5
*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(
x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x) + 567*int((sqrt(
x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 1
86*x**2 + 60*x + 8),x)*x**4 + 1890*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/
(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**3 +
 2331*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**
4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**2 + 1260*int((sqrt(x)*sqrt(3*x**
2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*
x + 8),x)*x + 252*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x*
*5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x))/(9*(9*x**4 + 30*x**...
```

3.223 $\int \frac{(2-5x)\sqrt{x}}{\left(2+5x+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{(2-5x)\sqrt{x}}{\left(2+5x+3x^2\right)^{5/2}}\,dx = -\frac{2\sqrt{x}(30+37x)}{3\left(2+5x+3x^2\right)^{3/2}} - \frac{198\sqrt{x}(2+3x)}{\sqrt{2+5x+3x^2}} + \frac{2\sqrt{x}(250+297x)}{\sqrt{2+5x+3x^2}} + \frac{198\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{\sqrt{1+x}\sqrt{2+3x}} - \frac{245\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}$$

output

```
-2/3*x^(1/2)*(30+37*x)/(3*x^2+5*x+2)^(3/2)-198*x^(1/2)*(2+3*x)/(3*x^2+5*x+
2)^(1/2)+2*x^(1/2)*(250+297*x)/(3*x^2+5*x+2)^(1/2)+198*2^(1/2)*(3*x^2+5*x+
2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^
(1/2)-245*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)
),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{5/2}}\,dx = -\frac{2(1188+4470x+5494x^2+2205x^3)}{3\sqrt{x}\,(2+5x+3x^2)^{3/2}} - \frac{198i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}xE\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)|\frac{3}{2}\right)}{\sqrt{2+5x+3x^2}} - \frac{47i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right),\frac{3}{2}\right)}{\sqrt{2+5x+3x^2}}$$

input
```
Integrate[((2 - 5*x)*Sqrt[x])/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(-2*(1188 + 4470*x + 5494*x^2 + 2205*x^3))/(3*Sqrt[x]*(2 + 5*x + 3*x^2)^(3
/2)) - ((198*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x*EllipticE[I*ArcSinh[Sqrt[2/3
]/Sqrt[x]], 3/2])/Sqrt[2 + 5*x + 3*x^2] - ((47*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2
/x]*x*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/Sqrt[2 + 5*x + 3*x^2]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1234, 27, 1235, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2-5x)\sqrt{x}}{(3x^2+5x+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1234}$$

$$-\frac{2}{3}\int -\frac{3(10-37x)}{2\sqrt{x}\,(3x^2+5x+2)^{3/2}}dx - \frac{2\sqrt{x}(37x+30)}{3\,(3x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\int \frac{10-37x}{\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}dx-\frac{2\sqrt{x}(37x+30)}{3\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1235$$

$$-\int \frac{297x+245}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{2\sqrt{x}(37x+30)}{3\left(3x^2+5x+2\right)^{3/2}}+\frac{2\sqrt{x}(297x+250)}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \ 1240$$

$$-2\int \frac{297x+245}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{2\sqrt{x}(37x+30)}{3\left(3x^2+5x+2\right)^{3/2}}+\frac{2\sqrt{x}(297x+250)}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \ 1503$$

$$-2\left(245\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+297\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2\sqrt{x}(37x+30)}{3\left(3x^2+5x+2\right)^{3/2}}+\frac{2\sqrt{x}(297x+250)}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \ 1413$$

$$-2\left(297\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{245(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{2\sqrt{x}(37x+30)}{3\left(3x^2+5x+2\right)^{3/2}}+\frac{2\sqrt{x}(297x+250)}{\sqrt{3x^2+5x+2}}$$

$$\Big\downarrow \ 1456$$

$$-2\left(\frac{245(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+297\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}E\left(\arctan\left(\sqrt{x}\right.\right.}{3\sqrt{3x^2+5x+2}}\right.\right.$$
$$\frac{2\sqrt{x}(37x+30)}{3\left(3x^2+5x+2\right)^{3/2}}+\frac{2\sqrt{x}(297x+250)}{\sqrt{3x^2+5x+2}}$$

input
```
Int[((2 - 5*x)*Sqrt[x])/(2 + 5*x + 3*x^2)^(5/2),x]
```

output
```
(-2*Sqrt[x]*(30 + 37*x))/(3*(2 + 5*x + 3*x^2)^(3/2)) + (2*Sqrt[x]*(250 + 2
97*x))/Sqrt[2 + 5*x + 3*x^2] - 2*(297*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x
 + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqr
t[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (245*(1 + x)*Sqrt[(2 + 3*x)/(1
+ x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(\frac{\left(-\frac{20}{9}-\frac{74x}{27}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(-\frac{250}{3}-99x\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{245\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3\sqrt{3x^3+5x^2+2x}}-\frac{99\sqrt{6x+4}\,\sqrt{3+3x}}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
default	$\frac{\left(156\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-297\,\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+260\sqrt{6x+4}\,\sqrt{3-}\right.}{}$

input
```
int((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*((-20/9-74/27*x)*(3*x^
3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/3)^2-2*x*(-250/3-99*x)*3^(1/2)/(x*(x^2+5/3
*x+2/3))^(1/2)-245/3*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2
+2*x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))-99*(6*x+4)^(1/2)*(3+3*x
)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*EllipticE(1/2*(6*x+4)^(1
/2),I*2^(1/2))-EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.70

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{5/2}}\,dx = -\frac{2\left(80\sqrt{3}(9x^4+30x^3+37x^2+20x+4)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-297\sqrt{3}(9x^4+30x^3+37\right.}{3}$$

input
```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(80*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*weierstrassPInverse(
28/27, 80/729, x + 5/9) - 297*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x + 5/9
)) - (2673*x^3 + 6705*x^2 + 5495*x + 1470)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/
(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
```

Sympy [F]

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{5/2}}\,dx = -\int \left(-\frac{2\sqrt{x}}{9x^4\sqrt{3x^2+5x+2}+30x^3\sqrt{3x^2+5x+2}+37x^2\sqrt{3x^2+5x+2}+20x\sqrt{3x^2+5x+2}+4\sqrt{3x^2+\cdots}}\right.$$
$$-\int \frac{5x^{\frac{3}{2}}}{9x^4\sqrt{3x^2+5x+2}+30x^3\sqrt{3x^2+5x+2}+37x^2\sqrt{3x^2+5x+2}+20x\sqrt{3x^2+5x+2}+4\sqrt{3x^2+5x\cdots}}$$

input
```
integrate((2-5*x)*x**(1/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
-Integral(-2*sqrt(x)/(9*x**4*sqrt(3*x**2 + 5*x + 2) + 30*x**3*sqrt(3*x**2
+ 5*x + 2) + 37*x**2*sqrt(3*x**2 + 5*x + 2) + 20*x*sqrt(3*x**2 + 5*x + 2)
+ 4*sqrt(3*x**2 + 5*x + 2)), x) - Integral(5*x**(3/2)/(9*x**4*sqrt(3*x**2
+ 5*x + 2) + 30*x**3*sqrt(3*x**2 + 5*x + 2) + 37*x**2*sqrt(3*x**2 + 5*x +
2) + 20*x*sqrt(3*x**2 + 5*x + 2) + 4*sqrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)\sqrt{x}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)*sqrt(x)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{(5\,x-2)\sqrt{x}}{(3\,x^2+5\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)*sqrt(x)/(3*x^2 + 5*x + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{5/2}}\,dx = -\int \frac{\sqrt{x}\,(5\,x-2)}{(3\,x^2+5\,x+2)^{5/2}}\,dx$$

input
```
int(-(x^(1/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2),x)
```

output
```
-int((x^(1/2)*(5*x - 2))/(5*x + 3*x^2 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(2-5x)\sqrt{x}}{(2+5x+3x^2)^{5/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{3x^2+5x+2}+18\left(\int \frac{\sqrt{3x^2+5x+2}}{27\sqrt{x}\,x^6+135\sqrt{x}\,x^5+279\sqrt{x}\,x^4+305\sqrt{x}\,x^3+186\sqrt{x}\,x^2+60\sqrt{x}\,x+8}\right.}{}$$

input
```
int((2-5*x)*x^(1/2)/(3*x^2+5*x+2)^(5/2),x)
```

output

```
(2*( - sqrt(x)*sqrt(3*x**2 + 5*x + 2) + 9*int(sqrt(3*x**2 + 5*x + 2)/(27*s
qrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186
*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**4 + 30*int(sqrt(3*x**2 + 5
*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(
x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**3 + 37*int(sq
rt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**
4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**
2 + 20*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 27
9*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sq
rt(x)),x)*x + 4*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*
x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)
*x + 8*sqrt(x)),x) - 180*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 +
 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**4 - 600*int((
sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**
3 + 186*x**2 + 60*x + 8),x)*x**3 - 740*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)
*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x*
*2 - 400*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*
x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x - 80*int((sqrt(x)*sqrt(3*x**2
+ 5*x + 2)*x)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x
+ 8),x)))/(5*(9*x**4 + 30*x**3 + 37*x**2 + 20*x + 4))
```

3.224 $\int \frac{2-5x}{\sqrt{x}(2+5x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 181

$$\int \frac{2-5x}{\sqrt{x}\left(2+5x+3x^2\right)^{5/2}}\,dx=\frac{2\sqrt{x}(38+45x)}{3\left(2+5x+3x^2\right)^{3/2}}+\frac{715\sqrt{x}(2+3x)}{3\sqrt{2+5x+3x^2}}-\frac{5\sqrt{x}(361+429x)}{3\sqrt{2+5x+3x^2}}-\frac{715\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x}\sqrt{2+3x}}+\frac{295\sqrt{2}\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2+5x+3x^2}}$$

output

```
2/3*x^(1/2)*(38+45*x)/(3*x^2+5*x+2)^(3/2)+715/3*x^(1/2)*(2+3*x)/(3*x^2+5*x
+2)^(1/2)-5/3*x^(1/2)*(361+429*x)/(3*x^2+5*x+2)^(1/2)-715/3*2^(1/2)*(3*x^2
+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+
3*x)^(1/2)+295*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^
(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.92

$$\int \frac{2-5x}{\sqrt{x}\,(2+5x+3x^2)^{5/2}}\,dx = \frac{2(1430+5383x+6615x^2+2655x^3)+715i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}(2+5x+3x$$

input
```
Integrate[(2 - 5*x)/(Sqrt[x]*(2 + 5*x + 3*x^2)^(5/2)),x]
```

output
```
(2*(1430 + 5383*x + 6615*x^2 + 2655*x^3) + (715*I)*Sqrt[2 + 2/x]*Sqrt[3 +
2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2
] + (170*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*Elliptic
F[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(3*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1235, 27, 1235, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{\sqrt{x}\,(3x^2+5x+2)^{5/2}}dx$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{2\sqrt{x}(45x+38)}{3\,(3x^2+5x+2)^{3/2}}-\frac{1}{3}\int \frac{5(7-27x)}{\sqrt{x}\,(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\sqrt{x}(45x+38)}{3\,(3x^2+5x+2)^{3/2}}-\frac{5}{3}\int \frac{7-27x}{\sqrt{x}\,(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{2\sqrt{x}(45x+38)}{3\left(3x^2+5x+2\right)^{3/2}}-\frac{5}{3}\left(\frac{\sqrt{x}(429x+361)}{\sqrt{3x^2+5x+2}}-\int\frac{3(143x+118)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)$$

$\Big\downarrow$ 27

$$\frac{2\sqrt{x}(45x+38)}{3\left(3x^2+5x+2\right)^{3/2}}-\frac{5}{3}\left(\frac{\sqrt{x}(429x+361)}{\sqrt{3x^2+5x+2}}-\frac{3}{2}\int\frac{143x+118}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)$$

$\Big\downarrow$ 1240

$$\frac{2\sqrt{x}(45x+38)}{3\left(3x^2+5x+2\right)^{3/2}}-\frac{5}{3}\left(\frac{\sqrt{x}(429x+361)}{\sqrt{3x^2+5x+2}}-3\int\frac{143x+118}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)$$

$\Big\downarrow$ 1503

$$\frac{2\sqrt{x}(45x+38)}{3\left(3x^2+5x+2\right)^{3/2}}-\frac{5}{3}\left(\frac{\sqrt{x}(429x+361)}{\sqrt{3x^2+5x+2}}-3\left(118\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+143\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)$$

$\Big\downarrow$ 1413

$$\frac{2\sqrt{x}(45x+38)}{3\left(3x^2+5x+2\right)^{3/2}}-\frac{5}{3}\left(\frac{\sqrt{x}(429x+361)}{\sqrt{3x^2+5x+2}}-3\left(143\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{59\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}\right)\right)$$

$\Big\downarrow$ 1456

$$\frac{2\sqrt{x}(45x+38)}{3\left(3x^2+5x+2\right)^{3/2}}-\frac{5}{3}\left(\frac{\sqrt{x}(429x+361)}{\sqrt{3x^2+5x+2}}-3\left(\frac{59\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+143\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+\right.\right.\right.$$

input

```
Int[(2 - 5*x)/(Sqrt[x]*(2 + 5*x + 3*x^2)^(5/2)),x]
```

output

```
(2*Sqrt[x]*(38 + 45*x))/(3*(2 + 5*x + 3*x^2)^(3/2)) - (5*((Sqrt[x]*(361 +
429*x))/Sqrt[2 + 5*x + 3*x^2] - 3*(143*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*
x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sq
rt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (59*Sqrt[2]*(1 + x)*Sqrt[(2 +
3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x + 3*x^2])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(\frac{\left(\frac{76}{27}+\frac{10x}{3}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(\frac{1805}{18}+\frac{715x}{6}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}+\frac{295\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)}{3\sqrt{3x^3+5x^2+2x}}+\frac{715\sqrt{6x+4}\,\sqrt{3+3x}}{}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
default	$-\frac{\left(1125\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-2145\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+1875\sqrt{6x+}\right.}{}$

input
```
int((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*((76/27+10/3*x)*(3*x^3
+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/3)^2-2*x*(1805/18+715/6*x)*3^(1/2)/(x*(x^2+
5/3*x+2/3))^(1/2)+295/3*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*
x^2+2*x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))+715/6*(6*x+4)^(1/2)*
(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*EllipticE(1/2*(6*x
+4)^(1/2),I*2^(1/2))-EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{2-5x}{\sqrt{x}\,(2+5x+3x^2)^{5/2}}\,dx = \frac{1735\,\sqrt{3}(9\,x^4+30\,x^3+37\,x^2+20\,x+4)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)}{}$$

input
```
integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(1735*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)*weierstrassPInvers
e(28/27, 80/729, x + 5/9) - 6435*sqrt(3)*(9*x^4 + 30*x^3 + 37*x^2 + 20*x +
 4)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/729, x +
5/9)) - 9*(6435*x^3 + 16140*x^2 + 13225*x + 3534)*sqrt(3*x^2 + 5*x + 2)*sq
rt(x))/(9*x^4 + 30*x^3 + 37*x^2 + 20*x + 4)
```

Sympy [F]

$$\int \frac{2-5x}{\sqrt{x}\,(2+5x+3x^2)^{5/2}}\,dx = -\int \frac{5\sqrt{x}}{9x^4\sqrt{3x^2+5x+2}+30x^3\sqrt{3x^2+5x+2}+37x^2\sqrt{3x^2+5x+2}+20x\sqrt{3x^2+5x+2}+4\sqrt{3x^2+5x}$$
$$-\int \left(-\frac{2}{9x^{\frac{9}{2}}\sqrt{3x^2+5x+2}+30x^{\frac{7}{2}}\sqrt{3x^2+5x+2}+37x^{\frac{5}{2}}\sqrt{3x^2+5x+2}+20x^{\frac{3}{2}}\sqrt{3x^2+5x+2}+4\sqrt{x}\sqrt{}}\right.$$

input
```
integrate((2-5*x)/x**(1/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
-Integral(5*sqrt(x)/(9*x**4*sqrt(3*x**2 + 5*x + 2) + 30*x**3*sqrt(3*x**2 +
 5*x + 2) + 37*x**2*sqrt(3*x**2 + 5*x + 2) + 20*x*sqrt(3*x**2 + 5*x + 2) +
 4*sqrt(3*x**2 + 5*x + 2)), x) - Integral(-2/(9*x**(9/2)*sqrt(3*x**2 + 5*x
 + 2) + 30*x**(7/2)*sqrt(3*x**2 + 5*x + 2) + 37*x**(5/2)*sqrt(3*x**2 + 5*x
 + 2) + 20*x**(3/2)*sqrt(3*x**2 + 5*x + 2) + 4*sqrt(x)*sqrt(3*x**2 + 5*x +
 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{\sqrt{x}\,(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{5\,x-2}{(3\,x^2+5\,x+2)^{\frac{5}{2}}\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/((3*x^2 + 5*x + 2)^(5/2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{2-5x}{\sqrt{x}\left(2+5x+3x^2\right)^{5/2}}\,dx=\int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{5}{2}}\sqrt{x}}\,dx$$

input
```
integrate((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/((3*x^2 + 5*x + 2)^(5/2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{\sqrt{x}\left(2+5x+3x^2\right)^{5/2}}\,dx=-\int \frac{5\,x-2}{\sqrt{x}\left(3\,x^2+5\,x+2\right)^{5/2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(1/2)*(5*x + 3*x^2 + 2)^(5/2)),x)
```

output
```
-int((5*x - 2)/(x^(1/2)*(5*x + 3*x^2 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{\sqrt{x}\left(2+5x+3x^2\right)^{5/2}}\,dx=\frac{-10\sqrt{x}\,\sqrt{3x^2+5x+2}\,x+108\left(\int \frac{\sqrt{3x^2+5x+2}}{27\sqrt{x}\,x^6+135\sqrt{x}\,x^5+279\sqrt{x}\,x^4+305\sqrt{x}\,x^3+186\sqrt{x}\,x^2+}\right.}{}$$

input
```
int((2-5*x)/x^(1/2)/(3*x^2+5*x+2)^(5/2),x)
```

output
```
( - 10*sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x + 108*int(sqrt(3*x**2 + 5*x + 2)/(
27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 +
 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**4 + 360*int(sqrt(3*x**
2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*
sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**3 + 444*
int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(
x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),
x)*x**2 + 240*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x*
*5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x
 + 8*sqrt(x)),x)*x + 48*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*
sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60
*sqrt(x)*x + 8*sqrt(x)),x) - 405*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2)
/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**4
- 1350*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2)/(27*x**6 + 135*x**5 + 279
*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x**3 - 1665*int((sqrt(x)*sqrt(3
*x**2 + 5*x + 2)*x**2)/(27*x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**
2 + 60*x + 8),x)*x**2 - 900*int((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2)/(27*
x**6 + 135*x**5 + 279*x**4 + 305*x**3 + 186*x**2 + 60*x + 8),x)*x - 180*in
t((sqrt(x)*sqrt(3*x**2 + 5*x + 2)*x**2)/(27*x**6 + 135*x**5 + 279*x**4 + 3
05*x**3 + 186*x**2 + 60*x + 8),x))/(6*(9*x**4 + 30*x**3 + 37*x**2 + 20*...
```

3.225 $\int \frac{2-5x}{x^{3/2}(2+5x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = -\frac{2}{\sqrt{x}\left(2+5x+3x^2\right)^{3/2}} - \frac{\sqrt{x}(115+123x)}{3\left(2+5x+3x^2\right)^{3/2}} - \frac{838\sqrt{x}(2+3x)}{3\sqrt{2+5x+3x^2}} + \frac{\sqrt{x}(2105+2514x)}{3\sqrt{2+5x+3x^2}} + \frac{838\sqrt{2}\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)\left|-\frac{1}{2}\right.\right)}{3\sqrt{1+x}\sqrt{2+3x}} - \frac{695\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x+3x^2}}$$

output
```
-2/x^(1/2)/(3*x^2+5*x+2)^(3/2)-1/3*x^(1/2)*(115+123*x)/(3*x^2+5*x+2)^(3/2)
-838/3*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+1/3*x^(1/2)*(2105+2514*x)/(3*x^
2+5*x+2)^(1/2)+838/3*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1+x)^(
1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-695/2*2^(1/2)*(1+x)^(1/2)*(2
+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*x+2)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \frac{-2(3358+12665x+15576x^2+6255x^3)-1676i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}x^{3/2}(2+5\ldots}{\ldots}$$

input
```
Integrate[(2 - 5*x)/(x^(3/2)*(2 + 5*x + 3*x^2)^(5/2)),x]
```

output
```
(-2*(3358 + 12665*x + 15576*x^2 + 6255*x^3) - (1676*I)*Sqrt[2 + 2/x]*Sqrt[
3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt[x]],
 3/2] - (409*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(3/2)*(2 + 5*x + 3*x^2)*Elli
pticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(6*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/
2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1235, 25, 1235, 27, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{x^{3/2}\left(3x^2+5x+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}-\frac{1}{3}\int -\frac{225x+41}{x^{3/2}\left(3x^2+5x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{225x+41}{x^{3/2}\left(3x^2+5x+2\right)^{3/2}}dx+\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{3}\left(-\int \frac{2085x+1676}{2x^{3/2}\sqrt{3x^2+5x+2}}dx-\frac{2085x+1717}{\sqrt{x}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{2085x+1676}{x^{3/2}\sqrt{3x^2+5x+2}}dx-\frac{2085x+1717}{\sqrt{x}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\int -\frac{3(838x+695)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{1676\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)-\frac{2085x+1717}{\sqrt{x}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{1676\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\int \frac{838x+695}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{2085x+1717}{\sqrt{x}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1240}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{1676\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\int \frac{838x+695}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{2085x+1717}{\sqrt{x}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{1676\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(695\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+838\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{2085x+1717}{\sqrt{x}\sqrt{3x^2+5x+}\right.$$

$$\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{1676\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(838\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{695(x+1)\sqrt{\frac{3x+2}{x+1}}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)\right.\right.$$

$$\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow 1456$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{1676\sqrt{3x^2+5x+2}}{\sqrt{x}}-6\left(\frac{695(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+838\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}\right.\right.\right.\right.$$
$$\frac{2(45x+38)}{3\sqrt{x}\left(3x^2+5x+2\right)^{3/2}}$$

input
```
Int[(2 - 5*x)/(x^(3/2)*(2 + 5*x + 3*x^2)^(5/2)),x]
```

output
```
(2*(38 + 45*x))/(3*Sqrt[x]*(2 + 5*x + 3*x^2)^(3/2)) + (-((1717 + 2085*x)/(
Sqrt[x]*Sqrt[2 + 5*x + 3*x^2])) + ((1676*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x] -
6*(838*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*S
qrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x +
 3*x^2])) + (695*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]]
, -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])))/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{3x^2+5x+2}{2\sqrt{(3x^2+5x+2)x}}+\frac{\left(-\frac{100}{27}-\frac{38x}{9}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(-\frac{4225}{36}-\frac{1679x}{12}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\frac{695\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{-6x}\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+}}{2}\right.}{6\sqrt{3x^3+5x^2+2x}}\right.}{\sqrt{x}\,\sqrt{3x^2+5x+2}}$
default	$\frac{1287\,\mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2-2514\,\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,x^2+2145\sqrt{6x+4}\,\sqrt{}}{}$

input

```
int((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(-1/2*(3*x^2+5*x+2)/((
3*x^2+5*x+2)*x)^(1/2)+(-100/27-38/9*x)*(3*x^3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+
2/3)^2-2*x*(-4225/36-1679/12*x)*3^(1/2)/(x*(x^2+5/3*x+2/3))^(1/2)-695/6*(6
*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*EllipticF(1
/2*(6*x+4)^(1/2),I*2^(1/2))-419/3*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)
/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))-Ellip
ticF(1/2*(6*x+4)^(1/2),I*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.69

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = -\frac{2065\sqrt{3}(9\,x^5+30\,x^4+37\,x^3+20\,x^2+4\,x)\mathrm{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-7542\sqrt{3}(9\,x^5+30\,x^4}{}$$

input

```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*(2065*sqrt(3)*(9*x^5 + 30*x^4 + 37*x^3 + 20*x^2 + 4*x)*weierstrassPI
nverse(28/27, 80/729, x + 5/9) - 7542*sqrt(3)*(9*x^5 + 30*x^4 + 37*x^3 + 2
0*x^2 + 4*x)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27, 80/
729, x + 5/9)) - 9*(7542*x^4 + 18885*x^3 + 15430*x^2 + 4095*x - 6)*sqrt(3*
x^2 + 5*x + 2)*sqrt(x))/(9*x^5 + 30*x^4 + 37*x^3 + 20*x^2 + 4*x)
```

Sympy [F]

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{5/2}}\,dx =$$
$$-\int \frac{5}{9x^{\frac{9}{2}}\sqrt{3x^2+5x+2}+30x^{\frac{7}{2}}\sqrt{3x^2+5x+2}+37x^{\frac{5}{2}}\sqrt{3x^2+5x+2}+20x^{\frac{3}{2}}\sqrt{3x^2+5x+2}+4\sqrt{x}\sqrt{3x^2}$$
$$-\int \left(-\frac{2}{9x^{\frac{11}{2}}\sqrt{3x^2+5x+2}+30x^{\frac{9}{2}}\sqrt{3x^2+5x+2}+37x^{\frac{7}{2}}\sqrt{3x^2+5x+2}+20x^{\frac{5}{2}}\sqrt{3x^2+5x+2}+4x^{\frac{3}{2}}\sqrt{}}\right.$$

input
```
integrate((2-5*x)/x**(3/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
-Integral(5/(9*x**(9/2)*sqrt(3*x**2 + 5*x + 2) + 30*x**(7/2)*sqrt(3*x**2 +
 5*x + 2) + 37*x**(5/2)*sqrt(3*x**2 + 5*x + 2) + 20*x**(3/2)*sqrt(3*x**2 +
 5*x + 2) + 4*sqrt(x)*sqrt(3*x**2 + 5*x + 2)), x) - Integral(-2/(9*x**(11/
2)*sqrt(3*x**2 + 5*x + 2) + 30*x**(9/2)*sqrt(3*x**2 + 5*x + 2) + 37*x**(7/
2)*sqrt(3*x**2 + 5*x + 2) + 20*x**(5/2)*sqrt(3*x**2 + 5*x + 2) + 4*x**(3/2
)*sqrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{x^{3/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{5}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/((3*x^2 + 5*x + 2)^(5/2)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{2-5x}{x^{3/2}(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{5\,x-2}{(3\,x^2+5\,x+2)^{\frac{5}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/((3*x^2 + 5*x + 2)^(5/2)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{3/2}(2+5x+3x^2)^{5/2}}\,dx = \int -\frac{5\,x-2}{x^{3/2}\,(3\,x^2+5\,x+2)^{5/2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(3/2)*(5*x + 3*x^2 + 2)^(5/2)),x)
```

output
```
int(-(5*x - 2)/(x^(3/2)*(5*x + 3*x^2 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{x^{3/2}(2+5x+3x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((2-5*x)/x^(3/2)/(3*x^2+5*x+2)^(5/2),x)
```

output
```
( - 14*sqrt(3*x**2 + 5*x + 2)*x**2 - 4*sqrt(3*x**2 + 5*x + 2) - 450*sqrt(x
)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqr
t(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)
),x)*x**4 - 1500*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135
*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 6
0*sqrt(x)*x + 8*sqrt(x)),x)*x**3 - 1850*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)
/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3
 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**2 - 1000*sqrt(x)*int
(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*
x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*
x - 200*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*
x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)
*x + 8*sqrt(x)),x) - 567*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x**3)/(27*sqr
t(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*s
qrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**4 - 1890*sqrt(x)*int((sqrt(3
*x**2 + 5*x + 2)*x**3)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x
**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x
**3 - 2331*sqrt(x)*int((sqrt(3*x**2 + 5*x + 2)*x**3)/(27*sqrt(x)*x**6 + 13
5*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 +
60*sqrt(x)*x + 8*sqrt(x)),x)*x**2 - 1260*sqrt(x)*int((sqrt(3*x**2 + 5*x...
```

3.226 $\int \frac{2-5x}{x^{5/2}(2+5x+3x^2)^{5/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 220

$$\begin{aligned}\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{5/2}} dx = &-\frac{2}{3x^{3/2}\left(2+5x+3x^2\right)^{3/2}} \\ &+\frac{15}{\sqrt{x}\left(2+5x+3x^2\right)^{3/2}}+\frac{\sqrt{x}(86+75x)}{\left(2+5x+3x^2\right)^{3/2}}+\frac{625\sqrt{x}(2+3x)}{2\sqrt{2+5x+3x^2}} \\ &-\frac{5\sqrt{x}(307+375x)}{2\sqrt{2+5x+3x^2}}-\frac{625\sqrt{2+5x+3x^2}E\left(\arctan\left(\sqrt{x}\right)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x}\sqrt{2+3x}} \\ &+\frac{795\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x+3x^2}}\end{aligned}$$

output
```
-2/3/x^(3/2)/(3*x^2+5*x+2)^(3/2)+15/x^(1/2)/(3*x^2+5*x+2)^(3/2)+x^(1/2)*(8
6+75*x)/(3*x^2+5*x+2)^(3/2)+625/2*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)-5/2*
x^(1/2)*(307+375*x)/(3*x^2+5*x+2)^(1/2)-625/2*2^(1/2)*(3*x^2+5*x+2)^(1/2)*
EllipticE(x^(1/2)/(1+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)+795
/2*2^(1/2)*(1+x)^(1/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I
*2^(1/2))/(3*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.77

$$\int \frac{2-5x}{x^{5/2}(2+5x+3x^2)^{5/2}}\,dx = \frac{-4+7590x+28806x^2+35550x^3+14310x^4+1875i\sqrt{2+\frac{2}{x}}\sqrt{3+\frac{2}{x}}x^{5/2}(2}{}$$

input
```
Integrate[(2 - 5*x)/(x^(5/2)*(2 + 5*x + 3*x^2)^(5/2)),x]
```

output
```
(-4 + 7590*x + 28806*x^2 + 35550*x^3 + 14310*x^4 + (1875*I)*Sqrt[2 + 2/x]*
Sqrt[3 + 2/x]*x^(5/2)*(2 + 5*x + 3*x^2)*EllipticE[I*ArcSinh[Sqrt[2/3]/Sqrt
[x]], 3/2] + (510*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(5/2)*(2 + 5*x + 3*x^2)
*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(6*x^(3/2)*(2 + 5*x + 3*x^2
)^(3/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1235, 27, 1235, 27, 1237, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{x^{5/2}(3x^2+5x+2)^{5/2}}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2(45x+38)}{3x^{3/2}(3x^2+5x+2)^{3/2}}-\frac{1}{3}\int -\frac{9(35x+13)}{x^{5/2}(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$3\int \frac{35x+13}{x^{5/2}(3x^2+5x+2)^{3/2}}dx+\frac{2(45x+38)}{3x^{3/2}(3x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \ 1235$$

$$3\left(-\int \frac{5(135x+106)}{2x^{5/2}\sqrt{3x^2+5x+2}}dx-\frac{225x+181}{x^{3/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{3/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$3\left(-\frac{5}{2}\int \frac{135x+106}{x^{5/2}\sqrt{3x^2+5x+2}}dx-\frac{225x+181}{x^{3/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{3/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1237$$

$$3\left(-\frac{5}{2}\left(-\frac{1}{3}\int \frac{159x+125}{x^{3/2}\sqrt{3x^2+5x+2}}dx-\frac{106\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{225x+181}{x^{3/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{3/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1237$$

$$3\left(-\frac{5}{2}\left(\frac{1}{3}\left(\int -\frac{3(125x+106)}{2\sqrt{x}\sqrt{3x^2+5x+2}}dx+\frac{125\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)-\frac{106\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{225x+181}{x^{3/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{3/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$3\left(-\frac{5}{2}\left(\frac{1}{3}\left(\frac{125\sqrt{3x^2+5x+2}}{\sqrt{x}}-\frac{3}{2}\int \frac{125x+106}{\sqrt{x}\sqrt{3x^2+5x+2}}dx\right)-\frac{106\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{225x+181}{x^{3/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{3/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1240$$

$$3\left(-\frac{5}{2}\left(\frac{1}{3}\left(\frac{125\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\int \frac{125x+106}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{106\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{225x+181}{x^{3/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{3/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1503$$

$$3\left(-\frac{5}{2}\left(\frac{1}{3}\left(\frac{125\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(106\int\frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+125\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)\right)-\frac{106\sqrt{3x^2+5}}{3x^{3/2}}\right.\right.$$
$$\frac{2(45x+38)}{3x^{3/2}\left(3x^2+5x+2\right)^{3/2}}$$

$\big\downarrow$ 1413

$$3\left(-\frac{5}{2}\left(\frac{1}{3}\left(\frac{125\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(125\int\frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{53\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),\right.}{\sqrt{3x^2+5x+2}}\right.\right.\right.\right.$$
$$\frac{2(45x+38)}{3x^{3/2}\left(3x^2+5x+2\right)^{3/2}}$$

$\big\downarrow$ 1456

$$3\left(-\frac{5}{2}\left(\frac{1}{3}\left(\frac{125\sqrt{3x^2+5x+2}}{\sqrt{x}}-3\left(\frac{53\sqrt{2}(x+1)\sqrt{\frac{3x+2}{x+1}}\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{3x^2+5x+2}}+125\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}\right.\right.\right.\right.\right.$$
$$\frac{2(45x+38)}{3x^{3/2}\left(3x^2+5x+2\right)^{3/2}}$$

input
```
Int[(2 - 5*x)/(x^(5/2)*(2 + 5*x + 3*x^2)^(5/2)),x]
```

output
```
(2*(38 + 45*x))/(3*x^(3/2)*(2 + 5*x + 3*x^2)^(3/2)) + 3*(-((181 + 225*x)/(
x^(3/2)*Sqrt[2 + 5*x + 3*x^2])) - (5*((-106*Sqrt[2 + 5*x + 3*x^2])/(3*x^(3
/2)) + ((125*Sqrt[2 + 5*x + 3*x^2])/Sqrt[x] - 3*(125*((Sqrt[x]*(2 + 3*x))/
(3*Sqrt[2 + 5*x + 3*x^2]) - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*Ellip
ticE[ArcTan[Sqrt[x]], -1/2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (53*Sqrt[2]*(1 +
 x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticF[ArcTan[Sqrt[x]], -1/2])/Sqrt[2 + 5*x
 + 3*x^2]))/3))/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{\sqrt{3x^3+5x^2+2x}}{6x^2}+\frac{\frac{55}{4}x^2+\frac{275}{12}x+\frac{55}{6}}{\sqrt{(3x^2+5x+2)x}}+\frac{\left(\frac{136}{27}+\frac{50x}{9}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(\frac{9479}{72}+\frac{3805x}{24}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}+\frac{265\sqrt{6x+4}\sqrt{3+3x}\sqrt{-6x}\ \mathrm{El}}{2\sqrt{3x^3+5x}}\right.}{\sqrt{x}\sqrt{3x^2+5x+2}}$
default	$-\frac{855\sqrt{6x+4}\sqrt{3+3x}\sqrt{6}\sqrt{-x}\ \mathrm{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^3-1875\,\mathrm{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)\sqrt{6x+4}\sqrt{3+3x}\sqrt{6}\sqrt{-x}\,x^3+1425\,\mathrm{Ellipticl}}{}$

input
```
int((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(-1/6/x^2*(3*x^3+5*x^2
+2*x)^(1/2)+55/12*(3*x^2+5*x+2)/((3*x^2+5*x+2)*x)^(1/2)+(136/27+50/9*x)*(3
*x^3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/3)^2-2*x*(9479/72+3805/24*x)*3^(1/2)/(x
*(x^2+5/3*x+2/3))^(1/2)+265/2*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*
x^3+5*x^2+2*x)^(1/2)*EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))+625/4*(6*x+4)^
(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*EllipticE(1/
2*(6*x+4)^(1/2),I*2^(1/2))-EllipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.68

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \frac{1645\sqrt{3}(9\,x^6+30\,x^5+37\,x^4+20\,x^3+4\,x^2)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x\right.}{}$$

input
```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/18*(1645*sqrt(3)*(9*x^6 + 30*x^5 + 37*x^4 + 20*x^3 + 4*x^2)*weierstrassP
Inverse(28/27, 80/729, x + 5/9) - 5625*sqrt(3)*(9*x^6 + 30*x^5 + 37*x^4 +
20*x^3 + 4*x^2)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/27,
80/729, x + 5/9)) - 3*(16875*x^5 + 41940*x^4 + 33825*x^3 + 8694*x^2 - 90*x
 + 4)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/(9*x^6 + 30*x^5 + 37*x^4 + 20*x^3 + 4
*x^2)
```

Sympy [F]

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{5/2}}\,dx =$$

$$-\int \frac{5}{9x^{\frac{11}{2}}\sqrt{3x^2+5x+2}+30x^{\frac{9}{2}}\sqrt{3x^2+5x+2}+37x^{\frac{7}{2}}\sqrt{3x^2+5x+2}+20x^{\frac{5}{2}}\sqrt{3x^2+5x+2}+4x^{\frac{3}{2}}\sqrt{3x^2}}$$

$$-\int \left(-\frac{2}{9x^{\frac{13}{2}}\sqrt{3x^2+5x+2}+30x^{\frac{11}{2}}\sqrt{3x^2+5x+2}+37x^{\frac{9}{2}}\sqrt{3x^2+5x+2}+20x^{\frac{7}{2}}\sqrt{3x^2+5x+2}+4x^{\frac{5}{2}}\sqrt{}}\right.$$

input
```
integrate((2-5*x)/x**(5/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
-Integral(5/(9*x**(11/2)*sqrt(3*x**2 + 5*x + 2) + 30*x**(9/2)*sqrt(3*x**2
+ 5*x + 2) + 37*x**(7/2)*sqrt(3*x**2 + 5*x + 2) + 20*x**(5/2)*sqrt(3*x**2
+ 5*x + 2) + 4*x**(3/2)*sqrt(3*x**2 + 5*x + 2)), x) - Integral(-2/(9*x**(1
3/2)*sqrt(3*x**2 + 5*x + 2) + 30*x**(11/2)*sqrt(3*x**2 + 5*x + 2) + 37*x**
(9/2)*sqrt(3*x**2 + 5*x + 2) + 20*x**(7/2)*sqrt(3*x**2 + 5*x + 2) + 4*x**(
5/2)*sqrt(3*x**2 + 5*x + 2)), x)
```

Maxima [F]

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{5}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/((3*x^2 + 5*x + 2)^(5/2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{5}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/((3*x^2 + 5*x + 2)^(5/2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \int -\frac{5\,x-2}{x^{5/2}\,\left(3\,x^2+5\,x+2\right)^{5/2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(5/2)*(5*x + 3*x^2 + 2)^(5/2)),x)
```

output
```
int(-(5*x - 2)/(x^(5/2)*(5*x + 3*x^2 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{x^{5/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((2-5*x)/x^(5/2)/(3*x^2+5*x+2)^(5/2),x)
```

output
```
(210*sqrt(3*x**2 + 5*x + 2)*x**3 + 60*sqrt(3*x**2 + 5*x + 2)*x - 8*sqrt(3*
x**2 + 5*x + 2) - 1080*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**7
 + 135*sqrt(x)*x**6 + 279*sqrt(x)*x**5 + 305*sqrt(x)*x**4 + 186*sqrt(x)*x*
*3 + 60*sqrt(x)*x**2 + 8*sqrt(x)*x),x)*x**5 - 3600*sqrt(x)*int(sqrt(3*x**2
 + 5*x + 2)/(27*sqrt(x)*x**7 + 135*sqrt(x)*x**6 + 279*sqrt(x)*x**5 + 305*s
qrt(x)*x**4 + 186*sqrt(x)*x**3 + 60*sqrt(x)*x**2 + 8*sqrt(x)*x),x)*x**4 -
4440*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**7 + 135*sqrt(x)*x**
6 + 279*sqrt(x)*x**5 + 305*sqrt(x)*x**4 + 186*sqrt(x)*x**3 + 60*sqrt(x)*x*
*2 + 8*sqrt(x)*x),x)*x**3 - 2400*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sq
rt(x)*x**7 + 135*sqrt(x)*x**6 + 279*sqrt(x)*x**5 + 305*sqrt(x)*x**4 + 186*
sqrt(x)*x**3 + 60*sqrt(x)*x**2 + 8*sqrt(x)*x),x)*x**2 - 480*sqrt(x)*int(sq
rt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**7 + 135*sqrt(x)*x**6 + 279*sqrt(x)*x**
5 + 305*sqrt(x)*x**4 + 186*sqrt(x)*x**3 + 60*sqrt(x)*x**2 + 8*sqrt(x)*x),x
)*x + 4428*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(
x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt
(x)*x + 8*sqrt(x)),x)*x**5 + 14760*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*
sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4 + 305*sqrt(x)*x**3 + 18
6*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x**4 + 18204*sqrt(x)*int(sqr
t(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**6 + 135*sqrt(x)*x**5 + 279*sqrt(x)*x**4
 + 305*sqrt(x)*x**3 + 186*sqrt(x)*x**2 + 60*sqrt(x)*x + 8*sqrt(x)),x)*x...
```

3.227 $\int \frac{2-5x}{x^{7/2}(2+5x+3x^2)^{5/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\begin{aligned}\int \frac{2-5x}{x^{7/2}(2+5x+3x^2)^{5/2}}\,dx &= -\frac{2}{5x^{5/2}(2+5x+3x^2)^{3/2}} \\ &+ \frac{13}{3x^{3/2}(2+5x+3x^2)^{3/2}} - \frac{292}{5\sqrt{x}(2+5x+3x^2)^{3/2}} \\ &- \frac{\sqrt{x}(6995+5241x)}{30(2+5x+3x^2)^{3/2}} - \frac{9521\sqrt{x}(2+3x)}{30\sqrt{2+5x+3x^2}} \\ &+ \frac{\sqrt{x}(21610+28563x)}{30\sqrt{2+5x+3x^2}} + \frac{9521\sqrt{2+5x+3x^2}\,E\left(\arctan\left(\sqrt{x}\right)\middle|-\frac{1}{2}\right)}{15\sqrt{2}\sqrt{1+x}\sqrt{2+3x}} \\ &- \frac{1733\sqrt{1+x}\sqrt{2+3x}\,\text{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+5x+3x^2}}\end{aligned}$$

output

```
-2/5/x^(5/2)/(3*x^2+5*x+2)^(3/2)+13/3/x^(3/2)/(3*x^2+5*x+2)^(3/2)-292/5/x^
(1/2)/(3*x^2+5*x+2)^(3/2)-1/30*x^(1/2)*(6995+5241*x)/(3*x^2+5*x+2)^(3/2)-9
521/30*x^(1/2)*(2+3*x)/(3*x^2+5*x+2)^(1/2)+1/30*x^(1/2)*(21610+28563*x)/(3
*x^2+5*x+2)^(1/2)+9521/30*2^(1/2)*(3*x^2+5*x+2)^(1/2)*EllipticE(x^(1/2)/(1
+x)^(1/2),1/2*I*2^(1/2))/(1+x)^(1/2)/(2+3*x)^(1/2)-1733/4*2^(1/2)*(1+x)^(1
/2)*(2+3*x)^(1/2)*InverseJacobiAM(arctan(x^(1/2)),1/2*I*2^(1/2))/(3*x^2+5*
x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.70

$$\int \frac{2-5x}{x^{7/2}(2+5x+3x^2)^{5/2}}\,dx = \frac{-2(12-130x+39836x^2+154195x^3+192342x^4+77985x^5)-19042i\sqrt{2}}{}$$

input
```
Integrate[(2 - 5*x)/(x^(7/2)*(2 + 5*x + 3*x^2)^(5/2)),x]
```

output
```
(-2*(12 - 130*x + 39836*x^2 + 154195*x^3 + 192342*x^4 + 77985*x^5) - (1904
2*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(7/2)*(2 + 5*x + 3*x^2)*EllipticE[I*Arc
Sinh[Sqrt[2/3]/Sqrt[x]], 3/2] - (6953*I)*Sqrt[2 + 2/x]*Sqrt[3 + 2/x]*x^(7/
2)*(2 + 5*x + 3*x^2)*EllipticF[I*ArcSinh[Sqrt[2/3]/Sqrt[x]], 3/2])/(60*x^(
5/2)*(2 + 5*x + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {1235, 25, 1235, 27, 1237, 1237, 27, 1237, 27, 1240, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-5x}{x^{7/2}(3x^2+5x+2)^{5/2}}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2(45x+38)}{3x^{5/2}(3x^2+5x+2)^{3/2}}-\frac{1}{3}\int -\frac{405x+193}{x^{7/2}(3x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{405x+193}{x^{7/2}(3x^2+5x+2)^{3/2}}dx+\frac{2(45x+38)}{3x^{5/2}(3x^2+5x+2)^{3/2}}$$

$$\big\downarrow \text{1235}$$

$$\frac{1}{3}\left(-\int \frac{3(3275x+2504)}{2x^{7/2}\sqrt{3x^2+5x+2}}dx-\frac{1965x+1541}{x^{5/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{3275x+2504}{x^{7/2}\sqrt{3x^2+5x+2}}dx-\frac{1965x+1541}{x^{5/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\big\downarrow \text{1237}$$

$$\frac{1}{3}\left(-\frac{3}{2}\left(-\frac{1}{5}\int \frac{11268x+8665}{x^{5/2}\sqrt{3x^2+5x+2}}dx-\frac{2504\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{1965x+1541}{x^{5/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\big\downarrow \text{1237}$$

$$\frac{1}{3}\left(-\frac{3}{2}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{25995x+19042}{2x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{8665\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2504\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{1965x+1541}{x^{5/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{3}{2}\left(\frac{1}{5}\left(\frac{1}{6}\int \frac{25995x+19042}{x^{3/2}\sqrt{3x^2+5x+2}}dx+\frac{8665\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2504\sqrt{3x^2+5x+2}}{5x^{5/2}}\right)-\frac{1965x+1541}{x^{5/2}\sqrt{3x^2+5x+2}}\right)+\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\big\downarrow \text{1237}$$

$$\frac{1}{3}\left(-\frac{3}{2}\left(\frac{1}{5}\left(\frac{1}{6}\left(-\int -\frac{3(9521x+8665)}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{19042\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{8665\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2504\sqrt{3x^2+5}}{5x^{5/2}}\right.\right.$$
$$\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{3}{2}\left(\frac{1}{5}\left(\frac{1}{6}\left(3\int \frac{9521x+8665}{\sqrt{x}\sqrt{3x^2+5x+2}}dx-\frac{19042\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{8665\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2504\sqrt{3x^2+5x}}{5x^{5/2}}\right.\right.$$
$$\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$\downarrow$ 1240

$$\frac{1}{3}\left(-\frac{3}{2}\left(\frac{1}{5}\left(\frac{1}{6}\left(6\int \frac{9521x+8665}{\sqrt{3x^2+5x+2}}d\sqrt{x}-\frac{19042\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{8665\sqrt{3x^2+5x+2}}{3x^{3/2}}\right)-\frac{2504\sqrt{3x^2+5x+}}{5x^{5/2}}\right.\right.$$
$$\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$\downarrow$ 1503

$$\frac{1}{3}\left(-\frac{3}{2}\left(\frac{1}{5}\left(\frac{1}{6}\left(6\left(8665\int \frac{1}{\sqrt{3x^2+5x+2}}d\sqrt{x}+9521\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}\right)-\frac{19042\sqrt{3x^2+5x+2}}{\sqrt{x}}\right)+\frac{866}{}\right.\right.\right.$$
$$\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$\downarrow$ 1413

$$\frac{1}{3}\left(-\frac{3}{2}\left(\frac{1}{5}\left(\frac{1}{6}\left(6\left(9521\int \frac{x}{\sqrt{3x^2+5x+2}}d\sqrt{x}+\frac{8665(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}\right)-\frac{19042\sqrt{}}{}\right.\right.\right.\right.$$
$$\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

$\downarrow$ 1456

$$\frac{1}{3}\left(-\frac{3}{2}\left(\frac{1}{5}\left(\frac{1}{6}\left(6\left(\frac{8665(x+1)\sqrt{\frac{3x+2}{x+1}}\operatorname{EllipticF}\left(\arctan\left(\sqrt{x}\right),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^2+5x+2}}+9521\left(\frac{\sqrt{x}(3x+2)}{3\sqrt{3x^2+5x+2}}-\frac{\sqrt{2}(x+1)\sqrt{\frac{3}{}}}{3\sqrt{}}\right.\right.\right.\right.\right.\right.$$
$$\frac{2(45x+38)}{3x^{5/2}\left(3x^2+5x+2\right)^{3/2}}$$

input

```
Int[(2 - 5*x)/(x^(7/2)*(2 + 5*x + 3*x^2)^(5/2)),x]
```

output
```
(2*(38 + 45*x))/(3*x^(5/2)*(2 + 5*x + 3*x^2)^(3/2)) + (-((1541 + 1965*x)/(
x^(5/2)*Sqrt[2 + 5*x + 3*x^2])) - (3*((-2504*Sqrt[2 + 5*x + 3*x^2])/(5*x^(
5/2)) + ((8665*Sqrt[2 + 5*x + 3*x^2])/(3*x^(3/2)) + ((-19042*Sqrt[2 + 5*x
+ 3*x^2])/Sqrt[x] + 6*(9521*((Sqrt[x]*(2 + 3*x))/(3*Sqrt[2 + 5*x + 3*x^2])
 - (Sqrt[2]*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*EllipticE[ArcTan[Sqrt[x]], -1/
2])/(3*Sqrt[2 + 5*x + 3*x^2])) + (8665*(1 + x)*Sqrt[(2 + 3*x)/(1 + x)]*Ell
ipticF[ArcTan[Sqrt[x]], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x + 3*x^2])))/6)/5))/2)
/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1240
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]),
x_Symbol] :> Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x,
 Sqrt[x]], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12

method	result
elliptic	$\frac{\sqrt{(3x^2+5x+2)x}\left(-\frac{\sqrt{3x^3+5x^2+2x}}{10x^3}+\frac{19\sqrt{3x^3+5x^2+2x}}{12x^2}-\frac{2591\left(3x^2+5x+2\right)}{120\sqrt{\left(3x^2+5x+2\right)x}}+\frac{\left(-\frac{190}{27}-\frac{68x}{9}\right)\sqrt{3x^3+5x^2+2x}}{\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)^2}-\frac{2x\left(-\frac{19765}{144}-\frac{8135x}{48}\right)\sqrt{3}}{\sqrt{x\left(x^2+\frac{5}{3}x+\frac{2}{3}\right)}}-\right.}{\sqrt{x}\sqrt{3x^2+5}}$
default	$\frac{7704\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^4-28563\sqrt{6x+4}\,\sqrt{3+3x}\,\sqrt{6}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\frac{\sqrt{6x+4}}{2},i\sqrt{2}\right)x^4+12840\sqrt{6x+4}}{}$

input
```
int((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x^2+5*x+2)*x)^(1/2)/x^(1/2)/(3*x^2+5*x+2)^(1/2)*(-1/10/x^3*(3*x^3+5*x^
2+2*x)^(1/2)+19/12/x^2*(3*x^3+5*x^2+2*x)^(1/2)-2591/120*(3*x^2+5*x+2)/((3*
x^2+5*x+2)*x)^(1/2)+(-190/27-68/9*x)*(3*x^3+5*x^2+2*x)^(1/2)/(x^2+5/3*x+2/
3)^2-2*x*(-19765/144-8135/48*x)*3^(1/2)/(x*(x^2+5/3*x+2/3))^(1/2)-1733/12*
(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(1/2)/(3*x^3+5*x^2+2*x)^(1/2)*EllipticF
(1/2*(6*x+4)^(1/2),I*2^(1/2))-9521/60*(6*x+4)^(1/2)*(3+3*x)^(1/2)*(-6*x)^(
1/2)/(3*x^3+5*x^2+2*x)^(1/2)*(1/3*EllipticE(1/2*(6*x+4)^(1/2),I*2^(1/2))-E
llipticF(1/2*(6*x+4)^(1/2),I*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.62

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{5/2}}\,dx =$$

$$-\frac{30380\sqrt{3}(9\,x^7+30\,x^6+37\,x^5+20\,x^4+4\,x^3)\text{weierstrassPInverse}\left(\frac{28}{27},\frac{80}{729},x+\frac{5}{9}\right)-85689\sqrt{3}(9\,x^7+30}{}$$

input
```
integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/270*(30380*sqrt(3)*(9*x^7 + 30*x^6 + 37*x^5 + 20*x^4 + 4*x^3)*weierstra
ssPInverse(28/27, 80/729, x + 5/9) - 85689*sqrt(3)*(9*x^7 + 30*x^6 + 37*x^
5 + 20*x^4 + 4*x^3)*weierstrassZeta(28/27, 80/729, weierstrassPInverse(28/
27, 80/729, x + 5/9)) - 9*(85689*x^6 + 207645*x^5 + 159935*x^4 + 36225*x^3
 - 1752*x^2 + 130*x - 12)*sqrt(3*x^2 + 5*x + 2)*sqrt(x))/(9*x^7 + 30*x^6 +
 37*x^5 + 20*x^4 + 4*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((2-5*x)/x**(7/2)/(3*x**2+5*x+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{5}{2}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="maxima")
```

output
```
-integrate((5*x - 2)/((3*x^2 + 5*x + 2)^(5/2)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{5/2}}\,dx = \int -\frac{5\,x-2}{\left(3\,x^2+5\,x+2\right)^{\frac{5}{2}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(5*x - 2)/((3*x^2 + 5*x + 2)^(5/2)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{5/2}}\,dx=\int -\frac{5\,x-2}{x^{7/2}\,\left(3\,x^2+5\,x+2\right)^{5/2}}\,dx$$

input
```
int(-(5*x - 2)/(x^(7/2)*(5*x + 3*x^2 + 2)^(5/2)),x)
```

output
```
int(-(5*x - 2)/(x^(7/2)*(5*x + 3*x^2 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{2-5x}{x^{7/2}\left(2+5x+3x^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((2-5*x)/x^(7/2)/(3*x^2+5*x+2)^(5/2),x)
```

output

```
(50*sqrt(3*x**2 + 5*x + 2)*x - 12*sqrt(3*x**2 + 5*x + 2) - 2160*sqrt(x)*in
t(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**8 + 135*sqrt(x)*x**7 + 279*sqrt(x)
*x**6 + 305*sqrt(x)*x**5 + 186*sqrt(x)*x**4 + 60*sqrt(x)*x**3 + 8*sqrt(x)*
x**2),x)*x**6 - 7200*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**8 +
 135*sqrt(x)*x**7 + 279*sqrt(x)*x**6 + 305*sqrt(x)*x**5 + 186*sqrt(x)*x**4
 + 60*sqrt(x)*x**3 + 8*sqrt(x)*x**2),x)*x**5 - 8880*sqrt(x)*int(sqrt(3*x**
2 + 5*x + 2)/(27*sqrt(x)*x**8 + 135*sqrt(x)*x**7 + 279*sqrt(x)*x**6 + 305*
sqrt(x)*x**5 + 186*sqrt(x)*x**4 + 60*sqrt(x)*x**3 + 8*sqrt(x)*x**2),x)*x**
4 - 4800*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**8 + 135*sqrt(x)
*x**7 + 279*sqrt(x)*x**6 + 305*sqrt(x)*x**5 + 186*sqrt(x)*x**4 + 60*sqrt(x
)*x**3 + 8*sqrt(x)*x**2),x)*x**3 - 960*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/
(27*sqrt(x)*x**8 + 135*sqrt(x)*x**7 + 279*sqrt(x)*x**6 + 305*sqrt(x)*x**5
+ 186*sqrt(x)*x**4 + 60*sqrt(x)*x**3 + 8*sqrt(x)*x**2),x)*x**2 + 4968*sqrt
(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x**7 + 135*sqrt(x)*x**6 + 279*s
qrt(x)*x**5 + 305*sqrt(x)*x**4 + 186*sqrt(x)*x**3 + 60*sqrt(x)*x**2 + 8*sq
rt(x)*x),x)*x**6 + 16560*sqrt(x)*int(sqrt(3*x**2 + 5*x + 2)/(27*sqrt(x)*x*
*7 + 135*sqrt(x)*x**6 + 279*sqrt(x)*x**5 + 305*sqrt(x)*x**4 + 186*sqrt(x)*
x**3 + 60*sqrt(x)*x**2 + 8*sqrt(x)*x),x)*x**5 + 20424*sqrt(x)*int(sqrt(3*x
**2 + 5*x + 2)/(27*sqrt(x)*x**7 + 135*sqrt(x)*x**6 + 279*sqrt(x)*x**5 + 30
5*sqrt(x)*x**4 + 186*sqrt(x)*x**3 + 60*sqrt(x)*x**2 + 8*sqrt(x)*x),x)*x...
```

3.228 $\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 240

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^3\,dx=\frac{a^3A(ex)^{1+m}}{e(1+m)}+\frac{a^2(3Ab+aB)(ex)^{2+m}}{e^2(2+m)}+\frac{3a(abB+A(b^2+ac))\,(ex)^{3+m}}{e^3(3+m)}+\frac{(3aB(b^2+ac)+A(b^3+6abc))\,(ex)^{4+m}}{e^4(4+m)}+\frac{(b^3B+3Ab^2c+6abBc+3aAc^2)\,(ex)^{5+m}}{e^5(5+m)}+\frac{3c(b^2B+Abc+aBc)\,(ex)^{6+m}}{e^6(6+m)}+\frac{c^2(3bB+Ac)(ex)^{7+m}}{e^7(7+m)}+\frac{Bc^3(ex)^{8+m}}{e^8(8+m)}$$

output

```
a^3*A*(e*x)^(1+m)/e/(1+m)+a^2*(3*A*b+B*a)*(e*x)^(2+m)/e^2/(2+m)+3*a*(a*b*B
+A*(a*c+b^2))*(e*x)^(3+m)/e^3/(3+m)+(3*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*(e*x
)^(4+m)/e^4/(4+m)+(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*(e*x)^(5+m)/e^5/(5
+m)+3*c*(A*b*c+B*a*c+B*b^2)*(e*x)^(6+m)/e^6/(6+m)+c^2*(A*c+3*B*b)*(e*x)^(7
+m)/e^7/(7+m)+B*c^3*(e*x)^(8+m)/e^8/(8+m)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 672 vs. 2(240) = 480.

Time = 2.37 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.80

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$

$$= \frac{(ex)^m\left(x(3bB+Ac(8+m)+Bc(7+m)x)(a+x(b+cx))^3+\frac{3\left(-x(ac(6+m)(bB(1+m)-2Ac(8+m))+2b(b^2B(4+\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
((e*x)^m*(x*(3*b*B + A*c*(8 + m) + B*c*(7 + m)*x)*(a + x*(b + c*x))^3 + (3
*(-(x*(a*c*(6 + m)*(b*B*(1 + m) - 2*A*c*(8 + m)) + 2*b*(b^2*B*(4 + m) - 2*
a*B*c*(7 + m) - A*b*c*(8 + m)) + c*(5 + m)*(b^2*B*(4 + m) - 2*a*B*c*(7 + m
) - A*b*c*(8 + m))*x)*(a + x*(b + c*x))^2) + (2*x*((-2*a^2*c*(4 + m)*(2*a*
c*(6 + m)*(b*B*(1 + m) - 2*A*c*(8 + m)) - b*(1 + m)*(b^2*B*(4 + m) - 2*a*B
*c*(7 + m) - A*b*c*(8 + m))))/(1 + m) + a*b*(a*b*c*(6 + m)*(b*B*(1 + m) -
2*A*c*(8 + m)) - (b^2*(3 + m) - 2*a*c*(5 + m))*(b^2*B*(4 + m) - 2*a*B*c*(7
 + m) - A*b*c*(8 + m))) - (a*b*c*(4 + m)*(2*a*c*(6 + m)*(b*B*(1 + m) - 2*A
*c*(8 + m)) - b*(1 + m)*(b^2*B*(4 + m) - 2*a*B*c*(7 + m) - A*b*c*(8 + m)))
*x)/(2 + m) + ((b^2*(2 + m) - 2*a*c*(3 + m))*(a*b*c*(6 + m)*(b*B*(1 + m) -
 2*A*c*(8 + m)) - (b^2*(3 + m) - 2*a*c*(5 + m))*(b^2*B*(4 + m) - 2*a*B*c*(
7 + m) - A*b*c*(8 + m)))*x)/(2 + m) + (-(a*c*(4 + m)*(2*a*c*(6 + m)*(b*B*(
1 + m) - 2*A*c*(8 + m)) - b*(1 + m)*(b^2*B*(4 + m) - 2*a*B*c*(7 + m) - A*b
*c*(8 + m)))) + b*(-(a*b*c*(6 + m)*(b*B*(1 + m) - 2*A*c*(8 + m))) + (b^2*(
3 + m) - 2*a*c*(5 + m))*(b^2*B*(4 + m) - 2*a*B*c*(7 + m) - A*b*c*(8 + m)))
 + c*(3 + m)*(-(a*b*c*(6 + m)*(b*B*(1 + m) - 2*A*c*(8 + m))) + (b^2*(3 + m
) - 2*a*c*(5 + m))*(b^2*B*(4 + m) - 2*a*B*c*(7 + m) - A*b*c*(8 + m)))*x)*(
a + x*(b + c*x))))/(c*(3 + m)*(4 + m))))/(c*(5 + m)*(6 + m))))/(c*(7 + m)*
(8 + m))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(ex)^m \left(a+bx+cx^2\right)^3 \, dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(a^3A(ex)^m+\frac{a^2(ex)^{m+1}(aB+3Ab)}{e}+\frac{3c(ex)^{m+5}\left(aBc+Abc+b^2B\right)}{e^5}+\frac{3a(ex)^{m+2}\left(A\left(ac+b^2\right)+abB\right)}{e^2}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3A(ex)^{m+1}}{e(m+1)}+\frac{a^2(ex)^{m+2}(aB+3Ab)}{e^2(m+2)}+\frac{3c(ex)^{m+6}\left(aBc+Abc+b^2B\right)}{e^6(m+6)}+\frac{3a(ex)^{m+3}\left(A\left(ac+b^2\right)+abB\right)}{e^3(m+3)}+\frac{(ex)^{m+5}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)}{e^5(m+5)}+\frac{(ex)^{m+4}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)}{e^4(m+4)}+\frac{c^2(ex)^{m+7}(Ac+3bB)}{e^7(m+7)}+\frac{Bc^3(ex)^{m+8}}{e^8(m+8)}$$

input
```
Int[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^3,x]
```

output
```
(a^3*A*(e*x)^(1 + m))/(e*(1 + m)) + (a^2*(3*A*b + a*B)*(e*x)^(2 + m))/(e^2
*(2 + m)) + (3*a*(a*b*B + A*(b^2 + a*c))*(e*x)^(3 + m))/(e^3*(3 + m)) + ((
3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*(e*x)^(4 + m))/(e^4*(4 + m)) + ((b^
3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2)*(e*x)^(5 + m))/(e^5*(5 + m)) + (3
*c*(b^2*B + A*b*c + a*B*c)*(e*x)^(6 + m))/(e^6*(6 + m)) + (c^2*(3*b*B + A*
c)*(e*x)^(7 + m))/(e^7*(7 + m)) + (B*c^3*(e*x)^(8 + m))/(e^8*(8 + m))
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.01

method	result
norman	$\frac{(3Aa\,c^2+3A\,b^2c+6Babc+B\,b^3)x^5\mathrm{e}^{m\ln(ex)}}{5+m}+\frac{(6Aabc+A\,b^3+3B\,a^2c+3Ba\,b^2)x^4\mathrm{e}^{m\ln(ex)}}{4+m}+\frac{B\,c^3x^8\mathrm{e}^{m\ln(ex)}}{8+m}+\frac{a^2(3Ab$
gosper	Expression too large to display
risch	Expression too large to display
orering	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)/(5+m)*x^5*exp(m*ln(e*x))+(6*A*a*b*c+
A*b^3+3*B*a^2*c+3*B*a*b^2)/(4+m)*x^4*exp(m*ln(e*x))+B*c^3/(8+m)*x^8*exp(m*
ln(e*x))+a^2*(3*A*b+B*a)/(2+m)*x^2*exp(m*ln(e*x))+a^3*A/(1+m)*x*exp(m*ln(e
*x))+c^2*(A*c+3*B*b)/(7+m)*x^7*exp(m*ln(e*x))+3*a*(A*a*c+A*b^2+B*a*b)/(3+m
)*x^3*exp(m*ln(e*x))+3*c*(A*b*c+B*a*c+B*b^2)/(6+m)*x^6*exp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1350 vs. $2(240) = 480$.

Time = 0.11 (sec) , antiderivative size = 1350, normalized size of antiderivative = 5.62

$$\int (ex)^m(A + Bx) \left(a + bx + cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
((B*c^3*m^7 + 28*B*c^3*m^6 + 322*B*c^3*m^5 + 1960*B*c^3*m^4 + 6769*B*c^3*m
^3 + 13132*B*c^3*m^2 + 13068*B*c^3*m + 5040*B*c^3)*x^8 + ((3*B*b*c^2 + A*c
^3)*m^7 + 29*(3*B*b*c^2 + A*c^3)*m^6 + 343*(3*B*b*c^2 + A*c^3)*m^5 + 2135*
(3*B*b*c^2 + A*c^3)*m^4 + 17280*B*b*c^2 + 5760*A*c^3 + 7504*(3*B*b*c^2 + A
*c^3)*m^3 + 14756*(3*B*b*c^2 + A*c^3)*m^2 + 14832*(3*B*b*c^2 + A*c^3)*m)*x
^7 + 3*((B*b^2*c + (B*a + A*b)*c^2)*m^7 + 30*(B*b^2*c + (B*a + A*b)*c^2)*m
^6 + 366*(B*b^2*c + (B*a + A*b)*c^2)*m^5 + 2340*(B*b^2*c + (B*a + A*b)*c^2
)*m^4 + 6720*B*b^2*c + 8409*(B*b^2*c + (B*a + A*b)*c^2)*m^3 + 6720*(B*a +
A*b)*c^2 + 16830*(B*b^2*c + (B*a + A*b)*c^2)*m^2 + 17144*(B*b^2*c + (B*a +
 A*b)*c^2)*m)*x^6 + ((B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*m^7 + 31*
(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*m^6 + 391*(B*b^3 + 3*A*a*c^2 +
 3*(2*B*a*b + A*b^2)*c)*m^5 + 2581*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2
)*c)*m^4 + 8064*B*b^3 + 24192*A*a*c^2 + 9544*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a
*b + A*b^2)*c)*m^3 + 19564*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*m^2
 + 24192*(2*B*a*b + A*b^2)*c + 20304*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b
^2)*c)*m)*x^5 + ((3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*m^7 + 32*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*m^6 + 418*(3*B*a*b^2 + A*b^3 + 3*(
B*a^2 + 2*A*a*b)*c)*m^5 + 2864*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)
*m^4 + 30240*B*a*b^2 + 10080*A*b^3 + 10993*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 +
 2*A*a*b)*c)*m^3 + 23312*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*m^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11116 vs. $2(230) = 460$.

Time = 0.94 (sec) , antiderivative size = 11116, normalized size of antiderivative = 46.32

$$\int (ex)^m(A + Bx)\left(a + bx + cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x+A)*(c*x**2+b*x+a)**3,x)
```

output
```
Piecewise(((-A*a**3/(7*x**7) - A*a**2*b/(2*x**6) - 3*A*a**2*c/(5*x**5) - 3
*A*a*b**2/(5*x**5) - 3*A*a*b*c/(2*x**4) - A*a*c**2/x**3 - A*b**3/(4*x**4)
- A*b**2*c/x**3 - 3*A*b*c**2/(2*x**2) - A*c**3/x - B*a**3/(6*x**6) - 3*B*a
**2*b/(5*x**5) - 3*B*a**2*c/(4*x**4) - 3*B*a*b**2/(4*x**4) - 2*B*a*b*c/x**
3 - 3*B*a*c**2/(2*x**2) - B*b**3/(3*x**3) - 3*B*b**2*c/(2*x**2) - 3*B*b*c*
*2/x + B*c**3*log(x))/e**8, Eq(m, -8)), ((-A*a**3/(6*x**6) - 3*A*a**2*b/(5
*x**5) - 3*A*a**2*c/(4*x**4) - 3*A*a*b**2/(4*x**4) - 2*A*a*b*c/x**3 - 3*A*
a*c**2/(2*x**2) - A*b**3/(3*x**3) - 3*A*b**2*c/(2*x**2) - 3*A*b*c**2/x + A
*c**3*log(x) - B*a**3/(5*x**5) - 3*B*a**2*b/(4*x**4) - B*a**2*c/x**3 - B*a
*b**2/x**3 - 3*B*a*b*c/x**2 - 3*B*a*c**2/x - B*b**3/(2*x**2) - 3*B*b**2*c/
x + 3*B*b*c**2*log(x) + B*c**3*x)/e**7, Eq(m, -7)), ((-A*a**3/(5*x**5) - 3
*A*a**2*b/(4*x**4) - A*a**2*c/x**3 - A*a*b**2/x**3 - 3*A*a*b*c/x**2 - 3*A*
a*c**2/x - A*b**3/(2*x**2) - 3*A*b**2*c/x + 3*A*b*c**2*log(x) + A*c**3*x -
 B*a**3/(4*x**4) - B*a**2*b/x**3 - 3*B*a**2*c/(2*x**2) - 3*B*a*b**2/(2*x**
2) - 6*B*a*b*c/x + 3*B*a*c**2*log(x) - B*b**3/x + 3*B*b**2*c*log(x) + 3*B*
b*c**2*x + B*c**3*x**2/2)/e**6, Eq(m, -6)), ((-A*a**3/(4*x**4) - A*a**2*b/
x**3 - 3*A*a**2*c/(2*x**2) - 3*A*a*b**2/(2*x**2) - 6*A*a*b*c/x + 3*A*a*c**
2*log(x) - A*b**3/x + 3*A*b**2*c*log(x) + 3*A*b*c**2*x + A*c**3*x**2/2 - B
*a**3/(3*x**3) - 3*B*a**2*b/(2*x**2) - 3*B*a**2*c/x - 3*B*a*b**2/x + 6*B*a
*b*c*log(x) + 3*B*a*c**2*x + B*b**3*log(x) + 3*B*b**2*c*x + 3*B*b*c**2*...
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.70

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \frac{Bc^3e^mx^8x^m}{m+8}+\frac{3\,Bbc^2e^mx^7x^m}{m+7}+\frac{Ac^3e^mx^7x^m}{m+7}+\frac{3\,Bb^2ce^mx^6x^m}{m+6}+\frac{3\,Bac^2e^mx^6x^m}{m+6}+\frac{3\,Abc^2e^mx^6x^m}{m+6}+\frac{Bb^3e^mx^5x^m}{m+5}+\frac{6\,Babce^mx^5x^m}{m+5}+\frac{3\,Ab^2ce^mx^5x^m}{m+5}+\frac{3\,Aac^2e^mx^5x^m}{m+5}+\frac{3\,Bab^2e^mx^4x^m}{m+4}+\frac{Ab^3e^mx^4x^m}{m+4}+\frac{3\,Ba^2ce^mx^4x^m}{m+4}+\frac{6\,Aabce^mx^4x^m}{m+4}+\frac{3\,Ba^2be^mx^3x^m}{m+3}+\frac{3\,Aab^2e^mx^3x^m}{m+3}+\frac{3\,Aa^2ce^mx^3x^m}{m+3}+\frac{Ba^3e^mx^2x^m}{m+2}+\frac{3\,Aa^2be^mx^2x^m}{m+2}+\frac{(ex)^{m+1}\,Aa^3}{e(m+1)}$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
B*c^3*e^m*x^8*x^m/(m + 8) + 3*B*b*c^2*e^m*x^7*x^m/(m + 7) + A*c^3*e^m*x^7*
x^m/(m + 7) + 3*B*b^2*c*e^m*x^6*x^m/(m + 6) + 3*B*a*c^2*e^m*x^6*x^m/(m + 6
) + 3*A*b*c^2*e^m*x^6*x^m/(m + 6) + B*b^3*e^m*x^5*x^m/(m + 5) + 6*B*a*b*c*
e^m*x^5*x^m/(m + 5) + 3*A*b^2*c*e^m*x^5*x^m/(m + 5) + 3*A*a*c^2*e^m*x^5*x^
m/(m + 5) + 3*B*a*b^2*e^m*x^4*x^m/(m + 4) + A*b^3*e^m*x^4*x^m/(m + 4) + 3*
B*a^2*c*e^m*x^4*x^m/(m + 4) + 6*A*a*b*c*e^m*x^4*x^m/(m + 4) + 3*B*a^2*b*e^
m*x^3*x^m/(m + 3) + 3*A*a*b^2*e^m*x^3*x^m/(m + 3) + 3*A*a^2*c*e^m*x^3*x^m/
(m + 3) + B*a^3*e^m*x^2*x^m/(m + 2) + 3*A*a^2*b*e^m*x^2*x^m/(m + 2) + (e*x
)^(m + 1)*A*a^3/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2736 vs. $2(240) = 480$.

Time = 0.27 (sec) , antiderivative size = 2736, normalized size of antiderivative = 11.40

$$\int (ex)^m(A + Bx)\left(a + bx + cx^2\right)^3 dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
((e*x)^m*B*c^3*m^7*x^8 + 3*(e*x)^m*B*b*c^2*m^7*x^7 + (e*x)^m*A*c^3*m^7*x^7
 + 28*(e*x)^m*B*c^3*m^6*x^8 + 3*(e*x)^m*B*b^2*c*m^7*x^6 + 3*(e*x)^m*B*a*c^
2*m^7*x^6 + 3*(e*x)^m*A*b*c^2*m^7*x^6 + 87*(e*x)^m*B*b*c^2*m^6*x^7 + 29*(e
*x)^m*A*c^3*m^6*x^7 + 322*(e*x)^m*B*c^3*m^5*x^8 + (e*x)^m*B*b^3*m^7*x^5 +
6*(e*x)^m*B*a*b*c*m^7*x^5 + 3*(e*x)^m*A*b^2*c*m^7*x^5 + 3*(e*x)^m*A*a*c^2*
m^7*x^5 + 90*(e*x)^m*B*b^2*c*m^6*x^6 + 90*(e*x)^m*B*a*c^2*m^6*x^6 + 90*(e*
x)^m*A*b*c^2*m^6*x^6 + 1029*(e*x)^m*B*b*c^2*m^5*x^7 + 343*(e*x)^m*A*c^3*m^
5*x^7 + 1960*(e*x)^m*B*c^3*m^4*x^8 + 3*(e*x)^m*B*a*b^2*m^7*x^4 + (e*x)^m*A
*b^3*m^7*x^4 + 3*(e*x)^m*B*a^2*c*m^7*x^4 + 6*(e*x)^m*A*a*b*c*m^7*x^4 + 31*
(e*x)^m*B*b^3*m^6*x^5 + 186*(e*x)^m*B*a*b*c*m^6*x^5 + 93*(e*x)^m*A*b^2*c*m
^6*x^5 + 93*(e*x)^m*A*a*c^2*m^6*x^5 + 1098*(e*x)^m*B*b^2*c*m^5*x^6 + 1098*
(e*x)^m*B*a*c^2*m^5*x^6 + 1098*(e*x)^m*A*b*c^2*m^5*x^6 + 6405*(e*x)^m*B*b*
c^2*m^4*x^7 + 2135*(e*x)^m*A*c^3*m^4*x^7 + 6769*(e*x)^m*B*c^3*m^3*x^8 + 3*
(e*x)^m*B*a^2*b*m^7*x^3 + 3*(e*x)^m*A*a*b^2*m^7*x^3 + 3*(e*x)^m*A*a^2*c*m^
7*x^3 + 96*(e*x)^m*B*a*b^2*m^6*x^4 + 32*(e*x)^m*A*b^3*m^6*x^4 + 96*(e*x)^m
*B*a^2*c*m^6*x^4 + 192*(e*x)^m*A*a*b*c*m^6*x^4 + 391*(e*x)^m*B*b^3*m^5*x^5
 + 2346*(e*x)^m*B*a*b*c*m^5*x^5 + 1173*(e*x)^m*A*b^2*c*m^5*x^5 + 1173*(e*x
)^m*A*a*c^2*m^5*x^5 + 7020*(e*x)^m*B*b^2*c*m^4*x^6 + 7020*(e*x)^m*B*a*c^2*
m^4*x^6 + 7020*(e*x)^m*A*b*c^2*m^4*x^6 + 22512*(e*x)^m*B*b*c^2*m^3*x^7 + 7
504*(e*x)^m*A*c^3*m^3*x^7 + 13132*(e*x)^m*B*c^3*m^2*x^8 + (e*x)^m*B*a^3...
```

Mupad [B] (verification not implemented)

Time = 11.65 (sec) , antiderivative size = 769, normalized size of antiderivative = 3.20

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^3\,dx$$
$$= \frac{x^4\,(e\,x)^m\,\left(3\,B\,c\,a^2+3\,B\,a\,b^2+6\,A\,c\,a\,b+A\,b^3\right)\,\left(m^7+32\,m^6+418\,m^5+2864\,m^4+10993\,m^3+23312\,m\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+}$$
$$+\frac{x^5\,(e\,x)^m\,\left(B\,b^3+3\,A\,b^2\,c+6\,B\,a\,b\,c+3\,A\,a\,c^2\right)\,\left(m^7+31\,m^6+391\,m^5+2581\,m^4+9544\,m^3+19564\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+}$$
$$+\frac{A\,a^3\,x\,(e\,x)^m\,\left(m^7+35\,m^6+511\,m^5+4025\,m^4+18424\,m^3+48860\,m^2+69264\,m+40320\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}$$
$$+\frac{3\,a\,x^3\,(e\,x)^m\,\left(A\,b^2+B\,a\,b+A\,a\,c\right)\,\left(m^7+33\,m^6+447\,m^5+3195\,m^4+12864\,m^3+28692\,m^2+32048\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+4032}$$
$$+\frac{3\,c\,x^6\,(e\,x)^m\,\left(B\,b^2+A\,c\,b+B\,a\,c\right)\,\left(m^7+30\,m^6+366\,m^5+2340\,m^4+8409\,m^3+16830\,m^2+17144\,m\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}$$
$$+\frac{B\,c^3\,x^8\,(e\,x)^m\,\left(m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}$$
$$+\frac{a^2\,x^2\,(e\,x)^m\,\left(3\,A\,b+B\,a\right)\,\left(m^7+34\,m^6+478\,m^5+3580\,m^4+15289\,m^3+36706\,m^2+44712\,m+201\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}$$
$$+\frac{c^2\,x^7\,(e\,x)^m\,\left(A\,c+3\,B\,b\right)\,\left(m^7+29\,m^6+343\,m^5+2135\,m^4+7504\,m^3+14756\,m^2+14832\,m+5760\right.}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^3,x)
```

output
```
(x^4*(e*x)^m*(A*b^3 + 3*B*a*b^2 + 3*B*a^2*c + 6*A*a*b*c)*(24876*m + 23312*
m^2 + 10993*m^3 + 2864*m^4 + 418*m^5 + 32*m^6 + m^7 + 10080))/(109584*m +
118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 4
0320) + (x^5*(e*x)^m*(B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*b*c)*(20304*m
+ 19564*m^2 + 9544*m^3 + 2581*m^4 + 391*m^5 + 31*m^6 + m^7 + 8064))/(10958
4*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m
^8 + 40320) + (A*a^3*x*(e*x)^m*(69264*m + 48860*m^2 + 18424*m^3 + 4025*m^4
 + 511*m^5 + 35*m^6 + m^7 + 40320))/(109584*m + 118124*m^2 + 67284*m^3 + 2
2449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (3*a*x^3*(e*x)^m*(
A*b^2 + A*a*c + B*a*b)*(32048*m + 28692*m^2 + 12864*m^3 + 3195*m^4 + 447*m
^5 + 33*m^6 + m^7 + 13440))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4
 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (3*c*x^6*(e*x)^m*(B*b^2 +
A*b*c + B*a*c)*(17144*m + 16830*m^2 + 8409*m^3 + 2340*m^4 + 366*m^5 + 30*m
^6 + m^7 + 6720))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^
5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (B*c^3*x^8*(e*x)^m*(13068*m + 13132*
m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(109584*m + 11
8124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 403
20) + (a^2*x^2*(e*x)^m*(3*A*b + B*a)*(44712*m + 36706*m^2 + 15289*m^3 + 35
80*m^4 + 478*m^5 + 34*m^6 + m^7 + 20160))/(109584*m + 118124*m^2 + 67284*m
^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (c^2*x^7*...
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1335, normalized size of antiderivative = 5.56

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^3\,dx = \text{Too large to display}$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^3,x)
```

output

```
(x**m*e**m*x*(a**4*m**7 + 35*a**4*m**6 + 511*a**4*m**5 + 4025*a**4*m**4 +
18424*a**4*m**3 + 48860*a**4*m**2 + 69264*a**4*m + 40320*a**4 + 4*a**3*b*m
**7*x + 136*a**3*b*m**6*x + 1912*a**3*b*m**5*x + 14320*a**3*b*m**4*x + 611
56*a**3*b*m**3*x + 146824*a**3*b*m**2*x + 178848*a**3*b*m*x + 80640*a**3*b
*x + 3*a**3*c*m**7*x**2 + 99*a**3*c*m**6*x**2 + 1341*a**3*c*m**5*x**2 + 95
85*a**3*c*m**4*x**2 + 38592*a**3*c*m**3*x**2 + 86076*a**3*c*m**2*x**2 + 96
144*a**3*c*m*x**2 + 40320*a**3*c*x**2 + 6*a**2*b**2*m**7*x**2 + 198*a**2*b
**2*m**6*x**2 + 2682*a**2*b**2*m**5*x**2 + 19170*a**2*b**2*m**4*x**2 + 771
84*a**2*b**2*m**3*x**2 + 172152*a**2*b**2*m**2*x**2 + 192288*a**2*b**2*m*x
**2 + 80640*a**2*b**2*x**2 + 9*a**2*b*c*m**7*x**3 + 288*a**2*b*c*m**6*x**3
 + 3762*a**2*b*c*m**5*x**3 + 25776*a**2*b*c*m**4*x**3 + 98937*a**2*b*c*m**
3*x**3 + 209808*a**2*b*c*m**2*x**3 + 223884*a**2*b*c*m*x**3 + 90720*a**2*b
*c*x**3 + 3*a**2*c**2*m**7*x**4 + 93*a**2*c**2*m**6*x**4 + 1173*a**2*c**2*
m**5*x**4 + 7743*a**2*c**2*m**4*x**4 + 28632*a**2*c**2*m**3*x**4 + 58692*a
**2*c**2*m**2*x**4 + 60912*a**2*c**2*m*x**4 + 24192*a**2*c**2*x**4 + 4*a*b
**3*m**7*x**3 + 128*a*b**3*m**6*x**3 + 1672*a*b**3*m**5*x**3 + 11456*a*b**
3*m**4*x**3 + 43972*a*b**3*m**3*x**3 + 93248*a*b**3*m**2*x**3 + 99504*a*b*
*3*m*x**3 + 40320*a*b**3*x**3 + 9*a*b**2*c*m**7*x**4 + 279*a*b**2*c*m**6*x
**4 + 3519*a*b**2*c*m**5*x**4 + 23229*a*b**2*c*m**4*x**4 + 85896*a*b**2*c*
m**3*x**4 + 176076*a*b**2*c*m**2*x**4 + 182736*a*b**2*c*m*x**4 + 72576*...
```

3.229 $\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{a^2A(ex)^{1+m}}{e(1+m)}+\frac{a(2Ab+aB)(ex)^{2+m}}{e^2(2+m)}+\frac{\left(2abB+A(b^2+2ac)\right)(ex)^{3+m}}{e^3(3+m)}+\frac{\left(b^2B+2Abc+2aBc\right)(ex)^{4+m}}{e^4(4+m)}+\frac{c(2bB+Ac)(ex)^{5+m}}{e^5(5+m)}+\frac{Bc^2(ex)^{6+m}}{e^6(6+m)}$$

output
```
a^2*A*(e*x)^(1+m)/e/(1+m)+a*(2*A*b+B*a)*(e*x)^(2+m)/e^2/(2+m)+(2*a*b*B+A*(
2*a*c+b^2))*(e*x)^(3+m)/e^3/(3+m)+(2*A*b*c+2*B*a*c+B*b^2)*(e*x)^(4+m)/e^4/
(4+m)+c*(A*c+2*B*b)*(e*x)^(5+m)/e^5/(5+m)+B*c^2*(e*x)^(6+m)/e^6/(6+m)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.86

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$
$$= \frac{(ex)^m\left(x(2bB+Ac(6+m)+Bc(5+m)x)(a+x(b+cx))^2+\frac{2x\left(-\frac{2a^2c(4+m)(bB(1+m)-2Ac(6+m))}{1+m}+ab(b^2B(3+m)\right.}{}\right.}{}$$

input
```
Integrate[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
((e*x)^m*(x*(2*b*B + A*c*(6 + m) + B*c*(5 + m)*x)*(a + x*(b + c*x))^2 + (2
*x*((-2*a^2*c*(4 + m)*(b*B*(1 + m) - 2*A*c*(6 + m)))/(1 + m) + a*b*(b^2*B*
(3 + m) - 2*a*B*c*(5 + m) - A*b*c*(6 + m)) - (a*b*c*(4 + m)*(b*B*(1 + m) -
 2*A*c*(6 + m))*x)/(2 + m) + ((b^2*(2 + m) - 2*a*c*(3 + m))*(b^2*B*(3 + m)
 - 2*a*B*c*(5 + m) - A*b*c*(6 + m))*x)/(2 + m) - (a*c*(4 + m)*(b*B*(1 + m)
 - 2*A*c*(6 + m)) + b*(b^2*B*(3 + m) - 2*a*B*c*(5 + m) - A*b*c*(6 + m)) +
c*(3 + m)*(b^2*B*(3 + m) - 2*a*B*c*(5 + m) - A*b*c*(6 + m))*x)*(a + x*(b +
 c*x))))/(c*(3 + m)*(4 + m))))/(c*(5 + m)*(6 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(ex)^m\left(a+bx+cx^2\right)^2\,dx$$
$$\Big\downarrow \text{ 1195}$$

$$\int \left(a^2A(ex)^m + \frac{(ex)^{m+3}\left(2aBc+2Abc+b^2B\right)}{e^3} + \frac{(ex)^{m+2}\left(A\left(2ac+b^2\right)+2abB\right)}{e^2} + \frac{a(ex)^{m+1}(aB+2Ab)}{e} + \right.$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2A(ex)^{m+1}}{e(m+1)} + \frac{(ex)^{m+4}\left(2aBc+2Abc+b^2B\right)}{e^4(m+4)} + \frac{(ex)^{m+3}\left(A\left(2ac+b^2\right)+2abB\right)}{e^3(m+3)} + \frac{a(ex)^{m+2}(aB+2Ab)}{e^2(m+2)} + \frac{c(ex)^{m+5}(Ac+2bB)}{e^5(m+5)} + \frac{Bc^2(ex)^{m+6}}{e^6(m+6)}$$

input
```
Int[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^2,x]
```

output
```
(a^2*A*(e*x)^(1 + m))/(e*(1 + m)) + (a*(2*A*b + a*B)*(e*x)^(2 + m))/(e^2*(
2 + m)) + ((2*a*b*B + A*(b^2 + 2*a*c))*(e*x)^(3 + m))/(e^3*(3 + m)) + ((b^
2*B + 2*A*b*c + 2*a*B*c)*(e*x)^(4 + m))/(e^4*(4 + m)) + (c*(2*b*B + A*c)*(
e*x)^(5 + m))/(e^5*(5 + m)) + (B*c^2*(e*x)^(6 + m))/(e^6*(6 + m))
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

method	result
norman	$\frac{(2Abc+2aBc+B\,b^2)x^4 e^{m\ln(ex)}}{4+m}+\frac{(2Aac+b^2A+2abB)x^3 e^{m\ln(ex)}}{3+m}+\frac{B\,c^2x^6 e^{m\ln(ex)}}{6+m}+\frac{a(2Ab+Ba)x^2 e^{m\ln(ex)}}{2+m}+\frac{a^2}{}$
gosper	$\frac{x(B\,c^2m^5x^5+A\,c^2m^5x^4+2Bbc\,m^5x^4+15B\,c^2m^4x^5+2Abc\,m^5x^3+16A\,c^2m^4x^4+2Bac\,m^5x^3+B\,b^2m^5x^3+32Bbc\,m^4x^4+85B}{}$
risch	$\frac{x(B\,c^2m^5x^5+A\,c^2m^5x^4+2Bbc\,m^5x^4+15B\,c^2m^4x^5+2Abc\,m^5x^3+16A\,c^2m^4x^4+2Bac\,m^5x^3+B\,b^2m^5x^3+32Bbc\,m^4x^4+85B}{}$
orering	$\frac{x(B\,c^2m^5x^5+A\,c^2m^5x^4+2Bbc\,m^5x^4+15B\,c^2m^4x^5+2Abc\,m^5x^3+16A\,c^2m^4x^4+2Bac\,m^5x^3+B\,b^2m^5x^3+32Bbc\,m^4x^4+85B}{}$
parallelrisch	Expression too large to display

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(2*A*b*c+2*B*a*c+B*b^2)/(4+m)*x^4*exp(m*ln(e*x))+(2*A*a*c+A*b^2+2*B*a*b)/(
3+m)*x^3*exp(m*ln(e*x))+B*c^2/(6+m)*x^6*exp(m*ln(e*x))+a*(2*A*b+B*a)/(2+m)
*x^2*exp(m*ln(e*x))+a^2*A/(1+m)*x*exp(m*ln(e*x))+c*(A*c+2*B*b)/(5+m)*x^5*e
xp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(155) = 310.

Time = 0.08 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.70

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$
$$=\frac{((Bc^2m^5+15\,Bc^2m^4+85\,Bc^2m^3+225\,Bc^2m^2+274\,Bc^2m+120\,Bc^2)x^6+((2\,Bbc+Ac^2)m^5+16\,(2}{}$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
((B*c^2*m^5 + 15*B*c^2*m^4 + 85*B*c^2*m^3 + 225*B*c^2*m^2 + 274*B*c^2*m +
120*B*c^2)*x^6 + ((2*B*b*c + A*c^2)*m^5 + 16*(2*B*b*c + A*c^2)*m^4 + 95*(2
*B*b*c + A*c^2)*m^3 + 288*B*b*c + 144*A*c^2 + 260*(2*B*b*c + A*c^2)*m^2 +
324*(2*B*b*c + A*c^2)*m)*x^5 + ((B*b^2 + 2*(B*a + A*b)*c)*m^5 + 17*(B*b^2
+ 2*(B*a + A*b)*c)*m^4 + 107*(B*b^2 + 2*(B*a + A*b)*c)*m^3 + 180*B*b^2 + 3
07*(B*b^2 + 2*(B*a + A*b)*c)*m^2 + 360*(B*a + A*b)*c + 396*(B*b^2 + 2*(B*a
 + A*b)*c)*m)*x^4 + ((2*B*a*b + A*b^2 + 2*A*a*c)*m^5 + 18*(2*B*a*b + A*b^2
 + 2*A*a*c)*m^4 + 121*(2*B*a*b + A*b^2 + 2*A*a*c)*m^3 + 480*B*a*b + 240*A*
b^2 + 480*A*a*c + 372*(2*B*a*b + A*b^2 + 2*A*a*c)*m^2 + 508*(2*B*a*b + A*b
^2 + 2*A*a*c)*m)*x^3 + ((B*a^2 + 2*A*a*b)*m^5 + 19*(B*a^2 + 2*A*a*b)*m^4 +
 137*(B*a^2 + 2*A*a*b)*m^3 + 360*B*a^2 + 720*A*a*b + 461*(B*a^2 + 2*A*a*b)
*m^2 + 702*(B*a^2 + 2*A*a*b)*m)*x^2 + (A*a^2*m^5 + 20*A*a^2*m^4 + 155*A*a^
2*m^3 + 580*A*a^2*m^2 + 1044*A*a^2*m + 720*A*a^2)*x)*(e*x)^m/(m^6 + 21*m^5
 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4027 vs. $2(144) = 288$.

Time = 0.60 (sec) , antiderivative size = 4027, normalized size of antiderivative = 25.98

$$\int (ex)^m (A + Bx) \left(a + bx + cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x+A)*(c*x**2+b*x+a)**2,x)
```

output
```
Piecewise(((-A*a**2/(5*x**5) - A*a*b/(2*x**4) - 2*A*a*c/(3*x**3) - A*b**2/
(3*x**3) - A*b*c/x**2 - A*c**2/x - B*a**2/(4*x**4) - 2*B*a*b/(3*x**3) - B*
a*c/x**2 - B*b**2/(2*x**2) - 2*B*b*c/x + B*c**2*log(x))/e**6, Eq(m, -6)),
((-A*a**2/(4*x**4) - 2*A*a*b/(3*x**3) - A*a*c/x**2 - A*b**2/(2*x**2) - 2*A
*b*c/x + A*c**2*log(x) - B*a**2/(3*x**3) - B*a*b/x**2 - 2*B*a*c/x - B*b**2
/x + 2*B*b*c*log(x) + B*c**2*x)/e**5, Eq(m, -5)), ((-A*a**2/(3*x**3) - A*a
*b/x**2 - 2*A*a*c/x - A*b**2/x + 2*A*b*c*log(x) + A*c**2*x - B*a**2/(2*x**
2) - 2*B*a*b/x + 2*B*a*c*log(x) + B*b**2*log(x) + 2*B*b*c*x + B*c**2*x**2/
2)/e**4, Eq(m, -4)), ((-A*a**2/(2*x**2) - 2*A*a*b/x + 2*A*a*c*log(x) + A*b
**2*log(x) + 2*A*b*c*x + A*c**2*x**2/2 - B*a**2/x + 2*B*a*b*log(x) + 2*B*a
*c*x + B*b**2*x + B*b*c*x**2 + B*c**2*x**3/3)/e**3, Eq(m, -3)), ((-A*a**2/
x + 2*A*a*b*log(x) + 2*A*a*c*x + A*b**2*x + A*b*c*x**2 + A*c**2*x**3/3 + B
*a**2*log(x) + 2*B*a*b*x + B*a*c*x**2 + B*b**2*x**2/2 + 2*B*b*c*x**3/3 + B
*c**2*x**4/4)/e**2, Eq(m, -2)), ((A*a**2*log(x) + 2*A*a*b*x + A*a*c*x**2 +
 A*b**2*x**2/2 + 2*A*b*c*x**3/3 + A*c**2*x**4/4 + B*a**2*x + B*a*b*x**2 +
2*B*a*c*x**3/3 + B*b**2*x**3/3 + B*b*c*x**4/2 + B*c**2*x**5/5)/e, Eq(m, -1
)), (A*a**2*m**5*x*(e*x)**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m
**2 + 1764*m + 720) + 20*A*a**2*m**4*x*(e*x)**m/(m**6 + 21*m**5 + 175*m**4
 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 155*A*a**2*m**3*x*(e*x)**m/(m**6
 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 580*A*a*...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.48

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^2\,dx=\frac{Bc^2e^mx^6x^m}{m+6}+\frac{2\,Bbce^mx^5x^m}{m+5}+\frac{Ac^2e^mx^5x^m}{m+5}+\frac{Bb^2e^mx^4x^m}{m+4}+\frac{2\,Bace^mx^4x^m}{m+4}+\frac{2\,Abce^mx^4x^m}{m+4}+\frac{2\,Babe^mx^3x^m}{m+3}+\frac{Ab^2e^mx^3x^m}{m+3}+\frac{2\,Aace^mx^3x^m}{m+3}+\frac{Ba^2e^mx^2x^m}{m+2}+\frac{2\,Aabe^mx^2x^m}{m+2}+\frac{(ex)^{m+1}Aa^2}{e(m+1)}$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
B*c^2*e^m*x^6*x^m/(m + 6) + 2*B*b*c*e^m*x^5*x^m/(m + 5) + A*c^2*e^m*x^5*x^
m/(m + 5) + B*b^2*e^m*x^4*x^m/(m + 4) + 2*B*a*c*e^m*x^4*x^m/(m + 4) + 2*A*
b*c*e^m*x^4*x^m/(m + 4) + 2*B*a*b*e^m*x^3*x^m/(m + 3) + A*b^2*e^m*x^3*x^m/
(m + 3) + 2*A*a*c*e^m*x^3*x^m/(m + 3) + B*a^2*e^m*x^2*x^m/(m + 2) + 2*A*a*
b*e^m*x^2*x^m/(m + 2) + (e*x)^(m + 1)*A*a^2/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(155) = 310$.

Time = 0.22 (sec) , antiderivative size = 1142, normalized size of antiderivative = 7.37

$$\int (ex)^m(A + Bx)\left(a + bx + cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
((e*x)^m*B*c^2*m^5*x^6 + 2*(e*x)^m*B*b*c*m^5*x^5 + (e*x)^m*A*c^2*m^5*x^5 +
 15*(e*x)^m*B*c^2*m^4*x^6 + (e*x)^m*B*b^2*m^5*x^4 + 2*(e*x)^m*B*a*c*m^5*x^
4 + 2*(e*x)^m*A*b*c*m^5*x^4 + 32*(e*x)^m*B*b*c*m^4*x^5 + 16*(e*x)^m*A*c^2*
m^4*x^5 + 85*(e*x)^m*B*c^2*m^3*x^6 + 2*(e*x)^m*B*a*b*m^5*x^3 + (e*x)^m*A*b
^2*m^5*x^3 + 2*(e*x)^m*A*a*c*m^5*x^3 + 17*(e*x)^m*B*b^2*m^4*x^4 + 34*(e*x)
^m*B*a*c*m^4*x^4 + 34*(e*x)^m*A*b*c*m^4*x^4 + 190*(e*x)^m*B*b*c*m^3*x^5 +
95*(e*x)^m*A*c^2*m^3*x^5 + 225*(e*x)^m*B*c^2*m^2*x^6 + (e*x)^m*B*a^2*m^5*x
^2 + 2*(e*x)^m*A*a*b*m^5*x^2 + 36*(e*x)^m*B*a*b*m^4*x^3 + 18*(e*x)^m*A*b^2
*m^4*x^3 + 36*(e*x)^m*A*a*c*m^4*x^3 + 107*(e*x)^m*B*b^2*m^3*x^4 + 214*(e*x
)^m*B*a*c*m^3*x^4 + 214*(e*x)^m*A*b*c*m^3*x^4 + 520*(e*x)^m*B*b*c*m^2*x^5
+ 260*(e*x)^m*A*c^2*m^2*x^5 + 274*(e*x)^m*B*c^2*m*x^6 + (e*x)^m*A*a^2*m^5*
x + 19*(e*x)^m*B*a^2*m^4*x^2 + 38*(e*x)^m*A*a*b*m^4*x^2 + 242*(e*x)^m*B*a*
b*m^3*x^3 + 121*(e*x)^m*A*b^2*m^3*x^3 + 242*(e*x)^m*A*a*c*m^3*x^3 + 307*(e
*x)^m*B*b^2*m^2*x^4 + 614*(e*x)^m*B*a*c*m^2*x^4 + 614*(e*x)^m*A*b*c*m^2*x^
4 + 648*(e*x)^m*B*b*c*m*x^5 + 324*(e*x)^m*A*c^2*m*x^5 + 120*(e*x)^m*B*c^2*
x^6 + 20*(e*x)^m*A*a^2*m^4*x + 137*(e*x)^m*B*a^2*m^3*x^2 + 274*(e*x)^m*A*a
*b*m^3*x^2 + 744*(e*x)^m*B*a*b*m^2*x^3 + 372*(e*x)^m*A*b^2*m^2*x^3 + 744*(
e*x)^m*A*a*c*m^2*x^3 + 396*(e*x)^m*B*b^2*m*x^4 + 792*(e*x)^m*B*a*c*m*x^4 +
 792*(e*x)^m*A*b*c*m*x^4 + 288*(e*x)^m*B*b*c*x^5 + 144*(e*x)^m*A*c^2*x^5 +
 155*(e*x)^m*A*a^2*m^3*x + 461*(e*x)^m*B*a^2*m^2*x^2 + 922*(e*x)^m*A*a*...
```

Mupad [B] (verification not implemented)

Time = 11.63 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.61

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$
$$= (e\,x)^m\left(\frac{x^3\,(A\,b^2+2\,B\,a\,b+2\,A\,a\,c)\,(m^5+18\,m^4+121\,m^3+372\,m^2+508\,m+240)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}+\frac{x^4\,(B\,b^2+2\,A\,c\,b+2\,B\,a\,c)\,(m^5+17\,m^4+107\,m^3+307\,m^2+396\,m+180)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}+\frac{A\,a^2\,x\,(m^5+20\,m^4+155\,m^3+580\,m^2+1044\,m+720)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}+\frac{a\,x^2\,(2\,A\,b+B\,a)\,(m^5+19\,m^4+137\,m^3+461\,m^2+702\,m+360)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}+\frac{c\,x^5\,(A\,c+2\,B\,b)\,(m^5+16\,m^4+95\,m^3+260\,m^2+324\,m+144)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}+\frac{B\,c^2\,x^6\,(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120)}{m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720}\right)$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^2,x)
```

output
```
(e*x)^m*((x^3*(A*b^2 + 2*A*a*c + 2*B*a*b)*(508*m + 372*m^2 + 121*m^3 + 18*
m^4 + m^5 + 240))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 +
720) + (x^4*(B*b^2 + 2*A*b*c + 2*B*a*c)*(396*m + 307*m^2 + 107*m^3 + 17*m^
4 + m^5 + 180))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 72
0) + (A*a^2*x*(1044*m + 580*m^2 + 155*m^3 + 20*m^4 + m^5 + 720))/(1764*m +
 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (a*x^2*(2*A*b + B*a)
*(702*m + 461*m^2 + 137*m^3 + 19*m^4 + m^5 + 360))/(1764*m + 1624*m^2 + 73
5*m^3 + 175*m^4 + 21*m^5 + m^6 + 720) + (c*x^5*(A*c + 2*B*b)*(324*m + 260*
m^2 + 95*m^3 + 16*m^4 + m^5 + 144))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4
 + 21*m^5 + m^6 + 720) + (B*c^2*x^6*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m
^5 + 120))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 590, normalized size of antiderivative = 3.81

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^2\,dx$$
$$= \frac{x^m e^m x(b\,c^2m^5x^5 + a\,c^2m^5x^4 + 2b^2c\,m^5x^4 + 15b\,c^2m^4x^5 + 4abc\,m^5x^3 + 16a\,c^2m^4x^4 + b^3m^5x^3 + 32b^2c\,m^4}{}$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^2,x)
```

output
```
(x**m*e**m*x*(a**3*m**5 + 20*a**3*m**4 + 155*a**3*m**3 + 580*a**3*m**2 + 1
044*a**3*m + 720*a**3 + 3*a**2*b*m**5*x + 57*a**2*b*m**4*x + 411*a**2*b*m*
*3*x + 1383*a**2*b*m**2*x + 2106*a**2*b*m*x + 1080*a**2*b*x + 2*a**2*c*m**
5*x**2 + 36*a**2*c*m**4*x**2 + 242*a**2*c*m**3*x**2 + 744*a**2*c*m**2*x**2
 + 1016*a**2*c*m*x**2 + 480*a**2*c*x**2 + 3*a*b**2*m**5*x**2 + 54*a*b**2*m
**4*x**2 + 363*a*b**2*m**3*x**2 + 1116*a*b**2*m**2*x**2 + 1524*a*b**2*m*x*
*2 + 720*a*b**2*x**2 + 4*a*b*c*m**5*x**3 + 68*a*b*c*m**4*x**3 + 428*a*b*c*
m**3*x**3 + 1228*a*b*c*m**2*x**3 + 1584*a*b*c*m*x**3 + 720*a*b*c*x**3 + a*
c**2*m**5*x**4 + 16*a*c**2*m**4*x**4 + 95*a*c**2*m**3*x**4 + 260*a*c**2*m*
*2*x**4 + 324*a*c**2*m*x**4 + 144*a*c**2*x**4 + b**3*m**5*x**3 + 17*b**3*m
**4*x**3 + 107*b**3*m**3*x**3 + 307*b**3*m**2*x**3 + 396*b**3*m*x**3 + 180
*b**3*x**3 + 2*b**2*c*m**5*x**4 + 32*b**2*c*m**4*x**4 + 190*b**2*c*m**3*x*
*4 + 520*b**2*c*m**2*x**4 + 648*b**2*c*m*x**4 + 288*b**2*c*x**4 + b*c**2*m
**5*x**5 + 15*b*c**2*m**4*x**5 + 85*b*c**2*m**3*x**5 + 225*b*c**2*m**2*x**
5 + 274*b*c**2*m*x**5 + 120*b*c**2*x**5))/(m**6 + 21*m**5 + 175*m**4 + 735
*m**3 + 1624*m**2 + 1764*m + 720)
```

3.230 $\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{aA(ex)^{1+m}}{e(1+m)}+\frac{(Ab+aB)(ex)^{2+m}}{e^2(2+m)}+\frac{(bB+Ac)(ex)^{3+m}}{e^3(3+m)}+\frac{Bc(ex)^{4+m}}{e^4(4+m)}$$

output
```
a*A*(e*x)^(1+m)/e/(1+m)+(A*b+B*a)*(e*x)^(2+m)/e^2/(2+m)+(A*c+B*b)*(e*x)^(3
+m)/e^3/(3+m)+B*c*(e*x)^(4+m)/e^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)\,dx$$
$$=\frac{x(ex)^m\left(a\left(12+7m+m^2\right)(A(2+m)+B(1+m)x)+(1+m)x(A(4+m)(b(3+m)+c(2+m)x)+B\right.}{(1+m)(2+m)(3+m)(4+m)}$$

input
```
Integrate[(e*x)^m*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(x*(e*x)^m*(a*(12 + 7*m + m^2)*(A*(2 + m) + B*(1 + m)*x) + (1 + m)*x*(A*(4
 + m)*(b*(3 + m) + c*(2 + m)*x) + B*(2 + m)*x*(b*(4 + m) + c*(3 + m)*x)))
/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(ex)^m \left(a+bx+cx^2\right) \, dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{(ex)^{m+1}(aB+Ab)}{e}+aA(ex)^m+\frac{(ex)^{m+2}(Ac+bB)}{e^2}+\frac{Bc(ex)^{m+3}}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+2}(aB+Ab)}{e^2(m+2)}+\frac{aA(ex)^{m+1}}{e(m+1)}+\frac{(ex)^{m+3}(Ac+bB)}{e^3(m+3)}+\frac{Bc(ex)^{m+4}}{e^4(m+4)}$$

input
```
Int[(e*x)^m*(A + B*x)*(a + b*x + c*x^2),x]
```

output
```
(a*A*(e*x)^(1 + m))/(e*(1 + m)) + ((A*b + a*B)*(e*x)^(2 + m))/(e^2*(2 + m)
) + ((b*B + A*c)*(e*x)^(3 + m))/(e^3*(3 + m)) + (B*c*(e*x)^(4 + m))/(e^4*(
4 + m))
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

method	result
norman	$\frac{(Ab+Ba)x^2 \mathrm{e}^{m\ln(ex)}}{2+m}+\frac{(Ac+Bb)x^3 \mathrm{e}^{m\ln(ex)}}{3+m}+\frac{Aax\,\mathrm{e}^{m\ln(ex)}}{1+m}+\frac{Bcx^4 \mathrm{e}^{m\ln(ex)}}{4+m}$
gosper	$\frac{x(Bcm^3x^3+Acm^3x^2+Bbm^3x^2+6Bcm^2x^3+Abm^3x+7Acm^2x^2+Bam^3x+7Bbm^2x^2+11Bcmx^3+Aam^3+8Abm^2x+14}{(4+m)(3+}$
risch	$\frac{x(Bcm^3x^3+Acm^3x^2+Bbm^3x^2+6Bcm^2x^3+Abm^3x+7Acm^2x^2+Bam^3x+7Bbm^2x^2+11Bcmx^3+Aam^3+8Abm^2x+14}{(4+m)(3+}$
orering	$\frac{x(Bcm^3x^3+Acm^3x^2+Bbm^3x^2+6Bcm^2x^3+Abm^3x+7Acm^2x^2+Bam^3x+7Bbm^2x^2+11Bcmx^3+Aam^3+8Abm^2x+14}{(4+m)(3+}$
parallelrisch	$6Bx^4(ex)^m c+8Ax^3(ex)^m c+8Bx^3(ex)^m b+12Ax^2(ex)^m b+12Bx^2(ex)^m a+24Ax(ex)^m a+Bx^4(ex)^m cm^3+Ax^3(ex)^m cm$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(A*b+B*a)/(2+m)*x^2*exp(m*ln(e*x))+(A*c+B*b)/(3+m)*x^3*exp(m*ln(e*x))+A*a/
(1+m)*x*exp(m*ln(e*x))+B*c/(4+m)*x^4*exp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(83) = 166.

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)\,dx$$
$$=\frac{((Bcm^3+6\,Bcm^2+11\,Bcm+6\,Bc)x^4+((Bb+Ac)m^3+7\,(Bb+Ac)m^2+8\,Bb+8\,Ac+14\,(Bb+A}{m^4}$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
((B*c*m^3 + 6*B*c*m^2 + 11*B*c*m + 6*B*c)*x^4 + ((B*b + A*c)*m^3 + 7*(B*b
+ A*c)*m^2 + 8*B*b + 8*A*c + 14*(B*b + A*c)*m)*x^3 + ((B*a + A*b)*m^3 + 8*
(B*a + A*b)*m^2 + 12*B*a + 12*A*b + 19*(B*a + A*b)*m)*x^2 + (A*a*m^3 + 9*A
*a*m^2 + 26*A*a*m + 24*A*a)*x)*(e*x)^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. 2(71) = 142.

Time = 0.33 (sec) , antiderivative size = 981, normalized size of antiderivative = 11.82

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x+A)*(c*x**2+b*x+a),x)
```

output
```
Piecewise(((-A*a/(3*x**3) - A*b/(2*x**2) - A*c/x - B*a/(2*x**2) - B*b/x +
B*c*log(x))/e**4, Eq(m, -4)), ((-A*a/(2*x**2) - A*b/x + A*c*log(x) - B*a/x
 + B*b*log(x) + B*c*x)/e**3, Eq(m, -3)), ((-A*a/x + A*b*log(x) + A*c*x + B
*a*log(x) + B*b*x + B*c*x**2/2)/e**2, Eq(m, -2)), ((A*a*log(x) + A*b*x + A
*c*x**2/2 + B*a*x + B*b*x**2/2 + B*c*x**3/3)/e, Eq(m, -1)), (A*a*m**3*x*(e
*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*A*a*m**2*x*(e*x)**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*A*a*m*x*(e*x)**m/(m**4 + 10*m**3 +
 35*m**2 + 50*m + 24) + 24*A*a*x*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m
 + 24) + A*b*m**3*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 8
*A*b*m**2*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 19*A*b*m*
x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 12*A*b*x**2*(e*x)**
m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + A*c*m**3*x**3*(e*x)**m/(m**4 +
10*m**3 + 35*m**2 + 50*m + 24) + 7*A*c*m**2*x**3*(e*x)**m/(m**4 + 10*m**3
+ 35*m**2 + 50*m + 24) + 14*A*c*m*x**3*(e*x)**m/(m**4 + 10*m**3 + 35*m**2
+ 50*m + 24) + 8*A*c*x**3*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24)
+ B*a*m**3*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 8*B*a*m*
*2*x**2*(e*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 19*B*a*m*x**2*(e
*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 12*B*a*x**2*(e*x)**m/(m**4
 + 10*m**3 + 35*m**2 + 50*m + 24) + B*b*m**3*x**3*(e*x)**m/(m**4 + 10*m**3
 + 35*m**2 + 50*m + 24) + 7*B*b*m**2*x**3*(e*x)**m/(m**4 + 10*m**3 + 35...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)\,dx=\frac{Bce^mx^4x^m}{m+4}+\frac{Bbe^mx^3x^m}{m+3}+\frac{Ace^mx^3x^m}{m+3}+\frac{Bae^mx^2x^m}{m+2}+\frac{Abe^mx^2x^m}{m+2}+\frac{(ex)^{m+1}Aa}{e(m+1)}$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
B*c*e^m*x^4*x^m/(m + 4) + B*b*e^m*x^3*x^m/(m + 3) + A*c*e^m*x^3*x^m/(m + 3
) + B*a*e^m*x^2*x^m/(m + 2) + A*b*e^m*x^2*x^m/(m + 2) + (e*x)^(m + 1)*A*a/
(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(83) = 166$.

Time = 0.23 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.07

$$\int (ex)^m (A + Bx) \left(a + bx + cx^2\right) dx = \frac{(ex)^m Bcm^3x^4 + (ex)^m Bbm^3x^3 + (ex)^m Acm^3x^3 + 6(ex)^m Bcm^2x^4 + (ex)^m Bam^3x^2 + (ex)^m Abm^3x^2}{}$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
((e*x)^m*B*c*m^3*x^4 + (e*x)^m*B*b*m^3*x^3 + (e*x)^m*A*c*m^3*x^3 + 6*(e*x)
^m*B*c*m^2*x^4 + (e*x)^m*B*a*m^3*x^2 + (e*x)^m*A*b*m^3*x^2 + 7*(e*x)^m*B*b
*m^2*x^3 + 7*(e*x)^m*A*c*m^2*x^3 + 11*(e*x)^m*B*c*m*x^4 + (e*x)^m*A*a*m^3*
x + 8*(e*x)^m*B*a*m^2*x^2 + 8*(e*x)^m*A*b*m^2*x^2 + 14*(e*x)^m*B*b*m*x^3 +
 14*(e*x)^m*A*c*m*x^3 + 6*(e*x)^m*B*c*x^4 + 9*(e*x)^m*A*a*m^2*x + 19*(e*x)
^m*B*a*m*x^2 + 19*(e*x)^m*A*b*m*x^2 + 8*(e*x)^m*B*b*x^3 + 8*(e*x)^m*A*c*x^
3 + 26*(e*x)^m*A*a*m*x + 12*(e*x)^m*B*a*x^2 + 12*(e*x)^m*A*b*x^2 + 24*(e*x
)^m*A*a*x)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Mupad [B] (verification not implemented)

Time = 11.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int (ex)^m (A + Bx) \left(a + bx + cx^2\right) dx = (e\,x)^m \left(\frac{x^2\,(A\,b + B\,a)\,\left(m^3 + 8\,m^2 + 19\,m + 12\right)}{m^4 + 10\,m^3 + 35\,m^2 + 50\,m + 24} + \frac{x^3\,(A\,c + B\,b)\,\left(m^3 + 7\,m^2 + 14\,m + 8\right)}{m^4 + 10\,m^3 + 35\,m^2 + 50\,m + 24} + \frac{A\,a\,x\,\left(m^3 + 9\,m^2 + 26\,m + 24\right)}{m^4 + 10\,m^3 + 35\,m^2 + 50\,m + 24} + \frac{B\,c\,x^4\,\left(m^3 + 6\,m^2 + 11\,m + 6\right)}{m^4 + 10\,m^3 + 35\,m^2 + 50\,m + 24}\right)$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2),x)
```

output
```
(e*x)^m*((x^2*(A*b + B*a)*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m
^3 + m^4 + 24) + (x^3*(A*c + B*b)*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2
 + 10*m^3 + m^4 + 24) + (A*a*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 +
 10*m^3 + m^4 + 24) + (B*c*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 +
10*m^3 + m^4 + 24))
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.27

$$\int (ex)^m (A+Bx) \left(a+bx+cx^2\right) dx$$
$$= \frac{x^m e^m x(bc\,m^3x^3 + ac\,m^3x^2 + b^2m^3x^2 + 6bc\,m^2x^3 + 2ab\,m^3x + 7ac\,m^2x^2 + 7b^2m^2x^2 + 11bcm\,x^3 + a^2m^3 +}{m^4 + 10m^3 +}$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a),x)
```

output
```
(x**m*e**m*x*(a**2*m**3 + 9*a**2*m**2 + 26*a**2*m + 24*a**2 + 2*a*b*m**3*x
 + 16*a*b*m**2*x + 38*a*b*m*x + 24*a*b*x + a*c*m**3*x**2 + 7*a*c*m**2*x**2
 + 14*a*c*m*x**2 + 8*a*c*x**2 + b**2*m**3*x**2 + 7*b**2*m**2*x**2 + 14*b**
2*m*x**2 + 8*b**2*x**2 + b*c*m**3*x**3 + 6*b*c*m**2*x**3 + 11*b*c*m*x**3 +
 6*b*c*x**3))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24)
```

3.231 $\int \frac{(ex)^m(A+Bx)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 196

$$\int \frac{(ex)^m(A+Bx)}{a+bx+cx^2}\,dx = -\frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)e(1+m)} - \frac{\left(2Ac-B\left(b+\sqrt{b^2-4ac}\right)\right)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)e(1+m)}$$

output
```
-(B*b-2*A*c-B*(-4*a*c+b^2)^(1/2))*(e*x)^(1+m)*hypergeom([1, 1+m],[2+m],-2*
c*x/(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))/e/(1
+m)-(2*A*c-B*(b+(-4*a*c+b^2)^(1/2)))*(e*x)^(1+m)*hypergeom([1, 1+m],[2+m],
-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/e
/(1+m)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int \frac{(ex)^m(A+Bx)}{a+bx+cx^2}\,dx = \frac{x(ex)^m\left(\left(-2aB+A\left(b+\sqrt{b^2-4ac}\right)\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)+\left(2aB+A(\right.\right.}{2a\sqrt{b^2-4ac}(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x))/(a + b*x + c*x^2),x]
```

output
```
(x*(e*x)^m*((-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*Hypergeometric2F1[1, 1 +
m, 2 + m, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (2*a*B + A*(-b + Sqrt[b^2 -
4*a*c]))*Hypergeometric2F1[1, 1 + m, 2 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c
])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(ex)^m}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{(ex)^m\left(\frac{2Ac-bB}{\sqrt{b^2-4ac}}+B\right)}{-\sqrt{b^2-4ac}+b+2cx}+\frac{(ex)^m\left(B-\frac{2Ac-bB}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b+2cx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+1}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{e(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(ex)^{m+1}\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[((e*x)^m*(A + B*x))/(a + b*x + c*x^2),x]
```

output
```
((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*(e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*
e*(1 + m)) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*(e*x)^(1 + m)*Hypergeo
metric2F1[1, 1 + m, 2 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b
^2 - 4*a*c])*e*(1 + m))
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (Bx+A)}{c\,x^2+bx+a}dx$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a),x)
```

output
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a),x)
```

Fricas [F]

$$\int \frac{(ex)^m(A+Bx)}{a+bx+cx^2}\,dx = \int \frac{(Bx+A)(ex)^m}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x)^m/(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{(ex)^m(A+Bx)}{a+bx+cx^2}\,dx = \int \frac{(ex)^m\,(A+Bx)}{a+bx+cx^2}\,dx$$

input
```
integrate((e*x)**m*(B*x+A)/(c*x**2+b*x+a),x)
```

output
```
Integral((e*x)**m*(A + B*x)/(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(ex)^m(A+Bx)}{a+bx+cx^2}\,dx = \int \frac{(Bx+A)(ex)^m}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x)^m/(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(ex)^m(A+Bx)}{a+bx+cx^2}\,dx=\int \frac{(Bx+A)(ex)^m}{cx^2+bx+a}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^m/(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{a+bx+cx^2}\,dx=\int \frac{(e\,x)^m\,(A+B\,x)}{c\,x^2+b\,x+a}\,dx$$

input
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2),x)
```

output
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m(A+Bx)}{a+bx+cx^2}\,dx=\frac{e^m\left(x^m a-\left(\int \frac{x^m}{c\,x^3+b\,x^2+ax}dx\right)a^2m-\left(\int \frac{x^m x}{c\,x^2+bx+a}dx\right)acm+\left(\int \frac{x^m x}{c\,x^2+bx+a}dx\right)b^2m\right)}{bm}$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a),x)
```

output
```
(e**m*(x**m*a - int(x**m/(a*x + b*x**2 + c*x**3),x)*a**2*m - int((x**m*x)/
(a + b*x + c*x**2),x)*a*c*m + int((x**m*x)/(a + b*x + c*x**2),x)*b**2*m))/
(b*m)
```

3.232 $\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 324

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^2}\,dx = \frac{(ex)^{1+m}\left(Ab^2-abB-2aAc+(Ab-2aB)cx\right)}{a\left(b^2-4ac\right)e\left(a+bx+cx^2\right)}$$
$$-\frac{c\left(Ab\left(b+\sqrt{b^2-4ac}\right)m-2a\left(bB-2Ac(1-m)+B\sqrt{b^2-4ac}\,m\right)\right)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\right.}{a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)e(1+m)}$$
$$+\frac{c\left(Ab\left(b-\sqrt{b^2-4ac}\right)m-2a\left(bB-2Ac(1-m)-B\sqrt{b^2-4ac}\,m\right)\right)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\right.}{a\left(b^2-4ac\right)^{3/2}\left(b+\sqrt{b^2-4ac}\right)e(1+m)}$$

output
```
(e*x)^(1+m)*(A*b^2-a*b*B-2*A*a*c+(A*b-2*B*a)*c*x)/a/(-4*a*c+b^2)/e/(c*x^2+
b*x+a)-c*(A*b*(b+(-4*a*c+b^2)^(1/2))*m-2*a*(B*b-2*A*c*(1-m)+B*(-4*a*c+b^2)
^(1/2)*m))*(e*x)^(1+m)*hypergeom([1, 1+m],[2+m],-2*c*x/(b-(-4*a*c+b^2)^(1/
2)))/a/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))/e/(1+m)+c*(A*b*(b-(-4*a*c
+b^2)^(1/2))*m-2*a*(B*b-2*A*c*(1-m)-B*(-4*a*c+b^2)^(1/2)*m))*(e*x)^(1+m)*h
ypergeom([1, 1+m],[2+m],-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2
)/(b+(-4*a*c+b^2)^(1/2))/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{(ex)^m\left(\frac{x(-aB(b+2cx)+A(b^2-2ac+bcx))}{a+x(b+cx)}+\frac{cx\left(-\frac{\left((Ab-2aB)m+\frac{-2a(bB+2Ac(-1+m))+Ab^2m}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}}\right.}{}\right.}{a(b^2-4ac)}$$

input
```
Integrate[((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^2,x]
```

output
```
((e*x)^m*((x*(-(a*B*(b + 2*c*x)) + A*(b^2 - 2*a*c + b*c*x)))/(a + x*(b + c
*x)) + (c*x*(-((((A*b - 2*a*B)*m + (-2*a*(b*B + 2*A*c*(-1 + m)) + A*b^2*m)
/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + m, 2 + m, (2*c*x)/(-b + Sqrt[
b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c])) - (((A*b - 2*a*B)*m + (2*a*b*B +
4*a*A*c*(-1 + m) - A*b^2*m)/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + m,
 2 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c])))/(1 +
m)))/(a*(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(ex)^m}{(a+bx+cx^2)^2}\,dx$$
$$\Big\downarrow \; 1235$$

$$\frac{(ex)^{m+1}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ae\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{e^2(ex)^m\left(Amb^2+a(2Ac(1-m)-bB(m+1))+(Ab-2aB)cmx\right)}{cx^2+bx+a}dx}{ae^2\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{(ex)^{m+1}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ae\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{(ex)^m\left(Amb^2+a(2Ac(1-m)-bB(m+1))+(Ab-2aB)cmx\right)}{cx^2+bx+a}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 1200$$

$$\frac{(ex)^{m+1}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ae\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \left(\frac{\left((Ab-2aB)cm+\frac{c\left(Amb^2-2aBb+4aAc-4aAcm\right)}{\sqrt{b^2-4ac}}\right)(ex)^m}{b+2cx-\sqrt{b^2-4ac}}+\frac{\left((Ab-2aB)cm-\frac{c\left(Amb^2-2aBb+4aAc-4aAcm\right)}{\sqrt{b^2-4ac}}\right)(ex)^m}{b+2cx+\sqrt{b^2-4ac}}\right)dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{(ex)^{m+1}\left(cx(Ab-2aB)-2aAc-abB+Ab^2\right)}{ae\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\frac{c(ex)^{m+1}\left(m(Ab-2aB)-\frac{2a(bB-2Ac(1-m))-Ab^2m}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{e(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{c(ex)^{m+1}\left(\frac{-4aAc(1-m)+2abB-Ab^2}{\sqrt{b^2-4ac}}\right.}{}}{a\left(b^2-4ac\right)}$$

input
```
Int[((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^2,x]
```

output
```
((e*x)^(1 + m)*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x))/(a*(b^2 - 4*
a*c)*e*(a + b*x + c*x^2)) - ((c*((A*b - 2*a*B)*m - (2*a*(b*B - 2*A*c*(1 -
m)) - A*b^2*m)/Sqrt[b^2 - 4*a*c])*(e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m
, 2 + m, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*e*(1
+ m)) + (c*((A*b - 2*a*B)*m + (2*a*b*B - 4*a*A*c*(1 - m) - A*b^2*m)/Sqrt[b
^2 - 4*a*c])*(e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-2*c*x)/(b
+ Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*e*(1 + m)))/(a*(b^2 - 4*a*
c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (Bx+A)}{(c\,x^2+bx+a)^2}dx$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^2,x)
```

output
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^2}\,dx=\int \frac{(Bx+A)(ex)^m}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x)^m/(c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*
x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**m*(B*x+A)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^2}\,dx=\int \frac{(Bx+A)(ex)^m}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x)^m/(c*x^2 + b*x + a)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^2}\,dx = \int \frac{(Bx+A)(ex)^m}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^m/(c*x^2 + b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^2}\,dx = \int \frac{(e\,x)^m\,(A+B\,x)}{(c\,x^2+b\,x+a)^2}\,dx$$

input
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^2,x)
```

output
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^2}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^2,x)
```

output

```
(e**m*(x**m*a - int(x**m/(a**2*m*x - a**2*x + 2*a*b*m*x**2 - 2*a*b*x**2 +
2*a*c*m*x**3 - 2*a*c*x**3 + b**2*m*x**3 - b**2*x**3 + 2*b*c*m*x**4 - 2*b*c
*x**4 + c**2*m*x**5 - c**2*x**5),x)*a**3*m**2 + int(x**m/(a**2*m*x - a**2*
x + 2*a*b*m*x**2 - 2*a*b*x**2 + 2*a*c*m*x**3 - 2*a*c*x**3 + b**2*m*x**3 -
b**2*x**3 + 2*b*c*m*x**4 - 2*b*c*x**4 + c**2*m*x**5 - c**2*x**5),x)*a**3*m
 - int(x**m/(a**2*m*x - a**2*x + 2*a*b*m*x**2 - 2*a*b*x**2 + 2*a*c*m*x**3
- 2*a*c*x**3 + b**2*m*x**3 - b**2*x**3 + 2*b*c*m*x**4 - 2*b*c*x**4 + c**2*
m*x**5 - c**2*x**5),x)*a**2*b*m**2*x + int(x**m/(a**2*m*x - a**2*x + 2*a*b
*m*x**2 - 2*a*b*x**2 + 2*a*c*m*x**3 - 2*a*c*x**3 + b**2*m*x**3 - b**2*x**3
 + 2*b*c*m*x**4 - 2*b*c*x**4 + c**2*m*x**5 - c**2*x**5),x)*a**2*b*m*x - in
t(x**m/(a**2*m*x - a**2*x + 2*a*b*m*x**2 - 2*a*b*x**2 + 2*a*c*m*x**3 - 2*a
*c*x**3 + b**2*m*x**3 - b**2*x**3 + 2*b*c*m*x**4 - 2*b*c*x**4 + c**2*m*x**
5 - c**2*x**5),x)*a**2*c*m**2*x**2 + int(x**m/(a**2*m*x - a**2*x + 2*a*b*m
*x**2 - 2*a*b*x**2 + 2*a*c*m*x**3 - 2*a*c*x**3 + b**2*m*x**3 - b**2*x**3 +
 2*b*c*m*x**4 - 2*b*c*x**4 + c**2*m*x**5 - c**2*x**5),x)*a**2*c*m*x**2 - i
nt((x**m*x)/(a**2*m - a**2 + 2*a*b*m*x - 2*a*b*x + 2*a*c*m*x**2 - 2*a*c*x*
*2 + b**2*m*x**2 - b**2*x**2 + 2*b*c*m*x**3 - 2*b*c*x**3 + c**2*m*x**4 - c
**2*x**4),x)*a**2*c*m**2 + 3*int((x**m*x)/(a**2*m - a**2 + 2*a*b*m*x - 2*a
*b*x + 2*a*c*m*x**2 - 2*a*c*x**2 + b**2*m*x**2 - b**2*x**2 + 2*b*c*m*x**3
- 2*b*c*x**3 + c**2*m*x**4 - c**2*x**4),x)*a**2*c*m - 2*int((x**m*x)/(a...
```

3.233 $\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{A(ex)^{1+m}\left(a+bx+cx^2\right)^{5/2}\operatorname{AppellF1}\left(1+m,-\frac{5}{2},-\frac{5}{2},2+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(1+m)\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{5/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{5/2}}+\frac{B(ex)^{2+m}\left(a+bx+cx^2\right)^{5/2}\operatorname{AppellF1}\left(2+m,-\frac{5}{2},-\frac{5}{2},3+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(2+m)\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{5/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{5/2}}$$

output
```
A*(e*x)^(1+m)*(c*x^2+b*x+a)^(5/2)*AppellF1(1+m,-5/2,-5/2,2+m,-2*c*x/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e/(1+m)/(1+2*c*x/(b-(-4*a*
c+b^2)^(1/2)))^(5/2)/(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(5/2)+B*(e*x)^(2+m)*
(c*x^2+b*x+a)^(5/2)*AppellF1(2+m,-5/2,-5/2,3+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e^2/(2+m)/(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)
))^(5/2)/(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(5/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 618 vs. $2(281) = 562$.

Time = 3.61 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.20

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx = \frac{x(ex)^m\sqrt{a+x(b+cx)}\left(a^2A(720+1044m+580m^2+155m^3+20m^4+m^5)\,\text{AppellF1}\left(1+\right.\right.}{}$$

input
```
Integrate[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(x*(e*x)^m*Sqrt[a + x*(b + c*x)]*(a^2*A*(720 + 1044*m + 580*m^2 + 155*m^3
+ 20*m^4 + m^5)*AppellF1[1 + m, -1/2, -1/2, 2 + m, (-2*c*x)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x*(a*(2*A*b + a*B)*
(360 + 342*m + 119*m^2 + 18*m^3 + m^4)*AppellF1[2 + m, -1/2, -1/2, 3 + m,
(-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (2 +
 m)*x*((2*a*b*B + A*(b^2 + 2*a*c))*(120 + 74*m + 15*m^2 + m^3)*AppellF1[3
+ m, -1/2, -1/2, 4 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sq
rt[b^2 - 4*a*c])] + (3 + m)*x*((b^2*B + 2*A*b*c + 2*a*B*c)*(30 + 11*m + m^
2)*AppellF1[4 + m, -1/2, -1/2, 5 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + c*(4 + m)*x*((2*b*B + A*c)*(6 + m)*Appel
lF1[5 + m, -1/2, -1/2, 6 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-
b + Sqrt[b^2 - 4*a*c])] + B*c*(5 + m)*x*AppellF1[6 + m, -1/2, -1/2, 7 + m,
 (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])]))))))
/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(ex)^m \left(a+bx+cx^2\right)^{5/2} dx$$

$$\Big\downarrow \ 1269$$

$$A\int (ex)^m \left(cx^2+bx+a\right)^{5/2} dx+\frac{B\int (ex)^{m+1} \left(cx^2+bx+a\right)^{5/2} dx}{e}$$

$$\Big\downarrow \ 1179$$

$$\frac{A\left(a+bx+cx^2\right)^{5/2}\int (ex)^m \left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^{5/2}d(ex)}{e\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{5/2}}+\frac{B\left(a+bx+cx^2\right)^{5/2}\int (ex)^{m+1} \left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^{5/2}d(ex)}{e^2\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{5/2}}$$

$$\Big\downarrow \ 150$$

$$\frac{A(ex)^{m+1}\left(a+bx+cx^2\right)^{5/2}\text{AppellF1}\left(m+1,-\frac{5}{2},-\frac{5}{2},m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(m+1)\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{5/2}}+\frac{B(ex)^{m+2}\left(a+bx+cx^2\right)^{5/2}\text{AppellF1}\left(m+2,-\frac{5}{2},-\frac{5}{2},m+3,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(m+2)\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{5/2}}$$

input
```
Int[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(A*(e*x)^(1 + m)*(a + b*x + c*x^2)^(5/2)*AppellF1[1 + m, -5/2, -5/2, 2 + m
, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(e*
(1 + m)*(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^(5/2)*(1 + (2*c*x)/(b + Sqrt
[b^2 - 4*a*c]))^(5/2)) + (B*(e*x)^(2 + m)*(a + b*x + c*x^2)^(5/2)*AppellF1
[2 + m, -5/2, -5/2, 3 + m, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b +
 Sqrt[b^2 - 4*a*c])])/(e^2*(2 + m)*(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^(
5/2)*(1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^(5/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
 - e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (ex)^m (Bx + A) \left(c\,x^2 + bx + a\right)^{\frac{5}{2}} dx$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

output
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

Fricas [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{5}{2}}(Bx+A)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
integral((B*c^2*x^5 + (2*B*b*c + A*c^2)*x^4 + (B*b^2 + 2*(B*a + A*b)*c)*x^
3 + A*a^2 + (2*B*a*b + A*b^2 + 2*A*a*c)*x^2 + (B*a^2 + 2*A*a*b)*x)*sqrt(c*
x^2 + b*x + a)*(e*x)^m, x)
```

Sympy [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int (ex)^m\,(A+Bx)\left(a+bx+cx^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x)**m*(B*x+A)*(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral((e*x)**m*(A + B*x)*(a + b*x + c*x**2)**(5/2), x)
```

Maxima [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{5}{2}}(Bx+A)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(5/2)*(B*x + A)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{5}{2}}(Bx+A)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(5/2)*(B*x + A)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int (e\,x)^m\,\left(A+B\,x\right)\,\left(c\,x^2+b\,x+a\right)^{5/2}\,dx$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int (ex)^m\,(Bx+A)\,\left(c\,x^2+bx+a\right)^{\frac{5}{2}}\,dx$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

output
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(5/2),x)
```

3.234 $\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{A(ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}\operatorname{AppellF1}\left(1+m,-\frac{3}{2},-\frac{3}{2},2+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(1+m)\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{3/2}}+\frac{B(ex)^{2+m}\left(a+bx+cx^2\right)^{3/2}\operatorname{AppellF1}\left(2+m,-\frac{3}{2},-\frac{3}{2},3+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(2+m)\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{3/2}}$$

output
```
A*(e*x)^(1+m)*(c*x^2+b*x+a)^(3/2)*AppellF1(1+m,-3/2,-3/2,2+m,-2*c*x/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e/(1+m)/(1+2*c*x/(b-(-4*a*
c+b^2)^(1/2)))^(3/2)/(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(3/2)+B*(e*x)^(2+m)*
(c*x^2+b*x+a)^(3/2)*AppellF1(2+m,-3/2,-3/2,3+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e^2/(2+m)/(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)
))^(3/2)/(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.08 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.44

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{x(ex)^m\sqrt{a+x(b+cx)}\left(aA(24+26m+9m^2+m^3)\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,-\frac{}{b+}\right.\right.}{}$$

input
```
Integrate[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(x*(e*x)^m*Sqrt[a + x*(b + c*x)]*(a*A*(24 + 26*m + 9*m^2 + m^3)*AppellF1[1
 + m, -1/2, -1/2, 2 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + S
qrt[b^2 - 4*a*c])] + (1 + m)*x*((A*b + a*B)*(12 + 7*m + m^2)*AppellF1[2 +
m, -1/2, -1/2, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt
[b^2 - 4*a*c])] + (2 + m)*x*((b*B + A*c)*(4 + m)*AppellF1[3 + m, -1/2, -1/
2, 4 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c
])] + B*c*(3 + m)*x*AppellF1[4 + m, -1/2, -1/2, 5 + m, (-2*c*x)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])]))))/((1 + m)*(2 + m)*(3 +
 m)*(4 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(ex)^m\left(a+bx+cx^2\right)^{3/2}dx$$

$$\Big\downarrow \; 1269$$

$$A\int (ex)^m\left(cx^2+bx+a\right)^{3/2}dx+\frac{B\int (ex)^{m+1}\left(cx^2+bx+a\right)^{3/2}dx}{e}$$

$$\Big\downarrow \text{1179}$$

$$\frac{A(a+bx+cx^2)^{3/2}\int (ex)^m \left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}d(ex)}{e\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{3/2}}+\frac{B(a+bx+cx^2)^{3/2}\int (ex)^{m+1} \left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}d(ex)}{e^2\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{3/2}}$$

$$\Big\downarrow \text{150}$$

$$\frac{A(ex)^{m+1}(a+bx+cx^2)^{3/2}\text{AppellF1}\left(m+1,-\frac{3}{2},-\frac{3}{2},m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(m+1)\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{3/2}}+\frac{B(ex)^{m+2}(a+bx+cx^2)^{3/2}\text{AppellF1}\left(m+2,-\frac{3}{2},-\frac{3}{2},m+3,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(m+2)\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{3/2}}$$

input
```
Int[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(A*(e*x)^(1 + m)*(a + b*x + c*x^2)^(3/2)*AppellF1[1 + m, -3/2, -3/2, 2 + m
, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(e*
(1 + m)*(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^(3/2)*(1 + (2*c*x)/(b + Sqrt
[b^2 - 4*a*c]))^(3/2)) + (B*(e*x)^(2 + m)*(a + b*x + c*x^2)^(3/2)*AppellF1
[2 + m, -3/2, -3/2, 3 + m, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b +
 Sqrt[b^2 - 4*a*c])])/(e^2*(2 + m)*(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^(
3/2)*(1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^(3/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (ex)^m (Bx+A) \left(c\,x^2+bx+a\right)^{\frac{3}{2}}dx$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

Fricas [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{3}{2}}(Bx+A)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*c*x^3 + (B*b + A*c)*x^2 + A*a + (B*a + A*b)*x)*sqrt(c*x^2 + b*
x + a)*(e*x)^m, x)
```

Sympy [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int (ex)^m\,(A+Bx)\left(a+bx+cx^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)**m*(B*x+A)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((e*x)**m*(A + B*x)*(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{3}{2}}(Bx+A)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)*(B*x + A)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{3}{2}}(Bx+A)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)*(B*x + A)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int (e\,x)^m\,\left(A+B\,x\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,dx$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int (ex)^m\,(Bx+A)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}dx$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(3/2),x)
```

3.235 $\int (ex)^m(A+Bx)\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\int (ex)^m(A+Bx)\sqrt{a+bx+cx^2}\,dx = \frac{A(ex)^{1+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(1+m)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}} + \frac{B(ex)^{2+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(2+m)\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}}$$

output
```
A*(e*x)^(1+m)*(c*x^2+b*x+a)^(1/2)*AppellF1(1+m,-1/2,-1/2,2+m,-2*c*x/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e/(1+m)/(1+2*c*x/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+B*(e*x)^(2+m)*
(c*x^2+b*x+a)^(1/2)*AppellF1(2+m,-1/2,-1/2,3+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2
)),-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e^2/(2+m)/(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.83

$$\int (ex)^m (A + Bx)\sqrt{a + bx + cx^2}\, dx$$
$$= \frac{x(ex)^m \sqrt{a + x(b + cx)}\left(A(2 + m)\,\text{AppellF1}\left(1 + m, -\frac{1}{2}, -\frac{1}{2}, 2 + m, -\frac{2cx}{b+\sqrt{b^2-4ac}}, \frac{2cx}{-b+\sqrt{b^2-4ac}}\right) + B(1 + \cdots\right.}{(1 + m)(2 + m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}\cdots}{b+\sqrt{b^2-\cdots}}}}$$

input
```
Integrate[(e*x)^m*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(x*(e*x)^m*Sqrt[a + x*(b + c*x)]*(A*(2 + m)*AppellF1[1 + m, -1/2, -1/2, 2
+ m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] +
 B*(1 + m)*x*AppellF1[2 + m, -1/2, -1/2, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 + m)*(2 + m)*Sqrt[(b - Sqr
t[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + Bx)(ex)^m \sqrt{a + bx + cx^2}\, dx$$

$$\Big\downarrow \text{1269}$$

$$A \int (ex)^m \sqrt{cx^2 + bx + a}dx + \frac{B \int (ex)^{m+1}\sqrt{cx^2 + bx + a}dx}{e}$$

$$\Big\downarrow \text{1179}$$

$$\frac{A\sqrt{a+bx+cx^2}\int (ex)^m\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{b+\sqrt{b^2-4ac}}+1}d(ex)}{e\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}}+\frac{B\sqrt{a+bx+cx^2}\int (ex)^{m+1}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{b+\sqrt{b^2-4ac}}+1}d(ex)}{e^2\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{A(ex)^{m+1}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(m+1)\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}}+\frac{B(ex)^{m+2}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(m+2)\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(e*x)^m*(A + B*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(A*(e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
(-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(e*(1
 + m)*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x)/(b + Sqrt
[b^2 - 4*a*c])]) + (B*(e*x)^(2 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[2 + m,
-1/2, -1/2, 3 + m, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^
2 - 4*a*c])])/(e^2*(2 + m)*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int \left(e x \right)^{m} \left(B x +A \right) \sqrt{c \,x^{2}+b x +a}d x$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

Fricas [F]

$$\int (ex)^m(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(Bx+A)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(B*x + A)*(e*x)^m, x)
```

Sympy [F]

$$\int (ex)^m(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int (ex)^m\,(A+Bx)\,\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((e*x)**m*(B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((e*x)**m*(A + B*x)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int (ex)^m(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(Bx+A)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(Bx+A)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(B*x + A)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int (e\,x)^m\,(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int (ex)^m(A+Bx)\sqrt{a+bx+cx^2}\,dx = \int (ex)^m\,(Bx+A)\,\sqrt{c\,x^2+bx+a}dx$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

3.236 $\int \frac{(ex)^m(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\int \frac{(ex)^m(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{A(ex)^{1+m}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(1+m)\sqrt{a+bx+cx^2}} + \frac{B(ex)^{2+m}\sqrt{1+\frac{2cx}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(2+m,\frac{1}{2},\frac{1}{2},3+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(2+m)\sqrt{a+bx+cx^2}}$$

output
```
A*(e*x)^(1+m)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b
^2)^(1/2)))^(1/2)*AppellF1(1+m,1/2,1/2,2+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e/(1+m)/(c*x^2+b*x+a)^(1/2)+B*(e*x)^(2+m)*(1
+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2
)*AppellF1(2+m,1/2,1/2,3+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-2*c*x/(b+(-4*a*c
+b^2)^(1/2)))/e^2/(2+m)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.72 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.83

$$\int \frac{(ex)^m(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{x(ex)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}}\left(A(2+m)\,\mathrm{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right.\right.}{(1+m)(2+m)\sqrt{a+x(b+cx}}$$

input
```
Integrate[((e*x)^m*(A + B*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(x*(e*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])]*(A*(2 + m)*Ap
pellF1[1 + m, 1/2, 1/2, 2 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(
-b + Sqrt[b^2 - 4*a*c])] + B*(1 + m)*x*AppellF1[2 + m, 1/2, 1/2, 3 + m, (-
2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 +
m)*(2 + m)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1269$$

$$A\int \frac{(ex)^m}{\sqrt{cx^2+bx+a}}dx+\frac{B\int \frac{(ex)^{m+1}}{\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \; 1179$$

$$\frac{A\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\int \frac{(ex)^m}{\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{b+\sqrt{b^2-4ac}}+1}}d(ex)}{e\sqrt{a+bx+cx^2}}+\frac{B\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\int \frac{(ex)^{m+1}}{\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{b+\sqrt{b^2-4ac}}+1}}d(ex)}{e^2\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 150$$

$$\frac{A(ex)^{m+1}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(m+1)\sqrt{a+bx+cx^2}}+\frac{B(ex)^{m+2}\sqrt{\frac{2cx}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(m+2,\frac{1}{2},\frac{1}{2},m+3,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(m+2)\sqrt{a+bx+cx^2}}$$

input
```
Int[((e*x)^m*(A + B*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(A*(e*x)^(1 + m)*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (-2*c*x)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(e*(1 + m)*Sqrt[a +
 b*x + c*x^2]) + (B*(e*x)^(2 + m)*Sqrt[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2 + m, 1/2, 1/2, 3 +
m, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(e
^2*(2 + m)*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c)))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
 - e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(ex)^m (Bx + A)}{\sqrt{c\,x^2 + bx + a}}dx$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m(A + Bx)}{\sqrt{a + bx + cx^2}}\,dx = \int \frac{(Bx + A)(ex)^m}{\sqrt{cx^2 + bx + a}}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F]

$$\int \frac{(ex)^m(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(ex)^m\,(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x)**m*(B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((e*x)**m*(A + B*x)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(ex)^m(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Bx+A)(ex)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x)^m/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(ex)^m(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Bx+A)(ex)^m}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^m/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(e\,x)^m\,(A+B\,x)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx = e^m\left(\left(\int \frac{x^m}{\sqrt{c\,x^2+bx+a}}dx\right)a+\left(\int \frac{x^m x}{\sqrt{c\,x^2+bx+a}}dx\right)b\right)$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
e**m*(int(x**m/sqrt(a + b*x + c*x**2),x)*a + int((x**m*x)/sqrt(a + b*x + c
*x**2),x)*b)
```

3.237 $\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{A(ex)^{1+m}\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{3/2}\text{AppellF1}\left(1+m,\frac{3}{2},\frac{3}{2},2+m,\right.}{e(1+m)\left(a+bx+cx^2\right)^{3/2}}$$
$$+\frac{B(ex)^{2+m}\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{3/2}\text{AppellF1}\left(2+m,\frac{3}{2},\frac{3}{2},3+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(2+m)\left(a+bx+cx^2\right)^{3/2}}$$

output
```
A*(e*x)^(1+m)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(3/2)*(1+2*c*x/(b+(-4*a*c+b
^2)^(1/2)))^(3/2)*AppellF1(1+m,3/2,3/2,2+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e/(1+m)/(c*x^2+b*x+a)^(3/2)+B*(e*x)^(2+m)*(1
+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(3/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(3/2
)*AppellF1(2+m,3/2,3/2,3+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-2*c*x/(b+(-4*a*c
+b^2)^(1/2)))/e^2/(2+m)/(c*x^2+b*x+a)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.39 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.97

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{x(ex)^m\left(-b+\sqrt{b^2-4ac}-2cx\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)^{3/2}\left(A(2+m)\right.}{\left(-b\right.}$$

input
```
Integrate[((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(x*(e*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt
[b^2 - 4*a*c]))^(3/2)*(A*(2 + m)*AppellF1[1 + m, 3/2, 3/2, 2 + m, (-2*c*x)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + B*(1 + m)*x*
AppellF1[2 + m, 3/2, 3/2, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)
/(-b + Sqrt[b^2 - 4*a*c])]))/((-b + Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)*(a
+ x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(ex)^m}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1269}$$

$$A\int \frac{(ex)^m}{(cx^2+bx+a)^{3/2}}dx+\frac{B\int \frac{(ex)^{m+1}}{(cx^2+bx+a)^{3/2}}dx}{e}$$

$$\Big\downarrow \text{1179}$$

$$\frac{A\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{3/2}\int \frac{(ex)^m}{\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}d(ex)}{e\left(a+bx+cx^2\right)^{3/2}}+\frac{B\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{3/2}\int \frac{(ex)^{m+1}}{\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}d(ex)}{e^2\left(a+bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{150}$$

$$\frac{A(ex)^{m+1}\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{3/2}\text{AppellF1}\left(m+1,\frac{3}{2},\frac{3}{2},m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(m+1)\left(a+bx+cx^2\right)^{3/2}}+\frac{B(ex)^{m+2}\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{3/2}\text{AppellF1}\left(m+2,\frac{3}{2},\frac{3}{2},m+3,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(m+2)\left(a+bx+cx^2\right)^{3/2}}$$

input
```
Int[((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(A*(e*x)^(1 + m)*(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^(3/2)*(1 + (2*c*x)/
(b + Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[1 + m, 3/2, 3/2, 2 + m, (-2*c*x)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(e*(1 + m)*(a +
 b*x + c*x^2)^(3/2)) + (B*(e*x)^(2 + m)*(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c
]))^(3/2)*(1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[2 + m, 3/2,
 3/2, 3 + m, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*
a*c])])/(e^2*(2 + m)*(a + b*x + c*x^2)^(3/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(ex)^{m} (Bx+A)}{(c\,x^{2}+bx+a)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)(ex)^m}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(B*x + A)*(e*x)^m/(c^2*x^4 + 2*b*c*x^3 + 2*
a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(ex)^m\,(A+Bx)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**m*(B*x+A)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((e*x)**m*(A + B*x)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)(ex)^m}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x)^m/(c*x^2 + b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)(ex)^m}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^m/(c*x^2 + b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(e\,x)^m\,(A+B\,x)}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{3/2}}\,dx = e^m\left(\left(\int \frac{x^m}{\sqrt{c\,x^2+bx+a}\,a+\sqrt{c\,x^2+bx+a}\,bx+\sqrt{c\,x^2+bx+a}\,c\,x^2}dx\right)a + \left(\int \frac{x^m x}{\sqrt{c\,x^2+bx+a}\,a+\sqrt{c\,x^2+bx+a}\,bx+\sqrt{c\,x^2+bx+a}\,c\,x^2}dx\right)b\right)$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
e**m*(int(x**m/(sqrt(a + b*x + c*x**2)*a + sqrt(a + b*x + c*x**2)*b*x + sq
rt(a + b*x + c*x**2)*c*x**2),x)*a + int((x**m*x)/(sqrt(a + b*x + c*x**2)*a
 + sqrt(a + b*x + c*x**2)*b*x + sqrt(a + b*x + c*x**2)*c*x**2),x)*b)
```

3.238 $\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 281

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{A(ex)^{1+m}\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{5/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{5/2}\text{AppellF1}\left(1+m,\frac{5}{2},\frac{5}{2},2+m,\right.}{e(1+m)\left(a+bx+cx^2\right)^{5/2}}$$
$$+\frac{B(ex)^{2+m}\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{5/2}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{5/2}\text{AppellF1}\left(2+m,\frac{5}{2},\frac{5}{2},3+m,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(2+m)\left(a+bx+cx^2\right)^{5/2}}$$

output
```
A*(e*x)^(1+m)*(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(5/2)*(1+2*c*x/(b+(-4*a*c+b
^2)^(1/2)))^(5/2)*AppellF1(1+m,5/2,5/2,2+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-
2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e/(1+m)/(c*x^2+b*x+a)^(5/2)+B*(e*x)^(2+m)*(1
+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^(5/2)*(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(5/2
)*AppellF1(2+m,5/2,5/2,3+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-2*c*x/(b+(-4*a*c
+b^2)^(1/2)))/e^2/(2+m)/(c*x^2+b*x+a)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.76 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.84

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{x(ex)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}}\left(A(2+m)\,\text{AppellF1}\left(1+m,\frac{5}{2},\frac{5}{2},2+m\right.\right.}{a^2(1+m}$$

input
```
Integrate[((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(x*(e*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])]*(A*(2 + m)*Ap
pellF1[1 + m, 5/2, 5/2, 2 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(
-b + Sqrt[b^2 - 4*a*c])] + B*(1 + m)*x*AppellF1[2 + m, 5/2, 5/2, 3 + m, (-
2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])]))/(a^2*(
1 + m)*(2 + m)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(ex)^m}{(a+bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1269}$$

$$A\int \frac{(ex)^m}{(cx^2+bx+a)^{5/2}}dx+\frac{B\int \frac{(ex)^{m+1}}{(cx^2+bx+a)^{5/2}}dx}{e}$$

$$\Big\downarrow \text{ 1179}$$

$$\frac{A\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{5/2}\int\frac{(ex)^m}{\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^{5/2}}d(ex)}{e\left(a+bx+cx^2\right)^{5/2}}+\frac{B\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{5/2}\int\frac{(ex)^{m+1}}{\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^{5/2}}d(ex)}{e^2\left(a+bx+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{A(ex)^{m+1}\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{5/2}\text{AppellF1}\left(m+1,\frac{5}{2},\frac{5}{2},m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e(m+1)\left(a+bx+cx^2\right)^{5/2}}+\frac{B(ex)^{m+2}\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{5/2}\text{AppellF1}\left(m+2,\frac{5}{2},\frac{5}{2},m+3,-\frac{2cx}{b-\sqrt{b^2-4ac}},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{e^2(m+2)\left(a+bx+cx^2\right)^{5/2}}$$

input
```
Int[((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(A*(e*x)^(1 + m)*(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^(5/2)*(1 + (2*c*x)/
(b + Sqrt[b^2 - 4*a*c]))^(5/2)*AppellF1[1 + m, 5/2, 5/2, 2 + m, (-2*c*x)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(e*(1 + m)*(a +
 b*x + c*x^2)^(5/2)) + (B*(e*x)^(2 + m)*(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c
]))^(5/2)*(1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^(5/2)*AppellF1[2 + m, 5/2,
 5/2, 3 + m, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*
a*c])])/(e^2*(2 + m)*(a + b*x + c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(ex)^m (Bx + A)}{(c\,x^2 + bx + a)^{\frac{5}{2}}} dx$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m(A + Bx)}{(a + bx + cx^2)^{5/2}}\, dx = \int \frac{(Bx + A)(ex)^m}{(cx^2 + bx + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(B*x + A)*(e*x)^m/(c^3*x^6 + 3*b*c^2*x^5 +
3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^3 + 3*(a*b^2 +
 a^2*c)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x+A)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)(ex)^m}{(cx^2+bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x)^m/(c*x^2 + b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)(ex)^m}{(cx^2+bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^m/(c*x^2 + b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{(e\,x)^m\,(A+B\,x)}{(c\,x^2+b\,x+a)^{5/2}}\,dx$$

input
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(5/2),x)
```

output
```
int(((e*x)^m*(A + B*x))/(a + b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m(A+Bx)}{(a+bx+cx^2)^{5/2}}\,dx = e^m\left(\left(\int \frac{x^m}{\sqrt{c\,x^2+bx+a}\,a^2+2\sqrt{c\,x^2+bx+a}\,abx+2\sqrt{c\,x^2+bx+a}\,ac\,x^2+\sqrt{c\right.\right.$$
$$+\left(\int \frac{x^m x}{\sqrt{c\,x^2+bx+a}\,a^2+2\sqrt{c\,x^2+bx+a}\,abx+2\sqrt{c\,x^2+bx+a}\,ac\,x^2+\sqrt{c\,x^2+bx+a}\,b^2x^2+2\sqrt{c\,x^2+\right.$$

input
```
int((e*x)^m*(B*x+A)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
e**m*(int(x**m/(sqrt(a + b*x + c*x**2)*a**2 + 2*sqrt(a + b*x + c*x**2)*a*b
*x + 2*sqrt(a + b*x + c*x**2)*a*c*x**2 + sqrt(a + b*x + c*x**2)*b**2*x**2
+ 2*sqrt(a + b*x + c*x**2)*b*c*x**3 + sqrt(a + b*x + c*x**2)*c**2*x**4),x)
*a + int((x**m*x)/(sqrt(a + b*x + c*x**2)*a**2 + 2*sqrt(a + b*x + c*x**2)*
a*b*x + 2*sqrt(a + b*x + c*x**2)*a*c*x**2 + sqrt(a + b*x + c*x**2)*b**2*x*
*2 + 2*sqrt(a + b*x + c*x**2)*b*c*x**3 + sqrt(a + b*x + c*x**2)*c**2*x**4)
,x)*b)
```

3.239 $\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 277

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$=\frac{A(ex)^{1+m}\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx+cx^2)^p\operatorname{AppellF1}\left(1+m,-p,-p,2+m,-\frac{}{b-\sqrt{}}\right.}{e(1+m)}$$
$$+\frac{B(ex)^{2+m}\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx+cx^2)^p\operatorname{AppellF1}\left(2+m,-p,-p,3+m,-\frac{}{b}\right.}{e^2(2+m)}$$

output
```
A*(e*x)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,-2*c*x/(b-(-4*a*c+b^2
)^(1/2)),-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/e/(1+m)/((1+2*c*x/(b-(-4*a*c+b^2)^
(1/2)))^p)/((1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))^p)+B*(e*x)^(2+m)*(c*x^2+b*x+a
)^p*AppellF1(2+m,-p,-p,3+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-2*c*x/(b+(-4*a*c
+b^2)^(1/2)))/e^2/(2+m)/((1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x/(b+
(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.84

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$=\frac{x(ex)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\left(A(2+m)\,\text{AppellF1}\left(1+m,-p,-p,2+\right.\right.}{(1+m)(}$$

input
```
Integrate[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output
```
(x*(e*x)^m*(a + x*(b + c*x))^p*(A*(2 + m)*AppellF1[1 + m, -p, -p, 2 + m, (
-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + B*(1
+ m)*x*AppellF1[2 + m, -p, -p, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 + m)*(2 + m)*((b - Sqrt[b^2 - 4*a*c]
 + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b +
 Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(ex)^m\left(a+bx+cx^2\right)^p\,dx$$
$$\Big\downarrow \text{1269}$$
$$A\int (ex)^m\left(cx^2+bx+a\right)^p dx+\frac{B\int (ex)^{m+1}\left(cx^2+bx+a\right)^p dx}{e}$$
$$\Big\downarrow \text{1179}$$

$$\frac{A\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{-p}(a+bx+cx^2)^p\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\int (ex)^m\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^p d(ex)}{e}+\frac{B\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{-p}(a+bx+cx^2)^p\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\int (ex)^{m+1}\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx}{b+\sqrt{b^2-4ac}}+1\right)^p d(ex)}{e^2}$$

$$\Big\downarrow \text{150}$$

$$\frac{A(ex)^{m+1}\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx+cx^2)^p \operatorname{AppellF1}\left(m+1,-p,-p,m+2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right.}{e(m+1)}$$
$$\frac{B(ex)^{m+2}\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx+cx^2)^p \operatorname{AppellF1}\left(m+2,-p,-p,m+3,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right.}{e^2(m+2)}$$

input
```
Int[(e*x)^m*(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output
```
(A*(e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (-2*c*
x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(e*(1 + m)*
(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x)/(b + Sqrt[b^2 - 4*a*c
]))^p) + (B*(e*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 +
m, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(e
^2*(2 + m)*(1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x)/(b + Sqrt[
b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (ex)^m (Bx+A) \left(c\,x^2+bx+a\right)^p dx$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

output
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int (ex)^m(A+Bx)\left(a+bx+cx^2\right)^p\,dx = \int (Bx+A)\left(cx^2+bx+a\right)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(c*x^2 + b*x + a)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m (A + Bx) \left(a + bx + cx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x+A)*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m (A + Bx) \left(a + bx + cx^2\right)^p \, dx = \int (Bx + A)(cx^2 + bx + a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m (A + Bx) \left(a + bx + cx^2\right)^p \, dx = \int (Bx + A)(cx^2 + bx + a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (A+Bx) \left(a+bx+cx^2\right)^p \, dx = \int (e\,x)^m\,(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^p \,dx$$

input
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^p,x)
```

output
```
int((e*x)^m*(A + B*x)*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int (ex)^m (A+Bx) \left(a+bx+cx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

output

```
(e**m*(2*x**m*(a + b*x + c*x**2)**p*a**2*c*m*p + 4*x**m*(a + b*x + c*x**2)
**p*a**2*c*p**2 + 4*x**m*(a + b*x + c*x**2)**p*a**2*c*p - x**m*(a + b*x +
c*x**2)**p*a*b**2*m*p - x**m*(a + b*x + c*x**2)**p*a*b**2*p + x**m*(a + b*
x + c*x**2)**p*a*b*c*m**2*x + 3*x**m*(a + b*x + c*x**2)**p*a*b*c*m*p*x + 2
*x**m*(a + b*x + c*x**2)**p*a*b*c*m*x + 2*x**m*(a + b*x + c*x**2)**p*a*b*c
*p**2*x + 2*x**m*(a + b*x + c*x**2)**p*a*b*c*p*x + x**m*(a + b*x + c*x**2)
**p*b**3*m*p*x + x**m*(a + b*x + c*x**2)**p*b**3*p**2*x + x**m*(a + b*x +
c*x**2)**p*b**2*c*m**2*x**2 + 3*x**m*(a + b*x + c*x**2)**p*b**2*c*m*p*x**2
 + x**m*(a + b*x + c*x**2)**p*b**2*c*m*x**2 + 2*x**m*(a + b*x + c*x**2)**p
*b**2*c*p**2*x**2 + x**m*(a + b*x + c*x**2)**p*b**2*c*p*x**2 - 2*int((x**m
*(a + b*x + c*x**2)**p*x)/(a*m**3 + 5*a*m**2*p + 3*a*m**2 + 8*a*m*p**2 + 9
*a*m*p + 2*a*m + 4*a*p**3 + 6*a*p**2 + 2*a*p + b*m**3*x + 5*b*m**2*p*x + 3
*b*m**2*x + 8*b*m*p**2*x + 9*b*m*p*x + 2*b*m*x + 4*b*p**3*x + 6*b*p**2*x +
 2*b*p*x + c*m**3*x**2 + 5*c*m**2*p*x**2 + 3*c*m**2*x**2 + 8*c*m*p**2*x**2
 + 9*c*m*p*x**2 + 2*c*m*x**2 + 4*c*p**3*x**2 + 6*c*p**2*x**2 + 2*c*p*x**2)
,x)*a**2*c**2*m**5*p - 18*int((x**m*(a + b*x + c*x**2)**p*x)/(a*m**3 + 5*a
*m**2*p + 3*a*m**2 + 8*a*m*p**2 + 9*a*m*p + 2*a*m + 4*a*p**3 + 6*a*p**2 +
2*a*p + b*m**3*x + 5*b*m**2*p*x + 3*b*m**2*x + 8*b*m*p**2*x + 9*b*m*p*x +
2*b*m*x + 4*b*p**3*x + 6*b*p**2*x + 2*b*p*x + c*m**3*x**2 + 5*c*m**2*p*x**
2 + 3*c*m**2*x**2 + 8*c*m*p**2*x**2 + 9*c*m*p*x**2 + 2*c*m*x**2 + 4*c*p...
```

3.240 $\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 390

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$= -\frac{(bB(4+p)-Ac(5+2p))x^2\left(a+bx+cx^2\right)^{1+p}}{2c^2(2+p)(5+2p)}+\frac{Bx^3\left(a+bx+cx^2\right)^{1+p}}{c(5+2p)}$$
$$+\frac{(2ac(3+2p)(bB(4+p)-Ac(5+2p))+b(2+p)(6aBc(2+p)-b(3+p)(bB(4+p)-Ac(5+2p)))}{4c^4(1+p)(2+p)(3+2p)}$$
$$+\frac{2^{-3-2p}\left(12a^2Bc^2-12ab^2Bc(3+p)+6aAbc^2(5+2p)+b^4B\left(12+7p+p^2\right)-Ab^3c\left(15+11p+2p^2\right)\right)(}{c^5(3+2p)(5+2p)}$$

output
```
-1/2*(b*B*(4+p)-A*c*(5+2*p))*x^2*(c*x^2+b*x+a)^(p+1)/c^2/(2+p)/(5+2*p)+B*x
^3*(c*x^2+b*x+a)^(p+1)/c/(5+2*p)+1/4*(2*a*c*(3+2*p)*(b*B*(4+p)-A*c*(5+2*p)
)+b*(2+p)*(6*a*B*c*(2+p)-b*(3+p)*(b*B*(4+p)-A*c*(5+2*p)))-2*c*(p+1)*(6*a*B
*c*(2+p)-b*(3+p)*(b*B*(4+p)-A*c*(5+2*p)))*x)*(c*x^2+b*x+a)^(p+1)/c^4/(p+1)
/(2+p)/(3+2*p)/(5+2*p)+2^(-3-2*p)*(12*B*a^2*c^2-12*a*b^2*B*c*(3+p)+6*a*A*b
*c^2*(5+2*p)+b^4*B*(p^2+7*p+12)-A*b^3*c*(2*p^2+11*p+15))*(2*c*x+b)*(c*x^2+
b*x+a)^p*hypergeom([1/2, -p],[3/2],(2*c*x+b)^2/(-4*a*c+b^2))/c^5/(3+2*p)/(
5+2*p)/((-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.54

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$=\frac{1}{20}x^4\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\left(5A\operatorname{AppellF1}\left(4,-p,-p,5,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)+4Bx\operatorname{AppellF1}\left(5,-p,-p,6,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[x^3*(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output
```
(x^4*(a + x*(b + c*x))^p*(5*A*AppellF1[4, -p, -p, 5, (-2*c*x)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + 4*B*x*AppellF1[5, -p, -p,
 6, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])]))/
(20*((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1236, 25, 1236, 25, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$\Big\downarrow \text{1236}$$
$$\frac{\int -x^2(3aB+(bB(p+4)-Ac(2p+5))x)\left(cx^2+bx+a\right)^p dx}{c(2p+5)}+\frac{Bx^3\left(a+bx+cx^2\right)^{p+1}}{c(2p+5)}$$

$$\Big\downarrow \; 25$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{p+1}}{c(2p+5)}-\frac{\int x^2(3aB+(bB(p+4)-Ac(2p+5))x)\left(cx^2+bx+a\right)^p dx}{c(2p+5)}$$

$$\Big\downarrow \; 1236$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{p+1}}{c(2p+5)}-\frac{\frac{\int -x\left(2a(bB(p+4)-Ac(2p+5))-\left(-B\left(p^2+7p+12\right)b^2+Ac\left(2p^2+11p+15\right)b+6aBc(p+2)\right)x\right)\left(cx^2+bx+a\right)^p dx}{2c(p+2)}+\frac{x^2\left(a+bx+cx^2\right)^{p+1}(bB(p+4)-A}{2c(p+2)}}{c(2p+5)}$$

$$\Big\downarrow \; 25$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{p+1}}{c(2p+5)}-\frac{\frac{x^2\left(a+bx+cx^2\right)^{p+1}(bB(p+4)-Ac(2p+5))}{2c(p+2)}-\frac{\int x\left(2a(bB(p+4)-Ac(2p+5))-\left(-B\left(p^2+7p+12\right)b^2+Ac\left(2p^2+11p+15\right)b+6aBc(p+2)\right)x\right)\left(cx^2+bx-\right.}{2c(p+2)}}{c(2p+5)}$$

$$\Big\downarrow \; 1225$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{p+1}}{c(2p+5)}-\frac{x^2\left(a+bx+cx^2\right)^{p+1}(bB(p+4)-Ac(2p+5))}{2c(p+2)}-\frac{(p+2)\left(12a^2Bc^2+6aAbc^2(2p+5)-12ab^2Bc(p+3)-Ab^3c\left(2p^2+11p+15\right)+b^4B\left(p^2+7p+12\right)\right)\int\left(cx^2+bx+a\right)^p d}{2c^2(2p+3)}$$

$$\Big\downarrow \; 1096$$

$$\frac{Bx^3\left(a+bx+cx^2\right)^{p+1}}{c(2p+5)}-\frac{x^2\left(a+bx+cx^2\right)^{p+1}(bB(p+4)-Ac(2p+5))}{2c(p+2)}-\frac{\left(a+bx+cx^2\right)^{p+1}\left(-2c(p+1)x\left(6aBc(p+2)+Abc\left(2p^2+11p+15\right)+b^2(-B)\left(p^2+7p+12\right)\right)+b(p+2)\left(6aBc(p+2)+\right.\right.}{2c^2(p+1)(2p+3)}$$

input
```
Int[x^3*(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output
```
(B*x^3*(a + b*x + c*x^2)^(1 + p))/(c*(5 + 2*p)) - (((b*B*(4 + p) - A*c*(5
+ 2*p))*x^2*(a + b*x + c*x^2)^(1 + p))/(2*c*(2 + p)) - (((2*a*c*(3 + 2*p)*
(b*B*(4 + p) - A*c*(5 + 2*p)) + b*(2 + p)*(6*a*B*c*(2 + p) - b^2*B*(12 + 7
*p + p^2) + A*b*c*(15 + 11*p + 2*p^2)) - 2*c*(1 + p)*(6*a*B*c*(2 + p) - b^
2*B*(12 + 7*p + p^2) + A*b*c*(15 + 11*p + 2*p^2))*x)*(a + b*x + c*x^2)^(1
+ p))/(2*c^2*(1 + p)*(3 + 2*p)) - (2^p*(2 + p)*(12*a^2*B*c^2 - 12*a*b^2*B*
c*(3 + p) + 6*a*A*b*c^2*(5 + 2*p) + b^4*B*(12 + 7*p + p^2) - A*b^3*c*(15 +
 11*p + 2*p^2))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1
 - p)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + S
qrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(
1 + p)*(3 + 2*p)))/(2*c*(2 + p)))/(c*(5 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [F]

$$\int x^3(Bx+A)\left(c\,x^2+bx+a\right)^p dx$$

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

output
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)\left(cx^2+bx+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((B*x^4 + A*x^3)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**3*(B*x+A)*(c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)\left(cx^2+bx+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p*x^3, x)
```

Giac [F]

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)\left(cx^2+bx+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int x^3\,(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

input
```
int(x^3*(A + B*x)*(a + b*x + c*x^2)^p,x)
```

output
```
int(x^3*(A + B*x)*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int x^3(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\text{too large to display}$$

input
```
int(x^3*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

output
```
(8*(a + b*x + c*x**2)**p*a**3*c**2*p**3 + 84*(a + b*x + c*x**2)**p*a**3*c*
*2*p**2 + 208*(a + b*x + c*x**2)**p*a**3*c**2*p + 138*(a + b*x + c*x**2)**
p*a**3*c**2 - 6*(a + b*x + c*x**2)**p*a**2*b**2*c*p**3 - 57*(a + b*x + c*x
**2)**p*a**2*b**2*c*p**2 - 159*(a + b*x + c*x**2)**p*a**2*b**2*c*p - 126*(
a + b*x + c*x**2)**p*a**2*b**2*c - 8*(a + b*x + c*x**2)**p*a**2*b*c**2*p**
4*x - 84*(a + b*x + c*x**2)**p*a**2*b*c**2*p**3*x - 208*(a + b*x + c*x**2)
**p*a**2*b*c**2*p**2*x - 138*(a + b*x + c*x**2)**p*a**2*b*c**2*p*x + 16*(a
 + b*x + c*x**2)**p*a**2*c**3*p**4*x**2 + 72*(a + b*x + c*x**2)**p*a**2*c*
*3*p**3*x**2 + 92*(a + b*x + c*x**2)**p*a**2*c**3*p**2*x**2 + 30*(a + b*x
+ c*x**2)**p*a**2*c**3*p*x**2 + (a + b*x + c*x**2)**p*a*b**4*p**3 + 9*(a +
 b*x + c*x**2)**p*a*b**4*p**2 + 26*(a + b*x + c*x**2)**p*a*b**4*p + 24*(a
+ b*x + c*x**2)**p*a*b**4 + 6*(a + b*x + c*x**2)**p*a*b**3*c*p**4*x + 57*(
a + b*x + c*x**2)**p*a*b**3*c*p**3*x + 159*(a + b*x + c*x**2)**p*a*b**3*c*
p**2*x + 126*(a + b*x + c*x**2)**p*a*b**3*c*p*x - 12*(a + b*x + c*x**2)**p
*a*b**2*c**2*p**4*x**2 - 84*(a + b*x + c*x**2)**p*a*b**2*c**2*p**3*x**2 -
141*(a + b*x + c*x**2)**p*a*b**2*c**2*p**2*x**2 - 51*(a + b*x + c*x**2)**p
*a*b**2*c**2*p*x**2 + 24*(a + b*x + c*x**2)**p*a*b*c**3*p**4*x**3 + 88*(a
+ b*x + c*x**2)**p*a*b*c**3*p**3*x**3 + 90*(a + b*x + c*x**2)**p*a*b*c**3*
p**2*x**3 + 26*(a + b*x + c*x**2)**p*a*b*c**3*p*x**3 + 16*(a + b*x + c*x**
2)**p*a*c**4*p**4*x**4 + 88*(a + b*x + c*x**2)**p*a*c**4*p**3*x**4 + 16...
```

3.241 $\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 247

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\frac{Bx^2\left(a+bx+cx^2\right)^{1+p}}{2c(2+p)}$$
$$-\frac{(2aBc(3+2p)+b(2+p)(2Ac(2+p)-bB(3+p))-2c(1+p)(2Ac(2+p)-bB(3+p))x)\left(a+bx+\right.}{4c^3(1+p)(2+p)(3+2p)}$$
$$+\frac{2^{-3-2p}\left(6abBc-4aAc^2+2Ab^2c(2+p)-b^3B(3+p)\right)(b+2cx)\left(a+bx+cx^2\right)^p\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{-p}\text{Hyp}}{c^4(3+2p)}$$

output
```
1/2*B*x^2*(c*x^2+b*x+a)^(p+1)/c/(2+p)-1/4*(2*a*B*c*(3+2*p)+b*(2+p)*(2*A*c*
(2+p)-b*B*(3+p))-2*c*(p+1)*(2*A*c*(2+p)-b*B*(3+p))*x)*(c*x^2+b*x+a)^(p+1)/
c^3/(p+1)/(2+p)/(3+2*p)+2^(-3-2*p)*(6*B*a*b*c-4*A*a*c^2+2*A*b^2*c*(2+p)-b^
3*B*(3+p))*(2*c*x+b)*(c*x^2+b*x+a)^p*hypergeom([1/2, -p],[3/2],(2*c*x+b)^2
/(-4*a*c+b^2))/c^4/(3+2*p)/((-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.72 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.85

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$=\frac{1}{12}x^3\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\left(4A\,\text{AppellF1}\left(3,-p,-p,4,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)+3Bx\,\text{AppellF1}\left(4,-p,-p,5,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[x^2*(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output
```
(x^3*(a + x*(b + c*x))^p*(4*A*AppellF1[3, -p, -p, 4, (-2*c*x)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + 3*B*x*AppellF1[4, -p, -p,
 5, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])]))/
(12*((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1236, 25, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$\Big\downarrow \text{1236}$$
$$\frac{\int -x(2aB-(2Ac(p+2)-bB(p+3))x)\left(cx^2+bx+a\right)^p dx}{2c(p+2)}+\frac{Bx^2\left(a+bx+cx^2\right)^{p+1}}{2c(p+2)}$$

$$\Big\downarrow \ 25$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{p+1}}{2c(p+2)}-\frac{\int x(2aB-(2Ac(p+2)-bB(p+3))x)\left(cx^2+bx+a\right)^p\,dx}{2c(p+2)}$$

$$\Big\downarrow \ 1225$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{p+1}}{2c(p+2)}-\frac{\frac{\left(a+bx+cx^2\right)^{p+1}(2aBc(2p+3)-2c(p+1)x(2Ac(p+2)-bB(p+3))+b(p+2)(2Ac(p+2)-bB(p+3)))}{2c^2(p+1)(2p+3)}-\frac{(p+2)\left(-4aAc^2+6abBc+2Ab^2c(p+2)+b^3(-\cdots\right.}{2c^2(2p+3)}}{2c(p+2)}$$

$$\Big\downarrow \ 1096$$

$$\frac{Bx^2\left(a+bx+cx^2\right)^{p+1}}{2c(p+2)}-\frac{\frac{2^p(p+2)\left(a+bx+cx^2\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(-4aAc^2+6abBc+2Ab^2c(p+2)+b^3(-B)(p+3)\right)\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx\cdots}{2\sqrt{\cdots}}\right.}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}}}{2c(p+2)}$$

input

```
Int[x^2*(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output

```
(B*x^2*(a + b*x + c*x^2)^(1 + p))/(2*c*(2 + p)) - (((2*a*B*c*(3 + 2*p) + b
*(2 + p)*(2*A*c*(2 + p) - b*B*(3 + p)) - 2*c*(1 + p)*(2*A*c*(2 + p) - b*B*
(3 + p))*x)*(a + b*x + c*x^2)^(1 + p))/(2*c^2*(1 + p)*(3 + 2*p)) + (2^p*(2
 + p)*(6*a*b*B*c - 4*a*A*c^2 + 2*A*b^2*c*(2 + p) - b^3*B*(3 + p))*(-((b -
Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^
(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x
)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p)))/(2*c*
(2 + p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [F]

$$\int x^{2}(Bx+A)\left(c\,x^{2}+bx+a\right)^{p}dx$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

output
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)\left(cx^2+bx+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((B*x^3 + A*x^2)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F]

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x+a)**p,x)
```

output
```
Integral(x**2*(A + B*x)*(a + b*x + c*x**2)**p, x)
```

Maxima [F]

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)\left(cx^2+bx+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)\left(cx^2+bx+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int x^2\,\left(A+B\,x\right)\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

input
```
int(x^2*(A + B*x)*(a + b*x + c*x^2)^p,x)
```

output
```
int(x^2*(A + B*x)*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int x^2(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\text{too large to display}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

output

```
( - 8*(a + b*x + c*x**2)**p*a**3*c**2*p**2 - 24*(a + b*x + c*x**2)**p*a**3
*c**2*p - 16*(a + b*x + c*x**2)**p*a**3*c**2 + 6*(a + b*x + c*x**2)**p*a**
2*b**2*c*p**2 + 28*(a + b*x + c*x**2)**p*a**2*b**2*c*p + 26*(a + b*x + c*x
**2)**p*a**2*b**2*c + 8*(a + b*x + c*x**2)**p*a**2*b*c**2*p**3*x + 24*(a +
 b*x + c*x**2)**p*a**2*b*c**2*p**2*x + 16*(a + b*x + c*x**2)**p*a**2*b*c**
2*p*x - (a + b*x + c*x**2)**p*a*b**4*p**2 - 5*(a + b*x + c*x**2)**p*a*b**4
*p - 6*(a + b*x + c*x**2)**p*a*b**4 - 6*(a + b*x + c*x**2)**p*a*b**3*c*p**
3*x - 28*(a + b*x + c*x**2)**p*a*b**3*c*p**2*x - 26*(a + b*x + c*x**2)**p*
a*b**3*c*p*x + 12*(a + b*x + c*x**2)**p*a*b**2*c**2*p**3*x**2 + 26*(a + b*
x + c*x**2)**p*a*b**2*c**2*p**2*x**2 + 10*(a + b*x + c*x**2)**p*a*b**2*c**
2*p*x**2 + 8*(a + b*x + c*x**2)**p*a*b*c**3*p**3*x**3 + 28*(a + b*x + c*x*
*2)**p*a*b*c**3*p**2*x**3 + 28*(a + b*x + c*x**2)**p*a*b*c**3*p*x**3 + 8*(
a + b*x + c*x**2)**p*a*b*c**3*x**3 + (a + b*x + c*x**2)**p*b**5*p**3*x + 5
*(a + b*x + c*x**2)**p*b**5*p**2*x + 6*(a + b*x + c*x**2)**p*b**5*p*x - 2*
(a + b*x + c*x**2)**p*b**4*c*p**3*x**2 - 7*(a + b*x + c*x**2)**p*b**4*c*p*
*2*x**2 - 3*(a + b*x + c*x**2)**p*b**4*c*p*x**2 + 4*(a + b*x + c*x**2)**p*
b**3*c**2*p**3*x**3 + 6*(a + b*x + c*x**2)**p*b**3*c**2*p**2*x**3 + 2*(a +
 b*x + c*x**2)**p*b**3*c**2*p*x**3 + 8*(a + b*x + c*x**2)**p*b**2*c**3*p**
3*x**4 + 24*(a + b*x + c*x**2)**p*b**2*c**3*p**2*x**4 + 22*(a + b*x + c*x*
*2)**p*b**2*c**3*p*x**4 + 6*(a + b*x + c*x**2)**p*b**2*c**3*x**4 + 64*i...
```

3.242 $\int x(A+Bx)\left(a+bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 175

$$\int x(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$=-\frac{(bB(2+p)-Ac(3+2p)-2Bc(1+p)x)\left(a+bx+cx^2\right)^{1+p}}{2c^2(1+p)(3+2p)}$$
$$-\frac{4^{-1-p}\left(2aBc-b^2B(2+p)+Abc(3+2p)\right)(b+2cx)\left(a+bx+cx^2\right)^p\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{-p}\text{Hypergeometri}}{c^3(3+2p)}$$

output
```
-1/2*(b*B*(2+p)-A*c*(3+2*p)-2*B*c*(p+1)*x)*(c*x^2+b*x+a)^(p+1)/c^2/(p+1)/(
3+2*p)-4^(-1-p)*(2*B*a*c-b^2*B*(2+p)+A*b*c*(3+2*p))*(2*c*x+b)*(c*x^2+b*x+a
)^p*hypergeom([1/2, -p],[3/2],(2*c*x+b)^2/(-4*a*c+b^2))/c^3/(3+2*p)/((-c*(
c*x^2+b*x+a)/(-4*a*c+b^2))^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.55 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.20

$$\int x(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$=\frac{1}{6}x^2\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}(a$$
$$+x(b+cx))^p\left(3A\,\mathrm{AppellF1}\left(2,-p,-p,3,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)\right.$$
$$\left.+2Bx\,\mathrm{AppellF1}\left(3,-p,-p,4,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)\right)$$

input
```
Integrate[x*(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output
```
(x^2*(a + x*(b + c*x))^p*(3*A*AppellF1[2, -p, -p, 3, (-2*c*x)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + 2*B*x*AppellF1[3, -p, -p,
 4, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])]))/
(6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[
b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$\Big\downarrow \; 1225$$

$$-\frac{\left(2aBc+Abc(2p+3)+b^2(-B)(p+2)\right)\int \left(cx^2+bx+a\right)^p dx}{2c^2(2p+3)}-\frac{\left(a+bx+cx^2\right)^{p+1}\left(-Ac(2p+3)+bB(p+2)-2Bc(p+1)x\right)}{2c^2(p+1)(2p+3)}$$

$$\Big\downarrow \text{1096}$$

$$\frac{2^p\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\left(2aBc+Abc(2p+3)+b^2(-B)(p+2)\right)\text{Hypergeometric2F1}\left(-p\right.}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}}$$
$$\frac{\left(a+bx+cx^2\right)^{p+1}\left(-Ac(2p+3)+bB(p+2)-2Bc(p+1)x\right)}{2c^2(p+1)(2p+3)}$$

input
```
Int[x*(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output
```
-1/2*((b*B*(2 + p) - A*c*(3 + 2*p) - 2*B*c*(1 + p)*x)*(a + b*x + c*x^2)^(1
 + p))/(c^2*(1 + p)*(3 + 2*p)) + (2^p*(2*a*B*c - b^2*B*(2 + p) + A*b*c*(3
+ 2*p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a
 + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2
- 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(
3 + 2*p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int x(Bx+A)\left(c\,x^{2}+bx+a\right)^{p}dx$$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

output
```
int(x*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int x(A+Bx)\left(a+bx+cx^{2}\right)^{p}\,dx=\int (Bx+A)\left(cx^{2}+bx+a\right)^{p}x\,dx$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A*x)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F]

$$\int x(A+Bx)\left(a+bx+cx^{2}\right)^{p}\,dx=\int x(A+Bx)\left(a+bx+cx^{2}\right)^{p}\,dx$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x+a)**p,x)
```

output
```
Integral(x*(A + B*x)*(a + b*x + c*x**2)**p, x)
```

Maxima [F]

$$\int x(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)\left(cx^2+bx+a\right)^p x\,dx$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p*x, x)
```

Giac [F]

$$\int x(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)\left(cx^2+bx+a\right)^p x\,dx$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int x\,(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

input
```
int(x*(A + B*x)*(a + b*x + c*x^2)^p,x)
```

output
```
int(x*(A + B*x)*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int x(A+Bx)\left(a+bx+cx^2\right)^p\,dx=\text{Too large to display}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x+a)^p,x)
```

output
```
( - 6*(a + b*x + c*x**2)**p*a**2*c*p - 7*(a + b*x + c*x**2)**p*a**2*c + (a
 + b*x + c*x**2)**p*a*b**2*p + 2*(a + b*x + c*x**2)**p*a*b**2 + 6*(a + b*x
 + c*x**2)**p*a*b*c*p**2*x + 7*(a + b*x + c*x**2)**p*a*b*c*p*x + 4*(a + b*
x + c*x**2)**p*a*c**2*p**2*x**2 + 8*(a + b*x + c*x**2)**p*a*c**2*p*x**2 +
3*(a + b*x + c*x**2)**p*a*c**2*x**2 - (a + b*x + c*x**2)**p*b**3*p**2*x -
2*(a + b*x + c*x**2)**p*b**3*p*x + 2*(a + b*x + c*x**2)**p*b**2*c*p**2*x**
2 + (a + b*x + c*x**2)**p*b**2*c*p*x**2 + 4*(a + b*x + c*x**2)**p*b*c**2*p
**2*x**3 + 6*(a + b*x + c*x**2)**p*b*c**2*p*x**3 + 2*(a + b*x + c*x**2)**p
*b*c**2*x**3 + 32*int(((a + b*x + c*x**2)**p*x)/(4*a*p**2 + 8*a*p + 3*a +
4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x)*a
**2*c**2*p**5 + 176*int(((a + b*x + c*x**2)**p*x)/(4*a*p**2 + 8*a*p + 3*a
+ 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x)
*a**2*c**2*p**4 + 328*int(((a + b*x + c*x**2)**p*x)/(4*a*p**2 + 8*a*p + 3*
a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),
x)*a**2*c**2*p**3 + 244*int(((a + b*x + c*x**2)**p*x)/(4*a*p**2 + 8*a*p +
3*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2
),x)*a**2*c**2*p**2 + 60*int(((a + b*x + c*x**2)**p*x)/(4*a*p**2 + 8*a*p +
 3*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**
2),x)*a**2*c**2*p - 24*int(((a + b*x + c*x**2)**p*x)/(4*a*p**2 + 8*a*p + 3
*a + 4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x*...
```

3.243 $\int (A+Bx)\left(a+bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int (A+Bx)\left(a+bx+cx^2\right)^p\,dx=\frac{B\left(a+bx+cx^2\right)^{1+p}}{2c(1+p)}-\frac{2^{-2(1+p)}(bB-2Ac)(b+2cx)\left(a+bx+cx^2\right)^p\left(-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}}{c^2}$$

output
```
1/2*B*(c*x^2+b*x+a)^(p+1)/c/(p+1)-(-2*A*c+B*b)*(2*c*x+b)*(c*x^2+b*x+a)^p*h
ypergeom([1/2, -p],[3/2],(2*c*x+b)^2/(-4*a*c+b^2))/(2^(2*p+2))/c^2/((-c*(c
*x^2+b*x+a)/(-4*a*c+b^2))^p)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.66 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.20

$$\int (A+Bx)\left(a+bx+cx^2\right)^p\,dx=\frac{1}{2}(a+x(b+cx))^p\left(Bx^2\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right)^{-p}\text{AppellF1}\left(2,-p,-p,3,-\frac{2cx}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)+\frac{2^pA\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right)}{c(1+p)}\right)$$

input
```
Integrate[(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output
```
((a + x*(b + c*x))^p*((B*x^2*AppellF1[2, -p, -p, 3, (-2*c*x)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])/((((b - Sqrt[b^2 - 4*a*c] +
2*c*x)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sq
rt[b^2 - 4*a*c]))^p) + (2^p*A*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Hypergeometr
ic2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a
*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)))
/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \; 1160$$

$$\frac{B\left(a+bx+cx^2\right)^{p+1}}{2c(p+1)}-\frac{(bB-2Ac)\int \left(cx^2+bx+a\right)^p\,dx}{2c}$$

$$\Big\downarrow \; 1096$$

$$\frac{2^p(bB-2Ac)\left(a+bx+cx^2\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}$$

$$\frac{B\left(a+bx+cx^2\right)^{p+1}}{2c(p+1)}$$

input

```
Int[(A + B*x)*(a + b*x + c*x^2)^p,x]
```

output

```
(B*(a + b*x + c*x^2)^(1 + p))/(2*c*(1 + p)) + (2^p*(b*B - 2*A*c)*(-((b - S
qrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(
1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/(2*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Defintions of rubi rules used

rule 1096

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int (Bx + A)\left(c\,x^2 + bx + a\right)^p dx$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^p,x)
```

output
```
int((B*x+A)*(c*x^2+b*x+a)^p,x)
```

Fricas [F]

$$\int (A + Bx)\left(a + bx + cx^2\right)^p\, dx = \int (Bx + A)\left(cx^2 + bx + a\right)^p dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(c*x^2 + b*x + a)^p, x)
```

Sympy [F]

$$\int (A + Bx)\left(a + bx + cx^2\right)^p\, dx = \int (A + Bx)\left(a + bx + cx^2\right)^p\, dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**p,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**p, x)
```

Maxima [F]

$$\int (A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)(cx^2+bx+a)^p\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p, x)
```

Giac [F]

$$\int (A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (Bx+A)(cx^2+bx+a)^p\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)\left(a+bx+cx^2\right)^p\,dx=\int (A+B\,x)\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

input
```
int((A + B*x)*(a + b*x + c*x^2)^p,x)
```

output
```
int((A + B*x)*(a + b*x + c*x^2)^p, x)
```

Reduce [F]

$$\int (A+Bx)\left(a+bx+cx^2\right)^p\,dx$$
$$= \frac{4(c\,x^2+bx+a)^p\,a^2cp+4(c\,x^2+bx+a)^p\,a^2c-(c\,x^2+bx+a)^p\,a\,b^2+2(c\,x^2+bx+a)^p\,abcpx+2(c\,x^2}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^p,x)
```

output
```
(4*(a + b*x + c*x**2)**p*a**2*c*p + 4*(a + b*x + c*x**2)**p*a**2*c - (a +
b*x + c*x**2)**p*a*b**2 + 2*(a + b*x + c*x**2)**p*a*b*c*p*x + 2*(a + b*x +
 c*x**2)**p*a*b*c*x + (a + b*x + c*x**2)**p*b**3*p*x + 2*(a + b*x + c*x**2
)**p*b**2*c*p*x**2 + (a + b*x + c*x**2)**p*b**2*c*x**2 - 16*int(((a + b*x
+ c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*a**2*
c**2*p**3 - 24*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x +
2*c*p*x**2 + c*x**2),x)*a**2*c**2*p**2 - 8*int(((a + b*x + c*x**2)**p*x)/(
2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*a**2*c**2*p + 12*int((
(a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2)
,x)*a*b**2*c*p**3 + 18*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x
+ b*x + 2*c*p*x**2 + c*x**2),x)*a*b**2*c*p**2 + 6*int(((a + b*x + c*x**2)*
*p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*a*b**2*c*p - 2*
int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*
x**2),x)*b**4*p**3 - 3*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x
+ b*x + 2*c*p*x**2 + c*x**2),x)*b**4*p**2 - int(((a + b*x + c*x**2)**p*x)/
(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**4*p)/(2*b*c*(2*p**
2 + 3*p + 1))
```

3.244 $\int \frac{(A+Bx)(a+bx+cx^2)^p}{x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 236

$$\int \frac{(A+Bx)(a+bx+cx^2)^p}{x} \, dx = \frac{2^{-1+2p}A\left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4}}{2cx}\right.}{p} + \frac{2^{-1-2p}B(b+2cx)(a+bx+cx^2)^p\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{c}$$

output
```
2^(-1+2*p)*A*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2
)^(1/2))/c/x,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x)/p/(((b-(-4*a*c+b^2)^(1/2)+2*
c*x)/c/x)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/x)^p)+2^(-1-2*p)*B*(2*c*x+b)
*(c*x^2+b*x+a)^p*hypergeom([1/2, -p],[3/2],(2*c*x+b)^2/(-4*a*c+b^2))/c/((-
c*(c*x^2+b*x+a)/(-4*a*c+b^2))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x}\,dx = \frac{1}{2}(a+x(b+cx))^p\left(\frac{4^pA\left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-4ac}}{2cx},\frac{-b+\sqrt{b^2}}{2cx}\right.}{p}\right.$$
$$+\frac{2^pB\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)^{-p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right)}{c(1+p)}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^p)/x,x]
```

output
```
((a + x*(b + c*x))^p*((4^p*A*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b + Sqr
t[b^2 - 4*a*c])/(c*x), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x)])/(p*((b - Sqrt[b^
2 - 4*a*c] + 2*c*x)/(c*x))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p) +
(2^p*B*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Hypergeometric2F1[-p, 1 + p, 2 + p,
 (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(c*(1 + p)*((b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)))/2
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1269, 1096, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x}\,dx$$
$$\Big\downarrow \ 1269$$
$$A\int \frac{\left(cx^2+bx+a\right)^p}{x}dx+B\int \left(cx^2+bx+a\right)^p dx$$

$$\Big\downarrow \ 1096$$

$$A\int \frac{(cx^2+bx+a)^p}{x}dx-\frac{B2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx+cx^2)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

$$\Big\downarrow \ 1178$$

$$-A4^p\left(\frac{1}{x}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx}+1\right)$$

$$\frac{B2^{p+1}(a+bx+cx^2)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

$$\Big\downarrow \ 150$$

$$\frac{A2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx},-\right.}{p}$$

$$\frac{B2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx+cx^2)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^p)/x,x]
```

output
```
(2^(-1 + 2*p)*A*(a + b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(
b - Sqrt[b^2 - 4*a*c])/(c*x), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x)])/(p*((b
- Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*
x))^p) - (2^(1 + p)*B*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]
*(1 + p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(Bx + A)\,(c\,x^2 + bx + a)^p}{x}dx$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^p/x,x)
```

output
```
int((B*x+A)*(c*x^2+b*x+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x}\,dx=\int \frac{(Bx+A)(cx^2+bx+a)^p}{x}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p/x,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(c*x^2 + b*x + a)^p/x, x)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x}\,dx=\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**p/x,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**p/x, x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x}\,dx=\int \frac{(Bx+A)(cx^2+bx+a)^p}{x}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p/x, x)
```

Giac [F]

$$\int \frac{(A+Bx)(a+bx+cx^2)^p}{x}\,dx = \int \frac{(Bx+A)(cx^2+bx+a)^p}{x}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p/x,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(a+bx+cx^2)^p}{x}\,dx = \int \frac{(A+B\,x)\,(c\,x^2+b\,x+a)^p}{x}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^p)/x,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^p)/x, x)
```

Reduce [F]

$$\int \frac{(A+Bx)(a+bx+cx^2)^p}{x}\,dx$$
$$= \frac{4(c\,x^2+bx+a)^p\,ap+(c\,x^2+bx+a)^p\,a+(c\,x^2+bx+a)^p\,bpx+4\left(\int \frac{(c\,x^2+bx+a)^p}{2cp\,x^3+2bp\,x^2+c\,x^3+2apx+b\,x^2+ax}dx\right)a}{}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^p/x,x)
```

output
```
(4*(a + b*x + c*x**2)**p*a*p + (a + b*x + c*x**2)**p*a + (a + b*x + c*x**2
)**p*b*p*x + 4*int((a + b*x + c*x**2)**p/(2*a*p*x + a*x + 2*b*p*x**2 + b*x
**2 + 2*c*p*x**3 + c*x**3),x)*a**2*p**3 + 4*int((a + b*x + c*x**2)**p/(2*a
*p*x + a*x + 2*b*p*x**2 + b*x**2 + 2*c*p*x**3 + c*x**3),x)*a**2*p**2 + int
((a + b*x + c*x**2)**p/(2*a*p*x + a*x + 2*b*p*x**2 + b*x**2 + 2*c*p*x**3 +
 c*x**3),x)*a**2*p - 12*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x
 + b*x + 2*c*p*x**2 + c*x**2),x)*a*c*p**3 - 8*int(((a + b*x + c*x**2)**p*x
)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*a*c*p**2 - int(((a
+ b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)
*a*c*p + 2*int(((a + b*x + c*x**2)**p*x)/(2*a*p + a + 2*b*p*x + b*x + 2*c*
p*x**2 + c*x**2),x)*b**2*p**3 + int(((a + b*x + c*x**2)**p*x)/(2*a*p + a +
 2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p**2)/(p*(2*p + 1))
```

3.245 $\int \frac{(A+Bx)(a+bx+cx^2)^p}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 274

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^2}\,dx = -\frac{A\left(a+bx+cx^2\right)^{1+p}}{ax} + \frac{2^{-1+2p}(aB+Abp)\left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(a+bx+cx^2\right)^p \text{AppellF1}\left(-2p,-p,-p,1-\right.}{ap} + \frac{2^{-1-2p}A(1+2p)(b+2cx)\left(a+bx+cx^2\right)^p\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{(b+2cx)^2}{b^2-4ac}\right)}{a}$$

output
```
-A*(c*x^2+b*x+a)^(p+1)/a/x+2^(-1+2*p)*(A*b*p+B*a)*(c*x^2+b*x+a)^p*AppellF1
(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2)^(1/2))/c/x,-1/2*(b+(-4*a*c+b^2)^(1/
2))/c/x)/a/p/(((b-(-4*a*c+b^2)^(1/2)+2*c*x)/c/x)^p)/(((b+(-4*a*c+b^2)^(1/2
)+2*c*x)/c/x)^p)+2^(-1-2*p)*A*(1+2*p)*(2*c*x+b)*(c*x^2+b*x+a)^p*hypergeom(
[1/2, -p],[3/2],(2*c*x+b)^2/(-4*a*c+b^2))/a/((-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^2}\,dx$$
$$= \frac{\left(1+\frac{b-\sqrt{b^2-4ac}}{2cx}\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}+2cx}{c}\right)^{p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}(a+x(b+cx))^p\left(2Ap\,\text{Appel}\right.}{}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^p)/x^2,x]
```

output
```
(((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/c)^p*(a + x*(b + c*x))^p*(2*A*p*AppellF1
[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x), (-b + Sqrt[
b^2 - 4*a*c])/(2*c*x)] + B*(-1 + 2*p)*x*AppellF1[-2*p, -p, -p, 1 - 2*p, -1
/2*(b + Sqrt[b^2 - 4*a*c])/(c*x), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x)]))/(2*p
*(-1 + 2*p)*(1 + (b - Sqrt[b^2 - 4*a*c])/(2*c*x))^p*x*((b - Sqrt[b^2 - 4*a
*c])/(2*c) + x)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1237, 25, 1269, 1096, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^2}\,dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int -\frac{(aB+Abp+Ac(2p+1)x)\left(cx^2+bx+a\right)^p}{x}dx}{a}-\frac{A\left(a+bx+cx^2\right)^{p+1}}{ax}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(aB+Abp+Ac(2p+1)x)\left(cx^2+bx+a\right)^p}{x}dx}{a}-\frac{A\left(a+bx+cx^2\right)^{p+1}}{ax}$$

$$\Big\downarrow \ 1269$$

$$\frac{(aB+Abp)\int \frac{(cx^2+bx+a)^p}{x}dx+Ac(2p+1)\int (cx^2+bx+a)^p\,dx}{a}-\frac{A(a+bx+cx^2)^{p+1}}{ax}$$

$$\Big\downarrow \ 1096$$

$$\frac{(aB+Abp)\int \frac{(cx^2+bx+a)^p}{x}dx-\frac{Ac2^{p+1}(2p+1)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx+cx^2)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+2cx+\cdots}{2\sqrt{b^2\cdots}}\right.}{(p+1)\sqrt{b^2-4ac}}}{a}$$

$$\frac{A(a+bx+cx^2)^{p+1}}{ax}$$

$$\Big\downarrow \ 1178$$

$$-4^p\left(\frac{1}{x}\right)^{2p}(aB+Abp)\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx}+1\right)^p\left(\frac{b+\sqrt{b^2-4ac}}{2cx}\cdots\right.$$

$$a$$

$$\frac{A(a+bx+cx^2)^{p+1}}{ax}$$

$$\Big\downarrow \ 150$$

$$\frac{\frac{2^{2p-1}(aB+Abp)\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx},-\frac{b+\sqrt{b^2-4ac}}{2cx}\right)}{p}-\frac{Ac\cdots}{\cdots}}{a}$$

$$\frac{A(a+bx+cx^2)^{p+1}}{ax}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^p)/x^2,x]
```

output
```
-((A*(a + b*x + c*x^2)^(1 + p))/(a*x)) + ((2^(-1 + 2*p)*(a*B + A*b*p)*(a +
 b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b - Sqrt[b^2 - 4*a*c
])/(c*x), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x)])/(p*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x)/(c*x))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p) - (2^(1 + p)*
A*c*(1 + 2*p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 -
 p)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqr
t[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)
))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(Bx+A)\left(c\,x^2+bx+a\right)^p}{x^2}dx$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^p/x^2,x)
```

output
```
int((B*x+A)*(c*x^2+b*x+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^2}\,dx = \int \frac{(Bx+A)(cx^2+bx+a)^p}{x^2}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(c*x^2 + b*x + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^2}\,dx = \int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^2}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**p/x**2,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^2}\,dx = \int \frac{(Bx+A)(cx^2+bx+a)^p}{x^2}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^2}\,dx = \int \frac{(Bx+A)(cx^2+bx+a)^p}{x^2}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^2}\,dx = \int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^p}{x^2}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^p)/x^2,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^p)/x^2, x)
```

Reduce [F]

$$\int \frac{(A+Bx)(a+bx+cx^2)^p}{x^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^p/x^2,x)
```

output
```
( - (a + b*x + c*x**2)**p*a*b*p**2 + (a + b*x + c*x**2)**p*a*b*p - 3*(a +
b*x + c*x**2)**p*a*c*p*x + (a + b*x + c*x**2)**p*a*c*x + (a + b*x + c*x**2
)**p*b**2*p*x - (a + b*x + c*x**2)**p*b**2*x + int((a + b*x + c*x**2)**p/(
a*p*x - a*x + b*p*x**2 - b*x**2 + c*p*x**3 - c*x**3),x)*a*b**2*p**4*x - in
t((a + b*x + c*x**2)**p/(a*p*x - a*x + b*p*x**2 - b*x**2 + c*p*x**3 - c*x*
*3),x)*a*b**2*p**3*x - int((a + b*x + c*x**2)**p/(a*p*x - a*x + b*p*x**2 -
 b*x**2 + c*p*x**3 - c*x**3),x)*a*b**2*p**2*x + int((a + b*x + c*x**2)**p/
(a*p*x - a*x + b*p*x**2 - b*x**2 + c*p*x**3 - c*x**3),x)*a*b**2*p*x + 2*in
t((a + b*x + c*x**2)**p/(a*p - a + b*p*x - b*x + c*p*x**2 - c*x**2),x)*a*b
*c*p**4*x - int((a + b*x + c*x**2)**p/(a*p - a + b*p*x - b*x + c*p*x**2 -
c*x**2),x)*a*b*c*p**3*x - 2*int((a + b*x + c*x**2)**p/(a*p - a + b*p*x - b
*x + c*p*x**2 - c*x**2),x)*a*b*c*p**2*x + int((a + b*x + c*x**2)**p/(a*p -
 a + b*p*x - b*x + c*p*x**2 - c*x**2),x)*a*b*c*p*x + 6*int(((a + b*x + c*x
**2)**p*x)/(a*p - a + b*p*x - b*x + c*p*x**2 - c*x**2),x)*a*c**2*p**3*x -
8*int(((a + b*x + c*x**2)**p*x)/(a*p - a + b*p*x - b*x + c*p*x**2 - c*x**2
),x)*a*c**2*p**2*x + 2*int(((a + b*x + c*x**2)**p*x)/(a*p - a + b*p*x - b*
x + c*p*x**2 - c*x**2),x)*a*c**2*p*x - int(((a + b*x + c*x**2)**p*x)/(a*p
- a + b*p*x - b*x + c*p*x**2 - c*x**2),x)*b**2*c*p**3*x + 2*int(((a + b*x
+ c*x**2)**p*x)/(a*p - a + b*p*x - b*x + c*p*x**2 - c*x**2),x)*b**2*c*p**2
*x - int(((a + b*x + c*x**2)**p*x)/(a*p - a + b*p*x - b*x + c*p*x**2 - ...
```

3.246 $\int \frac{(A+Bx)(a+bx+cx^2)^p}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 336

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^3}\,dx$$
$$= -\frac{A\left(a+bx+cx^2\right)^{1+p}}{2ax^2} - \frac{(2aB-Ab(1-p))\left(a+bx+cx^2\right)^{1+p}}{2a^2x}$$
$$+ \frac{4^{-1+p}\left(2abB+2aAc-Ab^2(1-p)\right)\left(\frac{b-\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}\left(a+bx+cx^2\right)^p \operatorname{AppellF1}\left(-\right.}{a^2}$$
$$+ \frac{4^{-1-p}(2aB-Ab(1-p))(1+2p)(b+2cx)\left(a+bx+cx^2\right)^p\left(-\frac{c(a+bx+cx^2)}{b^2-4ac}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\right.}{a^2}$$

output
```
-1/2*A*(c*x^2+b*x+a)^(p+1)/a/x^2-1/2*(2*B*a-A*b*(1-p))*(c*x^2+b*x+a)^(p+1)
/a^2/x+4^(-1+p)*(2*a*b*B+2*A*a*c-A*b^2*(1-p))*(c*x^2+b*x+a)^p*AppellF1(-2*
p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2)^(1/2))/c/x,-1/2*(b+(-4*a*c+b^2)^(1/2))/
c/x)/a^2/(((b-(-4*a*c+b^2)^(1/2)+2*c*x)/c/x)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*
c*x)/c/x)^p)+4^(-1-p)*(2*B*a-A*b*(1-p))*(1+2*p)*(2*c*x+b)*(c*x^2+b*x+a)^p*
hypergeom([1/2, -p],[3/2],(2*c*x+b)^2/(-4*a*c+b^2))/a^2/((-c*(c*x^2+b*x+a)
/(-4*a*c+b^2))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^3}\,dx$$
$$= \frac{\left(1+\frac{b-\sqrt{b^2-4ac}}{2cx}\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}+2cx}{c}\right)^{p}\left(\frac{b+\sqrt{b^2-4ac}+2cx}{cx}\right)^{-p}(a+x(b+cx))^p\left(2B(-1+p\right.}{}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^p)/x^3,x]
```

output
```
(((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/c)^p*(a + x*(b + c*x))^p*(2*B*(-1 + p)*x
*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x), (-
b + Sqrt[b^2 - 4*a*c])/(2*c*x)] + A*(-1 + 2*p)*AppellF1[2 - 2*p, -p, -p, 3
 - 2*p, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x), (-b + Sqrt[b^2 - 4*a*c])/(2*c*
x)]))/(2*(-1 + p)*(-1 + 2*p)*(1 + (b - Sqrt[b^2 - 4*a*c])/(2*c*x))^p*x^2*(
(b - Sqrt[b^2 - 4*a*c])/(2*c) + x)^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x
))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1237, 25, 1237, 25, 1269, 1096, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^p}{x^3}\,dx$$
$$\Big\downarrow \text{ 1237}$$
$$-\frac{\int -\frac{(2aB-A(b-bp)+2Acpx)\left(cx^2+bx+a\right)^p}{x^2}dx}{2a}-\frac{A\left(a+bx+cx^2\right)^{p+1}}{2ax^2}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(2aB-Ab(1-p)+2Acpx)(cx^2+bx+a)^p}{x^2}dx}{2a}-\frac{A(a+bx+cx^2)^{p+1}}{2ax^2}$$

$\Big\downarrow$ 1237

$$\frac{-\frac{\int -\frac{\left(\left(-A(1-p)b^2+2aBb+2aAc\right)p+c(2aB-Ab(1-p))(2p+1)x\right)\left(cx^2+bx+a\right)^p}{x}dx}{a}-\frac{(2aB-Ab(1-p))\left(a+bx+cx^2\right)^{p+1}}{ax}}{2a}-\frac{A\left(a+bx+cx^2\right)^{p+1}}{2ax^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\left(\left(-A(1-p)b^2+2aBb+2aAc\right)p+c(2aB-Ab(1-p))(2p+1)x\right)\left(cx^2+bx+a\right)^p}{x}dx}{a}-\frac{(2aB-Ab(1-p))\left(a+bx+cx^2\right)^{p+1}}{ax}}{2a}-\frac{A\left(a+bx+cx^2\right)^{p+1}}{2ax^2}$$

$\Big\downarrow$ 1269

$$\frac{\frac{p\left(2aAc+2abB-Ab^2(1-p)\right)\int \frac{\left(cx^2+bx+a\right)^p}{x}dx+c(2p+1)(2aB-Ab(1-p))\int \left(cx^2+bx+a\right)^p dx}{a}-\frac{(2aB-Ab(1-p))\left(a+bx+cx^2\right)^{p+1}}{ax}}{2a}-\frac{A\left(a+bx+cx^2\right)^{p+1}}{2ax^2}$$

$\Big\downarrow$ 1096

$$\frac{\frac{p\left(2aAc+2abB-Ab^2(1-p)\right)\int \frac{\left(cx^2+bx+a\right)^p}{x}dx-\frac{c2^{p+1}(2p+1)(2aB-Ab(1-p))\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+\right.}{(p+1)\sqrt{b^2-4ac}}}{a}}{2a}$$

$$\frac{A\left(a+bx+cx^2\right)^{p+1}}{2ax^2}$$

$\Big\downarrow$ 1178

$$\frac{\frac{-4^p p\left(\frac{1}{x}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p\left(2aAc+2abB-Ab^2(1-p)\right)\int \left(\frac{b-\sqrt{b^2-4ac}}{2cx}+1\right)^p\left(\frac{b+\sqrt{b^2-4ac}}{2cx}+1\right)^p\left(\frac{1}{x}\right)}{a}}{}$$

$$\frac{A\left(a+bx+cx^2\right)^{p+1}}{2ax^2}$$

$\Big\downarrow$ 150

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx}{cx}\right)^{-p}(a+bx+cx^2)^p\left(2aAc+2abB-Ab^2(1-p)\right)\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx},-\frac{b+\sqrt{}}{}\right.}{a}$$

$$\frac{A(a+bx+cx^2)^{p+1}}{2ax^2}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^p)/x^3,x]
```

output
```
-1/2*(A*(a + b*x + c*x^2)^(1 + p))/(a*x^2) + (-(((2*a*B - A*b*(1 - p))*(a
+ b*x + c*x^2)^(1 + p))/(a*x)) + ((2^(-1 + 2*p)*(2*a*b*B + 2*a*A*c - A*b^2
*(1 - p))*(a + b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b - Sq
rt[b^2 - 4*a*c])/(c*x), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x)])/(((b - Sqrt[b
^2 - 4*a*c] + 2*c*x)/(c*x))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(c*x))^p) -
 (2^(1 + p)*c*(2*a*B - A*b*(1 - p))*(1 + 2*p)*(-((b - Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)*Hypergeometr
ic2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*
c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(Bx + A)\,(c\,x^2 + bx + a)^p}{x^3}dx$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^p/x^3,x)
```

output
```
int((B*x+A)*(c*x^2+b*x+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^p}{x^3}\,dx = \int \frac{(Bx+A)(cx^2+bx+a)^p}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(c*x^2 + b*x + a)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^p}{x^3}\,dx = \int \frac{(A+Bx)\,(a+bx+cx^2)^p}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**p/x**3,x)
```

output
```
Integral((A + B*x)*(a + b*x + c*x**2)**p/x**3, x)
```

Maxima [F]

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^p}{x^3}\,dx = \int \frac{(Bx+A)(cx^2+bx+a)^p}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(A+Bx)(a+bx+cx^2)^p}{x^3}\,dx = \int \frac{(Bx+A)(cx^2+bx+a)^p}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x^2 + b*x + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(a+bx+cx^2)^p}{x^3}\,dx = \int \frac{(A+B\,x)\,(c\,x^2+b\,x+a)^p}{x^3}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^p)/x^3,x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^p)/x^3, x)
```

Reduce [F]

$$\int \frac{(A+Bx)(a+bx+cx^2)^p}{x^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(c*x^2+b*x+a)^p/x^3,x)
```

output

```
( - 2*(a + b*x + c*x**2)**p*a*p + (a + b*x + c*x**2)**p*a + 2*(a + b*x + c
*x**2)**p*b*x + 4*int((a + b*x + c*x**2)**p/(2*a*p**2*x**2 - 5*a*p*x**2 +
2*a*x**2 + 2*b*p**2*x**3 - 5*b*p*x**3 + 2*b*x**3 + 2*c*p**2*x**4 - 5*c*p*x
**4 + 2*c*x**4),x)*a*b*p**4*x**2 - 4*int((a + b*x + c*x**2)**p/(2*a*p**2*x
**2 - 5*a*p*x**2 + 2*a*x**2 + 2*b*p**2*x**3 - 5*b*p*x**3 + 2*b*x**3 + 2*c*
p**2*x**4 - 5*c*p*x**4 + 2*c*x**4),x)*a*b*p**3*x**2 - 11*int((a + b*x + c*
x**2)**p/(2*a*p**2*x**2 - 5*a*p*x**2 + 2*a*x**2 + 2*b*p**2*x**3 - 5*b*p*x*
*3 + 2*b*x**3 + 2*c*p**2*x**4 - 5*c*p*x**4 + 2*c*x**4),x)*a*b*p**2*x**2 +
6*int((a + b*x + c*x**2)**p/(2*a*p**2*x**2 - 5*a*p*x**2 + 2*a*x**2 + 2*b*p
**2*x**3 - 5*b*p*x**3 + 2*b*x**3 + 2*c*p**2*x**4 - 5*c*p*x**4 + 2*c*x**4),
x)*a*b*p*x**2 + 8*int((a + b*x + c*x**2)**p/(2*a*p**2*x - 5*a*p*x + 2*a*x
+ 2*b*p**2*x**2 - 5*b*p*x**2 + 2*b*x**2 + 2*c*p**2*x**3 - 5*c*p*x**3 + 2*c
*x**3),x)*a*c*p**4*x**2 - 24*int((a + b*x + c*x**2)**p/(2*a*p**2*x - 5*a*p
*x + 2*a*x + 2*b*p**2*x**2 - 5*b*p*x**2 + 2*b*x**2 + 2*c*p**2*x**3 - 5*c*p
*x**3 + 2*c*x**3),x)*a*c*p**3*x**2 + 18*int((a + b*x + c*x**2)**p/(2*a*p**
2*x - 5*a*p*x + 2*a*x + 2*b*p**2*x**2 - 5*b*p*x**2 + 2*b*x**2 + 2*c*p**2*x
**3 - 5*c*p*x**3 + 2*c*x**3),x)*a*c*p**2*x**2 - 4*int((a + b*x + c*x**2)**
p/(2*a*p**2*x - 5*a*p*x + 2*a*x + 2*b*p**2*x**2 - 5*b*p*x**2 + 2*b*x**2 +
2*c*p**2*x**3 - 5*c*p*x**3 + 2*c*x**3),x)*a*c*p*x**2 + 4*int((a + b*x + c*
x**2)**p/(2*a*p**2*x - 5*a*p*x + 2*a*x + 2*b*p**2*x**2 - 5*b*p*x**2 + 2...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file